Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/33-1.1.2.5

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May 18, 2024 Compiled on May 18, 2024 at 3:29pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**571**]. This is test number [33].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	74.26 (424)	25.74 (147)
Maple	66.73 (381)	33.27 (190)
Rubi	66.20 (378)	33.80 (193)
Fricas	40.11 (229)	59.89 (342)
Giac	29.42 (168)	70.58 (403)
Reduce	24.17 (138)	75.83 (433)
Mupad	10.51 (60)	89.49 (511)
Sympy	9.28 (53)	90.72 (518)
Maxima	7.71 (44)	92.29 (527)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

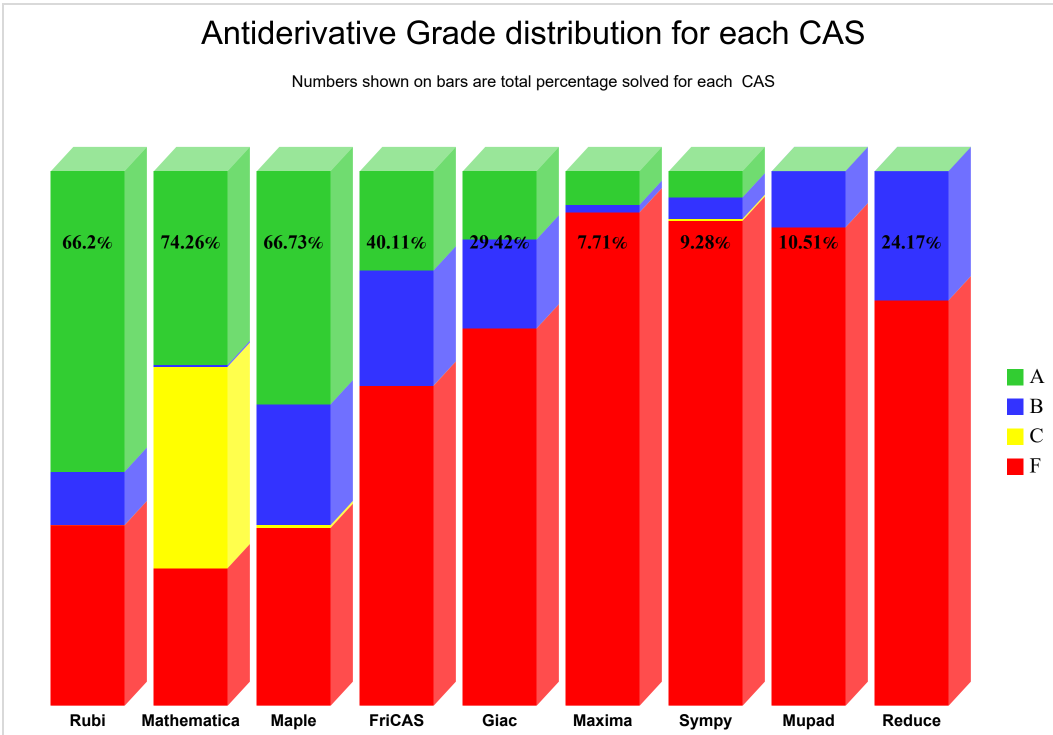
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

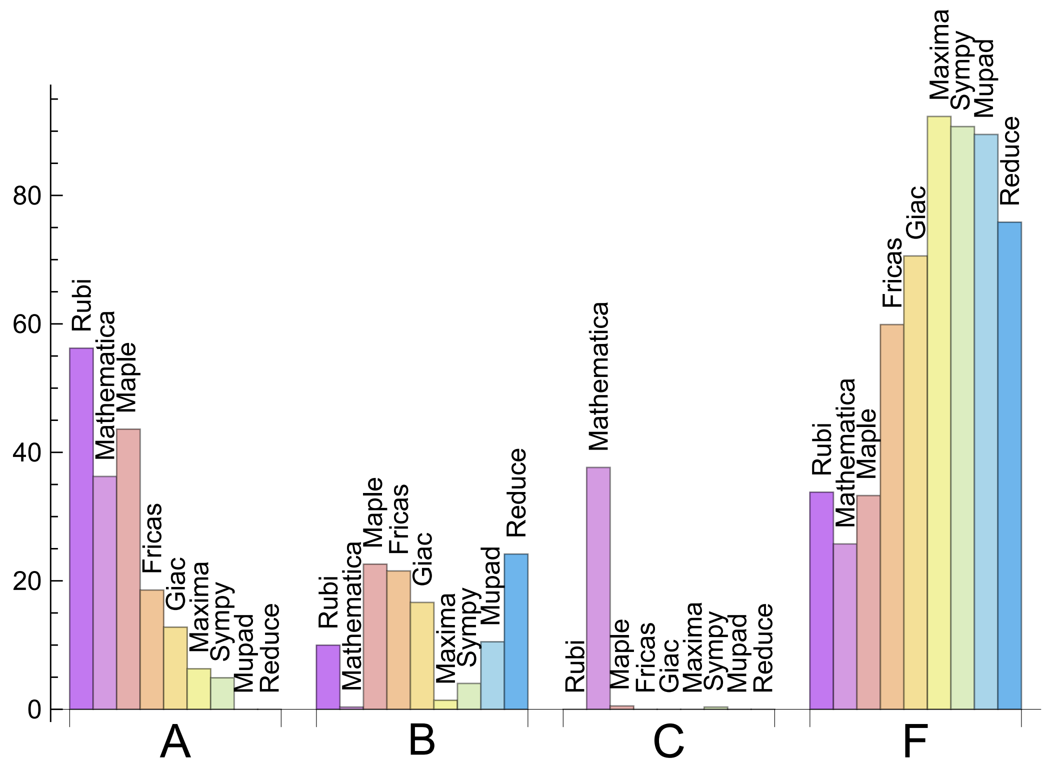
System	% A grade	% B grade	% C grade	% F grade
Rubi	56.217	9.982	0.000	33.800
Maple	43.608	22.592	0.525	33.275
Mathematica	36.252	0.350	37.653	25.744
Fricas	18.564	21.541	0.000	59.895
Giac	12.785	16.637	0.000	70.578
Maxima	6.305	1.401	0.000	92.294
Sympy	4.904	4.028	0.350	90.718
Mupad	0.000	10.508	0.000	89.492
Reduce	0.000	24.168	0.000	75.832

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	147	100.00	0.00	0.00
Maple	190	100.00	0.00	0.00
Rubi	193	100.00	0.00	0.00
Fricas	342	45.32	54.68	0.00
Giac	403	96.53	0.00	3.47
Reduce	433	100.00	0.00	0.00
Mupad	511	0.00	100.00	0.00
Sympy	518	62.36	37.64	0.00
Maxima	527	87.86	0.00	12.14

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Rubi	0.73
Reduce	1.55
Giac	2.72
Sympy	3.81
Mathematica	5.59
Mupad	6.82
Fricas	7.50
Maple	8.40

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	441.94	1.16	330.50	0.97
Maxima	461.34	1.39	412.00	1.32
Rubi	494.70	1.44	382.50	1.23
Sympy	606.62	2.15	483.00	1.84
Maple	898.10	1.86	461.00	1.34
Giac	1294.01	3.15	638.00	2.10
Fricas	1414.35	4.66	977.00	3.45
Reduce	2954.05	9.76	1282.00	5.45
Mupad	37089.42	100.60	811.00	2.21

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

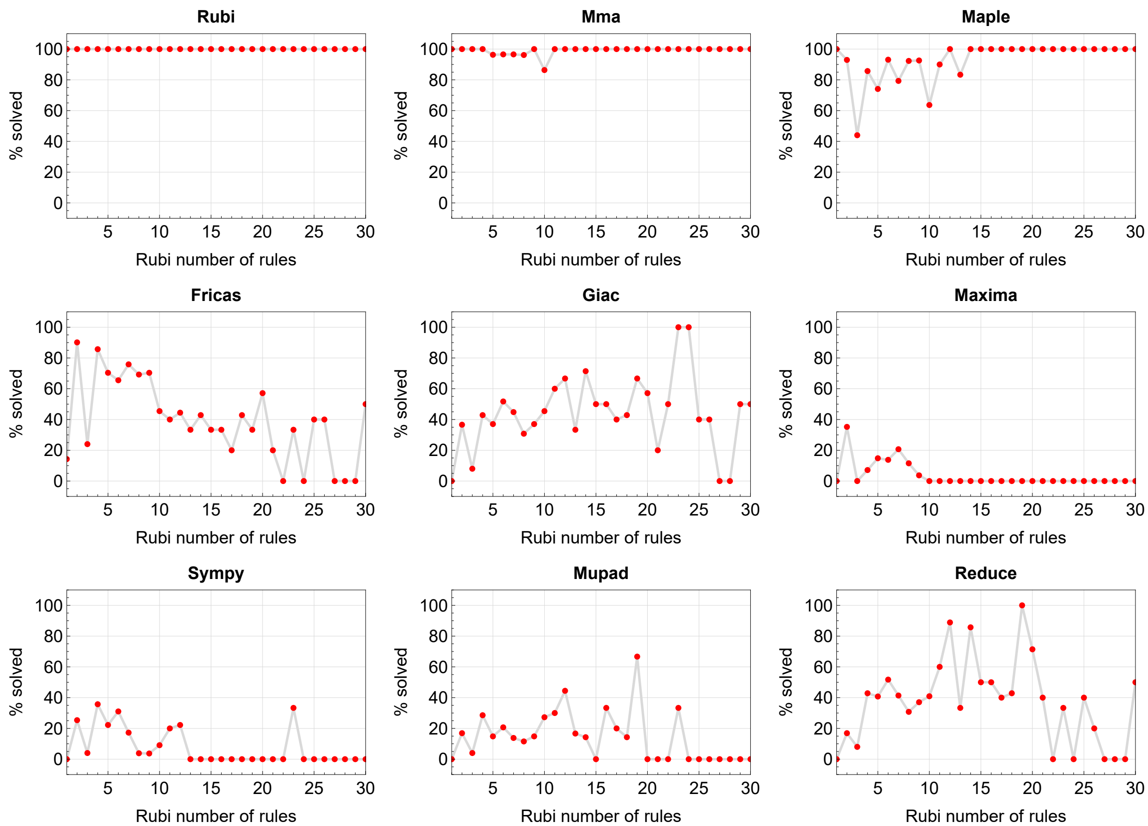


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

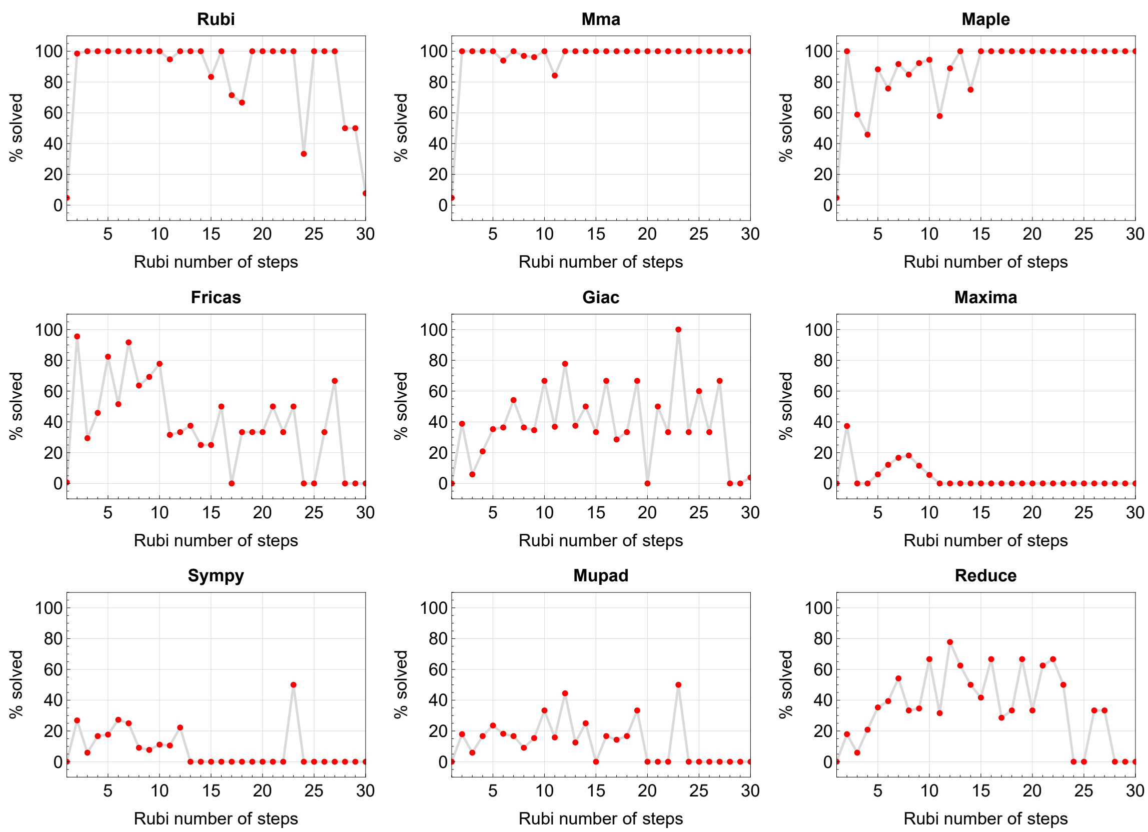


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

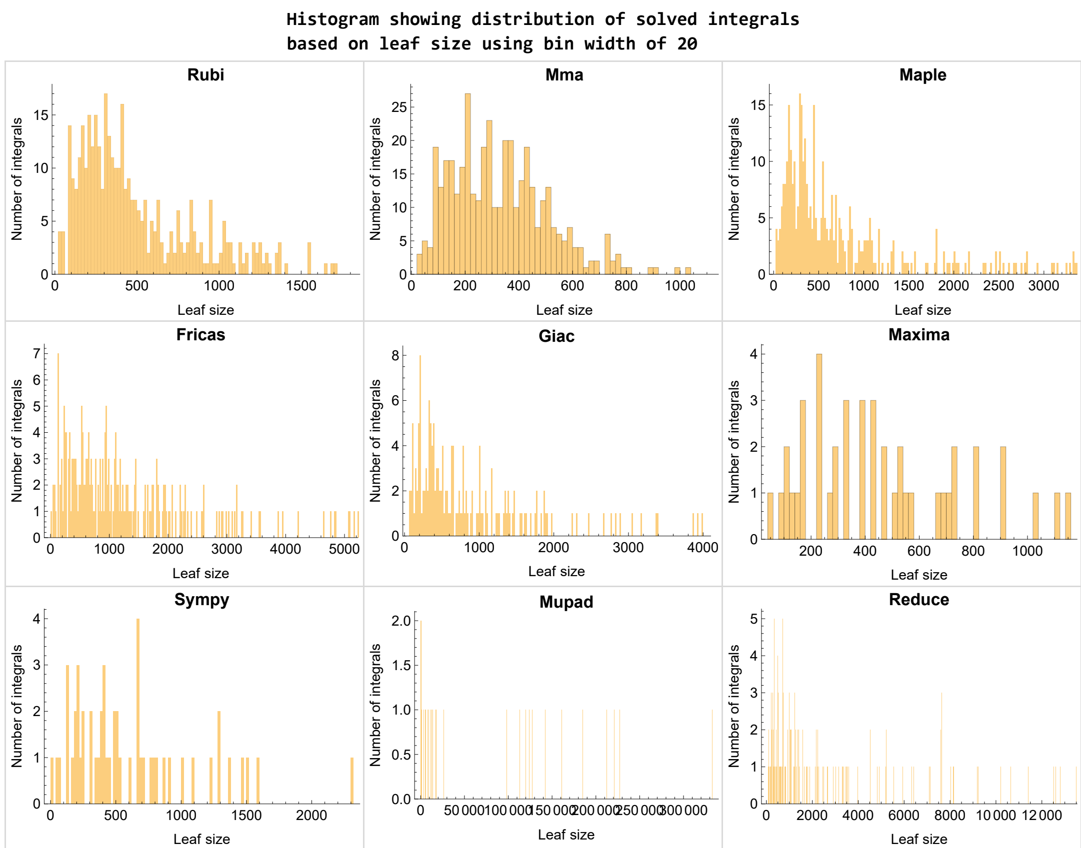


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

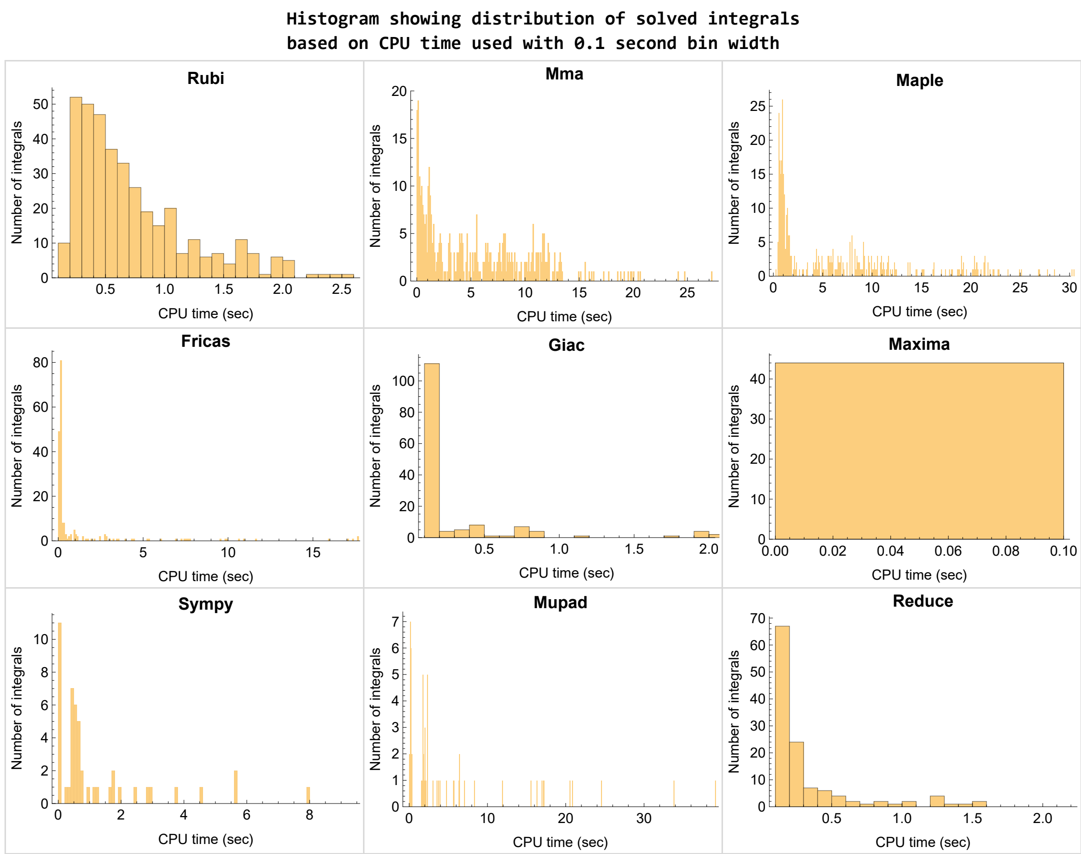


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

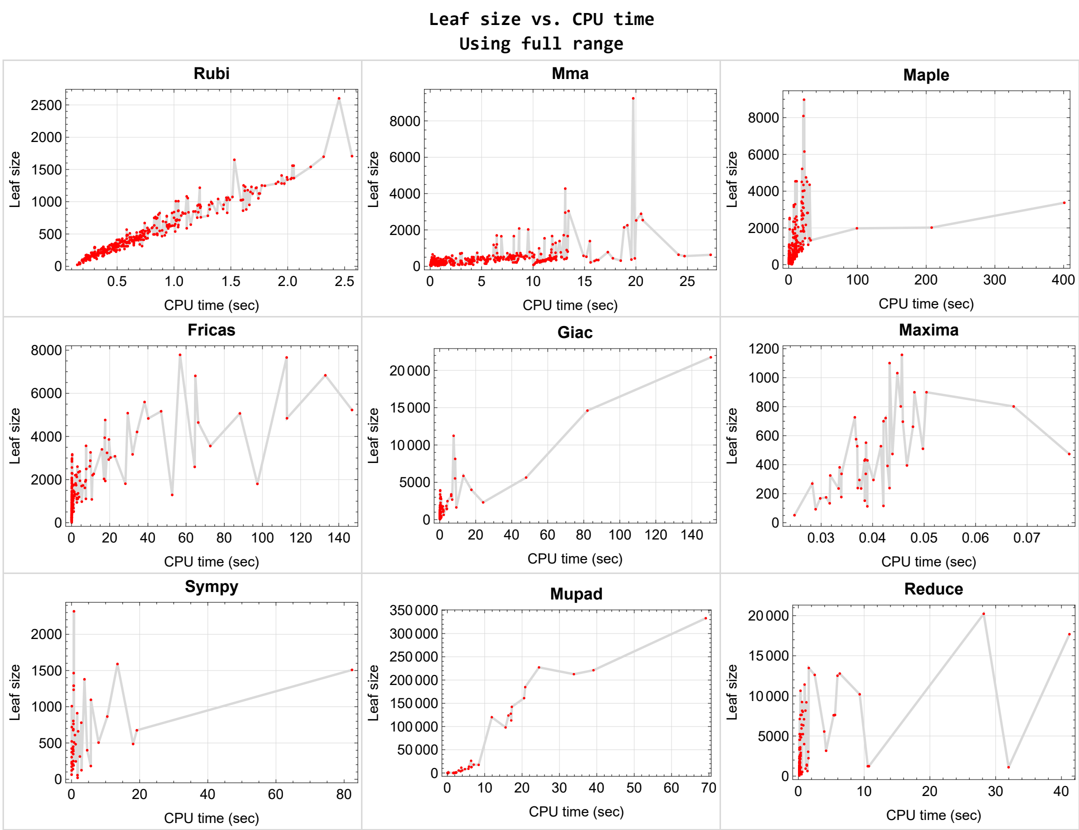


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {571}

Mathematica {265, 273, 296, 342, 347, 351, 353, 354, 355, 492, 562, 563, 564, 565, 566, 567, 568, 569, 570}

Maple $\{50, 51\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

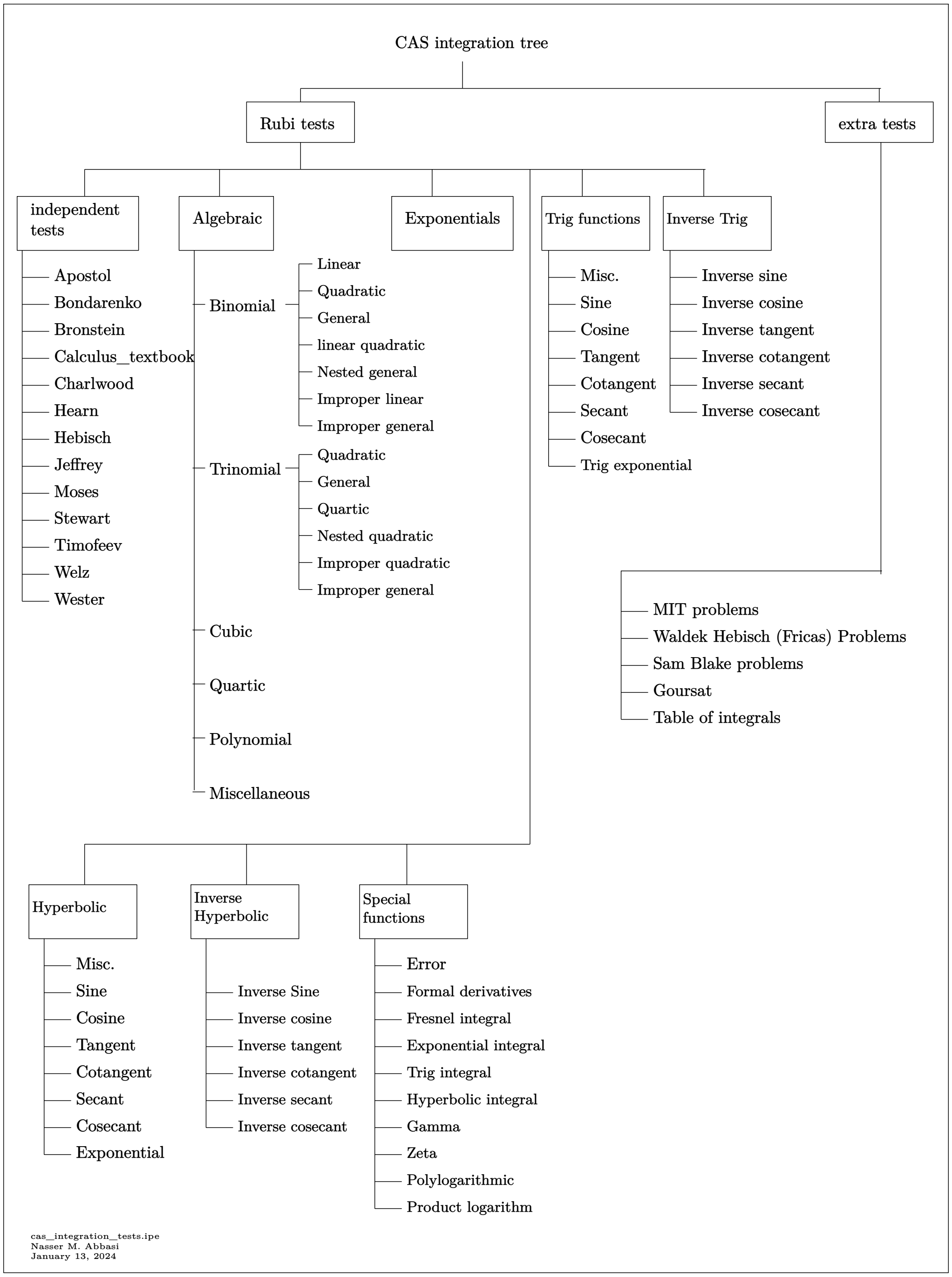
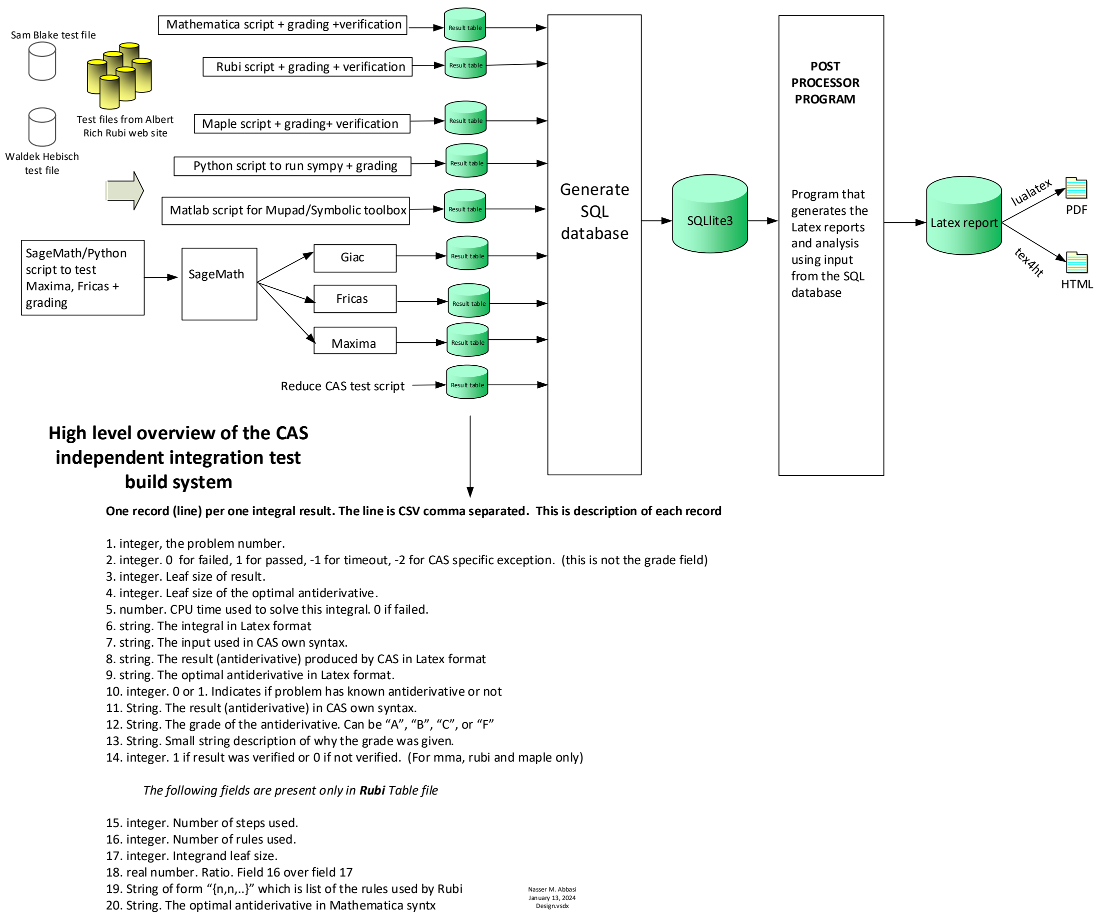


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 55, 56, 57, 58, 60, 61, 62, 63, 67, 68, 69, 70, 72, 73, 74, 75, 76, 80, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 142, 145, 146, 151, 152, 158, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 280, 281, 282, 283, 284, 287, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 308, 309, 310, 311, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 352, 353, 354, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 372, 374, 376, 377, 378, 384, 385, 403, 404, 405, 409, 410, 459, 460, 461, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571 }

B grade { 51, 52, 53, 54, 59, 64, 65, 66, 71, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 132, 141, 144, 147, 153, 157, 159, 244, 245, 246, 250, 252, 259, 271, 272, 273, 278, 279, 285, 288, 302, 303, 307, 315, 336, 345, 349, 350, 351, 364, 371, 373, 375, 511, 569 }

C grade { }

F normal fail { 129, 143, 148, 149, 150, 154, 155, 156, 160, 169, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 277, 286, 289, 301, 305, 306, 312, 313, 314, 355, 356, 357, 379, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 137, 138, 139, 140, 142, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 348, 349, 350, 352, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 384, 403, 404, 405, 410, 459, 460, 461, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 504, 508, 511, 565, 566, 570 }

B grade { 141, 332 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 342, 347, 351, 353, 354, 355, 497, 498, 499, 500, 501, 506, 507, 559, 560, 561, 562, 563, 564, 567, 568, 569

}

F normal fail { 382, 383, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 409, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 502, 503, 505, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 571 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 15, 16, 21, 22, 23, 24, 29, 30, 32, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 55, 56, 57, 58, 62, 63, 64, 68, 69, 70, 74, 75, 76, 77, 80, 82, 83, 85, 92, 94, 95, 96, 98, 100, 104, 105, 106, 111, 112, 113, 114, 115, 116, 118, 125, 126, 127, 133, 135, 136, 137, 138, 139, 140, 142, 145, 146, 161, 162, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380 }

B grade { 5, 6, 7, 11, 12, 13, 14, 17, 18, 19, 20, 25, 26, 27, 28, 31, 33, 34, 35, 38, 50, 51, 52, 53, 54, 59, 60, 61, 65, 66, 67, 71, 72, 73, 78, 79, 81, 84, 86, 87, 88, 89, 90, 91, 93, 97, 99, 101, 102, 103, 107, 108, 109, 110, 117, 119, 120, 121, 122, 123, 124, 128, 129, 132, 141, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 232, 233, 234, 243, 244, 245, 246, 249, 336, 357, 381 }

C grade { 130, 131, 134 }

F normal fail { 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 21, 22, 23, 24, 27, 28, 29, 30, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 68, 69, 70, 74, 75, 76, 77, 81, 82, 83, 138, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 214, 215, 219, 220, 225, 226, 227, 231, 232, 236, 242, 243, 260, 261, 262, 263, 267, 268, 269, 270, 274, 275, 276, 280, 290, 291, 292, 293, 294, 298, 299, 300, 316, 317, 318, 323, 324, 337, 338, 339, 343, 344, 348, 360 }

B grade { 5, 6, 7, 12, 13, 14, 18, 19, 20, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 50, 51, 59, 60, 61, 65, 66, 67, 71, 72, 73, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 211, 212, 213, 216, 217, 218, 221, 222, 223, 224, 228, 229, 230, 233, 234, 235, 237, 238, 239, 240, 244, 245, 246, 247, 248, 249, 250, 264, 265, 266, 271, 272, 273, 277, 278, 279, 295, 296, 297, 301, 302, 303, 305, 306, 307, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 340, 341, 342, 345, 346, 347, 349, 350, 351, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375 }

C grade { }

F normal fail { 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 382, 385, 386, 387, 388, 391, 392, 393, 394, 400, 401, 405, 406, 407, 408, 410, 411, 412, 415, 416, 417, 418, 420, 424, 425, 426, 429, 430, 431, 432, 433, 435, 436, 439, 444, 445, 446, 447, 448, 450, 451, 452, 455, 456, 457, 458, 459, 461, 462, 463, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 537, 538, 539, 540, 541, 543, 544, 545, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 565, 566, 567, 568, 569, 570, 571 }

F(-1) timedout fail { 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 241, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 283, 284, 285, 286, 287, 288, 289, 304, 308, 309, 310, 311, 312, 313, 314, 315, 332, 333, 334, 335, 336, 352, 353, 354, 355, 356, 357, 376, 377, 378, 379, 380, 381, 383, 384, 389, 390, 395, 396, 397, 398, 399, 402, 403, 404, 409, 413, 414, 419, 421, 422, 423, 427, 428, 434, 437, 438, 440, 441, 442, 443, 449, 453, 454, 460, 464, 466, 516, 521, 522, 523, 536, 542, 546, 547, 548, 562, 563, 564 }

F(-2) exception fail { }

Maxima

A grade { 200, 201, 202, 203, 208, 209, 214, 225, 226, 231, 242, 260, 261, 262, 267, 268, 269, 274, 275, 290, 291, 292, 298, 299, 316, 317, 318, 323, 324, 337, 338, 339, 343, 344, 348, 360 }

B grade { 358, 359, 364, 365, 366, 370, 371, 372 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 264, 265, 266, 271, 272, 273, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 294, 295, 296, 297, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 320, 321, 322, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 341, 342, 346, 347, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 368, 369, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571 }

F(-1) timedout fail { }

F(-2) exception fail { 204, 205, 206, 207, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 263, 270, 276, 293, 300, 304, 319, 325, 328, 340, 345, 349, 361, 367, 373 }

Giac

A grade { 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 225, 226, 229, 230, 231, 236, 237, 238, 239, 240, 242, 247, 248, 251, 253, 260, 261, 262, 267, 268, 269, 274, 275, 280, 290, 291, 292, 298, 299, 316, 317, 318, 323, 324, 331, 337, 338, 339, 343, 344, 348, 352, 358, 359, 360, 364, 365, 366, 371, 372 }

B grade { 216, 224, 227, 228, 232, 233, 234, 235, 241, 243, 244, 245, 246, 249, 250, 252, 254, 255, 256, 257, 258, 259, 264, 265, 266, 271, 272, 273, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 294, 295, 296, 297, 301, 302, 303, 305, 306, 307, 309, 310, 311, 312, 313, 314, 320, 321, 322, 326, 327, 329, 330, 332, 333, 334, 335, 336, 340, 341, 342, 346, 347, 350, 351, 353, 354, 355, 356, 357, 361, 362, 363, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 380, 381 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571 }

F(-1) timedout fail { }

F(-2) exception fail { 263, 270, 276, 293, 300, 304, 308, 315, 319, 325, 328, 345, 349, 373 }

Mupad

A grade { }

B grade { 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571 }

F(-2) exception fail { }

Sympy

A grade { 138, 200, 201, 202, 203, 206, 207, 208, 209, 214, 225, 226, 231, 242, 260, 261, 262, 267, 268, 269, 274, 275, 292, 316, 317, 318, 323, 324 }

B grade { 204, 205, 210, 211, 212, 213, 215, 216, 217, 227, 228, 229, 232, 233, 234, 243, 244, 290, 291, 298, 299, 339, 360 }

C grade { 560, 561 }

F normal fail { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 181, 263, 264, 270, 271, 272, 273, 276, 277, 278, 279, 280, 293, 294, 295, 300, 301, 302, 303, 304, 305, 306, 307, 308, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 334, 337, 338, 340, 343, 344, 345, 348, 349, 352, 353, 358, 359, 364, 365, 366, 370, 371, 372, 376, 377, 379, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 427, 428, 429, 440, 441, 442, 443, 444, 447, 448, 449, 450, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 521, 522, 527, 528, 529, 530, 531, 533, 536, 537, 538, 539, 540, 542, 543, 552, 553, 555, 559, 562, 565, 566 }

F(-1) timedout fail { 7, 13, 14, 19, 20, 32, 37, 38, 61, 66, 67, 71, 72, 73, 85, 86, 90, 91, 102, 103, 108, 109, 110, 129, 134, 149, 153, 154, 155, 156, 159, 160, 172, 173, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 218, 219, 220, 221, 222, 223, 224, 230, 235, 236, 237, 238, 239, 240, 241, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 296, 297, 309, 310, 311, 312, 313, 314, 315, 321, 322, 333, 335, 336, 341, 342, 346, 347, 350, 351, 354, 355, 356, 357, 361, 362, 363, 367, 368, 369, 373, 374, 375, 378, 380, 381, 388, 393, 394, 398, 399, 400, 401, 413, 424, 425, 426, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 445, 446, 451, 452, 453, 456, 457, 468, 469, 473, 474, 475, 484, 518, 519, 520, 523, 524, 525, 526, 532, 534, 535, 541, 544, 545, 546, 547, 548, 549, 550, 551, 554, 556, 557, 558, 563, 564, 567, 568, 569, 570, 571 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 284, 287, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 307, 308, 309, 310, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 332, 333, 337, 338, 339, 340, 341, 342, 345, 346, 347, 349, 350, 352, 353, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 375, 376 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 260, 267, 268, 274, 275, 283, 285, 286, 288, 289, 290, 297, 298, 299, 304, 305, 306, 311, 312, 313, 314, 323, 324, 334, 335, 336, 343, 344, 348, 351, 354, 355, 356, 357, 364, 365, 370, 371, 374, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	471	368	695	0	494	0	0	906	0
N.S.	1	0.90	0.70	1.32	0.00	0.94	0.00	0.00	1.72	0.00
time (sec)	N/A	0.672	4.028	8.278	0.000	0.101	0.000	0.000	0.783	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	350	269	431	0	313	0	0	520	0
N.S.	1	0.92	0.71	1.13	0.00	0.82	0.00	0.00	1.36	0.00
time (sec)	N/A	0.478	2.167	5.893	0.000	0.086	0.000	0.000	0.530	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	265	215	312	0	214	0	0	273	0
N.S.	1	0.93	0.76	1.10	0.00	0.75	0.00	0.00	0.96	0.00
time (sec)	N/A	0.361	1.933	7.903	0.000	0.105	0.000	0.000	0.377	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	259	208	378	0	237	0	0	840	0
N.S.	1	1.26	1.01	1.83	0.00	1.15	0.00	0.00	4.08	0.00
time (sec)	N/A	0.357	3.075	5.848	0.000	0.093	0.000	0.000	0.695	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	275	298	518	0	550	0	0	0	0
N.S.	1	1.02	1.11	1.93	0.00	2.04	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	4.305	6.000	0.000	0.094	0.000	0.000	1.339	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	400	375	754	0	1175	0	0	0	0
N.S.	1	1.06	0.99	1.99	0.00	3.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	4.610	6.049	0.000	0.140	0.000	0.000	1.865	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	553	518	1016	0	2070	0	0	0	0
N.S.	1	1.07	1.00	1.97	0.00	4.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	5.110	6.006	0.000	0.161	0.000	0.000	2.352	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	615	479	1164	0	679	0	0	1358	0
N.S.	1	0.90	0.70	1.71	0.00	1.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.939	8.158	10.806	0.000	0.106	0.000	0.000	1.183	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	471	367	695	0	493	0	0	906	0
N.S.	1	0.91	0.71	1.34	0.00	0.95	0.00	0.00	1.75	0.00
time (sec)	N/A	0.697	3.874	7.972	0.000	0.108	0.000	0.000	0.772	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	360	279	448	0	355	0	0	571	0
N.S.	1	0.92	0.71	1.14	0.00	0.90	0.00	0.00	1.45	0.00
time (sec)	N/A	0.523	3.723	11.819	0.000	0.105	0.000	0.000	0.621	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	340	262	543	0	459	0	0	1250	0
N.S.	1	1.16	0.89	1.85	0.00	1.56	0.00	0.00	4.25	0.00
time (sec)	N/A	0.505	5.585	11.565	0.000	0.099	0.000	0.000	1.147	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	349	297	559	0	535	0	0	0	0
N.S.	1	1.20	1.02	1.92	0.00	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	6.289	7.737	0.000	0.106	0.000	0.000	1.990	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	381	374	749	0	1130	0	0	0	0
N.S.	1	1.04	1.02	2.05	0.00	3.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.534	6.644	8.244	0.000	0.139	0.000	0.000	3.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	537	518	1021	0	2004	0	0	0	0
N.S.	1	1.06	1.03	2.02	0.00	3.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.786	7.148	8.247	0.000	0.165	0.000	0.000	4.077	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	616	482	1164	0	691	0	0	1358	0
N.S.	1	0.90	0.70	1.70	0.00	1.01	0.00	0.00	1.98	0.00
time (sec)	N/A	0.962	6.888	10.877	0.000	0.114	0.000	0.000	1.095	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	482	372	691	0	536	0	0	955	0
N.S.	1	0.91	0.70	1.30	0.00	1.01	0.00	0.00	1.80	0.00
time (sec)	N/A	0.730	4.911	14.564	0.000	0.110	0.000	0.000	0.884	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	449	354	793	0	714	0	0	0	0
N.S.	1	1.10	0.87	1.95	0.00	1.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.653	6.575	15.135	0.000	0.110	0.000	0.000	1.777	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	450	375	843	0	846	0	0	0	0
N.S.	1	1.16	0.97	2.18	0.00	2.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.692	7.370	16.155	0.000	0.115	0.000	0.000	2.938	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	473	378	846	0	1091	0	0	0	0
N.S.	1	1.18	0.94	2.10	0.00	2.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.733	7.095	9.971	0.000	0.139	0.000	0.000	5.515	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	512	509	1061	0	1935	0	0	0	0
N.S.	1	1.04	1.04	2.17	0.00	3.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.770	7.615	10.480	0.000	0.186	0.000	0.000	7.198	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	363	274	448	0	334	0	0	571	0
N.S.	1	0.94	0.71	1.16	0.00	0.87	0.00	0.00	1.48	0.00
time (sec)	N/A	0.487	3.778	10.265	0.000	0.096	0.000	0.000	0.625	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	268	214	312	0	192	0	0	273	0
N.S.	1	0.95	0.76	1.11	0.00	0.68	0.00	0.00	0.97	0.00
time (sec)	N/A	0.363	1.913	6.776	0.000	0.091	0.000	0.000	0.388	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	207	131	158	0	130	0	0	94	0
N.S.	1	1.00	0.64	0.77	0.00	0.63	0.00	0.00	0.46	0.00
time (sec)	N/A	0.292	3.773	5.628	0.000	0.091	0.000	0.000	0.222	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	260	0	0	142	0
N.S.	1	1.00	1.01	1.67	0.00	1.24	0.00	0.00	0.68	0.00
time (sec)	N/A	0.274	8.052	7.765	0.000	0.098	0.000	0.000	0.479	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	301	299	535	0	618	0	0	190	0
N.S.	1	1.05	1.04	1.86	0.00	2.15	0.00	0.00	0.66	0.00
time (sec)	N/A	0.380	10.761	9.917	0.000	0.103	0.000	0.000	1.004	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	428	374	761	0	1292	0	0	238	0
N.S.	1	1.08	0.94	1.92	0.00	3.25	0.00	0.00	0.60	0.00
time (sec)	N/A	0.546	11.288	12.279	0.000	0.134	0.000	0.000	1.332	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	453	344	795	0	693	0	0	0	0
N.S.	1	1.11	0.84	1.94	0.00	1.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	6.579	15.046	0.000	0.120	0.000	0.000	1.787	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	341	253	545	0	436	0	0	1250	0
N.S.	1	1.16	0.86	1.85	0.00	1.48	0.00	0.00	4.24	0.00
time (sec)	N/A	0.500	5.510	11.559	0.000	0.110	0.000	0.000	1.190	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	259	191	328	0	220	0	0	840	0
N.S.	1	1.26	0.93	1.59	0.00	1.07	0.00	0.00	4.08	0.00
time (sec)	N/A	0.356	3.050	5.699	0.000	0.098	0.000	0.000	0.714	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	253	0	0	142	0
N.S.	1	1.00	0.99	1.60	0.00	1.21	0.00	0.00	0.68	0.00
time (sec)	N/A	0.282	8.404	7.769	0.000	0.107	0.000	0.000	0.465	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	281	264	519	0	636	0	0	224	0
N.S.	1	1.04	0.97	1.92	0.00	2.35	0.00	0.00	0.83	0.00
time (sec)	N/A	0.370	10.768	10.278	0.000	0.118	0.000	0.000	0.444	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	389	405	670	0	1316	0	0	306	0
N.S.	1	1.05	1.10	1.82	0.00	3.57	0.00	0.00	0.83	0.00
time (sec)	N/A	0.546	11.549	12.252	0.000	0.161	0.000	0.000	1.081	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	456	372	845	0	899	0	0	0	0
N.S.	1	1.16	0.95	2.15	0.00	2.29	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	7.918	16.263	0.000	0.141	0.000	0.000	3.103	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	351	297	565	0	564	0	0	0	0
N.S.	1	1.20	1.01	1.93	0.00	1.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.485	6.319	7.755	0.000	0.121	0.000	0.000	2.095	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	277	296	518	0	534	0	0	0	0
N.S.	1	1.02	1.09	1.90	0.00	1.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	4.246	6.019	0.000	0.112	0.000	0.000	1.341	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	301	302	532	0	647	0	0	190	0
N.S.	1	1.04	1.04	1.84	0.00	2.24	0.00	0.00	0.66	0.00
time (sec)	N/A	0.379	10.700	10.063	0.000	0.118	0.000	0.000	0.989	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	403	412	669	0	1342	0	0	306	0
N.S.	1	1.05	1.08	1.75	0.00	3.50	0.00	0.00	0.80	0.00
time (sec)	N/A	0.521	12.181	12.403	0.000	0.184	0.000	0.000	1.054	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	531	436	1013	0	2280	0	0	0	0
N.S.	1	1.07	0.88	2.04	0.00	4.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	11.650	14.498	0.000	0.362	0.000	0.000	2.040	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	207	131	158	0	130	0	0	94	0
N.S.	1	1.00	0.64	0.77	0.00	0.63	0.00	0.00	0.46	0.00
time (sec)	N/A	0.290	0.082	5.596	0.000	0.094	0.000	0.000	0.223	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	135	161	0	130	0	0	100	0
N.S.	1	1.00	0.71	0.85	0.00	0.68	0.00	0.00	0.53	0.00
time (sec)	N/A	0.325	3.922	5.735	0.000	0.097	0.000	0.000	0.227	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	138	161	0	130	0	0	100	0
N.S.	1	1.00	0.73	0.85	0.00	0.68	0.00	0.00	0.53	0.00
time (sec)	N/A	0.323	3.858	5.648	0.000	0.088	0.000	0.000	0.233	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	138	160	0	126	0	0	102	0
N.S.	1	1.00	0.72	0.83	0.00	0.66	0.00	0.00	0.53	0.00
time (sec)	N/A	0.329	3.977	5.749	0.000	0.102	0.000	0.000	0.228	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	260	0	0	142	0
N.S.	1	1.00	1.01	1.67	0.00	1.24	0.00	0.00	0.68	0.00
time (sec)	N/A	0.272	0.288	7.793	0.000	0.102	0.000	0.000	0.469	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	257	220	349	0	257	0	0	148	0
N.S.	1	1.24	1.06	1.69	0.00	1.24	0.00	0.00	0.71	0.00
time (sec)	N/A	0.405	8.396	7.952	0.000	0.101	0.000	0.000	0.498	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	213	333	0	254	0	0	144	0
N.S.	1	1.05	0.90	1.41	0.00	1.07	0.00	0.00	0.61	0.00
time (sec)	N/A	0.393	8.239	7.912	0.000	0.115	0.000	0.000	0.482	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	255	221	338	0	257	0	0	150	0
N.S.	1	1.01	0.88	1.34	0.00	1.02	0.00	0.00	0.60	0.00
time (sec)	N/A	0.408	8.314	7.919	0.000	0.108	0.000	0.000	0.516	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	192	81	105	0	124	0	0	90	0
N.S.	1	1.01	0.42	0.55	0.00	0.65	0.00	0.00	0.47	0.00
time (sec)	N/A	0.285	3.408	4.227	0.000	0.093	0.000	0.000	0.214	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	250	142	282	0	181	0	0	259	0
N.S.	1	0.97	0.55	1.09	0.00	0.70	0.00	0.00	1.00	0.00
time (sec)	N/A	0.335	1.532	5.374	0.000	0.089	0.000	0.000	0.351	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	328	186	390	0	274	0	0	484	0
N.S.	1	0.92	0.52	1.10	0.00	0.77	0.00	0.00	1.36	0.00
time (sec)	N/A	0.437	1.608	4.934	0.000	0.094	0.000	0.000	0.467	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	104	2540	0	369	0	0	79	0
N.S.	1	1.00	0.92	22.48	0.00	3.27	0.00	0.00	0.70	0.00
time (sec)	N/A	0.321	3.138	1.408	0.000	0.117	0.000	0.000	200.029	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	224	557	100	2477	0	372	0	0	73	0
N.S.	1	2.49	0.45	11.06	0.00	1.66	0.00	0.00	0.33	0.00
time (sec)	N/A	0.729	3.088	1.259	0.000	0.119	0.000	0.000	200.025	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	132	84	130	0	79	0	0	84	0
N.S.	1	2.87	1.83	2.83	0.00	1.72	0.00	0.00	1.83	0.00
time (sec)	N/A	0.223	4.186	4.617	0.000	0.090	0.000	0.000	0.283	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	132	131	190	0	55	0	0	84	0
N.S.	1	2.87	2.85	4.13	0.00	1.20	0.00	0.00	1.83	0.00
time (sec)	N/A	0.262	1.136	4.075	0.000	0.080	0.000	0.000	0.299	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	132	84	156	0	79	0	0	84	0
N.S.	1	2.87	1.83	3.39	0.00	1.72	0.00	0.00	1.83	0.00
time (sec)	N/A	0.231	0.019	4.343	0.000	0.097	0.000	0.000	0.299	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	1352	584	1300	0	881	0	0	0	0
N.S.	1	1.59	0.69	1.53	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	2.027	7.783	11.600	0.000	0.131	0.000	0.000	1.330	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	1043	422	704	0	593	0	0	1140	0
N.S.	1	1.71	0.69	1.16	0.00	0.97	0.00	0.00	1.87	0.00
time (sec)	N/A	1.388	3.411	8.463	0.000	0.094	0.000	0.000	0.887	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	826	303	453	0	409	0	0	651	0
N.S.	1	1.86	0.68	1.02	0.00	0.92	0.00	0.00	1.46	0.00
time (sec)	N/A	0.983	2.963	10.546	0.000	0.108	0.000	0.000	0.668	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	595	281	532	0	423	0	0	1420	0
N.S.	1	1.94	0.92	1.74	0.00	1.38	0.00	0.00	4.64	0.00
time (sec)	N/A	0.766	4.124	11.667	0.000	0.092	0.000	0.000	1.174	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	840	370	693	0	934	0	0	0	0
N.S.	1	2.28	1.00	1.88	0.00	2.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.000	5.512	7.990	0.000	0.126	0.000	0.000	2.698	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	952	499	1037	0	1697	0	0	0	0
N.S.	1	1.89	0.99	2.05	0.00	3.36	0.00	0.00	0.00	0.00
time (sec)	N/A	1.228	5.567	8.274	0.000	0.147	0.000	0.000	3.762	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	1232	723	1440	0	2980	0	0	0	0
N.S.	1	1.67	0.98	1.96	0.00	4.05	0.00	0.00	0.00	0.00
time (sec)	N/A	1.626	6.511	8.397	0.000	0.216	0.000	0.000	4.706	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	847	1342	575	1300	0	880	0	0	0	0
N.S.	1	1.58	0.68	1.53	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	1.998	7.902	11.473	0.000	0.108	0.000	0.000	1.367	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	1062	438	723	0	652	0	0	1195	0
N.S.	1	1.67	0.69	1.14	0.00	1.03	0.00	0.00	1.88	0.00
time (sec)	N/A	1.476	5.228	13.681	0.000	0.101	0.000	0.000	1.058	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	951	384	846	0	771	0	0	0	0
N.S.	1	2.07	0.84	1.84	0.00	1.68	0.00	0.00	0.00	0.00
time (sec)	N/A	1.308	6.739	15.299	0.000	0.106	0.000	0.000	2.036	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	906	417	862	0	937	0	0	0	0
N.S.	1	2.18	1.00	2.08	0.00	2.26	0.00	0.00	0.00	0.00
time (sec)	N/A	1.231	7.551	16.205	0.000	0.121	0.000	0.000	4.294	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	1063	497	1072	0	1651	0	0	0	0
N.S.	1	2.01	0.94	2.03	0.00	3.12	0.00	0.00	0.00	0.00
time (sec)	N/A	1.460	7.623	10.141	0.000	0.142	0.000	0.000	6.364	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	1213	720	1474	0	2915	0	0	0	0
N.S.	1	1.65	0.98	2.00	0.00	3.96	0.00	0.00	0.00	0.00
time (sec)	N/A	1.720	8.420	10.527	0.000	0.208	0.000	0.000	8.527	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1122	1706	784	2046	0	1200	0	0	0	0
N.S.	1	1.52	0.70	1.82	0.00	1.07	0.00	0.00	0.00	0.00
time (sec)	N/A	2.565	9.676	13.898	0.000	0.114	0.000	0.000	1.859	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	874	1379	603	1214	0	941	0	0	0	0
N.S.	1	1.58	0.69	1.39	0.00	1.08	0.00	0.00	0.00	0.00
time (sec)	N/A	1.993	8.750	13.843	0.000	0.112	0.000	0.000	1.619	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	1231	545	1083	0	1184	0	0	0	0
N.S.	1	1.89	0.83	1.66	0.00	1.81	0.00	0.00	0.00	0.00
time (sec)	N/A	1.681	7.700	28.408	0.000	0.117	0.000	0.000	3.159	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	1218	560	1432	0	1459	0	0	0	0
N.S.	1	2.01	0.92	2.36	0.00	2.41	0.00	0.00	0.00	0.00
time (sec)	N/A	1.725	8.666	30.235	0.000	0.131	0.000	0.000	5.913	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	1152	535	1324	0	1648	0	0	0	0
N.S.	1	1.84	0.86	2.12	0.00	2.64	0.00	0.00	0.00	0.00
time (sec)	N/A	1.672	8.035	31.974	0.000	0.133	0.000	0.000	11.213	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	1366	724	1573	0	2837	0	0	0	0
N.S.	1	1.73	0.92	1.99	0.00	3.59	0.00	0.00	0.00	0.00
time (sec)	N/A	2.034	8.770	12.358	0.000	0.207	0.000	0.000	14.738	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	1074	423	723	0	618	0	0	1195	0
N.S.	1	1.70	0.67	1.15	0.00	0.98	0.00	0.00	1.89	0.00
time (sec)	N/A	1.514	5.170	13.625	0.000	0.107	0.000	0.000	1.024	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	832	291	453	0	382	0	0	651	0
N.S.	1	1.91	0.67	1.04	0.00	0.88	0.00	0.00	1.50	0.00
time (sec)	N/A	1.075	3.039	10.368	0.000	0.101	0.000	0.000	0.670	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	483	236	325	0	224	0	0	288	0
N.S.	1	1.60	0.78	1.08	0.00	0.74	0.00	0.00	0.95	0.00
time (sec)	N/A	0.593	6.339	8.905	0.000	0.096	0.000	0.000	0.471	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	695	246	449	0	432	0	0	657	0
N.S.	1	2.49	0.88	1.61	0.00	1.55	0.00	0.00	2.35	0.00
time (sec)	N/A	0.904	10.700	6.297	0.000	0.098	0.000	0.000	0.700	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	772	350	699	0	981	0	0	0	0
N.S.	1	2.35	1.06	2.12	0.00	2.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.937	11.726	8.723	0.000	0.125	0.000	0.000	1.563	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	995	478	1039	0	1811	0	0	0	0
N.S.	1	2.04	0.98	2.13	0.00	3.72	0.00	0.00	0.00	0.00
time (sec)	N/A	1.327	11.664	10.984	0.000	0.165	0.000	0.000	2.096	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	1252	535	1088	0	1226	0	0	0	0
N.S.	1	1.93	0.82	1.68	0.00	1.89	0.00	0.00	0.00	0.00
time (sec)	N/A	1.772	9.934	21.450	0.000	0.124	0.000	0.000	3.251	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	972	370	851	0	811	0	0	0	0
N.S.	1	2.13	0.81	1.87	0.00	1.78	0.00	0.00	0.00	0.00
time (sec)	N/A	1.336	6.755	19.022	0.000	0.112	0.000	0.000	1.977	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	633	273	532	0	442	0	0	1420	0
N.S.	1	2.08	0.90	1.75	0.00	1.45	0.00	0.00	4.67	0.00
time (sec)	N/A	0.827	4.119	9.234	0.000	0.103	0.000	0.000	1.193	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	862	249	448	0	500	0	0	657	0
N.S.	1	3.10	0.90	1.61	0.00	1.80	0.00	0.00	2.36	0.00
time (sec)	N/A	1.039	10.769	6.424	0.000	0.099	0.000	0.000	0.673	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	738	337	649	0	942	0	0	1497	0
N.S.	1	2.40	1.09	2.11	0.00	3.06	0.00	0.00	4.86	0.00
time (sec)	N/A	0.844	11.084	8.486	0.000	0.116	0.000	0.000	0.915	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	941	542	867	0	1949	0	0	0	0
N.S.	1	2.03	1.17	1.87	0.00	4.21	0.00	0.00	0.00	0.00
time (sec)	N/A	1.207	12.246	18.110	0.000	0.161	0.000	0.000	1.537	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	1247	539	1437	0	1456	0	0	0	0
N.S.	1	2.02	0.87	2.33	0.00	2.36	0.00	0.00	0.00	0.00
time (sec)	N/A	1.799	9.403	23.799	0.000	0.128	0.000	0.000	5.915	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	926	390	851	0	911	0	0	0	0
N.S.	1	2.23	0.94	2.05	0.00	2.19	0.00	0.00	0.00	0.00
time (sec)	N/A	1.235	7.942	20.513	0.000	0.119	0.000	0.000	4.372	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	857	338	699	0	922	0	0	0	0
N.S.	1	2.35	0.93	1.92	0.00	2.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.037	5.520	7.021	0.000	0.117	0.000	0.000	2.642	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	779	341	683	0	950	0	0	0	0
N.S.	1	2.35	1.03	2.06	0.00	2.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.936	12.247	8.533	0.000	0.133	0.000	0.000	1.550	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	953	518	868	0	1836	0	0	0	0
N.S.	1	2.17	1.18	1.97	0.00	4.17	0.00	0.00	0.00	0.00
time (sec)	N/A	1.217	12.762	10.968	0.000	0.174	0.000	0.000	1.532	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	1170	502	1267	0	3168	0	0	0	0
N.S.	1	1.91	0.82	2.07	0.00	5.17	0.00	0.00	0.00	0.00
time (sec)	N/A	1.622	12.065	20.425	0.000	0.372	0.000	0.000	3.724	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	715	1203	651	0	0	0	0	0	0
N.S.	1	1.27	2.14	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.047	6.483	9.612	0.000	0.000	0.000	0.000	13.763	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	382	742	831	0	0	0	0	779	0
N.S.	1	0.95	1.85	2.07	0.00	0.00	0.00	0.00	1.94	0.00
time (sec)	N/A	0.550	4.573	9.681	0.000	0.000	0.000	0.000	6.505	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	326	185	340	0	0	0	0	28	0
N.S.	1	1.01	0.57	1.06	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.451	2.384	1.234	0.000	0.000	0.000	0.000	0.445	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	43	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.195	2.214	4.194	0.000	0.000	0.000	0.000	0.313	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	207	286	0	0	0	0	67	0
N.S.	1	1.00	0.99	1.37	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.299	4.027	6.695	0.000	0.000	0.000	0.000	8.276	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	398	1179	1944	0	0	0	0	91	0
N.S.	1	0.87	2.57	4.24	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.567	6.171	9.260	0.000	0.000	0.000	0.000	21.874	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	715	451	659	0	0	0	0	1576	0
N.S.	1	1.25	0.79	1.15	0.00	0.00	0.00	0.00	2.75	0.00
time (sec)	N/A	1.016	8.146	9.715	0.000	0.000	0.000	0.000	13.600	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	385	345	835	0	0	0	0	779	0
N.S.	1	0.91	0.82	1.98	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.565	4.943	9.690	0.000	0.000	0.000	0.000	6.354	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	323	197	341	0	0	0	0	94	0
N.S.	1	0.89	0.54	0.94	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.434	3.873	4.316	0.000	0.000	0.000	0.000	1.182	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	224	304	592	0	0	0	0	142	0
N.S.	1	0.80	1.09	2.11	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.304	6.732	6.541	0.000	0.000	0.000	0.000	15.176	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	391	402	1122	0	0	0	0	190	0
N.S.	1	0.89	0.91	2.55	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.646	8.162	9.172	0.000	0.000	0.000	0.000	32.867	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	624	547	4505	0	0	0	0	238	0
N.S.	1	0.95	0.83	6.84	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.947	8.750	19.064	0.000	0.000	0.000	0.000	58.707	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	821	1126	2026	946	0	0	0	0	30	0
N.S.	1	1.37	2.47	1.15	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.691	9.528	19.292	0.000	0.000	0.000	0.000	200.017	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	716	454	656	0	0	0	0	0	0
N.S.	1	1.20	0.76	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.028	6.654	9.915	0.000	0.000	0.000	0.000	13.918	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	601	351	734	0	0	0	0	794	0
N.S.	1	1.42	0.83	1.73	0.00	0.00	0.00	0.00	1.87	0.00
time (sec)	N/A	0.810	5.623	19.013	0.000	0.000	0.000	0.000	6.738	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	822	350	1065	0	0	0	0	0	0
N.S.	1	1.86	0.79	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.379	7.637	6.855	0.000	0.000	0.000	0.000	28.165	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	759	449	1703	0	0	0	0	0	0
N.S.	1	1.65	0.98	3.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.236	9.652	9.171	0.000	0.000	0.000	0.000	68.387	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	630	535	2417	0	0	0	0	0	0
N.S.	1	0.96	0.82	3.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	8.873	12.028	0.000	0.000	0.000	0.000	103.979	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	951	919	8088	0	0	0	0	0	0
N.S.	1	0.96	0.92	8.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.663	11.783	21.438	0.000	0.000	0.000	0.000	143.296	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	323	197	341	0	0	0	0	94	0
N.S.	1	0.89	0.54	0.94	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.435	0.159	4.314	0.000	0.000	0.000	0.000	1.156	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	207	204	300	0	0	0	0	100	0
N.S.	1	0.68	0.67	0.98	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.340	4.032	4.415	0.000	0.000	0.000	0.000	1.137	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	409	206	341	0	0	0	0	97	0
N.S.	1	1.30	0.66	1.09	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.734	3.979	4.373	0.000	0.000	0.000	0.000	1.091	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	310	207	298	0	0	0	0	103	0
N.S.	1	0.97	0.65	0.93	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.577	3.951	4.247	0.000	0.000	0.000	0.000	1.057	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	100	101	118	0	0	0	0	75	0
N.S.	1	0.47	0.48	0.56	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.253	2.698	5.888	0.000	0.000	0.000	0.000	0.329	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	343	221	304	0	0	0	0	127	0
N.S.	1	1.00	0.65	0.89	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.453	4.582	8.655	0.000	0.000	0.000	0.000	0.628	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	436	1037	1818	0	0	0	0	179	0
N.S.	1	0.91	2.16	3.78	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.674	6.388	17.825	0.000	0.000	0.000	0.000	30.203	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	343	365	412	0	0	0	0	127	0
N.S.	1	1.00	1.07	1.20	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.452	4.826	8.436	0.000	0.000	0.000	0.000	0.618	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	523	407	956	0	0	0	0	215	0
N.S.	1	1.17	0.91	2.14	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.672	9.210	18.023	0.000	0.000	0.000	0.000	20.112	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	849	1660	2755	0	0	0	0	303	0
N.S.	1	1.29	2.52	4.19	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.171	13.370	20.361	0.000	0.000	0.000	0.000	28.817	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	440	437	1325	0	0	0	0	179	0
N.S.	1	0.92	0.91	2.76	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.665	8.003	10.927	0.000	0.000	0.000	0.000	30.873	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	786	1662	2127	0	0	0	0	303	0
N.S.	1	1.18	2.50	3.20	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.064	11.849	20.294	0.000	0.000	0.000	0.000	28.757	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	983	1031	744	8976	0	0	0	0	427	0
N.S.	1	1.05	0.76	9.13	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.606	12.184	22.339	0.000	0.000	0.000	0.000	34.478	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	861	720	1041	0	0	0	0	0	0
N.S.	1	1.91	1.60	2.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.607	7.498	6.770	0.000	0.000	0.000	0.000	23.001	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	494	511	630	0	0	0	0	144	0
N.S.	1	1.27	1.31	1.62	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.830	6.595	6.586	0.000	0.000	0.000	0.000	13.548	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	455	363	391	0	0	0	0	68	0
N.S.	1	1.27	1.01	1.09	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.746	4.209	7.002	0.000	0.000	0.000	0.000	7.958	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	484	382	411	0	0	0	0	128	0
N.S.	1	1.33	1.05	1.13	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.854	4.659	8.680	0.000	0.000	0.000	0.000	0.597	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	828	411	957	0	0	0	0	216	0
N.S.	1	1.60	0.80	1.85	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.468	8.564	11.030	0.000	0.000	0.000	0.000	20.137	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	0	1710	2794	0	0	0	0	304	0
N.S.	1	0.00	2.21	3.61	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.000	12.982	20.191	0.000	0.000	0.000	0.000	47.599	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	234	204	263	0	0	0	0	344	0
N.S.	1	0.97	0.84	1.09	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.379	2.959	6.381	0.000	0.000	0.000	0.000	0.513	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	188	78	120	0	0	0	0	24	0
N.S.	1	0.98	0.41	0.62	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.311	1.440	0.966	0.000	0.000	0.000	0.000	0.183	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	58	54	64	0	0	0	0	34	0
N.S.	1	2.52	2.35	2.78	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.166	2.326	3.344	0.000	0.000	0.000	0.000	0.172	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	121	78	147	0	0	0	0	46	0
N.S.	1	1.41	0.91	1.71	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.227	2.830	4.917	0.000	0.000	0.000	0.000	0.652	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	217	353	372	0	0	0	0	58	0
N.S.	1	0.92	1.50	1.58	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.357	3.524	6.846	0.000	0.000	0.000	0.000	0.928	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	302	134	293	0	0	0	0	28	0
N.S.	1	1.01	0.45	0.98	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.420	2.262	0.977	0.000	0.000	0.000	0.000	0.297	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0	43	0
N.S.	1	1.00	1.01	1.43	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.191	2.035	3.411	0.000	0.000	0.000	0.000	0.242	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	49	49	53	0	0	0	0	73	0
N.S.	1	0.25	0.25	0.27	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.174	2.239	4.689	0.000	0.000	0.000	0.000	0.256	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	14	19	0	41	0
N.S.	1	1.00	1.03	0.97	0.00	0.39	0.53	0.00	1.14	0.00
time (sec)	N/A	0.173	0.730	3.484	0.000	0.086	1.776	0.000	0.160	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	0	0	0	0	40	0
N.S.	1	1.00	1.00	1.26	0.00	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.152	0.124	4.525	0.000	0.000	0.000	0.000	0.188	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	0	0	0	0	40	0
N.S.	1	1.00	1.00	1.26	0.00	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.150	0.733	4.938	0.000	0.000	0.000	0.000	0.185	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	43	40	46	0	0	0	0	40	0
N.S.	1	2.26	2.11	2.42	0.00	0.00	0.00	0.00	2.11	0.00
time (sec)	N/A	0.174	0.888	4.503	0.000	0.000	0.000	0.000	0.178	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	43	36	0	0	0	0	38	0
N.S.	1	1.00	1.30	1.09	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.166	0.504	4.573	0.000	0.000	0.000	0.000	0.169	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	0	1697	1817	0	0	0	0	0	0
N.S.	1	0.00	2.86	3.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.471	21.039	0.000	0.000	0.000	0.000	71.799	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	1030	352	1075	0	0	0	0	0	0
N.S.	1	2.40	0.82	2.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.490	5.320	5.096	0.000	0.000	0.000	0.000	30.351	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	378	401	559	0	0	0	0	39	0
N.S.	1	0.92	0.97	1.35	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.540	4.102	1.470	0.000	0.000	0.000	0.000	9.070	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	565	496	725	0	0	0	0	67	0
N.S.	1	1.28	1.13	1.65	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.780	4.932	6.685	0.000	0.000	0.000	0.000	9.498	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	1179	277	873	0	0	0	0	108	0
N.S.	1	2.84	0.67	2.10	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.648	5.422	9.001	0.000	0.000	0.000	0.000	33.023	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	0	443	2933	0	0	0	0	149	0
N.S.	1	0.00	0.75	4.95	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	6.455	18.634	0.000	0.000	0.000	0.000	33.285	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	705	0	2953	1807	0	0	0	0	30	0
N.S.	1	0.00	4.19	2.56	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	13.133	22.218	0.000	0.000	0.000	0.000	200.029	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	0	2079	2013	0	0	0	0	0	0
N.S.	1	0.00	3.84	3.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.652	21.363	0.000	0.000	0.000	0.000	89.133	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	723	335	1168	0	0	0	0	0	0
N.S.	1	1.69	0.78	2.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.028	5.537	5.135	0.000	0.000	0.000	0.000	30.478	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	684	357	1481	0	0	0	0	142	0
N.S.	1	1.39	0.72	3.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.929	6.218	6.566	0.000	0.000	0.000	0.000	15.709	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	1407	318	1536	0	0	0	0	224	0
N.S.	1	2.64	0.60	2.89	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.946	8.507	9.296	0.000	0.000	0.000	0.000	51.256	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	0	414	2460	0	0	0	0	306	0
N.S.	1	0.00	0.73	4.32	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	8.090	18.898	0.000	0.000	0.000	0.000	48.532	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	962	0	4275	2166	0	0	0	0	30	0
N.S.	1	0.00	4.44	2.25	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	13.132	24.950	0.000	0.000	0.000	0.000	200.027	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	600	1811	0	0	0	0	30	0
N.S.	1	0.00	0.83	2.52	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	11.641	21.957	0.000	0.000	0.000	0.000	200.026	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	1127	448	1880	0	0	0	0	0	0
N.S.	1	2.08	0.83	3.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.755	7.556	21.337	0.000	0.000	0.000	0.000	71.790	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	1026	410	2343	0	0	0	0	0	0
N.S.	1	1.66	0.66	3.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.464	7.445	6.880	0.000	0.000	0.000	0.000	26.802	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	1650	493	2325	0	0	0	0	0	0
N.S.	1	2.49	0.74	3.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.531	10.074	9.131	0.000	0.000	0.000	0.000	145.640	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	0	481	3434	0	0	0	0	0	0
N.S.	1	0.00	0.71	5.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.099	18.741	0.000	0.000	0.000	0.000	90.155	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	360	422	574	0	0	0	0	40	0
N.S.	1	1.00	1.18	1.60	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.610	4.621	1.572	0.000	0.000	0.000	0.000	8.841	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	378	401	559	0	0	0	0	39	0
N.S.	1	0.92	0.97	1.35	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.567	4.197	1.552	0.000	0.000	0.000	0.000	9.103	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	448	587	973	0	0	0	0	127	0
N.S.	1	1.05	1.38	2.28	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.624	6.130	8.796	0.000	0.000	0.000	0.000	0.733	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	817	1652	1560	0	0	0	0	30	0
N.S.	1	1.57	3.18	3.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.100	6.917	11.191	0.000	0.000	0.000	0.000	200.025	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	760	1279	558	3335	0	0	0	0	30	0
N.S.	1	1.68	0.73	4.39	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.896	8.167	20.631	0.000	0.000	0.000	0.000	200.024	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	817	1659	1339	0	0	0	0	30	0
N.S.	1	1.58	3.21	2.59	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.082	8.135	11.271	0.000	0.000	0.000	0.000	200.027	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	1366	3040	3679	0	0	0	0	362	0
N.S.	1	1.93	4.29	5.20	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	2.054	13.451	20.606	0.000	0.000	0.000	0.000	28.241	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	747	1283	569	2509	0	0	0	0	30	0
N.S.	1	1.72	0.76	3.36	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.946	8.319	20.529	0.000	0.000	0.000	0.000	200.024	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1047	0	765	3094	0	0	0	0	509	0
N.S.	1	0.00	0.73	2.96	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.000	12.692	21.736	0.000	0.000	0.000	0.000	84.730	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	430	617	989	0	0	0	0	132	0
N.S.	1	1.01	1.45	2.32	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.693	8.846	8.817	0.000	0.000	0.000	0.000	0.724	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	447	587	973	0	0	0	0	127	0
N.S.	1	1.05	1.38	2.29	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.610	5.727	8.586	0.000	0.000	0.000	0.000	0.737	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	959	0	636	4513	0	0	0	0	30	0
N.S.	1	0.00	0.66	4.71	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	7.752	26.800	0.000	0.000	0.000	0.000	200.017	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	795	0	509	3242	0	0	0	0	0	0
N.S.	1	0.00	0.64	4.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.866	7.426	0.000	0.000	0.000	0.000	182.934	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	0	414	1895	0	0	0	0	0	0
N.S.	1	0.00	0.73	3.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.605	5.964	0.000	0.000	0.000	0.000	61.931	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	0	389	1928	0	0	0	0	50	0
N.S.	1	0.00	0.70	3.47	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	4.217	1.534	0.000	0.000	0.000	0.000	32.906	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	0	422	2505	0	0	0	0	91	0
N.S.	1	0.00	0.70	4.13	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	4.659	9.191	0.000	0.000	0.000	0.000	23.959	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	0	486	2670	0	0	0	0	149	0
N.S.	1	0.00	0.76	4.20	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	6.334	19.155	0.000	0.000	0.000	0.000	28.354	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	923	0	731	4339	0	0	0	0	207	0
N.S.	1	0.00	0.79	4.70	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	9.918	21.466	0.000	0.000	0.000	0.000	54.833	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	792	0	509	2813	0	0	0	0	30	0
N.S.	1	0.00	0.64	3.55	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	9.091	26.994	0.000	0.000	0.000	0.000	200.017	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	0	378	1688	0	0	0	0	30	0
N.S.	1	0.00	0.61	2.73	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	7.547	7.125	0.000	0.000	0.000	0.000	200.025	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	0	384	1790	0	0	0	0	0	0
N.S.	1	0.00	0.70	3.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.524	5.977	0.000	0.000	0.000	0.000	61.384	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	0	402	2021	0	0	0	0	190	0
N.S.	1	0.00	0.55	2.77	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	5.843	9.168	0.000	0.000	0.000	0.000	42.784	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	0	457	2179	0	0	0	0	306	0
N.S.	1	0.00	0.58	2.77	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.000	7.831	10.538	0.000	0.000	0.000	0.000	63.812	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	785	0	575	2691	0	0	0	0	422	0
N.S.	1	0.00	0.73	3.43	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	9.284	21.796	0.000	0.000	0.000	0.000	84.283	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	942	0	674	4349	0	0	0	0	30	0
N.S.	1	0.00	0.72	4.62	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	10.760	30.444	0.000	0.000	0.000	0.000	200.021	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	728	0	502	2817	0	0	0	0	30	0
N.S.	1	0.00	0.69	3.87	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	9.079	27.792	0.000	0.000	0.000	0.000	200.020	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	0	440	3151	0	0	0	0	0	0
N.S.	1	0.00	0.64	4.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.839	7.486	0.000	0.000	0.000	0.000	183.723	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	0	476	3288	0	0	0	0	0	0
N.S.	1	0.00	0.57	3.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.124	9.432	0.000	0.000	0.000	0.000	76.794	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	954	0	520	3358	0	0	0	0	0	0
N.S.	1	0.00	0.55	3.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.117	19.214	0.000	0.000	0.000	0.000	110.896	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	0	670	4004	0	0	0	0	0	0
N.S.	1	0.00	0.67	4.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	10.783	20.399	0.000	0.000	0.000	0.000	182.646	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1059	0	582	4537	0	0	0	0	30	0
N.S.	1	0.00	0.55	4.28	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	7.983	9.781	0.000	0.000	0.000	0.000	200.028	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1199	0	635	4539	0	0	0	0	30	0
N.S.	1	0.00	0.53	3.79	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	10.108	11.607	0.000	0.000	0.000	0.000	200.028	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	0	761	5213	0	0	0	0	30	0
N.S.	1	0.00	0.62	4.27	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	11.779	19.573	0.000	0.000	0.000	0.000	200.021	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	438	2616	0	0	0	0	179	0
N.S.	1	0.00	0.65	3.91	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	5.188	11.133	0.000	0.000	0.000	0.000	54.114	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	835	0	595	3337	0	0	0	0	30	0
N.S.	1	0.00	0.71	4.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	8.081	20.314	0.000	0.000	0.000	0.000	200.022	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	810	0	600	3117	0	0	0	0	30	0
N.S.	1	0.00	0.74	3.85	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	8.507	20.793	0.000	0.000	0.000	0.000	200.025	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1130	0	805	6155	0	0	0	0	30	0
N.S.	1	0.00	0.71	5.45	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	13.370	22.859	0.000	0.000	0.000	0.000	200.024	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1161	0	893	4030	0	0	0	0	30	0
N.S.	1	0.00	0.77	3.47	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	13.269	22.757	0.000	0.000	0.000	0.000	200.023	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1572	0	9241	4748	0	0	0	0	30	0
N.S.	1	0.00	5.88	3.02	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	19.737	25.178	0.000	0.000	0.000	0.000	200.029	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	175	175	236	218	221	182
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.37	1.27	1.28	1.06
time (sec)	N/A	0.382	0.086	0.489	0.031	0.078	0.033	0.125	0.150	0.111

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	134	134	173	165	169	143
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.33	1.27	1.30	1.10
time (sec)	N/A	0.315	0.067	0.453	0.032	0.073	0.027	0.122	0.156	1.661

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	93	93	121	114	117	99
N.S.	1	1.00	1.02	1.00	0.99	0.99	1.29	1.21	1.24	1.05
time (sec)	N/A	0.262	0.038	0.455	0.029	0.088	0.025	0.120	0.147	0.046

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	52	52	63	62	65	54
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.12	1.11	1.16	0.96
time (sec)	N/A	0.203	0.014	0.247	0.025	0.072	0.017	0.124	0.141	1.583

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	87	72	74	0	191	206	82	136	108
N.S.	1	1.18	0.97	1.00	0.00	2.58	2.78	1.11	1.84	1.46
time (sec)	N/A	0.227	0.055	0.516	0.000	0.091	0.299	0.110	0.142	0.123

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	111	95	97	0	318	190	103	266	95
N.S.	1	1.17	1.00	1.02	0.00	3.35	2.00	1.08	2.80	1.00
time (sec)	N/A	0.231	0.063	0.579	0.000	0.088	0.602	0.123	0.148	0.173

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	141	130	132	0	471	246	142	433	136
N.S.	1	1.07	0.98	1.00	0.00	3.57	1.86	1.08	3.28	1.03
time (sec)	N/A	0.266	0.079	0.490	0.000	0.097	1.257	0.117	0.149	0.225

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	168	171	163	0	642	313	195	618	176
N.S.	1	0.97	0.99	0.94	0.00	3.71	1.81	1.13	3.57	1.02
time (sec)	N/A	0.287	0.103	0.502	0.000	0.092	2.453	0.129	0.149	1.763

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	236	236	304	289	293	233
N.S.	1	1.00	1.00	1.05	1.04	1.04	1.35	1.28	1.30	1.03
time (sec)	N/A	0.446	0.086	0.510	0.033	0.070	0.035	0.127	0.148	0.105

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	168	168	216	202	205	158
N.S.	1	1.00	1.00	1.07	1.06	1.06	1.37	1.28	1.30	1.00
time (sec)	N/A	0.372	0.058	0.495	0.030	0.065	0.029	0.115	0.166	0.073

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	156	115	170	0	366	347	182	271	203
N.S.	1	1.34	0.99	1.47	0.00	3.16	2.99	1.57	2.34	1.75
time (sec)	N/A	0.377	0.063	0.528	0.000	0.083	0.498	0.123	0.161	1.716

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	172	134	182	0	552	483	203	481	257
N.S.	1	1.26	0.99	1.34	0.00	4.06	3.55	1.49	3.54	1.89
time (sec)	N/A	0.385	0.096	0.544	0.000	0.091	1.200	0.123	0.154	1.756

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	221	183	230	0	777	400	252	717	243
N.S.	1	1.21	1.00	1.26	0.00	4.25	2.19	1.38	3.92	1.33
time (sec)	N/A	0.397	0.128	0.553	0.000	0.094	4.589	0.119	0.162	1.832

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	262	242	289	0	1024	486	330	988	303
N.S.	1	1.07	0.99	1.18	0.00	4.20	1.99	1.35	4.05	1.24
time (sec)	N/A	0.456	0.152	0.540	0.000	0.098	18.069	0.127	0.157	0.246

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	326	326	423	409	417	335
N.S.	1	1.00	1.00	1.09	1.05	1.05	1.36	1.32	1.35	1.08
time (sec)	N/A	0.588	0.114	0.510	0.032	0.071	0.038	0.123	0.153	1.743

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	249	179	300	0	586	508	313	441	312
N.S.	1	1.38	0.99	1.67	0.00	3.26	2.82	1.74	2.45	1.73
time (sec)	N/A	0.522	0.090	0.535	0.000	0.085	0.725	0.126	0.157	1.713

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	255	176	308	0	834	661	334	723	389
N.S.	1	1.43	0.99	1.73	0.00	4.69	3.71	1.88	4.06	2.19
time (sec)	N/A	0.523	0.127	0.556	0.000	0.096	1.936	0.124	0.158	0.164

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	311	219	344	0	1102	865	389	1032	495
N.S.	1	1.41	0.99	1.56	0.00	4.99	3.91	1.76	4.67	2.24
time (sec)	N/A	0.561	0.163	0.529	0.000	0.100	10.487	0.120	0.156	1.864

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	374	295	417	0	1422	0	475	1373	444
N.S.	1	1.27	1.00	1.41	0.00	4.82	0.00	1.61	4.65	1.51
time (sec)	N/A	0.586	0.194	0.616	0.000	0.110	0.000	0.123	0.154	1.921

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	68	0	404	0	68	110	1245
N.S.	1	1.00	0.99	0.79	0.00	4.70	0.00	0.79	1.28	14.48
time (sec)	N/A	0.192	0.074	0.631	0.000	0.136	0.000	0.125	0.152	2.347

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	156	124	131	0	997	0	151	428	5567
N.S.	1	1.11	0.89	0.94	0.00	7.12	0.00	1.08	3.06	39.76
time (sec)	N/A	0.288	0.184	0.684	0.000	0.597	0.000	0.129	0.152	3.997

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	259	216	265	0	2339	0	301	1037	12640
N.S.	1	1.17	0.97	1.19	0.00	10.54	0.00	1.36	4.67	56.94
time (sec)	N/A	0.422	0.233	0.724	0.000	3.564	0.000	0.124	0.158	6.483

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	238	187	198	0	2381	0	299	1095	9047
N.S.	1	1.11	0.87	0.93	0.00	11.13	0.00	1.40	5.12	42.28
time (sec)	N/A	0.404	0.236	1.093	0.000	4.347	0.000	0.125	0.153	5.655

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	361	269	331	0	4763	0	443	2169	17499
N.S.	1	1.15	0.86	1.06	0.00	15.22	0.00	1.42	6.93	55.91
time (sec)	N/A	0.582	0.382	0.825	0.000	17.637	0.000	0.131	0.159	8.387

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	502	353	461	0	7784	0	906	3522	98059
N.S.	1	1.12	0.79	1.03	0.00	17.38	0.00	2.02	7.86	218.88
time (sec)	N/A	0.854	0.532	0.874	0.000	56.838	0.000	0.136	0.171	15.588

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	362	395	382	382	522	495	499	402
N.S.	1	1.00	1.00	1.09	1.06	1.06	1.44	1.37	1.38	1.11
time (sec)	N/A	0.648	0.122	0.543	0.034	0.063	0.054	0.125	0.151	0.152

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	244	286	270	270	372	349	352	271
N.S.	1	1.00	1.00	1.17	1.11	1.11	1.52	1.43	1.44	1.11
time (sec)	N/A	0.476	0.085	0.576	0.028	0.067	0.039	0.134	0.150	2.080

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	225	190	291	0	652	609	370	519	351
N.S.	1	1.24	1.05	1.61	0.00	3.60	3.36	2.04	2.87	1.94
time (sec)	N/A	0.491	0.106	0.712	0.000	0.079	1.000	0.120	0.159	2.034

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	347	208	372	0	934	780	393	844	455
N.S.	1	1.67	1.00	1.79	0.00	4.49	3.75	1.89	4.06	2.19
time (sec)	N/A	0.646	0.162	0.576	0.000	0.101	2.868	0.127	0.158	0.200

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	412	259	406	0	1238	675	460	1195	430
N.S.	1	1.57	0.99	1.55	0.00	4.73	2.58	1.76	4.56	1.64
time (sec)	N/A	0.677	0.178	0.568	0.000	0.110	19.169	0.123	0.163	0.272

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	502	337	498	0	1596	0	564	1595	523
N.S.	1	1.48	0.99	1.46	0.00	4.69	0.00	1.66	4.69	1.54
time (sec)	N/A	0.739	0.232	0.564	0.000	0.137	0.000	0.128	0.160	2.309

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	493	553	552	552	719	698	705	575
N.S.	1	1.00	1.00	1.12	1.12	1.12	1.46	1.42	1.43	1.17
time (sec)	N/A	0.820	0.167	0.530	0.039	0.069	0.081	0.124	0.147	2.105

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	338	292	609	0	1068	911	623	826	569
N.S.	1	1.17	1.01	2.10	0.00	3.68	3.14	2.15	2.85	1.96
time (sec)	N/A	0.699	0.144	0.556	0.000	0.084	1.661	0.124	0.147	2.084

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	523	275	614	0	1442	1096	638	1264	733
N.S.	1	1.90	1.00	2.23	0.00	5.24	3.99	2.32	4.60	2.67
time (sec)	N/A	0.931	0.213	0.675	0.000	0.102	5.686	0.124	0.157	0.258

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	585	332	646	0	1852	1508	709	1724	901
N.S.	1	1.76	1.00	1.94	0.00	5.56	4.53	2.13	5.18	2.71
time (sec)	N/A	0.949	0.244	0.693	0.000	0.127	82.315	0.128	0.154	0.327

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	704	423	724	0	2296	0	829	2236	764
N.S.	1	1.65	0.99	1.70	0.00	5.39	0.00	1.95	5.25	1.79
time (sec)	N/A	1.020	0.464	0.582	0.000	0.176	0.000	0.129	0.159	2.381

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	145	102	113	0	678	0	111	213	3684
N.S.	1	1.44	1.01	1.12	0.00	6.71	0.00	1.10	2.11	36.48
time (sec)	N/A	0.258	0.069	0.669	0.000	1.036	0.000	0.125	0.152	3.554

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	207	142	210	0	1702	0	211	710	8320
N.S.	1	1.34	0.92	1.36	0.00	11.05	0.00	1.37	4.61	54.03
time (sec)	N/A	0.432	0.167	0.733	0.000	5.247	0.000	0.127	0.159	4.715

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	295	270	432	0	3241	0	434	1587	13717
N.S.	1	1.08	0.99	1.58	0.00	11.83	0.00	1.58	5.79	50.06
time (sec)	N/A	0.498	0.333	0.844	0.000	18.599	0.000	0.132	0.160	5.789

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	413	203	316	0	3861	0	396	1759	12581
N.S.	1	1.60	0.79	1.22	0.00	14.97	0.00	1.53	6.82	48.76
time (sec)	N/A	0.631	0.327	0.760	0.000	19.590	0.000	0.126	0.160	6.435

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	639	321	540	0	6809	0	592	3303	127501
N.S.	1	1.56	0.78	1.32	0.00	16.65	0.00	1.45	8.08	311.74
time (sec)	N/A	0.938	0.487	0.851	0.000	64.858	0.000	0.136	0.161	16.998

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	882	441	761	0	0	0	1271	5234	161006
N.S.	1	1.50	0.75	1.29	0.00	0.00	0.00	2.16	8.90	273.82
time (sec)	N/A	1.315	0.691	0.930	0.000	0.000	0.000	0.132	0.186	20.546

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	653	767	727	727	1008	984	993	784
N.S.	1	1.00	1.00	1.17	1.11	1.11	1.54	1.51	1.52	1.20
time (sec)	N/A	1.011	0.238	0.538	0.037	0.076	0.083	0.123	0.151	0.224

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	479	427	927	0	1604	1379	1013	1293	838
N.S.	1	1.12	1.00	2.17	0.00	3.75	3.22	2.37	3.02	1.96
time (sec)	N/A	0.919	0.225	0.681	0.000	0.104	3.794	0.137	0.158	0.172

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	880	390	996	0	2076	1590	1022	1885	1177
N.S.	1	2.27	1.01	2.57	0.00	5.36	4.11	2.64	4.87	3.04
time (sec)	N/A	1.635	0.297	0.619	0.000	0.123	13.483	0.128	0.172	0.307

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	1127	450	1015	0	2560	0	1093	2493	1299
N.S.	1	2.48	0.99	2.23	0.00	5.63	0.00	2.40	5.48	2.85
time (sec)	N/A	1.747	0.351	0.661	0.000	0.200	0.000	0.132	0.353	2.317

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	1308	552	1095	0	3136	0	1230	3156	1132
N.S.	1	2.36	1.00	1.98	0.00	5.66	0.00	2.22	5.70	2.04
time (sec)	N/A	1.915	0.417	0.618	0.000	0.346	0.000	0.121	4.216	2.347

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	227	131	167	0	1081	0	191	354	5412
N.S.	1	1.72	0.99	1.27	0.00	8.19	0.00	1.45	2.68	41.00
time (sec)	N/A	0.407	0.089	0.684	0.000	10.627	0.000	0.126	0.165	3.040

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	312	169	297	0	2588	0	293	1064	11519
N.S.	1	1.85	1.00	1.76	0.00	15.31	0.00	1.73	6.30	68.16
time (sec)	N/A	0.719	0.184	0.775	0.000	64.524	0.000	0.128	0.162	3.745

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	434	283	599	0	4843	0	571	2204	18014
N.S.	1	1.52	0.99	2.09	0.00	16.93	0.00	2.00	7.71	62.99
time (sec)	N/A	0.768	0.408	0.899	0.000	112.850	0.000	0.134	0.163	7.068

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	639	217	448	0	5227	0	559	2451	142283
N.S.	1	2.14	0.73	1.50	0.00	17.48	0.00	1.87	8.20	475.86
time (sec)	N/A	1.039	0.421	0.820	0.000	146.919	0.000	0.127	0.158	17.258

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	953	338	752	0	0	0	750	4530	184741
N.S.	1	1.89	0.67	1.49	0.00	0.00	0.00	1.49	8.97	365.82
time (sec)	N/A	1.437	0.602	0.892	0.000	0.000	0.000	0.134	0.174	20.844

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	1540	453	1051	0	0	0	1605	7120	227222
N.S.	1	2.48	0.73	1.69	0.00	0.00	0.00	2.58	11.47	365.90
time (sec)	N/A	2.202	0.793	0.954	0.000	0.000	0.000	0.135	0.213	24.532

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	166	133	117	0	0	0	137	212	26278
N.S.	1	1.24	0.99	0.87	0.00	0.00	0.00	1.02	1.58	196.10
time (sec)	N/A	0.245	0.204	0.805	0.000	0.000	0.000	0.131	0.167	6.365

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	266	189	189	0	0	0	358	1214	120009
N.S.	1	1.31	0.93	0.93	0.00	0.00	0.00	1.76	5.98	591.18
time (sec)	N/A	0.433	0.856	1.103	0.000	0.000	0.000	0.127	0.187	11.908

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	464	329	470	0	0	0	787	3509	113015
N.S.	1	1.39	0.99	1.41	0.00	0.00	0.00	2.36	10.54	339.38
time (sec)	N/A	0.711	0.509	1.725	0.000	0.000	0.000	0.131	0.240	17.105

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	414	264	261	0	0	0	795	4534	123740
N.S.	1	1.53	0.98	0.97	0.00	0.00	0.00	2.94	16.79	458.30
time (sec)	N/A	0.660	1.091	1.723	0.000	0.000	0.000	0.136	0.242	16.350

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	624	395	544	0	0	0	1240	10641	212642
N.S.	1	1.56	0.99	1.36	0.00	0.00	0.00	3.10	26.60	531.60
time (sec)	N/A	0.975	1.021	3.013	0.000	0.000	0.000	0.154	0.324	33.865

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	1052	520	823	0	0	0	3373	20240	332957
N.S.	1	1.98	0.98	1.55	0.00	0.00	0.00	6.34	38.05	625.86
time (sec)	N/A	1.742	1.565	5.563	0.000	0.000	0.000	0.196	28.184	69.157

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	850	321	333	0	0	0	1680	10200	221098
N.S.	1	2.51	0.95	0.99	0.00	0.00	0.00	4.97	30.18	654.14
time (sec)	N/A	1.188	1.012	2.652	0.000	0.000	0.000	0.191	9.325	39.111

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	365	346	312	528	818	678	430	28	0
N.S.	1	0.94	0.89	0.80	1.35	2.10	1.74	1.10	0.07	0.00
time (sec)	N/A	0.602	1.095	0.700	0.042	0.421	0.490	0.149	200.020	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	247	231	208	337	538	413	273	491	0
N.S.	1	0.93	0.87	0.78	1.26	2.01	1.55	1.02	1.84	0.00
time (sec)	N/A	0.389	0.646	0.679	0.034	0.201	0.436	0.146	0.297	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	149	135	123	177	314	207	146	261	0
N.S.	1	0.96	0.87	0.79	1.13	2.01	1.33	0.94	1.67	0.00
time (sec)	N/A	0.261	0.315	0.540	0.034	0.124	0.387	0.133	0.169	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	134	141	115	0	777	0	0	323	0
N.S.	1	1.05	1.10	0.90	0.00	6.07	0.00	0.00	2.52	0.00
time (sec)	N/A	0.301	0.540	1.399	0.000	0.727	0.000	0.000	0.174	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	154	156	151	0	1267	0	349	929	0
N.S.	1	1.08	1.09	1.06	0.00	8.86	0.00	2.44	6.50	0.00
time (sec)	N/A	0.287	0.888	0.801	0.000	1.030	0.000	0.149	0.187	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	183	300	155	0	854	0	859	2655	0
N.S.	1	1.08	1.76	0.91	0.00	5.02	0.00	5.05	15.62	0.00
time (sec)	N/A	0.319	11.751	0.924	0.000	0.900	0.000	0.345	0.260	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	308	378	282	0	1618	0	1777	5233	0
N.S.	1	1.08	1.33	0.99	0.00	5.70	0.00	6.26	18.43	0.00
time (sec)	N/A	0.488	11.546	0.951	0.000	7.000	0.000	0.713	0.695	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	1009	587	546	900	1414	1290	787	30	0
N.S.	1	1.49	0.87	0.81	1.33	2.09	1.91	1.16	0.04	0.00
time (sec)	N/A	1.038	2.152	0.936	0.050	1.411	0.652	0.172	200.022	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	670	388	359	576	940	804	505	30	0
N.S.	1	1.53	0.89	0.82	1.32	2.15	1.84	1.15	0.07	0.00
time (sec)	N/A	0.742	1.288	0.773	0.037	0.495	0.517	0.151	200.024	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	247	230	208	337	550	413	273	491	0
N.S.	1	0.93	0.86	0.78	1.26	2.06	1.55	1.02	1.84	0.00
time (sec)	N/A	0.385	0.657	0.708	0.039	0.222	0.447	0.143	0.274	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	236	195	197	0	1192	0	0	644	0
N.S.	1	1.19	0.98	0.99	0.00	6.02	0.00	0.00	3.25	0.00
time (sec)	N/A	0.409	0.716	0.953	0.000	3.940	0.000	0.000	0.206	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	412	201	202	0	2125	0	509	1801	0
N.S.	1	2.11	1.03	1.04	0.00	10.90	0.00	2.61	9.24	0.00
time (sec)	N/A	0.727	1.081	0.971	0.000	2.752	0.000	0.160	0.238	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	593	258	282	0	3262	0	1368	5191	0
N.S.	1	2.22	0.97	1.06	0.00	12.22	0.00	5.12	19.44	0.00
time (sec)	N/A	0.849	10.853	0.963	0.000	9.949	0.000	0.180	0.329	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	947	524	364	0	1976	0	2774	7090	0
N.S.	1	2.70	1.49	1.04	0.00	5.63	0.00	7.90	20.20	0.00
time (sec)	N/A	1.217	15.187	1.074	0.000	7.462	0.000	0.754	0.861	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	1009	584	546	900	1432	1290	787	30	0
N.S.	1	1.49	0.86	0.81	1.33	2.12	1.91	1.16	0.04	0.00
time (sec)	N/A	1.017	2.129	0.912	0.048	1.440	0.605	0.172	200.025	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	365	342	312	528	846	678	430	28	0
N.S.	1	0.94	0.88	0.80	1.35	2.17	1.74	1.10	0.07	0.00
time (sec)	N/A	0.593	1.101	0.779	0.037	0.605	0.505	0.145	200.022	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	395	294	311	0	1809	0	0	1077	0
N.S.	1	1.27	0.94	1.00	0.00	5.80	0.00	0.00	3.45	0.00
time (sec)	N/A	0.634	1.202	0.982	0.000	28.216	0.000	0.000	0.387	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	0	285	295	0	3401	0	720	2906	0
N.S.	1	0.00	1.08	1.11	0.00	12.83	0.00	2.72	10.97	0.00
time (sec)	N/A	0.000	1.871	1.075	0.000	15.929	0.000	0.174	0.361	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	1024	285	419	0	5166	0	1875	8152	0
N.S.	1	3.18	0.89	1.30	0.00	16.04	0.00	5.82	25.32	0.00
time (sec)	N/A	1.588	11.119	1.183	0.000	46.913	0.000	0.195	0.577	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	1559	427	566	0	6833	0	3864	11416	0
N.S.	1	3.35	0.92	1.21	0.00	14.66	0.00	8.29	24.50	0.00
time (sec)	N/A	2.041	12.075	1.399	0.000	132.959	0.000	0.237	0.945	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	190	145	111	0	977	0	180	224	0
N.S.	1	1.58	1.21	0.92	0.00	8.14	0.00	1.50	1.87	0.00
time (sec)	N/A	0.337	0.536	1.618	0.000	5.367	0.000	0.465	0.476	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	260	203	210	0	0	0	377	1408	0
N.S.	1	1.41	1.10	1.14	0.00	0.00	0.00	2.05	7.65	0.00
time (sec)	N/A	0.478	1.377	1.352	0.000	0.000	0.000	0.868	1.280	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	406	279	349	0	0	0	1173	7591	0
N.S.	1	1.31	0.90	1.13	0.00	0.00	0.00	3.78	24.49	0.00
time (sec)	N/A	0.653	11.014	1.574	0.000	0.000	0.000	2.004	5.446	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	606	485	622	0	0	0	3384	30	0
N.S.	1	1.14	0.91	1.17	0.00	0.00	0.00	6.36	0.06	0.00
time (sec)	N/A	1.037	12.204	2.156	0.000	0.000	0.000	6.364	200.024	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	256	216	210	0	0	0	376	1408	0
N.S.	1	1.41	1.19	1.15	0.00	0.00	0.00	2.07	7.74	0.00
time (sec)	N/A	0.469	1.418	1.426	0.000	0.000	0.000	0.865	1.274	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	650	208	278	0	0	0	1081	30	0
N.S.	1	2.61	0.84	1.12	0.00	0.00	0.00	4.34	0.12	0.00
time (sec)	N/A	0.958	10.658	1.548	0.000	0.000	0.000	1.994	200.028	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	0	347	490	0	0	0	1461	30	0
N.S.	1	0.00	0.92	1.30	0.00	0.00	0.00	3.88	0.08	0.00
time (sec)	N/A	0.000	11.655	1.832	0.000	0.000	0.000	3.978	200.024	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	403	291	345	0	0	0	1174	7591	0
N.S.	1	1.31	0.94	1.12	0.00	0.00	0.00	3.81	24.65	0.00
time (sec)	N/A	0.666	11.036	1.617	0.000	0.000	0.000	2.064	5.364	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	1281	347	492	0	0	0	1462	30	0
N.S.	1	3.39	0.92	1.30	0.00	0.00	0.00	3.87	0.08	0.00
time (sec)	N/A	1.970	11.689	1.857	0.000	0.000	0.000	3.847	200.026	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	0	465	900	0	0	0	8143	30	0
N.S.	1	0.00	0.92	1.78	0.00	0.00	0.00	16.06	0.06	0.00
time (sec)	N/A	0.000	13.281	2.951	0.000	0.000	0.000	8.577	200.026	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	387	431	390	662	1026	1234	543	28	0
N.S.	1	0.80	0.89	0.81	1.37	2.13	2.56	1.13	0.06	0.00
time (sec)	N/A	0.632	1.581	0.903	0.048	0.783	0.610	0.153	200.019	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	268	297	270	436	698	763	357	627	0
N.S.	1	0.80	0.88	0.80	1.29	2.07	2.26	1.06	1.86	0.00
time (sec)	N/A	0.391	0.889	0.681	0.039	0.317	0.562	0.152	1.385	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	172	175	159	239	414	388	200	345	0
N.S.	1	0.83	0.85	0.77	1.15	2.00	1.87	0.97	1.67	0.00
time (sec)	N/A	0.275	0.481	0.582	0.037	0.149	0.467	0.136	0.174	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	199	186	167	0	1107	0	0	697	0
N.S.	1	1.08	1.01	0.90	0.00	5.98	0.00	0.00	3.77	0.00
time (sec)	N/A	0.414	0.647	0.905	0.000	7.529	0.000	0.000	0.197	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	206	198	212	0	1324	0	514	1227	0
N.S.	1	1.04	0.99	1.07	0.00	6.65	0.00	2.58	6.17	0.00
time (sec)	N/A	0.413	1.112	0.950	0.000	1.795	0.000	0.151	0.214	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	229	204	222	0	2206	0	883	3328	0
N.S.	1	1.10	0.98	1.07	0.00	10.61	0.00	4.25	16.00	0.00
time (sec)	N/A	0.395	10.390	1.314	0.000	1.950	0.000	0.178	0.269	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	277	414	241	0	1242	0	1754	3988	0
N.S.	1	1.22	1.82	1.06	0.00	5.47	0.00	7.73	17.57	0.00
time (sec)	N/A	0.465	13.236	0.921	0.000	1.600	0.000	0.709	0.884	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	416	508	392	0	2204	0	3048	28	0
N.S.	1	1.10	1.34	1.03	0.00	5.82	0.00	8.04	0.07	0.00
time (sec)	N/A	0.654	12.749	1.070	0.000	10.996	0.000	0.523	200.024	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	821	1217	722	667	1101	1740	2317	971	30	0
N.S.	1	1.48	0.88	0.81	1.34	2.12	2.82	1.18	0.04	0.00
time (sec)	N/A	1.227	3.074	0.996	0.043	2.402	0.710	0.190	200.026	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	824	483	449	722	1182	1464	638	30	0
N.S.	1	1.52	0.89	0.83	1.33	2.18	2.70	1.18	0.06	0.00
time (sec)	N/A	0.874	1.806	0.839	0.043	0.902	0.622	0.165	200.024	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	395	279	291	0	1801	0	0	1271	0
N.S.	1	1.40	0.99	1.03	0.00	6.39	0.00	0.00	4.51	0.00
time (sec)	N/A	0.646	1.161	1.155	0.000	97.361	0.000	0.000	0.222	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	0	302	306	0	2263	0	816	2161	0
N.S.	1	0.00	1.06	1.08	0.00	7.97	0.00	2.87	7.61	0.00
time (sec)	N/A	0.000	2.005	1.056	0.000	11.623	0.000	0.173	0.297	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	1024	270	313	0	3566	0	1394	5944	0
N.S.	1	3.31	0.87	1.01	0.00	11.54	0.00	4.51	19.24	0.00
time (sec)	N/A	1.596	10.647	1.129	0.000	7.604	0.000	0.195	0.476	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	1559	376	463	0	5081	0	2836	8163	0
N.S.	1	3.77	0.91	1.12	0.00	12.27	0.00	6.85	19.72	0.00
time (sec)	N/A	2.052	11.464	1.141	0.000	29.434	0.000	0.216	1.075	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	629	421	461	0	0	0	0	30	0
N.S.	1	1.44	0.97	1.06	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.987	1.975	1.075	0.000	0.000	0.000	0.000	200.023	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	0	426	457	0	3557	0	1167	30	0
N.S.	1	0.00	1.14	1.22	0.00	9.49	0.00	3.11	0.08	0.00
time (sec)	N/A	0.000	3.670	1.323	0.000	72.700	0.000	0.195	200.025	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	0	340	445	0	5597	0	1960	30	0
N.S.	1	0.00	0.86	1.13	0.00	14.21	0.00	4.97	0.08	0.00
time (sec)	N/A	0.000	10.943	1.339	0.000	38.260	0.000	0.224	200.022	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	2602	456	599	0	7661	0	3926	12619	0
N.S.	1	5.10	0.89	1.17	0.00	15.02	0.00	7.70	24.74	0.00
time (sec)	N/A	2.451	11.801	1.462	0.000	112.687	0.000	0.263	2.488	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	291	205	156	0	0	0	0	517	0
N.S.	1	1.85	1.31	0.99	0.00	0.00	0.00	0.00	3.29	0.00
time (sec)	N/A	0.489	1.048	1.427	0.000	0.000	0.000	0.000	0.565	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	267	205	222	0	0	0	458	1306	0
N.S.	1	1.46	1.12	1.21	0.00	0.00	0.00	2.50	7.14	0.00
time (sec)	N/A	0.565	1.172	1.457	0.000	0.000	0.000	0.871	1.294	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	402	274	310	0	0	0	1054	7621	0
N.S.	1	1.41	0.96	1.08	0.00	0.00	0.00	3.69	26.65	0.00
time (sec)	N/A	0.686	10.576	1.671	0.000	0.000	0.000	1.917	5.575	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	588	451	574	0	0	0	3177	30	0
N.S.	1	1.19	0.91	1.16	0.00	0.00	0.00	6.41	0.06	0.00
time (sec)	N/A	1.112	11.798	2.105	0.000	0.000	0.000	6.353	200.030	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	0	301	331	0	0	0	1331	30	0
N.S.	1	0.00	1.13	1.24	0.00	0.00	0.00	5.00	0.11	0.00
time (sec)	N/A	0.000	2.270	1.658	0.000	0.000	0.000	1.976	200.023	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	0	346	455	0	0	0	1411	30	0
N.S.	1	0.00	0.92	1.21	0.00	0.00	0.00	3.75	0.08	0.00
time (sec)	N/A	0.000	11.177	1.878	0.000	0.000	0.000	3.924	200.027	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	0	439	630	0	0	0	5515	30	0
N.S.	1	0.00	0.91	1.30	0.00	0.00	0.00	11.39	0.06	0.00
time (sec)	N/A	0.000	12.137	2.339	0.000	0.000	0.000	8.500	200.027	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F(-1)	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	429	236	193	0	0	0	0	924	0
N.S.	1	2.13	1.17	0.96	0.00	0.00	0.00	0.00	4.60	0.00
time (sec)	N/A	0.756	1.026	1.488	0.000	0.000	0.000	0.000	1.093	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	342	266	239	395	626	388	324	589	0
N.S.	1	1.14	0.89	0.80	1.32	2.09	1.29	1.08	1.96	0.00
time (sec)	N/A	0.555	0.693	0.668	0.047	0.254	0.540	0.145	0.242	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	223	173	156	239	396	250	198	359	0
N.S.	1	1.13	0.87	0.79	1.21	2.00	1.26	1.00	1.81	0.00
time (sec)	N/A	0.371	0.381	0.606	0.043	0.136	0.500	0.137	0.167	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	125	101	89	116	220	139	99	181	0
N.S.	1	1.11	0.89	0.79	1.03	1.95	1.23	0.88	1.60	0.00
time (sec)	N/A	0.248	0.185	0.513	0.042	0.099	0.409	0.133	0.151	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	108	77	0	741	0	0	289	0
N.S.	1	1.00	1.19	0.85	0.00	8.14	0.00	0.00	3.18	0.00
time (sec)	N/A	0.217	0.348	0.789	0.000	0.610	0.000	0.000	0.157	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	132	100	0	521	0	331	813	0
N.S.	1	1.00	1.15	0.87	0.00	4.53	0.00	2.88	7.07	0.00
time (sec)	N/A	0.239	0.698	0.820	0.000	1.129	0.000	0.337	0.186	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	215	291	197	0	1094	0	907	3591	0
N.S.	1	1.09	1.48	1.00	0.00	5.55	0.00	4.60	18.23	0.00
time (sec)	N/A	0.328	11.215	0.931	0.000	2.749	0.000	0.371	0.289	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	345	397	317	0	1936	0	1896	6329	0
N.S.	1	1.12	1.28	1.03	0.00	6.27	0.00	6.14	20.48	0.00
time (sec)	N/A	0.491	11.914	1.007	0.000	17.679	0.000	0.707	0.753	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	800	468	434	700	1110	690	611	30	0
N.S.	1	1.50	0.88	0.81	1.31	2.08	1.29	1.15	0.06	0.00
time (sec)	N/A	0.863	1.493	0.873	0.042	0.778	0.550	0.161	200.024	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	510	297	281	430	716	449	379	30	0
N.S.	1	1.52	0.88	0.84	1.28	2.13	1.34	1.13	0.09	0.00
time (sec)	N/A	0.612	0.823	0.718	0.039	0.271	0.489	0.142	200.023	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	168	148	133	0	1131	0	0	578	0
N.S.	1	1.24	1.10	0.99	0.00	8.38	0.00	0.00	4.28	0.00
time (sec)	N/A	0.320	0.452	0.921	0.000	1.043	0.000	0.000	0.210	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	225	183	193	0	1947	0	499	1649	0
N.S.	1	1.32	1.08	1.14	0.00	11.45	0.00	2.94	9.70	0.00
time (sec)	N/A	0.382	1.113	1.078	0.000	4.403	0.000	0.162	0.270	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	349	373	211	0	1318	0	1350	4829	0
N.S.	1	1.34	1.43	0.81	0.00	5.05	0.00	5.17	18.50	0.00
time (sec)	N/A	0.495	11.298	1.013	0.000	2.883	0.000	0.360	0.411	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	303	211	194	0	1729	0	0	984	0
N.S.	1	1.40	0.97	0.89	0.00	7.97	0.00	0.00	4.53	0.00
time (sec)	N/A	0.511	0.776	0.931	0.000	6.016	0.000	0.000	0.279	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	412	256	265	0	3081	0	652	2669	0
N.S.	1	1.91	1.19	1.23	0.00	14.26	0.00	3.02	12.36	0.00
time (sec)	N/A	0.708	1.552	1.079	0.000	22.756	0.000	0.170	0.343	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	593	409	373	0	4646	0	1863	7632	0
N.S.	1	1.92	1.32	1.21	0.00	15.04	0.00	6.03	24.70	0.00
time (sec)	N/A	0.806	11.494	1.134	0.000	66.376	0.000	0.182	0.490	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	147	96	0	1289	0	165	508	0
N.S.	1	1.00	1.20	0.79	0.00	10.57	0.00	1.35	4.16	0.00
time (sec)	N/A	0.242	0.544	1.342	0.000	52.722	0.000	0.158	0.586	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	203	498	231	0	0	0	459	3033	0
N.S.	1	1.00	2.44	1.13	0.00	0.00	0.00	2.25	14.87	0.00
time (sec)	N/A	0.355	12.793	1.447	0.000	0.000	0.000	0.842	1.491	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	383	433	360	0	0	0	1344	12786	0
N.S.	1	1.11	1.26	1.04	0.00	0.00	0.00	3.90	37.06	0.00
time (sec)	N/A	0.603	11.486	1.743	0.000	0.000	0.000	1.919	6.298	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	428	391	360	0	0	0	1772	30	0
N.S.	1	1.51	1.38	1.27	0.00	0.00	0.00	6.24	0.11	0.00
time (sec)	N/A	0.584	13.039	1.675	0.000	0.000	0.000	1.764	200.027	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	837	575	636	0	0	0	1833	30	0
N.S.	1	1.97	1.35	1.50	0.00	0.00	0.00	4.31	0.07	0.00
time (sec)	N/A	1.213	14.907	2.295	0.000	0.000	0.000	3.669	200.029	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	1697	777	1168	0	0	0	11237	30	0
N.S.	1	3.00	1.37	2.06	0.00	0.00	0.00	19.85	0.05	0.00
time (sec)	N/A	2.315	17.268	17.010	0.000	0.000	0.000	7.704	200.028	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	311	268	251	392	864	0	342	1098	0
N.S.	1	1.27	1.09	1.02	1.60	3.53	0.00	1.40	4.48	0.00
time (sec)	N/A	0.594	0.988	0.727	0.043	0.346	0.000	0.145	31.964	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	202	173	168	236	566	0	206	710	0
N.S.	1	1.23	1.05	1.02	1.44	3.45	0.00	1.26	4.33	0.00
time (sec)	N/A	0.370	0.555	0.639	0.038	0.229	0.000	0.136	0.178	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	117	104	99	113	324	182	101	376	0
N.S.	1	1.16	1.03	0.98	1.12	3.21	1.80	1.00	3.72	0.00
time (sec)	N/A	0.250	0.297	0.789	0.039	0.121	5.664	0.137	0.164	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	109	85	0	509	0	224	702	0
N.S.	1	1.00	1.18	0.92	0.00	5.53	0.00	2.43	7.63	0.00
time (sec)	N/A	0.226	0.473	0.819	0.000	0.991	0.000	0.137	0.184	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	189	188	192	0	1108	0	414	3450	0
N.S.	1	1.11	1.11	1.13	0.00	6.52	0.00	2.44	20.29	0.00
time (sec)	N/A	0.326	1.008	0.878	0.000	3.273	0.000	0.444	0.242	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	309	2140	311	0	2032	0	1012	6407	0
N.S.	1	1.10	7.64	1.11	0.00	7.26	0.00	3.61	22.88	0.00
time (sec)	N/A	0.489	18.851	0.984	0.000	17.039	0.000	0.693	0.492	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	765	467	446	697	1490	0	641	30	0
N.S.	1	1.79	1.09	1.04	1.63	3.48	0.00	1.50	0.07	0.00
time (sec)	N/A	0.837	1.774	0.911	0.046	0.916	0.000	0.156	200.025	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	486	299	286	429	994	0	398	30	0
N.S.	1	1.79	1.10	1.05	1.58	3.65	0.00	1.46	0.11	0.00
time (sec)	N/A	0.586	1.044	0.783	0.038	0.303	0.000	0.151	200.023	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	326	155	166	0	1822	0	0	1363	0
N.S.	1	2.43	1.16	1.24	0.00	13.60	0.00	0.00	10.17	0.00
time (sec)	N/A	0.647	0.564	0.892	0.000	2.139	0.000	0.000	0.190	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	300	223	222	0	1364	0	538	4917	0
N.S.	1	1.38	1.02	1.02	0.00	6.26	0.00	2.47	22.56	0.00
time (sec)	N/A	0.461	1.253	0.992	0.000	3.419	0.000	0.427	0.268	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	517	2542	408	0	2616	0	1451	9235	0
N.S.	1	1.21	5.95	0.96	0.00	6.13	0.00	3.40	21.63	0.00
time (sec)	N/A	0.722	20.660	1.135	0.000	9.584	0.000	0.713	0.552	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	1083	699	656	1032	2222	0	1013	30	0
N.S.	1	1.71	1.10	1.03	1.63	3.50	0.00	1.60	0.05	0.00
time (sec)	N/A	1.112	2.921	1.089	0.045	2.482	0.000	0.179	200.025	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	538	225	221	0	2877	0	0	2263	0
N.S.	1	3.04	1.27	1.25	0.00	16.25	0.00	0.00	12.79	0.00
time (sec)	N/A	1.010	1.215	1.071	0.000	9.832	0.000	0.000	0.230	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	645	324	350	0	4834	0	734	7621	0
N.S.	1	2.27	1.14	1.23	0.00	17.02	0.00	2.58	26.83	0.00
time (sec)	N/A	1.148	2.019	1.072	0.000	40.208	0.000	0.173	0.323	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	836	2523	460	0	3022	0	1902	30	0
N.S.	1	2.12	6.39	1.16	0.00	7.65	0.00	4.82	0.08	0.00
time (sec)	N/A	1.128	20.026	1.210	0.000	20.433	0.000	0.714	200.037	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	317	199	166	0	0	0	246	2231	0
N.S.	1	1.92	1.21	1.01	0.00	0.00	0.00	1.49	13.52	0.00
time (sec)	N/A	0.517	1.285	1.452	0.000	0.000	0.000	0.729	1.537	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	504	1386	335	0	0	0	632	12518	0
N.S.	1	1.73	4.76	1.15	0.00	0.00	0.00	2.17	43.02	0.00
time (sec)	N/A	0.851	15.527	1.613	0.000	0.000	0.000	2.195	5.947	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	782	2885	589	0	0	0	1630	30	0
N.S.	1	1.60	5.89	1.20	0.00	0.00	0.00	3.33	0.06	0.00
time (sec)	N/A	1.298	20.496	2.146	0.000	0.000	0.000	9.207	200.029	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	0	2255	561	0	0	0	2677	30	0
N.S.	1	0.00	5.39	1.34	0.00	0.00	0.00	6.40	0.07	0.00
time (sec)	N/A	0.000	19.168	2.096	0.000	0.000	0.000	6.913	200.037	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	0	433	1119	0	0	0	2307	30	0
N.S.	1	0.00	0.65	1.68	0.00	0.00	0.00	3.45	0.04	0.00
time (sec)	N/A	0.000	19.892	17.477	0.000	0.000	0.000	24.102	200.027	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	943	0	635	2018	0	0	0	14609	30	0
N.S.	1	0.00	0.67	2.14	0.00	0.00	0.00	15.49	0.03	0.00
time (sec)	N/A	0.000	24.138	208.007	0.000	0.000	0.000	82.012	200.024	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	310	258	251	474	978	0	336	1229	0
N.S.	1	1.37	1.14	1.11	2.09	4.31	0.00	1.48	5.41	0.00
time (sec)	N/A	0.537	0.891	0.839	0.044	0.396	0.000	0.152	10.557	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	204	167	169	295	616	0	200	738	0
N.S.	1	1.24	1.01	1.02	1.79	3.73	0.00	1.21	4.47	0.00
time (sec)	N/A	0.385	0.495	0.731	0.040	0.178	0.000	0.139	0.169	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	105	102	152	322	505	105	341	0
N.S.	1	1.02	0.93	0.90	1.35	2.85	4.47	0.93	3.02	0.00
time (sec)	N/A	0.240	0.308	0.609	0.038	0.116	7.934	0.126	0.182	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	168	162	156	0	940	0	507	1425	0
N.S.	1	1.11	1.07	1.03	0.00	6.18	0.00	3.34	9.38	0.00
time (sec)	N/A	0.306	0.768	0.938	0.000	2.804	0.000	0.138	0.210	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	296	213	283	0	1914	0	839	5210	0
N.S.	1	1.22	0.88	1.16	0.00	7.88	0.00	3.45	21.44	0.00
time (sec)	N/A	0.470	15.593	1.141	0.000	7.250	0.000	0.420	0.414	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	436	292	447	0	3168	0	1567	9199	0
N.S.	1	1.16	0.78	1.19	0.00	8.45	0.00	4.18	24.53	0.00
time (sec)	N/A	0.660	15.976	1.052	0.000	31.981	0.000	0.437	1.214	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	750	459	443	802	1732	0	640	30	0
N.S.	1	2.04	1.25	1.20	2.18	4.71	0.00	1.74	0.08	0.00
time (sec)	N/A	0.828	1.630	1.404	0.045	0.937	0.000	0.153	200.022	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	473	298	282	510	1140	0	394	30	0
N.S.	1	1.90	1.20	1.13	2.05	4.58	0.00	1.58	0.12	0.00
time (sec)	N/A	0.584	0.941	1.089	0.050	0.386	0.000	0.142	200.039	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	204	167	170	295	634	0	200	738	0
N.S.	1	1.24	1.01	1.03	1.79	3.84	0.00	1.21	4.47	0.00
time (sec)	N/A	0.383	0.466	0.929	0.037	0.158	0.000	0.136	0.246	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	251	168	163	0	1154	0	768	2122	0
N.S.	1	1.49	0.99	0.96	0.00	6.83	0.00	4.54	12.56	0.00
time (sec)	N/A	0.498	0.953	0.993	0.000	2.749	0.000	0.147	0.299	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	483	367	365	0	2492	0	1116	8023	0
N.S.	1	1.40	1.06	1.06	0.00	7.22	0.00	3.23	23.26	0.00
time (sec)	N/A	0.706	2.107	1.148	0.000	7.712	0.000	0.440	0.669	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	751	348	603	0	4208	0	2249	13481	0
N.S.	1	1.15	0.53	0.92	0.00	6.43	0.00	3.44	20.61	0.00
time (sec)	N/A	1.000	16.161	1.177	0.000	34.308	0.000	0.457	1.584	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	1056	690	649	1158	2602	0	1015	30	0
N.S.	1	1.98	1.29	1.22	2.17	4.88	0.00	1.90	0.06	0.00
time (sec)	N/A	1.121	2.950	1.108	0.046	3.092	0.000	0.177	200.022	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	750	461	444	802	1754	0	640	30	0
N.S.	1	2.04	1.25	1.21	2.18	4.77	0.00	1.74	0.08	0.00
time (sec)	N/A	0.826	1.657	1.335	0.067	1.192	0.000	0.157	200.022	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	310	257	251	474	1012	0	336	1229	0
N.S.	1	1.37	1.13	1.11	2.09	4.46	0.00	1.48	5.41	0.00
time (sec)	N/A	0.549	0.882	0.829	0.078	0.426	0.000	0.146	10.726	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	466	220	237	0	3941	0	0	3361	0
N.S.	1	2.26	1.07	1.15	0.00	19.13	0.00	0.00	16.32	0.00
time (sec)	N/A	0.895	1.389	1.150	0.000	17.377	0.000	0.000	0.247	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	753	433	421	0	2928	0	1390	30	0
N.S.	1	1.96	1.13	1.10	0.00	7.62	0.00	3.62	0.08	0.00
time (sec)	N/A	1.205	2.507	1.281	0.000	19.504	0.000	0.385	200.028	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	1253	340	746	0	5066	0	2892	17693	0
N.S.	1	2.04	0.55	1.21	0.00	8.24	0.00	4.70	28.77	0.00
time (sec)	N/A	1.611	16.341	1.304	0.000	88.180	0.000	0.486	41.219	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	325	265	236	0	0	0	1545	5555	0
N.S.	1	1.38	1.12	1.00	0.00	0.00	0.00	6.55	23.54	0.00
time (sec)	N/A	0.546	3.982	1.524	0.000	0.000	0.000	1.195	3.933	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	556	309	529	0	0	0	2471	30	0
N.S.	1	1.25	0.69	1.19	0.00	0.00	0.00	5.54	0.07	0.00
time (sec)	N/A	0.896	18.520	2.365	0.000	0.000	0.000	4.287	200.029	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	720	887	439	1063	0	0	0	3998	30	0
N.S.	1	1.23	0.61	1.48	0.00	0.00	0.00	5.55	0.04	0.00
time (sec)	N/A	1.432	17.757	4.099	0.000	0.000	0.000	17.587	200.026	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	0	368	1048	0	0	0	5859	30	0
N.S.	1	0.00	0.56	1.60	0.00	0.00	0.00	8.97	0.05	0.00
time (sec)	N/A	0.000	19.563	5.537	0.000	0.000	0.000	13.162	200.030	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	0	559	1980	0	0	0	5637	30	0
N.S.	1	0.00	0.54	1.91	0.00	0.00	0.00	5.45	0.03	0.00
time (sec)	N/A	0.000	24.722	99.270	0.000	0.000	0.000	47.966	200.025	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1441	0	630	3369	0	0	0	21752	30	0
N.S.	1	0.00	0.44	2.34	0.00	0.00	0.00	15.10	0.02	0.00
time (sec)	N/A	0.000	27.259	400.978	0.000	0.000	0.000	150.537	200.026	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	854	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	646	0	0	0	0	0	0	0	662	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.344	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	543	509	0	0	0	0	0	36	0
N.S.	1	1.02	0.95	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.749	4.063	0.000	0.000	0.000	0.000	0.000	0.521	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	478	433	0	0	0	0	0	0	30	0
N.S.	1	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.674	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	561	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	870	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	859	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	666	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.751	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	590	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	830	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1161	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	900	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	182.184	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	755	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	708	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	812	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	777	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	671	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.476	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	537	540	512	0	0	0	0	0	36	0
N.S.	1	1.01	0.95	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.745	4.648	0.000	0.000	0.000	0.000	0.000	0.510	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	159	0	0	0	0	0	51	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.290	3.103	0.000	0.000	0.000	0.000	0.000	0.451	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	256	148	0	0	0	0	0	75	0
N.S.	1	1.72	0.99	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.379	5.462	0.000	0.000	0.000	0.000	0.000	14.914	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	423	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	614	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	549	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	316	0	0	0	0	0	0	30	0
N.S.	1	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.483	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	75	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.249	5.314	0.000	0.000	0.000	0.000	0.000	15.187	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	395	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	716	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	629	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	786	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1146	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.019	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	850	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	650	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.407	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	557	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	835	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1296	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1147	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	866	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	155.057	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	694	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	691	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	813	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	735	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1218	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1171	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	934	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	862	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	877	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1161	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1134	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	858	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	147.162	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	680	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.472	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	0	0	0	0	0	0	0	110	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.791	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	504	0	0	0	0	0	0	0	158	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	32.873	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	583	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	901	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	686	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	575	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	0	0	0	0	0	0	0	158	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	28.537	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	401	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	718	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	683	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	585	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	528	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	711	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	648	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.420	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	545	503	0	0	0	0	0	36	0
N.S.	1	1.03	0.95	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.715	4.219	0.000	0.000	0.000	0.000	0.000	0.531	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	0	0	0	0	0	51	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.295	3.120	0.000	0.000	0.000	0.000	0.000	0.450	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	148	0	0	0	0	0	75	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.249	5.365	0.000	0.000	0.000	0.000	0.000	14.725	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	410	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	588	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	672	0	0	0	0	0	0	0	859	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.601	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	0	0	0	0	0	0	0	110	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.780	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	0	0	0	0	0	0	0	158	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	34.095	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	564	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	877	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	903	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	180.244	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	0	0	0	0	0	0	874	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.041	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	616	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	54.429	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	619	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	840	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	148	148	0	0	0	0	0	83	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.253	3.490	0.000	0.000	0.000	0.000	0.000	0.394	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	0	0	0	0	0	0	0	135	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.851	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	0	0	0	0	0	0	0	135	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.844	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	426	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	445	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	636	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	951	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	46	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.213	1.141	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	176	130	0	0	0	0	0	48	0
N.S.	1	1.89	1.40	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.233	1.217	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0	46	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.202	1.121	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	172	130	0	0	0	0	0	48	0
N.S.	1	1.81	1.37	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.234	1.190	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	0	0	0	0	0	46	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.206	1.132	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	48	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.204	1.109	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	46	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.204	1.068	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	171	113	0	0	0	0	0	48	0
N.S.	1	1.84	1.22	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.225	1.158	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	89	0	0	0	0	0	41	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.210	1.325	0.000	0.000	0.000	0.000	0.000	0.429	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	43	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.225	1.309	0.000	0.000	0.000	0.000	0.000	0.967	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	0	0	0	0	0	43	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.221	1.275	0.000	0.000	0.000	0.000	0.000	0.967	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	100	98	0	0	0	0	0	41	0
N.S.	1	1.02	1.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.214	1.283	0.000	0.000	0.000	0.000	0.000	0.438	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	293	96	0	0	0	0	0	41	0
N.S.	1	1.48	0.48	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.305	1.330	0.000	0.000	0.000	0.000	0.000	0.446	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	285	96	0	0	0	0	0	43	0
N.S.	1	1.47	0.49	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.302	1.320	0.000	0.000	0.000	0.000	0.000	0.981	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	297	105	0	0	0	0	0	43	0
N.S.	1	1.56	0.55	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.302	1.315	0.000	0.000	0.000	0.000	0.000	0.979	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	285	93	0	0	0	0	0	41	0
N.S.	1	1.47	0.48	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.301	1.262	0.000	0.000	0.000	0.000	0.000	0.436	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	334	301	0	0	0	0	0	36	0
N.S.	1	1.02	0.92	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.474	2.018	0.000	0.000	0.000	0.000	0.000	0.440	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	334	412	0	0	0	0	0	0	36	0
N.S.	1	1.23	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.521	0.000	0.000	0.000	0.000	0.000	0.000	0.432	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	408	0	0	0	0	0	0	36	0
N.S.	1	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.497	0.000	0.000	0.000	0.000	0.000	0.000	0.434	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	324	279	0	0	0	0	0	36	0
N.S.	1	1.04	0.89	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.483	2.147	0.000	0.000	0.000	0.000	0.000	0.426	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	411	0	0	0	0	0	0	38	0
N.S.	1	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.510	0.000	0.000	0.000	0.000	0.000	0.000	0.962	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	334	328	297	0	0	0	0	0	38	0
N.S.	1	0.98	0.89	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.477	1.508	0.000	0.000	0.000	0.000	0.000	0.986	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	334	289	0	0	0	0	0	38	0
N.S.	1	0.98	0.85	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.466	1.520	0.000	0.000	0.000	0.000	0.000	0.977	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	322	283	0	0	0	0	0	38	0
N.S.	1	1.03	0.90	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.481	1.405	0.000	0.000	0.000	0.000	0.000	0.982	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	481	748	0	0	0	0	0	0	30	0
N.S.	1	1.56	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.852	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	568	148	0	0	0	0	0	75	0
N.S.	1	3.84	1.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.584	5.439	0.000	0.000	0.000	0.000	0.000	15.217	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	354	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	691	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	0	0	0	0	0	0	0	158	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	32.891	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	397	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	521	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	789	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	925	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	700	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	55.066	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	607	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	555	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	784	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1188	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	0	0	0	0	0	0	0	135	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.903	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	637	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	434	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	614	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	967	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	666	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	946	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	715	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	586	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	532	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	798	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	688	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	526	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	714	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	812	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	716	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	658	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	560	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	732	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1028	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	643	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1002	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	651	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	960	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	995	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1446	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	278	268	193	0	0	0	0	0	231	0
N.S.	1	0.96	0.69	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.419	10.245	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	181	118	0	0	0	124	0	145	0
N.S.	1	0.95	0.62	0.00	0.00	0.00	0.65	0.00	0.76	0.00
time (sec)	N/A	0.291	10.139	0.000	0.000	0.000	2.918	0.000	0.172	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	119	74	0	0	0	56	0	69	0
N.S.	1	1.43	0.89	0.00	0.00	0.00	0.67	0.00	0.83	0.00
time (sec)	N/A	0.212	10.034	0.000	0.000	0.000	1.743	0.000	0.158	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	302	351	0	0	0	0	0	133	0
N.S.	1	1.18	1.37	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.497	10.425	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	358	407	987	0	0	0	0	0	217	0
N.S.	1	1.14	2.76	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.614	10.586	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	548	1540	0	0	0	0	0	301	0
N.S.	1	1.11	3.12	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.874	11.115	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	217	0	0	0	0	0	205	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.239	10.327	0.000	0.000	0.000	0.000	0.000	0.497	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	235	0	0	0	0	0	163	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.250	10.361	0.000	0.000	0.000	0.000	0.000	0.558	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	236	1207	0	0	0	0	0	168	0
N.S.	1	1.40	7.18	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.336	11.913	0.000	0.000	0.000	0.000	0.000	0.652	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	334	1290	0	0	0	0	0	422	0
N.S.	1	1.99	7.68	0.00	0.00	0.00	0.00	0.00	2.51	0.00
time (sec)	N/A	0.488	12.192	0.000	0.000	0.000	0.000	0.000	1.245	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	454	1387	0	0	0	0	0	766	0
N.S.	1	2.70	8.26	0.00	0.00	0.00	0.00	0.00	4.56	0.00
time (sec)	N/A	0.708	12.575	0.000	0.000	0.000	0.000	0.000	2.213	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.343	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	278	0	0	0	0	0	0	140	0
N.S.	1	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.436	0.000	0.000	0.000	0.000	0.000	0.000	0.569	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [307] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	0.90	30	0.233
2	A	6	6	0.92	30	0.200
3	A	5	5	0.93	30	0.167
4	A	6	6	1.26	30	0.200
5	A	5	5	1.02	30	0.167
6	A	7	7	1.06	30	0.233
7	A	9	9	1.07	30	0.300
8	A	9	9	0.90	30	0.300
9	A	8	8	0.91	30	0.267
10	A	7	7	0.92	30	0.233
11	A	7	7	1.16	30	0.233
12	A	8	8	1.20	30	0.267
13	A	7	7	1.04	30	0.233
14	A	9	9	1.06	30	0.300
15	A	9	9	0.90	30	0.300
16	A	8	8	0.91	30	0.267
17	A	8	8	1.10	30	0.267
18	A	9	9	1.16	30	0.300
19	A	10	10	1.18	30	0.333
20	A	9	9	1.04	30	0.300
21	A	6	6	0.94	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	5	0.95	30	0.167
23	A	4	4	1.00	30	0.133
24	A	3	3	1.00	30	0.100
25	A	4	4	1.05	30	0.133
26	A	5	5	1.08	30	0.167
27	A	10	10	1.11	30	0.333
28	A	8	8	1.16	30	0.267
29	A	6	6	1.26	30	0.200
30	A	3	3	1.00	30	0.100
31	A	4	4	1.04	30	0.133
32	A	5	5	1.05	30	0.167
33	A	9	9	1.16	30	0.300
34	A	8	8	1.20	30	0.267
35	A	5	5	1.02	30	0.167
36	A	5	5	1.04	30	0.167
37	A	8	8	1.05	30	0.267
38	A	8	8	1.07	30	0.267
39	A	4	4	1.00	30	0.133
40	A	7	7	1.00	31	0.226
41	A	7	7	1.00	31	0.226
42	A	7	7	1.00	32	0.219
43	A	3	3	1.00	30	0.100
44	A	9	9	1.24	31	0.290
45	A	8	8	1.05	31	0.258
46	A	9	9	1.01	32	0.281
47	A	4	4	1.01	30	0.133
48	A	6	6	0.97	30	0.200
49	A	7	7	0.92	30	0.233
50	A	2	2	1.00	87	0.023
51	B	5	5	2.49	81	0.062
52	B	3	3	2.87	30	0.100
53	B	4	4	2.87	38	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	B	1	1	2.87	47	0.021
55	A	2	2	1.59	32	0.062
56	A	2	2	1.71	32	0.062
57	A	2	2	1.86	32	0.062
58	A	2	2	1.94	32	0.062
59	B	2	2	2.28	32	0.062
60	A	2	2	1.89	32	0.062
61	A	2	2	1.67	32	0.062
62	A	2	2	1.58	32	0.062
63	A	2	2	1.67	32	0.062
64	B	2	2	2.07	32	0.062
65	B	2	2	2.18	32	0.062
66	B	2	2	2.01	32	0.062
67	A	2	2	1.65	32	0.062
68	A	2	2	1.52	32	0.062
69	A	2	2	1.58	32	0.062
70	A	2	2	1.89	32	0.062
71	B	2	2	2.01	32	0.062
72	A	2	2	1.84	32	0.062
73	A	2	2	1.73	32	0.062
74	A	2	2	1.70	32	0.062
75	A	2	2	1.91	32	0.062
76	A	2	2	1.60	32	0.062
77	B	2	2	2.49	32	0.062
78	B	2	2	2.35	32	0.062
79	B	2	2	2.04	32	0.062
80	A	2	2	1.93	32	0.062
81	B	2	2	2.13	32	0.062
82	B	2	2	2.08	32	0.062
83	B	2	2	3.10	32	0.062
84	B	2	2	2.40	32	0.062
85	B	2	2	2.03	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	B	2	2	2.02	32	0.062
87	B	2	2	2.23	32	0.062
88	B	2	2	2.35	32	0.062
89	B	2	2	2.35	32	0.062
90	B	2	2	2.17	32	0.062
91	A	2	2	1.91	32	0.062
92	A	17	17	1.27	32	0.531
93	A	9	9	0.95	32	0.281
94	A	6	6	1.01	32	0.188
95	A	1	1	1.00	32	0.031
96	A	3	3	1.00	32	0.094
97	A	9	9	0.87	32	0.281
98	A	17	17	1.25	32	0.531
99	A	9	9	0.91	32	0.281
100	A	6	6	0.89	32	0.188
101	A	3	3	0.80	32	0.094
102	A	9	9	0.89	32	0.281
103	A	13	13	0.95	32	0.406
104	A	27	27	1.37	32	0.844
105	A	18	18	1.20	32	0.562
106	A	13	13	1.42	32	0.406
107	A	20	20	1.86	32	0.625
108	A	18	18	1.65	32	0.562
109	A	13	13	0.96	32	0.406
110	A	21	21	0.96	32	0.656
111	A	6	6	0.89	32	0.188
112	A	5	5	0.68	33	0.152
113	A	15	15	1.30	33	0.455
114	A	11	11	0.97	34	0.324
115	A	3	3	0.47	32	0.094
116	A	6	6	1.00	32	0.188
117	A	10	10	0.91	32	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	6	1.00	32	0.188
119	A	9	9	1.17	32	0.281
120	A	16	16	1.29	32	0.500
121	A	10	10	0.92	32	0.312
122	A	16	16	1.18	32	0.500
123	A	20	20	1.05	32	0.625
124	A	29	29	1.91	33	0.879
125	A	17	17	1.27	33	0.515
126	A	15	15	1.27	33	0.455
127	A	16	16	1.33	33	0.485
128	A	28	28	1.60	33	0.848
129	F	0	0	N/A	0.000	N/A
130	A	8	8	0.97	28	0.286
131	A	6	6	0.98	28	0.214
132	B	1	1	2.52	28	0.036
133	A	3	3	1.41	28	0.107
134	A	8	8	0.92	28	0.286
135	A	6	6	1.01	32	0.188
136	A	1	1	1.00	32	0.031
137	A	1	1	0.25	32	0.031
138	A	3	3	1.00	30	0.100
139	A	1	1	1.00	32	0.031
140	A	1	1	1.00	32	0.031
141	B	2	2	2.26	32	0.062
142	A	2	2	1.00	32	0.062
143	F	0	0	N/A	0.000	N/A
144	B	21	21	2.40	32	0.656
145	A	8	8	0.92	32	0.250
146	A	12	12	1.28	32	0.375
147	B	22	22	2.84	32	0.688
148	F	0	0	N/A	0.000	N/A
149	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	F	0	0	N/A	0.000	N/A
151	A	15	15	1.69	32	0.469
152	A	14	14	1.39	32	0.438
153	B	26	26	2.64	32	0.812
154	F	0	0	N/A	0.000	N/A
155	F	0	0	N/A	0.000	N/A
156	F	0	0	N/A	0.000	N/A
157	B	25	25	2.08	32	0.781
158	A	21	21	1.66	32	0.656
159	B	30	30	2.49	32	0.938
160	F	0	0	N/A	0.000	N/A
161	A	11	11	1.00	33	0.333
162	A	8	8	0.92	32	0.250
163	A	8	8	1.05	32	0.250
164	A	15	15	1.57	32	0.469
165	A	26	26	1.68	32	0.812
166	A	15	15	1.58	32	0.469
167	A	25	25	1.93	32	0.781
168	A	26	26	1.72	32	0.812
169	F	0	0	N/A	0.000	N/A
170	A	11	11	1.01	33	0.333
171	A	8	8	1.05	32	0.250
172	F	0	0	N/A	0.000	N/A
173	F	0	0	N/A	0.000	N/A
174	F	0	0	N/A	0.000	N/A
175	F	0	0	N/A	0.000	N/A
176	F	0	0	N/A	0.000	N/A
177	F	0	0	N/A	0.000	N/A
178	F	0	0	N/A	0.000	N/A
179	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	F	0	0	N/A	0.000	N/A
181	F	0	0	N/A	0.000	N/A
182	F	0	0	N/A	0.000	N/A
183	F	0	0	N/A	0.000	N/A
184	F	0	0	N/A	0.000	N/A
185	F	0	0	N/A	0.000	N/A
186	F	0	0	N/A	0.000	N/A
187	F	0	0	N/A	0.000	N/A
188	F	0	0	N/A	0.000	N/A
189	F	0	0	N/A	0.000	N/A
190	F	0	0	N/A	0.000	N/A
191	F	0	0	N/A	0.000	N/A
192	F	0	0	N/A	0.000	N/A
193	F	0	0	N/A	0.000	N/A
194	F	0	0	N/A	0.000	N/A
195	F	0	0	N/A	0.000	N/A
196	F	0	0	N/A	0.000	N/A
197	F	0	0	N/A	0.000	N/A
198	F	0	0	N/A	0.000	N/A
199	F	0	0	N/A	0.000	N/A
200	A	2	2	1.00	24	0.083
201	A	2	2	1.00	24	0.083
202	A	2	2	1.00	24	0.083
203	A	2	2	1.00	22	0.091
204	A	4	4	1.18	24	0.167
205	A	4	4	1.17	24	0.167
206	A	4	4	1.07	24	0.167
207	A	5	5	0.97	24	0.208
208	A	2	2	1.00	26	0.077
209	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
210	A	6	6	1.34	26	0.231
211	A	6	6	1.26	26	0.231
212	A	5	5	1.21	26	0.192
213	A	6	6	1.07	26	0.231
214	A	2	2	1.00	26	0.077
215	A	7	7	1.38	26	0.269
216	A	7	7	1.43	26	0.269
217	A	6	6	1.41	26	0.231
218	A	7	7	1.27	26	0.269
219	A	2	2	1.00	26	0.077
220	A	4	4	1.11	26	0.154
221	A	5	5	1.17	26	0.192
222	A	6	6	1.11	26	0.231
223	A	7	7	1.15	26	0.269
224	A	10	10	1.12	26	0.385
225	A	2	2	1.00	28	0.071
226	A	2	2	1.00	28	0.071
227	A	9	9	1.24	28	0.321
228	A	11	11	1.67	28	0.393
229	A	10	10	1.57	28	0.357
230	A	9	9	1.48	28	0.321
231	A	2	2	1.00	28	0.071
232	A	10	10	1.17	28	0.357
233	A	12	12	1.90	28	0.429
234	A	11	11	1.76	28	0.393
235	A	11	11	1.65	28	0.393
236	A	5	5	1.44	28	0.179
237	A	8	8	1.34	28	0.286
238	A	8	8	1.08	28	0.286
239	A	9	9	1.60	28	0.321
240	A	10	10	1.56	28	0.357
241	A	14	14	1.50	28	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	A	2	2	1.00	28	0.071
243	A	12	12	1.12	28	0.429
244	B	23	23	2.27	28	0.821
245	B	16	16	2.48	28	0.571
246	B	16	16	2.36	28	0.571
247	A	8	8	1.72	28	0.286
248	A	10	10	1.85	28	0.357
249	A	13	13	1.52	28	0.464
250	B	18	18	2.14	28	0.643
251	A	19	19	1.89	28	0.679
252	B	17	17	2.48	28	0.607
253	A	3	3	1.24	28	0.107
254	A	6	6	1.31	28	0.214
255	A	9	9	1.39	28	0.321
256	A	10	10	1.53	28	0.357
257	A	12	12	1.56	28	0.429
258	A	19	19	1.98	28	0.679
259	B	12	12	2.51	28	0.429
260	A	8	7	0.94	28	0.250
261	A	7	6	0.93	28	0.214
262	A	6	5	0.96	26	0.192
263	A	8	7	1.05	28	0.250
264	A	8	7	1.08	28	0.250
265	A	7	6	1.08	28	0.214
266	A	9	8	1.08	28	0.286
267	A	2	2	1.49	30	0.067
268	A	2	2	1.53	30	0.067
269	A	7	6	0.93	28	0.214
270	A	13	12	1.19	30	0.400
271	B	21	20	2.11	30	0.667
272	B	16	15	2.22	30	0.500
273	B	15	14	2.70	30	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
274	A	2	2	1.49	30	0.067
275	A	8	7	0.94	28	0.250
276	A	19	18	1.27	30	0.600
277	F	0	0	N/A	0.000	N/A
278	B	26	25	3.18	30	0.833
279	B	21	20	3.35	30	0.667
280	A	7	6	1.58	30	0.200
281	A	15	14	1.41	30	0.467
282	A	16	15	1.31	30	0.500
283	A	24	23	1.14	30	0.767
284	A	16	15	1.41	30	0.500
285	B	19	18	2.61	30	0.600
286	F	0	0	N/A	0.000	N/A
287	A	17	16	1.31	30	0.533
288	B	30	29	3.39	30	0.967
289	F	0	0	N/A	0.000	N/A
290	A	9	8	0.80	28	0.286
291	A	8	7	0.80	28	0.250
292	A	7	6	0.83	26	0.231
293	A	9	8	1.08	28	0.286
294	A	10	9	1.04	28	0.321
295	A	10	9	1.10	28	0.321
296	A	9	8	1.22	28	0.286
297	A	10	9	1.10	28	0.321
298	A	2	2	1.48	30	0.067
299	A	2	2	1.52	30	0.067
300	A	19	18	1.40	30	0.600
301	F	0	0	N/A	0.000	N/A
302	B	26	25	3.31	30	0.833
303	B	21	20	3.77	30	0.667
304	A	26	25	1.44	30	0.833
305	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	F	0	0	N/A	0.000	N/A
307	B	31	30	5.10	30	1.000
308	A	13	12	1.85	30	0.400
309	A	15	14	1.46	30	0.467
310	A	16	15	1.41	30	0.500
311	A	25	24	1.19	30	0.800
312	F	0	0	N/A	0.000	N/A
313	F	0	0	N/A	0.000	N/A
314	F	0	0	N/A	0.000	N/A
315	B	21	20	2.13	30	0.667
316	A	7	6	1.14	28	0.214
317	A	6	5	1.13	28	0.179
318	A	5	4	1.11	26	0.154
319	A	6	5	1.00	28	0.179
320	A	5	4	1.00	28	0.143
321	A	6	5	1.09	28	0.179
322	A	7	6	1.12	28	0.214
323	A	2	2	1.50	30	0.067
324	A	2	2	1.52	30	0.067
325	A	10	9	1.24	30	0.300
326	A	11	10	1.32	30	0.333
327	A	10	9	1.34	30	0.300
328	A	15	14	1.40	30	0.467
329	A	21	20	1.91	30	0.667
330	A	16	15	1.92	30	0.500
331	A	4	3	1.00	30	0.100
332	A	8	7	1.00	30	0.233
333	A	12	11	1.11	30	0.367
334	A	11	10	1.51	30	0.333
335	A	23	22	1.97	30	0.733
336	B	25	24	3.00	30	0.800
337	A	10	9	1.27	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	8	7	1.23	28	0.250
339	A	6	5	1.16	26	0.192
340	A	6	5	1.00	28	0.179
341	A	7	6	1.11	28	0.214
342	A	8	7	1.10	28	0.250
343	A	2	2	1.79	30	0.067
344	A	2	2	1.79	30	0.067
345	B	16	15	2.43	30	0.500
346	A	11	10	1.38	30	0.333
347	A	12	11	1.21	30	0.367
348	A	2	2	1.71	30	0.067
349	B	22	21	3.04	30	0.700
350	B	27	26	2.27	30	0.867
351	B	16	15	2.12	30	0.500
352	A	13	12	1.92	30	0.400
353	A	22	21	1.73	30	0.700
354	A	25	24	1.60	30	0.800
355	F	0	0	N/A	0.000	N/A
356	F	0	0	N/A	0.000	N/A
357	F	0	0	N/A	0.000	N/A
358	A	9	8	1.37	28	0.286
359	A	8	7	1.24	28	0.250
360	A	6	5	1.02	26	0.192
361	A	7	6	1.11	28	0.214
362	A	10	9	1.22	28	0.321
363	A	11	10	1.16	28	0.357
364	B	2	2	2.04	30	0.067
365	A	2	2	1.90	30	0.067
366	A	8	7	1.24	28	0.250
367	A	14	13	1.49	30	0.433
368	A	13	12	1.40	30	0.400
369	A	15	14	1.15	30	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
370	A	2	2	1.98	30	0.067
371	B	2	2	2.04	30	0.067
372	A	9	8	1.37	28	0.286
373	B	20	19	2.26	30	0.633
374	A	27	26	1.96	30	0.867
375	B	18	17	2.04	30	0.567
376	A	12	11	1.38	30	0.367
377	A	19	18	1.25	30	0.600
378	A	24	23	1.23	30	0.767
379	F	0	0	N/A	0.000	N/A
380	F	0	0	N/A	0.000	N/A
381	F	0	0	N/A	0.000	N/A
382	F	0	0	N/A	0.000	N/A
383	F	0	0	N/A	0.000	N/A
384	A	8	7	1.02	34	0.206
385	A	8	7	0.91	34	0.206
386	F	0	0	N/A	0.000	N/A
387	F	0	0	N/A	0.000	N/A
388	F	0	0	N/A	0.000	N/A
389	F	0	0	N/A	0.000	N/A
390	F	0	0	N/A	0.000	N/A
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	F	0	0	N/A	0.000	N/A
394	F	0	0	N/A	0.000	N/A
395	F	0	0	N/A	0.000	N/A
396	F	0	0	N/A	0.000	N/A
397	F	0	0	N/A	0.000	N/A
398	F	0	0	N/A	0.000	N/A
399	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
400	F	0	0	N/A	0.000	N/A
401	F	0	0	N/A	0.000	N/A
402	F	0	0	N/A	0.000	N/A
403	A	8	7	1.01	34	0.206
404	A	3	2	1.00	34	0.059
405	A	5	4	1.72	34	0.118
406	F	0	0	N/A	0.000	N/A
407	F	0	0	N/A	0.000	N/A
408	F	0	0	N/A	0.000	N/A
409	A	6	5	0.66	34	0.147
410	A	3	2	1.00	34	0.059
411	F	0	0	N/A	0.000	N/A
412	F	0	0	N/A	0.000	N/A
413	F	0	0	N/A	0.000	N/A
414	F	0	0	N/A	0.000	N/A
415	F	0	0	N/A	0.000	N/A
416	F	0	0	N/A	0.000	N/A
417	F	0	0	N/A	0.000	N/A
418	F	0	0	N/A	0.000	N/A
419	F	0	0	N/A	0.000	N/A
420	F	0	0	N/A	0.000	N/A
421	F	0	0	N/A	0.000	N/A
422	F	0	0	N/A	0.000	N/A
423	F	0	0	N/A	0.000	N/A
424	F	0	0	N/A	0.000	N/A
425	F	0	0	N/A	0.000	N/A
426	F	0	0	N/A	0.000	N/A
427	F	0	0	N/A	0.000	N/A
428	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
429	F	0	0	N/A	0.000	N/A
430	F	0	0	N/A	0.000	N/A
431	F	0	0	N/A	0.000	N/A
432	F	0	0	N/A	0.000	N/A
433	F	0	0	N/A	0.000	N/A
434	F	0	0	N/A	0.000	N/A
435	F	0	0	N/A	0.000	N/A
436	F	0	0	N/A	0.000	N/A
437	F	0	0	N/A	0.000	N/A
438	F	0	0	N/A	0.000	N/A
439	F	0	0	N/A	0.000	N/A
440	F	0	0	N/A	0.000	N/A
441	F	0	0	N/A	0.000	N/A
442	F	0	0	N/A	0.000	N/A
443	F	0	0	N/A	0.000	N/A
444	F	0	0	N/A	0.000	N/A
445	F	0	0	N/A	0.000	N/A
446	F	0	0	N/A	0.000	N/A
447	F	0	0	N/A	0.000	N/A
448	F	0	0	N/A	0.000	N/A
449	F	0	0	N/A	0.000	N/A
450	F	0	0	N/A	0.000	N/A
451	F	0	0	N/A	0.000	N/A
452	F	0	0	N/A	0.000	N/A
453	F	0	0	N/A	0.000	N/A
454	F	0	0	N/A	0.000	N/A
455	F	0	0	N/A	0.000	N/A
456	F	0	0	N/A	0.000	N/A
457	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
458	F	0	0	N/A	0.000	N/A
459	A	8	7	1.03	34	0.206
460	A	3	2	1.00	34	0.059
461	A	3	2	1.00	34	0.059
462	F	0	0	N/A	0.000	N/A
463	F	0	0	N/A	0.000	N/A
464	F	0	0	N/A	0.000	N/A
465	F	0	0	N/A	0.000	N/A
466	F	0	0	N/A	0.000	N/A
467	F	0	0	N/A	0.000	N/A
468	F	0	0	N/A	0.000	N/A
469	F	0	0	N/A	0.000	N/A
470	F	0	0	N/A	0.000	N/A
471	F	0	0	N/A	0.000	N/A
472	F	0	0	N/A	0.000	N/A
473	F	0	0	N/A	0.000	N/A
474	F	0	0	N/A	0.000	N/A
475	F	0	0	N/A	0.000	N/A
476	A	3	2	1.02	34	0.059
477	F	0	0	N/A	0.000	N/A
478	F	0	0	N/A	0.000	N/A
479	F	0	0	N/A	0.000	N/A
480	F	0	0	N/A	0.000	N/A
481	F	0	0	N/A	0.000	N/A
482	F	0	0	N/A	0.000	N/A
483	F	0	0	N/A	0.000	N/A
484	F	0	0	N/A	0.000	N/A
485	A	4	3	1.00	34	0.088
486	A	4	3	1.89	34	0.088
487	A	4	3	1.00	34	0.088

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
488	A	4	3	1.81	34	0.088
489	A	4	3	1.00	34	0.088
490	A	4	3	1.00	34	0.088
491	A	4	3	1.00	34	0.088
492	A	4	3	1.84	34	0.088
493	A	4	3	1.00	34	0.088
494	A	4	3	1.00	34	0.088
495	A	4	3	1.00	34	0.088
496	A	4	3	1.02	34	0.088
497	A	6	5	1.48	34	0.147
498	A	6	5	1.47	34	0.147
499	A	6	5	1.56	34	0.147
500	A	6	5	1.47	34	0.147
501	A	11	10	1.02	34	0.294
502	A	11	10	1.23	34	0.294
503	A	11	10	1.20	34	0.294
504	A	11	10	1.04	34	0.294
505	A	11	10	1.25	34	0.294
506	A	11	10	0.98	34	0.294
507	A	11	10	0.98	34	0.294
508	A	11	10	1.03	34	0.294
509	F	0	0	N/A	0.000	N/A
510	A	9	8	1.56	34	0.235
511	B	6	5	3.84	34	0.147
512	F	0	0	N/A	0.000	N/A
513	F	0	0	N/A	0.000	N/A
514	F	0	0	N/A	0.000	N/A
515	F	0	0	N/A	0.000	N/A
516	F	0	0	N/A	0.000	N/A
517	F	0	0	N/A	0.000	N/A
518	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
519	F	0	0	N/A	0.000	N/A
520	F	0	0	N/A	0.000	N/A
521	F	0	0	N/A	0.000	N/A
522	F	0	0	N/A	0.000	N/A
523	F	0	0	N/A	0.000	N/A
524	F	0	0	N/A	0.000	N/A
525	F	0	0	N/A	0.000	N/A
526	F	0	0	N/A	0.000	N/A
527	F	0	0	N/A	0.000	N/A
528	F	0	0	N/A	0.000	N/A
529	F	0	0	N/A	0.000	N/A
530	F	0	0	N/A	0.000	N/A
531	F	0	0	N/A	0.000	N/A
532	F	0	0	N/A	0.000	N/A
533	F	0	0	N/A	0.000	N/A
534	F	0	0	N/A	0.000	N/A
535	F	0	0	N/A	0.000	N/A
536	F	0	0	N/A	0.000	N/A
537	F	0	0	N/A	0.000	N/A
538	F	0	0	N/A	0.000	N/A
539	F	0	0	N/A	0.000	N/A
540	F	0	0	N/A	0.000	N/A
541	F	0	0	N/A	0.000	N/A
542	F	0	0	N/A	0.000	N/A
543	F	0	0	N/A	0.000	N/A
544	F	0	0	N/A	0.000	N/A
545	F	0	0	N/A	0.000	N/A
546	F	0	0	N/A	0.000	N/A
547	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
548	F	0	0	N/A	0.000	N/A
549	F	0	0	N/A	0.000	N/A
550	F	0	0	N/A	0.000	N/A
551	F	0	0	N/A	0.000	N/A
552	F	0	0	N/A	0.000	N/A
553	F	0	0	N/A	0.000	N/A
554	F	0	0	N/A	0.000	N/A
555	F	0	0	N/A	0.000	N/A
556	F	0	0	N/A	0.000	N/A
557	F	0	0	N/A	0.000	N/A
558	F	0	0	N/A	0.000	N/A
559	A	8	8	0.96	28	0.286
560	A	6	6	0.95	26	0.231
561	A	4	4	1.43	19	0.211
562	A	10	9	1.18	28	0.321
563	A	12	11	1.14	28	0.393
564	A	14	13	1.11	28	0.464
565	A	3	3	1.00	30	0.100
566	A	3	3	1.00	30	0.100
567	A	5	5	1.40	30	0.167
568	A	7	7	1.99	30	0.233
569	B	9	9	2.70	30	0.300
570	A	7	7	1.00	26	0.269
571	A	6	6	1.41	32	0.188

CHAPTER 3

Listing of integrals

3.1 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 526

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx = \frac{\left(8a^3d^3f+3b^3c^2(7de-2cf)+ab^2cd(49de+9cf)-a^2bd^2(14de+19cf)\right)x\sqrt{c+dx^2}}{105b^2d^2\sqrt{a+bx^2}} - \frac{\left(4a^2d^2f-3b^2c(7de-2cf)-abd(7de+6cf)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^2d} + \frac{(7bde-2bcf+adf)x\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{35bd} + \frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{7d} - \frac{\sqrt{a}\left(8a^3d^3f+3b^3c^2(7de-2cf)+ab^2cd(49de+9cf)-a^2bd^2(14de+19cf)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right|}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\left(4a^2d^2f+3b^2c(21de-cf)-abd(7de+9cf)\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f+3*b^3*c^2*(-2*c*f+7*d*e)+a*b^2*c*d*(9*c*f+49*d*e)-a^2*b
*d^2*(19*c*f+14*d*e))*x*(d*x^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)-1/105*(4*a
^2*d^2*f-3*b^2*c*(-2*c*f+7*d*e)-a*b*d*(6*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b^2/d+1/35*(a*d*f-2*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(3/2)/b/d+1/7*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/d-1/105*a^(1/2)*(8*a^3
*d^3*f+3*b^3*c^2*(-2*c*f+7*d*e)+a*b^2*c*d*(9*c*f+49*d*e)-a^2*b*d^2*(19*c*f
+14*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1
-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+1/105*a^(3/2)*(4*a^2*d^2*f+3*b^2*c*(-c*f+21*d*e)-a*b*d*(9*c*f+7*d*e))*(
d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2)
)/b^(5/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.70

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f-abd(7de+9cf+3dfx^2)-3b^2(c^2f+2cd(7e+4fx^2)+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f - a*b*d*(7*d*e + 9*c
*f + 3*d*f*x^2) - 3*b^2*(c^2*f + 2*c*d*(7*e + 4*f*x^2) + d^2*x^2*(7*e + 5*
f*x^2)))) - I*c*(8*a^3*d^3*f + 3*b^3*c^2*(7*d*e - 2*c*f) + a*b^2*c*d*(49*d
*e + 9*c*f) - a^2*b*d^2*(14*d*e + 19*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*
d)*(4*a^2*d^2*f + 3*b^2*c*(-7*d*e + 2*c*f) - a*b*d*(7*d*e + 6*c*f))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(105*a^2*(b/a)^(5/2)*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {403, 403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left((7bde+3bcf-4adf)x^2+c(7be-af)\right)dx}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)x^2+c\left(4dfa^2-7bdea-8bcfa+35b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-4adf+3bcf+7bde)}{5b}}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{\int \frac{\left(3c^2(7de-2cf)b^3+acd(49de+9cf)b^2-a^2d^2(14de+19cf)b+8a^3d^3f\right)x^2+ac\left(3c(21de-cf)b^2-ad(7de+9cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15\right.}{3d}}{5b}}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{\frac{ac\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5b}}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(a\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5b}}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$\downarrow$ 388

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx\right.}{a\left(c+dx\right.}}}}{3d}}{5b}}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$\downarrow$ 313

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8\right.}{}}{5b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(7*b) + (((7*b*d*e + 3*b*c*f - 4
*a*d*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((8*a^2*d^2*f + 3*b^
2*c*(14*d*e + c*f) - a*b*d*(14*d*e + 15*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*d) + ((8*a^3*d^3*f + 3*b^3*c^2*(7*d*e - 2*c*f) + a*b^2*c*d*(49*d
*e + 9*c*f) - a^2*b*d^2*(14*d*e + 19*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2])) + (c^(3/2)*(4*a^2*d^2*f + 3*b^2*c*(21*d*e - c*f) - a*b*d*(7*d
*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
))/(3*d))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{df\,x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7}+\frac{\left(a\,d^2f+2bcdf+b\,d^2e-\frac{df(6ad+6bc)}{7}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(\frac{9acdf}{7}+a\,d^2e+b\,c^2 \right.}{}\right.}{}$
risch	$-\frac{x\left(-15b^2d^2f\,x^4-3ab\,d^2f\,x^2-24b^2cf\,x^2d-21b^2d^2e\,x^2+4f\,d^2a^2-9fdcba-7ab\,d^2e-3f\,c^2b^2-42d\,b^2ce\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{105d\,b^2}+\frac{\left(-\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*d*f*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*
(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(9/7*a*c*d*
f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+
6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c^
2*e-1/3*(9/7*a*c*d*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^
2*e-1/7*d*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c^2*f+2*a*c*d*e+b*c^2*e-3/5*
(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+6*b*c))/b/d*a*c-1/3*(9/7*a*c*d*f
+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.94

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(7\,(3\,b^3c^3d+7\,ab^2c^2d^2-2\,a^2bcd^3)e-(6\,b^3c^4-9\,ab^2c^3d+19\,a^2bc^2d^2-8\,a^3cd^3)f)x\sqrt{-\frac{c}{d}}E(\arcsin\Big(}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
-1/105*(sqrt(b*d)*(7*(3*b^3*c^3*d + 7*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3)*e - (
6*b^3*c^4 - 9*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 8*a^3*c*d^3)*f)*x*sqrt(-c/d
)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(3*b^3*c^3*d
+ 7*a*b^2*c^2*d^2 - a^2*b*d^4 - (2*a^2*b - 9*a*b^2)*c*d^3)*e - (6*b^3*c^4
- 9*a*b^2*c^3*d - 4*a^3*d^4 + (19*a^2*b + 3*a*b^2)*c^2*d^2 - (8*a^3 - 9*a^
2*b)*c*d^3)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(15*b^3*d^4*f*x^6 + 3*(7*b^3*d^4*e + (8*b^3*c*d^3 + a*b^2*d^4)*f)*x^4 + (7
*(6*b^3*c*d^3 + a*b^2*d^4)*e + (3*b^3*c^2*d^2 + 9*a*b^2*c*d^3 - 4*a^2*b*d^
4)*f)*x^2 + 7*(3*b^3*c^2*d^2 + 7*a*b^2*c*d^3 - 2*a^2*b*d^4)*e - (6*b^3*c^3
*d - 9*a*b^2*c^2*d^2 + 19*a^2*b*c*d^3 - 8*a^3*d^4)*f)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c))/(b^3*d^3*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 9*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*c*d*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x + 42*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b**2*c*d*e*x + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3 +
21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*d**2*f*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f - 19*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*
x**4),x)*a**2*b*c*d**2*f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e + 9*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*
a*b**2*c**2*d*f + 49*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a
*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e - 6*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c*
*3*f + 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*b**3*c**2*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f - 9*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)
*a**2*b*c**2*d*f - 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d...
```

3.2 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\ dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\ dx = \frac{\left(5bce+5ade+2acf-\frac{2bc^2f}{d}-\frac{2a^2df}{b}\right)x\sqrt{c+dx^2}}{15d\sqrt{a+bx^2}} + \frac{(5bde-2bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd} + \frac{fx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} + \frac{\sqrt{a}(2a^2d^2f-b^2c(5de-2cf)-abd(5de+2cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(10bde-bcf-adf)\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(5*b*c*e+5*a*d*e+2*a*c*f-2*b*c^2*f/d-2*a^2*d*f/b)*x*(d*x^2+c)^(1/2)/d
/(b*x^2+a)^(1/2)+1/15*(a*d*f-2*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/b/d+1/5*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d+1/15*a^(1/2)*(2*a^2*d^
2*f-b^2*c*(-2*c*f+5*d*e)-a*b*d*(2*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^(3/2)*(-a*d*f-b*c*f+10*b*d*e
)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)\;dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(adf+b(5de+cf+3dfx^2))+ic(2a^2d^2f+b^2c(-5de+2cf)-abd(5de+2cf}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(5*d*e + c*f + 3*d*f*x^2
)) + I*c*(2*a^2*d^2*f + b^2*c*(-5*d*e + 2*c*f) - a*b*d*(5*d*e + 2*c*f))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*c*(-(b*c) + a*d)*(5*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(15*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)\;dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left((5bde+bcf-2adf)x^2+c(5be-af)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \frac{ac(10bde-bcf-adf)-\left(-c(5de-2cf)b^2-ad(5de+2cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+bcf+5bde)}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 406

$$\frac{\frac{ac(-adf-bcf+10bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+b\cdots}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 320

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+x\sqrt{a+bx^2}\sqrt{\cdots}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 388

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3d}+\cdots}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$\big\downarrow$ 313

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cdots\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\cdots\right.}{a\left(c+\cdots\right.}}}\cdots\right.}{3d}\cdots}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((5*b*d*e + b*c*f - 2*a*d
*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (-((2*a^2*d^2*f - b^2*c*(5*
d*e - 2*c*f) - a*b*d*(5*d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))) + (c^(3/2)*(10*b*d*e - b*c*f - a*d*f)*Sqrt[a + b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*b)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5}+\frac{\left(adf+bcf+bde-\frac{f(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}+\frac{\left(ace-\frac{\left(adf+bcf+bde-\frac{f(4\cdots}{}\right.}{3bd}\right.}{}\right.}{}$
risch	$\frac{x\left(3bdf\,x^2+adf+bcf+5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15bd}-\frac{\left(-\frac{\left(2f\,d^2a^2-2fdcba-5ab\,d^2e+2f\,c^2b^2-5d\,b^2ce\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac\cdots}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3f\,x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3f\,x^5+4\sqrt{-\frac{b}{a}}\,b^2c\,d^2f\,x^5+5\sqrt{-\frac{b}{a}}\,b^2d^3e\,x^5+\sqrt{-\frac{b}{a}}\,a^2d^3f\,x^3+5\sqrt{-\frac{b}{a}}\,abc\,d^2f\,x^3\cdots\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f*x^3*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*d*f+b*c*f+b*d*e-1/5*f*(4*a*d+4*b*c
))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c*e-1/3*(a*d*f+b*c*f+b*d*e
-1/5*f*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-(2/5*a*c*f+a*d*e+b*c*e-1/3*(a*d*f+b*c*f+b*d*e-1/5*f*(4*
a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(5\left(b^2c^2d+abcd^2\right)e-2\left(b^2c^3-abc^2d+a^2cd^2\right)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(5\left(b^2c^2d+\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
-1/15*(sqrt(b*d)*(5*(b^2*c^2*d + a*b*c*d^2)*e - 2*(b^2*c^3 - a*b*c^2*d + a
^2*c*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(b*d)*(5*(b^2*c^2*d + a*b*c*d^2 + 2*a*b*d^3)*e - (2*b^2*c^3 - 2*a*b*c^2*
d + a^2*d^3 + (2*a^2 + a*b)*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(
-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*f*x^4 + (5*b^2*d^3*e + (b^2*c*d^2 + a*b*
d^3)*f)*x^2 + 5*(b^2*c*d^2 + a*b*d^3)*e - 2*(b^2*c^2*d - a*b*c*d^2 + a^2*d
^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx+5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdex+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f + 2*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**
4),x)*a*b*c*d*f + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*
d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e - 2*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2*f +
 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
 + b*d*x**4),x)*b**2*c*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f - int((sqrt(c + d*x**2)*sq
rt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f + 10*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a*b*c*d*e)/(15*b*d)
```

3.3 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{(3bde-2bcf+adf)x\sqrt{c+dx^2}}{3d^2\sqrt{a+bx^2}}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{\sqrt{a}(3bde-2bcf+adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(3de-cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
1/3*(a*d*f-2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d^2/(b*x^2+a)^(1/2)+1/3*f*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d-1/3*a^(1/2)*(a*d*f-2*b*c*f+3*b*d*e)*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2
))/b^(1/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(3/2)
*(-c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/b^(1/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dfx(a+bx^2)(c+dx^2) - ic(3bde-2bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i(-bc}{3\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2) - I*c*(3*b*d*e - 2*b*c*f + a*d*f)
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + I*(-(b*c) + a*d)*(-3*d*e + 2*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/
a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 403$$
$$\frac{\int \frac{(3bde-2bcf+adf)x^2+a(3de-cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$
$$\Big\downarrow \; 406$$

$$\frac{\frac{a(3de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}}{}$$

$$\Big\downarrow \text{320}$$

$$\frac{(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \text{388}$$

$$\frac{(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + ((3*b*d*e - 2*b*c*f + a*d*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - c*f)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 7.90 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{fx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3d}+\frac{\left(ae-\frac{acf}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\left(af+be-\frac{f(2ad+2bc)}{3d}\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
risch	$\frac{fx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3d}-\frac{\left(\frac{(adf-2bcf+3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}+\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,b\,d^2f\,x^5-\sqrt{-\frac{b}{a}}\,a\,d^2f\,x^3-\sqrt{-\frac{b}{a}}\,bcdf\,x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-3\sqrt{b}\right.}{}$

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*f/d*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*e-1/3*a*c*f/d)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*f+b*e-1/3*f/d*(2*a*d+2*b*c))*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(3\,bc^2de-(2\,bc^3-ac^2d)f)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(3\,(bc^2d+ad^3)e-(2\,bc^3-ac^2d-}{3\,bcd}$$

input

```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((3*b*c^2*d*e - (2*b*c^3 - a*c^2*d)*f)*sqrt(b*d)*x*sqrt(-c/d)*ellipti
c_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(3*(b*c^2*d + a*d^3)*e -
(2*b*c^3 - a*c^2*d + a*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (b*c*d^2*f*x^2 + 3*b*c*d^2*e - (2*b*c^2*d - a*c*d^2)*f)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)adf-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bcf+3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{}}{bd\,x^4+ad\,x^2}\right.}{3d}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f - 2*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),
x)*b*c*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
+ b*c*x**2 + b*d*x**4),x)*b*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f + 3*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*e)/(3*d)
```

3.4 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{fx\sqrt{a+bx^2}}{d\sqrt{c+dx^2}} + \frac{(de-2cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
f*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+(-2*c*f+d*e)*(b*x^2+a)^(1/2)*Ellipti
cE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(3/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(1/2)*f*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(3/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2) - ibc(-de+2cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{b}{a}}cd^2\sqrt{}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + 2*c*f)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*c*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d^2*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}} - \frac{\int -\frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd} + \frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-b(de-2cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) + (-(b*(d*e - 2*c*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(-\frac{(bd\,x^2+ad)(cf-de)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{(cf-de)(ad-bc)}{d^2c}+\frac{a(cf-de)}{dc}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\ldots}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}}{\sqrt{b\,x^2+a}\sqrt{x^2d\ldots}}$
default	$\frac{\sqrt{b\,x^2+a}\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,bcdf\,x^3+\sqrt{-\frac{b}{a}}\,b\,d^2e\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-2\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{Ellip}\ldots\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)*(c*f-d*e)/c/d^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+((a*d*f-b*c*f+b*d*e)/d
^2-(c*f-d*e)/d^2*(a*d-b*c)/c+a/d*(c*f-d*e)/c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f*b/d+(c*f-d*e)/d*b/c)*c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{((bcd^2e-2\,bc^2df)x^3+(bc^2de-2\,bc^3f)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(\ldots}{\ldots}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
(((b*c*d^2*e - 2*b*c^2*d*f)*x^3 + (b*c^2*d*e - 2*b*c^3*f)*x)*sqrt(b*d)*sqr
t(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b*c*d^2*e - (2*b*c
^2*d + a*d^3)*f)*x^3 + (b*c^2*d*e - (2*b*c^3 + a*c*d^2)*f)*x)*sqrt(b*d)*sq
rt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*c*d^2*f*x^2 - b*
c*d^2*e + 2*b*c^2*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^4*x^3 + b*c
^2*d^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,afx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bex-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2c}\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*e*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*
x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d*
f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 +
a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**2*f*x**2
 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 +
 a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*f -
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d*e + 2*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d*f*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x*
*4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*d**2*e*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 +
b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c**2*f - int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c*d*f*x**2 + int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*
b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**2*e + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x*...
```

3.5 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx &= \frac{(de-cf)x\sqrt{a+bx^2}}{3cd\,(c+dx^2)^{3/2}} \\ &- \frac{(ad(2de+cf)-bc(de+2cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{3/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{b(de-cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/3*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(3/2)-1/3*(a*d*(c*f+2*d*e)-
b*c*(2*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(3/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(d*x^2+c)^(1/2)+1/3*b*(-c*f+d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(
arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(3/2)/(-a*d+b*c)/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad^2(3ce+2dex^2+cfx^2)-bc(c^2f+d^2ex^2+2cd(e+fx^2))\right)}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d^2*(3*c*e + 2*d*e*x^2 + c*f*x^2) - b*c*(c^2
*f + d^2*e*x^2 + 2*c*d*(e + f*x^2))) + I*b*c*(a*d*(2*d*e + c*f) - b*c*(d*e
 + 2*c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*(d*e + 2*c*f)*S
qrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d^2*(-(b*c) + a*d)*Sqrt[a + b*x^
2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 401$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{3cd\,(c+dx^2)^{3/2}}-\frac{\int -\frac{b(de+2cf)x^2+a(2de+cf)}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3cd}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b(de+2cf)x^2+a(2de+cf)}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(cf+2de)-bc(2cf+de))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad(cf+2de)-bc(2cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(cf+2de)-bc(2cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(3*c*d*(c + d*x^2)^(3/2)) + (-(((a*d*(2*d*
e + c*f) - b*c*(d*e + 2*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(d*e - c*f)*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(
b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(246) = 492$.

Time $= 6.00$ (sec) , antiderivative size $= 518$, normalized size of antiderivative $= 1.93$

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(cf-de)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,d^3\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)x\left(acdf+2a\,d^2e-2b\,c^2f-bcde\right)}{3c^2d^2(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{fb}{d^2}-\frac{(cf-de)b}{3d^2c}+\frac{acdf+2a\,d^2e-2b}{3d^2c^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(c*f-d*e
)/c/d^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/3*(b*d*x^2+a*d
)/c^2/d^2/(a*d-b*c)*x*(a*c*d*f+2*a*d^2*e-2*b*c^2*f-b*c*d*e)/((x^2+c/d)*(b*
d*x^2+a*d))^(1/2)+(f*b/d^2-1/3*(c*f-d*e)/d^2*b/c+1/3/d^2*(a*c*d*f+2*a*d^2*
e-2*b*c^2*f-b*c*d*e)/c^2-1/3*a/d/c^2/(a*d-b*c)*(a*c*d*f+2*a*d^2*e-2*b*c^2*
f-b*c*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+
1/3*b/d^2*(a*c*d*f+2*a*d^2*e-2*b*c^2*f-b*c*d*e)/(a*d-b*c)/c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(246) = 492$.

Time = 0.09 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{(((b^2cd^3-2\,abd^4)e+(2\,b^2c^2d^2-abcd^3)f)x^4+2\,((b^2c^2d^2-2\,abcd^3)e+(2\,b^2c^3d-abc^2d^2)f)x^2+(b^2c^3d}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((((b^2*c*d^3 - 2*a*b*d^4)*e + (2*b^2*c^2*d^2 - a*b*c*d^3)*f)*x^4 + 2
*((b^2*c^2*d^2 - 2*a*b*c*d^3)*e + (2*b^2*c^3*d - a*b*c^2*d^2)*f)*x^2 + (b^
2*c^3*d - 2*a*b*c^2*d^2)*e + (2*b^2*c^4 - a*b*c^3*d)*f)*sqrt(a*c)*sqrt(-b/
a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((b^2*c*d^3 - (a^2 + 2*a
*b)*d^4)*e + (2*b^2*c^2*d^2 + (a^2 - a*b)*c*d^3)*f)*x^4 + 2*((b^2*c^2*d^2
- (a^2 + 2*a*b)*c*d^3)*e + (2*b^2*c^3*d + (a^2 - a*b)*c^2*d^2)*f)*x^2 + (b
^2*c^3*d - (a^2 + 2*a*b)*c^2*d^2)*e + (2*b^2*c^4 + (a^2 - a*b)*c^3*d)*f)*s
qrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((a*b*c
*d^3 - 2*a^2*d^4)*e + (2*a*b*c^2*d^2 - a^2*c*d^3)*f)*x^3 + (a*b*c^3*d*f +
(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*e)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b*
c^5*d^2 - a^2*c^4*d^3 + (a*b*c^3*d^4 - a^2*c^2*d^5)*x^4 + 2*(a*b*c^4*d^3 -
 a^2*c^3*d^4)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f*x - sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d +
3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2
*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*
b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a**2*b*c**
2*d**2*f + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3
*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*
a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b
**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a**2*b*c*d*
*3*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*
a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a
*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b*
*2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a**2*b*d**4*
f*x**4 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*a
**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a*
b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b**
2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a*b**2*c**3*d
*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c*
*2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3
*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c...
```

3.6 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 378

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \frac{(de-cf)x\sqrt{a+bx^2}}{5cd\,(c+dx^2)^{5/2}} - \frac{(ad(4de+cf)-bc(3de+2cf))x\sqrt{a+bx^2}}{15c^2d(bc-ad)\,(c+dx^2)^{3/2}} + \frac{(2a^2d^2(4de+cf)+b^2c^2(3de+2cf)-abcd(13de+2cf))\,\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b(bc(6de-cf)-ad(4de+cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
1/5*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(5/2)-1/15*(a*d*(c*f+4*d*e)
-b*c*(2*c*f+3*d*e))*x*(b*x^2+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x^2+c)^(3/2)+1/1
5*(2*a^2*d^2*(c*f+4*d*e)+b^2*c^2*(2*c*f+3*d*e)-a*b*c*d*(2*c*f+13*d*e))*(b*
x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/
2))/c^(5/2)/d^(3/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)+1/15*b*(b*c*(-c*f+6*d*e)-a*d*(c*f+4*d*e))*(b*x^2+a)^(1/2)*InverseJa
cobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(3/2)/(-a*d+
b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)+c(bc-ad)(-ad(4de+cf)+bc(3d\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f) + c*(b*c - a*d
)*(-(a*d*(4*d*e + c*f)) + b*c*(3*d*e + 2*c*f))*(c + d*x^2) + (2*a^2*d^2*(4
*d*e + c*f) + b^2*c^2*(3*d*e + 2*c*f) - a*b*c*d*(13*d*e + 2*c*f))*(c + d*x
^2)^2) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((2*a
^2*d^2*(4*d*e + c*f) + b^2*c^2*(3*d*e + 2*c*f) - a*b*c*d*(13*d*e + 2*c*f))
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(-(a*d*(4*d*
e + c*f)) + b*c*(3*d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)]))/(15*Sqrt[b/a]*c^3*d^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(5
/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {401, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\,(c+dx^2)^{5/2}}-\frac{\int -\frac{b(3de+2cf)x^2+a(4de+cf)}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{5cd}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b(3de+2cf)x^2+a(4de+cf)}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{5cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int -\frac{b(ad(4de+cf)-bc(3de+2cf))x^2+a(2ad(4de+cf)-bc(9de+cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(cf+4de)-bc(2cf+3de))}{3c(c+dx^2)^{3/2}(bc-ad)}}{5cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{b(ad(4de+cf)-bc(3de+2cf))x^2+a(2ad(4de+cf)-bc(9de+cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(cf+4de)-bc(2cf+3de))}{3c(c+dx^2)^{3/2}(bc-ad)}}{5cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{-\frac{-\frac{\left(2a^2d^2(cf+4de)-abcd(2cf+13de)+b^2c^2(2cf+3de)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab(bc(6de-cf)-ad(cf+4de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(cf+4de)}{3c(c+dx^2)^{3/2}}}{5cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{-\frac{-\frac{ab(bc(6de-cf)-ad(cf+4de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(cf+4de)-abcd(2cf+13de)+b^2c^2(2cf+3de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-x\sqrt{a+b}}{5cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 320$$

$$-\frac{-\frac{-\frac{\sqrt{a+bx^2}\left(2a^2d^2(cf+4de)-abcd(2cf+13de)+b^2c^2(2cf+3de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc(6de-cf)-ad(cf+4de))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}}{5cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(5*c*d*(c + d*x^2)^(5/2)) + (-1/3*((a*d*(4
*d*e + c*f) - b*c*(3*d*e + 2*c*f))*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c +
d*x^2)^(3/2)) - (-(((2*a^2*d^2*(4*d*e + c*f) + b^2*c^2*(3*d*e + 2*c*f) - a
*b*c*d*(13*d*e + 2*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) - (b*Sqrt[c]*(b*c*(6*d*e - c*f) - a*d*(
4*d*e + c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]))/(3*c*(b*c - a*d)))/(5*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(349) = 698$.

Time = 6.05 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(cf-de)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c\,d^4\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(acdf+4a\,d^2e-2b\,c^2f-3bcde\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15d^3c^2(ad-bc)\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)x\left(2a^2cf\right.}{15c^3}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(c*f-d*e
)/c/d^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/15*(a*c*d*f+4*
a*d^2*e-2*b*c^2*f-3*b*c*d*e)/d^3/c^2/(a*d-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d^2/(a*d-b*c)^2*x*(2*a^2*c*d
^2*f+8*a^2*d^3*e-2*a*b*c^2*d*f-13*a*b*c*d^2*e+2*b^2*c^3*f+3*b^2*c^2*d*e)/(
(x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/15*b*(a*c*d*f+4*a*d^2*e-2*b*c^2*f-3*b*c*
d*e)/(a*d-b*c)/c^2/d^2+1/15/d^2/(a*d-b*c)*(2*a^2*c*d^2*f+8*a^2*d^3*e-2*a*b
*c^2*d*f-13*a*b*c*d^2*e+2*b^2*c^3*f+3*b^2*c^2*d*e)/c^3-1/15*a/d/c^3/(a*d-b
*c)^2*(2*a^2*c*d^2*f+8*a^2*d^3*e-2*a*b*c^2*d*f-13*a*b*c*d^2*e+2*b^2*c^3*f+
3*b^2*c^2*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))+1/15*b/d^2*(2*a^2*c*d^2*f+8*a^2*d^3*e-2*a*b*c^2*d*f-13*a*b*c*d^2*e+2*b
^2*c^3*f+3*b^2*c^2*d*e)/(a*d-b*c)^2/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(349) = 698$.

Time = 0.14 (sec) , antiderivative size = 1175, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output

```
-1/15*((((3*b^3*c^2*d^4 - 13*a*b^2*c*d^5 + 8*a^2*b*d^6)*e + 2*(b^3*c^3*d^3
 - a*b^2*c^2*d^4 + a^2*b*c*d^5)*f)*x^6 + 3*((3*b^3*c^3*d^3 - 13*a*b^2*c^2*
d^4 + 8*a^2*b*c*d^5)*e + 2*(b^3*c^4*d^2 - a*b^2*c^3*d^3 + a^2*b*c^2*d^4)*f
)*x^4 + 3*((3*b^3*c^4*d^2 - 13*a*b^2*c^3*d^3 + 8*a^2*b*c^2*d^4)*e + 2*(b^3
*c^5*d - a*b^2*c^4*d^2 + a^2*b*c^3*d^3)*f)*x^2 + (3*b^3*c^5*d - 13*a*b^2*c
^4*d^2 + 8*a^2*b*c^3*d^3)*e + 2*(b^3*c^6 - a*b^2*c^5*d + a^2*b*c^4*d^2)*f)
*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((3*b
^3*c^2*d^4 - (6*a^2*b + 13*a*b^2)*c*d^5 + 4*(a^3 + 2*a^2*b)*d^6)*e + (2*b^
3*c^3*d^3 + (a^2*b - 2*a*b^2)*c^2*d^4 + (a^3 + 2*a^2*b)*c*d^5)*f)*x^6 + 3*
((3*b^3*c^3*d^3 - (6*a^2*b + 13*a*b^2)*c^2*d^4 + 4*(a^3 + 2*a^2*b)*c*d^5)*
e + (2*b^3*c^4*d^2 + (a^2*b - 2*a*b^2)*c^3*d^3 + (a^3 + 2*a^2*b)*c^2*d^4)*
f)*x^4 + 3*((3*b^3*c^4*d^2 - (6*a^2*b + 13*a*b^2)*c^3*d^3 + 4*(a^3 + 2*a^2
*b)*c^2*d^4)*e + (2*b^3*c^5*d + (a^2*b - 2*a*b^2)*c^4*d^2 + (a^3 + 2*a^2*b
)*c^3*d^3)*f)*x^2 + (3*b^3*c^5*d - (6*a^2*b + 13*a*b^2)*c^4*d^2 + 4*(a^3 +
 2*a^2*b)*c^3*d^3)*e + (2*b^3*c^6 + (a^2*b - 2*a*b^2)*c^5*d + (a^3 + 2*a^2
*b)*c^4*d^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - (((3*a*b^2*c^2*d^4 - 13*a^2*b*c*d^5 + 8*a^3*d^6)*e + 2*(a*b^2*c^3
*d^3 - a^2*b*c^2*d^4 + a^3*c*d^5)*f)*x^5 + ((9*a*b^2*c^3*d^3 - 33*a^2*b*c^
2*d^4 + 20*a^3*c*d^5)*e + (6*a*b^2*c^4*d^2 - 7*a^2*b*c^3*d^3 + 5*a^3*c^2*d
^4)*f)*x^3 + ((9*a*b^2*c^4*d^2 - 26*a^2*b*c^3*d^3 + 15*a^3*c^2*d^4)*e +...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input

```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(7/2),x)
```

output

```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f*x - sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*
d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 +
2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8
*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**
2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b
**2*c*d**4*x**10),x)*a**2*b*c**3*d**2*f + 6*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*
x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**
2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*
b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6
 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**2*b*c**2*d**3*f*x**2 +
 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c*
*3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x
**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**
3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c*
*4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x*
*10),x)*a**2*b*c*d**4*f*x**4 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a*
*2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*...
```

3.7 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 517

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx &= \frac{(de-cf)x\sqrt{a+bx^2}}{7cd\,(c+dx^2)^{7/2}} \\ &- \frac{(ad(6de+cf)-bc(5de+2cf))x\sqrt{a+bx^2}}{35c^2d(bc-ad)\,(c+dx^2)^{5/2}} \\ &+ \frac{(4a^2d^2(6de+cf)+3b^2c^2(5de+2cf)-abcd(43de+6cf))\,x\sqrt{a+bx^2}}{105c^3d(bc-ad)^2\,(c+dx^2)^{3/2}} \\ &- \frac{(8a^3d^3(6de+cf)-3b^3c^3(5de+2cf)+ab^2c^2d(103de+9cf)-a^2bcd^2(128de+19cf))\,\sqrt{a+bx^2}\,E\left(\text{arct}\right.}{105c^{7/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{b(3b^2c^2(15de-cf)+4a^2d^2(6de+cf)-abcd(61de+9cf))\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{5/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/7*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(7/2)-1/35*(a*d*(c*f+6*d*e)
-b*c*(2*c*f+5*d*e))*x*(b*x^2+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x^2+c)^(5/2)+1/1
05*(4*a^2*d^2*(c*f+6*d*e)+3*b^2*c^2*(2*c*f+5*d*e)-a*b*c*d*(6*c*f+43*d*e))*
x*(b*x^2+a)^(1/2)/c^3/d/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/105*(8*a^3*d^3*(c*f
+6*d*e)-3*b^3*c^3*(2*c*f+5*d*e)+a*b^2*c^2*d*(9*c*f+103*d*e)-a^2*b*c*d^2*(1
9*c*f+128*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))/c^(7/2)/d^(3/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)+1/105*b*(3*b^2*c^2*(-c*f+15*d*e)+4*a^2*d^2*(c*f+
6*d*e)-a*b*c*d*(9*c*f+61*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1
/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(3/2)/(-a*d+b*c)^3/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.11 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)+3c^2(bc-ad)^2(-ad(6de+cf)+b\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f) + 3*c^2*(b*c
- a*d)^2*(-(a*d*(6*d*e + c*f)) + b*c*(5*d*e + 2*c*f))*(c + d*x^2) + c*(b*c
 - a*d)*(4*a^2*d^2*(6*d*e + c*f) + 3*b^2*c^2*(5*d*e + 2*c*f) - a*b*c*d*(43
*d*e + 6*c*f))*(c + d*x^2)^2 + (-8*a^3*d^3*(6*d*e + c*f) + 3*b^3*c^3*(5*d*
e + 2*c*f) - a*b^2*c^2*d*(103*d*e + 9*c*f) + a^2*b*c*d^2*(128*d*e + 19*c*f
))*(c + d*x^2)^3) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^3*Sqrt[1 + (d*x^
2)/c]*((-8*a^3*d^3*(6*d*e + c*f) + 3*b^3*c^3*(5*d*e + 2*c*f) - a*b^2*c^2*d
*(103*d*e + 9*c*f) + a^2*b*c*d^2*(128*d*e + 19*c*f))*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(4*a^2*d^2*(6*d*e + c*f) + 3*b^2*c
^2*(5*d*e + 2*c*f) - a*b*c*d*(43*d*e + 6*c*f))*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)]))/(105*Sqrt[b/a]*c^4*d^2*(b*c - a*d)^3*Sqrt[a + b*x^2]
*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 402, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}-\frac{\int -\frac{b(5de+2cf)x^2+a(6de+cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{7/2}}dx}{7cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(5de+2cf)x^2+a(6de+cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{7/2}}dx}{7cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int -\frac{3b(ad(6de+cf)-bc(5de+2cf))x^2+a(4ad(6de+cf)-bc(25de+3cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(cf+6de)-bc(2cf+5de))}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}}{7cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{3b(ad(6de+cf)-bc(5de+2cf))x^2+a(4ad(6de+cf)-bc(25de+3cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(cf+6de)-bc(2cf+5de))}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}}{7cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{402}$$

$$-\frac{-\frac{\frac{\int -\frac{b\left(3b^2(5de+2cf)c^2-abd(43de+6cf)c+4a^2d^2(6de+cf)\right)x^2+a\left(3b^2(20de+cf)c^2-abd(104de+15cf)c+8a^2d^2(6de+cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}\left(4a^2d^2(cf+6de)-abc\dots\right.}{3c\left(c+dx^2\right)\dots}}{5c(bc-ad)}\dots}{7cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\downarrow$ 25

$$-\frac{-\frac{-\frac{\int \frac{b\left(3b^2(5de+2cf)c^2-abd(43de+6cf)c+4a^2d^2(6de+cf)\right)x^2+a\left(3b^2(20de+cf)c^2-abd(104de+15cf)c+8a^2d^2(6de+cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}\left(4a^2d^2(cf+6de)-abc\dots\right.}{3c\left(c+dx^2\right)\dots}}{5c(bc-ad)}\dots}{7cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\downarrow$ 400

$$-\frac{-\frac{\frac{ab\left(4a^2d^2(cf+6de)-abcd(9cf+61de)+3b^2c^2(15de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\left(8a^3d^3(cf+6de)-a^2bcd^2(19cf+128de)+ab^2c^2d(9cf+103de)-3b^3c^3(2cf-\dots\right.}{bc-ad}}{3c(bc-ad)}\dots}{5c(bc-ad)}\dots}{7cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\downarrow$ 313

$$-\frac{-\frac{\frac{ab\left(4a^2d^2(cf+6de)-abcd(9cf+61de)+3b^2c^2(15de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\left(8a^3d^3(cf+6de)-a^2bcd^2(19cf+128de)+ab^2c^2d(9cf+103de)-3b\dots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\dots\right.}{a\left(c+dx^2\dots\right.}}}}{3c(bc-ad)}\dots}{5c(bc-ad)}\dots}{7cd}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\downarrow$ 320

$$-\frac{-\frac{x\sqrt{a+bx^2}\left(4a^2d^2(cf+6de)-abcd(6cf+43de)+3b^2c^2(2cf+5de)\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(cf+6de)-abcd(9cf+61de)+3b^2c^2(15de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{}}{5c(bc-ad)}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(7*c*d*(c + d*x^2)^(7/2)) + (-1/5*((a*d*(6
*d*e + c*f) - b*c*(5*d*e + 2*c*f))*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c +
d*x^2)^(5/2)) - (-1/3*((4*a^2*d^2*(6*d*e + c*f) + 3*b^2*c^2*(5*d*e + 2*c*f
) - a*b*c*d*(43*d*e + 6*c*f))*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2
)^(3/2)) - (-(((8*a^3*d^3*(6*d*e + c*f) - 3*b^3*c^3*(5*d*e + 2*c*f) + a*b^
2*c^2*d*(103*d*e + 9*c*f) - a^2*b*c*d^2*(128*d*e + 19*c*f))*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d
]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b
*Sqrt[c]*(3*b^2*c^2*(15*d*e - c*f) + 4*a^2*d^2*(6*d*e + c*f) - a*b*c*d*(61
*d*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2]))/(3*c*(b*c - a*d)))/(5*c*(b*c - a*d)))/(7*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(482) = 964$.

Time = 6.01 (sec) , antiderivative size = 1016, normalized size of antiderivative = 1.97

method	result	size
elliptic	Expression too large to display	1016
default	Expression too large to display	5139

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*(c*f-d*e
)/c/d^5*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^4+1/35*(a*c*d*f+6*
a*d^2*e-2*b*c^2*f-5*b*c*d*e)/d^4/(a*d-b*c)/c^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/(x^2+c/d)^3+1/105*(4*a^2*c*d^2*f+24*a^2*d^3*e-6*a*b*c^2*d*f-43*
a*b*c*d^2*e+6*b^2*c^3*f+15*b^2*c^2*d*e)/(a*d-b*c)^2/c^3/d^3*x*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/105*(b*d*x^2+a*d)/c^4/d^2/(a*d-b*c)^
3*x*(8*a^3*c*d^3*f+48*a^3*d^4*e-19*a^2*b*c^2*d^2*f-128*a^2*b*c*d^3*e+9*a*b
^2*c^3*d*f+103*a*b^2*c^2*d^2*e-6*b^3*c^4*f-15*b^3*c^3*d*e)/((x^2+c/d)*(b*d
*x^2+a*d))^(1/2)+(1/105*b*(4*a^2*c*d^2*f+24*a^2*d^3*e-6*a*b*c^2*d*f-43*a*b
*c*d^2*e+6*b^2*c^3*f+15*b^2*c^2*d*e)/(a*d-b*c)^2/c^3/d^2+1/105/d^2/(a*d-b*
c)^2*(8*a^3*c*d^3*f+48*a^3*d^4*e-19*a^2*b*c^2*d^2*f-128*a^2*b*c*d^3*e+9*a*
b^2*c^3*d*f+103*a*b^2*c^2*d^2*e-6*b^3*c^4*f-15*b^3*c^3*d*e)/c^4-1/105*a/d/
c^4/(a*d-b*c)^3*(8*a^3*c*d^3*f+48*a^3*d^4*e-19*a^2*b*c^2*d^2*f-128*a^2*b*c
*d^3*e+9*a*b^2*c^3*d*f+103*a*b^2*c^2*d^2*e-6*b^3*c^4*f-15*b^3*c^3*d*e))/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/105*b/d^2*(8
*a^3*c*d^3*f+48*a^3*d^4*e-19*a^2*b*c^2*d^2*f-128*a^2*b*c*d^3*e+9*a*b^2*c^3
*d*f+103*a*b^2*c^2*d^2*e-6*b^3*c^4*f-15*b^3*c^3*d*e)/(a*d-b*c)^3/c^3/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2070 vs. $2(482) = 964$.

Time = 0.16 (sec) , antiderivative size = 2070, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output

```
-1/105*((((15*b^4*c^3*d^5 - 103*a*b^3*c^2*d^6 + 128*a^2*b^2*c*d^7 - 48*a^3
*b*d^8)*e + (6*b^4*c^4*d^4 - 9*a*b^3*c^3*d^5 + 19*a^2*b^2*c^2*d^6 - 8*a^3*
b*c*d^7)*f)*x^8 + 4*((15*b^4*c^4*d^4 - 103*a*b^3*c^3*d^5 + 128*a^2*b^2*c^2
*d^6 - 48*a^3*b*c*d^7)*e + (6*b^4*c^5*d^3 - 9*a*b^3*c^4*d^4 + 19*a^2*b^2*c
^3*d^5 - 8*a^3*b*c^2*d^6)*f)*x^6 + 6*((15*b^4*c^5*d^3 - 103*a*b^3*c^4*d^4
+ 128*a^2*b^2*c^3*d^5 - 48*a^3*b*c^2*d^6)*e + (6*b^4*c^6*d^2 - 9*a*b^3*c^5
*d^3 + 19*a^2*b^2*c^4*d^4 - 8*a^3*b*c^3*d^5)*f)*x^4 + 4*((15*b^4*c^6*d^2 -
 103*a*b^3*c^5*d^3 + 128*a^2*b^2*c^4*d^4 - 48*a^3*b*c^3*d^5)*e + (6*b^4*c^
7*d - 9*a*b^3*c^6*d^2 + 19*a^2*b^2*c^5*d^3 - 8*a^3*b*c^4*d^4)*f)*x^2 + (15
*b^4*c^7*d - 103*a*b^3*c^6*d^2 + 128*a^2*b^2*c^5*d^3 - 48*a^3*b*c^4*d^4)*e
 + (6*b^4*c^8 - 9*a*b^3*c^7*d + 19*a^2*b^2*c^6*d^2 - 8*a^3*b*c^5*d^3)*f)*s
qrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((15*b^
4*c^3*d^5 - (45*a^2*b^2 + 103*a*b^3)*c^2*d^6 + (61*a^3*b + 128*a^2*b^2)*c*
d^7 - 24*(a^4 + 2*a^3*b)*d^8)*e + (6*b^4*c^4*d^4 + 3*(a^2*b^2 - 3*a*b^3)*c
^3*d^5 + (9*a^3*b + 19*a^2*b^2)*c^2*d^6 - 4*(a^4 + 2*a^3*b)*c*d^7)*f)*x^8
+ 4*((15*b^4*c^4*d^4 - (45*a^2*b^2 + 103*a*b^3)*c^3*d^5 + (61*a^3*b + 128*
a^2*b^2)*c^2*d^6 - 24*(a^4 + 2*a^3*b)*c*d^7)*e + (6*b^4*c^5*d^3 + 3*(a^2*b
^2 - 3*a*b^3)*c^4*d^4 + (9*a^3*b + 19*a^2*b^2)*c^3*d^5 - 4*(a^4 + 2*a^3*b)
*c^2*d^6)*f)*x^6 + 6*((15*b^4*c^5*d^3 - (45*a^2*b^2 + 103*a*b^3)*c^4*d^4 +
 (61*a^3*b + 128*a^2*b^2)*c^3*d^5 - 24*(a^4 + 2*a^3*b)*c^2*d^6)*e + (6*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input

```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(9/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{{\left(d\,x^2+c\right)}^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f*x - sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b*e*x + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*
d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d**4*x*
*6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**
2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 +
 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x
**4 - 10*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x
**10 - b**2*c*d**5*x**12),x)*a**2*b*c**4*d**2*f + 12*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*
c**3*d**3*x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**
6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**
3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**
12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x**4 - 10*b**2*c**4*d**2*x**6 - 10*b**
2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x**10 - b**2*c*d**5*x**12),x)*a**2*b*c
**3*d**3*f*x**2 + 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*
c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d*
*4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*
d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x
**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**
5*d*x**4 - 10*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**...
```

3.8 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \left(e+fx^2\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 680

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \left(e+fx^2\right) \, dx = \frac{(8a^4d^4f+5ab^3c^2d(18de-5cf)-2b^4c^3(9de-4cf)+18a^2b^2cd^2(5de+cf)-a^3bd \cdots}{315b^2d^3\sqrt{a+bx^2}}$$
$$-\frac{(4a^3d^3f-3ab^2cd(27de-7cf)+2b^3c^2(9de-4cf)-9a^2bd^2(de+cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^2d^2}$$
$$-\frac{\left(18bce-72ade+17acf-\frac{8bc^2f}{d}-\frac{3a^2df}{b}\right)x\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{315d}$$
$$+\frac{(9bde-4bcf+3adf)x\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{63d^2}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{9d}$$
$$-\frac{\sqrt{a}(8a^4d^4f+5ab^3c^2d(18de-5cf)-2b^4c^3(9de-4cf)+18a^2b^2cd^2(5de+cf)-a^3bd^3(18de+25cf))\sqrt{\cdots}}{315b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(4a^3d^3f-b^3c^2(9de-4cf)+6ab^2cd(27de-2cf)-3a^2bd^2(3de+4cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{315b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/315*(8*a^4*d^4*f+5*a*b^3*c^2*d*(-5*c*f+18*d*e)-2*b^4*c^3*(-4*c*f+9*d*e)+
18*a^2*b^2*c*d^2*(c*f+5*d*e)-a^3*b*d^3*(25*c*f+18*d*e))*x*(d*x^2+c)^(1/2)/
b^2/d^3/(b*x^2+a)^(1/2)-1/315*(4*a^3*d^3*f-3*a*b^2*c*d*(-7*c*f+27*d*e)+2*b
^3*c^2*(-4*c*f+9*d*e)-9*a^2*b*d^2*(c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)/b^2/d^2-1/315*(18*b*c*e-72*a*d*e+17*a*c*f-8*b*c^2*f/d-3*a^2*d*f/b)*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d+1/63*(3*a*d*f-4*b*c*f+9*b*d*e)*x*(b*x^2+
a)^(1/2)*(d*x^2+c)^(5/2)/d^2+1/9*f*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/d-1/3
15*a^(1/2)*(8*a^4*d^4*f+5*a*b^3*c^2*d*(-5*c*f+18*d*e)-2*b^4*c^3*(-4*c*f+9*
d*e)+18*a^2*b^2*c*d^2*(c*f+5*d*e)-a^3*b*d^3*(25*c*f+18*d*e))*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2
)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/315*a^(3/2)*(4*a^3
*d^3*f-b^3*c^2*(-4*c*f+9*d*e)+6*a*b^2*c*d*(-2*c*f+27*d*e)-3*a^2*b*d^2*(4*c
*f+3*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*
d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.70

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \left(e+fx^2\right) \, dx = \frac{\sqrt{\frac{b}{a}} dx (a+bx^2) (c+dx^2) (-4a^3d^3f + 3a^2bd^2(3de+4cf+dfx^2) + b^3(-4c^3f + 3c}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*a^3*d^3*f + 3*a^2*b*d^2*(3*d*e
+ 4*c*f + d*f*x^2) + b^3*(-4*c^3*f + 3*c^2*d*(3*e + f*x^2) + 5*d^3*x^4*(9*
e + 7*f*x^2) + 2*c*d^2*x^2*(36*e + 25*f*x^2)) + a*b^2*d*(12*c^2*f + 2*d^2*
x^2*(36*e + 25*f*x^2) + c*d*(153*e + 83*f*x^2))) - I*c*(8*a^4*d^4*f + 5*a*
b^3*c^2*d*(18*d*e - 5*c*f) + 18*a^2*b^2*c*d^2*(5*d*e + c*f) + 2*b^4*c^3*(-
9*d*e + 4*c*f) - a^3*b*d^3*(18*d*e + 25*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) +
 a*d)*(4*a^3*d^3*f + 2*b^3*c^2*(9*d*e - 4*c*f) - 9*a^2*b*d^2*(d*e + c*f) +
 3*a*b^2*c*d*(-27*d*e + 7*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*a^2*(b/a)^(5/2)*d^3*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 615, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {403, 403, 403, 27, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \left(e+fx^2\right) \, dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \sqrt{dx^2+c} \left((9bde+3bcf-4adf)x^2+c(9be-af)\right)dx}{9b}+\frac{fx\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2} \left(\left(3c(24de+cf)b^2-ad(18de+17cf)b+8a^2d^2f\right)x^2+c\left(4dfa^2-9bdea-10bcfa+63b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{x\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(-4adf+3bcf+9bde)}{7b}}{9b}+\frac{fx\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{\int \frac{3\sqrt{bx^2+a}\left(\left(c^2(9de-4cf)b^3+9acd(9de+cf)b^2-3a^2d^2(6de+7cf)b+8a^3d^3f\right)x^2+ac\left(c(81de-cf)b^2-ad(9de+11cf)b+4a^2d^2f\right)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(8a}{}}{7b}}{9b}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{\sqrt{bx^2+a}\left(\left(c^2(9de-4cf)b^3+9acd(9de+cf)b^2-3a^2d^2(6de+7cf)b+8a^3d^3f\right)x^2+ac\left(c(81de-cf)b^2-ad(9de+11cf)b+4a^2d^2f\right)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(8a}{}}{7b}}{9b}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 403

$$\frac{\frac{3\left(\frac{\int \frac{\left(-2c^3(9de-4cf)b^4+5ac^2d(18de-5cf)b^3+18a^2cd^2(5de+cf)b^2-a^3d^3(18de+25cf)b+8a^4d^4f\right)x^2+ac\left(-c^2(9de-4cf)b^3+6acd(27de-2cf)b^2-3a^2d^2(3de+4c}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}\right.}{5d}}{}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 406

$$\frac{\frac{3\left(\frac{ac\left(4a^3d^3f-3a^2bd^2(4cf+3de)+6ab^2cd(27de-2cf)+b^3\left(-c^2\right)(9de-4cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^4d^4f-a^3bd^3(25cf+18de)+18a^2b^2cd^2(cf+5de)+5ab^3c^2}{3d}\right.}{5d}}{}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 320

$$\frac{3\left(\frac{\left(8a^4d^4f-a^3bd^3(25cf+18de)+18a^2b^2cd^2(cf+5de)+5ab^3c^2d(18de-5cf)-2b^4c^3(9de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^3d^3f-3a^2bd^2(4cf+3de\right.}{}}{3d}\right.}{}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 388

$$\frac{3\left(\frac{\left(8a^4d^4f-a^3bd^3(25cf+18de)+18a^2b^2cd^2(cf+5de)+5ab^3c^2d(18de-5cf)-2b^4c^3(9de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^3d^3f-3a^2\right.}{}}{3d}\right.}{}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 313

$$\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(17cf+18de)+3b^2c(cf+24de)\right)}{5d}+\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(7cf+6de)+9ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3d}+\right.}{}$$

$$\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{9b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(9*b) + (((9*b*d*e + 3*b*c*f - 4
*a*d*f)*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (((8*a^2*d^2*f + 3*b^
2*c*(24*d*e + c*f) - a*b*d*(18*d*e + 17*c*f))*x*(a + b*x^2)^(3/2)*Sqrt[c +
 d*x^2])/(5*d) + (3*(((8*a^3*d^3*f + b^3*c^2*(9*d*e - 4*c*f) + 9*a*b^2*c*d
*(9*d*e + c*f) - 3*a^2*b*d^2*(6*d*e + 7*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*d) + ((8*a^4*d^4*f + 5*a*b^3*c^2*d*(18*d*e - 5*c*f) - 2*b^4*c^3*
(9*d*e - 4*c*f) + 18*a^2*b^2*c*d^2*(5*d*e + c*f) - a^3*b*d^3*(18*d*e + 25*
c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*a^3*d^3*f
- b^3*c^2*(9*d*e - 4*c*f) + 6*a*b^2*c*d*(27*d*e - 2*c*f) - 3*a^2*b*d^2*(3*
d*e + 4*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]))/(3*d)))/(5*d))/(7*b))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 10.81 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.71

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1314
default	Expression too large to display	1846

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/315/b^2/d^2*x*(-35*b^3*d^3*f*x^6-50*a*b^2*d^3*f*x^4-50*b^3*c*d^2*f*x^4-
45*b^3*d^3*e*x^4-3*a^2*b*d^3*f*x^2-83*a*b^2*c*d^2*f*x^2-72*a*b^2*d^3*e*x^2
-3*b^3*c^2*d*f*x^2-72*b^3*c*d^2*e*x^2+4*a^3*d^3*f-12*a^2*b*c*d^2*f-9*a^2*b
*d^3*e-12*a*b^2*c^2*d*f-153*a*b^2*c*d^2*e+4*b^3*c^3*f-9*b^3*c^2*d*e)*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)+1/315/b^2/d^2*(-(8*a^4*d^4*f-25*a^3*b*c*d^3*f-1
8*a^3*b*d^4*e+18*a^2*b^2*c^2*d^2*f+90*a^2*b^2*c*d^3*e-25*a*b^3*c^3*d*f+90*
a*b^3*c^2*d^2*e+8*b^4*c^4*f-18*b^4*c^3*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2)))+4*a*b^3*c^4*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))+4*a^4*c*d^3*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-9*a*b^3*c^3*d*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))+162*a^2*b^2*c^2*d^2*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-12*a^2*b^2*c^3*d*f/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9*a^3*b*c*d^3*e/(-b/a)^(1/2)*(1...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} (e+fx^2) \, dx = \frac{\sqrt{bd}(18\,(b^4c^4d - 5\,ab^3c^3d^2 - 5\,a^2b^2c^2d^3 + a^3bcd^4)e - (8\,b^4c^5 - 25\,ab^3c^4d + 18\,a^2b^2}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/315*(sqrt(b*d)*(18*(b^4*c^4*d - 5*a*b^3*c^3*d^2 - 5*a^2*b^2*c^2*d^3 + a^
3*b*c*d^4)*e - (8*b^4*c^5 - 25*a*b^3*c^4*d + 18*a^2*b^2*c^3*d^2 - 25*a^3*b
*c^2*d^3 + 8*a^4*c*d^4)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a
*d/(b*c)) - sqrt(b*d)*(9*(2*b^4*c^4*d - 10*a*b^3*c^3*d^2 + a^3*b*d^5 - (10
*a^2*b^2 - a*b^3)*c^2*d^3 + 2*(a^3*b - 9*a^2*b^2)*c*d^4)*e - (8*b^4*c^5 -
25*a*b^3*c^4*d + 4*a^4*d^5 + 2*(9*a^2*b^2 + 2*a*b^3)*c^3*d^2 - (25*a^3*b +
 12*a^2*b^2)*c^2*d^3 + 4*(2*a^4 - 3*a^3*b)*c*d^4)*f)*x*sqrt(-c/d)*elliptic
_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (35*b^4*d^5*f*x^8 + 5*(9*b^4*d^5*e +
 10*(b^4*c*d^4 + a*b^3*d^5)*f)*x^6 + (72*(b^4*c*d^4 + a*b^3*d^5)*e + (3*b^
4*c^2*d^3 + 83*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*f)*x^4 + (9*(b^4*c^2*d^3 + 17*
a*b^3*c*d^4 + a^2*b^2*d^5)*e - 4*(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 - 3*a^2*b^
2*c*d^4 + a^3*b*d^5)*f)*x^2 - 18*(b^4*c^3*d^2 - 5*a*b^3*c^2*d^3 - 5*a^2*b^
2*c*d^4 + a^3*b*d^5)*e + (8*b^4*c^4*d - 25*a*b^3*c^3*d^2 + 18*a^2*b^2*c^2*
d^3 - 25*a^3*b*c*d^4 + 8*a^4*d^5)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3
*d^4*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)*(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Giac [F]

$$\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} (e+fx^2) \, dx = \int (bx^2+a)^{\frac{3}{2}} (dx^2+c)^{\frac{3}{2}} (fx^2+e) \, dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} (e+fx^2) \, dx = \int (b\,x^2+a)^{3/2} \, (d\,x^2+c)^{3/2} \, (f\,x^2+e) \, dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} (e+fx^2) \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f*x + 12*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*b*c*d**2*f*x + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a**2*b*d**3*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*x**3
+ 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f*x + 153*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*x + 83*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a*b**2*c*d**2*f*x**3 + 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d
**3*e*x**3 + 50*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f*x**5 - 4*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f*x + 9*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b**3*c**2*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2
*d*f*x**3 + 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*x**3 + 50*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f*x**5 + 45*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**3*d**3*e*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b
**3*d**3*f*x**7 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*
d*x**2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f - 25*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d*
*3*f - 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e + 18*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b**2*c**2*d**2
*f + 90*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c
*x**2 + b*d*x**4),x)*a**2*b**2*c*d**3*e - 25*int((sqrt(c + d*x**2)*sqrt...
```

3.9 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 519

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)\,dx = -\frac{(6a^3d^3f-ab^2cd(49de-19cf)+2b^3c^2(7de-4cf)-3a^2bd^2(7de+3cf))\,x\sqrt{c+dx^2}}{105bd^3\sqrt{a+bx^2}}$$
$$-\frac{\left(14bce-42ade+15acf-\frac{8bc^2f}{d}-\frac{3a^2df}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d}$$
$$+\frac{(7bde-4bcf+3adf)x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{35d^2}+\frac{fx(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{7d}$$
$$+\frac{\sqrt{a}(6a^3d^3f-ab^2cd(49de-19cf)+2b^3c^2(7de-4cf)-3a^2bd^2(7de+3cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{105b^{3/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(3a^2d^2f+b^2c(7de-4cf)-9abd(7de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/105*(6*a^3*d^3*f-a*b^2*c*d*(-19*c*f+49*d*e)+2*b^3*c^2*(-4*c*f+7*d*e)-3*
a^2*b*d^2*(3*c*f+7*d*e))*x*(d*x^2+c)^(1/2)/b/d^3/(b*x^2+a)^(1/2)-1/105*(14
*b*c*e-42*a*d*e+15*a*c*f-8*b*c^2*f/d-3*a^2*d*f/b)*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/d+1/35*(3*a*d*f-4*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/
2)/d^2+1/7*f*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/d+1/105*a^(1/2)*(6*a^3*d^3*
f-a*b^2*c*d*(-19*c*f+49*d*e)+2*b^3*c^2*(-4*c*f+7*d*e)-3*a^2*b*d^2*(3*c*f+7
*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/b^(3/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-
1/105*a^(3/2)*(3*a^2*d^2*f+b^2*c*(-4*c*f+7*d*e)-9*a*b*d*(-c*f+7*d*e))*(d*x
^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b
^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.71

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(3a^2d^2f+3abd(14de+3cf+8dfx^2)+b^2(-4c^2f+cd(7e+3fx^2)\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(3*a^2*d^2*f + 3*a*b*d*(14*d*e + 3*
c*f + 8*d*f*x^2) + b^2*(-4*c^2*f + c*d*(7*e + 3*f*x^2) + 3*d^2*x^2*(7*e +
5*f*x^2))) + I*c*(6*a^3*d^3*f + 2*b^3*c^2*(7*d*e - 4*c*f) - 3*a^2*b*d^2*(7
*d*e + 3*c*f) + a*b^2*c*d*(-49*d*e + 19*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) +
 a*d)*(3*a^2*d^2*f + 3*a*b*d*(14*d*e - 5*c*f) + 2*b^2*c*(-7*d*e + 4*c*f))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(105*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {403, 403, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left((7bde+bcf-2adf)x^2+c(7be-af)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(ac(28bde-bcf-3adf)-\left(-c(7de-4cf)b^2-3ad(7de+2cf)b+6a^2d^2f\right)x^2\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-2adf+bcf+7bde)}{5d}}{7b}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\frac{\int -\frac{\left(2c^2(7de-4cf)b^3-acd(49de-19cf)b^2-3a^2d^2(7de+3cf)b+6a^3d^3f\right)x^2+ac\left(c(7de-4cf)b^2-9ad(7de-cf)b+3a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(6a^2d^2f-3abd(}{3d}}{5d}}{7b}$$
$$\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{-\frac{\int \frac{\left(2c^2(7de-4cf)b^3-acd(49de-19cf)b^2-3a^2d^2(7de+3cf)b+6a^3d^3f\right)x^2+ac\left(c(7de-4cf)b^2-9ad(7de-cf)b+3a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(6a^2d^2f-3abd(}{3d}}{5d}}{7b}$$
$$\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{\frac{-ac\left(3a^2d^2f-9abd(7de-cf)+b^2c(7de-4cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(6a^3d^3f-3a^2bd^2(3cf+7de)-ab^2cd(49de-19cf)+2b^3c^2(7de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}}{7b}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\downarrow$ 320

$$\frac{\frac{-\frac{\left(6a^3d^3f-3a^2bd^2(3cf+7de)-ab^2cd(49de-19cf)+2b^3c^2(7de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-9abd(7de-cf)+b^2c(7de-4cf)\right)\text{EllipticF}\left(\ldots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}}{7b}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\downarrow$ 388

$$\frac{\frac{-\frac{\left(6a^3d^3f-3a^2bd^2(3cf+7de)-ab^2cd(49de-19cf)+2b^3c^2(7de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-9abd(7de-cf)+b^2c(7de-4cf)\right)\ldots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}}{7b}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

$\downarrow$ 313

$$\frac{-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(6a^2d^2f-3abd(2cf+7de)+b^2(-c)(7de-4cf)\right)}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-9abd(7de-cf)+b^2c(7de-4cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\ldots}{\ldots}}{5d}\ldots+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}$$

input
```
Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (((7*b*d*e + b*c*f - 2*a*d
*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) + (-1/3*((6*a^2*d^2*f - b^2
*c*(7*d*e - 4*c*f) - 3*a*b*d*(7*d*e + 2*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/d - ((6*a^3*d^3*f - a*b^2*c*d*(49*d*e - 19*c*f) + 2*b^3*c^2*(7*d*e
- 4*c*f) - 3*a^2*b*d^2*(7*d*e + 3*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d
*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])) + (c^(3/2)*(3*a^2*d^2*f + b^2*c*(7*d*e - 4*c*f) - 9*a*b*d*(7*d*e
 - c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(
3*d))/(5*d))/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 7.97 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.34

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{b f x^5 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{7}+\frac{\left(2 d f a b+b^2 c f+b^2 d e-\frac{b f (6 a d+6 b c)}{7}\right) x^3 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{5 b d}+\frac{\left(a^2 d f+\frac{9 a b c f}{7}+2 a b \cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{x(15 b^2 d^2 f x^4+24 a b d^2 f x^2+3 b^2 c f x^2 d+21 b^2 d^2 e x^2+3 f d^2 a^2+9 f d c b a+42 a b d^2 e-4 f c^2 b^2+7 d b^2 c e)\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{105 b d^2}-\frac{\left(-\frac{(6 f \cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*b*f*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2*d*f*a*b+b^2*c*f+b^2*d*e-1/7*b*f*
(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a^2*d*f+9/
7*a*b*c*f+2*a*b*d*e+c*e*b^2-1/5*(2*d*f*a*b+b^2*c*f+b^2*d*e-1/7*b*f*(6*a*d+
6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^2*
c*e-1/3*(a^2*d*f+9/7*a*b*c*f+2*a*b*d*e+c*e*b^2-1/5*(2*d*f*a*b+b^2*c*f+b^2*
d*e-1/7*b*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a^2*c*f+a^2*d*e+2*a*c*e*b-3/5*
(2*d*f*a*b+b^2*c*f+b^2*d*e-1/7*b*f*(6*a*d+6*b*c))/b/d*a*c-1/3*(a^2*d*f+9/7
*a*b*c*f+2*a*b*d*e+c*e*b^2-1/5*(2*d*f*a*b+b^2*c*f+b^2*d*e-1/7*b*f*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\frac{\sqrt{bd}(7\,(2\,b^3c^3d-7\,ab^2c^2d^2-3\,a^2bcd^3)e-(8\,b^3c^4-19\,ab^2c^3d+9\,a^2bc^2d^2-6\,a^3cd^3)f)x\sqrt{-\frac{c}{d}}E}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/105*(sqrt(b*d)*(7*(2*b^3*c^3*d - 7*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)*e - (8
*b^3*c^4 - 19*a*b^2*c^3*d + 9*a^2*b*c^2*d^2 - 6*a^3*c*d^3)*f)*x*sqrt(-c/d)
*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(2*b^3*c^3*d -
 7*a*b^2*c^2*d^2 - 9*a^2*b*d^4 - (3*a^2*b - a*b^2)*c*d^3)*e - (8*b^3*c^4 -
 19*a*b^2*c^3*d - 3*a^3*d^4 + (9*a^2*b + 4*a*b^2)*c^2*d^2 - 3*(2*a^3 + 3*a
^2*b)*c*d^3)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) +
 (15*b^3*d^4*f*x^6 + 3*(7*b^3*d^4*e + (b^3*c*d^3 + 8*a*b^2*d^4)*f)*x^4 + (
7*(b^3*c*d^3 + 6*a*b^2*d^4)*e - (4*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 3*a^2*b*d
^4)*f)*x^2 - 7*(2*b^3*c^2*d^2 - 7*a*b^2*c*d^3 - 3*a^2*b*d^4)*e + (8*b^3*c^
3*d - 19*a*b^2*c^2*d^2 + 9*a^2*b*c*d^3 - 6*a^3*d^4)*f)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c))/(b^2*d^4*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c} \left(fx^2+e\right) dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx = \int \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right) \, dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output

```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 9*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b*c*d*f*x + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2
*e*x + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 - 4*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*b**2*c*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3 + 21*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**2*d**2*f*x**5 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f + 9*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*a**2*b*c*d**2*f + 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e - 19*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b
**2*c**2*d*f + 49*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*
x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e + 8*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*
f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*
x**2 + b*d*x**4),x)*b**3*c**2*d*e - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f - 9*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*
*2*b*c**2*d*f + 63*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x...
```

3.10 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 393

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{(3a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf))\,x\sqrt{c+dx^2}}{15d^3\sqrt{a+bx^2}} + \frac{(5bde-4bcf+3adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{fx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} - \frac{\sqrt{a}(3a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15\sqrt{b}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(bc(5de-4cf)-3ad(5de-2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{b}cd^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(3*a^2*d^2*f+a*b*d*(-13*c*f+20*d*e)-2*b^2*c*(-4*c*f+5*d*e))*x*(d*x^2+
c)^(1/2)/d^3/(b*x^2+a)^(1/2)+1/15*(3*a*d*f-4*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/d^2+1/5*f*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-1/15*a^(
1/2)*(3*a^2*d^2*f+a*b*d*(-13*c*f+20*d*e)-2*b^2*c*(-4*c*f+5*d*e))*(d*x^2+c)
^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^
(1/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(b*
c*(-4*c*f+5*d*e)-3*a*d*(-2*c*f+5*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arc
tan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d^2/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.72 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(6adf+b(5de-4cf+3dfx^2)) - ic(3a^2d^2f+abd \cdots}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d*f + b*(5*d*e - 4*c*f + 3*d*f
*x^2)) - I*c*(3*a^2*d^2*f + a*b*d*(20*d*e - 13*c*f) + 2*b^2*c*(-5*d*e + 4*
c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*(2*b*c*(5*d*e - 4*c*f) + 3*a*d*(-5*d
*e + 3*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {403, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \frac{\sqrt{bx^2+a}((5bde-4bcf+3adf)x^2+a(5de-cf))}{\sqrt{dx^2+c}}dx}{5d}+\frac{fx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{a(bc(5de-4cf)-3ad(5de-2cf))-\left(-2c(5de-4cf)b^2+ad(20de-13cf)b+3a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d}}{5d}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

$\Big\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d}-\frac{\int \frac{a(bc(5de-4cf)-3ad(5de-2cf))-\left(-2c(5de-4cf)b^2+ad(20de-13cf)b+3a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

$\Big\downarrow$ 406

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d}-\frac{a(bc(5de-4cf)-3ad(5de-2cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(3a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf)\right)\int}{3d}}{5d}$$
$$\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

$\Big\downarrow$ 320

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc(5de-4cf)-3ad(5de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(3a^2d^2f+abd(20de-13cf)-2b^2\right.}{3d}}{5d}$$
$$\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

$\Big\downarrow$ 388

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc(5de-4cf)-3ad(5de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(3a^2d^2f+abd(20de-13cf)-2b^2\right.}{3d}}{5d}$$
$$\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

$\Big\downarrow$ 313

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3adf-4bcf+5bde)}{3d} - \frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc(5de-4cf)-3ad(5de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - (3a^2d^2f+abd(20de-13cf)-2b^2}{3d}}{5d}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) + (((5*b*d*e - 4*b*c*f + 3*a
*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (-((3*a^2*d^2*f + a*b*d*(
20*d*e - 13*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
 + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]))) + (Sqrt[c]*(b*c*(5*d*e - 4*c*f) - 3*a*d*(5*d*e - 2*c*f))*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 11.82 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)} \Bigg(\frac{f b x^3 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{5d} + \frac{\left(2afb+b^2 e-\frac{fb(4ad+4bc)}{5d}\right) x \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3bd} + \frac{\Bigg(a^2 e - \frac{\Big(2afb+b^2e-\frac{fb(4ad+}{5d}}{3bd}}{}}{}$
risch	$\frac{x(3bdf x^2+6adf-4bcf+5bde)\sqrt{b x^2+a}\sqrt{x^2 d+c}}{15d^2} - \frac{\Bigg(\frac{(3f d^2 a^2-13fdcba+20ab d^2 e+8f c^2 b^2-10d b^2 ce) c \sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}} \Big(\mathrm{EllipticF}\Big(}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+x^2 b c+}}}{}$
default	$-\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\Big(-3\sqrt{-\frac{b}{a}}\, b^2 d^3 f x^7-9\sqrt{-\frac{b}{a}}\, a b d^3 f x^5+\sqrt{-\frac{b}{a}}\, b^2 c d^2 f x^5-5\sqrt{-\frac{b}{a}}\, b^2 d^3 e x^5-6\sqrt{-\frac{b}{a}}\, a^2 d^3 f x^3-5\sqrt{-\frac{b}{a}}\, a b c d^2}{}$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f*b/d*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2*a*f*b+b^2*e-1/5*f*b/d*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^2*e-1/3*(2*a*f*b+b^2*e
-1/5*f*b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(a^2*f+2*a*e*b-3/5*a*b*c/d*f-1/3*(2*a*f*b+b^2*e-1/5
*f*b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bd}(10\,(b^2c^3d-2\,abc^2d^2)e-(8\,b^2c^4-13\,abc^3d+3\,a^2c^2d^2)f)x\sqrt{-\frac{c}{d}}E(\mathrm{arcs}}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(sqrt(b*d)*(10*(b^2*c^3*d - 2*a*b*c^2*d^2)*e - (8*b^2*c^4 - 13*a*b*c^
3*d + 3*a^2*c^2*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/
(b*c)) - sqrt(b*d)*(5*(2*b^2*c^3*d - 4*a*b*c^2*d^2 + a*b*c*d^3 - 3*a^2*d^4
)*e - (8*b^2*c^4 - 13*a*b*c^3*d - 6*a^2*c*d^3 + (3*a^2 + 4*a*b)*c^2*d^2)*f
)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*b^2*c*d^3*
f*x^4 + (5*b^2*c*d^3*e - 2*(2*b^2*c^2*d^2 - 3*a*b*c*d^3)*f)*x^2 - 10*(b^2*
c^2*d^2 - 2*a*b*c*d^3)*e + (8*b^2*c^3*d - 13*a*b*c^2*d^2 + 3*a^2*c*d^3)*f)
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{6\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx - 4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx + 5\sqrt{d\,x^2+c}\,\sqrt{b\,x}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output
```
(6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f - 13*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*a*b*c*d*f + 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e + 8*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*
*2*f - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*b**2*c*d*e - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f + 15*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2
*d**2*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*
x**2 + b*d*x**4),x)*a*b*c**2*f - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d*e)/(15*d**2)
```

3.11 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 294

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{(3bde-4bcf+3adf)x\sqrt{a+bx^2}}{3d^2\sqrt{c+dx^2}}+\frac{fx(a+bx^2)^{3/2}}{3d\sqrt{c+dx^2}} + \frac{(ad(3de-7cf)-2bc(3de-4cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(3bde-4bcf+3adf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
1/3*(3*a*d*f-4*b*c*f+3*b*d*e)*x*(b*x^2+a)^(1/2)/d^2/(d*x^2+c)^(1/2)+1/3*f*
x*(b*x^2+a)^(3/2)/d/(d*x^2+c)^(1/2)+1/3*(a*d*(-7*c*f+3*d*e)-2*b*c*(-4*c*f+
3*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+1/3*c^(1/2)*(3*a*d*f-4*b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM
(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(5/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.59 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3ad(de-cf)+bc(-3de+4cf+dfx^2)\right)-ibc(ad(-3de+$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*d*e + 4*c*f + d*f*
x^2)) - I*b*c*(a*d*(-3*d*e + 7*c*f) + b*(6*c*d*e - 8*c^2*f))*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- I*c*(-(b*c) + a*d)*(6*b*d*e - 8*b*c*f + 3*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b
/a]*c*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {401, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int -\frac{\sqrt{bx^2+a}\left(acf-b(3de-4cf)x^2\right)}{\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(acf-b(3de-4cf)x^2)}{\sqrt{dx^2+c}}dx}{cd}+\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 403

$$\frac{\frac{\int \frac{ac(3bde-4bcf+3adf)-b(ad(3de-7cf)-2bc(3de-4cf))x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}}{cd}+\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 406

$$\frac{\frac{ac(3adf-4bcf+3bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-b(ad(3de-7cf)-2bc(3de-4cf))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}}{cd}+\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 320

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf-4bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b(ad(3de-7cf)-2bc(3de-4cf))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-}{3d}}{cd}$$

$$\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 388

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf-4bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b(ad(3de-7cf)-2bc(3de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)}{3d}-bx\sqrt{a+bx^2}\sqrt{}}{cd}$$

$$\frac{x(a+bx^2)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf-4bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(ad(3de-7cf)-2bc(3de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}}{cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) + (-1/3*(b*(3*d*e
- 4*c*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d + (-(b*(a*d*(3*d*e - 7*c*f)
- 2*b*c*(3*d*e - 4*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c
^(3/2)*(3*b*d*e - 4*b*c*f + 3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(265) = 530$.

Time = 11.56 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.85

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{\left(b d\,x^2+a d\right)\left(a c d f-a\,d^2 e-b\,c^2 f+b c d e\right)x}{c\,d^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(b d\,x^2+a d\right)}}+\frac{f b x\sqrt{b d\,x^4+a d\,x^2+x^2 b c+a c}}{3 d^2}+\frac{\left(\frac{f\,d^2 a^2-2 f d c b a+2 a b\,d^2 e+f\,c^2 b^2-d\,b^2 c e}{d^3}\right.}{}\right.}{}$
risch	$\frac{f x\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,b}{3 d^2}+\frac{\left(\frac{\left(3 f\,d^2 a^2-7 f d c b a+6 a b\,d^2 e+3 f\,c^2 b^2-3 d\,b^2 c e\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{d\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^4+a d\,x^2+x^2 b c+a c}}-\frac{b(4 a d f-5}{}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\left(\sqrt{-\frac{b}{a}}\,b^2 c\,d^2 f\,x^5-2\sqrt{-\frac{b}{a}}\,a b c\,d^2 f\,x^3+3\sqrt{-\frac{b}{a}}\,a b\,d^3 e\,x^3+4\sqrt{-\frac{b}{a}}\,b^2 c^2 d f\,x^3-3\sqrt{-\frac{b}{a}}\,b^2 c\,d^2 e\,x^3+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2}{}}\right.}{}$

input

```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2
)+1/3*f*b/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+((a^2*d^2*f-2*a*b*c*d*
f+2*a*b*d^2*e+b^2*c^2*f-b^2*c*d*e)/d^3-(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d
^3*(a*d-b*c)/c+a/d^2*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c-1/3*a*b*c/d^2*f)/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b/d^2*(2*a*
d*f-b*c*f+b*d*e)+(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^2*b/c-1/3*f*b/d^2*(2*
a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((3\,(2\,b^2c^2d^2-abcd^3)e-(8\,b^2c^3d-7\,abc^2d^2)f)x^3+(3\,(2\,b^2c^3d-abc^2d^2)e-(8\,b^2c^4-7\,abc^3d)f)x)\sqrt{bd}\sqrt{}}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((3*(2*b^2*c^2*d^2 - a*b*c*d^3)*e - (8*b^2*c^3*d - 7*a*b*c^2*d^2)*f)
*x^3 + (3*(2*b^2*c^3*d - a*b*c^2*d^2)*e - (8*b^2*c^4 - 7*a*b*c^3*d)*f)*x)*
sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((3*(2*
b^2*c^2*d^2 - a*b*c*d^3 + a*b*d^4)*e - (8*b^2*c^3*d - 7*a*b*c^2*d^2 + 4*a*
b*c*d^3 - 3*a^2*d^4)*f)*x^3 + (3*(2*b^2*c^3*d - a*b*c^2*d^2 + a*b*c*d^3)*e
 - (8*b^2*c^4 - 7*a*b*c^3*d + 4*a*b*c^2*d^2 - 3*a^2*c*d^3)*f)*x)*sqrt(b*d)
*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b^2*c*d^3*f*x^4
 + (3*b^2*c*d^3*e - 4*(b^2*c^2*d^2 - a*b*c*d^3)*f)*x^2 + 3*(2*b^2*c^2*d^2
- a*b*c*d^3)*e - (8*b^2*c^3*d - 7*a*b*c^2*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c))/(b*c*d^5*x^3 + b*c^2*d^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x - 3*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*b*c*f*x + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x + 2*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*f*x**3 - 3*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2
+ 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c*d**2*f - 3*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2
 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*d**3*f*x**2 + 11*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2
*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c**2*d*f - 6*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**
2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c*d**2*e + 11*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c
**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c*d**2*f*x**2 - 6*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*d**3*e*x**2 - 8*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*
x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**3*c**3*f + 6*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**
4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**3*c**2*d*e - 8*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x...
```

3.12 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 291

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}} dx = \frac{(de-cf)x(a+bx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} - \frac{b(de-4cf)x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} + \frac{(2bc(de-4cf)+ad(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{b(de-4cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
1/3*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(3/2)-1/3*b*(-4*c*f+d*e)*x*
(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(1/2)+1/3*(2*b*c*(-4*c*f+d*e)+a*d*(c*f+2*d
*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/
a/d)^(1/2))/c^(3/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2
)-1/3*b*(-4*c*f+d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1
/2)),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d
*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad^2(3ce+2dex^2+cfx^2)+bc(-4c^2f+2d^2ex^2+cd(e-5\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d^2*(3*c*e + 2*d*e*x^2 + c*f*x^2) + b*c*(-4*
c^2*f + 2*d^2*e*x^2 + c*d*(e - 5*f*x^2))) + I*b*c*(2*b*c*(d*e - 4*c*f) + a
*d*(2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Elli
pticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b*c*(d*e - 4*c*f) +
a*d*(d*e + 5*c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Ell
ipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d^3*Sqrt[a +
 b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {401, 25, 401, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(a+bx^2)^{3/2}(de-cf)}{3cd(c+dx^2)^{3/2}} - \frac{\int -\frac{\sqrt{bx^2+a}\left(a(2de+cf)-b(de-4cf)x^2\right)}{(dx^2+c)^{3/2}}dx}{3cd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(a(2de+cf)-b(de-4cf)x^2\right)}{(dx^2+c)^{3/2}}dx}{3cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\big\downarrow$ 401

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{\int \frac{b\left((2bc(de-4cf)+ad(2de+cf))x^2+ac(de-4cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{b\int \frac{(2bc(de-4cf)+ad(2de+cf))x^2+ac(de-4cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\big\downarrow$ 406

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{b\left(ac(de-4cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(ad(cf+2de)+2bc(de-4cf))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\big\downarrow$ 320

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{b\left((ad(cf+2de)+2bc(de-4cf))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(de-4cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\big\downarrow$ 388

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{b\left((ad(cf+2de)+2bc(de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(de-4cf)\,\text{EllipticF}\left(\arctan\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{cd}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+2de)+bc(de-4cf))}{cd\sqrt{c+dx^2}}-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-4cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(ad(cf+2de)+2bc(de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{cd}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(3/2))/(3*c*d*(c + d*x^2)^(3/2)) + (((b*c*(d*e
- 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (
b*((2*b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[
c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2])) + (c^(3/2)*(d*e - 4*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])))/(c*d))/(3*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(262) = 524$.

Time = 7.74 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(acdf-a\,d^2e-b\,c^2f+bcde\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)\left(acdf+2a\,d^2e-5b\,c^2f+2bcde\right)x}{3c^2d^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{b(2adf-2bcf+}{d^3}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2
+c/d)^2+1/3*(b*d*x^2+a*d)*(a*c*d*f+2*a*d^2*e-5*b*c^2*f+2*b*c*d*e)/c^2/d^3*
x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(b*(2*a*d*f-2*b*c*f+b*d*e)/d^3-1/3*(a*c*
d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3*b/c+1/3*(a*c*d*f+2*a*d^2*e-5*b*c^2*f+2*b*
c*d*e)/d^3*(a*d-b*c)/c^2-1/3*a/d^2*(a*c*d*f+2*a*d^2*e-5*b*c^2*f+2*b*c*d*e)
/c^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b^2*f
/d^2-1/3*(a*c*d*f+2*a*d^2*e-5*b*c^2*f+2*b*c*d*e)/d^2*b/c^2)*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(262) = 524$.

Time = 0.11 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \frac{((2\,(bc^2d^3+acd^4)e-(8\,bc^3d^2-ac^2d^3)f)x^5+2\,(2\,(bc^3d^2+ac^2d^3)e-(8\,bc^4}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((2*(b*c^2*d^3 + a*c*d^4)*e - (8*b*c^3*d^2 - a*c^2*d^3)*f)*x^5 + 2*(2
*(b*c^3*d^2 + a*c^2*d^3)*e - (8*b*c^4*d - a*c^3*d^2)*f)*x^3 + (2*(b*c^4*d
+ a*c^3*d^2)*e - (8*b*c^5 - a*c^4*d)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((2*b*c^2*d^3 + 2*a*c*d^4 + a*d^5)*e
- (8*b*c^3*d^2 - a*c^2*d^3 + 4*a*c*d^4)*f)*x^5 + 2*((2*b*c^3*d^2 + 2*a*c^2
*d^3 + a*c*d^4)*e - (8*b*c^4*d - a*c^3*d^2 + 4*a*c^2*d^3)*f)*x^3 + ((2*b*c
^4*d + 2*a*c^3*d^2 + a*c^2*d^3)*e - (8*b*c^5 - a*c^4*d + 4*a*c^3*d^2)*f)*x
)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*b*
c^2*d^3*f*x^4 - ((3*b*c^2*d^3 + a*c*d^4)*e - 2*(6*b*c^3*d^2 - a*c^2*d^3)*f
)*x^2 - 2*(b*c^3*d^2 + a*c^2*d^3)*e + (8*b*c^4*d - a*c^3*d^2)*f)*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c))/(c^2*d^6*x^5 + 2*c^3*d^5*x^3 + c^4*d^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(e+fx^2)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x + 3*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b*c*f*x - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x + 2
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**3 - 2*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b**2*c*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**
6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**
2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**
8),x)*a**3*b*c**2*d**3*f + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6
 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2
*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8
),x)*a**3*b*c*d**4*f*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**
6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**
2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**
8),x)*a**3*b*d**5*f*x**4 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**
6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**
2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**
8),x)*a**2*b**2*c**3*d**2*f - 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)...
```

3.13 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 365

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \frac{(de-cf)x(a+bx^2)^{3/2}}{5cd(c+dx^2)^{5/2}} + \frac{(ad(4de+cf)-bc(de+4cf))x\sqrt{a+bx^2}}{15c^2d^2(c+dx^2)^{3/2}} + \frac{(3abcd(de-cf)-2a^2d^2(4de+cf)+2b^2c^2(de+4cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}d^{5/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b(ad(4de+cf)-bc(de+4cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{5/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
1/5*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(5/2)+1/15*(a*d*(c*f+4*d*e)
-b*c*(4*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/d^2/(d*x^2+c)^(3/2)+1/15*(3*a*b*c*
d*(-c*f+d*e)-2*a^2*d^2*(c*f+4*d*e)+2*b^2*c^2*(4*c*f+d*e))*(b*x^2+a)^(1/2)*
EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(5/2)/d
^(5/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/15*b*(
a*d*(c*f+4*d*e)-b*c*(4*c*f+d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^
(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(5/2)/(-a*d+b*c)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.64 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)-c(bc-ad)(bc(2de-7cf)+ad\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f) - c*(b*c - a
*d)*(b*c*(2*d*e - 7*c*f) + a*d*(4*d*e + c*f))*(c + d*x^2) - (3*a*b*c*d*(d*
e - c*f) - 2*a^2*d^2*(4*d*e + c*f) + 2*b^2*c^2*(d*e + 4*c*f))*(c + d*x^2)^
2)) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((3*a*b*
c*d*(d*e - c*f) - 2*a^2*d^2*(4*d*e + c*f) + 2*b^2*c^2*(d*e + 4*c*f))*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(a*d*(4*d*e + c*f)
 + 2*b*c*(d*e + 4*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(
15*Sqrt[b/a]*c^3*d^3*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {401, 25, 401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(a+bx^2)^{3/2}(de-cf)}{5cd(c+dx^2)^{5/2}} - \frac{\int -\frac{\sqrt{bx^2+a}(b(de+4cf)x^2+a(4de+cf))}{(dx^2+c)^{5/2}}dx}{5cd}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(b(de+4cf)x^2+a(4de+cf)\right)}{(dx^2+c)^{5/2}}dx}{5cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \ 401$$

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+4de)-bc(4cf+de))}{3cd(c+dx^2)^{3/2}}-\frac{\int -\frac{b(ad(4de+cf)+2bc(de+4cf))x^2+a(2ad(4de+cf)+bc(de+4cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}}{5cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{b(ad(4de+cf)+2bc(de+4cf))x^2+a(2ad(4de+cf)+bc(de+4cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(ad(cf+4de)-bc(4cf+de))}{3cd(c+dx^2)^{3/2}}}{5cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \ 400$$

$$\frac{\frac{\frac{\left(-2a^2d^2(cf+4de)+3abcd(de-cf)+2b^2c^2(4cf+de)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{ab(ad(cf+4de)-bc(4cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3cd}+\frac{x\sqrt{a+bx^2}(ad(cf+4de)-bc}{3cd(c+dx^2)^{3/2}}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \ 313$$

$$\frac{\frac{\frac{ab(ad(cf+4de)-bc(4cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{a+bx^2}\left(-2a^2d^2(cf+4de)+3abcd(de-cf)+2b^2c^2(4cf+de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3cd}+\frac{x\sqrt{a+bx^2}(a}{3}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \ 320$$

$$\frac{\frac{\frac{\sqrt{a+bx^2}\left(-2a^2d^2(cf+4de)+3abcd(de-cf)+2b^2c^2(4cf+de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(cf+4de)-bc(4cf+de))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2)) + (((a*d*(4*d*
e + c*f) - b*c*(d*e + 4*c*f))*x*Sqrt[a + b*x^2])/(3*c*d*(c + d*x^2)^(3/2))
 + (((3*a*b*c*d*(d*e - c*f) - 2*a^2*d^2*(4*d*e + c*f) + 2*b^2*c^2*(d*e + 4
*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2]) + (b*Sqrt[c]*(a*d*(4*d*e + c*f) - b*c*(d*e + 4*c*f))*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*
c*d))/(5*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(336) = 672$.

Time = 8.24 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.05

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(acdf-a d^2 e-b c^2 f+bcde\right)x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{5c d^5\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(acdf+4a d^2 e-7b c^2 f+2bcde\right)x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{15c^2 d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{(bc}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^5*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2
+c/d)^3+1/15*(a*c*d*f+4*a*d^2*e-7*b*c^2*f+2*b*c*d*e)/c^2/d^4*x*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d^3/(a*d-b*c)*
x*(2*a^2*c*d^2*f+8*a^2*d^3*e+3*a*b*c^2*d*f-3*a*b*c*d^2*e-8*b^2*c^3*f-2*b^2
*c^2*d*e)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(b^2*f/d^3+1/15*b*(a*c*d*f+4*a*d
^2*e-7*b*c^2*f+2*b*c*d*e)/d^3/c^2+1/15/d^3*(2*a^2*c*d^2*f+8*a^2*d^3*e+3*a*
b*c^2*d*f-3*a*b*c*d^2*e-8*b^2*c^3*f-2*b^2*c^2*d*e)/c^3-1/15*a/d^2/c^3/(a*d
-b*c)*(2*a^2*c*d^2*f+8*a^2*d^3*e+3*a*b*c^2*d*f-3*a*b*c*d^2*e-8*b^2*c^3*f-2
*b^2*c^2*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))+1/15/d^3*b*(2*a^2*c*d^2*f+8*a^2*d^3*e+3*a*b*c^2*d*f-3*a*b*c*d^2*e-8*b^2
*c^3*f-2*b^2*c^2*d*e)/(a*d-b*c)/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(336) = 672$.

Time = 0.14 (sec) , antiderivative size = 1130, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*((((2*b^3*c^2*d^4 + 3*a*b^2*c*d^5 - 8*a^2*b*d^6)*e + (8*b^3*c^3*d^3
- 3*a*b^2*c^2*d^4 - 2*a^2*b*c*d^5)*f)*x^6 + 3*((2*b^3*c^3*d^3 + 3*a*b^2*c^
2*d^4 - 8*a^2*b*c*d^5)*e + (8*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 2*a^2*b*c^2*
d^4)*f)*x^4 + 3*((2*b^3*c^4*d^2 + 3*a*b^2*c^3*d^3 - 8*a^2*b*c^2*d^4)*e + (
8*b^3*c^5*d - 3*a*b^2*c^4*d^2 - 2*a^2*b*c^3*d^3)*f)*x^2 + (2*b^3*c^5*d + 3
*a*b^2*c^4*d^2 - 8*a^2*b*c^3*d^3)*e + (8*b^3*c^6 - 3*a*b^2*c^5*d - 2*a^2*b
*c^4*d^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - (((2*b^3*c^2*d^4 + (a^2*b + 3*a*b^2)*c*d^5 - 4*(a^3 + 2*a^2*b)*d^6)*
e + (8*b^3*c^3*d^3 + (4*a^2*b - 3*a*b^2)*c^2*d^4 - (a^3 + 2*a^2*b)*c*d^5)*
f)*x^6 + 3*((2*b^3*c^3*d^3 + (a^2*b + 3*a*b^2)*c^2*d^4 - 4*(a^3 + 2*a^2*b)
*c*d^5)*e + (8*b^3*c^4*d^2 + (4*a^2*b - 3*a*b^2)*c^3*d^3 - (a^3 + 2*a^2*b)
*c^2*d^4)*f)*x^4 + 3*((2*b^3*c^4*d^2 + (a^2*b + 3*a*b^2)*c^3*d^3 - 4*(a^3
+ 2*a^2*b)*c^2*d^4)*e + (8*b^3*c^5*d + (4*a^2*b - 3*a*b^2)*c^4*d^2 - (a^3
+ 2*a^2*b)*c^3*d^3)*f)*x^2 + (2*b^3*c^5*d + (a^2*b + 3*a*b^2)*c^4*d^2 - 4*
(a^3 + 2*a^2*b)*c^3*d^3)*e + (8*b^3*c^6 + (4*a^2*b - 3*a*b^2)*c^5*d - (a^3
 + 2*a^2*b)*c^4*d^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a
)), a*d/(b*c)) - (((2*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - 8*a^3*d^6)*e + (8*a*
b^2*c^3*d^3 - 3*a^2*b*c^2*d^4 - 2*a^3*c*d^5)*f)*x^5 + (2*(3*a*b^2*c^3*d^3
+ 4*a^2*b*c^2*d^4 - 10*a^3*c*d^5)*e + (9*a*b^2*c^4*d^2 + 2*a^2*b*c^3*d^3 -
 5*a^3*c^2*d^4)*f)*x^3 + ((a*b^2*c^4*d^2 + 11*a^2*b*c^3*d^3 - 15*a^3*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input

```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x - 3*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b*c*f*x - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x - 4
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**3 + 2*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b**2*c*f*x**3 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*
c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3
*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 -
 b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2
*d**3*x**8 - b**2*c*d**4*x**10),x)*a**3*b*c**3*d**3*f - 18*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*
a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2
*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d
**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2
*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**3*b*c**
2*d**4*f*x**2 - 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c*
*4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6
 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4
+ 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*
x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8
- b**2*c*d**4*x**10),x)*a**3*b*c*d**5*f*x**4 - 6*int((sqrt(c + d*x**2)*...
```

3.14 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 505

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx &= \frac{(de-cf)x(a+bx^2)^{3/2}}{7cd(c+dx^2)^{7/2}} \\
&+ \frac{(ad(6de+cf)-bc(3de+4cf))x\sqrt{a+bx^2}}{35c^2d^2(c+dx^2)^{5/2}} \\
&+ \frac{(abcd(15de-cf)-4a^2d^2(6de+cf)+2b^2c^2(3de+4cf))\,x\sqrt{a+bx^2}}{105c^3d^2(bc-ad)(c+dx^2)^{3/2}} \\
&+ \frac{(ab^2c^2d(12de-5cf)+8a^3d^3(6de+cf)+2b^3c^3(3de+4cf)-a^2bcd^2(72de+5cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\right.\right.}{105c^{7/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&- \frac{b(4a^2d^2(6de+cf)-abcd(33de+2cf)+b^2c^2(3de+4cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{5/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

output
```
1/7*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(7/2)+1/35*(a*d*(c*f+6*d*e)
-b*c*(4*c*f+3*d*e))*x*(b*x^2+a)^(1/2)/c^2/d^2/(d*x^2+c)^(5/2)+1/105*(a*b*c
*d*(-c*f+15*d*e)-4*a^2*d^2*(c*f+6*d*e)+2*b^2*c^2*(4*c*f+3*d*e))*x*(b*x^2+a
)^(1/2)/c^3/d^2/(-a*d+b*c)/(d*x^2+c)^(3/2)+1/105*(a*b^2*c^2*d*(-5*c*f+12*d
*e)+8*a^3*d^3*(c*f+6*d*e)+2*b^3*c^3*(4*c*f+3*d*e)-a^2*b*c*d^2*(5*c*f+72*d*
e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))/c^(7/2)/d^(5/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d
*x^2+c)^(1/2)-1/105*b*(4*a^2*d^2*(c*f+6*d*e)-a*b*c*d*(2*c*f+33*d*e)+b^2*c^
2*(4*c*f+3*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2))
,(1-b*c/a/d)^(1/2))/c^(5/2)/d^(5/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)-3c^2(bc-ad)^2(bc(2de-9cf)\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f) - 3*c^2*(b*
c - a*d)^2*(b*c*(2*d*e - 9*c*f) + a*d*(6*d*e + c*f))*(c + d*x^2) - c*(b*c
- a*d)*(a*b*c*d*(15*d*e - c*f) - 4*a^2*d^2*(6*d*e + c*f) + 2*b^2*c^2*(3*d*
e + 4*c*f))*(c + d*x^2)^2 - (a*b^2*c^2*d*(12*d*e - 5*c*f) + 8*a^3*d^3*(6*d
*e + c*f) + 2*b^3*c^3*(3*d*e + 4*c*f) - a^2*b*c*d^2*(72*d*e + 5*c*f))*(c +
 d*x^2)^3)) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*
((a*b^2*c^2*d*(12*d*e - 5*c*f) + 8*a^3*d^3*(6*d*e + c*f) + 2*b^3*c^3*(3*d*
e + 4*c*f) - a^2*b*c*d^2*(72*d*e + 5*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - (b*c - a*d)*(a*b*c*d*(15*d*e - c*f) - 4*a^2*d^2*(6*d*e +
 c*f) + 2*b^2*c^2*(3*d*e + 4*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)]))/(105*Sqrt[b/a]*c^4*d^3*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)
^(7/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 401, 25, 402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{9/2}}dx$$

$$\Big\downarrow \ 401$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}-\frac{\int -\frac{\sqrt{bx^2+a}\left(b(3de+4cf)x^2+a(6de+cf)\right)}{\left(dx^2+c\right)^{7/2}}dx}{7cd}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(b(3de+4cf)x^2+a(6de+cf)\right)}{\left(dx^2+c\right)^{7/2}}dx}{7cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \ 401$$

$$\frac{\frac{x\sqrt{a+bx^2}(ad(cf+6de)-bc(4cf+3de))}{5cd\left(c+dx^2\right)^{5/2}}-\frac{\int -\frac{b(3ad(6de+cf)+2bc(3de+4cf))x^2+a(4ad(6de+cf)+bc(3de+4cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}}{7cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{b(3ad(6de+cf)+2bc(3de+4cf))x^2+a(4ad(6de+cf)+bc(3de+4cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}+\frac{x\sqrt{a+bx^2}(ad(cf+6de)-bc(4cf+3de))}{5cd\left(c+dx^2\right)^{5/2}}}{7cd}+\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\int -\frac{a\left(-b^2(3de+4cf)c^2-abd(48de+cf)c+8a^2d^2(6de+cf)\right)-b\left(2b^2(3de+4cf)c^2+abd(15de-cf)c-4a^2d^2(6de+cf)\right)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+abcd(15\ldots\right.}{3c\left(c+dx^2\right)^{3/2}(b\ldots}}{5cd}}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+abcd(15de-cf)+2b^2c^2(4cf+3de)\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\int \frac{a\left(-b^2(3de+4cf)c^2-abd(48de+cf)c+8a^2d^2(6de+cf)\right)-b\left(2b^2(3de+4cf)c^2+abd(15de-cf)\ldots\right.}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}}{3c(bc-ad)}}{5cd}}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\Big\downarrow$ 400

$$\frac{\frac{x\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+abcd(15de-cf)+2b^2c^2(4cf+3de)\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{ab\left(4a^2d^2(cf+6de)-abcd(2cf+33de)+b^2c^2(4cf+3de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\left(8a^3d^3(cf+\ldots\right.}{\ldots}}{3c(bc-ad)\ldots}}{5cd}}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\Big\downarrow$ 313

$$\frac{\frac{x\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+abcd(15de-cf)+2b^2c^2(4cf+3de)\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{ab\left(4a^2d^2(cf+6de)-abcd(2cf+33de)+b^2c^2(4cf+3de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\left(8a\ldots\right.}{\ldots}}{3c\ldots}}{5cd}}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$\Big\downarrow$ 320

$$\frac{\frac{x\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+abcd(15de-cf)+2b^2c^2(4cf+3de)\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(cf+6de)-abcd(2cf+33de)+b^2c^2(4cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7/2)) + (((a*d*(6*d*
e + c*f) - b*c*(3*d*e + 4*c*f))*x*Sqrt[a + b*x^2])/(5*c*d*(c + d*x^2)^(5/2
)) + (((a*b*c*d*(15*d*e - c*f) - 4*a^2*d^2*(6*d*e + c*f) + 2*b^2*c^2*(3*d*
e + 4*c*f))*x*Sqrt[a + b*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (-(((
a*b^2*c^2*d*(12*d*e - 5*c*f) + 8*a^3*d^3*(6*d*e + c*f) + 2*b^3*c^3*(3*d*e
+ 4*c*f) - a^2*b*c*d^2*(72*d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(4*a^2*d^2
*(6*d*e + c*f) - a*b*c*d*(33*d*e + 2*c*f) + b^2*c^2*(3*d*e + 4*c*f))*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*
c*(b*c - a*d)))/(5*c*d))/(7*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1020 vs. 2(470) = 940.

Time = 8.25 (sec) , antiderivative size = 1021, normalized size of antiderivative = 2.02

method	result	size
elliptic	Expression too large to display	1021
default	Expression too large to display	5139

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*(a*c*d*f
-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^6*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2
+c/d)^4+1/35*(a*c*d*f+6*a*d^2*e-9*b*c^2*f+2*b*c*d*e)/c^2/d^5*x*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/105*(4*a^2*c*d^2*f+24*a^2*d^3*e+a*b
*c^2*d*f-15*a*b*c*d^2*e-8*b^2*c^3*f-6*b^2*c^2*d*e)/d^4/(a*d-b*c)/c^3*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/105*(b*d*x^2+a*d)/c^4/d^3/(
a*d-b*c)^2*x*(8*a^3*c*d^3*f+48*a^3*d^4*e-5*a^2*b*c^2*d^2*f-72*a^2*b*c*d^3*
e-5*a*b^2*c^3*d*f+12*a*b^2*c^2*d^2*e+8*b^3*c^4*f+6*b^3*c^3*d*e)/((x^2+c/d)
*(b*d*x^2+a*d))^(1/2)+(1/105*b*(4*a^2*c*d^2*f+24*a^2*d^3*e+a*b*c^2*d*f-15*
a*b*c*d^2*e-8*b^2*c^3*f-6*b^2*c^2*d*e)/d^3/(a*d-b*c)/c^3+1/105/d^3/(a*d-b*
c)*(8*a^3*c*d^3*f+48*a^3*d^4*e-5*a^2*b*c^2*d^2*f-72*a^2*b*c*d^3*e-5*a*b^2*
c^3*d*f+12*a*b^2*c^2*d^2*e+8*b^3*c^4*f+6*b^3*c^3*d*e)/c^4-1/105*a/d^2/c^4/
(a*d-b*c)^2*(8*a^3*c*d^3*f+48*a^3*d^4*e-5*a^2*b*c^2*d^2*f-72*a^2*b*c*d^3*e
-5*a*b^2*c^3*d*f+12*a*b^2*c^2*d^2*e+8*b^3*c^4*f+6*b^3*c^3*d*e))/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/105/d^3*b*(8*a^3*c*d
^3*f+48*a^3*d^4*e-5*a^2*b*c^2*d^2*f-72*a^2*b*c*d^3*e-5*a*b^2*c^3*d*f+12*a*
b^2*c^2*d^2*e+8*b^3*c^4*f+6*b^3*c^3*d*e)/(a*d-b*c)^2/c^3/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2004 vs. $2(470) = 940$.

Time = 0.16 (sec) , antiderivative size = 2004, normalized size of antiderivative = 3.97

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(((6*(b^4*c^3*d^5 + 2*a*b^3*c^2*d^6 - 12*a^2*b^2*c*d^7 + 8*a^3*b*d^
8)*e + (8*b^4*c^4*d^4 - 5*a*b^3*c^3*d^5 - 5*a^2*b^2*c^2*d^6 + 8*a^3*b*c*d^
7)*f)*x^8 + 4*(6*(b^4*c^4*d^4 + 2*a*b^3*c^3*d^5 - 12*a^2*b^2*c^2*d^6 + 8*a
^3*b*c*d^7)*e + (8*b^4*c^5*d^3 - 5*a*b^3*c^4*d^4 - 5*a^2*b^2*c^3*d^5 + 8*a
^3*b*c^2*d^6)*f)*x^6 + 6*(6*(b^4*c^5*d^3 + 2*a*b^3*c^4*d^4 - 12*a^2*b^2*c^
3*d^5 + 8*a^3*b*c^2*d^6)*e + (8*b^4*c^6*d^2 - 5*a*b^3*c^5*d^3 - 5*a^2*b^2*
c^4*d^4 + 8*a^3*b*c^3*d^5)*f)*x^4 + 4*(6*(b^4*c^6*d^2 + 2*a*b^3*c^5*d^3 -
12*a^2*b^2*c^4*d^4 + 8*a^3*b*c^3*d^5)*e + (8*b^4*c^7*d - 5*a*b^3*c^6*d^2 -
 5*a^2*b^2*c^5*d^3 + 8*a^3*b*c^4*d^4)*f)*x^2 + 6*(b^4*c^7*d + 2*a*b^3*c^6*
d^2 - 12*a^2*b^2*c^5*d^3 + 8*a^3*b*c^4*d^4)*e + (8*b^4*c^8 - 5*a*b^3*c^7*d
 - 5*a^2*b^2*c^6*d^2 + 8*a^3*b*c^5*d^3)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e
(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((3*(2*b^4*c^3*d^5 + (a^2*b^2 + 4*a*b^
3)*c^2*d^6 - (11*a^3*b + 24*a^2*b^2)*c*d^7 + 8*(a^4 + 2*a^3*b)*d^8)*e + (8
*b^4*c^4*d^4 + (4*a^2*b^2 - 5*a*b^3)*c^3*d^5 - (2*a^3*b + 5*a^2*b^2)*c^2*d
^6 + 4*(a^4 + 2*a^3*b)*c*d^7)*f)*x^8 + 4*(3*(2*b^4*c^4*d^4 + (a^2*b^2 + 4*
a*b^3)*c^3*d^5 - (11*a^3*b + 24*a^2*b^2)*c^2*d^6 + 8*(a^4 + 2*a^3*b)*c*d^7
)*e + (8*b^4*c^5*d^3 + (4*a^2*b^2 - 5*a*b^3)*c^4*d^4 - (2*a^3*b + 5*a^2*b^
2)*c^3*d^5 + 4*(a^4 + 2*a^3*b)*c^2*d^6)*f)*x^6 + 6*(3*(2*b^4*c^5*d^3 + (a^
2*b^2 + 4*a*b^3)*c^4*d^4 - (11*a^3*b + 24*a^2*b^2)*c^3*d^5 + 8*(a^4 + 2*a^
3*b)*c^2*d^6)*e + (8*b^4*c^6*d^2 + (4*a^2*b^2 - 5*a*b^3)*c^5*d^3 - (2*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x - 3*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b*c*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x -
 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**3 + 2*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b**2*c*f*x**3 - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a
**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 -
2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*
c**2*d**4*x**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 -
 5*b**2*c**5*d*x**4 - 10*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*
b**2*c**2*d**4*x**10 - b**2*c*d**5*x**12),x)*a**3*b*c**4*d**3*f - 36*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2
*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x
**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x
**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x**10
+ 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x**4 - 10*b**2*c**4*d*
*2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x**10 - b**2*c*d**5*x*
*12),x)*a**3*b*c**3*d**4*f*x**2 - 54*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 +
 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c*
*6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 +...
```

3.15 $\int (a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)\ dx$

Optimal result

Integrand size = 30, antiderivative size = 685

$$\int (a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)\ dx =$$

$$-\frac{(10a^4d^4f+ab^3c^2d(99de-56cf)-6a^2b^2cd^2(29de-11cf)-8b^4c^3(3de-2cf)-5a^3bd^3(9de+4cf))\,x\sqrt{}}{315bd^4\sqrt{a+bx^2}}$$

$$+\frac{(5a^3d^3f-3ab^2cd(29de-16cf)+8b^3c^2(3de-2cf)+45a^2bd^2(3de-cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^3}$$

$$+\frac{(5a^2d^2f-4b^2c(3de-2cf)+15abd(2de-cf))\,x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{105d^3}$$

$$+\frac{(9bde-6bcf+5adf)x(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{63d^2}+\frac{fx(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{9d}$$

$$+\frac{\sqrt{a}(10a^4d^4f+ab^3c^2d(99de-56cf)-6a^2b^2cd^2(29de-11cf)-8b^4c^3(3de-2cf)-5a^3bd^3(9de+4cf))}{315b^{3/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(5a^3d^3f+3ab^2cd(16de-9cf)-4b^3c^2(3de-2cf)-30a^2bd^2(6de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\text{arctan}\right.}{315b^{3/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(10*a^4*d^4*f+a*b^3*c^2*d*(-56*c*f+99*d*e)-6*a^2*b^2*c*d^2*(-11*c*f
+29*d*e)-8*b^4*c^3*(-2*c*f+3*d*e)-5*a^3*b*d^3*(4*c*f+9*d*e))*x*(d*x^2+c)^(
1/2)/b/d^4/(b*x^2+a)^(1/2)+1/315*(5*a^3*d^3*f-3*a*b^2*c*d*(-16*c*f+29*d*e)
+8*b^3*c^2*(-2*c*f+3*d*e)+45*a^2*b*d^2*(-c*f+3*d*e))*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b/d^3+1/105*(5*a^2*d^2*f-4*b^2*c*(-2*c*f+3*d*e)+15*a*b*d*(-c*
f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d^3+1/63*(5*a*d*f-6*b*c*f+9*b*
d*e)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/d^2+1/9*f*x*(b*x^2+a)^(5/2)*(d*x^2+
c)^(3/2)/d+1/315*a^(1/2)*(10*a^4*d^4*f+a*b^3*c^2*d*(-56*c*f+99*d*e)-6*a^2*
b^2*c*d^2*(-11*c*f+29*d*e)-8*b^4*c^3*(-2*c*f+3*d*e)-5*a^3*b*d^3*(4*c*f+9*d
*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/
b/c)^(1/2))/b^(3/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/
315*a^(3/2)*(5*a^3*d^3*f+3*a*b^2*c*d*(-9*c*f+16*d*e)-4*b^3*c^2*(-2*c*f+3*d
*e)-30*a^2*b*d^2*(-c*f+6*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1
/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.70

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(5a^3d^3f+15a^2bd^2(9de+2cf+5dfx^2)+b^3(8c^3f-6c^2d(2e+fx^2)\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3*f + 15*a^2*b*d^2*(9*d*e
+ 2*c*f + 5*d*f*x^2) + b^3*(8*c^3*f - 6*c^2*d*(2*e + f*x^2) + c*d^2*x^2*(9
*e + 5*f*x^2) + 5*d^3*x^4*(9*e + 7*f*x^2)) + a*b^2*d*(-27*c^2*f + 4*c*d*(1
2*e + 5*f*x^2) + 5*d^2*x^2*(27*e + 19*f*x^2))) + I*c*(10*a^4*d^4*f + a*b^3
*c^2*d*(99*d*e - 56*c*f) + 8*b^4*c^3*(-3*d*e + 2*c*f) - 5*a^3*b*d^3*(9*d*e
 + 4*c*f) + 6*a^2*b^2*c*d^2*(-29*d*e + 11*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c)
 + a*d)*(5*a^3*d^3*f + 45*a^2*b*d^2*(3*d*e - c*f) - 8*b^3*c^2*(-3*d*e + 2*
c*f) + 3*a*b^2*c*d*(-29*d*e + 16*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^4*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 616, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {403, 403, 403, 27, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int \frac{\left(bx^2+a\right)^{5/2}\left((9bde+bcf-2adf)x^2+c(9be-af)\right)}{\sqrt{dx^2+c}}dx}{9b}+\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(ac(54bde-bcf-5adf)-\left(-3c(3de-2cf)b^2-5ad(9de+2cf)b+10a^2d^2f\right)x^2\right)}{\sqrt{dx^2+c}}dx}{7d}+\frac{x\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(-2adf+bcf+9bde)}{7d}}{9b}+\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int -\frac{3\sqrt{bx^2+a}\left(\left(4c^2(3de-2cf)b^3-3acd(13de-7cf)b^2-15a^2d^2(3de+cf)b+10a^3d^3f\right)x^2+ac\left(c(3de-2cf)b^2-5ad(15de-cf)b+5a^2d^2f\right)\right)}{\sqrt{dx^2+c}}dx}{5d} - \frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+d \cdots}}{\cdots}}{7d} \cdots}{9b}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{3\int \frac{\sqrt{bx^2+a}\left(\left(4c^2(3de-2cf)b^3-3acd(13de-7cf)b^2-15a^2d^2(3de+cf)b+10a^3d^3f\right)x^2+ac\left(c(3de-2cf)b^2-5ad(15de-cf)b+5a^2d^2f\right)\right)}{\sqrt{dx^2+c}}dx}{5d} - \frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+d \cdots}}{\cdots}}{7d} \cdots}{9b}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 403

$$-\frac{3\left(\frac{\int \frac{\left(-8c^3(3de-2cf)b^4+ac^2d(99de-56cf)b^3-6a^2cd^2(29de-11cf)b^2-5a^3d^3(9de+4cf)b+10a^4d^4f\right)x^2+ac\left(-4c^2(3de-2cf)b^3+3acd(16de-9cf)b^2-30a^2d^2(\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}} \cdots}{3d} \cdots\right.}{5d \cdots}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 406

$$-\frac{3\left(\frac{ac\left(5a^3d^3f-30a^2bd^2(6de-cf)+3ab^2cd(16de-9cf)-4b^3c^2(3de-2cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(10a^4d^4f-5a^3bd^3(4cf+9de)-6a^2b^2cd^2(29de-11cf)+ab^3c^2 \cdots\right.}{3d} \cdots\right.}{5d \cdots}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 320

$$-\frac{3\left(\frac{\left(10a^4d^4f-5a^3bd^3(4cf+9de)-6a^2b^2cd^2(29de-11cf)+ab^3c^2d(99de-56cf)-8b^4c^3(3de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(5a^3d^3f-30a^2bd^2(6d\right.}{}}{3d}\right.}{}}{}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 388

$$-\frac{3\left(\frac{\left(10a^4d^4f-5a^3bd^3(4cf+9de)-6a^2b^2cd^2(29de-11cf)+ab^3c^2d(99de-56cf)-8b^4c^3(3de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(5a^3d^3f\right.}{}}{3d}\right.}{}}{}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 313

$$-\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(10a^2d^2f-5abd(2cf+9de)-3b^2c(3de-2cf)\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(10a^3d^3f-15a^2bd^2(cf+3de)-3ab^2cd(13de-7cf)+4b^3c^2(3de-2c\right.}{3d}\right.}{}$$

$$\frac{fx\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{9b}$$

input `Int[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2),x]`

output
```
(f*x*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(9*b) + (((9*b*d*e + b*c*f - 2*a*d
*f)*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*d) + (-1/5*((10*a^2*d^2*f - 3*
b^2*c*(3*d*e - 2*c*f) - 5*a*b*d*(9*d*e + 2*c*f))*x*(a + b*x^2)^(3/2)*Sqrt[
c + d*x^2])/d - (3*(((10*a^3*d^3*f - 3*a*b^2*c*d*(13*d*e - 7*c*f) + 4*b^3*
c^2*(3*d*e - 2*c*f) - 15*a^2*b*d^2*(3*d*e + c*f))*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(3*d) + ((10*a^4*d^4*f + a*b^3*c^2*d*(99*d*e - 56*c*f) - 6*a^2*
b^2*c*d^2*(29*d*e - 11*c*f) - 8*b^4*c^3*(3*d*e - 2*c*f) - 5*a^3*b*d^3*(9*d
*e + 4*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(5*a^3
*d^3*f + 3*a*b^2*c*d*(16*d*e - 9*c*f) - 4*b^3*c^2*(3*d*e - 2*c*f) - 30*a^2
*b*d^2*(6*d*e - c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]))/(3*d)))/(5*d))/(7*d))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 10.88 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.70

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1186
default	Expression too large to display	1845

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/315/b*x*(35*b^3*d^3*f*x^6+95*a*b^2*d^3*f*x^4+5*b^3*c*d^2*f*x^4+45*b^3*d^
3*e*x^4+75*a^2*b*d^3*f*x^2+20*a*b^2*c*d^2*f*x^2+135*a*b^2*d^3*e*x^2-6*b^3*
c^2*d*f*x^2+9*b^3*c*d^2*e*x^2+5*a^3*d^3*f+30*a^2*b*c*d^2*f+135*a^2*b*d^3*e
-27*a*b^2*c^2*d*f+48*a*b^2*c*d^2*e+8*b^3*c^3*f-12*b^3*c^2*d*e)*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^3-1/315/b/d^3*(-(10*a^4*d^4*f-20*a^3*b*c*d^3*f-45*a
^3*b*d^4*e+66*a^2*b^2*c^2*d^2*f-174*a^2*b^2*c*d^3*e-56*a*b^3*c^3*d*f+99*a*
b^3*c^2*d^2*e+16*b^4*c^4*f-24*b^4*c^3*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2)))+8*a*b^3*c^4*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))+5*a^4*c*d^3*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-12*a*b^3*c^3*d*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))+48*a^2*b^2*c^2*d^2*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-27*a^2*b^2*c^3*d*f/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-180*a^3*b*c*d^3*e/(-b/a)^(1/2)*(...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(3\,(8\,b^4c^4d-33\,ab^3c^3d^2+58\,a^2b^2c^2d^3+15\,a^3bcd^4)e-2\,(8\,b^4c^5-28\,ab^3c^4d+33\,a^2b^2c^3d^2-10\,a^3bc^2d}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
-1/315*(sqrt(b*d)*(3*(8*b^4*c^4*d - 33*a*b^3*c^3*d^2 + 58*a^2*b^2*c^2*d^3
+ 15*a^3*b*c*d^4)*e - 2*(8*b^4*c^5 - 28*a*b^3*c^4*d + 33*a^2*b^2*c^3*d^2 -
 10*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c
/d)/x), a*d/(b*c)) - sqrt(b*d)*(3*(8*b^4*c^4*d - 33*a*b^3*c^3*d^2 + 60*a^3
*b*d^5 + 2*(29*a^2*b^2 + 2*a*b^3)*c^2*d^3 + (15*a^3*b - 16*a^2*b^2)*c*d^4)
*e - (16*b^4*c^5 - 56*a*b^3*c^4*d + 5*a^4*d^5 + 2*(33*a^2*b^2 + 4*a*b^3)*c
^3*d^2 - (20*a^3*b + 27*a^2*b^2)*c^2*d^3 + 10*(a^4 + 3*a^3*b)*c*d^4)*f)*x*
sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*d^5*f*x^8
 + 5*(9*b^4*d^5*e + (b^4*c*d^4 + 19*a*b^3*d^5)*f)*x^6 + (9*(b^4*c*d^4 + 15
*a*b^3*d^5)*e - (6*b^4*c^2*d^3 - 20*a*b^3*c*d^4 - 75*a^2*b^2*d^5)*f)*x^4 -
 (3*(4*b^4*c^2*d^3 - 16*a*b^3*c*d^4 - 45*a^2*b^2*d^5)*e - (8*b^4*c^3*d^2 -
 27*a*b^3*c^2*d^3 + 30*a^2*b^2*c*d^4 + 5*a^3*b*d^5)*f)*x^2 + 3*(8*b^4*c^3*
d^2 - 33*a*b^3*c^2*d^3 + 58*a^2*b^2*c*d^4 + 15*a^3*b*d^5)*e - 2*(8*b^4*c^4
*d - 28*a*b^3*c^3*d^2 + 33*a^2*b^2*c^2*d^3 - 10*a^3*b*c*d^4 + 5*a^4*d^5)*f
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \left(b\,x^2+a\right)^{5/2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output
```
(5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f*x + 30*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*b*c*d**2*f*x + 135*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a**2*b*d**3*e*x + 75*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*x**3
- 27*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f*x + 48*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*x + 20*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a*b**2*c*d**2*f*x**3 + 135*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d
**3*e*x**3 + 95*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f*x**5 + 8*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f*x - 12*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b**3*c**2*d*e*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**
2*d*f*x**3 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*x**3 + 5*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f*x**5 + 45*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**3*d**3*e*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*3*d**3*f*x**7 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*
d*x**2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f + 20*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d*
*3*f + 45*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e - 66*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b**2*c**2*d**2
*f + 174*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*
c*x**2 + b*d*x**4),x)*a**2*b**2*c*d**3*e + 56*int((sqrt(c + d*x**2)*sqr...
```

3.16 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 530

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{(15a^3d^3f - ab^2cd(161de-128cf) + a^2bd^2(161de-103cf) + 8b^3c^2(7de -}{105d^4\sqrt{a+bx^2}}$$

$$+ \frac{(15a^2d^2f + abd(56de-43cf) - 4b^2c(7de-6cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3}$$

$$+ \frac{(7bde-6bcf+5adf)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$- \frac{\sqrt{a}(15a^3d^3f - ab^2cd(161de-128cf) + a^2bd^2(161de-103cf) + 8b^3c^2(7de-6cf))\sqrt{c+dx^2}\,E\Big(\arctan}{105\sqrt{b}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$- \frac{a^{3/2}(abcd(77de-61cf) - 4b^2c^2(7de-6cf) - 15a^2d^2(7de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1\right.}{105\sqrt{b}cd^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(15*a^3*d^3*f-a*b^2*c*d*(-128*c*f+161*d*e)+a^2*b*d^2*(-103*c*f+161*d
*e)+8*b^3*c^2*(-6*c*f+7*d*e))*x*(d*x^2+c)^(1/2)/d^4/(b*x^2+a)^(1/2)+1/105*
(15*a^2*d^2*f+a*b*d*(-43*c*f+56*d*e)-4*b^2*c*(-6*c*f+7*d*e))*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^3+1/35*(5*a*d*f-6*b*c*f+7*b*d*e)*x*(b*x^2+a)^(3/2)*
(d*x^2+c)^(1/2)/d^2+1/7*f*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/d-1/105*a^(1/2
)*(15*a^3*d^3*f-a*b^2*c*d*(-128*c*f+161*d*e)+a^2*b*d^2*(-103*c*f+161*d*e)+
8*b^3*c^2*(-6*c*f+7*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b
*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-1/105*a^(3/2)*(a*b*c*d*(-61*c*f+77*d*e)-4*b^2*c^2*(-6*c
*f+7*d*e)-15*a^2*d^2*(-3*c*f+7*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arcta
n(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d^3/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(45a^2d^2f+abd(77de-61cf+45dfx^2)+b^2(24c^2 \cdots}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(45*a^2*d^2*f + a*b*d*(77*d*e - 61*
c*f + 45*d*f*x^2) + b^2*(24*c^2*f + 3*d^2*x^2*(7*e + 5*f*x^2) - 2*c*d*(14*
e + 9*f*x^2))) - I*c*(15*a^3*d^3*f + a^2*b*d^2*(161*d*e - 103*c*f) - 8*b^3
*c^2*(-7*d*e + 6*c*f) + a*b^2*c*d*(-161*d*e + 128*c*f))*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(
-(b*c) + a*d)*(a*b*c*d*(133*d*e - 104*c*f) + 15*a^2*d^2*(-7*d*e + 4*c*f) +
 8*b^2*c^2*(-7*d*e + 6*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {403, 403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left((7bde-6bcf+5adf)x^2+a(7de-cf)\right)}{\sqrt{dx^2+c}}dx}{7d}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(a(bc(7de-6cf)-5ad(7de-2cf))-\left(-4c(7de-6cf)b^2+ad(56de-43cf)b+15a^2d^2f\right)x^2\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}}{7d}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\int \frac{\sqrt{bx^2+a}\left(a(bc(7de-6cf)-5ad(7de-2cf))-\left(-4c(7de-6cf)b^2+ad(56de-43cf)b+15a^2d^2f\right)x^2\right)}{\sqrt{dx^2+c}}dx}{5d}}{7d}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\frac{\int \frac{a\left(-4b^2(7de-6cf)c^2+abd(77de-61cf)c-15a^2d^2(7de-3cf)\right)-\left(8c^2(7de-6cf)b^3-acd(161de-128cf)b^2+a^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}}{5d}}{7d}+\frac{fx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\frac{a\left(-15a^2d^2(7de-3cf)+abcd(77de-61cf)-4b^2c^2(7de-6cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(15a^3d^3f+a^2bd^2(161de\right.}{3d}}{7d}$$

$$\frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

$\downarrow$ 320

$$\frac{\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}\left(-15a^2d^2(7de-3cf)+abcd(77de-61cf)-4b^2c^2(7de-6cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(1\right.}{3d}}{7}$$

$$\frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

$\downarrow$ 388

$$\frac{\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}\left(-15a^2d^2(7de-3cf)+abcd(77de-61cf)-4b^2c^2(7de-6cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(1\right.}{3d}}{}$$

$$\frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

$\downarrow$ 313

$$\frac{\frac{x(a+bx^2)^{3/2}\sqrt{c+dx^2}(5adf-6bcf+7bde)}{5d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}\left(-15a^2d^2(7de-3cf)+abcd(77de-61cf)-4b^2c^2(7de-6cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(1\right.}{}}{}$$

$$\frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*d) + (((7*b*d*e - 6*b*c*f + 5*a
*d*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (-1/3*((15*a^2*d^2*f +
a*b*d*(56*d*e - 43*c*f) - 4*b^2*c*(7*d*e - 6*c*f))*x*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/d + (-((15*a^3*d^3*f - a*b^2*c*d*(161*d*e - 128*c*f) + a^2*b*d
^2*(161*d*e - 103*c*f) + 8*b^3*c^2*(7*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*(a*b*c*d*(77*d*e - 61*c*f) - 4*b^2*c^2*(
7*d*e - 6*c*f) - 15*a^2*d^2*(7*d*e - 3*c*f))*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d))/(7*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 14.56 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{b^2 f x^5\sqrt{b d x^4+a d x^2+x^2 b c+a c}}{7d}+\frac{\left(3 a b^2 f+b^3 e-\frac{b^2 f(6 a d+6 b c)}{7d}\right)x^3\sqrt{b d x^4+a d x^2+x^2 b c+a c}}{5 b d}+\frac{\left(3a^2 b f+3 a b^2 e-\frac{5 a b^2 c}{7d}\right.}{}\right.}{}$
risch	$\frac{x(15b^2d^2 f x^4+45ab d^2 f x^2-18b^2 c f x^2 d+21b^2 d^2 e x^2+45 f d^2 a^2-61 f d c b a+77 a b d^2 e+24 f c^2 b^2-28 d b^2 c e)\sqrt{b x^2+a}\sqrt{x^2 d+c}}{105 d^3}-$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*b^2/d*f*x
^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(3*a*b^2*f+b^3*e-1/7*b^2/d*f*(6
*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(3*a^2*b*f+3*
a*b^2*e-5/7*a*b^2*c/d*f-1/5*(3*a*b^2*f+b^3*e-1/7*b^2/d*f*(6*a*d+6*b*c))/b/
d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^3*e-1/3*(3*a
^2*b*f+3*a*b^2*e-5/7*a*b^2*c/d*f-1/5*(3*a*b^2*f+b^3*e-1/7*b^2/d*f*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(a^3*f+3*a^2*b*e-3/5*(3*a*b^2*f+b^3*e-1/7*b^2/d*f*(
6*a*d+6*b*c))/b/d*a*c-1/3*(3*a^2*b*f+3*a*b^2*e-5/7*a*b^2*c/d*f-1/5*(3*a*b^
2*f+b^3*e-1/7*b^2/d*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{\sqrt{bd}(7\,(8\,b^3c^4d-23\,ab^2c^3d^2+23\,a^2bc^2d^3)e-(48\,b^3c^5-128\,ab^2c^4d+103\,a^2bc^3d^2-15\,a^3c^2d^3)f)x\sqrt{-\frac{c}{d}}E}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(sqrt(b*d)*(7*(8*b^3*c^4*d - 23*a*b^2*c^3*d^2 + 23*a^2*b*c^2*d^3)*e
 - (48*b^3*c^5 - 128*a*b^2*c^4*d + 103*a^2*b*c^3*d^2 - 15*a^3*c^2*d^3)*f)*
x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(8
*b^3*c^4*d - 23*a*b^2*c^3*d^2 - 11*a^2*b*c*d^4 + 15*a^3*d^5 + (23*a^2*b +
4*a*b^2)*c^2*d^3)*e - (48*b^3*c^5 - 128*a*b^2*c^4*d + 45*a^3*c*d^4 + (103*
a^2*b + 24*a*b^2)*c^3*d^2 - (15*a^3 + 61*a^2*b)*c^2*d^3)*f)*x*sqrt(-c/d)*e
lliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (15*b^3*c*d^4*f*x^6 + 3*(7*b^
3*c*d^4*e - 3*(2*b^3*c^2*d^3 - 5*a*b^2*c*d^4)*f)*x^4 - (7*(4*b^3*c^2*d^3 -
 11*a*b^2*c*d^4)*e - (24*b^3*c^3*d^2 - 61*a*b^2*c^2*d^3 + 45*a^2*b*c*d^4)*
f)*x^2 + 7*(8*b^3*c^3*d^2 - 23*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4)*e - (48*b^3
*c^4*d - 128*a*b^2*c^3*d^2 + 103*a^2*b*c^2*d^3 - 15*a^3*c*d^4)*f)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b*c*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output

```
(45*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x - 61*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b*c*d*f*x + 77*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x + 45*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 + 24*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x - 28*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**2*c*d*e*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3
+ 21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b**2*d**2*f*x**5 + 15*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f - 103*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a**2*b*c*d**2*f + 161*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e + 128*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x*
*4),x)*a*b**2*c**2*d*f - 161*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e - 48*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x
)*b**3*c**3*f + 56*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d
*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**2*d*e - 45*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f +
 105*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a**3*d**3*e + 61*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(...
```

3.17 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 407

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf))\,x\sqrt{a+bx^2}}{15d^3\sqrt{c+dx^2}}$$
$$+\frac{(5bde-6bcf+5adf)x(a+bx^2)^{3/2}}{15d^2\sqrt{c+dx^2}}+\frac{fx(a+bx^2)^{5/2}}{5d\sqrt{c+dx^2}}$$
$$-\frac{(abcd(65de-88cf)-a^2d^2(15de-38cf)-8b^2c^2(5de-6cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{c}(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/15*(15*a^2*d^2*f+a*b*d*(-41*c*f+30*d*e)-4*b^2*c*(-6*c*f+5*d*e))*x*(b*x^2
+a)^(1/2)/d^3/(d*x^2+c)^(1/2)+1/15*(5*a*d*f-6*b*c*f+5*b*d*e)*x*(b*x^2+a)^(
3/2)/d^2/(d*x^2+c)^(1/2)+1/5*f*x*(b*x^2+a)^(5/2)/d/(d*x^2+c)^(1/2)-1/15*(a
*b*c*d*(-88*c*f+65*d*e)-a^2*d^2*(-38*c*f+15*d*e)-8*b^2*c^2*(-6*c*f+5*d*e))
*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)
^(1/2))/c^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/
15*c^(1/2)*(15*a^2*d^2*f+a*b*d*(-41*c*f+30*d*e)-4*b^2*c*(-6*c*f+5*d*e))*(b
*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))
/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.58 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15a^2d^2(de-cf)+abcd(-30de+41cf+11dfx^2)+b^2c(-\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*a^2*d^2*(d*e - c*f) + a*b*c*d*(-30*d*e + 41
*c*f + 11*d*f*x^2) + b^2*c*(-24*c^2*f + c*d*(20*e - 6*f*x^2) + d^2*x^2*(5*
e + 3*f*x^2))) - I*b*c*(a*b*c*d*(65*d*e - 88*c*f) + 8*b^2*c^2*(-5*d*e + 6*
c*f) + a^2*d^2*(-15*d*e + 38*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(15*a
^2*d^2*f + a*b*d*(45*d*e - 64*c*f) + 8*b^2*c*(-5*d*e + 6*c*f))*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
])/(15*Sqrt[b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {401, 25, 403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int -\frac{(bx^2+a)^{3/2}(acf-b(5de-6cf)x^2)}{\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(acf-b(5de-6cf)x^2)}{\sqrt{dx^2+c}}dx}{cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(ac(5bde-6bcf+5adf)-b(ad(15de-23cf)-4bc(5de-6cf))x^2\right)}{\sqrt{dx^2+c}}dx}{5d}-\frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(5de-6cf)}{5d}}{cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\frac{\int \frac{b\left(-8b^2(5de-6cf)c^2+abd(65de-88cf)c-a^2d^2(15de-38cf)\right)x^2+ac\left(-4c(5de-6cf)b^2+ad(30de-41cf)b+15a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(ad(15de-23cf)-}{3d}}{5d}}{cd}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\frac{\frac{b\left(-a^2d^2(15de-38cf)+abcd(65de-88cf)-8b^2c^2(5de-6cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac\left(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{bx\cdots}{\cdots}}{5d}\cdots}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 320

$$\frac{\frac{\frac{b\left(-a^2d^2(15de-38cf)+abcd(65de-88cf)-8b^2c^2(5de-6cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf)\right)\text{EllipticF}\left(\arctan\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\cdots}{5d}\cdots}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 388

$$\frac{\frac{\frac{b\left(-a^2d^2(15de-38cf)+abcd(65de-88cf)-8b^2c^2(5de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf)\right)\text{El}\cdots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\cdots}{5d}\cdots}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(15a^2d^2f+abd(30de-41cf)-4b^2c(5de-6cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(-a^2d^2(15de-38cf)+abcd(65de-88cf)-8b^2c^2(5de-6cf)\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right)}{3d}\cdots}{5d}\cdots}{cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{cd\sqrt{c+dx^2}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(5/2))/(c*d*Sqrt[c + d*x^2]) + (-1/5*(b*(5*d*e
- 6*c*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/d + (-1/3*(b*(a*d*(15*d*e -
23*c*f) - 4*b*c*(5*d*e - 6*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d + (b
*(a*b*c*d*(65*d*e - 88*c*f) - a^2*d^2*(15*d*e - 38*c*f) - 8*b^2*c^2*(5*d*e
 - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(15*a^2*
d^2*f + a*b*d*(30*d*e - 41*c*f) - 4*b^2*c*(5*d*e - 6*c*f))*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(372) = 744$.

Time = 15.14 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.95

method	result
risch	$\frac{bx(3bdf\,x^2+11adf-9bcf+5bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15d^3}+\frac{\left(\frac{\left(15f\,d^3a^3-56a^2bc\,d^2f+45a^2b\,d^3e+54a\,b^2c^2df-50a\,b^2c\,d^2e-15b^3c^3f+15b^3c^2de\right)\cdots}{d\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+\cdots}}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(bd\,x^2+ad\right)\left(a^2cf\,d^2-a^2d^3e-2ab\,c^2df+2abc\,d^2e+b^2c^3f-b^2c^2de\right)x}{c\,d^4\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{b^2f\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5d^2}+\frac{\left(\frac{b^2(3adf-b\cdots}{d^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*b*x*(3*b*d*f*x^2+11*a*d*f-9*b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/d^3+1/15/d^3*((15*a^3*d^3*f-56*a^2*b*c*d^2*f+45*a^2*b*d^3*e+54*a*b^2
*c^2*d*f-50*a*b^2*c*d^2*e-15*b^3*c^3*f+15*b^3*c^2*d*e)/d/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b*(23*a^2*d^2*f-58*a*b*c*d*f+
35*a*b*d^2*e+33*b^2*c^2*f-25*b^2*c*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2)))-15*(a^3*c*d^3*f-a^3*d^4*e-3*a^2*b*c^2*d^2*f+3*a^2*b*c*d^3*e+3*a
*b^2*c^3*d*f-3*a*b^2*c^2*d^2*e-b^3*c^4*f+b^3*c^3*d*e)/d*((b*d*x^2+a*d)/c/(
a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/(a*d-b*c)/c*a*d)/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^
(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{((5\,(8\,b^3c^3d^2-13\,ab^2c^2d^3+3\,a^2bcd^4)e-2\,(24\,b^3c^4d-44\,ab^2c^3d^2+19\,a^2bc^2}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(((5*(8*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4)*e - 2*(24*b^3
*c^4*d - 44*a*b^2*c^3*d^2 + 19*a^2*b*c^2*d^3)*f)*x^3 + (5*(8*b^3*c^4*d - 1
3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3)*e - 2*(24*b^3*c^5 - 44*a*b^2*c^4*d + 19
*a^2*b*c^3*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x)
, a*d/(b*c)) - ((5*(8*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 - 6*a^2*b*d^5 + (3*a^
2*b + 4*a*b^2)*c*d^4)*e - (48*b^3*c^4*d - 88*a*b^2*c^3*d^2 - 41*a^2*b*c*d^
4 + 15*a^3*d^5 + 2*(19*a^2*b + 12*a*b^2)*c^2*d^3)*f)*x^3 + (5*(8*b^3*c^4*d
 - 13*a*b^2*c^3*d^2 - 6*a^2*b*c*d^4 + (3*a^2*b + 4*a*b^2)*c^2*d^3)*e - (48
*b^3*c^5 - 88*a*b^2*c^4*d - 41*a^2*b*c^2*d^3 + 15*a^3*c*d^4 + 2*(19*a^2*b
+ 12*a*b^2)*c^3*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) + (3*b^3*c*d^4*f*x^6 + (5*b^3*c*d^4*e - (6*b^3*c^2*d^3 -
 11*a*b^2*c*d^4)*f)*x^4 - (5*(4*b^3*c^2*d^3 - 7*a*b^2*c*d^4)*e - (24*b^3*c
^3*d^2 - 47*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4)*f)*x^2 - 5*(8*b^3*c^3*d^2 - 13
*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4)*e + 2*(24*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 1
9*a^2*b*c^2*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^6*x^3 + b*c^2*
d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
(15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**2*f*x - 33*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*b*c*d*f*x + 45*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
2*b*d**2*e*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*f*x - 15*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d*e*x + 22*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b**2*c*d*f*x**3 - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3
*c**2*f*x**3 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*e*x**3 + 6*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*f*x**5 - 15*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2
 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*c*d**3*f - 15*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x*
*2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*d**4*f*x**2 + 79*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c*
*2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b**2*c**2*d**2*f - 45*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**
4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b**2*c*d**3*e + 79*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b**2*c*d**3*f*x
**2 - 45*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x*
*2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b**2*
d**4*e*x**2 - 112*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 ...
```

3.18 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 387

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \frac{(de-cf)x(a+bx^2)^{5/2}}{3cd(c+dx^2)^{3/2}}$$
$$-\frac{b(ad(de-7cf)-4bc(de-2cf))x\sqrt{a+bx^2}}{3cd^3\sqrt{c+dx^2}}-\frac{b(de-2cf)x(a+bx^2)^{3/2}}{3cd^2\sqrt{c+dx^2}}$$
$$+\frac{(abcd(3de-16cf)-8b^2c^2(de-2cf)+a^2d^2(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{b(ad(de-7cf)-4bc(de-2cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-c*f+d*e)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(3/2)-1/3*b*(a*d*(-7*c*f+d*
e)-4*b*c*(-2*c*f+d*e))*x*(b*x^2+a)^(1/2)/c/d^3/(d*x^2+c)^(1/2)-1/3*b*(-2*c
*f+d*e)*x*(b*x^2+a)^(3/2)/c/d^2/(d*x^2+c)^(1/2)+1/3*(a*b*c*d*(-16*c*f+3*d*
e)-8*b^2*c^2*(-2*c*f+d*e)+a^2*d^2*(c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d
^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(7/2)/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(a*d*(-7*c*f+d*e)-4*b*c*
(-2*c*f+d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1
-b*c/a/d)^(1/2))/c^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.37 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(a^2d^3(3ce+2dex^2+cfx^2)+abcd(-7c^2f+3d^2ex^2+cd(e\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a^2*d^3*(3*c*e + 2*d*e*x^2 + c*f*x^2) + a*b*c*
d*(-7*c^2*f + 3*d^2*e*x^2 + c*d*(e - 9*f*x^2)) + b^2*c^2*(8*c^2*f + d^2*x^
2*(-5*e + f*x^2) + c*d*(-4*e + 10*f*x^2))) + I*b*c*(a*b*c*d*(3*d*e - 16*c*
f) + a^2*d^2*(2*d*e + c*f) + 8*b^2*c^2*(-(d*e) + 2*c*f))*Sqrt[1 + (b*x^2)/
a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] - I*b*c*(-(b*c) + a*d)*(8*b*c*(d*e - 2*c*f) + a*d*(d*e + 8*c*f))*S
qrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d^4*Sqrt[a + b*x^2]*(c + d*x^2)^
(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 401, 27, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(a(2de+cf)-3b(de-2cf)x^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{3cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(a(2de+cf)-3b(de-2cf)x^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{\int \frac{3b\sqrt{bx^2+a}\left((4bc(de-2cf)+ad(2de+cf))x^2+ac(de-2cf)\right)}{\sqrt{dx^2+c}}dx}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{3b\int \frac{\sqrt{bx^2+a}\left((4bc(de-2cf)+ad(2de+cf))x^2+ac(de-2cf)\right)}{\sqrt{dx^2+c}}dx}{cd}}{3cd}+\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{3b\left(\frac{\int \frac{\left(-8b^2(de-2cf)c^2+abd(3de-16cf)c+a^2d^2(2de+cf)\right)x^2+ac(ad(de-7cf)-4bc(de-2cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx\cdots}}{\cdots}\cdots\right)}{cd}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 406

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{3b\left(\frac{\left(a^2d^2(cf+2de)+abcd(3de-16cf)-8b^2c^2(de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(ad(de-7cf)-4bc(de-2c\cdots}{3d}\cdots\right)}{cd}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 320

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{3b\left(\frac{\left(a^2d^2(cf+2de)+abcd(3de-16cf)-8b^2c^2(de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(ad(de-7cf)-\cdots}{\sqrt{c\cdots}}}{3d}\cdots\right)}{cd}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 388

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}}-\frac{3b\left(\frac{\left(a^2d^2(cf+2de)+abcd(3de-16cf)-8b^2c^2(de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(\cdots}{\cdots}}{3d}\cdots\right)}{\cdots}}{3cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{3cd\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 313

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+2de)+3bc(de-2cf))}{cd\sqrt{c+dx^2}} - \frac{3b\left(\frac{\left(a^2d^2(cf+2de)+abcd(3de-16cf)-8b^2c^2(de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{3d}\right.}{}}{3cd}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{3cd(c+dx^2)^{3/2}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(5/2))/(3*c*d*(c + d*x^2)^(3/2)) + (((3*b*c*(d*
e - 2*c*f) + a*d*(2*d*e + c*f))*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2])
 - (3*b*(((4*b*c*(d*e - 2*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(3*d) + ((a*b*c*d*(3*d*e - 16*c*f) - 8*b^2*c^2*(d*e - 2*c*f)
+ a^2*d^2*(2*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^
(3/2)*(a*d*(d*e - 7*c*f) - 4*b*c*(d*e - 2*c*f))*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)))/(c*d))/(3*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(352) = 704$.

Time = 16.16 (sec) , antiderivative size = 843, normalized size of antiderivative = 2.18

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)} \left(-\frac{\left(a^2 c f d^2 - a^2 d^3 e - 2 a b c^2 d f + 2 a b c d^2 e + b^2 c^3 f - b^2 c^2 d e\right) x \sqrt{b d x^4 + a d x^2 + x^2 b c + a c}}{3 c d^5 \left(x^2 + \frac{c}{d}\right)^2} + \frac{\left(b d x^2 + a d\right)\left(a^2 c f d^2 + 2 a^2 d^3 e - 9 a \right.}{3 c^2 d^4 \sqrt{\left(x^2 + \right.}} \right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a^2*c*d
^2*f-a^2*d^3*e-2*a*b*c^2*d*f+2*a*b*c*d^2*e+b^2*c^3*f-b^2*c^2*d*e)/c/d^5*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/3*(b*d*x^2+a*d)*(a^2*c*d
^2*f+2*a^2*d^3*e-9*a*b*c^2*d*f+3*a*b*c*d^2*e+8*b^2*c^3*f-5*b^2*c^2*d*e)/c^
2/d^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*b^2*f/d^3*x*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+(b*(3*a^2*d^2*f-6*a*b*c*d*f+3*a*b*d^2*e+3*b^2*c^2*f-2*b^
2*c*d*e)/d^4-1/3*(a^2*c*d^2*f-a^2*d^3*e-2*a*b*c^2*d*f+2*a*b*c*d^2*e+b^2*c^
3*f-b^2*c^2*d*e)/d^4*b/c+1/3*(a^2*c*d^2*f+2*a^2*d^3*e-9*a*b*c^2*d*f+3*a*b*
c*d^2*e+8*b^2*c^3*f-5*b^2*c^2*d*e)/d^4*(a*d-b*c)/c^2-1/3*a/d^3*(a^2*c*d^2*
f+2*a^2*d^3*e-9*a*b*c^2*d*f+3*a*b*c*d^2*e+8*b^2*c^3*f-5*b^2*c^2*d*e)/c^2-1
/3*a*b^2*c/d^3*f)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-(1/d^3*b^2*(3*a*d*f-2*b*c*f+b*d*e)-1/3*(a^2*c*d^2*f+2*a^2*d^3*e-9*a*
b*c^2*d*f+3*a*b*c*d^2*e+8*b^2*c^3*f-5*b^2*c^2*d*e)/d^3*b/c^2-1/3*b^2*f/d^3
*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(356) = 712$.

Time = 0.11 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((((8*b^2*c^3*d^3 - 3*a*b*c^2*d^4 - 2*a^2*c*d^5)*e - (16*b^2*c^4*d^2
- 16*a*b*c^3*d^3 + a^2*c^2*d^4)*f)*x^5 + 2*((8*b^2*c^4*d^2 - 3*a*b*c^3*d^3
 - 2*a^2*c^2*d^4)*e - (16*b^2*c^5*d - 16*a*b*c^4*d^2 + a^2*c^3*d^3)*f)*x^3
 + ((8*b^2*c^5*d - 3*a*b*c^4*d^2 - 2*a^2*c^3*d^3)*e - (16*b^2*c^6 - 16*a*b
*c^5*d + a^2*c^4*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c
/d)/x), a*d/(b*c)) - (((8*b^2*c^3*d^3 - 3*a*b*c^2*d^4 - a^2*d^6 - 2*(a^2 -
 2*a*b)*c*d^5)*e - (16*b^2*c^4*d^2 - 16*a*b*c^3*d^3 - 7*a^2*c*d^5 + (a^2 +
 8*a*b)*c^2*d^4)*f)*x^5 + 2*((8*b^2*c^4*d^2 - 3*a*b*c^3*d^3 - a^2*c*d^5 -
2*(a^2 - 2*a*b)*c^2*d^4)*e - (16*b^2*c^5*d - 16*a*b*c^4*d^2 - 7*a^2*c^2*d^
4 + (a^2 + 8*a*b)*c^3*d^3)*f)*x^3 + ((8*b^2*c^5*d - 3*a*b*c^4*d^2 - a^2*c^
2*d^4 - 2*(a^2 - 2*a*b)*c^3*d^3)*e - (16*b^2*c^6 - 16*a*b*c^5*d - 7*a^2*c^
3*d^3 + (a^2 + 8*a*b)*c^4*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsi
n(sqrt(-c/d)/x), a*d/(b*c)) - (b^2*c^2*d^4*f*x^6 + (3*b^2*c^2*d^4*e - (6*b
^2*c^3*d^3 - 7*a*b*c^2*d^4)*f)*x^4 + ((12*b^2*c^3*d^3 - 5*a*b*c^2*d^4 - a^
2*c*d^5)*e - (24*b^2*c^4*d^2 - 25*a*b*c^3*d^3 + 2*a^2*c^2*d^4)*f)*x^2 + (8
*b^2*c^4*d^2 - 3*a*b*c^3*d^3 - 2*a^2*c^2*d^4)*e - (16*b^2*c^5*d - 16*a*b*c
^4*d^2 + a^2*c^3*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*d^7*x^5 + 2
*c^3*d^6*x^3 + c^4*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(e+fx^2)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(5/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x + 18*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b*c*f*x - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x
+ 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**3 - 12*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c*f*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*
d*e*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*f*x**5 + 15*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**
2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b
*d**3*x**8),x)*a**2*b*c**2*d**2*f + 30*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c*
*3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*a**2*b*c*d**
3*f*x**2 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3 + 3*a*c
**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4
 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*a**2*b*d**4*f*x**4 - 48*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**
4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3
*x**8),x)*a*b**2*c**3*d*f + 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 +
 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*a*b**2*c**2*d**2*e -
96*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2
+ 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c...
```

3.19 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 402

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx=\frac{(de-cf)x(a+bx^2)^{5/2}}{5cd(c+dx^2)^{5/2}}$$
$$+\frac{(bc(de-6cf)+ad(4de+cf))x(a+bx^2)^{3/2}}{15c^2d^2(c+dx^2)^{3/2}}$$
$$-\frac{b(4bc(de-6cf)+ad(4de+cf))x\sqrt{a+bx^2}}{15c^2d^3\sqrt{c+dx^2}}$$
$$+\frac{(8b^2c^2(de-6cf)+2a^2d^2(4de+cf)+abcd(7de+8cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{b(4bc(de-6cf)+ad(4de+cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/5*(-c*f+d*e)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(5/2)+1/15*(b*c*(-6*c*f+d*e
)+a*d*(c*f+4*d*e))*x*(b*x^2+a)^(3/2)/c^2/d^2/(d*x^2+c)^(3/2)-1/15*b*(4*b*c
*(-6*c*f+d*e)+a*d*(c*f+4*d*e))*x*(b*x^2+a)^(1/2)/c^2/d^3/(d*x^2+c)^(1/2)+1
/15*(8*b^2*c^2*(-6*c*f+d*e)+2*a^2*d^2*(c*f+4*d*e)+a*b*c*d*(8*c*f+7*d*e))*(
b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))/c^(5/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15
*b*(4*b*c*(-6*c*f+d*e)+a*d*(c*f+4*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(ar
ctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(7/2)/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\Big(3c^2(bc-ad)^2(de-cf)-c(bc-ad)(bc(7de-12cf)+ad}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f) - c*(b*c - a*d
)*(b*c*(7*d*e - 12*c*f) + a*d*(4*d*e + c*f))*(c + d*x^2) + (b^2*c^2*(8*d*e
 - 33*c*f) + 2*a^2*d^2*(4*d*e + c*f) + a*b*c*d*(7*d*e + 8*c*f))*(c + d*x^2
)^2) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((8*b^2
*c^2*(d*e - 6*c*f) + 2*a^2*d^2*(4*d*e + c*f) + a*b*c*d*(7*d*e + 8*c*f))*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(a^2*d^2*(4*d*e + c*f)) +
 8*b^2*c^2*(-(d*e) + 6*c*f) - a*b*c*d*(3*d*e + 32*c*f))*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^4*Sqrt[a + b*x^2]*(c +
d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {401, 25, 401, 401, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}-\frac{\int -\frac{(bx^2+a)^{3/2}(a(4de+cf)-b(de-6cf)x^2)}{(dx^2+c)^{5/2}}dx}{5cd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(a(4de+cf)-b(de-6cf)x^2)}{(dx^2+c)^{5/2}}dx}{5cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{\int \frac{\sqrt{bx^2+a}\left(b(4bc(de-6cf)+ad(4de+cf))x^2+a(bc(de-6cf)-2ad(4de+cf))\right)}{\left(dx^2+c\right)^{3/2}}dx}{3cd}}{5cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{-\frac{\int -\frac{b\left(\left(8b^2(de-6cf)c^2+abd(7de+8cf)c+2a^2d^2(4de+cf)\right)x^2+ac(4bc(de-6cf)+ad(4de+cf))\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx}}{}}{3cd}}{5cd}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{\frac{\int \frac{b\left(\left(8b^2(de-6cf)c^2+abd(7de+8cf)c+2a^2d^2(4de+cf)\right)x^2+ac(4bc(de-6cf)+ad(4de+cf))\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}\left(2\right.}{}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{\frac{b\int \frac{\left(8b^2(de-6cf)c^2+abd(7de+8cf)c+2a^2d^2(4de+cf)\right)x^2+ac(4bc(de-6cf)+ad(4de+cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}\left(2a\right.}{}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 406

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{\frac{b\left(\left(2a^2d^2(cf+4de)+abcd(8cf+7de)+8b^2c^2(de-6cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(ad(cf+4de)+4bc(de-6cf))\int\right.}{cd}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 320

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{\frac{b\left(\left(2a^2d^2(cf+4de)+abcd(8cf+7de)+8b^2c^2(de-6cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(ad(cf+4de)+4bc}{\sqrt{d}\sqrt{c}}\right.}{cd}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{5cd\left(c+dx^2\right)^{5/2}}$$

$\Big\downarrow$ 388

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{b\left(\left(2a^2d^2(cf+4de)+abcd(8cf+7de)+8b^2c^2(de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(ad(\cdots}{\cdots}\right)}{cd}}{5cd}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}$$

$\big\downarrow$ 313

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+4de)+bc(de-6cf))}{3cd(c+dx^2)^{3/2}}-\frac{b\left(\left(2a^2d^2(cf+4de)+abcd(8cf+7de)+8b^2c^2(de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\cdots\right)}{cd}}{5cd}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{5cd(c+dx^2)^{5/2}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(5/2))/(5*c*d*(c + d*x^2)^(5/2)) + (((b*c*(d*e
- 6*c*f) + a*d*(4*d*e + c*f))*x*(a + b*x^2)^(3/2))/(3*c*d*(c + d*x^2)^(3/2
)) - (-((((4*b^2*c*(d*e - 6*c*f))/d + (2*a^2*d*(4*d*e + c*f))/c + a*b*(3*d
*e + 7*c*f))*x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2]) + (b*((8*b^2*c^2*(d*e - 6
*c*f) + 2*a^2*d^2*(4*d*e + c*f) + a*b*c*d*(7*d*e + 8*c*f))*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b*c*(d*e - 6*c*f) + a*d*(4*d*e
+ c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(
c*d))/(3*c*d))/(5*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(367) = 734$.

Time = 9.97 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.10

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(a^2cf\,d^2-a^2d^3e-2ab\,c^2df+2abc\,d^2e+b^2c^3f-b^2c^2de\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c\,d^6\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(a^2cf\,d^2+4a^2d^3e-13ab\,c^2df+3ab\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(a^2*c*d
^2*f-a^2*d^3*e-2*a*b*c^2*d*f+2*a*b*c*d^2*e+b^2*c^3*f-b^2*c^2*d*e)/c/d^6*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/15*(a^2*c*d^2*f+4*a^2*d^
3*e-13*a*b*c^2*d*f+3*a*b*c*d^2*e+12*b^2*c^3*f-7*b^2*c^2*d*e)/c^2/d^5*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)*(2*a^2*c*d
^2*f+8*a^2*d^3*e+8*a*b*c^2*d*f+7*a*b*c*d^2*e-33*b^2*c^3*f+8*b^2*c^2*d*e)/c
^3/d^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(b^2*(3*a*d*f-3*b*c*f+b*d*e)/d^4+
1/15*b*(a^2*c*d^2*f+4*a^2*d^3*e-13*a*b*c^2*d*f+3*a*b*c*d^2*e+12*b^2*c^3*f-
7*b^2*c^2*d*e)/c^2/d^4+1/15*(2*a^2*c*d^2*f+8*a^2*d^3*e+8*a*b*c^2*d*f+7*a*b
*c*d^2*e-33*b^2*c^3*f+8*b^2*c^2*d*e)/d^4*(a*d-b*c)/c^3-1/15*a/d^3*(2*a^2*c
*d^2*f+8*a^2*d^3*e+8*a*b*c^2*d*f+7*a*b*c*d^2*e-33*b^2*c^3*f+8*b^2*c^2*d*e)
/c^3)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b^3*f
/d^3-1/15*(2*a^2*c*d^2*f+8*a^2*d^3*e+8*a*b*c^2*d*f+7*a*b*c*d^2*e-33*b^2*c^
3*f+8*b^2*c^2*d*e)/d^3*b/c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(367) = 734$.

Time = 0.14 (sec) , antiderivative size = 1091, normalized size of antiderivative = 2.71

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*((((8*b^2*c^3*d^4 + 7*a*b*c^2*d^5 + 8*a^2*c*d^6)*e - 2*(24*b^2*c^4*d^
3 - 4*a*b*c^3*d^4 - a^2*c^2*d^5)*f)*x^7 + 3*((8*b^2*c^4*d^3 + 7*a*b*c^3*d^
4 + 8*a^2*c^2*d^5)*e - 2*(24*b^2*c^5*d^2 - 4*a*b*c^4*d^3 - a^2*c^3*d^4)*f)
*x^5 + 3*((8*b^2*c^5*d^2 + 7*a*b*c^4*d^3 + 8*a^2*c^3*d^4)*e - 2*(24*b^2*c^
6*d - 4*a*b*c^5*d^2 - a^2*c^4*d^3)*f)*x^3 + ((8*b^2*c^6*d + 7*a*b*c^5*d^2
+ 8*a^2*c^4*d^3)*e - 2*(24*b^2*c^7 - 4*a*b*c^6*d - a^2*c^5*d^2)*f)*x)*sqrt
(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((8*b^2*c^
3*d^4 + 7*a*b*c^2*d^5 + 4*a^2*d^7 + 4*(2*a^2 + a*b)*c*d^6)*e - (48*b^2*c^4
*d^3 - 8*a*b*c^3*d^4 - a^2*c*d^6 - 2*(a^2 - 12*a*b)*c^2*d^5)*f)*x^7 + 3*((
8*b^2*c^4*d^3 + 7*a*b*c^3*d^4 + 4*a^2*c*d^6 + 4*(2*a^2 + a*b)*c^2*d^5)*e -
 (48*b^2*c^5*d^2 - 8*a*b*c^4*d^3 - a^2*c^2*d^5 - 2*(a^2 - 12*a*b)*c^3*d^4)
*f)*x^5 + 3*((8*b^2*c^5*d^2 + 7*a*b*c^4*d^3 + 4*a^2*c^2*d^5 + 4*(2*a^2 + a
*b)*c^3*d^4)*e - (48*b^2*c^6*d - 8*a*b*c^5*d^2 - a^2*c^3*d^4 - 2*(a^2 - 12
*a*b)*c^4*d^3)*f)*x^3 + ((8*b^2*c^6*d + 7*a*b*c^5*d^2 + 4*a^2*c^3*d^4 + 4*
(2*a^2 + a*b)*c^4*d^3)*e - (48*b^2*c^7 - 8*a*b*c^6*d - a^2*c^4*d^3 - 2*(a^
2 - 12*a*b)*c^5*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) + (15*b^2*c^3*d^4*f*x^6 - ((15*b^2*c^3*d^4 + 4*a*b*c^2*d
^5 + 4*a^2*c*d^6)*e - (90*b^2*c^4*d^3 - 21*a*b*c^3*d^4 - a^2*c^2*d^5)*f)*x
^4 - ((20*b^2*c^4*d^3 + 17*a*b*c^3*d^4 + 9*a^2*c^2*d^5)*e - (120*b^2*c^5*d
^2 - 23*a*b*c^4*d^3 - 6*a^2*c^3*d^4)*f)*x^2 - (8*b^2*c^5*d^2 + 7*a*b*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{7/2}}\,dx=\int \frac{\left(b\,x^2+a\right)^{5/2}\left(f\,x^2+e\right)}{\left(d\,x^2+c\right)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{7/2}}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(7/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**2*f*x - 9*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*b*c*d*f*x - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*
b*d**2*e*x - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**2*f*x**3 + 18*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*f*x - 3*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*b**2*c*d*e*x + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2
*c*d*f*x**3 - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*e*x**3 + 4*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*f*x**5 - 12*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**3*c**2*f*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*3*c*d*e*x**3 - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*f*x**5 - 30*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d
**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8
- a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x*
*6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d
*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10)
,x)*a**4*b*c**3*d**4*f - 90*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*
d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d
**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b
**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d
**3*x**8 - b**2*c*d**4*x**10),x)*a**4*b*c**2*d**5*f*x**2 - 90*int((sqrt...
```

3.20 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 490

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \frac{(de-cf)x(a+bx^2)^{5/2}}{7cd\,(c+dx^2)^{7/2}}$$
$$+\frac{(ad(6de+cf)-bc(de+6cf))x(a+bx^2)^{3/2}}{35c^2d^2\,(c+dx^2)^{5/2}}$$
$$-\frac{(5abcd(de-cf)-4a^2d^2(6de+cf)+4b^2c^2(de+6cf))\,x\sqrt{a+bx^2}}{105c^3d^3\,(c+dx^2)^{3/2}}$$
$$+\frac{(ab^2c^2d(9de-16cf)+a^2bcd^2(16de-9cf)-8a^3d^3(6de+cf)+8b^3c^3(de+6cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\right.\right.}{105c^{7/2}d^{7/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{b(5abcd(de-cf)-4a^2d^2(6de+cf)+4b^2c^2(de+6cf))\sqrt{a+bx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{5/2}d^{7/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/7*(-c*f+d*e)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(7/2)+1/35*(a*d*(c*f+6*d*e)
-b*c*(6*c*f+d*e))*x*(b*x^2+a)^(3/2)/c^2/d^2/(d*x^2+c)^(5/2)-1/105*(5*a*b*c
*d*(-c*f+d*e)-4*a^2*d^2*(c*f+6*d*e)+4*b^2*c^2*(6*c*f+d*e))*x*(b*x^2+a)^(1/
2)/c^3/d^3/(d*x^2+c)^(3/2)+1/105*(a*b^2*c^2*d*(-16*c*f+9*d*e)+a^2*b*c*d^2*
(-9*c*f+16*d*e)-8*a^3*d^3*(c*f+6*d*e)+8*b^3*c^3*(6*c*f+d*e))*(b*x^2+a)^(1/
2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(7/2
)/d^(7/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105
*b*(5*a*b*c*d*(-c*f+d*e)-4*a^2*d^2*(c*f+6*d*e)+4*b^2*c^2*(6*c*f+d*e))*(b*x
^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c
^(5/2)/d^(7/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)-3c^2(bc-ad)^2(bc(9de-16cf)\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f) - 3*c^2*(b*c
- a*d)^2*(b*c*(9*d*e - 16*c*f) + a*d*(6*d*e + c*f))*(c + d*x^2) + c*(b*c -
 a*d)*(b^2*c^2*(8*d*e - 57*c*f) + 4*a^2*d^2*(6*d*e + c*f) + a*b*c*d*(13*d*
e + 8*c*f))*(c + d*x^2)^2 + (a*b^2*c^2*d*(9*d*e - 16*c*f) + a^2*b*c*d^2*(1
6*d*e - 9*c*f) - 8*a^3*d^3*(6*d*e + c*f) + 8*b^3*c^3*(d*e + 6*c*f))*(c + d
*x^2)^3) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*((a
*b^2*c^2*d*(9*d*e - 16*c*f) + a^2*b*c*d^2*(16*d*e - 9*c*f) - 8*a^3*d^3*(6*
d*e + c*f) + 8*b^3*c^3*(d*e + 6*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - (b*c - a*d)*(4*a^2*d^2*(6*d*e + c*f) + 8*b^2*c^2*(d*e + 6*c*f
) + a*b*c*d*(13*d*e + 8*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)]))/(105*Sqrt[b/a]*c^4*d^4*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {401, 25, 401, 25, 401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(a+bx^2)^{5/2}(de-cf)}{7cd(c+dx^2)^{7/2}}-\frac{\int -\frac{(bx^2+a)^{3/2}(b(de+6cf)x^2+a(6de+cf))}{(dx^2+c)^{7/2}}dx}{7cd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(b(de+6cf)x^2+a(6de+cf))}{(dx^2+c)^{7/2}}dx}{7cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x(a+bx^2)^{3/2}(ad(cf+6de)-bc(6cf+de))}{5cd(c+dx^2)^{5/2}}-\frac{\int -\frac{\sqrt{bx^2+a}\left(b(ad(6de+cf)+4bc(de+6cf))x^2+a(4ad(6de+cf)+bc(de+6cf))\right)}{\left(dx^2+c\right)^{5/2}}dx}{5cd}}{7cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(b(ad(6de+cf)+4bc(de+6cf))x^2+a(4ad(6de+cf)+bc(de+6cf))\right)}{\left(dx^2+c\right)^{5/2}}dx}{5cd}+\frac{x(a+bx^2)^{3/2}(ad(cf+6de)-bc(6cf+de))}{5cd(c+dx^2)^{5/2}}}{7cd}+\frac{x(a+bx^2)^{5/2}(de-cf)}{7cd(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{-\frac{-\frac{\int -\frac{b\left(8b^2(de+6cf)c^2+abd(13de+8cf)c+4a^2d^2(6de+cf)\right)x^2+a\left(4b^2(de+6cf)c^2+abd(8de+13cf)c+8a^2d^2(6de+cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}\left(-\frac{4a^2d(cf+6de)}{c}+5ab(de\cdots\right.}{3\left(c+dx^2\right)^{3/2}}}{5cd}}{7cd}\cdots$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\downarrow 25$$

$$\frac{\frac{\frac{\int \frac{b\left(8b^2(de+6cf)c^2+abd(13de+8cf)c+4a^2d^2(6de+cf)\right)x^2+a\left(4b^2(de+6cf)c^2+abd(8de+13cf)c+8a^2d^2(6de+cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}\left(-\frac{4a^2d(cf+6de)}{c}+5ab(de-cf\cdots\right.}{3\left(c+dx^2\right)^{3/2}}}{5cd}}{7cd}\cdots$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\downarrow 400$$

$$\frac{\frac{\frac{\frac{\left(-8a^3d^3(cf+6de)+a^2bcd^2(16de-9cf)+ab^2c^2d(9de-16cf)+8b^3c^3(6cf+de)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(-4a^2d^2(cf+6de)+5abcd(de-cf)+4b^2c^2(6cf+de)\right)\int \frac{\cdots}{\sqrt{bx\cdots}}}{bc-ad}}{3cd}\cdots}{5cd}\cdots}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\downarrow 313$$

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(-8a^3d^3(cf+6de)+a^2bcd^2(16de-9cf)+ab^2c^2d(9de-16cf)+8b^3c^3(6cf+de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab\left(-4a^2d^2(cf+6de)+5abcd(de-cf)+4b^2c^2(\cdots\right.}{bc-ad}}{3cd}\cdots}{5cd}\cdots}{7cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

$$\downarrow 320$$

$$\frac{\frac{\frac{\sqrt{a+bx^2}\left(-8a^3d^3(cf+6de)+a^2bcd^2(16de-9cf)+ab^2c^2d(9de-16cf)+8b^3c^3(6cf+de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(-4a^2d^2(cf+6de)+5abcd(de-c\ldots\right.}{\sqrt{d}\sqrt{c+dx^2}(\ldots}}{3cd}}{5cd}$$

$$\frac{x\left(a+bx^2\right)^{5/2}(de-cf)}{7cd\left(c+dx^2\right)^{7/2}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x]
```

output
```
((d*e - c*f)*x*(a + b*x^2)^(5/2))/(7*c*d*(c + d*x^2)^(7/2)) + (((a*d*(6*d*
e + c*f) - b*c*(d*e + 6*c*f))*x*(a + b*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2
)) + (-1/3*((5*a*b*(d*e - c*f) - (4*a^2*d*(6*d*e + c*f))/c + (4*b^2*c*(d*e
 + 6*c*f))/d)*x*Sqrt[a + b*x^2])/(c + d*x^2)^(3/2) + (((a*b^2*c^2*d*(9*d*e
 - 16*c*f) + a^2*b*c*d^2*(16*d*e - 9*c*f) - 8*a^3*d^3*(6*d*e + c*f) + 8*b^
3*c^3*(d*e + 6*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(5*a*b*c*d*(d*e - c*f) - 4*a^2*
d^2*(6*d*e + c*f) + 4*b^2*c^2*(d*e + 6*c*f))*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*d))/(5*c*d))/(7*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(455) = 910$.

Time = 10.48 (sec) , antiderivative size = 1061, normalized size of antiderivative = 2.17

method	result	size
elliptic	Expression too large to display	1061
default	Expression too large to display	5139

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*(a^2*c*d
^2*f-a^2*d^3*e-2*a*b*c^2*d*f+2*a*b*c*d^2*e+b^2*c^3*f-b^2*c^2*d*e)/c/d^7*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^4+1/35*(a^2*c*d^2*f+6*a^2*d^
3*e-17*a*b*c^2*d*f+3*a*b*c*d^2*e+16*b^2*c^3*f-9*b^2*c^2*d*e)/c^2/d^6*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/105*(4*a^2*c*d^2*f+24*a^2*d
^3*e+8*a*b*c^2*d*f+13*a*b*c*d^2*e-57*b^2*c^3*f+8*b^2*c^2*d*e)/c^3/d^5*x*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/105*(b*d*x^2+a*d)/c^4/d^4/
(a*d-b*c)*x*(8*a^3*c*d^3*f+48*a^3*d^4*e+9*a^2*b*c^2*d^2*f-16*a^2*b*c*d^3*e
+16*a*b^2*c^3*d*f-9*a*b^2*c^2*d^2*e-48*b^3*c^4*f-8*b^3*c^3*d*e)/((x^2+c/d)
*(b*d*x^2+a*d))^(1/2)+(b^3*f/d^4+1/105*b*(4*a^2*c*d^2*f+24*a^2*d^3*e+8*a*b
*c^2*d*f+13*a*b*c*d^2*e-57*b^2*c^3*f+8*b^2*c^2*d*e)/c^3/d^4+1/105/d^4*(8*a
^3*c*d^3*f+48*a^3*d^4*e+9*a^2*b*c^2*d^2*f-16*a^2*b*c*d^3*e+16*a*b^2*c^3*d*
f-9*a*b^2*c^2*d^2*e-48*b^3*c^4*f-8*b^3*c^3*d*e)/c^4-1/105*a/d^3/c^4/(a*d-b
*c)*(8*a^3*c*d^3*f+48*a^3*d^4*e+9*a^2*b*c^2*d^2*f-16*a^2*b*c*d^3*e+16*a*b^
2*c^3*d*f-9*a*b^2*c^2*d^2*e-48*b^3*c^4*f-8*b^3*c^3*d*e))/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/105/d^4*b*(8*a^3*c*d^3*f+48
*a^3*d^4*e+9*a^2*b*c^2*d^2*f-16*a^2*b*c*d^3*e+16*a*b^2*c^3*d*f-9*a*b^2*c^2
*d^2*e-48*b^3*c^4*f-8*b^3*c^3*d*e)/(a*d-b*c)/c^3/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(455) = 910$.

Time = 0.19 (sec) , antiderivative size = 1935, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*((((8*b^4*c^3*d^5 + 9*a*b^3*c^2*d^6 + 16*a^2*b^2*c*d^7 - 48*a^3*b*d
^8)*e + (48*b^4*c^4*d^4 - 16*a*b^3*c^3*d^5 - 9*a^2*b^2*c^2*d^6 - 8*a^3*b*c
*d^7)*f)*x^8 + 4*((8*b^4*c^4*d^4 + 9*a*b^3*c^3*d^5 + 16*a^2*b^2*c^2*d^6 -
48*a^3*b*c*d^7)*e + (48*b^4*c^5*d^3 - 16*a*b^3*c^4*d^4 - 9*a^2*b^2*c^3*d^5
 - 8*a^3*b*c^2*d^6)*f)*x^6 + 6*((8*b^4*c^5*d^3 + 9*a*b^3*c^4*d^4 + 16*a^2*
b^2*c^3*d^5 - 48*a^3*b*c^2*d^6)*e + (48*b^4*c^6*d^2 - 16*a*b^3*c^5*d^3 - 9
*a^2*b^2*c^4*d^4 - 8*a^3*b*c^3*d^5)*f)*x^4 + 4*((8*b^4*c^6*d^2 + 9*a*b^3*c
^5*d^3 + 16*a^2*b^2*c^4*d^4 - 48*a^3*b*c^3*d^5)*e + (48*b^4*c^7*d - 16*a*b
^3*c^6*d^2 - 9*a^2*b^2*c^5*d^3 - 8*a^3*b*c^4*d^4)*f)*x^2 + (8*b^4*c^7*d +
9*a*b^3*c^6*d^2 + 16*a^2*b^2*c^5*d^3 - 48*a^3*b*c^4*d^4)*e + (48*b^4*c^8 -
 16*a*b^3*c^7*d - 9*a^2*b^2*c^6*d^2 - 8*a^3*b*c^5*d^3)*f)*sqrt(a*c)*sqrt(-
b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((8*b^4*c^3*d^5 + (4*a
^2*b^2 + 9*a*b^3)*c^2*d^6 + (5*a^3*b + 16*a^2*b^2)*c*d^7 - 24*(a^4 + 2*a^3
*b)*d^8)*e + (48*b^4*c^4*d^4 + 8*(3*a^2*b^2 - 2*a*b^3)*c^3*d^5 - (5*a^3*b
+ 9*a^2*b^2)*c^2*d^6 - 4*(a^4 + 2*a^3*b)*c*d^7)*f)*x^8 + 4*((8*b^4*c^4*d^4
 + (4*a^2*b^2 + 9*a*b^3)*c^3*d^5 + (5*a^3*b + 16*a^2*b^2)*c^2*d^6 - 24*(a^
4 + 2*a^3*b)*c*d^7)*e + (48*b^4*c^5*d^3 + 8*(3*a^2*b^2 - 2*a*b^3)*c^4*d^4
- (5*a^3*b + 9*a^2*b^2)*c^3*d^5 - 4*(a^4 + 2*a^3*b)*c^2*d^6)*f)*x^6 + 6*((
8*b^4*c^5*d^3 + (4*a^2*b^2 + 9*a*b^3)*c^4*d^4 + (5*a^3*b + 16*a^2*b^2)*c^3
*d^5 - 24*(a^4 + 2*a^3*b)*c^2*d^6)*e + (48*b^4*c^6*d^2 + 8*(3*a^2*b^2 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input

```
int((b*x^2+a)^(5/2)*(f*x^2+e)/(d*x^2+c)^(9/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**2*f*x - 3*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**2*b*c*d*f*x - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
2*b*d**2*e*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**2*f*x**3 - 18
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*f*x - 3*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b**2*c*d*e*x - 34*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**
2*c*d*f*x**3 - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*e*x**3 - 18
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*f*x**5 + 12*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**3*c**2*f*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c*d*e*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*f*x**5 + 45
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**
4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*
d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*
d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*
x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x**4 - 10*b**2*c
**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x**10 - b**2*c*d
**5*x**12),x)*a**4*b*c**4*d**4*f + 180*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4
 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*
c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 +
25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c...
```

3.21 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 386

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{\left(20bce-10ade-13acf+\frac{3bc^2f}{d}+\frac{8a^2df}{b}\right)x\sqrt{c+dx^2}}{15b\sqrt{a+bx^2}} \\ &+ \frac{(5bde+3bcf-4adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2} + \frac{fx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5b} \\ &- \frac{\sqrt{a}(8a^2d^2f+b^2c(20de+3cf)-abd(10de+13cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{\sqrt{a}(15b^2ce-5abde-6abcf+4a^2df)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/15*(20*b*c*e-10*a*d*e-13*a*c*f+3*b*c^2*f/d+8*a^2*d*f/b)*x*(d*x^2+c)^(1/2
)/b/(b*x^2+a)^(1/2)+1/15*(-4*a*d*f+3*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)/b^2+1/5*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/b-1/15*a^(1/2)*(8*
a^2*d^2*f+b^2*c*(3*c*f+20*d*e)-a*b*d*(13*c*f+10*d*e))*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^(1/2)*(4*a^2*d*f-6*a*b
*c*f-5*a*b*d*e+15*b^2*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*
x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(4adf-b(5de+6cf+3dfx^2))-ic(8a^2d^2f+b}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d*f - b*(5*d*e + 6*c*f + 3*d
*f*x^2))) - I*c*(8*a^2*d^2*f + b^2*c*(20*d*e + 3*c*f) - a*b*d*(10*d*e + 13
*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(-5*b*d*e - 3*b*c*f + 4*a*d*f)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{\sqrt{a+bx^2}}\,dx$$

$\Big\downarrow$ 403

$$\frac{\int \frac{\sqrt{dx^2+c}\left((5bde+3bcf-4adf)x^2+c(5be-af)\right)}{\sqrt{bx^2+a}}dx}{5b}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 403

$$\frac{\frac{\int \frac{\left(c(20de+3cf)b^2-ad(10de+13cf)b+8a^2d^2f\right)x^2+c\left(4dfa^2-5bdea-6bcfa+15b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+3bcf+5bde)}{3b}}{5b}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 406

$$\frac{\frac{\left(8a^2d^2f-abd(13cf+10de)+b^2c(3cf+20de)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+c\left(4a^2df-6abcf-5abde+15b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{5b}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 320

$$\frac{\frac{\left(8a^2d^2f-abd(13cf+10de)+b^2c(3cf+20de)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2df-6abcf-5abde+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\cdots}{5b}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 388

$$\frac{\frac{\left(8a^2d^2f-abd(13cf+10de)+b^2c(3cf+20de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2df-6abcf-5abde+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\cdots\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}\cdots}{5b}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2df-6abcf-5abde+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^2d^2f-abd(13cf+10de)+b^2c(3cf+20de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E}{b\sqrt{d}\sqrt{c}}\right.}{3b}}{5b}$$

$$\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5b}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*b) + (((5*b*d*e + 3*b*c*f - 4*a
*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) + ((8*a^2*d^2*f + b^2*c*(20
*d*e + 3*c*f) - a*b*d*(10*d*e + 13*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c +
d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (c^(3/2)*(15*b^2*c*e - 5*a*b*d*e - 6*a*b*c*f + 4*a^2*d*f)*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/(5*b
)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 10.26 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{df\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5b}+\frac{\left(2cdf+d^2e-\frac{df(4ad+4bc)}{5b}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}+\frac{\left(c^2e-\frac{\left(2cdf+d^2e-\frac{df(4ad+4b}{5b}\right.}{3bd}\right.}{}\right.}{}$
risch	$-\frac{x\left(-3bdf\,x^2+4adf-6bcf-5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^2}+\frac{\left(-\frac{\left(8f\,d^2a^2-13f\,dcba-10ab\,d^2e+3f\,c^2b^2+20d\,b^2ce\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{Ellip}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+}}\right.}{}$
default	$-\frac{\sqrt{x^2d+c}\,\sqrt{b\,x^2+a}\,\left(-3\sqrt{-\frac{b}{a}}\,b^2d^3f\,x^7+\sqrt{-\frac{b}{a}}\,ab\,d^3f\,x^5-9\sqrt{-\frac{b}{a}}\,b^2c\,d^2f\,x^5-5\sqrt{-\frac{b}{a}}\,b^2d^3e\,x^5+4\sqrt{-\frac{b}{a}}\,a^2d^3f\,x^3-5\sqrt{-\frac{b}{a}}\,abc\,d^2\right.}{}$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b*d*f*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2*c*d*f+d^2*e-1/5/b*d*f*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c^2*e-1/3*(2*c*d*f+d^2*e
-1/5/b*d*f*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(c^2*f+2*c*e*d-3/5*a/b*c*d*f-1/3*(2*c*d*f+d^2*e-1/5
/b*d*f*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{\sqrt{bd}(10\,(2\,b^2c^2d-abcd^2)e+(3\,b^2c^3-13\,abc^2d+8\,a^2cd^2)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(5\,(4\,b^2c}{}$$

input

```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output

```
-1/15*(sqrt(b*d)*(10*(2*b^2*c^2*d - a*b*c*d^2)*e + (3*b^2*c^3 - 13*a*b*c^2
*d + 8*a^2*c*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - sqrt(b*d)*(5*(4*b^2*c^2*d - a*b*d^3 - (2*a*b - 3*b^2)*c*d^2)*e + (3*
b^2*c^3 - 13*a*b*c^2*d + 4*a^2*d^3 + 2*(4*a^2 - 3*a*b)*c*d^2)*f)*x*sqrt(-c
/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*f*x^4 + (5*b^
2*d^3*e + 2*(3*b^2*c*d^2 - 2*a*b*d^3)*f)*x^2 + 10*(2*b^2*c*d^2 - a*b*d^3)*
e + (3*b^2*c^2*d - 13*a*b*c*d^2 + 8*a^2*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/(b^3*d^2*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx+6\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx+5\sqrt{d\,x^2+c}\,\sqrt{b}}{}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + 6*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f - 13
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
 b*d*x**4),x)*a*b*c*d*f - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e + 3*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2
*c**2*f + 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
+ b*c*x**2 + b*d*x**4),x)*b**2*c*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f - 6*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*
b*c**2*f - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c
*x**2 + b*d*x**4),x)*a*b*c*d*e + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2*e)/(15*b**2)
```

3.22 $\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 281

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(3bde+bcf-2adf)x\sqrt{c+dx^2}}{3bd\sqrt{a+bx^2}}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} - \frac{\sqrt{a}(3bde+bcf-2adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}(3be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(-2*a*d*f+b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)+1/3*f*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b-1/3*a^(1/2)*(-2*a*d*f+b*c*f+3*b*d*e)*(d
*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)
*(-a*f+3*b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/b^(3/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{ic(2adf-b(3de+cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+f\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)-ic(\right.}{3b\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(I*c*(2*a*d*f - b*(3*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + f*(Sqrt[b/a]*d*x*(a + b*x^
2)*(c + d*x^2) - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*Sqrt[b/a]*d*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \frac{(3bde+bcf-2adf)x^2+c(3be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{c(3be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-2adf+bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{}$$

$$\Big\downarrow \; 320$$

$$\frac{(-2adf+bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3b}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$$\Big\downarrow \; 388$$

$$\frac{(-2adf+bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3b}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-2adf+bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) + ((3*b*d*e + b*c*f - 2*a*d*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*e - a*f)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3b}+\frac{\left(ce-\frac{acf}{3b}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\left(cf+de-\frac{f(2ad+2bc)}{3b}\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
risch	$\frac{f x\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3b}-\frac{\left(-\frac{(2adf-bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	$\frac{\sqrt{x^2d+c}\,\sqrt{b\,x^2+a}\left(\sqrt{-\frac{b}{a}}\,b\,d^2f\,x^5+\sqrt{-\frac{b}{a}}\,a\,d^2f\,x^3+\sqrt{-\frac{b}{a}}\,bcdf\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-\sqrt{\frac{b\,x^2+a}{a}}\right.}{}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*f/b*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c*e-1/3*a*c*f/b)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(c*f+d*e-1/3*f/b*(2*a*d+2*b*c))*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{(3\,bcde+(bc^2-2\,acd)f)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(3\,(bcd+bd^2)e+(bc^2-2\,acd-ad}{3\,b^2d^2x}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((3*b*c*d*e + (b*c^2 - 2*a*c*d)*f)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(
arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(3*(b*c*d + b*d^2)*e + (b*c^2
 - 2*a*c*d - a*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(
b*c)) - (b*d^2*f*x^2 + 3*b*d^2*e + (b*c*d - 2*a*d^2)*f)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c))/(b^2*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx - 2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right) adf + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right) bcf + 3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{}}{bd\,x^4+ad\,x^2}\right.}{3b}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f + int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),
x)*b*c*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
+ b*c*x**2 + b*d*x**4),x)*b*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f + 3*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*c*e)/(3*b)
```

3.23 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{fx\sqrt{c+dx^2}}{d\sqrt{a+bx^2}} - \frac{\sqrt{a}f\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}e\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f*x*(d*x^2+c)^(1/2)/d/(b*x^2+a)^(1/2)-a^(1/2)*f*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*e*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(de-cf)\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{406}$$

$$e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 388$$

$$f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf+\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf\right)\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{d\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\,\bigg(\frac{e\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{fc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\bigg(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{El}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f+EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*d*e+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f)*((d*x^2
+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(-b/a)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}c^2fx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}cdf - (d^2e+c^2f)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\cdots\right.}{bcd^2x}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c^2*f*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*d*f - (d^2*e + c^2*f)*sqrt(b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*e
```

3.24 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=-\frac{(de-cf)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{\sqrt{c}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-(-c*f+d*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))/c^(1/2)/d^(1/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+c^(1/2)*(-a*f+b*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(1/2)/(-a*d+b*c)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)-ibc(-de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\cdots}{\sqrt{\frac{b}{a}}cd(-bc+ad\cdots}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{400}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \text{313}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
-(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [A] (verified)

Time = 7.76 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bcdf\, x^3+\sqrt{-\frac{b}{a}}\, b\, d^2 e\, x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) acdf-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.}{\sqrt{-\frac{b}{a}}\, c}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{(bd\,x^2+ad)x(cf-de)}{cd(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*c*d*f*x^3+(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f-((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*b*c^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e-(-b/a)^(1/2)*a*c*d*f*x+(-b/a)^(1
/2)*a*d^2*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/d/c/(a*d-b*c)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.24

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{(abd^2e-abcdf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)}{ab^2c^3d-a^2bc^2d^2}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-((a*b*d^2*e - a*b*c*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (b^2*c*d*e -
 b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((a*b + b^2)*c*d*e + ((a*b + b^2)*d^2*e
 - (b^2*c*d + a^2*d^2)*f)*x^2 - (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(-b/a
)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*b^2*c^3*d - a^2*b*c^2*d^
2 + (a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right) f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right) e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*e
```

3.25 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}}{3c(bc-ad)\,(c+dx^2)^{3/2}} - \frac{(bc(4de-cf)-ad(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b(3bce-ade-2acf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-
c*f+4*d*e)-a*d*(c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(1/2)/(-a*d+b*c)^2/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b*(-2*a*c*f-a*d*e+3*b*c*e)*(b*x^
2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/
c^(1/2)/d^(1/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.76 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.04

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(ad^2(3ce+2dex^2+cfx^2)+bc(-5cde+2c^2f-4d^2ex^2+c$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d^2*(3*c*e + 2*d*e*x^2 + c*f*x^2) + b*c*(-5*
c*d*e + 2*c^2*f - 4*d^2*e*x^2 + c*d*f*x^2)) + I*b*c*(b*c*(-4*d*e + c*f) +
a*d*(2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Ell
ipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*(-(d*e)
 + c*f)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*(b*c - a*d)^2*Sqrt[a
+ b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{\int \frac{-b(de-cf)x^2+3bce-2ade-acf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{3c\,(c+dx^2)^{3/2}\,(bc-ad)}$$

$$\Big\downarrow \text{400}$$

$$\frac{\frac{b(-2acf-ade+3bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(bc(4de-cf)-ad(cf+2de))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 313$$

$$\frac{\frac{b(-2acf-ade+3bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(bc(4de-cf)-ad(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 320$$

$$\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-2acf-ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc(4de-cf)-ad(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*((d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) + (
-(((b*c*(4*d*e - c*f) - a*d*(2*d*e + c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(3*b*c*e
 - a*d*e - 2*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(265) = 530$.

Time = 9.92 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{x(cf-de)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,d^2(ad-bc)\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)x\left(acdf+2a\,d^2e+b\,c^2f-4bcde\right)}{3c^2d(ad-bc)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(-\frac{b(cf-de)}{3d(ad-bc)c}+\frac{acdf+2a\,d^2e+b\,c^2}{3(ad-bc)d}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c/d^2/(a
*d-b*c)*x*(c*f-d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/3*(b
*d*x^2+a*d)/c^2/d/(a*d-b*c)^2*x*(a*c*d*f+2*a*d^2*e+b*c^2*f-4*b*c*d*e)/((x^
2+c/d)*(b*d*x^2+a*d))^(1/2)+(-1/3*b/d*(c*f-d*e)/(a*d-b*c)/c+1/3/(a*d-b*c)/
d*(a*c*d*f+2*a*d^2*e+b*c^2*f-4*b*c*d*e)/c^2-1/3*a/c^2/(a*d-b*c)^2*(a*c*d*f
+2*a*d^2*e+b*c^2*f-4*b*c*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))+1/3*b*(a*c*d*f+2*a*d^2*e+b*c^2*f-4*b*c*d*e)/(a*d-b*c)^2
/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(265) = 530$.

Time = 0.10 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.15

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \frac{((2\,(2\,b^2cd^3-abd^4)e-(b^2c^2d^2+abcd^3)f)x^4+2\,(2\,(2\,b^2c^2d^2-abcd^3)e-(b}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((2*(2*b^2*c*d^3 - a*b*d^4)*e - (b^2*c^2*d^2 + a*b*c*d^3)*f)*x^4 + 2*
(2*(2*b^2*c^2*d^2 - a*b*c*d^3)*e - (b^2*c^3*d + a*b*c^2*d^2)*f)*x^2 + 2*(2
*b^2*c^3*d - a*b*c^2*d^2)*e - (b^2*c^4 + a*b*c^3*d)*f)*sqrt(a*c)*sqrt(-b/a
)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((((3*a*b + 4*b^2)*c*d^3 -
 (a^2 + 2*a*b)*d^4)*e - (b^2*c^2*d^2 + (2*a^2 + a*b)*c*d^3)*f)*x^4 + 2*(((
3*a*b + 4*b^2)*c^2*d^2 - (a^2 + 2*a*b)*c*d^3)*e - (b^2*c^3*d + (2*a^2 + a*
b)*c^2*d^2)*f)*x^2 + ((3*a*b + 4*b^2)*c^3*d - (a^2 + 2*a*b)*c^2*d^2)*e - (
b^2*c^4 + (2*a^2 + a*b)*c^3*d)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x
*sqrt(-b/a)), a*d/(b*c)) - ((2*(2*a*b*c*d^3 - a^2*d^4)*e - (a*b*c^2*d^2 +
a^2*c*d^3)*f)*x^3 - (2*a*b*c^3*d*f - (5*a*b*c^2*d^2 - 3*a^2*c*d^3)*e)*x)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d
^3 + (a*b^2*c^4*d^3 - 2*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^4 + 2*(a*b^2*c^5*d^
2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^3x^8+a\,d^3x^6+3bc\,d^2x^6+3ac\,d^2x^4+3b\,c^2d\,x^4+3a\,c^2d\,x^2+b\,c^3x^2+}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^3x^8+a\,d^3x^6+3bc\,d^2x^6+3ac\,d^2x^4+3b\,c^2d\,x^4+3a\,c^2d\,x^2+b\,c^3x^2+a\,c^3}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**3 + 3*a*c**2*d*x**2 + 3
*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*
x**6 + b*d**3*x**8),x)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**3
 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**
2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*e
```

3.26 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 397

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}\,dx &= -\frac{(de-cf)x\sqrt{a+bx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}}\\ &- \frac{(bc(8de-3cf)-ad(4de+cf))x\sqrt{a+bx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}\\ &- \frac{(b^2c^2(23de-3cf)+2a^2d^2(4de+cf)-abcd(23de+7cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &+ \frac{b(15b^2c^2e+a^2d(4de+cf)-abc(11de+9cf))\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15ac^{3/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/5*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(5/2)-1/15*(b*c*(
-3*c*f+8*d*e)-a*d*(c*f+4*d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c
)^(3/2)-1/15*(b^2*c^2*(-3*c*f+23*d*e)+2*a^2*d^2*(c*f+4*d*e)-a*b*c*d*(7*c*f
+23*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/c^(5/2)/d^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+1/15*b*(15*b^2*c^2*e+a^2*d*(c*f+4*d*e)-a*b*c*(9*c*f+11
*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/
d)^(1/2))/a/c^(3/2)/d^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.94

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\Big(3c^2(bc-ad)^2(de-cf)-c(bc-ad)(ad(4de+cf)+bc(}{}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f) - c*(b*c - a
*d)*(a*d*(4*d*e + c*f) + b*c*(-8*d*e + 3*c*f))*(c + d*x^2) + (b^2*c^2*(23*
d*e - 3*c*f) + 2*a^2*d^2*(4*d*e + c*f) - a*b*c*d*(23*d*e + 7*c*f))*(c + d*
x^2)^2)) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((-
2*a^2*d^2*(4*d*e + c*f) + b^2*c^2*(-23*d*e + 3*c*f) + a*b*c*d*(23*d*e + 7*
c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(a*d*(4
*d*e + c*f) + b*c*(-8*d*e + 3*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(15*Sqrt[b/a]*c^3*d*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(
5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {402, 402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}dx$$

$$\Big\downarrow \; 402$$

$$\frac{\int \frac{-3b(de-cf)x^2+5bce-4ade-acf}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{5c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{2d(4de+cf)a^2-bc(19de+6cf)a-b(bc(8de-3cf)-ad(4de+cf))x^2+15b^2c^2e}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(bc(8de-3cf)-ad(cf+4de))}{3c(c+dx^2)^{3/2}(bc-ad)}}{5c(bc-ad)}-\frac{x\sqrt{a+bx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\frac{b\left(a^2d(cf+4de)-abc(9cf+11de)+15b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\left(2a^2d^2(cf+4de)-abcd(7cf+23de)+b^2c^2(23de-3cf)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c(bc-ad)}-\frac{x\sqrt{a+bx^2}(bc}{3c(c}}{5c(bc-ad)}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{b\left(a^2d(cf+4de)-abc(9cf+11de)+15b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(cf+4de)-abcd(7cf+23de)+b^2c^2(23de-3cf)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-}{5c(bc-ad)}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(a^2d(cf+4de)-abc(9cf+11de)+15b^2c^2e\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(cf+4de)-abcd(7cf+23de)+b^2c^2(23de-3cf)\right)E\left(\arc\ldots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}}{5c(bc-ad)}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
-1/5*((d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) + (
-1/3*((b*c*(8*d*e - 3*c*f) - a*d*(4*d*e + c*f))*x*Sqrt[a + b*x^2])/(c*(b*c
 - a*d)*(c + d*x^2)^(3/2)) + (-(((b^2*c^2*(23*d*e - 3*c*f) + 2*a^2*d^2*(4*
d*e + c*f) - a*b*c*d*(23*d*e + 7*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(15*b^2*c^2*
e + a^2*d*(4*d*e + c*f) - a*b*c*(11*d*e + 9*c*f))*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d)))/(
5*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(368) = 736$.

Time = 12.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{x(cf-de)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c\,d^3(ad-bc)\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(acdf+4a\,d^2e+3b\,c^2f-8bcde\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15(ad-bc)^2c^2d^2\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)x\left(2a^2cf\,c\right.}{15c}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/c/d^3/(a
*d-b*c)*x*(c*f-d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/15*(
a*c*d*f+4*a*d^2*e+3*b*c^2*f-8*b*c*d*e)/(a*d-b*c)^2/c^2/d^2*x*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d/(a*d-b*c)^3*x*
(2*a^2*c*d^2*f+8*a^2*d^3*e-7*a*b*c^2*d*f-23*a*b*c*d^2*e-3*b^2*c^3*f+23*b^2
*c^2*d*e)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/15*b*(a*c*d*f+4*a*d^2*e+3*b*c
^2*f-8*b*c*d*e)/c^2/d/(a*d-b*c)^2+1/15/(a*d-b*c)^2/d*(2*a^2*c*d^2*f+8*a^2*
d^3*e-7*a*b*c^2*d*f-23*a*b*c*d^2*e-3*b^2*c^3*f+23*b^2*c^2*d*e)/c^3-1/15*a/
c^3/(a*d-b*c)^3*(2*a^2*c*d^2*f+8*a^2*d^3*e-7*a*b*c^2*d*f-23*a*b*c*d^2*e-3*
b^2*c^3*f+23*b^2*c^2*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))+1/15*b*(2*a^2*c*d^2*f+8*a^2*d^3*e-7*a*b*c^2*d*f-23*a*b*c*d^
2*e-3*b^2*c^3*f+23*b^2*c^2*d*e)/(a*d-b*c)^3/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(368) = 736$.

Time = 0.13 (sec) , antiderivative size = 1292, normalized size of antiderivative = 3.25

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*((((23*b^3*c^2*d^4 - 23*a*b^2*c*d^5 + 8*a^2*b*d^6)*e - (3*b^3*c^3*d^3
 + 7*a*b^2*c^2*d^4 - 2*a^2*b*c*d^5)*f)*x^6 + 3*((23*b^3*c^3*d^3 - 23*a*b^2
*c^2*d^4 + 8*a^2*b*c*d^5)*e - (3*b^3*c^4*d^2 + 7*a*b^2*c^3*d^3 - 2*a^2*b*c
^2*d^4)*f)*x^4 + 3*((23*b^3*c^4*d^2 - 23*a*b^2*c^3*d^3 + 8*a^2*b*c^2*d^4)*
e - (3*b^3*c^5*d + 7*a*b^2*c^4*d^2 - 2*a^2*b*c^3*d^3)*f)*x^2 + (23*b^3*c^5
*d - 23*a*b^2*c^4*d^2 + 8*a^2*b*c^3*d^3)*e - (3*b^3*c^6 + 7*a*b^2*c^5*d -
2*a^2*b*c^4*d^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)),
a*d/(b*c)) - ((((15*a*b^2 + 23*b^3)*c^2*d^4 - (11*a^2*b + 23*a*b^2)*c*d^5
+ 4*(a^3 + 2*a^2*b)*d^6)*e - (3*b^3*c^3*d^3 + (9*a^2*b + 7*a*b^2)*c^2*d^4
- (a^3 + 2*a^2*b)*c*d^5)*f)*x^6 + 3*(((15*a*b^2 + 23*b^3)*c^3*d^3 - (11*a^
2*b + 23*a*b^2)*c^2*d^4 + 4*(a^3 + 2*a^2*b)*c*d^5)*e - (3*b^3*c^4*d^2 + (9
*a^2*b + 7*a*b^2)*c^3*d^3 - (a^3 + 2*a^2*b)*c^2*d^4)*f)*x^4 + 3*(((15*a*b^
2 + 23*b^3)*c^4*d^2 - (11*a^2*b + 23*a*b^2)*c^3*d^3 + 4*(a^3 + 2*a^2*b)*c^
2*d^4)*e - (3*b^3*c^5*d + (9*a^2*b + 7*a*b^2)*c^4*d^2 - (a^3 + 2*a^2*b)*c^
3*d^3)*f)*x^2 + ((15*a*b^2 + 23*b^3)*c^5*d - (11*a^2*b + 23*a*b^2)*c^4*d^2
 + 4*(a^3 + 2*a^2*b)*c^3*d^3)*e - (3*b^3*c^6 + (9*a^2*b + 7*a*b^2)*c^5*d -
 (a^3 + 2*a^2*b)*c^4*d^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt
(-b/a)), a*d/(b*c)) - (((23*a*b^2*c^2*d^4 - 23*a^2*b*c*d^5 + 8*a^3*d^6)*e
- (3*a*b^2*c^3*d^3 + 7*a^2*b*c^2*d^4 - 2*a^3*c*d^5)*f)*x^5 + (2*(27*a*b^2*
c^3*d^3 - 29*a^2*b*c^2*d^4 + 10*a^3*c*d^5)*e - (9*a*b^2*c^4*d^2 + 12*a^...
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx=\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

Maxima [F]

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{7/2}}\,dx = \int \frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{7/2}}\,dx = \int \frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{7/2}}\,dx = \int \frac{f\,x^2 + e}{\sqrt{b\,x^2 + a}\,{(d\,x^2 + c)}^{7/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^4x^{10}+a\,d^4x^8+4bc\,d^3x^8+4ac\,d^3x^6+6b\,c^2d^2x^6+6a\,c^2d^2x^4+4b\,c^3d\,x^4+4a\,c^3d\,x^2+b\,c^4x^2+a\,c^4}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^4x^{10}+a\,d^4x^8+4bc\,d^3x^8+4ac\,d^3x^6+6b\,c^2d^2x^6+6a\,c^2d^2x^4+4b\,c^3d\,x^4+4a\,c^3d\,x^2+b\,c^4x^2+a\,c^4}\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x**2 + 6
*a*c**2*d**2*x**4 + 4*a*c*d**3*x**6 + a*d**4*x**8 + b*c**4*x**2 + 4*b*c**3
*d*x**4 + 6*b*c**2*d**2*x**6 + 4*b*c*d**3*x**8 + b*d**4*x**10),x)*f + int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**4 + 4*a*c**3*d*x**2 + 6*a*c**2*d
**2*x**4 + 4*a*c*d**3*x**6 + a*d**4*x**8 + b*c**4*x**2 + 4*b*c**3*d*x**4 +
 6*b*c**2*d**2*x**6 + 4*b*c*d**3*x**8 + b*d**4*x**10),x)*e
```

3.27 $\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 409

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(24a^2d^2f+15b^2c(2de+cf)-abd(20de+41cf))\,x\sqrt{c+dx^2}}{15b^3\sqrt{a+bx^2}}$$
$$-\frac{(6adf-5b(de+cf))x(c+dx^2)^{3/2}}{15b^2\sqrt{a+bx^2}}+\frac{fx(c+dx^2)^{5/2}}{5b\sqrt{a+bx^2}}$$
$$+\frac{(15b^3c^2e-48a^3d^2f+8a^2bd(5de+11cf)-ab^2c(65de+38cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15\sqrt{a}b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{\sqrt{a}(24a^2d^2f+15b^2c(2de+cf)-abd(20de+41cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(24*a^2*d^2*f+15*b^2*c*(c*f+2*d*e)-a*b*d*(41*c*f+20*d*e))*x*(d*x^2+c)
^(1/2)/b^3/(b*x^2+a)^(1/2)-1/15*(6*a*d*f-5*b*(c*f+d*e))*x*(d*x^2+c)^(3/2)/
b^2/(b*x^2+a)^(1/2)+1/5*f*x*(d*x^2+c)^(5/2)/b/(b*x^2+a)^(1/2)+1/15*(15*b^3
*c^2*e-48*a^3*d^2*f+8*a^2*b*d*(11*c*f+5*d*e)-a*b^2*c*(38*c*f+65*d*e))*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2
))/a^(1/2)/b^(7/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^
(1/2)*(24*a^2*d^2*f+15*b^2*c*(c*f+2*d*e)-a*b*d*(41*c*f+20*d*e))*(d*x^2+c)^
(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.58 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(15b^3c^2e-24a^3d^2f+a^2bd(20de+41cf-6dfx^2)+ab\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(c + d*x^2)*(15*b^3*c^2*e - 24*a^3*d^2*f + a^2*b*d
*(20*d*e + 41*c*f - 6*d*f*x^2) + a*b^2*(-15*c^2*f + d^2*x^2*(5*e + 3*f*x^2
) + c*d*(-30*e + 11*f*x^2))) - I*c*(-15*b^3*c^2*e + 48*a^3*d^2*f - 8*a^2*b
*d*(5*d*e + 11*c*f) + a*b^2*c*(65*d*e + 38*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c
) + a*d)*(15*b^2*c*e + 24*a^2*d*f - a*b*(20*d*e + 23*c*f))*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/
(15*b^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {401, 25, 403, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{(dx^2+c)^{3/2}(acf-d(5be-6af)x^2)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)^{3/2}(acf-d(5be-6af)x^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{\sqrt{dx^2+c}\left(d\left(24dfa^2-b(20de+23cf)a+15b^2ce\right)x^2+ac(6adf-5b(de+cf))\right)}{\sqrt{bx^2+a}}dx}{5b}-\frac{dx\sqrt{a+bx^2}(c+dx^2)^{3/2}(5be-6af)}{5b}}{ab}+\frac{x(c+dx^2)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\sqrt{dx^2+c}\left(d\left(24dfa^2-b(20de+23cf)a+15b^2ce\right)x^2+ac(6adf-5b(de+cf))\right)}{\sqrt{bx^2+a}}dx}{5b}-\frac{dx\sqrt{a+bx^2}(c+dx^2)^{3/2}(5be-6af)}{5b}}{ab}+\frac{x(c+dx^2)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{403}$$

$$-\frac{\frac{\frac{\int -\frac{ac\left(15c(2de+cf)b^2-ad(20de+41cf)b+24a^2d^2f\right)-d\left(-48d^2fa^3+8bd(5de+11cf)a^2-b^2c(65de+38cf)a+15b^3c^2e\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23a\right.}{3b}}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23abcf-20abde+15b^2ce\right)}{3b}-\frac{\int \frac{ac\left(15c(2de+cf)b^2-ad(20de+41cf)b+24a^2d^2f\right)-d\left(-48d^2fa^3+8bd(5de+11cf)a^2-b^2c(65de+38c\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3b}}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$-\frac{\frac{\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23abcf-20abde+15b^2ce\right)}{3b}-\frac{ac\left(24a^2d^2f-abd(41cf+20de)+15b^2c(cf+2de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d\left(-48a^3d^2f+8a^2bd(11cf+\right.}{3b}}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$-\frac{\frac{\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23abcf-20abde+15b^2ce\right)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(41cf+20de)+15b^2c(cf+2de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d\left(-\right.}{3b}}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$-\frac{\frac{\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23abcf-20abde+15b^2ce\right)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(41cf+20de)+15b^2c(cf+2de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d\left(-\right.}{3b}}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a^2df-23abcf-20abde+15b^2ce\right)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(41cf+20de)+15b^2c(cf+2de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d\Big(-\cdots}{5b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ab\sqrt{a+bx^2}}$$

input

```
Int[((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output

```
((b*e - a*f)*x*(c + d*x^2)^(5/2))/(a*b*Sqrt[a + b*x^2]) + (-1/5*(d*(5*b*e
- 6*a*f)*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/b - ((d*(15*b^2*c*e - 20*a*b
*d*e - 23*a*b*c*f + 24*a^2*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) -
 (-(d*(15*b^3*c^2*e - 48*a^3*d^2*f + 8*a^2*b*d*(5*d*e + 11*c*f) - a*b^2*c*
(65*d*e + 38*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)
*(24*a^2*d^2*f + 15*b^2*c*(2*d*e + c*f) - a*b*d*(20*d*e + 41*c*f))*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/(5*b))/(a*
b)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. 2(374) = 748.

Time = 15.05 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{dx(-3bdf\,x^2+9adf-11bcf-5bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^3}+\frac{\left(-\frac{\left(33f\,d^2a^2-58fdcba-25ab\,d^2e+23f\,c^2b^2+35d\,b^2ce\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,(\ldots}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+a\ldots}}\ldots\right.}{\ldots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(bd\,x^2+bc\right)\left(a^3d^2f-2a^2bcdf-a^2b\,d^2e+b^2c^2fa+2a\,b^2cde-b^3c^2e\right)x}{a\,b^4\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{d^2f\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5b^2}+\frac{\left(-\frac{d^2(adf-3\ldots}{b\ldots}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/b^3*d*x*(-3*b*d*f*x^2+9*a*d*f-11*b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)+1/15/b^3*(-(33*a^2*d^2*f-58*a*b*c*d*f-25*a*b*d^2*e+23*b^2*c^2*
f+35*b^2*c*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+15*(a^4*d^3*f-3
*a^3*b*c*d^2*f-a^3*b*d^3*e+3*a^2*b^2*c^2*d*f+3*a^2*b^2*c*d^2*e-a*b^3*c^3*f
-3*a*b^3*c^2*d*e+b^4*c^3*e)/b*(-(b*d*x^2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*
d*x^2+b*c))^(1/2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))))-(15*a^3*d^3*f-54*a^2*b*c*d^2*f-15*a^2*b*d^3*e+56*a*b^2*c^2*d*f
+50*a*b^2*c*d^2*e-15*b^3*c^3*f-45*b^3*c^2*d*e)/b/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a
)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{((5(3b^4c^3-13ab^3c^2d+8a^2b^2cd^2)e-2(19ab^3c^3-44a^2b^2c^2d+24a^3bcd^2)}{}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(((5*(3*b^4*c^3 - 13*a*b^3*c^2*d + 8*a^2*b^2*c*d^2)*e - 2*(19*a*b^3*c
^3 - 44*a^2*b^2*c^2*d + 24*a^3*b*c*d^2)*f)*x^3 + (5*(3*a*b^3*c^3 - 13*a^2*
b^2*c^2*d + 8*a^3*b*c*d^2)*e - 2*(19*a^2*b^2*c^3 - 44*a^3*b*c^2*d + 24*a^4
*c*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - ((5*(3*b^4*c^3 - 13*a*b^3*c^2*d + 4*a^2*b^2*d^3 + 2*(4*a^2*b^2 - 3*
a*b^3)*c*d^2)*e - (38*a*b^3*c^3 + 24*a^3*b*d^3 - (88*a^2*b^2 - 15*a*b^3)*c
^2*d + (48*a^3*b - 41*a^2*b^2)*c*d^2)*f)*x^3 + (5*(3*a*b^3*c^3 - 13*a^2*b^
2*c^2*d + 4*a^3*b*d^3 + 2*(4*a^3*b - 3*a^2*b^2)*c*d^2)*e - (38*a^2*b^2*c^3
 + 24*a^4*d^3 - (88*a^3*b - 15*a^2*b^2)*c^2*d + (48*a^4 - 41*a^3*b)*c*d^2)
*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) +
(3*a*b^3*d^3*f*x^6 + (5*a*b^3*d^3*e + (11*a*b^3*c*d^2 - 6*a^2*b^2*d^3)*f)*
x^4 + (5*(7*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (23*a*b^3*c^2*d - 47*a^2*b^2*
c*d^2 + 24*a^3*b*d^3)*f)*x^2 - 5*(3*a*b^3*c^2*d - 13*a^2*b^2*c*d^2 + 8*a^3
*b*d^3)*e + 2*(19*a^2*b^2*c^2*d - 44*a^3*b*c*d^2 + 24*a^4*d^3)*f)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(a*b^5*d*x^3 + a^2*b^4*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(5/2)*(e + f*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x)
```

output
```
(18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d**2*f*x - 12*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*d**3*f*x**3 - 33*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*c**2*d*f*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*e*x + 22
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*f*x**3 + 10*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*d**3*e*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*
b*d**3*f*x**5 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**3*f*x + 45*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*e*x + 48*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**
4 + b**2*c*x**4 + b**2*d*x**6),x)*a**4*d**4*f - 112*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4
 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*c*d**3*f - 40*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*d**4*e + 48*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*d**4*f*x**2 + 79*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b
*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b**2*c**2*d**2*f + 85*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b**2*c*d**3*e - 112*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a...
```

3.28 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 295

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{(4adf-3b(de+cf))x\sqrt{c+dx^2}}{3b^2\sqrt{a+bx^2}}+\frac{fx(c+dx^2)^{3/2}}{3b\sqrt{a+bx^2}} + \frac{(3b^2ce-6abde-7abcf+8a^2df)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}(4adf-3b(de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*(4*a*d*f-3*b*(c*f+d*e))*x*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)+1/3*f*x
*(d*x^2+c)^(3/2)/b/(b*x^2+a)^(1/2)+1/3*(8*a^2*d*f-7*a*b*c*f-6*a*b*d*e+3*b^
2*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/a^(1/2)/b^(5/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1
/2)-1/3*a^(1/2)*(4*a*d*f-3*b*(c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(ar
ctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/(b*x^2+a)^(1/2)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.51 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(3b^2ce+4a^2df+ab(-3de-3cf+dfx^2)\right)+ic(3b^2ce+\right.}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(c + d*x^2)*(3*b^2*c*e + 4*a^2*d*f + a*b*(-3*d*e -
 3*c*f + d*f*x^2)) + I*c*(3*b^2*c*e + 8*a^2*d*f - a*b*(6*d*e + 7*c*f))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] - I*c*(-(b*c) + a*d)*(-3*b*e + 4*a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b^3*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {401, 25, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \; 401$$

$$\frac{x(c+dx^2)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{\sqrt{dx^2+c}(acf-d(3be-4af)x^2)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{dx^2+c}\left(acf-d(3be-4af)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 403

$$\frac{\frac{\int -\frac{d\left(8dfa^2-b(6de+7cf)a+3b^2ce\right)x^2+ac(4adf-3b(de+cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}(3be-4af)}{3b}}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{d\left(8dfa^2-b(6de+7cf)a+3b^2ce\right)x^2+ac(4adf-3b(de+cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}(3be-4af)}{3b}}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 406

$$\frac{-\frac{d(8a^2df-ab(7cf+6de)+3b^2ce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(4adf-3b(cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}(3be-4af)}{3b}}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 320

$$\frac{-\frac{d(8a^2df-ab(7cf+6de)+3b^2ce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(4adf-3b(cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}(3be}{3b}}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 388

$$-\frac{\frac{d\left(8a^2df-ab(7cf+6de)+3b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(4adf-3b(cf+de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}-\frac{dx\sqrt{a+bx^2}\ldots}{}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$-\frac{\frac{d\left(8a^2df-ab(7cf+6de)+3b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(4adf-3b(cf+de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}}{ab}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ab\sqrt{a+bx^2}}$$

input

```
Int[((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output

```
((b*e - a*f)*x*(c + d*x^2)^(3/2))/(a*b*Sqrt[a + b*x^2]) + (-1/3*(d*(3*b*e
 - 4*a*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/b - (d*(3*b^2*c*e + 8*a^2*d*f
 - a*b*(6*d*e + 7*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3
/2)*(4*a*d*f - 3*b*(d*e + c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))/(3*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(266) = 532$.

Time = 11.56 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.85

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(\frac{(bd\,x^2+bc)(a^2df-abcf-abde+ce\,b^2)x}{a\,b^3\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}+\frac{dfx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3b^2}+\frac{\left(\frac{f\,d^2a^2-2fdcba-ab\,d^2e+f\,c^2b^2+2d\,b^2ce}{b^3}-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{x^2d+c}\,\sqrt{b\,x^2+a}\left(\sqrt{-\frac{b}{a}}\,ab\,d^2f\,x^5+4\sqrt{-\frac{b}{a}}\,a^2d^2f\,x^3-2\sqrt{-\frac{b}{a}}\,abcdf\,x^3-3\sqrt{-\frac{b}{a}}\,ab\,d^2e\,x^3+3\sqrt{-\frac{b}{a}}\,b^2cde\,x^3+4\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\cdots\right.}{\cdots}$
risch	$\frac{fx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,d}{3b^2}-\frac{\left(-\frac{(5adf-4bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\cdots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\cdots\right.}{\cdots}$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a/b^3*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)
+1/3*d*f/b^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+((a^2*d^2*f-2*a*b*c*d*f
-a*b*d^2*e+b^2*c^2*f+2*b^2*c*d*e)/b^3-(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
3*(a*d-b*c)/a-1/b^2*c*(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a-1/3*a/b^2*c*d*f)
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-1/b^2*d*(
a*d*f-2*b*c*f-b*d*e)-(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2*d/a-1/3*d*f/b^2
*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{((3(b^3c^2-2ab^2cd)e-(7ab^2c^2-8a^2bcd)f)x^3+(3(ab^2c^2-2a^2bcd)e-(7}{}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((3*(b^3*c^2 - 2*a*b^2*c*d)*e - (7*a*b^2*c^2 - 8*a^2*b*c*d)*f)*x^3 +
(3*(a*b^2*c^2 - 2*a^2*b*c*d)*e - (7*a^2*b*c^2 - 8*a^3*c*d)*f)*x)*sqrt(b*d)
*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((3*(b^3*c^2 - 2
*a*b^2*c*d - a*b^2*d^2)*e - (7*a*b^2*c^2 - 4*a^2*b*d^2 - (8*a^2*b - 3*a*b^
2)*c*d)*f)*x^3 + (3*(a*b^2*c^2 - 2*a^2*b*c*d - a^2*b*d^2)*e - (7*a^2*b*c^2
 - 4*a^3*d^2 - (8*a^3 - 3*a^2*b)*c*d)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_
f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b^2*d^2*f*x^4 + (3*a*b^2*d^2*e + 4
*(a*b^2*c*d - a^2*b*d^2)*f)*x^2 - 3*(a*b^2*c*d - 2*a^2*b*d^2)*e + (7*a^2*b
*c*d - 8*a^3*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*d*x^3 + a^2*b
^3*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x + 2*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*d**2*f*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f
*x + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x - 8*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x
**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3*f + 11*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**
4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**2*f + 6*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*e - 8*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**
4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*f*x**2 - 3*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d
*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c**2*d*f - 6*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*
d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*e + 11*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*
b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*f*x**2 + 6*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
+ 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*d**3*e*x**2 - 3*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*...
```

3.29 $\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{fx\sqrt{c+dx^2}}{b\sqrt{a+bx^2}} + \frac{(be-2af)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}f\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f*x*(d*x^2+c)^(1/2)/b/(b*x^2+a)^(1/2)+(-2*a*f+b*e)*(d*x^2+c)^(1/2)*Ellipti
cE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*f*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{-ic(-be+2af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(be-af)\left(\sqrt{\frac{b}{a}}x(c+dx^2)-ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((-I)*c*(-(b*e) + 2*a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*e - a*f)*(Sqrt[b/a]*x*(c + d*x^
2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{acf-d(be-2af)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{acf-d(be-2af)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(be-2af)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(be-2af)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(be-2af)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{ab}+\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(be-2af)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ab}+\frac{x\sqrt{c+dx^2}(be-af)}{ab\sqrt{a+bx^2}}$$

input

```
Int[(Sqrt[c + d*x^2]*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output

```
((b*e - a*f)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (-(d*(b*e - 2*a*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, adf\, x^{3}+\sqrt{-\frac{b}{a}}\, bde\, x^{3}-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) acf+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) bc\right.}{b(bd\, x^{4}}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)(x^{2} d+c)}\, \left(-\frac{\left(bd\, x^{2}+bc\right)(af-be)x}{a\, b^{2} \sqrt{\left(x^{2}+\frac{a}{b}\right)\left(bd\, x^{2}+bc\right)}}+\frac{\left(-\frac{adf-bcf-bde}{b^{2}}+\frac{(af-be)(ad-bc)}{b^{2} a}+\frac{c(af-be)}{ba}\right) \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}},\right.}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d}}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*a*d*f*x^3+(-b/a)^(1/2)*b*d*e*x^3-((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*f+((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*
e+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a*c*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*b*c*e-(-b/a)^(1/2)*a*c*f*x+(-b/a)^(1/2)*b*c*e*x)
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{((b^2ce-2\,abcf)x^3+(abce-2\,a^2cf)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((b^2}{}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(((b^2*c*e - 2*a*b*c*f)*x^3 + (a*b*c*e - 2*a^2*c*f)*x)*sqrt(b*d)*sqrt(-c/d
)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c*e - (2*a*b*c + a*b
*d)*f)*x^3 + (a*b*c*e - (2*a^2*c + a^2*d)*f)*x)*sqrt(b*d)*sqrt(-c/d)*ellip
tic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d*f*x^2 - a*b*d*e + 2*a^2*d*
f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^3*d*x^3 + a^2*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}(fx^2+e)}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{3/2}}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,cfx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,dex+2\Big(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+c}}{}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(3/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*f*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*d*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d
*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*d
**2*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*d*f -
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*
b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*e + 2*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*
x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*f*x**2 - int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x
**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*d*f*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x*
*2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*d**2*e*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*c**2*f + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*c*d*e - int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a*b*c**2*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x*...
```

3.30 $\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be-af)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}(de-cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(-a*f+b*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-a^(1/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arct
an(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}(-be+af)x(c+dx^2)+ic(-be+af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsi}\right.\right.}{b(-bc+a\ldots}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*(-(b*e) + a*f)*x*(c + d*x^2) + I*c*(-(b*e) + a*f)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*(-(b*c) + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*(-(b*c) + a*d)*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 400$$

$$\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx}{bc-ad}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^
2))/(c*(a + b*x^2))]) - (Sqrt[c]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [A] (verified)

Time = 7.77 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.60

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, adf\, x^3-\sqrt{-\frac{b}{a}}\, bde\, x^3+\sqrt{\frac{b x^2+a}{a}}\, \sqrt{\frac{x^2 d+c}{c}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) ade-\sqrt{\frac{b x^2+a}{a}}\, \sqrt{\frac{x^2 d+c}{c}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) bce-\right.}{\sqrt{-\frac{b}{a}}\, a(ad-}$
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(bd\, x^2+bc\right) x(af-be)}{ba(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\, x^2+bc\right)}}+\frac{\left(\frac{f}{b}-\frac{af-be}{ba}-\frac{c(af-be)}{a(ad-bc)}\right)\sqrt{1+\frac{b x^2}{a}}\, \sqrt{1+\frac{d x^2}{c}}\, \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^4+ad\, x^2+x^2 bc+ac}}+\right.}{\sqrt{b x^2+a}\, \sqrt{x^2 d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*a*d*f*x^3-(-b/a)^(1/2)*b*d*e*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*e-((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e
-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*c*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b*c*e+(-b/a)^(1/2)*a*c*f*x-(-b/a)^(1/2)*b*c*e*x)*(d
*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/a/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(ab^2ce-a^2bcf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(ab^2ce-a^2bcf+(b^3ce-ab^2cf)x^2)\sqrt{}}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
((a*b^2*c*e - a^2*b*c*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (a*b^2*c*e -
a^2*b*c*f + (b^3*c*e - a*b^2*c*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arc
sin(x*sqrt(-b/a)), a*d/(b*c)) - ((a^3 + a^2*b)*c*f + ((a^2*b + a*b^2)*c*f
- (b^3*c + a^2*b*d)*e)*x^2 - (a*b^2*c + a^3*d)*e)*sqrt(a*c)*sqrt(-b/a)*ell
iptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^3*b^2*c^2 - a^4*b*c*d + (a^2*
b^3*c^2 - a^3*b^2*c*d)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b
*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*e
```

3.31 $\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 271

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{(be-af)x}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(bce+ade-2acf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}(2bde-bcf-adf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(-a*f+b*e)*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+d^(1/2)*(-2*a*c*
f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)-c^(1/2)*(-a*d*f-b*c*f+2*b*d*e)*(b*x^2+a)^(1/2)*InverseJ
acobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(1/2)/(-a*d+b*c)^
2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(a^2d(de-cf)+b^2ce(c+dx^2)+ab(-c^2f+d^2ex^2-2cdfx^2))\right.}{}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d*(d*e - c*f) + b^2*c*e*(c + d*x^2) + a*b*(-(
c^2*f) + d^2*e*x^2 - 2*c*d*f*x^2)) - I*b*c*(-(b*c*e) - a*d*e + 2*a*c*f)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c -
a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{a(de-cf)-d(be-af)x^2}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{400}$$

$$\frac{\frac{x(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{a(-adf-bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(-2acf+ade+bce)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{a(bc-ad)}}{}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{a(-adf-bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(-2acf+ade+bce)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(-2acf+ade+bce)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
((b*e - a*f)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (-((Sqrt
[d]*(b*c*e + a*d*e - 2*a*c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(a*
(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(254) = 508$.

Time = 10.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{2bd\left(\frac{(2acf-ade-bce)x^3}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(a^2cfd-a^2d^2e+ab\,c^2f-b^2c^2e\right)x}{2abcd\left(a^2d^2-2abcd+b^2c^2\right)}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{db}+\frac{ac}{db}\right)bd}}+\frac{\left(\frac{e}{ac}+\frac{a^2cfd-a^2d^2e+ab\,c^2f-b^2c^2e}{ac\left(a^2d^2-2abcd+b^2c^2\right)}\right)\sqrt{1+\frac{b\,x}{a}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+c}}\right.}{\sqrt{}}$
default	$\left(-2\sqrt{-\frac{b}{a}}\,abcdf\,x^3+\sqrt{-\frac{b}{a}}\,ab\,d^2e\,x^3+\sqrt{-\frac{b}{a}}\,b^2cde\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cdf-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{Elli}\right.$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-2*b*d*(1/2*(
2*a*c*f-a*d*e-b*c*e)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^3+1/2*(a^2*c*d*f-a^
2*d^2*e+a*b*c^2*f-b^2*c^2*e)/a/b/c/d/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x)/((x^4+
(a*d+b*c)/d/b*x^2+a*c/d/b)*b*d)^(1/2)+(e/a/c+(a^2*c*d*f-a^2*d^2*e+a*b*c^2*
f-b^2*c^2*e)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b*(2*a*c*f-a*d*e-b*c*e)/a/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(254) = 508$.

Time = 0.12 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.35

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \frac{(2\,a^2b^2c^2f+(2\,ab^3cdf-(b^4cd+ab^3d^2)e)x^4-((b^4c^2+2\,ab^3cd+a^2b^2d^2)e}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
((2*a^2*b^2*c^2*f + (2*a*b^3*c*d*f - (b^4*c*d + a*b^3*d^2)*e)*x^4 - ((b^4*
c^2 + 2*a*b^3*c*d + a^2*b^2*d^2)*e - 2*(a*b^3*c^2 + a^2*b^2*c*d)*f)*x^2 -
(a*b^3*c^2 + a^2*b^2*c*d)*e)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt
(-b/a)), a*d/(b*c)) + (((b^4*c*d + (2*a^2*b^2 + a*b^3)*d^2)*e - (a^3*b*d^2
 + (a^2*b^2 + 2*a*b^3)*c*d)*f)*x^4 + ((b^4*c^2 + 2*(a^2*b^2 + a*b^3)*c*d +
 (2*a^3*b + a^2*b^2)*d^2)*e - (a^4*d^2 + (a^2*b^2 + 2*a*b^3)*c^2 + 2*(a^3*
b + a^2*b^2)*c*d)*f)*x^2 + (a*b^3*c^2 + (2*a^3*b + a^2*b^2)*c*d)*e - (a^4*
c*d + (a^3*b + 2*a^2*b^2)*c^2)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x
*sqrt(-b/a)), a*d/(b*c)) - ((2*a^2*b^2*c*d*f - (a*b^3*c*d + a^2*b^2*d^2)*e
)*x^3 - ((a*b^3*c^2 + a^3*b*d^2)*e - (a^2*b^2*c^2 + a^3*b*c*d)*f)*x)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2
 + (a^2*b^4*c^3*d - 2*a^3*b^3*c^2*d^2 + a^4*b^2*c*d^3)*x^4 + (a^2*b^4*c^4
- a^3*b^3*c^3*d - a^4*b^2*c^2*d^2 + a^5*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\int \frac{e+fx^2}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2d^2x^8+2ab\,d^2x^6+2b^2cd\,x^6+a^2d^2x^4+4abcd\,x^4+b^2c^2x^4+2a^2cd\,\cdots}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d^2x^8+2ab\,d^2x^6+2b^2cd\,x^6+a^2d^2x^4+4abcd\,x^4+b^2c^2x^4+2a^2cd\,x^2+2ab\,c^2x^2+a^2c^2}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2
+ a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b*
*2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)*f + int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*
b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c
*d*x**6 + b**2*d**2*x**8),x)*e
```

3.32 $\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}} dx = \frac{(be-af)x}{a(bc-ad)\sqrt{a+bx^2}(c+dx^2)^{3/2}} + \frac{d(3bce+ade-4acf)x\sqrt{a+bx^2}}{3ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{\sqrt{d}(3b^2c^2e+7abc(de-cf)-a^2d(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{b(3bc(3de-cf)-ad(de+5cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(-a*f+b*e)*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*d*(-4*a*c*f+
a*d*e+3*b*c*e)*x*(b*x^2+a)^(1/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/3*d^(1
/2)*(3*b^2*c^2*e+7*a*b*c*(-c*f+d*e)-a^2*d*(c*f+2*d*e))*(b*x^2+a)^(1/2)*Ell
ipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(3/2)/(-
a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(3*b*c*(-
c*f+3*d*e)-a*d*(5*c*f+d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)
*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.10

$$\int \frac{e+fx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x\left(-3b^3c^2e\left(c+dx^2\right)^2+a^3d^3\left(3ce+2dex^2+cfx^2\right)+a^2bd(5c^3f\right.\right.}{}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*x*(-3*b^3*c^2*e*(c + d*x^2)^2 + a^3*d^3*(3*c*e + 2*d
*e*x^2 + c*f*x^2) + a^2*b*d*(5*c^3*f + 2*d^3*e*x^4 + c*d^2*x^2*(-4*e + f*x
^2) + c^2*d*(-8*e + 4*f*x^2)) + a*b^2*c*(3*c^3*f + 11*c^2*d*f*x^2 - 7*d^3*
e*x^4 + c*d^2*x^2*(-8*e + 7*f*x^2))) + I*b*c*(-3*b^2*c^2*e + 7*a*b*c*(-(d*
e) + c*f) + a^2*d*(2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c)
+ a*d)*(-3*b*c*e - a*d*e + 4*a*c*f)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^2*(-
(b*c) + a*d)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {402, 402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\int \frac{a(de-cf)-3d(be-af)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(bc-ad)}-\frac{\frac{\int \frac{a(3bc(2de-cf)-ad(2de+cf))-bd(3bce+ade-4acf)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(-4acf+ade+3bce)}{3c(c+dx^2)^{3/2}(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \; 400$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(bc-ad)}-\frac{\frac{\frac{ab(3bc(3de-cf)-ad(5cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(a^2(-d)(cf+2de)+7abc(de-cf)+3b^2c^2e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(-4acf+ade+3bce)}{3c(c+dx^2)^{3/2}(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(bc-ad)}-\frac{\frac{\frac{ab(3bc(3de-cf)-ad(5cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(a^2(-d)(cf+2de)+7abc(de-cf)+3b^2c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(-4ac}{3c(c+dx^2)^{3/}}}{a(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{x(be-af)}{a\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(3bc(3de-cf)-ad(5cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(a^2(-d)(cf+2de)+7abc(de-cf)+3b^2c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-}{a(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
((b*e - a*f)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) - (-1/3*
(d*(3*b*c*e + a*d*e - 4*a*c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^
2)^(3/2)) + (-((Sqrt[d]*(3*b^2*c^2*e + 7*a*b*c*(d*e - c*f) - a^2*d*(2*d*e
+ c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])) + (b*Sqrt[c]*(3*b*c*(3*d*e - c*f) - a*d*(d*e + 5*c*f))*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(
b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b
*c - a*d)))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 12.25 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.82

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\left(bd\,x^2+bc\right)bx(af-be)}{a(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}-\frac{x(cf-de)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3cd(ad-bc)^2\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(bd\,x^2+ad\right)x\left(acdf+2a\,d^2e+4b\,c^2f-7bcde\right)}{3c^2(ad-bc)^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+ \cdots\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*b/a/(a*d-b*c)^3*x*(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/3/c/d/(a*d-
b*c)^2*x*(c*f-d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/3*(b*
d*x^2+a*d)/c^2/(a*d-b*c)^3*x*(a*c*d*f+2*a*d^2*e+4*b*c^2*f-7*b*c*d*e)/((x^2
+c/d)*(b*d*x^2+a*d))^(1/2)+(-b*(a*f-b*e)/(a*d-b*c)^2/a-b^2*c/a/(a*d-b*c)^3
*(a*f-b*e)-1/3*b*(c*f-d*e)/c/(a*d-b*c)^2+1/3/(a*d-b*c)^2*(a*c*d*f+2*a*d^2*
e+4*b*c^2*f-7*b*c*d*e)/c^2-1/3*a*d/c^2/(a*d-b*c)^3*(a*c*d*f+2*a*d^2*e+4*b*
c^2*f-7*b*c*d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-(-(a*f-b*e)*b^2*d/(a*d-b*c)^3/a-1/3*b*d*(a*c*d*f+2*a*d^2*e+4*b*c^2*f
-7*b*c*d*e)/(a*d-b*c)^3/c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. 2(342) = 684.

Time = 0.16 (sec) , antiderivative size = 1316, normalized size of antiderivative = 3.57

$$\int \frac{e+fx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((((3*b^4*c^2*d^2 + 7*a*b^3*c*d^3 - 2*a^2*b^2*d^4)*e - (7*a*b^3*c^2*d
^2 + a^2*b^2*c*d^3)*f)*x^6 + ((6*b^4*c^3*d + 17*a*b^3*c^2*d^2 + 3*a^2*b^2*
c*d^3 - 2*a^3*b*d^4)*e - (14*a*b^3*c^3*d + 9*a^2*b^2*c^2*d^2 + a^3*b*c*d^3
)*f)*x^4 + ((3*b^4*c^4 + 13*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3)*e - (7*a*b^3*c^4 + 15*a^2*b^2*c^3*d + 2*a^3*b*c^2*d^2)*f)*x^2 + (3*a*b
^3*c^4 + 7*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2)*e - (7*a^2*b^2*c^4 + a^3*b*c^3
*d)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
(((3*b^4*c^2*d^2 + (9*a^2*b^2 + 7*a*b^3)*c*d^3 - (a^3*b + 2*a^2*b^2)*d^4)*
e - ((3*a^2*b^2 + 7*a*b^3)*c^2*d^2 + (5*a^3*b + a^2*b^2)*c*d^3)*f)*x^6 + (
(6*b^4*c^3*d + (18*a^2*b^2 + 17*a*b^3)*c^2*d^2 + (7*a^3*b + 3*a^2*b^2)*c*d
^3 - (a^4 + 2*a^3*b)*d^4)*e - (2*(3*a^2*b^2 + 7*a*b^3)*c^3*d + (13*a^3*b +
 9*a^2*b^2)*c^2*d^2 + (5*a^4 + a^3*b)*c*d^3)*f)*x^4 + ((3*b^4*c^4 + (9*a^2
*b^2 + 13*a*b^3)*c^3*d + (17*a^3*b + 12*a^2*b^2)*c^2*d^2 - 2*(a^4 + 2*a^3*
b)*c*d^3)*e - ((3*a^2*b^2 + 7*a*b^3)*c^4 + (11*a^3*b + 15*a^2*b^2)*c^3*d +
 2*(5*a^4 + a^3*b)*c^2*d^2)*f)*x^2 + (3*a*b^3*c^4 + (9*a^3*b + 7*a^2*b^2)*
c^3*d - (a^4 + 2*a^3*b)*c^2*d^2)*e - ((3*a^3*b + 7*a^2*b^2)*c^4 + (5*a^4 +
 a^3*b)*c^3*d)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*
d/(b*c)) - (((3*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*e - (7*a^2*
b^2*c^2*d^2 + a^3*b*c*d^3)*f)*x^5 + (2*(3*a*b^3*c^3*d + 4*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3 - a^4*d^4)*e - (11*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 + a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{f\,x^2+e}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x}}{b^2d^3x^{10}+2ab\,d^3x^8+3b^2c\,d^2x^8+a^2d^3x^6+6abc\,d^2x^6+3b^2c^2d\,x^6+3}\right.$$

$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d^3x^{10}+2ab\,d^3x^8+3b^2c\,d^2x^8+a^2d^3x^6+6abc\,d^2x^6+3b^2c^2d\,x^6+3a^2c\,d^2x^4+6ab\,c^2d\,x^4+b^2c^3x^4+}\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c**3 + 3*a**2*c**2*d*x*
*2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*
x**4 + 6*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*
d*x**6 + 3*b**2*c*d**2*x**8 + b**2*d**3*x**10),x)*f + int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 +
 a**2*d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6
+ 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x*
*8 + b**2*d**3*x**10),x)*e
```

3.33 $\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 393

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{d(b^2ce-4abde-7abcf+8a^2df)\,x\sqrt{c+dx^2}}{3ab^3\sqrt{a+bx^2}} - \frac{d(be-2af)x(c+dx^2)^{3/2}}{3ab^2\sqrt{a+bx^2}} + \frac{(be-af)x(c+dx^2)^{5/2}}{3ab\,(a+bx^2)^{3/2}} + \frac{(2b^3c^2e+16a^3d^2f+ab^2c(3de+cf)-8a^2bd(de+2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{d(b^2ce-4abde-7abcf+8a^2df)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*d*(8*a^2*d*f-7*a*b*c*f-4*a*b*d*e+b^2*c*e)*x*(d*x^2+c)^(1/2)/a/b^3/(b*
x^2+a)^(1/2)-1/3*d*(-2*a*f+b*e)*x*(d*x^2+c)^(3/2)/a/b^2/(b*x^2+a)^(1/2)+1/
3*(-a*f+b*e)*x*(d*x^2+c)^(5/2)/a/b/(b*x^2+a)^(3/2)+1/3*(2*b^3*c^2*e+16*a^3
*d^2*f+a*b^2*c*(c*f+3*d*e)-8*a^2*b*d*(2*c*f+d*e))*(d*x^2+c)^(1/2)*Elliptic
E(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(7/2)/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*d*(8*a^2*d*f-7*a*b*c*f-
4*a*b*d*e+b^2*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2
)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(7/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.92 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{b}{a}\right)^{3/2}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(8a^4d^2f+2b^4c^2ex^2+ab^3c(3ce+3dex^2+cfx^2)+a\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b/a)^(3/2)*(Sqrt[b/a]*x*(c + d*x^2)*(8*a^4*d^2*f + 2*b^4*c^2*e*x^2 + a*b
^3*c*(3*c*e + 3*d*e*x^2 + c*f*x^2) + a^3*b*d*(-4*d*e - 7*c*f + 10*d*f*x^2)
 + a^2*b^2*d*(-5*d*e*x^2 + d*f*x^4 + c*(e - 9*f*x^2))) + I*c*(2*b^3*c^2*e
+ 16*a^3*d^2*f + a*b^2*c*(3*d*e + c*f) - 8*a^2*b*d*(d*e + 2*c*f))*(a + b*x
^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-2*b^2*c*e + 8*a^2*d*f - a*b*(4*d*e
 + c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 25, 401, 27, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{\left(dx^2+c\right)^{3/2}\left(c(2be+af)-3d(be-2af)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(dx^2+c\right)^{3/2}\left(c(2be+af)-3d(be-2af)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{\int \frac{3d\sqrt{dx^2+c}\left(\left(-8dfa^2+b(4de+cf)a+2b^2ce\right)x^2+ac(be-2af)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\int \frac{\sqrt{dx^2+c}\left(\left(-8dfa^2+b(4de+cf)a+2b^2ce\right)x^2+ac(be-2af)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\left(\frac{\int\frac{\left(16d^2fa^3-8bd(de+2cf)a^2+b^2c(3de+cf)a+2b^3c^2e\right)x^2+ac\left(8dfa^2-4bdea-7bcfa+b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{x\sqrt{a+b\cdots}}{\cdots}\right.}{ab}}{3ab}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{x(c+dx^2)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\left(\frac{ac\left(8a^2df-7abcf-4abde+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(16a^3d^2f-8a^2bd(2cf+de)+ab^2c(cf+3de)+2b^3\cdots\right.}{3b}\right.}{ab}}{3ab}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{x(c+dx^2)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\left(\frac{\left(16a^3d^2f-8a^2bd(2cf+de)+ab^2c(cf+3de)+2b^3c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(8a^2df-7abc\cdots\right.}{\cdots}}{3b}\right.}{ab}}{3ab}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{x(c+dx^2)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\left(\frac{\left(16a^3d^2f-8a^2bd(2cf+de)+ab^2c(cf+3de)+2b^3c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\cdots}{\cdots}}{3b}\right.}{\cdots}}{3ab}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{x(c+dx^2)^{3/2}(bc(af+2be)+3ad(be-2af))}{ab\sqrt{a+bx^2}}-\frac{3d\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-8a^2df+ab(cf+4de)+2b^2ce\right)}{3b}+\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(8a^2df-7abcf-4abde+b^2ce\right)\text{EllipticF}}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{}\right)}{}}{3ab}$$

$$\frac{x(c+dx^2)^{5/2}(be-af)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2)^(5/2))/(3*a*b*(a + b*x^2)^(3/2)) + (((3*a*d*(b*
e - 2*a*f) + b*c*(2*b*e + a*f))*x*(c + d*x^2)^(3/2))/(a*b*Sqrt[a + b*x^2])
 - (3*d*(((2*b^2*c*e - 8*a^2*d*f + a*b*(4*d*e + c*f))*x*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(3*b) + ((2*b^3*c^2*e + 16*a^3*d^2*f + a*b^2*c*(3*d*e + c*f
) - 8*a^2*b*d*(d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (S
qrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) +
 (c^(3/2)*(b^2*c*e - 4*a*b*d*e - 7*a*b*c*f + 8*a^2*d*f)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b)))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. 2(358) = 716.

Time = 16.26 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.15

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(a^3 d^2 f-2a^2 bcdf-a^2 b d^2 e+b^2 c^2 f a+2a b^2 cde-b^3 c^2 e\right) x \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3a b^5 \left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)\left(8a^3 d^2 f-9a^2 bcdf-5a^{2}\right.}{3b^4 a^2 \sqrt{\left(x^2+\right.}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a^3*d^2
*f-2*a^2*b*c*d*f-a^2*b*d^2*e+a*b^2*c^2*f+2*a*b^2*c*d*e-b^3*c^2*e)/a/b^5*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)*(8*a^3*d
^2*f-9*a^2*b*c*d*f-5*a^2*b*d^2*e+a*b^2*c^2*f+3*a*b^2*c*d*e+2*b^3*c^2*e)/b^
4/a^2*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/3*d^2*f/b^3*x*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+(d*(3*a^2*d^2*f-6*a*b*c*d*f-2*a*b*d^2*e+3*b^2*c^2*f+3*b^
2*c*d*e)/b^4-1/3*(a^3*d^2*f-2*a^2*b*c*d*f-a^2*b*d^2*e+a*b^2*c^2*f+2*a*b^2*
c*d*e-b^3*c^2*e)/b^4*d/a-1/3*(8*a^3*d^2*f-9*a^2*b*c*d*f-5*a^2*b*d^2*e+a*b^
2*c^2*f+3*a*b^2*c*d*e+2*b^3*c^2*e)/b^4*(a*d-b*c)/a^2-1/3/b^3*c*(8*a^3*d^2*
f-9*a^2*b*c*d*f-5*a^2*b*d^2*e+a*b^2*c^2*f+3*a*b^2*c*d*e+2*b^3*c^2*e)/a^2-1
/3*a/b^3*c*d^2*f)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-(-1/b^3*d^2*(2*a*d*f-3*b*c*f-b*d*e)-1/3*(8*a^3*d^2*f-9*a^2*b*c*d*f-5
*a^2*b*d^2*e+a*b^2*c^2*f+3*a*b^2*c*d*e+2*b^3*c^2*e)/b^3*d/a^2-1/3*d^2*f/b^
3*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(358) = 716$.

Time = 0.14 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.29

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((((2*b^5*c^3 + 3*a*b^4*c^2*d - 8*a^2*b^3*c*d^2)*e + (a*b^4*c^3 - 16*a
^2*b^3*c^2*d + 16*a^3*b^2*c*d^2)*f)*x^5 + 2*((2*a*b^4*c^3 + 3*a^2*b^3*c^2*
d - 8*a^3*b^2*c*d^2)*e + (a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 16*a^4*b*c*d^2)
*f)*x^3 + ((2*a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 8*a^4*b*c*d^2)*e + (a^3*b^2*
c^3 - 16*a^4*b*c^2*d + 16*a^5*c*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((2*b^5*c^3 + 3*a*b^4*c^2*d - 4*a^2*b
^3*d^3 - (8*a^2*b^3 - a*b^4)*c*d^2)*e + (a*b^4*c^3 - 16*a^2*b^3*c^2*d + 8*
a^3*b^2*d^3 + (16*a^3*b^2 - 7*a^2*b^3)*c*d^2)*f)*x^5 + 2*((2*a*b^4*c^3 + 3
*a^2*b^3*c^2*d - 4*a^3*b^2*d^3 - (8*a^3*b^2 - a^2*b^3)*c*d^2)*e + (a^2*b^3
*c^3 - 16*a^3*b^2*c^2*d + 8*a^4*b*d^3 + (16*a^4*b - 7*a^3*b^2)*c*d^2)*f)*x
^3 + ((2*a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 4*a^4*b*d^3 - (8*a^4*b - a^3*b^2)
*c*d^2)*e + (a^3*b^2*c^3 - 16*a^4*b*c^2*d + 8*a^5*d^3 + (16*a^5 - 7*a^4*b)
*c*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) + (a^2*b^3*d^3*f*x^6 + (3*a^2*b^3*d^3*e + (7*a^2*b^3*c*d^2 - 6*a^3*b^
2*d^3)*f)*x^4 - ((a*b^4*c^2*d + 5*a^2*b^3*c*d^2 - 12*a^3*b^2*d^3)*e + (2*a
^2*b^3*c^2*d - 25*a^3*b^2*c*d^2 + 24*a^4*b*d^3)*f)*x^2 - (2*a^2*b^3*c^2*d
+ 3*a^3*b^2*c*d^2 - 8*a^4*b*d^3)*e - (a^3*b^2*c^2*d - 16*a^4*b*c*d^2 + 16*
a^5*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^6*d*x^5 + 2*a^3*b^5*d*
x^3 + a^4*b^4*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}(e+fx^2)}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**(5/2)*(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output
```
(18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x - 12*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*d**2*f*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f*
x - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x + 14*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b*c*d*f*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*e
*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*f*x**5 + 48*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2*b*c*x**2
+ 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x**6 + b**3
*d*x**8),x)*a**4*d**3*f - 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a**3*c + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**
4 + 3*a*b**2*d*x**6 + b**3*c*x**6 + b**3*d*x**8),x)*a**3*b*c*d**2*f - 24*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2
*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x
**6 + b**3*d*x**8),x)*a**3*b*d**3*e + 96*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*
a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x**6 + b**3*d*x**8),x)*a**3*b*d**
3*f*x**2 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*
d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*
x**6 + b**3*c*x**6 + b**3*d*x**8),x)*a**2*b**2*c**2*d*f + 9*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3
*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x**6 + b**3...
```

3.34 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 293

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{d(be-4af)x\sqrt{c+dx^2}}{3ab^2\sqrt{a+bx^2}}+\frac{(be-af)x(c+dx^2)^{3/2}}{3ab(a+bx^2)^{3/2}}$$
$$+\frac{(2b^2ce-8a^2df+ab(2de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{d(be-4af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
-1/3*d*(-4*a*f+b*e)*x*(d*x^2+c)^(1/2)/a/b^2/(b*x^2+a)^(1/2)+1/3*(-a*f+b*e)
*x*(d*x^2+c)^(3/2)/a/b/(b*x^2+a)^(3/2)+1/3*(2*b^2*c*e-8*a^2*d*f+a*b*(c*f+2
*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/a^(3/2)/b^(5/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1
/2)-1/3*d*(-4*a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(5/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{b}{a}\right)^{3/2}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(-4a^3df+2b^3cex^2+a^2bd(e-5fx^2)+ab^2(3ce+2e\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b/a)^(3/2)*(Sqrt[b/a]*x*(c + d*x^2)*(-4*a^3*d*f + 2*b^3*c*e*x^2 + a^2*b*
d*(e - 5*f*x^2) + a*b^2*(3*c*e + 2*d*e*x^2 + c*f*x^2)) - I*c*(-2*b^2*c*e +
 8*a^2*d*f - a*b*(2*d*e + c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-2*b^2*c*e
 + 4*a^2*d*f - a*b*(d*e + c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b^4*(a + b*x
^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {401, 25, 401, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(c+dx^2)^{3/2}(be-af)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{\sqrt{dx^2+c}\,(c(2be+af)-d(be-4af)x^2)}{(bx^2+a)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{dx^2+c}\left(c(2be+af)-d(be-4af)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 401

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{\int \frac{d\left(\left(-8dfa^2+b(2de+cf)a+2b^2ce\right)x^2+ac(be-4af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{d\int \frac{\left(-8dfa^2+b(2de+cf)a+2b^2ce\right)x^2+ac(be-4af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 406

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\left(-8a^2df+ab(cf+2de)+2b^2ce\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(be-4af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 320

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\left(-8a^2df+ab(cf+2de)+2b^2ce\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-4af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 388

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\left(-8a^2df+ab(cf+2de)+2b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-4af)\text{EllipticF}\left(\arctan\left(\ldots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+b\ldots\right)}{a\left(c+d\ldots\right)}}}\right)}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 313

$$\frac{\frac{x\sqrt{c+dx^2}(bc(af+2be)+ad(be-4af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\left(-8a^2df+ab(cf+2de)+2b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-\ldots}{\sqrt{\ldots}}\right)}{ab}}{3ab}$$

$$\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2)^(3/2))/(3*a*b*(a + b*x^2)^(3/2)) + (((a*d*(b*e
 - 4*a*f) + b*c*(2*b*e + a*f))*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) - (
d*((2*b^2*c*e - 8*a^2*d*f + a*b*(2*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])) + (c^(3/2)*(b*e - 4*a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(264) = 528$.

Time = 7.76 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.93

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\left(a^2df-abcf-abde+ce\,b^2\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,b^4\left(x^2+\frac{a}{b}\right)^2}-\frac{\left(bd\,x^2+bc\right)\left(5a^2df-abcf-2abde-2ce\,b^2\right)x}{3a^2b^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(-\frac{d(2adf-2bcf-}{b^3}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/a/b^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+
a/b)^2-1/3*(b*d*x^2+b*c)*(5*a^2*d*f-a*b*c*f-2*a*b*d*e-2*b^2*c*e)/a^2/b^3*x
/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-d*(2*a*d*f-2*b*c*f-b*d*e)/b^3+1/3*(a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*d/a+1/3*(5*a^2*d*f-a*b*c*f-2*a*b*d*e-2*b^
2*c*e)/b^3*(a*d-b*c)/a^2+1/3/b^2*c*(5*a^2*d*f-a*b*c*f-2*a*b*d*e-2*b^2*c*e)
/a^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(d^2*f
/b^2+1/3*(5*a^2*d*f-a*b*c*f-2*a*b*d*e-2*b^2*c*e)/b^2*d/a^2)*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(264) = 528$.

Time = 0.12 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{((2(b^4c^2+ab^3cd)e+(ab^3c^2-8a^2b^2cd)f)x^5+2(2(ab^3c^2+a^2b^2cd)e+(a^2b}{}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((2*(b^4*c^2 + a*b^3*c*d)*e + (a*b^3*c^2 - 8*a^2*b^2*c*d)*f)*x^5 + 2*
(2*(a*b^3*c^2 + a^2*b^2*c*d)*e + (a^2*b^2*c^2 - 8*a^3*b*c*d)*f)*x^3 + (2*(
a^2*b^2*c^2 + a^3*b*c*d)*e + (a^3*b*c^2 - 8*a^4*c*d)*f)*x)*sqrt(b*d)*sqrt(
-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((2*b^4*c^2 + 2*a*b^3
*c*d + a*b^3*d^2)*e + (a*b^3*c^2 - 8*a^2*b^2*c*d - 4*a^2*b^2*d^2)*f)*x^5 +
 2*((2*a*b^3*c^2 + 2*a^2*b^2*c*d + a^2*b^2*d^2)*e + (a^2*b^2*c^2 - 8*a^3*b
*c*d - 4*a^3*b*d^2)*f)*x^3 + ((2*a^2*b^2*c^2 + 2*a^3*b*c*d + a^3*b*d^2)*e
+ (a^3*b*c^2 - 8*a^4*c*d - 4*a^4*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_
f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*a^2*b^2*d^2*f*x^4 - ((a*b^3*c*d +
3*a^2*b^2*d^2)*e + 2*(a^2*b^2*c*d - 6*a^3*b*d^2)*f)*x^2 - 2*(a^2*b^2*c*d +
 a^3*b*d^2)*e - (a^3*b*c*d - 8*a^4*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
)/(a^2*b^5*d*x^5 + 2*a^3*b^4*d*x^3 + a^4*b^3*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x + 2*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*d**2*f*x**3 + sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f*x
 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x - 2*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b*c*d*f*x**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d
**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x
**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**
8),x)*a**5*d**4*f + 17*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*
c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**
4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2
*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a
**4*b*c*d**3*f + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d
+ a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 -
3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b
**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**4*
b*d**4*e - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**
4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**
2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c
*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**4*b*d**
4*f*x**2 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + ...
```

3.35 $\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 272

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(be-af)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}} + \frac{(2b^2ce-2a^2df-ab(de-cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{d(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(3/2)+1/3*(2*b^2*c*e-2*a^2*
d*f-a*b*(-c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a
)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(3/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*d*(-a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobi
AM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/(-a*d+b*c)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.25 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(a^3df-2b^3cex^2+2a^2bd(e+fx^2)+ab^2(dex^2-c(3e+fx^2))\right)\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(a^3*d*f - 2*b^3*c*e*x^2 + 2*a^2*b*d*(e + f*x^2)
+ a*b^2*(d*e*x^2 - c*(3*e + f*x^2))) + I*c*(-2*b^2*c*e + 2*a^2*d*f + a*b*(
d*e - c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(2*b*e + a*f)*(a
 + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)])/(3*a^3*(b/a)^(3/2)*(-(b*c) + a*d)*(a + b*x^2)^(3/2
)*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{3ab\,(a+bx^2)^{3/2}}-\frac{\int -\frac{d(be+2af)x^2+c(2be+af)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(be+2af)x^2+c(2be+af)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3ab}+\frac{x\sqrt{c+dx^2}(be-af)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{(bc(af+2be)-ad(2af+be))\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx}{bc-ad}-\frac{cd(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3ab}+\frac{x\sqrt{c+dx^2}(be-af)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{c+dx^2}(bc(af+2be)-ad(2af+be))E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{cd(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3ab}+\frac{x\sqrt{c+dx^2}(be-af)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\sqrt{c+dx^2}(bc(af+2be)-ad(2af+be))E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3ab}+\frac{x\sqrt{c+dx^2}(be-af)}{3ab\,(a+bx^2)^{3/2}}$$

input

```
Int[(Sqrt[c + d*x^2]*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output

```
((b*e - a*f)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (((b*c*(2*b*e
 + a*f) - a*d*(b*e + 2*a*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/S
qrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sq
rt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(b*e - a*f)*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c
 - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(249) = 498$.

Time $= 6.02$ (sec) , antiderivative size $= 518$, normalized size of antiderivative $= 1.90$

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(af-be)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)x\left(2a^2df-abcf+abde-2ce\,b^2\right)}{3b^2a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{df}{b^2}-\frac{(af-be)d}{3b^2a}-\frac{2a^2df-abcf+abd}{3b^2a^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a*f-b*e
)/a/b^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c
)/b^2/a^2/(a*d-b*c)*x*(2*a^2*d*f-a*b*c*f+a*b*d*e-2*b^2*c*e)/((x^2+a/b)*(b*
d*x^2+b*c))^(1/2)+(d*f/b^2-1/3*(a*f-b*e)/b^2*d/a-1/3/b^2*(2*a^2*d*f-a*b*c*
f+a*b*d*e-2*b^2*c*e)/a^2-1/3/b*c/a^2/(a*d-b*c)*(2*a^2*d*f-a*b*c*f+a*b*d*e-
2*b^2*c*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+
1/3/b*(2*a^2*d*f-a*b*c*f+a*b*d*e-2*b^2*c*e)/(a*d-b*c)/a^2*c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(249) = 498$.

Time = 0.11 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{(((2\,b^5c-ab^4d)e+(ab^4c-2\,a^2b^3d)f)x^4+2\,((2\,ab^4c-a^2b^3d)e+(a^2b^3c-2\,a^3b^2d)f)x^2+(2\,a^2b^3c-a^3b}{}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((((2*b^5*c - a*b^4*d)*e + (a*b^4*c - 2*a^2*b^3*d)*f)*x^4 + 2*((2*a*b
^4*c - a^2*b^3*d)*e + (a^2*b^3*c - 2*a^3*b^2*d)*f)*x^2 + (2*a^2*b^3*c - a^
3*b^2*d)*e + (a^3*b^2*c - 2*a^4*b*d)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(ar
csin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*b^5*c + (a^2*b^3 - a*b^4)*d)*e + (a*
b^4*c - (a^3*b^2 + 2*a^2*b^3)*d)*f)*x^4 + 2*((2*a*b^4*c + (a^3*b^2 - a^2*b
^3)*d)*e + (a^2*b^3*c - (a^4*b + 2*a^3*b^2)*d)*f)*x^2 + (2*a^2*b^3*c + (a^
4*b - a^3*b^2)*d)*e + (a^3*b^2*c - (a^5 + 2*a^4*b)*d)*f)*sqrt(a*c)*sqrt(-b
/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*a*b^4*c - a^2*b^3*d
)*e + (a^2*b^3*c - 2*a^3*b^2*d)*f)*x^3 - (a^4*b*d*f - (3*a^2*b^3*c - 2*a^3
*b^2*d)*e)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^3*c - a^6*b^2*d + (a
^3*b^5*c - a^4*b^4*d)*x^4 + 2*(a^4*b^4*c - a^5*b^3*d)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*f*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*d*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4
*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2
*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*
d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**4*d**3*f
 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x*
*2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c*
*2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 +
 a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**3*b*c*d**2*f + i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a
**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**
2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**
3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**3*b*d**3*e + 4*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*
c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*
a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2
*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**3*b*d**3*f*x**2 + int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**
2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**
2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2...
```

3.36 $\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 289

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\frac{(be-af)x\sqrt{c+dx^2}}{3a(bc-ad)(a+bx^2)^{3/2}}+\frac{(2b^2ce+a^2df-ab(4de-cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d(bce-3ade+2acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}\sqrt{b}c(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+1/3*(2*b^2*c
*e+a^2*d*f-a*b*(-c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/(-a*d+b*c)^2/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*d*(2*a*c*f-3*a*d*e+b*c*e)*(d*x^
2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^
(1/2)/b^(1/2)/c/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(2a^3df+2b^3cex^2+a^2bd(-5e+fx^2)+ab^2(3ce-4dex^2+c\right.}{}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*x*(c + d*x^2)*(2*a^3*d*f + 2*b^3*c*e*x^2 + a^2*b*d*(-5*e + f*x^
2) + a*b^2*(3*c*e - 4*d*e*x^2 + c*f*x^2)) + I*c*(2*b^2*c*e + a^2*d*f + a*b
*(-4*d*e + c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*(2*b*c*e - 3*
a*d*e + a*c*f)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*(a
 + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {402, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x\sqrt{c+dx^2}(be-af)}{3a\,(a+bx^2)^{3/2}\,(bc-ad)}-\frac{\int -\frac{d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 400$$

$$\frac{\frac{\left(a^2df-ab(4de-cf)+2b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{d(2acf-3ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{c+dx^2}\left(a^2df-ab(4de-cf)+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d(2acf-3ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\sqrt{c+dx^2}\left(a^2df-ab(4de-cf)+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2acf-3ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\frac{x\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (((2
*b^2*c*e + a^2*d*f - a*b*(4*d*e - c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a
 + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c*e
 - 3*a*d*e + 2*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2]))/(3*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 10.06 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{x(af-be)\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3b^2 a(ad-bc)\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)x\left(a^2 df+abcf-4abde+2ce b^2\right)}{3b a^2(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(\frac{d(af-be)}{3b(ad-bc)a}-\frac{a^2 df+abcf-4abde+2c}{3(ad-bc)b a^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/b^2/a/(a*
d-b*c)*x*(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*
d*x^2+b*c)/b/a^2/(a*d-b*c)^2*x*(a^2*d*f+a*b*c*f-4*a*b*d*e+2*b^2*c*e)/((x^2
+a/b)*(b*d*x^2+b*c))^(1/2)+(1/3*d/b*(a*f-b*e)/(a*d-b*c)/a-1/3/(a*d-b*c)/b*
(a^2*d*f+a*b*c*f-4*a*b*d*e+2*b^2*c*e)/a^2-1/3*c/a^2/(a*d-b*c)^2*(a^2*d*f+a
*b*c*f-4*a*b*d*e+2*b^2*c*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))+1/3*(a^2*d*f+a*b*c*f-4*a*b*d*e+2*b^2*c*e)/(a*d-b*c)^2/a^2
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(266) = 532.

Time = 0.12 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.24

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{((2\,(b^5c^2-2\,ab^4cd)e+(ab^4c^2+a^2b^3cd)f)x^4+2\,(2\,(ab^4c^2-2\,a^2b^3cd)e+(a^2b^3c^2+a^3b^2cd)f)x^2+2\,(a^2b^3}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(((2*(b^5*c^2 - 2*a*b^4*c*d)*e + (a*b^4*c^2 + a^2*b^3*c*d)*f)*x^4 + 2
*(2*(a*b^4*c^2 - 2*a^2*b^3*c*d)*e + (a^2*b^3*c^2 + a^3*b^2*c*d)*f)*x^2 + 2
*(a^2*b^3*c^2 - 2*a^3*b^2*c*d)*e + (a^3*b^2*c^2 + a^4*b*c*d)*f)*sqrt(a*c)*
sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*b^5*c^2 - 3*
a^3*b^2*d^2 + (a^2*b^3 - 4*a*b^4)*c*d)*e + (a*b^4*c^2 + (2*a^3*b^2 + a^2*b
^3)*c*d)*f)*x^4 + 2*((2*a*b^4*c^2 - 3*a^4*b*d^2 + (a^3*b^2 - 4*a^2*b^3)*c*
d)*e + (a^2*b^3*c^2 + (2*a^4*b + a^3*b^2)*c*d)*f)*x^2 + (2*a^2*b^3*c^2 - 3
*a^5*d^2 + (a^4*b - 4*a^3*b^2)*c*d)*e + (a^3*b^2*c^2 + (2*a^5 + a^4*b)*c*d
)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((
2*(a*b^4*c^2 - 2*a^2*b^3*c*d)*e + (a^2*b^3*c^2 + a^3*b^2*c*d)*f)*x^3 + (2*
a^4*b*c*d*f + (3*a^2*b^3*c^2 - 5*a^3*b^2*c*d)*e)*x)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c))/(a^5*b^3*c^3 - 2*a^6*b^2*c^2*d + a^7*b*c*d^2 + (a^3*b^5*c^3 - 2
*a^4*b^4*c^2*d + a^5*b^3*c*d^2)*x^4 + 2*(a^4*b^4*c^3 - 2*a^5*b^3*c^2*d + a
^6*b^2*c*d^2)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^3d\,x^8+3a\,b^2d\,x^6+b^3c\,x^6+3a^2bd\,x^4+3a\,b^2c\,x^4+a^3d\,x^2+3a^2bc\,x^2+}\right.$$

$$\left.+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^3d\,x^8+3a\,b^2d\,x^6+b^3c\,x^6+3a^2bd\,x^4+3a\,b^2c\,x^4+a^3d\,x^2+3a^2bc\,x^2+a^3c}dx\right)e\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**3*c + a**3*d*x**2 + 3*a**
2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*
x**6 + b**3*d*x**8),x)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c
 + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a
*b**2*d*x**6 + b**3*c*x**6 + b**3*d*x**8),x)*e
```

3.37 $\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 383

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{(be-af)x}{3a(bc-ad)(a+bx^2)^{3/2}\sqrt{c+dx^2}} + \frac{(2b^2ce+3a^2df-ab(6de-cf))x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(2b^2c^2e-a^2d(3de-7cf)-abc(7de-cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}(b^2ce+3a^2df-ab(9de-5cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*f+b*e)*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+1/3*(2*b^2*c
*e+3*a^2*d*f-a*b*(-c*f+6*d*e))*x/a^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)+1/3*d^(1/2)*(2*b^2*c^2*e-a^2*d*(-7*c*f+3*d*e)-a*b*c*(-c*f+7*d*e))*
(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^
(1/2))/a^2/c^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^
(1/2)-1/3*c^(1/2)*d^(1/2)*(b^2*c*e+3*a^2*d*f-a*b*(-5*c*f+9*d*e))*(b*x^2+a)
^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/(-
a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.18 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.08

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}x(3a^4d^2(-de+cf)+2b^4c^2ex^2(c+dx^2)+ab^3c(c+dx^2)(3ce-7de}{}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(-(Sqrt[b/a]*x*(3*a^4*d^2*(-(d*e) + c*f) + 2*b^4*c^2*e*x^2*(c + d*x^2) + a
*b^3*c*(c + d*x^2)*(3*c*e - 7*d*e*x^2 + c*f*x^2) + a^3*b*d*(5*c^2*f - 6*d^
2*e*x^2 + 11*c*d*f*x^2) + a^2*b^2*d*(-3*d^2*e*x^4 + c^2*(-8*e + 4*f*x^2) +
 c*d*(-8*e*x^2 + 7*f*x^4)))) - I*b*c*(2*b^2*c^2*e + a*b*c*(-7*d*e + c*f) +
 a^2*d*(-3*d*e + 7*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(2*
b^2*c*e + 3*a^2*d*f + a*b*(-6*d*e + c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2
*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {402, 25, 402, 25, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \; 402$$

$$\frac{x(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)} - \frac{\int -\frac{3d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{3a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{3a(bc-ad)}+\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{x(3a^2df-ab(6de-cf)+2b^2ce)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{d\left(\left(3dfa^2-b(6de-cf)a+2b^2ce\right)x^2+a(bce+3ade-4acf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d\left(\left(3dfa^2-b(6de-cf)a+2b^2ce\right)x^2+a(bce+3ade-4acf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{x(3a^2df-ab(6de-cf)+2b^2ce)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\int \frac{\left(3dfa^2-b(6de-cf)a+2b^2ce\right)x^2+a(bce+3ade-4acf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{x(3a^2df-ab(6de-cf)+2b^2ce)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\Big\downarrow$ 400

$$\frac{\frac{d\left(\frac{\left(a^2(-d)(3de-7cf)-abc(7de-cf)+2b^2c^2e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{a\left(3a^2df-ab(9de-5cf)+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{x(3a^2df-ab(6de-cf)+2b^2c}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\Big\downarrow$ 313

$$\frac{\frac{d\left(\frac{\sqrt{a+bx^2}\left(a^2(-d)(3de-7cf)-abc(7de-cf)+2b^2c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\left(3a^2df-ab(9de-5cf)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{x(3a^2df-ab(}{a\sqrt{a+bx^2}\sqrt{}}}{3a(bc-ad)}$$
$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

$\downarrow$ 320

$$\frac{\frac{d\left(\frac{\sqrt{a+bx^2}\left(a^2(-d)(3de-7cf)-abc(7de-cf)+2b^2c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(3a^2df-ab(9de-5cf)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a(bc-ad)}}{3a(bc-ad)}$$
$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
((b*e - a*f)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (((2
*b^2*c*e + 3*a^2*d*f - a*b*(6*d*e - c*f))*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2]) + (d*(((2*b^2*c^2*e - a^2*d*(3*d*e - 7*c*f) - a*b*c*(7*
d*e - c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b^2*c*e + 3*a^2*d*f - a*b*(9*d*e - 5*c*f)
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])))/(a*(b*c - a*d)))/(3*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 12.40 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{x(af-be)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3ba(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}-\frac{\left(bd\,x^2+bc\right)x\left(4a^2df+abcf-7abde+2ce\,b^2\right)}{3a^2(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}-\frac{\left(bd\,x^2+ad\right)dx(cf-de)}{c(ad-bc)^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/b/a/(a*d
-b*c)^2*x*(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-1/3*(b
*d*x^2+b*c)/a^2/(a*d-b*c)^3*x*(4*a^2*d*f+a*b*c*f-7*a*b*d*e+2*b^2*c*e)/((x^
2+a/b)*(b*d*x^2+b*c))^(1/2)-(b*d*x^2+a*d)/c*d/(a*d-b*c)^3*x*(c*f-d*e)/((x^
2+c/d)*(b*d*x^2+a*d))^(1/2)+(-1/3*(a*f-b*e)*d/a/(a*d-b*c)^2+1/3/(a*d-b*c)^
2*(4*a^2*d*f+a*b*c*f-7*a*b*d*e+2*b^2*c*e)/a^2+1/3*b*c/a^2/(a*d-b*c)^3*(4*a
^2*d*f+a*b*c*f-7*a*b*d*e+2*b^2*c*e)-(c*f-d*e)*d/(a*d-b*c)^2/c+a*d^2/c/(a*d
-b*c)^3*(c*f-d*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-(1/3*b*d*(4*a^2*d*f+a*b*c*f-7*a*b*d*e+2*b^2*c*e)/(a*d-b*c)^3/a^2+b*
d^2*(c*f-d*e)/(a*d-b*c)^3/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. 2(354) = 708.

Time = 0.18 (sec) , antiderivative size = 1342, normalized size of antiderivative = 3.50

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((((2*b^6*c^2*d - 7*a*b^5*c*d^2 - 3*a^2*b^4*d^3)*e + (a*b^5*c^2*d + 7
*a^2*b^4*c*d^2)*f)*x^6 + ((2*b^6*c^3 - 3*a*b^5*c^2*d - 17*a^2*b^4*c*d^2 -
6*a^3*b^3*d^3)*e + (a*b^5*c^3 + 9*a^2*b^4*c^2*d + 14*a^3*b^3*c*d^2)*f)*x^4
 + ((4*a*b^5*c^3 - 12*a^2*b^4*c^2*d - 13*a^3*b^3*c*d^2 - 3*a^4*b^2*d^3)*e
+ (2*a^2*b^4*c^3 + 15*a^3*b^3*c^2*d + 7*a^4*b^2*c*d^2)*f)*x^2 + (2*a^2*b^4
*c^3 - 7*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2)*e + (a^3*b^3*c^3 + 7*a^4*b^2*c^2
*d)*f)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
(((2*b^6*c^2*d + (a^2*b^4 - 7*a*b^5)*c*d^2 - 3*(3*a^3*b^3 + a^2*b^4)*d^3)*
e + (a*b^5*c^2*d + 3*a^4*b^2*d^3 + (5*a^3*b^3 + 7*a^2*b^4)*c*d^2)*f)*x^6 +
 ((2*b^6*c^3 + (a^2*b^4 - 3*a*b^5)*c^2*d - (7*a^3*b^3 + 17*a^2*b^4)*c*d^2
- 6*(3*a^4*b^2 + a^3*b^3)*d^3)*e + (a*b^5*c^3 + 6*a^5*b*d^3 + (5*a^3*b^3 +
 9*a^2*b^4)*c^2*d + (13*a^4*b^2 + 14*a^3*b^3)*c*d^2)*f)*x^4 + ((4*a*b^5*c^
3 + 2*(a^3*b^3 - 6*a^2*b^4)*c^2*d - (17*a^4*b^2 + 13*a^3*b^3)*c*d^2 - 3*(3
*a^5*b + a^4*b^2)*d^3)*e + (2*a^2*b^4*c^3 + 3*a^6*d^3 + 5*(2*a^4*b^2 + 3*a
^3*b^3)*c^2*d + (11*a^5*b + 7*a^4*b^2)*c*d^2)*f)*x^2 + (2*a^2*b^4*c^3 + (a
^4*b^2 - 7*a^3*b^3)*c^2*d - 3*(3*a^5*b + a^4*b^2)*c*d^2)*e + (a^3*b^3*c^3
+ 3*a^6*c*d^2 + (5*a^5*b + 7*a^4*b^2)*c^2*d)*f)*sqrt(a*c)*sqrt(-b/a)*ellip
tic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*a*b^5*c^2*d - 7*a^2*b^4*c*d^
2 - 3*a^3*b^3*d^3)*e + (a^2*b^4*c^2*d + 7*a^3*b^3*c*d^2)*f)*x^5 + (2*(a*b^
5*c^3 - 2*a^2*b^4*c^2*d - 4*a^3*b^3*c*d^2 - 3*a^4*b^2*d^3)*e + (a^2*b^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x}}{b^3d^2x^{10}+3a\,b^2d^2x^8+2b^3cd\,x^8+3a^2b\,d^2x^6+6a\,b^2cd\,x^6+b^3c^2x^6+c}\right.$$

$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^3d^2x^{10}+3a\,b^2d^2x^8+2b^3cd\,x^8+3a^2b\,d^2x^6+6a\,b^2cd\,x^6+b^3c^2x^6+a^3d^2x^4+6a^2bcd\,x^4+3a\,b^2c^2x^4+}\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**3*c**2 + 2*a**3*c*d*x**2
+ a**3*d**2*x**4 + 3*a**2*b*c**2*x**2 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*
x**6 + 3*a*b**2*c**2*x**4 + 6*a*b**2*c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*
c**2*x**6 + 2*b**3*c*d*x**8 + b**3*d**2*x**10),x)*f + int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2
*b*c**2*x**2 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4
 + 6*a*b**2*c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x*
*8 + b**3*d**2*x**10),x)*e
```

3.38 $\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 496

$$
\begin{aligned}
\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx &= \frac{(be-af)x}{3a(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^{3/2}} \\
&+ \frac{(2b^2ce+5a^2df-ab(8de-cf))\,x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)^{3/2}} \\
&+ \frac{d(2b^2c^2e-a^2d(de-7cf)-abc(9de-cf))\,x\sqrt{a+bx^2}}{3a^2c(bc-ad)^3(c+dx^2)^{3/2}} \\
&+ \frac{\sqrt{d}(2b^3c^3e-2a^2bcd(5de-7cf)-ab^2c^2(10de-cf)+a^3d^2(2de+cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}(bc-ad)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&- \frac{b\sqrt{d}(b^2c^2e-2abc(9de-4cf)+a^2d(de+8cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}(bc-ad)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

output
```
1/3*(-a*f+b*e)*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)+1/3*(2*b^2*c
*e+5*a^2*d*f-a*b*(-c*f+8*d*e))*x/a^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c
)^(3/2)+1/3*d*(2*b^2*c^2*e-a^2*d*(-7*c*f+d*e)-a*b*c*(-c*f+9*d*e))*x*(b*x^2
+a)^(1/2)/a^2/c/(-a*d+b*c)^3/(d*x^2+c)^(3/2)+1/3*d^(1/2)*(2*b^3*c^3*e-2*a^
2*b*c*d*(-7*c*f+5*d*e)-a*b^2*c^2*(-c*f+10*d*e)+a^3*d^2*(c*f+2*d*e))*(b*x^2
+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
/a^2/c^(3/2)/(-a*d+b*c)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
1/3*b*d^(1/2)*(b^2*c^2*e-2*a*b*c*(-4*c*f+9*d*e)+a^2*d*(8*c*f+d*e))*(b*x^2+
a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/
c^(1/2)/(-a*d+b*c)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.65 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.88

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}x\Big(a^2cd^2(-bc+ad)(-de+cf)\left(a+bx^2\right)^2 - a^2d^2(ad(2de+cf)+bc$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-(Sqrt[b/a]*x*(a^2*c*d^2*(-(b*c) + a*d)*(-(d*e) + c*f)*(a + b*x^2)^2 - a^
2*d^2*(a*d*(2*d*e + c*f) + b*c*(-10*d*e + 7*c*f))*(a + b*x^2)^2*(c + d*x^2
) - a*b^2*c^2*(-(b*c) + a*d)*(-(b*e) + a*f)*(c + d*x^2)^2 - b^2*c^2*(2*b^2
*c*e + 7*a^2*d*f + a*b*(-10*d*e + c*f))*(a + b*x^2)*(c + d*x^2)^2)) + I*b*
c*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*((2*b^3*
c^3*e + a*b^2*c^2*(-10*d*e + c*f) + a^3*d^2*(2*d*e + c*f) + 2*a^2*b*c*d*(-
5*d*e + 7*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*
d)*(2*b^2*c^2*e + a*b*c*(-9*d*e + c*f) + a^2*d*(-(d*e) + 7*c*f))*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*a^2*Sqrt[b/a]*c^2*(b*c - a*d)^4
*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {402, 25, 402, 27, 402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{5d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5d(be-af)x^2+2bce-3ade+acf}{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}dx}{3a(bc-ad)}+\frac{x(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x(5a^2df-ab(8de-cf)+2b^2ce)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{3d\left(\left(5dfa^2-b(8de-cf)a+2b^2ce\right)x^2+a(bce+ade-2acf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3d\int \frac{\left(5dfa^2-b(8de-cf)a+2b^2ce\right)x^2+a(bce+ade-2acf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}+\frac{x(5a^2df-ab(8de-cf)+2b^2ce)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}}{3a(bc-ad)}+\frac{x(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{3d\left(\frac{\int \frac{b\left(-d(de-7cf)a^2-bc(9de-cf)a+2b^2c^2e\right)x^2+a\left(-d(2de+cf)a^2+bc(9de-7cf)a+b^2c^2e\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(a^2(-d)(de-7cf)-abc(9de-cf)+2b^2c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{a(bc-ad)}+$$

$$3a(bc-ad)$$

$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 400$$

$$\frac{3d\left(\frac{\frac{\left(a^3d^2(cf+2de)-2a^2bcd(5de-7cf)-ab^2c^2(10de-cf)+2b^3c^3e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(a^2d(8cf+de)-2abc(9de-4cf)+b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}+\frac{x\sqrt{}}{}\right.}{a(bc-ad)}$$

$$3a(bc-ad)$$

$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{3d\left(\frac{\frac{\sqrt{a+bx^2}\left(a^3d^2(cf+2de)-2a^2bcd(5de-7cf)-ab^2c^2(10de-cf)+2b^3c^3e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab\left(a^2d(8cf+de)-2abc(9de-4cf)+b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a\sqrt{}}}}{bc-ad}}{3c(bc-ad)}\right.}{a(bc-ad)}$$

$$3a(bc-ad)$$

$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{x(5a^2df-ab(8de-cf)+2b^2ce)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(a^2(-d)(de-7cf)-abc(9de-cf)+2b^2c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(a^3d^2(cf+2de)-2a^2bcd(5de-7cf)-ab^2c^2(10de-cf)\cdots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+\cdots\right.}{a\left(c+\cdots\right.}}\cdots}\cdots\right.}{a(\cdots}}{3a(bc-ad)}$$

$$\frac{x(be-af)}{3a\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
((b*e - a*f)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)) + ((
(2*b^2*c*e + 5*a^2*d*f - a*b*(8*d*e - c*f))*x)/(a*(b*c - a*d)*Sqrt[a + b*x
^2]*(c + d*x^2)^(3/2)) + (3*d*(((2*b^2*c^2*e - a^2*d*(d*e - 7*c*f) - a*b*c
*(9*d*e - c*f))*x*Sqrt[a + b*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) + (
((2*b^3*c^3*e - 2*a^2*b*c*d*(5*d*e - 7*c*f) - a*b^2*c^2*(10*d*e - c*f) + a
^3*d^2*(2*d*e + c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b^2*c^2*e - 2*a*b*c*(9*d*e -
4*c*f) + a^2*d*(d*e + 8*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d))))/(a*(b*c - a*d)))/(3*a*
(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(461) = 922$.

Time = 14.50 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.04

method	result	size
elliptic	Expression too large to display	1013
default	Expression too large to display	3687

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((-1/3/b/d*(2*
a*c*f-a*d*e-b*c*e)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^3-1/3*(a^2*c*d*f-a^2*
d^2*e+a*b*c^2*f-b^2*c^2*e)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^2*x)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^4+(a*d+b*c)/d/b*x^2+a*c/d/b)^2-2*b*d*(-
1/6*(a^3*c*d^2*f+2*a^3*d^3*e+14*a^2*b*c^2*d*f-10*a^2*b*c*d^2*e+a*b^2*c^3*f
-10*a*b^2*c^2*d*e+2*b^3*c^3*e)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2*x^3-1
/6*(a^4*c*d^3*f+2*a^4*d^4*e+7*a^3*b*c^2*d^2*f-9*a^3*b*c*d^3*e+7*a^2*b^2*c^
3*d*f-2*a^2*b^2*c^2*d^2*e+a*b^3*c^4*f-9*a*b^3*c^3*d*e+2*b^4*c^4*e)/a^2/c^2
/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2/b/d*x)/((x^4+(a*d+b*c)/d/b*x^2+a*c/d/b)*b*d
)^(1/2)+(1/3/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(a^2*c*d*f+2*a^2*d^2*e+a*b*c^2*f-
6*a*b*c*d*e+2*b^2*c^2*e)/a^2/c^2-1/3*(a^4*c*d^3*f+2*a^4*d^4*e+7*a^3*b*c^2*
d^2*f-9*a^3*b*c*d^3*e+7*a^2*b^2*c^3*d*f-2*a^2*b^2*c^2*d^2*e+a*b^3*c^4*f-9*
a*b^3*c^3*d*e+2*b^4*c^4*e)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2)/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*b*(a^3*c*d^2*f+2
*a^3*d^3*e+14*a^2*b*c^2*d*f-10*a^2*b*c*d^2*e+a*b^2*c^3*f-10*a*b^2*c^2*d*e+
2*b^3*c^3*e)/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2/a^2/c/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2280 vs. $2(461) = 922$.

Time = 0.36 (sec) , antiderivative size = 2280, normalized size of antiderivative = 4.60

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(((2*(b^6*c^3*d^2 - 5*a*b^5*c^2*d^3 - 5*a^2*b^4*c*d^4 + a^3*b^3*d^5)*
e + (a*b^5*c^3*d^2 + 14*a^2*b^4*c^2*d^3 + a^3*b^3*c*d^4)*f)*x^8 + 2*(2*(b^
6*c^4*d - 4*a*b^5*c^3*d^2 - 10*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2
*d^5)*e + (a*b^5*c^4*d + 15*a^2*b^4*c^3*d^2 + 15*a^3*b^3*c^2*d^3 + a^4*b^2
*c*d^4)*f)*x^6 + (2*(b^6*c^5 - a*b^5*c^4*d - 24*a^2*b^4*c^3*d^2 - 24*a^3*b
^3*c^2*d^3 - a^4*b^2*c*d^4 + a^5*b*d^5)*e + (a*b^5*c^5 + 18*a^2*b^4*c^4*d
+ 58*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + a^5*b*c*d^4)*f)*x^4 + 2*(2*(a*
b^5*c^5 - 4*a^2*b^4*c^4*d - 10*a^3*b^3*c^3*d^2 - 4*a^4*b^2*c^2*d^3 + a^5*b
*c*d^4)*e + (a^2*b^4*c^5 + 15*a^3*b^3*c^4*d + 15*a^4*b^2*c^3*d^2 + a^5*b*c
^2*d^3)*f)*x^2 + 2*(a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - 5*a^4*b^2*c^3*d^2 + a^
5*b*c^2*d^3)*e + (a^3*b^3*c^5 + 14*a^4*b^2*c^4*d + a^5*b*c^3*d^2)*f)*sqrt(
a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*b^6*c^3
*d^2 + (a^2*b^4 - 10*a*b^5)*c^2*d^3 - 2*(9*a^3*b^3 + 5*a^2*b^4)*c*d^4 + (a
^4*b^2 + 2*a^3*b^3)*d^5)*e + (a*b^5*c^3*d^2 + 2*(4*a^3*b^3 + 7*a^2*b^4)*c^
2*d^3 + (8*a^4*b^2 + a^3*b^3)*c*d^4)*f)*x^8 + 2*((2*b^6*c^4*d + (a^2*b^4 -
 8*a*b^5)*c^3*d^2 - (17*a^3*b^3 + 20*a^2*b^4)*c^2*d^3 - (17*a^4*b^2 + 8*a^
3*b^3)*c*d^4 + (a^5*b + 2*a^4*b^2)*d^5)*e + (a*b^5*c^4*d + (8*a^3*b^3 + 15
*a^2*b^4)*c^3*d^2 + (16*a^4*b^2 + 15*a^3*b^3)*c^2*d^3 + (8*a^5*b + a^4*b^2
)*c*d^4)*f)*x^6 + ((2*b^6*c^5 + (a^2*b^4 - 2*a*b^5)*c^4*d - 2*(7*a^3*b^3 +
 24*a^2*b^4)*c^3*d^2 - 2*(35*a^4*b^2 + 24*a^3*b^3)*c^2*d^3 - 2*(7*a^5*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e + fx^2}{(a + bx^2)^{5/2} (c + dx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e + fx^2}{(a + bx^2)^{5/2} (c + dx^2)^{5/2}} \, dx = \int \frac{fx^2 + e}{(bx^2 + a)^{\frac{5}{2}} (dx^2 + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{e + fx^2}{(a + bx^2)^{5/2} (c + dx^2)^{5/2}} \, dx = \int \frac{fx^2 + e}{(bx^2 + a)^{\frac{5}{2}} (dx^2 + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e + fx^2}{(a + bx^2)^{5/2} (c + dx^2)^{5/2}} \, dx = \int \frac{f\,x^2 + e}{(b\,x^2 + a)^{5/2}\,(d\,x^2 + c)^{5/2}} \, dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**6)/(a**3*c**3 + 3*a**3*c**2*d*x**2 + 3*a**3*c*d**2*x**4 + a*
*3*d**3*x**6 + 3*a**2*b*c**3*x**2 + 9*a**2*b*c**2*d*x**4 + 9*a**2*b*c*d**2
*x**6 + 3*a**2*b*d**3*x**8 + 3*a*b**2*c**3*x**4 + 9*a*b**2*c**2*d*x**6 + 9
*a*b**2*c*d**2*x**8 + 3*a*b**2*d**3*x**10 + b**3*c**3*x**6 + 3*b**3*c**2*d
*x**8 + 3*b**3*c*d**2*x**10 + b**3*d**3*x**12),x)*a**2*b*c**2*d*f + 6*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**3*c**3 + 3*a**3*c**2*d*x**2 +
 3*a**3*c*d**2*x**4 + a**3*d**3*x**6 + 3*a**2*b*c**3*x**2 + 9*a**2*b*c**2*
d*x**4 + 9*a**2*b*c*d**2*x**6 + 3*a**2*b*d**3*x**8 + 3*a*b**2*c**3*x**4 +
9*a*b**2*c**2*d*x**6 + 9*a*b**2*c*d**2*x**8 + 3*a*b**2*d**3*x**10 + b**3*c
**3*x**6 + 3*b**3*c**2*d*x**8 + 3*b**3*c*d**2*x**10 + b**3*d**3*x**12),x)*
a**2*b*c*d**2*f*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**
3*c**3 + 3*a**3*c**2*d*x**2 + 3*a**3*c*d**2*x**4 + a**3*d**3*x**6 + 3*a**2
*b*c**3*x**2 + 9*a**2*b*c**2*d*x**4 + 9*a**2*b*c*d**2*x**6 + 3*a**2*b*d**3
*x**8 + 3*a*b**2*c**3*x**4 + 9*a*b**2*c**2*d*x**6 + 9*a*b**2*c*d**2*x**8 +
 3*a*b**2*d**3*x**10 + b**3*c**3*x**6 + 3*b**3*c**2*d*x**8 + 3*b**3*c*d**2
*x**10 + b**3*d**3*x**12),x)*a**2*b*d**3*f*x**4 + 6*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**6)/(a**3*c**3 + 3*a**3*c**2*d*x**2 + 3*a**3*c*d**2*x**
4 + a**3*d**3*x**6 + 3*a**2*b*c**3*x**2 + 9*a**2*b*c**2*d*x**4 + 9*a**2*b*
c*d**2*x**6 + 3*a**2*b*d**3*x**8 + 3*a*b**2*c**3*x**4 + 9*a*b**2*c**2*d...
```

3.39 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{fx\sqrt{c+dx^2}}{d\sqrt{a+bx^2}} - \frac{\sqrt{a}f\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}e\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f*x*(d*x^2+c)^(1/2)/d/(b*x^2+a)^(1/2)-a^(1/2)*f*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*e*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(de-cf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{406}$$

$$e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+ f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf+\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf\right)\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{d\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{e\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{fc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{Ell}\ldots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f+EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*d*e+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f)*((d*x^2
+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(-b/a)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}c^2fx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}cdf - (d^2e+c^2f)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc})}{bcd^2x}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c^2*f*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*d*f - (d^2*e + c^2*f)*sqrt(b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*e
```

3.40 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}f\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output
```
c^(1/2)*f*(b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-
b*c/a/d)^(1/2))/b/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+c^(1/2)*(-a*f
+b*e)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*
c/a/d)^(1/2))/b/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\left(-cfE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)+(de+cf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),-\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*(-(c*f*EllipticE[I*ArcSinh[S
qrt[b/a]*x], -((a*d)/(b*c))]) + (d*e + c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], -((a*d)/(b*c))]))/(Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \ 399$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{b}+\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \ 323$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}+\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(be-af)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{a+bx^2}\sqrt{c-dx^2}}+\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{321}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{b}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{f\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{f\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}f\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

input

```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output

```
(Sqrt[c]*f*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x
)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2
]) + (Sqrt[c]*(b*e - a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]
*Sqrt[c - d*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

Maple [A] (verified)

Time = 5.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)af+\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)be+\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)af\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-x^2d+c}{c}}\,\sqrt{b\,x^2+a}\,\sqrt{-x^2}}{b\sqrt{\frac{d}{c}}\,(-bd\,x^4-ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(-x^2d+c)(b\,x^2+a)}\left(\frac{e\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}-\frac{fa\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+}}\right.}{\sqrt{-x^2d+c}\,\sqrt{b\,x^2+a}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*f+EllipticF(x*(1/c*d)^(1/2
),(-b*c/a/d)^(1/2))*b*e+EllipticE(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*f)*(
(b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/b
/(1/c*d)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}c^2fx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{bx^2+a}\sqrt{-dx^2+c}cdf - (d^2e+c^2f)\sqrt{-bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\right.}{bcd^2x}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
-(sqrt(-b*d)*c^2*f*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c))
 + sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*c*d*f - (d^2*e + c^2*f)*sqrt(-b*d)*x*s
qrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)f+\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 -
b*d*x**4),x)*f + int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 +
 b*c*x**2 - b*d*x**4),x)*e
```

3.41 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
a^(1/2)*f*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+a^(1/2)*(-c*f
+d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*
d/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)+(de-cf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)\right)}{\sqrt{-\frac{b}{a}}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + (d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))]))/(Sqrt[-(b/a)]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 399$$

$$\frac{(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{321}$$

$$\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{f\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input

```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output

```
(Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x
)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c
]) + (Sqrt[a]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)cf+\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)cf\right)\sqrt{\frac{x^2d+c}{c}}\sqrt{\frac{-b\,x^2+a}{a}}\sqrt{-b\,x^2+a}\,\sqrt{x^2}}{d\sqrt{\frac{b}{a}}\,(-bd\,x^4+ad\,x^2-x^2bc+ac)}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(\frac{e\sqrt{1-\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{fc\sqrt{1-\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\operatorname{Elli}\right.}{\sqrt{\frac{b}{a}}\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}\,d}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f+EllipticF(x*(b/a)^(1/2),(-
a*d/b/c)^(1/2))*d*e+EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f)*((d*x^2
+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(b/a)
^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}a^2fx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2+c}abf - (b^2e+a^2f)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\right.}{ab^2dx}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
-(sqrt(-b*d)*a^2*f*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
 + sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*f - (b^2*e + a^2*f)*sqrt(-b*d)*x*s
qrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)f+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 -
b*d*x**4),x)*f + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 -
 b*c*x**2 - b*d*x**4),x)*e
```

3.42 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 192

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx &= -\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}} \\ &\quad+\frac{\sqrt{a}(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-a^(1/2)*f*(1-b*x^2/a)^(1/2)*(-d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),
(a*d/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)+a^(1/2)*(c*f
+d*e)*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(a*d
/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\left(-cfE\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(de+cf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{-\frac{b}{a}}d\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*(-(c*f*EllipticE[I*ArcSinh[S
qrt[-(b/a)]*x], (a*d)/(b*c)]) + (d*e + c*f)*EllipticF[I*ArcSinh[Sqrt[-(b/a
)]*x], (a*d)/(b*c)]))/(Sqrt[-(b/a)]*d*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \ 399$$

$$\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{b}-\frac{f\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \ 323$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}-\frac{f\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{f\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{f\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{b}$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{f\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{f\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}f\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
-((Sqrt[c]*f*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]
)) + (Sqrt[c]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[a - b*x^2]*Sq
rt[c - d*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

method	result
default	$\frac{\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)af+\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)be-\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)af\right)\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-x^2d+c}{c}}\,\sqrt{-b\,x^2+a}\,\sqrt{-x^2d+c}}{b\sqrt{\frac{d}{c}}\,(bd\,x^4-ad\,x^2-x^2bc+ac)}$
elliptic	$\frac{\sqrt{(-x^2d+c)(-b\,x^2+a)}\left(\frac{e\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-x^2bc+ac}}+\frac{fa\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-x^2bc}}\right.}{\sqrt{-x^2d+c}\,\sqrt{-b\,x^2+a}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*f+EllipticF(x*(1/c*d)^(1/2),
(b*c/a/d)^(1/2))*b*e-EllipticE(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*f)*((-b*
x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/b/(
1/c*d)^(1/2)/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$
$$=\frac{\sqrt{bd}a^2fx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})+\sqrt{-bx^2+a}\sqrt{-dx^2+c}abf-(b^2e+a^2f)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)}{ab^2dx}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
(sqrt(b*d)*a^2*f*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*a*b*f - (b^2*e + a^2*f)*sqrt(b*d)*x*sqrt
(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx\right)f + \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c - a*d*x**2 - b*c*x**2 +
b*d*x**4),x)*f + int((sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c - a*d*x**2 -
 b*c*x**2 + b*d*x**4),x)*e
```

3.43 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{(de-cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-(-c*f+d*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))/c^(1/2)/d^(1/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+c^(1/2)*(-a*f+b*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(1/2)/(-a*d+b*c)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)-ibc(-de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\ldots}{\sqrt{\frac{b}{a}}cd(-bc+ad\ldots}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 400$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \; 313$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
-(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [A] (verified)

Time = 7.79 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bcdf\, x^{3}+\sqrt{-\frac{b}{a}}\, b\, d^{2} e\, x^{3}+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) acdf-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right.\right.}{\sqrt{-\frac{b}{a}}\, c}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(bd\, x^{2}+ad\right)x(cf-de)}{cd(ad-bc)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\, x^{2}+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad-bc)}\right)\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}-\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*c*d*f*x^3+(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f-((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*b*c^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e-(-b/a)^(1/2)*a*c*d*f*x+(-b/a)^(1
/2)*a*d^2*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/d/c/(a*d-b*c)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.24

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{(abd^2e-abcdf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)}{ab^2c^3d-a^2bc^2d^2}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-((a*b*d^2*e - a*b*c*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (b^2*c*d*e -
 b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((a*b + b^2)*c*d*e + ((a*b + b^2)*d^2*e
 - (b^2*c*d + a^2*d^2)*f)*x^2 - (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(-b/a
)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*b^2*c^3*d - a^2*b*c^2*d^
2 + (a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)f+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*e
```

3.44 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 207

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{(de-cf)\sqrt{a-bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1+\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{\frac{c(a-bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(be+af)\sqrt{a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1+\frac{bc}{ad}\right)}{a\sqrt{d}(bc+ad)\sqrt{\frac{c(a-bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
(-c*f+d*e)*(-b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),
(1+b*c/a/d)^(1/2))/c^(1/2)/d^(1/2)/(a*d+b*c)/(c*(-b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+c^(1/2)*(a*f+b*e)*(-b*x^2+a)^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(1+b*c/a/d)^(1/2))/a/d^(1/2)/(a*d+b*c)/(c*(-b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de-cf)x(a-bx^2)+ibc(-de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}}{\sqrt{-\frac{b}{a}}cd(bc}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c +
 a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {402, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}-\frac{\int -\frac{b(de-cf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(de-cf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{399}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{321}$$

$$\frac{\frac{b(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{331}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{330}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + ((Sqrt[a
]*Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2
)/c]) + (Sqrt[a]*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a -
 b*x^2]*Sqrt[c + d*x^2]))/(c*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bcdf\, x^{3}-\sqrt{\frac{b}{a}}\, b\, d^{2} e\, x^{3}+\sqrt{\frac{-b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) acdf+\sqrt{\frac{-b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) b \cdots}{\sqrt{\frac{b}{a}}\, dc(\cdots}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(-bd\,x^{2}+ad\right) x (cf-de)}{cd(ad+bc)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(-bd\,x^{2}+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad+bc)}\right)\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-1-\frac{ad-bc}{cb}}\right) \cdots}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}} \cdots\right)}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b*c*d*f*x^3-(b/a)^(1/2)*b*d^2*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*f+((-b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))
*b*c^2*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),
(-a*d/b/c)^(1/2))*b*c^2*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e-(b/a)^(1/2)*a*c*d*f*x+(b/a)^(1/
2)*a*d^2*e*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/d/c/(a*d+b*c)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{(abd^2e-abcdf)\sqrt{-bx^2+a}\sqrt{dx^2+c}x+(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2}{}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
((a*b*d^2*e - a*b*c*d*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x + (b^2*c*d*e -
 b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(b/a)*elliptic_e(a
rcsin(x*sqrt(b/a)), -a*d/(b*c)) + ((a*b - b^2)*c*d*e + ((a*b - b^2)*d^2*e
+ (b^2*c*d + a^2*d^2)*f)*x^2 + (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(b/a)*
elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a*b^2*c^3*d + a^2*b*c^2*d^2
+ (a*b^2*c^2*d^2 + a^2*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-b\,d^2x^6+a\,d^2x^4-2bcd\,x^4+2acd\,x^2-b\,c^2x^2+a\,c^2}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-b\,d^2x^6+a\,d^2x^4-2bcd\,x^4+2acd\,x^2-b\,c^2x^2+a\,c^2}dx\right)e$$

input

```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 - b*c**2*x**2 - 2*b*c*d*x**4 - b*d**2*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 - b*c**2*x**
2 - 2*b*c*d*x**4 - b*d**2*x**6),x)*e
```

3.45 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 237

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output
```
(c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(a*d+b*c)/(-d*x^2+c)^(1/2)-(c*f+d*e)*(b*x^2+
a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/c
^(1/2)/d^(1/2)/(a*d+b*c)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*(1+b*x^2/a)^
(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/c^(1
/2)/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.90

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de+cf)x(a+bx^2)-ibc(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{b}{a}}cd(bc+ad}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]
 + I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a +
b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {402, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{\int \frac{c(be-af)-b(de+cf)x^2}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \; 399$$

$$\frac{e(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{e\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt{c}e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{c(ad+bc)}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) + (-((Sqrt
[c]*(d*e + c*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*
x^2])) + (Sqrt[c]*(b*c + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*
x^2]*Sqrt[c - d*x^2]))/(c*(b*c + a*d))
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\, bcf\, x^{3}+\sqrt{\frac{d}{c}}\, bde\, x^{3}+\sqrt{\frac{-x^{2} d+c}{c}}\, \sqrt{\frac{b\,x^{2}+a}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{bc}{ad}}\right) ade+\sqrt{\frac{-x^{2} d+c}{c}}\, \sqrt{\frac{b\,x^{2}+a}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{bc}{ad}}\right) bce-\right.}{\sqrt{\frac{d}{c}}\, c(ad+b}$
elliptic	$\frac{\sqrt{(-x^{2} d+c)(b\,x^{2}+a)}\, \left(-\frac{\left(-bd\,x^{2}-ad\right) x (cf+de)}{cd(ad+bc) \sqrt{\left(x^{2}-\frac{c}{d}\right)\left(-bd\,x^{2}-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad+bc)}\right) \sqrt{1-\frac{d\,x^{2}}{c}}\, \sqrt{1+\frac{b\,x^{2}}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-ad+}{ad}}\right.}{\sqrt{\frac{d}{c}}\, \sqrt{-bd\,x^{4}-ad\,x^{2}+x^{2} bc+ac}}\right.}{\sqrt{-x^{2} d+c}\, \sqrt{b\,x^{2}+a}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((1/c*d)^(1/2)*b*c*f*x^3+(1/c*d)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((b*
x^2+a)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*d*e+((-d*x^2
+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2
))*b*c*e-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(1/c*d)^(1/2
),(-b*c/a/d)^(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*Ellipti
cE(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*d*e+(1/c*d)^(1/2)*a*c*f*x+(1/c*d)^(
1/2)*a*d*e*x)*(-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(1/c*d)^(1/2)/c/(a*d+b*c)/(
-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{(acd^2e+ac^2df)\sqrt{bx^2+a}\sqrt{-dx^2+c}x-(acd^2e+ac^2df-(ad^3e+acd^2f)x}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
((a*c*d^2*e + a*c^2*d*f)*sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e +
 a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(arc
sin(x*sqrt(d/c)), -b*c/(a*d)) - (((b*c^2*d + a*d^3)*e - (a*c^2*d - a*c*d^2
)*f)*x^2 - (b*c^3 + a*c*d^2)*e + (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)*
elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)))/(a*b*c^4*d + a^2*c^3*d^2 - (a
*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(-d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^2x^6+a\,d^2x^4-2bcd\,x^4-2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)f + \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^2x^6+a\,d^2x^4-2bcd\,x^4-2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 - 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 - 2*b*c*d*x**4 + b*d**2*x**6),x)*f + int((sqrt(c - d
*x**2)*sqrt(a + b*x**2))/(a*c**2 - 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 - 2*b*c*d*x**4 + b*d**2*x**6),x)*e
```

3.46 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c-dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 252

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt{a}\sqrt{b}(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{cd(bc-ad)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}} - \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

output

```
-(c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a^(1/2)*b^(1/2
)*(c*f+d*e)*(1-b*x^2/a)^(1/2)*(-d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
,(a*d/b/c)^(1/2))/c/d/(-a*d+b*c)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)-a^(1/2
)*f*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(a*d/b
/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.88

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de+cf)x(a-bx^2)+ibc(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(}{\sqrt{-\frac{b}{a}}cd(-bc+}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*
c)] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*
d)*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {402, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 402$$

$$-\frac{\int -\frac{c(be+af)-b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c(be+af)-b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{399}$$

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+e(bc-ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{323}$$

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{e\sqrt{1-\frac{dx^2}{c}}(bc-ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{323}$$

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{321}$$

$$\frac{(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

output
```
-(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((Sqr
t[c]*(d*e + c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^
2]) + (Sqrt[c]*(b*c - a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Ellip
ticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*S
qrt[c - d*x^2]))/(c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.34

method	result
default	$\frac{\left(-\sqrt{\frac{d}{c}}\, bcf\, x^3-\sqrt{\frac{d}{c}}\, bde\, x^3+\sqrt{\frac{-x^2 d+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right) ade-\sqrt{\frac{-x^2 d+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right) bce}{\sqrt{\frac{d}{c}}\, c(ad-}$
elliptic	$\frac{\sqrt{(-x^2 d+c)(-b\,x^2+a)}\left(-\frac{\left(bd\,x^2-ad\right)x(cf+de)}{cd(ad-bc)\sqrt{\left(x^2-\frac{c}{d}\right)\left(bd\,x^2-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad-bc)}\right)\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-}{ad}}\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-x^2bc+ac}}\right.}{\sqrt{-x^2 d+c}\,\sqrt{-b\,x^2+a}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(1/c*d)^(1/2)*b*c*f*x^3-(1/c*d)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*d*e-((-d*x^
2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/
2))*b*c*e-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(1/c*d)^(1
/2),(b*c/a/d)^(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticE(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*d*e+(1/c*d)^(1/2)*a*c*f*x+(1/c*d)^
(1/2)*a*d*e*x)*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(1/c*d)^(1/2)/c/(a*d-b*c)
/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx =$$

$$-\frac{(acd^2e+ac^2df)\sqrt{-bx^2+a}\sqrt{-dx^2+c}x-(acd^2e+ac^2df-(ad^3e+acd^2f)x^2)\sqrt{ac}\sqrt{\frac{d}{c}}E(\arcsin\left(x\sqrt{\frac{d}{c}}\right)}{abc^4d-a^2c^3d^2-}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
-((a*c*d^2*e + a*c^2*d*f)*sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e
 + a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(a
rcsin(x*sqrt(d/c)), b*c/(a*d)) + (((b*c^2*d - a*d^3)*e + (a*c^2*d - a*c*d^
2)*f)*x^2 - (b*c^3 - a*c*d^2)*e - (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)
*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d)))/(a*b*c^4*d - a^2*c^3*d^2 - (a
*b*c^3*d^2 - a^2*c^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx=\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-b\,d^2x^6+a\,d^2x^4+2bcd\,x^4-2acd\,x^2-b\,c^2x^2+a\,c^2}dx\right)f+\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-b\,d^2x^6+a\,d^2x^4+2bcd\,x^4-2acd\,x^2-b\,c^2x^2+a\,c^2}dx\right)e$$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c**2 - 2*a*c*d*x**2 + a*d*
*2*x**4 - b*c**2*x**2 + 2*b*c*d*x**4 - b*d**2*x**6),x)*f + int((sqrt(c - d
*x**2)*sqrt(a - b*x**2))/(a*c**2 - 2*a*c*d*x**2 + a*d**2*x**4 - b*c**2*x**
2 + 2*b*c*d*x**4 - b*d**2*x**6),x)*e
```

3.47 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{bx\sqrt{3+fx^2}}{f\sqrt{2+dx^2}} - \frac{\sqrt{3}b\sqrt{3+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{d}f\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} + \frac{a\sqrt{3+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output

```
b*x*(f*x^2+3)^(1/2)/f/(d*x^2+2)^(1/2)-3^(1/2)*b*(f*x^2+3)^(1/2)*EllipticE(
d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^(1/2),1/3*(9-6*f/d)^(1/2))/d^(1/2)/f/(d*x^2+
2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)+1/3*a*(f*x^2+3)^(1/2)*InverseJacobiAM
(arctan(1/2*d^(1/2)*x*2^(1/2)),1/3*(9-6*f/d)^(1/2))*3^(1/2)/d^(1/2)/(d*x^2
+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = -\frac{i\left(3bE\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-3b+af)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}\sqrt{d}f}$$

input
```
Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-3*b
+ a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))/(Sqrt[3]*S
qrt[d]*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx$$

$$\Big\downarrow \text{406}$$

$$a\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+b\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx$$

$$\Big\downarrow \text{320}$$

$$b\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{a\sqrt{dx^2+2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

$$\Big\downarrow \text{388}$$

$$b\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int\frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+b\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)$$

input
```
Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
b*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 +
d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTa
n[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2
)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

method	result
default	$\frac{\left(\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)ad-2\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)b+2\operatorname{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)b\right)\sqrt{2}}{2d\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(x^2d+2)}\left(\frac{a\sqrt{3f\,x^2+9}\,\sqrt{2x^2d+4}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{2\sqrt{-3f}\,\sqrt{df\,x^4+3x^2d+2f\,x^2+6}}-\frac{b\sqrt{3f\,x^2+9}\,\sqrt{2x^2d+4}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right.\right.}{\sqrt{-3f}\,\sqrt{df\,x^4+3x^2d+2f}}\right.}{\sqrt{f\,x^2+3}\,\sqrt{x^2d+2}}$

input
```
int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))
*a*d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2
))*b+2*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2
))*b)*2^(1/2)/d/(-f)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx =$$
$$-\frac{9\sqrt{3}\sqrt{df}bx\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2f}{3d}) - \sqrt{3}(af^2+9b)\sqrt{df}x\sqrt{-\frac{1}{f}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2f}{3d}) - 3}{3df^2x}$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(9*sqrt(3)*sqrt(d*f)*b*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1
/f)/x), 2/3*f/d) - sqrt(3)*(a*f^2 + 9*b)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f
(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) - 3*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3
)*b*f)/(d*f^2*x)
```

Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{b\,x^2+a}{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\left(\int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,x^2}{df\,x^4+3d\,x^2+2f\,x^2+6}dx\right)b+\left(\int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}{df\,x^4+3d\,x^2+2f\,x^2+6}dx\right)a$$

input
```
int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

output
```
int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x*
*2 + 6),x)*b + int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(d*f*x**4 + 3*d*x**
2 + 2*f*x**2 + 6),x)*a
```

3.48 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 259

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = -\frac{(6bd-2bf-3adf)x\sqrt{3+fx^2}}{3f^2\sqrt{2+dx^2}}+\frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} + \frac{(6bd-2bf-3adf)\sqrt{3+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}\sqrt{d}f^2\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} - \frac{2(b-af)\sqrt{3+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}\sqrt{d}f\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output
```
-1/3*(-3*a*d*f+6*b*d-2*b*f)*x*(f*x^2+3)^(1/2)/f^2/(d*x^2+2)^(1/2)+1/3*b*x*
(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/f+1/3*(-3*a*d*f+6*b*d-2*b*f)*(f*x^2+3)^(1/
2)*EllipticE(d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^(1/2),1/3*(9-6*f/d)^(1/2))*3^(1
/2)/d^(1/2)/f^2/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)-2/3*(-a*f+b)*(
f*x^2+3)^(1/2)*InverseJacobiAM(arctan(1/2*d^(1/2)*x*2^(1/2)),1/3*(9-6*f/d)
^(1/2))*3^(1/2)/d^(1/2)/f/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$$
$$= \frac{b\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}+i\sqrt{3}(6bd-2bf-3adf)E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}(3d-2f)(-2b+af)}{3\sqrt{d}f^2}$$

input
```
Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

output
```
(b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d - 2*b*f
- 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt
[3]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*
f)/(3*d)])/(3*Sqrt[d]*f^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}(a+bx^2)}{\sqrt{fx^2+3}}dx$$
$$\Big\downarrow \; 403$$
$$\frac{\int -\frac{(6bd-3afd-2bf)x^2+6(b-af)}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}+\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$
$$\Big\downarrow \; 25$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{\int \frac{(6bd-3afd-2bf)x^2+6(b-af)}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{6(b-af)\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+(-3adf+6bd-2bf)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{(-3adf+6bd-2bf)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}}{3f}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{(-3adf+6bd-2bf)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}}{3f}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}-\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(b-af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+(-3adf+6bd-2bf)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{3f}$$

input
```
Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

output
```
(b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) - ((6*b*d - 2*b*f - 3*a*d*f)*(
(x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*Ellipti
cE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (3*Sqrt[2]*(b - a*f)*Sqrt[2 + d*x^2]*
EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[f]*Sqrt[(2
+ d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]))/(3*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(f x^2+3)(x^2 d+2)}\,\Big(\frac{b x\sqrt{d f x^4+3x^2 d+2 f x^2+6}}{3f}+\frac{(2a-\frac{2b}{f})\sqrt{3f x^2+9}\,\sqrt{2x^2 d+4}\ \mathrm{EllipticF}\big(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\big)}{2\sqrt{-3f}\,\sqrt{d f x^4+3x^2 d+2 f x^2+6}}-\frac{(a d+2b-\frac{b(6d+4f)}{3f})\cdots}{\cdots}}{\sqrt{f x^2+3}\,\sqrt{x^2 d+2}\cdots}$
risch	$\frac{b x\sqrt{x^2 d+2}\,\sqrt{f x^2+3}}{3f}+\frac{\Big(-\frac{(3adf-6bd+2fb)\sqrt{3f x^2+9}\,\sqrt{2x^2 d+4}\,\big(\mathrm{EllipticF}(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2})-\mathrm{EllipticE}(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\cdots}{\sqrt{-3f}\,\sqrt{d f x^4+3x^2 d+2 f x^2+6}\,d}\cdots}{\cdots}$
default	$\frac{\sqrt{x^2 d+2}\,\sqrt{f x^2+3}\,\Big(b d^2 f x^5\sqrt{-f}+3\sqrt{2}\ \mathrm{EllipticE}\big(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\big)\,a d f\sqrt{x^2 d+2}\,\sqrt{f x^2+3}+3b d^2 x^3\sqrt{-f}+2b d f x^3\sqrt{-f}-6\cdots}{\cdots}$

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/3*b/f*x*(d*
f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(2*a-2*b/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1
/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-
3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-(a*d+2*b-1/3*b/f*(6*d+4*f))/(-3*f
)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1
/2)/d*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-Elliptic
E(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \frac{3\sqrt{3}(6\,bd-(3\,ad+2\,b)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2\,f}{3\,d})+\sqrt{3}(2\,af^3-2\,bf^2-18\,bd+3\,(3\,ad+}{3\,df^3x}$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(3)*(6*b*d - (3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_e
(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) + sqrt(3)*(2*a*f^3 - 2*b*f^2 - 18*
b*d + 3*(3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*
sqrt(-1/f)/x), 2/3*f/d) + (b*d*f^2*x^2 - 6*b*d*f + (3*a*d + 2*b)*f^2)*sqrt
(d*x^2 + 2)*sqrt(f*x^2 + 3))/(d*f^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+2}}{\sqrt{f\,x^2+3}}\,dx$$

input
```
int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2),x)
```

output
```
int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$$
$$= \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,bx+3\left(\int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,x^2}{df\,x^4+3d\,x^2+2f\,x^2+6}dx\right)adf-6\left(\int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,x^2}{df\,x^4+3d\,x^2+2f\,x^2+6}dx\right)bd+2\left(\int \frac{\sqrt{f\,x^2+3}\,\sqrt{}}{df\,x^4+3d\,x^2}\right.}{3f}$$

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

output
```
(sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*b*x + 3*int((sqrt(f*x**2 + 3)*sqrt(d*x*
*2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*a*d*f - 6*int((sqrt(
f*x**2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)
*b*d + 2*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2
 + 2*f*x**2 + 6),x)*b*f + 6*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(d*f*x
**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*a*f - 6*int((sqrt(f*x**2 + 3)*sqrt(d*x**
2 + 2))/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*b)/(3*f)
```

3.49 $\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 356

$$\begin{aligned}\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx &= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{3+fx^2}}{15df^2\sqrt{2+dx^2}} \\ &\quad - \frac{(6bd-2bf-5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx\sqrt{2+dx^2}(3+fx^2)^{3/2}}{5f} \\ &\quad - \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,\sqrt{3+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{5\sqrt{3}d^{3/2}f^2\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} \\ &\quad - \frac{2(3bd+2bf-10adf)\sqrt{3+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{5\sqrt{3}d^{3/2}f\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}\end{aligned}$$

output
```
1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*x*(f*x^2+3)^(1/2)/d/f^2/(
d*x^2+2)^(1/2)-1/15*(-5*a*d*f+6*b*d-2*b*f)*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/
2)/d/f+1/5*b*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(3/2)/f-1/15*(5*a*d*f*(3*d+2*f)-2
*b*(9*d^2-6*d*f+4*f^2))*(f*x^2+3)^(1/2)*EllipticE(d^(1/2)*x*2^(1/2)/(2*d*x
^2+4)^(1/2),1/3*(9-6*f/d)^(1/2))*3^(1/2)/d^(3/2)/f^2/(d*x^2+2)^(1/2)/((f*x
^2+3)/(d*x^2+2))^(1/2)-2/15*(-10*a*d*f+3*b*d+2*b*f)*(f*x^2+3)^(1/2)*Invers
eJacobiAM(arctan(1/2*d^(1/2)*x*2^(1/2)),1/3*(9-6*f/d)^(1/2))*3^(1/2)/d^(3/
2)/f/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \frac{\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}(2bf+5adf+3bd(1+fx^2))+i\sqrt{3}(-5adf(3d+2f)+2b(9d^2-6df+4f^2))}{15d^{3/2}f^2}$$

input
```
Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

output
```
(Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3*b*d*(1 +
 f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2 - 6*d*f + 4*f^2))*
EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3]*(3*d -
2*f)*(-6*b*d + 2*b*f + 5*a*d*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]],
(2*f)/(3*d)])/(15*d^(3/2)*f^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {403, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx^2+2}\sqrt{fx^2+3}\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int -\frac{\sqrt{dx^2+2}\left(3(2b-5ad)-(3bd+5afd-4bf)x^2\right)}{\sqrt{fx^2+3}}dx}{5d}+\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\int \frac{\sqrt{dx^2+2}\left(3(2b-5ad)-(3bd+5afd-4bf)x^2\right)}{\sqrt{fx^2+3}}dx}{5d}$$

$$\Big\downarrow \; 403$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\int \frac{6(3bd-10afd+2bf)-\left(5adf(3d+2f)-2b\left(9d^2-6fd+4f^2\right)\right)x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{3f}}{5d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{6(-10adf+3bd+2bf)\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx-\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{3f}}{5d}$$

$$\Big\downarrow \; 320$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx}{3f}-\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5\ldots}{3f}}{5d}$$

$$\Big\downarrow \; 388$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{\left(fx^2+3\right)^{3/2}}dx}{d}\right)}{3f}-\frac{x\sqrt{dx^2+\ldots}}{}}{5d}$$

$$\Big\downarrow \text{313}$$

$$\frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}-\frac{\frac{\frac{3\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right.}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right.}{3f}}{5d}$$

input
```
Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

output
```
(b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (-1/3*((3*b*d - 4*b*f + 5*
a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/f + (-((5*a*d*f*(3*d + 2*f) - 2*
b*(9*d^2 - 6*d*f + 4*f^2))*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqr
t[2]*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(d*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]))) + (3*Sqrt[
2]*(3*b*d + 2*b*f - 10*a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)
/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3
 + f*x^2]))/(3*f))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(f x^2+3)(x^2 d+2)}\left(\frac{b x^3\sqrt{d f x^4+3x^2 d+2 f x^2+6}}{5}+\frac{\left(a d f+3 b d+2 f b-\frac{b(12 d+8 f)}{5}\right)x\sqrt{d f x^4+3x^2 d+2 f x^2+6}}{3 d f}+\frac{\left(6a-\frac{2\left(a d f+3 b d+2 f b-\frac{b(12d-}{5}\right.}{d f}\right.}{}\right.}{}$ …
risch	$\frac{x(3 b d f x^2+5 a d f+3 b d+2 f b)\sqrt{f x^2+3}\sqrt{x^2 d+2}}{15 d f}+\frac{\left(-\frac{(15 a d^2 f+10 a d f^2-18 b d^2+12 b d f-8 b f^2)\sqrt{3 f x^2+9}\sqrt{2x^2 d+4}\left(\text{EllipticF}\left(\frac{x\sqrt{-3f}}{3}\right.\right.}{\sqrt{-3f}\sqrt{d f x^4+3x^2 d+2 f x^2+}}\right.}{}$ …
default	$\frac{\sqrt{f x^2+3}\sqrt{x^2 d+2}\left(3 b d^3 f^2 x^7\sqrt{-f}+5 a d^3 f^2 x^5\sqrt{-f}+12 b d^3 f x^5\sqrt{-f}+8 b d^2 f^2 x^5\sqrt{-f}+15 a d^3 f x^3\sqrt{-f}+10 a d^2 f^2 x^3\sqrt{-f}+15\sqrt{}\right.}{}$ …

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/5*b*x^3*(d*
f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/3*(a*d*f+3*b*d+2*f*b-1/5*b*(12*d+8*f))/d/
f*x*(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(6*a-2*(a*d*f+3*b*d+2*f*b-1/5*b*
(12*d+8*f))/d/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4
+3*d*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)
/f)^(1/2))-(3*a*d+2*a*f+12/5*b-1/3*(a*d*f+3*b*d+2*f*b-1/5*b*(12*d+8*f))/d/
f*(6*d+4*f))/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d
*x^2+2*f*x^2+6)^(1/2)/d*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/
f)^(1/2))-EllipticE(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$
$$= \frac{3\sqrt{3}(18\,bd^2-2\,(5\,ad-4\,b)f^2-3\,(5\,ad^2+4\,bd)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2\,f}{3\,d})+\sqrt{3}(4\,(5\,ad-}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*sqrt(3)*(18*b*d^2 - 2*(5*a*d - 4*b)*f^2 - 3*(5*a*d^2 + 4*b*d)*f)*s
qrt(d*f)*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) +
sqrt(3)*(4*(5*a*d - b)*f^3 - 54*b*d^2 + 6*((5*a - b)*d - 4*b)*f^2 + 9*(5*a
*d^2 + 4*b*d)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/
f)/x), 2/3*f/d) + (3*b*d^2*f^3*x^4 - 18*b*d^2*f + 2*(5*a*d - 4*b)*f^3 + 3*
(5*a*d^2 + 4*b*d)*f^2 + (3*b*d^2*f^2 + (5*a*d^2 + 2*b*d)*f^3)*x^2)*sqrt(d*
x^2 + 2)*sqrt(f*x^2 + 3))/(d^2*f^3*x)
```

Sympy [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int (a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
```

output
```
Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

Maxima [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Giac [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

input
```
integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx=\int \left(b\,x^2+a\right)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}\,dx$$

input
```
int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2),x)
```

output
```
int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$
$$=\frac{5\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,adfx+3\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,bdf\,x^3+3\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,bdx+2\sqrt{f\,x^2+3}\,\sqrt{}}{}$$

input
```
int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x)
```

output
```
(5*sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*a*d*f*x + 3*sqrt(f*x**2 + 3)*sqrt(d*x
**2 + 2)*b*d*f*x**3 + 3*sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*b*d*x + 2*sqrt(f
*x**2 + 3)*sqrt(d*x**2 + 2)*b*f*x + 15*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 +
 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*a*d**2*f + 10*int((sqrt(
f*x**2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)
*a*d*f**2 - 18*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*
d*x**2 + 2*f*x**2 + 6),x)*b*d**2 + 12*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 +
2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*b*d*f - 8*int((sqrt(f*x**
2 + 3)*sqrt(d*x**2 + 2)*x**2)/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*b*f*
*2 + 60*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(d*f*x**4 + 3*d*x**2 + 2*f
*x**2 + 6),x)*a*d*f - 18*int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(d*f*x**4
 + 3*d*x**2 + 2*f*x**2 + 6),x)*b*d - 12*int((sqrt(f*x**2 + 3)*sqrt(d*x**2
+ 2))/(d*f*x**4 + 3*d*x**2 + 2*f*x**2 + 6),x)*b*f)/(15*d*f)
```

3.50 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 87, antiderivative size = 113

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

output
```
-1/2*(b-(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*EllipticE(2^(1/2)
*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$$
$$= -2i\sqrt{2}a\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt
[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt
[b^2 - 4*a*c])]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {281, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b^2-4ac}-b+2cx^2}{\sqrt{\frac{2cx^2}{-\sqrt{b^2-4ac}-b}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}}dx$$
$$\Big\downarrow \text{ 281}$$
$$-\left(\left(\sqrt{b^2-4ac}+b\right)\int \frac{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}dx\right)$$
$$\Big\downarrow \text{ 327}$$
$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

input
```
Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b
 + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2539 vs. $2(92) = 184$.

Time = 1.41 (sec) , antiderivative size = 2540, normalized size of antiderivative = 22.48

method	result	size
elliptic	Expression too large to display	2540

input
```
int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/
2)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2
)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(
-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2+4*(-(2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c-(-(2*c*x^2+(-4
*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2-((-2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(
1/2)*b)*(-1/2*b/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-
b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(
3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a
*b*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c/(
-b+(-4*a*c+b^2)^(1/2))*x^2+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2))+4*c^2/(-b+(-4*a
*c+b^2)^(1/2))*x^4/(-b-(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(1/2*x*(-2*((-4
*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/(b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/
(-b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*
c)/(b+(-4*a*c+b^2)^(1/2))/a/c^2*(-b-(-4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.27

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}+abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right)\mid -\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2+2\,ac}{2\,ac})+\sqrt{\frac{1}{2}}\Big(\sqrt{}\cdots}{\cdots}$$

input
```
integrate((-b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) + a*b*x)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2
*a*c)/(a*c)) + sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b - b^2)*x - ((2*a
+ b)*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 +
 2*a*c)/(a*c)) + 2*a*c*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)*sqrt
(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c*x)
```

Sympy [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{-b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{-b-\sqrt{-4ac+b^2}}}\sqrt{\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((-b-(-4*a*c+b**2)**(1/2)+2*c*x**2)/(1+2*c*x**2/(-b-(-4*a*c+b**2)
**(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x**2 - sqrt
(-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b + 2*c*x**2 + sqrt(-4
*a*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

Maxima [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((-b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
 4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Giac [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((-b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
 4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int -\frac{b-2\,c\,x^2+\sqrt{b^2-4\,a\,c}}{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

input
```
int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c
)^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)),x)
```

output
```
int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c
)^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)), x)
```

Reduce [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{-b-\sqrt{-4ac+b^2}+2c\,x^2}{\sqrt{1+\frac{2c\,x^2}{-b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2c\,x^2}{-b+\sqrt{-4ac+b^2}}}}dx$$

input
```
int((-b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1
/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

output
```
int((-b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1
/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

3.51 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 81, antiderivative size = 224

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}}$$

output
```
1/2*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*EllipticE(2^(1/2)
*c^(1/2)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*
c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-b+(-4*a
*c+b^2)^(1/2))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x/(-b+(-4*a*c+b^2)^(1/2))^(
1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.45

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = -2i\sqrt{2}a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 557 vs. 2(224) = 448.

Time = 0.73 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {281, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}dx$$

$$\Big\downarrow \text{281}$$

$$\left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx$$

$$\Big\downarrow \; 324$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx+\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \; 320$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\ldots}{\sqrt{\ldots}}\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}\ldots}}\right.$$

$$\Big\downarrow \; 388$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{2c}\right)}{b-\sqrt{b^2-4ac}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{\ldots}{b-\ldots}}\ldots}{\ldots}\right.$$

$$\Big\downarrow \; 313$$

$$\left(b-\sqrt{b^2-4ac}\right)\left(\frac{2c\left(\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right|\ldots}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\ldots\right.}{b-\sqrt{b^2-4ac}}\ldots\right.$$

input
```
Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

output
```
(b - Sqrt[b^2 - 4*a*c])*((2*c*(((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]) - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2
*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]))
)/(b - Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt
[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]))
```

Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. $2(183) = 366$.

Time = 1.26 (sec) , antiderivative size = 2477, normalized size of antiderivative = 11.06

method	result	size
elliptic	Expression too large to display	2477

input
```
int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*((4*a*c-b^2)*(-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*(-(-2*c*x^2+(-4*a*c+b^2)
^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2
)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*((4*a*c-b^2)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)
-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*c*x^2+4*(-(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c-(-(-2*c*x^2+(-
4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((4*a*c-
b^2)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(
1/2)*b)*(1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16
*a^2*b*c^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*
a*c-b^2))^(1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c
^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)
*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4
*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2
)^(1/2)/(-4*a*c+b^2-8*c^2*x^2/(b+(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b+(-4*a*c+
b^2)^(1/2))*b^2-8*c^2*x^2/(b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b-(-4*a*c+b^2)
^(1/2))*b^2-16*c^3/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*a+4*c
^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*b^2)^(1/2)*EllipticF(
1/2*x*(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(183) = 366.

Time = 0.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.66

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}-abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\Bigg(\sqrt{b}}{}$$

input
```
integrate((b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*x)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*
a*c)/(a*c)) - sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b + b^2)*x + ((2*a -
 b)*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^
2 - 4*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt
((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2
*a*c)/(a*c)) + 2*a*c*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b
*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c*x)
```

Sympy [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

input
```
integrate((b-(-4*a*c+b**2)**(1/2)+2*c*x**2)/(1+2*c*x**2/(b-(-4*a*c+b**2)**
(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

output
```
Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2 - sqrt(-
4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*x**2 + sqrt(-4*a*c
 + b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

Maxima [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Giac [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

input
```
integrate((b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}\,\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

input
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/
2)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)),x)
```

output
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/
2)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b-\sqrt{-4ac+b^2}+2c\,x^2}{\sqrt{1+\frac{2c\,x^2}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2c\,x^2}{b+\sqrt{-4ac+b^2}}}}dx$$

input
```
int((b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

output
```
int((b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

3.52 $\int \frac{7+10x^2}{\sqrt{2+3x^2}(5+7x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \frac{1}{5}\sqrt{\frac{2}{7}}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right) + \frac{\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}}$$

output
```
1/35*14^(1/2)*EllipticE(35^(1/2)*x/(35*x^2+25)^(1/2),1/14*I*14^(1/2))+1/14
*InverseJacobiAM(arctan(1/5*35^(1/2)*x),1/14*I*14^(1/2))*14^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \frac{1}{70}\left(\frac{14x\sqrt{2+3x^2}}{\sqrt{5+7x^2}} + 2i\sqrt{14}E\left(i\text{arcsinh}\left(\sqrt{\frac{7}{5}}x\right)|\frac{15}{14}\right) - 7i\sqrt{14}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{7}{5}}x\right),\frac{15}{14}\right)\right)$$

input
```
Integrate[(7 + 10*x^2)/(Sqrt[2 + 3*x^2]*(5 + 7*x^2)^(3/2)),x]
```

output
```
((14*x*Sqrt[2 + 3*x^2])/Sqrt[5 + 7*x^2] + (2*I)*Sqrt[14]*EllipticE[I*ArcSi
nh[Sqrt[7/5]*x], 15/14] - (7*I)*Sqrt[14]*EllipticF[I*ArcSinh[Sqrt[7/5]*x],
 15/14])/70
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. $2(46) = 92$.

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{10x^2+7}{\sqrt{3x^2+2}\left(7x^2+5\right)^{3/2}}dx$$

$$\Big\downarrow \text{400}$$

$$\int \frac{\sqrt{3x^2+2}}{\left(7x^2+5\right)^{3/2}}dx+\int \frac{1}{\sqrt{3x^2+2}\sqrt{7x^2+5}}dx$$

$$\Big\downarrow \text{313}$$

$$\int \frac{1}{\sqrt{3x^2+2}\sqrt{7x^2+5}}dx+\frac{\sqrt{\frac{2}{7}}\sqrt{3x^2+2}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right)}{5\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}+\frac{\sqrt{\frac{2}{7}}\sqrt{3x^2+2}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right)}{5\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}$$

input
```
Int[(7 + 10*x^2)/(Sqrt[2 + 3*x^2]*(5 + 7*x^2)^(3/2)),x]
```

output
```
(Sqrt[2/7]*Sqrt[2 + 3*x^2]*EllipticE[ArcTan[Sqrt[7/5]*x], -1/14])/(5*Sqrt[
(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 + 7*x^2]) + (Sqrt[2 + 3*x^2]*EllipticF[Arc
Tan[Sqrt[7/5]*x], -1/14])/(Sqrt[14]*Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 +
 7*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(46) = 92$.

Time = 4.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}\,\left(7i\sqrt{7}\,\sqrt{2}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{35}\,\sqrt{6}}{14}\right)\sqrt{3x^2+2}\,\sqrt{7x^2+5}-2i\sqrt{7}\,\sqrt{2}\ \mathrm{EllipticE}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{35}\,\sqrt{6}}{14}\right)\sqrt{3x^2+2}\,\sqrt{7x^2+5}\right.}{70(21x^4+29x^2+10)}$
elliptic	$\frac{\sqrt{(3x^2+2)(7x^2+5)}\left(\frac{\left(21x^2+14\right)x}{35\sqrt{\left(x^2+\frac{5}{7}\right)\left(21x^2+14\right)}}-\frac{i\sqrt{35}\,\sqrt{35x^2+25}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{210}}{14}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{210}}{14}\right)\right)}{175\sqrt{21x^4+29x^2+10}}-\frac{i\sqrt{35}\,\sqrt{3}}{}\right.}{\sqrt{3x^2+2}\,\sqrt{7x^2+5}}$

input
```
int((10*x^2+7)/(3*x^2+2)^(1/2)/(7*x^2+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70*(3*x^2+2)^(1/2)*(7*x^2+5)^(1/2)*(7*I*7^(1/2)*2^(1/2)*EllipticF(1/5*I
*x*35^(1/2),1/14*35^(1/2)*6^(1/2))*(3*x^2+2)^(1/2)*(7*x^2+5)^(1/2)-2*I*7^(
1/2)*2^(1/2)*EllipticE(1/5*I*x*35^(1/2),1/14*35^(1/2)*6^(1/2))*(3*x^2+2)^(
1/2)*(7*x^2+5)^(1/2)-42*x^3-28*x)/(21*x^4+29*x^2+10)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = -\frac{14\,\sqrt{10}\sqrt{-\frac{7}{5}}(7\,x^2+5)E(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14})+11\,\sqrt{10}\sqrt{-\frac{7}{5}}(7\,x^2+5)F(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14})-70\,\sqrt{}}{350\,(7\,x^2+5)}$$

input
```
integrate((10*x^2+7)/(3*x^2+2)^(1/2)/(7*x^2+5)^(3/2),x, algorithm="fricas"
)
```

output
```
-1/350*(14*sqrt(10)*sqrt(-7/5)*(7*x^2 + 5)*elliptic_e(arcsin(sqrt(-7/5)*x)
, 15/14) + 11*sqrt(10)*sqrt(-7/5)*(7*x^2 + 5)*elliptic_f(arcsin(sqrt(-7/5)
*x), 15/14) - 70*sqrt(7*x^2 + 5)*sqrt(3*x^2 + 2)*x)/(7*x^2 + 5)
```

Sympy [F]

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \int \frac{10x^2+7}{\sqrt{3x^2+2}\,(7x^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((10*x**2+7)/(3*x**2+2)**(1/2)/(7*x**2+5)**(3/2),x)
```

output
```
Integral((10*x**2 + 7)/(sqrt(3*x**2 + 2)*(7*x**2 + 5)**(3/2)), x)
```

Maxima [F]

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \int \frac{10\,x^2+7}{(7\,x^2+5)^{\frac{3}{2}}\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((10*x^2+7)/(3*x^2+2)^(1/2)/(7*x^2+5)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((10*x^2 + 7)/((7*x^2 + 5)^(3/2)*sqrt(3*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \int \frac{10\,x^2+7}{(7\,x^2+5)^{\frac{3}{2}}\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate((10*x^2+7)/(3*x^2+2)^(1/2)/(7*x^2+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((10*x^2 + 7)/((7*x^2 + 5)^(3/2)*sqrt(3*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = \int \frac{10\,x^2+7}{\sqrt{3\,x^2+2}\,(7\,x^2+5)^{3/2}}\,dx$$

input
```
int((10*x^2 + 7)/((3*x^2 + 2)^(1/2)*(7*x^2 + 5)^(3/2)),x)
```

output
```
int((10*x^2 + 7)/((3*x^2 + 2)^(1/2)*(7*x^2 + 5)^(3/2)), x)
```

Reduce [F]

$$\int \frac{7+10x^2}{\sqrt{2+3x^2}\,(5+7x^2)^{3/2}}\,dx = 10\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}\,x^2}{147x^6+308x^4+215x^2+50}dx\right) + 7\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}}{147x^6+308x^4+215x^2+50}dx\right)$$

input

```
int((10*x^2+7)/(3*x^2+2)^(1/2)/(7*x^2+5)^(3/2),x)
```

output

```
10*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5)*x**2)/(147*x**6 + 308*x**4 + 215
*x**2 + 50),x) + 7*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5))/(147*x**6 + 308
*x**4 + 215*x**2 + 50),x)
```

3.53 $\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}(5+7x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 46

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \frac{1}{5}\sqrt{\frac{2}{7}}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)\middle|-\frac{1}{14}\right) + \frac{\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}}$$

output
```
1/35*14^(1/2)*EllipticE(35^(1/2)*x/(35*x^2+25)^(1/2),1/14*I*14^(1/2))+1/14
*InverseJacobiAM(arctan(1/5*35^(1/2)*x),1/14*I*14^(1/2))*14^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \frac{\sqrt{\frac{2+3x^2}{5+7x^2}}\left(28x+42x^3+2i\sqrt{5+7x^2}\sqrt{28+42x^2}E\left(i\,\text{arcsinh}\left(\sqrt{\frac{7}{5}}x\right)\middle|\frac{15}{14}\right)-7i\sqrt{5+7x^2}\sqrt{28+42x^2}\,\text{Ellipt}\right.}{70\,(2+3x^2)}$$

input
```
Integrate[(7 + 10*x^2)/(Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*(5 + 7*x^2)^2),x]
```

output
```
(Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*(28*x + 42*x^3 + (2*I)*Sqrt[5 + 7*x^2]*Sqrt
[28 + 42*x^2]*EllipticE[I*ArcSinh[Sqrt[7/5]*x], 15/14] - (7*I)*Sqrt[5 + 7*
x^2]*Sqrt[28 + 42*x^2]*EllipticF[I*ArcSinh[Sqrt[7/5]*x], 15/14]))/(70*(2 +
 3*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2050, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{10x^2+7}{\sqrt{\frac{3x^2+2}{7x^2+5}}\left(7x^2+5\right)^2}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{10x^2+7}{\sqrt{3x^2+2}\left(7x^2+5\right)^{3/2}}dx$$

$$\Big\downarrow \text{400}$$

$$\int \frac{\sqrt{3x^2+2}}{\left(7x^2+5\right)^{3/2}}dx+\int \frac{1}{\sqrt{3x^2+2}\sqrt{7x^2+5}}dx$$

$$\Big\downarrow \text{313}$$

$$\int \frac{1}{\sqrt{3x^2+2}\sqrt{7x^2+5}}dx+\frac{\sqrt{\frac{2}{7}}\sqrt{3x^2+2}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right)}{5\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}+\frac{\sqrt{\frac{2}{7}}\sqrt{3x^2+2}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right)}{5\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}$$

input
```
Int[(7 + 10*x^2)/(Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*(5 + 7*x^2)^2),x]
```

output
```
(Sqrt[2/7]*Sqrt[2 + 3*x^2]*EllipticE[ArcTan[Sqrt[7/5]*x], -1/14])/(5*Sqrt[
(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 + 7*x^2]) + (Sqrt[2 + 3*x^2]*EllipticF[Arc
Tan[Sqrt[7/5]*x], -1/14])/(Sqrt[14]*Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 +
 7*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(46) = 92.

Time = 4.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.13

method	result
default	$-\frac{7i\sqrt{35}\,\sqrt{35x^2+25}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{210}}{14}\right)\sqrt{(3x^2+2)(7x^2+5)}-2i\sqrt{35}\,\sqrt{35x^2+25}\,\sqrt{6x^2+4}\,\sqrt{(3x^2+2)(7x^2+5)}\,\operatorname{Elliptic}}{350\sqrt{\frac{3x^2+2}{7x^2+5}}\,(7x^2+5)\sqrt{21x^4+29x^2+10}}$

input
```
int((10*x^2+7)/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)^2,x,method=_RETURNVER
BOSE)
```

output
```
-1/350*(7*I*35^(1/2)*(35*x^2+25)^(1/2)*(6*x^2+4)^(1/2)*EllipticF(1/5*I*x*3
5^(1/2),1/14*210^(1/2))*((3*x^2+2)*(7*x^2+5))^(1/2)-2*I*35^(1/2)*(35*x^2+2
5)^(1/2)*(6*x^2+4)^(1/2)*((3*x^2+2)*(7*x^2+5))^(1/2)*EllipticE(1/5*I*x*35^
(1/2),1/14*210^(1/2))-210*(21*x^4+29*x^2+10)^(1/2)*x^3-140*x*(21*x^4+29*x^
2+10)^(1/2))/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)/(21*x^4+29*x^2+10)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = -\frac{1}{25}\,\sqrt{10}\sqrt{-\frac{7}{5}}E(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14}) - \frac{11}{350}\,\sqrt{10}\sqrt{-\frac{7}{5}}F(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14}) + \frac{1}{5}\,x\sqrt{\frac{3\,x^2+2}{7\,x^2+5}}$$

input
```
integrate((10*x^2+7)/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)^2,x, algorithm=
"fricas")
```

output
```
-1/25*sqrt(10)*sqrt(-7/5)*elliptic_e(arcsin(sqrt(-7/5)*x), 15/14) - 11/350
*sqrt(10)*sqrt(-7/5)*elliptic_f(arcsin(sqrt(-7/5)*x), 15/14) + 1/5*x*sqrt(
(3*x^2 + 2)/(7*x^2 + 5))
```

Sympy [F]

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \int \frac{10x^2+7}{\sqrt{\frac{3x^2+2}{7x^2+5}}\,(7x^2+5)^2}\,dx$$

input
```
integrate((10*x**2+7)/((3*x**2+2)/(7*x**2+5))**(1/2)/(7*x**2+5)**2,x)
```

output
```
Integral((10*x**2 + 7)/(sqrt((3*x**2 + 2)/(7*x**2 + 5))*(7*x**2 + 5)**2),
x)
```

Maxima [F]

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \int \frac{10\,x^2+7}{(7\,x^2+5)^2\sqrt{\frac{3\,x^2+2}{7\,x^2+5}}}\,dx$$

input
```
integrate((10*x^2+7)/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)^2,x, algorithm=
"maxima")
```

output
```
integrate((10*x^2 + 7)/((7*x^2 + 5)^2*sqrt((3*x^2 + 2)/(7*x^2 + 5))), x)
```

Giac [F]

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \int \frac{10\,x^2+7}{(7\,x^2+5)^2\sqrt{\frac{3\,x^2+2}{7\,x^2+5}}}\,dx$$

input
```
integrate((10*x^2+7)/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)^2,x, algorithm=
"giac")
```

output
```
integrate((10*x^2 + 7)/((7*x^2 + 5)^2*sqrt((3*x^2 + 2)/(7*x^2 + 5))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = \int \frac{10\,x^2+7}{(7\,x^2+5)^2\,\sqrt{\frac{3\,x^2+2}{7\,x^2+5}}}\,dx$$

input
```
int((10*x^2 + 7)/((7*x^2 + 5)^2*((3*x^2 + 2)/(7*x^2 + 5))^(1/2)),x)
```

output
```
int((10*x^2 + 7)/((7*x^2 + 5)^2*((3*x^2 + 2)/(7*x^2 + 5))^(1/2)), x)
```

Reduce [F]

$$\int \frac{7+10x^2}{\sqrt{\frac{2+3x^2}{5+7x^2}}\,(5+7x^2)^2}\,dx = 10\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}\,x^2}{147x^6+308x^4+215x^2+50}dx\right) + 7\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}}{147x^6+308x^4+215x^2+50}dx\right)$$

input
```
int((10*x^2+7)/((3*x^2+2)/(7*x^2+5))^(1/2)/(7*x^2+5)^2,x)
```

output
```
10*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5)*x**2)/(147*x**6 + 308*x**4 + 215
*x**2 + 50),x) + 7*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5))/(147*x**6 + 308
*x**4 + 215*x**2 + 50),x)
```

3.54 $\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right) dx$

Optimal result

Integrand size = 47, antiderivative size = 46

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right) dx=\frac{1}{5}\sqrt{\frac{2}{7}}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)\left|-\frac{1}{14}\right.\right)+\frac{\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}}$$

output
```
1/35*14^(1/2)*EllipticE(35^(1/2)*x/(35*x^2+25)^(1/2),1/14*I*14^(1/2))+1/14
*InverseJacobiAM(arctan(1/5*35^(1/2)*x),1/14*I*14^(1/2))*14^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right) dx=\frac{1}{70}\left(\frac{14x\sqrt{2+3x^2}}{\sqrt{5+7x^2}}+2i\sqrt{14}E\left(i\text{arcsinh}\left(\sqrt{\frac{7}{5}}x\right)\left|\frac{15}{14}\right.\right)-7i\sqrt{14}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{7}{5}}x\right),\frac{15}{14}\right)\right)$$

input
```
Integrate[Sqrt[2 + 3*x^2]/(5 + 7*x^2)^(3/2) + 1/(Sqrt[2 + 3*x^2]*Sqrt[5 +
7*x^2]),x]
```

output
```
((14*x*Sqrt[2 + 3*x^2])/Sqrt[5 + 7*x^2] + (2*I)*Sqrt[14]*EllipticE[I*ArcSi
nh[Sqrt[7/5]*x], 15/14] - (7*I)*Sqrt[14]*EllipticF[I*ArcSinh[Sqrt[7/5]*x],
 15/14])/70
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. $2(46) = 92$.

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.87, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\sqrt{3x^2+2}}{(7x^2+5)^{3/2}}+\frac{1}{\sqrt{7x^2+5}\sqrt{3x^2+2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{7}{5}}x\right),-\frac{1}{14}\right)}{\sqrt{14}\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}+\frac{\sqrt{\frac{2}{7}}\sqrt{3x^2+2}E\left(\arctan\left(\sqrt{\frac{7}{5}}x\right)|-\frac{1}{14}\right)}{5\sqrt{\frac{3x^2+2}{7x^2+5}}\sqrt{7x^2+5}}$$

input
```
Int[Sqrt[2 + 3*x^2]/(5 + 7*x^2)^(3/2) + 1/(Sqrt[2 + 3*x^2]*Sqrt[5 + 7*x^2]
),x]
```

output
```
(Sqrt[2/7]*Sqrt[2 + 3*x^2]*EllipticE[ArcTan[Sqrt[7/5]*x], -1/14])/(5*Sqrt[
(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 + 7*x^2]) + (Sqrt[2 + 3*x^2]*EllipticF[Arc
Tan[Sqrt[7/5]*x], -1/14])/(Sqrt[14]*Sqrt[(2 + 3*x^2)/(5 + 7*x^2)]*Sqrt[5 +
 7*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(46) = 92$.

Time = 4.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.39

method	result
default	$-\frac{i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{35}\,\sqrt{6}}{14}\right)\sqrt{2}\,\sqrt{7}}{14}+\frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}\,\left(i\sqrt{7}\,\sqrt{2}\,\mathrm{EllipticE}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{35}\,\sqrt{6}}{14}\right)\sqrt{3x^2+2}\,\sqrt{7x^2+5}-i\sqrt{7}\,\sqrt{2}\,\mathrm{Ellipti}\right.}{735x^4+1015x^2+350}$
elliptic	$\frac{\sqrt{(3x^2+2)(7x^2+5)}\left(\frac{(21x^2+14)x}{35\sqrt{\left(x^2+\frac{5}{7}\right)(21x^2+14)}}-\frac{i\sqrt{35}\,\sqrt{35x^2+25}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{210}}{14}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{35}}{5},\frac{\sqrt{210}}{14}\right)\right)}{175\sqrt{21x^4+29x^2+10}}-\frac{i\sqrt{35}\,\sqrt{3}}{}\right.}{\sqrt{3x^2+2}\,\sqrt{7x^2+5}}$

input
```
int((3*x^2+2)^(1/2)/(7*x^2+5)^(3/2)+1/(3*x^2+2)^(1/2)/(7*x^2+5)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-1/14*I*EllipticF(1/5*I*x*35^(1/2),1/14*35^(1/2)*6^(1/2))*2^(1/2)*7^(1/2)+
1/35*(3*x^2+2)^(1/2)*(7*x^2+5)^(1/2)*(I*EllipticE(1/5*I*x*35^(1/2),1/14*35
^(1/2)*6^(1/2))*7^(1/2)*2^(1/2)*(3*x^2+2)^(1/2)*(7*x^2+5)^(1/2)-I*Elliptic
F(1/5*I*x*35^(1/2),1/14*35^(1/2)*6^(1/2))*7^(1/2)*2^(1/2)*(3*x^2+2)^(1/2)*
(7*x^2+5)^(1/2)+21*x^3+14*x)/(21*x^4+29*x^2+10)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int\left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx =$$
$$-\frac{14\sqrt{10}\sqrt{-\frac{7}{5}}(7x^2+5)E(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14})+11\sqrt{10}\sqrt{-\frac{7}{5}}(7x^2+5)F(\arcsin\left(\sqrt{-\frac{7}{5}}x\right)\mid\frac{15}{14})-70\sqrt{}}{350\,(7x^2+5)}$$

input
```
integrate((3*x^2+2)^(1/2)/(7*x^2+5)^(3/2)+1/(3*x^2+2)^(1/2)/(7*x^2+5)^(1/2
),x, algorithm="fricas")
```

output
```
-1/350*(14*sqrt(10)*sqrt(-7/5)*(7*x^2 + 5)*elliptic_e(arcsin(sqrt(-7/5)*x)
, 15/14) + 11*sqrt(10)*sqrt(-7/5)*(7*x^2 + 5)*elliptic_f(arcsin(sqrt(-7/5)
*x), 15/14) - 70*sqrt(7*x^2 + 5)*sqrt(3*x^2 + 2)*x)/(7*x^2 + 5)
```

Sympy [F]

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx=\int \frac{10x^2+7}{\sqrt{3x^2+2}\,(7x^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)**(1/2)/(7*x**2+5)**(3/2)+1/(3*x**2+2)**(1/2)/(7*x**2+
5)**(1/2),x)
```

output
```
Integral((10*x**2 + 7)/(sqrt(3*x**2 + 2)*(7*x**2 + 5)**(3/2)), x)
```

Maxima [F]

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx=\int \frac{1}{\sqrt{7\,x^2+5}\sqrt{3\,x^2+2}}+\frac{\sqrt{3\,x^2+2}}{(7\,x^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)^(1/2)/(7*x^2+5)^(3/2)+1/(3*x^2+2)^(1/2)/(7*x^2+5)^(1/2
),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(7*x^2 + 5)*sqrt(3*x^2 + 2)) + sqrt(3*x^2 + 2)/(7*x^2 + 5
)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx=\int \frac{1}{\sqrt{7\,x^2+5}\sqrt{3\,x^2+2}}+\frac{\sqrt{3\,x^2+2}}{(7\,x^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)^(1/2)/(7*x^2+5)^(3/2)+1/(3*x^2+2)^(1/2)/(7*x^2+5)^(1/2
),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(7*x^2 + 5)*sqrt(3*x^2 + 2)) + sqrt(3*x^2 + 2)/(7*x^2 + 5
)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx=\int \frac{1}{\sqrt{3\,x^2+2}\,\sqrt{7\,x^2+5}}+\frac{\sqrt{3\,x^2+2}}{(7\,x^2+5)^{3/2}}\,dx$$

input
```
int(1/((3*x^2 + 2)^(1/2)*(7*x^2 + 5)^(1/2)) + (3*x^2 + 2)^(1/2)/(7*x^2 + 5
)^(3/2),x)
```

output
```
int(1/((3*x^2 + 2)^(1/2)*(7*x^2 + 5)^(1/2)) + (3*x^2 + 2)^(1/2)/(7*x^2 + 5
)^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{\sqrt{2+3x^2}}{(5+7x^2)^{3/2}}+\frac{1}{\sqrt{2+3x^2}\sqrt{5+7x^2}}\right)dx=10\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}\,x^2}{147x^6+308x^4+215x^2+50}dx\right)+7\left(\int \frac{\sqrt{3x^2+2}\,\sqrt{7x^2+5}}{147x^6+308x^4+215x^2+50}dx\right)$$

input
```
int((3*x^2+2)^(1/2)/(7*x^2+5)^(3/2)+1/(3*x^2+2)^(1/2)/(7*x^2+5)^(1/2),x)
```

output
```
10*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5)*x**2)/(147*x**6 + 308*x**4 + 215
*x**2 + 50),x) + 7*int((sqrt(3*x**2 + 2)*sqrt(7*x**2 + 5))/(147*x**6 + 308
*x**4 + 215*x**2 + 50),x)
```

3.55 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 849

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx =$$

$$-\frac{(16a^4d^4f^2-16a^3bd^3f(3de+2cf)-ab^3cd(147d^2e^2+54cdef-7c^2f^2)+3a^2b^2d^2(14d^2e^2+38cdef+3c^2}{315b^3d^3\sqrt{a+bx^2}}$$

$$+\frac{(8a^3d^3f^2-3a^2bd^2f(8de+5cf)+2b^3c(63d^2e^2+9cdef-2c^2f^2)+3ab^2d(7d^2e^2+18cdef+c^2f^2))\,x\sqrt{a-}}{315b^3d^2}$$

$$-\frac{(6a^2d^2f^2-abdf(18de+11cf)-3b^2(21d^2e^2+48cdef+c^2f^2))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^2d}$$

$$+\frac{f(18bde+10bcf+adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63b}+\frac{1}{9}df^2x^7\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{\sqrt{a}(16a^4d^4f^2-16a^3bd^3f(3de+2cf)-ab^3cd(147d^2e^2+54cdef-7c^2f^2)+3a^2b^2d^2(14d^2e^2+38cdef+}{315b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(8a^3d^3f^2-3a^2bd^2f(8de+5cf)+3ab^2d(7d^2e^2+18cdef+c^2f^2)-b^3c(189d^2e^2-18cdef+4c^2f^2))}{315b^{7/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(16*a^4*d^4*f^2-16*a^3*b*d^3*f*(2*c*f+3*d*e)-a*b^3*c*d*(-7*c^2*f^2+
54*c*d*e*f+147*d^2*e^2)+3*a^2*b^2*d^2*(3*c^2*f^2+38*c*d*e*f+14*d^2*e^2)-b^
4*c^2*(8*c^2*f^2-36*c*d*e*f+63*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d^3/(b*x^2+
a)^(1/2)+1/315*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(5*c*f+8*d*e)+2*b^3*c*(-2*c^2*
f^2+9*c*d*e*f+63*d^2*e^2)+3*a*b^2*d*(c^2*f^2+18*c*d*e*f+7*d^2*e^2))*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^2-1/315*(6*a^2*d^2*f^2-a*b*d*f*(11*c*f+1
8*d*e)-3*b^2*(c^2*f^2+48*c*d*e*f+21*d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/b^2/d+1/63*f*(a*d*f+10*b*c*f+18*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/b+1/9*d*f^2*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/315*a^(1/2)*(16
*a^4*d^4*f^2-16*a^3*b*d^3*f*(2*c*f+3*d*e)-a*b^3*c*d*(-7*c^2*f^2+54*c*d*e*f
+147*d^2*e^2)+3*a^2*b^2*d^2*(3*c^2*f^2+38*c*d*e*f+14*d^2*e^2)-b^4*c^2*(8*c
^2*f^2-36*c*d*e*f+63*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(5*c*f+
8*d*e)+3*a*b^2*d*(c^2*f^2+18*c*d*e*f+7*d^2*e^2)-b^3*c*(4*c^2*f^2-18*c*d*e*
f+189*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.78 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f^2-3a^2bd^2f(8de+5cf+2dfx^2)+ab^2d(3c^2f^2+cdf(54e+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2*f*(8*d
*e + 5*c*f + 2*d*f*x^2) + a*b^2*d*(3*c^2*f^2 + c*d*f*(54*e + 11*f*x^2) + d
^2*(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4)) + b^3*(-4*c^3*f^2 + 3*c^2*d*f*(6*e +
 f*x^2) + 2*c*d^2*(63*e^2 + 72*e*f*x^2 + 25*f^2*x^4) + d^3*x^2*(63*e^2 + 9
0*e*f*x^2 + 35*f^2*x^4))) + I*c*(16*a^4*d^4*f^2 - 16*a^3*b*d^3*f*(3*d*e +
2*c*f) + b^4*c^2*(-63*d^2*e^2 + 36*c*d*e*f - 8*c^2*f^2) + 3*a^2*b^2*d^2*(1
4*d^2*e^2 + 38*c*d*e*f + 3*c^2*f^2) + a*b^3*c*d*(-147*d^2*e^2 - 54*c*d*e*f
 + 7*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(b*c - a*d)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2
*f*(8*d*e + 3*c*f) + 3*a*b^2*d*(7*d^2*e^2 + 12*c*d*e*f - c^2*f^2) + b^3*c*
(63*d^2*e^2 - 36*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[b/a]*d^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(e^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}+2efx^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}+f^2x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}f^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x^5+\frac{(3bc+ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{63b}+\frac{2}{7}ef\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x^3+\frac{\left(-\frac{6da^2}{b}+11ca+\frac{3bc^2}{d}\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{315b}+\frac{2(3bc+ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35b}+\frac{de^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x}{5b}+\frac{2(3bc-ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15b}-\frac{\left(4b^3c^3-3ab^2dc^2+15a^2bd^2c-8a^3d^3\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{315b^3d^2}+\frac{2\left(-\frac{4da^2}{b}+9ca+\frac{3bc^2}{d}\right)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105b}+\frac{\left(3b^2c^2+7abdc-2a^2d^2\right)e^2\sqrt{bx^2+a}x}{15b^2\sqrt{dx^2+c}}+\frac{\left(8b^4c^4-7ab^3dc^3-9a^2b^2d^2c^2+32a^3bd^3c-16a^4d^4\right)f^2\sqrt{bx^2+a}x}{315b^4d^2\sqrt{dx^2+c}}-\frac{2(2bc-ad)\left(3b^2c^2-3abdc+8a^2d^2\right)ef\sqrt{bx^2+a}x}{105b^3d\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(3b^2c^2+7abdc-2a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(8b^4c^4-7ab^3dc^3-9a^2b^2d^2c^2+32a^3bd^3c-16a^4d^4\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^4d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(2bc-ad)\left(3b^2c^2-3abdc+8a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105b^3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(9bc-ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}\left(4b^3c^3-3ab^2dc^2+15a^2bd^2c-8a^3d^3\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315b^3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}\left(3b^2c^2+9abdc-4a^2d^2\right)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105b^2d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c
 + d*x^2]) - (2*(2*b*c - a*d)*(3*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*e*f*x*Sq
rt[a + b*x^2])/(105*b^3*d*Sqrt[c + d*x^2]) + ((8*b^4*c^4 - 7*a*b^3*c^3*d -
 9*a^2*b^2*c^2*d^2 + 32*a^3*b*c*d^3 - 16*a^4*d^4)*f^2*x*Sqrt[a + b*x^2])/(
315*b^4*d^2*Sqrt[c + d*x^2]) + (2*(3*b*c - a*d)*e^2*x*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(15*b) + (2*(9*a*c + (3*b*c^2)/d - (4*a^2*d)/b)*e*f*x*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(105*b) - ((4*b^3*c^3 - 3*a*b^2*c^2*d + 15*a^2*b
*c*d^2 - 8*a^3*d^3)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*b^3*d^2) +
 (2*(3*b*c + a*d)*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b) + ((11*a
*c + (3*b*c^2)/d - (6*a^2*d)/b)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(
315*b) + ((3*b*c + a*d)*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(63*b) +
(d*e^2*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (2*e*f*x^3*Sqrt[a + b*
x^2]*(c + d*x^2)^(3/2))/7 + (f^2*x^5*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/9
- (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*e^2*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(2*b*c - a*d)*
(3*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b^3*d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(8*b^4*c^4 - 7*a*b^3*c^3*d
 - 9*a^2*b^2*c^2*d^2 + 32*a^3*b*c*d^3 - 16*a^4*d^4)*f^2*Sqrt[a + b*x^2]...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 1300, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1300
risch	Expression too large to display	1494
default	Expression too large to display	2491

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*f^2*d*x^7
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*
f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5
*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7*(a*d^2*f^
2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*
x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e
^2+2*b*c^2*e*f+2*b*c*d*e^2-5/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*
d*(8*a*d+8*b*c))/b/d*a*c-1/5*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d
*e*f+b*d^2*e^2-1/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b
*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+(a*c^2*e^2-1/3*(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e^2+2*b*c^2*e*f+2*b
*c*d*e^2-5/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b
/d*a*c-1/5*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7
*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*
b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-(2*a*c^2*e*f+2*a*c*e^2*d+b*c^2*e^2-3/5*(11/9*a*c*d*f
^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7*(a*d^2*f^2+2*b*c*d*f^2+
2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a*c^2
*f^2+4*a*c*e*f*d+a*d^2*e^2+2*b*c^2*e*f+2*b*c*d*e^2-5/7*(a*d^2*f^2+2*b*c...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
-1/315*((21*(3*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 2*a^2*b^2*c*d^4)*e^2 - 6*(6
*b^4*c^4*d - 9*a*b^3*c^3*d^2 + 19*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4)*e*f + (
8*b^4*c^5 - 7*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 32*a^3*b*c^2*d^3 - 16*a^4*
c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (21*(3*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - a^2*b^2*d^5 - (2*a^2*b^2 - 9
*a*b^3)*c*d^4)*e^2 - 6*(6*b^4*c^4*d - 9*a*b^3*c^3*d^2 - 4*a^3*b*d^5 + (19*
a^2*b^2 + 3*a*b^3)*c^2*d^3 - (8*a^3*b - 9*a^2*b^2)*c*d^4)*e*f + (8*b^4*c^5
 - 7*a*b^3*c^4*d - 8*a^4*d^5 - (9*a^2*b^2 - 4*a*b^3)*c^3*d^2 + (32*a^3*b -
 3*a^2*b^2)*c^2*d^3 - (16*a^4 - 15*a^3*b)*c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/
d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*d^5*f^2*x^8 + 5*(
18*b^4*d^5*e*f + (10*b^4*c*d^4 + a*b^3*d^5)*f^2)*x^6 + (63*b^4*d^5*e^2 + 1
8*(8*b^4*c*d^4 + a*b^3*d^5)*e*f + (3*b^4*c^2*d^3 + 11*a*b^3*c*d^4 - 6*a^2*
b^2*d^5)*f^2)*x^4 + 21*(3*b^4*c^2*d^3 + 7*a*b^3*c*d^4 - 2*a^2*b^2*d^5)*e^2
 - 6*(6*b^4*c^3*d^2 - 9*a*b^3*c^2*d^3 + 19*a^2*b^2*c*d^4 - 8*a^3*b*d^5)*e*
f + (8*b^4*c^4*d - 7*a*b^3*c^3*d^2 - 9*a^2*b^2*c^2*d^3 + 32*a^3*b*c*d^4 -
16*a^4*d^5)*f^2 + (21*(6*b^4*c*d^4 + a*b^3*d^5)*e^2 + 6*(3*b^4*c^2*d^3 + 9
*a*b^3*c*d^4 - 4*a^2*b^2*d^5)*e*f - (4*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 15*
a^2*b^2*c*d^4 - 8*a^3*b*d^5)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b
^4*d^4*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2\,dx = \int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x - 15*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x - 24*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a**2*b*d**3*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f
**2*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x + 54*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x + 11*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a*b**2*d**3*e**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d*
*3*e*f*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**5 -
4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x + 18*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*b**3*c**2*d*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c**2*d*f**2*x**3 + 126*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*
e**2*x + 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**3 + 50*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*f
**2*x**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 + 32*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3*
f**2 + 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e*f - 9*int((sqrt(c + d*x**2)*sqrt(a...
```

3.56 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 609

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

$$= \frac{(8a^3d^3f^2 - a^2bd^2f(28de+5cf) + ab^2d(35d^2e^2+28cdef-5c^2f^2) + b^3c(35d^2e^2-28cdef+8c^2f^2))\,x\sqrt{}}{105b^2d^3\sqrt{a+bx^2}}$$

$$- \frac{\left(\frac{4a^2df^2}{b} - 2af(7de+cf) - b\left(35de^2+14cef-\frac{4c^2f^2}{d}\right)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105bd}$$

$$+ \frac{f(14bde+bcf+adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd} + \frac{1}{7}f^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$- \frac{\sqrt{a}(8a^3d^3f^2 - a^2bd^2f(28de+5cf) + ab^2d(35d^2e^2+28cdef-5c^2f^2) + b^3c(35d^2e^2-28cdef+8c^2f^2}{105b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+ \frac{2a^{3/2}(2a^2d^2f^2 - abdf(7de+cf) + b^2(35d^2e^2-7cdef+2c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1\right.}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f^2-a^2*b*d^2*f*(5*c*f+28*d*e)+a*b^2*d*(-5*c^2*f^2+28*c*d
*e*f+35*d^2*e^2)+b^3*c*(8*c^2*f^2-28*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)^(1/2
)/b^2/d^3/(b*x^2+a)^(1/2)-1/105*(4*a^2*d*f^2/b-2*a*f*(c*f+7*d*e)-b*(35*d*e
^2+14*c*e*f-4*c^2*f^2/d))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/35*f*(a*
d*f+b*c*f+14*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/7*f^2*x^5*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/105*a^(1/2)*(8*a^3*d^3*f^2-a^2*b*d^2*f*(5*
c*f+28*d*e)+a*b^2*d*(-5*c^2*f^2+28*c*d*e*f+35*d^2*e^2)+b^3*c*(8*c^2*f^2-28
*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)+2/105*a^(3/2)*(2*a^2*d^2*f^2-a*b*d*f*(c*f+7*d*e)+b^2*(2*c^2
*f^2-7*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f^2-abdf\left(14de+2cf+3dfx^2\right)-b^2\left(-4c^2f^2+cdf\left(14e+3fx^2\right)+d^2\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2 - a*b*d*f*(14*d*e
+ 2*c*f + 3*d*f*x^2) - b^2*(-4*c^2*f^2 + c*d*f*(14*e + 3*f*x^2) + d^2*(35*
e^2 + 42*e*f*x^2 + 15*f^2*x^4)))) - I*c*(8*a^3*d^3*f^2 - a^2*b*d^2*f*(28*d
*e + 5*c*f) + a*b^2*d*(35*d^2*e^2 + 28*c*d*e*f - 5*c^2*f^2) + b^3*c*(35*d^
2*e^2 - 28*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(4*a^2*
d^2*f^2 + a*b*d*f*(-14*d*e + c*f) + b^2*(-35*d^2*e^2 + 28*c*d*e*f - 8*c^2*
f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)])/(105*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(e^2\sqrt{a+bx^2}\sqrt{c+dx^2}+2efx^2\sqrt{a+bx^2}\sqrt{c+dx^2}+f^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{(bc+ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35bd}+\frac{2}{5}ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3+\frac{1}{3}e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x-\frac{2(2b^2c^2-abdc+2a^2d^2)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105b^2d^2}+\frac{2(bc+ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15bd}+\frac{(bc+ad)e^2\sqrt{bx^2+a}x}{3b\sqrt{dx^2+c}}+\frac{(bc+ad)(8b^2c^2-13abdc+8a^2d^2)f^2\sqrt{bx^2+a}x}{105b^3d^2\sqrt{dx^2+c}}-\frac{4(b^2c^2-abdc+a^2d^2)ef\sqrt{bx^2+a}x}{15b^2d\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc+ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc+ad)(8b^2c^2-13abdc+8a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105b^3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{4\sqrt{c}(b^2c^2-abdc+a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}(2b^2c^2-abdc+2a^2d^2)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105b^2d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}(bc+ad)ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
((b*c + a*d)*e^2*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) - (4*(b^2*c^2 -
a*b*c*d + a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(15*b^2*d*Sqrt[c + d*x^2]) + ((b
*c + a*d)*(8*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(105
*b^3*d^2*Sqrt[c + d*x^2]) + (e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 + (2
*(b*c + a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b*d) - (2*(2*b^2*c
^2 - a*b*c*d + 2*a^2*d^2)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*b^2*
d^2) + (2*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/5 + ((b*c + a*d)*f^2*x^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b*d) + (f^2*x^5*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/7 - (Sqrt[c]*(b*c + a*d)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (4*Sqrt[c]*(b^2*c^2 - a*b*c*d + a^2*
d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (Sqrt[c]*(b*c + a*d)*(8*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*f^2*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b
^3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(
3/2)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
- (2*c^(3/2)*(b*c + a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + ...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.46 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x^5 \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{7}+\frac{\left(ad f^2+bc f^2+2dbef-\frac{f^2(6ad+6bc)}{7}\right)x^3\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{5bd}+\frac{\left(\frac{2ac f^2}{7}+2adef+\right.}{}\right.}{}$
risch	$-\frac{x(-15 f^2 x^4 b^2 d^2-3ab d^2 f^2 x^2-3b^2 cd f^2 x^2-42 b^2 d^2 ef x^2+4a^2 d^2 f^2-2abcd f^2-14ab d^2 ef+4b^2 c^2 f^2-14 b^2 cdef-35 b^2 d^2 e^2)\sqrt{b x^2}}{105 b^2 d^2}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f^2*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*
(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2/7*a*c*f^
2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6*a*
d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*
c*e^2-1/3*(2/7*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*
d*b*e*f-1/7*f^2*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*c*e*f+a*d*e^2+b*c*e^2-
3/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6*a*d+6*b*c))/b/d*a*c-1/3*(2/7*a*c
*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6
*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx =$$

$$-\frac{(35\,(b^3c^2d^2+ab^2cd^3)e^2-28\,(b^3c^3d-ab^2c^2d^2+a^2bcd^3)ef+(8\,b^3c^4-5\,ab^2c^3d-5\,a^2bc^2d^2+8\,a^3cd^3)}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
-1/105*((35*(b^3*c^2*d^2 + a*b^2*c*d^3)*e^2 - 28*(b^3*c^3*d - a*b^2*c^2*d^
2 + a^2*b*c*d^3)*e*f + (8*b^3*c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 8*a^
3*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/
(b*c)) - (35*(b^3*c^2*d^2 + a*b^2*c*d^3 + 2*a*b^2*d^4)*e^2 - 14*(2*b^3*c^3
*d - 2*a*b^2*c^2*d^2 + a^2*b*d^4 + (2*a^2*b + a*b^2)*c*d^3)*e*f + (8*b^3*c
^4 - 5*a*b^2*c^3*d + 4*a^3*d^4 - (5*a^2*b - 4*a*b^2)*c^2*d^2 + 2*(4*a^3 -
a^2*b)*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x),
 a*d/(b*c)) - (15*b^3*d^4*f^2*x^6 + 3*(14*b^3*d^4*e*f + (b^3*c*d^3 + a*b^2
*d^4)*f^2)*x^4 + 35*(b^3*c*d^3 + a*b^2*d^4)*e^2 - 28*(b^3*c^2*d^2 - a*b^2*
c*d^3 + a^2*b*d^4)*e*f + (8*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 +
8*a^3*d^4)*f^2 + (35*b^3*d^4*e^2 + 14*(b^3*c*d^3 + a*b^2*d^4)*e*f - 2*(2*b
^3*c^2*d^2 - a*b^2*c*d^3 + 2*a^2*b*d^4)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x + 2*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*d**2*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 -
 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x + 14*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*c*d*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x +
 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f**2
- 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**
2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 - 28*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f - 5*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a*b**2*c**2*d*f**2 + 28*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f + 35*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*a*b**2*d**3*e**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 - 28*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b**3*c**2*d*e*f + 35*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/...
```

3.57 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 445

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx &= -\frac{(2a^2d^2f^2-abdf(10de-3cf)-b^2(15d^2e^2-20cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15bd^3\sqrt{a+bx^2}} \\ &\quad+\frac{f(10bde-4bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2}+\frac{f^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} \\ &\quad+\frac{\sqrt{a}(2a^2d^2f^2-abdf(10de-3cf)-b^2(15d^2e^2-20cdef+8c^2f^2))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad-\frac{a^{3/2}(acdf^2-b(15d^2e^2-10cdef+4c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cd^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/15*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+10*d*e)-b^2*(8*c^2*f^2-20*c*d*e*f+15*
d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d^3/(b*x^2+a)^(1/2)+1/15*f*(a*d*f-4*b*c*f+10
*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2+1/5*f^2*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/d+1/15*a^(1/2)*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+10*d*e)-b^
2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a
^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^3/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(a*c*d*f^2-b*(4*c^2*f^2-10*c*d*
e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/b^(3/2)/c/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dfx(a+bx^2)\,(c+dx^2)\,(adf+b(10de-4cf+3dfx^2))+ic(2a^2d^2f^2+abdf(-10de+3cf)+b^2(-15}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(10*d*e - 4*c*f + 3*d*
f*x^2)) + I*c*(2*a^2*d^2*f^2 + a*b*d*f*(-10*d*e + 3*c*f) + b^2*(-15*d^2*e^
2 + 20*c*d*e*f - 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d)*(a*c*d*f^2 +
b*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 433$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{2efx^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{f^2x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}-\frac{(4bc-ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15bd^2}+\frac{2ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3d}+\frac{e^2\sqrt{bx^2+a}x}{\sqrt{dx^2+c}}+\frac{\left(8b^2c^2-3abdc-2a^2d^2\right)f^2\sqrt{bx^2+a}x}{15b^2d^2\sqrt{dx^2+c}}-\frac{2(2bc-ad)ef\sqrt{bx^2+a}x}{3bd\sqrt{dx^2+c}}-\frac{\sqrt{c}e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(8b^2c^2-3abdc-2a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(2bc-ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(4bc-ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(e^2*x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (2*(2*b*c - a*d)*e*f*x*Sqrt[a +
b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*f^2
*x*Sqrt[a + b*x^2])/(15*b^2*d^2*Sqrt[c + d*x^2]) + (2*e*f*x*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(3*d) - ((4*b*c - a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b*d^2) + (f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqr
t[c]*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
(2*Sqrt[c]*(2*b*c - a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*f^2*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(1
5*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sq
rt[c]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) -
 (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]) + (c^(3/2)*(4*b*c - a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.55 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.02

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5d}+\frac{\left(a\,f^2+2bef-\frac{f^2(4ad+4bc)}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}+\frac{\left(a\,e^2-\frac{\left(a\,f^2+2bef-\frac{f^2(4ac}{5}\right.}{3bd}\right.}{}\right.}{}$
risch	$\frac{fx(3bdf\,x^2+adf-4bcf+10bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b\,d^2}-\frac{\left(-\frac{(2a^2d^2f^2+3abcd\,f^2-10ab\,d^2ef-8b^2c^2f^2+20b^2cdef-15b^2d^2e^2)c\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1}}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ac}}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f^2/d*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*f^2+2*b*e*f-1/5*f^2/d*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*e^2-1/3*(a*f^2+2*b*e*f
-1/5*f^2/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(2*a*e*f+b*e^2-3/5*a*c/d*f^2-1/3*(a*f^2+2*b*e*f-1/5
*f^2/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(15\,b^2c^2d^2e^2-10\,(2\,b^2c^3d-abc^2d^2)ef+(8\,b^2c^4-3\,abc^3d-2\,a^2c^2d^2)f^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/15*((15*b^2*c^2*d^2*e^2 - 10*(2*b^2*c^3*d - a*b*c^2*d^2)*e*f + (8*b^2*c
^4 - 3*a*b*c^3*d - 2*a^2*c^2*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), a*d/(b*c)) - (15*(b^2*c^2*d^2 + a*b*d^4)*e^2 - 10*(2*
b^2*c^3*d - a*b*c^2*d^2 + a*b*c*d^3)*e*f + (8*b^2*c^4 - 3*a*b*c^3*d - a^2*
c*d^3 - 2*(a^2 - 2*a*b)*c^2*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(ar
csin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*c*d^3*f^2*x^4 + 15*b^2*c*d^3*e^2 -
 10*(2*b^2*c^2*d^2 - a*b*c*d^3)*e*f + (8*b^2*c^3*d - 3*a*b*c^2*d^2 - 2*a^2
*c*d^3)*f^2 + (10*b^2*c*d^3*e*f - (4*b^2*c^2*d^2 - a*b*c*d^3)*f^2)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c*d^4*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,f^2x-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc\,f^2x+10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdefx+3\sqrt{d\,x^2+c}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x - 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b*c*f**2*x + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f*x + 3*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*f**2*x**3 - 2*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2
*f**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a*b*c*d*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e*f + 8*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*d*e*f + 15*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*
b**2*d**2*e**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*a**2*c*d*f**2 + 4*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f**2 - 10*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4)
,x)*a*b*c*d*e*f + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e**2)/(15*b*d**2)
```

3.58 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 306

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{f(6bde-4bcf+adf)x\sqrt{a+bx^2}}{3bd^2\sqrt{c+dx^2}} + \frac{f^2x^3\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} - \frac{(acdf^2-b(3d^2e^2-12cdef+8c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2\sqrt{c}f(3de-2cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*f*(a*d*f-4*b*c*f+6*b*d*e)*x*(b*x^2+a)^(1/2)/b/d^2/(d*x^2+c)^(1/2)+1/3*
f^2*x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)-1/3*(a*c*d*f^2-b*(8*c^2*f^2-12*c
*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))/b/c^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+2/3*c^(1/2)*f*(-2*c*f+3*d*e)*(b*x^2+a)^(1/2)*InverseJaco
biAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(5/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3d^2e^2+4c^2f^2+cdf(-6e+fx^2)\right)-ic(acdf^2+b(-3d^2e^2+$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*d^2*e^2 + 4*c^2*f^2 + c*d*f*(-6*e + f*x^2))
- I*c*(a*c*d*f^2 + b*(-3*d^2*e^2 + 12*c*d*e*f - 8*c^2*f^2))*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 I*c*(a*d*f*(-6*d*e + 5*c*f) + b*(-3*d^2*e^2 + 12*c*d*e*f - 8*c^2*f^2))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}+\frac{2efx^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}+\frac{f^2x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c^{3/2}f^2\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}ef\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4\sqrt{c}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}f^2\sqrt{a+bx^2}(8bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{e^2\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d^2}-\frac{f^2x\sqrt{a+bx^2}(8bc-ad)}{3bd^2\sqrt{c+dx^2}}+\frac{2efx\sqrt{a+bx^2}}{d\sqrt{c+dx^2}}-\frac{f^2x^3\sqrt{a+bx^2}}{d\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(2*e*f*x*Sqrt[a + b*x^2])/(d*Sqrt[c + d*x^2]) - ((8*b*c - a*d)*f^2*x*Sqrt[
a + b*x^2])/(3*b*d^2*Sqrt[c + d*x^2]) - (f^2*x^3*Sqrt[a + b*x^2])/(d*Sqrt[
c + d*x^2]) + (4*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d^2) + (e^2*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*S
qrt[c]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
+ (Sqrt[c]*(8*b*c - a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]) - (4*c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 11.67 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)} \Big(\frac{(bd x^2+ad)(c^2 f^2-2cdef+d^2 e^2)x}{c d^3 \sqrt{(x^2+\frac{c}{d})(bd x^2+ad)}} + \frac{f^2 x \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3d^2} + \frac{\Big(-\frac{acd f^2-2a d^2 ef-b c^2 f^2+2bcdef-b d^2 e^2}{d^3}+\frac{(c^2}{}}{}}{}$
risch	$\frac{f^2 x\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{3d^2}+\frac{\Big(-\frac{f(adf-5bcf+6bde)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\Big(\mathrm{EllipticF}\Big(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\Big)-\mathrm{EllipticE}\Big(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\Big)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2 bc+ac}\,d}}{}$
default	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,\Big(-\sqrt{-\frac{b}{a}}\,bc d^2 f^2 x^5-\sqrt{-\frac{b}{a}}\,ac d^2 f^2 x^3-4\sqrt{-\frac{b}{a}}\,b c^2 d f^2 x^3+6\sqrt{-\frac{b}{a}}\,bc d^2 ef x^3-3\sqrt{-\frac{b}{a}}\,b d^3 e^2 x^3+5\sqrt{\frac{b x^2+a}{a}}\,\sqrt{}}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)
*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*f
^2/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-(a*c*d*f^2-2*a*d^2*e*f-b*c^
2*f^2+2*b*c*d*e*f-b*d^2*e^2)/d^3+(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^3*(a*d-b*c)
/c-a/d^2*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c-1/3*a*c/d^2*f^2)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/d^2*f*(a*d*f-b*c*f+2*b*d*e)
-(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^2*b/c-1/3*f^2/d^2*(2*a*d+2*b*c))*c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{((3\,bcd^3e^2 - 12\,bc^2d^2ef + (8\,bc^3d - ac^2d^2)f^2)x^3 + (3\,bc^2d^2e^2 - 12\,bc^3def + (8}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
1/3*(((3*b*c*d^3*e^2 - 12*b*c^2*d^2*e*f + (8*b*c^3*d - a*c^2*d^2)*f^2)*x^3
 + (3*b*c^2*d^2*e^2 - 12*b*c^3*d*e*f + (8*b*c^4 - a*c^3*d)*f^2)*x)*sqrt(b*
d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((3*b*c*d^3*e^
2 - 6*(2*b*c^2*d^2 + a*d^4)*e*f + (8*b*c^3*d - a*c^2*d^2 + 4*a*c*d^3)*f^2)
*x^3 + (3*b*c^2*d^2*e^2 - 6*(2*b*c^3*d + a*c*d^3)*e*f + (8*b*c^4 - a*c^3*d
 + 4*a*c^2*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) + (b*c*d^3*f^2*x^4 - 3*b*c*d^3*e^2 + 12*b*c^2*d^2*e*f - (8*
b*c^3*d - a*c^2*d^2)*f^2 + (6*b*c*d^3*e*f - (4*b*c^2*d^2 - a*c*d^3)*f^2)*x
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^5*x^3 + b*c^2*d^4*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\,(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\,(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\,(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x + 6*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*e*f*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x*
*3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**2*x + 5*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x
**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**2*d*f**2 - 6*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*
x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*e*f + 5*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*
x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*f**2*x**2 - 6*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**3*e*f*x**2 - 8*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
+ b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**3*f**2 + 12*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*e*f - 8*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**
4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*f**2*x**2 - 3
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d
**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d**2*e**2 +
 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2...
```

3.59 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 369

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(d^2e^2-2cdef+4c^2f^2)\,x\sqrt{a+bx^2}}{3cd^2\,(c+dx^2)^{3/2}}+\frac{f^2x^3\sqrt{a+bx^2}}{d\,(c+dx^2)^{3/2}}$$
$$+\frac{(bc(d^2e^2+4cdef-8c^2f^2)-ad(2d^2e^2+2cdef-7c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{5/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{(3acdf^2-b(d^2e^2-2cdef+4c^2f^2))\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(4*c^2*f^2-2*c*d*e*f+d^2*e^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(3/2)+
f^2*x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(3/2)+1/3*(b*c*(-8*c^2*f^2+4*c*d*e*f+d
^2*e^2)-a*d*(-7*c^2*f^2+2*c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^
(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(5/2)/(-a*d
+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(3*a*c*d*f^2-b*(
4*c^2*f^2-2*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/
2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(5/2)/(-a*d+b*c)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.51 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)\left(ad(3c^2f+2d^2ex^2+cd(3e+4fx^2))-bc(4c^2f+d\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2)*(a*d*(3*c^2*f + 2*d^2*e*x^2 + c*d*(
3*e + 4*f*x^2)) - b*c*(4*c^2*f + d^2*e*x^2 + c*d*(2*e + 5*f*x^2))) - I*b*c
*(b*c*(d^2*e^2 + 4*c*d*e*f - 8*c^2*f^2) + a*d*(-2*d^2*e^2 - 2*c*d*e*f + 7*
c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(3*a*c*d*f^2 + b*(
d^2*e^2 + 4*c*d*e*f - 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c
^2*d^3*(-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 840 vs. 2(369) = 738.

Time = 1.00 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{(c+dx^2)^{5/2}}+\frac{2efx^2\sqrt{a+bx^2}}{(c+dx^2)^{5/2}}+\frac{f^2x^4\sqrt{a+bx^2}}{(c+dx^2)^{5/2}}\right)dx$$

$\downarrow$ 2009

$$-\frac{f^2\sqrt{bx^2+a}x^3}{3d\left(dx^2+c\right)^{3/2}}+\frac{(8bc-7ad)f^2\sqrt{bx^2+a}x}{3d^2(bc-ad)\sqrt{dx^2+c}}-\frac{(4bc-3ad)f^2\sqrt{bx^2+a}x}{3d^2(bc-ad)\sqrt{dx^2+c}}+\frac{e^2\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}-\frac{2ef\sqrt{bx^2+a}x}{3d\left(dx^2+c\right)^{3/2}}+\frac{(bc-2ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(8bc-7ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(2bc-ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{be^2\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}(4bc-3ad)f^2\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b\sqrt{c}ef\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(e^2*x*Sqrt[a + b*x^2])/(3*c*(c + d*x^2)^(3/2)) - (2*e*f*x*Sqrt[a + b*x^2]
)/(3*d*(c + d*x^2)^(3/2)) - (f^2*x^3*Sqrt[a + b*x^2])/(3*d*(c + d*x^2)^(3/
2)) + ((8*b*c - 7*a*d)*f^2*x*Sqrt[a + b*x^2])/(3*d^2*(b*c - a*d)*Sqrt[c +
d*x^2]) - ((4*b*c - 3*a*d)*f^2*x*Sqrt[a + b*x^2])/(3*d^2*(b*c - a*d)*Sqrt[
c + d*x^2]) + ((b*c - 2*a*d)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(2*b*c - a*d)*e*f*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[
c]*d^(3/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2]) - (Sqrt[c]*(8*b*c - 7*a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*e^2*Sqrt[a + b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*(b*c - a*
d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*e
*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(3*d^(3/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (Sqrt[c]*(4*b*c - 3*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*(b*c - a*d)*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(342) = 684$.

Time = 7.99 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.88

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\left(c^2f^2-2cdef+d^2e^2\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,d^4\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(bd\,x^2+ad\right)x\left(4a\,c^2d\,f^2-2acef\,d^2-2a\,d^3e^2-5b\,c^3f^2+4b\,c^2def+bc\,d^2\right.}{3c^2d^3(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(c^2*f^2-
2*c*d*e*f+d^2*e^2)/c/d^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2
-1/3*(b*d*x^2+a*d)/c^2/d^3/(a*d-b*c)*x*(4*a*c^2*d*f^2-2*a*c*d^2*e*f-2*a*d^
3*e^2-5*b*c^3*f^2+4*b*c^2*d*e*f+b*c*d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d))^(1/
2)+(f*(a*d*f-2*b*c*f+2*b*d*e)/d^3+1/3*(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^3*b/c-
1/3/d^3*(4*a*c^2*d*f^2-2*a*c*d^2*e*f-2*a*d^3*e^2-5*b*c^3*f^2+4*b*c^2*d*e*f
+b*c*d^2*e^2)/c^2+1/3*a/d^2/c^2/(a*d-b*c)*(4*a*c^2*d*f^2-2*a*c*d^2*e*f-2*a
*d^3*e^2-5*b*c^3*f^2+4*b*c^2*d*e*f+b*c*d^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b*f^2/d^2+1/3/d^2*b*(4*a*c^2*d*f^2-2
*a*c*d^2*e*f-2*a*d^3*e^2-5*b*c^3*f^2+4*b*c^2*d*e*f+b*c*d^2*e^2)/(a*d-b*c)/
c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-El
lipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(342) = 684$.

Time = 0.13 (sec) , antiderivative size = 934, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
1/3*((((b^2*c^2*d^4 - 2*a*b*c*d^5)*e^2 + 2*(2*b^2*c^3*d^3 - a*b*c^2*d^4)*e
*f - (8*b^2*c^4*d^2 - 7*a*b*c^3*d^3)*f^2)*x^5 + 2*((b^2*c^3*d^3 - 2*a*b*c^
2*d^4)*e^2 + 2*(2*b^2*c^4*d^2 - a*b*c^3*d^3)*e*f - (8*b^2*c^5*d - 7*a*b*c^
4*d^2)*f^2)*x^3 + ((b^2*c^4*d^2 - 2*a*b*c^3*d^3)*e^2 + 2*(2*b^2*c^5*d - a*
b*c^4*d^2)*e*f - (8*b^2*c^6 - 7*a*b*c^5*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*el
liptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((b^2*c^2*d^4 - 2*a*b*c*d^5 -
 a*b*d^6)*e^2 + 2*(2*b^2*c^3*d^3 - a*b*c^2*d^4 + a*b*c*d^5)*e*f - (8*b^2*c
^4*d^2 - 7*a*b*c^3*d^3 + 4*a*b*c^2*d^4 - 3*a^2*c*d^5)*f^2)*x^5 + 2*((b^2*c
^3*d^3 - 2*a*b*c^2*d^4 - a*b*c*d^5)*e^2 + 2*(2*b^2*c^4*d^2 - a*b*c^3*d^3 +
 a*b*c^2*d^4)*e*f - (8*b^2*c^5*d - 7*a*b*c^4*d^2 + 4*a*b*c^3*d^3 - 3*a^2*c
^2*d^4)*f^2)*x^3 + ((b^2*c^4*d^2 - 2*a*b*c^3*d^3 - a*b*c^2*d^4)*e^2 + 2*(2
*b^2*c^5*d - a*b*c^4*d^2 + a*b*c^3*d^3)*e*f - (8*b^2*c^6 - 7*a*b*c^5*d + 4
*a*b*c^4*d^2 - 3*a^2*c^3*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcs
in(sqrt(-c/d)/x), a*d/(b*c)) + (3*(b^2*c^3*d^3 - a*b*c^2*d^4)*f^2*x^4 - (b
^2*c^3*d^3 - 2*a*b*c^2*d^4)*e^2 - 2*(2*b^2*c^4*d^2 - a*b*c^3*d^3)*e*f + (8
*b^2*c^5*d - 7*a*b*c^4*d^2)*f^2 + (a*b*c*d^5*e^2 - 2*(3*b^2*c^3*d^3 - 2*a*
b*c^2*d^4)*e*f + (12*b^2*c^4*d^2 - 11*a*b*c^3*d^3)*f^2)*x^2)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c))/((b^2*c^3*d^6 - a*b*c^2*d^7)*x^5 + 2*(b^2*c^4*d^5 - a*
b*c^3*d^6)*x^3 + (b^2*c^5*d^4 - a*b*c^4*d^5)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x)
```

output

```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x - 2*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*d*e*f*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x**3
- 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**3 - sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b*d*e**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6
 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2
*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8
),x)*a**3*c**2*d**3*f**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6
 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2
*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8
),x)*a**3*c*d**4*f**2*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x*
*6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b*
*2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x*
*8),x)*a**3*d**5*f**2*x**4 - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x*
*6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b*
*2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x*
*8),x)*a**2*b*c**3*d**2*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)...
```

3.60 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 505

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \frac{(d^2e^2-2cdef-4c^2f^2)\,x\sqrt{a+bx^2}}{5cd^2\left(c+dx^2\right)^{5/2}} - \frac{f^2x^3\sqrt{a+bx^2}}{d\left(c+dx^2\right)^{5/2}}$$
$$+\frac{(bc(3d^2e^2+4cdef+8c^2f^2)-ad(4d^2e^2+2cdef+9c^2f^2))\,x\sqrt{a+bx^2}}{15c^2d^2(bc-ad)\left(c+dx^2\right)^{3/2}}$$
$$+\frac{(a^2d^2(8d^2e^2+4cdef+3c^2f^2)+b^2c^2(3d^2e^2+4cdef+8c^2f^2)-abcd(13d^2e^2+4cdef+13c^2f^2))\,\sqrt{a+b}{15c^{5/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{2b(de-cf)(bc(3de+2cf)-ad(2de+3cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/5*(-4*c^2*f^2-2*c*d*e*f+d^2*e^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(5/2)
-f^2*x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(5/2)+1/15*(b*c*(8*c^2*f^2+4*c*d*e*f+
3*d^2*e^2)-a*d*(9*c^2*f^2+2*c*d*e*f+4*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^2/d^2/
(-a*d+b*c)/(d*x^2+c)^(3/2)+1/15*(a^2*d^2*(3*c^2*f^2+4*c*d*e*f+8*d^2*e^2)+b
^2*c^2*(8*c^2*f^2+4*c*d*e*f+3*d^2*e^2)-a*b*c*d*(13*c^2*f^2+4*c*d*e*f+13*d^
2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(5/2)/d^(5/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+2/15*b*(-c*f+d*e)*(b*c*(2*c*f+3*d*e)-a*d*(3*c*f+2*d*e))*
(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2
))/c^(3/2)/d^(5/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(de-cf)(-2ad(2de+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c - a
*d)*(d*e - c*f)*(-2*a*d*(2*d*e + 3*c*f) + b*c*(3*d*e + 7*c*f))*(c + d*x^2)
 + (a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(3*d^2*e^2 + 4*c
*d*e*f + 8*c^2*f^2) - a*b*c*d*(13*d^2*e^2 + 4*c*d*e*f + 13*c^2*f^2))*(c +
d*x^2)^2) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((
a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(3*d^2*e^2 + 4*c*d*e
*f + 8*c^2*f^2) - a*b*c*d*(13*d^2*e^2 + 4*c*d*e*f + 13*c^2*f^2))*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*c*(3*d^2*e^2 + 4*c*
d*e*f + 8*c^2*f^2) - a*d*(4*d^2*e^2 + 2*c*d*e*f + 9*c^2*f^2))*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^3*(b*c - a*d)^2*S
qrt[a + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{7/2}}+\frac{2efx^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{7/2}}+\frac{f^2x^4\sqrt{a+bx^2}}{\left(c+dx^2\right)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2\sqrt{bx^2+ax^3}}{5d\left(dx^2+c\right)^{5/2}}+\frac{(3bc-4ad)e^2\sqrt{bx^2+ax}}{15c^2(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{(4bc-3ad)f^2\sqrt{bx^2+ax}}{15d^2(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{2(2bc-ad)ef\sqrt{bx^2+ax}}{15cd(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{e^2\sqrt{bx^2+ax}}{5c\left(dx^2+c\right)^{5/2}}-\frac{2ef\sqrt{bx^2+ax}}{5d\left(dx^2+c\right)^{5/2}}+\frac{\left(3b^2c^2-13abdc+8a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}\sqrt{d}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\left(8b^2c^2-13abdc+3a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{5/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{4\left(b^2c^2-abdc+a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{3/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2b(3bc-2ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b\sqrt{c}(2bc-3ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b(bc+ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{3/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output
```
(e^2*x*Sqrt[a + b*x^2])/(5*c*(c + d*x^2)^(5/2)) - (2*e*f*x*Sqrt[a + b*x^2]
)/(5*d*(c + d*x^2)^(5/2)) - (f^2*x^3*Sqrt[a + b*x^2])/(5*d*(c + d*x^2)^(5/
2)) + ((3*b*c - 4*a*d)*e^2*x*Sqrt[a + b*x^2])/(15*c^2*(b*c - a*d)*(c + d*x
^2)^(3/2)) + (2*(2*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(15*c*d*(b*c - a*d)*(
c + d*x^2)^(3/2)) - ((4*b*c - 3*a*d)*f^2*x*Sqrt[a + b*x^2])/(15*d^2*(b*c -
 a*d)*(c + d*x^2)^(3/2)) + ((3*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*e^2*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*c^
(5/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) + (4*(b^2*c^2 - a*b*c*d + a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*c^(3/2)*d^(3/2)*(b*c -
a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + ((8*b^2*c^
2 - 13*a*b*c*d + 3*a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*Sqrt[c]*d^(5/2)*(b*c - a*d)^2*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*b*(3*b*c - 2*a*d)*e^2*S
qrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(1
5*c^(3/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) - (2*b*(b*c + a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*Sqrt[c]*d^(3/2)*(b*c - a*d)^2*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*(2*b*c -
3*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(472) = 944.

Time = 8.27 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.05

method	result	size
elliptic	Expression too large to display	1037
default	Expression too large to display	4450

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*(c^2*f^2-
2*c*d*e*f+d^2*e^2)/c/d^5*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3
-1/15*(6*a*c^2*d*f^2-2*a*c*d^2*e*f-4*a*d^3*e^2-7*b*c^3*f^2+4*b*c^2*d*e*f+3
*b*c*d^2*e^2)/c^2/d^4/(a*d-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2
+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d^3/(a*d-b*c)^2*x*(3*a^2*c^2*d^2*f^2+4*a^2*
c*d^3*e*f+8*a^2*d^4*e^2-13*a*b*c^3*d*f^2-4*a*b*c^2*d^2*e*f-13*a*b*c*d^3*e^
2+8*b^2*c^4*f^2+4*b^2*c^3*d*e*f+3*b^2*c^2*d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d
))^(1/2)+(b*f^2/d^3-1/15*b*(6*a*c^2*d*f^2-2*a*c*d^2*e*f-4*a*d^3*e^2-7*b*c^
3*f^2+4*b*c^2*d*e*f+3*b*c*d^2*e^2)/d^3/(a*d-b*c)/c^2+1/15/d^3/(a*d-b*c)*(3
*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2-13*a*b*c^3*d*f^2-4*a*b*c^2*
d^2*e*f-13*a*b*c*d^3*e^2+8*b^2*c^4*f^2+4*b^2*c^3*d*e*f+3*b^2*c^2*d^2*e^2)/
c^3-1/15*a/d^2/c^3/(a*d-b*c)^2*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^
4*e^2-13*a*b*c^3*d*f^2-4*a*b*c^2*d^2*e*f-13*a*b*c*d^3*e^2+8*b^2*c^4*f^2+4*
b^2*c^3*d*e*f+3*b^2*c^2*d^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))+1/15/d^3*b*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*
d^4*e^2-13*a*b*c^3*d*f^2-4*a*b*c^2*d^2*e*f-13*a*b*c*d^3*e^2+8*b^2*c^4*f^2+
4*b^2*c^3*d*e*f+3*b^2*c^2*d^2*e^2)/(a*d-b*c)^2/c^2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(472) = 944$.

Time = 0.15 (sec) , antiderivative size = 1697, normalized size of antiderivative = 3.36

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="fricas
")
```

output
```
-1/15*((((3*b^3*c^2*d^5 - 13*a*b^2*c*d^6 + 8*a^2*b*d^7)*e^2 + 4*(b^3*c^3*d
^4 - a*b^2*c^2*d^5 + a^2*b*c*d^6)*e*f + (8*b^3*c^4*d^3 - 13*a*b^2*c^3*d^4
+ 3*a^2*b*c^2*d^5)*f^2)*x^6 + 3*((3*b^3*c^3*d^4 - 13*a*b^2*c^2*d^5 + 8*a^2
*b*c*d^6)*e^2 + 4*(b^3*c^4*d^3 - a*b^2*c^3*d^4 + a^2*b*c^2*d^5)*e*f + (8*b
^3*c^5*d^2 - 13*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4)*f^2)*x^4 + (3*b^3*c^5*d^2
 - 13*a*b^2*c^4*d^3 + 8*a^2*b*c^3*d^4)*e^2 + 4*(b^3*c^6*d - a*b^2*c^5*d^2
+ a^2*b*c^4*d^3)*e*f + (8*b^3*c^7 - 13*a*b^2*c^6*d + 3*a^2*b*c^5*d^2)*f^2
+ 3*((3*b^3*c^4*d^3 - 13*a*b^2*c^3*d^4 + 8*a^2*b*c^2*d^5)*e^2 + 4*(b^3*c^5
*d^2 - a*b^2*c^4*d^3 + a^2*b*c^3*d^4)*e*f + (8*b^3*c^6*d - 13*a*b^2*c^5*d^
2 + 3*a^2*b*c^4*d^3)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sq
rt(-b/a)), a*d/(b*c)) - (((3*b^3*c^2*d^5 - (6*a^2*b + 13*a*b^2)*c*d^6 + 4*
(a^3 + 2*a^2*b)*d^7)*e^2 + 2*(2*b^3*c^3*d^4 + (a^2*b - 2*a*b^2)*c^2*d^5 +
(a^3 + 2*a^2*b)*c*d^6)*e*f + (8*b^3*c^4*d^3 + (4*a^2*b - 13*a*b^2)*c^3*d^4
 - 3*(2*a^3 - a^2*b)*c^2*d^5)*f^2)*x^6 + 3*((3*b^3*c^3*d^4 - (6*a^2*b + 13
*a*b^2)*c^2*d^5 + 4*(a^3 + 2*a^2*b)*c*d^6)*e^2 + 2*(2*b^3*c^4*d^3 + (a^2*b
 - 2*a*b^2)*c^3*d^4 + (a^3 + 2*a^2*b)*c^2*d^5)*e*f + (8*b^3*c^5*d^2 + (4*a
^2*b - 13*a*b^2)*c^4*d^3 - 3*(2*a^3 - a^2*b)*c^3*d^4)*f^2)*x^4 + (3*b^3*c^
5*d^2 - (6*a^2*b + 13*a*b^2)*c^4*d^3 + 4*(a^3 + 2*a^2*b)*c^3*d^4)*e^2 + 2*
(2*b^3*c^6*d + (a^2*b - 2*a*b^2)*c^5*d^2 + (a^3 + 2*a^2*b)*c^4*d^3)*e*f +
(8*b^3*c^7 + (4*a^2*b - 13*a*b^2)*c^6*d - 3*(2*a^3 - a^2*b)*c^5*d^2)*f^...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(7/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x - 2*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*e*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x*
*3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**3 - sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b*d*e**2*x - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**
2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c*
*3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10
 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c*
*2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**3*c**3*d**3*f**2 - 24*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 +
12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5
- 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*
c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b
**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**3*c*
*2*d**4*f**2*x**2 - 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**
2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*
x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x
**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c
**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x
**8 - b**2*c*d**4*x**10),x)*a**3*c*d**5*f**2*x**4 - 8*int((sqrt(c + d*x...
```

3.61 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 736

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{(3d^2e^2-6cdef-4c^2f^2)\,x\sqrt{a+bx^2}}{21cd^2\,(c+dx^2)^{7/2}} - \frac{f^2x^3\sqrt{a+bx^2}}{3d\,(c+dx^2)^{7/2}}$$

$$+ \frac{(bc(15d^2e^2+12cdef+8c^2f^2)-ad(18d^2e^2+6cdef+11c^2f^2))\,x\sqrt{a+bx^2}}{105c^2d^2(bc-ad)\,(c+dx^2)^{5/2}}$$

$$+ \frac{(a^2d^2(24d^2e^2+8cdef+3c^2f^2)+b^2c^2(15d^2e^2+12cdef+8c^2f^2)-abcd(43d^2e^2+12cdef+15c^2f^2))\,x\sqrt{}}{105c^3d^2(bc-ad)^2\,(c+dx^2)^{3/2}}$$

$$- \frac{(2a^3d^3(24d^2e^2+8cdef+3c^2f^2)-b^3c^3(15d^2e^2+12cdef+8c^2f^2)-a^2bcd^2(128d^2e^2+38cdef+9c^2f^2)}{105c^{7/2}d^{5/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c}}$$

$$- \frac{b(abcd(61d^2e^2+18cdef-9c^2f^2)-b^2c^2(45d^2e^2-6cdef-4c^2f^2)-a^2d^2(24d^2e^2+8cdef+3c^2f^2))\sqrt{a}}{105c^{5/2}d^{5/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/21*(-4*c^2*f^2-6*c*d*e*f+3*d^2*e^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(7
/2)-1/3*f^2*x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(7/2)+1/105*(b*c*(8*c^2*f^2+12
*c*d*e*f+15*d^2*e^2)-a*d*(11*c^2*f^2+6*c*d*e*f+18*d^2*e^2))*x*(b*x^2+a)^(1
/2)/c^2/d^2/(-a*d+b*c)/(d*x^2+c)^(5/2)+1/105*(a^2*d^2*(3*c^2*f^2+8*c*d*e*f
+24*d^2*e^2)+b^2*c^2*(8*c^2*f^2+12*c*d*e*f+15*d^2*e^2)-a*b*c*d*(15*c^2*f^2
+12*c*d*e*f+43*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d^2/(-a*d+b*c)^2/(d*x^2+c)^
(3/2)-1/105*(2*a^3*d^3*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2)-b^3*c^3*(8*c^2*f^2
+12*c*d*e*f+15*d^2*e^2)-a^2*b*c*d^2*(9*c^2*f^2+38*c*d*e*f+128*d^2*e^2)+a*b
^2*c^2*d*(19*c^2*f^2+18*c*d*e*f+103*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^
(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(7/2)/d^(5/2)/(-a*d
+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*b*(a*b*c*d*(
-9*c^2*f^2+18*c*d*e*f+61*d^2*e^2)-b^2*c^2*(-4*c^2*f^2-6*c*d*e*f+45*d^2*e^2
)-a^2*d^2*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiA
M(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(5/2)/(-a*d+b*c)^
3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 723, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)^2+3c^2(bc-ad)^2(de-cf)(-2ad(3\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f)^2 + 3*c^2*(b*
c - a*d)^2*(d*e - c*f)*(-2*a*d*(3*d*e + 4*c*f) + b*c*(5*d*e + 9*c*f))*(c +
 d*x^2) + c*(b*c - a*d)*(a^2*d^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + b^
2*c^2*(15*d^2*e^2 + 12*c*d*e*f + 8*c^2*f^2) - a*b*c*d*(43*d^2*e^2 + 12*c*d
*e*f + 15*c^2*f^2))*(c + d*x^2)^2 + (-2*a^3*d^3*(24*d^2*e^2 + 8*c*d*e*f +
3*c^2*f^2) + b^3*c^3*(15*d^2*e^2 + 12*c*d*e*f + 8*c^2*f^2) + a^2*b*c*d^2*(
128*d^2*e^2 + 38*c*d*e*f + 9*c^2*f^2) - a*b^2*c^2*d*(103*d^2*e^2 + 18*c*d*
e*f + 19*c^2*f^2))*(c + d*x^2)^3) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^
3*Sqrt[1 + (d*x^2)/c]*((-2*a^3*d^3*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) +
b^3*c^3*(15*d^2*e^2 + 12*c*d*e*f + 8*c^2*f^2) + a^2*b*c*d^2*(128*d^2*e^2 +
 38*c*d*e*f + 9*c^2*f^2) - a*b^2*c^2*d*(103*d^2*e^2 + 18*c*d*e*f + 19*c^2*
f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(a^2*d^
2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(15*d^2*e^2 + 12*c*d*e*f
+ 8*c^2*f^2) - a*b*c*d*(43*d^2*e^2 + 12*c*d*e*f + 15*c^2*f^2))*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(105*Sqrt[b/a]*c^4*d^3*(b*c - a*d)^3
*Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 1232, normalized size of antiderivative = 1.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{(c+dx^2)^{9/2}}+\frac{2efx^2\sqrt{a+bx^2}}{(c+dx^2)^{9/2}}+\frac{f^2x^4\sqrt{a+bx^2}}{(c+dx^2)^{9/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2\sqrt{bx^2+a}x^3}{7d(dx^2+c)^{7/2}}+\frac{(15b^2c^2-43abdc+24a^2d^2)e^2\sqrt{bx^2+a}x}{105c^3(bc-ad)^2(dx^2+c)^{3/2}}+\frac{(8b^2c^2-15abdc+3a^2d^2)f^2\sqrt{bx^2+a}x}{105cd^2(bc-ad)^2(dx^2+c)^{3/2}}+\frac{4(3b^2c^2-3abdc+2a^2d^2)ef\sqrt{bx^2+a}x}{105c^2d(bc-ad)^2(dx^2+c)^{3/2}}+\frac{(5bc-6ad)e^2\sqrt{bx^2+a}x}{35c^2(bc-ad)(dx^2+c)^{5/2}}-\frac{(4bc-3ad)f^2\sqrt{bx^2+a}x}{35d^2(bc-ad)(dx^2+c)^{5/2}}+\frac{2(2bc-ad)ef\sqrt{bx^2+a}x}{35cd(bc-ad)(dx^2+c)^{5/2}}+\frac{e^2\sqrt{bx^2+a}x}{7c(dx^2+c)^{7/2}}-\frac{2ef\sqrt{bx^2+a}x}{7d(dx^2+c)^{7/2}}+\frac{(15b^3c^3-103ab^2dc^2+128a^2bd^2c-48a^3d^3)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{7/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{(bc-2ad)(8b^2c^2-3abdc+3a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{3/2}d^{5/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(2bc-ad)(3b^2c^2-3abdc+8a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{5/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{b(45b^2c^2-61abdc+24a^2d^2)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{5/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b(4b^2c^2-9abdc-3a^2d^2)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105\sqrt{c}d^{5/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b(3b^2c^2+9abdc-4a^2d^2)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{3/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
(e^2*x*Sqrt[a + b*x^2])/(7*c*(c + d*x^2)^(7/2)) - (2*e*f*x*Sqrt[a + b*x^2]
)/(7*d*(c + d*x^2)^(7/2)) - (f^2*x^3*Sqrt[a + b*x^2])/(7*d*(c + d*x^2)^(7/
2)) + ((5*b*c - 6*a*d)*e^2*x*Sqrt[a + b*x^2])/(35*c^2*(b*c - a*d)*(c + d*x
^2)^(5/2)) + (2*(2*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(35*c*d*(b*c - a*d)*(
c + d*x^2)^(5/2)) - ((4*b*c - 3*a*d)*f^2*x*Sqrt[a + b*x^2])/(35*d^2*(b*c -
 a*d)*(c + d*x^2)^(5/2)) + ((15*b^2*c^2 - 43*a*b*c*d + 24*a^2*d^2)*e^2*x*S
qrt[a + b*x^2])/(105*c^3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (4*(3*b^2*c^2
- 3*a*b*c*d + 2*a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(105*c^2*d*(b*c - a*d)^2*(
c + d*x^2)^(3/2)) + ((8*b^2*c^2 - 15*a*b*c*d + 3*a^2*d^2)*f^2*x*Sqrt[a + b
*x^2])/(105*c*d^2*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + ((15*b^3*c^3 - 103*a*
b^2*c^2*d + 128*a^2*b*c*d^2 - 48*a^3*d^3)*e^2*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*c^(7/2)*Sqrt[d]*(b*c - a
*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(2*b*c -
 a*d)*(3*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*c^(5/2)*d^(3/2)*(b*c - a
*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + ((b*c - 2*a
*d)*(8*b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*c^(3/2)*d^(5/2)*(b*c - a*d
)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(45*b^2*c^
2 - 61*a*b*c*d + 24*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1439 vs. $2(695) = 1390$.

Time = 8.40 (sec), antiderivative size = 1440, normalized size of antiderivative = 1.96

method	result	size
elliptic	Expression too large to display	1440
default	Expression too large to display	7943

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*(c^2*f^2-
2*c*d*e*f+d^2*e^2)/c/d^6*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^4
-1/35*(8*a*c^2*d*f^2-2*a*c*d^2*e*f-6*a*d^3*e^2-9*b*c^3*f^2+4*b*c^2*d*e*f+5
*b*c*d^2*e^2)/d^5/(a*d-b*c)/c^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2
+c/d)^3+1/105*(3*a^2*c^2*d^2*f^2+8*a^2*c*d^3*e*f+24*a^2*d^4*e^2-15*a*b*c^3
*d*f^2-12*a*b*c^2*d^2*e*f-43*a*b*c*d^3*e^2+8*b^2*c^4*f^2+12*b^2*c^3*d*e*f+
15*b^2*c^2*d^2*e^2)/d^4/(a*d-b*c)^2/c^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(x^2+c/d)^2+1/105*(b*d*x^2+a*d)/c^4/d^3/(a*d-b*c)^3*x*(6*a^3*c^2*d^3*f
^2+16*a^3*c*d^4*e*f+48*a^3*d^5*e^2-9*a^2*b*c^3*d^2*f^2-38*a^2*b*c^2*d^3*e*
f-128*a^2*b*c*d^4*e^2+19*a*b^2*c^4*d*f^2+18*a*b^2*c^3*d^2*e*f+103*a*b^2*c^
2*d^3*e^2-8*b^3*c^5*f^2-12*b^3*c^4*d*e*f-15*b^3*c^3*d^2*e^2)/((x^2+c/d)*(b
*d*x^2+a*d))^(1/2)+(1/105*b*(3*a^2*c^2*d^2*f^2+8*a^2*c*d^3*e*f+24*a^2*d^4*
e^2-15*a*b*c^3*d*f^2-12*a*b*c^2*d^2*e*f-43*a*b*c*d^3*e^2+8*b^2*c^4*f^2+12*
b^2*c^3*d*e*f+15*b^2*c^2*d^2*e^2)/d^3/(a*d-b*c)^2/c^3+1/105/d^3/(a*d-b*c)^
2*(6*a^3*c^2*d^3*f^2+16*a^3*c*d^4*e*f+48*a^3*d^5*e^2-9*a^2*b*c^3*d^2*f^2-3
8*a^2*b*c^2*d^3*e*f-128*a^2*b*c*d^4*e^2+19*a*b^2*c^4*d*f^2+18*a*b^2*c^3*d^
2*e*f+103*a*b^2*c^2*d^3*e^2-8*b^3*c^5*f^2-12*b^3*c^4*d*e*f-15*b^3*c^3*d^2*
e^2)/c^4-1/105*a/d^2/c^4/(a*d-b*c)^3*(6*a^3*c^2*d^3*f^2+16*a^3*c*d^4*e*f+4
8*a^3*d^5*e^2-9*a^2*b*c^3*d^2*f^2-38*a^2*b*c^2*d^3*e*f-128*a^2*b*c*d^4*e^2
+19*a*b^2*c^4*d*f^2+18*a*b^2*c^3*d^2*e*f+103*a*b^2*c^2*d^3*e^2-8*b^3*c^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. $2(693) = 1386$.

Time = 0.22 (sec) , antiderivative size = 2980, normalized size of antiderivative = 4.05

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="fricas
")
```

output
```
-1/105*((((15*b^4*c^3*d^6 - 103*a*b^3*c^2*d^7 + 128*a^2*b^2*c*d^8 - 48*a^3
*b*d^9)*e^2 + 2*(6*b^4*c^4*d^5 - 9*a*b^3*c^3*d^6 + 19*a^2*b^2*c^2*d^7 - 8*
a^3*b*c*d^8)*e*f + (8*b^4*c^5*d^4 - 19*a*b^3*c^4*d^5 + 9*a^2*b^2*c^3*d^6 -
 6*a^3*b*c^2*d^7)*f^2)*x^8 + 4*((15*b^4*c^4*d^5 - 103*a*b^3*c^3*d^6 + 128*
a^2*b^2*c^2*d^7 - 48*a^3*b*c*d^8)*e^2 + 2*(6*b^4*c^5*d^4 - 9*a*b^3*c^4*d^5
 + 19*a^2*b^2*c^3*d^6 - 8*a^3*b*c^2*d^7)*e*f + (8*b^4*c^6*d^3 - 19*a*b^3*c
^5*d^4 + 9*a^2*b^2*c^4*d^5 - 6*a^3*b*c^3*d^6)*f^2)*x^6 + 6*((15*b^4*c^5*d^
4 - 103*a*b^3*c^4*d^5 + 128*a^2*b^2*c^3*d^6 - 48*a^3*b*c^2*d^7)*e^2 + 2*(6
*b^4*c^6*d^3 - 9*a*b^3*c^5*d^4 + 19*a^2*b^2*c^4*d^5 - 8*a^3*b*c^3*d^6)*e*f
 + (8*b^4*c^7*d^2 - 19*a*b^3*c^6*d^3 + 9*a^2*b^2*c^5*d^4 - 6*a^3*b*c^4*d^5
)*f^2)*x^4 + (15*b^4*c^7*d^2 - 103*a*b^3*c^6*d^3 + 128*a^2*b^2*c^5*d^4 - 4
8*a^3*b*c^4*d^5)*e^2 + 2*(6*b^4*c^8*d - 9*a*b^3*c^7*d^2 + 19*a^2*b^2*c^6*d
^3 - 8*a^3*b*c^5*d^4)*e*f + (8*b^4*c^9 - 19*a*b^3*c^8*d + 9*a^2*b^2*c^7*d^
2 - 6*a^3*b*c^6*d^3)*f^2 + 4*((15*b^4*c^6*d^3 - 103*a*b^3*c^5*d^4 + 128*a^
2*b^2*c^4*d^5 - 48*a^3*b*c^3*d^6)*e^2 + 2*(6*b^4*c^7*d^2 - 9*a*b^3*c^6*d^3
 + 19*a^2*b^2*c^5*d^4 - 8*a^3*b*c^4*d^5)*e*f + (8*b^4*c^8*d - 19*a*b^3*c^7
*d^2 + 9*a^2*b^2*c^6*d^3 - 6*a^3*b*c^5*d^4)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)
*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((15*b^4*c^3*d^6 - (45*a^2
*b^2 + 103*a*b^3)*c^2*d^7 + (61*a^3*b + 128*a^2*b^2)*c*d^8 - 24*(a^4 + 2*a
^3*b)*d^9)*e^2 + 2*(6*b^4*c^4*d^5 + 3*(a^2*b^2 - 3*a*b^3)*c^3*d^6 + (9*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x - 6*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x*
*3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**3 - 3*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b*d*e**2*x - 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 3
0*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6
 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a
*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**
2 - 5*b**2*c**5*d*x**4 - 10*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 -
 5*b**2*c**2*d**4*x**10 - b**2*c*d**5*x**12),x)*a**3*c**4*d**3*f**2 - 72*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*
d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d*
*5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d*
*2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x*
*10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x**4 - 10*b**2*c**
4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x**10 - b**2*c*d**
5*x**12),x)*a**3*c**3*d**4*f**2*x**2 - 108*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*
x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 -
a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*...
```

3.62 $\int (a+bx^2)^{3/2} \sqrt{c+dx^2}(e+fx^2)^2 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 847

$$\int (a+bx^2)^{3/2} \sqrt{c+dx^2}(e+fx^2)^2 \, dx = \frac{(8a^4d^4f^2 - a^3bd^3f(36de+7cf) + 9a^2b^2d^2(7d^2e^2+6cdef-c^2f^2) - 2b^4c^2(21d^2e^2-24cdef+}{315b^2d^4\sqrt{a+bx^2}}$$
$$- \frac{(4a^3d^3f^2 - 3a^2bd^2f(6de+cf) - 3ab^2d(42d^2e^2+18cdef-5c^2f^2) - b^3c(21d^2e^2-24cdef+8c^2f^2))\, x\sqrt{}}{315b^2d^3}$$
$$+ \frac{(3a^2d^2f^2 + abdf(144de+11cf) + 3b^2(21d^2e^2+6cdef-2c^2f^2))\, x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^2}$$
$$+ \frac{f(18bde+bcf+10adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d} + \frac{1}{9}bf^2x^7\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$- \frac{\sqrt{a}(8a^4d^4f^2 - a^3bd^3f(36de+7cf) + 9a^2b^2d^2(7d^2e^2+6cdef-c^2f^2) - 2b^4c^2(21d^2e^2-24cdef+8c^2f^2)}{315b^{5/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+ \frac{a^{3/2}(4a^3d^3f^2 - 3a^2bd^2f(6de+cf) + 3ab^2d(63d^2e^2-18cdef+5c^2f^2) - b^3c(21d^2e^2-24cdef+8c^2f^2))}{315b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/315*(8*a^4*d^4*f^2-a^3*b*d^3*f*(7*c*f+36*d*e)+9*a^2*b^2*d^2*(-c^2*f^2+6*
c*d*e*f+7*d^2*e^2)-2*b^4*c^2*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2)+a*b^3*c*d*(
32*c^2*f^2-114*c*d*e*f+147*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^2/d^4/(b*x^2+a)^(
1/2)-1/315*(4*a^3*d^3*f^2-3*a^2*b*d^2*f*(c*f+6*d*e)-3*a*b^2*d*(-5*c^2*f^2+
18*c*d*e*f+42*d^2*e^2)-b^3*c*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2))*x*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^3+1/315*(3*a^2*d^2*f^2+a*b*d*f*(11*c*f+144*d
*e)+3*b^2*(-2*c^2*f^2+6*c*d*e*f+21*d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/b/d^2+1/63*f*(10*a*d*f+b*c*f+18*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/d+1/9*b*f^2*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/315*a^(1/2)*(8*a
^4*d^4*f^2-a^3*b*d^3*f*(7*c*f+36*d*e)+9*a^2*b^2*d^2*(-c^2*f^2+6*c*d*e*f+7*
d^2*e^2)-2*b^4*c^2*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2)+a*b^3*c*d*(32*c^2*f^2
-114*c*d*e*f+147*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+
b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)+1/315*a^(3/2)*(4*a^3*d^3*f^2-3*a^2*b*d^2*f*(c*f+6*d*e)
+3*a*b^2*d*(5*c^2*f^2-18*c*d*e*f+63*d^2*e^2)-b^3*c*(8*c^2*f^2-24*c*d*e*f+2
1*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a
*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.90 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.68

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4a^3d^3f^2+3a^2bd^2f(6de+cf+dfx^2)+ab^2d(-15c^2f^2+cdf(54e$$

input
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*a^3*d^3*f^2 + 3*a^2*b*d^2*f*(6*
d*e + c*f + d*f*x^2) + a*b^2*d*(-15*c^2*f^2 + c*d*f*(54*e + 11*f*x^2) + 2*
d^2*(63*e^2 + 72*e*f*x^2 + 25*f^2*x^4)) + b^3*(8*c^3*f^2 - 6*c^2*d*f*(4*e
+ f*x^2) + c*d^2*(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4) + d^3*x^2*(63*e^2 + 90*
e*f*x^2 + 35*f^2*x^4))) - I*c*(8*a^4*d^4*f^2 - a^3*b*d^3*f*(36*d*e + 7*c*f
) + 9*a^2*b^2*d^2*(7*d^2*e^2 + 6*c*d*e*f - c^2*f^2) - 2*b^4*c^2*(21*d^2*e^
2 - 24*c*d*e*f + 8*c^2*f^2) + a*b^3*c*d*(147*d^2*e^2 - 114*c*d*e*f + 32*c^
2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*(-9*a^2*b*d^3*e*f + 2*a^3*d^
3*f^2 - 3*a*b^2*d*(21*d^2*e^2 - 15*c*d*e*f + 4*c^2*f^2) + b^3*c*(21*d^2*e^
2 - 24*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*a^2*(b/a)^(5/2)*d^4*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 1342, normalized size of antiderivative = 1.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(e^2(a+bx^2)^{3/2}\sqrt{c+dx^2}+2efx^2(a+bx^2)^{3/2}\sqrt{c+dx^2}+f^2x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\begin{aligned}
&\frac{1}{9}f^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5+\frac{(bc+3ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{63d}+\\
&\frac{2}{7}ef\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^3+\frac{\left(\frac{3da^2}{b}+11ca-\frac{6bc^2}{d}\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3}{315d}+\\
&\frac{2(bc+3ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3}{35d}+\frac{be^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x}{5d}-\\
&\frac{2(bc-3ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{15d}+\\
&\frac{\left(8b^3c^3-15ab^2dc^2+3a^2bd^2c-4a^3d^3\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{315b^2d^3}+\\
&\frac{2\left(\frac{3da^2}{b}+9ca-\frac{4bc^2}{d}\right)ef\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{105d}+\frac{\left(\frac{3da^2}{b}+7ca-\frac{2bc^2}{d}\right)e^2\sqrt{bx^2+a}\,x}{15\sqrt{dx^2+c}}-\\
&\frac{\left(16b^4c^4-32ab^3dc^3+9a^2b^2d^2c^2+7a^3bd^3c-8a^4d^4\right)f^2\sqrt{bx^2+a}\,x}{315b^3d^3\sqrt{dx^2+c}}+\\
&\frac{2(bc-2ad)\left(8b^2c^2-3abdc+3a^2d^2\right)ef\sqrt{bx^2+a}\,x}{105b^2d^2\sqrt{dx^2+c}}+\\
&\frac{\sqrt{c}\left(2b^2c^2-7abdc-3a^2d^2\right)e^2\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\\
&\frac{\sqrt{c}\left(16b^4c^4-32ab^3dc^3+9a^2b^2d^2c^2+7a^3bd^3c-8a^4d^4\right)f^2\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^3d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\\
&\frac{2\sqrt{c}(bc-2ad)\left(8b^2c^2-3abdc+3a^2d^2\right)ef\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105b^2d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\\
&\frac{c^{3/2}(bc-9ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\\
&\frac{c^{3/2}\left(8b^3c^3-15ab^2dc^2+3a^2bd^2c-4a^3d^3\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315b^2d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\\
&\frac{2c^{3/2}\left(4b^2c^2-9abdc-3a^2d^2\right)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105bd^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}
\end{aligned}
$$

input
```
Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*e^2*x*Sqrt[a + b*x^2])/(15*Sqrt[c + d
*x^2]) + (2*(b*c - 2*a*d)*(8*b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*e*f*x*Sqrt[a
 + b*x^2])/(105*b^2*d^2*Sqrt[c + d*x^2]) - ((16*b^4*c^4 - 32*a*b^3*c^3*d +
 9*a^2*b^2*c^2*d^2 + 7*a^3*b*c*d^3 - 8*a^4*d^4)*f^2*x*Sqrt[a + b*x^2])/(31
5*b^3*d^3*Sqrt[c + d*x^2]) - (2*(b*c - 3*a*d)*e^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(15*d) + (2*(9*a*c - (4*b*c^2)/d + (3*a^2*d)/b)*e*f*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(105*d) + ((8*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - 4*a^3*d^3)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*b^2*d^3) + (
2*(b*c + 3*a*d)*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d) + ((11*a*c
 - (6*b*c^2)/d + (3*a^2*d)/b)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(31
5*d) + ((b*c + 3*a*d)*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(63*d) + (2
*e*f*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/7 + (f^2*x^5*(a + b*x^2)^(3/2)
*Sqrt[c + d*x^2])/9 + (b*e^2*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) +
(Sqrt[c]*(2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(3/2)*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*Sqrt[c]*(b*c - 2*a*d)*(8*b
^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*(16*b^4*c^4 - 32*a*b^3*c^3*d +
 9*a^2*b^2*c^2*d^2 + 7*a^3*b*c*d^3 - 8*a^4*d^4)*f^2*Sqrt[a + b*x^2]*Ell...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 11.47 (sec) , antiderivative size = 1300, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1300
risch	Expression too large to display	1494
default	Expression too large to display	2488

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*b*f^2*x^7
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(2*a*f^2*d*b+b^2*c*f^2+2*b^2*d*e*
f-1/9*b*f^2*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5
*(a^2*d*f^2+11/9*a*b*c*f^2+4*a*b*d*e*f+2*b^2*c*e*f+b^2*d*e^2-1/7*(2*a*f^2*
d*b+b^2*c*f^2+2*b^2*d*e*f-1/9*b*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*
x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a^2*c*f^2+2*a^2*e*f*d+4*a*c*e
*f*b+2*a*b*e^2*d+b^2*e^2*c-5/7*(2*a*f^2*d*b+b^2*c*f^2+2*b^2*d*e*f-1/9*b*f^
2*(8*a*d+8*b*c))/b/d*a*c-1/5*(a^2*d*f^2+11/9*a*b*c*f^2+4*a*b*d*e*f+2*b^2*c
*e*f+b^2*d*e^2-1/7*(2*a*f^2*d*b+b^2*c*f^2+2*b^2*d*e*f-1/9*b*f^2*(8*a*d+8*b
*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+(a^2*c*e^2-1/3*(a^2*c*f^2+2*a^2*e*f*d+4*a*c*e*f*b+2*a*b*e^2*d+b
^2*e^2*c-5/7*(2*a*f^2*d*b+b^2*c*f^2+2*b^2*d*e*f-1/9*b*f^2*(8*a*d+8*b*c))/b
/d*a*c-1/5*(a^2*d*f^2+11/9*a*b*c*f^2+4*a*b*d*e*f+2*b^2*c*e*f+b^2*d*e^2-1/7
*(2*a*f^2*d*b+b^2*c*f^2+2*b^2*d*e*f-1/9*b*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*
b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-(2*a^2*c*e*f+a^2*e^2*d+2*a*c*e^2*b-3/5*(a^2*d*f^2+11
/9*a*b*c*f^2+4*a*b*d*e*f+2*b^2*c*e*f+b^2*d*e^2-1/7*(2*a*f^2*d*b+b^2*c*f^2+
2*b^2*d*e*f-1/9*b*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a^2*c
*f^2+2*a^2*e*f*d+4*a*c*e*f*b+2*a*b*e^2*d+b^2*e^2*c-5/7*(2*a*f^2*d*b+b^2...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
1/315*((21*(2*b^4*c^3*d^2 - 7*a*b^3*c^2*d^3 - 3*a^2*b^2*c*d^4)*e^2 - 6*(8*
b^4*c^4*d - 19*a*b^3*c^3*d^2 + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4)*e*f + (1
6*b^4*c^5 - 32*a*b^3*c^4*d + 9*a^2*b^2*c^3*d^2 + 7*a^3*b*c^2*d^3 - 8*a^4*c
*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - (21*(2*b^4*c^3*d^2 - 7*a*b^3*c^2*d^3 - 9*a^2*b^2*d^5 - (3*a^2*b^2 -
a*b^3)*c*d^4)*e^2 - 6*(8*b^4*c^4*d - 19*a*b^3*c^3*d^2 - 3*a^3*b*d^5 + (9*a
^2*b^2 + 4*a*b^3)*c^2*d^3 - 3*(2*a^3*b + 3*a^2*b^2)*c*d^4)*e*f + (16*b^4*c
^5 - 32*a*b^3*c^4*d - 4*a^4*d^5 + (9*a^2*b^2 + 8*a*b^3)*c^3*d^2 + (7*a^3*b
 - 15*a^2*b^2)*c^2*d^3 - (8*a^4 - 3*a^3*b)*c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c
/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (35*b^4*d^5*f^2*x^8 + 5*
(18*b^4*d^5*e*f + (b^4*c*d^4 + 10*a*b^3*d^5)*f^2)*x^6 + (63*b^4*d^5*e^2 +
18*(b^4*c*d^4 + 8*a*b^3*d^5)*e*f - (6*b^4*c^2*d^3 - 11*a*b^3*c*d^4 - 3*a^2
*b^2*d^5)*f^2)*x^4 - 21*(2*b^4*c^2*d^3 - 7*a*b^3*c*d^4 - 3*a^2*b^2*d^5)*e^
2 + 6*(8*b^4*c^3*d^2 - 19*a*b^3*c^2*d^3 + 9*a^2*b^2*c*d^4 - 6*a^3*b*d^5)*e
*f - (16*b^4*c^4*d - 32*a*b^3*c^3*d^2 + 9*a^2*b^2*c^2*d^3 + 7*a^3*b*c*d^4
- 8*a^4*d^5)*f^2 + (21*(b^4*c*d^4 + 6*a*b^3*d^5)*e^2 - 6*(4*b^4*c^2*d^3 -
9*a*b^3*c*d^4 - 3*a^2*b^2*d^5)*e*f + (8*b^4*c^3*d^2 - 15*a*b^3*c^2*d^3 + 3
*a^2*b^2*c*d^4 - 4*a^3*b*d^5)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
b^3*d^5*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int {\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x + 3*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a**2*b*d**3*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3
*f**2*x**3 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x + 5
4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x + 11*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 126*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*b**2*d**3*e**2*x + 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
*2*d**3*e*f*x**3 + 50*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x
**5 + 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x - 24*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**3*c**2*d*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*
d**2*e**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**3 +
5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**
3*f**2*x**7 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 - 7*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3
*f**2 - 36*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 +
b*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e*f - 9*int((sqrt(c + d*x**2)*sqrt(...
```

3.63 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 636

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{2(3a^3d^3f^2-3a^2bd^2f(7de-2cf)+b^3c(35d^2e^2-56cdef+24c^2f^2)-ab^2d(70d^2e^2-91cdef+36c^2f^2))}{105bd^4\sqrt{a+bx^2}}$$

$$+\frac{(3a^2d^2f^2+3abdf(28de-11cf)+b^2(35d^2e^2-56cdef+24c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105bd^3}$$

$$+\frac{2f(7bde-3bcf+4adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2}+\frac{bf^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$$+\frac{2\sqrt{a}(3a^3d^3f^2-3a^2bd^2f(7de-2cf)+b^3c(35d^2e^2-56cdef+24c^2f^2)-ab^2d(70d^2e^2-91cdef+36c^2f^2}{105b^{3/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(3a^2cd^2f^2-3abd(35d^2e^2-28cdef+11c^2f^2)+b^2c(35d^2e^2-56cdef+24c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}}{105b^{3/2}cd^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/105*(3*a^3*d^3*f^2-3*a^2*b*d^2*f*(-2*c*f+7*d*e)+b^3*c*(24*c^2*f^2-56*c*
d*e*f+35*d^2*e^2)-a*b^2*d*(36*c^2*f^2-91*c*d*e*f+70*d^2*e^2))*x*(d*x^2+c)^
(1/2)/b/d^4/(b*x^2+a)^(1/2)+1/105*(3*a^2*d^2*f^2+3*a*b*d*f*(-11*c*f+28*d*e
)+b^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/b/d^3+2/35*f*(4*a*d*f-3*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/
2)/d^2+1/7*b*f^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+2/105*a^(1/2)*(3*a^
3*d^3*f^2-3*a^2*b*d^2*f*(-2*c*f+7*d*e)+b^3*c*(24*c^2*f^2-56*c*d*e*f+35*d^2
*e^2)-a*b^2*d*(36*c^2*f^2-91*c*d*e*f+70*d^2*e^2))*(d*x^2+c)^(1/2)*Elliptic
E(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^4/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/105*a^(3/2)*(3*a^2*c*d^2*f^2-
3*a*b*d*(11*c^2*f^2-28*c*d*e*f+35*d^2*e^2)+b^2*c*(24*c^2*f^2-56*c*d*e*f+35
*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*
d/b/c)^(1/2))/b^(3/2)/c/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(3a^2d^2f^2+3abdf(28de-11cf+8dfx^2)+b^2(24}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(3*a^2*d^2*f^2 + 3*a*b*d*f*(28*d*e
- 11*c*f + 8*d*f*x^2) + b^2*(24*c^2*f^2 - 2*c*d*f*(28*e + 9*f*x^2) + d^2*(
35*e^2 + 42*e*f*x^2 + 15*f^2*x^4))) + (2*I)*c*(3*a^3*d^3*f^2 + 3*a^2*b*d^2
*f*(-7*d*e + 2*c*f) + a*b^2*d*(-70*d^2*e^2 + 91*c*d*e*f - 36*c^2*f^2) + b^
3*c*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d
)*(3*a^2*c*d^2*f^2 + 3*a*b*d*(35*d^2*e^2 - 42*c*d*e*f + 16*c^2*f^2) - 2*b^
2*c*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*b*Sqrt[b/a]
*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 1062, normalized size of antiderivative = 1.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}+\frac{2efx^2\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}+\frac{f^2x^4\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bf^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}-\frac{2(3bc-4ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35d^2}+\frac{2bef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}+\frac{be^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3d}+\frac{\left(8b^2c^2-11abdc+a^2d^2\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{35bd^3}-\frac{4(2bc-3ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15d^2}-\frac{2(bc-2ad)e^2\sqrt{bx^2+a}x}{3d\sqrt{dx^2+c}}-\frac{2(2bc-ad)\left(4b^2c^2-4abdc-a^2d^2\right)f^2\sqrt{bx^2+a}x}{35b^2d^3\sqrt{dx^2+c}}-\frac{2\left(-\frac{3da^2}{b}+13ca-\frac{8bc^2}{d}\right)ef\sqrt{bx^2+a}x}{15d\sqrt{dx^2+c}}+\frac{2\sqrt{c}(bc-2ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(2bc-ad)\left(4b^2c^2-4abdc-a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2\sqrt{c}\left(8b^2c^2-13abdc+3a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc-3ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}\left(8b^2c^2-11abdc+a^2d^2\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{4c^{3/2}(2bc-3ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(-2*(b*c - 2*a*d)*e^2*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) - (2*(13*a*
c - (8*b*c^2)/d - (3*a^2*d)/b)*e*f*x*Sqrt[a + b*x^2])/(15*d*Sqrt[c + d*x^2
]) - (2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*f^2*x*Sqrt[a + b*x
^2])/(35*b^2*d^3*Sqrt[c + d*x^2]) + (b*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(3*d) - (4*(2*b*c - 3*a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*
d^2) + ((8*b^2*c^2 - 11*a*b*c*d + a^2*d^2)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(35*b*d^3) + (2*b*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) -
 (2*(3*b*c - 4*a*d)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b
*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (2*Sqrt[c]*(b*c - 2*a*d)
*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2
*Sqrt[c]*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*e*f*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(2*b*c - a*d)*(4*
b^2*c^2 - 4*a*b*c*d - a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*e^2*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (4*c^(3/2)*(2*b*c - 3
*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.68 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{b\,f^2x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7d}+\frac{\left(2ba\,f^2+2b^2ef-\frac{b\,f^2(6ad+6bc)}{7d}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(a^2f^2+4abfe+b^2e^2\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*b/d*f^2*x
^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2*b*a*f^2+2*b^2*e*f-1/7*b/d*f^
2*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a^2*f^2+
4*a*b*f*e+b^2*e^2-5/7*a*b*c/d*f^2-1/5*(2*b*a*f^2+2*b^2*e*f-1/7*b/d*f^2*(6*
a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(
a^2*e^2-1/3*(a^2*f^2+4*a*b*f*e+b^2*e^2-5/7*a*b*c/d*f^2-1/5*(2*b*a*f^2+2*b^
2*e*f-1/7*b/d*f^2*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^2*e*f+2*a*b*e^2-3/5*
(2*b*a*f^2+2*b^2*e*f-1/7*b/d*f^2*(6*a*d+6*b*c))/b/d*a*c-1/3*(a^2*f^2+4*a*b
*f*e+b^2*e^2-5/7*a*b*c/d*f^2-1/5*(2*b*a*f^2+2*b^2*e*f-1/7*b/d*f^2*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2\,(35\,(b^3c^3d^2-2\,ab^2c^2d^3)e^2-7\,(8\,b^3c^4d-13\,ab^2c^3d^2+3\,a^2bc^2d^3)ef+3\,(8}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
1/105*(2*(35*(b^3*c^3*d^2 - 2*a*b^2*c^2*d^3)*e^2 - 7*(8*b^3*c^4*d - 13*a*b
^2*c^3*d^2 + 3*a^2*b*c^2*d^3)*e*f + 3*(8*b^3*c^5 - 12*a*b^2*c^4*d + 2*a^2*
b*c^3*d^2 + a^3*c^2*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - (35*(2*b^3*c^3*d^2 - 4*a*b^2*c^2*d^3 + a*b^2*c*d^
4 - 3*a^2*b*d^5)*e^2 - 14*(8*b^3*c^4*d - 13*a*b^2*c^3*d^2 - 6*a^2*b*c*d^4
+ (3*a^2*b + 4*a*b^2)*c^2*d^3)*e*f + 3*(16*b^3*c^5 - 24*a*b^2*c^4*d + a^3*
c*d^4 + 4*(a^2*b + 2*a*b^2)*c^3*d^2 + (2*a^3 - 11*a^2*b)*c^2*d^3)*f^2)*sqr
t(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^3*
c*d^4*f^2*x^6 + 6*(7*b^3*c*d^4*e*f - (3*b^3*c^2*d^3 - 4*a*b^2*c*d^4)*f^2)*
x^4 - 70*(b^3*c^2*d^3 - 2*a*b^2*c*d^4)*e^2 + 14*(8*b^3*c^3*d^2 - 13*a*b^2*
c^2*d^3 + 3*a^2*b*c*d^4)*e*f - 6*(8*b^3*c^4*d - 12*a*b^2*c^3*d^2 + 2*a^2*b
*c^2*d^3 + a^3*c*d^4)*f^2 + (35*b^3*c*d^4*e^2 - 28*(2*b^3*c^2*d^3 - 3*a*b^
2*c*d^4)*e*f + 3*(8*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 + a^2*b*c*d^4)*f^2)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x - 33*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a*b*c*d*f**2*x + 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b*d**2*e*f*x + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 +
24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x - 56*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*c*d*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b
**2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x
+ 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 - 6*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f**2
 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x
**2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 + 42*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f +
72*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
 + b*d*x**4),x)*a*b**2*c**2*d*f**2 - 182*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f +
140*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**
2 + b*d*x**4),x)*a*b**2*d**3*e**2 - 48*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 + 112*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*b**3*c**2*d*e*f - 70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)...
```

3.64 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\, dx$

Optimal result

Integrand size = 32, antiderivative size = 459

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\, dx = \frac{(3a^2d^2f^2 + abdf(40de-27cf) + b^2(15d^2e^2-40cdef+24c^2f^2))\, x\sqrt{a+bx}}{15bd^3\sqrt{c+dx^2}}$$

$$+ \frac{2f(5bde-3bcf+3adf)x^3\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} + \frac{bf^2x^5\sqrt{a+bx^2}}{5d\sqrt{c+dx^2}}$$

$$- \frac{(3a^2cd^2f^2 + 2b^2c(15d^2e^2-40cdef+24c^2f^2) - abd(15d^2e^2-70cdef+48c^2f^2))\sqrt{a+bx^2}\, E\left(\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{15b\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+ \frac{\sqrt{c}(3adf(10de-7cf) + b(15d^2e^2-40cdef+24c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), 1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/15*(3*a^2*d^2*f^2+a*b*d*f*(-27*c*f+40*d*e)+b^2*(24*c^2*f^2-40*c*d*e*f+15
*d^2*e^2))*x*(b*x^2+a)^(1/2)/b/d^3/(d*x^2+c)^(1/2)+2/15*f*(3*a*d*f-3*b*c*f
+5*b*d*e)*x^3*(b*x^2+a)^(1/2)/d^2/(d*x^2+c)^(1/2)+1/5*b*f^2*x^5*(b*x^2+a)^
(1/2)/d/(d*x^2+c)^(1/2)-1/15*(3*a^2*c*d^2*f^2+2*b^2*c*(24*c^2*f^2-40*c*d*e
*f+15*d^2*e^2)-a*b*d*(48*c^2*f^2-70*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*E
llipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b/c^(1/2)/
d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/15*c^(1/2)*(3*a*
d*f*(-7*c*f+10*d*e)+b*(24*c^2*f^2-40*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(7/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3ad(5d^2e^2+7c^2f^2+2cdf(-5e+fx^2))+bc(-24c^2f^2+2\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d*(5*d^2*e^2 + 7*c^2*f^2 + 2*c*d*f*(-5*e +
 f*x^2)) + b*c*(-24*c^2*f^2 + 2*c*d*f*(20*e - 3*f*x^2) + d^2*(-15*e^2 + 10
*e*f*x^2 + 3*f^2*x^4))) - I*c*(3*a^2*c*d^2*f^2 + a*b*d*(-15*d^2*e^2 + 70*c
*d*e*f - 48*c^2*f^2) + 2*b^2*c*(15*d^2*e^2 - 40*c*d*e*f + 24*c^2*f^2))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*(3*a*d*f*(-5*d*e + 4*c*f) + b*(-15*d^2*
e^2 + 40*c*d*e*f - 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*c*d^4*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 951 vs. 2(459) = 918.

Time = 1.31 (sec) , antiderivative size = 951, normalized size of antiderivative = 2.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}+\frac{2efx^2(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}+\frac{f^2x^4(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6bf^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d^2} - \frac{f^2\left(bx^2+a\right)^{3/2}x^3}{d\sqrt{dx^2+c}} - \frac{(8bc-7ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{5d^3} + \frac{8bef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3d^2} - \frac{2ef\left(bx^2+a\right)^{3/2}x}{d\sqrt{dx^2+c}} - \frac{(bc-ad)e^2\sqrt{bx^2+a}x}{cd\sqrt{dx^2+c}} + \frac{(2bc-ad)e^2\sqrt{bx^2+a}x}{cd\sqrt{dx^2+c}} - \frac{\left(-\frac{da^2}{b}+16ca-\frac{16bc^2}{d}\right)f^2\sqrt{bx^2+a}x}{5d^2\sqrt{dx^2+c}} - \frac{2(8bc-7ad)ef\sqrt{bx^2+a}x}{3d^2\sqrt{dx^2+c}} - \frac{(2bc-ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{\sqrt{c}\left(16b^2c^2-16abdc+a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{2\sqrt{c}(8bc-7ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{b\sqrt{c}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{c^{3/2}(8bc-7ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{2\sqrt{c}(4bc-3ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*e^2*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a
*d)*e^2*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (2*(8*b*c - 7*a*d)*e*f*
x*Sqrt[a + b*x^2])/(3*d^2*Sqrt[c + d*x^2]) - ((16*a*c - (16*b*c^2)/d - (a^
2*d)/b)*f^2*x*Sqrt[a + b*x^2])/(5*d^2*Sqrt[c + d*x^2]) - (2*e*f*x*(a + b*x
^2)^(3/2))/(d*Sqrt[c + d*x^2]) - (f^2*x^3*(a + b*x^2)^(3/2))/(d*Sqrt[c + d
*x^2]) + (8*b*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d^2) - ((8*b*c - 7
*a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d^3) + (6*b*f^2*x^3*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(5*d^2) - ((2*b*c - a*d)*e^2*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*d^(3/2)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(8*b*c -
 7*a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) - (Sqrt[c]*(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*f^2*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*e^2*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*Sqrt[c]*(4*b*c -
 3*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) + (c^(3/2)*(8*b*c - 7*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 15.30 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.84

method	result
risch	$\frac{fx(3bdf\,x^2+6adf-9bcf+10bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15d^3}+\frac{\left(-\frac{(3a^2d^2f^2-33abcd\,f^2+40ab\,d^2ef+33b^2c^2f^2-50b^2cdef+15b^2d^2e^2)c\sqrt{1+\frac{b\,x^2}{a}}\cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+\cdots}}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{(bd\,x^2+ad)(a\,c^2d\,f^2-2acef\,d^2+a\,d^3e^2-b\,c^3f^2+2b\,c^2def-bc\,d^2e^2)x}{c\,d^4\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{b\,f^2x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5d^2}+\frac{\left(\frac{fb(2adf-b\cdots}{d^2\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*f*x*(3*b*d*f*x^2+6*a*d*f-9*b*c*f+10*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/d^3+1/15/d^3*(-(3*a^2*d^2*f^2-33*a*b*c*d*f^2+40*a*b*d^2*e*f+33*b^2*c
^2*f^2-50*b^2*c*d*e*f+15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2)))+15*(a^2*c^2*d^2*f^2-2*a^2*c*d^3*e*f+a^2*d^4*e^2-2*a*b*c^3*d*f^2+4
*a*b*c^2*d^2*e*f-2*a*b*c*d^3*e^2+b^2*c^4*f^2-2*b^2*c^3*d*e*f+b^2*c^2*d^2*e
^2)/d*((b*d*x^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/
(a*d-b*c)/c*a*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))+b/(a*d-b*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-(21*a^2*c*d^2*f
^2-30*a^2*d^3*e*f-39*a*b*c^2*d*f^2+70*a*b*c*d^2*e*f-30*a*b*d^3*e^2+15*b^2*
c^3*f^2-30*b^2*c^2*d*e*f+15*b^2*c*d^2*e^2)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1
/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
-1/15*(((15*(2*b^2*c^2*d^3 - a*b*c*d^4)*e^2 - 10*(8*b^2*c^3*d^2 - 7*a*b*c^
2*d^3)*e*f + 3*(16*b^2*c^4*d - 16*a*b*c^3*d^2 + a^2*c^2*d^3)*f^2)*x^3 + (1
5*(2*b^2*c^3*d^2 - a*b*c^2*d^3)*e^2 - 10*(8*b^2*c^4*d - 7*a*b*c^3*d^2)*e*f
 + 3*(16*b^2*c^5 - 16*a*b*c^4*d + a^2*c^3*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d
)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((15*(2*b^2*c^2*d^3 - a*b*
c*d^4 + a*b*d^5)*e^2 - 10*(8*b^2*c^3*d^2 - 7*a*b*c^2*d^3 + 4*a*b*c*d^4 - 3
*a^2*d^5)*e*f + 3*(16*b^2*c^4*d - 16*a*b*c^3*d^2 - 7*a^2*c*d^4 + (a^2 + 8*
a*b)*c^2*d^3)*f^2)*x^3 + (15*(2*b^2*c^3*d^2 - a*b*c^2*d^3 + a*b*c*d^4)*e^2
 - 10*(8*b^2*c^4*d - 7*a*b*c^3*d^2 + 4*a*b*c^2*d^3 - 3*a^2*c*d^4)*e*f + 3*
(16*b^2*c^5 - 16*a*b*c^4*d - 7*a^2*c^2*d^3 + (a^2 + 8*a*b)*c^3*d^2)*f^2)*x
)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^
2*c*d^4*f^2*x^6 + 2*(5*b^2*c*d^4*e*f - 3*(b^2*c^2*d^3 - a*b*c*d^4)*f^2)*x^
4 + 15*(2*b^2*c^2*d^3 - a*b*c*d^4)*e^2 - 10*(8*b^2*c^3*d^2 - 7*a*b*c^2*d^3
)*e*f + 3*(16*b^2*c^4*d - 16*a*b*c^3*d^2 + a^2*c^2*d^3)*f^2 + (15*b^2*c*d^
4*e^2 - 40*(b^2*c^2*d^3 - a*b*c*d^4)*e*f + 3*(8*b^2*c^3*d^2 - 9*a*b*c^2*d^
3 + a^2*c*d^4)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^6*x^3 + b
*c^2*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output
```
( - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x + 15*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**2*d**2*e*f*x + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*c**2*f**2*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x + 6*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 + 15*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b*d**2*e**2*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**
2*c**2*f**2*x**3 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x**3
+ 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**5 + 12*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*
c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c**2*d**2*f**2 - 15*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c*d**3*e*f + 12*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c*d**3*f**2*
x**2 - 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x
**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*d*
*4*e*f*x**2 - 36*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*
a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*
b**2*c**3*d*f**2 + 55*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2
 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),
x)*a*b**2*c**2*d**2*e*f - 36*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**...
```

3.65 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 415

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(ad(de-cf)^2-bc(d^2e^2-2cdef+2c^2f^2))\,x\sqrt{a+bx^2}}{3cd^3(c+dx^2)^{3/2}} + \frac{bf^2x^5\sqrt{a+bx^2}}{3d(c+dx^2)^{3/2}} + \frac{2f(3bde-3bcf+2adf)x\sqrt{a+bx^2}}{3d^3\sqrt{c+dx^2}} + \frac{2(ad(d^2e^2+cdef-4c^2f^2)+bc(d^2e^2-8cdef+8c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(3acdf^2-b(d^2e^2-8cdef+8c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(a*d*(-c*f+d*e)^2-b*c*(2*c^2*f^2-2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)
/c/d^3/(d*x^2+c)^(3/2)+1/3*b*f^2*x^5*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(3/2)+2/3
*f*(2*a*d*f-3*b*c*f+3*b*d*e)*x*(b*x^2+a)^(1/2)/d^3/(d*x^2+c)^(1/2)+2/3*(a*
d*(-4*c^2*f^2+c*d*e*f+d^2*e^2)+b*c*(8*c^2*f^2-8*c*d*e*f+d^2*e^2))*(b*x^2+a
)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c
^(3/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(3*a*c*
d*f^2-b*(8*c^2*f^2-8*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.55 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad(de-cf)\left(3c^2f+2d^2ex^2+cd(3e+4fx^2)\right)+bc(8c^3f^2\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*(d*e - c*f)*(3*c^2*f + 2*d^2*e*x^2 + c*d*(
3*e + 4*f*x^2)) + b*c*(8*c^3*f^2 + 2*d^3*e^2*x^2 + 2*c^2*d*f*(-4*e + 5*f*x
^2) + c*d^2*(e^2 - 10*e*f*x^2 + f^2*x^4))) - (2*I)*b*c*(-(a*d*(d^2*e^2 + c
*d*e*f - 4*c^2*f^2)) - b*c*(d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*
x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*c*(3*a^2*c*d^2*f^2 + a*b*d*(d^2*e^2 + 10*c*d*e*f - 16*c^2
*f^2) + 2*b^2*c*(d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c
+ d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
])/(3*Sqrt[b/a]*c^2*d^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 906 vs. 2(415) = 830.

Time = 1.23 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 433$$

$$\int \left(\frac{e^2(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}}+\frac{2efx^2(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}}+\frac{f^2x^4(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(2bc-ad)f^2\sqrt{bx^2+a}x^3}{cd^2\sqrt{dx^2+c}}-\frac{f^2(bx^2+a)^{3/2}x^3}{3d(dx^2+c)^{3/2}}+\frac{(8bc-3ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3cd^3}-\frac{8(2bc-ad)f^2\sqrt{bx^2+a}x}{3d^3\sqrt{dx^2+c}}-\frac{2(4bc-ad)ef\sqrt{bx^2+a}x}{3cd^2\sqrt{dx^2+c}}+\frac{2(8bc-ad)ef\sqrt{bx^2+a}x}{3cd^2\sqrt{dx^2+c}}-\frac{2ef(bx^2+a)^{3/2}x}{3d(dx^2+c)^{3/2}}-\frac{(bc-ad)e^2\sqrt{bx^2+a}x}{3cd(dx^2+c)^{3/2}}+\frac{2(bc+ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{8\sqrt{c}(2bc-ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2(8bc-ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{be^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(8bc-3ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{8b\sqrt{c}ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*e^2*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^(3/2)) - (2*e*f*
x*(a + b*x^2)^(3/2))/(3*d*(c + d*x^2)^(3/2)) - (f^2*x^3*(a + b*x^2)^(3/2))
/(3*d*(c + d*x^2)^(3/2)) - (2*(4*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(3*c*d^
2*Sqrt[c + d*x^2]) + (2*(8*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(3*c*d^2*Sqrt
[c + d*x^2]) - (8*(2*b*c - a*d)*f^2*x*Sqrt[a + b*x^2])/(3*d^3*Sqrt[c + d*x
^2]) - ((2*b*c - a*d)*f^2*x^3*Sqrt[a + b*x^2])/(c*d^2*Sqrt[c + d*x^2]) + (
(8*b*c - 3*a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^3) + (2*(b*c
 + a*d)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*c^(3/2)*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - (2*(8*b*c - a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (8*Sqrt[c]*(2*b*c - a*d)*f^2*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(7/2)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*e^2*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*
d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (8*b*Sqrt
[c]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) -
 (Sqrt[c]*(8*b*c - 3*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(380) = 760$.

Time = 16.20 (sec) , antiderivative size = 862, normalized size of antiderivative = 2.08

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(a c^2 d f^2-2 a c e f d^2+a d^3 e^2-b c^3 f^2+2 b c^2 d e f-b c d^2 e^2\right) x \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3 c d^5\left(x^2+\frac{c}{d}\right)^2}-\frac{2\left(b d x^2+a d\right)\left(2 a c^2 d f^2-a c e f d^2-\cdots\right.}{3 c^2 d^4 \sqrt{\left(x^2\cdots\right.}}\cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(a*c^2*d*
f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f^2+2*b*c^2*d*e*f-b*c*d^2*e^2)/c/d^5*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2-2/3*(b*d*x^2+a*d)*(2*a*c^2*
d*f^2-a*c*d^2*e*f-a*d^3*e^2-4*b*c^3*f^2+5*b*c^2*d*e*f-b*c*d^2*e^2)/c^2/d^4
*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*b*f^2/d^3*x*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)+((a^2*d^2*f^2-4*a*b*c*d*f^2+4*a*b*d^2*e*f+3*b^2*c^2*f^2-4*b^2
*c*d*e*f+b^2*d^2*e^2)/d^4+1/3*(a*c^2*d*f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f
^2+2*b*c^2*d*e*f-b*c*d^2*e^2)/d^4*b/c-2/3*(2*a*c^2*d*f^2-a*c*d^2*e*f-a*d^3
*e^2-4*b*c^3*f^2+5*b*c^2*d*e*f-b*c*d^2*e^2)/d^4*(a*d-b*c)/c^2+2/3*a/d^3*(2
*a*c^2*d*f^2-a*c*d^2*e*f-a*d^3*e^2-4*b*c^3*f^2+5*b*c^2*d*e*f-b*c*d^2*e^2)/
c^2-1/3*a*b*c/d^3*f^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-(2*b/d^3*f*(a*d*f-b*c*f+b*d*e)+2/3*(2*a*c^2*d*f^2-a*c*d^2*e*f-a
*d^3*e^2-4*b*c^3*f^2+5*b*c^2*d*e*f-b*c*d^2*e^2)/d^3*b/c^2-1/3*b*f^2/d^3*(2
*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(380) = 760$.

Time = 0.12 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
1/3*(2*(((b^2*c^2*d^4 + a*b*c*d^5)*e^2 - (8*b^2*c^3*d^3 - a*b*c^2*d^4)*e*f
 + 4*(2*b^2*c^4*d^2 - a*b*c^3*d^3)*f^2)*x^5 + 2*((b^2*c^3*d^3 + a*b*c^2*d^
4)*e^2 - (8*b^2*c^4*d^2 - a*b*c^3*d^3)*e*f + 4*(2*b^2*c^5*d - a*b*c^4*d^2)
*f^2)*x^3 + ((b^2*c^4*d^2 + a*b*c^3*d^3)*e^2 - (8*b^2*c^5*d - a*b*c^4*d^2)
*e*f + 4*(2*b^2*c^6 - a*b*c^5*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((2*b^2*c^2*d^4 + 2*a*b*c*d^5 + a*b*d^6
)*e^2 - 2*(8*b^2*c^3*d^3 - a*b*c^2*d^4 + 4*a*b*c*d^5)*e*f + (16*b^2*c^4*d^
2 - 8*a*b*c^3*d^3 + 8*a*b*c^2*d^4 - 3*a^2*c*d^5)*f^2)*x^5 + 2*((2*b^2*c^3*
d^3 + 2*a*b*c^2*d^4 + a*b*c*d^5)*e^2 - 2*(8*b^2*c^4*d^2 - a*b*c^3*d^3 + 4*
a*b*c^2*d^4)*e*f + (16*b^2*c^5*d - 8*a*b*c^4*d^2 + 8*a*b*c^3*d^3 - 3*a^2*c
^2*d^4)*f^2)*x^3 + ((2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + a*b*c^2*d^4)*e^2 - 2*
(8*b^2*c^5*d - a*b*c^4*d^2 + 4*a*b*c^3*d^3)*e*f + (16*b^2*c^6 - 8*a*b*c^5*
d + 8*a*b*c^4*d^2 - 3*a^2*c^3*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b^2*c^2*d^4*f^2*x^6 + 2*(3*b^2*c^2*d^
4*e*f - (3*b^2*c^3*d^3 - 2*a*b*c^2*d^4)*f^2)*x^4 - 2*(b^2*c^3*d^3 + a*b*c^
2*d^4)*e^2 + 2*(8*b^2*c^4*d^2 - a*b*c^3*d^3)*e*f - 8*(2*b^2*c^5*d - a*b*c^
4*d^2)*f^2 - ((3*b^2*c^2*d^4 + a*b*c*d^5)*e^2 - 4*(6*b^2*c^3*d^3 - a*b*c^2
*d^4)*e*f + (24*b^2*c^4*d^2 - 13*a*b*c^3*d^3)*f^2)*x^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c))/(b*c^2*d^7*x^5 + 2*b*c^3*d^6*x^3 + b*c^4*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)**2/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x)
```

output
```
(6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x - 3*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*d**2*e*f*x + 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
2*d**2*f**2*x**3 - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*f**2*x + 9
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x - 10*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*b*c*d*f**2*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
d**2*e**2*x + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*f*x**3 + sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**5 + 6*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b**2*c**2*f**2*x**3 - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**
2*c*d*e*f*x**3 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**5 + 3*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d*
*2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x*
*2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x*
*4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a**4*c**2*d**4*f**2 + 6*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d*
*2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x*
*2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x*
*4 - 3*b**2*c**2*d**2*x**6 - b**2*c*d**3*x**8),x)*a**4*c*d**5*f**2*x**2 +
3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*
d**2*x**2 + 3*a**2*c*d**3*x**4 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*
x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3...
```

3.66 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 529

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \frac{(ad(de-cf)^2 - bc(d^2e^2 - 2cdef + 6c^2f^2))\,x\sqrt{a+bx^2}}{5cd^3(c+dx^2)^{5/2}}$$
$$+ \frac{bf^2x^5\sqrt{a+bx^2}}{d(c+dx^2)^{5/2}} + \frac{2(ad(2d^2e^2 + cdef - 3c^2f^2) + bc(d^2e^2 - 7cdef + 21c^2f^2))\,x\sqrt{a+bx^2}}{15c^2d^3(c+dx^2)^{3/2}}$$
$$+ \frac{(2b^2c^2(d^2e^2 + 8cdef - 24c^2f^2) - a^2d^2(8d^2e^2 + 4cdef + 3c^2f^2) + 3abcd(d^2e^2 - 2cdef + 16c^2f^2))\sqrt{a+b}}{15c^{5/2}d^{7/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{b(bc(d^2e^2 + 8cdef - 24c^2f^2) - ad(4d^2e^2 + 2cdef - 21c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), 1 - \frac{bc}{ad}\right)}{15c^{3/2}d^{7/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/5*(a*d*(-c*f+d*e)^2-b*c*(6*c^2*f^2-2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)
/c/d^3/(d*x^2+c)^(5/2)+b*f^2*x^5*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(5/2)+2/15*(a
*d*(-3*c^2*f^2+c*d*e*f+2*d^2*e^2)+b*c*(21*c^2*f^2-7*c*d*e*f+d^2*e^2))*x*(b
*x^2+a)^(1/2)/c^2/d^3/(d*x^2+c)^(3/2)+1/15*(2*b^2*c^2*(-24*c^2*f^2+8*c*d*e
*f+d^2*e^2)-a^2*d^2*(3*c^2*f^2+4*c*d*e*f+8*d^2*e^2)+3*a*b*c*d*(16*c^2*f^2-
2*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(7/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*b*(b*c*(-24*c^2*f^2+8*c*d*e*f+d^2*e^2)-a*
d*(-21*c^2*f^2+2*c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arcta
n(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(7/2)/(-a*d+b*c)/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2-2c(bc-ad)(de-cf)(bc(de\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 - 2*c*(b*c
 - a*d)*(d*e - c*f)*(b*c*(d*e - 6*c*f) + a*d*(2*d*e + 3*c*f))*(c + d*x^2)
+ (a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) - 3*a*b*c*d*(d^2*e^2 - 2*c*
d*e*f + 11*c^2*f^2) + b^2*c^2*(-2*d^2*e^2 - 16*c*d*e*f + 33*c^2*f^2))*(c +
 d*x^2)^2)) - I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*
((a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) - 3*a*b*c*d*(d^2*e^2 - 2*c*d
*e*f + 16*c^2*f^2) + 2*b^2*c^2*(-(d^2*e^2) - 8*c*d*e*f + 24*c^2*f^2))*Elli
pticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 2*(b*c - a*d)*(-(a*d*(2*d^2*e
^2 + c*d*e*f + 12*c^2*f^2)) + b*c*(-(d^2*e^2) - 8*c*d*e*f + 24*c^2*f^2))*E
llipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^4*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1063 vs. 2(529) = 1058.

Time = 1.46 (sec) , antiderivative size = 1063, normalized size of antiderivative = 2.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}+\frac{2efx^2\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}+\frac{f^2x^4\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(2bc-ad)f^2\sqrt{bx^2+a}x^3}{5cd^2(dx^2+c)^{3/2}}-\frac{f^2(bx^2+a)^{3/2}x^3}{5d(dx^2+c)^{5/2}}+\frac{(16b^2c^2-16abdc+a^2d^2)f^2\sqrt{bx^2+a}x}{5cd^3(bc-ad)\sqrt{dx^2+c}}-\frac{b(8bc-7ad)f^2\sqrt{bx^2+a}x}{5d^3(bc-ad)\sqrt{dx^2+c}}+\frac{2(bc+2ad)e^2\sqrt{bx^2+a}x}{15c^2d(dx^2+c)^{3/2}}-\frac{2(4bc-ad)ef\sqrt{bx^2+a}x}{15cd^2(dx^2+c)^{3/2}}-\frac{2ef(bx^2+a)^{3/2}x}{5d(dx^2+c)^{5/2}}-\frac{(bc-ad)e^2\sqrt{bx^2+a}x}{5cd(dx^2+c)^{5/2}}+\frac{(2b^2c^2+3abdc-8a^2d^2)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}d^{3/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(16b^2c^2-16abdc+a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5\sqrt{c}d^{7/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(8b^2c^2-3abdc-2a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{3/2}d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b(bc-4ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{3/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{b\sqrt{c}(8bc-7ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5d^{7/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b(4bc-ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output

```
-1/5*((b*c - a*d)*e^2*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^(5/2)) - (2*e*f*
x*(a + b*x^2)^(3/2))/(5*d*(c + d*x^2)^(5/2)) - (f^2*x^3*(a + b*x^2)^(3/2))
/(5*d*(c + d*x^2)^(5/2)) + (2*(b*c + 2*a*d)*e^2*x*Sqrt[a + b*x^2])/(15*c^2
*d*(c + d*x^2)^(3/2)) - (2*(4*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(15*c*d^2*
(c + d*x^2)^(3/2)) - ((2*b*c - a*d)*f^2*x^3*Sqrt[a + b*x^2])/(5*c*d^2*(c +
 d*x^2)^(3/2)) - (b*(8*b*c - 7*a*d)*f^2*x*Sqrt[a + b*x^2])/(5*d^3*(b*c - a
*d)*Sqrt[c + d*x^2]) + ((16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*f^2*x*Sqrt[a +
 b*x^2])/(5*c*d^3*(b*c - a*d)*Sqrt[c + d*x^2]) + ((2*b^2*c^2 + 3*a*b*c*d -
 8*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(15*c^(5/2)*d^(3/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]) + (2*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*e*f*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(1
5*c^(3/2)*d^(5/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - ((16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*f^2*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*Sqrt[c]*d^(7/2)*(b
*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*(b*c
 - 4*a*d)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*c^(3/2)*d^(3/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2]) - (2*b*(4*b*c - a*d)*e*f*Sqrt[a + b*x^2]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*Sqrt[c]*d^(5/2)*(...
```

Defintions of rubi rules used

rule 433

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(496) = 992.

Time = 10.14 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.03

method	result	size
elliptic	Expression too large to display	1072
default	Expression too large to display	4450

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*(a*c^2*d*
f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f^2+2*b*c^2*d*e*f-b*c*d^2*e^2)/c/d^6*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3-2/15*(3*a*c^2*d*f^2-a*c*d^2
*e*f-2*a*d^3*e^2-6*b*c^3*f^2+7*b*c^2*d*e*f-b*c*d^2*e^2)/c^2/d^5*x*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d^4/(a*d-b*
c)*x*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2-33*a*b*c^3*d*f^2+6*a
*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+33*b^2*c^4*f^2-16*b^2*c^3*d*e*f-2*b^2*c^2*d
^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(f*b*(2*a*d*f-3*b*c*f+2*b*d*e)/d^4
-2/15*b*(3*a*c^2*d*f^2-a*c*d^2*e*f-2*a*d^3*e^2-6*b*c^3*f^2+7*b*c^2*d*e*f-b
*c*d^2*e^2)/c^2/d^4+1/15/d^4*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*
e^2-33*a*b*c^3*d*f^2+6*a*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+33*b^2*c^4*f^2-16*b
^2*c^3*d*e*f-2*b^2*c^2*d^2*e^2)/c^3-1/15*a/d^3/c^3/(a*d-b*c)*(3*a^2*c^2*d^
2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2-33*a*b*c^3*d*f^2+6*a*b*c^2*d^2*e*f-3*a
*b*c*d^3*e^2+33*b^2*c^4*f^2-16*b^2*c^3*d*e*f-2*b^2*c^2*d^2*e^2))/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b^2*f^2/d^3-1/15/d^3
*b*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2-33*a*b*c^3*d*f^2+6*a*b
*c^2*d^2*e*f-3*a*b*c*d^3*e^2+33*b^2*c^4*f^2-16*b^2*c^3*d*e*f-2*b^2*c^2*d^2
*e^2)/(a*d-b*c)/c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1651 vs. $2(496) = 992$.

Time = 0.14 (sec) , antiderivative size = 1651, normalized size of antiderivative = 3.12

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="fricas
")
```

output
```
1/15*((((2*b^2*c^3*d^5 + 3*a*b*c^2*d^6 - 8*a^2*c*d^7)*e^2 + 2*(8*b^2*c^4*d
^4 - 3*a*b*c^3*d^5 - 2*a^2*c^2*d^6)*e*f - 3*(16*b^2*c^5*d^3 - 16*a*b*c^4*d
^4 + a^2*c^3*d^5)*f^2)*x^7 + 3*((2*b^2*c^4*d^4 + 3*a*b*c^3*d^5 - 8*a^2*c^2
*d^6)*e^2 + 2*(8*b^2*c^5*d^3 - 3*a*b*c^4*d^4 - 2*a^2*c^3*d^5)*e*f - 3*(16*
b^2*c^6*d^2 - 16*a*b*c^5*d^3 + a^2*c^4*d^4)*f^2)*x^5 + 3*((2*b^2*c^5*d^3 +
 3*a*b*c^4*d^4 - 8*a^2*c^3*d^5)*e^2 + 2*(8*b^2*c^6*d^2 - 3*a*b*c^5*d^3 - 2
*a^2*c^4*d^4)*e*f - 3*(16*b^2*c^7*d - 16*a*b*c^6*d^2 + a^2*c^5*d^3)*f^2)*x
^3 + ((2*b^2*c^6*d^2 + 3*a*b*c^5*d^3 - 8*a^2*c^4*d^4)*e^2 + 2*(8*b^2*c^7*d
 - 3*a*b*c^6*d^2 - 2*a^2*c^5*d^3)*e*f - 3*(16*b^2*c^8 - 16*a*b*c^7*d + a^2
*c^6*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*
d/(b*c)) - (((2*b^2*c^3*d^5 + 3*a*b*c^2*d^6 - 4*a^2*d^8 - (8*a^2 - a*b)*c*
d^7)*e^2 + 2*(8*b^2*c^4*d^4 - 3*a*b*c^3*d^5 - a^2*c*d^7 - 2*(a^2 - 2*a*b)*
c^2*d^6)*e*f - 3*(16*b^2*c^5*d^3 - 16*a*b*c^4*d^4 - 7*a^2*c^2*d^6 + (a^2 +
 8*a*b)*c^3*d^5)*f^2)*x^7 + 3*((2*b^2*c^4*d^4 + 3*a*b*c^3*d^5 - 4*a^2*c*d^
7 - (8*a^2 - a*b)*c^2*d^6)*e^2 + 2*(8*b^2*c^5*d^3 - 3*a*b*c^4*d^4 - a^2*c^
2*d^6 - 2*(a^2 - 2*a*b)*c^3*d^5)*e*f - 3*(16*b^2*c^6*d^2 - 16*a*b*c^5*d^3
- 7*a^2*c^3*d^5 + (a^2 + 8*a*b)*c^4*d^4)*f^2)*x^5 + 3*((2*b^2*c^5*d^3 + 3*
a*b*c^4*d^4 - 4*a^2*c^2*d^6 - (8*a^2 - a*b)*c^3*d^5)*e^2 + 2*(8*b^2*c^6*d^
2 - 3*a*b*c^5*d^3 - a^2*c^3*d^5 - 2*(a^2 - 2*a*b)*c^4*d^4)*e*f - 3*(16*b^2
*c^7*d - 16*a*b*c^6*d^2 - 7*a^2*c^4*d^4 + (a^2 + 8*a*b)*c^5*d^3)*f^2)*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**2/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x - sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**2*d**2*e*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*
*2*d**2*f**2*x**3 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*f**2*x -
3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x + 14*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b*c*d*f**2*x**3 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d
**2*e**2*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*f*x**3 + 2*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**5 - 6*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b**2*c**2*f**2*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*2*c*d*e*f*x**3 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**5 - 1
2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**
3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x*
*8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3
*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**
4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**
10),x)*a**4*c**3*d**4*f**2 - 36*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**
2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c*
*3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10
 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c*
*2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**4*c**2*d**5*f**2*x**2 - 36*int(...
```

3.67 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 737

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{(ad(de-cf)^2-bc(d^2e^2-2cdef-6c^2f^2))\,x\sqrt{a+bx^2}}{7cd^3\,(c+dx^2)^{7/2}}$$

$$-\frac{bf^2x^5\sqrt{a+bx^2}}{d\,(c+dx^2)^{7/2}}$$

$$+\frac{2(bc(d^2e^2-9cdef-27c^2f^2)+ad(3d^2e^2+cdef-4c^2f^2))\,x\sqrt{a+bx^2}}{35c^2d^3\,(c+dx^2)^{5/2}}$$

$$+\frac{(abcd(15d^2e^2-2cdef-48c^2f^2)-a^2d^2(24d^2e^2+8cdef+3c^2f^2)+2b^2c^2(3d^2e^2+8cdef+24c^2f^2))\,x\sqrt{c}}{105c^3d^3(bc-ad)\,(c+dx^2)^{3/2}}$$

$$+\frac{2(ab^2c^2d(6d^2e^2-5cdef-36c^2f^2)-a^2bcd^2(36d^2e^2+5cdef-6c^2f^2)+a^3d^3(24d^2e^2+8cdef+3c^2f^2)+}{105c^{7/2}d^{7/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx}}$$

$$-\frac{b(a^2d^2(24d^2e^2+8cdef+3c^2f^2)+b^2c^2(3d^2e^2+8cdef+24c^2f^2)-abcd(33d^2e^2+4cdef+33c^2f^2))\sqrt{a}}{105c^{5/2}d^{7/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/7*(a*d*(-c*f+d*e)^2-b*c*(-6*c^2*f^2-2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2
)/c/d^3/(d*x^2+c)^(7/2)-b*f^2*x^5*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(7/2)+2/35*(
b*c*(-27*c^2*f^2-9*c*d*e*f+d^2*e^2)+a*d*(-4*c^2*f^2+c*d*e*f+3*d^2*e^2))*x*
(b*x^2+a)^(1/2)/c^2/d^3/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-48*c^2*f^2-2*c*d*
e*f+15*d^2*e^2)-a^2*d^2*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2)+2*b^2*c^2*(24*c^2
*f^2+8*c*d*e*f+3*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d^3/(-a*d+b*c)/(d*x^2+c)^
(3/2)+2/105*(a*b^2*c^2*d*(-36*c^2*f^2-5*c*d*e*f+6*d^2*e^2)-a^2*b*c*d^2*(-6
*c^2*f^2+5*c*d*e*f+36*d^2*e^2)+a^3*d^3*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2)+b^
3*c^3*(24*c^2*f^2+8*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*
x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(7/2)/d^(7/2)/(-a*d+b*c)^
2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*b*(a^2*d^2*(3*c^2*
f^2+8*c*d*e*f+24*d^2*e^2)+b^2*c^2*(24*c^2*f^2+8*c*d*e*f+3*d^2*e^2)-a*b*c*d
*(33*c^2*f^2+4*c*d*e*f+33*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan
(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(7/2)/(-a*d+b*c)^2/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.42 (sec) , antiderivative size = 720, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)^2-6c^2(bc-ad)^2(de-cf)(bc(\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f)^2 - 6*c^2*(
b*c - a*d)^2*(d*e - c*f)*(b*c*(d*e - 8*c*f) + a*d*(3*d*e + 4*c*f))*(c + d*
x^2) + c*(b*c - a*d)*(a*b*c*d*(-15*d^2*e^2 + 2*c*d*e*f - 57*c^2*f^2) + a^2
*d^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(-6*d^2*e^2 - 16*c*d*e
*f + 57*c^2*f^2))*(c + d*x^2)^2 - 2*(a*b^2*c^2*d*(6*d^2*e^2 - 5*c*d*e*f -
36*c^2*f^2) + a^3*d^3*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + a^2*b*c*d^2*(
-36*d^2*e^2 - 5*c*d*e*f + 6*c^2*f^2) + b^3*c^3*(3*d^2*e^2 + 8*c*d*e*f + 24
*c^2*f^2))*(c + d*x^2)^3)) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^3*Sqrt[
1 + (d*x^2)/c]*(2*(a*b^2*c^2*d*(6*d^2*e^2 - 5*c*d*e*f - 36*c^2*f^2) + a^3*
d^3*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + a^2*b*c*d^2*(-36*d^2*e^2 - 5*c*
d*e*f + 6*c^2*f^2) + b^3*c^3*(3*d^2*e^2 + 8*c*d*e*f + 24*c^2*f^2))*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(a*b*c*d*(15*d^2*e^2
 - 2*c*d*e*f - 48*c^2*f^2) - a^2*d^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2)
+ 2*b^2*c^2*(3*d^2*e^2 + 8*c*d*e*f + 24*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/(105*Sqrt[b/a]*c^4*d^4*(b*c - a*d)^2*Sqrt[a + b*x
^2]*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 1213, normalized size of antiderivative = 1.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}}+\frac{2efx^2(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}}+\frac{f^2x^4(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(2bc-ad)f^2\sqrt{bx^2+a}x^3}{35cd^2(dx^2+c)^{5/2}}-\frac{f^2(bx^2+a)^{3/2}x^3}{7d(dx^2+c)^{7/2}}+\frac{(2b^2c^2+5abdc-8a^2d^2)e^2\sqrt{bx^2+a}x}{35c^3d(bc-ad)(dx^2+c)^{3/2}}-\frac{(8b^2c^2-5abdc-2a^2d^2)f^2\sqrt{bx^2+a}x}{35cd^3(bc-ad)(dx^2+c)^{3/2}}+\frac{2(8b^2c^2-abdc-4a^2d^2)ef\sqrt{bx^2+a}x}{105c^2d^2(bc-ad)(dx^2+c)^{3/2}}+\frac{2(bc+3ad)e^2\sqrt{bx^2+a}x}{35c^2d(dx^2+c)^{5/2}}-\frac{2(4bc-ad)ef\sqrt{bx^2+a}x}{35cd^2(dx^2+c)^{5/2}}-\frac{2ef(bx^2+a)^{3/2}x}{7d(dx^2+c)^{7/2}}-\frac{(bc-ad)e^2\sqrt{bx^2+a}x}{7cd(dx^2+c)^{7/2}}+\frac{2(bc-2ad)(b^2c^2+4abdc-4a^2d^2)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35c^{7/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(2bc-ad)(4b^2c^2-4abdc-a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35c^{3/2}d^{7/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(bc+ad)(8b^2c^2-13abdc+8a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{5/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b(b^2c^2-11abdc+8a^2d^2)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35c^{5/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b(8b^2c^2-11abdc+a^2d^2)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35\sqrt{c}d^{7/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4b(2b^2c^2-abdc+2a^2d^2)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{3/2}d^{5/2}(bc-ad)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
-1/7*((b*c - a*d)*e^2*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^(7/2)) - (2*e*f*
x*(a + b*x^2)^(3/2))/(7*d*(c + d*x^2)^(7/2)) - (f^2*x^3*(a + b*x^2)^(3/2))
/(7*d*(c + d*x^2)^(7/2)) + (2*(b*c + 3*a*d)*e^2*x*Sqrt[a + b*x^2])/(35*c^2
*d*(c + d*x^2)^(5/2)) - (2*(4*b*c - a*d)*e*f*x*Sqrt[a + b*x^2])/(35*c*d^2*
(c + d*x^2)^(5/2)) - (3*(2*b*c - a*d)*f^2*x^3*Sqrt[a + b*x^2])/(35*c*d^2*(
c + d*x^2)^(5/2)) + ((2*b^2*c^2 + 5*a*b*c*d - 8*a^2*d^2)*e^2*x*Sqrt[a + b*
x^2])/(35*c^3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + (2*(8*b^2*c^2 - a*b*c*d -
 4*a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(105*c^2*d^2*(b*c - a*d)*(c + d*x^2)^(3
/2)) - ((8*b^2*c^2 - 5*a*b*c*d - 2*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(35*c*d
^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (2*(b*c - 2*a*d)*(b^2*c^2 + 4*a*b*c*d
- 4*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(35*c^(7/2)*d^(3/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(b*c + a*d)*(8*b^2*c^2 - 13*a*b*c*d +
8*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(105*c^(5/2)*d^(5/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d -
a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(35*c^(3/2)*d^(7/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2]) - (b*(b^2*c^2 - 11*a*b*c*d + 8*a^2*d^2)*e^2*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(698) = 1396$.

Time = 10.53 (sec) , antiderivative size = 1474, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1474
default	Expression too large to display	7943

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*(a*c^2*d*
f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f^2+2*b*c^2*d*e*f-b*c*d^2*e^2)/c/d^7*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^4-2/35*(4*a*c^2*d*f^2-a*c*d^2
*e*f-3*a*d^3*e^2-8*b*c^3*f^2+9*b*c^2*d*e*f-b*c*d^2*e^2)/c^2/d^6*x*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/105*(3*a^2*c^2*d^2*f^2+8*a^2*c*d
^3*e*f+24*a^2*d^4*e^2-57*a*b*c^3*d*f^2+2*a*b*c^2*d^2*e*f-15*a*b*c*d^3*e^2+
57*b^2*c^4*f^2-16*b^2*c^3*d*e*f-6*b^2*c^2*d^2*e^2)/d^5/(a*d-b*c)/c^3*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+2/105*(b*d*x^2+a*d)/c^4/d^4/(
a*d-b*c)^2*x*(3*a^3*c^2*d^3*f^2+8*a^3*c*d^4*e*f+24*a^3*d^5*e^2+6*a^2*b*c^3
*d^2*f^2-5*a^2*b*c^2*d^3*e*f-36*a^2*b*c*d^4*e^2-36*a*b^2*c^4*d*f^2-5*a*b^2
*c^3*d^2*e*f+6*a*b^2*c^2*d^3*e^2+24*b^3*c^5*f^2+8*b^3*c^4*d*e*f+3*b^3*c^3*
d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(b^2*f^2/d^4+1/105*b*(3*a^2*c^2*d
^2*f^2+8*a^2*c*d^3*e*f+24*a^2*d^4*e^2-57*a*b*c^3*d*f^2+2*a*b*c^2*d^2*e*f-1
5*a*b*c*d^3*e^2+57*b^2*c^4*f^2-16*b^2*c^3*d*e*f-6*b^2*c^2*d^2*e^2)/d^4/(a*
d-b*c)/c^3+2/105/d^4/(a*d-b*c)*(3*a^3*c^2*d^3*f^2+8*a^3*c*d^4*e*f+24*a^3*d
^5*e^2+6*a^2*b*c^3*d^2*f^2-5*a^2*b*c^2*d^3*e*f-36*a^2*b*c*d^4*e^2-36*a*b^2
*c^4*d*f^2-5*a*b^2*c^3*d^2*e*f+6*a*b^2*c^2*d^3*e^2+24*b^3*c^5*f^2+8*b^3*c^
4*d*e*f+3*b^3*c^3*d^2*e^2)/c^4-2/105*a/d^3/c^4/(a*d-b*c)^2*(3*a^3*c^2*d^3*
f^2+8*a^3*c*d^4*e*f+24*a^3*d^5*e^2+6*a^2*b*c^3*d^2*f^2-5*a^2*b*c^2*d^3*e*f
-36*a^2*b*c*d^4*e^2-36*a*b^2*c^4*d*f^2-5*a*b^2*c^3*d^2*e*f+6*a*b^2*c^2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2915 vs. $2(698) = 1396$.

Time = 0.21 (sec) , antiderivative size = 2915, normalized size of antiderivative = 3.96

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="fricas
")
```

output
```
-1/105*(2*((3*(b^4*c^3*d^6 + 2*a*b^3*c^2*d^7 - 12*a^2*b^2*c*d^8 + 8*a^3*b*
d^9)*e^2 + (8*b^4*c^4*d^5 - 5*a*b^3*c^3*d^6 - 5*a^2*b^2*c^2*d^7 + 8*a^3*b*
c*d^8)*e*f + 3*(8*b^4*c^5*d^4 - 12*a*b^3*c^4*d^5 + 2*a^2*b^2*c^3*d^6 + a^3
*b*c^2*d^7)*f^2)*x^8 + 4*(3*(b^4*c^4*d^5 + 2*a*b^3*c^3*d^6 - 12*a^2*b^2*c^
2*d^7 + 8*a^3*b*c*d^8)*e^2 + (8*b^4*c^5*d^4 - 5*a*b^3*c^4*d^5 - 5*a^2*b^2*
c^3*d^6 + 8*a^3*b*c^2*d^7)*e*f + 3*(8*b^4*c^6*d^3 - 12*a*b^3*c^5*d^4 + 2*a
^2*b^2*c^4*d^5 + a^3*b*c^3*d^6)*f^2)*x^6 + 6*(3*(b^4*c^5*d^4 + 2*a*b^3*c^4
*d^5 - 12*a^2*b^2*c^3*d^6 + 8*a^3*b*c^2*d^7)*e^2 + (8*b^4*c^6*d^3 - 5*a*b^
3*c^5*d^4 - 5*a^2*b^2*c^4*d^5 + 8*a^3*b*c^3*d^6)*e*f + 3*(8*b^4*c^7*d^2 -
12*a*b^3*c^6*d^3 + 2*a^2*b^2*c^5*d^4 + a^3*b*c^4*d^5)*f^2)*x^4 + 3*(b^4*c^
7*d^2 + 2*a*b^3*c^6*d^3 - 12*a^2*b^2*c^5*d^4 + 8*a^3*b*c^4*d^5)*e^2 + (8*b
^4*c^8*d - 5*a*b^3*c^7*d^2 - 5*a^2*b^2*c^6*d^3 + 8*a^3*b*c^5*d^4)*e*f + 3*
(8*b^4*c^9 - 12*a*b^3*c^8*d + 2*a^2*b^2*c^7*d^2 + a^3*b*c^6*d^3)*f^2 + 4*(
3*(b^4*c^6*d^3 + 2*a*b^3*c^5*d^4 - 12*a^2*b^2*c^4*d^5 + 8*a^3*b*c^3*d^6)*e
^2 + (8*b^4*c^7*d^2 - 5*a*b^3*c^6*d^3 - 5*a^2*b^2*c^5*d^4 + 8*a^3*b*c^4*d^
5)*e*f + 3*(8*b^4*c^8*d - 12*a*b^3*c^7*d^2 + 2*a^2*b^2*c^6*d^3 + a^3*b*c^5
*d^4)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - ((3*(2*b^4*c^3*d^6 + (a^2*b^2 + 4*a*b^3)*c^2*d^7 - (11*a^3*b + 24
*a^2*b^2)*c*d^8 + 8*(a^4 + 2*a^3*b)*d^9)*e^2 + 2*(8*b^4*c^4*d^5 + (4*a^2*b
^2 - 5*a*b^3)*c^3*d^6 - (2*a^3*b + 5*a^2*b^2)*c^2*d^7 + 4*(a^4 + 2*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**2/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*e*f*x - 9*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*b*c**2*f**2*x - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b*c*d*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 - 3
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e**2*x - 6*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b*d**2*e*f*x**3 - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
*d**2*f**2*x**5 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x**3
+ 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x**3 + 3*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**5 + 27*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**4)/(3*a**2*c**5*d + 15*a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3
*x**4 + 30*a**2*c**2*d**4*x**6 + 15*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 -
 a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x*
*6 + 25*a*b*c**2*d**4*x**8 + 14*a*b*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2
*c**6*x**2 - 5*b**2*c**5*d*x**4 - 10*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d*
*3*x**8 - 5*b**2*c**2*d**4*x**10 - b**2*c*d**5*x**12),x)*a**4*c**4*d**4*f*
*2 + 108*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(3*a**2*c**5*d + 15*
a**2*c**4*d**2*x**2 + 30*a**2*c**3*d**3*x**4 + 30*a**2*c**2*d**4*x**6 + 15
*a**2*c*d**5*x**8 + 3*a**2*d**6*x**10 - a*b*c**6 - 2*a*b*c**5*d*x**2 + 5*a
*b*c**4*d**2*x**4 + 20*a*b*c**3*d**3*x**6 + 25*a*b*c**2*d**4*x**8 + 14*a*b
*c*d**5*x**10 + 3*a*b*d**6*x**12 - b**2*c**6*x**2 - 5*b**2*c**5*d*x**4 - 1
0*b**2*c**4*d**2*x**6 - 10*b**2*c**3*d**3*x**8 - 5*b**2*c**2*d**4*x**10...
```

3.68 $\int (a+bx^2)^{5/2} \sqrt{c+dx^2} (e+fx^2)^2 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 1122

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \left(e+fx^2\right)^2 \, dx = \text{Too large to display}$$

output
```
1/3465*(40*a^5*d^5*f^2-5*a^4*b*d^4*f*(9*c*f+44*d*e)+5*a^3*b^2*d^3*(-14*c^2
*f^2+88*c*d*e*f+99*d^2*e^2)+8*b^5*c^3*(16*c^2*f^2-44*c*d*e*f+33*d^2*e^2)+a
^2*b^3*c*d^2*(403*c^2*f^2-1452*c*d*e*f+1914*d^2*e^2)-a*b^4*c^2*d*(408*c^2*
f^2-1232*c*d*e*f+1089*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^2/d^5/(b*x^2+a)^(1/2)-
1/3465*(20*a^4*d^4*f^2-10*a^3*b*d^3*f*(2*c*f+11*d*e)-15*a^2*b^2*d^2*(-12*c
^2*f^2+44*c*d*e*f+99*d^2*e^2)+4*b^4*c^2*(16*c^2*f^2-44*c*d*e*f+33*d^2*e^2)
-2*a*b^3*c*d*(98*c^2*f^2-297*c*d*e*f+264*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b^2/d^4+1/3465*(15*a^3*d^3*f^2+10*a^2*b*d^2*f*(13*c*f+165*d*e)+
5*a*b^2*d*(-29*c^2*f^2+88*c*d*e*f+297*d^2*e^2)+3*b^3*c*(16*c^2*f^2-44*c*d*
e*f+33*d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^3+1/693*(113*a^2*
d^2*f^2+2*a*b*d*f*(12*c*f+209*d*e)+b^2*(-8*c^2*f^2+22*c*d*e*f+99*d^2*e^2))
*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2+1/99*b*f*(23*a*d*f+b*c*f+22*b*d*e
)*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+1/11*b^2*f^2*x^9*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)-1/3465*a^(1/2)*(40*a^5*d^5*f^2-5*a^4*b*d^4*f*(9*c*f+44*d*e)
+5*a^3*b^2*d^3*(-14*c^2*f^2+88*c*d*e*f+99*d^2*e^2)+8*b^5*c^3*(16*c^2*f^2-4
4*c*d*e*f+33*d^2*e^2)+a^2*b^3*c*d^2*(403*c^2*f^2-1452*c*d*e*f+1914*d^2*e^2
)-a*b^4*c^2*d*(408*c^2*f^2-1232*c*d*e*f+1089*d^2*e^2))*(d*x^2+c)^(1/2)*Ell
ipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^5/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+2/3465*a^(3/2)*(10*a^4*d^4
*f^2-5*a^3*b*d^3*f*(2*c*f+11*d*e)+30*a^2*b^2*d^2*(3*c^2*f^2-11*c*d*e*f+...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.68 (sec) , antiderivative size = 784, normalized size of antiderivative = 0.70

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\big(e+fx^2\big)^2\,dx=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(-20a^4d^4f^2+5a^3bd^3f(22de+4cf+3dfx^2)+b^4(-64c^4f^2+16c^3 \ldots}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-20*a^4*d^4*f^2 + 5*a^3*b*d^3*f*(2
2*d*e + 4*c*f + 3*d*f*x^2) + b^4*(-64*c^4*f^2 + 16*c^3*d*f*(11*e + 3*f*x^2
) - 4*c^2*d^2*(33*e^2 + 33*e*f*x^2 + 10*f^2*x^4) + c*d^3*x^2*(99*e^2 + 110
*e*f*x^2 + 35*f^2*x^4) + 5*d^4*x^4*(99*e^2 + 154*e*f*x^2 + 63*f^2*x^4)) +
5*a^2*b^2*d^2*(-36*c^2*f^2 + 2*c*d*f*(66*e + 13*f*x^2) + d^2*(297*e^2 + 33
0*e*f*x^2 + 113*f^2*x^4)) + a*b^3*d*(196*c^3*f^2 - c^2*d*f*(594*e + 145*f*
x^2) + 8*c*d^2*(66*e^2 + 55*e*f*x^2 + 15*f^2*x^4) + 5*d^3*x^2*(297*e^2 + 4
18*e*f*x^2 + 161*f^2*x^4))) - I*c*(40*a^5*d^5*f^2 - 5*a^4*b*d^4*f*(44*d*e
+ 9*c*f) + a*b^4*c^2*d*(-1089*d^2*e^2 + 1232*c*d*e*f - 408*c^2*f^2) + 5*a^
3*b^2*d^3*(99*d^2*e^2 + 88*c*d*e*f - 14*c^2*f^2) + 8*b^5*c^3*(33*d^2*e^2 -
 44*c*d*e*f + 16*c^2*f^2) + a^2*b^3*c*d^2*(1914*d^2*e^2 - 1452*c*d*e*f + 4
03*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + I*c*(b*c - a*d)*(-20*a^4*d^4*f^2 + 5*a^3*b*d^3
*f*(22*d*e + c*f) + a*b^3*c*d*(-957*d^2*e^2 + 1056*c*d*e*f - 344*c^2*f^2)
+ 8*b^4*c^2*(33*d^2*e^2 - 44*c*d*e*f + 16*c^2*f^2) + 15*a^2*b^2*d^2*(99*d^
2*e^2 - 66*c*d*e*f + 17*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3465*a^2*(b/a)^(5/2)*d^5*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 1706, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \left(e+fx^2\right)^2 \, dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(e^2 \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}+2efx^2 \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}+f^2x^4 \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{11}f^2\left(bx^2+a\right)^{5/2}\sqrt{dx^2+c}x^5+\frac{(bc+5ad)f^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{99d}-$$

$$\frac{\left(8b^2c^2-17abdc-15a^2d^2\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{693d^2}+\frac{2}{9}ef\left(bx^2+a\right)^{5/2}\sqrt{dx^2+c}x^3+$$

$$\frac{2(bc+5ad)ef\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{63d}+$$

$$\frac{\left(48b^3c^3-145ab^2dc^2+130a^2bd^2c+15a^3d^3\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{3465bd^3}-$$

$$\frac{2\left(2b^2c^2-5abdc-5a^2d^2\right)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{105d^2}+\frac{be^2\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x}{7d}-$$

$$\frac{2(2bc-5ad)e^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x}{35d}-\frac{\left(4b^2c^2-13abdc-15a^2d^2\right)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105d^2}-$$

$$\frac{4\left(16b^4c^4-49ab^3dc^3+45a^2b^2d^2c^2-5a^3bd^3c+5a^4d^4\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3465b^2d^4}+$$

$$\frac{2\left(8b^3c^3-27ab^2dc^2+30a^2bd^2c+5a^3d^3\right)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{315bd^3}+$$

$$\frac{\left(8b^3c^3-33ab^2dc^2+58a^2bd^2c+15a^3d^3\right)e^2\sqrt{bx^2+a}x}{105bd^2\sqrt{dx^2+c}}+$$

$$\frac{\left(128b^5c^5-408ab^4dc^4+403a^2b^3d^2c^3-70a^3b^2d^3c^2-45a^4bd^4c+40a^5d^5\right)f^2\sqrt{bx^2+a}x}{3465b^3d^4\sqrt{dx^2+c}}-$$

$$\frac{4\left(8b^4c^4-28ab^3dc^3+33a^2b^2d^2c^2-10a^3bd^3c+5a^4d^4\right)ef\sqrt{bx^2+a}x}{315b^2d^3\sqrt{dx^2+c}}-$$

$$\frac{\sqrt{c}\left(8b^3c^3-33ab^2dc^2+58a^2bd^2c+15a^3d^3\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-$$

$$\frac{\sqrt{c}\left(128b^5c^5-408ab^4dc^4+403a^2b^3d^2c^3-70a^3b^2d^3c^2-45a^4bd^4c+40a^5d^5\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3465b^3d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

$$\frac{4\sqrt{c}\left(8b^4c^4-28ab^3dc^3+33a^2b^2d^2c^2-10a^3bd^3c+5a^4d^4\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+$$

$$\frac{4c^{3/2}\left(b^2c^2-4abdc+15a^2d^2\right)e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+$$

$$\frac{4c^{3/2}\left(16b^4c^4-49ab^3dc^3+45a^2b^2d^2c^2-5a^3bd^3c+5a^4d^4\right)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3465b^2d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-$$

$$\frac{2c^{3/2}\left(8b^3c^3-27ab^2dc^2+30a^2bd^2c+5a^3d^3\right)ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315bd^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
((8*b^3*c^3 - 33*a*b^2*c^2*d + 58*a^2*b*c*d^2 + 15*a^3*d^3)*e^2*x*Sqrt[a +
 b*x^2])/(105*b*d^2*Sqrt[c + d*x^2]) - (4*(8*b^4*c^4 - 28*a*b^3*c^3*d + 33
*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*e*f*x*Sqrt[a + b*x^2])/(315
*b^2*d^3*Sqrt[c + d*x^2]) + ((128*b^5*c^5 - 408*a*b^4*c^4*d + 403*a^2*b^3*
c^3*d^2 - 70*a^3*b^2*c^2*d^3 - 45*a^4*b*c*d^4 + 40*a^5*d^5)*f^2*x*Sqrt[a +
 b*x^2])/(3465*b^3*d^4*Sqrt[c + d*x^2]) - ((4*b^2*c^2 - 13*a*b*c*d - 15*a^
2*d^2)*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^2) + (2*(8*b^3*c^3 -
27*a*b^2*c^2*d + 30*a^2*b*c*d^2 + 5*a^3*d^3)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(315*b*d^3) - (4*(16*b^4*c^4 - 49*a*b^3*c^3*d + 45*a^2*b^2*c^2*d
^2 - 5*a^3*b*c*d^3 + 5*a^4*d^4)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(34
65*b^2*d^4) - (2*(2*b^2*c^2 - 5*a*b*c*d - 5*a^2*d^2)*e*f*x^3*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(105*d^2) + ((48*b^3*c^3 - 145*a*b^2*c^2*d + 130*a^2*b
*c*d^2 + 15*a^3*d^3)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3465*b*d^3)
 - ((8*b^2*c^2 - 17*a*b*c*d - 15*a^2*d^2)*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(693*d^2) - (2*(2*b*c - 5*a*d)*e^2*x*(a + b*x^2)^(3/2)*Sqrt[c + d
*x^2])/(35*d) + (2*(b*c + 5*a*d)*e*f*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]
)/(63*d) + ((b*c + 5*a*d)*f^2*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(99*d
) + (2*e*f*x^3*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/9 + (f^2*x^5*(a + b*x^2)
^(5/2)*Sqrt[c + d*x^2])/11 + (b*e^2*x*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))
/(7*d) - (Sqrt[c]*(8*b^3*c^3 - 33*a*b^2*c^2*d + 58*a^2*b*c*d^2 + 15*a^3...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.90 (sec) , antiderivative size = 2046, normalized size of antiderivative = 1.82

method	result	size
risch	Expression too large to display	2046
elliptic	Expression too large to display	2349
default	Expression too large to display	3341

input

```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/3465/b^2*x*(-315*b^4*d^4*f^2*x^8-805*a*b^3*d^4*f^2*x^6-35*b^4*c*d^3*f^2
*x^6-770*b^4*d^4*e*f*x^6-565*a^2*b^2*d^4*f^2*x^4-120*a*b^3*c*d^3*f^2*x^4-2
090*a*b^3*d^4*e*f*x^4+40*b^4*c^2*d^2*f^2*x^4-110*b^4*c*d^3*e*f*x^4-495*b^4
*d^4*e^2*x^4-15*a^3*b*d^4*f^2*x^2-130*a^2*b^2*c*d^3*f^2*x^2-1650*a^2*b^2*d
^4*e*f*x^2+145*a*b^3*c^2*d^2*f^2*x^2-440*a*b^3*c*d^3*e*f*x^2-1485*a*b^3*d^
4*e^2*x^2-48*b^4*c^3*d*f^2*x^2+132*b^4*c^2*d^2*e*f*x^2-99*b^4*c*d^3*e^2*x^
2+20*a^4*d^4*f^2-20*a^3*b*c*d^3*f^2-110*a^3*b*d^4*e*f+180*a^2*b^2*c^2*d^2*
f^2-660*a^2*b^2*c*d^3*e*f-1485*a^2*b^2*d^4*e^2-196*a*b^3*c^3*d*f^2+594*a*b
^3*c^2*d^2*e*f-528*a*b^3*c*d^3*e^2+64*b^4*c^4*f^2-176*b^4*c^3*d*e*f+132*b^
4*c^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4+1/3465/b^2/d^4*(-(40*a^
5*d^5*f^2-45*a^4*b*c*d^4*f^2-220*a^4*b*d^5*e*f-70*a^3*b^2*c^2*d^3*f^2+440*
a^3*b^2*c*d^4*e*f+495*a^3*b^2*d^5*e^2+403*a^2*b^3*c^3*d^2*f^2-1452*a^2*b^3
*c^2*d^3*e*f+1914*a^2*b^3*c*d^4*e^2-408*a*b^4*c^4*d*f^2+1232*a*b^4*c^3*d^2
*e*f-1089*a*b^4*c^2*d^3*e^2+128*b^5*c^5*f^2-352*b^5*c^4*d*e*f+264*b^5*c^3*
d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+64*a*b^4*c^5*f^2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+20*a^5*c*d^4*f^
2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1200, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
-1/3465*((33*(8*b^5*c^4*d^2 - 33*a*b^4*c^3*d^3 + 58*a^2*b^3*c^2*d^4 + 15*a
^3*b^2*c*d^5)*e^2 - 44*(8*b^5*c^5*d - 28*a*b^4*c^4*d^2 + 33*a^2*b^3*c^3*d^
3 - 10*a^3*b^2*c^2*d^4 + 5*a^4*b*c*d^5)*e*f + (128*b^5*c^6 - 408*a*b^4*c^5
*d + 403*a^2*b^3*c^4*d^2 - 70*a^3*b^2*c^3*d^3 - 45*a^4*b*c^2*d^4 + 40*a^5*
c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (33*(8*b^5*c^4*d^2 - 33*a*b^4*c^3*d^3 + 60*a^3*b^2*d^6 + 2*(29*a^2*
b^3 + 2*a*b^4)*c^2*d^4 + (15*a^3*b^2 - 16*a^2*b^3)*c*d^5)*e^2 - 22*(16*b^5
*c^5*d - 56*a*b^4*c^4*d^2 + 5*a^4*b*d^6 + 2*(33*a^2*b^3 + 4*a*b^4)*c^3*d^3
 - (20*a^3*b^2 + 27*a^2*b^3)*c^2*d^4 + 10*(a^4*b + 3*a^3*b^2)*c*d^5)*e*f +
 (128*b^5*c^6 - 408*a*b^4*c^5*d + 20*a^5*d^6 + (403*a^2*b^3 + 64*a*b^4)*c^
4*d^2 - 14*(5*a^3*b^2 + 14*a^2*b^3)*c^3*d^3 - 45*(a^4*b - 4*a^3*b^2)*c^2*d
^4 + 20*(2*a^5 - a^4*b)*c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcs
in(sqrt(-c/d)/x), a*d/(b*c)) - (315*b^5*d^6*f^2*x^10 + 35*(22*b^5*d^6*e*f
+ (b^5*c*d^5 + 23*a*b^4*d^6)*f^2)*x^8 + 5*(99*b^5*d^6*e^2 + 22*(b^5*c*d^5
+ 19*a*b^4*d^6)*e*f - (8*b^5*c^2*d^4 - 24*a*b^4*c*d^5 - 113*a^2*b^3*d^6)*f
^2)*x^6 + (99*(b^5*c*d^5 + 15*a*b^4*d^6)*e^2 - 22*(6*b^5*c^2*d^4 - 20*a*b^
4*c*d^5 - 75*a^2*b^3*d^6)*e*f + (48*b^5*c^3*d^3 - 145*a*b^4*c^2*d^4 + 130*
a^2*b^3*c*d^5 + 15*a^3*b^2*d^6)*f^2)*x^4 + 33*(8*b^5*c^3*d^3 - 33*a*b^4*c^
2*d^4 + 58*a^2*b^3*c*d^5 + 15*a^3*b^2*d^6)*e^2 - 44*(8*b^5*c^4*d^2 - 28*a*
b^4*c^3*d^3 + 33*a^2*b^3*c^2*d^4 - 10*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*e*f ...
```

Sympy [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \left(b\,x^2+a\right)^{5/2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
( - 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*d**4*f**2*x + 20*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**3*b*c*d**3*f**2*x + 110*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a**3*b*d**4*e*f*x + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*
d**4*f**2*x**3 - 180*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2
*f**2*x + 660*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*e*f*x + 1
30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*f**2*x**3 + 1485*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*e**2*x + 1650*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a**2*b**2*d**4*e*f*x**3 + 565*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a**2*b**2*d**4*f**2*x**5 + 196*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b**3*c**3*d*f**2*x - 594*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*
d**2*e*f*x - 145*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*f**2*x
**3 + 528*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e**2*x + 440*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e*f*x**3 + 120*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*b**3*c*d**3*f**2*x**5 + 1485*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*b**3*d**4*e**2*x**3 + 2090*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b**3*d**4*e*f*x**5 + 805*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*f
**2*x**7 - 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**4*f**2*x + 176*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**3*d*e*f*x + 48*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b**4*c**3*d*f**2*x**3 - 132*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*b**4*c**2*d**2*e**2*x - 132*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c*...
```

3.69 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 874

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(10a^4d^4f^2-5a^3bd^3f(18de-5cf)-8b^4c^2(21d^2e^2-36cdef+16c^2f^2)-3a^2b^2d^2(161d^2e^2-206cdef+8}{315bd^5\sqrt{a+bx^2}}$$

$$+\frac{(5a^3d^3f^2+15a^2bd^2f(18de-7cf)-4b^3c(21d^2e^2-36cdef+16c^2f^2)+3ab^2d(77d^2e^2-122cdef+52c^2}{315bd^4}$$

$$+\frac{(75a^2d^2f^2+5abdf(54de-23cf)+3b^2(21d^2e^2-36cdef+16c^2f^2))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315d^3}$$

$$+\frac{bf(18bde-8bcf+19adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d^2}+\frac{b^2f^2x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{9d}$$

$$+\frac{\sqrt{a}(10a^4d^4f^2-5a^3bd^3f(18de-5cf)-8b^4c^2(21d^2e^2-36cdef+16c^2f^2)-3a^2b^2d^2(161d^2e^2-206cdef}{315b^{3/2}d^5\sqrt{a+bx^2}\sqrt{\frac{a(c}{c(a}}}$$

$$-\frac{a^{3/2}(5a^3cd^3f^2-15a^2bd^2(21d^2e^2-18cdef+7c^2f^2)-4b^3c^2(21d^2e^2-36cdef+16c^2f^2)+3ab^2cd(77d^2e}{315b^{3/2}cd^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(10*a^4*d^4*f^2-5*a^3*b*d^3*f*(-5*c*f+18*d*e)-8*b^4*c^2*(16*c^2*f^2
-36*c*d*e*f+21*d^2*e^2)-3*a^2*b^2*d^2*(81*c^2*f^2-206*c*d*e*f+161*d^2*e^2)
+a*b^3*c*d*(328*c^2*f^2-768*c*d*e*f+483*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d^5/
(b*x^2+a)^(1/2)+1/315*(5*a^3*d^3*f^2+15*a^2*b*d^2*f*(-7*c*f+18*d*e)-4*b^3*
c*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2)+3*a*b^2*d*(52*c^2*f^2-122*c*d*e*f+77*
d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^4+1/315*(75*a^2*d^2*f^2+5*
a*b*d*f*(-23*c*f+54*d*e)+3*b^2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2))*x^3*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3+1/63*b*f*(19*a*d*f-8*b*c*f+18*b*d*e)*x^5*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2+1/9*b^2*f^2*x^7*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/d+1/315*a^(1/2)*(10*a^4*d^4*f^2-5*a^3*b*d^3*f*(-5*c*f+18*d*e)-8*
b^4*c^2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2)-3*a^2*b^2*d^2*(81*c^2*f^2-206*c
*d*e*f+161*d^2*e^2)+a*b^3*c*d*(328*c^2*f^2-768*c*d*e*f+483*d^2*e^2))*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2)
)/b^(3/2)/d^5/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2
)*(5*a^3*c*d^3*f^2-15*a^2*b*d^2*(7*c^2*f^2-18*c*d*e*f+21*d^2*e^2)-4*b^3*c^
2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2)+3*a*b^2*c*d*(52*c^2*f^2-122*c*d*e*f+7
7*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a
*d/b/c)^(1/2))/b^(3/2)/c/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.75 (sec) , antiderivative size = 603, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(5a^3d^3f^2+15a^2bd^2f(18de-7cf+5dfx^2)+b^3(}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3*f^2 + 15*a^2*b*d^2*f*(18
*d*e - 7*c*f + 5*d*f*x^2) + b^3*(-64*c^3*f^2 + 48*c^2*d*f*(3*e + f*x^2) -
4*c*d^2*(21*e^2 + 27*e*f*x^2 + 10*f^2*x^4) + d^3*x^2*(63*e^2 + 90*e*f*x^2
+ 35*f^2*x^4)) + a*b^2*d*(156*c^2*f^2 - c*d*f*(366*e + 115*f*x^2) + d^2*(2
31*e^2 + 270*e*f*x^2 + 95*f^2*x^4))) + I*c*(10*a^4*d^4*f^2 + 5*a^3*b*d^3*f
*(-18*d*e + 5*c*f) - 8*b^4*c^2*(21*d^2*e^2 - 36*c*d*e*f + 16*c^2*f^2) - 3*
a^2*b^2*d^2*(161*d^2*e^2 - 206*c*d*e*f + 81*c^2*f^2) + a*b^3*c*d*(483*d^2*
e^2 - 768*c*d*e*f + 328*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(b*c - a*d)*(5*a^3*c*d^
3*f^2 + 45*a^2*b*d^2*(7*d^2*e^2 - 8*c*d*e*f + 3*c^2*f^2) + 8*b^3*c^2*(21*d
^2*e^2 - 36*c*d*e*f + 16*c^2*f^2) - 3*a*b^2*c*d*(133*d^2*e^2 - 208*c*d*e*f
 + 88*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 1379, normalized size of antiderivative = 1.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}+\frac{2efx^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}+\frac{f^2x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bf^2(bx^2+a)^{3/2}\sqrt{dx^2+c}x^5}{9d} - \frac{4b(2bc-3ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{63d^2} + \frac{2bef(bx^2+a)^{3/2}\sqrt{dx^2+c}x^3}{7d} + \frac{(48b^2c^2-115abdc+75a^2d^2)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{315d^3} - \frac{4b(3bc-5ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35d^2} + \frac{be^2(bx^2+a)^{3/2}\sqrt{dx^2+c}x}{5d} - \frac{4b(bc-2ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15d^2} - \frac{(64b^3c^3-156ab^2dc^2+105a^2bd^2c-5a^3d^3)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{315bd^4} + \frac{2(24b^2c^2-61abdc+45a^2d^2)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105d^3} + \frac{(8b^2c^2-23abdc+23a^2d^2)e^2\sqrt{bx^2+a}x}{15d^2\sqrt{dx^2+c}} + \frac{(128b^4c^4-328ab^3dc^3+243a^2b^2d^2c^2-25a^3bd^3c-10a^4d^4)f^2\sqrt{bx^2+a}x}{315b^2d^4\sqrt{dx^2+c}} - \frac{2(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3)ef\sqrt{bx^2+a}x}{105bd^3\sqrt{dx^2+c}} - \frac{\sqrt{c}(8b^2c^2-23abdc+23a^2d^2)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{\sqrt{c}(128b^4c^4-328ab^3dc^3+243a^2b^2d^2c^2-25a^3bd^3c-10a^4d^4)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{2\sqrt{c}(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{\sqrt{c}(4b^2c^2-11abdc+15a^2d^2)e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{c^{3/2}(64b^3c^3-156ab^2dc^2+105a^2bd^2c-5a^3d^3)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{2c^{3/2}(24b^2c^2-61abdc+45a^2d^2)ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(15*d^2*Sqrt
[c + d*x^2]) - (2*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3
*d^3)*e*f*x*Sqrt[a + b*x^2])/(105*b*d^3*Sqrt[c + d*x^2]) + ((128*b^4*c^4 -
 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*c*d^3 - 10*a^4*d^4)*f^2*
x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^2]) - (4*b*(b*c - 2*a*d)*e^2*
x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) + (2*(24*b^2*c^2 - 61*a*b*c*d
+ 45*a^2*d^2)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - ((64*b^3*
c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*f^2*x*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])/(315*b*d^4) - (4*b*(3*b*c - 5*a*d)*e*f*x^3*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(35*d^2) + ((48*b^2*c^2 - 115*a*b*c*d + 75*a^2*d^2)*f^
2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*d^3) - (4*b*(2*b*c - 3*a*d)*f^
2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(63*d^2) + (b*e^2*x*(a + b*x^2)^(3/
2)*Sqrt[c + d*x^2])/(5*d) + (2*b*e*f*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]
)/(7*d) + (b*f^2*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*d) - (Sqrt[c]*(
8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(48*b^3*c^3 - 128*a*b^2*c^2*
d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.84 (sec) , antiderivative size = 1214, normalized size of antiderivative = 1.39

method	result	size
elliptic	Expression too large to display	1214
risch	Expression too large to display	1591
default	Expression too large to display	2551

input

```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*b^2/d*f^2
*x^7*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(3*f^2*a*b^2+2*e*f*b^3-1/9*b^
2/d*f^2*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(3*
a^2*b*f^2+6*a*b^2*e*f+e^2*b^3-7/9*a*b^2*c/d*f^2-1/7*(3*f^2*a*b^2+2*e*f*b^3
-1/9*b^2/d*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)+1/3*(a^3*f^2+6*a^2*b*e*f+3*a*b^2*e^2-5/7*(3*f^2*a*b^2+2
*e*f*b^3-1/9*b^2/d*f^2*(8*a*d+8*b*c))/b/d*a*c-1/5*(3*a^2*b*f^2+6*a*b^2*e*f
+e^2*b^3-7/9*a*b^2*c/d*f^2-1/7*(3*f^2*a*b^2+2*e*f*b^3-1/9*b^2/d*f^2*(8*a*d
+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)+(a^3*e^2-1/3*(a^3*f^2+6*a^2*b*e*f+3*a*b^2*e^2-5/7*(3*f^2*a*
b^2+2*e*f*b^3-1/9*b^2/d*f^2*(8*a*d+8*b*c))/b/d*a*c-1/5*(3*a^2*b*f^2+6*a*b^
2*e*f+e^2*b^3-7/9*a*b^2*c/d*f^2-1/7*(3*f^2*a*b^2+2*e*f*b^3-1/9*b^2/d*f^2*(
8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^3*e*f+3*a^2*b*e^2-3/
5*(3*a^2*b*f^2+6*a*b^2*e*f+e^2*b^3-7/9*a*b^2*c/d*f^2-1/7*(3*f^2*a*b^2+2*e*
f*b^3-1/9*b^2/d*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a^3*f^2
+6*a^2*b*e*f+3*a*b^2*e^2-5/7*(3*f^2*a*b^2+2*e*f*b^3-1/9*b^2/d*f^2*(8*a*d+8
*b*c))/b/d*a*c-1/5*(3*a^2*b*f^2+6*a*b^2*e*f+e^2*b^3-7/9*a*b^2*c/d*f^2-1/7*
(3*f^2*a*b^2+2*e*f*b^3-1/9*b^2/d*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{5/2} (e+fx^2)^2}{\sqrt{c+dx^2}} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/315*((21*(8*b^4*c^4*d^2 - 23*a*b^3*c^3*d^3 + 23*a^2*b^2*c^2*d^4)*e^2 -
6*(48*b^4*c^5*d - 128*a*b^3*c^4*d^2 + 103*a^2*b^2*c^3*d^3 - 15*a^3*b*c^2*d
^4)*e*f + (128*b^4*c^6 - 328*a*b^3*c^5*d + 243*a^2*b^2*c^4*d^2 - 25*a^3*b*
c^3*d^3 - 10*a^4*c^2*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) - (21*(8*b^4*c^4*d^2 - 23*a*b^3*c^3*d^3 - 11*a^2*b
^2*c*d^5 + 15*a^3*b*d^6 + (23*a^2*b^2 + 4*a*b^3)*c^2*d^4)*e^2 - 6*(48*b^4*
c^5*d - 128*a*b^3*c^4*d^2 + 45*a^3*b*c*d^5 + (103*a^2*b^2 + 24*a*b^3)*c^3*
d^3 - (15*a^3*b + 61*a^2*b^2)*c^2*d^4)*e*f + (128*b^4*c^6 - 328*a*b^3*c^5*
d - 5*a^4*c*d^5 + (243*a^2*b^2 + 64*a*b^3)*c^4*d^2 - (25*a^3*b + 156*a^2*b
^2)*c^3*d^3 - 5*(2*a^4 - 21*a^3*b)*c^2*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*c*d^5*f^2*x^8 + 5*(18*
b^4*c*d^5*e*f - (8*b^4*c^2*d^4 - 19*a*b^3*c*d^5)*f^2)*x^6 + (63*b^4*c*d^5*
e^2 - 54*(2*b^4*c^2*d^4 - 5*a*b^3*c*d^5)*e*f + (48*b^4*c^3*d^3 - 115*a*b^3
*c^2*d^4 + 75*a^2*b^2*c*d^5)*f^2)*x^4 + 21*(8*b^4*c^3*d^3 - 23*a*b^3*c^2*d
^4 + 23*a^2*b^2*c*d^5)*e^2 - 6*(48*b^4*c^4*d^2 - 128*a*b^3*c^3*d^3 + 103*a
^2*b^2*c^2*d^4 - 15*a^3*b*c*d^5)*e*f + (128*b^4*c^5*d - 328*a*b^3*c^4*d^2
+ 243*a^2*b^2*c^3*d^3 - 25*a^3*b*c^2*d^4 - 10*a^4*c*d^5)*f^2 - (21*(4*b^4*
c^2*d^4 - 11*a*b^3*c*d^5)*e^2 - 6*(24*b^4*c^3*d^3 - 61*a*b^3*c^2*d^4 + 45*
a^2*b^2*c*d^5)*e*f + (64*b^4*c^4*d^2 - 156*a*b^3*c^3*d^3 + 105*a^2*b^2*c^2
*d^4 - 5*a^3*b*c*d^5)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output

```
(5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x - 105*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x + 270*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a**2*b*d**3*e*f*x + 75*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**
3*f**2*x**3 + 156*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x -
 366*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x - 115*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 231*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b**2*d**3*e**2*x + 270*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b**2*d**3*e*f*x**3 + 95*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f*
*2*x**5 - 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x + 144*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*e*f*x + 48*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b**3*c**2*d*f**2*x**3 - 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c*d**2*e**2*x - 108*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f
*x**3 - 40*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*b**3*d**3*f**2*x**7 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 - 25*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*
*3*b*c*d**3*f**2 + 90*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c +
a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e*f + 243*int((sqrt(c + ...
```

3.70 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 653

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(15a^3d^3f^2+a^2bd^2f(322de-219cf)-4b^3c(35d^2e^2-84cdef+48c^2f^2)+}{105bd^4\sqrt{c+dx^2}}$$
$$+\frac{(45a^2d^2f^2+abdf(154de-93cf)+b^2(35d^2e^2-84cdef+48c^2f^2))\,x^3\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}}$$
$$+\frac{bf(14bde-8bcf+15adf)x^5\sqrt{a+bx^2}}{35d^2\sqrt{c+dx^2}}+\frac{b^2f^2x^7\sqrt{a+bx^2}}{7d\sqrt{c+dx^2}}$$
$$-\frac{(15a^3cd^3f^2-8b^3c^2(35d^2e^2-84cdef+48c^2f^2)-a^2bd^2(105d^2e^2-532cdef+369c^2f^2)+ab^2cd(455d^2e^2}{105b\sqrt{c}d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{2\sqrt{c}(15a^2d^2f(7de-5cf)-2b^2c(35d^2e^2-84cdef+48c^2f^2)+abd(105d^2e^2-287cdef+174c^2f^2))\sqrt{a-}}{105d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/105*(15*a^3*d^3*f^2+a^2*b*d^2*f*(-219*c*f+322*d*e)-4*b^3*c*(48*c^2*f^2-8
4*c*d*e*f+35*d^2*e^2)+a*b^2*d*(396*c^2*f^2-658*c*d*e*f+245*d^2*e^2))*x*(b*
x^2+a)^(1/2)/b/d^4/(d*x^2+c)^(1/2)+1/105*(45*a^2*d^2*f^2+a*b*d*f*(-93*c*f+
154*d*e)+b^2*(48*c^2*f^2-84*c*d*e*f+35*d^2*e^2))*x^3*(b*x^2+a)^(1/2)/d^3/(
d*x^2+c)^(1/2)+1/35*b*f*(15*a*d*f-8*b*c*f+14*b*d*e)*x^5*(b*x^2+a)^(1/2)/d^
2/(d*x^2+c)^(1/2)+1/7*b^2*f^2*x^7*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)-1/105*
(15*a^3*c*d^3*f^2-8*b^3*c^2*(48*c^2*f^2-84*c*d*e*f+35*d^2*e^2)-a^2*b*d^2*(
369*c^2*f^2-532*c*d*e*f+105*d^2*e^2)+a*b^2*c*d*(744*c^2*f^2-1232*c*d*e*f+4
55*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))/b/c^(1/2)/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+2/105*c^(1/2)*(15*a^2*d^2*f*(-5*c*f+7*d*e)-2*b^2*c*(48*c^2*f^2
-84*c*d*e*f+35*d^2*e^2)+a*b*d*(174*c^2*f^2-287*c*d*e*f+105*d^2*e^2))*(b*x^
2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^
(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.70 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15a^2d^2(7d^2e^2+10c^2f^2+cdf(-14e+3fx^2))+b^2c(192c^3\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*a^2*d^2*(7*d^2*e^2 + 10*c^2*f^2 + c*d*f*(-1
4*e + 3*f*x^2)) + b^2*c*(192*c^3*f^2 + 48*c^2*d*f*(-7*e + f*x^2) + 4*c*d^2
*(35*e^2 - 21*e*f*x^2 - 6*f^2*x^4) + d^3*x^2*(35*e^2 + 42*e*f*x^2 + 15*f^2
*x^4)) + a*b*c*d*(-348*c^2*f^2 + c*d*f*(574*e - 93*f*x^2) + d^2*(-210*e^2
+ 154*e*f*x^2 + 45*f^2*x^4))) - I*c*(15*a^3*c*d^3*f^2 + a^2*b*d^2*(-105*d^
2*e^2 + 532*c*d*e*f - 369*c^2*f^2) - 8*b^3*c^2*(35*d^2*e^2 - 84*c*d*e*f +
48*c^2*f^2) + a*b^2*c*d*(455*d^2*e^2 - 1232*c*d*e*f + 744*c^2*f^2))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] + I*c*(-(b*c) + a*d)*(15*a^2*d^2*f*(-14*d*e + 11*c*f) + a*b*d*(-315
*d^2*e^2 + 896*c*d*e*f - 552*c^2*f^2) + 8*b^2*c*(35*d^2*e^2 - 84*c*d*e*f +
 48*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[b/a]*c*d^5*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 1231, normalized size of antiderivative = 1.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}+\frac{2efx^2(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}+\frac{f^2x^4(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8bf^2(bx^2+a)^{3/2}\sqrt{dx^2+c}x^3}{7d^2}-\frac{3b(16bc-15ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35d^3}-\frac{f^2(bx^2+a)^{5/2}x^3}{d\sqrt{dx^2+c}}+\frac{12bef(bx^2+a)^{3/2}\sqrt{dx^2+c}x}{5d^2}+\frac{b(4bc-3ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3cd^2}+\frac{2(32b^2c^2-58abdc+25a^2d^2)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{35d^4}-\frac{2b(24bc-23ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15d^3}-\frac{2ef(bx^2+a)^{5/2}x}{d\sqrt{dx^2+c}}-\frac{(bc-ad)e^2(bx^2+a)^{3/2}x}{cd\sqrt{dx^2+c}}-\frac{(8b^2c^2-13abdc+3a^2d^2)e^2\sqrt{bx^2+a}x}{3cd^2\sqrt{dx^2+c}}-\frac{(128b^3c^3-248ab^2dc^2+123a^2bd^2c-5a^3d^3)f^2\sqrt{bx^2+a}x}{35bd^4\sqrt{dx^2+c}}+\frac{4(24b^2c^2-44abdc+19a^2d^2)ef\sqrt{bx^2+a}x}{15d^3\sqrt{dx^2+c}}+\frac{(8b^2c^2-13abdc+3a^2d^2)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}(128b^3c^3-248ab^2dc^2+123a^2bd^2c-5a^3d^3)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4\sqrt{c}(24b^2c^2-44abdc+19a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b\sqrt{c}(2bc-3ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}(32b^2c^2-58abdc+25a^2d^2)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(24b^2c^2-41abdc+15a^2d^2)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
-1/3*((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(c*d^2*S
qrt[c + d*x^2]) + (4*(24*b^2*c^2 - 44*a*b*c*d + 19*a^2*d^2)*e*f*x*Sqrt[a +
 b*x^2])/(15*d^3*Sqrt[c + d*x^2]) - ((128*b^3*c^3 - 248*a*b^2*c^2*d + 123*
a^2*b*c*d^2 - 5*a^3*d^3)*f^2*x*Sqrt[a + b*x^2])/(35*b*d^4*Sqrt[c + d*x^2])
 - ((b*c - a*d)*e^2*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) - (2*e*f*x*
(a + b*x^2)^(5/2))/(d*Sqrt[c + d*x^2]) - (f^2*x^3*(a + b*x^2)^(5/2))/(d*Sq
rt[c + d*x^2]) + (b*(4*b*c - 3*a*d)*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(3*c*d^2) - (2*b*(24*b*c - 23*a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(15*d^3) + (2*(32*b^2*c^2 - 58*a*b*c*d + 25*a^2*d^2)*f^2*x*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(35*d^4) - (3*b*(16*b*c - 15*a*d)*f^2*x^3*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(35*d^3) + (12*b*e*f*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^
2])/(5*d^2) + (8*b*f^2*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*d^2) + ((
8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*Sqrt[c]*(24*b^2*c^2 - 44*a*b*c
*d + 19*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(15*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (Sqrt[c]*(128*b^3*c^3 - 248*a*b^2*c^2*d + 123*a^2*b*c*d^2 -
 5*a^3*d^3)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(35*b*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 28.41 (sec) , antiderivative size = 1083, normalized size of antiderivative = 1.66

method	result	size
risch	Expression too large to display	1083
elliptic	Expression too large to display	1639
default	Expression too large to display	2029

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(15*b^2*d^2*f^2*x^4+45*a*b*d^2*f^2*x^2-39*b^2*c*d*f^2*x^2+42*b^2*d
^2*e*f*x^2+45*a^2*d^2*f^2-138*a*b*c*d*f^2+154*a*b*d^2*e*f+87*b^2*c^2*f^2-1
26*b^2*c*d*e*f+35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4+1/105/d
^4*(-(15*a^3*d^3*f^2-264*a^2*b*c*d^2*f^2+322*a^2*b*d^3*e*f+534*a*b^2*c^2*d
*f^2-812*a*b^2*c*d^2*e*f+245*a*b^2*d^3*e^2-279*b^3*c^3*f^2+462*b^3*c^2*d*e
*f-175*b^3*c*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+105*(a^
3*c^2*d^3*f^2-2*a^3*c*d^4*e*f+a^3*d^5*e^2-3*a^2*b*c^3*d^2*f^2+6*a^2*b*c^2*
d^3*e*f-3*a^2*b*c*d^4*e^2+3*a*b^2*c^4*d*f^2-6*a*b^2*c^3*d^2*e*f+3*a*b^2*c^
2*d^3*e^2-b^3*c^5*f^2+2*b^3*c^4*d*e*f-b^3*c^3*d^2*e^2)/d*((b*d*x^2+a*d)/c/
(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/(a*d-b*c)/c*a*d)/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-(150*a^3*c*d^3*f^2-210*a^3*d^4*e*f-453
*a^2*b*c^2*d^2*f^2+784*a^2*b*c*d^3*e*f-315*a^2*b*d^4*e^2+402*a*b^2*c^3*d*f
^2-756*a*b^2*c^2*d^2*e*f+350*a*b^2*c*d^3*e^2-105*b^3*c^4*f^2+210*b^3*c^3*d
*e*f-105*b^3*c^2*d^2*e^2)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1184, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
1/105*(((35*(8*b^3*c^3*d^3 - 13*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5)*e^2 - 28*(2
4*b^3*c^4*d^2 - 44*a*b^2*c^3*d^3 + 19*a^2*b*c^2*d^4)*e*f + 3*(128*b^3*c^5*
d - 248*a*b^2*c^4*d^2 + 123*a^2*b*c^3*d^3 - 5*a^3*c^2*d^4)*f^2)*x^3 + (35*
(8*b^3*c^4*d^2 - 13*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4)*e^2 - 28*(24*b^3*c^5*
d - 44*a*b^2*c^4*d^2 + 19*a^2*b*c^3*d^3)*e*f + 3*(128*b^3*c^6 - 248*a*b^2*
c^5*d + 123*a^2*b*c^4*d^2 - 5*a^3*c^3*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*el
liptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((35*(8*b^3*c^3*d^3 - 13*a*b^2
*c^2*d^4 - 6*a^2*b*d^6 + (3*a^2*b + 4*a*b^2)*c*d^5)*e^2 - 14*(48*b^3*c^4*d
^2 - 88*a*b^2*c^3*d^3 - 41*a^2*b*c*d^5 + 15*a^3*d^6 + 2*(19*a^2*b + 12*a*b
^2)*c^2*d^4)*e*f + 3*(128*b^3*c^5*d - 248*a*b^2*c^4*d^2 + 50*a^3*c*d^5 + (
123*a^2*b + 64*a*b^2)*c^3*d^3 - (5*a^3 + 116*a^2*b)*c^2*d^4)*f^2)*x^3 + (3
5*(8*b^3*c^4*d^2 - 13*a*b^2*c^3*d^3 - 6*a^2*b*c*d^5 + (3*a^2*b + 4*a*b^2)*
c^2*d^4)*e^2 - 14*(48*b^3*c^5*d - 88*a*b^2*c^4*d^2 - 41*a^2*b*c^2*d^4 + 15
*a^3*c*d^5 + 2*(19*a^2*b + 12*a*b^2)*c^3*d^3)*e*f + 3*(128*b^3*c^6 - 248*a
*b^2*c^5*d + 50*a^3*c^2*d^4 + (123*a^2*b + 64*a*b^2)*c^4*d^2 - (5*a^3 + 11
6*a^2*b)*c^3*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d
)/x), a*d/(b*c)) + (15*b^3*c*d^5*f^2*x^8 + 3*(14*b^3*c*d^5*e*f - (8*b^3*c^
2*d^4 - 15*a*b^2*c*d^5)*f^2)*x^6 + (35*b^3*c*d^5*e^2 - 14*(6*b^3*c^2*d^4 -
 11*a*b^2*c*d^5)*e*f + 3*(16*b^3*c^3*d^3 - 31*a*b^2*c^2*d^4 + 15*a^2*b*c*d
^5)*f^2)*x^4 - 35*(8*b^3*c^3*d^3 - 13*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5)*e^...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output

```
( - 135*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c*d**2*f**2*x + 210*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*a**3*d**3*e*f*x + 279*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a**2*b*c**2*d*f**2*x - 462*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2
*b*c*d**2*e*f*x + 90*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*
x**3 + 315*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*e**2*x - 144*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*f**2*x + 252*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*b**2*c**2*d*e*f*x - 186*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b**2*c**2*d*f**2*x**3 - 105*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*
d**2*e**2*x + 308*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x**3
 + 90*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**5 + 96*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x**3 - 168*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b**3*c**2*d*e*f*x**3 - 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*b**3*c**2*d*f**2*x**5 + 70*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*
e**2*x**3 + 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**5 + 30
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**7 + 165*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*c**2*d**3*f**2 - 210*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2
*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*c*d**4*e*f + 1
65*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 ...
```

3.71 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 606

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx =\ & -\frac{(10abcd(de-cf)^2-5a^2d^2(de-cf)^2-b^2c^2(5d^2e^2-10cdef+8c^2f^2))\,x\sqrt{a+bx^2}}{15cd^4(c+dx^2)^{3/2}}\\
&+\frac{b^2f^2x^7\sqrt{a+bx^2}}{5d(c+dx^2)^{3/2}}\\
&+\frac{(23a^2d^2f^2+2abdf(35de-41cf)+b^2(15d^2e^2-70cdef+56c^2f^2))\,x\sqrt{a+bx^2}}{15d^4\sqrt{c+dx^2}}\\
&+\frac{bf(10bde-8bcf+11adf)x^3\sqrt{a+bx^2}}{15d^3\sqrt{c+dx^2}}\\
&+\frac{(a^2d^2(10d^2e^2+10cdef-43c^2f^2)-8b^2c^2(5d^2e^2-20cdef+16c^2f^2)+abcd(15d^2e^2-160cdef+168c^2f}{15c^{3/2}d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\
&+\frac{(15a^2cd^2f^2+4b^2c(5d^2e^2-20cdef+16c^2f^2)-abd(5d^2e^2-70cdef+76c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\text{arct}\right.}{15\sqrt{c}d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/15*(10*a*b*c*d*(-c*f+d*e)^2-5*a^2*d^2*(-c*f+d*e)^2-b^2*c^2*(8*c^2*f^2-1
0*c*d*e*f+5*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d^4/(d*x^2+c)^(3/2)+1/5*b^2*f^2*
x^7*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(3/2)+1/15*(23*a^2*d^2*f^2+2*a*b*d*f*(-41*
c*f+35*d*e)+b^2*(56*c^2*f^2-70*c*d*e*f+15*d^2*e^2))*x*(b*x^2+a)^(1/2)/d^4/
(d*x^2+c)^(1/2)+1/15*b*f*(11*a*d*f-8*b*c*f+10*b*d*e)*x^3*(b*x^2+a)^(1/2)/d
^3/(d*x^2+c)^(1/2)+1/15*(a^2*d^2*(-43*c^2*f^2+10*c*d*e*f+10*d^2*e^2)-8*b^2
*c^2*(16*c^2*f^2-20*c*d*e*f+5*d^2*e^2)+a*b*c*d*(168*c^2*f^2-160*c*d*e*f+15
*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))/c^(3/2)/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)+1/15*(15*a^2*c*d^2*f^2+4*b^2*c*(16*c^2*f^2-20*c*d*e*f+5*d^2*e^2)-a
*b*d*(76*c^2*f^2-70*c*d*e*f+5*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(ar
ctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(9/2)/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.67 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(5a^2d^2(de-cf)\left(3c^2f+2d^2ex^2+cd(3e+4fx^2)\right)+b^2c^2(\cdots\right.}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(5*a^2*d^2*(d*e - c*f)*(3*c^2*f + 2*d^2*e*x^2 +
 c*d*(3*e + 4*f*x^2)) + b^2*c^2*(-64*c^3*f^2 + 80*c^2*d*f*(e - f*x^2) - 4*
c*d^2*(5*e^2 - 25*e*f*x^2 + 2*f^2*x^4) + d^3*x^2*(-25*e^2 + 10*e*f*x^2 + 3
*f^2*x^4)) + a*b*c*d*(76*c^3*f^2 + 15*d^3*e^2*x^2 + c^2*d*f*(-70*e + 97*f*
x^2) + c*d^2*(5*e^2 - 90*e*f*x^2 + 11*f^2*x^4))) - I*b*c*(a*b*c*d*(-15*d^2
*e^2 + 160*c*d*e*f - 168*c^2*f^2) + 8*b^2*c^2*(5*d^2*e^2 - 20*c*d*e*f + 16
*c^2*f^2) + a^2*d^2*(-10*d^2*e^2 - 10*c*d*e*f + 43*c^2*f^2))*Sqrt[1 + (b*x
^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(15*a^2*c*d^2*f^2 + a*b*d*(5*d^2*e^2 + 80
*c*d*e*f - 104*c^2*f^2) + 8*b^2*c*(5*d^2*e^2 - 20*c*d*e*f + 16*c^2*f^2))*S
qrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*c^2*d^5*Sqrt[a + b*x^2]*(c + d*x^2)
^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1218 vs. $2(606) = 1212$.

Time = 1.73 (sec) , antiderivative size = 1218, normalized size of antiderivative = 2.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}}+\frac{2efx^2(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}}+\frac{f^2x^4(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b(16bc-5ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5cd^3}-\frac{(8bc-3ad)f^2\left(bx^2+a\right)^{3/2}x^3}{3cd^2\sqrt{dx^2+c}}-\frac{f^2\left(bx^2+a\right)^{5/2}x^3}{3d\left(dx^2+c\right)^{3/2}}-\frac{(16bc-15ad)(4bc-ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15cd^4}+\frac{2b(8bc-ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3cd^3}-\frac{2(6bc-ad)ef\left(bx^2+a\right)^{3/2}x}{3cd^2\sqrt{dx^2+c}}-\frac{2(bc-ad)(2bc+ad)e^2\sqrt{bx^2+a}x}{3c^2d^2\sqrt{dx^2+c}}-\frac{\left(\frac{2da^2}{c}+3ba-\frac{8b^2c}{d}\right)e^2\sqrt{bx^2+a}x}{3cd\sqrt{dx^2+c}}+\frac{\left(128b^2c^2-168abdc+43a^2d^2\right)f^2\sqrt{bx^2+a}x}{15d^4\sqrt{dx^2+c}}-\frac{2\left(16b^2c^2-16abdc+a^2d^2\right)ef\sqrt{bx^2+a}x}{3cd^3\sqrt{dx^2+c}}-\frac{2ef\left(bx^2+a\right)^{5/2}x}{3d\left(dx^2+c\right)^{3/2}}-\frac{(bc-ad)e^2\left(bx^2+a\right)^{3/2}x}{3cd\left(dx^2+c\right)^{3/2}}-\frac{\left(8b^2c^2-3abdc-2a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(128b^2c^2-168abdc+43a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\left(16b^2c^2-16abdc+a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{b(4bc-ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}(16bc-15ad)(4bc-ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b\sqrt{c}(8bc-7ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*e^2*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^(3/2)) - (2*e*
f*x*(a + b*x^2)^(5/2))/(3*d*(c + d*x^2)^(3/2)) - (f^2*x^3*(a + b*x^2)^(5/2
))/(3*d*(c + d*x^2)^(3/2)) - (2*(b*c - a*d)*(2*b*c + a*d)*e^2*x*Sqrt[a + b
*x^2])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((3*a*b - (8*b^2*c)/d + (2*a^2*d)/c)*
e^2*x*Sqrt[a + b*x^2])/(3*c*d*Sqrt[c + d*x^2]) - (2*(16*b^2*c^2 - 16*a*b*c
*d + a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(3*c*d^3*Sqrt[c + d*x^2]) + ((128*b^2
*c^2 - 168*a*b*c*d + 43*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(15*d^4*Sqrt[c + d
*x^2]) - (2*(6*b*c - a*d)*e*f*x*(a + b*x^2)^(3/2))/(3*c*d^2*Sqrt[c + d*x^2
]) - ((8*b*c - 3*a*d)*f^2*x^3*(a + b*x^2)^(3/2))/(3*c*d^2*Sqrt[c + d*x^2])
 + (2*b*(8*b*c - a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^3) - (
(16*b*c - 15*a*d)*(4*b*c - a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15
*c*d^4) + (b*(16*b*c - 5*a*d)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*
c*d^3) - ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*e^2*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*d^(5/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(16*b^2*c^2 - 16*a*b
*c*d + a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) - (Sqrt[c]*(128*b^2*c^2 - 168*a*b*c*d + 43*a^2*d^2)*f^2
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(15*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. 2(565) = 1130.

Time = 30.24 (sec) , antiderivative size = 1432, normalized size of antiderivative = 2.36

method	result	size
elliptic	Expression too large to display	1432
risch	Expression too large to display	1464
default	Expression too large to display	3237

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(a^2*c^2*
d^2*f^2-2*a^2*c*d^3*e*f+a^2*d^4*e^2-2*a*b*c^3*d*f^2+4*a*b*c^2*d^2*e*f-2*a*
b*c*d^3*e^2+b^2*c^4*f^2-2*b^2*c^3*d*e*f+b^2*c^2*d^2*e^2)/c/d^6*x*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2-1/3*(b*d*x^2+a*d)*(4*a^2*c^2*d^2*f^
2-2*a^2*c*d^3*e*f-2*a^2*d^4*e^2-15*a*b*c^3*d*f^2+18*a*b*c^2*d^2*e*f-3*a*b*
c*d^3*e^2+11*b^2*c^4*f^2-16*b^2*c^3*d*e*f+5*b^2*c^2*d^2*e^2)/d^5/c^2*x/((x
^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/5*b^2*f^2/d^3*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+1/3*(b^2/d^3*f*(3*a*d*f-2*b*c*f+2*b*d*e)-1/5*b^2*f^2/d^3*(4*a*d
+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+((a^3*d^3*f^2-6*a^2*b*c
*d^2*f^2+6*a^2*b*d^3*e*f+9*a*b^2*c^2*d*f^2-12*a*b^2*c*d^2*e*f+3*a*b^2*d^3*
e^2-4*b^3*c^3*f^2+6*b^3*c^2*d*e*f-2*b^3*c*d^2*e^2)/d^5+1/3*(a^2*c^2*d^2*f^
2-2*a^2*c*d^3*e*f+a^2*d^4*e^2-2*a*b*c^3*d*f^2+4*a*b*c^2*d^2*e*f-2*a*b*c*d^
3*e^2+b^2*c^4*f^2-2*b^2*c^3*d*e*f+b^2*c^2*d^2*e^2)/d^5*b/c-1/3*(4*a^2*c^2*
d^2*f^2-2*a^2*c*d^3*e*f-2*a^2*d^4*e^2-15*a*b*c^3*d*f^2+18*a*b*c^2*d^2*e*f-
3*a*b*c*d^3*e^2+11*b^2*c^4*f^2-16*b^2*c^3*d*e*f+5*b^2*c^2*d^2*e^2)/d^5*(a*
d-b*c)/c^2+1/3*a/d^4*(4*a^2*c^2*d^2*f^2-2*a^2*c*d^3*e*f-2*a^2*d^4*e^2-15*a
*b*c^3*d*f^2+18*a*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+11*b^2*c^4*f^2-16*b^2*c^3*
d*e*f+5*b^2*c^2*d^2*e^2)/c^2-1/3*(b^2/d^3*f*(3*a*d*f-2*b*c*f+2*b*d*e)-1/5*
b^2*f^2/d^3*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(567) = 1134$.

Time = 0.13 (sec) , antiderivative size = 1459, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
-1/15*(((5*(8*b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 - 2*a^2*b*c*d^6)*e^2 - 10*(16*
b^3*c^4*d^3 - 16*a*b^2*c^3*d^4 + a^2*b*c^2*d^5)*e*f + (128*b^3*c^5*d^2 - 1
68*a*b^2*c^4*d^3 + 43*a^2*b*c^3*d^4)*f^2)*x^5 + 2*(5*(8*b^3*c^4*d^3 - 3*a*
b^2*c^3*d^4 - 2*a^2*b*c^2*d^5)*e^2 - 10*(16*b^3*c^5*d^2 - 16*a*b^2*c^4*d^3
 + a^2*b*c^3*d^4)*e*f + (128*b^3*c^6*d - 168*a*b^2*c^5*d^2 + 43*a^2*b*c^4*
d^3)*f^2)*x^3 + (5*(8*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 - 2*a^2*b*c^3*d^4)*e^2
 - 10*(16*b^3*c^6*d - 16*a*b^2*c^5*d^2 + a^2*b*c^4*d^3)*e*f + (128*b^3*c^7
 - 168*a*b^2*c^6*d + 43*a^2*b*c^5*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*ellipt
ic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((5*(8*b^3*c^3*d^4 - 3*a*b^2*c^2*d
^5 - a^2*b*d^7 - 2*(a^2*b - 2*a*b^2)*c*d^6)*e^2 - 10*(16*b^3*c^4*d^3 - 16*
a*b^2*c^3*d^4 - 7*a^2*b*c*d^6 + (a^2*b + 8*a*b^2)*c^2*d^5)*e*f + (128*b^3*
c^5*d^2 - 168*a*b^2*c^4*d^3 - 76*a^2*b*c^2*d^5 + 15*a^3*c*d^6 + (43*a^2*b
+ 64*a*b^2)*c^3*d^4)*f^2)*x^5 + 2*(5*(8*b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 - a^
2*b*c*d^6 - 2*(a^2*b - 2*a*b^2)*c^2*d^5)*e^2 - 10*(16*b^3*c^5*d^2 - 16*a*b
^2*c^4*d^3 - 7*a^2*b*c^2*d^5 + (a^2*b + 8*a*b^2)*c^3*d^4)*e*f + (128*b^3*c
^6*d - 168*a*b^2*c^5*d^2 - 76*a^2*b*c^3*d^4 + 15*a^3*c^2*d^5 + (43*a^2*b +
 64*a*b^2)*c^4*d^3)*f^2)*x^3 + (5*(8*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 - a^2*b
*c^2*d^5 - 2*(a^2*b - 2*a*b^2)*c^3*d^4)*e^2 - 10*(16*b^3*c^6*d - 16*a*b^2*
c^5*d^2 - 7*a^2*b*c^3*d^4 + (a^2*b + 8*a*b^2)*c^4*d^3)*e*f + (128*b^3*c^7
- 168*a*b^2*c^6*d - 76*a^2*b*c^4*d^3 + 15*a^3*c^3*d^4 + (43*a^2*b + 64*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**2/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)^2}{(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(5/2),x)
```

output
```
(69*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x - 30*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*d**2*e*f*x + 46*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a**2*d**2*f**2*x**3 - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*f**2*
x + 180*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x - 142*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 - 45*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a*b*d**2*e**2*x + 140*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*f*
x**3 + 22*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**5 + 96*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x**3 - 120*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b**2*c*d*e*f*x**3 - 16*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**
2*c*d*f**2*x**5 + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x**3
 + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**5 + 6*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**7 + 30*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x*
*6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*a**
3*c**2*d**3*f**2 + 60*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3
 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**
2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)*a**3*c*d**4*f**2*x**2 + 30*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**3 + 3*a*c**2*d*x**2 + 3*a
*c*d**2*x**4 + a*d**3*x**6 + b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x*
*6 + b*d**3*x**8),x)*a**3*d**5*f**2*x**4 - 225*int((sqrt(c + d*x**2)*sq...
```

3.72 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 625

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx =$$

$$-\frac{(6abcd(de-cf)^2-3a^2d^2(de-cf)^2-b^2c^2(3d^2e^2-6cdef+8c^2f^2))\,x\sqrt{a+bx^2}}{15cd^4\,(c+dx^2)^{5/2}}$$

$$+\frac{b^2f^2x^7\sqrt{a+bx^2}}{3d\,(c+dx^2)^{5/2}}$$

$$+\frac{(2a^2d^2(2d^2e^2+cdef-3c^2f^2)+abcd(3d^2e^2-26cdef+23c^2f^2)-b^2c^2(7d^2e^2-24cdef+32c^2f^2))\,x\sqrt{a}}{15c^2d^4\,(c+dx^2)^{3/2}}$$

$$+\frac{bf(6bde-8bcf+7adf)x\sqrt{a+bx^2}}{3d^4\sqrt{c+dx^2}}$$

$$+\frac{(abcd(7d^2e^2+16cdef-88c^2f^2)+a^2d^2(8d^2e^2+4cdef+3c^2f^2)+8b^2c^2(d^2e^2-12cdef+16c^2f^2))\,\sqrt{a+}}{15c^{5/2}d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$-\frac{2b(ad(2d^2e^2+cdef-18c^2f^2)+2bc(d^2e^2-12cdef+16c^2f^2))\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{15c^{3/2}d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/15*(6*a*b*c*d*(-c*f+d*e)^2-3*a^2*d^2*(-c*f+d*e)^2-b^2*c^2*(8*c^2*f^2-6*
c*d*e*f+3*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d^4/(d*x^2+c)^(5/2)+1/3*b^2*f^2*x^
7*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(5/2)+1/15*(2*a^2*d^2*(-3*c^2*f^2+c*d*e*f+2*
d^2*e^2)+a*b*c*d*(23*c^2*f^2-26*c*d*e*f+3*d^2*e^2)-b^2*c^2*(32*c^2*f^2-24*
c*d*e*f+7*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^2/d^4/(d*x^2+c)^(3/2)+1/3*b*f*(7*a
*d*f-8*b*c*f+6*b*d*e)*x*(b*x^2+a)^(1/2)/d^4/(d*x^2+c)^(1/2)+1/15*(a*b*c*d*
(-88*c^2*f^2+16*c*d*e*f+7*d^2*e^2)+a^2*d^2*(3*c^2*f^2+4*c*d*e*f+8*d^2*e^2)
+8*b^2*c^2*(16*c^2*f^2-12*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1
/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(9/2)/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/15*b*(a*d*(-18*c^2*f^2+c*d*e*f+
2*d^2*e^2)+2*b*c*(16*c^2*f^2-12*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJ
acobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(9/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.03 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2-c(bc-ad)(de-cf)(bc(7de-\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 - c*(b*c - a
*d)*(d*e - c*f)*(b*c*(7*d*e - 17*c*f) + 2*a*d*(2*d*e + 3*c*f))*(c + d*x^2)
 + (a*b*c*d*(7*d^2*e^2 + 16*c*d*e*f - 53*c^2*f^2) + a^2*d^2*(8*d^2*e^2 + 4
*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(8*d^2*e^2 - 66*c*d*e*f + 73*c^2*f^2))*(c
+ d*x^2)^2 + 5*b^2*c^3*f^2*(c + d*x^2)^3) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c +
 d*x^2)^2*Sqrt[1 + (d*x^2)/c]*((a*b*c*d*(7*d^2*e^2 + 16*c*d*e*f - 88*c^2*f
^2) + a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) + 8*b^2*c^2*(d^2*e^2 - 1
2*c*d*e*f + 16*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
(a*b*c*d*(3*d^2*e^2 + 64*c*d*e*f - 152*c^2*f^2) + 8*b^2*c^2*(d^2*e^2 - 12*
c*d*e*f + 16*c^2*f^2) + a^2*d^2*(4*d^2*e^2 + 2*c*d*e*f + 39*c^2*f^2))*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^5*Sqrt[a
 + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 1152, normalized size of antiderivative = 1.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}}+\frac{2efx^2(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}}+\frac{f^2x^4(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b(16bc-11ad)f^2\sqrt{bx^2+a}x^3}{5cd^3\sqrt{dx^2+c}}-\frac{(8bc-3ad)f^2\left(bx^2+a\right)^{3/2}x^3}{15cd^2\left(dx^2+c\right)^{3/2}}-\frac{f^2\left(bx^2+a\right)^{5/2}x^3}{5d\left(dx^2+c\right)^{5/2}}+\frac{4b(16bc-9ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15cd^4}-\frac{\left(128b^2c^2-88abdc+3a^2d^2\right)f^2\sqrt{bx^2+a}x}{15cd^4\sqrt{dx^2+c}}-\frac{4\left(\frac{da^2}{c}+4ba-\frac{24b^2c}{d}\right)ef\sqrt{bx^2+a}x}{15cd^2\sqrt{dx^2+c}}+\frac{2\left(\frac{2da^2}{c}+7ba-\frac{24b^2c}{d}\right)ef\sqrt{bx^2+a}x}{15cd^2\sqrt{dx^2+c}}-\frac{2(6bc-ad)ef\left(bx^2+a\right)^{3/2}x}{15cd^2\left(dx^2+c\right)^{3/2}}+\frac{4\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)e^2\sqrt{bx^2+a}x}{15\left(dx^2+c\right)^{3/2}}-\frac{2ef\left(bx^2+a\right)^{5/2}x}{5d\left(dx^2+c\right)^{5/2}}-\frac{(bc-ad)e^2\left(bx^2+a\right)^{3/2}x}{5cd\left(dx^2+c\right)^{5/2}}+\frac{\left(8b^2c^2+7abdc+8a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{5/2}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\left(128b^2c^2-88abdc+3a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4\left(24b^2c^2-4abdc-a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15c^{3/2}d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4b(bc+ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15c^{3/2}d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4b\sqrt{c}(16bc-9ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{9/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2b(24bc-ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x]
```

output
```
-1/5*((b*c - a*d)*e^2*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^(5/2)) - (2*e*
f*x*(a + b*x^2)^(5/2))/(5*d*(c + d*x^2)^(5/2)) - (f^2*x^3*(a + b*x^2)^(5/2
))/(5*d*(c + d*x^2)^(5/2)) + (4*(a^2/c^2 - b^2/d^2)*e^2*x*Sqrt[a + b*x^2])
/(15*(c + d*x^2)^(3/2)) - (2*(6*b*c - a*d)*e*f*x*(a + b*x^2)^(3/2))/(15*c*
d^2*(c + d*x^2)^(3/2)) - ((8*b*c - 3*a*d)*f^2*x^3*(a + b*x^2)^(3/2))/(15*c
*d^2*(c + d*x^2)^(3/2)) - (4*(4*a*b - (24*b^2*c)/d + (a^2*d)/c)*e*f*x*Sqrt
[a + b*x^2])/(15*c*d^2*Sqrt[c + d*x^2]) + (2*(7*a*b - (24*b^2*c)/d + (2*a^
2*d)/c)*e*f*x*Sqrt[a + b*x^2])/(15*c*d^2*Sqrt[c + d*x^2]) - ((128*b^2*c^2
- 88*a*b*c*d + 3*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(15*c*d^4*Sqrt[c + d*x^2]
) - (b*(16*b*c - 11*a*d)*f^2*x^3*Sqrt[a + b*x^2])/(5*c*d^3*Sqrt[c + d*x^2]
) + (4*b*(16*b*c - 9*a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*c*d^4
) + ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*c^(5/2)*d^(5/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*(24*b^2*c^2 - 4*a*b*c*d
- a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(15*c^(3/2)*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) + ((128*b^2*c^2 - 88*a*b*c*d + 3*a^2*d^2)*f^2*Sqrt[a + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*Sqrt[c]*d^
(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*b*(b*c +
 a*d)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(584) = 1168.

Time = 31.97 (sec) , antiderivative size = 1324, normalized size of antiderivative = 2.12

method	result	size
elliptic	Expression too large to display	1324
risch	Expression too large to display	2001
default	Expression too large to display	4465

input

```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*(a^2*c^2*
d^2*f^2-2*a^2*c*d^3*e*f+a^2*d^4*e^2-2*a*b*c^3*d*f^2+4*a*b*c^2*d^2*e*f-2*a*
b*c*d^3*e^2+b^2*c^4*f^2-2*b^2*c^3*d*e*f+b^2*c^2*d^2*e^2)/c/d^7*x*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3-1/15*(6*a^2*c^2*d^2*f^2-2*a^2*c*d^3
*e*f-4*a^2*d^4*e^2-23*a*b*c^3*d*f^2+26*a*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+17*
b^2*c^4*f^2-24*b^2*c^3*d*e*f+7*b^2*c^2*d^2*e^2)/c^2/d^6*x*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)*(3*a^2*c^2*d^2*f^2+4*a^
2*c*d^3*e*f+8*a^2*d^4*e^2-53*a*b*c^3*d*f^2+16*a*b*c^2*d^2*e*f+7*a*b*c*d^3*
e^2+73*b^2*c^4*f^2-66*b^2*c^3*d*e*f+8*b^2*c^2*d^2*e^2)/c^3/d^5*x/((x^2+c/d
)*(b*d*x^2+a*d))^(1/2)+1/3*b^2*f^2/d^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)+(b*(3*a^2*d^2*f^2-9*a*b*c*d*f^2+6*a*b*d^2*e*f+6*b^2*c^2*f^2-6*b^2*c*d*e
*f+b^2*d^2*e^2)/d^5-1/15*b*(6*a^2*c^2*d^2*f^2-2*a^2*c*d^3*e*f-4*a^2*d^4*e^
2-23*a*b*c^3*d*f^2+26*a*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+17*b^2*c^4*f^2-24*b^
2*c^3*d*e*f+7*b^2*c^2*d^2*e^2)/c^2/d^5+1/15*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3
*e*f+8*a^2*d^4*e^2-53*a*b*c^3*d*f^2+16*a*b*c^2*d^2*e*f+7*a*b*c*d^3*e^2+73*
b^2*c^4*f^2-66*b^2*c^3*d*e*f+8*b^2*c^2*d^2*e^2)/d^5*(a*d-b*c)/c^3-1/15*a/d
^4*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2-53*a*b*c^3*d*f^2+16*a*
b*c^2*d^2*e*f+7*a*b*c*d^3*e^2+73*b^2*c^4*f^2-66*b^2*c^3*d*e*f+8*b^2*c^2*d^
2*e^2)/c^3-1/3*b^2*f^2/d^4*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. $2(584) = 1168$.

Time = 0.13 (sec) , antiderivative size = 1648, normalized size of antiderivative = 2.64

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="fricas
")
```

output
```
1/15*((((8*b^2*c^3*d^5 + 7*a*b*c^2*d^6 + 8*a^2*c*d^7)*e^2 - 4*(24*b^2*c^4*
d^4 - 4*a*b*c^3*d^5 - a^2*c^2*d^6)*e*f + (128*b^2*c^5*d^3 - 88*a*b*c^4*d^4
 + 3*a^2*c^3*d^5)*f^2)*x^7 + 3*((8*b^2*c^4*d^4 + 7*a*b*c^3*d^5 + 8*a^2*c^2
*d^6)*e^2 - 4*(24*b^2*c^5*d^3 - 4*a*b*c^4*d^4 - a^2*c^3*d^5)*e*f + (128*b^
2*c^6*d^2 - 88*a*b*c^5*d^3 + 3*a^2*c^4*d^4)*f^2)*x^5 + 3*((8*b^2*c^5*d^3 +
 7*a*b*c^4*d^4 + 8*a^2*c^3*d^5)*e^2 - 4*(24*b^2*c^6*d^2 - 4*a*b*c^5*d^3 -
a^2*c^4*d^4)*e*f + (128*b^2*c^7*d - 88*a*b*c^6*d^2 + 3*a^2*c^5*d^3)*f^2)*x
^3 + ((8*b^2*c^6*d^2 + 7*a*b*c^5*d^3 + 8*a^2*c^4*d^4)*e^2 - 4*(24*b^2*c^7*
d - 4*a*b*c^6*d^2 - a^2*c^5*d^3)*e*f + (128*b^2*c^8 - 88*a*b*c^7*d + 3*a^2
*c^6*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*
d/(b*c)) - (((8*b^2*c^3*d^5 + 7*a*b*c^2*d^6 + 4*a^2*d^8 + 4*(2*a^2 + a*b)*
c*d^7)*e^2 - 2*(48*b^2*c^4*d^4 - 8*a*b*c^3*d^5 - a^2*c*d^7 - 2*(a^2 - 12*a
*b)*c^2*d^6)*e*f + (128*b^2*c^5*d^3 - 88*a*b*c^4*d^4 - 36*a^2*c^2*d^6 + (3
*a^2 + 64*a*b)*c^3*d^5)*f^2)*x^7 + 3*((8*b^2*c^4*d^4 + 7*a*b*c^3*d^5 + 4*a
^2*c*d^7 + 4*(2*a^2 + a*b)*c^2*d^6)*e^2 - 2*(48*b^2*c^5*d^3 - 8*a*b*c^4*d^
4 - a^2*c^2*d^6 - 2*(a^2 - 12*a*b)*c^3*d^5)*e*f + (128*b^2*c^6*d^2 - 88*a*
b*c^5*d^3 - 36*a^2*c^3*d^5 + (3*a^2 + 64*a*b)*c^4*d^4)*f^2)*x^5 + 3*((8*b^
2*c^5*d^3 + 7*a*b*c^4*d^4 + 4*a^2*c^2*d^6 + 4*(2*a^2 + a*b)*c^3*d^5)*e^2 -
 2*(48*b^2*c^6*d^2 - 8*a*b*c^5*d^3 - a^2*c^3*d^5 - 2*(a^2 - 12*a*b)*c^4*d^
4)*e*f + (128*b^2*c^7*d - 88*a*b*c^6*d^2 - 36*a^2*c^4*d^4 + (3*a^2 + 64...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**2/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(7/2),x)
```

output

```
( - 27*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c*d**2*f**2*x - 6*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a**3*d**3*e*f*x - 36*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a**3*d**3*f**2*x**3 + 147*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*
*2*d*f**2*x - 54*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*e*f*x + 2
14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x**3 - 9*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*e**2*x - 72*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a**2*b*d**3*e*f*x**3 + 28*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*
b*d**3*f**2*x**5 - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*f**2*
x + 108*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*e*f*x - 290*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x**3 - 9*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b**2*c*d**2*e**2*x + 180*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a*b**2*c*d**2*e*f*x**3 - 46*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*
d**2*f**2*x**5 - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e**2*x**
3 + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e*f*x**5 + 4*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**7 + 96*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b**3*c**3*f**2*x**3 - 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*3*c**2*d*e*f*x**3 + 16*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f**2
*x**5 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e**2*x**3 - 12*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**5 - 2*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**3*c*d**2*f**2*x**7 - 120*int((sqrt(c + d*x**2)*sqrt(...
```

3.73 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 791

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx =$$

$$-\frac{(2abcd(de-cf)^2-a^2d^2(de-cf)^2-b^2c^2(d^2e^2-2cdef+8c^2f^2))\,x\sqrt{a+bx^2}}{7cd^4(c+dx^2)^{7/2}}$$

$$+\frac{b^2f^2x^7\sqrt{a+bx^2}}{d(c+dx^2)^{7/2}}$$

$$+\frac{(2a^2d^2(3d^2e^2+cdef-4c^2f^2)+abcd(3d^2e^2-34cdef+31c^2f^2)-b^2c^2(9d^2e^2-32cdef+128c^2f^2))\,x\sqrt{c}}{35c^2d^4(c+dx^2)^{5/2}}$$

$$+\frac{(abcd(13d^2e^2+16cdef-99c^2f^2)+a^2d^2(24d^2e^2+8cdef+3c^2f^2)+2b^2c^2(4d^2e^2-57cdef+228c^2f^2))\,:}{105c^3d^4(c+dx^2)^{3/2}}$$

$$+\frac{(8b^3c^3(d^2e^2+12cdef-48c^2f^2)+a^2bcd^2(16d^2e^2-18cdef-33c^2f^2)-2a^3d^3(24d^2e^2+8cdef+3c^2f^2)\,-}{105c^{7/2}d^{9/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+}}$$

$$-\frac{b(4b^2c^2(d^2e^2+12cdef-48c^2f^2)-a^2d^2(24d^2e^2+8cdef+3c^2f^2)+5abcd(d^2e^2-2cdef+36c^2f^2))\,\sqrt{a}}{105c^{5/2}d^{9/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/7*(2*a*b*c*d*(-c*f+d*e)^2-a^2*d^2*(-c*f+d*e)^2-b^2*c^2*(8*c^2*f^2-2*c*d
*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d^4/(d*x^2+c)^(7/2)+b^2*f^2*x^7*(b*x^2+
a)^(1/2)/d/(d*x^2+c)^(7/2)+1/35*(2*a^2*d^2*(-4*c^2*f^2+c*d*e*f+3*d^2*e^2)+
a*b*c*d*(31*c^2*f^2-34*c*d*e*f+3*d^2*e^2)-b^2*c^2*(128*c^2*f^2-32*c*d*e*f+
9*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^2/d^4/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-99*
c^2*f^2+16*c*d*e*f+13*d^2*e^2)+a^2*d^2*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2)+2*
b^2*c^2*(228*c^2*f^2-57*c*d*e*f+4*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d^4/(d*x
^2+c)^(3/2)+1/105*(8*b^3*c^3*(-48*c^2*f^2+12*c*d*e*f+d^2*e^2)+a^2*b*c*d^2*
(-33*c^2*f^2-18*c*d*e*f+16*d^2*e^2)-2*a^3*d^3*(3*c^2*f^2+8*c*d*e*f+24*d^2*
e^2)+a*b^2*c^2*d*(408*c^2*f^2-32*c*d*e*f+9*d^2*e^2))*(b*x^2+a)^(1/2)*Ellip
ticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(7/2)/d^(9/2
)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*b*(4*b^
2*c^2*(-48*c^2*f^2+12*c*d*e*f+d^2*e^2)-a^2*d^2*(3*c^2*f^2+8*c*d*e*f+24*d^2
*e^2)+5*a*b*c*d*(36*c^2*f^2-2*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJac
obiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/c^(5/2)/d^(9/2)/(-a*d+b
*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.77 (sec) , antiderivative size = 724, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)^2-3c^2(bc-ad)^2(de-cf)(bc(9c\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f)^2 - 3*c^2*(b*
c - a*d)^2*(d*e - c*f)*(b*c*(9*d*e - 23*c*f) + 2*a*d*(3*d*e + 4*c*f))*(c +
 d*x^2) + c*(b*c - a*d)*(a*b*c*d*(13*d^2*e^2 + 16*c*d*e*f - 99*c^2*f^2) +
a^2*d^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(8*d^2*e^2 - 114*c*
d*e*f + 141*c^2*f^2))*(c + d*x^2)^2 - (a*b^2*c^2*d*(-9*d^2*e^2 + 32*c*d*e*
f - 303*c^2*f^2) + 2*a^3*d^3*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + a^2*b*
c*d^2*(-16*d^2*e^2 + 18*c*d*e*f + 33*c^2*f^2) + b^3*c^3*(-8*d^2*e^2 - 96*c
*d*e*f + 279*c^2*f^2))*(c + d*x^2)^3) - I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x
^2)^3*Sqrt[1 + (d*x^2)/c]*((a*b^2*c^2*d*(-9*d^2*e^2 + 32*c*d*e*f - 408*c^2
*f^2) + 2*a^3*d^3*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2) + a^2*b*c*d^2*(-16*
d^2*e^2 + 18*c*d*e*f + 33*c^2*f^2) + 8*b^3*c^3*(-(d^2*e^2) - 12*c*d*e*f +
48*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*
(-(a^2*d^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2)) + 8*b^2*c^2*(-(d^2*e^2) -
 12*c*d*e*f + 48*c^2*f^2) - a*b*c*d*(13*d^2*e^2 + 16*c*d*e*f + 216*c^2*f^2
))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(105*Sqrt[b/a]*c^4*d^5
*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 1366, normalized size of antiderivative = 1.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}}+\frac{2efx^2(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}}+\frac{f^2x^4(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{2da^2}{c}+9ba-\frac{16b^2c}{d}\right)f^2\sqrt{bx^2+a}x^3}{35cd^2\left(dx^2+c\right)^{3/2}}-\frac{(8bc-3ad)f^2\left(bx^2+a\right)^{3/2}x^3}{35cd^2\left(dx^2+c\right)^{5/2}}-\frac{f^2\left(bx^2+a\right)^{5/2}x^3}{7d\left(dx^2+c\right)^{7/2}}-\frac{b\left(64b^2c^2-60abdc+a^2d^2\right)f^2\sqrt{bx^2+a}x}{35cd^4(bc-ad)\sqrt{dx^2+c}}+\frac{\left(128b^3c^3-136ab^2dc^2+11a^2bd^2c+2a^3d^3\right)f^2\sqrt{bx^2+a}x}{35c^2d^4(bc-ad)\sqrt{dx^2+c}}+\frac{\left(8b^2c^2+13abdc+24a^2d^2\right)e^2\sqrt{bx^2+a}x}{105c^3d^2\left(dx^2+c\right)^{3/2}}+\frac{2\left(\frac{4da^2}{c}+5ba-\frac{24b^2c}{d}\right)ef\sqrt{bx^2+a}x}{105cd^2\left(dx^2+c\right)^{3/2}}-\frac{2(6bc-ad)ef\left(bx^2+a\right)^{3/2}x}{35cd^2\left(dx^2+c\right)^{5/2}}-\frac{2(bc-ad)(2bc+3ad)e^2\sqrt{bx^2+a}x}{35c^2d^2\left(dx^2+c\right)^{5/2}}-\frac{2ef\left(bx^2+a\right)^{5/2}x}{7d\left(dx^2+c\right)^{7/2}}-\frac{(bc-ad)e^2\left(bx^2+a\right)^{3/2}x}{7cd\left(dx^2+c\right)^{7/2}}+\frac{\left(8b^3c^3+9ab^2dc^2+16a^2bd^2c-48a^3d^3\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{7/2}d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\left(128b^3c^3-136ab^2dc^2+11a^2bd^2c+2a^3d^3\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35c^{3/2}d^{9/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\left(48b^3c^3-16ab^2dc^2-9a^2bd^2c-8a^3d^3\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105c^{5/2}d^{7/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\left(4b^2c^2+5abdc-24a^2d^2\right)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{5/2}d^{5/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{b\left(64b^2c^2-60abdc+a^2d^2\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35\sqrt{c}d^{9/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b\left(24b^2c^2-5abdc-4a^2d^2\right)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105c^{3/2}d^{7/2}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x]
```

output
```
-1/7*((b*c - a*d)*e^2*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^(7/2)) - (2*e*
f*x*(a + b*x^2)^(5/2))/(7*d*(c + d*x^2)^(7/2)) - (f^2*x^3*(a + b*x^2)^(5/2
))/(7*d*(c + d*x^2)^(7/2)) - (2*(b*c - a*d)*(2*b*c + 3*a*d)*e^2*x*Sqrt[a +
 b*x^2])/(35*c^2*d^2*(c + d*x^2)^(5/2)) - (2*(6*b*c - a*d)*e*f*x*(a + b*x^
2)^(3/2))/(35*c*d^2*(c + d*x^2)^(5/2)) - ((8*b*c - 3*a*d)*f^2*x^3*(a + b*x
^2)^(3/2))/(35*c*d^2*(c + d*x^2)^(5/2)) + ((8*b^2*c^2 + 13*a*b*c*d + 24*a^
2*d^2)*e^2*x*Sqrt[a + b*x^2])/(105*c^3*d^2*(c + d*x^2)^(3/2)) + (2*(5*a*b
- (24*b^2*c)/d + (4*a^2*d)/c)*e*f*x*Sqrt[a + b*x^2])/(105*c*d^2*(c + d*x^2
)^(3/2)) + ((9*a*b - (16*b^2*c)/d + (2*a^2*d)/c)*f^2*x^3*Sqrt[a + b*x^2])/
(35*c*d^2*(c + d*x^2)^(3/2)) - (b*(64*b^2*c^2 - 60*a*b*c*d + a^2*d^2)*f^2*
x*Sqrt[a + b*x^2])/(35*c*d^4*(b*c - a*d)*Sqrt[c + d*x^2]) + ((128*b^3*c^3
- 136*a*b^2*c^2*d + 11*a^2*b*c*d^2 + 2*a^3*d^3)*f^2*x*Sqrt[a + b*x^2])/(35
*c^2*d^4*(b*c - a*d)*Sqrt[c + d*x^2]) + ((8*b^3*c^3 + 9*a*b^2*c^2*d + 16*a
^2*b*c*d^2 - 48*a^3*d^3)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(105*c^(7/2)*d^(5/2)*(b*c - a*d)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(48*b^3*c^3 - 16*a*b^2*c^2*
d - 9*a^2*b*c*d^2 - 8*a^3*d^3)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*c^(5/2)*d^(7/2)*(b*c - a*d)*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - ((128*b^3*c^3 - 136*a*b^
2*c^2*d + 11*a^2*b*c*d^2 + 2*a^3*d^3)*f^2*Sqrt[a + b*x^2]*EllipticE[Arc...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. 2(752) = 1504.

Time = 12.36 (sec) , antiderivative size = 1573, normalized size of antiderivative = 1.99

method	result	size
elliptic	Expression too large to display	1573
default	Expression too large to display	7943

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*(a^2*c^2*
d^2*f^2-2*a^2*c*d^3*e*f+a^2*d^4*e^2-2*a*b*c^3*d*f^2+4*a*b*c^2*d^2*e*f-2*a*
b*c*d^3*e^2+b^2*c^4*f^2-2*b^2*c^3*d*e*f+b^2*c^2*d^2*e^2)/c/d^8*x*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^4-1/35*(8*a^2*c^2*d^2*f^2-2*a^2*c*d^3
*e*f-6*a^2*d^4*e^2-31*a*b*c^3*d*f^2+34*a*b*c^2*d^2*e*f-3*a*b*c*d^3*e^2+23*
b^2*c^4*f^2-32*b^2*c^3*d*e*f+9*b^2*c^2*d^2*e^2)/c^2/d^7*x*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3+1/105*(3*a^2*c^2*d^2*f^2+8*a^2*c*d^3*e*f+2
4*a^2*d^4*e^2-99*a*b*c^3*d*f^2+16*a*b*c^2*d^2*e*f+13*a*b*c*d^3*e^2+141*b^2
*c^4*f^2-114*b^2*c^3*d*e*f+8*b^2*c^2*d^2*e^2)/c^3/d^6*x*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+1/105*(b*d*x^2+a*d)/c^4/d^5/(a*d-b*c)*x*(6*a
^3*c^2*d^3*f^2+16*a^3*c*d^4*e*f+48*a^3*d^5*e^2+33*a^2*b*c^3*d^2*f^2+18*a^2
*b*c^2*d^3*e*f-16*a^2*b*c*d^4*e^2-303*a*b^2*c^4*d*f^2+32*a*b^2*c^3*d^2*e*f
-9*a*b^2*c^2*d^3*e^2+279*b^3*c^5*f^2-96*b^3*c^4*d*e*f-8*b^3*c^3*d^2*e^2)/(
(x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(b^2*f*(3*a*d*f-4*b*c*f+2*b*d*e)/d^5+1/105*
b*(3*a^2*c^2*d^2*f^2+8*a^2*c*d^3*e*f+24*a^2*d^4*e^2-99*a*b*c^3*d*f^2+16*a*
b*c^2*d^2*e*f+13*a*b*c*d^3*e^2+141*b^2*c^4*f^2-114*b^2*c^3*d*e*f+8*b^2*c^2
*d^2*e^2)/c^3/d^5+1/105/d^5*(6*a^3*c^2*d^3*f^2+16*a^3*c*d^4*e*f+48*a^3*d^5
*e^2+33*a^2*b*c^3*d^2*f^2+18*a^2*b*c^2*d^3*e*f-16*a^2*b*c*d^4*e^2-303*a*b^
2*c^4*d*f^2+32*a*b^2*c^3*d^2*e*f-9*a*b^2*c^2*d^3*e^2+279*b^3*c^5*f^2-96*b^
3*c^4*d*e*f-8*b^3*c^3*d^2*e^2)/c^4-1/105*a/d^4/c^4/(a*d-b*c)*(6*a^3*c^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2837 vs. $2(752) = 1504$.

Time = 0.21 (sec) , antiderivative size = 2837, normalized size of antiderivative = 3.59

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="fricas
")
```

output
```
1/105*((((8*b^3*c^4*d^6 + 9*a*b^2*c^3*d^7 + 16*a^2*b*c^2*d^8 - 48*a^3*c*d^
9)*e^2 + 2*(48*b^3*c^5*d^5 - 16*a*b^2*c^4*d^6 - 9*a^2*b*c^3*d^7 - 8*a^3*c^
2*d^8)*e*f - 3*(128*b^3*c^6*d^4 - 136*a*b^2*c^5*d^5 + 11*a^2*b*c^4*d^6 + 2
*a^3*c^3*d^7)*f^2)*x^9 + 4*((8*b^3*c^5*d^5 + 9*a*b^2*c^4*d^6 + 16*a^2*b*c^
3*d^7 - 48*a^3*c^2*d^8)*e^2 + 2*(48*b^3*c^6*d^4 - 16*a*b^2*c^5*d^5 - 9*a^2
*b*c^4*d^6 - 8*a^3*c^3*d^7)*e*f - 3*(128*b^3*c^7*d^3 - 136*a*b^2*c^6*d^4 +
 11*a^2*b*c^5*d^5 + 2*a^3*c^4*d^6)*f^2)*x^7 + 6*((8*b^3*c^6*d^4 + 9*a*b^2*
c^5*d^5 + 16*a^2*b*c^4*d^6 - 48*a^3*c^3*d^7)*e^2 + 2*(48*b^3*c^7*d^3 - 16*
a*b^2*c^6*d^4 - 9*a^2*b*c^5*d^5 - 8*a^3*c^4*d^6)*e*f - 3*(128*b^3*c^8*d^2
- 136*a*b^2*c^7*d^3 + 11*a^2*b*c^6*d^4 + 2*a^3*c^5*d^5)*f^2)*x^5 + 4*((8*b
^3*c^7*d^3 + 9*a*b^2*c^6*d^4 + 16*a^2*b*c^5*d^5 - 48*a^3*c^4*d^6)*e^2 + 2*
(48*b^3*c^8*d^2 - 16*a*b^2*c^7*d^3 - 9*a^2*b*c^6*d^4 - 8*a^3*c^5*d^5)*e*f
- 3*(128*b^3*c^9*d - 136*a*b^2*c^8*d^2 + 11*a^2*b*c^7*d^3 + 2*a^3*c^6*d^4)
*f^2)*x^3 + ((8*b^3*c^8*d^2 + 9*a*b^2*c^7*d^3 + 16*a^2*b*c^6*d^4 - 48*a^3*
c^5*d^5)*e^2 + 2*(48*b^3*c^9*d - 16*a*b^2*c^8*d^2 - 9*a^2*b*c^7*d^3 - 8*a^
3*c^6*d^4)*e*f - 3*(128*b^3*c^10 - 136*a*b^2*c^9*d + 11*a^2*b*c^8*d^2 + 2*
a^3*c^7*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x),
 a*d/(b*c)) - (((8*b^3*c^4*d^6 + 9*a*b^2*c^3*d^7 - 24*a^3*d^10 + 4*(4*a^2*
b + a*b^2)*c^2*d^8 - (48*a^3 - 5*a^2*b)*c*d^9)*e^2 + 2*(48*b^3*c^5*d^5 - 1
6*a*b^2*c^4*d^6 - 4*a^3*c*d^9 - 3*(3*a^2*b - 8*a*b^2)*c^3*d^7 - (8*a^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**2/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^2/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^2)/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^2}{(c+dx^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^2/(d*x^2+c)^(9/2),x)
```

output
```
(9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c*d**2*f**2*x - 6*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a**3*d**3*e*f*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a**3*d**3*f**2*x**3 - 81*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**2*d*
f**2*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*e*f*x - 168*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x**3 - 9*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*b*d**3*e**2*x - 12*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a**2*b*d**3*e*f*x**3 - 54*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3
*f**2*x**5 + 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*f**2*x - 36
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*e*f*x + 342*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x**3 - 3*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a*b**2*c*d**2*e**2*x - 68*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**
2*c*d**2*e*f*x**3 + 162*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f*
*2*x**5 - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e**2*x**3 - 36*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e*f*x**5 + 18*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**7 - 96*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**3*c**3*f**2*x**3 + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2
*d*e*f*x**3 - 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f**2*x**5 +
 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e**2*x**3 + 12*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**5 - 6*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b**3*c*d**2*f**2*x**7 + 270*int((sqrt(c + d*x**2)*sqrt(a + b*x...
```

3.74 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 631

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2(24a^3d^3f^2-4a^2bd^2f(14de+9cf)-b^3c(70d^2e^2+21cdef-3c^2f^2)+ab^2d(35d^2e^2+91cdef+6c^2f^2))}{105b^3d^2\sqrt{a+bx^2}}$$

$$+\frac{(24a^2d^2f^2-abdf(56de+33cf)+b^2(35d^2e^2+84cdef+3c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^3d}$$

$$+\frac{2f(7bde+4bcf-3adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35b^2}+\frac{df^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7b}$$

$$+\frac{2\sqrt{a}(24a^3d^3f^2-4a^2bd^2f(14de+9cf)-b^3c(70d^2e^2+21cdef-3c^2f^2)+ab^2d(35d^2e^2+91cdef+6c^2f^2}{105b^{7/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{a}(105b^3cde^2-24a^3d^2f^2+a^2bdf(56de+33cf)-ab^2(35d^2e^2+84cdef+3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\right.}{105b^{7/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/105*(24*a^3*d^3*f^2-4*a^2*b*d^2*f*(9*c*f+14*d*e)-b^3*c*(-3*c^2*f^2+21*c
*d*e*f+70*d^2*e^2)+a*b^2*d*(6*c^2*f^2+91*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)^
(1/2)/b^3/d^2/(b*x^2+a)^(1/2)+1/105*(24*a^2*d^2*f^2-a*b*d*f*(33*c*f+56*d*e
)+b^2*(3*c^2*f^2+84*c*d*e*f+35*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b^3/d+2/35*f*(-3*a*d*f+4*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/
2)/b^2+1/7*d*f^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b+2/105*a^(1/2)*(24*a
^3*d^3*f^2-4*a^2*b*d^2*f*(9*c*f+14*d*e)-b^3*c*(-3*c^2*f^2+21*c*d*e*f+70*d^
2*e^2)+a*b^2*d*(6*c^2*f^2+91*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*Elliptic
E(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(1/2)*(105*b^3*c*d*e^2-
24*a^3*d^2*f^2+a^2*b*d*f*(33*c*f+56*d*e)-a*b^2*(3*c^2*f^2+84*c*d*e*f+35*d^
2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/b^(7/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(24a^2d^2f^2-abdf(56de+33cf+18dfx^2)+b^2(3\cdots}{\cdots}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(24*a^2*d^2*f^2 - a*b*d*f*(56*d*e +
 33*c*f + 18*d*f*x^2) + b^2*(3*c^2*f^2 + 12*c*d*f*(7*e + 2*f*x^2) + d^2*(3
5*e^2 + 42*e*f*x^2 + 15*f^2*x^4))) + (2*I)*c*(24*a^3*d^3*f^2 - 4*a^2*b*d^2
*f*(14*d*e + 9*c*f) + b^3*c*(-70*d^2*e^2 - 21*c*d*e*f + 3*c^2*f^2) + a*b^2
*d*(35*d^2*e^2 + 91*c*d*e*f + 6*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d
)*(24*a^2*d^2*f^2 - a*b*d*f*(56*d*e + 15*c*f) + b^2*(35*d^2*e^2 + 42*c*d*e
*f - 6*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*b^3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}+\frac{2efx^2(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}+\frac{f^2x^4(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{df^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7b}+\frac{2(4bc-3ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35b^2}+\frac{2def\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5b}+\frac{de^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\left(b^2c^2-11abdc+8a^2d^2\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{35b^3d}+\frac{4(3bc-2ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15b^2}+\frac{2d(2bc-ad)e^2\sqrt{bx^2+a}x}{3b^2\sqrt{dx^2+c}}-\frac{2(bc-2ad)\left(b^2c^2+4abdc-4a^2d^2\right)f^2\sqrt{bx^2+a}x}{35b^4d\sqrt{dx^2+c}}+\frac{2\left(3b^2c^2-13abdc+8a^2d^2\right)ef\sqrt{bx^2+a}x}{15b^3\sqrt{dx^2+c}}-\frac{2\sqrt{c}\sqrt{d}(2bc-ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(bc-2ad)\left(b^2c^2+4abdc-4a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^4d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2\sqrt{c}\left(3b^2c^2-13abdc+8a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(3bc-ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}\left(b^2c^2-11abdc+8a^2d^2\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35b^3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4c^{3/2}(3bc-2ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(2*d*(2*b*c - a*d)*e^2*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (2*(3*
b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(15*b^3*Sqrt[c +
d*x^2]) - (2*(b*c - 2*a*d)*(b^2*c^2 + 4*a*b*c*d - 4*a^2*d^2)*f^2*x*Sqrt[a
+ b*x^2])/(35*b^4*d*Sqrt[c + d*x^2]) + (d*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*b) + (4*(3*b*c - 2*a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(
15*b^2) + ((b^2*c^2 - 11*a*b*c*d + 8*a^2*d^2)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(35*b^3*d) + (2*d*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b
) + (2*(4*b*c - 3*a*d)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b^2) +
 (d*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*b) - (2*Sqrt[c]*Sqrt[d]*(2
*b*c - a*d)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) - (2*Sqrt[c]*(3*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*e*f*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(b*c - 2
*a*d)*(b^2*c^2 + 4*a*b*c*d - 4*a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^4*d^(3/2)*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*d)*e^2*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*c^(3/
2)*(3*b*c - 2*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.62 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{d f^2 x^5 \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{7b}+\frac{\left(2cd f^2+2d^2 ef-\frac{d f^2(6ad+6bc)}{7b}\right)x^3\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{5bd}+\frac{\left(c^2 f^2+4cdef+d^2 e^2\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7/b*d*f^2*x
^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2*c*d*f^2+2*d^2*e*f-1/7/b*d*f^
2*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(c^2*f^2+
4*c*d*e*f+d^2*e^2-5/7*a/b*c*d*f^2-1/5*(2*c*d*f^2+2*d^2*e*f-1/7/b*d*f^2*(6*
a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(
c^2*e^2-1/3*(c^2*f^2+4*c*d*e*f+d^2*e^2-5/7*a/b*c*d*f^2-1/5*(2*c*d*f^2+2*d^
2*e*f-1/7/b*d*f^2*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*c^2*e*f+2*c*d*e^2-3/5*
(2*c*d*f^2+2*d^2*e*f-1/7/b*d*f^2*(6*a*d+6*b*c))/b/d*a*c-1/3*(c^2*f^2+4*c*d
*e*f+d^2*e^2-5/7*a/b*c*d*f^2-1/5*(2*c*d*f^2+2*d^2*e*f-1/7/b*d*f^2*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2\,(35\,(2\,b^3c^2d^2-ab^2cd^3)e^2+7\,(3\,b^3c^3d-13\,ab^2c^2d^2+8\,a^2bcd^3)ef-3\,(b^3c^4+2\,ab^2c^3d-12\,a^2bc^2d^2+8\,a}{}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
-1/105*(2*(35*(2*b^3*c^2*d^2 - a*b^2*c*d^3)*e^2 + 7*(3*b^3*c^3*d - 13*a*b^
2*c^2*d^2 + 8*a^2*b*c*d^3)*e*f - 3*(b^3*c^4 + 2*a*b^2*c^3*d - 12*a^2*b*c^2
*d^2 + 8*a^3*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (35*(4*b^3*c^2*d^2 - a*b^2*d^4 - (2*a*b^2 - 3*b^3)*c*d
^3)*e^2 + 14*(3*b^3*c^3*d - 13*a*b^2*c^2*d^2 + 4*a^2*b*d^4 + 2*(4*a^2*b -
3*a*b^2)*c*d^3)*e*f - 3*(2*b^3*c^4 + 4*a*b^2*c^3*d + 8*a^3*d^4 - (24*a^2*b
 - a*b^2)*c^2*d^2 + (16*a^3 - 11*a^2*b)*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)
*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (15*b^3*d^4*f^2*x^6 + 6*(7*
b^3*d^4*e*f + (4*b^3*c*d^3 - 3*a*b^2*d^4)*f^2)*x^4 + 70*(2*b^3*c*d^3 - a*b
^2*d^4)*e^2 + 14*(3*b^3*c^2*d^2 - 13*a*b^2*c*d^3 + 8*a^2*b*d^4)*e*f - 6*(b
^3*c^3*d + 2*a*b^2*c^2*d^2 - 12*a^2*b*c*d^3 + 8*a^3*d^4)*f^2 + (35*b^3*d^4
*e^2 + 28*(3*b^3*c*d^3 - 2*a*b^2*d^4)*e*f + 3*(b^3*c^2*d^2 - 11*a*b^2*c*d^
3 + 8*a^2*b*d^4)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^4*d^3*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**2/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x)
```

output
```
(24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x - 33*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x - 56*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b*d**2*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 +
 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x + 84*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*c*d*e*f*x + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b
**2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x
+ 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 - 48*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f**
2 + 72*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*
x**2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 + 112*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f
- 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x*
*2 + b*d*x**4),x)*a*b**2*c**2*d*f**2 - 182*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f
- 70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x*
*2 + b*d*x**4),x)*a*b**2*d**3*e**2 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 + 42*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*b**3*c**2*d*e*f + 140*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)...
```

3.75 $\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 435

$$\begin{aligned}
&\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx \\
&= \frac{\left(\frac{8a^2df^2}{b}-af(20de+3cf)+b\left(15de^2+10cef-\frac{2c^2f^2}{d}\right)\right)x\sqrt{c+dx^2}}{15bd\sqrt{a+bx^2}} \\
&\quad+\frac{f(10bde+bcf-4adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d}+\frac{f^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b} \\
&\quad-\frac{\sqrt{a}(8a^2d^2f^2-abdf(20de+3cf)+b^2(15d^2e^2+10cdef-2c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{\sqrt{a}(15b^2de^2+4a^2df^2-abf(10de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/15*(8*a^2*d*f^2/b-a*f*(3*c*f+20*d*e)+b*(15*d*e^2+10*c*e*f-2*c^2*f^2/d))*
x*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)+1/15*f*(-4*a*d*f+b*c*f+10*b*d*e)*x*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d+1/5*f^2*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/b-1/15*a^(1/2)*(8*a^2*d^2*f^2-a*b*d*f*(3*c*f+20*d*e)+b^2*(-2*c^2*f^
2+10*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b
*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)+1/15*a^(1/2)*(15*b^2*d*e^2+4*a^2*d*f^2-a*b*f*(c*f+10*d*
e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^
(1/2))/b^(5/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-ic(8a^2d^2f^2 - abdf(20de+3cf) + b^2(15d^2e^2+10cdef-2c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
((-I)*c*(8*a^2*d^2*f^2 - a*b*d*f*(20*d*e + 3*c*f) + b^2*(15*d^2*e^2 + 10*c
*d*e*f - 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + f*(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*
x^2)*(4*a*d*f - b*(10*d*e + c*f + 3*d*f*x^2))) + (2*I)*c*(-(b*c) + a*d)*(-
5*b*d*e + b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}+\frac{2efx^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}+\frac{f^2x^4\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5b}+\frac{(bc-4ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15b^2d}+\frac{2ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{de^2\sqrt{bx^2+a}x}{b\sqrt{dx^2+c}}-\frac{\left(2b^2c^2+3abdc-8a^2d^2\right)f^2\sqrt{bx^2+a}x}{15b^3d\sqrt{dx^2+c}}+\frac{2(bc-2ad)ef\sqrt{bx^2+a}x}{3b^2\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{d}e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}\left(2b^2c^2+3abdc-8a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2\sqrt{c}(bc-2ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}(bc-4ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(d*e^2*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) + (2*(b*c - 2*a*d)*e*f*x*Sqr
t[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) - ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^
2)*f^2*x*Sqrt[a + b*x^2])/(15*b^3*d*Sqrt[c + d*x^2]) + (2*e*f*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*b) + ((b*c - 4*a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(15*b^2*d) + (f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b) -
(Sqrt[c]*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2]) - (2*Sqrt[c]*(b*c - 2*a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^
2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*b^3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) + (c^(3/2)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]) - (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*c - 4*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x^3 \sqrt{bd\, x^4+ad\, x^2+x^2 bc+ac}}{5b}+\frac{\left(c f^2+2def-\frac{f^2(4ad+4bc)}{5b}\right) x \sqrt{bd\, x^4+ad\, x^2+x^2 bc+ac}}{3bd}+\frac{\left(c e^2-\frac{\left(c f^2+2def-\frac{f^2(4ad}{5}\right.}{3bd}\right.}{}\right.}{}$
risch	$-\frac{fx(-3bdf\, x^2+4adf-bcf-10bde)\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{15b^2 d}+\frac{\left(-\frac{\left(8a^2d^2f^2-3abcd\, f^2-20ab\, d^2ef-2b^2c^2f^2+10b^2cdef+15b^2d^2e^2\right)c\sqrt{1+\frac{b x^2}{a}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\, x}}\right.}{}$
default	$-\frac{\sqrt{x^2 d+c}\,\sqrt{b x^2+a}\left(-3\sqrt{-\frac{b}{a}}\, b^2d^3f^2x^7+\sqrt{-\frac{b}{a}}\, ab\, d^3f^2x^5-4\sqrt{-\frac{b}{a}}\, b^2c\, d^2f^2x^5-10\sqrt{-\frac{b}{a}}\, b^2d^3ef\, x^5+4\sqrt{-\frac{b}{a}}\, a^2d^3f^2x^3-10\sqrt{-\frac{b}{a}}\right.}{}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b*f^2*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(c*f^2+2*d*e*f-1/5/b*f^2*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c*e^2-1/3*(c*f^2+2*d*e*f
-1/5/b*f^2*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(2*c*e*f+d*e^2-3/5*a/b*c*f^2-1/3*(c*f^2+2*d*e*f-1/5
/b*f^2*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx =$$

$$-\frac{(15\,b^2cd^2e^2+10\,(b^2c^2d-2\,abcd^2)ef-(2\,b^2c^3+3\,abc^2d-8\,a^2cd^2)f^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{a}{b}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
-1/15*((15*b^2*c*d^2*e^2 + 10*(b^2*c^2*d - 2*a*b*c*d^2)*e*f - (2*b^2*c^3 +
 3*a*b*c^2*d - 8*a^2*c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(
sqrt(-c/d)/x), a*d/(b*c)) - (15*(b^2*c*d^2 + b^2*d^3)*e^2 + 10*(b^2*c^2*d
- 2*a*b*c*d^2 - a*b*d^3)*e*f - (2*b^2*c^3 + 3*a*b*c^2*d - 4*a^2*d^3 - (8*a
^2 - a*b)*c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - (3*b^2*d^3*f^2*x^4 + 15*b^2*d^3*e^2 + 10*(b^2*c*d^2 - 2*a
*b*d^3)*e*f - (2*b^2*c^2*d + 3*a*b*c*d^2 - 8*a^2*d^3)*f^2 + (10*b^2*d^3*e*
f + (b^2*c*d^2 - 4*a*b*d^3)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^
3*d^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**2/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx=\int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx=\int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx=\int \frac{\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,f^2x+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc\,f^2x+10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdef\,x+3\sqrt{d\,x^2+}}{}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x + sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b*c*f**2*x + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f*x +
 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*f**2*x**3 + 8*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d
**2*f**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
+ b*c*x**2 + b*d*x**4),x)*a*b*c*d*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e*f - 2*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*b**2*c**2*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*d*e*f + 15*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),
x)*b**2*d**2*e**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x
**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f**2 - int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f**2 - 10*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x*
*4),x)*a*b*c*d*e*f + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d
*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*d*e**2)/(15*b**2*d)
```

3.76 $\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 302

$$\begin{aligned}\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{2f(3bde-bcf-adf)x\sqrt{c+dx^2}}{3bd^2\sqrt{a+bx^2}}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}\\ &\quad-\frac{2\sqrt{a}f(3bde-bcf-adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad+\frac{\sqrt{a}(3bde^2-acf^2)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
2/3*f*(-a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b/d^2/(b*x^2+a)^(1/2)+1/3*f
^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-2/3*a^(1/2)*f*(-a*d*f-b*c*f+3*b*d
*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b
/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3
*a^(1/2)*(-a*c*f^2+3*b*d*e^2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/
2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}df^2x(a+bx^2)(c+dx^2)+2icf(-3bde+bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(\ldots}{3b\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+\ldots}}$$

input
```
Integrate[(e + f*x^2)^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*f*(-3*b*d*e + b*c*f +
 a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - I*(a*c*d*f^2 + b*(3*d^2*e^2 - 6*c*d*e*f + 2*c^2*f^2
))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{2efx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{f^2x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{c}f^2\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}e^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{c}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2f^2x\sqrt{a+bx^2}(ad+bc)}{3b^2d\sqrt{c+dx^2}}+\frac{2efx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(2*e*f*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (2*(b*c + a*d)*f^2*x*Sqrt[
a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])/(3*b*d) - (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(b*c + a*d)*f^2*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*e^2*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*f^2*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.90 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x\sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3bd}+\frac{\left(e^2-\frac{a c f^2}{3bd}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+x^2 b c+a c}}-\frac{\left(2ef-\frac{f^2(2ad+2b}{3bd}\right.}{}\right.}{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}$
risch	$\frac{f^2 x\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{3bd}-\frac{\left(-\frac{2f(adf+bcf-3bde)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right.}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+x^2 b c+a c}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b d^2 f^2 x^5+\sqrt{-\frac{b}{a}}\,a d^2 f^2 x^3+\sqrt{-\frac{b}{a}}\,b c d f^2 x^3+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a c d f^2+2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{Elli}\right.}{}$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*f^2/b/d*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(e^2-1/3*a/b*c/d*f^2)/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*e*f-1/3*f^2/b/d*(2*a*d+2
*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left(3\,bc^2def-(bc^3+ac^2d)f^2\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(3\,bd^3e^2+6\,bc^2def-(2\,bc^3+2\,ac^2}{}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(2*(3*b*c^2*d*e*f - (b*c^3 + a*c^2*d)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b*d^3*e^2 + 6*b*c^2*d*e*f -
(2*b*c^3 + 2*a*c^2*d + a*c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arc
sin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d^2*f^2*x^2 + 6*b*c*d^2*e*f - 2*(b*c^
2*d + a*c*d^2)*f^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c*d^3*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f^2x - 2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad\,f^2 - 2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc\,f^2 + 6\left(\int \frac{\sqrt{d}}{bd\,x}\right.}{3bd}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f**2*x - 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f**2 - 2*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*b*c*f**2 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*d*e*f - int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f**2 + 3*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x
)*b*d*e**2)/(3*b*d)
```

3.77 $\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 279

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx = \frac{f^2x\sqrt{a+bx^2}}{bd\sqrt{c+dx^2}} + \frac{(2bcf(de-cf)-d(bde^2-acf^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c}d^{3/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(bde^2-af(2de-cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ad^{3/2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
f^2*x*(b*x^2+a)^(1/2)/b/d/(d*x^2+c)^(1/2)+(2*b*c*f*(-c*f+d*e)-d*(-a*c*f^2+
b*d*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/b/c^(1/2)/d^(3/2)/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+c^(1/2)*(b*d*e^2-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*Inv
erseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(3/2)/(-a*d+
b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.88

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{-ic(acdf^2-b(d^2e^2-2cdef+2c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.}{}$$

input

```
Integrate[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output

```
((-I)*c*(a*c*d*f^2 - b*(d^2*e^2 - 2*c*d*e*f + 2*c^2*f^2))*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
d*e - c*f)*(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - (2*I)*c*(-(b*c) + a*d)
*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 695 vs. 2(279) = 558.

Time = 0.90 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{e^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}+\frac{2efx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}+\frac{f^2x^4}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}f^2\sqrt{a+bx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{bd^{3/2}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{c}e^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}e^2\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{c}ef\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{f^2x\sqrt{a+bx^2}(2bc-ad)}{bd\sqrt{c+dx^2}(bc-ad)}-\frac{cf^2x\sqrt{a+bx^2}}{d\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
-((c*f^2*x*Sqrt[a + b*x^2])/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + ((2*b*c - a
*d)*f^2*x*Sqrt[a + b*x^2])/(b*d*(b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[d]*e^
2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) + (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(2*b*c - a*d)*f^2*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*d^(3/2)*(b*c - a*d)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*e^2*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sq
rt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) -
 (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2]) + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{(bd x^2+ad)x(c^2 f^2-2cdef+d^2e^2)}{c d^2(ad-bc)\sqrt{(x^2+\frac{c}{d})(bd x^2+ad)}}+\frac{\left(-\frac{f(cf-2de)}{d^2}+\frac{c^2f^2-2cdef+d^2e^2}{d^2 c}-\frac{a(c^2f^2-2cdef+d^2e^2)}{dc(ad-bc)}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x}{c}}}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b c^2 d f^2 x^3-2\sqrt{-\frac{b}{a}}\,bc d^2 ef x^3+\sqrt{-\frac{b}{a}}\,b d^3 e^2 x^3-2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a c^2 d f^2+2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d-}{c}}\right.}{}$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)
/c/d^2/(a*d-b*c)*x*(c^2*f^2-2*c*d*e*f+d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d))^(
1/2)+(-f*(c*f-2*d*e)/d^2+(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^2/c-a/d/c/(a*d-b*c)
*(c^2*f^2-2*c*d*e*f+d^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-(f^2/d-(c^2*f^2-2*c*d*e*f+d^2*e^2)/d/(a*d-b*c)*b/c)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((bcd^3e^2-2\,bc^2d^2ef+(2\,bc^3d-ac^2d^2)f^2)x^3+(bc^2d^2e^2-2\,bc^3def+(2\,bc^4-ac^3d)f^2)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\mathrm{ar}}{}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
-(((b*c*d^3*e^2 - 2*b*c^2*d^2*e*f + (2*b*c^3*d - a*c^2*d^2)*f^2)*x^3 + (b*
c^2*d^2*e^2 - 2*b*c^3*d*e*f + (2*b*c^4 - a*c^3*d)*f^2)*x)*sqrt(b*d)*sqrt(-
c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((b*c*d^3 + b*d^4)*e^2
 - 2*(b*c^2*d^2 + a*d^4)*e*f + (2*b*c^3*d - a*c^2*d^2 + a*c*d^3)*f^2)*x^3
+ ((b*c^2*d^2 + b*c*d^3)*e^2 - 2*(b*c^3*d + a*c*d^3)*e*f + (2*b*c^4 - a*c^
3*d + a*c^2*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (b*c*d^3*e^2 - 2*b*c^2*d^2*e*f + (b*c^2*d^2 - a*c*d^3)*f
^2*x^2 + (2*b*c^3*d - a*c^2*d^2)*f^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((b
^2*c^2*d^4 - a*b*c*d^5)*x^3 + (b^2*c^3*d^3 - a*b*c^2*d^4)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,efx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)b\,c^2f^2}{}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d
*x**4 + b*d**2*x**6),x)*b*c**2*f**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4
 + b*d**2*x**6),x)*b*c*d*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d*
*2*x**6),x)*b*c*d*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**
2*x**6),x)*b*d**2*e*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*
*2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6
),x)*a*c**2*e*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*
d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*c*d*
e*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2
+ a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b*c**2*e**2 +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b*c*d*e**2*x**2)/(b*c*(
c + d*x**2))
```

3.78 $\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 329

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = -\frac{(de-cf)^2x\sqrt{a+bx^2}}{3cd(bc-ad)\,(c+dx^2)^{3/2}} - \frac{2(de-cf)(bc(2de+cf)-ad(de+2cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{3/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(3b^2cde^2+3a^2cdf^2-ab(d^2e^2+4cdef+c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}d^{3/2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/c/d/(-a*d+b*c)/(d*x^2+c)^(3/2)-2/3*(-c
*f+d*e)*(b*c*(c*f+2*d*e)-a*d*(2*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2
)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(3/2)/(-a*d+b*c
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(3*b^2*c*d*e^2+3*a
^2*c*d*f^2-a*b*(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiA
M(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(3/2)/(-a*d+b*c
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.73 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)\left(-bc(c^2f+4d^2ex^2+cd(5e+2fx^2))+ad(3c^2f+\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2)*(-(b*c*(c^2*f + 4*d^2*e*x^2 + c*d*(
5*e + 2*f*x^2))) + a*d*(3*c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 4*f*x^2))) - (2
*I)*b*c*(-(d*e) + c*f)*(-(b*c*(2*d*e + c*f)) + a*d*(d*e + 2*c*f))*Sqrt[1 +
 (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(3*a*c*d*f^2 + b*(d^2*e^2 - 2*c*d*e*
f - 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d^2*(b*c - a*d)
^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 772 vs. 2(329) = 658.

Time = 0.94 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}+\frac{2efx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}+\frac{f^2x^4}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\right)dx$$

$\downarrow$ 2009

$$-\frac{2\sqrt{d}e^2\sqrt{a+bx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}f^2\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}f^2\sqrt{a+bx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{be^2\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4b\sqrt{c}ef\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2ef\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{de^2x\sqrt{a+bx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{2efx\sqrt{a+bx^2}}{3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{cf^2x\sqrt{a+bx^2}}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*(d*e^2*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) + (2*e*f*
x*Sqrt[a + b*x^2])/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) - (c*f^2*x*Sqrt[a + b
*x^2])/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - (2*Sqrt[d]*(2*b*c - a*d)*e^2*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
3*c^(3/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) + (2*(b*c + a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(b*c - 2*a*d)*f^2*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^
(3/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
 + (b*(3*b*c - a*d)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*b*Sqrt[c]*e*f*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*(b*c -
 a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*
(b*c - 3*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*d^(3/2)*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(306) = 612$.

Time = 8.72 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.12

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{x\left(c^2f^2-2cdef+d^2e^2\right)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,d^3(ad-bc)\left(x^2+\frac{c}{d}\right)^2}-\frac{2\left(bd\,x^2+ad\right)x\left(2a\,c^2d\,f^2-acef\,d^2-a\,d^3e^2-b\,c^3f^2-b\,c^2def+2bc\,d^2e^2\right.}{3c^2d^2(ad-bc)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/c/d^3/(a*
d-b*c)*x*(c^2*f^2-2*c*d*e*f+d^2*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(
x^2+c/d)^2-2/3*(b*d*x^2+a*d)/c^2/d^2/(a*d-b*c)^2*x*(2*a*c^2*d*f^2-a*c*d^2*
e*f-a*d^3*e^2-b*c^3*f^2-b*c^2*d*e*f+2*b*c*d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d
))^(1/2)+(f^2/d^2+1/3/d^2*b*(c^2*f^2-2*c*d*e*f+d^2*e^2)/(a*d-b*c)/c-2/3/d^
2/(a*d-b*c)*(2*a*c^2*d*f^2-a*c*d^2*e*f-a*d^3*e^2-b*c^3*f^2-b*c^2*d*e*f+2*b
*c*d^2*e^2)/c^2+2/3*a/d/c^2/(a*d-b*c)^2*(2*a*c^2*d*f^2-a*c*d^2*e*f-a*d^3*e
^2-b*c^3*f^2-b*c^2*d*e*f+2*b*c*d^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*b/d^2*(2*a*c^2*d*f^2-a*c*d^2*e*f-a*d^3*e^
2-b*c^3*f^2-b*c^2*d*e*f+2*b*c*d^2*e^2)/(a*d-b*c)^2/c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(306) = 612$.

Time = 0.12 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.98

$$\int \frac{\left(e+fx^2\right)^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
1/3*(2*(((2*b^3*c*d^4 - a*b^2*d^5)*e^2 - (b^3*c^2*d^3 + a*b^2*c*d^4)*e*f -
 (b^3*c^3*d^2 - 2*a*b^2*c^2*d^3)*f^2)*x^4 + (2*b^3*c^3*d^2 - a*b^2*c^2*d^3
)*e^2 - (b^3*c^4*d + a*b^2*c^3*d^2)*e*f - (b^3*c^5 - 2*a*b^2*c^4*d)*f^2 +
2*((2*b^3*c^2*d^3 - a*b^2*c*d^4)*e^2 - (b^3*c^3*d^2 + a*b^2*c^2*d^3)*e*f -
 (b^3*c^4*d - 2*a*b^2*c^3*d^2)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(a
rcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((((3*a*b^2 + 4*b^3)*c*d^4 - (a^2*b + 2*
a*b^2)*d^5)*e^2 - 2*(b^3*c^2*d^3 + (2*a^2*b + a*b^2)*c*d^4)*e*f - (2*b^3*c
^3*d^2 - 3*a^3*c*d^4 + (a^2*b - 4*a*b^2)*c^2*d^3)*f^2)*x^4 + ((3*a*b^2 + 4
*b^3)*c^3*d^2 - (a^2*b + 2*a*b^2)*c^2*d^3)*e^2 - 2*(b^3*c^4*d + (2*a^2*b +
 a*b^2)*c^3*d^2)*e*f - (2*b^3*c^5 - 3*a^3*c^3*d^2 + (a^2*b - 4*a*b^2)*c^4*
d)*f^2 + 2*(((3*a*b^2 + 4*b^3)*c^2*d^3 - (a^2*b + 2*a*b^2)*c*d^4)*e^2 - 2*
(b^3*c^3*d^2 + (2*a^2*b + a*b^2)*c^2*d^3)*e*f - (2*b^3*c^4*d - 3*a^3*c^2*d
^3 + (a^2*b - 4*a*b^2)*c^3*d^2)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*((2*a*b^2*c*d^4 - a^2*b*d^5)*e^2 - (
a*b^2*c^2*d^3 + a^2*b*c*d^4)*e*f - (a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3)*f^2)*
x^3 - (4*a*b^2*c^3*d^2*e*f - (5*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*e^2 + (a*b^
2*c^4*d - 3*a^2*b*c^3*d^2)*f^2)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^3
*c^6*d^2 - 2*a^2*b^2*c^5*d^3 + a^3*b*c^4*d^4 + (a*b^3*c^4*d^4 - 2*a^2*b^2*
c^3*d^5 + a^3*b*c^2*d^6)*x^4 + 2*(a*b^3*c^5*d^3 - 2*a^2*b^2*c^4*d^4 + a^3*
b*c^3*d^5)*x^2)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4
 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b
*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 -
 b**2*c*d**3*x**8),x)*a**2*c**2*d**2*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4
 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b
*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 -
 b**2*c*d**3*x**8),x)*a**2*c*d**3*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4
 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b
*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 -
 b**2*c*d**3*x**8),x)*a**2*d**4*f**2*x**4 - 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4
 + a**2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b
*d**4*x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 -
 b**2*c*d**3*x**8),x)*a*b*c**3*d*f**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**2 + 3*a**2*c*d**3*x**4 + a**
2*d**4*x**6 - a*b*c**4 - 2*a*b*c**3*d*x**2 + 2*a*b*c*d**3*x**6 + a*b*d**4*
x**8 - b**2*c**4*x**2 - 3*b**2*c**3*d*x**4 - 3*b**2*c**2*d**2*x**6 - b*...
```

3.79 $\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 487

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = -\frac{(de-cf)^2x\sqrt{a+bx^2}}{5cd(bc-ad)\,(c+dx^2)^{5/2}}$$
$$-\frac{2(de-cf)(bc(4de+cf)-ad(2de+3cf))x\sqrt{a+bx^2}}{15c^2d(bc-ad)^2\,(c+dx^2)^{3/2}}$$
$$+\frac{(abcd(23d^2e^2+14cdef-7c^2f^2)-b^2c^2(23d^2e^2-6cdef-2c^2f^2)-a^2d^2(8d^2e^2+4cdef+3c^2f^2))\sqrt{a+}}{15c^{5/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{b(15b^2c^2de^2-abc(11d^2e^2+18cdef+c^2f^2)+a^2d(4d^2e^2+2cdef+9c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{15ac^{3/2}d^{3/2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/5*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/c/d/(-a*d+b*c)/(d*x^2+c)^(5/2)-2/15*(-
c*f+d*e)*(b*c*(c*f+4*d*e)-a*d*(3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)/c^2/d/(-a*d
+b*c)^2/(d*x^2+c)^(3/2)+1/15*(a*b*c*d*(-7*c^2*f^2+14*c*d*e*f+23*d^2*e^2)-b
^2*c^2*(-2*c^2*f^2-6*c*d*e*f+23*d^2*e^2)-a^2*d^2*(3*c^2*f^2+4*c*d*e*f+8*d^
2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(5/2)/d^(3/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+1/15*b*(15*b^2*c^2*d*e^2-a*b*c*(c^2*f^2+18*c*d*e*f+11*d^
2*e^2)+a^2*d*(9*c^2*f^2+2*c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacob
iAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(3/2)/d^(3/2)/(-a*d+b
*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.66 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2+2c(bc-ad)(de-cf)(bc(4de\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + 2*c*(b*c
 - a*d)*(d*e - c*f)*(b*c*(4*d*e + c*f) - a*d*(2*d*e + 3*c*f))*(c + d*x^2)
+ (b^2*c^2*(23*d^2*e^2 - 6*c*d*e*f - 2*c^2*f^2) + a^2*d^2*(8*d^2*e^2 + 4*c
*d*e*f + 3*c^2*f^2) + a*b*c*d*(-23*d^2*e^2 - 14*c*d*e*f + 7*c^2*f^2))*(c +
 d*x^2)^2)) + I*b*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*
((a*b*c*d*(23*d^2*e^2 + 14*c*d*e*f - 7*c^2*f^2) + b^2*c^2*(-23*d^2*e^2 + 6
*c*d*e*f + 2*c^2*f^2) - a^2*d^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2))*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 2*(b*c - a*d)*(-(d*e) + c*f)*(
b*c*(4*d*e + c*f) - a*d*(2*d*e + 3*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^2*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c + d
*x^2)^(5/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 995 vs. 2(487) = 974.

Time = 1.33 (sec) , antiderivative size = 995, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}+\frac{2efx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}+\frac{f^2x^4}{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d}\left(23b^2c^2-23abdc+8a^2d^2\right)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{15c^{5/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{b\left(15b^2c^2-11abdc+4a^2d^2\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{15ac^{3/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4d(2bc-ad)x\sqrt{bx^2+a}e^2}{15c^2(bc-ad)^2\left(dx^2+c\right)^{3/2}}-\frac{dx\sqrt{bx^2+a}e^2}{5c(bc-ad)\left(dx^2+c\right)^{5/2}}+\frac{2\left(3b^2c^2+7abdc-2a^2d^2\right)f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{15c^{3/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2b(9bc-ad)f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{15\sqrt{c}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(3bc+ad)fx\sqrt{bx^2+a}e}{15c(bc-ad)^2\left(dx^2+c\right)^{3/2}}+\frac{2fx\sqrt{bx^2+a}e}{5(bc-ad)\left(dx^2+c\right)^{5/2}}+\frac{\left(2b^2c^2-7abdc-3a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{3/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(bc-9ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(bc-3ad)f^2x\sqrt{bx^2+a}}{15d(bc-ad)^2\left(dx^2+c\right)^{3/2}}-\frac{cf^2x\sqrt{bx^2+a}}{5d(bc-ad)\left(dx^2+c\right)^{5/2}}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
-1/5*(d*e^2*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) + (2*e*f*
x*Sqrt[a + b*x^2])/(5*(b*c - a*d)*(c + d*x^2)^(5/2)) - (c*f^2*x*Sqrt[a + b
*x^2])/(5*d*(b*c - a*d)*(c + d*x^2)^(5/2)) - (4*d*(2*b*c - a*d)*e^2*x*Sqrt
[a + b*x^2])/(15*c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (2*(3*b*c + a*d)*e
*f*x*Sqrt[a + b*x^2])/(15*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (2*(b*c - 3
*a*d)*f^2*x*Sqrt[a + b*x^2])/(15*d*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (Sqr
t[d]*(23*b^2*c^2 - 23*a*b*c*d + 8*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*c^(5/2)*(b*c - a*d)^3*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*(3*b^2*c^2 + 7*a
*b*c*d - 2*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(15*c^(3/2)*Sqrt[d]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^
2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*Sqrt[c]*d^(3/2)*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]) + (b*(15*b^2*c^2 - 11*a*b*c*d + 4*a^2*d^2)*e^2*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a*c
^(3/2)*Sqrt[d]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) - (2*b*(9*b*c - a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*Sqrt[c]*Sqrt[d]*(b*c - a*d)^3*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b*c - 9*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(458) = 916$.

Time = 10.98 (sec) , antiderivative size = 1039, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1039
default	Expression too large to display	4448

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/c/d^4/(a*
d-b*c)*x*(c^2*f^2-2*c*d*e*f+d^2*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(
x^2+c/d)^3-2/15*(3*a*c^2*d*f^2-a*c*d^2*e*f-2*a*d^3*e^2-b*c^3*f^2-3*b*c^2*d
*e*f+4*b*c*d^2*e^2)/d^3/(a*d-b*c)^2/c^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(x^2+c/d)^2+1/15*(b*d*x^2+a*d)/c^3/d^2/(a*d-b*c)^3*x*(3*a^2*c^2*d^2*f^
2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2+7*a*b*c^3*d*f^2-14*a*b*c^2*d^2*e*f-23*a*b*
c*d^3*e^2-2*b^2*c^4*f^2-6*b^2*c^3*d*e*f+23*b^2*c^2*d^2*e^2)/((x^2+c/d)*(b*
d*x^2+a*d))^(1/2)+(-2/15*b*(3*a*c^2*d*f^2-a*c*d^2*e*f-2*a*d^3*e^2-b*c^3*f^
2-3*b*c^2*d*e*f+4*b*c*d^2*e^2)/c^2/d^2/(a*d-b*c)^2+1/15/d^2/(a*d-b*c)^2*(3
*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4*e^2+7*a*b*c^3*d*f^2-14*a*b*c^2*
d^2*e*f-23*a*b*c*d^3*e^2-2*b^2*c^4*f^2-6*b^2*c^3*d*e*f+23*b^2*c^2*d^2*e^2)
/c^3-1/15*a/d/c^3/(a*d-b*c)^3*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*d^4
*e^2+7*a*b*c^3*d*f^2-14*a*b*c^2*d^2*e*f-23*a*b*c*d^3*e^2-2*b^2*c^4*f^2-6*b
^2*c^3*d*e*f+23*b^2*c^2*d^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))+1/15*b/d^2*(3*a^2*c^2*d^2*f^2+4*a^2*c*d^3*e*f+8*a^2*
d^4*e^2+7*a*b*c^3*d*f^2-14*a*b*c^2*d^2*e*f-23*a*b*c*d^3*e^2-2*b^2*c^4*f^2-
6*b^2*c^3*d*e*f+23*b^2*c^2*d^2*e^2)/(a*d-b*c)^3/c^2/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(457) = 914$.

Time = 0.16 (sec) , antiderivative size = 1811, normalized size of antiderivative = 3.72

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas
")
```

output
```
1/15*(((((23*b^3*c^2*d^5 - 23*a*b^2*c*d^6 + 8*a^2*b*d^7)*e^2 - 2*(3*b^3*c^3
*d^4 + 7*a*b^2*c^2*d^5 - 2*a^2*b*c*d^6)*e*f - (2*b^3*c^4*d^3 - 7*a*b^2*c^3
*d^4 - 3*a^2*b*c^2*d^5)*f^2)*x^6 + 3*((23*b^3*c^3*d^4 - 23*a*b^2*c^2*d^5 +
 8*a^2*b*c*d^6)*e^2 - 2*(3*b^3*c^4*d^3 + 7*a*b^2*c^3*d^4 - 2*a^2*b*c^2*d^5
)*e*f - (2*b^3*c^5*d^2 - 7*a*b^2*c^4*d^3 - 3*a^2*b*c^3*d^4)*f^2)*x^4 + (23
*b^3*c^5*d^2 - 23*a*b^2*c^4*d^3 + 8*a^2*b*c^3*d^4)*e^2 - 2*(3*b^3*c^6*d +
7*a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3)*e*f - (2*b^3*c^7 - 7*a*b^2*c^6*d - 3*a^
2*b*c^5*d^2)*f^2 + 3*((23*b^3*c^4*d^3 - 23*a*b^2*c^3*d^4 + 8*a^2*b*c^2*d^5
)*e^2 - 2*(3*b^3*c^5*d^2 + 7*a*b^2*c^4*d^3 - 2*a^2*b*c^3*d^4)*e*f - (2*b^3
*c^6*d - 7*a*b^2*c^5*d^2 - 3*a^2*b*c^4*d^3)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)
*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((((15*a*b^2 + 23*b^3)*c^2*
d^5 - (11*a^2*b + 23*a*b^2)*c*d^6 + 4*(a^3 + 2*a^2*b)*d^7)*e^2 - 2*(3*b^3*
c^3*d^4 + (9*a^2*b + 7*a*b^2)*c^2*d^5 - (a^3 + 2*a^2*b)*c*d^6)*e*f - (2*b^
3*c^4*d^3 + (a^2*b - 7*a*b^2)*c^3*d^4 - 3*(3*a^3 + a^2*b)*c^2*d^5)*f^2)*x^
6 + 3*(((15*a*b^2 + 23*b^3)*c^3*d^4 - (11*a^2*b + 23*a*b^2)*c^2*d^5 + 4*(a
^3 + 2*a^2*b)*c*d^6)*e^2 - 2*(3*b^3*c^4*d^3 + (9*a^2*b + 7*a*b^2)*c^3*d^4
- (a^3 + 2*a^2*b)*c^2*d^5)*e*f - (2*b^3*c^5*d^2 + (a^2*b - 7*a*b^2)*c^4*d^
3 - 3*(3*a^3 + a^2*b)*c^3*d^4)*f^2)*x^4 + ((15*a*b^2 + 23*b^3)*c^5*d^2 - (
11*a^2*b + 23*a*b^2)*c^4*d^3 + 4*(a^3 + 2*a^2*b)*c^3*d^4)*e^2 - 2*(3*b^3*c
^6*d + (9*a^2*b + 7*a*b^2)*c^5*d^2 - (a^3 + 2*a^2*b)*c^4*d^3)*e*f - (2*...
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + 4*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d
**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d
*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 +
2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*
x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**2*c**3*d**2*f**2 +
 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c
**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*
x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d*
*3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**5*x**10 - b**2*c**5*x**2 - 4*b**2*c
**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*b**2*c**2*d**3*x**8 - b**2*c*d**4*x
**10),x)*a**2*c**2*d**3*f**2*x**2 + 12*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2*x**2 + 12*a**2*c**2*d**3*x**4
+ 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a*b*c**5 - 2*a*b*c**4*d*x**2 + 2
*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x**6 + 7*a*b*c*d**4*x**8 + 2*a*b*d**
5*x**10 - b**2*c**5*x**2 - 4*b**2*c**4*d*x**4 - 6*b**2*c**3*d**2*x**6 - 4*
b**2*c**2*d**3*x**8 - b**2*c*d**4*x**10),x)*a**2*c*d**4*f**2*x**4 + 4*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**2*c**4*d + 8*a**2*c**3*d**2
*x**2 + 12*a**2*c**2*d**3*x**4 + 8*a**2*c*d**4*x**6 + 2*a**2*d**5*x**8 - a
*b*c**5 - 2*a*b*c**4*d*x**2 + 2*a*b*c**3*d**2*x**4 + 8*a*b*c**2*d**3*x*...
```

3.80 $\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 649

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx =$$

$$-\frac{(192a^3d^3f^2-12a^2bd^2f(28de+33cf)-b^3c(245d^2e^2+322cdef+15c^2f^2)+ab^2d(140d^2e^2+658cdef+}{105b^4d\sqrt{a+bx^2}}$$

$$+\frac{(48a^2d^2f^2-3abdf(28de+31cf)+b^2(35d^2e^2+154cdef+45c^2f^2))\,x^3\sqrt{c+dx^2}}{105b^3\sqrt{a+bx^2}}$$

$$+\frac{df(14bde+15bcf-8adf)x^5\sqrt{c+dx^2}}{35b^2\sqrt{a+bx^2}}+\frac{d^2f^2x^7\sqrt{c+dx^2}}{7b\sqrt{a+bx^2}}$$

$$+\frac{(105b^4c^2de^2+384a^4d^3f^2-24a^3bd^2f(28de+31cf)-ab^3c(455d^2e^2+532cdef+15c^2f^2)+a^2b^2d(280d^2e}{105\sqrt{a}b^{9/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{2\sqrt{a}(96a^3d^2f^2-105b^3ce(de+cf)-6a^2bdf(28de+29cf)+ab^2(70d^2e^2+287cdef+75c^2f^2))\sqrt{c+dx}}{105b^{9/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/105*(192*a^3*d^3*f^2-12*a^2*b*d^2*f*(33*c*f+28*d*e)-b^3*c*(15*c^2*f^2+3
22*c*d*e*f+245*d^2*e^2)+a*b^2*d*(219*c^2*f^2+658*c*d*e*f+140*d^2*e^2))*x*(
d*x^2+c)^(1/2)/b^4/d/(b*x^2+a)^(1/2)+1/105*(48*a^2*d^2*f^2-3*a*b*d*f*(31*c
*f+28*d*e)+b^2*(45*c^2*f^2+154*c*d*e*f+35*d^2*e^2))*x^3*(d*x^2+c)^(1/2)/b^
3/(b*x^2+a)^(1/2)+1/35*d*f*(-8*a*d*f+15*b*c*f+14*b*d*e)*x^5*(d*x^2+c)^(1/2
)/b^2/(b*x^2+a)^(1/2)+1/7*d^2*f^2*x^7*(d*x^2+c)^(1/2)/b/(b*x^2+a)^(1/2)+1/
105*(105*b^4*c^2*d*e^2+384*a^4*d^3*f^2-24*a^3*b*d^2*f*(31*c*f+28*d*e)-a*b^
3*c*(15*c^2*f^2+532*c*d*e*f+455*d^2*e^2)+a^2*b^2*d*(369*c^2*f^2+1232*c*d*e
*f+280*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(
1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(9/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)-2/105*a^(1/2)*(96*a^3*d^2*f^2-105*b^3*c*e*(c*f+d*e)-6*a^2*
b*d*f*(29*c*f+28*d*e)+a*b^2*(75*c^2*f^2+287*c*d*e*f+70*d^2*e^2))*(d*x^2+c)
^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(9/2
)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(105b^4c^2e^2+192a^4d^2f^2+12a^3bdf(-28de-29cf+\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(c + d*x^2)*(105*b^4*c^2*e^2 + 192*a^4*d^2*f^2 +
 12*a^3*b*d*f*(-28*d*e - 29*c*f + 4*d*f*x^2) + a^2*b^2*(150*c^2*f^2 + c*d*
f*(574*e - 93*f*x^2) + 4*d^2*(35*e^2 - 21*e*f*x^2 - 6*f^2*x^4)) + a*b^3*(1
5*c^2*f*(-14*e + 3*f*x^2) + d^2*x^2*(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4) + c
*d*(-210*e^2 + 154*e*f*x^2 + 45*f^2*x^4))) + I*c*(105*b^4*c^2*d*e^2 + 384*
a^4*d^3*f^2 - 24*a^3*b*d^2*f*(28*d*e + 31*c*f) - a*b^3*c*(455*d^2*e^2 + 53
2*c*d*e*f + 15*c^2*f^2) + a^2*b^2*d*(280*d^2*e^2 + 1232*c*d*e*f + 369*c^2*
f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-105*b^3*c*d*e^2 + 192*a^3*d^2*f^
2 - 12*a^2*b*d*f*(28*d*e + 17*c*f) + a*b^2*(140*d^2*e^2 + 322*c*d*e*f + 15
*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)]))/(105*b^5*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 1252, normalized size of antiderivative = 1.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^2}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}+\frac{2efx^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}+\frac{f^2x^4\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{f^2\left(dx^2+c\right)^{5/2}x^3}{b\sqrt{bx^2+a}}+\frac{8df^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x^3}{7b^2}+\frac{3d(15bc-16ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35b^3}-\frac{2ef\left(dx^2+c\right)^{5/2}x}{b\sqrt{bx^2+a}}+\frac{12def\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x}{5b^2}+\frac{(bc-ad)e^2\left(dx^2+c\right)^{3/2}x}{ab\sqrt{bx^2+a}}-\frac{d(3bc-4ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3ab^2}+\frac{2\left(25b^2c^2-58abdc+32a^2d^2\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{35b^4}+\frac{2d(23bc-24ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15b^3}-\frac{d\left(3b^2c^2-13abdc+8a^2d^2\right)e^2\sqrt{bx^2+a}x}{3ab^3\sqrt{dx^2+c}}+\frac{\left(5b^3c^3-123ab^2dc^2+248a^2bd^2c-128a^3d^3\right)f^2\sqrt{bx^2+a}x}{35b^5\sqrt{dx^2+c}}+\frac{4d\left(19b^2c^2-44abdc+24a^2d^2\right)ef\sqrt{bx^2+a}x}{15b^4\sqrt{dx^2+c}}+\frac{\sqrt{c}\sqrt{d}\left(3b^2c^2-13abdc+8a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(5b^3c^3-123ab^2dc^2+248a^2bd^2c-128a^3d^3\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^5\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4\sqrt{c}\sqrt{d}\left(19b^2c^2-44abdc+24a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}\sqrt{d}(3bc-2ad)e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}\left(25b^2c^2-58abdc+32a^2d^2\right)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35b^4\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}\left(15b^2c^2-41abdc+24a^2d^2\right)ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15ab^3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}
$$

input
```
Int[((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
-1/3*(d*(3*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(a*b^3
*Sqrt[c + d*x^2]) + (4*d*(19*b^2*c^2 - 44*a*b*c*d + 24*a^2*d^2)*e*f*x*Sqrt
[a + b*x^2])/(15*b^4*Sqrt[c + d*x^2]) + ((5*b^3*c^3 - 123*a*b^2*c^2*d + 24
8*a^2*b*c*d^2 - 128*a^3*d^3)*f^2*x*Sqrt[a + b*x^2])/(35*b^5*Sqrt[c + d*x^2
]) - (d*(3*b*c - 4*a*d)*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a*b^2) +
 (2*d*(23*b*c - 24*a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b^3) +
(2*(25*b^2*c^2 - 58*a*b*c*d + 32*a^2*d^2)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(35*b^4) + (3*d*(15*b*c - 16*a*d)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(35*b^3) + ((b*c - a*d)*e^2*x*(c + d*x^2)^(3/2))/(a*b*Sqrt[a + b*x
^2]) + (12*d*e*f*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*b^2) + (8*d*f^2*x
^3*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(7*b^2) - (2*e*f*x*(c + d*x^2)^(5/2)
)/(b*Sqrt[a + b*x^2]) - (f^2*x^3*(c + d*x^2)^(5/2))/(b*Sqrt[a + b*x^2]) +
(Sqrt[c]*Sqrt[d]*(3*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*e^2*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^3*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*Sqrt[c]*Sqrt[d]*(19*b^
2*c^2 - 44*a*b*c*d + 24*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^4*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(5*b^3*c^3 - 123*a*b^2*c^2*d + 248*a^2
*b*c*d^2 - 128*a^3*d^3)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(35*b^5*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 21.45 (sec) , antiderivative size = 1088, normalized size of antiderivative = 1.68

method	result	size
risch	Expression too large to display	1088
elliptic	Expression too large to display	1638
default	Expression too large to display	1983

input

```
int((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/105/b^4*x*(15*b^2*d^2*f^2*x^4-39*a*b*d^2*f^2*x^2+45*b^2*c*d*f^2*x^2+42*b
^2*d^2*e*f*x^2+87*a^2*d^2*f^2-138*a*b*c*d*f^2-126*a*b*d^2*e*f+45*b^2*c^2*f
^2+154*b^2*c*d*e*f+35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/105/b
^4*(-(279*a^3*d^3*f^2-534*a^2*b*c*d^2*f^2-462*a^2*b*d^3*e*f+264*a*b^2*c^2*
d*f^2+812*a*b^2*c*d^2*e*f+175*a*b^2*d^3*e^2-15*b^3*c^3*f^2-322*b^3*c^2*d*e
*f-245*b^3*c*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+105*(a^
5*d^3*f^2-3*a^4*b*c*d^2*f^2-2*a^4*b*d^3*e*f+3*a^3*b^2*c^2*d*f^2+6*a^3*b^2*
c*d^2*e*f+a^3*b^2*d^3*e^2-a^2*b^3*c^3*f^2-6*a^2*b^3*c^2*d*e*f-3*a^2*b^3*c*
d^2*e^2+2*a*b^4*c^3*e*f+3*a*b^4*c^2*d*e^2-b^5*c^3*e^2)/b*(-(b*d*x^2+b*c)/a
/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-(105*a^4*d^3*f^2-402*a^3*b*c*d^2*f^
2-210*a^3*b*d^3*e*f+453*a^2*b^2*c^2*d*f^2+756*a^2*b^2*c*d^2*e*f+105*a^2*b^
2*d^3*e^2-150*a*b^3*c^3*f^2-784*a*b^3*c^2*d*e*f-350*a*b^3*c*d^2*e^2+210*b^
4*c^3*e*f+315*b^4*c^2*d*e^2)/b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(608) = 1216$.

Time = 0.12 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.89

$$\int \frac{(c + dx^2)^{5/2} (e + fx^2)^2}{(a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
1/105*(((35*(3*b^5*c^3*d - 13*a*b^4*c^2*d^2 + 8*a^2*b^3*c*d^3)*e^2 - 28*(1
9*a*b^4*c^3*d - 44*a^2*b^3*c^2*d^2 + 24*a^3*b^2*c*d^3)*e*f - 3*(5*a*b^4*c^
4 - 123*a^2*b^3*c^3*d + 248*a^3*b^2*c^2*d^2 - 128*a^4*b*c*d^3)*f^2)*x^3 +
(35*(3*a*b^4*c^3*d - 13*a^2*b^3*c^2*d^2 + 8*a^3*b^2*c*d^3)*e^2 - 28*(19*a^
2*b^3*c^3*d - 44*a^3*b^2*c^2*d^2 + 24*a^4*b*c*d^3)*e*f - 3*(5*a^2*b^3*c^4
- 123*a^3*b^2*c^3*d + 248*a^4*b*c^2*d^2 - 128*a^5*c*d^3)*f^2)*x)*sqrt(b*d)
*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((35*(3*b^5*c^3*
d - 13*a*b^4*c^2*d^2 + 4*a^2*b^3*d^4 + 2*(4*a^2*b^3 - 3*a*b^4)*c*d^3)*e^2
- 14*(38*a*b^4*c^3*d + 24*a^3*b^2*d^4 - (88*a^2*b^3 - 15*a*b^4)*c^2*d^2 +
(48*a^3*b^2 - 41*a^2*b^3)*c*d^3)*e*f - 3*(5*a*b^4*c^4 - 123*a^2*b^3*c^3*d
- 64*a^4*b*d^4 + 2*(124*a^3*b^2 - 25*a^2*b^3)*c^2*d^2 - 4*(32*a^4*b - 29*a
^3*b^2)*c*d^3)*f^2)*x^3 + (35*(3*a*b^4*c^3*d - 13*a^2*b^3*c^2*d^2 + 4*a^3*
b^2*d^4 + 2*(4*a^3*b^2 - 3*a^2*b^3)*c*d^3)*e^2 - 14*(38*a^2*b^3*c^3*d + 24
*a^4*b*d^4 - (88*a^3*b^2 - 15*a^2*b^3)*c^2*d^2 + (48*a^4*b - 41*a^3*b^2)*c
*d^3)*e*f - 3*(5*a^2*b^3*c^4 - 123*a^3*b^2*c^3*d - 64*a^5*d^4 + 2*(124*a^4
*b - 25*a^3*b^2)*c^2*d^2 - 4*(32*a^5 - 29*a^4*b)*c*d^3)*f^2)*x)*sqrt(b*d)*
sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*a*b^4*d^4*f^2
*x^8 + 3*(14*a*b^4*d^4*e*f + (15*a*b^4*c*d^3 - 8*a^2*b^3*d^4)*f^2)*x^6 + (
35*a*b^4*d^4*e^2 + 14*(11*a*b^4*c*d^3 - 6*a^2*b^3*d^4)*e*f + 3*(15*a*b^4*c
^2*d^2 - 31*a^2*b^3*c*d^3 + 16*a^3*b^2*d^4)*f^2)*x^4 - 35*(3*a*b^4*c^2*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(5/2)*(e + f*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

output

```
( - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c*d**2*f**2*x + 96*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x**3 + 279*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a**2*b*c**2*d*f**2*x + 252*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
**2*b*c*d**2*e*f*x - 186*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f
**2*x**3 - 168*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*e*f*x**3 - 48
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f**2*x**5 - 135*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b**2*c**3*f**2*x - 462*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b**2*c**2*d*e*f*x + 90*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*
c**2*d*f**2*x**3 - 105*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e**
2*x + 308*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x**3 + 90*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**5 + 70*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b**2*d**3*e**2*x**3 + 84*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b**2*d**3*e*f*x**5 + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2
*d**3*f**2*x**7 + 210*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*e*f*x +
315*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*e**2*x - 384*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2
*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**5*d**4*f**2 + 936*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**4*b*c*d**3*f**2 + 672*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b...
```

3.81 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 456

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{(24a^2d^2f^2-abdf(40de+27cf)+b^2(15d^2e^2+40cdef+3c^2f^2))\,x\sqrt{c+dx}}{15b^3d\sqrt{a+bx^2}}$$

$$+\frac{2f(5bde+3bcf-3adf)x^3\sqrt{c+dx^2}}{15b^2\sqrt{a+bx^2}}+\frac{df^2x^5\sqrt{c+dx^2}}{5b\sqrt{a+bx^2}}$$

$$+\frac{(15b^3cde^2-48a^3d^2f^2+16a^2bdf(5de+3cf)-ab^2(30d^2e^2+70cdef+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{15\sqrt{a}b^{7/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{a}(24a^2df^2+15b^2e(de+2cf)-abf(40de+21cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(24*a^2*d^2*f^2-a*b*d*f*(27*c*f+40*d*e)+b^2*(3*c^2*f^2+40*c*d*e*f+15*
d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d/(b*x^2+a)^(1/2)+2/15*f*(-3*a*d*f+3*b*c*f
+5*b*d*e)*x^3*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)+1/5*d*f^2*x^5*(d*x^2+c)^
(1/2)/b/(b*x^2+a)^(1/2)+1/15*(15*b^3*c*d*e^2-48*a^3*d^2*f^2+16*a^2*b*d*f*(
3*c*f+5*d*e)-a*b^2*(3*c^2*f^2+70*c*d*e*f+30*d^2*e^2))*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(7/
2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^(1/2)*(24*a^2*
d*f^2+15*b^2*e*(2*c*f+d*e)-a*b*f*(21*c*f+40*d*e))*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/(b*x^2+a)^(1/
2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(15b^3ce^2-24a^3df^2+a^2bf(40de+21cf-6dfx^2)+\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(c + d*x^2)*(15*b^3*c*e^2 - 24*a^3*d*f^2 + a^2*b
*f*(40*d*e + 21*c*f - 6*d*f*x^2) + a*b^2*(6*c*f*(-5*e + f*x^2) + d*(-15*e^
2 + 10*e*f*x^2 + 3*f^2*x^4))) - I*c*(-15*b^3*c*d*e^2 + 48*a^3*d^2*f^2 - 16
*a^2*b*d*f*(5*d*e + 3*c*f) + a*b^2*(30*d^2*e^2 + 70*c*d*e*f + 3*c^2*f^2))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(15*b^2*d*e^2 + 24*a^2*d*f^2 - a*b*f*(40
*d*e + 3*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b^4*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. 2(456) = 912.

Time = 1.34 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}+\frac{2efx^2(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}+\frac{f^2x^4(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2\left(dx^2+c\right)^{3/2}x^3}{b\sqrt{bx^2+a}}+\frac{6df^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5b^2}-\frac{2ef\left(dx^2+c\right)^{3/2}x}{b\sqrt{bx^2+a}}+\frac{(7bc-8ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{5b^3}+\frac{8def\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b^2}+\frac{(bc-ad)e^2\sqrt{dx^2+c}x}{ab\sqrt{bx^2+a}}-\frac{d(bc-2ad)e^2\sqrt{bx^2+a}x}{ab^2\sqrt{dx^2+c}}+\frac{\left(b^2c^2-16abdc+16a^2d^2\right)f^2\sqrt{bx^2+a}x}{5b^4\sqrt{dx^2+c}}+\frac{2d(7bc-8ad)ef\sqrt{bx^2+a}x}{3b^3\sqrt{dx^2+c}}+\frac{\sqrt{c}\sqrt{d}(bc-2ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(b^2c^2-16abdc+16a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2\sqrt{c}\sqrt{d}(7bc-8ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}\sqrt{d}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}(7bc-8ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}(3bc-4ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
-((d*(b*c - 2*a*d)*e^2*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) + (2*d*
(7*b*c - 8*a*d)*e*f*x*Sqrt[a + b*x^2])/(3*b^3*Sqrt[c + d*x^2]) + ((b^2*c^2
 - 16*a*b*c*d + 16*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(5*b^4*Sqrt[c + d*x^2])
 + ((b*c - a*d)*e^2*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (8*d*e*f*x*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b^2) + ((7*b*c - 8*a*d)*f^2*x*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(5*b^3) + (6*d*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(5*b^2) - (2*e*f*x*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2]) - (f^2*x^
3*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*
e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*Sqrt
[c]*Sqrt[d]*(7*b*c - 8*a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*f^2*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^
4*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2
)*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(a*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
 (2*c^(3/2)*(3*b*c - 4*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(7*b*c - 8*a*d)*f^2*Sqrt[a + b*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(421) = 842$.

Time = 19.02 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{fx(-3bdf\,x^2+9adf-6bcf-10bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^3}+\frac{\left(-\frac{\left(33a^2d^2f^2-33abcd\,f^2-50ab\,d^2ef+3b^2c^2f^2+40b^2cdef+15b^2d^2e^2\right)c\sqrt{1+\cdots}}{\sqrt{-\frac{b}{a}}\,\sqrt{b\cdots}}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(bd\,x^2+bc\right)\left(a^3d\,f^2-a^2c\,f^2b-2a^2bdef+2acef\,b^2+a\,b^2d\,e^2-b^3c\,e^2\right)x}{a\,b^4\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{d\,f^2x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5b^2}+\frac{\left(-\frac{df(adf-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*f/b^3*x*(-3*b*d*f*x^2+9*a*d*f-6*b*c*f-10*b*d*e)*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)+1/15/b^3*(-(33*a^2*d^2*f^2-33*a*b*c*d*f^2-50*a*b*d^2*e*f+3*b^2
*c^2*f^2+40*b^2*c*d*e*f+15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2)))+15*(a^4*d^2*f^2-2*a^3*b*c*d*f^2-2*a^3*b*d^2*e*f+a^2*b^2*c^2*f^2
+4*a^2*b^2*c*d*e*f+a^2*b^2*d^2*e^2-2*a*b^3*c^2*e*f-2*a*b^3*c*d*e^2+b^4*c^2
*e^2)/b*(-(b*d*x^2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/a
+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-(15*a^3*d^
2*f^2-39*a^2*b*c*d*f^2-30*a^2*b*d^2*e*f+21*a*b^2*c^2*f^2+70*a*b^2*c*d*e*f+
15*a*b^2*d^2*e^2-30*b^3*c^2*e*f-30*b^3*c*d*e^2)/b/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+
a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
1/15*(((15*(b^4*c^2*d - 2*a*b^3*c*d^2)*e^2 - 10*(7*a*b^3*c^2*d - 8*a^2*b^2
*c*d^2)*e*f - 3*(a*b^3*c^3 - 16*a^2*b^2*c^2*d + 16*a^3*b*c*d^2)*f^2)*x^3 +
 (15*(a*b^3*c^2*d - 2*a^2*b^2*c*d^2)*e^2 - 10*(7*a^2*b^2*c^2*d - 8*a^3*b*c
*d^2)*e*f - 3*(a^2*b^2*c^3 - 16*a^3*b*c^2*d + 16*a^4*c*d^2)*f^2)*x)*sqrt(b
*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((15*(b^4*c^2
*d - 2*a*b^3*c*d^2 - a*b^3*d^3)*e^2 - 10*(7*a*b^3*c^2*d - 4*a^2*b^2*d^3 -
(8*a^2*b^2 - 3*a*b^3)*c*d^2)*e*f - 3*(a*b^3*c^3 - 16*a^2*b^2*c^2*d + 8*a^3
*b*d^3 + (16*a^3*b - 7*a^2*b^2)*c*d^2)*f^2)*x^3 + (15*(a*b^3*c^2*d - 2*a^2
*b^2*c*d^2 - a^2*b^2*d^3)*e^2 - 10*(7*a^2*b^2*c^2*d - 4*a^3*b*d^3 - (8*a^3
*b - 3*a^2*b^2)*c*d^2)*e*f - 3*(a^2*b^2*c^3 - 16*a^3*b*c^2*d + 8*a^4*d^3 +
 (16*a^4 - 7*a^3*b)*c*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(
sqrt(-c/d)/x), a*d/(b*c)) + (3*a*b^3*d^3*f^2*x^6 + 2*(5*a*b^3*d^3*e*f + 3*
(a*b^3*c*d^2 - a^2*b^2*d^3)*f^2)*x^4 - 15*(a*b^3*c*d^2 - 2*a^2*b^2*d^3)*e^
2 + 10*(7*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*e*f + 3*(a^2*b^2*c^2*d - 16*a^3*b*c
*d^2 + 16*a^4*d^3)*f^2 + (15*a*b^3*d^3*e^2 + 40*(a*b^3*c*d^2 - a^2*b^2*d^3
)*e*f + 3*(a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f^2)*x^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(a*b^5*d^2*x^3 + a^2*b^4*d^2*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

output

```
(9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x - 6*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*d**2*f**2*x**3 - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*c**2*f**2*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x + 6*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 + 10*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b*d**2*e*f*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b*d**2*f**2*x**5 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*e*f*x +
 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e**2*x + 24*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b
*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**4*d**3*f**2 - 36*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*
b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*c*d**2*f**2 - 40*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*d**3*e*f + 24*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*d**3*f**2*x**2 + 12
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*
b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b**2*c**2*d*f
**2 + 55*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**
2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b**2*
c*d**2*e*f - 36*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + ...
```

3.82 $\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 304

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx &= \frac{f(6bde+bcf-4adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{a+bx^2}}+\frac{f^2x^3\sqrt{c+dx^2}}{3b\sqrt{a+bx^2}} \\ &+ \frac{(3b^2de^2+8a^2df^2-abf(12de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{2\sqrt{a}f(3be-2af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
1/3*f*(-4*a*d*f+b*c*f+6*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(b*x^2+a)^(1/2)+1/3
*f^2*x^3*(d*x^2+c)^(1/2)/b/(b*x^2+a)^(1/2)+1/3*(3*b^2*d*e^2+8*a^2*d*f^2-a*
b*f*(c*f+12*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(5/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)+2/3*a^(1/2)*f*(-2*a*f+3*b*e)*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(3b^2e^2+4a^2f^2+abf(-6e+fx^2)\right)+ic(3b^2de^2+8a^2df\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(c + d*x^2)*(3*b^2*e^2 + 4*a^2*f^2 + a*b*f*(-6*e
 + f*x^2)) + I*c*(3*b^2*d*e^2 + 8*a^2*d*f^2 - a*b*f*(12*d*e + c*f))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*c*(3*b^2*d*e^2 + 4*a^2*d*f^2 - a*b*f*(6*d*e + c*f))*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
]))/(3*b^3*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 633 vs. 2(304) = 608.

Time = 0.83 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}+\frac{2efx^2\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}+\frac{f^2x^4\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\begin{aligned}
&-\frac{\sqrt{c}f^2\sqrt{a+bx^2}(bc-8ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\\
&\frac{4c^{3/2}f^2\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\\
&\frac{4\sqrt{c}\sqrt{d}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\\
&\frac{2c^{3/2}ef\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{e^2\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\\
&\frac{f^2x\sqrt{a+bx^2}(bc-8ad)}{3b^3\sqrt{c+dx^2}}+\frac{4defx\sqrt{a+bx^2}}{b^2\sqrt{c+dx^2}}+\frac{4f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b^2}-\frac{2efx\sqrt{c+dx^2}}{b\sqrt{a+bx^2}}-\\
&\frac{f^2x^3\sqrt{c+dx^2}}{b\sqrt{a+bx^2}}
\end{aligned}
$$

input

```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output

```
(4*d*e*f*x*Sqrt[a + b*x^2])/(b^2*Sqrt[c + d*x^2]) + ((b*c - 8*a*d)*f^2*x*S
qrt[a + b*x^2])/(3*b^3*Sqrt[c + d*x^2]) - (2*e*f*x*Sqrt[c + d*x^2])/(b*Sqr
t[a + b*x^2]) - (f^2*x^3*Sqrt[c + d*x^2])/(b*Sqrt[a + b*x^2]) + (4*f^2*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b^2) + (e^2*Sqrt[c + d*x^2]*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b*
x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (4*Sqrt[c]*Sqrt[d]*e*f*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b^2*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 8*
a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(3*b^3*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2]) + (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(a*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]) - (4*c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 9.23 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(b d x^2+bc\right)\left(a^2 f^2-2abfe+b^2 e^2\right)x}{a b^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(b d x^2+bc\right)}}+\frac{f^2 x\sqrt{b d x^4+a d x^2+x^2 bc+ac}}{3b^2}+\frac{\left(\frac{a^2 d f^2-abc f^2-2abdef+2b^2 cef+b^2 d e^2}{b^3}-\frac{(a^2 f^2 \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{x^2 d+c}\sqrt{b x^2+a}\left(\sqrt{-\frac{b}{a}}\, ab d^2 f^2 x^5+4\sqrt{-\frac{b}{a}}\, a^2 d^2 f^2 x^3+\sqrt{-\frac{b}{a}}\, abcd f^2 x^3-6\sqrt{-\frac{b}{a}}\, ab d^2 ef x^3+3\sqrt{-\frac{b}{a}}\, b^2 d^2 e^2 x^3+4\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2 \cdots}{\cdots}}\right.}{\cdots}$
risch	$\frac{f^2 x\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{3b^2}-\frac{\left(-\frac{f(5adf-bcf-6bde)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\cdots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+x^2 bc+ac}\, d}\cdots\right.}{\cdots}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*(a^2*f^2-2*a*b*e*f+b^2*e^2)/a/b^3*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/3*f
^2/b^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+((a^2*d*f^2-a*b*c*f^2-2*a*b*d
*e*f+2*b^2*c*e*f+b^2*d*e^2)/b^3-(a^2*f^2-2*a*b*e*f+b^2*e^2)/b^3*(a*d-b*c)/
a-1/b^2*c*(a^2*f^2-2*a*b*e*f+b^2*e^2)/a-1/3*a/b^2*c*f^2)/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-f/b^2*(a*d*f-b*c*f-2*b*d*e)
-(a^2*f^2-2*a*b*e*f+b^2*e^2)/b^2*d/a-1/3*f^2/b^2*(2*a*d+2*b*c))*c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{((3\,b^3cde^2 - 12\,ab^2cdef - (ab^2c^2 - 8\,a^2bcd)\,f^2)\,x^3 + (3\,ab^2cde^2 - 12\,a^2bcdef -}{}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
1/3*(((3*b^3*c*d*e^2 - 12*a*b^2*c*d*e*f - (a*b^2*c^2 - 8*a^2*b*c*d)*f^2)*x
^3 + (3*a*b^2*c*d*e^2 - 12*a^2*b*c*d*e*f - (a^2*b*c^2 - 8*a^3*c*d)*f^2)*x)
*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((3*b^
3*c*d*e^2 - 6*(2*a*b^2*c*d + a*b^2*d^2)*e*f - (a*b^2*c^2 - 8*a^2*b*c*d - 4
*a^2*b*d^2)*f^2)*x^3 + (3*a*b^2*c*d*e^2 - 6*(2*a^2*b*c*d + a^2*b*d^2)*e*f
- (a^2*b*c^2 - 8*a^3*c*d - 4*a^3*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b^2*d^2*f^2*x^4 - 3*a*b^2*d^2*e^
2 + 12*a^2*b*d^2*e*f + (a^2*b*c*d - 8*a^3*d^2)*f^2 + (6*a*b^2*d^2*e*f + (a
*b^2*c*d - 4*a^2*b*d^2)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*
d^2*x^3 + a^2*b^3*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x + 2*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*f**2*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*f
*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**2*x - 8*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*
x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**2*f**2 + 5*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*
x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d*f**2 + 12*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*
d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*e*f - 8*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b
*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*f**2*x**2 - 6*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
+ 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d*e*f + 5*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
+ 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d*f**2*x**2 - 3*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c
*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*d**2*e**2 + 12
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*
b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*d**2*e*f*x*
*2 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x*...
```

3.83 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 278

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{a+bx^2}} + \frac{(b^2de^2+2a^2df^2-abf(2de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}d(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}(acf^2+be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f^2*x*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)+(b^2*d*e^2+2*a^2*d*f^2-a*b*f*(c*
f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1
-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)-a^(1/2)*(a*c*f^2+b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*Inv
erseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/(-a*d+
b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}d(be-af)^2x(c+dx^2) - ic(b^2de^2+2a^2df^2-abf(2de+cf))\sqrt{1+\frac{bx}{a}}}{}$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*(b*e - a*f)^2*x*(c + d*x^2)) - I*c*(b^2*d*e^2 + 2*a^2*d*f^2
 - a*b*f*(2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*(-(b*d*e^2) + a*c*
f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])/(a^2*(b/a)^(3/2)*d*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 862 vs. 2(278) = 556.

Time = 1.04 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{2efx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{f^2x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{a(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{a(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{bx\sqrt{dx^2+c}e^2}{a(bc-ad)\sqrt{bx^2+a}}-\frac{dx\sqrt{bx^2+a}e^2}{a(bc-ad)\sqrt{dx^2+c}}-\frac{2\sqrt{c}\sqrt{d}f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{b(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2fx\sqrt{dx^2+c}e}{(bc-ad)\sqrt{bx^2+a}}+\frac{2dfx\sqrt{bx^2+a}e}{b(bc-ad)\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc-2ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b^2\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{b\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{af^2x\sqrt{dx^2+c}}{b(bc-ad)\sqrt{bx^2+a}}+\frac{(bc-2ad)f^2x\sqrt{bx^2+a}}{b^2(bc-ad)\sqrt{dx^2+c}}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-((d*e^2*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (2*d*e*f*x*
Sqrt[a + b*x^2])/(b*(b*c - a*d)*Sqrt[c + d*x^2]) + ((b*c - 2*a*d)*f^2*x*Sq
rt[a + b*x^2])/(b^2*(b*c - a*d)*Sqrt[c + d*x^2]) + (b*e^2*x*Sqrt[c + d*x^2
])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (2*e*f*x*Sqrt[c + d*x^2])/((b*c - a*d
)*Sqrt[a + b*x^2]) + (a*f^2*x*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x
^2]) + (Sqrt[c]*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2]) - (2*Sqrt[c]*Sqrt[d]*e*f*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*(b*c - a*d)*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 2*a*d)*f^2*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b^
2*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) - (Sqrt[c]*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*f^2*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*(b*c - a
*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.42 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(-\frac{(bd\,x^2+bc)x\left(a^2f^2-2abfe+b^2e^2\right)}{b^2a(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(-\frac{f(af-2be)}{b^2}+\frac{a^2f^2-2abfe+b^2e^2}{b^2a}+\frac{c\left(a^2f^2-2abfe+b^2e^2\right)}{ba(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+d}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a^2d^2f^2x^3+2\sqrt{-\frac{b}{a}}\,ab\,d^2ef\,x^3-\sqrt{-\frac{b}{a}}\,b^2d^2e^2x^3-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cd\,f^2+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+}{c}}\right.}{}$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2/a/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^
(1/2)+(-f*(a*f-2*b*e)/b^2+(a^2*f^2-2*a*b*e*f+b^2*e^2)/b^2/a+1/b*c/a/(a*d-b
*c)*(a^2*f^2-2*a*b*e*f+b^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-(f^2/b+(a^2*f^2-2*a*b*e*f+b^2*e^2)/b/(a*d-b*c)*d/a)*c
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.80

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((b^3c^2de^2-2\,ab^2c^2def-(ab^2c^3-2\,a^2bc^2d)f^2)x^3+(ab^2c^2de^2-2\,a^2bc^2def}{}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
(((b^3*c^2*d*e^2 - 2*a*b^2*c^2*d*e*f - (a*b^2*c^3 - 2*a^2*b*c^2*d)*f^2)*x^
3 + (a*b^2*c^2*d*e^2 - 2*a^2*b*c^2*d*e*f - (a^2*b*c^3 - 2*a^3*c^2*d)*f^2)*
x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((b
^3*c^2*d + a*b^2*d^3)*e^2 - 2*(a*b^2*c^2*d + a*b^2*c*d^2)*e*f - (a*b^2*c^3
 - 2*a^2*b*c^2*d - a^2*b*c*d^2)*f^2)*x^3 + ((a*b^2*c^2*d + a^2*b*d^3)*e^2
- 2*(a^2*b*c^2*d + a^2*b*c*d^2)*e*f - (a^2*b*c^3 - 2*a^3*c^2*d - a^3*c*d^2
)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - (a*b^2*c*d^2*e^2 - 2*a^2*b*c*d^2*e*f - (a*b^2*c^2*d - a^2*b*c*d^2)*f^2*
x^2 - (a^2*b*c^2*d - 2*a^3*c*d^2)*f^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((
a*b^4*c^2*d^2 - a^2*b^3*c*d^3)*x^3 + (a^2*b^3*c^2*d^2 - a^3*b^2*c*d^3)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,efx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)a^2d\,f^2$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*d*f**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4
 + b**2*d*x**6),x)*a*b*d*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*a*b*d*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*
d*x**6),x)*b**2*d*e*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2
*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6
),x)*a**2*c*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d
*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*d
*e**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*
a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*e*f*x**2 +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x
**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*e**2*x**2)/(a*d*(
a + b*x**2))
```

3.84 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 308

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{(be-af)^2x}{ab(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{(b^2cde^2+a^2cdf^2+ab(d^2e^2-4cdef+c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{c}\sqrt{d}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{2\sqrt{c}(be-af)(de-cf)\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(-a*f+b*e)^2*x/a/b/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+(b^2*c*d*e^2
+a^2*c*d*f^2+a*b*(c^2*f^2-4*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^
(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/b/c^(1/2)/d^(1/2)/(
-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2*c^(1/2)*(-a*
f+b*e)*(-c*f+d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)
),(1-b*c/a/d)^(1/2))/a/d^(1/2)/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(b^2ce^2(c+dx^2)+ab(d^2e^2x^2-4cdefx^2+c^2f(-2e+fx^2))+\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(b^2*c*e^2*(c + d*x^2) + a*b*(d^2*e^2*x^2 - 4*c*
d*e*f*x^2 + c^2*f*(-2*e + f*x^2)) + a^2*(d^2*e^2 + 2*c^2*f^2 + c*d*f*(-2*e
 + f*x^2))) + I*c*(b^2*c*d*e^2 + a^2*c*d*f^2 + a*b*(d^2*e^2 - 4*c*d*e*f +
c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(b*d*e^2 + a*f*(-2*d*e + c*f))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)]))/(b*c*d*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 738 vs. 2(308) = 616.

Time = 0.84 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}+\frac{2efx^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}+\frac{f^2x^4}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2b\sqrt{c}\sqrt{d}e^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}e^2\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}ef\sqrt{a+bx^2}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4\sqrt{c}\sqrt{d}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}f^2\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{be^2x}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{2efx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{af^2x}{b\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(b*e^2*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (2*e*f*x)/((b*
c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (a*f^2*x)/(b*(b*c - a*d)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(b*c + a*d)*e^2*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*Sqrt[c]*S
qrt[d]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/((b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (Sqrt[c]*(b*c + a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*Sqrt[d]*e^2*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c -
a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]
*(b*c + a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (2*c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(291) = 582$.

Time = 8.49 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.11

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)} \left(-\frac{2bd\left(-\frac{\left(a^2cd f^2+ab c^2 f^2-4abcdef+ab d^2 e^2+b^2 d e^2 c\right)x^3}{2bdac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2a^2c^2f^2-2a^2cdef+a^2d^2e^2-2ab c^2ef+b^2c^2e^2\right)x}{2bdac\left(a^2d^2-2abcd+b^2c^2\right)}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{db}+\frac{ac}{db}\right)bd}}+ \cdots\right.}{\cdots}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, a^2 c d^2 f^2 x^3+\sqrt{-\frac{b}{a}}\, ab c^2 d f^2 x^3-4\sqrt{-\frac{b}{a}}\, abc d^2 ef x^3+\sqrt{-\frac{b}{a}}\, ab d^3 e^2 x^3+\sqrt{-\frac{b}{a}}\, b^2 c d^2 e^2 x^3-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\, \mathrm{EllipticF}\left(x \cdots\right.\right.}{\cdots}$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-2*b*d*(-1/2/
b/d*(a^2*c*d*f^2+a*b*c^2*f^2-4*a*b*c*d*e*f+a*b*d^2*e^2+b^2*c*d*e^2)/a/c/(a
^2*d^2-2*a*b*c*d+b^2*c^2)*x^3-1/2/b/d*(2*a^2*c^2*f^2-2*a^2*c*d*e*f+a^2*d^2
*e^2-2*a*b*c^2*e*f+b^2*c^2*e^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x)/((x^4+(
a*d+b*c)/d/b*x^2+a*c/d/b)*b*d)^(1/2)+(f^2/b/d-1/b/d*(a*c*f^2-b*d*e^2)/a/c-
(2*a^2*c^2*f^2-2*a^2*c*d*e*f+a^2*d^2*e^2-2*a*b*c^2*e*f+b^2*c^2*e^2)/a/c/(a
^2*d^2-2*a*b*c*d+b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))+(a^2*c*d*f^2+a*b*c^2*f^2-4*a*b*c*d*e*f+a*b*d^2*e^2+b^2*c*d*
e^2)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(291) = 582$.

Time = 0.12 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.06

$$\int \frac{\left(e+fx^2\right)^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
((4*a^2*b^2*c^2*d*e*f + (4*a*b^3*c*d^2*e*f - (b^4*c*d^2 + a*b^3*d^3)*e^2 -
 (a*b^3*c^2*d + a^2*b^2*c*d^2)*f^2)*x^4 - (a*b^3*c^2*d + a^2*b^2*c*d^2)*e^
2 - (a^2*b^2*c^3 + a^3*b*c^2*d)*f^2 - ((b^4*c^2*d + 2*a*b^3*c*d^2 + a^2*b^
2*d^3)*e^2 - 4*(a*b^3*c^2*d + a^2*b^2*c*d^2)*e*f + (a*b^3*c^3 + 2*a^2*b^2*
c^2*d + a^3*b*c*d^2)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sq
rt(-b/a)), a*d/(b*c)) + (((b^4*c*d^2 + (2*a^2*b^2 + a*b^3)*d^3)*e^2 - 2*(a
^3*b*d^3 + (a^2*b^2 + 2*a*b^3)*c*d^2)*e*f + (a*b^3*c^2*d + (2*a^3*b + a^2*
b^2)*c*d^2)*f^2)*x^4 + (a*b^3*c^2*d + (2*a^3*b + a^2*b^2)*c*d^2)*e^2 - 2*(
a^4*c*d^2 + (a^3*b + 2*a^2*b^2)*c^2*d)*e*f + (a^2*b^2*c^3 + (2*a^4 + a^3*b
)*c^2*d)*f^2 + ((b^4*c^2*d + 2*(a^2*b^2 + a*b^3)*c*d^2 + (2*a^3*b + a^2*b^
2)*d^3)*e^2 - 2*(a^4*d^3 + (a^2*b^2 + 2*a*b^3)*c^2*d + 2*(a^3*b + a^2*b^2)
*c*d^2)*e*f + (a*b^3*c^3 + 2*(a^3*b + a^2*b^2)*c^2*d + (2*a^4 + a^3*b)*c*d
^2)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b
*c)) - ((4*a^2*b^2*c*d^2*e*f - (a*b^3*c*d^2 + a^2*b^2*d^3)*e^2 - (a^2*b^2*
c^2*d + a^3*b*c*d^2)*f^2)*x^3 - (2*a^3*b*c^2*d*f^2 + (a*b^3*c^2*d + a^3*b*
d^3)*e^2 - 2*(a^2*b^2*c^2*d + a^3*b*c*d^2)*e*f)*x)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c))/(a^3*b^3*c^4*d - 2*a^4*b^2*c^3*d^2 + a^5*b*c^2*d^3 + (a^2*b^4*c^
3*d^2 - 2*a^3*b^3*c^2*d^3 + a^4*b^2*c*d^4)*x^4 + (a^2*b^4*c^4*d - a^3*b^3*
c^3*d^2 - a^4*b^2*c^2*d^3 + a^5*b*c*d^4)*x^2)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)**2/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*f**2*x + 2*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*
c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d
*x**6 + b**2*d**2*x**8),x)*a*b*c*d*e*f + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x
**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6
+ b**2*d**2*x**8),x)*a*b*d**2*e*f*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x
**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6
+ b**2*d**2*x**8),x)*b**2*c*d*e*f*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x
**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6
+ b**2*d**2*x**8),x)*b**2*d**2*e*f*x**4 + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 +
4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2
*d**2*x**8),x)*a**2*c**2*f**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*
x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8
),x)*a**2*c*d*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*
*2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**...
```

3.85 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 463

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \frac{(be-af)^2x}{ab(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}$$
$$+\frac{(3b^2cde^2+3a^2cdf^2+ab(d^2e^2-8cdef+c^2f^2))\,x\sqrt{a+bx^2}}{3abc(bc-ad)^2\,(c+dx^2)^{3/2}}$$
$$+\frac{(3b^2c^2de^2-a^2d(2d^2e^2+2cdef-7c^2f^2)+abc(7d^2e^2-14cdef+c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3ac^{3/2}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{(3a^2cdf^2+3b^2ce(3de-2cf)-ab(d^2e^2+10cdef-5c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(-a*f+b*e)^2*x/a/b/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*(3*b^2*c
*d*e^2+3*a^2*c*d*f^2+a*b*(c^2*f^2-8*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/a/
b/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/3*(3*b^2*c^2*d*e^2-a^2*d*(-7*c^2*f^2+2*
c*d*e*f+2*d^2*e^2)+a*b*c*(c^2*f^2-14*c*d*e*f+7*d^2*e^2))*(b*x^2+a)^(1/2)*E
llipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(3/2)/
d^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(
3*a^2*c*d*f^2+3*b^2*c*e*(-2*c*f+3*d*e)-a*b*(-5*c^2*f^2+10*c*d*e*f+d^2*e^2)
)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1
/2))/a/c^(1/2)/d^(1/2)/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.25 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.17

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx\left(-3b^3c^2e^2(c+dx^2)^2+a^3d(de-cf)\left(3c^2f+2d^2ex^2+cd(3e\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(-3*b^3*c^2*e^2*(c + d*x^2)^2 + a^3*d*(d*e - c*f
)*(3*c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 4*f*x^2)) + a*b^2*c*(-7*d^3*e^2*x^4
+ c^2*d*f*x^2*(22*e - f*x^2) - 2*c^3*f*(-3*e + f*x^2) + 2*c*d^2*e*x^2*(-4*
e + 7*f*x^2)) + a^2*b*(-5*c^4*f^2 + 2*d^4*e^2*x^4 + 10*c^3*d*f*(e - f*x^2)
 + 2*c*d^3*e*x^2*(-2*e + f*x^2) + c^2*d^2*(-8*e^2 + 8*e*f*x^2 - 7*f^2*x^4)
)) - I*b*c*(3*b^2*c^2*d*e^2 + a*b*c*(7*d^2*e^2 - 14*c*d*e*f + c^2*f^2) + a
^2*d*(-2*d^2*e^2 - 2*c*d*e*f + 7*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*
(-(b*c) + a*d)*(3*b^2*c*d*e^2 + 3*a^2*c*d*f^2 + a*b*(d^2*e^2 - 8*c*d*e*f +
 c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^2*d*(-(b*c) + a*d)^3*Sqrt[a +
 b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 941 vs. 2(463) = 926.

Time = 1.21 (sec) , antiderivative size = 941, normalized size of antiderivative = 2.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}+\frac{2efx^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}+\frac{f^2x^4}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d}(3b^2c^2+7abdc-2a^2d^2)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{3ac^{3/2}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{d}(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{3a\sqrt{c}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d(3bc+ad)x\sqrt{bx^2+a}e^2}{3ac(bc-ad)^2(dx^2+c)^{3/2}}+\frac{bxe^2}{a(bc-ad)\sqrt{bx^2+a}(dx^2+c)^{3/2}}-\frac{2\sqrt{d}(7bc+ad)f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{3\sqrt{c}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2b\sqrt{c}(3bc+5ad)f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{3a\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{8dfx\sqrt{bx^2+a}e}{3(bc-ad)^2(dx^2+c)^{3/2}}-\frac{2fxe}{(bc-ad)\sqrt{bx^2+a}(dx^2+c)^{3/2}}+\frac{\sqrt{c}(bc+7ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(5bc+3ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{(bc+3ad)f^2x\sqrt{bx^2+a}}{3b(bc-ad)^2(dx^2+c)^{3/2}}+\frac{af^2x}{b(bc-ad)\sqrt{bx^2+a}(dx^2+c)^{3/2}}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(b*e^2*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) - (2*e*f*x)/((
b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) + (a*f^2*x)/(b*(b*c - a*d)*S
qrt[a + b*x^2]*(c + d*x^2)^(3/2)) + (d*(3*b*c + a*d)*e^2*x*Sqrt[a + b*x^2]
)/(3*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (8*d*e*f*x*Sqrt[a + b*x^2])/(3
*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + ((b*c + 3*a*d)*f^2*x*Sqrt[a + b*x^2])/
(3*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (Sqrt[d]*(3*b^2*c^2 + 7*a*b*c*d -
2*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a*c^(3/2)*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (2*Sqrt[d]*(7*b*c + a*d)*e*f*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*(b*c - a*d)
^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*(b*c
+ 7*a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*Sqrt[d]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) - (b*Sqrt[d]*(9*b*c - a*d)*e^2*Sqrt[a + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*(b*c - a*d)^3
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*b*Sqrt[c]*(3*
b*c + 5*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*Sqrt[d]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(5*b*c + 3*a*d)*f^2*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*(b*c -...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 18.11 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{\left(bd\,x^2+bc\right)x\left(a^2 f^2-2abfe+b^2 e^2\right)}{a(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{x\left(c^2 f^2-2cdef+d^2 e^2\right)\sqrt{bd\,x^4+ad\,x^2+x^2 bc+ac}}{3c\,d^2(ad-bc)^2\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(bd\,x^2+ad\right)x\left(4a\,c^2 d\,f^2-2ace\right.}{3c^2 d(ad-bc}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/a/(a*d-b*c)^3*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(1
/2)+1/3/c/d^2/(a*d-b*c)^2*x*(c^2*f^2-2*c*d*e*f+d^2*e^2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/(x^2+c/d)^2-1/3*(b*d*x^2+a*d)/c^2/d/(a*d-b*c)^3*x*(4*a*c
^2*d*f^2-2*a*c*d^2*e*f-2*a*d^3*e^2+b*c^3*f^2-8*b*c^2*d*e*f+7*b*c*d^2*e^2)/
((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+((a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)^2/a
+b*c/a/(a*d-b*c)^3*(a^2*f^2-2*a*b*e*f+b^2*e^2)+1/3*b/d*(c^2*f^2-2*c*d*e*f+
d^2*e^2)/(a*d-b*c)^2/c-1/3/(a*d-b*c)^2/d*(4*a*c^2*d*f^2-2*a*c*d^2*e*f-2*a*
d^3*e^2+b*c^3*f^2-8*b*c^2*d*e*f+7*b*c*d^2*e^2)/c^2+1/3*a/c^2/(a*d-b*c)^3*(
4*a*c^2*d*f^2-2*a*c*d^2*e*f-2*a*d^3*e^2+b*c^3*f^2-8*b*c^2*d*e*f+7*b*c*d^2*
e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b*d*(
a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)^3/a+1/3*b*(4*a*c^2*d*f^2-2*a*c*d^2*e*
f-2*a*d^3*e^2+b*c^3*f^2-8*b*c^2*d*e*f+7*b*c*d^2*e^2)/(a*d-b*c)^3/c^2)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1949 vs. $2(436) = 872$.

Time = 0.16 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.21

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
-1/3*((((3*b^5*c^2*d^3 + 7*a*b^4*c*d^4 - 2*a^2*b^3*d^5)*e^2 - 2*(7*a*b^4*c
^2*d^3 + a^2*b^3*c*d^4)*e*f + (a*b^4*c^3*d^2 + 7*a^2*b^3*c^2*d^3)*f^2)*x^6
 + ((6*b^5*c^3*d^2 + 17*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - 2*a^3*b^2*d^5)*e
^2 - 2*(14*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e*f + (2*a*b
^4*c^4*d + 15*a^2*b^3*c^3*d^2 + 7*a^3*b^2*c^2*d^3)*f^2)*x^4 + (3*a*b^4*c^4
*d + 7*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3)*e^2 - 2*(7*a^2*b^3*c^4*d + a^3
*b^2*c^3*d^2)*e*f + (a^2*b^3*c^5 + 7*a^3*b^2*c^4*d)*f^2 + ((3*b^5*c^4*d +
13*a*b^4*c^3*d^2 + 12*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4)*e^2 - 2*(7*a*b^4*
c^4*d + 15*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3)*e*f + (a*b^4*c^5 + 9*a^2*b
^3*c^4*d + 14*a^3*b^2*c^3*d^2)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(a
rcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((3*b^5*c^2*d^3 + (9*a^2*b^3 + 7*a*b^4)
*c*d^4 - (a^3*b^2 + 2*a^2*b^3)*d^5)*e^2 - 2*((3*a^2*b^3 + 7*a*b^4)*c^2*d^3
 + (5*a^3*b^2 + a^2*b^3)*c*d^4)*e*f + (a*b^4*c^3*d^2 + 3*a^4*b*c*d^4 + (5*
a^3*b^2 + 7*a^2*b^3)*c^2*d^3)*f^2)*x^6 + ((6*b^5*c^3*d^2 + (18*a^2*b^3 + 1
7*a*b^4)*c^2*d^3 + (7*a^3*b^2 + 3*a^2*b^3)*c*d^4 - (a^4*b + 2*a^3*b^2)*d^5
)*e^2 - 2*(2*(3*a^2*b^3 + 7*a*b^4)*c^3*d^2 + (13*a^3*b^2 + 9*a^2*b^3)*c^2*
d^3 + (5*a^4*b + a^3*b^2)*c*d^4)*e*f + (2*a*b^4*c^4*d + 3*a^5*c*d^4 + 5*(2
*a^3*b^2 + 3*a^2*b^3)*c^3*d^2 + (11*a^4*b + 7*a^3*b^2)*c^2*d^3)*f^2)*x^4 +
 (3*a*b^4*c^4*d + (9*a^3*b^2 + 7*a^2*b^3)*c^3*d^2 - (a^4*b + 2*a^3*b^2)*c^
2*d^3)*e^2 - 2*((3*a^3*b^2 + 7*a^2*b^3)*c^4*d + (5*a^4*b + a^3*b^2)*c^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a*
*2*d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2
*a*b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8
+ b**2*d**3*x**10),x)*a**2*c**2*d*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*
d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2*a*
b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8 + b
**2*d**3*x**10),x)*a**2*c*d**2*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*
d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2*a*
b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8 + b
**2*d**3*x**10),x)*a**2*d**3*f**2*x**4 - 3*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*
d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2*a*
b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8 + b
**2*d**3*x**10),x)*a*b*c**2*d*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**4)/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*d**3*x*
*6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2*a*b*d**3*
x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8 + b**2*...
```

3.86 $\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 618

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(5b^4c^2e^2+8a^4d^2f^2-10ab^3ce(de+cf)-10a^3bdf(de+cf)+5a^2b^2(d^2e^2 -}{15ab^4(a+bx^2)^{3/2}}$$

$$+\frac{d^2f^2x^7\sqrt{c+dx^2}}{5b(a+bx^2)^{3/2}}$$

$$+\frac{(56a^2d^2f^2-2abdf(35de+41cf)+b^2(15d^2e^2+70cdef+23c^2f^2))\,x\sqrt{c+dx^2}}{15b^4\sqrt{a+bx^2}}$$

$$+\frac{df(10bde+11bcf-8adf)x^3\sqrt{c+dx^2}}{15b^3\sqrt{a+bx^2}}$$

$$+\frac{(10b^4c^2e^2-128a^4d^2f^2+5ab^3ce(3de+2cf)+8a^3bdf(20de+21cf)-a^2b^2(40d^2e^2+160cdef+43c^2f^2))}{15a^{3/2}b^{9/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{(5b^3cde^2-64a^3d^2f^2+4a^2bdf(20de+19cf)-5ab^2(4d^2e^2+14cdef+3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\text{arcta}\right.}{15\sqrt{a}b^{9/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(5*b^4*c^2*e^2+8*a^4*d^2*f^2-10*a*b^3*c*e*(c*f+d*e)-10*a^3*b*d*f*(c*f
+d*e)+5*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(d*x^2+c)^(1/2)/a/b^4/(b*x^
2+a)^(3/2)+1/5*d^2*f^2*x^7*(d*x^2+c)^(1/2)/b/(b*x^2+a)^(3/2)+1/15*(56*a^2*
d^2*f^2-2*a*b*d*f*(41*c*f+35*d*e)+b^2*(23*c^2*f^2+70*c*d*e*f+15*d^2*e^2))*
x*(d*x^2+c)^(1/2)/b^4/(b*x^2+a)^(1/2)+1/15*d*f*(-8*a*d*f+11*b*c*f+10*b*d*e
)*x^3*(d*x^2+c)^(1/2)/b^3/(b*x^2+a)^(1/2)+1/15*(10*b^4*c^2*e^2-128*a^4*d^2
*f^2+5*a*b^3*c*e*(2*c*f+3*d*e)+8*a^3*b*d*f*(21*c*f+20*d*e)-a^2*b^2*(43*c^2
*f^2+160*c*d*e*f+40*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(9/2)/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*(5*b^3*c*d*e^2-64*a^3*d^2*f^2+4*a^2*b*d*f*
(19*c*f+20*d*e)-5*a*b^2*(3*c^2*f^2+14*c*d*e*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(9/
2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.40 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{b}{a}\right)^{3/2}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(-64a^5d^2f^2+10b^5c^2e^2x^2+5ab^4ce(3ce+3dex^2+\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
((b/a)^(3/2)*(Sqrt[b/a]*x*(c + d*x^2)*(-64*a^5*d^2*f^2 + 10*b^5*c^2*e^2*x^
2 + 5*a*b^4*c*e*(3*c*e + 3*d*e*x^2 + 2*c*f*x^2) + 4*a^4*b*d*f*(19*c*f + 20
*d*(e - f*x^2)) - a^3*b^2*(15*c^2*f^2 + c*d*f*(70*e - 97*f*x^2) + 4*d^2*(5
*e^2 - 25*e*f*x^2 + 2*f^2*x^4)) + a^2*b^3*(-20*c^2*f^2*x^2 + d^2*x^2*(-25*
e^2 + 10*e*f*x^2 + 3*f^2*x^4) + c*d*(5*e^2 - 90*e*f*x^2 + 11*f^2*x^4))) -
I*c*(-10*b^4*c^2*e^2 + 128*a^4*d^2*f^2 - 5*a*b^3*c*e*(3*d*e + 2*c*f) - 8*a
^3*b*d*f*(20*d*e + 21*c*f) + a^2*b^2*(40*d^2*e^2 + 160*c*d*e*f + 43*c^2*f^
2))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*(5*b^3*c*e^2 + 32*a^
3*d*f^2 + 5*a*b^2*e*(2*d*e + c*f) - 2*a^2*b*f*(20*d*e + 7*c*f))*(a + b*x^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]))/(15*b^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1247 vs. 2(618) = 1236.

Time = 1.80 (sec) , antiderivative size = 1247, normalized size of antiderivative = 2.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2(c+dx^2)^{5/2}}{(a+bx^2)^{5/2}}+\frac{2efx^2(c+dx^2)^{5/2}}{(a+bx^2)^{5/2}}+\frac{f^2x^4(c+dx^2)^{5/2}}{(a+bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2(dx^2+c)^{5/2}x^3}{3b(bx^2+a)^{3/2}}+\frac{(3bc-8ad)f^2(dx^2+c)^{3/2}x^3}{3ab^2\sqrt{bx^2+a}}-\frac{d(5bc-16ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5ab^3}-\frac{2ef(dx^2+c)^{5/2}x}{3b(bx^2+a)^{3/2}}+\frac{2(bc-6ad)ef(dx^2+c)^{3/2}x}{3ab^2\sqrt{bx^2+a}}+\frac{(bc-ad)e^2(dx^2+c)^{3/2}x}{3ab(bx^2+a)^{3/2}}-\frac{(15bc-16ad)(bc-4ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15ab^4}-\frac{2d(bc-8ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3ab^3}+\frac{2(bc-ad)(bc+2ad)e^2\sqrt{dx^2+c}x}{3a^2b^2\sqrt{bx^2+a}}-\frac{d(2b^2c^2+3abdc-8a^2d^2)e^2\sqrt{bx^2+a}x}{3a^2b^3\sqrt{dx^2+c}}+\frac{d(43b^2c^2-168abdc+128a^2d^2)f^2\sqrt{bx^2+a}x}{15b^5\sqrt{dx^2+c}}-\frac{2d(b^2c^2-16abdc+16a^2d^2)ef\sqrt{bx^2+a}x}{3ab^4\sqrt{dx^2+c}}+\frac{\sqrt{c}\sqrt{d}(2b^2c^2+3abdc-8a^2d^2)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{d}(43b^2c^2-168abdc+128a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^5\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}\sqrt{d}(b^2c^2-16abdc+16a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}\sqrt{d}(bc-4ad)e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(15bc-16ad)(bc-4ad)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15ab^4\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}\sqrt{d}(7bc-8ad)ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(a^2*b^
3*Sqrt[c + d*x^2]) - (2*d*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*f*x*Sqrt[a
 + b*x^2])/(3*a*b^4*Sqrt[c + d*x^2]) + (d*(43*b^2*c^2 - 168*a*b*c*d + 128*
a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(15*b^5*Sqrt[c + d*x^2]) + (2*(b*c - a*d)*
(b*c + 2*a*d)*e^2*x*Sqrt[c + d*x^2])/(3*a^2*b^2*Sqrt[a + b*x^2]) - (2*d*(b
*c - 8*a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*a*b^3) - ((15*b*c -
16*a*d)*(b*c - 4*a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*a*b^4) -
(d*(5*b*c - 16*a*d)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*a*b^3) + (
(b*c - a*d)*e^2*x*(c + d*x^2)^(3/2))/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c -
 6*a*d)*e*f*x*(c + d*x^2)^(3/2))/(3*a*b^2*Sqrt[a + b*x^2]) + ((3*b*c - 8*a
*d)*f^2*x^3*(c + d*x^2)^(3/2))/(3*a*b^2*Sqrt[a + b*x^2]) - (2*e*f*x*(c + d
*x^2)^(5/2))/(3*b*(a + b*x^2)^(3/2)) - (f^2*x^3*(c + d*x^2)^(5/2))/(3*b*(a
 + b*x^2)^(3/2)) + (Sqrt[c]*Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*e^
2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(3*a^2*b^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sq
rt[c]*Sqrt[d]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*f*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^4*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*(43*b^2*c^2
- 168*a*b*c*d + 128*a^2*d^2)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^5*Sqrt[(c*(a + b*x^2))/(a*(c + d*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(577) = 1154$.

Time = 23.80 (sec) , antiderivative size = 1437, normalized size of antiderivative = 2.33

method	result	size
elliptic	Expression too large to display	1437
risch	Expression too large to display	1471
default	Expression too large to display	3084

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(a^4*d^2*
f^2-2*a^3*b*c*d*f^2-2*a^3*b*d^2*e*f+a^2*b^2*c^2*f^2+4*a^2*b^2*c*d*e*f+a^2*
b^2*d^2*e^2-2*a*b^3*c^2*e*f-2*a*b^3*c*d*e^2+b^4*c^2*e^2)/a/b^6*x*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-1/3*(b*d*x^2+b*c)*(11*a^4*d^2*f^2-1
5*a^3*b*c*d*f^2-16*a^3*b*d^2*e*f+4*a^2*b^2*c^2*f^2+18*a^2*b^2*c*d*e*f+5*a^
2*b^2*d^2*e^2-2*a*b^3*c^2*e*f-3*a*b^3*c*d*e^2-2*b^4*c^2*e^2)/a^2/b^5*x/((x
^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/5/b^3*d^2*f^2*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+1/3*(-1/b^3*d^2*f*(2*a*d*f-3*b*c*f-2*b*d*e)-1/5/b^3*d^2*f^2*(4*
a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-(4*a^3*d^3*f^2-9*a
^2*b*c*d^2*f^2-6*a^2*b*d^3*e*f+6*a*b^2*c^2*d*f^2+12*a*b^2*c*d^2*e*f+2*a*b^
2*d^3*e^2-b^3*c^3*f^2-6*b^3*c^2*d*e*f-3*b^3*c*d^2*e^2)/b^5+1/3*(a^4*d^2*f^
2-2*a^3*b*c*d*f^2-2*a^3*b*d^2*e*f+a^2*b^2*c^2*f^2+4*a^2*b^2*c*d*e*f+a^2*b^
2*d^2*e^2-2*a*b^3*c^2*e*f-2*a*b^3*c*d*e^2+b^4*c^2*e^2)/b^5*d/a+1/3*(11*a^4
*d^2*f^2-15*a^3*b*c*d*f^2-16*a^3*b*d^2*e*f+4*a^2*b^2*c^2*f^2+18*a^2*b^2*c*
d*e*f+5*a^2*b^2*d^2*e^2-2*a*b^3*c^2*e*f-3*a*b^3*c*d*e^2-2*b^4*c^2*e^2)/b^5
*(a*d-b*c)/a^2+1/3/b^4*c*(11*a^4*d^2*f^2-15*a^3*b*c*d*f^2-16*a^3*b*d^2*e*f
+4*a^2*b^2*c^2*f^2+18*a^2*b^2*c*d*e*f+5*a^2*b^2*d^2*e^2-2*a*b^3*c^2*e*f-3*
a*b^3*c*d*e^2-2*b^4*c^2*e^2)/a^2-1/3*(-1/b^3*d^2*f*(2*a*d*f-3*b*c*f-2*b*d*
e)-1/5/b^3*d^2*f^2*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(577) = 1154$.

Time = 0.13 (sec) , antiderivative size = 1456, normalized size of antiderivative = 2.36

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas
")
```

output
```
1/15*(((5*(2*b^6*c^3 + 3*a*b^5*c^2*d - 8*a^2*b^4*c*d^2)*e^2 + 10*(a*b^5*c^
3 - 16*a^2*b^4*c^2*d + 16*a^3*b^3*c*d^2)*e*f - (43*a^2*b^4*c^3 - 168*a^3*b
^3*c^2*d + 128*a^4*b^2*c*d^2)*f^2)*x^5 + 2*(5*(2*a*b^5*c^3 + 3*a^2*b^4*c^2
*d - 8*a^3*b^3*c*d^2)*e^2 + 10*(a^2*b^4*c^3 - 16*a^3*b^3*c^2*d + 16*a^4*b^
2*c*d^2)*e*f - (43*a^3*b^3*c^3 - 168*a^4*b^2*c^2*d + 128*a^5*b*c*d^2)*f^2)
*x^3 + (5*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 8*a^4*b^2*c*d^2)*e^2 + 10*(a^
3*b^3*c^3 - 16*a^4*b^2*c^2*d + 16*a^5*b*c*d^2)*e*f - (43*a^4*b^2*c^3 - 168
*a^5*b*c^2*d + 128*a^6*c*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcs
in(sqrt(-c/d)/x), a*d/(b*c)) - ((5*(2*b^6*c^3 + 3*a*b^5*c^2*d - 4*a^2*b^4*
d^3 - (8*a^2*b^4 - a*b^5)*c*d^2)*e^2 + 10*(a*b^5*c^3 - 16*a^2*b^4*c^2*d +
8*a^3*b^3*d^3 + (16*a^3*b^3 - 7*a^2*b^4)*c*d^2)*e*f - (43*a^2*b^4*c^3 + 64
*a^4*b^2*d^3 - 3*(56*a^3*b^3 - 5*a^2*b^4)*c^2*d + 4*(32*a^4*b^2 - 19*a^3*b
^3)*c*d^2)*f^2)*x^5 + 2*(5*(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 4*a^3*b^3*d^3
- (8*a^3*b^3 - a^2*b^4)*c*d^2)*e^2 + 10*(a^2*b^4*c^3 - 16*a^3*b^3*c^2*d +
8*a^4*b^2*d^3 + (16*a^4*b^2 - 7*a^3*b^3)*c*d^2)*e*f - (43*a^3*b^3*c^3 + 64
*a^5*b*d^3 - 3*(56*a^4*b^2 - 5*a^3*b^3)*c^2*d + 4*(32*a^5*b - 19*a^4*b^2)*
c*d^2)*f^2)*x^3 + (5*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 4*a^4*b^2*d^3 - (8
*a^4*b^2 - a^3*b^3)*c*d^2)*e^2 + 10*(a^3*b^3*c^3 - 16*a^4*b^2*c^2*d + 8*a^
5*b*d^3 + (16*a^5*b - 7*a^4*b^2)*c*d^2)*e*f - (43*a^4*b^2*c^3 + 64*a^6*d^3
 - 3*(56*a^5*b - 5*a^4*b^2)*c^2*d + 4*(32*a^6 - 19*a^5*b)*c*d^2)*f^2)*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output

```
( - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x + 96*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x**3 + 69*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b*c**2*f**2*x + 180*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*
f*x - 142*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 - 120*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*f*x**3 - 16*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*b*d**2*f**2*x**5 - 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*
c**2*e*f*x + 46*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x**3 - 45
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e**2*x + 140*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c*d*e*f*x**3 + 22*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**2*c*d*f**2*x**5 + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x
**3 + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**5 + 6*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**7 - 384*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*
d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x**6 + b**3*d*x**8),x)
*a**5*d**3*f**2 + 504*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c
 + a**3*d*x**2 + 3*a**2*b*c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a
*b**2*d*x**6 + b**3*c*x**6 + b**3*d*x**8),x)*a**4*b*c*d**2*f**2 + 480*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**3*c + a**3*d*x**2 + 3*a**2*b*
c*x**2 + 3*a**2*b*d*x**4 + 3*a*b**2*c*x**4 + 3*a*b**2*d*x**6 + b**3*c*x**6
 + b**3*d*x**8),x)*a**4*b*d**3*e*f - 768*int((sqrt(c + d*x**2)*sqrt(a +...
```

3.87 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 416

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(b^3ce^2-2a^3df^2+a^2bf(2de+cf)-ab^2e(de+2cf))\,x\sqrt{c+dx^2}}{3ab^3(a+bx^2)^{3/2}} + \frac{df^2x^5\sqrt{c+dx^2}}{3b(a+bx^2)^{3/2}} + \frac{2f(3bde+2bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{a+bx^2}} + \frac{2(b^3ce^2+8a^3df^2+ab^2e(de+cf)-4a^2bf(2de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{(b^2de^2+8a^2df^2-abf(8de+3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{7/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(b^3*c*e^2-2*a^3*d*f^2+a^2*b*f*(c*f+2*d*e)-a*b^2*e*(2*c*f+d*e))*x*(d*x
^2+c)^(1/2)/a/b^3/(b*x^2+a)^(3/2)+1/3*d*f^2*x^5*(d*x^2+c)^(1/2)/b/(b*x^2+a
)^(3/2)+2/3*f*(-3*a*d*f+2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b^3/(b*x^2+a)^(
1/2)+2/3*(b^3*c*e^2+8*a^3*d*f^2+a*b^2*e*(c*f+d*e)-4*a^2*b*f*(c*f+2*d*e))*(
d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(
1/2))/a^(3/2)/b^(7/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*
(b^2*d*e^2+8*a^2*d*f^2-a*b*f*(3*c*f+8*d*e))*(d*x^2+c)^(1/2)*InverseJacobiA
M(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(7/2)/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.94 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{b}{a}\right)^{3/2}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(8a^4df^2+2b^4ce^2x^2+ab^3e(3ce+2dex^2+2cfx^2)+\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
((b/a)^(3/2)*(Sqrt[b/a]*x*(c + d*x^2)*(8*a^4*d*f^2 + 2*b^4*c*e^2*x^2 + a*b
^3*e*(3*c*e + 2*d*e*x^2 + 2*c*f*x^2) + a^3*b*f*(-8*d*e - 3*c*f + 10*d*f*x^
2) + a^2*b^2*(-4*c*f^2*x^2 + d*(e^2 - 10*e*f*x^2 + f^2*x^4))) + (2*I)*c*(b
^3*c*e^2 + 8*a^3*d*f^2 + a*b^2*e*(d*e + c*f) - 4*a^2*b*f*(2*d*e + c*f))*(a
 + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c*e^2 + 8*a^3*d*f^2 + a*b^2*e*(d*e + 2
*c*f) - a^2*b*f*(8*d*e + 5*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b^5*(a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 926 vs. 2(416) = 832.

Time = 1.23 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{e^2(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}+\frac{2efx^2(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}+\frac{f^2x^4(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{f^2\left(dx^2+c\right)^{3/2}x^3}{3b\left(bx^2+a\right)^{3/2}}+\frac{(bc-2ad)f^2\sqrt{dx^2+c}x^3}{ab^2\sqrt{bx^2+a}}-\frac{2ef\left(dx^2+c\right)^{3/2}x}{3b\left(bx^2+a\right)^{3/2}}-\frac{(3bc-8ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3ab^3}+\frac{2(bc-4ad)ef\sqrt{dx^2+c}x}{3ab^2\sqrt{bx^2+a}}+\frac{(bc-ad)e^2\sqrt{dx^2+c}x}{3ab\left(bx^2+a\right)^{3/2}}+\frac{8d(bc-2ad)f^2\sqrt{bx^2+a}x}{3b^4\sqrt{dx^2+c}}-\frac{2d(bc-8ad)ef\sqrt{bx^2+a}x}{3ab^3\sqrt{dx^2+c}}+\frac{2(bc+ad)e^2\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{8\sqrt{c}\sqrt{d}(bc-2ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}\sqrt{d}(bc-8ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}\sqrt{d}e^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(3bc-8ad)f^2\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{8c^{3/2}\sqrt{d}ef\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*d*(b*c - 8*a*d)*e*f*x*Sqrt[a + b*x^2])/(3*a*b^3*Sqrt[c + d*x^2]) + (8*
d*(b*c - 2*a*d)*f^2*x*Sqrt[a + b*x^2])/(3*b^4*Sqrt[c + d*x^2]) + ((b*c - a
*d)*e^2*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c - 4*a*d)*e*
f*x*Sqrt[c + d*x^2])/(3*a*b^2*Sqrt[a + b*x^2]) + ((b*c - 2*a*d)*f^2*x^3*Sq
rt[c + d*x^2])/(a*b^2*Sqrt[a + b*x^2]) - ((3*b*c - 8*a*d)*f^2*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*a*b^3) - (2*e*f*x*(c + d*x^2)^(3/2))/(3*b*(a + b
*x^2)^(3/2)) - (f^2*x^3*(c + d*x^2)^(3/2))/(3*b*(a + b*x^2)^(3/2)) + (2*(b
*c + a*d)*e^2*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (
a*d)/(b*c)])/(3*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) + (2*Sqrt[c]*Sqrt[d]*(b*c - 8*a*d)*e*f*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^3*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (8*Sqrt[c]*Sqrt[d]*(b*c - 2*a*d
)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*
d)])/(3*b^4*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3
/2)*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*a^2*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) + (8*c^(3/2)*Sqrt[d]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - 8*a*d)*f^2*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^3*Sqrt[d]*Sqrt[...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(381) = 762$.

Time = 20.51 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.05

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(a^3 d f^2-a^2 c f^2 b-2a^2 bdef+2acef b^2+a b^2 d e^2-b^3 c e^2\right)x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3a b^5\left(x^2+\frac{a}{b}\right)^2}+\frac{2\left(bd x^2+bc\right)\left(4a^3 d f^2-2a^2 c f^2 b \cdots\right.}{3a^2 b^4\sqrt{\left(x^2 \cdots\right.}} \cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a^3*d*f
^2-a^2*b*c*f^2-2*a^2*b*d*e*f+2*a*b^2*c*e*f+a*b^2*d*e^2-b^3*c*e^2)/a/b^5*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+2/3*(b*d*x^2+b*c)*(4*a^3*d
*f^2-2*a^2*b*c*f^2-5*a^2*b*d*e*f+a*b^2*c*e*f+a*b^2*d*e^2+b^3*c*e^2)/a^2/b^
4*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/3/b^3*d*f^2*x*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+((3*a^2*d^2*f^2-4*a*b*c*d*f^2-4*a*b*d^2*e*f+b^2*c^2*f^2+4*b^
2*c*d*e*f+b^2*d^2*e^2)/b^4-1/3*(a^3*d*f^2-a^2*b*c*f^2-2*a^2*b*d*e*f+2*a*b^
2*c*e*f+a*b^2*d*e^2-b^3*c*e^2)/b^4*d/a-2/3*(4*a^3*d*f^2-2*a^2*b*c*f^2-5*a^
2*b*d*e*f+a*b^2*c*e*f+a*b^2*d*e^2+b^3*c*e^2)/b^4*(a*d-b*c)/a^2-2/3/b^3*c*(
4*a^3*d*f^2-2*a^2*b*c*f^2-5*a^2*b*d*e*f+a*b^2*c*e*f+a*b^2*d*e^2+b^3*c*e^2)
/a^2-1/3/b^3*d*f^2*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-(-2/b^3*d*f*(a*d*f-b*c*f-b*d*e)-2/3*(4*a^3*d*f^2-2*a^2*b*c*f^2
-5*a^2*b*d*e*f+a*b^2*c*e*f+a*b^2*d*e^2+b^3*c*e^2)/b^3*d/a^2-1/3/b^3*d*f^2*
(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(381) = 762$.

Time = 0.12 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.19

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas
")
```

output
```
1/3*(2*(((b^5*c^2 + a*b^4*c*d)*e^2 + (a*b^4*c^2 - 8*a^2*b^3*c*d)*e*f - 4*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d)*f^2)*x^5 + 2*((a*b^4*c^2 + a^2*b^3*c*d)*e^2 +
 (a^2*b^3*c^2 - 8*a^3*b^2*c*d)*e*f - 4*(a^3*b^2*c^2 - 2*a^4*b*c*d)*f^2)*x^
3 + ((a^2*b^3*c^2 + a^3*b^2*c*d)*e^2 + (a^3*b^2*c^2 - 8*a^4*b*c*d)*e*f - 4
*(a^4*b*c^2 - 2*a^5*c*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) - (((2*b^5*c^2 + 2*a*b^4*c*d + a*b^4*d^2)*e^2 + 2*
(a*b^4*c^2 - 8*a^2*b^3*c*d - 4*a^2*b^3*d^2)*e*f - (8*a^2*b^3*c^2 - 8*a^3*b
^2*d^2 - (16*a^3*b^2 - 3*a^2*b^3)*c*d)*f^2)*x^5 + 2*((2*a*b^4*c^2 + 2*a^2*
b^3*c*d + a^2*b^3*d^2)*e^2 + 2*(a^2*b^3*c^2 - 8*a^3*b^2*c*d - 4*a^3*b^2*d^
2)*e*f - (8*a^3*b^2*c^2 - 8*a^4*b*d^2 - (16*a^4*b - 3*a^3*b^2)*c*d)*f^2)*x
^3 + ((2*a^2*b^3*c^2 + 2*a^3*b^2*c*d + a^3*b^2*d^2)*e^2 + 2*(a^3*b^2*c^2 -
 8*a^4*b*c*d - 4*a^4*b*d^2)*e*f - (8*a^4*b*c^2 - 8*a^5*d^2 - (16*a^5 - 3*a
^4*b)*c*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a
*d/(b*c)) + (a^2*b^3*d^2*f^2*x^6 + 2*(3*a^2*b^3*d^2*e*f + (2*a^2*b^3*c*d -
 3*a^3*b^2*d^2)*f^2)*x^4 - 2*(a^2*b^3*c*d + a^3*b^2*d^2)*e^2 - 2*(a^3*b^2*
c*d - 8*a^4*b*d^2)*e*f + 8*(a^4*b*c*d - 2*a^5*d^2)*f^2 - ((a*b^4*c*d + 3*a
^2*b^3*d^2)*e^2 + 4*(a^2*b^3*c*d - 6*a^3*b^2*d^2)*e*f - (13*a^3*b^2*c*d -
24*a^4*b*d^2)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^6*d*x^5 +
2*a^3*b^5*d*x^3 + a^4*b^4*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx=\int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output

```
(9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x - 6*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*d**2*f**2*x**3 - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*c**2*f**2*x - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x + 10*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 + 6*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b*d**2*e*f*x**3 + sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
d**2*f**2*x**5 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*e*f*x - 4*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x**3 + 3*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c*d*e**2*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**
2*c*d*e*f*x**3 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**5 + 24
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 -
 a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x
**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b
**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**6*d**4*f**2 - 60*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**
3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2
+ 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*
d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*a**5*b*c*d**3*f**2 - 24*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**
3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2
+ 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b*...
```

3.88 $\int \frac{\sqrt{c+dx^2}\,(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 364

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(b^2e^2-2abef+4a^2f^2)\,x\sqrt{c+dx^2}}{3ab^2\,(a+bx^2)^{3/2}} + \frac{f^2x^3\sqrt{c+dx^2}}{b\,(a+bx^2)^{3/2}}$$
$$+ \frac{(2b^3ce^2+8a^3df^2-ab^2e(de-2cf)-a^2bf(4de+7cf))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{5/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{(b^2de^2+4a^2df^2-abf(2de+3cf))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{5/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(4*a^2*f^2-2*a*b*e*f+b^2*e^2)*x*(d*x^2+c)^(1/2)/a/b^2/(b*x^2+a)^(3/2)+
f^2*x^3*(d*x^2+c)^(1/2)/b/(b*x^2+a)^(3/2)+1/3*(2*b^3*c*e^2+8*a^3*d*f^2-a*b
^2*e*(-2*c*f+d*e)-a^2*b*f*(7*c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(5/2)/(-a*d+b*c)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*(b^2*d*e^2+4*a^2*d*f^
2-a*b*f*(3*c*f+2*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(5/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{b}{a}\right)^{3/2}\left(-ic(2b^3ce^2+8a^3df^2+ab^2e(-de+2cf)-a^2bf(4de+7cf))\,(a+bx\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
((b/a)^(3/2)*((-I)*c*(2*b^3*c*e^2 + 8*a^3*d*f^2 + a*b^2*e*(-(d*e) + 2*c*f)
 - a^2*b*f*(4*d*e + 7*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (-(b*e) + a*f)*(Sqr
t[b/a]*x*(c + d*x^2)*(4*a^3*d*f - 2*b^3*c*e*x^2 + a*b^2*(-3*c*e + d*e*x^2
- 4*c*f*x^2) + a^2*b*(2*d*e - 3*c*f + 5*d*f*x^2)) - (2*I)*c*(-(b*c) + a*d)
*(b*e + 2*a*f)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b^4*(-(b*c) + a*d)*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 857 vs. 2(364) = 728.

Time = 1.04 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 433$$

$$\int \left(\frac{e^2\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}+\frac{2efx^2\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}+\frac{f^2x^4\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f^2\sqrt{dx^2+c}x^3}{3b\left(bx^2+a\right)^{3/2}}-\frac{(3bc-4ad)f^2\sqrt{dx^2+c}x}{3b^2(bc-ad)\sqrt{bx^2+a}}+\frac{e^2\sqrt{dx^2+c}x}{3a\left(bx^2+a\right)^{3/2}}-\frac{2ef\sqrt{dx^2+c}x}{3b\left(bx^2+a\right)^{3/2}}+\frac{d(7bc-8ad)f^2\sqrt{bx^2+a}x}{3b^3(bc-ad)\sqrt{dx^2+c}}+\frac{(2bc-ad)e^2\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{2(bc-2ad)ef\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}\sqrt{d}(7bc-8ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{c^{3/2}\sqrt{d}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(3bc-4ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}\sqrt{d}ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(d*(7*b*c - 8*a*d)*f^2*x*Sqrt[a + b*x^2])/(3*b^3*(b*c - a*d)*Sqrt[c + d*x^
2]) + (e^2*x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) - (2*e*f*x*Sqrt[c +
d*x^2])/(3*b*(a + b*x^2)^(3/2)) - (f^2*x^3*Sqrt[c + d*x^2])/(3*b*(a + b*x^
2)^(3/2)) - ((3*b*c - 4*a*d)*f^2*x*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*Sqr
t[a + b*x^2]) + ((2*b*c - a*d)*e^2*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[
b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*Sqrt[b]*(b*c - a*d)*Sqrt[a +
b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) + (2*(b*c - 2*a*d)*e*f*Sqrt[
c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*Sqr
t[a]*b^(3/2)*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^
2))]) - (Sqrt[c]*Sqrt[d]*(7*b*c - 8*a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^3*(b*c - a*d)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[d]*e^2*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(
b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(
3/2)*Sqrt[d]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*b*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2]) + (c^(3/2)*(3*b*c - 4*a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*(b*c - a*d)
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(337) = 674$.

Time = 7.02 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.92

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{(a^2f^2-2abfe+b^2e^2)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,b^4\left(x^2+\frac{a}{b}\right)^2}-\frac{(bd\,x^2+bc)x\left(5a^3d\,f^2-4a^2c\,f^2b-4a^2bdef+2acef\,b^2-a\,b^2d\,e^2+2b^3c\right.}{3b^3a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*(a^2*f^2-
2*a*b*e*f+b^2*e^2)/a/b^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2
-1/3*(b*d*x^2+b*c)/b^3/a^2/(a*d-b*c)*x*(5*a^3*d*f^2-4*a^2*b*c*f^2-4*a^2*b*
d*e*f+2*a*b^2*c*e*f-a*b^2*d*e^2+2*b^3*c*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(1/
2)+(-f*(2*a*d*f-b*c*f-2*b*d*e)/b^3+1/3*(a^2*f^2-2*a*b*e*f+b^2*e^2)/b^3*d/a
+1/3/b^3*(5*a^3*d*f^2-4*a^2*b*c*f^2-4*a^2*b*d*e*f+2*a*b^2*c*e*f-a*b^2*d*e^
2+2*b^3*c*e^2)/a^2+1/3/b^2*c/a^2/(a*d-b*c)*(5*a^3*d*f^2-4*a^2*b*c*f^2-4*a^
2*b*d*e*f+2*a*b^2*c*e*f-a*b^2*d*e^2+2*b^3*c*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f^2*d/b^2+1/3/b^2*d*(5*a^3*d*f^2-4*
a^2*b*c*f^2-4*a^2*b*d*e*f+2*a*b^2*c*e*f-a*b^2*d*e^2+2*b^3*c*e^2)/(a*d-b*c)
/a^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-E
llipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(337) = 674$.

Time = 0.12 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas
")
```

output
```
1/3*((((2*b^5*c^2 - a*b^4*c*d)*e^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d)*e*f - (
7*a^2*b^3*c^2 - 8*a^3*b^2*c*d)*f^2)*x^5 + 2*((2*a*b^4*c^2 - a^2*b^3*c*d)*e
^2 + 2*(a^2*b^3*c^2 - 2*a^3*b^2*c*d)*e*f - (7*a^3*b^2*c^2 - 8*a^4*b*c*d)*f
^2)*x^3 + ((2*a^2*b^3*c^2 - a^3*b^2*c*d)*e^2 + 2*(a^3*b^2*c^2 - 2*a^4*b*c*
d)*e*f - (7*a^4*b*c^2 - 8*a^5*c*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((2*b^5*c^2 - a*b^4*c*d + a*b^4*d^2)*
e^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d - a^2*b^3*d^2)*e*f - (7*a^2*b^3*c^2 - 4
*a^3*b^2*d^2 - (8*a^3*b^2 - 3*a^2*b^3)*c*d)*f^2)*x^5 + 2*((2*a*b^4*c^2 - a
^2*b^3*c*d + a^2*b^3*d^2)*e^2 + 2*(a^2*b^3*c^2 - 2*a^3*b^2*c*d - a^3*b^2*d
^2)*e*f - (7*a^3*b^2*c^2 - 4*a^4*b*d^2 - (8*a^4*b - 3*a^3*b^2)*c*d)*f^2)*x
^3 + ((2*a^2*b^3*c^2 - a^3*b^2*c*d + a^3*b^2*d^2)*e^2 + 2*(a^3*b^2*c^2 - 2
*a^4*b*c*d - a^4*b*d^2)*e*f - (7*a^4*b*c^2 - 4*a^5*d^2 - (8*a^5 - 3*a^4*b)
*c*d)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) + (3*(a^2*b^3*c*d - a^3*b^2*d^2)*f^2*x^4 - (2*a^2*b^3*c*d - a^3*b^2*d
^2)*e^2 - 2*(a^3*b^2*c*d - 2*a^4*b*d^2)*e*f + (7*a^4*b*c*d - 8*a^5*d^2)*f^
2 - (a*b^4*c*d*e^2 + 2*(2*a^2*b^3*c*d - 3*a^3*b^2*d^2)*e*f - (11*a^3*b^2*c
*d - 12*a^4*b*d^2)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((a^2*b^6*c*
d - a^3*b^5*d^2)*x^5 + 2*(a^3*b^5*c*d - a^4*b^4*d^2)*x^3 + (a^4*b^4*c*d -
a^5*b^3*d^2)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^2/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x + 2*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*f**2*x**3 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*f
*x - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**3 + sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b*d*e**2*x - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b
*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2
*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x
**8),x)*a**5*d**3*f**2 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**
2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**
4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8)
,x)*a**4*b*c*d**2*f**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2
*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4
 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),
x)*a**4*b*d**3*e*f - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**4
*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**2*x*
*4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**4 -
2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8),x)*
a**4*b*d**3*f**2*x**2 - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/...
```

3.89 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 331

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{(be-af)^2x\sqrt{c+dx^2}}{3ab(bc-ad)\,(a+bx^2)^{3/2}} + \frac{2(be-af)\,(b^2ce-a^2df-2ab(de-cf))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{(b^2cde^2+a^2cdf^2-ab(3d^2e^2-4cdef+3c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}c(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(-a*f+b*e)^2*x*(d*x^2+c)^(1/2)/a/b/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*(-a*
f+b*e)*(b^2*c*e-a^2*d*f-2*a*b*(-c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(3/2)/(-a*d+b*c
)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*(b^2*c*d*e^2+a^2*c
*d*f^2-a*b*(3*c^2*f^2-4*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiA
M(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/c/(-a*d+b*c
)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.25 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}(-be+af)x(c+dx^2)\left(a^3df-2b^3cex^2+ab^2(-3ce+4dex^2-4cfx^2)+\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(b*e) + a*f)*x*(c + d*x^2)*(a^3*d*f - 2*b^3*c*e*x^2 + a*b^2*(
-3*c*e + 4*d*e*x^2 - 4*c*f*x^2) + a^2*b*(5*d*e - 3*c*f + 2*d*f*x^2)) + (2*
I)*c*(-(b*e) + a*f)*(-(b^2*c*e) + a^2*d*f + 2*a*b*(d*e - c*f))*(a + b*x^2)
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*(-(b*c) + a*d)*(-2*b^2*c*e^2 + a^2*c*f^2 + a*b*e*(3*d*e
- 2*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^3*(b/a)^(3/2)*(b*c - a*d)^2*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 779 vs. 2(331) = 662.

Time = 0.94 (sec) , antiderivative size = 779, normalized size of antiderivative = 2.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}+\frac{2efx^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}+\frac{f^2x^4}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{2\sqrt{b}e^2\sqrt{c+dx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}e^2\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{a}f^2\sqrt{c+dx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{4c^{3/2}\sqrt{d}ef\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}f^2\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2ef\sqrt{c+dx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{be^2x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{2efx\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{af^2x\sqrt{c+dx^2}}{3b\left(a+bx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*e^2*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) - (2*e*f*x*S
qrt[c + d*x^2])/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (a*f^2*x*Sqrt[c + d*x^
2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sqrt[b]*(b*c - 2*a*d)*e^2*Sqr
t[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a
^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]
) + (2*(b*c + a*d)*e*f*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]], 1 - (a*d)/(b*c)])/(3*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqr
t[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (2*Sqrt[a]*(2*b*c - a*d)*f^2*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*b^(3/
2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
(Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*c^(3/2)*Sqrt[d]*e*f*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*
b*c - a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a*b*Sqrt[d]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(308) = 616$.

Time = 8.53 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.06

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(-\frac{x\left(a^2f^2-2abfe+b^2e^2\right)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3b^3a(ad-bc)\left(x^2+\frac{a}{b}\right)^2}+\frac{2\left(bd\,x^2+bc\right)x\left(a^3d\,f^2-2a^2c\,f^2b+a^2bdef+acef\,b^2-2a\,b^2d\,e^2+b^3c\,e\right.}{3b^2a^2(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/b^3/a/(a
*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
(x^2+a/b)^2+2/3*(b*d*x^2+b*c)/b^2/a^2/(a*d-b*c)^2*x*(a^3*d*f^2-2*a^2*b*c*f
^2+a^2*b*d*e*f+a*b^2*c*e*f-2*a*b^2*d*e^2+b^3*c*e^2)/((x^2+a/b)*(b*d*x^2+b*
c))^(1/2)+(f^2/b^2-1/3/b^2*d*(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)/a-2/3/(
a*d-b*c)/b^2*(a^3*d*f^2-2*a^2*b*c*f^2+a^2*b*d*e*f+a*b^2*c*e*f-2*a*b^2*d*e^
2+b^3*c*e^2)/a^2-2/3/b*c/a^2/(a*d-b*c)^2*(a^3*d*f^2-2*a^2*b*c*f^2+a^2*b*d*
e*f+a*b^2*c*e*f-2*a*b^2*d*e^2+b^3*c*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3/b*(a^3*d*f^2-2*a^2*b*c*f^2+a^2*b*d*e*f+a
*b^2*c*e*f-2*a*b^2*d*e^2+b^3*c*e^2)/(a*d-b*c)^2/a^2*c/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(308) = 616$.

Time = 0.13 (sec) , antiderivative size = 950, normalized size of antiderivative = 2.87

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(2*(((b^6*c^2 - 2*a*b^5*c*d)*e^2 + (a*b^5*c^2 + a^2*b^4*c*d)*e*f - (2
*a^2*b^4*c^2 - a^3*b^3*c*d)*f^2)*x^4 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d)*e^2 +
 (a^3*b^3*c^2 + a^4*b^2*c*d)*e*f - (2*a^4*b^2*c^2 - a^5*b*c*d)*f^2 + 2*((a
*b^5*c^2 - 2*a^2*b^4*c*d)*e^2 + (a^2*b^4*c^2 + a^3*b^3*c*d)*e*f - (2*a^3*b
^3*c^2 - a^4*b^2*c*d)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*s
qrt(-b/a)), a*d/(b*c)) - (((2*b^6*c^2 - 3*a^3*b^3*d^2 + (a^2*b^4 - 4*a*b^5
)*c*d)*e^2 + 2*(a*b^5*c^2 + (2*a^3*b^3 + a^2*b^4)*c*d)*e*f - ((3*a^3*b^3 +
 4*a^2*b^4)*c^2 - (a^4*b^2 + 2*a^3*b^3)*c*d)*f^2)*x^4 + (2*a^2*b^4*c^2 - 3
*a^5*b*d^2 + (a^4*b^2 - 4*a^3*b^3)*c*d)*e^2 + 2*(a^3*b^3*c^2 + (2*a^5*b +
a^4*b^2)*c*d)*e*f - ((3*a^5*b + 4*a^4*b^2)*c^2 - (a^6 + 2*a^5*b)*c*d)*f^2
+ 2*((2*a*b^5*c^2 - 3*a^4*b^2*d^2 + (a^3*b^3 - 4*a^2*b^4)*c*d)*e^2 + 2*(a^
2*b^4*c^2 + (2*a^4*b^2 + a^3*b^3)*c*d)*e*f - ((3*a^4*b^2 + 4*a^3*b^3)*c^2
- (a^5*b + 2*a^4*b^2)*c*d)*f^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsi
n(x*sqrt(-b/a)), a*d/(b*c)) - (2*((a*b^5*c^2 - 2*a^2*b^4*c*d)*e^2 + (a^2*b
^4*c^2 + a^3*b^3*c*d)*e*f - (2*a^3*b^3*c^2 - a^4*b^2*c*d)*f^2)*x^3 + (4*a^
4*b^2*c*d*e*f + (3*a^2*b^4*c^2 - 5*a^3*b^3*c*d)*e^2 - (3*a^4*b^2*c^2 - a^5
*b*c*d)*f^2)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^4*c^3 - 2*a^6*b^3*
c^2*d + a^7*b^2*c*d^2 + (a^3*b^6*c^3 - 2*a^4*b^5*c^2*d + a^5*b^4*c*d^2)*x^
4 + 2*(a^4*b^5*c^3 - 2*a^5*b^4*c^2*d + a^6*b^3*c*d^2)*x^2)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2
 + 3*a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*
a*b**3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 -
 b**4*c*d*x**8),x)*a**4*d**2*f**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*
a**3*b*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**
3*c**2*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4
*c*d*x**8),x)*a**3*b*c*d*f**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b
*d**2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2
*x**4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x
**8),x)*a**3*b*d**2*e*f + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**
2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**
4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x**8)
,x)*a**3*b*d**2*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a**4*c*d + a**4*d**2*x**2 - a**3*b*c**2 + 2*a**3*b*c*d*x**2 + 3*a**3*b*d**
2*x**4 - 3*a**2*b**2*c**2*x**2 + 3*a**2*b**2*d**2*x**6 - 3*a*b**3*c**2*x**
4 - 2*a*b**3*c*d*x**6 + a*b**3*d**2*x**8 - b**4*c**2*x**6 - b**4*c*d*x*...
```

3.90 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 440

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{(be-af)^2x}{3ab(bc-ad)(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$
$$+\frac{2(be-af)(bce-3ade+2acf)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{d}(2b^3c^2e^2-a^3cdf^2-ab^2ce(7de-2cf)-a^2b(3d^2e^2-14cdef+7c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3a^2b\sqrt{c}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{c}(b^2cde^2+a^2df(6de-5cf)-ab(9d^2e^2-10cdef+3c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right.}{3a^2\sqrt{d}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*f+b*e)^2*x/a/b/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+2/3*(-a*
f+b*e)*(2*a*c*f-3*a*d*e+b*c*e)*x/a^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)+1/3*d^(1/2)*(2*b^3*c^2*e^2-a^3*c*d*f^2-a*b^2*c*e*(-2*c*f+7*d*e)-a^
2*b*(7*c^2*f^2-14*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/
c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^2/b/c^(1/2)/(-a*d+b*c)^3/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*c^(1/2)*(b^2*c*d*e^2+a^2
*d*f*(-5*c*f+6*d*e)-a*b*(3*c^2*f^2-10*c*d*e*f+9*d^2*e^2))*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/d^(1/2)/(
-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.76 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(-2b^4c^2e^2x^2(c+dx^2)-ab^3ce(c+dx^2)(3ce-7dex^2+2cfx^2)+a^4$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*x*(-2*b^4*c^2*e^2*x^2*(c + d*x^2) - a*b^3*c*e*(c + d*x^2)*(3*c*
e - 7*d*e*x^2 + 2*c*f*x^2) + a^4*d*(3*d^2*e^2 + 5*c^2*f^2 + 2*c*d*f*(-3*e
+ f*x^2)) + a^3*b*(3*c^3*f^2 + 6*d^3*e^2*x^2 + c*d^2*f*x^2*(-22*e + f*x^2)
 + 10*c^2*d*f*(-e + f*x^2)) + a^2*b^2*(4*c^3*f^2*x^2 + 3*d^3*e^2*x^4 + 2*c
*d^2*e*x^2*(4*e - 7*f*x^2) + c^2*d*(8*e^2 - 8*e*f*x^2 + 7*f^2*x^4))) + I*c
*(-2*b^3*c^2*e^2 + a^3*c*d*f^2 + a*b^2*c*e*(7*d*e - 2*c*f) + a^2*b*(3*d^2*
e^2 - 14*c*d*e*f + 7*c^2*f^2))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c)
+ a*d)*(-(b*e) + a*f)*(b*c*e - 3*a*d*e + 2*a*c*f)*(a + b*x^2)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
)/(3*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 953 vs. 2(440) = 880.

Time = 1.22 (sec) , antiderivative size = 953, normalized size of antiderivative = 2.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}+\frac{2efx^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}+\frac{f^2x^4}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d}\left(2b^2c^2-7abdc-3a^2d^2\right)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{3a^2\sqrt{c}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}\sqrt{d}(bc-9ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{3a^2(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2b(bc-3ad)xe^2}{3a^2(bc-ad)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{bxe^2}{3a(bc-ad)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}+\frac{2\sqrt{c}\sqrt{d}(bc+7ad)f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{3a(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2\sqrt{c}\sqrt{d}(5bc+3ad)f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{3a(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2(bc+3ad)fxe}{3a(bc-ad)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{2fxe}{3(bc-ad)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{d}(7bc+ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(3bc+5ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{d}(bc-ad)^3\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{4cf^2x}{3(bc-ad)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{af^2x}{3b(bc-ad)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(b*e^2*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) - (2*e*f*x)/
(3*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (a*f^2*x)/(3*b*(b*c -
a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*(b*c - 3*a*d)*e^2*x)/(3*a^2
*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (2*(b*c + 3*a*d)*e*f*x)/
(3*a*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (4*c*f^2*x)/(3*(b*c
- a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(2*b^2*c^2 - 7*a*b*c*
d - 3*a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*Sqrt[d]*(b*c + 7*a*d)*e*f*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*(b*
c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[d]*(7*b*c + a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*e^2*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(
b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*S
qrt[c]*Sqrt[d]*(5*b*c + 3*a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c + 5*a*d)*f^2*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sq...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(411) = 822$.

Time = 10.97 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.97

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{x\left(a^2f^2-2abfe+b^2e^2\right)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3b^2a(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)x\left(a^3d\,f^2+4a^2c\,f^2b-8a^2bdef-2acef\,b^2+7a\,b^2d\,e^2-2b^3c\right.}{3b\,a^2(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}\right.}{}$
default	Expression too large to display

input

```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/b^2/a/(a*
d-b*c)^2*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)/b/a^2/(a*d-b*c)^3*x*(a^3*d*f^2+4*a^2*b*c*f^
2-8*a^2*b*d*e*f-2*a*b^2*c*e*f+7*a*b^2*d*e^2-2*b^3*c*e^2)/((x^2+a/b)*(b*d*x
^2+b*c))^(1/2)+(b*d*x^2+a*d)/c/(a*d-b*c)^3*x*(c^2*f^2-2*c*d*e*f+d^2*e^2)/(
(x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/3*d/b*(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b
*c)^2/a-1/3/(a*d-b*c)^2/b*(a^3*d*f^2+4*a^2*b*c*f^2-8*a^2*b*d*e*f-2*a*b^2*c
*e*f+7*a*b^2*d*e^2-2*b^3*c*e^2)/a^2-1/3*c/a^2/(a*d-b*c)^3*(a^3*d*f^2+4*a^2
*b*c*f^2-8*a^2*b*d*e*f-2*a*b^2*c*e*f+7*a*b^2*d*e^2-2*b^3*c*e^2)+1/(a*d-b*c
)^2*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c-a*d/c/(a*d-b*c)^3*(c^2*f^2-2*c*d*e*f+d^2
*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-1/3
*d*(a^3*d*f^2+4*a^2*b*c*f^2-8*a^2*b*d*e*f-2*a*b^2*c*e*f+7*a*b^2*d*e^2-2*b^
3*c*e^2)/(a*d-b*c)^3/a^2-b*d*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c/(a*d-b*c)^3)*c/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(411) = 822$.

Time = 0.17 (sec) , antiderivative size = 1836, normalized size of antiderivative = 4.17

$$\int \frac{(e + fx^2)^2}{(a + bx^2)^{5/2} (c + dx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
-1/3*((((2*b^6*c^2*d - 7*a*b^5*c*d^2 - 3*a^2*b^4*d^3)*e^2 + 2*(a*b^5*c^2*d
 + 7*a^2*b^4*c*d^2)*e*f - (7*a^2*b^4*c^2*d + a^3*b^3*c*d^2)*f^2)*x^6 + ((2
*b^6*c^3 - 3*a*b^5*c^2*d - 17*a^2*b^4*c*d^2 - 6*a^3*b^3*d^3)*e^2 + 2*(a*b^
5*c^3 + 9*a^2*b^4*c^2*d + 14*a^3*b^3*c*d^2)*e*f - (7*a^2*b^4*c^3 + 15*a^3*
b^3*c^2*d + 2*a^4*b^2*c*d^2)*f^2)*x^4 + (2*a^2*b^4*c^3 - 7*a^3*b^3*c^2*d -
 3*a^4*b^2*c*d^2)*e^2 + 2*(a^3*b^3*c^3 + 7*a^4*b^2*c^2*d)*e*f - (7*a^4*b^2
*c^3 + a^5*b*c^2*d)*f^2 + ((4*a*b^5*c^3 - 12*a^2*b^4*c^2*d - 13*a^3*b^3*c*
d^2 - 3*a^4*b^2*d^3)*e^2 + 2*(2*a^2*b^4*c^3 + 15*a^3*b^3*c^2*d + 7*a^4*b^2
*c*d^2)*e*f - (14*a^3*b^3*c^3 + 9*a^4*b^2*c^2*d + a^5*b*c*d^2)*f^2)*x^2)*s
qrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*b^6
*c^2*d + (a^2*b^4 - 7*a*b^5)*c*d^2 - 3*(3*a^3*b^3 + a^2*b^4)*d^3)*e^2 + 2*
(a*b^5*c^2*d + 3*a^4*b^2*d^3 + (5*a^3*b^3 + 7*a^2*b^4)*c*d^2)*e*f - ((3*a^
3*b^3 + 7*a^2*b^4)*c^2*d + (5*a^4*b^2 + a^3*b^3)*c*d^2)*f^2)*x^6 + ((2*b^6
*c^3 + (a^2*b^4 - 3*a*b^5)*c^2*d - (7*a^3*b^3 + 17*a^2*b^4)*c*d^2 - 6*(3*a
^4*b^2 + a^3*b^3)*d^3)*e^2 + 2*(a*b^5*c^3 + 6*a^5*b*d^3 + (5*a^3*b^3 + 9*a
^2*b^4)*c^2*d + (13*a^4*b^2 + 14*a^3*b^3)*c*d^2)*e*f - ((3*a^3*b^3 + 7*a^2
*b^4)*c^3 + (11*a^4*b^2 + 15*a^3*b^3)*c^2*d + 2*(5*a^5*b + a^4*b^2)*c*d^2)
*f^2)*x^4 + (2*a^2*b^4*c^3 + (a^4*b^2 - 7*a^3*b^3)*c^2*d - 3*(3*a^5*b + a^
4*b^2)*c*d^2)*e^2 + 2*(a^3*b^3*c^3 + 3*a^6*c*d^2 + (5*a^5*b + 7*a^4*b^2)*c
^2*d)*e*f - ((3*a^5*b + 7*a^4*b^2)*c^3 + (5*a^6 + a^5*b)*c^2*d)*f^2 + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2*b*
c**2*x**2 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4 +
6*a*b**2*c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x**8
+ b**3*d**2*x**10),x)*a**2*b*c**2*f**2 - 3*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2*b*c**
2*x**2 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4 + 6*a
*b**2*c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x**8 + b
**3*d**2*x**10),x)*a**2*b*c*d*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**4)/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2*b*c**2*x**2
+ 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4 + 6*a*b**2*c
*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x**8 + b**3*d**
2*x**10),x)*a**2*b*c*d*f**2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2*b*c**2*x**2
 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4 + 6*a*b**2*
c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x**8 + b**3*d*
*2*x**10),x)*a**2*b*d**2*e*f*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a**3*c**2 + 2*a**3*c*d*x**2 + a**3*d**2*x**4 + 3*a**2*b*c**2*x**
2 + 6*a**2*b*c*d*x**4 + 3*a**2*b*d**2*x**6 + 3*a*b**2*c**2*x**4 + 6*a*b**2
*c*d*x**6 + 3*a*b**2*d**2*x**8 + b**3*c**2*x**6 + 2*b**3*c*d*x**8 + b**...
```

3.91 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 613

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{(be-af)^2x}{3ab(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^{3/2}}$$
$$+\frac{2(be-af)(b^2ce+a^2df-ab(4de-2cf))\,x}{3a^2b(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}$$
$$+\frac{d(2b^3c^2e^2-3a^3cdf^2-ab^2ce(9de-2cf)-a^2b(d^2e^2-14cdef+5c^2f^2))\,x\sqrt{a+bx^2}}{3a^2bc(bc-ad)^3(c+dx^2)^{3/2}}$$
$$+\frac{2\sqrt{d}(b^3c^3e^2-ab^2c^2e(5de-cf)+a^3d(d^2e^2+cdef-4c^2f^2)-a^2bc(5d^2e^2-14cdef+4c^2f^2))\sqrt{a+bx^2}\,E}{3a^2c^{3/2}(bc-ad)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{(b^3c^2de^2-3a^3cd^2f^2+a^2bd(d^2e^2+16cdef-10c^2f^2)-ab^2c(18d^2e^2-16cdef+3c^2f^2))\sqrt{a+bx^2}\,\text{Ellipt}}{3a^2\sqrt{c}\sqrt{d}(bc-ad)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*f+b*e)^2*x/a/b/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)+2/3*(-a*
f+b*e)*(b^2*c*e+a^2*d*f-a*b*(-2*c*f+4*d*e))*x/a^2/b/(-a*d+b*c)^2/(b*x^2+a)
^(1/2)/(d*x^2+c)^(3/2)+1/3*d*(2*b^3*c^2*e^2-3*a^3*c*d*f^2-a*b^2*c*e*(-2*c*
f+9*d*e)-a^2*b*(5*c^2*f^2-14*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/a^2/b/c/(
-a*d+b*c)^3/(d*x^2+c)^(3/2)+2/3*d^(1/2)*(b^3*c^3*e^2-a*b^2*c^2*e*(-c*f+5*d
*e)+a^3*d*(-4*c^2*f^2+c*d*e*f+d^2*e^2)-a^2*b*c*(4*c^2*f^2-14*c*d*e*f+5*d^2
*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*
c/a/d)^(1/2))/a^2/c^(3/2)/(-a*d+b*c)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*
x^2+c)^(1/2)-1/3*(b^3*c^2*d*e^2-3*a^3*c*d^2*f^2+a^2*b*d*(-10*c^2*f^2+16*c*
d*e*f+d^2*e^2)-a*b^2*c*(3*c^2*f^2-16*c*d*e*f+18*d^2*e^2))*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/d
^(1/2)/(-a*d+b*c)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.07 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}x\Big(a^2cd(bc-ad)(de-cf)^2(a+bx^2)^2 - 2a^2d(de-cf)(bc(-5de+$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-(Sqrt[b/a]*x*(a^2*c*d*(b*c - a*d)*(d*e - c*f)^2*(a + b*x^2)^2 - 2*a^2*d*
(d*e - c*f)*(b*c*(-5*d*e + 2*c*f) + a*d*(d*e + 2*c*f))*(a + b*x^2)^2*(c +
d*x^2) + a*b*c^2*(-(b*c) + a*d)*(b*e - a*f)^2*(c + d*x^2)^2 - 2*b*c^2*(b*e
 - a*f)*(b^2*c*e + 2*a^2*d*f + a*b*(-5*d*e + 2*c*f))*(a + b*x^2)*(c + d*x^
2)^2)) - I*c*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/
c]*(-2*b*(b^3*c^3*e^2 + a*b^2*c^2*e*(-5*d*e + c*f) + a^3*d*(d^2*e^2 + c*d*
e*f - 4*c^2*f^2) + a^2*b*c*(-5*d^2*e^2 + 14*c*d*e*f - 4*c^2*f^2))*Elliptic
E[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(2*b^3*c^2*e^2 - 3*a^
3*c*d*f^2 + a*b^2*c*e*(-9*d*e + 2*c*f) - a^2*b*(d^2*e^2 - 14*c*d*e*f + 5*c
^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*a^2*Sqrt[b/a]
*c^2*(b*c - a*d)^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 1170, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{e^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}+\frac{2efx^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}+\frac{f^2x^4}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\sqrt{d}(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{3a^2c^{3/2}(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{d}\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{3a^2\sqrt{c}(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(2b^2c^2-9abdc-a^2d^2\right)x\sqrt{bx^2+a}e^2}{3a^2c(bc-ad)^3\left(dx^2+c\right)^{3/2}}+\frac{2b(bc-4ad)xe^2}{3a^2(bc-ad)^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{bxe^2}{3a(bc-ad)\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}+\frac{2\sqrt{d}\left(b^2c^2+14abdc+a^2d^2\right)f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{3a\sqrt{c}(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{16b\sqrt{c}\sqrt{d}(bc+ad)f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{3a(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2d(bc+7ad)fx\sqrt{bx^2+a}e}{3a(bc-ad)^3\left(dx^2+c\right)^{3/2}}+\frac{2(bc+5ad)fxe}{3a(bc-ad)^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}-\frac{2fxe}{3(bc-ad)\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}-\frac{8\sqrt{c}\sqrt{d}(bc+ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}(3bc+ad)(bc+3ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{d}(bc-ad)^4\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{d(5bc+3ad)f^2x\sqrt{bx^2+a}}{3b(bc-ad)^3\left(dx^2+c\right)^{3/2}}-\frac{2(2bc+ad)f^2x}{3b(bc-ad)^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{af^2x}{3b(bc-ad)\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(b*e^2*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)) - (2*e*f*x
)/(3*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)) + (a*f^2*x)/(3*b*(b*
c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)) + (2*b*(b*c - 4*a*d)*e^2*x)/
(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) + (2*(b*c + 5*a*d)
*e*f*x)/(3*a*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) - (2*(2*b*c
+ a*d)*f^2*x)/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) + (d*(
2*b^2*c^2 - 9*a*b*c*d - a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(3*a^2*c*(b*c - a*
d)^3*(c + d*x^2)^(3/2)) + (2*d*(b*c + 7*a*d)*e*f*x*Sqrt[a + b*x^2])/(3*a*(
b*c - a*d)^3*(c + d*x^2)^(3/2)) - (d*(5*b*c + 3*a*d)*f^2*x*Sqrt[a + b*x^2]
)/(3*b*(b*c - a*d)^3*(c + d*x^2)^(3/2)) + (2*Sqrt[d]*(b*c + a*d)*(b^2*c^2
- 6*a*b*c*d + a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(3*a^2*c^(3/2)*(b*c - a*d)^4*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[d]*(b^2*c^2 + 14*a*b*c*d + a
^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*a*Sqrt[c]*(b*c - a*d)^4*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) - (8*Sqrt[c]*Sqrt[d]*(b*c + a*d)*f^2*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*(b*c - a*d)^4*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*(b^2*c^2
 - 18*a*b*c*d + a^2*d^2)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^4*Sqrt[(c*(a + b...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(578) = 1156$.

Time = 20.42 (sec) , antiderivative size = 1267, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	1267
default	Expression too large to display	5222

input
```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((1/3/b^2/d^2*
(a^2*c*d*f^2+a*b*c^2*f^2-4*a*b*c*d*e*f+a*b*d^2*e^2+b^2*c*d*e^2)/a/c/(a^2*d
^2-2*a*b*c*d+b^2*c^2)*x^3+1/3/b^2/d^2*(2*a^2*c^2*f^2-2*a^2*c*d*e*f+a^2*d^2
*e^2-2*a*b*c^2*e*f+b^2*c^2*e^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^4+(a*d+b*c)/d/b*x^2+a*c/d/b)^2-2*b*d*(1/3*
(4*a^3*c^2*d*f^2-a^3*c*d^2*e*f-a^3*d^3*e^2+4*a^2*b*c^3*f^2-14*a^2*b*c^2*d*
e*f+5*a^2*b*c*d^2*e^2-a*b^2*c^3*e*f+5*a*b^2*c^2*d*e^2-b^3*c^3*e^2)/a^2/c^2
/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2*x^3+1/6*(5*a^4*c^2*d^2*f^2-2*a^4*c*d^3*e*f-
2*a^4*d^4*e^2+6*a^3*b*c^3*d*f^2-14*a^3*b*c^2*d^2*e*f+9*a^3*b*c*d^3*e^2+5*a
^2*b^2*c^4*f^2-14*a^2*b^2*c^3*d*e*f+2*a^2*b^2*c^2*d^2*e^2-2*a*b^3*c^4*e*f+
9*a*b^3*c^3*d*e^2-2*b^4*c^4*e^2)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2/b/d
*x)/((x^4+(a*d+b*c)/d/b*x^2+a*c/d/b)*b*d)^(1/2)+(-2/3*(a^2*c^2*f^2-a^2*c*d
*e*f-a^2*d^2*e^2-a*b*c^2*e*f+3*a*b*c*d*e^2-b^2*c^2*e^2)/(a^2*d^2-2*a*b*c*d
+b^2*c^2)/a^2/c^2+1/3*(5*a^4*c^2*d^2*f^2-2*a^4*c*d^3*e*f-2*a^4*d^4*e^2+6*a
^3*b*c^3*d*f^2-14*a^3*b*c^2*d^2*e*f+9*a^3*b*c*d^3*e^2+5*a^2*b^2*c^4*f^2-14
*a^2*b^2*c^3*d*e*f+2*a^2*b^2*c^2*d^2*e^2-2*a*b^3*c^4*e*f+9*a*b^3*c^3*d*e^2
-2*b^4*c^4*e^2)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2)/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*b*(4*a^3*c^2*d*f^2-a^3*c*d^
2*e*f-a^3*d^3*e^2+4*a^2*b*c^3*f^2-14*a^2*b*c^2*d*e*f+5*a^2*b*c*d^2*e^2-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3168 vs. 2(578) = 1156.

Time = 0.37 (sec) , antiderivative size = 3168, normalized size of antiderivative = 5.17

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4
 + a**4*d**4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d*
*2*x**4 + 10*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x*
*2 + 12*a**2*b**2*c**3*d*x**4 + 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2
*c*d**3*x**8 + 3*a**2*b**2*d**4*x**10 + 3*a*b**3*c**4*x**4 + 10*a*b**3*c**
3*d*x**6 + 12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 + a*b**3*d**4*
x**12 + b**4*c**4*x**6 + 3*b**4*c**3*d*x**8 + 3*b**4*c**2*d**2*x**10 + b**
4*c*d**3*x**12),x)*a**4*c**2*d**2*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 +
a**4*d**4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d**2*
x**4 + 10*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x**2
+ 12*a**2*b**2*c**3*d*x**4 + 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2*c*
d**3*x**8 + 3*a**2*b**2*d**4*x**10 + 3*a*b**3*c**4*x**4 + 10*a*b**3*c**3*d
*x**6 + 12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 + a*b**3*d**4*x**
12 + b**4*c**4*x**6 + 3*b**4*c**3*d*x**8 + 3*b**4*c**2*d**2*x**10 + b**4*c
*d**3*x**12),x)*a**4*c*d**3*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 + a**
4*d**4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d**2*x**
4 + 10*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x**2 ...
```

3.92 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 561

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2} dx = \\
&-\frac{(2a^2d^2f^2+abdf(5de-12cf)-b^2(15d^2e^2-35cdef+23c^2f^2))\,x\sqrt{c+dx^2}}{15bf^3\sqrt{a+bx^2}} \\
&-\frac{d(5bde-11bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bf^2}+\frac{d^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f} \\
&+\frac{\sqrt{a}(2a^2d^2f^2+abdf(5de-12cf)-b^2(15d^2e^2-35cdef+23c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{a^{3/2}d(acdf^2-b(15d^2e^2-40cdef+34c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{a^{3/2}(de-cf)^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(2*a^2*d^2*f^2+a*b*d*f*(-12*c*f+5*d*e)-b^2*(23*c^2*f^2-35*c*d*e*f+15
*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/f^3/(b*x^2+a)^(1/2)-1/15*d*(-a*d*f-11*b*c*f
+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/f^2+1/5*d^2*x^3*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/f+1/15*a^(1/2)*(2*a^2*d^2*f^2+a*b*d*f*(-12*c*f+5*d*e)-
b^2*(23*c^2*f^2-35*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/f^3/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*d*(a*c*d*f^2-b*(34*c^2*f^2-4
0*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)-a^(3/2)*(-c*f+d*e)^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^3/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 1203, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2),x]
```

output

```
(-5*a*b*Sqrt[b/a]*c*d^2*e^2*f^2*x + 11*a*b*Sqrt[b/a]*c^2*d*e*f^3*x + a^2*S
qrt[b/a]*c*d^2*e*f^3*x - 5*a*b*(b/a)^(3/2)*c*d^2*e^2*f^2*x^3 - 5*a*b*Sqrt[
b/a]*d^3*e^2*f^2*x^3 + 11*a*b*(b/a)^(3/2)*c^2*d*e*f^3*x^3 + 15*a*b*Sqrt[b/
a]*c*d^2*e*f^3*x^3 + a^2*Sqrt[b/a]*d^3*e*f^3*x^3 - 5*a*b*(b/a)^(3/2)*d^3*e
^2*f^2*x^5 + 14*a*b*(b/a)^(3/2)*c*d^2*e*f^3*x^5 + 4*a*b*Sqrt[b/a]*d^3*e*f^
3*x^5 + 3*a*b*(b/a)^(3/2)*d^3*e*f^3*x^7 + I*c*e*f*(2*a^2*d^2*f^2 + a*b*d*f
*(5*d*e - 12*c*f) + b^2*(-15*d^2*e^2 + 35*c*d*e*f - 23*c^2*f^2))*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)] - I*e*(a^2*c*d^2*f^3 + a*b*d*f*(15*d^2*e^2 - 35*c*d*e*f + 22*c^2*f^2)
- b^2*(15*d^3*e^3 - 30*c*d^2*e^2*f + 10*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] - (15*I)*b^2*d^3*e^4*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (45*I)*b^2*c*d^2*e^
3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*a*b*d^3*e^3*f*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - (45*I)*b^2*c^2*d*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (45*I)*a*b*
c*d^2*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*
e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*b^2*c^3*e*f^3*Sqrt[1 ...
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.27, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {420, 318, 403, 27, 406, 320, 388, 313, 418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{d\int \sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{d\left(\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 403

$$\frac{d\left(\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{\frac{1}{3}\int \frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 406

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+}}{5b}\right.}{f}$$

$$\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 320

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\right.}{f}$$

$$\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\downarrow$ 388

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(}{5b}\right.}{f}$$

$$\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\downarrow$ 313

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\downarrow$ 418

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}+\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{\int\frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int\frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\downarrow$ 406

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a}\right.}{f^2}\right.}{f}$$

$\downarrow$ 320

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}}{f}$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{f}$$

$\downarrow$ 388

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right.}{f}\cdots$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a\cdots\right.}{a\left(c\cdots\right.}}}\cdots\right.\right.}{f^2}\cdots\right.}{f}\cdots$$

$\Big\downarrow$ 313

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right.}{f}\cdots$$

$$\frac{(de-cf)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\cdots\right.\right.\right.}{f^2}\cdots\right.}{f}\cdots$$

$\Big\downarrow$ 414

$$\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right.}{f}$$

$$\frac{(de-cf)\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-\cdots\right.\right.}{\cdots}\right.}{f}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2),x]
```

output
```
(d*((d*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + ((2*(3*b*c - a*d)*x*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/3 + ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*((
x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*b)))/f - ((
d*e - c*f)*(-((d*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e -
 4*b*c*f - a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(
3*d*e - 5*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2]))/3))/f^2) + (a^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f
)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f^2*S
qrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 9.61 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.16

method	result
risch	$\frac{dx\left(3bdf\,x^2+adf+11bcf-5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b\,f^2}-\frac{\left(\frac{\left(a^2c\,d^2f^3-34ab\,c^2d\,f^3+40abc\,d^2e\,f^2-15ab\,d^3e^2f-15b^2c^3f^3+45b^2c^2de\,f^2-45b^2\right.}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*d*x*(3*b*d*f*x^2+a*d*f+11*b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/b/f^2-1/15/b/f^2*((a^2*c*d^2*f^3-34*a*b*c^2*d*f^3+40*a*b*c*d^2*e*f^2-1
5*a*b*d^3*e^2*f-15*b^2*c^3*f^3+45*b^2*c^2*d*e*f^2-45*b^2*c*d^2*e^2*f+15*b^
2*d^3*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))-1/f*(2*a^2*d^2*f^2-12*a*b*c*d*f^2+5*a*b*d^2*e*f-23*b^2*c^2*f^2+35*b^2*c
*d*e*f-15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-15*(a*c^
3*f^4-3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3+3*b*c^2*d*
e^2*f^2-3*b*c*d^2*e^3*f+b*d^3*e^4)*b/f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^
(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b
*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{e+fx^2}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{5}{2}}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{5/2}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{e+fx^2}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d**2*f*x + 11*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b*c*d*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*e*x + 3*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*f*x**3 - 2*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c
*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**3*f**2 + 12*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*
c*d**2*f**2 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*
f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
b*d*f*x**6),x)*a*b*d**3*e*f + 23*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**
4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c**2*d*f**2 - 35*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b
*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d**2*e*f + 15*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b
**2*d**3*e**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*
f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
b*d*f*x**6),x)*a**2*c*d**2*f**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c...
```

3.93 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 402

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx &= -\frac{(3bde-4bcf-adf)x\sqrt{c+dx^2}}{3f^2\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f}\\
&+\frac{\sqrt{a}(3bde-4bcf-adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&-\frac{a^{3/2}d(3de-5cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{a^{3/2}(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/3*(-a*d*f-4*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/f^2/(b*x^2+a)^(1/2)+1/3*d*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3*a^(1/2)*(-a*d*f-4*b*c*f+3*b*d*e)*(
d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(
1/2))/b^(1/2)/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3
/2)*d*(-5*c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1
/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+a^(3/2)*(-c*f+d*e)^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^2/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.57 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{a\sqrt{\frac{b}{a}}cdef^2x+b\sqrt{\frac{b}{a}}cdef^2x^3+a\sqrt{\frac{b}{a}}d^2ef^2x^3+b\sqrt{\frac{b}{a}}d^2ef^2x^5-icef(-3bde+}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(a*Sqrt[b/a]*c*d*e*f^2*x + b*Sqrt[b/a]*c*d*e*f^2*x^3 + a*Sqrt[b/a]*d^2*e*f
^2*x^3 + b*Sqrt[b/a]*d^2*e*f^2*x^5 - I*c*e*f*(-3*b*d*e + 4*b*c*f + a*d*f)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*e*(a*d*f*(-3*d*e + 4*c*f) + b*(3*d^2*e^2 - 3*c*d*e*f - c^
2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + (3*I)*b*d^2*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (6*I)*
b*c*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e)
, I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*d^2*e^2*f*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + (3*I)*b*c^2*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (6*I)*a*c*d*e
*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c^2*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*Sqrt[b/a]*e*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

$$\Big\downarrow \text{418}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}+\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{\sqrt{dx^2+c}}dx}{f^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\int \frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{\sqrt{dx^2+c}}dx}{f^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{\int \frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int \frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{406}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)-\frac{1}{3}fx\sqrt{a+bx^2}\right.}{f^2}$$

$\downarrow$ 388

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)-\frac{1}{3}f\right.}{f^2}$$

$\downarrow$ 313

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.\right.\right.}{f^2}$$

$\downarrow$ 414

$$\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.\right.\right.}{f^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
-((d*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e - 4*b*c*f - a
*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - 5*c*
f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3))/f^2
) + (a^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), Arc
Tan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f^2*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(372) = 744$.

Time = 9.68 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.07

method	result
risch	$\frac{dx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3f}+\frac{\left(-\frac{f(adf+4bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3/f*(1/f^2*(-f*(a*d*f+4*b*c*f-
3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*b*c^2*f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b*d^2*e^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5*a*c*d*f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*d^2*e*f/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*b*c*d*e*f/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*(a*c^2*f^3-2*a*c*d*
e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^2/e/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(
d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{(d\,x^2+c)}^{3/2}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,dx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*d*x + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d**2*f + 4*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*f - 3*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d**2*e +
5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*a*c*d*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*d**2*e + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*b*c**2*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f
*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*e + 3*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
 b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c**2*f - int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*d*e)/(3*f)
```

3.94 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx &= \frac{bx\sqrt{c+dx^2}}{f\sqrt{a+bx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{f\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad + \frac{a^{3/2}d\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad - \frac{a^{3/2}(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
b*x*(d*x^2+c)^(1/2)/f/(b*x^2+a)^(1/2)-a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/f/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*d*(d*x^2+c)^(1/2)*InverseJaco
biAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*Ell
ipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b
^(1/2)/c/e/f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(bcefE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-(be-af)\left(de\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)+\right.\right.}{\sqrt{\frac{b}{a}}ef^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*e*f*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*e - a*f)*(d*e*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] + (-(d*e) + c*f)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/a]*e*f^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {410, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}dx$$

$$\Big\downarrow \text{410}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(b*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (c^(3/2)*(b*e -
a*f)*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adef-\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd\,e^2+\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) \cdots\right.}{(bd \cdots}$
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)ad}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac}}-\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)bde}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac}}+ \cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*e*f-EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b*d*e^2+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*f
+EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c*f^2-El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d*e*f-Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c*e*f+Elliptic
Pi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d*e^2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/f^2/(-b/a)^(1/2)/e
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e + f*x**2),x)
```

3.95 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
a^(3/2)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a
*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(be\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)+(-be+af)\,\text{EllipticPi}\left(\frac{af}{be},i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.\right.}{\sqrt{\frac{b}{a}}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*e*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (-(b*e) + a*f)*EllipticPi[(a*f)/(b*e), I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*e*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \ 414$$

$$\frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(a^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sq
rt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2)
)/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)be+\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)af-\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)be\right)\sqrt{\frac{x^2d+c}{c}}\sqrt{\frac{b\,x^2+a}{a}}\sqrt{x^2d+c}}{fe\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{b\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)a}{e\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*e+EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*f-EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*e)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/
f*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 + d*e*x**2 + d*f*x
**4),x)
```

3.96 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
d^(1/2)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(1/2)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)-a^(3/2)*f*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-c*f+d*e)/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{-\sqrt{\frac{b}{a}}dex(a+bx^2) - ibce\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - ic(}{\sqrt{\frac{b}{a}}ce(-de+cf)\sqrt{a+}}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(-(Sqrt[b/a]*d*e*x*(a + b*x^2)) - I*b*c*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*e) + a*f
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*e*(-(d*e) + c*f)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \text{416}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf} - \frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) - (a^(3/2)*f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[
(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[a +
b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

Maple [A] (verified)

Time = 6.70 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.37

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bde\, x^3+\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\ \mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right) bce+\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\ \mathrm{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right) acf-\sqrt{\frac{b x^2}{a}}}{ce\sqrt{-\frac{b}{a}}\,(cf-de)(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(bd\,x^2+ad\right)x}{c(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{b\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\ \mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{(cf-de)\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{f\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\ \mathrm{Ell}}{(cf-de)e\sqrt{-\frac{b}{a}}\,\sqrt{}}\right.}{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*e*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c*f-
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c*e-(-b/a)^(1/2)*a*d*e*x)*(d*x^2+c)^(1/2)*(
b*x^2+a)^(1/2)/c/e/(-b/a)^(1/2)/(c*f-d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)} \,d x$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}}{d^{2} f\,x^{6}+2 c d f\,x^{4}+d^{2} e\,x^{4}+c^{2} f\,x^{2}+2 c d e\,x^{2}+c^{2} e}d x$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e + c**2*f*x**2 + 2*c*d*e*x*
*2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.97 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 458

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \frac{dx\sqrt{a+bx^2}}{3c(de-cf)(c+dx^2)^{3/2}} - \frac{\sqrt{d}(ad(2de-5cf)-bc(de-4cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}(bc-ad)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(3b^2c^3f^2+3a^2cd^2f^2-abd(d^2e^2-2cdef+7c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}(bc-ad)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}f^2(be-af)\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*d^(1/2)*(a*d*(-5*
c*f+2*d*e)-b*c*(-4*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/(-a*d+b*c)/(-c*f+d*e)^2/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(3*b^2*c^3*f^2+3*a^2*c*d^2*f
^2-a*b*d*(7*c^2*f^2-2*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(ar
ctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/(-a*d+b*c)/(-
c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^2*(-a
*f+b*e)*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c
*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(-2*a*b*Sqrt[b/a]*c^2*d^2*e^2*x + 3*a^2*Sqrt[b/a]*c*d^3*e^2*x + 5*a*b*Sqrt
[b/a]*c^3*d*e*f*x - 6*a^2*Sqrt[b/a]*c^2*d^2*e*f*x - 2*a*b*(b/a)^(3/2)*c^2*
d^2*e^2*x^3 + 2*a*b*Sqrt[b/a]*c*d^3*e^2*x^3 + 2*a^2*Sqrt[b/a]*d^4*e^2*x^3
+ 5*a*b*(b/a)^(3/2)*c^3*d*e*f*x^3 - 2*a*b*Sqrt[b/a]*c^2*d^2*e*f*x^3 - 5*a^
2*Sqrt[b/a]*c*d^3*e*f*x^3 - a*b*(b/a)^(3/2)*c*d^3*e^2*x^5 + 2*a*b*Sqrt[b/a
]*d^4*e^2*x^5 + 4*a*b*(b/a)^(3/2)*c^2*d^2*e*f*x^5 - 5*a*b*Sqrt[b/a]*c*d^3*
e*f*x^5 - I*b*c*e*(b*c*(d*e - 4*c*f) + a*d*(-2*d*e + 5*c*f))*Sqrt[1 + (b*x
^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*(c
 + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - (3*I)*b^2*c^4*e*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[
(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*b*c^3*d*e*f*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*b*c^4*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
(3*I)*a^2*c^3*d*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*b^2*c^3*d*e*f*x^2*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*b*c^2*d^2*e*f*x^2*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*...
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {421, 25, 401, 25, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}-\frac{\int -\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow 320$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow 414$$

$$\frac{a^{3/2}f^2\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}}\right)}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(3*c*(c + d*x^2)^(3/2)) + (-(((a*d*(2*
d*e - 5*c*f) - b*c*(d*e - 4*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(d*e - c*f)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*
c)))/(d*e - c*f)^2 + (a^(3/2)*f^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*
e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f
)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. $2(434) = 868$.

Time = 9.26 (sec) , antiderivative size = 1944, normalized size of antiderivative = 4.24

method	result	size
default	Expression too large to display	1944
elliptic	Expression too large to display	2040

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*(-b/a)^(1/2)*a*b*d^4*e^2*x^5+(-b/a)^(1/2)*b^2*c*d^3*e^2*x^5+2*(-b
/a)^(1/2)*b^2*c^2*d^2*e^2*x^3-3*(-b/a)^(1/2)*a^2*c*d^3*e^2*x+((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c
^2*d^2*e^2*x^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^2*c^2*d^2*e^2*x^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
a^2*c^2*d^2*f^2*x^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d^2*e^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d^2*e^2-2*(-
b/a)^(1/2)*a^2*d^4*e^2*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d^3*e^2*x^2-((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3*d*e*f
*x^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a*b*c*d^3*e^2*x^2+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3*d*e*f*x^2+3*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1
/2)/(-b/a)^(1/2))*a*b*c^3*d*f^2*x^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c^3
*d*e*f*x^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*b*c^3*d*e*f-5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{5/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^3f\,x^8+3c\,d^2f\,x^6+d^3e\,x^6+3c^2df\,x^4+3c\,d^2e\,x^4+c^3f\,x^2+3c^2de\,x^2+}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**3*e + c**3*f*x**2 + 3*c**2*d*e
*x**2 + 3*c**2*d*f*x**4 + 3*c*d**2*e*x**4 + 3*c*d**2*f*x**6 + d**3*e*x**6
+ d**3*f*x**8),x)
```

3.98 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 573

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{(3a^2d^2f^2 - abdf(20de-27cf) + b^2(15d^2e^2 - 20cdef + 3c^2f^2))\,x\sqrt{c+d\cdots}}{15df^3\sqrt{a+bx^2}}$$
$$- \frac{(5bde-6bcf-6adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15f^2} + \frac{bdx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f}$$
$$- \frac{\sqrt{a}(3a^2d^2f^2 - abdf(20de-27cf) + b^2(15d^2e^2-20cdef+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15\sqrt{b}df^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{a^{3/2}(3adf(5de-8cf) - b(15d^2e^2-25cdef+9c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{b}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{a^{3/2}(be-af)(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(3*a^2*d^2*f^2-a*b*d*f*(-27*c*f+20*d*e)+b^2*(3*c^2*f^2-20*c*d*e*f+15*
d^2*e^2))*x*(d*x^2+c)^(1/2)/d/f^3/(b*x^2+a)^(1/2)-1/15*(-6*a*d*f-6*b*c*f+5
*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/5*b*d*x^3*(b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/f-1/15*a^(1/2)*(3*a^2*d^2*f^2-a*b*d*f*(-27*c*f+20*d*e)+b^2
*(3*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/f^3/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(3*a*d*f*(-8*c*f+5*d*e)-b*(9*c
^2*f^2-25*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1
/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-a*f+b*e)*(-c*f+d*e)^2*(d*x^2+c)^(1/2)*Elli
pticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^
(1/2)/c/e/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.15 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{-icef(3a^2d^2f^2+abdf(-20de+27cf)+b^2(15d^2e^2-20cdef+3c^2f^2))\ \sqrt{}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
((-I)*c*e*f*(3*a^2*d^2*f^2 + a*b*d*f*(-20*d*e + 27*c*f) + b^2*(15*d^2*e^2
- 20*c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e*(3*a^2*d^2*f^2*(-5*d*e + 7*c
*f) + a*b*d*f*(30*d^2*e^2 - 35*c*d*e*f - 3*c^2*f^2) + b^2*(-15*d^3*e^3 + 1
5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 3*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*e*
f^2*x*(a + b*x^2)*(c + d*x^2)*(6*a*d*f + b*(-5*d*e + 6*c*f + 3*d*f*x^2)) -
 (15*I)*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[
b/a]*d*e*f^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {420, 318, 403, 27, 406, 320, 388, 313, 418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{b\left(\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{\frac{1}{3}\int \frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+}}{5b}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3}{5b}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\ }}{5b}}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\big\downarrow$ 418

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\ }}{5b}}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}+\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}$$

$\big\downarrow$ 25

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\ }}{5b}}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}$$

$\big\downarrow$ 403

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{\int\frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int\frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a}}{f^2}\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(a\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a\cdots\right.}{a\left(c\cdots\right.}}}\cdots\right.\right.}{f^2}\cdots\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5b}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\cdots\right.\right.\right.}{f^2}\cdots\right)}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{}}{5b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-\right.\right.}{}\right.}{f}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(b*((d*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + ((2*(3*b*c - a*d)*x*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/3 + ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*((
x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*b)))/f - ((
b*e - a*f)*(-((d*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e -
 4*b*c*f - a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(
3*d*e - 5*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2]))/3))/f^2) + (a^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f
)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f^2*S
qrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.15

method	result
risch	$\frac{x\left(3bdf\,x^{2}+6adf+6bcf-5bde\right)\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{15f^{2}}+\frac{\left(\frac{\left(24a^{2}cd\,f^{3}-15a^{2}d^{2}e\,f^{2}+24ab\,c^{2}f^{3}-55abcde\,f^{2}+30ab\,d^{2}e^{2}f-15b^{2}c^{2}e\,f^{2}+30b^{2}cd\,e\right.}{f^{2}\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*f*x^2+6*a*d*f+6*b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/
2)/f^2+1/15/f^2*((24*a^2*c*d*f^3-15*a^2*d^2*e*f^2+24*a*b*c^2*f^3-55*a*b*c*
d*e*f^2+30*a*b*d^2*e^2*f-15*b^2*c^2*e*f^2+30*b^2*c*d*e^2*f-15*b^2*d^2*e^3)
/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(3*
a^2*d^2*f^2+27*a*b*c*d*f^2-20*a*b*d^2*e*f+3*b^2*c^2*f^2-20*b^2*c*d*e*f+15*
b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+15*(a^2*c^2*f^4-
2*a^2*c*d*e*f^3+a^2*d^2*e^2*f^2-2*a*b*c^2*e*f^3+4*a*b*c*d*e^2*f^2-2*a*b*d^
2*e^3*f+b^2*c^2*e^2*f^2-2*b^2*c*d*e^3*f+b^2*d^2*e^4)/f^2/e/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*
x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^{2}\right)^{3/2}\left(c+dx^{2}\right)^{3/2}}{e+fx^{2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output

```
(6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + 6*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 27*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x
**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2 -
 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d
*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),
x)*a*b*d**2*e*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e +
a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**
4 + b*d*f*x**6),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c
*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 +
 b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e**2 + 24
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a**2*c*d*f**2 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x...
```

3.99 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 421

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = -\frac{b(3bde-bcf-4adf)x\sqrt{c+dx^2}}{3df^2\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f}+\frac{\sqrt{a}\sqrt{b}(3bde-bcf-4adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3df^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a^{3/2}(3bde-2bcf-3adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a^{3/2}(be-af)(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*b*(-4*a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d/f^2/(b*x^2+a)^(1/2)+1/
3*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3*a^(1/2)*b^(1/2)*(-4*a*d*f-b*c*
f+3*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/d/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/
3*a^(3/2)*(-3*a*d*f-2*b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arcta
n(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(-a*f+b*e)*(-c*f+d*e)*(d*x^2+c)^(1/2)*
EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2)
)/b^(1/2)/c/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{-ibcef(-3bde+bcf+4adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
((-I)*b*c*e*f*(-3*b*d*e + b*c*f + 4*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e*(3*a^2*d^2*f
^2 + a*b*d*f*(-6*d*e + c*f) + b^2*(3*d^2*e^2 - c^2*f^2))*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*
(b*Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2) - (3*I)*(b*e - a*f)^2*(-(d*e)
 + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/a]*d*e*f^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}dx$$

$$\Big\downarrow \text{ 418}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}+\frac{b\int -\frac{\sqrt{dx^2+c}(-bfx^2+be-2af)}{\sqrt{bx^2+a}}dx}{f^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\int \frac{\sqrt{dx^2+c}(-bfx^2+be-2af)}{\sqrt{bx^2+a}}dx}{f^2}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\left(\frac{\int \frac{b\left((3bde-bcf-4adf)x^2+c(3be-5af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\left(\frac{1}{3}\int \frac{(3bde-bcf-4adf)x^2+c(3be-5af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(c(3be-5af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-4adf-bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-$$
$$\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\frac{1}{3}fx\sqrt{a+bx}\right.}{f^2}$$

$\Big\downarrow$ 388

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-$$
$$\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\frac{1}{3}\right.}{f^2}$$

$\Big\downarrow$ 313

$$\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-$$
$$\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right|}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.\right.}{f^2}$$

$\Big\downarrow$ 414

$$\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$
$$\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right|}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.\right.}{f^2}$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
-((b*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e - b*c*f - 4*a
*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*e - 5*a*
f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3))/f
^2) + (c^(3/2)*(b*e - a*f)^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*f^2*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(391) = 782$.

Time = 9.69 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.98

method	result
risch	$\frac{bx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3f}+\frac{\left(-\frac{fb(4adf+bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3/f*(1/f^2*(-f*b*(4*a*d*f+b*c*
f-3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*a^2*d*f^2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b^2*d*e^2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5*a*b*c*f^2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*b^2*c*e*f/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*a*b*d*e*f/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*(a^2*c*f^3-a^2*
d*e*f^2-2*a*b*c*e*f^2+2*a*b*d*e^2*f+b^2*c*e^2*f-b^2*d*e^3)/f^2/e/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+
a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}{f\,x^2+e} \,d x$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bx+4\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}d \right.}{}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*x + 4*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
 b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d*f + int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c
*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*f - 3*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d*e +
3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*a**2*d*f + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*
x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*
d*f*x**6),x)*a*b*c*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*a*b*d*e - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*
x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*e + 3*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
 b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*c*f - int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*e)/(3*f)
```

3.100 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 363

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{bx\sqrt{a+bx^2}}{f\sqrt{c+dx^2}} \\ &- \frac{b\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{c}(b^2ce-2abcf+a^2df)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}f(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{c^{3/2}(be-af)^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
b*x*(b*x^2+a)^(1/2)/f/(d*x^2+c)^(1/2)-b*c^(1/2)*(b*x^2+a)^(1/2)*EllipticE(
d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/d^(1/2)/f/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(1/2)*(a^2*d*f-2*a*b*c*f+b^2*c*e)
*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/
2))/a/d^(1/2)/f/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
-c^(3/2)*(-a*f+b*e)^2*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^
2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/f/(-c*f+d*e)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(b^2cefE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-be(bde+bcf-2adf)\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\right.\right.}{\sqrt{\frac{b}{a}}def^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b^2*c*e*f*EllipticE[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e*(b*d*e + b*c*f - 2*a*d*f)*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(b*e - a*f)^2*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d*e*f^2*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input

```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output

```
(b*(b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (a^(3/2)*(b*e - a*
f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]]
, 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c
*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(\text{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b^2 cef + \text{EllipticPi}\left(x\sqrt{-\frac{b}{a}}, \frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right) a^2 d f^2 - 2\,\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}}, \frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right) abdef + \text{EllipticPi}\left(x\sqrt{\right.\right.}{e}$
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(\frac{2b\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)a}{f\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+x^2bc+ac}} - \frac{b^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)e}{f^2\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+x^2bc+ac}} - \right.}{}$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e*f+EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d*f^2-2*EllipticPi(x*(-b/a)^
(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*d*e*f+EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d*e^2+2*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a*b*d*e*f-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b^2*c*e*f-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*d*e^2)*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/f^2/d/(-b/a
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)b + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e + c*f*x**2 + d*e*x**2 +
d*f*x**4),x)*b + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 +
 d*e*x**2 + d*f*x**4),x)*a
```

3.101 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 280

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{(bc-ad)^2x}{cd(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}\sqrt{b}(bc-ad)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{cd(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(-a*d+b*c)^2*x/c/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*b^(1
/2)*(-a*d+b*c)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/c/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)+a^(3/2)*(-a*f+b*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-c*f+d*e)/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.73 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}d(bc-ad)efx(a+bx^2)-ibc(-bc+ad)ef\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\mathrm{arcsi}}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*(b*c - a*d)*e*f*x*(a + b*x^2) - I*b*c*(-(b*c) + a*d)*e*f*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*b^2*c*e*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*d*(b*e - a*f)^2*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*e*f*(-(d*e) + c*f)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {417, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \; 417$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}$$

$$\Big\downarrow \; 313$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{414}$$

$$\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
-(((b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])) + (a^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi
[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*
c*e*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(274) = 548$.

Time = 6.54 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.11

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, ab\, d^{2} ef\, x^{3}+\sqrt{-\frac{b}{a}}\, b^{2} cdef\, x^{3}+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b^{2} c^{2} ef-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\right.\right.}{}$
elliptic	$\frac{\sqrt{\left(b\, x^{2}+a\right)\left(x^{2} d+c\right)}\, \left(-\frac{\left(bd\, x^{2}+ad\right)(ad-bc)x}{cd(cf-de)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\, x^{2}+ad\right)}}+\frac{\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right) b^{2}}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}\, df}+\frac{b\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \mathrm{E}}{(cf-de)\sqrt{-\frac{b}{a}}}\right.}{}$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*a*b*d^2*e*f*x^3+(-b/a)^(1/2)*b^2*c*d*e*f*x^3+((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^
2*e*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*b^2*c*d*e^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*e*f-((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*e*f+((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a^2*c*d*f^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d*e*
f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c*d*e^2-(-b/a)^(1/2)*a^2*d^2*e*f*x+(-b/
a)^(1/2)*a*b*c*d*e*f*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f/e/(-b/a)^(1/2)/d
/c/(c*f-d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx\right)b + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e + c**2*f*x**2 + 2*c*d
*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)*b + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(c**2*e + c**2*f*x**2 + 2*c*d*e*x**2 + 2*c*d*f*x*
*4 + d**2*e*x**4 + d**2*f*x**6),x)*a
```

3.102 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 440

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)} \, dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{3c(de-cf)(c+dx^2)^{3/2}}$$
$$+\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{(3b^2c^2ef+3a^2cdf^2-ab(d^2e^2+cdef+4c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}f(be-af)^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/3*(a*d*(-
5*c*f+2*d*e)+b*c*(c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(1/2)/(-c*f+d*e)^2/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(3*b^2*c^2*e*f+3*a^2*c*d*f^2-a
*b*(4*c^2*f^2+c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(
1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/(-c*f+d*e)^3/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*f*(-a*f+b*e)^2*(b*x^2+a)^(
1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^
(1/2))/a/d^(1/2)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}dex(a+bx^2)\left(ad(-6c^2f+2d^2ex^2+cd(3e-5fx^2))+bc(2c^2f+2d^2ex^2\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*e*x*(a + b*x^2)*(a*d*(-6*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 5*f
*x^2)) + b*c*(2*c^2*f + 2*d^2*e*x^2 + c*d*(e + f*x^2))) - I*b*c*e*(-(b*c*(
2*d*e + c*f)) + a*d*(-2*d*e + 5*c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(
b*c) + a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*c^2*d*(b*e -
 a*f)^2*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*e*(d*e
- c*f)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {419, 25, 401, 25, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$-\frac{-\frac{\int -\frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{\frac{(ad(2de-5cf)+bc(cf+2de))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx-ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 414}$$

$$\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{a^{3/2}f\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(-1/3*((b*c - a*d)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)^(3/2)) +
(((a*d*(2*d*e - 5*c*f) + b*c*(2*d*e + c*f))*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(d*e - c*f)*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c))/(d*e - c*f)
^2 - (a^(3/2)*f*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), Ar
cTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)^2*Sq
rt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(416) = 832$.

Time = 9.17 (sec) , antiderivative size = 1122, normalized size of antiderivative = 2.55

method	result	size
elliptic	Expression too large to display	1122
default	Expression too large to display	1791

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a*d-b*c
)/c/d^2/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2-1/3*(b
*d*x^2+a*d)*(5*a*c*d*f-2*a*d^2*e-b*c^2*f-2*b*c*d*e)/d/(c*f-d*e)^2/c^2*x/((
x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))*b^2/d/(c*f-d*e)+1/(c*f-d*e)^2*e/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2-1/3*b^2/(c*f-d*e
)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)/d*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*f-2/
3*b^2/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))*e+5/3*b/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))*a*f-2/3*b/(c*f-d*e)^2/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))*a*e-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))*b/(c*f-d*e)/c*a+1/(c*f-d*e)^2/e*f^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^3f\,x^8+3c\,d^2f\,x^6+d^3e\,x^6+3c^2df\,x^4+3c\,d^2e\,x^4+c^3f\,x^2+3c^2de\,x^2}\right.$$

$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^3f\,x^8+3c\,d^2f\,x^6+d^3e\,x^6+3c^2df\,x^4+3c\,d^2e\,x^4+c^3f\,x^2+3c^2de\,x^2+c^3e}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**3*e + c**3*f*x**2 + 3*c**
2*d*e*x**2 + 3*c**2*d*f*x**4 + 3*c*d**2*e*x**4 + 3*c*d**2*f*x**6 + d**3*e*
x**6 + d**3*f*x**8),x)*b + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**3*e
 + c**3*f*x**2 + 3*c**2*d*e*x**2 + 3*c**2*d*f*x**4 + 3*c*d**2*e*x**4 + 3*c
*d**2*f*x**6 + d**3*e*x**6 + d**3*f*x**8),x)*a
```

3.103 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 659

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{5c(de-cf)(c+dx^2)^{5/2}}$$

$$+\frac{(ad(4de-9cf)+bc(2de+3cf))x\sqrt{a+bx^2}}{15c^2(de-cf)^2(c+dx^2)^{3/2}}$$

$$+\frac{(b^2c^2(2d^2e^2-14cdef-3c^2f^2)+3abcd(d^2e^2-2cdef+11c^2f^2)-a^2d^2(8d^2e^2-26cdef+33c^2f^2))\sqrt{a+}}{15c^{5/2}\sqrt{d}(bc-ad)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$-\frac{(15b^3c^4ef^2-15a^3c^2d^2f^3-a^2bd(4d^3e^3-17cd^2e^2f+7c^2def^2-39c^3f^3)+ab^2c(d^3e^3-8cd^2e^2f-17c^2de}{15ac^{3/2}\sqrt{d}(bc-ad)(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+d}}$$

$$+\frac{c^{3/2}f^2(be-af)^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/5*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/15*(a*d*(
-9*c*f+4*d*e)+b*c*(3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)/c^2/(-c*f+d*e)^2/(d*x^2
+c)^(3/2)+1/15*(b^2*c^2*(-3*c^2*f^2-14*c*d*e*f+2*d^2*e^2)+3*a*b*c*d*(11*c^
2*f^2-2*c*d*e*f+d^2*e^2)-a^2*d^2*(33*c^2*f^2-26*c*d*e*f+8*d^2*e^2))*(b*x^2
+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
/c^(5/2)/d^(1/2)/(-a*d+b*c)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)-1/15*(15*b^3*c^4*e*f^2-15*a^3*c^2*d^2*f^3-a^2*b*d*(-39*c^3*
f^3+7*c^2*d*e*f^2-17*c*d^2*e^2*f+4*d^3*e^3)+a*b^2*c*(-21*c^3*f^3-17*c^2*d*
e*f^2-8*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/
2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(3/2)/d^(1/2)/(-a*d+b*c)/(-c*f+d*e)^4
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^2*(-a*f+b*e)^2*
(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(
1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.75 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \frac{-\sqrt{\frac{b}{a}}dex(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(ad(4c\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x]
```

output
```
(-(Sqrt[b/a]*d*e*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c
 - a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 9*c*f) + b*c*(2*d*e + 3*c*f))*(c + d*
x^2) + (b^2*c^2*(-2*d^2*e^2 + 14*c*d*e*f + 3*c^2*f^2) - 3*a*b*c*d*(d^2*e^2
 - 2*c*d*e*f + 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2 - 26*c*d*e*f + 33*c^2*f^2)
)*(c + d*x^2)^2)) - I*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)
/c]*(b*e*(b^2*c^2*(-2*d^2*e^2 + 14*c*d*e*f + 3*c^2*f^2) - 3*a*b*c*d*(d^2*e
^2 - 2*c*d*e*f + 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2 - 26*c*d*e*f + 33*c^2*f^
2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(-(d
*e) + c*f)*(a*d*(4*d*e - 9*c*f) + b*c*(2*d*e + 3*c*f))*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + 15*c^2*d*f*(b*e - a*f)^2*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(15*Sqrt[b/a]*c^3*d*(b*c - a*
d)*e*(d*e - c*f)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {419, 25, 401, 25, 27, 402, 27, 400, 313, 320, 416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{7/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{7/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$-\frac{-\frac{\int -\frac{d\left(b(ad(3de-8cf)+bc(2de+3cf))x^2+a(ad(4de-9cf)+bc(de+4cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d\left(b(ad(3de-8cf)+bc(2de+3cf))x^2+a(ad(4de-9cf)+bc(de+4cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{b(ad(3de-8cf)+bc(2de+3cf))x^2+a(ad(4de-9cf)+bc(de+4cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{(bc-ad)\left(b(ad(4de-9cf)+bc(2de+3cf))x^2+a(2ad(4de-9cf)+bc(de+9cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{x\sqrt{a+bx^2}(ad(4de-9cf)+bc(3cf+2de))}{3c\left(c+dx^2\right)^{3/2}}}{5c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{b(ad(4de-9cf)+bc(2de+3cf))x^2+a(2ad(4de-9cf)+bc(de+9cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(ad(4de-9cf)+bc(3cf+2de))}{3c\left(c+dx^2\right)^{3/2}}}{5c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{5c(c+dx^2)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 400

$$\frac{\frac{\frac{ab(ad(4de-9cf)-bc(de-6cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(ad(4de-9cf)+bc(3c}{3c\left(c+dx^2\right)^{3/2}}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{ab(ad(4de-9cf)-bc(de-6cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}+\frac{x\sqrt{a+bx^2}(ad}{3}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 320

$$\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)-bc(de-6cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 416

$$\frac{\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)-bc(de-6cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{d\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)-bc(de-6cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)-bc(de-6cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)-3abcd(de-6cf)-b^2c^2(3cf+2de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x]
```

output
```
(-1/5*((b*c - a*d)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)^(5/2)) +
(((a*d*(4*d*e - 9*c*f) + b*c*(2*d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(3*c*(c +
 d*x^2)^(3/2)) + (-(((2*a^2*d^2*(4*d*e - 9*c*f) - 3*a*b*c*d*(d*e - 6*c*f)
- b^2*c^2*(2*d*e + 3*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(a*d*(4*d*e - 9*c*f) - b
*c*(d*e - 6*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))/(3*c))/(5*c))/(d*e - c*f)^2 - (f*(b*e - a*f)*((Sqrt[d]
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(Sqrt[c]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
) - (a^(3/2)*f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]
*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[a + b*x^2]*S
qrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4504 vs. 2(629) = 1258.

Time = 19.06 (sec) , antiderivative size = 4505, normalized size of antiderivative = 6.84

method	result	size
elliptic	Expression too large to display	4505
default	Expression too large to display	6065

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-2/5/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d/c/(c*f-d*e)^3*a*b*
e*f-26/15/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a^
3/c^2/(a*d-b*c)/(c*f-d*e)^3*e*f*d^3-1/5*(a*d-b*c)/d^3/(c*f-d*e)/c*x*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+c/d)^3-1/15*(9*a*c*d*f-4*a*d^2*e-3*b*c^
2*f-2*b*c*d*e)/(c*f-d*e)^2/c^2/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(
x^2+c/d)^2-11/5*b^2/(a*d-b*c)*c/(c*f-d*e)^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*f^2-1/5*b^2/(a*d-b*c)/c/(c*f-d*e)^3/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*d^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*e^2-14/15*
b^3/(a*d-b*c)*c/(c*f-d*e)^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))*e*f+11/5*b/(a*d-b*c)/(c*f-d*e)^3/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a^2*f^2+8/15*b/(a*d-b*c)/c^2/(c*f-d
*e)^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*d^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(7/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{7/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^4f\,x^{10}+4c\,d^3f\,x^8+d^4e\,x^8+6c^2d^2f\,x^6+4c\,d^3e\,x^6+4c^3df\,x^4+6c^2d^2$$

$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^4f\,x^{10}+4c\,d^3f\,x^8+d^4e\,x^8+6c^2d^2f\,x^6+4c\,d^3e\,x^6+4c^3df\,x^4+6c^2d^2e\,x^4+c^4f\,x^2+4c^3de\,x^2+c^4$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**4*e + c**4*f*x**2 + 4*c**
3*d*e*x**2 + 4*c**3*d*f*x**4 + 6*c**2*d**2*e*x**4 + 6*c**2*d**2*f*x**6 + 4
*c*d**3*e*x**6 + 4*c*d**3*f*x**8 + d**4*e*x**8 + d**4*f*x**10),x)*b + int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**4*e + c**4*f*x**2 + 4*c**3*d*e*x**
2 + 4*c**3*d*f*x**4 + 6*c**2*d**2*e*x**4 + 6*c**2*d**2*f*x**6 + 4*c*d**3*e
*x**6 + 4*c*d**3*f*x**8 + d**4*e*x**8 + d**4*f*x**10),x)*a
```

3.104 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 821

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{(15a^3d^3f^3 - a^2bd^2f^2(161de-219cf) + ab^2df(245d^2e^2 - 329cdef + 51c^2}{105d^2f^4\sqrt{a+}}$$

$$+\frac{(45a^2d^2f^2 - abdf(77de-93cf) + b^2(35d^2e^2 - 42cdef + 3c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105df^3}$$

$$-\frac{b(7bde-8bcf-15adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35f^2} + \frac{b^2dx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7f}$$

$$-\frac{\sqrt{a}(15a^3d^3f^3 - a^2bd^2f^2(161de-219cf) + ab^2df(245d^2e^2 - 329cdef + 51c^2f^2) - b^3(105d^3e^3 - 140cd^2e}{105\sqrt{b}d^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(15a^2d^2f^2(7de-11cf) - abdf(210d^2e^2 - 343cdef + 117c^2f^2) + b^2(105d^3e^3 - 175cd^2e^2f + 63c^2def}{105\sqrt{b}cdf^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}(be-af)^2(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be}, \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1-\frac{ad}{bc}\right)}{\sqrt{b}cef^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(15*a^3*d^3*f^3-a^2*b*d^2*f^2*(-219*c*f+161*d*e)+a*b^2*d*f*(51*c^2*f
^2-329*c*d*e*f+245*d^2*e^2)-b^3*(6*c^3*f^3+21*c^2*d*e*f^2-140*c*d^2*e^2*f+
105*d^3*e^3))*x*(d*x^2+c)^(1/2)/d^2/f^4/(b*x^2+a)^(1/2)+1/105*(45*a^2*d^2*
f^2-a*b*d*f*(-93*c*f+77*d*e)+b^2*(3*c^2*f^2-42*c*d*e*f+35*d^2*e^2))*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f^3-1/35*b*(-15*a*d*f-8*b*c*f+7*b*d*e)*x^3*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/7*b^2*d*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/f-1/105*a^(1/2)*(15*a^3*d^3*f^3-a^2*b*d^2*f^2*(-219*c*f+161*d*e)+a*
b^2*d*f*(51*c^2*f^2-329*c*d*e*f+245*d^2*e^2)-b^3*(6*c^3*f^3+21*c^2*d*e*f^2
-140*c*d^2*e^2*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d^2/f^4/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)-1/105*a^(3/2)*(15*a^2*d^2*f^2*(-11*c*f+7*d*e)-
a*b*d*f*(117*c^2*f^2-343*c*d*e*f+210*d^2*e^2)+b^2*(3*c^3*f^3+63*c^2*d*e*f^
2-175*c*d^2*e^2*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(
1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d/f^4/(b*x^2+a)^(1/2)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(-a*f+b*e)^2*(-c*f+d*e)^2*(d*x^2+c)^(1/2)
*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2
))/b^(1/2)/c/e/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 2026, normalized size of antiderivative = 2.47

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(35*a*b^2*Sqrt[b/a]*c*d^3*e^3*f^2*x - 42*a*b^2*Sqrt[b/a]*c^2*d^2*e^2*f^3*x
 - 77*a^3*(b/a)^(3/2)*c*d^3*e^2*f^3*x + 3*a*b^2*Sqrt[b/a]*c^3*d*e*f^4*x +
93*a^3*(b/a)^(3/2)*c^2*d^2*e*f^4*x + 45*a^3*Sqrt[b/a]*c*d^3*e*f^4*x + 35*b
^3*Sqrt[b/a]*c*d^3*e^3*f^2*x^3 + 35*a*b^2*Sqrt[b/a]*d^4*e^3*f^2*x^3 - 42*b
^3*Sqrt[b/a]*c^2*d^2*e^2*f^3*x^3 - 140*a*b^2*Sqrt[b/a]*c*d^3*e^2*f^3*x^3 -
 77*a^3*(b/a)^(3/2)*d^4*e^2*f^3*x^3 + 3*b^3*Sqrt[b/a]*c^3*d*e*f^4*x^3 + 12
0*a*b^2*Sqrt[b/a]*c^2*d^2*e*f^4*x^3 + 183*a^3*(b/a)^(3/2)*c*d^3*e*f^4*x^3
+ 45*a^3*Sqrt[b/a]*d^4*e*f^4*x^3 + 35*b^3*Sqrt[b/a]*d^4*e^3*f^2*x^5 - 63*b
^3*Sqrt[b/a]*c*d^3*e^2*f^3*x^5 - 98*a*b^2*Sqrt[b/a]*d^4*e^2*f^3*x^5 + 27*b
^3*Sqrt[b/a]*c^2*d^2*e*f^4*x^5 + 177*a*b^2*Sqrt[b/a]*c*d^3*e*f^4*x^5 + 90*
a^3*(b/a)^(3/2)*d^4*e*f^4*x^5 - 21*b^3*Sqrt[b/a]*d^4*e^2*f^3*x^7 + 39*b^3*
Sqrt[b/a]*c*d^3*e*f^4*x^7 + 60*a*b^2*Sqrt[b/a]*d^4*e*f^4*x^7 + 15*b^3*Sqrt
[b/a]*d^4*e*f^4*x^9 - I*c*e*f*(15*a^3*d^3*f^3 + a^2*b*d^2*f^2*(-161*d*e +
219*c*f) + a*b^2*d*f*(245*d^2*e^2 - 329*c*d*e*f + 51*c^2*f^2) - b^3*(105*d
^3*e^3 - 140*c*d^2*e^2*f + 21*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e
*(15*a^3*d^3*f^3*(-7*d*e + 10*c*f) + a^2*b*d^2*f^2*(315*d^2*e^2 - 392*c*d*
e*f + 3*c^2*f^2) + a*b^2*d*f*(-315*d^3*e^3 + 350*c*d^2*e^2*f + 56*c^2*d*e*
f^2 - 54*c^3*f^3) + b^3*(105*d^4*e^4 - 105*c*d^3*e^3*f - 35*c^2*d^2*e^2*f^
2 + 21*c^3*d*e*f^3 + 6*c^4*f^4))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c...
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 1126, normalized size of antiderivative = 1.37, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.844$, Rules used = {420, 318, 403, 27, 403, 25, 406, 320, 388, 313, 420, 318, 403, 27, 406, 320, 388, 313, 418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(2d(4bc-ad)x^2+c(7bc-ad)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{\frac{\int \frac{3d\sqrt{bx^2+a}\left(\left(b^2c^2+9abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5d}+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\frac{3}{5}\int \frac{\sqrt{bx^2+a}\left(\left(b^2c^2+9abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{dx^2+c}}dx+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{7b}+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{\int -\frac{2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}\right)+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad}{7b}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{\int\frac{2(bc+ad)\left(b^2c^2-6abdc+a^2d^2\right)x^2+ac\left(b^2c^2-18abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)+\frac{2}{5}x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{7b}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{ac\left(a^2d^2-18abcd+b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{7b}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)\text{EllipticF}\left(\text{arct}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{7b}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 388

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)E}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c}{c}}}}{3d}}{7b}\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d\right.}{}}{3d}}{7b}\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}$$

$\downarrow$ 420

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2\cdots\right.}{\cdots}}{3d}\cdots\right.}{7b}\cdots\right.$$
$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad}$$

$$\frac{(be-af)\left(\frac{b\int\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 318$$

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^{\cdots}\right.}{\cdots}}{3d}\right.}{7b}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2\cdots\right.}{}}{3d}\right.}{7b}\right.$$

$$\frac{}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\frac{\int\frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx\cdots\right.}{fx^2+\cdots}}{f}\right.}{f}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d\right.}{}}{3d}\right.}{7b}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\frac{1}{3}\int\frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+\right.}{fx^2+e}}{f}\right.}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2\cdots\right.}{\cdots}}{3d}\cdots\right)}{7b}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\cdots}{5b}\cdots\right)}{f}\cdots\right)}{f}$$

$$\Big\downarrow \; 320$$

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2\right.}{}}{3d}\right.}{7b}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-}{5b}\right.}{f}\right.}{f}$$

↓ 388

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d\cdots\right.}{\cdots}}{3d}\cdots\right.}{7b}\cdots\right.$$

$$\overline{\qquad\qquad\qquad f \qquad\qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\cdots}{5b}\cdots\right.}{f}\cdots\right.$$

$$\overline{\qquad\qquad\qquad f \qquad\qquad\qquad}$$

↓ 313

$$b\left(\frac{\frac{3}{5}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2d^2+9abcd+b^2c^2\right)}{3d}-\frac{2(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2\cdots\right.}{\cdots}}{3d}\cdots\right.}{7b}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\cdots}\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\cdots}$$

$\downarrow$ 418

$$\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Ellipti}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{7b\cdots}\cdots\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\cdots\right.\right.}{5b\cdots}\cdots\right)}{f\cdots}\cdots\right.}{f\cdots}$$

$\downarrow$ 25

$$\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Ellipti}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{7b\cdots}\cdots\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\cdots\right.\right.}{5b\cdots}\cdots\right)}{f\cdots}\cdots\right.}{f\cdots}$$

↓ 403

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Ellipti}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{7b}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\right.\right.}{5b}\right.}{f}\right.$$

$\downarrow$ 27

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\mathrm{Elliptic}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{7b}\cdots\right.$$

$$\overline{\qquad\qquad f\qquad\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

↓ 406

$$\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\mathrm{Ellipti}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{7b\cdots}\cdots\right.}{f\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\cdots\right.\right.}{5b\cdots}\cdots\right)}{f\cdots}\cdots\right.$$

↓ 320

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Elliptic}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{7b}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\right.\right.}{5b}\right.}{f}\right.$$

↓ 388

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Elliptic}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{7b}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\right.\right.}{5b}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Elliptic}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{7b}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\right.\right.}{5b}\right)}{f}\right.$$

$\downarrow$ 414

$$b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{5/2}}{7b}+\frac{\frac{2}{5}(4bc-ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}+\frac{3}{5}\left(\frac{\left(b^2c^2+9abdc-2a^2d^2\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\left(b^2c^2-18abdc+a^2d^2\right)\sqrt{bx^2+a}\,\text{Ellipti}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{7b}\right.$$

$$\overline{\qquad\qquad\qquad f \qquad\qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}cx+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2\right.\right.}{5b}\right.}{f}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(b*((d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + ((2*(4*b*c - a*d)*x*(a
 + b*x^2)^(3/2)*Sqrt[c + d*x^2])/5 + (3*(((b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2
)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (2*(b*c + a*d)*(b^2*c^2 - 6*a
*b*c*d + a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(
b^2*c^2 - 18*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))/(3*d)))/5)/(7*b)))/f - ((b*e - a*f)*((b*((d*x*(a + b
*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + ((2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/3 + ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*((x*Sqrt[a + b*x^
2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*b)))/f - ((b*e - a*f)*(-((
d*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e - 4*b*c*f - a*d*
f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - 5*c*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 19.29 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.15

method	result
risch	$\frac{x\left(15f^2x^4b^2d^2+45ab\,d^2f^2x^2+24b^2cd\,f^2x^2-21b^2d^2ef\,x^2+45a^2d^2f^2+93abcd\,f^2-77ab\,d^2ef+3b^2c^2f^2-42b^2cdef+35b^2d^2e^2\right)\sqrt{b\,x^2}}{105d\,f^3}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/105/d*x*(15*b^2*d^2*f^2*x^4+45*a*b*d^2*f^2*x^2+24*b^2*c*d*f^2*x^2-21*b^2
*d^2*e*f*x^2+45*a^2*d^2*f^2+93*a*b*c*d*f^2-77*a*b*d^2*e*f+3*b^2*c^2*f^2-42
*b^2*c*d*e*f+35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3+1/105/f^3
/d*((165*a^3*c*d^2*f^4-105*a^3*d^3*e*f^3+222*a^2*b*c^2*d*f^4-553*a^2*b*c*d
^2*e*f^3+315*a^2*b*d^3*e^2*f^2-3*a*b^2*c^3*f^4-273*a*b^2*c^2*d*e*f^3+595*a
*b^2*c*d^2*e^2*f^2-315*a*b^2*d^3*e^3*f+105*b^3*c^2*d*e^2*f^2-210*b^3*c*d^2
*e^3*f+105*b^3*d^3*e^4)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-1/f*(15*a^3*d^3*f^3+219*a^2*b*c*d^2*f^3-161*a^2*b*d^3*e*f^
2+51*a*b^2*c^2*d*f^3-329*a*b^2*c*d^2*e*f^2+245*a*b^2*d^3*e^2*f-6*b^3*c^3*f
^3-21*b^3*c^2*d*e*f^2+140*b^3*c*d^2*e^2*f-105*b^3*d^3*e^3)*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*
(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2)))+105*(a^3*c^2*f^5-2*a^3*c*d*e*f^4+a^3*d^2*e^2
*f^3-3*a^2*b*c^2*e*f^4+6*a^2*b*c*d*e^2*f^3-3*a^2*b*d^2*e^3*f^2+3*a*b^2*c^2
*e^2*f^3-6*a*b^2*c*d*e^3*f^2+3*a*b^2*d^2*e^4*f-b^3*c^2*e^3*f^2+2*b^3*c*d*e
^4*f-b^3*d^2*e^5)*d/f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1
/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*
x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

3.105 $\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 595

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{b(23a^2d^2f^2-abdf(35de-12cf)+b^2(15d^2e^2-5cdef-2c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f^3\sqrt{a+bx^2}}$$
$$-\frac{b(5bde-bcf-11adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15df^2}+\frac{b^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f}$$
$$-\frac{\sqrt{a}\sqrt{b}(23a^2d^2f^2-abdf(35de-12cf)+b^2(15d^2e^2-5cdef-2c^2f^2))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15d^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(15a^2d^2f^2-abdf(30de-19cf)+b^2(15d^2e^2-10cdef-c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1\ldots\right)}{15\sqrt{b}cdf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(be-af)^2(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*b*(23*a^2*d^2*f^2-a*b*d*f*(-12*c*f+35*d*e)+b^2*(-2*c^2*f^2-5*c*d*e*f+
15*d^2*e^2))*x*(d*x^2+c)^(1/2)/d^2/f^3/(b*x^2+a)^(1/2)-1/15*b*(-11*a*d*f-b
*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f^2+1/5*b^2*x^3*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/f-1/15*a^(1/2)*b^(1/2)*(23*a^2*d^2*f^2-a*b*d*f*(-1
2*c*f+35*d*e)+b^2*(-2*c^2*f^2-5*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*Ellip
ticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d^2/f^3/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^(3/2)*(15*a^2*d^2*f^2-a*b
*d*f*(-19*c*f+30*d*e)+b^2*(-c^2*f^2-10*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2
)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d
/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-a*f+b*e)^2*
(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),
1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{-ibcef(23a^2d^2f^2+abdf(-35de+12cf)+b^2(15d^2e^2-5cdef-2c^2f^2))\sqrt{}}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
((-I)*b*c*e*f*(23*a^2*d^2*f^2 + a*b*d*f*(-35*d*e + 12*c*f) + b^2*(15*d^2*e
^2 - 5*c*d*e*f - 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e*(15*a^3*d^3*f^3 + a^2*b*d^
2*f^2*(-45*d*e + 11*c*f) + a*b^2*d*f*(45*d^2*e^2 - 5*c*d*e*f - 13*c^2*f^2)
 + b^3*(-15*d^3*e^3 + 5*c^2*d*e*f^2 + 2*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(b*Sqrt
[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(11*a*d*f + b*(-5*d*e + c*f + 3*d*f*
x^2)) + (15*I)*d*(b*e - a*f)^3*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))
/(15*Sqrt[b/a]*d^2*e*f^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.20, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {420, 318, 25, 403, 27, 406, 320, 388, 313, 418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int (bx^2+a)^{3/2}\sqrt{dx^2+c}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{\int -\frac{\sqrt{dx^2+c}\left(2b(bc-3ad)x^2+a(bc-5ad)\right)}{\sqrt{bx^2+a}}dx}{5d}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}-\frac{\int \frac{\sqrt{dx^2+c}\left(2b(bc-3ad)x^2+a(bc-5ad)\right)}{\sqrt{bx^2+a}}dx}{5d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}-\frac{\frac{\int \frac{b\left(\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{5d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\int\frac{\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{5d}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-9ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3}{5d}\right)}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}}{5d}\right)}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$\downarrow$ 388

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{5d}\right)}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}$$

$\downarrow$ 418

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}+\frac{b\int-\frac{\sqrt{dx^2+c}\left(-bfx^2+be-2af\right)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\int\frac{\sqrt{dx^2+c}\left(-bfx^2+be-2af\right)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+b\right.}{a\left(c+\right.}}}\right.}{5d}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{\int\frac{b\left((3bde-bcf-4adf)x^2+c(3be-5af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+b\right.}{a\left(c+\right.}}}\right.}{5d}\right.}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\int\frac{(3bde-bcf-4adf)x^2+c(3be-5af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{ EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(c(3be-5af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-4adf-bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a}\right.}{f^2}}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\text{ EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}}{f}$$

$\downarrow$ 388

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(\left(-4adf-bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(\right.}{a\left(\right.}}}\right.}{f^2}}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{5d}}{f}$$

$$\frac{(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}\right.\right.\right.}{f^2}}{f}$$

$$\Big\downarrow \; 414$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{3}\left(\left(-3a^2d^2-7abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\operatorname{EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+b\right.}{a\left(c+d\right.}}}}{5d}}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(\right.\right.}{}\right)}{f}$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(b*((b*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) - ((2*(b*c - 3*a*d)*x*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/3 + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*((
x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c - 9*a*d)*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3)/(5*d)))/f - ((
b*e - a*f)*(-((b*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e -
 b*c*f - 4*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(
3*b*e - 5*a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]))/3))/f^2) + (c^(3/2)*(b*e - a*f)^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c
*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*f^2
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 9.92 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.10

method	result
risch	$\frac{bx(3bdf\,x^2+11adf+bcf-5bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15d\,f^2}+\frac{\left(\frac{\left(15a^3d^2f^3+34a^2bcd\,f^3-45a^2b\,d^2e\,f^2-a\,b^2c^2f^3-40a\,b^2cde\,f^2+45a\,b^2d^2e^2f+15b\right.}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*b*x*(3*b*d*f*x^2+11*a*d*f+b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/d/f^2+1/15/f^2/d*((15*a^3*d^2*f^3+34*a^2*b*c*d*f^3-45*a^2*b*d^2*e*f^2-
a*b^2*c^2*f^3-40*a*b^2*c*d*e*f^2+45*a*b^2*d^2*e^2*f+15*b^3*c*d*e^2*f-15*b^
3*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))-1/f*b*(23*a^2*d^2*f^2+12*a*b*c*d*f^2-35*a*b*d^2*e*f-2*b^2*c^2*f^2-5*b^2
*c*d*e*f+15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+15*(
a^3*c*f^4-a^3*d*e*f^3-3*a^2*b*c*e*f^3+3*a^2*b*d*e^2*f^2+3*a*b^2*c*e^2*f^2-
3*a*b^2*d*e^3*f-b^3*c*e^3*f+b^3*d*e^4)*d/f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b
/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2
)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
(11*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x + sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b**2*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*e*x + 3*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*f*x**3 + 23*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*b*d**2*f**2 + 12*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a
*b**2*c*d*f**2 - 35*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e +
a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**
4 + b*d*f*x**6),x)*a*b**2*d**2*e*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c
*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c**2*f**2 - 5*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*d*e*f + 15*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b
**3*d**2*e**2 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4
 + b*d*f*x**6),x)*a**3*d**2*f**2 + 34*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b...
```

3.106 $\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 424

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = & -\frac{b^2(3bde+2bcf-7adf)x\sqrt{c+dx^2}}{3d^2f^2\sqrt{a+bx^2}}+\frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3df} \\
& +\frac{\sqrt{a}b^{3/2}(3bde+2bcf-7adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3d^2f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
& -\frac{a^{3/2}\sqrt{b}(3bde+bcf-6adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3cdf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
& +\frac{a^{3/2}(be-af)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/3*b^2*(-7*a*d*f+2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d^2/f^2/(b*x^2+a)^(1
/2)+1/3*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f+1/3*a^(1/2)*b^(3/2)*(-7*
a*d*f+2*b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/d^2/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)-1/3*a^(3/2)*b^(1/2)*(-6*a*d*f+b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*I
nverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/d/f^2/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(-a*f+b*e)^2*(d*x^2+c)^(
1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^
(1/2))/b^(1/2)/c/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-ib^2cef(-3bde-2bcf+7adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*b^2*c*e*f*(-3*b*d*e - 2*b*c*f + 7*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*e*(9*a^2
*d^2*f^2 - a*b*d*f*(9*d*e + 8*c*f) + b^2*(3*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^
2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] + d*(a*b*(b/a)^(3/2)*e*f^2*x*(a + b*x^2)*(c + d*x^2) + (3
*I)*d*(b*e - a*f)^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/a]*d^2*e*f^3*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {420, 318, 25, 406, 320, 388, 313, 420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{\int -\frac{2b(bc-2ad)x^2+a(bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{2b(bc-2ad)x^2+a(bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{a(bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2b(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2b(bc-2ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}\right)}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$\downarrow$ 388

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}\right)}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}\right)}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)}{f}$$

$\downarrow$ 324

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(a\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}\right)}{f}$$

$$(be-af)\left(\frac{b\left(b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 388

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}\right)}{f}$$

$$(be-af)\left(\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 313

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+\right.}}{f}\right.}{f}$$

$$\downarrow \; 414$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2b(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{Ell}}{\sqrt{b}cef\sqrt{}}\right.}{f}$$

input
```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*((b*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (2*b*(b*c - 2*a*d)*((x*S
qrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*c - 3*a*d)*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)))/f - ((b*e -
a*f)*((b*(b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (a^(3/2)*(b*
e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sq
rt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^
2))/(c*(a + b*x^2))])))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 19.01 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.73

method	result
risch	$\frac{b^{2} x \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{3 d f}+\frac{b\left(-\frac{f b(7adf-2bcf-3bde)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+}{c}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b^{3}d^{2}e\,f^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a\,b^{2}d^{2}e\,f^{2}x^{3}+\sqrt{-\frac{b}{a}}\,b^{3}cde\,f^{2}x^{3}+9\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^{2}b\,d^{2}e\,f^{2}-8\sqrt{\frac{b\,x^{2}+a}{a}}\right.}{}$
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f+1/3/d/f*(b/f^2*(-f*b*(7*a*d*
f-2*b*c*f-3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+9*a^2*d*f^
2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b^2*d*e^
2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-a*b*c*f^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9*a*b*d*e*f/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*(a^3*f^3-
3*a^2*b*e*f^2+3*a*b^2*e^2*f-b^3*e^3)*d/f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a
)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/
(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2x+7\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right.}{}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*x + 7*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**
2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*d*f - 2*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x*
*4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*f - 3*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**
3*d*e + 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*a**2*b*d*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e
+ a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x
**4 + b*d*f*x**6),x)*a*b**2*c*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*
x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*d*e - 2*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e
*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*e + 3*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**3*d*f - int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*
x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*c*e...
```

3.107 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 441

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{b^2x\sqrt{a+bx^2}}{df\sqrt{c+dx^2}} - \frac{(2abcdf-a^2d^2f+b^2c(de-2cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}f(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{c}(b^3cde^2-a^3d^2f^2-ab^2cf(4de-cf)+a^2bdf(2de+cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ad^{3/2}f(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}(be-af)^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
b^2*x*(b*x^2+a)^(1/2)/d/f/(d*x^2+c)^(1/2)-(2*a*b*c*d*f-a^2*d^2*f+b^2*c*(-2
*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))/c^(1/2)/d^(3/2)/f/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)+c^(1/2)*(b^3*c*d*e^2-a^3*d^2*f^2-a*b^2*c*f*(-c*f+4*d*
e)+a^2*b*d*f*(c*f+2*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x
/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(3/2)/f/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*(-a*f+b*e)^3*(b*x^2+a)^(1/2)*EllipticP
i(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/
2)/e/f/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.64 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{-ibcef(-2abcdf+a^2d^2f+b^2c(-de+2cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((-I)*b*c*e*f*(-2*a*b*c*d*f + a^2*d^2*f + b^2*c*(-(d*e) + 2*c*f))*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] - I*b^2*c*e*(-(d*e) + c*f)*(3*a*d*f - b*(d*e + 2*c*f))*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 d*(Sqrt[b/a]*(b*c - a*d)^2*e*f^2*x*(a + b*x^2) + I*c*d*(-(b*e) + a*f)^3*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*e*f^2*(-(d*e) + c*f)*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.86, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {419, 25, 401, 25, 27, 403, 25, 406, 320, 388, 313, 418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{3/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{3/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$-\frac{\frac{\int -\frac{d\sqrt{bx^2+a}\left(b(bc(4de-3cf)-ad(3de-2cf))x^2+acd(be-af)\right)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)(de-cf)}{c\sqrt{c+dx^2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d\sqrt{bx^2+a}\left(b(bc(4de-3cf)-ad(3de-2cf))x^2+acd(be-af)\right)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)(de-cf)}{c\sqrt{c+dx^2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(b(bc(4de-3cf)-ad(3de-2cf))x^2+acd(be-af)\right)}{\sqrt{dx^2+c}}dx}{c}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{c\sqrt{c+dx^2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\frac{\int -\frac{ac\left(c(4de-3cf)b^2-2ad(3de-cf)b+3a^2d^2f\right)-b\left(-2b^2(4de-3cf)c^2+abd(13de-7cf)c-a^2d^2(3de+cf)\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\int \frac{ac\left(c(4de-3cf)b^2-2ad(3de-cf)b+3a^2d^2f\right)-b\left(-2b^2(4de-3cf)c^2+abd(13de-7cf)c-a^2d^2(3de+cf)\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{ac\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-b\left(-a^2d^2(cf+3de)+abcd(13de-7cf)-2b\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b\left(-a^2d^2(cf+3de)\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$\Big\downarrow$ 388

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de)\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int\frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de)\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int\frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}$$

$\Big\downarrow$ 418

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de)\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}+\frac{b\int-\frac{\sqrt{dx^2+c}\left(-bfx^2+be-2af\right)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\int\frac{\sqrt{dx^2+c}(-bfx^2+be-2af)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 403

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\left(\frac{\int\frac{b\left((3bde-bcf-4adf)x^2+c(3be-5af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f^2}-\frac{b\left(\frac{1}{3}\int\frac{(3bde-bcf-4adf)x^2+c(3be-5af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 406

$$\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}$$

$$(de-cf)^2$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(c(3be-5af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-4adf-bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{\cdots}\right.}{f^2}\right.}{(de-cf)^2}$$

↓ 320

$$\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}$$

$$(de-cf)^2$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{c}\cdots}\right.\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{(de-cf)^2}$$

↓ 388

$$\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}$$

$$(de-cf)^2$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\text{EllipticF}}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\ldots}}\right.\right.}{f^2}\right.}{(de-cf)^2}$$

$\downarrow$ 313

$$\frac{\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc(4de-3cf)-ad(3de-2cf))}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(3a^2d^2f-2abd(3de-cf)+b^2c(4de-3cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b\left(-a^2d^2(cf+3de\right.}{3d}}{c}$$

$$(de-cf)^2$$

$$\frac{f(be-af)\left(\frac{(be-af)^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}\ldots\right.\right.\right.}{f^2}\right.}{(de-cf)^2}$$

$\downarrow$ 414

$$\frac{\frac{b(bc(4de-3cf)-ad(3de-2cf))x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d}-\frac{\frac{c^{3/2}\left(c(4de-3cf)b^2-2ad(3de-cf)b+3a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-b\left(-2b^2(4de-3cf)\right.}{3d}}{c}$$

$$(de-cf)^2$$

$$\frac{f(be-af)\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\right.\right.}{}\right.}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(-(((b*c - a*d)*(d*e - c*f)*x*(a + b*x^2)^(3/2))/(c*Sqrt[c + d*x^2])) + ((
b*(b*c*(4*d*e - 3*c*f) - a*d*(3*d*e - 2*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*d) - (-(b*(a*b*c*d*(13*d*e - 7*c*f) - 2*b^2*c^2*(4*d*e - 3*c*f)
- a^2*d^2*(3*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c
^(3/2)*(3*a^2*d^2*f + b^2*c*(4*d*e - 3*c*f) - 2*a*b*d*(3*d*e - c*f))*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/c)/(d*e
- c*f)^2 - (f*(b*e - a*f)*(-((b*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
) + ((3*b*d*e - b*c*f - 4*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)) + (c^(3/2)*(3*b*e - 5*a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))/3))/f^2) + (c^(3/2)*(b*e - a*f)^2*Sqrt[a + b*x^2]*El
lipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
a*Sqrt[d]*e*f^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(
d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(423) = 846$.

Time = 6.86 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	1065
elliptic	Expression too large to display	1254

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*a^2*b*d^3*e*f^2*x^3+2*(-b/a)^(1/2)*a*b^2*c*d^2*e*f^2*x^3-(-
b/a)^(1/2)*b^3*c^2*d*e*f^2*x^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*e*f^2-3*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2
*c*d^2*e^2*f-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^3*c^3*e*f^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^2*d*e^2*f+((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c
*d^2*e^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a^2*b*c*d^2*e*f^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*e*f^2+2*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
^3*c^3*e*f^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b^3*c^2*d*e^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*c*d^
2*f^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),
a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*b*c*d^2*e*f^2+3*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a*b^2*c*d^2*e^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^3*c*d^2*e^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(5/2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output

```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*x - 6*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**6)/(2*a*c**3*e*f + 2*a*c**3*f**2*x**2 + a*c**2*d*e**2 + 5*a
*c**2*d*e*f*x**2 + 4*a*c**2*d*f**2*x**4 + 2*a*c*d**2*e**2*x**2 + 4*a*c*d**
2*e*f*x**4 + 2*a*c*d**2*f**2*x**6 + a*d**3*e**2*x**4 + a*d**3*e*f*x**6 + 2
*b*c**3*e*f*x**2 + 2*b*c**3*f**2*x**4 + b*c**2*d*e**2*x**2 + 5*b*c**2*d*e*
f*x**4 + 4*b*c**2*d*f**2*x**6 + 2*b*c*d**2*e**2*x**4 + 4*b*c*d**2*e*f*x**6
 + 2*b*c*d**2*f**2*x**8 + b*d**3*e**2*x**6 + b*d**3*e*f*x**8),x)*a*b**2*c*
*2*d*f**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a*c**3*e*f +
 2*a*c**3*f**2*x**2 + a*c**2*d*e**2 + 5*a*c**2*d*e*f*x**2 + 4*a*c**2*d*f**
2*x**4 + 2*a*c*d**2*e**2*x**2 + 4*a*c*d**2*e*f*x**4 + 2*a*c*d**2*f**2*x**6
 + a*d**3*e**2*x**4 + a*d**3*e*f*x**6 + 2*b*c**3*e*f*x**2 + 2*b*c**3*f**2*
x**4 + b*c**2*d*e**2*x**2 + 5*b*c**2*d*e*f*x**4 + 4*b*c**2*d*f**2*x**6 + 2
*b*c*d**2*e**2*x**4 + 4*b*c*d**2*e*f*x**6 + 2*b*c*d**2*f**2*x**8 + b*d**3*
e**2*x**6 + b*d**3*e*f*x**8),x)*a*b**2*c*d**2*e*f - 6*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(2*a*c**3*e*f + 2*a*c**3*f**2*x**2 + a*c**2*d*e**
2 + 5*a*c**2*d*e*f*x**2 + 4*a*c**2*d*f**2*x**4 + 2*a*c*d**2*e**2*x**2 + 4*
a*c*d**2*e*f*x**4 + 2*a*c*d**2*f**2*x**6 + a*d**3*e**2*x**4 + a*d**3*e*f*x
**6 + 2*b*c**3*e*f*x**2 + 2*b*c**3*f**2*x**4 + b*c**2*d*e**2*x**2 + 5*b*c*
*2*d*e*f*x**4 + 4*b*c**2*d*f**2*x**6 + 2*b*c*d**2*e**2*x**4 + 4*b*c*d**2*e
*f*x**6 + 2*b*c*d**2*f**2*x**8 + b*d**3*e**2*x**6 + b*d**3*e*f*x**8),x)...
```

3.108 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)} dx$

Optimal result

Integrand size = 32, antiderivative size = 460

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)} dx = \frac{(bc-ad)^2 x\sqrt{a+bx^2}}{3cd(de-cf)(c+dx^2)^{3/2}} - \frac{(bc-ad)(ad(2de-5cf)+bc(5de-2cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}d^{3/2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(bc-ad)\left(3b^2cde^2+3a^2cdf^2-ab(d^2e^2+4cdef+c^2f^2)\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}d^{3/2}(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(be-af)^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*(-a*
d+b*c)*(a*d*(-5*c*f+2*d*e)+b*c*(-2*c*f+5*d*e))*(b*x^2+a)^(1/2)*EllipticE(d
^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(3/2)/(-c*
f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(-a*d+b*c)*(3
*b^2*c*d*e^2+3*a^2*c*d*f^2-a*b*(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2
)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d
^(3/2)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2
)*(-a*f+b*e)^3*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1
/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.65 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \frac{-ibc(-bc+ad)ef(bc(-5de+2cf)+ad(-2de+5cf))\sqrt{1+\frac{bx^2}{a}}(c+dx^2)\ \cdots}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
((-I)*b*c*(-(b*c) + a*d)*e*f*(b*c*(-5*d*e + 2*c*f) + a*d*(-2*d*e + 5*c*f))
*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*(-(d*e) + c*f)*(-2*a*b*c*d*f + a^2*d^2
*f + b^2*c*(3*d*e - 2*c*f))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*(b*c -
 a*d)*e*f*x*(a + b*x^2)*(b*c*(-4*c*d*e + c^2*f - 5*d^2*e*x^2 + 2*c*d*f*x^2
) + a*d*(-3*c*d*e + 6*c^2*f - 2*d^2*e*x^2 + 5*c*d*f*x^2)) + (3*I)*c^2*d*(b
*e - a*f)^3*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/a]*c^2*d^2*
e*f*(d*e - c*f)^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.65, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {419, 25, 401, 25, 27, 401, 25, 27, 406, 320, 388, 313, 420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{-\frac{\int -\frac{d\sqrt{bx^2+a}\left(b(bc(4de-cf)-ad(de+2cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{(dx^2+c)^{3/2}}dx}{3cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{d\sqrt{bx^2+a}\left(b(bc(4de-cf)-ad(de+2cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{(dx^2+c)^{3/2}}dx}{3cd}-\frac{x(a+bx^2)^{3/2}(bc-ad)(de-cf)}{3c(c+dx^2)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(b(bc(4de-cf)-ad(de+2cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{-\frac{\int -\frac{b\left(ac(bc(4de-cf)-ad(de+2cf))-\left(-2b^2(4de-cf)c^2+3abd(de+2cf)c+a^2d^2(2de-5cf)\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(2de-5cf)+bc(4de-cf))}{cd\sqrt{c+dx^2}}}{3c}-\frac{x(a+b}{}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{b\left(ac(bc(4de-cf)-ad(de+2cf))-\left(-2b^2(4de-cf)c^2+3abd(de+2cf)c+a^2d^2(2de-5cf)\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(2de-5cf)+bc(4de-cf))}{cd\sqrt{c+dx^2}}}{3c}-\frac{x(a+bx^2)}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{b\int \frac{ac(bc(4de-cf)-ad(de+2cf))-\left(-2b^2(4de-cf)c^2+3abd(de+2cf)c+a^2d^2(2de-5cf)\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(2de-5cf)+bc(4de-cf))}{cd\sqrt{c+dx^2}}}{3c}-\frac{x\left(a+bx^2\right)^{3/}}{3c(}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\frac{\frac{b\left(ac(bc(4de-cf)-ad(2cf+de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(2de \cdots}{cd\sqrt{c+ \cdots}}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}-\frac{x \cdots}{\cdots}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/ \cdots}}}{b} \cdots\right.\right.}{cd}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+ \cdots}} \cdots\right.\right.}{cd}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 420$$

$$\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+\cdots}}\right.\cdots}{cd}}{3c}\cdots}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 324$$

$$\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+\cdots}}\right.\cdots}{cd}}{3c}\cdots}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\left(a\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+\cdots}}\right.\cdots}{cd}}{3c}\cdots}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\left(b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 388

$$\frac{\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\Big(}{b\sqrt{d}\sqrt{c+}}\right.\right.}{cd}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\Big(}{b\sqrt{d}\sqrt{c+}}\right.\right.}{cd}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2}}{f}\right.}{(de-cf)^2}$$

$\Big\downarrow$ 414

$$\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(bc(4de-cf)-ad(2cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(a^2d^2(2de-5cf)+3abcd(2cf+de)-2b^2c^2(4de-cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\right.}{b\sqrt{d}\sqrt{c+}}\right.\right.}{cd}}{3c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{E}}{\sqrt{b}cef}\right.}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(-1/3*((b*c - a*d)*(d*e - c*f)*x*(a + b*x^2)^(3/2))/(c*(c + d*x^2)^(3/2))
+ (-(((b*c - a*d)*(a*d*(2*d*e - 5*c*f) + b*c*(4*d*e - c*f))*x*Sqrt[a + b*x
^2])/(c*d*Sqrt[c + d*x^2])) + (b*(-((a^2*d^2*(2*d*e - 5*c*f) - 2*b^2*c^2*(
4*d*e - c*f) + 3*a*b*c*d*(d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d
*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]))) + (c^(3/2)*(b*c*(4*d*e - c*f) - a*d*(d*e + 2*c*f))*Sqrt[a + b*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(c*d))/(3*c))/(d*e -
c*f)^2 - (f*(b*e - a*f)*((b*(b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
 + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
))/f - (a^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), Ar
cTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f*Sqrt[a + b*x^2
]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(436) = 872$.

Time = 9.17 (sec) , antiderivative size = 1703, normalized size of antiderivative = 3.70

method	result	size
elliptic	Expression too large to display	1703
default	Expression too large to display	2855

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/c/d^3/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
(x^2+c/d)^2-1/3*(b*d*x^2+a*d)*(5*a^2*c*d^2*f-2*a^2*d^3*e-3*a*b*c^2*d*f-3*a
*b*c*d^2*e-2*b^2*c^3*f+5*b^2*c^2*d*e)/d^2/c^2/(c*f-d*e)^2*x/((x^2+c/d)*(b*
d*x^2+a*d))^(1/2)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))*b^3/d^2/f-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))*b/(c*f-d*e)/c*a^2+2/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))/d*b^2/(c*f-d*e)*a-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))/d^2*b^3/(c*f-d*e)*c+f^2/(c*f-d*e)^2/e/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3-3*
f/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/
(-b/a)^(1/2))*a^2*b+3/(c*f-d*e)^2*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b^2-1/(c*f-d*e)^2*e^2/f/(-b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 2*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**6)/(2*a**2*c**3*d*e*f + 2*a**2*c**3*d*f**2*x**2 + 6*a
**2*c**2*d**2*e*f*x**2 + 6*a**2*c**2*d**2*f**2*x**4 + 6*a**2*c*d**3*e*f*x*
*4 + 6*a**2*c*d**3*f**2*x**6 + 2*a**2*d**4*e*f*x**6 + 2*a**2*d**4*f**2*x**
8 - 2*a*b*c**4*e*f - 2*a*b*c**4*f**2*x**2 + a*b*c**3*d*e**2 - 3*a*b*c**3*d
*e*f*x**2 - 4*a*b*c**3*d*f**2*x**4 + 3*a*b*c**2*d**2*e**2*x**2 + 3*a*b*c**
2*d**2*e*f*x**4 + 3*a*b*c*d**3*e**2*x**4 + 7*a*b*c*d**3*e*f*x**6 + 4*a*b*c
*d**3*f**2*x**8 + a*b*d**4*e**2*x**6 + 3*a*b*d**4*e*f*x**8 + 2*a*b*d**4*f*
*2*x**10 - 2*b**2*c**4*e*f*x**2 - 2*b**2*c**4*f**2*x**4 + b**2*c**3*d*e**2
*x**2 - 5*b**2*c**3*d*e*f*x**4 - 6*b**2*c**3*d*f**2*x**6 + 3*b**2*c**2*d**
2*e**2*x**4 - 3*b**2*c**2*d**2*e*f*x**6 - 6*b**2*c**2*d**2*f**2*x**8 + 3*b
**2*c*d**3*e**2*x**6 + b**2*c*d**3*e*f*x**8 - 2*b**2*c*d**3*f**2*x**10 + b
**2*d**4*e**2*x**8 + b**2*d**4*e*f*x**10),x)*a**2*b**3*c**2*d**2*f**2 - 4*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c**3*d*e*f + 2*a**2*c
**3*d*f**2*x**2 + 6*a**2*c**2*d**2*e*f*x**2 + 6*a**2*c**2*d**2*f**2*x**4 +
 6*a**2*c*d**3*e*f*x**4 + 6*a**2*c*d**3*f**2*x**6 + 2*a**2*d**4*e*f*x**6 +
 2*a**2*d**4*f**2*x**8 - 2*a*b*c**4*e*f - 2*a*b*c**4*f**2*x**2 + a*b*c**3*
d*e**2 - 3*a*b*c**3*d*e*f*x**2 - 4*a*b*c**3*d*f**2*x**4 + 3*a*b*c**2*d**2*
e**2*x**2 + 3*a*b*c**2*d**2*e*f*x**4 + 3*a*b*c*d**3*e**2*x**4 + 7*a*b*c*d*
*3*e*f*x**6 + 4*a*b*c*d**3*f**2*x**8 + a*b*d**4*e**2*x**6 + 3*a*b*d**4*...
```

3.109 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 655

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{5cd(de-cf)(c+dx^2)^{5/2}}$$

$$-\frac{(bc-ad)(ad(4de-9cf)+bc(7de-2cf))x\sqrt{a+bx^2}}{15c^2d(de-cf)^2(c+dx^2)^{3/2}}$$

$$+\frac{(abcd(7d^2e^2-29cdef-8c^2f^2)+b^2c^2(8d^2e^2+9cdef-2c^2f^2)+a^2d^2(8d^2e^2-26cdef+33c^2f^2))\sqrt{a+}}{15c^{5/2}d^{3/2}(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+\frac{(15b^3c^3de^2f-15a^3c^2d^2f^3-a^2bd(4d^3e^3-17cd^2e^2f-8c^2def^2-24c^3f^3)-ab^2c(4d^3e^3+8cd^2e^2f+32c^2}{15ac^{3/2}d^{3/2}(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$-\frac{c^{3/2}f(be-af)^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/5*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(5/2)-1/15*(-a
*d+b*c)*(a*d*(-9*c*f+4*d*e)+b*c*(-2*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/c^2/d/(-
c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*(a*b*c*d*(-8*c^2*f^2-29*c*d*e*f+7*d^2*e^2)
+b^2*c^2*(-2*c^2*f^2+9*c*d*e*f+8*d^2*e^2)+a^2*d^2*(33*c^2*f^2-26*c*d*e*f+8
*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))/c^(5/2)/d^(3/2)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)+1/15*(15*b^3*c^3*d*e^2*f-15*a^3*c^2*d^2*f^3-a^2*b*d*(
-24*c^3*f^3-8*c^2*d*e*f^2-17*c*d^2*e^2*f+4*d^3*e^3)-a*b^2*c*(c^3*f^3+32*c^
2*d*e*f^2+8*c*d^2*e^2*f+4*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan
(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(3/2)/d^(3/2)/(-c*f+d*e)^4/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*f*(-a*f+b*e)^3*(b*x^2+
a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a
/d)^(1/2))/a/d^(1/2)/e/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.87 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}dex(a+bx^2)\left(3c^2(bc-ad)^2(de-cf)^2-c(bc-ad)(-de+cf)(bc(-7d\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*e*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 - c*(b*c -
 a*d)*(-(d*e) + c*f)*(b*c*(-7*d*e + 2*c*f) + a*d*(-4*d*e + 9*c*f))*(c + d*
x^2) + (a*b*c*d*(7*d^2*e^2 - 29*c*d*e*f - 8*c^2*f^2) + b^2*c^2*(8*d^2*e^2
+ 9*c*d*e*f - 2*c^2*f^2) + a^2*d^2*(8*d^2*e^2 - 26*c*d*e*f + 33*c^2*f^2))*
(c + d*x^2)^2) + I*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]
*(b*e*(a*b*c*d*(7*d^2*e^2 - 29*c*d*e*f - 8*c^2*f^2) + b^2*c^2*(8*d^2*e^2 +
 9*c*d*e*f - 2*c^2*f^2) + a^2*d^2*(8*d^2*e^2 - 26*c*d*e*f + 33*c^2*f^2))*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*(b*c - a*d)*e*(-(d*e) +
c*f)*(b*c*(-7*d*e + 2*c*f) + a*d*(-4*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] - 15*c^2*d^2*(b*e - a*f)^3*EllipticPi[(a*f)/(b*e),
 I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*Sqrt[b/a]*c^3*d^2*e*(d*e - c*f
)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {419, 25, 401, 25, 27, 401, 25, 400, 313, 320, 417, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{7/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{7/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{-\frac{\int -\frac{d\sqrt{bx^2+a}\left(b(ad(de-6cf)+bc(4de+cf))x^2+a(ad(4de-9cf)+bc(de+4cf))\right)}{\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{5c\left(c+dx^2\right)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{d\sqrt{bx^2+a}\left(b(ad(de-6cf)+bc(4de+cf))x^2+a(ad(4de-9cf)+bc(de+4cf))\right)}{\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{5c\left(c+dx^2\right)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(b(ad(de-6cf)+bc(4de+cf))x^2+a(ad(4de-9cf)+bc(de+4cf))\right)}{\left(dx^2+c\right)^{5/2}}dx}{5c}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{5c\left(c+dx^2\right)^{5/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 401$$

$$\frac{\frac{-\frac{\int -\frac{b\left(2b^2(4de+cf)c^2+abd(3de-8cf)c+a^2d^2(4de-9cf)\right)x^2+a\left(b^2(4de+cf)c^2+abd(3de+2cf)c+2a^2d^2(4de-9cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(4de-9cf)+bc(cf}{3cd\left(c+dx^2\right)^{3/2}}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{b\left(2b^2(4de+cf)c^2+abd(3de-8cf)c+a^2d^2(4de-9cf)\right)x^2+a\left(b^2(4de+cf)c^2+abd(3de+2cf)c+2a^2d^2(4de-9cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(4de-9cf)+bc(cf+4d}{3cd\left(c+dx^2\right)^{3/2}}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 400

$$\frac{\frac{\frac{\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx-ab(ad(4de-9cf)+bc(cf+4de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(4de-9cf)+}{3cd\left(c+dx^2\right)^{3/2}}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 313

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-ab(ad(4de-9cf)+bc(cf+4de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-}{3}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 320

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)+bc(cf+4de))\ \text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3cd}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 417

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)+bc(cf+4de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3cd}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}\right)}{(de-cf)^2}$$

$\downarrow$ 313

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)+bc(cf+4de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3cd}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}$$

$\downarrow$ 414

$$\frac{\frac{\frac{\frac{\sqrt{a+bx^2}\left(2a^2d^2(4de-9cf)+7abcd(de-cf)+2b^2c^2(cf+4de)\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(4de-9cf)+bc(cf+4de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3cd}}{5c}}{(de-cf)^2}$$

$$\frac{f(be-af)\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x]
```

output
```
(-1/5*((b*c - a*d)*(d*e - c*f)*x*(a + b*x^2)^(3/2))/(c*(c + d*x^2)^(5/2))
+ (-1/3*((b*c - a*d)*(a*d*(4*d*e - 9*c*f) + b*c*(4*d*e + c*f))*x*Sqrt[a +
b*x^2])/(c*d*(c + d*x^2)^(3/2)) + (((2*a^2*d^2*(4*d*e - 9*c*f) + 7*a*b*c*d
*(d*e - c*f) + 2*b^2*c^2*(4*d*e + c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(a*d*(4*d*e - 9*c*f) + b
*c*(4*d*e + c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))/(3*c*d))/(5*c))/(d*e - c*f)^2 - (f*(b*e - a*f)*(-(((b*c - a*d)*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[c]*Sqrt[d]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2])) + (a^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e),
ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sq
rt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2416 vs. $2(625) = 1250$.

Time = 12.03 (sec) , antiderivative size = 2417, normalized size of antiderivative = 3.69

method	result	size
elliptic	Expression too large to display	2417
default	Expression too large to display	5577

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/c/d^4/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
(x^2+c/d)^3-1/15*(9*a^2*c*d^2*f-4*a^2*d^3*e-7*a*b*c^2*d*f-3*a*b*c*d^2*e-2*
b^2*c^3*f+7*b^2*c^2*d*e)/d^3/(c*f-d*e)^2/c^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/(x^2+c/d)^2-1/15*(b*d*x^2+a*d)*(33*a^2*c^2*d^2*f^2-26*a^2*c*d^3*e
*f+8*a^2*d^4*e^2-8*a*b*c^3*d*f^2-29*a*b*c^2*d^2*e*f+7*a*b*c*d^3*e^2-2*b^2*
c^4*f^2+9*b^2*c^3*d*e*f+8*b^2*c^2*d^2*e^2)/d^2/(c*f-d*e)^3/c^3*x/((x^2+c/d
)*(b*d*x^2+a*d))^(1/2)-7/15/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))*b^3/d/(c*f-d*e)^2*e+1/(c*f-d*e)^3/e*f^3/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3-3/(c*f-d*e)^3*
f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))*a^2*b+4/15/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))*b/c^2*d/(c*f-d*e)^2*a^2*e-26/15*d*b/(c*f-d*e)^3/c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a^2*e*f-29/15*b^2/(c*f-d*e)^3/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(7/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{7/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 20*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(4*a**2*c**4*d*e*f + 4*a**2*c**4*d*f**2*x**2 + 16
*a**2*c**3*d**2*e*f*x**2 + 16*a**2*c**3*d**2*f**2*x**4 + 24*a**2*c**2*d**3
*e*f*x**4 + 24*a**2*c**2*d**3*f**2*x**6 + 16*a**2*c*d**4*e*f*x**6 + 16*a**
2*c*d**4*f**2*x**8 + 4*a**2*d**5*e*f*x**8 + 4*a**2*d**5*f**2*x**10 - 2*a*b
*c**5*e*f - 2*a*b*c**5*f**2*x**2 + 3*a*b*c**4*d*e**2 - a*b*c**4*d*e*f*x**2
 - 4*a*b*c**4*d*f**2*x**4 + 12*a*b*c**3*d**2*e**2*x**2 + 16*a*b*c**3*d**2*
e*f*x**4 + 4*a*b*c**3*d**2*f**2*x**6 + 18*a*b*c**2*d**3*e**2*x**4 + 34*a*b
*c**2*d**3*e*f*x**6 + 16*a*b*c**2*d**3*f**2*x**8 + 12*a*b*c*d**4*e**2*x**6
 + 26*a*b*c*d**4*e*f*x**8 + 14*a*b*c*d**4*f**2*x**10 + 3*a*b*d**5*e**2*x**
8 + 7*a*b*d**5*e*f*x**10 + 4*a*b*d**5*f**2*x**12 - 2*b**2*c**5*e*f*x**2 -
2*b**2*c**5*f**2*x**4 + 3*b**2*c**4*d*e**2*x**2 - 5*b**2*c**4*d*e*f*x**4 -
 8*b**2*c**4*d*f**2*x**6 + 12*b**2*c**3*d**2*e**2*x**4 - 12*b**2*c**3*d**2
*f**2*x**8 + 18*b**2*c**2*d**3*e**2*x**6 + 10*b**2*c**2*d**3*e*f*x**8 - 8*
b**2*c**2*d**3*f**2*x**10 + 12*b**2*c*d**4*e**2*x**8 + 10*b**2*c*d**4*e*f*
x**10 - 2*b**2*c*d**4*f**2*x**12 + 3*b**2*d**5*e**2*x**10 + 3*b**2*d**5*e*
f*x**12),x)*a**2*b**3*c**3*d**2*f**2 - 60*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**6)/(4*a**2*c**4*d*e*f + 4*a**2*c**4*d*f**2*x**2 + 16*a**2*c**3*d
**2*e*f*x**2 + 16*a**2*c**3*d**2*f**2*x**4 + 24*a**2*c**2*d**3*e*f*x**4 +
24*a**2*c**2*d**3*f**2*x**6 + 16*a**2*c*d**4*e*f*x**6 + 16*a**2*c*d**4*...
```

3.110 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 995

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{7cd(de-cf)(c+dx^2)^{7/2}}$$

$$-\frac{(bc-ad)(ad(6de-13cf)+bc(9de-2cf))x\sqrt{a+bx^2}}{35c^2d(de-cf)^2(c+dx^2)^{5/2}}$$

$$+\frac{(abcd(13d^2e^2-47cdef-36c^2f^2)+b^2c^2(8d^2e^2+33cdef-6c^2f^2)+a^2d^2(24d^2e^2-76cdef+87c^2f^2))\,x}{105c^3d(de-cf)^3(c+dx^2)^{3/2}}$$

$$+\frac{(a^2bcd^2(16d^3e^3-55cd^2e^2f+27c^2def^2-303c^3f^3)-a^3d^3(48d^3e^3-200cd^2e^2f+326c^2def^2-279c^3f^3)}{105c^{7/2}d^{3/}}$$

$$-\frac{(105b^4c^5de^2f^2+105a^4c^3d^3f^4+ab^3c^2(4d^4e^4-44cd^3e^3f-137c^2d^2e^2f^2-240c^3def^3-3c^4f^4)+5a^2b^2cd}{}$$

$$+\frac{c^{3/2}f^2(be-af)^3\sqrt{a+bx^2}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^5\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/7*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(7/2)-1/35*(-a
*d+b*c)*(a*d*(-13*c*f+6*d*e)+b*c*(-2*c*f+9*d*e))*x*(b*x^2+a)^(1/2)/c^2/d/(
-c*f+d*e)^2/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-36*c^2*f^2-47*c*d*e*f+13*d^2*
e^2)+b^2*c^2*(-6*c^2*f^2+33*c*d*e*f+8*d^2*e^2)+a^2*d^2*(87*c^2*f^2-76*c*d*
e*f+24*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d/(-c*f+d*e)^3/(d*x^2+c)^(3/2)+1/10
5*(a^2*b*c*d^2*(-303*c^3*f^3+27*c^2*d*e*f^2-55*c*d^2*e^2*f+16*d^3*e^3)-a^3
*d^3*(-279*c^3*f^3+326*c^2*d*e*f^2-200*c*d^2*e^2*f+48*d^3*e^3)+b^3*c^3*(6*
c^3*f^3-39*c^2*d*e*f^2-80*c*d^2*e^2*f+8*d^3*e^3)+a*b^2*c^2*d*(33*c^3*f^3+2
93*c^2*d*e*f^2-20*c*d^2*e^2*f+9*d^3*e^3))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2
)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(7/2)/d^(3/2)/(-a*d+b*c
)/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*(105*
b^4*c^5*d*e^2*f^2+105*a^4*c^3*d^3*f^4+a*b^3*c^2*(-3*c^4*f^4-240*c^3*d*e*f^
3-137*c^2*d^2*e^2*f^2-44*c*d^3*e^3*f+4*d^4*e^4)+5*a^2*b^2*c*d*(36*c^4*f^4+
52*c^3*d*e*f^3+41*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+d^4*e^4)-a^3*b*d^2*(297*c^
4*f^4-40*c^3*d*e*f^3+263*c^2*d^2*e^2*f^2-124*c*d^3*e^3*f+24*d^4*e^4))*(b*x
^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a
/c^(5/2)/d^(3/2)/(-a*d+b*c)/(-c*f+d*e)^5/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)+c^(3/2)*f^2*(-a*f+b*e)^3*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)
*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-c*
f+d*e)^5/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.78 (sec) , antiderivative size = 919, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}dex(a+bx^2)\left(15c^3(bc-ad)^3(de-cf)^3+3c^2(bc-ad)^2(de-cf)^2(bc(\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*e*x*(a + b*x^2)*(15*c^3*(b*c - a*d)^3*(d*e - c*f)^3 + 3*c^2*(
b*c - a*d)^2*(d*e - c*f)^2*(b*c*(-9*d*e + 2*c*f) + a*d*(-6*d*e + 13*c*f))*
(c + d*x^2) + c*(b*c - a*d)*(d*e - c*f)*(a*b*c*d*(13*d^2*e^2 - 47*c*d*e*f
- 36*c^2*f^2) + b^2*c^2*(8*d^2*e^2 + 33*c*d*e*f - 6*c^2*f^2) + a^2*d^2*(24
*d^2*e^2 - 76*c*d*e*f + 87*c^2*f^2))*(c + d*x^2)^2 + (a^2*b*c*d^2*(16*d^3*
e^3 - 55*c*d^2*e^2*f + 27*c^2*d*e*f^2 - 303*c^3*f^3) + b^3*c^3*(8*d^3*e^3
- 80*c*d^2*e^2*f - 39*c^2*d*e*f^2 + 6*c^3*f^3) + a*b^2*c^2*d*(9*d^3*e^3 -
20*c*d^2*e^2*f + 293*c^2*d*e*f^2 + 33*c^3*f^3) + a^3*d^3*(-48*d^3*e^3 + 20
0*c*d^2*e^2*f - 326*c^2*d*e*f^2 + 279*c^3*f^3))*(c + d*x^2)^3) + I*c*Sqrt[
1 + (b*x^2)/a]*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*(b*e*(a^2*b*c*d^2*(16*d^3
*e^3 - 55*c*d^2*e^2*f + 27*c^2*d*e*f^2 - 303*c^3*f^3) + b^3*c^3*(8*d^3*e^3
 - 80*c*d^2*e^2*f - 39*c^2*d*e*f^2 + 6*c^3*f^3) + a*b^2*c^2*d*(9*d^3*e^3 -
 20*c*d^2*e^2*f + 293*c^2*d*e*f^2 + 33*c^3*f^3) + a^3*d^3*(-48*d^3*e^3 + 2
00*c*d^2*e^2*f - 326*c^2*d*e*f^2 + 279*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(d*e - c*f)*(a*b*c*d*(13*d^2*e^2
- 47*c*d*e*f - 36*c^2*f^2) + b^2*c^2*(8*d^2*e^2 + 33*c*d*e*f - 6*c^2*f^2)
+ a^2*d^2*(24*d^2*e^2 - 76*c*d*e*f + 87*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + 105*c^3*d^2*f*(-(b*e) + a*f)^3*EllipticPi[(a*f)/(
b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(105*Sqrt[b/a]*c^4*d^2*(b*c
- a*d)*e*(d*e - c*f)^4*Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 951, normalized size of antiderivative = 0.96, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.656, Rules used = {419, 25, 401, 25, 27, 401, 25, 402, 27, 400, 313, 320, 419, 25, 401, 25, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}dx$$

$$\Big\downarrow \ 419$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{9/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(bfc^2+d^2(be-af)x^2+ad(de-2cf))}{(dx^2+c)^{9/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{-\frac{\int -\frac{d\sqrt{bx^2+a}\left(b(ad(3de-10cf)+bc(4de+3cf))x^2+a(ad(6de-13cf)+bc(de+6cf))\right)}{\left(dx^2+c\right)^{7/2}}dx}{7cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{7c(c+dx^2)^{7/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d\sqrt{bx^2+a}\left(b(ad(3de-10cf)+bc(4de+3cf))x^2+a(ad(6de-13cf)+bc(de+6cf))\right)}{\left(dx^2+c\right)^{7/2}}dx}{7cd}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{7c(c+dx^2)^{7/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(b(ad(3de-10cf)+bc(4de+3cf))x^2+a(ad(6de-13cf)+bc(de+6cf))\right)}{\left(dx^2+c\right)^{7/2}}dx}{7c}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)(de-cf)}{7c(c+dx^2)^{7/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{-\frac{\int -\frac{b\left(2b^2(4de+3cf)c^2+abd(9de-2cf)c+3a^2d^2(6de-13cf)\right)x^2+a\left(b^2(4de+3cf)c^2+7abd(de+2cf)c+4a^2d^2(6de-13cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(6de-13cf)}{5cd\left(c+dx^2\right)^{5/2}}}{7c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{b\left(2b^2(4de+3cf)c^2+abd(9de-2cf)c+3a^2d^2(6de-13cf)\right)x^2+a\left(b^2(4de+3cf)c^2+7abd(de+2cf)c+4a^2d^2(6de-13cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{5cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(ad(6de-13cf)+bc}{5cd\left(c+dx^2\right)^{5/2}}}{7c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{(bc-ad)\left(b\left(2b^2(4de+3cf)c^2+abd(13de+cf)c+4a^2d^2(6de-13cf)\right)x^2+a\left(b^2(4de+3cf)c^2+abd(8de+41cf)c+8a^2d^2(6de-13cf)\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13\right.}{}}{5cd}}{7c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{b\left(2b^2(4de+3cf)c^2+abd(13de+cf)c+4a^2d^2(6de-13cf)\right)x^2+a\left(b^2(4de+3cf)c^2+abd(8de+41cf)c+8a^2d^2(6de-13cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(\right.}{3c\left(c+dx^2\right)}}{5cd}}{7c}}{(de-cf)^2}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 400

$$\frac{\frac{\frac{\frac{ab\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\left(-8a^3d^3(6de-13cf)+a^2bcd^2(16de-93cf)+ab^2c^2d(9de-2cf)+2b^3c^3(3cf+4de\right.}{bc-ad}}{3c}}{5cd}}{7c}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(de-cf)^2}$$

$\downarrow$ 313

$$\frac{\frac{\frac{\frac{ab\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{a+bx^2}\left(-8a^3d^3(6de-13cf)+a^2bcd^2(16de-93cf)+ab^2c^2d(9de-2cf)+2b^3c^3\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}}{5cd}}{7c}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$\downarrow$ 320

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}$$

$$\frac{f(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{(de-cf)^2}$$

$\downarrow$ 419

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}$$

$$\frac{f(be-af)\left(-\frac{\int -\frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{\int\frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{-\frac{\int-\frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}d}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{\int\frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{\int\frac{b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\big\downarrow$ 400

$$\frac{\frac{\frac{\frac{x\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)+abcd(cf+13de)+2b^2c^2(3cf+4de)\right)}{3c\left(c+dx^2\right)^{3/2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}\left(4a^2d^2(6de-13cf)-5abcd(de-8cf)-b^2c^2(3cf+4de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5cd}}{}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{(ad(2de-5cf)+bc(cf+2de))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx-ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2+}}{\sqrt{dx^2+c}\left(fx\right.}}{(de-cf)^2}\right.}{(de-cf)^2}$$

$\downarrow$ 313

$$\frac{\frac{\frac{\frac{\left(2b^2(4de+3cf)c^2+abd(13de+cf)c+4a^2d^2(6de-13cf)\right)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\left(2b^3(4de+3cf)c^3+ab^2d(9de-2cf)c^2+a^2bd^2(16de-93cf)c-8a^3d^3(6de-13cf)\right)\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{5cd}}{}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-a}{}\right.}{(de-cf)^2}$$

$\downarrow$ 320

$$\frac{\frac{\frac{\left(2b^2(4de+3cf)c^2+abd(13de+cf)c+4a^2d^2(6de-13cf)\right)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{\left(2b^3(4de+3cf)c^3+ab^2d(9de-2cf)c^2+a^2bd^2(16de-93cf)c-8a^3d^3(6de-13cf)\right)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x}{3c\left(dx^2+c\right)^3}}{(de-cf)^2}\right.}{(de-cf)^2}$$

$\downarrow$ 414

$$\frac{\frac{\frac{\left(2b^2(4de+3cf)c^2+abd(13de+cf)c+4a^2d^2(6de-13cf)\right)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{\left(2b^3(4de+3cf)c^3+ab^2d(9de-2cf)c^2+a^2bd^2(16de-93cf)c-8a^3d^3(6de-13cf)\right)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}}{5cd}}{}$$

$$\frac{f(be-af)\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x}{3c\left(dx^2+c\right)^3}}{(de-cf)^2}\right.}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)),x]
```

output
```
(-1/7*((b*c - a*d)*(d*e - c*f)*x*(a + b*x^2)^(3/2))/(c*(c + d*x^2)^(7/2))
+ (-1/5*((b*c - a*d)*(a*d*(6*d*e - 13*c*f) + b*c*(4*d*e + 3*c*f))*x*Sqrt[a
 + b*x^2])/(c*d*(c + d*x^2)^(5/2)) + (((4*a^2*d^2*(6*d*e - 13*c*f) + a*b*c
*d*(13*d*e + c*f) + 2*b^2*c^2*(4*d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(3*c*(c
+ d*x^2)^(3/2)) + (((a^2*b*c*d^2*(16*d*e - 93*c*f) - 8*a^3*d^3*(6*d*e - 13
*c*f) + a*b^2*c^2*d*(9*d*e - 2*c*f) + 2*b^3*c^3*(4*d*e + 3*c*f))*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*S
qrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
+ (b*Sqrt[c]*(4*a^2*d^2*(6*d*e - 13*c*f) - 5*a*b*c*d*(d*e - 8*c*f) - b^2*c
^2*(4*d*e + 3*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))/(3*c))/(5*c*d))/(7*c))/(d*e - c*f)^2 - (f*(b*e - a*f
)*((-1/3*((b*c - a*d)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)^(3/2))
 + (((a*d*(2*d*e - 5*c*f) + b*c*(2*d*e + c*f))*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(d*e - c*f)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c))/(d*e - c
*f)^2 - (a^(3/2)*f*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e),
 ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8087 vs. $2(959) = 1918$.

Time = 21.44 (sec) , antiderivative size = 8088, normalized size of antiderivative = 8.13

method	result	size
elliptic	Expression too large to display	8088
default	Expression too large to display	13660

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(9/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx=\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{9/2}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 54*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(6*a**2*c**5*d*e*f + 6*a**2*c**5*d*f**2*x**2 + 30
*a**2*c**4*d**2*e*f*x**2 + 30*a**2*c**4*d**2*f**2*x**4 + 60*a**2*c**3*d**3
*e*f*x**4 + 60*a**2*c**3*d**3*f**2*x**6 + 60*a**2*c**2*d**4*e*f*x**6 + 60*
a**2*c**2*d**4*f**2*x**8 + 30*a**2*c*d**5*e*f*x**8 + 30*a**2*c*d**5*f**2*x
**10 + 6*a**2*d**6*e*f*x**10 + 6*a**2*d**6*f**2*x**12 - 2*a*b*c**6*e*f - 2
*a*b*c**6*f**2*x**2 + 5*a*b*c**5*d*e**2 + a*b*c**5*d*e*f*x**2 - 4*a*b*c**5
*d*f**2*x**4 + 25*a*b*c**4*d**2*e**2*x**2 + 35*a*b*c**4*d**2*e*f*x**4 + 10
*a*b*c**4*d**2*f**2*x**6 + 50*a*b*c**3*d**3*e**2*x**4 + 90*a*b*c**3*d**3*e
*f*x**6 + 40*a*b*c**3*d**3*f**2*x**8 + 50*a*b*c**2*d**4*e**2*x**6 + 100*a*
b*c**2*d**4*e*f*x**8 + 50*a*b*c**2*d**4*f**2*x**10 + 25*a*b*c*d**5*e**2*x*
*8 + 53*a*b*c*d**5*e*f*x**10 + 28*a*b*c*d**5*f**2*x**12 + 5*a*b*d**6*e**2*
x**10 + 11*a*b*d**6*e*f*x**12 + 6*a*b*d**6*f**2*x**14 - 2*b**2*c**6*e*f*x*
*2 - 2*b**2*c**6*f**2*x**4 + 5*b**2*c**5*d*e**2*x**2 - 5*b**2*c**5*d*e*f*x
**4 - 10*b**2*c**5*d*f**2*x**6 + 25*b**2*c**4*d**2*e**2*x**4 + 5*b**2*c**4
*d**2*e*f*x**6 - 20*b**2*c**4*d**2*f**2*x**8 + 50*b**2*c**3*d**3*e**2*x**6
 + 30*b**2*c**3*d**3*e*f*x**8 - 20*b**2*c**3*d**3*f**2*x**10 + 50*b**2*c**
2*d**4*e**2*x**8 + 40*b**2*c**2*d**4*e*f*x**10 - 10*b**2*c**2*d**4*f**2*x*
*12 + 25*b**2*c*d**5*e**2*x**10 + 23*b**2*c*d**5*e*f*x**12 - 2*b**2*c*d**5
*f**2*x**14 + 5*b**2*d**6*e**2*x**12 + 5*b**2*d**6*e*f*x**14),x)*a**2*b...
```

3.111 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 363

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{bx\sqrt{a+bx^2}}{f\sqrt{c+dx^2}} - \frac{b\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(b^2ce-2abcf+a^2df)\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}f(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}(be-af)^2\sqrt{a+bx^2}\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
b*x*(b*x^2+a)^(1/2)/f/(d*x^2+c)^(1/2)-b*c^(1/2)*(b*x^2+a)^(1/2)*EllipticE(
d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/d^(1/2)/f/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(1/2)*(a^2*d*f-2*a*b*c*f+b^2*c*e)
*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/
2))/a/d^(1/2)/f/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
-c^(3/2)*(-a*f+b*e)^2*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^
2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/f/(-c*f+d*e)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$

$$-\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(b^2cefE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-be(bde+bcf-2adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\right.\right.}{\sqrt{\frac{b}{a}}def^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b^2*c*e*f*EllipticE[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e*(b*d*e + b*c*f - 2*a*d*f)*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(b*e - a*f)^2*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d*e*f^2*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {420, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*(b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (a^(3/2)*(b*e - a*
f)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]]
, 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c
*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^{2}cef+\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)a^{2}d\,f^{2}-2\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)abdef+\operatorname{EllipticPi}\left(x\sqrt{\right.\right.}{e}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\left(\frac{2b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)a}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\frac{b^{2}\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)e}{f^{2}\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\right.}{}$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e*f+EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d*f^2-2*EllipticPi(x*(-b/a)^
(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*d*e*f+EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d*e^2+2*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a*b*d*e*f-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b^2*c*e*f-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*d*e^2)*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/f^2/d/(-b/a
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)b + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e + c*f*x**2 + d*e*x**2 +
d*f*x**4),x)*b + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 +
 d*e*x**2 + d*f*x**4),x)*a
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}(e+fx^2)} dx$

Optimal result

Integrand size = 33, antiderivative size = 305

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \frac{b\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} - \frac{b\sqrt{c}(be-af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}f^2\sqrt{a+bx^2}\sqrt{c-dx^2}} + \frac{\sqrt{c}(be-af)^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{cf}{de},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}ef^2\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output

```
b*c^(1/2)*(b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-
b*c/a/d)^(1/2))/d^(1/2)/f/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)-b*c^(1/2)*(-a
*f+b*e)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-
b*c/a/d)^(1/2))/d^(1/2)/f^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)+c^(1/2)*(-a*f
+b*e)^2*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2),-
c*f/d/e,(-b*c/a/d)^(1/2))/d^(1/2)/e/f^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\left(-b^2cefE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-be(bde-bcf-2adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.\right.}{\sqrt{\frac{b}{a}}def^2\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c - d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*(-(b^2*c*e*f*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], -((a*d)/(b*c))]) - b*e*(b*d*e - b*c*f - 2*a*d*f)*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] + d*(b*e - a*f)^2*EllipticPi[(
a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(Sqrt[b/a]*d*e*f^2*S
qrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {420, 331, 330, 327, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{ 331}$$

$$\frac{b\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{b\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{327}$$

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}-\frac{a^{3/2}\sqrt{c-dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}+1\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}}$$

input

```
Int[(a + b*x^2)^(3/2)/(Sqrt[c - d*x^2]*(e + f*x^2)),x]
```

output

```
(b*Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x
)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*f*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2
]) - (a^(3/2)*(b*e - a*f)*Sqrt[c - d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcT
an[(Sqrt[b]*x)/Sqrt[a]], 1 + (a*d)/(b*c)])/(Sqrt[b]*c*e*f*Sqrt[a + b*x^2]*
Sqrt[(a*(c - d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.98

method	result
default	$\frac{\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)abef-\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b^2e^2+\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)abef+\operatorname{EllipticPi}\left(x\sqrt{\frac{d}{c}},-\frac{cf}{de},\frac{\sqrt{-\frac{b}{a}}}{\sqrt{\frac{d}{c}}}\right)a\cdots}{e\sqrt{\frac{d}{c}}\,f^2(-bd\,x^4-ad\,x\cdots}$
elliptic	$\frac{\sqrt{(-x^2d+c)(b\,x^2+a)}\left(\frac{b\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)a}{f\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}-\frac{b^2\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)e}{f^2\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}-\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*b*e*f-EllipticF(x*(1/c*d)^(
1/2),(-b*c/a/d)^(1/2))*b^2*e^2+EllipticE(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))
*a*b*e*f+EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(-b/a)^(1/2)/(1/c*d)^(1/2))*a
^2*f^2-2*EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(-b/a)^(1/2)/(1/c*d)^(1/2))*a
*b*e*f+EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(-b/a)^(1/2)/(1/c*d)^(1/2))*b^2
*e^2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(-d*x^2+c)^(1/2)*(b*x^2+a)^
(1/2)/e/(1/c*d)^(1/2)/f^2/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(-d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c - d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima"
)
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(-d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(-d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{\sqrt{c-d\,x^2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c - d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c - d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx=\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-df\,x^4+cf\,x^2-de\,x^2+ce}dx\right)b+\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-df\,x^4+cf\,x^2-de\,x^2+ce}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e + c*f*x**2 - d*e*x**2 -
d*f*x**4),x)*b + int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 -
 d*e*x**2 - d*f*x**4),x)*a
```

3.113 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 314

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{a}b^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{df\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}\sqrt{b}(bde+bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{df^2\sqrt{a-bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{a}(be+af)^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
a^(1/2)*b^(3/2)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1
/2),(-a*d/b/c)^(1/2))/d/f/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-a^(1/2)*b^(1/
2)*(2*a*d*f+b*c*f+b*d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(
1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/d/f^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+a^
(1/2)*(a*f+b*e)^2*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x
/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/f^2/(-b*x^2+a)^(1/2)/(d*x^2+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(b^2cefE\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-be(bde+bcf+2adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\right.\right.\right.}{\sqrt{-\frac{b}{a}}def^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a - b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b^2*c*e*f*EllipticE[I*ArcSi
nh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - b*e*(b*d*e + b*c*f + 2*a*d*f)*Ellipt
icF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + d*(b*e + a*f)^2*EllipticP
i[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]))/(Sqrt[-(b/a
)]*d*e*f^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {420, 326, 323, 323, 321, 331, 330, 327, 415, 323, 323, 321, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 420$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}}dx}{f}$$

$$\Big\downarrow \; 326$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{f}$$

$$\Big\downarrow \text{323}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{f}$$

$$\Big\downarrow \text{323}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{f}$$

$$\Big\downarrow \text{321}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{f}$$

$$\Big\downarrow \text{331}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$$\Big\downarrow \text{ 415}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{f}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$\downarrow$ 413

$$\frac{(af+be)\left(\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$\downarrow$ 413

$$\frac{(af+be)\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

$\downarrow$ 412

$$\frac{(af+be)\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{f}-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{f}$$

input
```
Int[(a - b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
-((b*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*
x^2)/c])) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a -
 b*x^2]*Sqrt[c + d*x^2])))/f) + ((b*e + a*f)*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])) + (Sqrt[a]*(b*e + a*f)*S
qrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), ArcSin[(
Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*e*f*Sqrt[a - b*x^2]*Sqrt[c
+ d*x^2])))/f
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(-2\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)abdef-\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)b^{2}cef-\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)b^{2}de^{2}+\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(-bx^{2}+a)(x^{2}d+c)}\left(-\frac{2b\sqrt{1-\frac{bx^{2}}{a}}\sqrt{1+\frac{dx^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)a}{f\sqrt{\frac{b}{a}}\sqrt{-bdx^{4}+adx^{2}-x^{2}bc+ac}}-\frac{b^{2}\sqrt{1-\frac{bx^{2}}{a}}\sqrt{1+\frac{dx^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)e}{f^{2}\sqrt{\frac{b}{a}}\sqrt{-bdx^{4}+adx^{2}-x^{2}bc+ac}}-\right.}{}$

input
```
int((-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-2*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*d*e*f-EllipticF(x*(b/a)^
(1/2),(-a*d/b/c)^(1/2))*b^2*c*e*f-EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)
)*b^2*d*e^2+EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c*e*f+EllipticPi
(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a^2*d*f^2+2*EllipticPi
(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b*d*e*f+EllipticPi(x
*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b^2*d*e^2)*((d*x^2+c)/c)
^(1/2)*((-b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/e/f^2/d/(b/a)
^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima"
)
```

output
```
integrate((-b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int((a - b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a - b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)b+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)a$$

input
```
int((-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
- int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(c*e + c*f*x**2 + d*e*x**2
 + d*f*x**4),x)*b + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(c*e + c*f*x**
2 + d*e*x**2 + d*f*x**4),x)*a
```

3.114 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 319

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = -\frac{\sqrt{a}b^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{df\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$
$$-\frac{\sqrt{a}\sqrt{b}(bde-bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{df^2\sqrt{a-bx^2}\sqrt{c-dx^2}}$$
$$+\frac{\sqrt{a}(be+af)^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef^2\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

output
```
-a^(1/2)*b^(3/2)*(1-b*x^2/a)^(1/2)*(-d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2),(a*d/b/c)^(1/2))/d/f/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)-a^(1/2)*b^(1
/2)*(2*a*d*f-b*c*f+b*d*e)*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(b^
(1/2)*x/a^(1/2),(a*d/b/c)^(1/2))/d/f^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)+a
^(1/2)*(a*f+b*e)^2*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*
x/a^(1/2),-a*f/b/e,(a*d/b/c)^(1/2))/b^(1/2)/e/f^2/(-b*x^2+a)^(1/2)/(-d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.65

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\left(-b^2cefE\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-be(bde-bcf+2adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\right.\right.\right.}{\sqrt{-\frac{b}{a}}def^2\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(a - b*x^2)^(3/2)/(Sqrt[c - d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*(-(b^2*c*e*f*EllipticE[I*Arc
Sinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)]) - b*e*(b*d*e - b*c*f + 2*a*d*f)*Ellipt
icF[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)] + d*(b*e + a*f)^2*EllipticPi[-
((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)]))/(Sqrt[-(b/a)]*d*e
*f^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {420, 331, 330, 327, 415, 323, 323, 321, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{420}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}-\frac{b\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{f}$$

$$\Big\downarrow \text{331}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}-\frac{b\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}-\frac{b\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{(af+be)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}(fx^2+e)}dx}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{415}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{f}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{a-bx^2}\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$\Big\downarrow$ 321

$$\frac{(af+be)\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}(fx^2+e)}dx}{f}-\frac{b\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{a-bx^2}\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$\Big\downarrow$ 413

$$\frac{(af+be)\left(\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(fx^2+e)}dx}{f\sqrt{a-bx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{a-bx^2}\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$\Big\downarrow$ 413

$$\frac{(af+be)\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(fx^2+e)}dx}{f\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{a-bx^2}\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

$\Big\downarrow$ 412

$$\frac{(af+be)\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(af+be)\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{a-bx^2}\sqrt{c-dx^2}}\right)}{f}-\frac{b\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

input
```
Int[(a - b*x^2)^(3/2)/(Sqrt[c - d*x^2]*(e + f*x^2)),x]
```

output
```
-((b*Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]
*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]
)) + ((b*e + a*f)*(-((b*Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*f*Sqrt[a - b*x
^2]*Sqrt[c - d*x^2])) + (Sqrt[a]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt[b]*x)/Sqrt[a]], (a*d)/(
b*c)])/(Sqrt[b]*e*f*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])))/f
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)abef-\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b^2e^2-\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)abef+\operatorname{EllipticPi}\left(x\sqrt{\frac{d}{c}},-\frac{cf}{de},\frac{\sqrt{\frac{b}{a}}}{\sqrt{\frac{d}{c}}}\right)a^2f^2+\right.}{e\sqrt{\frac{d}{c}}\,f^2(bd\,x^4-ad\,x^2-}$
elliptic	$\frac{\sqrt{(-x^2d+c)(-b\,x^2+a)}\left(-\frac{b\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)a}{f\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-x^2bc+ac}}-\frac{b^2\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right.}{f^2\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-x^2bc+ac}}\right.}{}$

input
```
int((-b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*b*e*f-EllipticF(x*(1/c*d)^(
1/2),(b*c/a/d)^(1/2))*b^2*e^2-EllipticE(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a
*b*e*f+EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(b/a)^(1/2)/(1/c*d)^(1/2))*a^2*
f^2+2*EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(b/a)^(1/2)/(1/c*d)^(1/2))*a*b*e
*f+EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(b/a)^(1/2)/(1/c*d)^(1/2))*b^2*e^2)
*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/
2)/e/(1/c*d)^(1/2)/f^2/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(-d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((a - b*x**2)**(3/2)/(sqrt(c - d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(sqrt(-d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/(sqrt(-d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{\sqrt{c-d\,x^2}\,(f\,x^2+e)}\,dx$$

input
```
int((a - b*x^2)^(3/2)/((c - d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a - b*x^2)^(3/2)/((c - d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{c-dx^2}\,(e+fx^2)}\,dx = -\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-df\,x^4+cf\,x^2-de\,x^2+ce}dx\right)b+\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-df\,x^4+cf\,x^2-de\,x^2+ce}dx\right)a$$

input
```
int((-b*x^2+a)^(3/2)/(-d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
- int((sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(c*e + c*f*x**2 - d*e*x**2
 - d*f*x**4),x)*b + int((sqrt(c - d*x**2)*sqrt(a - b*x**2))/(c*e + c*f*x**
2 - d*e*x**2 - d*f*x**4),x)*a
```

3.115 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 212

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/c/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)-a^(3/2)*f*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(
1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{af}{be},i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}e\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[-a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)\sqrt{\frac{x^{2}d+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{x^{2}d+c}\,\sqrt{b\,x^{2}+a}}{e\sqrt{-\frac{b}{a}}\,(bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac)}$	118
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}$	133

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
 a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
```

3.116 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 342

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = -\frac{d^{3/2}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{d}(bde-2bcf+adf)\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}f^2\sqrt{a+bx^2}\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-d^(3/2)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-
b*c/a/d)^(1/2))/c^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+c^(1/2)*d^(1/2)*(a*d*f-2*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/(-a*d+b*c)/
(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^2*(
b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1
-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \frac{-\sqrt{\frac{b}{a}}d^2ex(a+bx^2)-ibcde\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right.\right.}{\sqrt{\frac{b}{a}}c(-bc+ad)e}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(-(Sqrt[b/a]*d^2*e*x*(a + b*x^2)) - I*b*c*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) +
 a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*(-(b*c) + a*d)*e*(-(d*e)
+ c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {421, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)}dx$$

$$\Big\downarrow \ 421$$

$$\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 400$$

$$\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 313$$

$$\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 320$$

$$\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 414$$

$$\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d
]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(d*
e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*(d*e - c*f)^2*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.89

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, b\, d^{2} e\, x^{3}+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b c d e+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticPi}\left(x \sqrt{-\frac{b}{a}}, \frac{a f}{b e}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right) a c d f-\sqrt{\cdots}\right.}{c(ad-bc)e\sqrt{-\frac{b}{a}}\,(cf-de)(bd\, x^{4}+ad\, x^{2}+x^{2}bc+\cdots}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(bd\, x^{2}+ad\right) dx}{c(ad-bc)(cf-de)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\, x^{2}+ad\right)}}-\frac{\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right) d}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2}bc+ac}\, c(cf-de)}+\frac{\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d \cdots}{}}}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x \cdots}}\right.}{\cdots}$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*
c*d*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c^2*f-(-b/a)^(1/2)*a*d^2*e*x)*(d*x^2+
c)^(1/2)*(b*x^2+a)^(1/2)/c/(a*d-b*c)/e/(-b/a)^(1/2)/(c*f-d*e)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{}}{b\,d^2f\,x^8+a\,d^2f\,x^6+2bcdf\,x^6+b\,d^2e\,x^6+2acdf\,x^4+a\,d^2e\,x^4}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2*e + a*c**2*f*x**2 + 2*a*c*
d*e*x**2 + 2*a*c*d*f*x**4 + a*d**2*e*x**4 + a*d**2*f*x**6 + b*c**2*e*x**2
+ b*c**2*f*x**4 + 2*b*c*d*e*x**4 + 2*b*c*d*f*x**6 + b*d**2*e*x**6 + b*d**2
*f*x**8),x)
```

3.117 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 481

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = -\frac{d^2x\sqrt{a+bx^2}}{3c(bc-ad)(de-cf)\,(c+dx^2)^{3/2}}$$

$$-\frac{d^{3/2}(bc(4de-7cf)-ad(2de-5cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{d}(3a^2cd^2f^2+3b^2c(d^2e^2-3cdef+3c^2f^2)-abd(d^2e^2-5cdef+10c^2f^2))\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{3a\sqrt{c}(bc-ad)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$-\frac{c^{3/2}f^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*d^2*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*d^(
3/2)*(b*c*(-7*c*f+4*d*e)-a*d*(-5*c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(
1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/(-a*d+b*c)^2/(
-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*d^(1/2)*(3
*a^2*c*d^2*f^2+3*b^2*c*(3*c^2*f^2-3*c*d*e*f+d^2*e^2)-a*b*d*(10*c^2*f^2-5*c
*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2))
,(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*f^3*(b*x^2+a)^(1/2)*EllipticPi(d^(1/
2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-
c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.16

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(-5*a*b*Sqrt[b/a]*c^2*d^3*e^2*x + 3*a^2*Sqrt[b/a]*c*d^4*e^2*x + 8*a*b*Sqrt
[b/a]*c^3*d^2*e*f*x - 6*a^2*Sqrt[b/a]*c^2*d^3*e*f*x - 5*a*b*(b/a)^(3/2)*c^
2*d^3*e^2*x^3 - a*b*Sqrt[b/a]*c*d^4*e^2*x^3 + 2*a^2*Sqrt[b/a]*d^5*e^2*x^3
+ 8*a*b*(b/a)^(3/2)*c^3*d^2*e*f*x^3 + a*b*Sqrt[b/a]*c^2*d^3*e*f*x^3 - 5*a^
2*Sqrt[b/a]*c*d^4*e*f*x^3 - 4*a*b*(b/a)^(3/2)*c*d^4*e^2*x^5 + 2*a*b*Sqrt[b
/a]*d^5*e^2*x^5 + 7*a*b*(b/a)^(3/2)*c^2*d^3*e*f*x^5 - 5*a*b*Sqrt[b/a]*c*d^
4*e*f*x^5 - I*b*c*d*e*(b*c*(4*d*e - 7*c*f) + a*d*(-2*d*e + 5*c*f))*Sqrt[1
+ (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] + I*b*c*d*(-(b*c) + a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (b*x^
2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - (3*I)*b^2*c^5*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (6*I)*a*b*c^4*
d*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a^2*c^3*d^2*f^2*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] - (3*I)*b^2*c^4*d*f^2*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (6*I)*a*b
*c^3*d^2*f^2*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/
(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a^2*c^2*d^3*f^2*x^2*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh...
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {421, 25, 402, 25, 400, 313, 320, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int \frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+$$
$$\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{d\left(-\frac{\frac{b(ad(de-4cf)-3bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}\right)}{(de-cf)^2}$$
$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx}{3c(}\right.}{(de-cf)^2}$$

$\downarrow$ 413

$$\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{\sqrt{a+bx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx}{3c(}\right.}{(de-cf)^2}$$

$\downarrow$ 413

$$\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx}{3c(}\right.}{(de-cf)^2}$$

$\downarrow$ 412

$$\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+$$

$$\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx}{3c(}\right.}{(de-cf)^2}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(d*(-1/3*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2
)) - ((Sqrt[d]*(b*c*(4*d*e - 7*c*f) - a*d*(2*d*e - 5*c*f))*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c -
a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*(
a*d*(d*e - 4*c*f) - 3*b*c*(d*e - 2*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d))))/(d*e - c*
f)^2 + (Sqrt[-a]*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a
*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*(d*e - c
*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1817 vs. $2(457) = 914$.

Time = 17.82 (sec) , antiderivative size = 1818, normalized size of antiderivative = 3.78

method	result	size
default	Expression too large to display	1818
elliptic	Expression too large to display	2008

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2)
,a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^3*d^2*f^2+6*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b
/a)^(1/2))*a*b*c^3*d^2*f^2*x^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2*d^3*
f^2*x^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2
),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c^4*d*f^2*x^2+6*(-b/a)^(1/2)*a^
2*c^2*d^3*e*f*x+6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b
/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c^4*d*f^2-3*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1
/2)/(-b/a)^(1/2))*b^2*f^2*c^5+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3*d^2*e^2-4*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3*d^
2*e^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*a*b*c^2*d^3*e*f*x^2-5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d^3*e*f*x^2-((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*
b*c*d^4*e^2*x^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*b^2*c^3*d^2*e*f*x^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d^4*e^2*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{}{b\,d^3f\,x^{10}+a\,d^3f\,x^8+3bc\,d^2f\,x^8+b\,d^3e\,x^8+3ac\,d^2f\,x^6+a\,d^3}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**3*e + a*c**3*f*x**2 + 3*a*c*
*2*d*e*x**2 + 3*a*c**2*d*f*x**4 + 3*a*c*d**2*e*x**4 + 3*a*c*d**2*f*x**6 +
a*d**3*e*x**6 + a*d**3*f*x**8 + b*c**3*e*x**2 + b*c**3*f*x**4 + 3*b*c**2*d
*e*x**4 + 3*b*c**2*d*f*x**6 + 3*b*c*d**2*e*x**6 + 3*b*c*d**2*f*x**8 + b*d*
*3*e*x**8 + b*d**3*f*x**10),x)
```

3.118 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 342

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^{3/2}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}\sqrt{b}(bde+bcf-2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c(bc-ad)(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
b^(3/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/a^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-a^(1/2)*b^(1/2)*(-2*a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/
(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*f^2*(
d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1
-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}\left(ab\left(\frac{b}{a}\right)^{3/2}cex + ab\left(\frac{b}{a}\right)^{3/2}dex^3 + ib^2ce\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(a*b*(b/a)^(3/2)*c*e*x + a*b*(b/a)^(3/2)*d*e*x^3 + I*b^2*c*e*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + I*b*(-(b*c) + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*b*c*f*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - I*a^2*d*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*(b*c - a*d)*e*(b*e
 - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {421, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2} - \frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*((Sqrt[b]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt
[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c +
d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(
b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(b*e -
 a*f)^2 + (a^(3/2)*f^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[
(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(b*e - a*f)^2*Sqrt[a
+ b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 8.44 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, b^{2} d e\, x^{3}-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right) a b d e+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right) b^{2} c e-\sqrt{\frac{b\,x^{2}+a}{a}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2} d+c)}\left(\frac{\left(b d\,x^{2}+b c\right) b x}{a(a d-b c)(a f-b e)\sqrt{\left(x^{2}+\frac{a}{b}\right)\left(b d\,x^{2}+b c\right)}}-\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right) b}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}+x^{2} b c+a c}\, a(a f-b e)}-\frac{b^{2} c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+}}{a(a d-b c)(a f-}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b^2*d*e*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*d*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e-((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e+
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*f+
(-b/a)^(1/2)*b^2*c*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2)/a/(
a*d-b*c)/(a*f-b*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4
 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)
```

3.119 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 447

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx &= \frac{b^2x}{a(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{d}\left(abd^2e-a^2d^2f+b^2c(de-cf)\right)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{\sqrt{c}d^{3/2}(2bde-3bcf+adf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{c^{3/2}f^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+d^(1/2)*(a*b
*d^2*e-a^2*d^2*f+b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/(-a*f+b*e
)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(1/2)*d^(3/
2)*(a*d*f-3*b*c*f+2*b*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*
x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/(-a*d+b*c)^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*f^3*(b*x^2+a)^(1/2)*EllipticPi(d^(
1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/
(-a*f+b*e)/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.21 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{\frac{bex(a^3d^3f-ab^2d^3ex^2+b^3c(-de+cf)(c+dx^2)+a^2bd^3(-e+fx^2))}{a}+ib\sqrt{\frac{b}{a}}ce(-a$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((b*e*x*(a^3*d^3*f - a*b^2*d^3*e*x^2 + b^3*c*(-(d*e) + c*f)*(c + d*x^2) +
a^2*b*d^3*(-e + f*x^2)))/a + I*b*Sqrt[b/a]*c*e*(-(a*b*d^2*e) + a^2*d^2*f +
 b^2*c*(-(d*e) + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*b*(b/a)^(3/2)*c*(-(b*c) + a*d)*e
*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*Sqrt[b/a]*c*(b*c - a*d)^2*f^2*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])/(b*c*(b*c - a*d)^2*e*(b*e - a*f)*(-(d*e) + c*f)*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {421, 25, 402, 400, 313, 320, 416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{a(bde+bcf-2adf)-bd(be-af)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(-2a^2df+abde+b^2ce\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2df+abde+b^2ce\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 320

$$\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2df+abde+b^2ce\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 416

$$\frac{f^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2df+abde+b^2ce\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 313

$$\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(-2a^2df+abde+b^2ce)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 414

$$\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(-2a^2df+abde+b^2ce)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{a(bc-ad)}\right.}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e - a*f)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (-
((Sqrt[d]*(b^2*c*e + a*b*d*e - 2*a^2*d*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(2*b*d*e + b*c*f
- 3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2]))/(a*(b*c - a*d))))/(b*e - a*f)^2 + (f^2*((Sqrt[d]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(d*e
 - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (a^(3/2)*
f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]],
 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))])))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(429) = 858$.

Time = 18.02 (sec) , antiderivative size = 956, normalized size of antiderivative = 2.14

method	result
default	$\left(-\sqrt{-\frac{b}{a}}\, a^{2} b\, d^{3} e f\, x^{3}+\sqrt{-\frac{b}{a}}\, a\, b^{2} d^{3} e^{2} x^{3}-\sqrt{-\frac{b}{a}}\, b^{3} c^{2} d e f\, x^{3}+\sqrt{-\frac{b}{a}}\, b^{3} c\, d^{2} e^{2} x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) a\, b^{2} c$
elliptic	Expression too large to display

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*a^2*b*d^3*e*f*x^3+(-b/a)^(1/2)*a*b^2*d^3*e^2*x^3-(-b/a)^(1/
2)*b^3*c^2*d*e*f*x^3+(-b/a)^(1/2)*b^3*c*d^2*e^2*x^3+((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*e*
f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a*b^2*c*d^2*e^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*e*f+((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^2*d*e^2+((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b*c*d^2*e*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c*d^2*e^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*e*f-((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*
c^2*d*e^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2
),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*c*d^2*f^2-2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/
a)^(1/2))*a^2*b*c^2*d*f^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
Pi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b^2*c^3*f^2-(-b/a
)^(1/2)*a^3*d^3*e*f*x+(-b/a)^(1/2)*a^2*b*d^3*e^2*x-(-b/a)^(1/2)*b^3*c^3*e*
f*x+(-b/a)^(1/2)*b^3*c^2*d*e^2*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/a/(a*d
-b*c)^2/e/(-b/a)^(1/2)/(a*f-b*e)/(c*f-d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{}{b^2d^2f\,x^{10}+2ab\,d^2f\,x^8+2b^2cdf\,x^8+b^2d^2e\,x^8+a^2d^2f\,x^6+4}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2*e + a**2*c**2*f*x**2 +
2*a**2*c*d*e*x**2 + 2*a**2*c*d*f*x**4 + a**2*d**2*e*x**4 + a**2*d**2*f*x**
6 + 2*a*b*c**2*e*x**2 + 2*a*b*c**2*f*x**4 + 4*a*b*c*d*e*x**4 + 4*a*b*c*d*f
*x**6 + 2*a*b*d**2*e*x**6 + 2*a*b*d**2*f*x**8 + b**2*c**2*e*x**4 + b**2*c*
*2*f*x**6 + 2*b**2*c*d*e*x**6 + 2*b**2*c*d*f*x**8 + b**2*d**2*e*x**8 + b**
2*d**2*f*x**10),x)
```

3.120 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 658

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx &= -\frac{d^2x}{3c(bc-ad)(de-cf)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}} \\
&+ \frac{b(abd^2e-a^2d^2f+3b^2c(de-cf))\,x}{3ac(bc-ad)^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{d}(ab^2cd^2e(7de-10cf)+a^3d^3f(2de-5cf)+3b^3c^2(de-cf)^2-2a^2bd^2(d^2e^2+cdef-5c^2f^2))\sqrt{a+b}}{3ac^{3/2}(bc-ad)^3(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&- \frac{d^{3/2}(3a^2cd^2f^2+3b^2c(3d^2e^2-8cdef+6c^2f^2)-abd(d^2e^2-8cdef+13c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\text{arcta}\right.}{3a\sqrt{c}(bc-ad)^3(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&+ \frac{c^{3/2}f^4\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

output
```
-1/3*d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*b*(
a*b*d^2*e-a^2*d^2*f+3*b^2*c*(-c*f+d*e))*x/a/c/(-a*d+b*c)^2/(-a*f+b*e)/(-c*
f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*d^(1/2)*(a*b^2*c*d^2*e*(-10*c*f
+7*d*e)+a^3*d^3*f*(-5*c*f+2*d*e)+3*b^3*c^2*(-c*f+d*e)^2-2*a^2*b*d^2*(-5*c^
2*f^2+c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(3/2)/(-a*d+b*c)^3/(-a*f+b*e)/(-c*f+d*e
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*d^(3/2)*(3*a^2*c*d
^2*f^2+3*b^2*c*(6*c^2*f^2-8*c*d*e*f+3*d^2*e^2)-a*b*d*(13*c^2*f^2-8*c*d*e*f
+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*
c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^3/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^4*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c
^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-a*f+b*
e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.37 (sec) , antiderivative size = 1660, normalized size of antiderivative = 2.52

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*((-I)*b*c*e*(-3*b^3*c^2*(d*e - c*f)^2 + a^3*d^3*f*(-2*d*e + 5*c
*f) + a*b^2*c*d^2*e*(-7*d*e + 10*c*f) + 2*a^2*b*d^2*(d^2*e^2 + c*d*e*f - 5
*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (Sqrt[b/a]*(3*b^5*c^4*d^2*e^3*x + 8*
a^2*b^3*c^2*d^4*e^3*x - 3*a^3*b^2*c*d^5*e^3*x - 6*b^5*c^5*d*e^2*f*x - 11*a
^2*b^3*c^3*d^3*e^2*f*x - 2*a^3*b^2*c^2*d^4*e^2*f*x + 3*a^4*b*c*d^5*e^2*f*x
 + 3*b^5*c^6*e*f^2*x + 11*a^3*b^2*c^3*d^3*e*f^2*x - 6*a^4*b*c^2*d^4*e*f^2*
x + 6*b^5*c^3*d^3*e^3*x^3 + 8*a*b^4*c^2*d^4*e^3*x^3 + 4*a^2*b^3*c*d^5*e^3*
x^3 - 2*a^3*b^2*d^6*e^3*x^3 - 12*b^5*c^4*d^2*e^2*f*x^3 - 11*a*b^4*c^3*d^3*
e^2*f*x^3 - 12*a^2*b^3*c^2*d^4*e^2*f*x^3 + a^3*b^2*c*d^5*e^2*f*x^3 + 2*a^4
*b*d^6*e^2*f*x^3 + 6*b^5*c^5*d*e*f^2*x^3 + 11*a^2*b^3*c^3*d^3*e*f^2*x^3 +
4*a^3*b^2*c^2*d^4*e*f^2*x^3 - 5*a^4*b*c*d^5*e*f^2*x^3 + 3*b^5*c^2*d^4*e^3*
x^5 + 7*a*b^4*c*d^5*e^3*x^5 - 2*a^2*b^3*d^6*e^3*x^5 - 6*b^5*c^3*d^3*e^2*f*
x^5 - 10*a*b^4*c^2*d^4*e^2*f*x^5 - 2*a^2*b^3*c*d^5*e^2*f*x^5 + 2*a^3*b^2*d
^6*e^2*f*x^5 + 3*b^5*c^4*d^2*e*f^2*x^5 + 10*a^2*b^3*c^2*d^4*e*f^2*x^5 - 5*
a^3*b^2*c*d^5*e*f^2*x^5 + I*a*b*Sqrt[b/a]*c*(-(b*c) + a*d)*e*(-(d*e) + c*f
)*(-(a*b*d^2*e) + a^2*d^2*f + 3*b^2*c*(-(d*e) + c*f))*Sqrt[1 + (b*x^2)/a]*
(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] + ((3*I)*b^5*c^6*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(b/a)^(3/2) - (9*...
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.29, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {421, 25, 402, 402, 400, 313, 320, 421, 25, 401, 25, 27, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\int \frac{a(bde+bcf-2adf)-3bd(be-af)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\int \frac{a\left(3c(2de+cf)b^2-ad(2de+11cf)b+4a^2d^2f\right)-bd\left(-2dfa^2+b(de-2cf)a+3b^2ce\right)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}\left(-2a^2df+ab(de-\cdots\right.}{3c\left(c+dx^2\right)^{3/2}(bc-\cdots}}{a(bc-ad)}\right.}{(be-af)^2}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{ab\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(4a^3d^2f-a^2bd(13cf+2de)+ab^2c(cf+7de)+3b^3\cdots\right.}{bc-ad}}{3c(bc-ad)}}{a(bc-ad)}\right.}{(be-af)^2}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{313}$$

$$b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{ab\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(13cf+2de)+ab^2c(cf\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}}{a(bc-ad)}\right.$$

$$(be-af)^2$$

$$\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 320

$$\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}+$$

$$b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.$$

$$(be-af)^2$$

$\downarrow$ 421

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.$$

$$(be-af)^2$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 401

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int-\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 27

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int\frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 400

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\ldots\right.}{\ldots}}{3c(bc-ad)}}{a(bc-ad)\ldots}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{f^2\left(\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\ldots\right.}\ldots}{(de-cf)^2}\right)}{(be-af)^2}$$

$$\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\ldots\right.}{\ldots}}{3c(bc-ad)}}{a(bc-ad)\ldots}\right)}{(be-af)^2}$$

$\downarrow$ 320

$$f^2\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}\right.}{(de-cf)^2}\right.$$

$$\overline{(be-af)^2}$$

$$b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}\left(2a^2d^2f-abd(13cf+de)+3b^2c(cf+3de)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(4a^3d^2f-a^2bd(\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right.$$

$$\overline{(be-af)^2}$$

$\downarrow$ 414

$$\frac{\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-}{}}{3c}\right)}{(de-cf)^2}\right)}{(be-af)^2}$$

$$\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}(dx^2+c)^{3/2}}-\frac{\frac{\frac{b\sqrt{c}\left(3c(3de+cf)b^2-ad(de+13cf)b+2a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}\left(4d^2fa^3-bd(2de+13cf)\right.}{}}{3c(bc-ad)}}{a(bc-ad)}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e - a*f)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) -
(-1/3*(d*(3*b^2*c*e - 2*a^2*d*f + a*b*(d*e - 2*c*f))*x*Sqrt[a + b*x^2])/(c
*(b*c - a*d)*(c + d*x^2)^(3/2)) + (-((Sqrt[d]*(3*b^3*c^2*e + 4*a^3*d^2*f +
 a*b^2*c*(7*d*e + c*f) - a^2*b*d*(2*d*e + 13*c*f))*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(2*a^2*d
^2*f + 3*b^2*c*(3*d*e + c*f) - a*b*d*(d*e + 13*c*f))*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d)))/
(a*(b*c - a*d))))/(b*e - a*f)^2 + (f^2*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2]
)/(3*c*(c + d*x^2)^(3/2)) + (-(((a*d*(2*d*e - 5*c*f) - b*c*(d*e - 4*c*f))*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])) + (b*Sqrt[c]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c)))/(d*e - c*f)^2 + (a^(3/2)*f^2*
Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1
 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))])))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2754 vs. 2(628) = 1256.

Time = 20.36 (sec) , antiderivative size = 2755, normalized size of antiderivative = 4.19

method	result	size
elliptic	Expression too large to display	2755
default	Expression too large to display	4117

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c*d/(a*d
-b*c)*x/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(x^2+c/d)^2-1/3*(b*d*x^2+a*d)/c^2*d^2/(a*d-b*c)^2*x*(5*a*c*d*f-2*a*d^2
*e-10*b*c^2*f+7*b*c*d*e)/(c*f-d*e)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/((x^2
+c/d)*(b*d*x^2+a*d))^(1/2)+(b*d*x^2+b*c)*b^3/a/(a*d-b*c)^3*x/(a*f-b*e)/((x
^2+a/b)*(b*d*x^2+b*c))^(1/2)+3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))*a*d^4/c/(a*d-b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e-b*c^2*f+
b*c*d*e)*b*e+5/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^3*b/(a*d-b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e-b
*c^2*f+b*c*d*e)*a*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3
/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*d^4*b/(a*d-b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e
)*a*e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/3/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/c*d^2/(a*d-b*c)*b/(a*c*d*
f-a*d^2*e-b*c^2*f+b*c*d*e)-5/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))*d^3/(a*d-b*c)/c/(c*f-d*e)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*
d*e)*a*f+2/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{5}{2}}(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(5/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{}{b^2d^3f\,x^{12}+2ab\,d^3f\,x^{10}+3b^2c\,d^2f\,x^{10}+b^2d^3e\,x^{10}+a^2d^3f\,x}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**3*e + a**2*c**3*f*x**2 +
3*a**2*c**2*d*e*x**2 + 3*a**2*c**2*d*f*x**4 + 3*a**2*c*d**2*e*x**4 + 3*a**
2*c*d**2*f*x**6 + a**2*d**3*e*x**6 + a**2*d**3*f*x**8 + 2*a*b*c**3*e*x**2
+ 2*a*b*c**3*f*x**4 + 6*a*b*c**2*d*e*x**4 + 6*a*b*c**2*d*f*x**6 + 6*a*b*c*
d**2*e*x**6 + 6*a*b*c*d**2*f*x**8 + 2*a*b*d**3*e*x**8 + 2*a*b*d**3*f*x**10
 + b**2*c**3*e*x**4 + b**2*c**3*f*x**6 + 3*b**2*c**2*d*e*x**6 + 3*b**2*c**
2*d*f*x**8 + 3*b**2*c*d**2*e*x**8 + 3*b**2*c*d**2*f*x**10 + b**2*d**3*e*x*
*10 + b**2*d**3*f*x**12),x)
```

3.121 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 480

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^2x\sqrt{c+dx^2}}{3a(bc-ad)(be-af)\,(a+bx^2)^{3/2}}$$
$$+\frac{b^{3/2}(2b^2ce-4abde-5abcf+7a^2df)\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{\sqrt{b}(b^3cde^2-9a^3d^2f^2+a^2bdf(9de+10cf)-ab^2(3d^2e^2+5cdef+3c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{3\sqrt{a}c(bc-ad)^2(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}f^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*b^2*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/3*b^(3
/2)*(7*a^2*d*f-5*a*b*c*f-4*a*b*d*e+2*b^2*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/(-a*d+b*c)^2/
(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*b^(1/2)*(
b^3*c*d*e^2-9*a^3*d^2*f^2+a^2*b*d*f*(10*c*f+9*d*e)-a*b^2*(3*c^2*f^2+5*c*d*
e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/a^(1/2)/c/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*f^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a
*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.00 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{ab\left(\frac{b}{a}\right)^{3/2}ex(c+dx^2)\left(8a^3df+2b^3cex^2+ab^2(3ce-4dex^2-5cfx\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(a*b*(b/a)^(3/2)*e*x*(c + d*x^2)*(8*a^3*d*f + 2*b^3*c*e*x^2 + a*b^2*(3*c*e
 - 4*d*e*x^2 - 5*c*f*x^2) + a^2*b*(-5*d*e - 6*c*f + 7*d*f*x^2)) + I*b^2*c*
e*(2*b^2*c*e + 7*a^2*d*f - a*b*(4*d*e + 5*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 I*b*(-(b*c) + a*d)*e*(2*b^2*c*e + 6*a^2*d*f - a*b*(3*d*e + 5*c*f))*(a + b
*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - (3*I)*a^2*(b*c - a*d)^2*f^2*(a + b*x^2)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*e*(b*e - a*f)^2*(a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {421, 25, 402, 25, 400, 313, 320, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx\sqrt{c+dx^2}(be-af)}{3a(a+bx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a(a+bx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{\frac{b\left(7a^2df-5abcf-4abde+2b^2ce\right)\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a(a+bx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a(a+bx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}$$

$$\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3a(bc-ad)}+\frac{b}{3a}\right.}{(be-af)^2}$$

$$\Big\downarrow \text{413}$$

$$\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{\sqrt{a+bx^2}(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3a(bc-ad)}+\frac{b}{3a}\right.}{(be-af)^2}$$

$$\Big\downarrow 413$$

$$\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\frac{b}{3a}\right.}{(be-af)^2}$$

$$\Big\downarrow 412$$

$$\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\frac{b}{3a}\right.}{(be-af)^2}$$

$$\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)^2}$$

input

```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output

```
(b*((b*(b*e - a*f)*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2))
+ ((Sqrt[b]*(2*b^2*c*e - 4*a*b*d*e - 5*a*b*c*f + 7*a^2*d*f)*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*(b*c -
 a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sq
rt[d]*(b^2*c*e + 6*a^2*d*f - a*b*(3*d*e + 4*c*f))*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a*(b*c - a*d))))/(b*e - a
*f)^2 + (Sqrt[-a]*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(
a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*(b*e -
a*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(456) = 912$.

Time = 10.93 (sec) , antiderivative size = 1325, normalized size of antiderivative = 2.76

method	result	size
elliptic	Expression too large to display	1325
default	Expression too large to display	2062

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/a/(a*d-b*
c)*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^
2+b*c)*b/a^2/(a*d-b*c)^2*x*(7*a^2*d*f-5*a*b*c*f-4*a*b*d*e+2*b^2*c*e)/(a*f-
b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d*b/(a*d-b*c)/a/(a*f-b*e)-7/3/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/(a*d-b*c)*b/(a*f-b*e)^2*d
*f+5/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/(a*d-
b*c)*b^2/a/(a*f-b*e)^2*c*f+4/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))/(a*d-b*c)*b^2/a/(a*f-b*e)^2*d*e-2/3/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/(a*d-b*c)*b^3/a^2/(a*f-b*e)^2*c
*e-7/3*b^2/(a*d-b*c)^2/(a*f-b*e)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))*d*f+5/3*b^3/a/(a*d-b*c)^2/(a*f-b*e)^2*c^2/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*f+4/3*b^3/a/(a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{}{b^3df\,x^{10}+3a\,b^2df\,x^8+b^3cf\,x^8+b^3de\,x^8+3a^2bdf\,x^6+3a\,b^2cf}$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*e + a**3*c*f*x**2 + a**3*d
*e*x**2 + a**3*d*f*x**4 + 3*a**2*b*c*e*x**2 + 3*a**2*b*c*f*x**4 + 3*a**2*b
*d*e*x**4 + 3*a**2*b*d*f*x**6 + 3*a*b**2*c*e*x**4 + 3*a*b**2*c*f*x**6 + 3*
a*b**2*d*e*x**6 + 3*a*b**2*d*f*x**8 + b**3*c*e*x**6 + b**3*c*f*x**8 + b**3
*d*e*x**8 + b**3*d*f*x**10),x)
```

3.122 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 665

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{b^2x}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$
$$+\frac{b^2(2b^2ce-6abde-5abcf+9a^2df)\,x}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{d}(6a^3bd^3ef-3a^4d^3f^2+2b^4c^2e(de-cf)-ab^3c(7d^2e^2-2cdef-5c^2f^2)-a^2b^2d(3d^2e^2-10cdef+10}{3a^2\sqrt{c}(bc-ad)^3(be-af)^2(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{c}\sqrt{d}(3a^3d^3f^2-3ab^2d^2e(3de-4cf)+6a^2bd^2f(de-2cf)+b^3c(de-cf)^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\text{arctan}\right.}{3a^2(bc-ad)^3(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}f^4\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+1/3*b^2*
(9*a^2*d*f-5*a*b*c*f-6*a*b*d*e+2*b^2*c*e)*x/a^2/(-a*d+b*c)^2/(-a*f+b*e)^2/
(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*d^(1/2)*(6*a^3*b*d^3*e*f-3*a^4*d^3*f^2
+2*b^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-5*c^2*f^2-2*c*d*e*f+7*d^2*e^2)-a^2*b^2*d
*(10*c^2*f^2-10*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^3/(-a*f+
b*e)^2/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*c^(1
/2)*d^(1/2)*(3*a^3*d^3*f^2-3*a*b^2*d^2*e*(-4*c*f+3*d*e)+6*a^2*b*d^2*f*(-2*
c*f+d*e)+b^3*c*(-c*f+d*e)^2)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2
)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/(-a*d+b*c)^3/(-a*f+b*e)/(-c*f+d*e)^2/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^4*(b*x^2+a)^(1/2)
*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2
))/a/d^(1/2)/e/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.85 (sec) , antiderivative size = 1662, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((-I)*b*c*e*(-6*a^3*b*d^3*e*f + 3*a^4*d^3*f^2 + 2*b^4*c^2*e*(-(d*e) + c*f)
 + a*b^3*c*(7*d^2*e^2 - 2*c*d*e*f - 5*c^2*f^2) + a^2*b^2*d*(3*d^2*e^2 - 10
*c*d*e*f + 10*c^2*f^2))*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (Sqrt[b/a]*(3*a*b^6*c^3
*d*e^3*x - 8*a^2*b^5*c^2*d^2*e^3*x - 3*a^4*b^3*d^4*e^3*x - 3*a*b^6*c^4*e^2
*f*x + 2*a^2*b^5*c^3*d*e^2*f*x + 11*a^3*b^4*c^2*d^2*e^2*f*x + 6*a^5*b^2*d^
4*e^2*f*x + 6*a^2*b^5*c^4*e*f^2*x - 11*a^3*b^4*c^3*d*e*f^2*x - 3*a^6*b*d^4
*e*f^2*x + 2*b^7*c^3*d*e^3*x^3 - 4*a*b^6*c^2*d^2*e^3*x^3 - 8*a^2*b^5*c*d^3
*e^3*x^3 - 6*a^3*b^4*d^4*e^3*x^3 - 2*b^7*c^4*e^2*f*x^3 - a*b^6*c^3*d*e^2*f
*x^3 + 12*a^2*b^5*c^2*d^2*e^2*f*x^3 + 11*a^3*b^4*c*d^3*e^2*f*x^3 + 12*a^4*
b^3*d^4*e^2*f*x^3 + 5*a*b^6*c^4*e*f^2*x^3 - 4*a^2*b^5*c^3*d*e*f^2*x^3 - 11
*a^3*b^4*c^2*d^2*e*f^2*x^3 - 6*a^5*b^2*d^4*e*f^2*x^3 + 2*b^7*c^2*d^2*e^3*x
^5 - 7*a*b^6*c*d^3*e^3*x^5 - 3*a^2*b^5*d^4*e^3*x^5 - 2*b^7*c^3*d*e^2*f*x^5
 + 2*a*b^6*c^2*d^2*e^2*f*x^5 + 10*a^2*b^5*c*d^3*e^2*f*x^5 + 6*a^3*b^4*d^4*
e^2*f*x^5 + 5*a*b^6*c^3*d*e*f^2*x^5 - 10*a^2*b^5*c^2*d^2*e*f^2*x^5 - 3*a^4
*b^3*d^4*e*f^2*x^5 - I*a*b^2*Sqrt[b/a]*c*(-(b*c) + a*d)*e*(-(d*e) + c*f)*(
2*b^2*c*e + 9*a^2*d*f - a*b*(6*d*e + 5*c*f))*(a + b*x^2)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3
*I)*a^4*b^3*Sqrt[b/a]*c^4*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (9*I)*a^7*(b...
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {421, 25, 402, 25, 402, 25, 27, 400, 313, 320, 421, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{\int \frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{\frac{d\int \frac{b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{\frac{d\left(\frac{\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-\cdots\right)}{a\sqrt{a+bx^2}\sqrt{c+\cdots}}}{3a(bc-ad)}\cdots\right)}{(be-af)^2}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\cdots}{3a(bc-ad)}\cdots\right)}{(be-af)^2}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 320

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\right.}{\sqrt{d}\sqrt{}}\right)}{a(bc-ad)}}{3a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 421

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\right.}{\sqrt{d}\sqrt{}}\right)}{a(bc-ad)}}{3a(bc-ad)}\right.}{(be-af)^2}$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\cdots\right.}{\sqrt{d}\sqrt{\cdots}}\right)}{a(bc-ad)}}{3a(bc-ad)}\cdots\right)}{(be-af)^2}$$

$\downarrow$ 400

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\cdots\right.}{\sqrt{d}\sqrt{\cdots}}\right)}{a(bc-ad)}}{3a(bc-ad)}\cdots\right)}{(be-af)^2}$$

$\downarrow$ 313

$$\frac{f^2\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\cdots\right.}{\sqrt{d}\sqrt{\cdots}}\right.}{a(bc-ad)}}{3a(bc-ad)}\cdots\right.}{(be-af)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{(be-af)^2}\cdots$$

$$\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\cdots\right.}{\sqrt{d}\sqrt{\cdots}}\right.}{a(bc-ad)}}{3a(bc-ad)}\cdots\right.}{(be-af)^2}$$

$$\Big\downarrow \; 414$$

$$\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\left(15a^2df-7abc\right.}{\sqrt{d}\sqrt{}}\right)}{a(bc-ad)}}{3a(bc-ad)}\right)}{(be-af)^2}$$

$$\frac{f^2\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf}{\sqrt{c}\sqrt{c+dx^2}(}\right)}{(de-cf)^2}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e - a*f)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
 + ((b*(2*b^2*c*e - 6*a*b*d*e - 5*a*b*c*f + 9*a^2*d*f)*x)/(a*(b*c - a*d)*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2]) + (d*(((2*b^3*c^2*e + 6*a^3*d^2*f - a^2*b*d
*(3*d*e - 7*c*f) - a*b^2*c*(7*d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*(b^2*c*e
 - 9*a*b*d*e - 7*a*b*c*f + 15*a^2*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*(b*c - a*d)))/(3*a*(b*c - a*d
))))/(b*e - a*f)^2 + (f^2*((d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e
- 2*b*c*f + a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])))/(d*e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*Ellip
ticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*S
qrt[d]*e*(d*e - c*f)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2])))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. $2(635) = 1270$.

Time = 20.29 (sec) , antiderivative size = 2127, normalized size of antiderivative = 3.20

method	result	size
elliptic	Expression too large to display	2127
default	Expression too large to display	4115

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)/c*d^3/(a*d-b*c)^3*x/(c*f-d*e)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)-1/3*b/a/(a
*d-b*c)*x/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)/(x^2+a/b)^2-1/3*(b*d*x^2+b*c)*b^2/a^2/(a*d-b*c)^2*x*(10*a^2*d*f-5*a*
b*c*f-7*a*b*d*e+2*b^2*c*e)/(a*f-b*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/((x
^2+a/b)*(b*d*x^2+b*c))^(1/2)+10/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))*b^2/(a*d-b*c)/(a*f-b*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)*d*f-5/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)*b^3/(a*d-b*c)/a/(a*f-b*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*c*f-7/3/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^3/(a*d-b*c)/a/
(a*f-b*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*d*e+2/3/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^4/(a*d-b*c)/a^2/(a*f-b*e)/(a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)*c*e+10/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^3/(a*d-b*c)^2/(a*f-b
*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))+f^3/(a*f-b*e)^2/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{}{b^3d^2f\,x^{12}+3a\,b^2d^2f\,x^{10}+2b^3cdf\,x^{10}+b^3d^2e\,x^{10}+3a^2b\,d^2f}$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c**2*e + a**3*c**2*f*x**2 +
2*a**3*c*d*e*x**2 + 2*a**3*c*d*f*x**4 + a**3*d**2*e*x**4 + a**3*d**2*f*x**
6 + 3*a**2*b*c**2*e*x**2 + 3*a**2*b*c**2*f*x**4 + 6*a**2*b*c*d*e*x**4 + 6*
a**2*b*c*d*f*x**6 + 3*a**2*b*d**2*e*x**6 + 3*a**2*b*d**2*f*x**8 + 3*a*b**2
*c**2*e*x**4 + 3*a*b**2*c**2*f*x**6 + 6*a*b**2*c*d*e*x**6 + 6*a*b**2*c*d*f
*x**8 + 3*a*b**2*d**2*e*x**8 + 3*a*b**2*d**2*f*x**10 + b**3*c**2*e*x**6 +
b**3*c**2*f*x**8 + 2*b**3*c*d*e*x**8 + 2*b**3*c*d*f*x**10 + b**3*d**2*e*x*
*10 + b**3*d**2*f*x**12),x)
```

3.123 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 983

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \frac{b^2x}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}(c+dx^2)^{3/2}}$$

$$+\frac{d(abd^2e-a^2d^2f+b^2c(de-cf))\,x}{3ac(bc-ad)^2(be-af)(de-cf)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}$$

$$+\frac{b(2a^3bd^3ef-a^4d^3f^2+2b^4c^2e(de-cf)-ab^3c(9d^2e^2-4cdef-5c^2f^2)-a^2b^2d(d^2e^2-12cdef+12c^2f^2}{3a^2c(bc-ad)^3(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{d}(a^5d^4f^2(2de-5cf)+2b^5c^3e(de-cf)^2-5ab^4c^2(de-cf)^2(2de+cf)+a^3b^2d^3e(2d^2e^2+15cdef-26}{3a^2c^{3/2}(bc-ad)^4(be}$$

$$-\frac{\sqrt{d}(3a^4cd^4f^3-a^3bd^3f(de-4cf)^2+b^4c^2(de-cf)^3-3ab^3cd^2e(6d^2e^2-15cdef+10c^2f^2)+a^2b^2d^2(d^3e}{3a^2\sqrt{c}(bc-ad)^4(be-af)(de-cf)^3\sqrt{\frac{c}{a}}}$$

$$-\frac{c^{3/2}f^5\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)+1/3*d*(a
*b*d^2*e-a^2*d^2*f+b^2*c*(-c*f+d*e))*x/a/c/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d
*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*b*(2*a^3*b*d^3*e*f-a^4*d^3*f^2+2*b
^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-5*c^2*f^2-4*c*d*e*f+9*d^2*e^2)-a^2*b^2*d*(12
*c^2*f^2-12*c*d*e*f+d^2*e^2))*x/a^2/c/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)
/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*d^(1/2)*(a^5*d^4*f^2*(-5*c*f+2*d*e)+2
*b^5*c^3*e*(-c*f+d*e)^2-5*a*b^4*c^2*(-c*f+d*e)^2*(c*f+2*d*e)+a^3*b^2*d^3*e
*(-26*c^2*f^2+15*c*d*e*f+2*d^2*e^2)-a^4*b*d^3*f*(-13*c^2*f^2+4*d^2*e^2)-a^
2*b^3*c*d*(-13*c^3*f^3+26*c^2*d*e*f^2-26*c*d^2*e^2*f+10*d^3*e^3))*(b*x^2+a
)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a
^2/c^(3/2)/(-a*d+b*c)^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)-1/3*d^(1/2)*(3*a^4*c*d^4*f^3-a^3*b*d^3*f*(-4*c*f+d
*e)^2+b^4*c^2*(-c*f+d*e)^3-3*a*b^3*c*d^2*e*(10*c^2*f^2-15*c*d*e*f+6*d^2*e^
2)+a^2*b^2*d^2*(30*c^3*f^3-29*c^2*d*e*f^2+7*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a
)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c
^(1/2)/(-a*d+b*c)^4/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)-c^(3/2)*f^5*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)
/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/(-a*f+b*e)^2/(
-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.18 (sec) , antiderivative size = 744, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \frac{-\sqrt{\frac{b}{a}}ex\Big(a^2cd^4(-bc+ad)(be-af)^2(-de+cf)\,(a+bx^2)^2+a$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(-(Sqrt[b/a]*e*x*(a^2*c*d^4*(-(b*c) + a*d)*(b*e - a*f)^2*(-(d*e) + c*f)*(a
 + b*x^2)^2 + a^2*d^4*(b*e - a*f)^2*(b*c*(10*d*e - 13*c*f) + a*d*(-2*d*e +
 5*c*f))*(a + b*x^2)^2*(c + d*x^2) - a*b^4*c^2*(-(b*c) + a*d)*(-(b*e) + a*
f)*(d*e - c*f)^2*(c + d*x^2)^2 - b^4*c^2*(d*e - c*f)^2*(2*b^2*c*e + 13*a^2
*d*f - 5*a*b*(2*d*e + c*f))*(a + b*x^2)*(c + d*x^2)^2)) - I*c*(a + b*x^2)*
Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(-(b*e*(a^5*d^4*f^2*(2
*d*e - 5*c*f) + 2*b^5*c^3*e*(d*e - c*f)^2 - 5*a*b^4*c^2*(d*e - c*f)^2*(2*d
*e + c*f) + a^3*b^2*d^3*e*(2*d^2*e^2 + 15*c*d*e*f - 26*c^2*f^2) + a^4*b*d^
3*f*(-4*d^2*e^2 + 13*c^2*f^2) + a^2*b^3*c*d*(-10*d^3*e^3 + 26*c*d^2*e^2*f
- 26*c^2*d*e*f^2 + 13*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]) + (b*c - a*d)*(b*e*(-(d*e) + c*f)*(-2*a^3*b*d^3*e*f + a^4*d^3*f^2 +
2*b^4*c^2*e*(-(d*e) + c*f) + a*b^3*c*(9*d^2*e^2 - 4*c*d*e*f - 5*c^2*f^2) +
 a^2*b^2*d*(d^2*e^2 - 12*c*d*e*f + 12*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - 3*a^2*c*(-(b*c) + a*d)^3*f^4*EllipticPi[(a*f)/(b*e)
, I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(3*a^2*Sqrt[b/a]*c^2*(b*c - a*d)
^4*e*(b*e - a*f)^2*(d*e - c*f)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 1031, normalized size of antiderivative = 1.05, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {421, 25, 402, 25, 402, 27, 402, 400, 313, 320, 421, 25, 402, 25, 400, 313, 320, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}(dx^2+c)^{5/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}(dx^2+c)^{5/2}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+5bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+5bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}dx}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\int -\frac{3d\left(b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x^2+a\left(-2dfa^2+bdea+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{\frac{3d\int \frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x^2+a\left(-2dfa^2+bdea+b^2ce\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{a(bc-ad)}+\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a(a+bx^2)^{3/2}(c+dx^2)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{3d\left(\frac{\int \frac{b\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)x^2+a\left(4d^2fa^3-bd(2de+17cf)a^2+b^2c(9de+5cf)a+b^3c^2e\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11\cdots\right.}{3c\left(c+dx^2\right)^{3\cdots}}\cdots\right)}{a(bc-ad)}\cdots}{3a(bc-ad)}\cdots\right)}{(be-af)^2\cdots}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{3d\left(\frac{\frac{\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd(5de-3cf)-5ab^3c^2(cf+2de)+2b^4c^3e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(-2a^3d^2f+a^2bd(28cf+de)-2ab^2c(5cf+9de)+b^3\cdots\right.}{bc-ad}\cdots}{3c(bc-ad)}\cdots\right)}{a(bc-ad)}\cdots}{3a(bc-ad)}\cdots\right)}{\cdots}$$

$$\frac{f^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 313

$$b\left(\frac{\frac{3d\left(\frac{\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd(5de-3cf)-5ab^3c^2(cf+2de)+2b^4c^3e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ab\left(-2a^3d^2f+a^2bd(28cf+de)-2ab\right.}{}}{3c(bc-ad)}}{a(bc-ad)}}{3a(bc-a}\right.$$

$$\frac{f^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 320

$$\frac{f^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{(be-af)^2}+$$

$$b\left(\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}(c+dx^2)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11cf)-ab^2c(5cf+9de)+2b^3c^2e\right)}{3c(c+dx^2)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd\right.}{\sqrt{c}\sqrt{d}\sqrt{c+}}\right.}{}\right.$$

$\downarrow$ 421

$$\frac{f^2\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$b\left(\frac{\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11cf)-ab^2c(5cf+9de)+2b^3c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd(\right.}{\sqrt{c}\sqrt{d}\sqrt{c+}}\right.}{}}{}\right.$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$b\left(\frac{\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11cf)-ab^2c(5cf+9de)+2b^3c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd(\right.}{\sqrt{c}\sqrt{d}\sqrt{c+}}\right.}{}}{}\right.$$

$\downarrow$ 402

$$\frac{f^2\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$b\left(\frac{\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11cf)-ab^2c(5cf+9de)+2b^3c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd\right.}{\sqrt{c}\sqrt{d}\sqrt{c+}}\right.}{}}{}\right.$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{(be-af)^2}+$$

$$b\left(\frac{\frac{bx\left(11a^2df-5abcf-8abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3d\left(\frac{x\sqrt{a+bx^2}\left(2a^3d^2f-a^2bd(de-11cf)-ab^2c(5cf+9de)+2b^3c^2e\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{a+bx^2}\left(-4a^4d^3f+a^3bd^2(19cf+2de)-2a^2b^2cd\cdots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+\cdots}}\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 400

$$\frac{\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int \frac{}{\sqrt{b}}}{bc-ad}}{3c(bc-ad)}}{(de-cf)^2}\right)}{(be-af)^2}$$

$$b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{\left(-4d^3\right.}{}\right)}{}}{}\right)$$

↓ 313

$$\frac{\frac{\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{b(ad(de-4cf)-3}{}}{3c(bc-ad)}\right.}{(de-cf)^2}}{(be-af)^2}$$

$$b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{\left(-4d^3\right.}{}\right.}{}}{}\right.$$

↓ 320

$$\frac{\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx f^2}{(de-cf)^2} + \frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} - \frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \frac{b\sqrt{c}(ad(de-4cf}{\cdots}}{3c(bc-ad)}\cdots}{(de-cf)^2}\cdots\right.}{(be-af)^2}$$

$$b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}} + \frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}} + \frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} + \frac{\left(-4d^3\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

↓ 413

$$\left(\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}} + \frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} - \frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \frac{b\sqrt{c}(}{}}{3c(bc-}\right.}{(de-cf)^2} \right.$$

$$(be-af)^2$$

$$b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}} + \frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}} + \frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} + \frac{\left(-4d^3\right.}{}\right.}{}}{} \right.$$

↓ 413

$$\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{(de-cf)^2}\right.$$

$$(be-af)^2$$

$$b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{\left(-4d^3\right.}{}\right.}{}}{}\right.$$

↓ 412

$$\frac{\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{(}\right.}{(be-af)^2}$$

$$\frac{b\left(\frac{b(be-af)x}{3a(bc-ad)(bx^2+a)^{3/2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(11dfa^2-8bdea-5bcfa+2b^2ce\right)x}{a(bc-ad)\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}+\frac{3d\left(\frac{\left(2d^2fa^3-bd(de-11cf)a^2-b^2c(9de+5cf)a+2b^3c^2e\right)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}+\frac{\left(-4d^3\right.}{}\right.}{}}{}\right.}{}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e - a*f)*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)
) + ((b*(2*b^2*c*e - 8*a*b*d*e - 5*a*b*c*f + 11*a^2*d*f)*x)/(a*(b*c - a*d)
*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)) + (3*d*(((2*b^3*c^2*e + 2*a^3*d^2*f -
a^2*b*d*(d*e - 11*c*f) - a*b^2*c*(9*d*e + 5*c*f))*x*Sqrt[a + b*x^2])/(3*c*
(b*c - a*d)*(c + d*x^2)^(3/2)) + (((2*b^4*c^3*e - 4*a^4*d^3*f - 2*a^2*b^2*
c*d*(5*d*e - 3*c*f) - 5*a*b^3*c^2*(2*d*e + c*f) + a^3*b*d^2*(2*d*e + 19*c*
f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) - (b*Sqrt[c]*(b^3*c^2*e - 2*a^3*d^2*f - 2*a*b^2*c*(9*d*e + 5*
c*f) + a^2*b*d*(d*e + 28*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d))))/(a*(b*c - a*d)))/(3*a
*(b*c - a*d))))/(b*e - a*f)^2 + (f^2*((d*(-1/3*(d*(d*e - c*f)*x*Sqrt[a + b
*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - ((Sqrt[d]*(b*c*(4*d*e - 7*c*f)
- a*d*(2*d*e - 5*c*f))*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*(a*d*(d*e - 4*c*f) - 3*b*c*(d*e - 2*c
*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2]))/(3*c*(b*c - a*d))))/(d*e - c*f)^2 + (Sqrt[-a]*f^2*Sqrt[1 + (b*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8975 vs. 2(947) = 1894.

Time = 22.34 (sec) , antiderivative size = 8976, normalized size of antiderivative = 9.13

method	result	size
elliptic	Expression too large to display	8976
default	Expression too large to display	11693

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{5}{2}}(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(5/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \int \frac{1}{{(b\,x^2+a)}^{5/2}\,{(d\,x^2+c)}^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{}{b^3d^3f\,x^{14}+3a\,b^2d^3f\,x^{12}+3b^3c\,d^2f\,x^{12}+b^3d^3e\,x^{12}+3a^2b\,d^3}$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c**3*e + a**3*c**3*f*x**2 +
3*a**3*c**2*d*e*x**2 + 3*a**3*c**2*d*f*x**4 + 3*a**3*c*d**2*e*x**4 + 3*a**
3*c*d**2*f*x**6 + a**3*d**3*e*x**6 + a**3*d**3*f*x**8 + 3*a**2*b*c**3*e*x*
*2 + 3*a**2*b*c**3*f*x**4 + 9*a**2*b*c**2*d*e*x**4 + 9*a**2*b*c**2*d*f*x**
6 + 9*a**2*b*c*d**2*e*x**6 + 9*a**2*b*c*d**2*f*x**8 + 3*a**2*b*d**3*e*x**8
 + 3*a**2*b*d**3*f*x**10 + 3*a*b**2*c**3*e*x**4 + 3*a*b**2*c**3*f*x**6 + 9
*a*b**2*c**2*d*e*x**6 + 9*a*b**2*c**2*d*f*x**8 + 9*a*b**2*c*d**2*e*x**8 +
9*a*b**2*c*d**2*f*x**10 + 3*a*b**2*d**3*e*x**10 + 3*a*b**2*d**3*f*x**12 +
b**3*c**3*e*x**6 + b**3*c**3*f*x**8 + 3*b**3*c**2*d*e*x**8 + 3*b**3*c**2*d
*f*x**10 + 3*b**3*c*d**2*e*x**10 + 3*b**3*c*d**2*f*x**12 + b**3*d**3*e*x**
12 + b**3*d**3*f*x**14),x)
```

3.124 $\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 450

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \frac{(bc+ad)^2x\sqrt{c+dx^2}}{ab(be+af)\sqrt{a-bx^2}}$$
$$-\frac{(b^2c^2f+2a^2d^2f+abd(de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}f(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{(b^2c^3f^2+a^2cd^2f^2+abd(d^2e^2-2cdef+3c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}f^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{a}(de-cf)^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(a*d+b*c)^2*x*(d*x^2+c)^(1/2)/a/b/(a*f+b*e)/(-b*x^2+a)^(1/2)-(b^2*c^2*f+2*
a^2*d^2*f+a*b*d*(2*c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/f/(a*f+b*e)/(-b*x^2+a)^
(1/2)/(1+d*x^2/c)^(1/2)+(b^2*c^3*f^2+a^2*c*d^2*f^2+a*b*d*(3*c^2*f^2-2*c*d*
e*f+d^2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1
/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/f^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^
2+c)^(1/2)-a^(1/2)*(-c*f+d*e)^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/f^2/(a*f+b*e)/
(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.50 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \frac{b\left(b^2\sqrt{-\frac{b}{a}}c^3ef^2x+2ab\sqrt{-\frac{b}{a}}c^2def^2x+a^2\sqrt{-\frac{b}{a}}cd^2ef^2x+b^2\sqrt{-\frac{b}{a}}c^2def^2x^3\right.}{}$$

input
```
Integrate[(c + d*x^2)^(5/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*(b^2*Sqrt[-(b/a)]*c^3*e*f^2*x + 2*a*b*Sqrt[-(b/a)]*c^2*d*e*f^2*x + a^2*
Sqrt[-(b/a)]*c*d^2*e*f^2*x + b^2*Sqrt[-(b/a)]*c^2*d*e*f^2*x^3 + 2*a*b*Sqrt
[-(b/a)]*c*d^2*e*f^2*x^3 + a^2*Sqrt[-(b/a)]*d^3*e*f^2*x^3 + I*c*e*f*(b^2*c
^2*f + 2*a^2*d^2*f + a*b*d*(d*e + 2*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*e*(b^2*c^
3*f^2 + a^2*c*d^2*f^2 + a*b*d*(d^2*e^2 - 2*c*d*e*f + 3*c^2*f^2))*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))] + I*a*b*d^3*e^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (3*I)*a*b*
c*d^2*e^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*
e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*a*b*c^2*d*e*f^2*Sq
rt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh
[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*b*c^3*f^3*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))]))/(a^3*(-(b/a))^(5/2)*e*f^2*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c
 + d*x^2])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.91, number of steps used = 29, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.879$, Rules used = {419, 25, 401, 25, 27, 403, 25, 399, 323, 323, 321, 331, 330, 327, 420, 319, 27, 399, 323, 323, 321, 331, 330, 327, 410, 331, 330, 327, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a-bx^2\right)^{3/2}\left(e+fx^2\right)}dx$$

$$\Big\downarrow \text{ 419}$$

$$-\frac{\int -\frac{\left(dx^2+c\right)^{3/2}\left(dfa^2+2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(a-bx^2\right)^{3/2}}dx}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(dx^2+c\right)^{3/2}\left(dfa^2+2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(a-bx^2\right)^{3/2}}dx}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{\int -\frac{b\sqrt{dx^2+c}\left(d\left(3dfa^2+2b(2de+cf)a+3b^2ce\right)x^2+abc(de-cf)\right)}{\sqrt{a-bx^2}}dx}{ab}+\frac{x\left(c+dx^2\right)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\int \frac{b\sqrt{dx^2+c}\left(d\left(3dfa^2+2b(2de+cf)a+3b^2ce\right)x^2+abc(de-cf)\right)}{\sqrt{a-bx^2}}dx}{ab}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\int\frac{\sqrt{dx^2+c}\left(d\left(3dfa^2+2b(2de+cf)a+3b^2ce\right)x^2+abc(de-cf)\right)}{\sqrt{a-bx^2}}dx}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 403

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{-\frac{\int-\frac{d\left(6d^2fa^3+bd(8de+7cf)a^2+b^2c(13de-cf)a+3b^3c^2e\right)x^2+ac\left(3c(2de-cf)b^2+2ad(2de+cf)b+3a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3b}-\frac{dx\sqrt{a-b\ldots}}{\ldots}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\int\frac{d\left(6d^2fa^3+bd(8de+7cf)a^2+b^2c(13de-cf)a+3b^3c^2e\right)x^2+ac\left(3c(2de-cf)b^2+2ad(2de+cf)b+3a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3b}-\frac{dx\sqrt{a-bx^2}\sqrt{\ldots}}{\ldots}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 399

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-c(ad+bc)\left(3a^2df+2ab(cf+2de)+3b^2ce\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{d\ldots}}}{3b}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 323

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(3a^2df+2ab(cf+2de)+3b^2ce\right)\int\frac{\ldots}{\sqrt{\ldots}}}{\sqrt{c+dx^2}}}{3b}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(3a^2df+2ab(cf+2de)+3b\cdots\right.}{\sqrt{a-bx^2}\sqrt{c+dx^2}\cdots}\cdots}{3b}}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(3a^2df+2ab(cf+2de)-\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+\cdots}}\cdots}{3b}}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(3a^2df+2a\cdots\right.}{\sqrt{b}\sqrt{a\cdots}}\cdots}{3b}}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(3a\cdots\right.}{\cdots}\cdots}{3b}}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{3b}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}(dx^2+c)^{3/2}}{fx^2+e}dx}{(af+be)^2}$$

$\downarrow$ 420

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{3b}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\int\sqrt{a-bx^2}\sqrt{dx^2+c}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}\right)}{(af+be)^2}$$

$\downarrow$ 319

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{3b}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{2}{3}\int\frac{2ac-(bc-ad)x^2}{2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}\right)}{(af+be)^2}$$

$\downarrow$ 27

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{3b}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\int\frac{2ac-(bc-ad)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}\right)}{(af+be)^2}$$

$\downarrow$ 399

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}-\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{c(ad+bc)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}\right)}{(af+be)^2}$$

$\big\downarrow$ 323

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}-\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}d\cdots}{f}\cdots\right)}{(af+be)^2}$$

$\big\downarrow$ 323

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}-\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{a-bx^2}\cdots}{fx^2\cdots}\cdots}{f}\cdots\right)}{(af+be)^2}$$

$\big\downarrow$ 321

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+}}{}}{3b}}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(de-cf)\int}{}\right.}{(af+be)^2}$$

$$\Big\downarrow \ \textbf{331}$$

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+}}{}}{3b}}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(bc-ad)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}-\frac{(}{}\right.}{(af+be)^2}$$

$$\Big\downarrow \ \textbf{330}$$

$$\frac{\frac{x\left(c+dx^2\right)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+}}{}}{3b}}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(bc-ad)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)+\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\right)}{f}\right.}{(af+be)^2}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}\cdots$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)+\frac{1}{3}x\sqrt{a-\cdots}\cdots\right)}{f}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \ 410$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}\cdots$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)+\frac{1}{3}x\sqrt{a-\cdots}\cdots\right)}{f}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \ 331$$

$$\frac{\frac{x(c+dx^2)^{3/2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(6a^3d^2f+a^2bd(7cf+8de)+ab^2c(13de-cf)+3b^3c^2e\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+\cdots}}{\cdots}}{3b}}{(af+be)^2}\cdots$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)+\frac{1}{3}x\sqrt{a-\cdots}\cdots\right)}{f}\cdots\right)}{(af+be)^2}$$

$\downarrow$ 330

$$\frac{\frac{(bc+ad)(be+af)x(dx^2+c)^{3/2}}{a\sqrt{a-bx^2}} - \frac{\frac{\sqrt{a}\left(6d^2fa^3+bd(8de+7cf)a^2+b^2c(13de-cf)a+3b^3c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c(bc+ad)\left(3dfa^2+\right.}{}}{3b}}{(be+af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\sqrt{a-bx^2}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{\sqrt{a}c(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}} - \frac{\sqrt{a}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{c}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.\right.}{f}\right.}{(be+af)^2}$$

$\downarrow$ 327

$$\frac{\frac{(bc+ad)(be+af)x(dx^2+c)^{3/2}}{a\sqrt{a-bx^2}} - \frac{\frac{\sqrt{a}\left(6d^2fa^3+bd(8de+7cf)a^2+b^2c(13de-cf)a+3b^3c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c(bc+ad)\left(3dfa^2+\right.}{}}{3b}}{(be+af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\sqrt{a-bx^2}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{\sqrt{a}c(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}} - \frac{\sqrt{a}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{c}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.\right.}{f}\right.}{(be+af)^2}$$

$\downarrow$ 414

$$\frac{\frac{(bc+ad)(be+af)x(dx^2+c)^{3/2}}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\left(6d^2fa^3+bd(8de+7cf)a^2+b^2c(13de-cf)a+3b^3c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)\left(3dfa^2+\right.}{}}{3b}}{(be+af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{1}{3}\sqrt{a-bx^2}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{\sqrt{a}c(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.\right.}{f}\right.}{(be+af}$$

input
```
Int[(c + d*x^2)^(5/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(((b*c + a*d)*(b*e + a*f)*x*(c + d*x^2)^(3/2))/(a*Sqrt[a - b*x^2]) - (-1/3
*(d*(3*b^2*c*e + 3*a^2*d*f + 2*a*b*(2*d*e + c*f))*x*Sqrt[a - b*x^2]*Sqrt[c
 + d*x^2])/b + ((Sqrt[a]*(3*b^3*c^2*e + 6*a^3*d^2*f + a*b^2*c*(13*d*e - c*
f) + a^2*b*d*(8*d*e + 7*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*S
qrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(b*c + a*d)*(3*b^2*c*e + 3*a^2*d*f + 2*a*
b*(2*d*e + c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[
(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d
*x^2]))/(3*b))/a)/(b*e + a*f)^2 - (f*(d*e - c*f)*((d*((x*Sqrt[a - b*x^2]*S
qrt[c + d*x^2])/3 + (-((Sqrt[a]*(b*c - a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d
*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*S
qrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c + a*d)*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)
/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/3))/f - ((d*e - c*f
)*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2
)/c])) + (c^(3/2)*(b*e + a*f)*Sqrt[a - b*x^2]*EllipticPi[1 - (c*f)/(d*e),
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 + (b*c)/(a*d)])/(a*Sqrt[d]*e*f*Sqrt[(c*(a -
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f))/(b*e + a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(401) = 802$.

Time = 6.77 (sec) , antiderivative size = 1041, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1041
elliptic	Expression too large to display	1700

input
```
int((d*x^2+c)^(5/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a^2*d^3*e*f^2*x^3+2*(b/a)^(1/2)*a*b*c*d^2*e*f^2*x^3+(b/a)^(1/
2)*b^2*c^2*d*e*f^2*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a^2*c*d^2*e*f^2+3*((-b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*c^2*d*e*f^2
-2*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/
b/c)^(1/2))*a*b*c*d^2*e^2*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*d^3*e^3+((-b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c^3*e*f^2-2*
((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c
)^(1/2))*a^2*c*d^2*e*f^2-2*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*c^2*d*e*f^2-((-b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*c*d^2*e^2
*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/
b/c)^(1/2))*b^2*c^3*e*f^2+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b*c^3*f^3-3*((-b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/
c*d)^(1/2)/(b/a)^(1/2))*a*b*c^2*d*e*f^2+3*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b
*c*d^2*e^2*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(
1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b*d^3*e^3+(b/a)^(1/2)*a^2*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(5/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a-bx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/(-b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**(5/2)/((a - b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima"
)
```

output
```
integrate((d*x^2 + c)^(5/2)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(5/2)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{(a-b\,x^2)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^(5/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^(5/2)/((a - b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(5/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output

```
(3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*c*d*x + 4*int((sqrt(c + d*x**2)*sqrt(
a - b*x**2)*x**6)/(2*a**3*c*e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 +
 2*a**3*d*f**2*x**4 - a**2*b*c*e**2 - 5*a**2*b*c*e*f*x**2 - 4*a**2*b*c*f**
2*x**4 - a**2*b*d*e**2*x**2 - 5*a**2*b*d*e*f*x**4 - 4*a**2*b*d*f**2*x**6 +
 2*a*b**2*c*e**2*x**2 + 4*a*b**2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**6 + 2*a*b
**2*d*e**2*x**4 + 4*a*b**2*d*e*f*x**6 + 2*a*b**2*d*f**2*x**8 - b**3*c*e**2
*x**4 - b**3*c*e*f*x**6 - b**3*d*e**2*x**6 - b**3*d*e*f*x**8),x)*a**3*d**3
*f**2 + 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(2*a**3*c*e*f + 2*a
**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 + 2*a**3*d*f**2*x**4 - a**2*b*c*e**2 -
 5*a**2*b*c*e*f*x**2 - 4*a**2*b*c*f**2*x**4 - a**2*b*d*e**2*x**2 - 5*a**2*
b*d*e*f*x**4 - 4*a**2*b*d*f**2*x**6 + 2*a*b**2*c*e**2*x**2 + 4*a*b**2*c*e*
f*x**4 + 2*a*b**2*c*f**2*x**6 + 2*a*b**2*d*e**2*x**4 + 4*a*b**2*d*e*f*x**6
 + 2*a*b**2*d*f**2*x**8 - b**3*c*e**2*x**4 - b**3*c*e*f*x**6 - b**3*d*e**2
*x**6 - b**3*d*e*f*x**8),x)*a**2*b*c*d**2*f**2 - 4*int((sqrt(c + d*x**2)*s
qrt(a - b*x**2)*x**6)/(2*a**3*c*e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x*
*2 + 2*a**3*d*f**2*x**4 - a**2*b*c*e**2 - 5*a**2*b*c*e*f*x**2 - 4*a**2*b*c
*f**2*x**4 - a**2*b*d*e**2*x**2 - 5*a**2*b*d*e*f*x**4 - 4*a**2*b*d*f**2*x*
*6 + 2*a*b**2*c*e**2*x**2 + 4*a*b**2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**6 + 2
*a*b**2*d*e**2*x**4 + 4*a*b**2*d*e*f*x**6 + 2*a*b**2*d*f**2*x**8 - b**3*c*
e**2*x**4 - b**3*c*e*f*x**6 - b**3*d*e**2*x**6 - b**3*d*e*f*x**8),x)*a*...
```

3.125 $\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)} dx$

Optimal result

Integrand size = 33, antiderivative size = 389

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)} dx &= \frac{(bc+ad)x\sqrt{c+dx^2}}{a(be+af)\sqrt{a-bx^2}} \\ &- \frac{(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &+ \frac{(bc^2f-ad(de-2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}f(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{a}(de-cf)^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
(a*d+b*c)*x*(d*x^2+c)^(1/2)/a/(a*f+b*e)/(-b*x^2+a)^(1/2)-(a*d+b*c)*(1-b*x^
2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a
^(1/2)/b^(1/2)/(a*f+b*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(b*c^2*f-a*d*(
-2*c*f+d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/f/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)+a^(1/2)*(-c*f+d*e)^2*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticP
i(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/f/(a*f+b*e)/(-b*x
^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.59 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \frac{b\sqrt{-\frac{b}{a}}c^2efx + a\sqrt{-\frac{b}{a}}cdefx + b\sqrt{-\frac{b}{a}}cdefx^3 + a\sqrt{-\frac{b}{a}}d^2efx^3 + ic(bc+ad}{}$$

input
```
Integrate[(c + d*x^2)^(3/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*Sqrt[-(b/a)]*c^2*e*f*x + a*Sqrt[-(b/a)]*c*d*e*f*x + b*Sqrt[-(b/a)]*c*d*
e*f*x^3 + a*Sqrt[-(b/a)]*d^2*e*f*x^3 + I*c*(b*c + a*d)*e*f*Sqrt[1 - (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c
))] - I*e*(b*c^2*f + a*d*(-(d*e) + 2*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*d^2*e^
2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*Arc
Sinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (2*I)*a*c*d*e*f*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x]
, -((a*d)/(b*c))] - I*a*c^2*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*Sq
rt[-(b/a)]*e*f*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.27, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {419, 25, 401, 25, 27, 399, 323, 323, 321, 331, 330, 327, 410, 331, 330, 327, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{\sqrt{dx^2+c}\left(dfa^2+2bcfa+b^2(de-cf)x^2+b^2ce\right)}{(a-bx^2)^{3/2}}dx}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\sqrt{dx^2+c}\left(dfa^2+2bcfa+b^2(de-cf)x^2+b^2ce\right)}{(a-bx^2)^{3/2}}dx}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 401

$$\frac{\frac{\int -\frac{b\left(d\left(dfa^2+2bdea+b^2ce\right)x^2+abc(de-cf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\int \frac{b\left(d\left(dfa^2+2bdea+b^2ce\right)x^2+abc(de-cf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ab}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\int \frac{d\left(dfa^2+2bdea+b^2ce\right)x^2+abc(de-cf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 399

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\left(a^2df+2abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-c(ad+bc)(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 323

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\left(a^2df+2abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}}{a}}{(af+be)^2}-\frac{f(de-cf)\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 323

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{(a^2df+2abde+b^2ce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 321

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{(a^2df+2abde+b^2ce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 331

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(a^2df+2abde+b^2ce\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 330

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}-\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 327

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$\Big\downarrow$ 410

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{f}\right)}{(af+be)^2}$$

$\Big\downarrow$ 331

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$\Big\downarrow$ 330

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a}\cdots}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}$$

$\Big\downarrow$ 327

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{\frac{x\sqrt{c+dx^2}(ad+bc)(af+be)}{a\sqrt{a-bx^2}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2df+2abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{a}}{(af+be)^2}$$

$$\frac{f(de-cf)\left(\frac{c^{3/2}\sqrt{a-bx^2}(af+be)\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}+1\right)}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c\left(a-bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}$$

input

```
Int[(c + d*x^2)^(3/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output

```
(((b*c + a*d)*(b*e + a*f)*x*Sqrt[c + d*x^2])/(a*Sqrt[a - b*x^2]) - ((Sqrt[
a]*(b^2*c*e + 2*a*b*d*e + a^2*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ell
ipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x
^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(b*c + a*d)*(b*e + a*f)*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)
/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/a)/(b*e + a*f)^2 - (f
*(d*e - c*f)*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ellip
ticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt
[1 + (d*x^2)/c])) + (c^(3/2)*(b*e + a*f)*Sqrt[a - b*x^2]*EllipticPi[1 - (c
*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 + (b*c)/(a*d)])/(a*Sqrt[d]*e*f*S
qrt[(c*(a - b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(b*e + a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.62

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, a\, d^{2} e f\, x^{3}+\sqrt{\frac{b}{a}}\, b c d e f\, x^{3}+2\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a c d e f-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\right.\right.}{}$
elliptic	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a*d^2*e*f*x^3+(b/a)^(1/2)*b*c*d*e*f*x^3+2*((-b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*e*f-
((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c
)^(1/2))*a*d^2*e^2+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b
/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2*e*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*e*f-((-b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2*e*
f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b
/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c^2*f^2-2*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*
c*d*e*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),
-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*d^2*e^2+(b/a)^(1/2)*a*c*d*e*f*x+(b/
a)^(1/2)*b*c^2*e*f*x)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/e/(b/a)^(1/2)/a/(
a*f+b*e)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a-bx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(-b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**(3/2)/((a - b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima"
)
```

output
```
integrate((d*x^2 + c)^(3/2)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/2)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{(d\,x^2+c)^{3/2}}{(a-b\,x^2)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^(3/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^(3/2)/((a - b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2f\,x^6-2abf\,x^4+b^2e\,x^4+a^2f\,x^2-2abe\,x^2+a^2e}dx\right)d+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{b^2f\,x^6-2abf\,x^4+b^2e\,x^4+a^2f\,x^2-2abe\,x^2+a^2e}dx\right)c$$

input
```
int((d*x^2+c)^(3/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*e + a**2*f*x**2 - 2*a*b
*e*x**2 - 2*a*b*f*x**4 + b**2*e*x**4 + b**2*f*x**6),x)*d + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2))/(a**2*e + a**2*f*x**2 - 2*a*b*e*x**2 - 2*a*b*f*x*
*4 + b**2*e*x**4 + b**2*f*x**6),x)*c
```

3.126 $\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 358

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \frac{bx\sqrt{c+dx^2}}{a(be+af)\sqrt{a-bx^2}} - \frac{\sqrt{b}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} + \frac{(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} - \frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
b*x*(d*x^2+c)^(1/2)/a/(a*f+b*e)/(-b*x^2+a)^(1/2)-b^(1/2)*c*(-b*x^2+a)^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/
(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+(a*d+b*c)*(-b*x^2+a)^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/b^(1/
2)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*(-c*f+d*e)*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/
c)^(1/2))/b^(1/2)/e/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \frac{b\sqrt{-\frac{b}{a}}cex + b\sqrt{-\frac{b}{a}}dex^3 + ibce\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\right.}{}$$

input
```
Integrate[Sqrt[c + d*x^2]/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*Sqrt[-(b/a)]*c*e*x + b*Sqrt[-(b/a)]*d*e*x^3 + I*b*c*e*Sqrt[1 - (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))
] - I*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*a*d*e*Sqrt[1 - (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*
d)/(b*c))] - I*a*c*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-(
(a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*Sqrt[-(b/a)]*
e*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {421, 401, 27, 399, 323, 323, 321, 331, 330, 327, 410, 331, 330, 327, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\int \frac{\sqrt{dx^2+c}(-bfx^2+be+2af)}{(a-bx^2)^{3/2}}dx}{(af+be)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{b\left(acf-bdex^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 399$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{c(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 321$$

$$\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{331}$$

$$\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{330}$$

$$\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \text{410}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{f}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \text{331}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \text{330}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 414$$

$$\frac{f^2\left(\frac{c^{3/2}\sqrt{a-bx^2}(af+be)\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}+1\right)}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c\left(a-bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}}\right)}{(af+be)^2}$$

input
```
Int[Sqrt[c + d*x^2]/((a - b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*(((b*e + a*f)*x*Sqrt[c + d*x^2])/(a*Sqrt[a - b*x^2]) + (-((Sqrt[a]*Sqrt
[b]*e*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqr
t[a]], -((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*
c*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^
2]))/a))/(b*e + a*f)^2 + (f^2*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt
[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqr
t[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (c^(3/2)*(b*e + a*f)*Sqrt[a - b*x^2]*
EllipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 + (b*c)/(a*d)])
/(a*Sqrt[d]*e*f*Sqrt[(c*(a - b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(
b*e + a*f)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bde\, x^{3}+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) ade+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) bce-\sqrt{\frac{-b\, x^{2}+a}{a}}\right.}{}$
elliptic	$\frac{\sqrt{(-b\, x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(-bd\, x^{2}-bc\right) x}{a(af+be)\sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-bd\, x^{2}-bc\right)}}+\frac{d\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{ad-bc}{cb}}\right)}{(af+be)\sqrt{\frac{b}{a}}\, \sqrt{-bd\, x^{4}+ad\, x^{2}-x^{2} bc+ac}}+\frac{bc\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}}{a(af+be)\sqrt{\frac{b}{a}}\, \sqrt{}}\right.}{}$

input
```
int((d*x^2+c)^(1/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b*d*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*d*e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e-((-b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e+((-b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c
*d)^(1/2)/(b/a)^(1/2))*a*c*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*d*e+(b/a)^(1/2
)*b*c*e*x)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(b/a)^(1/2)/a/(a*f+b*e)/(-b*
d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{\frac{3}{2}}\,(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(c + d*x**2)/((a - b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{dx^2+c}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(d*x^2 + c)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{dx^2+c}}{(-bx^2+a)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)/((-b*x^2 + a)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(a-b\,x^2)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^(1/2)/((a - b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^(1/2)/((a - b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}}{(a-bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{b^2f\,x^6-2abf\,x^4+b^2e\,x^4+a^2f\,x^2-2abe\,x^2+a^2e}dx$$

input
```
int((d*x^2+c)^(1/2)/(-b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*e + a**2*f*x**2 - 2*a*b*e*x*
*2 - 2*a*b*f*x**4 + b**2*e*x**4 + b**2*f*x**6),x)
```

3.127 $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 363

$$
\begin{aligned}
\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{b^2x\sqrt{c+dx^2}}{a(bc+ad)(be+af)\sqrt{a-bx^2}} \\
&- \frac{b^{3/2}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{a^{3/2}(bc+ad)(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}
$$

output
```
b^2*x*(d*x^2+c)^(1/2)/a/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)-b^(3/2)*c*(-b
*x^2+a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/
2))/a^(3/2)/(a*d+b*c)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+b^(1/2)*
(-b*x^2+a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^
(1/2))/a^(3/2)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+a^(1/2)*f*(1-b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d
/b/c)^(1/2))/b^(1/2)/e/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.66 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^2\sqrt{-\frac{b}{a}}cex + b^2\sqrt{-\frac{b}{a}}dex^3 + ib^2ce\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b^2*Sqrt[-(b/a)]*c*e*x + b^2*Sqrt[-(b/a)]*d*e*x^3 + I*b^2*c*e*Sqrt[1 - (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] - I*b*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*b*c*f*Sqrt[1 - (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]
*x], -((a*d)/(b*c))] - I*a^2*d*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*S
qrt[-(b/a)]*(b*c + a*d)*e*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {421, 402, 399, 323, 323, 321, 331, 330, 327, 415, 323, 323, 321, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\int \frac{-bfx^2+be+2af}{(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx}{(af+be)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{a(bde+bcf+2adf)-bd(be+af)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 399$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 321$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 331

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 330

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 327

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\Big\downarrow$ 415

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{f}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \ \mathbf{323}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \ \mathbf{323}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \ \mathbf{321}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\downarrow$ 413

$$\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\downarrow$ 413

$$\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\downarrow$ 412

$$\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

input
```
Int[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e + a*f)*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (-
((Sqrt[a]*Sqrt[b]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 +
(d*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*(b*e + 2*a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(S
qrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(a*(b*c + a*d))))/(b*e + a*f)^2 +
 (f^2*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt[c
+ d*x^2])) + (Sqrt[a]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(
Sqrt[b]*e*f*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(b*e + a*f)^2
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 8.68 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, b^{2} d e\, x^{3}+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a b d e+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) b^{2} c e-\sqrt{\frac{-b\, x^{2}+}{a}}\right.}{}$
elliptic	$\frac{\sqrt{\left(-b\, x^{2}+a\right)\left(x^{2} d+c\right)}\, \left(-\frac{\left(-b d\, x^{2}-b c\right) b x}{a \left(a d+b c\right)\left(a f+b e\right) \sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d\, x^{2}-b c\right)}}+\frac{\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right) b}{\sqrt{\frac{b}{a}}\, \sqrt{-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c}\, a \left(a f+b e\right)}-\frac{b^{2} c \sqrt{1-\frac{b\, x^{2}}{a}}}{a \left(a d+b c\right)\left(a\right.}\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b^2*d*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*d*e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c*e-((-b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c*
e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b
/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a^2*d*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b*c*
f+(b/a)^(1/2)*b^2*c*e*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/e/(b/a)^(1/2)/a/
(a*f+b*e)/(a*d+b*c)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxim
a")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac"
)
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-}}{b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4-2abcf\,x^4-}$$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4
 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)
```

3.128 $\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)} dx$

Optimal result

Integrand size = 33, antiderivative size = 516

$$
\begin{aligned}
\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx &= \frac{b^2x}{a(bc+ad)(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&+ \frac{d(abd^2e+a^2d^2f-b^2c(de-cf))\,x\sqrt{a-bx^2}}{ac(bc+ad)^2(be+af)(de-cf)\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{b}(abd^2e+a^2d^2f-b^2c(de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}c(bc+ad)^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&+ \frac{b^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}(bc+ad)(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&- \frac{\sqrt{a}f^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}
$$

output
```
b^2*x/a/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+d*(a*b*d^2*e+
a^2*d^2*f-b^2*c*(-c*f+d*e))*x*(-b*x^2+a)^(1/2)/a/c/(a*d+b*c)^2/(a*f+b*e)/(
-c*f+d*e)/(d*x^2+c)^(1/2)+b^(1/2)*(a*b*d^2*e+a^2*d^2*f-b^2*c*(-c*f+d*e))*(
1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1
/2))/a^(1/2)/c/(a*d+b*c)^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/
c)^(1/2)+b^(3/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a
^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x
^2+c)^(1/2)-a^(1/2)*f^2*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(
1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/(a*f+b*e)/(-c*f+d*e)/(
-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.56 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{ibce(abd^2e+a^2d^2f+b^2c(-de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(ia}{}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(I*b*c*e*(a*b*d^2*e + a^2*d^2*f + b^2*c*(-(d*e) + c*f))*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
 - (a*(-(b/a))^(3/2)*e*x*(-(a^3*d^3*f) + a*b^2*d^3*e*x^2 + b^3*c*(-(d*e) +
 c*f)*(c + d*x^2) + a^2*b*d^3*(-e + f*x^2)) + I*b^3*c*(b*c + a*d)*e*(-(d*e
) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
-(b/a)]*x], -((a*d)/(b*c))] + I*a*b*c*(b*c + a*d)^2*f^2*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x]
, -((a*d)/(b*c))])/b)/(a*Sqrt[-(b/a)]*c*(b*c + a*d)^2*e*(b*e + a*f)*(-(d*e
) + c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.60, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.848, Rules used = {421, 402, 402, 25, 27, 399, 323, 323, 321, 331, 330, 327, 421, 25, 401, 27, 399, 323, 323, 321, 331, 330, 327, 410, 331, 330, 327, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \ 421$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}+\frac{b\int \frac{-bfx^2+be+2af}{(a-bx^2)^{3/2}(dx^2+c)^{3/2}}dx}{(af+be)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{bd(be+af)x^2+a(bde+bcf+2adf)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{-\frac{\int -\frac{b\left(ac(2bde+bcf+3adf)-d\left(-2dfa^2-bdea+b^2ce\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\frac{\int \frac{b\left(ac(2bde+bcf+3adf)-d\left(-2dfa^2-bdea+b^2ce\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\frac{b\int \frac{ac(2bde+bcf+3adf)-d\left(-2dfa^2-bdea+b^2ce\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$$\Big\downarrow \text{399}$$

$$\frac{b\left(\frac{\frac{b\left(c(ad+bc)(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-\left(-2a^2df-abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$$\Big\downarrow \text{323}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-\left(-2a^2df-abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+be\cdots}{a\sqrt{a-bx^2}\sqrt{c+dx\cdots}}\cdots\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$\big\downarrow$ 323

$$\frac{b\left(\frac{\frac{b\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-\left(-2a^2df-abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\frac{b\cdots}{a\sqrt{a-bx\cdots}}\cdots\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$\big\downarrow$ 321

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\left(-2a^2df-abde+b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde+b^2ce\right)}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}+\cdots\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$\big\downarrow$ 331

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-2a^2df-abde+b^2ce\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-abde\cdots\right.}{c\sqrt{c+dx^2}(ad+bc)}}{a(ad+bc)}\cdots\right.}{(af+be)^2}$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 330

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2\cdots\right.}{c\sqrt{c+dx^2}\cdots}}{a(ad+bc)}\cdots\right.}{(af+be)^2}$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 327

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a\cdots}}{\cdots}}{a(ad+bc)}\cdots\right.}{(af+be)^2}$$

$\Big\downarrow$ 421

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}-\frac{d\int-\frac{\sqrt{a-bx^2}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}}{a(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}\right)}{(af+be)^2}$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\int\frac{\sqrt{a-bx^2}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}}{a(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}\right)}{(af+be)^2}$$

$\downarrow$ 401

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\int\frac{d\left(acf-bdex^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{cd}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}}{a(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}\right)}{(af+be)^2}$$

$\downarrow$ 27

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\int\frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$\downarrow$ 399

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{c(af+be)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{c}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$\downarrow$ 323

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{c}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \text{323}$$

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{c}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{f^2\left(\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{c}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\ \cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \text{331}$$

$$\frac{f^2\left(\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{c}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}\right)}{(af+be)^2}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\ \cdots}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2}$$

$$\Big\downarrow \text{330}$$

$$\frac{f^2\left(\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{c}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2\cdots}}{fx^2+e}\cdots}{(de-cf)^2\cdots}\cdots\right)}{(af+be)^2\cdots}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a\cdots}}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2\cdots}$$

$$\Big\downarrow \; 327$$

$$\frac{f^2\left(\frac{f^2\int\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcs\cdots\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+\cdots}}}{c}\cdots\right)}{(de-cf)^2\cdots}\cdots\right)}{(af+be)^2\cdots}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a\cdots}}{\cdots}}{a(ad+bc)}\cdots\right)}{(af+be)^2\cdots}$$

$\Big\downarrow$ 410

$$\frac{f^2\left(\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{f}\right)}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\cdots}{c}\cdots\right.}{(de-cf)^2}\cdots\right.}{(af+be)^2}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right.}{(af+be)^2}$$

$\Big\downarrow$ 331

$$\frac{f^2\left(\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}}\right)}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{\cdots}\right.}{(de-cf)^2}\cdots\right.}{(af+be)^2}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}\cdots}{\cdots}}{a(ad+bc)}\cdots\right.}{(af+be)^2}$$

$\Big\downarrow$ 330

$$\frac{f^2\left(\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}(fx^2+e)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}}{}\right.}{(de-}\right.}{(af+be)^2}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}}{}}{a(ad+bc)}\right.}{(af+be)^2}$$

$\downarrow$ 327

$$\frac{f^2\left(\frac{f^2\left(\frac{(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}(fx^2+e)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}}-\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{E}}{\sqrt{b}\sqrt{a-bx^2}}}{}\right.}{}\right.}{(af+be)^2}$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2df-abde+b^2ce\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}-\frac{dx\sqrt{a}}{}}{a(ad+bc)}\right.}{(af+be)^2}$$

$\downarrow$ 414

$$\frac{\left(\frac{c^{3/2}(be+af)\sqrt{a-bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}+1\right)}{a\sqrt{d}ef\sqrt{\frac{c(a-bx^2)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)f^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)x\sqrt{a-bx^2}}{c\sqrt{dx^2+c}}-\frac{\sqrt{a}c(\ldots}{\ldots}\right.}{\ldots}$$

$$(be+af)^2$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}\sqrt{dx^2+c}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-2dfa^2-bdea+b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\ldots}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}\ldots}\right.}{c(bc+ad)}}{a(bc+ad)}\right.$$

$$(be+af)^2$$

input

```
Int[1/((a - b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e + a*f)*x)/(a*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]) + (-
((d*(b^2*c*e - a*b*d*e - 2*a^2*d*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt
[c + d*x^2])) + (b*(-((Sqrt[a]*(b^2*c*e - a*b*d*e - 2*a^2*d*f)*Sqrt[1 - (b
*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b
*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c +
a*d)*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[
(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d
*x^2])))/(c*(b*c + a*d)))/(a*(b*c + a*d))))/(b*e + a*f)^2 + (f^2*((d*(((d*
e - c*f)*x*Sqrt[a - b*x^2])/(c*Sqrt[c + d*x^2]) - (-((Sqrt[a]*Sqrt[b]*e*Sq
rt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -
((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*e +
 a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)
/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/c))
/(d*e - c*f)^2 + (f^2*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x
^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqrt[a - b*
x^2]*Sqrt[1 + (d*x^2)/c])) + (c^(3/2)*(b*e + a*f)*Sqrt[a - b*x^2]*Elliptic
Pi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 + (b*c)/(a*d)])/(a*Sqrt
[d]*e*f*Sqrt[(c*(a - b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(d*e - c*
f)^2))/(b*e + a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(463) = 926.

Time = 11.03 (sec) , antiderivative size = 957, normalized size of antiderivative = 1.85

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, a^{2} b\, d^{3} e f\, x^{3}+\sqrt{\frac{b}{a}}\, a\, b^{2} d^{3} e^{2} x^{3}+\sqrt{\frac{b}{a}}\, b^{3} c^{2} d e f\, x^{3}-\sqrt{\frac{b}{a}}\, b^{3} c\, d^{2} e^{2} x^{3}+\sqrt{\frac{-b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a\, b^{2} c^{2} d e f-\right.}{}$
elliptic	Expression too large to display

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a^2*b*d^3*e*f*x^3+(b/a)^(1/2)*a*b^2*d^3*e^2*x^3+(b/a)^(1/2)*b
^3*c^2*d*e*f*x^3-(b/a)^(1/2)*b^3*c*d^2*e^2*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b^2*c^2*d*e*f-((
-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^
(1/2))*a*b^2*c*d^2*e^2+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^3*c^3*e*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^3*c^2*d*e^2-((-b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)
)*a^2*b*c*d^2*e*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/
a)^(1/2),(-a*d/b/c)^(1/2))*a*b^2*c*d^2*e^2-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^3*c^3*e*f+((-b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b
^3*c^2*d*e^2+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(
1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a^3*c*d^2*f^2+2*((-b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/
(b/a)^(1/2))*a^2*b*c^2*d*f^2+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b^2*c^3*f^2-(b
/a)^(1/2)*a^3*d^3*e*f*x-(b/a)^(1/2)*a^2*b*d^3*e^2*x+(b/a)^(1/2)*b^3*c^3*e*
f*x-(b/a)^(1/2)*b^3*c^2*d*e^2*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/c/e/(c*f
-d*e)/(b/a)^(1/2)/a/(a*d+b*c)^2/(a*f+b*e)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxim
a")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac"
)
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{1}{\left(a-b\,x^2\right)^{3/2}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{}{b^2d^2f\,x^{10}-2ab\,d^2f\,x^8+2b^2cdf\,x^8+b^2d^2e\,x^8+a^2d^2f\,x^6-4}$$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c**2*e + a**2*c**2*f*x**2 +
2*a**2*c*d*e*x**2 + 2*a**2*c*d*f*x**4 + a**2*d**2*e*x**4 + a**2*d**2*f*x**
6 - 2*a*b*c**2*e*x**2 - 2*a*b*c**2*f*x**4 - 4*a*b*c*d*e*x**4 - 4*a*b*c*d*f
*x**6 - 2*a*b*d**2*e*x**6 - 2*a*b*d**2*f*x**8 + b**2*c**2*e*x**4 + b**2*c*
*2*f*x**6 + 2*b**2*c*d*e*x**6 + 2*b**2*c*d*f*x**8 + b**2*d**2*e*x**8 + b**
2*d**2*f*x**10),x)
```

3.129 $\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 774

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \frac{d^2x}{3c(bc+ad)(de-cf)\sqrt{a-bx^2}(c+dx^2)^{3/2}}$$

$$-\frac{b(abd^2e+a^2d^2f-3b^2c(de-cf))\,x}{3ac(bc+ad)^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$+\frac{d(ab^2cd^2e(7de-10cf)+a^3d^3f(2de-5cf)-3b^3c^2(de-cf)^2+2a^2bd^2(d^2e^2+cdef-5c^2f^2))\,x\sqrt{a-bx}}{3ac^2(bc+ad)^3(be+af)(de-cf)^2\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{b}(ab^2cd^2e(7de-10cf)+a^3d^3f(2de-5cf)-3b^3c^2(de-cf)^2+2a^2bd^2(d^2e^2+cdef-5c^2f^2))\sqrt{1-b}}{3\sqrt{a}c^2(bc+ad)^3(be+af)(de-cf)^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$-\frac{\sqrt{b}(abd^2e+a^2d^2f-3b^2c(de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3\sqrt{a}c(bc+ad)^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{a}f^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e(be+af)(de-cf)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/3*d^2*x/c/(a*d+b*c)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)-1/3*b*(a
*b*d^2*e+a^2*d^2*f-3*b^2*c*(-c*f+d*e))*x/a/c/(a*d+b*c)^2/(a*f+b*e)/(-c*f+d
*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*d*(a*b^2*c*d^2*e*(-10*c*f+7*d*e)+
a^3*d^3*f*(-5*c*f+2*d*e)-3*b^3*c^2*(-c*f+d*e)^2+2*a^2*b*d^2*(-5*c^2*f^2+c*
d*e*f+d^2*e^2))*x*(-b*x^2+a)^(1/2)/a/c^2/(a*d+b*c)^3/(a*f+b*e)/(-c*f+d*e)^
2/(d*x^2+c)^(1/2)+1/3*b^(1/2)*(a*b^2*c*d^2*e*(-10*c*f+7*d*e)+a^3*d^3*f*(-5
*c*f+2*d*e)-3*b^3*c^2*(-c*f+d*e)^2+2*a^2*b*d^2*(-5*c^2*f^2+c*d*e*f+d^2*e^2
))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c
)^(1/2))/a^(1/2)/c^2/(a*d+b*c)^3/(a*f+b*e)/(-c*f+d*e)^2/(-b*x^2+a)^(1/2)/(
1+d*x^2/c)^(1/2)-1/3*b^(1/2)*(a*b*d^2*e+a^2*d^2*f-3*b^2*c*(-c*f+d*e))*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/
2))/a^(1/2)/c/(a*d+b*c)^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^
(1/2)+a^(1/2)*f^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x
/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/(a*f+b*e)/(-c*f+d*e)^2/(-b*x
^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.98 (sec) , antiderivative size = 1710, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a-bx^2)^{3/2}\,(c+dx^2)^{5/2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(I*b*c*e*(3*b^3*c^2*(d*e - c*f)^2 + a^3*d^3*f*(-2*d*e + 5*c*f) + a*b^2*c*d
^2*e*(-7*d*e + 10*c*f) - 2*a^2*b*d^2*(d^2*e^2 + c*d*e*f - 5*c^2*f^2))*Sqrt
[1 - (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-
(b/a)]*x], -((a*d)/(b*c))] + (-3*b^5*c^4*d^2*e^3*x - 8*a^2*b^3*c^2*d^4*e^3
*x - 3*a^3*b^2*c*d^5*e^3*x + 6*b^5*c^5*d*e^2*f*x + 11*a^2*b^3*c^3*d^3*e^2*
f*x - 2*a^3*b^2*c^2*d^4*e^2*f*x - 3*a^4*b*c*d^5*e^2*f*x - 3*b^5*c^6*e*f^2*
x + 11*a^3*b^2*c^3*d^3*e*f^2*x + 6*a^4*b*c^2*d^4*e*f^2*x - 6*b^5*c^3*d^3*e
^3*x^3 + 8*a*b^4*c^2*d^4*e^3*x^3 - 4*a^2*b^3*c*d^5*e^3*x^3 - 2*a^3*b^2*d^6
*e^3*x^3 + 12*b^5*c^4*d^2*e^2*f*x^3 - 11*a*b^4*c^3*d^3*e^2*f*x^3 + 12*a^2*
b^3*c^2*d^4*e^2*f*x^3 + a^3*b^2*c*d^5*e^2*f*x^3 - 2*a^4*b*d^6*e^2*f*x^3 -
6*b^5*c^5*d*e*f^2*x^3 - 11*a^2*b^3*c^3*d^3*e*f^2*x^3 + 4*a^3*b^2*c^2*d^4*e
*f^2*x^3 + 5*a^4*b*c*d^5*e*f^2*x^3 - 3*b^5*c^2*d^4*e^3*x^5 + 7*a*b^4*c*d^5
*e^3*x^5 + 2*a^2*b^3*d^6*e^3*x^5 + 6*b^5*c^3*d^3*e^2*f*x^5 - 10*a*b^4*c^2*
d^4*e^2*f*x^5 + 2*a^2*b^3*c*d^5*e^2*f*x^5 + 2*a^3*b^2*d^6*e^2*f*x^5 - 3*b^
5*c^4*d^2*e*f^2*x^5 - 10*a^2*b^3*c^2*d^4*e*f^2*x^5 - 5*a^3*b^2*c*d^5*e*f^2
*x^5 - I*a*b*Sqrt[-(b/a)]*c*(b*c + a*d)*e*(-(d*e) + c*f)*(a*b*d^2*e + a^2*
d^2*f + 3*b^2*c*(-(d*e) + c*f))*Sqrt[1 - (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + ((3*I)*a*
b^4*c^6*f^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*
e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/Sqrt[-(b/a)] + (9*I)*a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}+\frac{b\int \frac{-bfx^2+be+2af}{(a-bx^2)^{3/2}(dx^2+c)^{5/2}}dx}{(af+be)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{3bd(be+af)x^2+a(bde+bcf+2adf)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{5/2}}dx}{a(ad+bc)}+\frac{bx(af+be)}{a\sqrt{a-bx^2}(c+dx^2)^{3/2}(ad+bc)}\right)}{(af+be)^2}$$

$\downarrow$ 402

$$b\left(\frac{-\frac{\int -\frac{bd\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)x^2+a\left(3c(2de+cf)b^2+ad(2de+11cf)b+4a^2d^2f\right)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{3c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-ab(de-2cf)+3b^2ce\right)}{3c\left(c+dx^2\right)^{3/2}(ad+bc)}}{a(ad+bc)}+\frac{bx(a}{a\sqrt{a-bx^2}(c+c}\right.$$

$$\overline{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 25

$$b\left(\frac{\frac{\int \frac{bd\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)x^2+a\left(3c(2de+cf)b^2+ad(2de+11cf)b+4a^2d^2f\right)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{3c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-2a^2df-ab(de-2cf)+3b^2ce\right)}{3c\left(c+dx^2\right)^{3/2}(ad+bc)}}{a(ad+bc)}+\frac{bx(af+b}{a\sqrt{a-bx^2}(c+dx^2)}\right.$$

$$\overline{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 402

$$b\left(\frac{\frac{-\frac{\int -\frac{b\left(ac\left(3c(3de+cf)b^2+ad(de+13cf)b+2a^2d^2f\right)-d\left(-4d^2fa^3-bd(2de+13cf)a^2-b^2c(7de+cf)a+3b^3c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-4a^3d^2f-a^2bd(13cf+2d\right.}{c\sqrt{c+dx^2}(ad-}}{3c(ad+bc)}}{a(ad+bc)}\right.$$

$$\overline{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{\frac{\int \frac{b\left(ac\left(3c(3de+cf)b^2+ad(de+13cf)b+2a^2d^2f\right)-d\left(-4d^2fa^3-bd(2de+13cf)a^2-b^2c(7de+cf)a+3b^3c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-\right.}{c\sqrt{c+dx^2}(ad+bc)}}{3c(ad+bc)}}{a(ad+bc)}\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{\frac{\frac{b\int \frac{ac\left(3c(3de+cf)b^2+ad(de+13cf)b+2a^2d^2f\right)-d\left(-4d^2fa^3-bd(2de+13cf)a^2-b^2c(7de+cf)a+3b^3c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-a\right.}{c\sqrt{c+dx^2}(ad+bc)}}{3c(ad+bc)}}{a(ad+bc)}\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 399

$$\frac{b\left(\frac{\frac{\frac{b\left(c(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf+7de)+3b^3c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{a-bx^2}\left(-4\right.}{}}{3c(ad+bc)}}{a(ad+bc)}\right.}{(af+be)^2}$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 323

$$\frac{b\left(\frac{\frac{b\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf+7de)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx\right)}{c(ad+bc)}-\frac{dx\sqrt{\cdots}}{\cdots}}{3c(ad+bc)}}{a(ad+bc)}\cdots\right.}{(af+be)^{\cdots}}$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 323

$$\frac{b\left(\frac{\frac{b\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf+7de)+3b^3c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}d\cdots\right.}{c(ad+bc)}\cdots}{3c(ad+bc)}}{a(ad+bc)}\cdots\right.}{(af+\cdots}$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 321

$$b\left(\frac{\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf+7de)+3b^3c^2e\right)\int\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(ad+bc)}\right.$$

$$(a$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 331

$$b\left(\frac{\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf+7de)+3\right.}{\sqrt{a-bx^2}}\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(ad+bc)}\right.$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 330

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c(cf\right.}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{c(ad+bc)}}{3c(ad+bc)}}{a(ad+bc)}\right.$$

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}$$

$\Big\downarrow$ 327

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{(af+be)^2}+$$

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right)}{c(ad+bc)}}{3c(ad+bc)}}{a(a}\right.$$

$\Big\downarrow$ 421

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{a-bx^2}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(a}$$

$\Big\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{a-bx^2}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(a}$$

$\Big\downarrow$ 401

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{d\left(a(2de-5cf)-b(de-4cf)x^2\right)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(a}\right.$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{d\left(a(2de-5cf)-b(de-4cf)x^2\right)}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{c(ad+bc)}}{3c(ad+bc)}}{a(a}\right.$$

$\downarrow$ 27

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{a(2de-5cf)-b(de-4cf)x^2}{\sqrt{a-bx^2}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(af+be)^2}+$$

$$b\left(\frac{\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\ \cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}\cdots}}\cdots\right)}{c(ad+bc)}}{3c(ad+bc)}\cdots}{a(a\cdots}\cdots\right.$$

$$\Big\downarrow \; 402$$

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{x\sqrt{a-bx^2}(ad(2de-5cf)+bc(de-4cf))}{c\sqrt{c+dx^2}(ad+bc)}-\frac{\int -\frac{b\left((ad(2de-5cf)+bc(de-4cf))x^2+ac(de-cf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}}{3c}+\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\cdots\right.}{(af+be)^2}$$

$$b\left(\frac{\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\ \cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx}{c}\cdots}}\cdots\right)}{c(ad+bc)}}{3c(ad+bc)}\cdots}{a(a\cdots}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{\int \frac{b\left((ad(2de-5cf)+bc(de-4cf))x^2+ac(de-cf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(ad(2de-5cf)+bc(de-4cf))}{c\sqrt{c+dx^2}(ad+bc)}}{3c}+\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(af+be)^2}$$

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\right)}{c(ad+bc)}}{3c(ad+bc)}\cdots}{a(a\cdots}\right.$$

$\downarrow$ 27

$$\frac{f^2\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{b\int \frac{(ad(2de-5cf)+bc(de-4cf))x^2+ac(de-cf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{c(ad+bc)}+\frac{x\sqrt{a-bx^2}(ad(2de-5cf)+bc(de-4cf))}{c\sqrt{c+dx^2}(ad+bc)}}{3c}+\frac{x\sqrt{a-bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{(af+be)^2}$$

$$b\left(\frac{\frac{b\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(-2a^2df-ab(de-2cf)+3b^2ce\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-4a^3d^2f-a^2bd(13cf+2de)-ab^2c\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\right)}{c(ad+bc)}}{3c(ad+bc)}\cdots}{a(a\cdots}\right.$$

$\downarrow$ 399

$$\frac{\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}}+\frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(bc+ad)(de\cdots}{\cdots}\right.}{c(bc+ad)}}{3c}\right.}{(de-cf)^2}\right.}{(be+af)^2}$$

$$\frac{b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 323

$$\frac{\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx f^2}{(de-cf)^2} + \frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}} + \frac{\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}} + \frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d} - \frac{c(bc+ad)(de\cdots}{\cdots}\right.}{c(bc+ad)}}{3c}\right.}{(de-cf)^2}\right.}{(be+af)^2}$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}} + \frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}} - \frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)}}{\cdots}\right.$$

$\downarrow$ 323

$$\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx f^2}{(de-cf)^2} + \frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}} + \frac{\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}} + \frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d} - \frac{c(bc+ad)(de\cdots}{\cdots}\right.}{c(bc+ad)\cdots}}{3c}\right.}{(de-cf)^2}\right.$$

$$\overline{(be+af)^2}$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}} + \frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}} - \frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)\cdots}}{\cdots}\right.$$

$$\downarrow 321$$

$$\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}}+\frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c(bc+ad)(\cdots}{\cdots}\right.}{c(bc\cdots}}{3c}\right.}{(de-cf)^2}\right.$$

$$(be+af)^2$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)}}{\cdots}\right.$$

$\downarrow$ 331

$$\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx f^2}{(de-cf)^2} + \frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}} + \frac{\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}} + \frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}} - \frac{\sqrt{c}\cdots}{\cdots}\right.}{\cdots}}{3c}\right.}{(de-cf)^2}\right.$$

$$(be+af)^2$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}} + \frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}} - \frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)}}{\cdots}\right.$$

$$\downarrow 330$$

$$\frac{\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}}+\frac{b\left(\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{}}{3c}\right.}{(de-cf)^2}\right.}{(be+af)^2}$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-4d^2fa^3-bd(\right.}{}\right.}{c(bc+ad)}}{}\right.$$

$\downarrow$ 327

$$\frac{\left(\frac{\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}}+\frac{b\left(\frac{\sqrt{a}(ad(2de-5cf)+bc(de-4cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{3c}\right.}{(de-cf)^2}\right.}{(be+af)^2}$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-4d^2fa^3-bd\right.}{}\right.}{c(bc+ad)}}{}\right.$$

$$\downarrow 415$$

$$\left(\frac{\left(\frac{(be+af)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{f}\right)f^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{a-bx^2}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{(ad(2de-5cf)+bc(de-4cf))\sqrt{a-bx^2}x}{c(bc+ad)\sqrt{dx^2+c}}+\frac{b\left(\frac{\sqrt{a}(ad(2\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad(be+af\cdots}$$

$$b\left(\frac{b(be+af)x}{a(bc+ad)\sqrt{a-bx^2}(dx^2+c)^{3/2}}+\frac{\frac{b\left(\frac{\sqrt{a}c(bc+ad)\left(-2dfa^2-b(de-2cf)a+3b^2ce\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-4d^2fa^3-bd\cdots\right.}{\cdots}\right.}{c(bc+ad)\cdots}}{\cdots}\right.$$

input
```
Int[1/((a - b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2793 vs. $2(707) = 1414$.

Time = 20.19 (sec) , antiderivative size = 2794, normalized size of antiderivative = 3.61

method	result	size
elliptic	Expression too large to display	2794
default	Expression too large to display	4122

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c*d/(a
*d+b*c)*x/(a*c*d*f-a*d^2*e+b*c^2*f-b*c*d*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)/(x^2+c/d)^2-1/3*(-b*d*x^2+a*d)/c^2*d^2/(a*d+b*c)^2*x*(5*a*c*d*f-2*a
*d^2*e+10*b*c^2*f-7*b*c*d*e)/(c*f-d*e)/(a*c*d*f-a*d^2*e+b*c^2*f-b*c*d*e)/(
(x^2+c/d)*(-b*d*x^2+a*d))^(1/2)-(-b*d*x^2-b*c)*b^3/a/(a*d+b*c)^3*x/(a*f+b*
e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+5/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(
-1-(a*d-b*c)/c/b)^(1/2))*a*d^3/(a*d+b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e+b*c^
2*f-b*c*d*e)*b*f+10/3*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-
b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*d^2*b^2/(a*d+b*c)^2/(c*f-d*e)/(a*c*d*f-
a*d^2*e+b*c^2*f-b*c*d*e)*f*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2
))-10/3*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)*d^2*b^2/(a*d+b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e+b*c^2*
f-b*c*d*e)*f*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-7/3/(b/a)^(
1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1
/2)*d^3*b^2/(a*d+b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e+b*c^2*f-b*c*d*e)*e*Elli
pticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+7/3/(b/a)^(1/2)*(1-b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*d^3*b^2/(a*d+
b*c)^2/(c*f-d*e)/(a*c*d*f-a*d^2*e+b*c^2*f-b*c*d*e)*e*EllipticE(x*(b/a)^(1/
2),(-1-(a*d-b*c)/c/b)^(1/2))-5/3/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="maxim
a")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x, algorithm="giac"
)
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{3/2}\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{}{b^2d^3f\,x^{12}-2ab\,d^3f\,x^{10}+3b^2c\,d^2f\,x^{10}+b^2d^3e\,x^{10}+a^2d^3f\,x}$$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c**3*e + a**2*c**3*f*x**2 +
3*a**2*c**2*d*e*x**2 + 3*a**2*c**2*d*f*x**4 + 3*a**2*c*d**2*e*x**4 + 3*a**
2*c*d**2*f*x**6 + a**2*d**3*e*x**6 + a**2*d**3*f*x**8 - 2*a*b*c**3*e*x**2
- 2*a*b*c**3*f*x**4 - 6*a*b*c**2*d*e*x**4 - 6*a*b*c**2*d*f*x**6 - 6*a*b*c*
d**2*e*x**6 - 6*a*b*c*d**2*f*x**8 - 2*a*b*d**3*e*x**8 - 2*a*b*d**3*f*x**10
 + b**2*c**3*e*x**4 + b**2*c**3*f*x**6 + 3*b**2*c**2*d*e*x**6 + 3*b**2*c**
2*d*f*x**8 + 3*b**2*c*d**2*e*x**8 + 3*b**2*c*d**2*f*x**10 + b**2*d**3*e*x*
*10 + b**2*d**3*f*x**12),x)
```

3.130 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 242

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(3a-7b)\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{(a-2b)(a-b)\sqrt{2+x^2}\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

output

```
-(a-2*b)*x*(x^2+2)^(1/2)/b^2/(x^2+1)^(1/2)+1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/
2)/b+2^(1/2)*(a-2*b)*(x^2+2)^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))/
b^2/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)-1/6*(3*a-7*b)*(x^2+2)^(1/2)*Inve
rseJacobiAM(arctan(x),1/2*2^(1/2))*2^(1/2)/b^2/(x^2+1)^(1/2)/((x^2+2)/(x^2
+1))^(1/2)+1/2*(a-2*b)*(a-b)*(x^2+2)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/
a,1/2*2^(1/2))*2^(1/2)/a/b^2/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{ab^2x\sqrt{1+x^2}\sqrt{2+x^2}+3ia(a-2b)bE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-ia(3a^2-9ab+7}{}$$

input
```
Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
(a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*EllipticE[I*Arc
Sinh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*EllipticF[I*ArcSinh[x/Sq
rt[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (12*
I)*a^2*b*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (15*I)*a*b^2*Ellip
ticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*
ArcSinh[x/Sqrt[2]], 2])/(3*a*b^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {418, 25, 403, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^{3/2}\sqrt{x^2+2}}{a+bx^2}dx$$

$$\Big\downarrow \text{418}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}+\frac{\int -\frac{\sqrt{x^2+2}(-bx^2+a-2b)}{\sqrt{x^2+1}}dx}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\int \frac{\sqrt{x^2+2}(-bx^2+a-2b)}{\sqrt{x^2+1}}dx}{b^2}$$

$$\Big\downarrow \; 403$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\int \frac{3(a-2b)x^2+2(3a-5b)}{\sqrt{x^2+1}\sqrt{x^2+2}}dx-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \; 406$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(2(3a-5b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+3(a-2b)\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \; 320$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(3(a-2b)\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \; 388$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx\right)+\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \; 313$$

$$\frac{(a-b)^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{b^2}-\frac{\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

$$\Big\downarrow \; 414$$

$$\frac{2\sqrt{x^2+1}(a-b)^2\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{ab^2\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}-$$

$$\frac{\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+3(a-2b)\left(\frac{x\sqrt{x^2+2}}{\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)\right)-\frac{1}{3}bx\sqrt{x^2+1}\sqrt{x^2+2}}{b^2}$$

input
```
Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
-((-1/3*(b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]) + (3*(a - 2*b)*((x*Sqrt[2 + x^2]
)/Sqrt[1 + x^2] - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/(Sqrt[
1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])) + (Sqrt[2]*(3*a - 5*b)*Sqrt[2 + x^2]*
EllipticF[ArcTan[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/3)/b
^2) + (2*(a - b)^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]]
, -1])/(a*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x\sqrt{x^2+1}\,\sqrt{x^2+2}}{3b} - \frac{\left(-\frac{3i\left(a^3-4a^2b+5a\,b^2-2b^3\right)\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\ \mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^2a\sqrt{x^4+3x^2+2}}+\frac{i\left(3a^2-12ab+13b^2\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\ \mathrm{E}}{2b^2\sqrt{x^4+3x^2+2}}\right.}{3b\sqrt{x^2+}}$
default	$-\frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\left(-a\,b^2x^5+3i\sqrt{x^2+1}\,\sqrt{x^2+2}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\,\sqrt{x^2+2}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2b+7i\sqrt{x^2+1}\,\sqrt{x\right.}{}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{x\sqrt{x^4+3x^2+2}}{3b}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\ \mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{b\sqrt{x^4+3x^2+2}}-\frac{4ia\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\ \mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^2\sqrt{x^4+3x^2+2}}+\frac{ia^2\sqrt{2}\,\sqrt{}}{}\right.}{}$

input
```
int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/2)/b-1/3/b*(-3*I*(a^3-4*a^2*b+5*a*b^2-2*b^3
)/b^2/a*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipt
icPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))+1/2*I*(3*a^2-12*a*b+13*b^2)/b^2*2^(1/2
)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1
/2),2^(1/2))+3/2*I*(a-2*b)/b*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*
x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2)
,2^(1/2))))*((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)
```

Fricas [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{(x^2+1)^{\frac{3}{2}}\sqrt{x^2+2}}{a+bx^2}\,dx$$

input
```
integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((x**2 + 1)**(3/2)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{(x^2+1)^{3/2}\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

input
```
int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\,x-3\left(\int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\,x^4}{b\,x^6+a\,x^4+3b\,x^4+3a\,x^2+2b\,x^2+2a}dx\right)a+6\left(\int \frac{\sqrt{x^2+1}}{b\,x^6+a\,x^4+3b\,x}\right.}{}$$

input
```
int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(x**2 + 1)*sqrt(x**2 + 2)*x - 3*int((sqrt(x**2 + 1)*sqrt(x**2 + 2)*x*
*4)/(a*x**4 + 3*a*x**2 + 2*a + b*x**6 + 3*b*x**4 + 2*b*x**2),x)*a + 6*int(
(sqrt(x**2 + 1)*sqrt(x**2 + 2)*x**4)/(a*x**4 + 3*a*x**2 + 2*a + b*x**6 + 3
*b*x**4 + 2*b*x**2),x)*b - 6*int((sqrt(x**2 + 1)*sqrt(x**2 + 2)*x**2)/(a*x
**4 + 3*a*x**2 + 2*a + b*x**6 + 3*b*x**4 + 2*b*x**2),x)*a + 13*int((sqrt(x
**2 + 1)*sqrt(x**2 + 2)*x**2)/(a*x**4 + 3*a*x**2 + 2*a + b*x**6 + 3*b*x**4
 + 2*b*x**2),x)*b - 2*int((sqrt(x**2 + 1)*sqrt(x**2 + 2))/(a*x**4 + 3*a*x*
*2 + 2*a + b*x**6 + 3*b*x**4 + 2*b*x**2),x)*a + 6*int((sqrt(x**2 + 1)*sqrt
(x**2 + 2))/(a*x**4 + 3*a*x**2 + 2*a + b*x**6 + 3*b*x**4 + 2*b*x**2),x)*b)
/(3*b)
```

3.131 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 192

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+x^2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

output
```
x*(x^2+2)^(1/2)/b/(x^2+1)^(1/2)-2^(1/2)*(x^2+2)^(1/2)*EllipticE(x/(x^2+1)^
(1/2),1/2*2^(1/2))/b/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)+1/2*(x^2+2)^(1/
2)*InverseJacobiAM(arctan(x),1/2*2^(1/2))*2^(1/2)/b/(x^2+1)^(1/2)/((x^2+2)
/(x^2+1))^(1/2)-1/2*(a-2*b)*(x^2+2)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a
,1/2*2^(1/2))*2^(1/2)/a/b/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{i\left(-abE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(a-2b)\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+(-a+b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsi}\right.\right.\right.}{ab^2}$$

input
```
Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
(I*(-(a*b*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]) + (a - 2*b)*(a*EllipticF[I*A
rcSinh[x/Sqrt[2]], 2] + (-a + b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]],
 2]))/(a*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {409, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

$$\Big\downarrow \; 409$$

$$\frac{\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}dx}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \; 324$$

$$\frac{\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \; 320$$

$$\frac{\int \frac{x^2}{\sqrt{x^2+1}\sqrt{x^2+2}}dx+\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{-\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx+\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{(a-2b)\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{\sqrt{x^2+1}}}{b}-\frac{\sqrt{x^2+2}(a-2b)\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

input
```
Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

output
```
((x*Sqrt[2 + x^2])/Sqrt[1 + x^2] - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan
[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*Elli
pticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/
b - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 409
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f/e, 0] &&  !Sim
plerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

method	result
default	$-\frac{i\left(\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba+a^{2}\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)-3\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ba+2\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b^{2}-\mathrm{EllipticF}\left(\right.\right.}{a\,b^{2}}$
elliptic	$\frac{\sqrt{(x^{2}+1)(x^{2}+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b\sqrt{x^{4}+3x^{2}+2}}-\frac{ia\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^{2}\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{El}}{b\sqrt{x^{4}+3x}}\right.}{}$

input
```
int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-I*(EllipticE(1/2*I*x*2^(1/2),2^(1/2))*b*a+a^2*EllipticPi(1/2*I*x*2^(1/2),
2*b/a,2^(1/2))-3*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*a+2*EllipticP
i(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2-EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^
2+2*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*b*a)/a/b^2
```

Fricas [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

input
```
integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

input
```
integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

input
```
int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
```

output
```
int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^2+a}dx$$

input
```
int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

output
```
int((sqrt(x**2 + 1)*sqrt(x**2 + 2))/(a + b*x**2),x)
```

3.132 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\frac{2\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a}$$

output
```
2*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/2),1-2*b/a,I)/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = -\frac{i\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)-(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{ab}$$

input
```
Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

output
```
((-I)*(a*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (a - 2*b)*EllipticPi[(2*b)/a
, I*ArcSinh[x/Sqrt[2]], 2]))/(a*b)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{414\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,(a+bx^2)}\,dx$$
$$\Big\downarrow \text{ 414}$$
$$\frac{2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

input
```
Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

output
```
(2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(
1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 3.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

method	result
default	$-\frac{i\left(a\,\mathrm{EllipticF}\left(\frac{i x\sqrt{2}}{2},\sqrt{2}\right)-a\,\mathrm{EllipticPi}\left(\frac{i x\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+2b\,\mathrm{EllipticPi}\left(\frac{i x\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)\right)}{ab}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i x\sqrt{2}}{2},\sqrt{2}\right)}{2b\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{i x\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{Ell}}{a\sqrt{x^4+3}}\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-I*(a*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-a*EllipticPi(1/2*I*x*2^(1/2),2*b/
a,2^(1/2))+2*b*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))/a/b
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^4 + (a + b)*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(a+bx^2)\,\sqrt{x^2+1}}\,dx$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,(b\,x^2+a)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^4+a\,x^2+b\,x^2+a}dx$$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x)
```

output
```
int((sqrt(x**2 + 1)*sqrt(x**2 + 2))/(a*x**2 + a + b*x**4 + b*x**2),x)
```

3.133 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 86

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \frac{\sqrt{2}\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a(a-b)}$$

output
```
2^(1/2)*(x^2+2)^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))/(a-b)/(x^2+1)
^(1/2)/((x^2+2)/(x^2+1))^(1/2)-2*b*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/2),1-
2*b/a,I)/a/(a-b)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \frac{\frac{x}{\sqrt{\frac{1+x^2}{2+x^2}}} + iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right) - \frac{i(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{a}}{a-b}$$

input
```
Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

output
```
(x/Sqrt[(1 + x^2)/(2 + x^2)] + I*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (I*(
a - 2*b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/a)/(a - b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{3/2}\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{416}$$

$$\frac{\int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{3/2}}dx}{a-b}-\frac{b\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\left(bx^2+a\right)}dx}{a-b}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{b\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\left(bx^2+a\right)}dx}{a-b}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

input
```
Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

output
```
(Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*S
qrt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, Arc
Tan[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

method	result
default	$\frac{\left(i\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a\sqrt{x^2+1}\,\sqrt{x^2+2}-i\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)a\sqrt{x^2+1}\,\sqrt{x^2+2}+2i\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b\sqrt{x^2+1}\,\sqrt{x^2+2}}{a(x^4+3x^2+2)(a-b)}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{(x^2+2)x}{(a-b)\sqrt{(x^2+1)(x^2+2)}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{(a-b)\sqrt{x^4+3x^2+2}}+\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)-I*Elli
pticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)+2*I*El
lipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+1)^(1/2)*(x^2+2)^(1/2)+a*x^
3+2*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/a/(x^4+3*x^2+2)/(a-b)
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)} \, dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^
2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)} \, dx = \int \frac{\sqrt{x^2+2}}{(a+bx^2)(x^2+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(x**2 + 2)/((a + b*x**2)*(x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx = \int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{3/2}\left(b\,x^2+a\right)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^6+a\,x^4+2b\,x^4+2a\,x^2+b\,x^2+a}dx$$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x)
```

output
```
int((sqrt(x**2 + 1)*sqrt(x**2 + 2))/(a*x**4 + 2*a*x**2 + a + b*x**6 + 2*b*
x**4 + b*x**2),x)
```

3.134 $\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2} \left(a+b x^2\right)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 235

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2} \left(a+b x^2\right)} \, dx=\frac{x \sqrt{2+x^2}}{3 (a-b) \left(1+x^2\right)^{3/2}}+\frac{\sqrt{2} (a-2 b) \sqrt{2+x^2} E\left(\arctan (x)\left|\frac{1}{2}\right.\right)}{(a-b)^2 \sqrt{1+x^2} \sqrt{\frac{2+x^2}{1+x^2}}}-\frac{\left(2 a^2-4 a b-b^2\right) \sqrt{2+x^2} \operatorname{EllipticF}\left(\arctan (x),\frac{1}{2}\right)}{3 \sqrt{2} (a-b)^3 \sqrt{1+x^2} \sqrt{\frac{2+x^2}{1+x^2}}}+\frac{(a-2 b) b^2 \sqrt{2+x^2} \operatorname{EllipticPi}\left(1-\frac{b}{a},\arctan (x),\frac{1}{2}\right)}{\sqrt{2} a (a-b)^3 \sqrt{1+x^2} \sqrt{\frac{2+x^2}{1+x^2}}}$$

output
```
1/3*x*(x^2+2)^(1/2)/(a-b)/(x^2+1)^(3/2)+2^(1/2)*(a-2*b)*(x^2+2)^(1/2)*Elli
pticE(x/(x^2+1)^(1/2),1/2*2^(1/2))/(a-b)^2/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))
^(1/2)-1/6*(2*a^2-4*a*b-b^2)*(x^2+2)^(1/2)*InverseJacobiAM(arctan(x),1/2*2
^(1/2))*2^(1/2)/(a-b)^3/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)+1/2*(a-2*b)*
b^2*(x^2+2)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*2^(1/2)/a/
(a-b)^3/(x^2+1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \frac{4a^2x\sqrt{1+x^2}\sqrt{2+x^2} - 7abx\sqrt{1+x^2}\sqrt{2+x^2} + 3a^2x^3\sqrt{1+x^2}\sqrt{2+x^2} - 6a$$

input
```
Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

output
```
(4*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 7*a*b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]
 + 3*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 6*a*b*x^3*Sqrt[1 + x^2]*Sqrt[2
+ x^2] + (3*I)*a*(a - 2*b)*(1 + x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]
- I*a*(a - b)*(1 + x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3*I)*a*b*E
llipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2*EllipticPi[(2*b)/a
, I*ArcSinh[x/Sqrt[2]], 2] + (6*I)*a*b*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x
/Sqrt[2]], 2] - (12*I)*b^2*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2
] + (3*I)*a*b*x^4*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2
*x^4*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/(3*a*(a - b)^2*(1 + x^2
)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {421, 25, 401, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2}}{(x^2+1)^{5/2}(a+bx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2} - \frac{\int -\frac{\sqrt{x^2+2}(-bx^2+a-2b)}{(x^2+1)^{5/2}}dx}{(a-b)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\int \frac{\sqrt{x^2+2}(-bx^2+a-2b)}{(x^2+1)^{5/2}}dx}{(a-b)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{x\sqrt{x^2+2}(a-b)}{3(x^2+1)^{3/2}}-\frac{1}{3}\int -\frac{(a-4b)x^2+2(2a-5b)}{(x^2+1)^{3/2}\sqrt{x^2+2}}dx}{(a-b)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\int \frac{(a-4b)x^2+2(2a-5b)}{(x^2+1)^{3/2}\sqrt{x^2+2}}dx+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{400}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\left(3(a-2b)\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx-2(a-b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-2(a-b)\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}dx\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}+\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{b^2\int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}(bx^2+a)}dx}{(a-b)^2}+\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}(a-b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{2b^2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+$$
$$\frac{\frac{1}{3}\left(\frac{3\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}(a-b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}\right)+\frac{\sqrt{x^2+2}x(a-b)}{3(x^2+1)^{3/2}}}{(a-b)^2}$$

input
```
Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

output
```
(((a - b)*x*Sqrt[2 + x^2])/(3*(1 + x^2)^(3/2)) + ((3*Sqrt[2]*(a - 2*b)*Sqr
t[2 + x^2]*EllipticE[ArcTan[x], 1/2])/(Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x
^2)]) - (Sqrt[2]*(a - b)*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[1
+ x^2]*Sqrt[(2 + x^2)/(1 + x^2)]))/3)/(a - b)^2 + (2*b^2*Sqrt[1 + x^2]*Ell
ipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(
2 + x^2)]*Sqrt[2 + x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{x\sqrt{x^4+3x^2+2}}{3(a-b)(x^2+1)^2}+\frac{(x^2+2)x(a-2b)}{(a-b)^2\sqrt{(x^2+1)(x^2+2)}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}\,(a-b)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,a\,\mathrm{Elliptic}}{2(a-b)^2\sqrt{x^4+3x^2}}\right.}{}$
default	$\frac{3i\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2x^2\sqrt{x^2+1}\,\sqrt{x^2+2}-i\,\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2x^2\sqrt{x^2+1}\,\sqrt{x^2+2}+3i\,\mathrm{EllipticPi}\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ab\sqrt{x^2+1}\,\sqrt{x}}{}$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)*(1/3*x/(a-b)*(x^4+3*x^
2+2)^(1/2)/(x^2+1)^2+(x^2+2)*x*(a-2*b)/(a-b)^2/((x^2+1)*(x^2+2))^(1/2)-1/6
*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2
*I*x*2^(1/2),2^(1/2))/(a-b)+1/2*I/(a-b)^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*a*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-I/(a-b)^2*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*b*EllipticE(1/2*I
*x*2^(1/2),2^(1/2))+I/(a-b)^2*b*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x
^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))-2*I/(a-b)^2*b^
2/a*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}\,(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^8 + (a + 3*b)*x^6 + 3*(a + b)*x^
4 + (3*a + b)*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(x^2+1)^{5/2}\,(b\,x^2+a)}\,dx$$

input
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)),x)
```

output
```
int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^8+a\,x^6+3b\,x^6+3a\,x^4+3b\,x^4+3a\,x^2+b\,x^2+a}dx$$

input
```
int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x)
```

output
```
int((sqrt(x**2 + 1)*sqrt(x**2 + 2))/(a*x**6 + 3*a*x**4 + 3*a*x**2 + a + b*
x**8 + 3*b*x**6 + 3*b*x**4 + b*x**2),x)
```

3.135 $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 298

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2} dx = \frac{dx\sqrt{3+fx^2}}{b\sqrt{2+dx^2}} - \frac{\sqrt{3}\sqrt{d}\sqrt{3+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{b\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} + \frac{2f\sqrt{3+fx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}b\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} + \frac{2(3b-af)\sqrt{3+fx^2}\operatorname{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}ab\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output
```
d*x*(f*x^2+3)^(1/2)/b/(d*x^2+2)^(1/2)-3^(1/2)*d^(1/2)*(f*x^2+3)^(1/2)*Elli
pticE(d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^(1/2),1/3*(9-6*f/d)^(1/2))/b/(d*x^2+2)
^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)+2/3*f*(f*x^2+3)^(1/2)*InverseJacobiAM(a
rctan(1/2*d^(1/2)*x*2^(1/2)),1/3*(9-6*f/d)^(1/2))*3^(1/2)/b/d^(1/2)/(d*x^2
+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)+2/3*(-a*f+3*b)*(f*x^2+3)^(1/2)*Ellip
ticPi(d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^(1/2),1-2*b/a/d,1/3*(9-6*f/d)^(1/2))*3
^(1/2)/a/b/d^(1/2)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$$
$$= \frac{i\left(-3abdE\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-2b+ad)\left(af\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(3b-af)\operatorname{EllipticP}\right.\right.}{\sqrt{3}ab^2\sqrt{d}}$$

input
```
Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

output
```
(I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-2*
b + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (3*
b - a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*
d)])))/(Sqrt[3]*a*b^2*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {410, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}dx$$
$$\Big\downarrow \text{410}$$
$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\int \frac{\sqrt{fx^2+3}}{\sqrt{dx^2+2}}dx}{b}$$
$$\Big\downarrow \text{324}$$
$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(3\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+f\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx\right)}{b}$$

$$\Big\downarrow \text{320}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(f\int \frac{x^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}dx+\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{b}$$

$$\Big\downarrow \text{388}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{3\int \frac{\sqrt{dx^2+2}}{(fx^2+3)^{3/2}}dx}{d}\right)+\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{(2b-ad)\int \frac{\sqrt{fx^2+3}}{(bx^2+a)\sqrt{dx^2+2}}dx}{b}+\frac{d\left(\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)\right)}{b}$$

$$\Big\downarrow \text{414}$$

$$\frac{3\sqrt{dx^2+2}(2b-ad)\,\text{EllipticPi}\left(1-\frac{3b}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{d\left(\frac{3\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+f\left(\frac{x\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}\right)\right)}{b}$$

input
```
Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

output
```
(d*(f*((x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[2 + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(
2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])) + (3*Sqrt[2 + d*x^2]*EllipticF[A
rcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])))/b + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*E
llipticPi[1 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/
(Sqrt[2]*a*b*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\left(\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)a^{2}df-a^{2}\operatorname{EllipticPi}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{fa},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)df-3\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)dba-2\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(f\,x^{2}+3)(x^{2}d+2)}\left(-\frac{\sqrt{3f\,x^{2}+9}\,\sqrt{2x^{2}d+4}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)adf}{2b^{2}\sqrt{-3f}\,\sqrt{df\,x^{4}+3x^{2}d+2f\,x^{2}+6}}+\frac{3\sqrt{3f\,x^{2}+9}\,\sqrt{2x^{2}d+4}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+\cdots}{f}}}{2}\right.\cdots}{2b\sqrt{-3f}\,\sqrt{df\,x^{4}+3x^{2}d+2f\,x^{2}+6}}\cdots\right.}{\cdots}$

input
```
int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2)
)*a^2*d*f-a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^(1/2)*(-d)
^(1/2)*3^(1/2)/(-f)^(1/2))*d*f-3*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^
(1/2)*2^(1/2)*(1/f*d)^(1/2))*d*b*a-2*f*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),
1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*a+3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/
2),3/f*b/a,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a+2*EllipticPi(1
/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))
*f*b*a-6*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^(1/2)*(-d)^(1/2
)*3^(1/2)/(-f)^(1/2))*b^2)*2^(1/2)/a/b^2/(-f)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}{b\,x^2+a}\,dx$$

input
```
int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2),x)
```

output
```
int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}{b\,x^2+a}dx$$

input
```
int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x)
```

output
```
int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(a + b*x**2),x)
```

3.136 $\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 93

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\frac{2\sqrt{3+fx^2}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output
```
2/3*(f*x^2+3)^(1/2)*EllipticPi(d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^(1/2),1-2*b/a
/d,1/3*(9-6*f/d)^(1/2))*3^(1/2)/a/d^(1/2)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^
2+2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = -\frac{i\left(ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(2b-ad)\,\text{EllipticPi}\left(\frac{2b}{ad},i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}ab\sqrt{d}}$$

input
```
Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

output
```
((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (2*b -
 a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
))/(Sqrt[3]*a*b*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2+2}}{\sqrt{fx^2+3}\,(a+bx^2)}dx$$

$$\Big\downarrow \; 414$$

$$\frac{2\sqrt{fx^2+3}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

input
```
Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

output
```
(2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]]
, 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2
 + d*x^2)])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{d}{f}}}{2}\right)ad-\operatorname{EllipticPi}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{fa},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)ad+2\operatorname{EllipticPi}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{fa},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)b\right)\sqrt{2}}{2ab\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(x^2d+2)}\left(\frac{\sqrt{3f\,x^2+9}\,\sqrt{2x^2d+4}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\,\sqrt{df\,x^4+3x^2d+2f\,x^2+6}}-\frac{\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{x^2d}{2}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{fa},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)d}{b\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3x^2d+2f\,x^2+6}}+ \cdots\right.}{\sqrt{f\,x^2+3}\,\sqrt{x^2d+2}}$

input
```
int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))
*a*d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^(1/2)*(-d)^(1/2)*3^
(1/2)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^
(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*b)*2^(1/2)/a/b/(-f)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*f*x^4 + (a*f + 3*b)*x^2 + 3*a)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{dx^2+2}}{(a+bx^2)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

input
```
integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{d\,x^2+2}}{(b\,x^2+a)\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx=\int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}{bf\,x^4+af\,x^2+3b\,x^2+3a}dx$$

input
```
int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x)
```

output
```
int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(a*f*x**2 + 3*a + b*f*x**4 + 3*b*x
**2),x)
```

3.137 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 194

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = -\frac{\sqrt{d}\sqrt{3+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}(2b-ad)\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}} + \frac{2b\sqrt{3+fx^2}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}(2b-ad)\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

output
```
-1/3*d^(1/2)*(f*x^2+3)^(1/2)*InverseJacobiAM(arctan(1/2*d^(1/2)*x*2^(1/2))
,1/3*(9-6*f/d)^(1/2))*3^(1/2)/(-a*d+2*b)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2
+2))^(1/2)+2/3*b*(f*x^2+3)^(1/2)*EllipticPi(d^(1/2)*x*2^(1/2)/(2*d*x^2+4)^
(1/2),1-2*b/a/d,1/3*(9-6*f/d)^(1/2))*3^(1/2)/a/d^(1/2)/(-a*d+2*b)/(d*x^2+2
)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3
]*a*Sqrt[-d])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^2+2}\sqrt{fx^2+3}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{412}$$

$$\frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

output
```
EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3
]*a*Sqrt[-d])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{\text{EllipticPi}\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{3b}{fa},\frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)\sqrt{2}}{2a\sqrt{-f}}$	53
elliptic	$\frac{\sqrt{(f\,x^2+3)(x^2d+2)}\,\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{x^2d}{2}}\,\text{EllipticPi}\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{fa},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)}{\sqrt{f\,x^2+3}\,\sqrt{x^2d+2}\,a\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3x^2d+2f\,x^2+6}}$	115

input
```
int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3/f*b/a,1/2*2^(1/2)*(-d)^(1/2)*3^(
1/2)/(-f)^(1/2))*2^(1/2)/a/(-f)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

input
```
int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}{bdf\,x^6+adf\,x^4+3bd\,x^4+2bf\,x^4+3ad\,x^2+2af\,x^2+6b\,x^2+6a}dx$$

input
```
int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

output
```
int((sqrt(f*x**2 + 3)*sqrt(d*x**2 + 2))/(a*d*f*x**4 + 3*a*d*x**2 + 2*a*f*x
**2 + 6*a + b*d*f*x**6 + 3*b*d*x**4 + 2*b*f*x**4 + 6*b*x**2),x)
```

3.138 $\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{1+\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

output
```
-(1+b*x^2/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{\frac{a+bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {281, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x^2-1)\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{281}$$
$$-\int \frac{1}{\sqrt{1-x^2}\sqrt{bx^2+a}}dx$$
$$\Big\downarrow \text{323}$$
$$-\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{1-x^2}\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{321}$$
$$-\frac{\sqrt{\frac{bx^2}{a}+1}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{b\,x^{2}+a}{a}}\,\operatorname{EllipticF}\left(x,\sqrt{-\frac{b}{a}}\right)}{\sqrt{b\,x^{2}+a}}$	35
elliptic	$-\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(x^{2}-1\right)}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x,\sqrt{-1-\frac{-a+b}{a}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{4}-a\,x^{2}+b\,x^{2}+a}}$	77

input
```
int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\,\sqrt{a+bx^2}}\,dx=-\frac{F(\arcsin{(x)}\mid -\frac{b}{a})}{\sqrt{a}}$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-elliptic_f(arcsin(x), -b/a)/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{F\left(\operatorname{asin}(x)\middle|-\frac{b}{a}\right)}{\sqrt{a}} & \text{for } x>-1 \wedge x<1 \end{cases}$$

input
```
integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-elliptic_f(asin(x), -b/a)/sqrt(a), (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(-1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)),x)
```

output
```
-int(1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{-x^2+1}}{b\,x^4+a\,x^2-b\,x^2-a}dx$$

input
```
int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**2)*sqrt( - x**2 + 1))/(a*x**2 - a + b*x**4 - b*x**2),x)
```

3.139 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}(1+3x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 23

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx=\frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{3}{5}\right)}{\sqrt{10}}$$

output
```
1/10*EllipticPi(1/3*15^(1/2)*x,-9/5,1/5*I*15^(1/2))*10^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx=\frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{3}{5}\right)}{\sqrt{10}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[2 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], -3/5]/Sqrt[10]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+2}\,(3x^2+1)}\,dx$$

$$\Big\downarrow \; 412$$

$$\frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{3}{5}\right)}{\sqrt{10}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[2 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], -3/5]/Sqrt[10]
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\operatorname{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{i\sqrt{5}\,\sqrt{3}}{5}\right)\sqrt{5}\,\sqrt{2}}{10}$	29
elliptic	$\frac{\sqrt{-(5x^2-3)(x^2+1)}\,\sqrt{3}\,\sqrt{5}\,\sqrt{1-\frac{5x^2}{3}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{i\sqrt{5}\,\sqrt{3}}{5}\right)}{5\sqrt{-5x^2+3}\,\sqrt{-10x^4-4x^2+6}}$	77

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+2)^(1/2)/(3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/10*EllipticPi(1/3*5^(1/2)*3^(1/2)*x,-9/5,1/5*I*5^(1/2)*3^(1/2))*5^(1/2)*
2^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx=\int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+2)^(1/2)/(3*x^2+1),x, algorithm="frica
s")
```

output
```
integral(-1/2*sqrt(2*x^2 + 2)*sqrt(-5*x^2 + 3)/(15*x^6 + 11*x^4 - 7*x^2 -
3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx=\frac{\sqrt{2}\int \frac{1}{3x^2\sqrt{3-5x^2}\sqrt{x^2+1}+\sqrt{3-5x^2}\sqrt{x^2+1}}\,dx}{2}$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(2*x**2+2)**(1/2)/(3*x**2+1),x)
```

output
```
sqrt(2)*Integral(1/(3*x**2*sqrt(3 - 5*x**2)*sqrt(x**2 + 1) + sqrt(3 - 5*x*
*2)*sqrt(x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+2)^(1/2)/(3*x^2+1),x, algorithm="maxim
a")
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 + 2)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+2)^(1/2)/(3*x^2+1),x, algorithm="giac"
)
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 + 2)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\,\left(3\,x^2+1\right)\,\sqrt{3-5\,x^2}}\,dx$$

input
```
int(1/((2*x^2 + 2)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 2)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2+2x^2}\,(1+3x^2)}\,dx = -\frac{\sqrt{2}\left(\int \frac{\sqrt{-5x^2+3}\,\sqrt{x^2+1}}{15x^6+11x^4-7x^2-3}dx\right)}{2}$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+2)^(1/2)/(3*x^2+1),x)
```

output
```
( - sqrt(2)*int((sqrt( - 5*x**2 + 3)*sqrt(x**2 + 1))/(15*x**6 + 11*x**4 -
7*x**2 - 3),x))/2
```

3.140 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}(1+3x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 23

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{6}{5}\right)}{\sqrt{5}}$$

output
```
1/5*EllipticPi(1/3*15^(1/2)*x,-9/5,1/5*I*30^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{6}{5}\right)}{\sqrt{5}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], -6/5]/Sqrt[5]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+1}\,(3x^2+1)}\,dx$$

$$\Big\downarrow \ 412$$

$$\frac{\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),-\frac{6}{5}\right)}{\sqrt{5}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], -6/5]/Sqrt[5]
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{5}\,\mathrm{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{i\sqrt{5}\,\sqrt{2}\,\sqrt{3}}{5}\right)}{5}$	29
elliptic	$\frac{\sqrt{-(5x^2-3)(2x^2+1)}\,\sqrt{3}\,\sqrt{5}\,\sqrt{1-\frac{5x^2}{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{\sqrt{-2}\,\sqrt{5}\,\sqrt{3}}{5}\right)}{5\sqrt{-5x^2+3}\,\sqrt{-10x^4+x^2+3}}$	79

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*5^(1/2)*EllipticPi(1/3*5^(1/2)*3^(1/2)*x,-9/5,1/5*I*5^(1/2)*2^(1/2)*3^
(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(3*x^2+1),x, algorithm="frica
s")
```

output
```
integral(-sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/(30*x^6 + 7*x^4 - 10*x^2 - 3),
x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+1}\cdot(3x^2+1)}\,dx$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(2*x**2+1)**(1/2)/(3*x**2+1),x)
```

output
```
Integral(1/(sqrt(3 - 5*x**2)*sqrt(2*x**2 + 1)*(3*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(3*x^2+1),x, algorithm="maxim
a")
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(3*x^2+1),x, algorithm="giac"
)
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2+1}\,\left(3\,x^2+1\right)\,\sqrt{3-5\,x^2}}\,dx$$

input
```
int(1/((2*x^2 + 1)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 1)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\,(1+3x^2)}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{-5x^2+3}}{30x^6+7x^4-10x^2-3}dx\right)$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(3*x^2+1),x)
```

output
```
 - int((sqrt(2*x**2 + 1)*sqrt( - 5*x**2 + 3))/(30*x**6 + 7*x**4 - 10*x**2
- 3),x)
```

3.141 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}(1+3x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 19

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = -\frac{5}{14}\,\text{EllipticPi}\left(\frac{9}{14},\arccos\left(\sqrt{\frac{5}{3}}x\right),6\right)$$

output
```
-5/14*EllipticPi(1/3*(-15*x^2+9)^(1/2),9/14,6^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \frac{\sqrt{1-2x^2}\,\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),\frac{6}{5}\right)}{\sqrt{-5+10x^2}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[-1 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
(Sqrt[1 - 2*x^2]*EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], 6/5])/Sqrt[-5 + 10*
x^2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2-1}\left(3x^2+1\right)}dx$$

$$\Big\downarrow \text{ 413}$$

$$\frac{\sqrt{1-2x^2}\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}(3x^2+1)}dx}{\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{1-2x^2}\,\text{EllipticPi}\left(-\frac{3}{2},\arcsin\left(\sqrt{2}x\right),\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^2-1}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[-1 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
(Sqrt[1 - 2*x^2]*EllipticPi[-3/2, ArcSin[Sqrt[2]*x], 5/6])/(Sqrt[6]*Sqrt[-
1 + 2*x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 4.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

method	result	size
default	$\frac{\text{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{\sqrt{5}\,\sqrt{2}\,\sqrt{3}}{5}\right)\sqrt{-2x^2+1}\,\sqrt{5}}{5\sqrt{2x^2-1}}$	46
elliptic	$\frac{\sqrt{-(5x^2-3)(2x^2-1)}\,\sqrt{3}\,\sqrt{5}\,\sqrt{1-\frac{5x^2}{3}}\,\sqrt{-2x^2+1}\ \text{EllipticPi}\left(\frac{\sqrt{5}\,\sqrt{3}\,x}{3},-\frac{9}{5},\frac{\sqrt{5}\,\sqrt{2}\,\sqrt{3}}{5}\right)}{5\sqrt{-5x^2+3}\,\sqrt{2x^2-1}\,\sqrt{-10x^4+11x^2-3}}$	99

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2-1)^(1/2)/(3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*EllipticPi(1/3*5^(1/2)*3^(1/2)*x,-9/5,1/5*5^(1/2)*2^(1/2)*3^(1/2))*(-2
*x^2+1)^(1/2)*5^(1/2)/(2*x^2-1)^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-1)^(1/2)/(3*x^2+1),x, algorithm="frica
s")
```

output
```
integral(-sqrt(2*x^2 - 1)*sqrt(-5*x^2 + 3)/(30*x^6 - 23*x^4 - 2*x^2 + 3),
x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2-1}\cdot(3x^2+1)}\,dx$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(2*x**2-1)**(1/2)/(3*x**2+1),x)
```

output
```
Integral(1/(sqrt(3 - 5*x**2)*sqrt(2*x**2 - 1)*(3*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-1)^(1/2)/(3*x^2+1),x, algorithm="maxim
a")
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 - 1)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-1)^(1/2)/(3*x^2+1),x, algorithm="giac"
)
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 - 1)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\,\left(3\,x^2+1\right)\,\sqrt{3-5\,x^2}}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 - 1)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-1+2x^2}\,(1+3x^2)}\,dx = -\left(\int \frac{\sqrt{2x^2-1}\,\sqrt{-5x^2+3}}{30x^6-23x^4-2x^2+3}dx\right)$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2-1)^(1/2)/(3*x^2+1),x)
```

output
```
- int((sqrt(2*x**2 - 1)*sqrt( - 5*x**2 + 3))/(30*x**6 - 23*x**4 - 2*x**2
+ 3),x)
```

3.142 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}(1+3x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 33

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticPi}\left(-3,\arcsin(x),\frac{5}{3}\right)}{\sqrt{6}\sqrt{-1+x^2}}$$

output
```
1/6*(-x^2+1)^(1/2)*EllipticPi(x,-3,1/3*15^(1/2))*6^(1/2)/(x^2-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticPi}\left(-\frac{9}{5},\arcsin\left(\sqrt{\frac{5}{3}}x\right),\frac{3}{5}\right)}{\sqrt{10}\sqrt{-1+x^2}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[-2 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
(Sqrt[1 - x^2]*EllipticPi[-9/5, ArcSin[Sqrt[5/3]*x], 3/5])/(Sqrt[10]*Sqrt[
-1 + x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2-2}\,(3x^2+1)}dx$$
$$\Big\downarrow \ 413$$
$$\frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-x^2}(3x^2+1)}dx}{\sqrt{2}\sqrt{x^2-1}}$$
$$\Big\downarrow \ 412$$
$$\frac{\sqrt{1-x^2}\,\text{EllipticPi}\left(-3,\arcsin(x),\frac{5}{3}\right)}{\sqrt{6}\sqrt{x^2-1}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[-2 + 2*x^2]*(1 + 3*x^2)),x]
```

output
```
(Sqrt[1 - x^2]*EllipticPi[-3, ArcSin[x], 5/3])/(Sqrt[6]*Sqrt[-1 + x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\operatorname{EllipticPi}\left(x,-3,\frac{\sqrt{3}\,\sqrt{5}}{3}\right)\sqrt{3}\,\sqrt{-x^{2}+1}\,\sqrt{2}}{6\sqrt{x^{2}-1}}$	36
elliptic	$\frac{\sqrt{-\left(5x^{2}-3\right)\left(x^{2}-1\right)}\,\sqrt{-x^{2}+1}\,\sqrt{1-\frac{5x^{2}}{3}}\,\operatorname{EllipticPi}\left(x,-3,\frac{\sqrt{3}\,\sqrt{5}}{3}\right)}{\sqrt{-5x^{2}+3}\,\sqrt{x^{2}-1}\,\sqrt{-10x^{4}+16x^{2}-6}}$	77

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2-2)^(1/2)/(3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*EllipticPi(x,-3,1/3*3^(1/2)*5^(1/2))*3^(1/2)*(-x^2+1)^(1/2)/(x^2-1)^(1
/2)*2^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-2)^(1/2)/(3*x^2+1),x, algorithm="frica
s")
```

output
```
integral(-1/2*sqrt(2*x^2 - 2)*sqrt(-5*x^2 + 3)/(15*x^6 - 19*x^4 + x^2 + 3)
, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = \frac{\sqrt{2}\int \frac{1}{3x^2\sqrt{3-5x^2}\sqrt{x^2-1}+\sqrt{3-5x^2}\sqrt{x^2-1}}\,dx}{2}$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(2*x**2-2)**(1/2)/(3*x**2+1),x)
```

output
```
sqrt(2)*Integral(1/(3*x**2*sqrt(3 - 5*x**2)*sqrt(x**2 - 1) + sqrt(3 - 5*x*
*2)*sqrt(x**2 - 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-2)^(1/2)/(3*x^2+1),x, algorithm="maxim
a")
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 - 2)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{(3\,x^2+1)\sqrt{2\,x^2-2}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2-2)^(1/2)/(3*x^2+1),x, algorithm="giac"
)
```

output
```
integrate(1/((3*x^2 + 1)*sqrt(2*x^2 - 2)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2-2}\,\left(3\,x^2+1\right)\,\sqrt{3-5\,x^2}}\,dx$$

input
```
int(1/((2*x^2 - 2)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 - 2)^(1/2)*(3*x^2 + 1)*(3 - 5*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{-2+2x^2}\,(1+3x^2)}\,dx = -\frac{\sqrt{2}\left(\int \frac{\sqrt{-5x^2+3}\,\sqrt{x^2-1}}{15x^6-19x^4+x^2+3}dx\right)}{2}$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2-2)^(1/2)/(3*x^2+1),x)
```

output
```
( - sqrt(2)*int((sqrt( - 5*x**2 + 3)*sqrt(x**2 - 1))/(15*x**6 - 19*x**4 +
x**2 + 3),x))/2
```

3.143 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 594

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^2}\,dx &= \frac{\left(2ad^2ef-b(15d^2e^2-20cdef+3c^2f^2)\right)x\sqrt{c+dx^2}}{6ef^3\sqrt{a+bx^2}} \\
&+ \frac{d^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^2} + \frac{(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^2\left(e+fx^2\right)} \\
&- \frac{\sqrt{a}\left(2ad^2ef-b(15d^2e^2-20cdef+3c^2f^2)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{b}ef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}\left(4ad^2ef(3de-4cf)-b(15d^3e^3-25cd^2e^2f+9c^2def^2-3c^3f^3)\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\right.}{6\sqrt{b}cef^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}(de-cf)^2\left(5bde^2-af(4de+cf)\right)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
1/6*(2*a*d^2*e*f-b*(3*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*x*(d*x^2+c)^(1/2)/e/
f^3/(b*x^2+a)^(1/2)+1/3*d^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/2*(-c*
f+d*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)-1/6*a^(1/2)*(2*
a*d^2*e*f-b*(3*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/e/f^3/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/6*a^(3/2)*(4*a*d^2*e*f*(-4*c*f
+3*d*e)-b*(-3*c^3*f^3+9*c^2*d*e*f^2-25*c*d^2*e^2*f+15*d^3*e^3))*(d*x^2+c)^
(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)
/c/e/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^
(3/2)*(-c*f+d*e)^2*(5*b*d*e^2-a*f*(c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticPi(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c
/e^2/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 1697, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
(5*a*Sqrt[b/a]*c*d^2*e^3*f^2*x - 6*a*Sqrt[b/a]*c^2*d*e^2*f^3*x + 3*a*Sqrt[
b/a]*c^3*e*f^4*x + 5*b*Sqrt[b/a]*c*d^2*e^3*f^2*x^3 + 5*a*Sqrt[b/a]*d^3*e^3
*f^2*x^3 - 6*b*Sqrt[b/a]*c^2*d*e^2*f^3*x^3 - 4*a*Sqrt[b/a]*c*d^2*e^2*f^3*x
^3 + 3*b*Sqrt[b/a]*c^3*e*f^4*x^3 + 3*a*Sqrt[b/a]*c^2*d*e*f^4*x^3 + 5*b*Sqr
t[b/a]*d^3*e^3*f^2*x^5 - 4*b*Sqrt[b/a]*c*d^2*e^2*f^3*x^5 + 2*a*Sqrt[b/a]*d
^3*e^2*f^3*x^5 + 3*b*Sqrt[b/a]*c^2*d*e*f^4*x^5 + 2*b*Sqrt[b/a]*d^3*e^2*f^3
*x^7 - I*c*e*f*(2*a*d^2*e*f + b*(-15*d^2*e^2 + 20*c*d*e*f - 3*c^2*f^2))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] - I*e*(2*a*d^2*e*f*(-6*d*e + 7*c*f) + b*(15*d^3*e^3
 - 15*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 3*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 (15*I)*b*d^3*e^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (30*I)*b*c*d^2*e^4*f*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - (12*I)*a*d^3*e^4*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15
*I)*b*c^2*d*e^3*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (21*I)*a*c*d^2*e^3*f^2*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] - (6*I)*a*c^2*d*e^2*f^3*Sqrt[1 + (b*x^2)/a]*Sq...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 420$$

$$\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{d\left(\frac{\int \frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{b\left(\frac{d\left(\frac{c(3bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{d\left(\frac{2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(\frac{d\left(\frac{2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{(}{\sqrt{b}}}{}\right)}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\right)}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 420

$$b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 324$$

$$b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 388

$$\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 414

$$b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.}{}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\cdots\right)}{\;}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 324

$$
b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\cdots\right.
$$

$$
\frac{(be-af)\left(\frac{d\left(\frac{d\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}
$$

$$\Big\downarrow \text{320}$$

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\cdots\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx\cdots\right.}{\sqrt{bx^2+\cdots}}}{f}\cdots\right.$$

$$\overline{\qquad\qquad\qquad f\qquad\qquad\qquad}$$

↓ 388

$$\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-\cdots}{\cdots}\right)}{f}$$

$$\downarrow \; 313$$

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\cdots\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}\cdots\right)}{f}\cdots\right.}{f}$$

↓ 414

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f} -$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\ldots$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

↓ 425

$$
\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\cdots\right)}{\cdots}
$$

$$
\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\right)}{f}\cdots\right)}{f}
$$

↓ 414

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

$$f$$

↓ 425

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f} -$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f} -$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}$$

↓ 413

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+cx}}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}}{f}\right.$$

$\downarrow$ 412

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}-\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

↓ 424

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f} - \cdots\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.$$

↓ 406

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

↓ 320

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f} -$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\ldots\right)}{f}\ldots\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\cdots\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\right)}{f}\cdots\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1816 vs. 2(556) = 1112.

Time = 21.04 (sec) , antiderivative size = 1817, normalized size of antiderivative = 3.06

method	result	size
elliptic	Expression too large to display	1817
risch	Expression too large to display	1876
default	Expression too large to display	2675

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(c^2*f^2-
2*c*d*e*f+d^2*e^2)/e/f^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1
/3*d^2/f^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+7/3/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^2/f^2*a*c-2/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^3/f^3*a*e-5/6/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d/f^2*b*c^2+5/2/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^3/f^4*b*e^2+1/3*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*d^2/f^2*a*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+10/3*
c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*d/f^2*b*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+
1/e/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(
1/2))*a*c^2*d+5*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d
)^(1/2)/(-b/a)^(1/2))*b*c*d^2-7/2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{5}{2}}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output

```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*d**2*e*x + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f*x - 4
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b*d**2*e*x**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a
*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4
+ a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**
2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d**2*e*f**2 - 8*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2
*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2
*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*
x**8),x)*a*b*c*d**2*f**3*x**2 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*
f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 +
b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**3*e**2*f + 7*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c
*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**
2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*
f**2*x**8),x)*a*b*d**3*e*f**2*x**2 - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a
*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2...
```

3.144 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 429

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \frac{d(3de-cf)x\sqrt{a+bx^2}}{2ef^2\sqrt{c+dx^2}} - \frac{(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef\,(e+fx^2)}$$
$$- \frac{\sqrt{c}\sqrt{d}(3de-cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2ef^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{c^{3/2}\sqrt{d}(3be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2aef^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{c^{3/2}(3bde^2-af(2de+cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2f^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/2*d*(-c*f+3*d*e)*x*(b*x^2+a)^(1/2)/e/f^2/(d*x^2+c)^(1/2)-1/2*(-c*f+d*e)*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)-1/2*c^(1/2)*d^(1/2)*(-c*f+
3*d*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*
c/a/d)^(1/2))/e/f^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/2*c^
(3/2)*d^(1/2)*(-a*f+3*b*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*
x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e/f^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)-1/2*c^(3/2)*(3*b*d*e^2-a*f*(c*f+2*d*e))*(b*x^2+a)^(1/2)*Ellipt
icPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^
(1/2)/e^2/f^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \frac{ibcef(-3de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ie}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(I*b*c*e*f*(-3*d*e + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e*(-3*b*d^2*e^2 + 2
*a*d^2*e*f + b*c^2*f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2
)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(d*e) + c*f)*(Sqrt[b/
a]*e*f^2*x*(a + b*x^2)*(c + d*x^2) - I*(-3*b*d*e^2 + a*f*(2*d*e + c*f))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e),
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*e^2*f^3*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1030 vs. $2(429) = 858$.

Time = 1.49 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.40, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.656$, Rules used = {425, 420, 324, 320, 388, 313, 414, 425, 414, 425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(\frac{d\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{d\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 388

$$\frac{b\left(\frac{d\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int\frac{}{\sqrt{bx^2+}}}{}\right.}{f}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\cdots}{\cdots}\right.}{f}\right.}{f}$$

$\downarrow$ 412

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\cdots\right.}{f}\right.}{f}$$

$\downarrow$ 424

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{f}\cdots\right)}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{f}\cdots\right)}{\cdots}$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\right.$$

$$f$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

$\downarrow$ 388

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{f\left((de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

↓ 313

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a}{a(dx^2+c}}\cdots}\right.}{f}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

$\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right)}{a\left(dx^2+c\cdots\right)}}}\cdots\right.}{f}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow\ 413$$

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\right.}{f}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

↓ 412

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
-(((b*e - a*f)*(-(((d*e - c*f)*((Sqrt[-a]*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c
)])/(Sqrt[b]*e*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - ((d*e - c*f)*((f^2*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2))
- (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*
f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b
]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])))/f))/f) + (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
a*e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f) + (b*((d
*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (c^(3/2)*(d*e -...
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(397) = 794$.

Time = 5.10 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.51

method	result	size
elliptic	Expression too large to display	1075
default	Expression too large to display	1572

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(c*f-d*e)
/e/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^2/f^2*a+1/2/e^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c^2-3/2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/f^3*e+3/2
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*d*b/f^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3
/2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^
(1/2))*b*c*d+3/2*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2))*b*d^2-1/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d^2+1/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*Ell...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*
c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**
6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d
*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e*f - int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*f**
2*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*e*f - 2*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*f**
2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*b**2*d**2*e**2 + 3*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2...
```

3.145 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 413

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx &= -\frac{bx\sqrt{c+dx^2}}{2ef\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)}+\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad-\frac{a^{3/2}\sqrt{b}(de-cf)\sqrt{c+dx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad+\frac{a^{3/2}\left(bde^2-acf^2\right)\sqrt{c+dx^2}\,\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/2*b*x*(d*x^2+c)^(1/2)/e/f/(b*x^2+a)^(1/2)+1/2*x*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/e/(f*x^2+e)+1/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/f/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*b^(1/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*I
nverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/f/(-a*f+b*
e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-a*c*f^2+b
*d*e^2)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a
*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \frac{\frac{acx}{e+fx^2}+\frac{bcx^3}{e+fx^2}+\frac{adx^3}{e+fx^2}+\frac{bdx^5}{e+fx^2}+\frac{ia\sqrt{\frac{b}{a}}c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\,\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{f}-\frac{ia\sqrt{\frac{b}{a}}(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{Elli}}{f^2}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
((a*c*x)/(e + f*x^2) + (b*c*x^3)/(e + f*x^2) + (a*d*x^3)/(e + f*x^2) + (b*
d*x^5)/(e + f*x^2) + (I*a*Sqrt[b/a]*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f - (I*a*Sqrt[b/a]*(d*
e + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)])/f^2 - (I*a*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]
*e) + (I*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticP
i[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f^2)/(2*e*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{423}$$

$$\frac{bd\int \frac{e-fx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{406}$$

$$\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{320}$$

$$\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{388}$$

$$\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \ 313$$

$$\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \ 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2\sqrt{a+bx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \ 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \ 412$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+$$

$$\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(e + f*x^2)) + (b*d*(-(f*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*e*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*f^2) + (Sqrt[-a]*((a*c)/e
- (b*d*e)/f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.35

method	result
elliptic	$\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2e\left(f\,x^2+e\right)}+\frac{bd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}}{2ef\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}$
default	$\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(\sqrt{-\frac{b}{a}}\,bde\,f^2x^5+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce\,f^2x^2+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*x/e*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/2*b*d/f^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/e/f*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*b/e/f*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(
-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-1/2/f^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f^2x^4+2ef\,x^2+e^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e**2 + 2*e*f*x**2 + f**2*x**4),x)
```

3.146 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 440

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx \\
&= \frac{bx\sqrt{c+dx^2}}{2e(de-cf)\sqrt{a+bx^2}} - \frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)} \\
&\quad - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2e(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{a^{3/2}(bde^2-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/2*f*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1
/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/(-c
*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*b^(1/2
)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/c/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^
(3/2)*(b*d*e^2-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-a*f+b*e
)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.93 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\frac{acfx}{e+fx^2} - \frac{bcfx^3}{e+fx^2} - \frac{adfx^3}{e+fx^2} - \frac{bdfx^5}{e+fx^2} - ia\sqrt{\frac{b}{a}}c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) + \frac{ia\sqrt{\frac{b}{a}}(-de+cf)\sqrt{1-}}{}}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-((a*c*f*x)/(e + f*x^2)) - (b*c*f*x^3)/(e + f*x^2) - (a*d*f*x^3)/(e + f*x
^2) - (b*d*f*x^5)/(e + f*x^2) - I*a*Sqrt[b/a]*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (I*a*Sqrt[b
/a]*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f - ((2*I)*a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
)/Sqrt[b/a] + (I*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f + (I*a*c*f*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e))/(2*e*(d*e - c*f)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{424}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x}{2e(e+f}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(f}}{2e(be-af)(de-cf)}\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{}{\sqrt{bx^2}}}{2e(be-af)(de-c}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-$$

$$\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}}{}\right.\right.}{2e(be-af)(de-cf)}\right.}{f}$$

$\big\downarrow$ 413

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a}}{b\sqrt{c}}\right.\right.}{2e(be-af)(de-}\right.}{f}$$

$\big\downarrow$ 413

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{2e(be}\right.}{f}$$

$\big\downarrow$ 412

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{}\right.}{f}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[-a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(e*f*Sqrt[a + b*x^2]*S
qrt[c + d*x^2]) - ((b*e - a*f)*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2
*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(
3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*S
qrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 6.68 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.65

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\,\Bigg(\frac{f x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2(cf-de)e(f\,x^2+e)}-\frac{bd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2(cf-de)f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{Elliptic}}{2e(cf-de)\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4}}}{}$
default	$\frac{\Bigg(\sqrt{-\frac{b}{a}}\,bde\,f^2x^5+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce\,f^2x^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd\,e^2f\,x^2-}{}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f/(c*f-d*
e)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2*b*d/(c*f-d*e)/f/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/e/(c*f-
d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*b
/e/(c*f-d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))+1/2/e^2*f/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2))*a*c-1/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1
/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+1/2/(c*f-d*e)/f/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**2 + 2*c*e*f*x**2 + c*f**2*x*
*4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)
```

3.147 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 415

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx &= -\frac{fx\sqrt{a+bx^2}}{2e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\ &+ \frac{\sqrt{d}(2de+cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2\sqrt{c}e(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{c}\sqrt{d}f(3bce-4ade+acf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2ae(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{c^{3/2}f(3bde^2-af(4de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/2*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*d^(1/2
)*(c*f+2*d*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))/c^(1/2)/e/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+1/2*c^(1/2)*d^(1/2)*f*(a*c*f-4*a*d*e+3*b*c*e)*(b*x^2+a)^
(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e/(-c
*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f*(3
*b*d*e^2-a*f*(-c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1
+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-c*f+d*e)^3/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(cf^2(c+dx^2)+2d^2e(e+fx^2)\right)+ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+\cdots}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(c*f^2*(c + d*x^2) + 2*d^2*e*(e + f*x^2)) + I*c
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(2*d*e + c*f)*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*e*(d*e - c*f)*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*b*d*e^2 + a*f*(-4*d*e + c*f))*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*c
*e^2*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1179 vs. 2(415) = 830.

Time = 1.65 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.84, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {425, 421, 25, 400, 313, 320, 414, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 313

$$\frac{b\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 320

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 414

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 426

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 421

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 400

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+\right.}}{de-cf}\right.}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{}{\sqrt{bx^2}}}{}\right.}{f}$$

$\downarrow$ 320

$$b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{b}{a}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 414

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{c}\sqrt{}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}$$

$\downarrow$ 424

$$b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\downarrow 406$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\downarrow 320$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$\downarrow$ 388

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(b}\right.}{(de-cf)^2}\right)}{de-cf}\right.}{}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{}}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}$$

$\downarrow$ 413

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(b}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$\downarrow$ 413

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(b}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$\downarrow$ 412

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(b*((d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sq
rt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))
/(d*e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e),
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*(d*e - c*f)^2*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - ((b*e - a*f)*
(-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f
)*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sq
rt[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))
/(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d
*e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
 ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d
*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(d*e - c*f)) + (d*((d*(-((Sqr
t[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(389) = 778.

Time = 9.00 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.10

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{2(cf-de)^2 e\left(f x^2+e\right)}+\frac{\left(b d x^2+a d\right) d x}{c(cf-de)^2 \sqrt{\left(x^2+\frac{c}{d}\right)\left(b d x^2+a d\right)}}-\frac{\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.}{2\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+x^2 b c+a c}\,(cf-de)^2}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^2/(c*f-
d*e)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+(b*d*x^2+a*d)/c*d
*x/(c*f-d*e)^2/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)-1/2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d*b/(c*f-d*e)^2+1/2*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f*
b/(c*f-d*e)^2/e*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*f*b/(c*f-d*e)^2/e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*d*b/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))+1/2/(c*f-d*e)^2/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-2/(c*f-d*e)^2/e*f/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+3/2/(c*f-d*e)
^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2
))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f^2x^8+2cd\,f^2x^6+2d^2ef\,x^6+c^2f^2x^4+4cdef\,x^4+d^2e^2x^4+2c^2ef\,x^2$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e**2 + 2*c**2*e*f*x**2 + c**
2*f**2*x**4 + 2*c*d*e**2*x**2 + 4*c*d*e*f*x**4 + 2*c*d*f**2*x**6 + d**2*e*
*2*x**4 + 2*d**2*e*f*x**6 + d**2*f**2*x**8),x)
```

3.148 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 593

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{d(2de+3cf)x\sqrt{a+bx^2}}{6ce(de-cf)^2(c+dx^2)^{3/2}}$$
$$-\frac{fx\sqrt{a+bx^2}}{2e(de-cf)(c+dx^2)^{3/2}(e+fx^2)}$$
$$-\frac{\sqrt{d}(ad(4d^2e^2-16cdef-3c^2f^2)-bc(2d^2e^2-14cdef-3c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{6c^{3/2}(bc-ad)e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{d}(15b^2c^3ef^2+3a^2cdf^2(6de-cf)-ab(2d^3e^3-4cd^2e^2f+35c^2def^2-3c^3f^3))\sqrt{a+bx^2}\,\text{EllipticF}\left(\text{arc}\right.}{6a\sqrt{c}(bc-ad)e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}f^2(5bde^2-af(6de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/6*d*(3*c*f+2*d*e)*x*(b*x^2+a)^(1/2)/c/e/(-c*f+d*e)^2/(d*x^2+c)^(3/2)-1/2
*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)-1/6*d^(1/2)*(a
*d*(-3*c^2*f^2-16*c*d*e*f+4*d^2*e^2)-b*c*(-3*c^2*f^2-14*c*d*e*f+2*d^2*e^2)
)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d
)^(1/2))/c^(3/2)/(-a*d+b*c)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)-1/6*d^(1/2)*(15*b^2*c^3*e*f^2+3*a^2*c*d*f^2*(-c*f+6*d*e)-
a*b*(-3*c^3*f^3+35*c^2*d*e*f^2-4*c*d^2*e^2*f+2*d^3*e^3))*(b*x^2+a)^(1/2)*I
nverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*
d+b*c)/e/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/2*
c^(3/2)*f^2*(5*b*d*e^2-a*f*(-c*f+6*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2
)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(
-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}ex(a+bx^2)\left(3c^2(bc-ad)f^3(c+dx^2)^2+2cd^2(bc-ad)e(-de+cf)\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*x*(a + b*x^2)*(3*c^2*(b*c - a*d)*f^3*(c + d*x^2)^2 + 2*c*d^
2*(b*c - a*d)*e*(-(d*e) + c*f)*(e + f*x^2) + 2*d^2*e*(2*a*d*(d*e - 4*c*f)
+ b*c*(-(d*e) + 7*c*f))*(c + d*x^2)*(e + f*x^2))) - I*c*Sqrt[1 + (b*x^2)/a
]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(a*d*(4*d^2*e^2 - 16*c*
d*e*f - 3*c^2*f^2) + b*c*(-2*d^2*e^2 + 14*c*d*e*f + 3*c^2*f^2))*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(-(b*e*(2*d^2*e^2 + c*d
*e*f - 3*c^2*f^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + 3*c*f*
(5*b*d*e^2 + a*f*(-6*d*e + c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)])))/(6*Sqrt[b/a]*c^2*(b*c - a*d)*e^2*(d*e - c*f)^3*Sqrt
[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx\ldots}{3c(\ldots}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{d\left(-\frac{\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2d}{\sqrt{c}\sqrt{c+dx^2}(bc-}}{3c(bc-ad)}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7}{\sqrt{c}\sqrt{}}}{3c(bc-ad)}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx}}{}}{3c(bc-ad)}\right)}{(de-cf)}\right)}{f}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 412

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+}{3c(bc-ad)}\right)}{}\right)}{f}$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 426

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right.\cdots}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 421

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right.\cdots}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)}\cdots\right)}{\cdots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}(\cdots}}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)}\cdots\right)}{\cdots}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(\cdots\right.}\cdots}{\cdots}\cdots\right)}{f}$$

$\downarrow$ 400

$$\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right.\cdots}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2\cdots}}}{bc-ad}}{3c(bc-ad)}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{f}$$

↓ 313

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{b(\cdots}{\cdots}}{3c(bc-ad)\cdots}\right)}{(de-cf)^2\cdots}\right)}{de-cf\cdots}\cdots\right)}{f\cdots}$$

↓ 320

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{\cdots}\right)}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{b\cdots}{\cdots}}{3c(bc\cdots}\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{f\cdots}$$

↓ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right)}{}\right)}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right)}{(de-cf)}\right)}{de-cf}\right)$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

↓ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right)}{f}$$

$$(be-af)\frac{\left(\frac{d\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right)}{de-cf}\right.}{f}$$

↓ 412

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}\cdots}}{\cdots}\right.}{\cdots}\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{\cdots}\right.}{\cdots}\right.}{de-cf\cdots}\right.}{f\cdots}$$

↓ 426

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{}\right.}{}\right)}{de-cf}\right.$$

↓ 421

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\ }}}{}\right.}{}\right)}{de-cf}\right.}{}$$

↓ 25

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{}\right.}{}\right)}{de-cf}\right.$$

$\downarrow$ 400

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\right.}}}{}\right.}{}\right.}{de-cf}\right.$$

↓ 313

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\right.}}{}\right.}{}\right.}{de-cf}\right.$$

↓ 320

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\left.\right.}}}{}\right.}{}\right.}{de-cf}\right.$$

↓ 414

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right)$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{}\right.}{}\right)}{de-cf}\right.$$

$\downarrow$ 424

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{}\right.}{}\right)}{de-cf}\right.$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2932 vs. 2(561) = 1122.

Time = 18.63 (sec) , antiderivative size = 2933, normalized size of antiderivative = 4.95

method	result	size
elliptic	Expression too large to display	2933
default	Expression too large to display	5111

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^3*d/e/(
c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/(d*f*x^2+d*e)+1/3/(c*f-d*e)^2/c*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^
2+c/d)^2+1/3*(b*d*x^2+a*d)/c^2*d/(a*d-b*c)*x*(8*a*c*d*f-2*a*d^2*e-7*b*c^2*
f+b*c*d*e)/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)/((x^2+c/d)*(b*d*x^2+a*d))
^(1/2)+8/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d
^2/c/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)*a*f-2/3/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^3/c^2/(c*f-d*e)/(c^2*f^2-2*c*d*e*
f+d^2*e^2)*a*e+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*d^2/c/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)*b*e+1/2/e^2/(c^2*f^2-2*
c*d*e*f+d^2*e^2)/(c*f-d*e)*f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-3/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)
*f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^
(1/2))*a*d-17/6/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{d^3f^2x^{10}+3c\,d^2f^2x^8+2d^3ef\,x^8+3c^2d\,f^2x^6+6c\,d^2ef\,x^6+d^3e^2x^6+c^3}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**3*e**2 + 2*c**3*e*f*x**2 + c**
3*f**2*x**4 + 3*c**2*d*e**2*x**2 + 6*c**2*d*e*f*x**4 + 3*c**2*d*f**2*x**6
+ 3*c*d**2*e**2*x**4 + 6*c*d**2*e*f*x**6 + 3*c*d**2*f**2*x**8 + d**3*e**2*
x**6 + 2*d**3*e*f*x**8 + d**3*f**2*x**10),x)
```

3.149 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 705

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \frac{(6a^2d^2ef^2 - abf(95d^2e^2 - 124cdef + 15c^2f^2) + b^2e(105d^2e^2 - 170cdef}{30ef^4\sqrt{a+bx^2}}$$

$$- \frac{d(10bde - 11bcf - 6adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15f^3}$$

$$+ \frac{bd^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f^2} - \frac{(be-af)(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^3(e+fx^2)}$$

$$- \frac{\sqrt{a}(6a^2d^2ef^2 - abf(95d^2e^2 - 124cdef + 15c^2f^2) + b^2e(105d^2e^2 - 170cdef + 61c^2f^2))\sqrt{c+dx^2}\,E\Big(\mathrm{arct}}{30\sqrt{b}ef^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$- \frac{a^{3/2}(6ad^2ef(10de - 13cf) - b(105d^3e^3 - 205cd^2e^2f + 113c^2def^2 - 15c^3f^3))\sqrt{c+dx^2}\,\mathrm{EllipticF}\Big(\mathrm{arctan}}{30\sqrt{b}cef^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$- \frac{a^{3/2}(de-cf)^2(be(7de-2cf) - af(4de+cf))\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be}, \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/30*(6*a^2*d^2*e*f^2-a*b*f*(15*c^2*f^2-124*c*d*e*f+95*d^2*e^2)+b^2*e*(61*
c^2*f^2-170*c*d*e*f+105*d^2*e^2))*x*(d*x^2+c)^(1/2)/e/f^4/(b*x^2+a)^(1/2)-
1/15*d*(-6*a*d*f-11*b*c*f+10*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3+
1/5*b*d^2*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2-1/2*(-a*f+b*e)*(-c*f+d*e
)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^3/(f*x^2+e)-1/30*a^(1/2)*(6*a^2*
d^2*e*f^2-a*b*f*(15*c^2*f^2-124*c*d*e*f+95*d^2*e^2)+b^2*e*(61*c^2*f^2-170*
c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/e/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)-1/30*a^(3/2)*(6*a*d^2*e*f*(-13*c*f+10*d*e)-b*(-15*c^3*f^3
+113*c^2*d*e*f^2-205*c*d^2*e^2*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJaco
biAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^4/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-c*f+d*e)^2*(b*e*(-2*
c*f+7*d*e)-a*f*(c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f^4/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.13 (sec) , antiderivative size = 2953, normalized size of antiderivative = 4.19

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
(-35*a*b*Sqrt[b/a]*c*d^2*e^4*f^2*x + 52*a*b*Sqrt[b/a]*c^2*d*e^3*f^3*x + 27
*a^2*Sqrt[b/a]*c*d^2*e^3*f^3*x - 15*a*b*Sqrt[b/a]*c^3*e^2*f^4*x - 30*a^2*S
qrt[b/a]*c^2*d*e^2*f^4*x + 15*a^2*Sqrt[b/a]*c^3*e*f^5*x - 35*a*b*(b/a)^(3/
2)*c*d^2*e^4*f^2*x^3 - 35*a*b*Sqrt[b/a]*d^3*e^4*f^2*x^3 + 52*a*b*(b/a)^(3/
2)*c^2*d*e^3*f^3*x^3 + 65*a*b*Sqrt[b/a]*c*d^2*e^3*f^3*x^3 + 27*a^2*Sqrt[b/
a]*d^3*e^3*f^3*x^3 - 15*a*b*(b/a)^(3/2)*c^3*e^2*f^4*x^3 - 23*a*b*Sqrt[b/a]
*c^2*d*e^2*f^4*x^3 - 18*a^2*Sqrt[b/a]*c*d^2*e^2*f^4*x^3 + 15*a*b*Sqrt[b/a]
*c^3*e*f^5*x^3 + 15*a^2*Sqrt[b/a]*c^2*d*e*f^5*x^3 - 35*a*b*(b/a)^(3/2)*d^3
*e^4*f^2*x^5 + 38*a*b*(b/a)^(3/2)*c*d^2*e^3*f^3*x^5 + 13*a*b*Sqrt[b/a]*d^3
*e^3*f^3*x^5 + 7*a*b*(b/a)^(3/2)*c^2*d*e^2*f^4*x^5 + 10*a*b*Sqrt[b/a]*c*d^
2*e^2*f^4*x^5 + 12*a^2*Sqrt[b/a]*d^3*e^2*f^4*x^5 + 15*a*b*Sqrt[b/a]*c^2*d*
e*f^5*x^5 - 14*a*b*(b/a)^(3/2)*d^3*e^3*f^3*x^7 + 28*a*b*(b/a)^(3/2)*c*d^2*
e^2*f^4*x^7 + 18*a*b*Sqrt[b/a]*d^3*e^2*f^4*x^7 + 6*a*b*(b/a)^(3/2)*d^3*e^2
*f^4*x^9 - I*c*e*f*(6*a^2*d^2*e*f^2 + a*b*f*(-95*d^2*e^2 + 124*c*d*e*f - 1
5*c^2*f^2) + b^2*e*(105*d^2*e^2 - 170*c*d*e*f + 61*c^2*f^2))*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - I*e*(12*a^2*d^2*e*f^2*(-5*d*e + 6*c*f) + b^2*e*(-105*d^3*e^3
 + 135*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 31*c^3*f^3) + a*b*f*(165*d^3*e^3 - 21
5*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\int \sqrt{bx^2+a}(dx^2+c)^{3/2}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 318$$

$$\frac{b\left(\frac{d\left(\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 403$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\int \frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 406$$

$$\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+\right.}{}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 388

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 313

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 418

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a\cdots}}{5b}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 25

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a\cdots}}{5b}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 403

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c}\cdots}{5b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 27

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c}\cdots}{5b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 406

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\,(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\,(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 388

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c\cdots}}{5b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c\cdots}}{5b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 414$$

$$b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{c \cdots}}{5b}\right.}{f}\right. \cdots$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx \cdots}}{b\sqrt{dx \cdots}}\right.\right.}{5b}\right.}{f}\right. \cdots$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 420

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right)}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 318

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\int\frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 406

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{c(3bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\right.}{f}$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(d\right.}{\sqrt{bx^2}}}{f}\right.}{f}\right.}{f}$$

↓ 388

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx\cdots}}{b\sqrt{dx\cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3b}\right)}{f}-\frac{(de\cdots}{\cdots}\cdots\right.}{f}$$

↓ 313

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2}}\right.}{3b}\right.}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 420

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2}}\right.}{3b}\right.}{f}\right.}{f}\right.}{f}$$

↓ 324

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2}}\right.}{3b}\right.}{f}\right.}{f}\right.$$

↓ 320

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx}}{b\sqrt{dx}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2}}\right.}{3b}\right.}{f}\right.}{}\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx\cdots}}{b\sqrt{dx\cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2\cdots}}\cdots\right.}{3b}\cdots\right.}{f}\cdots\right.\right.$$

$\downarrow$ 313

$$b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx\cdots}}{b\sqrt{dx\cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2\cdots}}\cdots\right.}{3b}\cdots\right.}{f}\cdots\right.\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(661) = 1322.

Time = 22.22 (sec) , antiderivative size = 1807, normalized size of antiderivative = 2.56

method	result	size
risch	Expression too large to display	1807
elliptic	Expression too large to display	3196
default	Expression too large to display	4748

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*d*x*(3*b*d*f*x^2+6*a*d*f+11*b*c*f-10*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/f^3+1/15/f^3*((39*a^2*c*d^2*f^3-30*a^2*d^3*e*f^2+79*a*b*c^2*d*f^3-1
70*a*b*c*d^2*e*f^2+90*a*b*d^3*e^2*f+15*b^2*c^3*f^3-90*b^2*c^2*d*e*f^2+135*
b^2*c*d^2*e^2*f-60*b^2*d^3*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))+15/f^2*(3*a^2*c^2*d*f^4-6*a^2*c*d^2*e*f^3+3*a^2*d^
3*e^2*f^2+2*a*b*c^3*f^4-12*a*b*c^2*d*e*f^3+18*a*b*c*d^2*e^2*f^2-8*a*b*d^3*
e^3*f-2*b^2*c^3*e*f^3+9*b^2*c^2*d*e^2*f^2-12*b^2*c*d^2*e^3*f+5*b^2*d^3*e^4
)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))+15*(a^2*c^3*f^5-3*a^2*c^2*d*e*f^4+3*a^2*c*d^2*e^2*f^3-a^2*d^3*e^3*f^2-
2*a*b*c^3*e*f^4+6*a*b*c^2*d*e^2*f^3-6*a*b*c*d^2*e^3*f^2+2*a*b*d^3*e^4*f+b^
2*c^3*e^2*f^3-3*b^2*c^2*d*e^3*f^2+3*b^2*c*d^2*e^4*f-b^2*d^3*e^5)/f^2*(1/2*
f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e
*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}{(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

3.150 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 542

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = -\frac{b(be(15de-11cf)-af(11de-3cf))x\sqrt{c+dx^2}}{6ef^3\sqrt{a+bx^2}}$$
$$+\frac{bdx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^2}+\frac{(be-af)(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^2(e+fx^2)}$$
$$+\frac{\sqrt{a}\sqrt{b}(be(15de-11cf)-af(11de-3cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6ef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(6ad^2ef-b(15d^2e^2-16cdef+3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6\sqrt{b}cef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(de-cf)(be(5de-2cf)-af(2de+cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/6*b*(b*e*(-11*c*f+15*d*e)-a*f*(-3*c*f+11*d*e))*x*(d*x^2+c)^(1/2)/e/f^3/
(b*x^2+a)^(1/2)+1/3*b*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/2*(-a*f+b*
e)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)+1/6*a^(1/2
)*b^(1/2)*(b*e*(-11*c*f+15*d*e)-a*f*(-3*c*f+11*d*e))*(d*x^2+c)^(1/2)*Ellip
ticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/f^3/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/6*a^(3/2)*(6*a*d^2*e*f-b*(3*c^2*
f^2-16*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-c*f+d*e)*(b*e*(-2*c*f+5*d*e)-a*f*(c*f+2
*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*
f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.65 (sec) , antiderivative size = 2079, normalized size of antiderivative = 3.84

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(5*a*b*Sqrt[b/a]*c*d*e^3*f^2*x - 3*a*b*Sqrt[b/a]*c^2*e^2*f^3*x - 3*a^2*Sqr
t[b/a]*c*d*e^2*f^3*x + 3*a^2*Sqrt[b/a]*c^2*e*f^4*x + 5*a*b*(b/a)^(3/2)*c*d
*e^3*f^2*x^3 + 5*a*b*Sqrt[b/a]*d^2*e^3*f^2*x^3 - 3*a*b*(b/a)^(3/2)*c^2*e^2
*f^3*x^3 - 4*a*b*Sqrt[b/a]*c*d*e^2*f^3*x^3 - 3*a^2*Sqrt[b/a]*d^2*e^2*f^3*x
^3 + 3*a*b*Sqrt[b/a]*c^2*e*f^4*x^3 + 3*a^2*Sqrt[b/a]*c*d*e*f^4*x^3 + 5*a*b
*(b/a)^(3/2)*d^2*e^3*f^2*x^5 - a*b*(b/a)^(3/2)*c*d*e^2*f^3*x^5 - a*b*Sqrt[
b/a]*d^2*e^2*f^3*x^5 + 3*a*b*Sqrt[b/a]*c*d*e*f^4*x^5 + 2*a*b*(b/a)^(3/2)*d
^2*e^2*f^3*x^7 + I*b*c*e*f*(b*e*(15*d*e - 11*c*f) + a*f*(-11*d*e + 3*c*f))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] - I*e*(6*a^2*d^2*e*f^2 + b^2*e*(15*d^2*e^2 - 6*c
*d*e*f - 5*c^2*f^2) + a*b*f*(-21*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2))*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] + (15*I)*b^2*d^2*e^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (21*I)
*b^2*c*d*e^4*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (21*I)*a*b*d^2*e^4*f*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] + (6*I)*b^2*c^2*e^3*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (24
*I)*a*b*c*d*e^3*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 418$$

$$\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}+\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{\int \frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int \frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+d}\right.}{f^2}}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 320$$

$$b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\left(-adf-4bcf+3bde\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 388

$$b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(-adf-4bcf+3bde\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E}{b\sqrt{d}\sqrt{c}}\right.\right.\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2 \text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 324

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2 \text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx}{\sqrt{bx^2+}}}{f}\right.}{f}$$

$\big\downarrow$ 388

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-\cdots}{}\right.}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+}}{f}\right)}{f}\right.}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2 \operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-adf-4bcf+\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\operatorname{Ellip}}{a\sqrt{d}ef\sqrt{c}}\right.}{f}\right.}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.\right.\right.}{\cdots}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\cdots\right)}{f}\cdots\right)}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.}{f}$$

$\downarrow$ 425

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.$$

↓ 413

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.$$

$$\downarrow 413$$

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.$$

↓ 412

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}}{f}\right.$$

↓ 424

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

↓ 406

$$
\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left(\left(3bde-4bcf-adf\right)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.\right.\right.}{\cdots}\right.}{f}
$$

$$
(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.
$$

↓ 320

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.\right.}{\cdots}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\right)}{f}\right.$$

↓ 388

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

↓ 313

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right.}{f}\right.$$

$\Big\downarrow$ 413

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left(\left(3bde-4bcf-adf\right)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}\right)}{f}\right.$$

↓ 413

$$b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.\right.}{\ldots}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2012 vs. $2(504) = 1008$.

Time = 21.36 (sec) , antiderivative size = 2013, normalized size of antiderivative = 3.71

method	result	size
risch	Expression too large to display	2013
elliptic	Expression too large to display	2160
default	Expression too large to display	3186

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/3/f^2*(1/f^2*(-2*b*f*(2*a*
d*f+2*b*c*f-3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*a^2*d^2*
f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b^2*c^
2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+9*b^2*
d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+11*a
*b*c*d*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1
2*a*b*d^2*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-12*b^2*c*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2)))+6/f^2*(a^2*c*d*f^3-a^2*d^2*e*f^2+a*b*c^2*f^3-4*a*b*c*d*e*f^2+3*a*b*d
^2*e^2*f-b^2*c^2*e*f^2+3*b^2*c*d*e^2*f-2*b^2*d^2*e^3)/e/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+3*(a^2*c^2*f^4-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}{\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}{\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}}{\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 11*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b*c*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2
*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x - 4*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b**2*c*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*2*d**2*e*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2
+ 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f*
*2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 +
 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a**2*b*d**3*e*f**2 - 3*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 +
 a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*
f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x
)*a**2*b*d**3*f**3*x**2 - 11*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/
(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**
4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e
**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b**2*c*d**2*e*f**2 - 11*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*
c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x*
*2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d
*f**2*x**8),x)*a*b**2*c*d**2*f**3*x**2 + 16*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x*...
```

3.151 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 429

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx &= \frac{b(3be-af)x\sqrt{c+dx^2}}{2ef^2\sqrt{a+bx^2}} - \frac{(be-af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef(e+fx^2)} \\
&- \frac{\sqrt{a}\sqrt{b}(3be-af)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}\sqrt{b}(3de-cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}(acf^2-be(3de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/2*b*(-a*f+3*b*e)*x*(d*x^2+c)^(1/2)/e/f^2/(b*x^2+a)^(1/2)-1/2*(-a*f+b*e)*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(-a*f+
3*b*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^
(3/2)*b^(1/2)*(-c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*
x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+1/2*a^(3/2)*(a*c*f^2-b*e*(-2*c*f+3*d*e))*(d*x^2+c)^(1/2)*Ellip
ticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(
1/2)/c/e^2/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \frac{ibcef(-3be+af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(I*b*c*e*f*(-3*b*e + a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(b*e - a*f)*(I*Sqrt
[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2) + b*e*(3*d*e + c*f)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] + (a*c*f^2 + b*e*(-3*d*e + 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(2*Sqrt[b/a]*e^2*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2
))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.69, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {425, 410, 324, 320, 388, 313, 414, 423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{410}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{b\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 414$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{def}\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 423$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{def}\sqrt{c+dx^2}\sqrt{\quad}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{bd\int\frac{e-fx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{def}\sqrt{c+dx^2}\sqrt{\quad}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$(be-af)\left(\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\right.$$
$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 388

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$(be-af)\left(\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(f\right.$$
$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{def}\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\right.}{b\sqrt{d}\sqrt{c+}}\right.\right.}{2ef^2}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{def}\sqrt{c+dx^2}\sqrt{}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2\sqrt{a+bx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}}{b\sqrt{d}\sqrt{}}\right.\right.}{2ef^2}\right.}{f}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.$$
$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\right.\right.}{2ef^2}\right.$$
$$\overline{\qquad\qquad f \qquad\qquad}$$

$\downarrow$ 412

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{}}\right.$$
$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{2ef^2}\right.$$
$$\overline{\qquad\qquad f \qquad\qquad}$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
-(((b*e - a*f)*((x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(e + f*x^2)) + (b
*d*(-(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*e*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d
]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*f^2) + (Sq
rt[-a]*((a*c)/e - (b*d*e)/f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b
]*e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/f) + (b*((b*(d*((x*Sqrt[a + b*x^2])
/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])))/f - (c^(3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*Ellip
ticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*S
qrt[d]*e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(397) = 794$.

Time = 5.14 (sec) , antiderivative size = 1168, normalized size of antiderivative = 2.72

method	result	size
elliptic	Expression too large to display	1168
default	Expression too large to display	1689

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a*f-b*e)
/e/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2/f^2*c+1/2/e^2/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c-1/f^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))
*b^2*c+3/2*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/
2)/(-b/a)^(1/2))*b^2*d+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*a*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*a*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+3/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))*b/f^2*a*d-3/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))*d*b^2/f^3*e+3/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/f^2*EllipticE(x*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
(2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x + sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*
a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x
**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b
*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e*f - 2*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*
x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b
*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2
*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*
c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**
6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d
*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*e*f - int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*f**
2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*b**2*d**2*e**2 + 3*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2...
```

3.152 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 493

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{b(be-af)x\sqrt{c+dx^2}}{2ef(de-cf)\sqrt{a+bx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(de-cf)} - \frac{fx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)} + \frac{\sqrt{a}\sqrt{b}(be-af)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ef(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(be(de-2cf)+af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/2*b*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/f/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/2*b*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)-1/2*f*x*(b*x^2+a)^(3/2)*(d*x
^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)+1/2*a^(1/2)*b^(1/2)*(-a*f+b*e)*(d*x^2+c
)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e
/f/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*
b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/c/e/f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/
2)*(b*e*(-2*c*f+d*e)+a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f/(
-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{-ibcef(-be+af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ib}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((-I)*b*c*e*f*(-(b*e) + a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e +
f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*e*(b*e + a*f)*
(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*e - a*f)*(Sqrt[b/a]*e*f^2*x*(
a + b*x^2)*(c + d*x^2) - I*(a*f*(-2*d*e + c*f) + b*e*(-(d*e) + 2*c*f))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*e^2*f^2*(d*e - c*f)*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.39, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {425, 414, 425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\downarrow \text{412}$$

$$\frac{\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\downarrow \text{424}$$

$$\frac{\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}d}{2e(be-af)(de-cf)}\right.}{f}\right.}{f}$$

$$\downarrow \text{406}$$

$$\frac{\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\right.}{f}\right.}{f}$$

$$\downarrow \text{320}$$

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\text{arc}\ldots\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\ldots}}\right.}{2e(be-af)(de-cf)}\ldots}{\ldots}\right.}{f\ldots}$$

$\downarrow$ 388

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\ldots}{a\sqrt{\ldots}}\right.}{2e(be-af)(de-cf)}\ldots}{\ldots}\right.}{f\ldots}$$

$\downarrow$ 313

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}d}{2e(be-af)(de-cf)}\right.}{}\right.}{f}$$

$\downarrow$ 413

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(}}{2e\sqrt{a+bx^2}(be-af)(de-cf)}\right.}{}\right.$$

$\downarrow$ 413

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int\frac{}{\sqrt{\frac{bx^2}{a}+1}}}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}\right.}{}\right.$$

$$\Big\downarrow \text{412}$$

$$\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{Elliptic}}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de}\right.}{}\right.$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
-(((b*e - a*f)*((Sqrt[-a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(e*f*S
qrt[a + b*x^2]*Sqrt[c + d*x^2]) - ((b*e - a*f)*((f^2*x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2)) - (b*d*(f*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*f)*(d*e - c*f)) +
(Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a
*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])))/f))/f) + (a^(3/2)*Sqrt[b]*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)
/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(c*e*f*Sqrt[a + b*x
^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. $2(455) = 910$.

Time = 6.57 (sec) , antiderivative size = 1481, normalized size of antiderivative = 3.00

method	result	size
elliptic	Expression too large to display	1481
default	Expression too large to display	1842

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a*f-b*e)
/(c*f-d*e)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/2*b/(c*f-d*
e)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a-1/2
*b/(c*f-d*e)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))*a-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d
*b/f/(c*f-d*e)*a+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))*d*b^2/f^2/(c*f-d*e)*e-1/2*b^2/(c*f-d*e)/f*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b^2/(c*f-d*e)/f*c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f/(c*f-d*e)/e^2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2
*c+1/2/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*a*b*c+1/2/(c*f-d*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx\right)b + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e**2 + 2*c*e*f*x**2 + c*f*
*2*x**4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)*b + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(c*e**2 + 2*c*e*f*x**2 + c*f**2*x**4 + d*e**2*x**
2 + 2*d*e*f*x**4 + d*f**2*x**6),x)*a
```

3.153 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 532

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx &= -\frac{(bc-ad)(2de+cf)x\sqrt{a+bx^2}}{2ce(de-cf)^2\sqrt{c+dx^2}} \\
&+\frac{b(3bce-2ade-acf)x\sqrt{c+dx^2}}{2ce(de-cf)^2\sqrt{a+bx^2}}-\frac{fx(a+bx^2)^{3/2}}{2e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\
&-\frac{\sqrt{a}\sqrt{b}(3bce-2ade-acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ce(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{a^{3/2}(af(4de-cf)-be(de+2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/2*(-a*d+b*c)*(c*f+2*d*e)*x*(b*x^2+a)^(1/2)/c/e/(-c*f+d*e)^2/(d*x^2+c)^(
1/2)+1/2*b*(-a*c*f-2*a*d*e+3*b*c*e)*x*(d*x^2+c)^(1/2)/c/e/(-c*f+d*e)^2/(b*
x^2+a)^(1/2)-1/2*f*x*(b*x^2+a)^(3/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e
)-1/2*a^(1/2)*b^(1/2)*(-a*c*f-2*a*d*e+3*b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/e/(-c*f+d*e)^2/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*b^(1/2)*(d*x^2+c)
^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/(-
c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(a*f*
(-c*f+4*d*e)-b*e*(2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-c*f+d*e)^2
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.51 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}efx(a+bx^2)\left(cf(be-af)\left(c+dx^2\right)+2d(bc-ad)e(e+fx^2)\right)-ic\sqrt{}}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*f*x*(a + b*x^2)*(c*f*(b*e - a*f)*(c + d*x^2) + 2*d*(b*c - a
*d)*e*(e + f*x^2))) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*(b*e*f*(3*b*c*e - a*(2*d*e + c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + (b*e - a*f)*(b*e*(d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - (a*f*(-4*d*e + c*f) + b*e*(d*e + 2*c*f))*EllipticPi[(a*f
)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(2*Sqrt[b/a]*c*e^2*f*(d*e
 - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(532) = 1064$.

Time = 1.95 (sec) , antiderivative size = 1407, normalized size of antiderivative = 2.64, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {425, 416, 313, 414, 425, 421, 25, 400, 313, 320, 414, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{416}$$

$$\frac{b\left(\frac{d\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\ldots}{\sqrt{bx^2+a}(dx^2+\ldots}}{f}\ldots\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\ldots}{\ldots}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{b}{a}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\Big/ f$$

$\downarrow$ 414

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(\cdots}{\sqrt{c}\sqrt{c\cdots}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\Big/ f$$

$\downarrow$ 426

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(\ldots}{\sqrt{c}\sqrt{\ldots}}\right)}{(de-cf)^2}\right)}{f}\ldots\right)$$

$$\ldots f$$

$\downarrow$ 421

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(\ldots}{\sqrt{c}\sqrt{\ldots}}\right)}{(de-cf)^2}\right)}{f}\ldots\right)$$

$\downarrow$ 25

$$\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(\cdots}{\sqrt{c}\sqrt{c\cdots}}\right.}{(de-cf)^2}\cdots}{f}\cdots\right.$$

$\downarrow$ 400

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\cdots}}{\sqrt{c}(bc\cdots}\right)}{(de-cf)^2}\right)}{f}\right.$$

↓ 313

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\cdots}}{\sqrt{c}(bc\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)$$

↓ 320

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(bc\cdots}\right.}{(de-cf)^2}\cdots}{f}\cdots\right.$$

↓ 414

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(bc\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{\cdots}$$

↓ 424

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(bc\cdots}\right.}{(de-cf)^2}\cdots}{f}\cdots\right.$$

↓ 406

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\cdots}}{\sqrt{c}(bc\cdots}\right)}{(de-cf)^2}\right)}{f}\right.$$

↓ 320

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\ldots}}{\sqrt{c}(bc\ldots}\right.}{(de-cf)^2}}{f}\right.$$

↓ 388

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\cdots}}{\sqrt{c}(bc\cdots}\right.}{(de-cf)^2}}{f}\right.$$

↓ 313

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\ldots}}{\sqrt{c}(bc\ldots}\right.}{(de-cf)^2}\ldots}{f}\ldots\right.$$

↓ 413

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(bc\cdots}\right.}{(de-cf)^2}\right.}{f}\right.$$

↓ 413

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} - \frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f} -$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} - \frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(bc\cdots}\right.}{(de-cf)^2}\right.}{f}\right.$$

↓ 412

$$\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{\cdots}}{\sqrt{c}(bc\cdots}\right)}{(de-cf)^2}\right)}{f}\right.$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(b*((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(Sqrt[c]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]) - (a^(3/2)*f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), Arc
Tan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[
a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/f - ((b*e - a*f)*((b*(
(d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d
]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(d*
e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*(d*e - c*f)^2*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - ((b*e - a*f)*(-((
f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e
 + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c
]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*
e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(494) = 988$.

Time = 9.30 (sec) , antiderivative size = 1536, normalized size of antiderivative = 2.89

method	result	size
elliptic	Expression too large to display	1536
default	Expression too large to display	2155

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a*f-b*e)
*f/e/(c*f-d*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+(b*d*x^2+
a*d)*(a*d-b*c)/c/(c*f-d*e)^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/2*f/(c*f-
d*e)^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a
)^(1/2))*a*b*c+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*b^2*d/(c*f-d*e)^2/f*e-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b/(c*f-d*e)^2*a*EllipticE(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/e^2*f^2/(c*f-d*e)^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c-2*f/(c*
f-d*e)^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b
/a)^(1/2))*a^2*d-1/2*e/(c*f-d*e)^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),
a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d/(c*f-d*e)^2*a+3/2*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f^2x^8+2cd\,f^2x^6+2d^2ef\,x^6+c^2f^2x^4+4cdef\,x^4+d^2e^2x^4+2c^2ef}\right.$$

$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f^2x^8+2cd\,f^2x^6+2d^2ef\,x^6+c^2f^2x^4+4cdef\,x^4+d^2e^2x^4+2c^2ef\,x^2+2cd\,e^2x^2+c^2e^2}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e**2 + 2*c**2*e*f*x**2
+ c**2*f**2*x**4 + 2*c*d*e**2*x**2 + 4*c*d*e*f*x**4 + 2*c*d*f**2*x**6 + d*
*2*e**2*x**4 + 2*d**2*e*f*x**6 + d**2*f**2*x**8),x)*b + int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(c**2*e**2 + 2*c**2*e*f*x**2 + c**2*f**2*x**4 + 2*c*
d*e**2*x**2 + 4*c*d*e*f*x**4 + 2*c*d*f**2*x**6 + d**2*e**2*x**4 + 2*d**2*e
*f*x**6 + d**2*f**2*x**8),x)*a
```

3.154 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 570

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = -\frac{(bc-ad)(2de+3cf)x\sqrt{a+bx^2}}{6ce(de-cf)^2(c+dx^2)^{3/2}} - \frac{fx(a+bx^2)^{3/2}}{2e(de-cf)(c+dx^2)^{3/2}(e+fx^2)} + \frac{\sqrt{d}(bce(4de+11cf)+a(4d^2e^2-16cdef-3c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{6c^{3/2}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(3a^2cdf^2(6de-cf)+3b^2c^2ef(3de+2cf)-2abde(d^2e^2+4cdef+10c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\right.}{6a\sqrt{c}\sqrt{d}e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}f(be-af)(af(6de-cf)-be(3de+2cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/6*(-a*d+b*c)*(3*c*f+2*d*e)*x*(b*x^2+a)^(1/2)/c/e/(-c*f+d*e)^2/(d*x^2+c)
^(3/2)-1/2*f*x*(b*x^2+a)^(3/2)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)+1/6*
d^(1/2)*(b*c*e*(11*c*f+4*d*e)+a*(-3*c^2*f^2-16*c*d*e*f+4*d^2*e^2))*(b*x^2+
a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/
c^(3/2)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/6
*(3*a^2*c*d*f^2*(-c*f+6*d*e)+3*b^2*c^2*e*f*(2*c*f+3*d*e)-2*a*b*d*e*(10*c^2
*f^2+4*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/
c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/e/(-c*f+d*e)^4/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/2*c^(3/2)*f*(-a*f+b*e)*(a*f*(-c*f+6*d
*e)-b*e*(2*c*f+3*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x
^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-c*f+d*e)^4/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.09 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(3c^2f^2(be-af)\left(c+dx^2\right)^2+2cd(bc-ad)e(-de+cf)\left(e\right.\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(3*c^2*f^2*(b*e - a*f)*(c + d*x^2)^2 + 2*c*d*(b
*c - a*d)*e*(-(d*e) + c*f)*(e + f*x^2) + 4*d*e*(a*d*(d*e - 4*c*f) + b*c*(d
*e + 2*c*f))*(c + d*x^2)*(e + f*x^2)) + I*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2
)*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(b*c*e*(4*d*e + 11*c*f) + a*(4*d^2*
e^2 - 16*c*d*e*f - 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)] + b*e*(d*e - c*f)*(5*b*c*e - 2*a*d*e - 3*a*c*f)*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] - 3*c*(b*e - a*f)*(a*f*(-6*d*e + c*f) + b*e*(3*d*
e + 2*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))
/(6*Sqrt[b/a]*c^2*e^2*(d*e - c*f)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e +
 f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \textcolor{blue}{400}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \textcolor{blue}{313}$$

$$\frac{b\left(\frac{d\left(\frac{\frac{ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}\right)}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 414

$$b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}\right.}{(de-cf)^2}\right.$$
$$\overline{\qquad f \qquad}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}\right.}{(de-cf)^2}\right.$$
$$\overline{\qquad f \qquad}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 421

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}}{(de-cf)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}}{(de-cf)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 402

$$b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}\right.}{(de-cf)^2}\right.$$
$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int-}{\sqrt{}}\right.$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}}{3c}}{(de-cf)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int -}{\sqrt{}}\right)}{f}$$

$$\Big\downarrow \; 400$$

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d\left(\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}}\right.}{3c}}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2}}}{bc-ad}}{3c(bc-ad)}\right.}{(de-cf)^2}\right.}{f}\right.}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(a\cdots}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\cdots}}{3c}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(-\frac{\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+b\cdots}}{3c\left(c+dx^2\cdots\right.}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\cdots}{(de-cf)^2\cdots}\cdots\right)}{f\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{b\cdots}{\cdots}}{3c(bc\cdots}\cdots\right)}{(de-cf)^2\cdots}\cdots\right)}{f\cdots}\cdots\right)}{f\cdots}$$

↓ 413

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{\cdots}\right.}{(de-cf)\cdots}\right.}{f}\right.$$

$$\overline{\qquad f \qquad}$$

↓ 413

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de}{}}{3c}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.}{}\right.}{f}\right.$$

$$\overline{\qquad f \qquad}$$

↓ 412

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de}{}}{3c}\right)}{(de-cf)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{c}}\right)}{}\right)}{f}\right)}{f}$$

↓ 426

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de}{}}{3c}\right.}{(de-cf)^2}\right.$$

$$f$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{c}}\right.}{}\right.}{f}\right.$$

↓ 421

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\right)}{(de-cf)^2\cdots}\right)}{f}$$

$$(be-af)\frac{\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{\cdots}\right)}{\cdots}\right)}{f}\cdots\right)}{\cdots}$$

↓ 25

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\right)}{(de-cf)^2\cdots}\right)}{f\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{f\cdots}\cdots\right)$$

↓ 402

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\right)}{(de-cf)^2\cdots}\right)}{f\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{\cdots}\right)}{f\cdots}\cdots\right)$$

↓ 25

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de}{}}{3c}\right)}{(de-cf)^2}\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E(}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{}\right)}{f}\right.$$

↓ 400

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de}{}}{3c}\right)}{(de-cf)^2}\right)$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{c}}\right)}{}\right)}{f}\right)$$

↓ 313

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\cdots\right)}{(de-cf)^2\cdots}\cdots\right)}{f\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{f\cdots}\cdots\right.$$

↓ 320

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\right)}{(de-cf)^2\cdots}\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{\cdots}\right)}{\cdots}\right)}{f}\cdots\right.$$

↓ 413

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de\cdots}{\cdots}}{3c\cdots}\right.}{(de-cf)^2\cdots}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{\cdots}\right.}{\cdots}\right.}{f}\right.$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2459 vs. $2(538) = 1076$.

Time = 18.90 (sec) , antiderivative size = 2460, normalized size of antiderivative = 4.32

method	result	size
elliptic	Expression too large to display	2460
default	Expression too large to display	4556

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a*f-b*e)
*f^2*d/e/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/(d*f*x^2+d*e)+1/3*(a*d-b*c)/(c*f-d*e)^2/c/d*x*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/(x^2+c/d)^2+2/3*(b*d*x^2+a*d)*(4*a*c*d*f-a*d^2*e-2*b*c^
2*f-b*c*d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)/c^2*x/((x^2+c/d)*(b*d*x
^2+a*d))^(1/2)-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2)-8/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b/(c^2*f^2-2*c*d*e
*f+d^2*e^2)/(c*f-d*e)*a*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))+2/3/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*d^2*b/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*a*e*Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(c^2*f^2-2*c*d*e*f+d^2*e^
2)/(c*f-d*e)/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*a*b*c-3/2*e/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d
-1/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{d^3f^2x^{10}+3c\,d^2f^2x^8+2d^3ef\,x^8+3c^2d\,f^2x^6+6c\,d^2ef\,x^6+d^3e^2x^6+}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^3f^2x^{10}+3c\,d^2f^2x^8+2d^3ef\,x^8+3c^2d\,f^2x^6+6c\,d^2ef\,x^6+d^3e^2x^6+c^3f^2x^4+6c^2def\,x^4+3c\,d^2e^2x}\right.$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**3*e**2 + 2*c**3*e*f*x**2
+ c**3*f**2*x**4 + 3*c**2*d*e**2*x**2 + 6*c**2*d*e*f*x**4 + 3*c**2*d*f**2*
x**6 + 3*c*d**2*e**2*x**4 + 6*c*d**2*e*f*x**6 + 3*c*d**2*f**2*x**8 + d**3*
e**2*x**6 + 2*d**3*e*f*x**8 + d**3*f**2*x**10),x)*b + int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(c**3*e**2 + 2*c**3*e*f*x**2 + c**3*f**2*x**4 + 3*c**2
*d*e**2*x**2 + 6*c**2*d*e*f*x**4 + 3*c**2*d*f**2*x**6 + 3*c*d**2*e**2*x**4
 + 6*c*d**2*e*f*x**6 + 3*c*d**2*f**2*x**8 + d**3*e**2*x**6 + 2*d**3*e*f*x*
*8 + d**3*f**2*x**10),x)*a
```

3.155 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 962

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \frac{(30a^3d^3ef^3 - a^2bdf^2(749d^2e^2 - 970cdef + 105c^2f^2) + 2ab^2def(840d^2e^2 \ldots}{2\ldots}$$
$$+ \frac{(45a^2d^2f^2 - 2abdf(77de - 85cf) + b^2(105d^2e^2 - 154cdef + 45c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105f^4}$$
$$- \frac{bd(14bde - 15bcf - 15adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35f^3}$$
$$+ \frac{b^2d^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7f^2} + \frac{(be-af)^2(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^4(e+fx^2)}$$
$$- \frac{\sqrt{a}(30a^3d^3ef^3 - a^2bdf^2(749d^2e^2 - 970cdef + 105c^2f^2) + 2ab^2def(840d^2e^2 - 1358cdef + 485c^2f^2) - b^3 \ldots}{210\sqrt{b}def^5\sqrt{a+bx^2}\sqrt{\frac{a(\ldots}{c(\ldots}}}$$
$$- \frac{a^{3/2}(60a^2d^2ef^2(7de-9cf) + b^2e(945d^3e^3 - 1995cd^2e^2f + 1267c^2def^2 - 225c^3f^3) - abf(1365d^3e^3 - 25 \ldots}{210\sqrt{b}cef^5\sqrt{a+bx^2}\sqrt{\frac{a(c+\ldots}{c(a+\ldots}}}$$
$$+ \frac{a^{3/2}(be-af)(de-cf)^2(be(9de-4cf) - af(4de+cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be}, \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1 - \ldots\right.}{2\sqrt{b}ce^2f^5\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/210*(30*a^3*d^3*e*f^3-a^2*b*d*f^2*(105*c^2*f^2-970*c*d*e*f+749*d^2*e^2)+
2*a*b^2*d*e*f*(485*c^2*f^2-1358*c*d*e*f+840*d^2*e^2)-b^3*e*(-30*c^3*f^3+74
9*c^2*d*e*f^2-1680*c*d^2*e^2*f+945*d^3*e^3))*x*(d*x^2+c)^(1/2)/d/e/f^5/(b*
x^2+a)^(1/2)+1/105*(45*a^2*d^2*f^2-2*a*b*d*f*(-85*c*f+77*d*e)+b^2*(45*c^2*
f^2-154*c*d*e*f+105*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^4-1/35*b
*d*(-15*a*d*f-15*b*c*f+14*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3+1
/7*b^2*d^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/2*(-a*f+b*e)^2*(-c*f+
d*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^4/(f*x^2+e)-1/210*a^(1/2)*(30
*a^3*d^3*e*f^3-a^2*b*d*f^2*(105*c^2*f^2-970*c*d*e*f+749*d^2*e^2)+2*a*b^2*d
*e*f*(485*c^2*f^2-1358*c*d*e*f+840*d^2*e^2)-b^3*e*(-30*c^3*f^3+749*c^2*d*e
*f^2-1680*c*d^2*e^2*f+945*d^3*e^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/e/f^5/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/210*a^(3/2)*(60*a^2*d^2*e*f^2*(-9*c*f+7
*d*e)+b^2*e*(-225*c^3*f^3+1267*c^2*d*e*f^2-1995*c*d^2*e^2*f+945*d^3*e^3)-a
*b*f*(-105*c^3*f^3+1235*c^2*d*e*f^2-2527*c*d^2*e^2*f+1365*d^3*e^3))*(d*x^2
+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(
1/2)/c/e/f^5/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(
-a*f+b*e)*(-c*f+d*e)^2*(b*e*(-4*c*f+9*d*e)-a*f*(c*f+4*d*e))*(d*x^2+c)^(1/2
)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/
2))/b^(1/2)/c/e^2/f^5/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.13 (sec) , antiderivative size = 4275, normalized size of antiderivative = 4.44

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
(315*a*b^2*Sqrt[b/a]*c*d^3*e^5*f^2*x - 518*a*b^2*Sqrt[b/a]*c^2*d^2*e^4*f^3
*x - 518*a^3*(b/a)^(3/2)*c*d^3*e^4*f^3*x + 195*a*b^2*Sqrt[b/a]*c^3*d*e^3*f
^4*x + 760*a^3*(b/a)^(3/2)*c^2*d^2*e^3*f^4*x + 195*a^3*Sqrt[b/a]*c*d^3*e^3
*f^4*x - 210*a^3*(b/a)^(3/2)*c^3*d*e^2*f^5*x - 210*a^3*Sqrt[b/a]*c^2*d^2*e
^2*f^5*x + 105*a^3*Sqrt[b/a]*c^3*d*e*f^6*x + 315*b^3*Sqrt[b/a]*c*d^3*e^5*f
^2*x^3 + 315*a*b^2*Sqrt[b/a]*d^4*e^5*f^2*x^3 - 518*b^3*Sqrt[b/a]*c^2*d^2*e
^4*f^3*x^3 - 910*a*b^2*Sqrt[b/a]*c*d^3*e^4*f^3*x^3 - 518*a^3*(b/a)^(3/2)*d
^4*e^4*f^3*x^3 + 195*b^3*Sqrt[b/a]*c^3*d*e^3*f^4*x^3 + 737*a*b^2*Sqrt[b/a]
*c^2*d^2*e^3*f^4*x^3 + 737*a^3*(b/a)^(3/2)*c*d^3*e^3*f^4*x^3 + 195*a^3*Sqr
t[b/a]*d^4*e^3*f^4*x^3 - 120*a*b^2*Sqrt[b/a]*c^3*d*e^2*f^5*x^3 - 80*a^3*(b
/a)^(3/2)*c^2*d^2*e^2*f^5*x^3 - 120*a^3*Sqrt[b/a]*c*d^3*e^2*f^5*x^3 + 105*
a^3*(b/a)^(3/2)*c^3*d*e*f^6*x^3 + 105*a^3*Sqrt[b/a]*c^2*d^2*e*f^6*x^3 + 31
5*b^3*Sqrt[b/a]*d^4*e^5*f^2*x^5 - 392*b^3*Sqrt[b/a]*c*d^3*e^4*f^3*x^5 - 39
2*a*b^2*Sqrt[b/a]*d^4*e^4*f^3*x^5 - 23*b^3*Sqrt[b/a]*c^2*d^2*e^3*f^4*x^5 +
 270*a*b^2*Sqrt[b/a]*c*d^3*e^3*f^4*x^5 - 23*a^3*(b/a)^(3/2)*d^4*e^3*f^4*x^
5 + 90*b^3*Sqrt[b/a]*c^3*d*e^2*f^5*x^5 + 310*a*b^2*Sqrt[b/a]*c^2*d^2*e^2*f
^5*x^5 + 310*a^3*(b/a)^(3/2)*c*d^3*e^2*f^5*x^5 + 90*a^3*Sqrt[b/a]*d^4*e^2*
f^5*x^5 + 105*a^3*(b/a)^(3/2)*c^2*d^2*e*f^6*x^5 + 126*b^3*Sqrt[b/a]*d^4*e^
4*f^3*x^7 - 272*b^3*Sqrt[b/a]*c*d^3*e^3*f^4*x^7 - 272*a*b^2*Sqrt[b/a]*d^4*
e^3*f^4*x^7 + 180*b^3*Sqrt[b/a]*c^2*d^2*e^2*f^5*x^7 + 550*a*b^2*Sqrt[b/...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \textbf{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \textbf{420}$$

$$\frac{b\left(\frac{b\int \sqrt{bx^2+a}(dx^2+c)^{5/2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 318

$$\frac{b\left(\frac{b\left(\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(2d(5bc-2ad)x^2+c(7bc-ad)\right)dx}{7b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 403

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(d\left(45b^2c^2-29abdc+8a^2d^2\right)x^2+c\left(35b^2c^2-15abdc+4a^2d^2\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{2dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(5bc-2ad)}{5b}}{7b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}\right)}{f}-\frac{(\ldots}{}\right.}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 403

$$\frac{b\left(\frac{b\left(\frac{\frac{\frac{\int \frac{d\left(\left(15b^3c^3+58ab^2dc^2-33a^2bd^2c+8a^3d^3\right)x^2+4ac\left(15b^2c^2-4abdc+a^2d^2\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)}{5b}+\frac{2dx\left(a+bx^2\right)^{3/2}\sqrt{c+\ldots}}{5b}}{7b}\ldots\right.}{f}\ldots\right.}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 27

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}\int \frac{\left(15b^3c^3+58ab^2dc^2-33a^2bd^2c+8a^3d^3\right)x^2+4ac\left(15b^2c^2-4abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)}{5b}+\frac{2dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx}}{5b}}{7b}}{f}}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 406

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}\left(4ac\left(a^2d^2-4abcd+15b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3-33a^2bcd^2+58ab^2c^2d+15b^3c^3\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29\right.}{5b}}{7b}}{f}}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}\left(\left(8a^3d^3-33a^2bcd^2+58ab^2c^2d+15b^3c^3\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{1}{3}x\sqrt{a+}}{5b}}{7b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 388

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}\left(\left(8a^3d^3-33a^2bcd^2+58ab^2c^2d+15b^3c^3\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}}{7b}}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\cdots\right.\right.}{5b}}{7b}\cdots\right.}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\cdots\right.\right.}{5b}}{7b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 318

$$b\left(\frac{b\left(\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\cdots\right.\right.}{5b}}{7b}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 403

$$b\left(\frac{b\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\right.\right.}{5b}}{7b}}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\cdots\right.}{5b}}{7b}\right.}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 406

$$b\left(\frac{b\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\cdots\right.}{5b}}{7b}\right.}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\right.\right.}{5b}}{7b}}{f}\right.$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2-29abcd+45b^2c^2\right)+\frac{1}{3}\left(\frac{4c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-4abcd+15b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8a^3d^3-33a^2bcd^2+58ab^2\right.\right.}{5b}}{7b}\right.}{f}\right.$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2\cdots}}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.}{\cdots}\right.}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 418

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{}\right)}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 25

$$b \left(\frac{b \left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b} + \frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b} + \frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}\cdots}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{\cdots} \cdots \right.}{f \cdots} \cdots \right.$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 403

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2+\cdots}}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{\cdots}\right)}{f}\cdots\right.}{\cdots}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 27

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{}\right.}{f}\right.$$

$$\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 406

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2\ldots}}{\sqrt{d}\sqrt{\frac{c(\ldots}{a(\ldots}}}\ldots\right.}{\ldots}\right.}{f}\ldots\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 320

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{}\right)}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 388

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{}\right.}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow\ 313$$

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2\cdots}}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{\cdots}}{\cdots}\right.}{f}\cdots\right.}{\cdots}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 414

$$b \left(\frac{b \left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b} + \frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b} + \frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{} \right.}{f} \right.$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2\cdots}}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{\cdots}\right)}{f}\cdots\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2\cdots}}{\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{\cdots}\right.}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 318

$$b\left(\frac{b\left(\frac{dx\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{7b}+\frac{2d(5bc-2ad)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{1}{3}\left(45b^2c^2-29abdc+8a^2d^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{4\left(15b^2c^2-4abdc+a^2d^2\right)\sqrt{bx^2-}}{\sqrt{d}\sqrt{\frac{c(}{a(}}}\right.}{}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\int\frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+}}{(f}}{f}\right.}{f}$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2165 vs. $2(912) = 1824$.

Time = 24.95 (sec) , antiderivative size = 2166, normalized size of antiderivative = 2.25

method	result	size
risch	Expression too large to display	2166
elliptic	Expression too large to display	4779
default	Expression too large to display	7152

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(15*b^2*d^2*f^2*x^4+45*a*b*d^2*f^2*x^2+45*b^2*c*d*f^2*x^2-42*b^2*d
^2*e*f*x^2+45*a^2*d^2*f^2+170*a*b*c*d*f^2-154*a*b*d^2*e*f+45*b^2*c^2*f^2-1
54*b^2*c*d*e*f+105*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^4+1/105/
f^4*((270*a^3*c*d^2*f^4-210*a^3*d^3*e*f^3+775*a^2*b*c^2*d*f^4-1736*a^2*b*c
*d^2*e*f^3+945*a^2*b*d^3*e^2*f^2+270*a*b^2*c^3*f^4-1736*a*b^2*c^2*d*e*f^3+
2730*a*b^2*c*d^2*e^2*f^2-1260*a*b^2*d^3*e^3*f-210*b^3*c^3*e*f^3+945*b^3*c^
2*d*e^2*f^2-1260*b^3*c*d^2*e^3*f+525*b^3*d^3*e^4)/f^2/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(15*a^3*d^3*f^3+380*a^2*b*c*
d^2*f^3-322*a^2*b*d^3*e*f^2+380*a*b^2*c^2*d*f^3-1148*a*b^2*c*d^2*e*f^2+735
*a*b^2*d^3*e^2*f+15*b^3*c^3*f^3-322*b^3*c^2*d*e*f^2+735*b^3*c*d^2*e^2*f-42
0*b^3*d^3*e^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+315/f^2*(a^3*c
^2*d*f^5-2*a^3*c*d^2*e*f^4+a^3*d^3*e^2*f^3+a^2*b*c^3*f^5-6*a^2*b*c^2*d*e*f
^4+9*a^2*b*c*d^2*e^2*f^3-4*a^2*b*d^3*e^3*f^2-2*a*b^2*c^3*e*f^4+9*a*b^2*c^2
*d*e^2*f^3-12*a*b^2*c*d^2*e^3*f^2+5*a*b^2*d^3*e^4*f+b^3*c^3*e^2*f^3-4*b^3*
c^2*d*e^3*f^2+5*b^3*c*d^2*e^4*f-2*b^3*d^3*e^5)/e/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(
-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+105*(a^3*c^3*f^6-3*a^3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}{(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

3.156 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 719

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = -\frac{b(2abdef(85de-62cf)-a^2df^2(61de-15cf)-b^2e(105d^2e^2-95cdef+6c^2f^2))\,x\sqrt{c+dx^2}}{30def^4\sqrt{a+bx^2}}$$
$$-\frac{b(10bde-6bcf-11adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15f^3}$$
$$+\frac{b^2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f^2}-\frac{(be-af)^2(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^3(e+fx^2)}$$
$$+\frac{\sqrt{a}\sqrt{b}(2abdef(85de-62cf)-a^2df^2(61de-15cf)-b^2e(105d^2e^2-95cdef+6c^2f^2))\sqrt{c+dx^2}\,E\Big(\text{arcta}}{30def^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(30a^2d^2ef^2-abf(135d^2e^2-128cdef+15c^2f^2)+b^2e(105d^2e^2-130cdef+33c^2f^2))\sqrt{c+dx^2}\,\text{Ellip}}{30\sqrt{b}cef^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(be-af)(de-cf)(be(7de-4cf)-af(2de+cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{2\sqrt{b}ce^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/30*b*(2*a*b*d*e*f*(-62*c*f+85*d*e)-a^2*d*f^2*(-15*c*f+61*d*e)-b^2*e*(6*
c^2*f^2-95*c*d*e*f+105*d^2*e^2))*x*(d*x^2+c)^(1/2)/d/e/f^4/(b*x^2+a)^(1/2)
-1/15*b*(-11*a*d*f-6*b*c*f+10*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3
+1/5*b^2*d*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2-1/2*(-a*f+b*e)^2*(-c*f+
d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^3/(f*x^2+e)+1/30*a^(1/2)*b^(1/2
)*(2*a*b*d*e*f*(-62*c*f+85*d*e)-a^2*d*f^2*(-15*c*f+61*d*e)-b^2*e*(6*c^2*f^
2-95*c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+
b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/e/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)+1/30*a^(3/2)*(30*a^2*d^2*e*f^2-a*b*f*(15*c^2*f^2-128*c*d*e
*f+135*d^2*e^2)+b^2*e*(33*c^2*f^2-130*c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2
)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e
/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-a*f+b*e
)*(-c*f+d*e)*(b*e*(-4*c*f+7*d*e)-a*f*(c*f+2*d*e))*(d*x^2+c)^(1/2)*Elliptic
Pi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2
)/c/e^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.64 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \frac{ibcef(2abdef(85de-62cf)+a^2df^2(-61de+15cf)+b^2e(-105d^2e^2+9}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(I*b*c*e*f*(2*a*b*d*e*f*(85*d*e - 62*c*f) + a^2*d*f^2*(-61*d*e + 15*c*f) +
 b^2*e*(-105*d^2*e^2 + 95*c*d*e*f - 6*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*e*(30*a^3*d^3*e*f^3 + 2*a*b^2*d*e*f*(120*d^2*e^2 - 70*c*d*e*f - 23*c^2*
f^2) + a^2*b*d*f^2*(-165*d^2*e^2 + 82*c*d*e*f + 15*c^2*f^2) + b^3*e*(-105*
d^3*e^3 + 60*c*d^2*e^2*f + 35*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] + d*(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(15*a^2*f^2*(-(d*e)
+ c*f) + 2*a*b*e*f*(26*d*e - 15*c*f + 11*d*f*x^2) + b^2*e*(3*c*f*(9*e + 4*
f*x^2) + d*(-35*e^2 - 14*e*f*x^2 + 6*f^2*x^4))) - (15*I)*(b*e - a*f)^2*(-(
d*e) + c*f)*(a*f*(2*d*e + c*f) + b*e*(-7*d*e + 4*c*f))*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)]))/(30*Sqrt[b/a]*d*e^2*f^5*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\left(\frac{b\int \sqrt{bx^2+a}(dx^2+c)^{3/2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{\frac{1}{3}\int \frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f} -}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a-\right.}{}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 418

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}}{f}$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 403

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}\cdots}{5b}\cdots\right)}{f}\cdots\right)$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 27

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}\cdots}{5b}\cdots\right)}{f}\cdots\right)$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 406

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}\right)}{f}\right)$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 320

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}\right)}{f}\right)$$

$$\frac{(be-af)\int\frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}$$

$\big\downarrow$ 388

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}\cdots}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 313

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}\cdots}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 414

$$b\left(\frac{b\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a}}{5b}}{f}\right.$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 425

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2}}{b\sqrt{dx^2}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 418

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2}}{b\sqrt{dx^2}}\right.\right.}{5b}\right)}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx(de-cf)^2}{f^2}+\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2}}{b\sqrt{dx^2}}\right.\right.}{5b}\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}$$

$$\Big\downarrow \text{403}$$

$$f$$

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2 \ldots}}{b\sqrt{dx^2 \ldots}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{\int\frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{bx^2+a}\sqrt{dx^2+c}\right)}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx \ldots\right.}{\left(fx^2+ \ldots\right.}}{f}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2 \cdots}}{b\sqrt{dx^2 \cdots}}\cdots\right)\right)}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int\frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{bx^2+a}\sqrt{dx^2+c}\right)}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+\cdots\right)}{\left(fx^2+e\right)\cdots}}{f}\cdots\right)}{f}$$

↓ 406

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2\cdots}}{b\sqrt{dx^2\cdots}}\right.\right.}{5b}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(3bde-4bcf-adf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{bx^2+a}\sqrt{dx\cdots}\right.}{f^2}\right.}{f}\right.}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^{\cdots}}}{b\sqrt{dx^{\cdots}}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}(3de-5cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-4bcf-adf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}d\cdots\right.\right.}{f^2}\cdots\right.}{f}\cdots\right.}{f}$$

↓ 388

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2 \cdots}}{b\sqrt{dx^2 \cdots}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}(3de-5cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{\cdots}}{\left(d \cdots\right)}}{\cdots}\right.\right.\right.}{f^2}\right.}{f}\right.}{f}$$

↓ 313

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2 \cdots}}{b\sqrt{dx^2 \cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3de-5cf)\sqrt{bx^2+a}\cdots}{\sqrt{d}\sqrt{\frac{c \cdots}{a \cdots}}\cdots}\cdots\right.\right.}{f^2}\cdots\right.}{f}\cdots\right.}{f}$$

↓ 414

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2}}{b\sqrt{dx^2}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.}{f}\right.$$

f

↓ 425

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2\cdots}}{b\sqrt{dx^2\cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.\right.\right.}{\cdots}\right.}{f}\cdots\right.$$

↓ 420

$$b\left(\frac{b\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{bx^2}}{b\sqrt{dx^2}}\right.\right.}{5b}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.\right.\right.}{}\right.}{f}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1810 vs. $2(675) = 1350$.

Time = 21.96 (sec) , antiderivative size = 1811, normalized size of antiderivative = 2.52

method	result	size
risch	Expression too large to display	1811
elliptic	Expression too large to display	3298
default	Expression too large to display	4969

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*b*x*(3*b*d*f*x^2+11*a*d*f+6*b*c*f-10*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/f^3+1/15/f^3*((15*a^3*d^2*f^3+79*a^2*b*c*d*f^3-90*a^2*b*d^2*e*f^2+3
9*a*b^2*c^2*f^3-170*a*b^2*c*d*e*f^2+135*a*b^2*d^2*e^2*f-30*b^3*c^2*e*f^2+9
0*b^3*c*d*e^2*f-60*b^3*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))+15/f^2*(2*a^3*c*d*f^4-2*a^3*d^2*e*f^3+3*a^2*b*c^2*
f^4-12*a^2*b*c*d*e*f^3+9*a^2*b*d^2*e^2*f^2-6*a*b^2*c^2*e*f^3+18*a*b^2*c*d*
e^2*f^2-12*a*b^2*d^2*e^3*f+3*b^3*c^2*e^2*f^2-8*b^3*c*d*e^3*f+5*b^3*d^2*e^4
)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))+15*(a^3*c^2*f^5-2*a^3*c*d*e*f^4+a^3*d^2*e^2*f^3-3*a^2*b*c^2*e*f^4+6*a^
2*b*c*d*e^2*f^3-3*a^2*b*d^2*e^3*f^2+3*a*b^2*c^2*e^2*f^3-6*a*b^2*c*d*e^3*f^
2+3*a*b^2*d^2*e^4*f-b^3*c^2*e^3*f^2+2*b^3*c*d*e^4*f-b^3*d^2*e^5)/f^2*(1/2*
f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e
*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.157 $\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 541

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx &= \frac{b(20abdef-3a^2df^2-b^2e(15de-2cf))\,x\sqrt{c+dx^2}}{6def^3\sqrt{a+bx^2}} \\ &+ \frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^2} + \frac{(be-af)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef^2\,(e+fx^2)} \\ &- \frac{\sqrt{a}\sqrt{b}(20abdef-3a^2df^2-b^2e(15de-2cf))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6def^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}\sqrt{b}(be(15de-7cf)-3af(5de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6cef^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}(be-af)\,(acf^2-be(5de-4cf))\,\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/6*b*(20*a*b*d*e*f-3*a^2*d*f^2-b^2*e*(-2*c*f+15*d*e))*x*(d*x^2+c)^(1/2)/d
/e/f^3/(b*x^2+a)^(1/2)+1/3*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/2*(
-a*f+b*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)-1/6*a^(1/2)*
b^(1/2)*(20*a*b*d*e*f-3*a^2*d*f^2-b^2*e*(-2*c*f+15*d*e))*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/e/f^3/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/6*a^(3/2)*b^(1/2)*(b*e*(-7
*c*f+15*d*e)-3*a*f*(-c*f+5*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^
(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-a*f+b*e)*(a*c*f^2-b*e*(-4*c*f+5*d*e))*(d*x
^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*
d/b/c)^(1/2))/b^(1/2)/c/e^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.56 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \frac{ibcef(-20abdef+3a^2df^2+b^2e(15de-2cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(I*b*c*e*f*(-20*a*b*d*e*f + 3*a^2*d*f^2 + b^2*e*(15*d*e - 2*c*f))*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*b*e*(3*a^2*d*f^2*(5*d*e + c*f) - 2*a*b*d*e*f*(15*d*e
+ 2*c*f) + b^2*e*(15*d^2*e^2 + 3*c*d*e*f - 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] + d*(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(-6*a*b*e*f + 3*a^2*f
^2 + b^2*e*(5*e + 2*f*x^2)) - (3*I)*(b*e - a*f)^2*(a*c*f^2 + b*e*(-5*d*e +
 4*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a
*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(6*Sqrt[b/a]*d*e^2*f^4*S
qrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1127 vs. 2(541) = 1082.

Time = 1.75 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.08, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.781$, Rules used = {425, 418, 25, 403, 27, 406, 320, 388, 313, 414, 425, 410, 324, 320, 388, 313, 414, 423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{418}$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}+\frac{b\int -\frac{\sqrt{dx^2+c}\left(-bfx^2+be-2af\right)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\int \frac{\sqrt{dx^2+c}\left(-bfx^2+be-2af\right)}{\sqrt{bx^2+a}}dx}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{\int \frac{b\left(\left(3bde-bcf-4adf\right)x^2+c(3be-5af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\int \frac{(3bde-bcf-4adf)x^2+c(3be-5af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(c(3be-5af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-4adf-bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+d}\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left((-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{(be-af)^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f^2}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}}{b\sqrt{d}\sqrt{}}\right.\right.\right.}{f^2}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 425

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\right.\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 410

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\right.\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 324

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-4adf-bcf\right.\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}\right)}{f}$$

$\big\downarrow$ 320

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-4adf-bcf\right.\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+}}{(fx}}{f}\right.}{f}$$

$\big\downarrow$ 388

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\cdots\right)\right)}{\cdots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-\cdots}{\cdots}\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+\right.}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 414

$$b\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\left(\frac{1}{3}\left(\frac{c^{3/2}\sqrt{a+bx^2}(3be-5af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-4adf-bcf\right.\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{Ellip}}{a\sqrt{d}ef\sqrt{c}}}{f}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 423

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{}\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{\cdots}}\right)}{f}\right)$$

$$\frac{\qquad}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{f}}{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{\ }}}{f}\right.}$$

$\downarrow$ 320

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{f}\right.}{}$$

$$\frac{}{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}\ldots}{a\sqrt{d}ef\sqrt{\ldots}}\right)}{f}\right.}$$

$\downarrow$ 388

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}}{f}\right.}$$

$\downarrow$ 313

$$b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{}}}{f}\right.$$

↓ 413

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{\cdots}}\right.}{f}\right.}$$

↓ 413

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\cdots\right.\right.}{f}\cdots\right)}{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}\cdots}{a\sqrt{d}ef\sqrt{\cdots}}\cdots\right)}{f}\cdots\right)}$$

$\downarrow$ 412

$$\frac{b\left(\frac{c^{3/2}(be-af)^2\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\left(\frac{1}{3}\left(\frac{(3be-5af)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(3bde-bcf-\right.\right.}{f}\right)}{}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{Ellip}}{a\sqrt{d}ef\sqrt{\cdots}}\right)}{f}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(b*(-((b*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e - b*c*f -
 4*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*e -
5*a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3
))/f^2) + (c^(3/2)*(b*e - a*f)^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e
), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*f^2*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - ((b*e - a*f)*(-(((b*
e - a*f)*((x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(e + f*x^2)) + (b*d*(-(
f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*e*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*f^2) + (Sqrt[-a]
*((a*c)/e - (b*d*e)/f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticP
i[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])))/f) + (b*((b*(d*((x*Sqrt[a + b*x^2])/(b*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && !SimplerSqrtQ[-f/e, -d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1879 vs. $2(503) = 1006$.

Time = 21.34 (sec) , antiderivative size = 1880, normalized size of antiderivative = 3.48

method	result	size
risch	Expression too large to display	1880
elliptic	Expression too large to display	1921
default	Expression too large to display	2874

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2+1/3/f^2*(b/f^2*(-f*b*(7*a*d*
f+b*c*f-6*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+9*a^2*d*f^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+9*b^2*d*e^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+8*a*b*c*f^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*b^2*c*e*f/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-18*a*b*d*e*f
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3/f^2*(a^3
*d*f^3+3*a^2*b*c*f^3-6*a^2*b*d*e*f^2-6*a*b^2*c*e*f^2+9*a*b^2*d*e^2*f+3*b^3
*c*e^2*f-4*b^3*d*e^3)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c
*d)^(1/2)/(-b/a)^(1/2))+3*(a^3*c*f^4-a^3*d*e*f^3-3*a^2*b*c*e*f^3+3*a^2*b*d
*e^2*f^2+3*a*b^2*c*e^2*f^2-3*a*b^2*d*e^3*f-b^3*c*e^3*f+b^3*d*e^4)/f^2*(1/2
*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}\sqrt{d\,x^2+c}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
(9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x + 8*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*b*c*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x - 4*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*e*x + 3*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b**2*d*e*x**3 - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a
*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4
+ a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**
2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a**2*b*d**2*e*f**2 - 9*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**
2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 +
2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2
*x**8),x)*a**2*b*d**2*f**3*x**2 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*
e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6
+ b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b**2*c*d*e*f**2 - 8
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
 a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2
*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 +
b*d*f**2*x**8),x)*a*b**2*c*d*f**3*x**2 + 25*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
+ 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c...
```

3.158 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 617

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{b(2abdef-a^2df^2-b^2e(3de-2cf))\,x\sqrt{c+dx^2}}{2def^2(de-cf)\sqrt{a+bx^2}} - \frac{b(be-af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef(de-cf)} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2e(de-cf)} - \frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{2e(de-cf)(e+fx^2)} + \frac{\sqrt{a}\sqrt{b}(2abdef-a^2df^2-b^2e(3de-2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2def^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{b}(3be-af)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(be-af)(be(3de-4cf)+af(2de-cf))\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/2*b*(2*a*b*d*e*f-a^2*d*f^2-b^2*e*(-2*c*f+3*d*e))*x*(d*x^2+c)^(1/2)/d/e/
f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/2*b*(-a*f+b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/e/f/(-c*f+d*e)+1/2*b*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e
)-1/2*f*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)+1/2*a^(1/
2)*b^(1/2)*(2*a*b*d*e*f-a^2*d*f^2-b^2*e*(-2*c*f+3*d*e))*(d*x^2+c)^(1/2)*El
lipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/e/f^2/(-c
*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*b^(1/2
)*(-a*f+3*b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(
1-a*d/b/c)^(1/2))/c/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-
1/2*a^(3/2)*(-a*f+b*e)*(b*e*(-4*c*f+3*d*e)+a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/
2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1
/2))/b^(1/2)/c/e^2/f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.44 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{-ibcef(-2abdef+a^2df^2+b^2e(3de-2cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\big(}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((-I)*b*c*e*f*(-2*a*b*d*e*f + a^2*d*f^2 + b^2*e*(3*d*e - 2*c*f))*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] + I*b*e*(-(d*e) + c*f)*(4*a*b*d*e*f + a^2*d*f^2 - b^2*e*(3
*d*e + 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - d*(b*e - a*f)^2*(Sqrt[b/a]*e*f^2
*x*(a + b*x^2)*(c + d*x^2) - I*(a*f*(-2*d*e + c*f) + b*e*(-3*d*e + 4*c*f))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*d*e^2*f^3*(d*e - c*
f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 1026, normalized size of antiderivative = 1.66, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.656$, Rules used = {425, 420, 324, 320, 388, 313, 414, 425, 414, 425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(\frac{b\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{b\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 414$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\ldots\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(\ldots}{c(\ldots}}}\ldots\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 425

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\ldots\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(\ldots}{c(\ldots}}}\ldots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\big\downarrow$ 414

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\ldots\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(\ldots}{c(\ldots}}}\ldots\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\big\downarrow$ 425

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{f}\right.}{f}\right.}{f}$$

$\big\downarrow$ 413

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int}{\sqrt{bx^2+}}\right.}{f}\right.}{f}$$

$\big\downarrow$ 413

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)}{}\right.}{f}\right.}{f}$$

$\downarrow$ 412

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\right.}{f}\right.}{f}$$

$\downarrow$ 424

$$\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\right.}{}\right.}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.}{}$$

$\downarrow$ 320

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$f$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

↓ 388

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

↓ 313

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

↓ 413

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$f$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

↓ 413

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$f$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

↓ 412

$$b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\Big(1-}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(}{c(}}}\right.$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{}\right.$$

input

```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
-(((b*e - a*f)*(-(((b*e - a*f)*((Sqrt[-a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d
)/(b*c)])/(e*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - ((b*e - a*f)*((f^2*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2)) -
(b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*f)
*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*
x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])))/f))/f) + (a^(3/2)*Sqrt[b]*Sqrt[c + d*x^2]*Elli
pticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(c*
e*f*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/f) + (b*((b*(
b*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (a^(3/2)*(b*e - a*f...
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. $2(573) = 1146$.

Time = 6.88 (sec) , antiderivative size = 2343, normalized size of antiderivative = 3.80

method	result	size
elliptic	Expression too large to display	2343
default	Expression too large to display	2976

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a^2*f^2-
2*a*b*e*f+b^2*e^2)/e/f/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*
x^2+e)+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*b/e/(c*f-d*e)*a^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/e/(c*f-d*e)*a^2*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b^2/(c*f-d*e)*a*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b^2/(c*f-d*e)*a*Elli
pticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f^2*b^3*e/
(c*f-d*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/f^2*b^3*e/(c*f-d*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
+2/f^2*e/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*a*b^2*d+1/2*f/e^2/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b
/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*c+1/(c*f-d*e)/e/(-b/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*x - int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 +
 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**
2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b**2*d*e*f -
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
 a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2
*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 +
b*d*f**2*x**8),x)*a*b**2*d*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a
*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x*
*6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**3*d*e**2 + int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*
f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2
 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f
**2*x**8),x)*b**3*d*e*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2
)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x
**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d
*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a**2*b*c*e*f + int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x*
*4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2...
```

3.159 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 662

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx &= \frac{(bc-ad)^2(2de+cf)x\sqrt{a+bx^2}}{2cde(de-cf)^2\sqrt{c+dx^2}} \\
&+ \frac{b(6abcdef-a^2df(2de+cf)-b^2ce(de+2cf))\,x\sqrt{c+dx^2}}{2cdef(de-cf)^2\sqrt{a+bx^2}} \\
&+ \frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2de(de-cf)} - \frac{fx(a+bx^2)^{5/2}}{2e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\
&- \frac{\sqrt{a}\sqrt{b}(6abcdef-a^2df(2de+cf)-b^2ce(de+2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2cdef(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{a^{3/2}\sqrt{b}(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}(be-af)(be(de-4cf)+af(4de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/2*(-a*d+b*c)^2*(c*f+2*d*e)*x*(b*x^2+a)^(1/2)/c/d/e/(-c*f+d*e)^2/(d*x^2+c
)^(1/2)+1/2*b*(6*a*b*c*d*e*f-a^2*d*f*(c*f+2*d*e)-b^2*c*e*(2*c*f+d*e))*x*(d
*x^2+c)^(1/2)/c/d/e/f/(-c*f+d*e)^2/(b*x^2+a)^(1/2)+1/2*b^2*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/d/e/(-c*f+d*e)-1/2*f*x*(b*x^2+a)^(5/2)/e/(-c*f+d*e)/(d*
x^2+c)^(1/2)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(6*a*b*c*d*e*f-a^2*d*f*(c*f+2*d
*e)-b^2*c*e*(2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/d/e/f/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*b^(1/2)*(-a*f+b*e)*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/f/(-c*f+d
*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-a*f+b*e)
*(b*e*(-4*c*f+d*e)+a*f*(-c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f/(-c
*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{ibcef(-6abcdef+a^2df(2de+cf)+b^2ce(de+2cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(I*b*c*e*f*(-6*a*b*c*d*e*f + a^2*d*f*(2*d*e + c*f) + b^2*c*e*(d*e + 2*c*f)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*e*(-(d*e) + c*f)*(-2*a*b*d*e*f + a^2*d*
f^2 + b^2*e*(-(d*e) + 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e +
 f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*e*f^
2*x*(a + b*x^2)*(b^2*c*e*(3*c*e + d*e*x^2 + 2*c*f*x^2) - 2*a*b*c*e*(2*d*e
+ c*f + 3*d*f*x^2) + a^2*(c^2*f^2 + c*d*f^2*x^2 + 2*d^2*e*(e + f*x^2))) -
I*c*(b*e - a*f)^2*(a*f*(-4*d*e + c*f) + b*e*(-(d*e) + 4*c*f))*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*c*d*e^2*f^2*(d*e - c*f)^2*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1650 vs. 2(662) = 1324.

Time = 1.53 (sec) , antiderivative size = 1650, normalized size of antiderivative = 2.49, number of steps used = 30, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {425, 417, 313, 414, 425, 416, 313, 414, 425, 421, 25, 400, 313, 320, 414, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{417}$$

$$\frac{b\left(\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-$$
$$\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 416$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}}{f}\cdots\right)}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/\cdots}}}{f}\cdots\right)\right)}{f}$$

$\downarrow$ 421

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+\right.}}{(de-cf)^2}\right.}{}\right.}{}\right.}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+\right.}}{(de-cf)^2}\right.}{}\right.}{}\right.}{f}$$

$\downarrow$ 400

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{(de-cf)^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 313

$$\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(d\left(\frac{(adf-2bcf+bde)\cdots}{\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{f}$$

↓ 320

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}}{(de-cf)^2}\cdots\right.\right.}{\cdots}\right.$$

↓ 414

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.\right.}{\ldots}\right.$$

↓ 426

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.\right.}{\ldots}\right.$$

↓ 421

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\cdots}{a\sqrt{d}e(de-\cdots}\right.\right.}{\cdots}\right.$$

↓ 25

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 400

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 313

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.\right.}{\ldots}\right.$$

↓ 320

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}}{\ldots}\right.$$

↓ 414

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\cdots}{a\sqrt{d}e(de-\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 424

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}}{\ldots}\right.$$

↓ 406

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}}{a\sqrt{d}e(de-}\right.}{}\right.}{}\right.$$

↓ 320

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 388

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 313

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.\right.}{\ldots}\right.$$

↓ 413

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}}{\ldots}\right.$$

↓ 413

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\right.}{\ldots}}{\ldots}\right.$$

↓ 412

$$\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{Ellipti}\ldots}{a\sqrt{d}e(de-\ldots}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(b*(-(((b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (a^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*Ellipt
icPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt
[b]*c*e*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
))/f - ((b*e - a*f)*((b*((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (a^(3/2)*f*Sqrt[c + d*x^2]*EllipticPi
[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*
c*e*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/f
 - ((b*e - a*f)*((b*((d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*
c*f + a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])))/(d*e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[
1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]
*e*(d*e - c*f)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/
f - ((b*e - a*f)*(-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e
- a*f)*(d*e - c*f)*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2324 vs. $2(618) = 1236$.

Time = 9.13 (sec) , antiderivative size = 2325, normalized size of antiderivative = 3.51

method	result	size
elliptic	Expression too large to display	2325
default	Expression too large to display	3312

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(a^2*f^2-
2*a*b*e*f+b^2*e^2)/(c*f-d*e)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*
x^2+e)+(b*d*x^2+a*d)*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d/(c*f-d*e)^2*x/((x^2+c
/d)*(b*d*x^2+a*d))^(1/2)+1/(c*f-d*e)^2*f/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^
(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*b*c-1/(c*f-d*e)^2*e/f/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b^2
*d-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d*b^3
/f^2/(c*f-d*e)^2*e^2+3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/(c*f-d*e)^2*a*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b/(c*f-d*e)^2*a^2*EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/(c*f-d*e)^2/e*
f*a^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
b/(c*f-d*e)^2/e*f*a^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 2*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**6)/(2*a**2*c**2*e**2*f + 4*a**2*c**2*e*f**2*x**2 + 2*
a**2*c**2*f**3*x**4 + 4*a**2*c*d*e**2*f*x**2 + 8*a**2*c*d*e*f**2*x**4 + 4*
a**2*c*d*f**3*x**6 + 2*a**2*d**2*e**2*f*x**4 + 4*a**2*d**2*e*f**2*x**6 + 2
*a**2*d**2*f**3*x**8 - a*b*c**2*e**3 + 3*a*b*c**2*e*f**2*x**4 + 2*a*b*c**2
*f**3*x**6 - 2*a*b*c*d*e**3*x**2 + 6*a*b*c*d*e*f**2*x**6 + 4*a*b*c*d*f**3*
x**8 - a*b*d**2*e**3*x**4 + 3*a*b*d**2*e*f**2*x**8 + 2*a*b*d**2*f**3*x**10
 - b**2*c**2*e**3*x**2 - 2*b**2*c**2*e**2*f*x**4 - b**2*c**2*e*f**2*x**6 -
 2*b**2*c*d*e**3*x**4 - 4*b**2*c*d*e**2*f*x**6 - 2*b**2*c*d*e*f**2*x**8 -
b**2*d**2*e**3*x**6 - 2*b**2*d**2*e**2*f*x**8 - b**2*d**2*e*f**2*x**10),x)
*a**2*b**3*c*d*e*f**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*
a**2*c**2*e**2*f + 4*a**2*c**2*e*f**2*x**2 + 2*a**2*c**2*f**3*x**4 + 4*a**
2*c*d*e**2*f*x**2 + 8*a**2*c*d*e*f**2*x**4 + 4*a**2*c*d*f**3*x**6 + 2*a**2
*d**2*e**2*f*x**4 + 4*a**2*d**2*e*f**2*x**6 + 2*a**2*d**2*f**3*x**8 - a*b*
c**2*e**3 + 3*a*b*c**2*e*f**2*x**4 + 2*a*b*c**2*f**3*x**6 - 2*a*b*c*d*e**3
*x**2 + 6*a*b*c*d*e*f**2*x**6 + 4*a*b*c*d*f**3*x**8 - a*b*d**2*e**3*x**4 +
 3*a*b*d**2*e*f**2*x**8 + 2*a*b*d**2*f**3*x**10 - b**2*c**2*e**3*x**2 - 2*
b**2*c**2*e**2*f*x**4 - b**2*c**2*e*f**2*x**6 - 2*b**2*c*d*e**3*x**4 - 4*b
**2*c*d*e**2*f*x**6 - 2*b**2*c*d*e*f**2*x**8 - b**2*d**2*e**3*x**6 - 2*b**
2*d**2*e**2*f*x**8 - b**2*d**2*e*f**2*x**10),x)*a**2*b**3*c*d*f**3*x**2...
```

3.160 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 677

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{(bc-ad)^2(2de+3cf)x\sqrt{a+bx^2}}{6cde(de-cf)^2(c+dx^2)^{3/2}}$$

$$+\frac{b^2x\sqrt{a+bx^2}}{2de(de-cf)\sqrt{c+dx^2}}-\frac{fx(a+bx^2)^{5/2}}{2e(de-cf)(c+dx^2)^{3/2}(e+fx^2)}$$

$$-\frac{(b^2c^2e(13de+2cf)-6abcde(de+4cf)-a^2d(4d^2e^2-16cdef-3c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{6c^{3/2}\sqrt{d}e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+\frac{(3a^3cdf^2(6de-cf)-3b^3c^2e^2(de+4cf)+ab^2ce(8d^2e^2+23cdef+14c^2f^2)-a^2bde(2d^2e^2+20cdef+2}{6a\sqrt{c}\sqrt{d}e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$-\frac{c^{3/2}(be-af)^2(af(6de-cf)-be(de+4cf))\sqrt{a+bx^2}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/6*(-a*d+b*c)^2*(3*c*f+2*d*e)*x*(b*x^2+a)^(1/2)/c/d/e/(-c*f+d*e)^2/(d*x^2
+c)^(3/2)+1/2*b^2*x*(b*x^2+a)^(1/2)/d/e/(-c*f+d*e)/(d*x^2+c)^(1/2)-1/2*f*x
*(b*x^2+a)^(5/2)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)-1/6*(b^2*c^2*e*(2*
c*f+13*d*e)-6*a*b*c*d*e*(4*c*f+d*e)-a^2*d*(-3*c^2*f^2-16*c*d*e*f+4*d^2*e^2
))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))/c^(3/2)/d^(1/2)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)+1/6*(3*a^3*c*d*f^2*(-c*f+6*d*e)-3*b^3*c^2*e^2*(4*c*f+d*e)+a
*b^2*c*e*(14*c^2*f^2+23*c*d*e*f+8*d^2*e^2)-a^2*b*d*e*(23*c^2*f^2+20*c*d*e*
f+2*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1
-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/e/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*(-a*f+b*e)^2*(a*f*(-c*f+6*d*e)-b*e*(4*
c*f+d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1
-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}defx(a+bx^2)\left(3c^2f(be-af)^2\,(c+dx^2)^2+2c(bc-ad)^2e(-de+cf$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*d*e*f*x*(a + b*x^2)*(3*c^2*f*(b*e - a*f)^2*(c + d*x^2)^2 + 2*
c*(b*c - a*d)^2*e*(-(d*e) + c*f)*(e + f*x^2) + 2*(b*c - a*d)*e*(2*a*d*(d*e
 - 4*c*f) + b*c*(5*d*e + c*f))*(c + d*x^2)*(e + f*x^2))) + I*c*Sqrt[1 + (b
*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(-(b*e*f*(b^2*c^2*e*(
13*d*e + 2*c*f) - 6*a*b*c*d*e*(d*e + 4*c*f) + a^2*d*(-4*d^2*e^2 + 16*c*d*e
*f + 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*(-(
d*e) + c*f)*(-10*a*b*c*d*e*f + b^2*c*e*(3*d*e + 2*c*f) + a^2*d*f*(2*d*e +
3*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*c*d*(b*e - a*f)
^2*(a*f*(-6*d*e + c*f) + b*e*(d*e + 4*c*f))*EllipticPi[(a*f)/(b*e), I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(6*Sqrt[b/a]*c^2*d*e^2*f*(d*e - c*f)^3*Sq
rt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 419$$

$$\frac{b\left(-\frac{\int -\frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\int \frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 401$$

$$\frac{b\left(\frac{-\frac{\int -\frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\frac{\int \frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{\int \frac{b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 400$$

$$\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(cf+2de))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx-ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx\cdots}}{\sqrt{dx^2+c}\cdots}}{(de-cf)^2\cdots}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f\cdots}{\cdots}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{a\cdots}{\cdots}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\downarrow 401$$

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\ldots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int-\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\ldots\right)}{f}$$

$\downarrow$ 25

$$b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{a}{}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{a}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int\frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 400

$$b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{a}{\ldots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}\right.}{f}$$

$\downarrow$ 313

$$
\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}\,(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}\,(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}\,(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\cdots\right.}{f}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}\,(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\,(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}+\frac{x\sqrt{a+bx^2}\,(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\cdots}{\sqrt{dx^2\cdots}}}{(de\cdots}\right.}{f}\cdots\right.}{f}
$$

$\downarrow$ 320

$$
\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a\cdots}{\cdots}\right)}{f}\cdots
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5cf)-bc(de-4cf))\sqrt{\cdots}}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\cdots}}}{3c}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{f}
$$

↓ 414

$$b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\frac{a}{\cdots}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\overline{\qquad f \qquad}$$

↓ 425

$$\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a\cdots}{\cdots}\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)$$

↓ 421

$$
\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a\cdots}{\cdots}\right)}{f}
$$

$$
(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)
$$

↓ 25

$$\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2}\cdots\right.}{f}\cdots\right.$$

↓ 402

$$\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2}\cdots\right.}{f}\cdots\right.$$

↓ 25

$$b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\ \cdots\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\ \cdots}{\cdots}\right.}{(de-cf)^2\ \cdots}\right.}{f}\ \cdots\right.$$

↓ 400

$$\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\ \cdots\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\ \cdots}{\cdots}\right.}{(de-cf)^2\ \cdots}\right.}{f\ \cdots}\right.$$

↓ 313

$$\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{f}\cdots\right.$$

↓ 320

$$
b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\frac{a\cdots}{\cdots}\right.
$$

$$
\overline{\qquad\qquad f \qquad\qquad}
$$

$$
(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{f}\cdots\right.
$$

$\downarrow$ 413

$$\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right.}{(de-cf)^2}\cdots\right.}{f}\cdots\right.$$

↓ 413

$$\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right.$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3433 vs. $2(639) = 1278$.

Time = 18.74 (sec) , antiderivative size = 3434, normalized size of antiderivative = 5.07

method	result	size
elliptic	Expression too large to display	3434
default	Expression too large to display	6707

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(f^2/(c^2*f^2-
2*c*d*e*f+d^2*e^2)/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*b*c+4*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/(c*f-d*e)/(c^2*f
^2-2*c*d*e*f+d^2*e^2)*a*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))-8/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*d*b/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)*a^2*f*Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2/(c^2*f^2-2*c*d*e*f+d^2*e^2)*a+1/3*(
a^2*d^2-2*a*b*c*d+b^2*c^2)/(c*f-d*e)^2/c/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/(x^2+c/d)^2+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e
^2)*a*e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3/c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*d^2*b/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)*a^2*e*EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*(b*d*x^2+a*d)*(8*a^2*c*d^2*f-2*a^2*d^3*e-
9*a*b*c^2*d*f-3*a*b*c*d^2*e+b^2*c^3*f+5*b^2*c^2*d*e)/d/(c*f-d*e)/(c^2*f^2-
2*c*d*e*f+d^2*e^2)/c^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3/(-b/a)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 20*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(4*a**2*c**3*e**2*f + 8*a**2*c**3*e*f**2*x**2 + 4
*a**2*c**3*f**3*x**4 + 12*a**2*c**2*d*e**2*f*x**2 + 24*a**2*c**2*d*e*f**2*
x**4 + 12*a**2*c**2*d*f**3*x**6 + 12*a**2*c*d**2*e**2*f*x**4 + 24*a**2*c*d
**2*e*f**2*x**6 + 12*a**2*c*d**2*f**3*x**8 + 4*a**2*d**3*e**2*f*x**6 + 8*a
**2*d**3*e*f**2*x**8 + 4*a**2*d**3*f**3*x**10 + a*b*c**3*e**3 + 6*a*b*c**3
*e**2*f*x**2 + 9*a*b*c**3*e*f**2*x**4 + 4*a*b*c**3*f**3*x**6 + 3*a*b*c**2*
d*e**3*x**2 + 18*a*b*c**2*d*e**2*f*x**4 + 27*a*b*c**2*d*e*f**2*x**6 + 12*a
*b*c**2*d*f**3*x**8 + 3*a*b*c*d**2*e**3*x**4 + 18*a*b*c*d**2*e**2*f*x**6 +
 27*a*b*c*d**2*e*f**2*x**8 + 12*a*b*c*d**2*f**3*x**10 + a*b*d**3*e**3*x**6
 + 6*a*b*d**3*e**2*f*x**8 + 9*a*b*d**3*e*f**2*x**10 + 4*a*b*d**3*f**3*x**1
2 + b**2*c**3*e**3*x**2 + 2*b**2*c**3*e**2*f*x**4 + b**2*c**3*e*f**2*x**6
+ 3*b**2*c**2*d*e**3*x**4 + 6*b**2*c**2*d*e**2*f*x**6 + 3*b**2*c**2*d*e*f*
*2*x**8 + 3*b**2*c*d**2*e**3*x**6 + 6*b**2*c*d**2*e**2*f*x**8 + 3*b**2*c*d
**2*e*f**2*x**10 + b**2*d**3*e**3*x**8 + 2*b**2*d**3*e**2*f*x**10 + b**2*d
**3*e*f**2*x**12),x)*a**2*b**3*c**2*d*e*f**2 - 20*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**6)/(4*a**2*c**3*e**2*f + 8*a**2*c**3*e*f**2*x**2 + 4*a**
2*c**3*f**3*x**4 + 12*a**2*c**2*d*e**2*f*x**2 + 24*a**2*c**2*d*e*f**2*x**4
 + 12*a**2*c**2*d*f**3*x**6 + 12*a**2*c*d**2*e**2*f*x**4 + 24*a**2*c*d**2*
e*f**2*x**6 + 12*a**2*c*d**2*f**3*x**8 + 4*a**2*d**3*e**2*f*x**6 + 8*a*...
```

3.161 $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 359

$$\begin{aligned}\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx &= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}\\ &\quad-\frac{\sqrt{c}\sqrt{d}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\\ &\quad+\frac{\sqrt{c}(b^2ce+a^2df)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}\end{aligned}$$

output

```
1/2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*c^(1/2)*d^(1/2)*(1-
d*x^2/c)^(1/2)*(f*x^2+e)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-c*f/d/e)^(1/2
))/a/b/(-d*x^2+c)^(1/2)/(1+f*x^2/e)^(1/2)-1/2*c^(1/2)*d^(1/2)*(a*f+b*e)*(1
-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-c*f/d/e)^(
1/2))/a/b^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/2*c^(1/2)*(a^2*d*f+b^2*c*e)
*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2),-b*c/a/d
,(-c*f/d/e)^(1/2))/a^2/b^2/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.62 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{cex}{a+bx^2} - \frac{dex^3}{a+bx^2} + \frac{cfx^3}{a+bx^2} - \frac{dfx^5}{a+bx^2} + \frac{ic\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{b} - \frac{ic\sqrt{-\frac{d}{c}}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+f\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) - (d*
f*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b - (I*c*Sqrt
[-(d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x],
-((c*f)/(d*e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)
]*x], -((c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {423, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 423$$

$$-\frac{df\int \frac{a-bx^2}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 399$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{(af+be)\int \frac{1}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{f}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 323$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{\frac{fx^2}{e}+1}(af+be)\int \frac{1}{\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 323$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 321$$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 331$$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{fx^2+e}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{330}$$

$$-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\int\frac{\sqrt{\frac{fx^2}{e}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{fx^2+e}}dx}{2\sqrt{c-dx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\int\frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{2\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(\frac{adf}{b^2}+\frac{ce}{a}\right)\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{df\left(\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}\right)}{2ab^2}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

input
```
Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
(x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) - (d*f*(-((b*Sqrt[c]
*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]]
, -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e])) + (Sqr
t[c]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[
(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sqrt[e +
 f*x^2])))/(2*a*b^2) + (Sqrt[c]*((c*e)/a + (a*d*f)/b^2)*Sqrt[1 - (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((c*f)/(d*e))])/(2*a*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(-x^2d+c)(f\,x^2+e)}\left(\frac{x\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}{2a\left(b\,x^2+a\right)}-\frac{df\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2b^2\sqrt{\frac{d}{c}}\,\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}-\frac{de\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{Ellipt}}{2ab\sqrt{\frac{d}{c}}\,\sqrt{-df\,x^4+}}\right.}{}$
default	$-\frac{\sqrt{-x^2d+c}\,\sqrt{f\,x^2+e}\,\left(\sqrt{\frac{d}{c}}\,a\,b^2df\,x^5+\sqrt{\frac{-x^2d+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a^2bdf\,x^2+\sqrt{\frac{-x^2d+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\operatorname{EllipticF}\left(x\right.\right.}{}$

input
```
int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(-d
*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(b*x^2+a)-1/2*d*f/b^2/(1/c*d)^(1/2)*(1-d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Ellip
ticF(x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))-1/2*d/a/b*e/(1/c*d)^(1/2)*(
1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*El
lipticF(x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*d/a/b*e/(1/c*d)^(1/2
)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)
*EllipticE(x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2/b^2/(1/c*d)^(1/2)
*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*
EllipticPi(x*(1/c*d)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(1/c*d)^(1/2))*d*f+1/2/a^
2/(1/c*d)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*
x^2+c*e)^(1/2)*EllipticPi(x*(1/c*d)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(1/c*d)^(1
/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c - d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac"
)
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.162 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 413

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx &= -\frac{dx\sqrt{e+fx^2}}{2ab\sqrt{c+dx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)}+\frac{\sqrt{c}\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{2ab\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad -\frac{c^{3/2}\sqrt{d}(be-af)\sqrt{e+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{2ab(bc-ad)e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad +\frac{c^{3/2}(b^2ce-a^2df)\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{2a^2b\sqrt{d}(bc-ad)e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
\end{aligned}$$

output
```
-1/2*d*x*(f*x^2+e)^(1/2)/a/b/(d*x^2+c)^(1/2)+1/2*x*(d*x^2+c)^(1/2)*(f*x^2+
e)^(1/2)/a/(b*x^2+a)+1/2*c^(1/2)*d^(1/2)*(f*x^2+e)^(1/2)*EllipticE(d^(1/2)
*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))/a/b/(d*x^2+c)^(1/2)/(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)-1/2*c^(3/2)*d^(1/2)*(-a*f+b*e)*(f*x^2+e)^(1/2)*I
nverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-c*f/d/e)^(1/2))/a/b/(-a*d+b*c)
/e/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+1/2*c^(3/2)*(-a^2*d*f+b
^2*c*e)*(f*x^2+e)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-b
*c/a/d,(1-c*f/d/e)^(1/2))/a^2/b/d^(1/2)/(-a*d+b*c)/e/(d*x^2+c)^(1/2)/(c*(f
*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{cex}{a+bx^2}+\frac{dex^3}{a+bx^2}+\frac{cfx^3}{a+bx^2}+\frac{dfx^5}{a+bx^2}+\frac{ic\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{b}-\frac{ic\sqrt{\frac{d}{c}}(be+af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{Ellip}}{b^2}}{2}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) + (d*
f*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b - (I*c*Sqrt[d/c]*(b*
e + a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/b^2 - (I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/
c]) + (I*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticP
i[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 423$$

$$\frac{df\int \frac{a-bx^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 406$$

$$\frac{df\left(a\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 320$$

$$\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \ 388$$

$$\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{(fx^2+e)^{3/2}}dx}{d}\right)\right)}{2ab^2}+\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 313$$

$$\frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{2\sqrt{c+dx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{2\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2ab^2}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

output
```
(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (d*f*(-(b*((x*Sqrt
[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2]))) + (a*Sqrt[e]*Sqrt[c + d*x^2]*Elliptic
F[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(2*a*b^2) + (Sqrt[-c]*((c*e)/a
- (a*d*f)/b^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a*Sqrt[d]*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(x^2d+c)(f\,x^2+e)}\left(\frac{x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{2a\left(b\,x^2+a\right)}+\frac{df\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2b^2\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{de\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{Elliptic}}{2ab\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+c}}$
default	$\frac{\sqrt{x^2d+c}\,\sqrt{f\,x^2+e}\left(\sqrt{-\frac{d}{c}}\,a\,b^2df\,x^5+\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2bdf\,x^2+\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}}\right.\right.}{}$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(d*f*
x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^2+a)+1/2*d*f/b^2/(-1/c*d)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*d/a/b*e/(-1/c*d)^(1/2)*(1
+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elli
pticF(x*(-1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2*d/a/b*e/(-1/c*d)^(1/2
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*
EllipticE(x*(-1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2/b^2/(-1/c*d)^(1/2
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*
EllipticPi(x*(-1/c*d)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*d*f+1/2/a
^2/(-1/c*d)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-1/c*d)^
(1/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.163 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 426

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= -\frac{fx\sqrt{a+bx^2}}{2e(be-af)\sqrt{c+dx^2}\,(e+fx^2)}+\frac{\sqrt{c}\sqrt{d}f\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2e(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &\quad+\frac{\sqrt{c}\sqrt{d}(2de-cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2ae(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &\quad-\frac{c^{3/2}f(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/2*f*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*c^(1/2
)*d^(1/2)*f*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))/e/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)+1/2*c^(1/2)*d^(1/2)*(-c*f+2*d*e)*(b*x^2+a)^(1/2)*InverseJ
acobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e/(-c*f+d*e)^2/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f*(b*e*(-2*c*f+3*d
*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^
2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.13 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\frac{acf^2x}{e+fx^2}+\frac{bcf^2x^3}{e+fx^2}+\frac{adf^2x^3}{e+fx^2}+\frac{bdf^2x^5}{e+fx^2}+ia\sqrt{\frac{b}{a}}cf\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ia\sqrt{\frac{b}{a}}(-de+c}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((a*c*f^2*x)/(e + f*x^2) + (b*c*f^2*x^3)/(e + f*x^2) + (a*d*f^2*x^3)/(e +
f*x^2) + (b*d*f^2*x^5)/(e + f*x^2) + I*a*Sqrt[b/a]*c*f*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*
Sqrt[b/a]*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (2*I)*a*Sqrt[b/a]*c*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + ((2*I)
*a*d*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/Sqrt[b/a] - (I*a*c*f^2*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(Sqrt[b/a]*e))/(2*e*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{424}$$

$$\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \text{406}$$

$$-\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \text{320}$$

$$-\frac{bd\left(f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \text{388}$$

$$-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \; 313$$

$$\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

input

```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output

```
(f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e +
f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(
b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin
[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*
f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(400) = 800$.

Time = 8.80 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.28

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{2\left(a c f^2-a d e f-b c e f+b d e^2\right) e\left(f x^2+e\right)}-\frac{d b \sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{2\left(a c f^2-a d e f-b c e f+b d e^2\right)\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2+x^2 b c+a c}}+\frac{f b c\sqrt{1+\frac{b x^2}{a}}}{2\left(a c f^2-a d e f-\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^
2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*f
*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*f^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))
*a*c-1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a
)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-1/(a*c*f^2-a*d*e*f-b*c*e*
f+b*d*e^2)/e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)
/(-b/a)^(1/2))*b*c+3/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4+2bcef\,x^4+bd\,e^2x^4+2acef\,x^2+ad\,e}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f
**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2
+ 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f*
*2*x**8),x)
```

3.164 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 520

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \frac{f^2x\sqrt{a+bx^2}}{2e(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{d}(adf(2de+cf)-b(2d^2e^2+c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2\sqrt{c}(bc-ad)e(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{c}\sqrt{d}(adf(4de-cf)+b(2d^2e^2-6cdef+c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a(bc-ad)e(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}f^2(be(5de-2cf)-af(4de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/2*f^2*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e
)+1/2*d^(1/2)*(a*d*f*(c*f+2*d*e)-b*(c^2*f^2+2*d^2*e^2))*(b*x^2+a)^(1/2)*El
lipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(1/2)/(-a
*d+b*c)/e/(-a*f+b*e)/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)+1/2*c^(1/2)*d^(1/2)*(a*d*f*(-c*f+4*d*e)+b*(c^2*f^2-6*c*d*e*f+2*d^2
*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/
d)^(1/2))/a/(-a*d+b*c)/e/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+1/2*c^(3/2)*f^2*(b*e*(-2*c*f+5*d*e)-a*f*(-c*f+4*d*e))*(b*x^2+a
)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/
d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.92 (sec) , antiderivative size = 1652, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-2*a*b*Sqrt[b/a]*d^3*e^4*x + 2*a^2*Sqrt[b/a]*d^3*e^3*f*x - a*b*Sqrt[b/a]*
c^3*e*f^3*x + a^2*Sqrt[b/a]*c^2*d*e*f^3*x - 2*a*b*(b/a)^(3/2)*d^3*e^4*x^3
+ 2*a^2*Sqrt[b/a]*d^3*e^2*f^2*x^3 - a*b*(b/a)^(3/2)*c^3*e*f^3*x^3 + a^2*Sq
rt[b/a]*c*d^2*e*f^3*x^3 - 2*a*b*(b/a)^(3/2)*d^3*e^3*f*x^5 + 2*a*b*Sqrt[b/a
]*d^3*e^2*f^2*x^5 - a*b*(b/a)^(3/2)*c^2*d*e*f^3*x^5 + a*b*Sqrt[b/a]*c*d^2*
e*f^3*x^5 + I*b*c*e*(a*d*f*(2*d*e + c*f) - b*(2*d^2*e^2 + c^2*f^2))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*e*f*(-(d*e) + c*f)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + (5*I)*b^2*c^2*d*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (5*I)*a*b
*c*d^2*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b^2*c^3*e^2*f^2*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - (2*I)*a*b*c^2*d*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4
*I)*a^2*c*d^2*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(
a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*b*c^3*e*f^3*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - I*a^2*c^2*d*e*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d...
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.57, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{421}$$

$$\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{ 400}$$

$$\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$\downarrow$ 414

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}}{de-cf}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}$$

$\downarrow$ 424

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}}{de-cf}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{de-cf}$$

$\downarrow$ 406

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c}}{2e(e+fx^2)(be-af}\right.}{de-cf}$$

$\downarrow$ 320

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}\right.}{de-cf}$$

$\downarrow$ 388

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(}}{2e(be-af)(de-cf)}\right.}{de-cf}$$

$\downarrow$ 313

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E(}{b\sqrt{d}\sqrt{c+}}\right.\right.}{2e(be-af)(de-cf)}\right.}{de-cf}$$

$\downarrow$ 413

$$\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{2e(be-af)(de-cf)}\right.}{de-cf}$$

$\downarrow$ 413

$$\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{}}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\cdots\right.}{de-cf}$$

$\downarrow$ 412

$$\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{\ldots}}}\right.}{(de-cf)^2}}{de-cf}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\ldots\right.}{de-cf}$$

input

```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output

```
-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)
*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqr
t[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/
(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*
e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*
e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(d*e - c*f)) + (d*((d*(-((Sqrt
[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*
d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(d*e - c*f)^2
+ (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*(d*e - c*f)^2*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(d*e - c*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(494) = 988$.

Time = 11.19 (sec) , antiderivative size = 1560, normalized size of antiderivative = 3.00

method	result	size
elliptic	Expression too large to display	1560
default	Expression too large to display	2442

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^3/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(c*f-d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/(f*x^2+e)+(b*d*x^2+a*d)/c*d^2/(a*d-b*c)*x/(c*f-d*e)^2/((x^2+c/d)*(b*d
*x^2+a*d))^(1/2)-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))*b*d*f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(c*f-d*e)+1/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/c*d^2/(c*f-d*e)^2-1/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*d^3/c/(a*d-b*c)
/(c*f-d*e)^2+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(c
*f-d*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(c*f-d*e)*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*b/(a*d-b*c)/(c*f-d*e)^2*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*b/(a*d-b
*c)/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.165 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 760

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx = \frac{d(adf(2de+3cf)-b(2d^2e^2+3c^2f^2))\,x\sqrt{a+bx^2}}{6c(bc-ad)e(be-af)(de-cf)^2\left(c+dx^2\right)^{3/2}}$$
$$+\frac{f^2x\sqrt{a+bx^2}}{2e(be-af)(de-cf)\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}$$
$$-\frac{\sqrt{d}(a^2d^2f(4d^2e^2-16cdef-3c^2f^2)+b^2c(8d^3e^3-20cd^2e^2f-3c^3f^3)-2abd(2d^3e^3-4cd^2e^2f-10c^2de}{6c^{3/2}(bc-ad)^2e(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{d}(3a^2cd^2f^2(6de-cf)-2abd(d^3e^3-8cd^2e^2f+25c^2def^2-3c^3f^3)+3b^2c(2d^3e^3-8cd^2e^2f+12c^2def^2}{6a\sqrt{c}(bc-ad)^2e(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}f^3(be(7de-2cf)-af(6de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/6*d*(a*d*f*(3*c*f+2*d*e)-b*(3*c^2*f^2+2*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/(-
a*d+b*c)/e/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/2*f^2*x*(b*x^2+a)^(1/
2)/e/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)-1/6*d^(1/2)*(a^2*d^2*
f*(-3*c^2*f^2-16*c*d*e*f+4*d^2*e^2)+b^2*c*(-3*c^3*f^3-20*c*d^2*e^2*f+8*d^3
*e^3)-2*a*b*d*(-3*c^3*f^3-10*c^2*d*e*f^2-4*c*d^2*e^2*f+2*d^3*e^3))*(b*x^2+
a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/
c^(3/2)/(-a*d+b*c)^2/e/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)+1/6*d^(1/2)*(3*a^2*c*d^2*f^2*(-c*f+6*d*e)-2*a*b*d*(-3
*c^3*f^3+25*c^2*d*e*f^2-8*c*d^2*e^2*f+d^3*e^3)+3*b^2*c*(-c^3*f^3+12*c^2*d*
e*f^2-8*c*d^2*e^2*f+2*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(
1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/e/(-c*f+d*e)^4/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f^3*(b*e*(-2*c*
f+7*d*e)-a*f*(-c*f+6*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1
+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)/(-c*
f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(3c^2(bc-ad)^2f^4(c+dx^2)^2+2cd^3(bc-ad)e(be\right.}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(3*c^2*(b*c - a*d)^2*f^4*(c + d*x^2)^2 + 2*c*d^
3*(b*c - a*d)*e*(b*e - a*f)*(-(d*e) + c*f)*(e + f*x^2) + 4*d^3*e*(b*e - a*
f)*(a*d*(d*e - 4*c*f) + b*c*(-2*d*e + 5*c*f))*(c + d*x^2)*(e + f*x^2)) + I
*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(a
^2*d^2*f*(-4*d^2*e^2 + 16*c*d*e*f + 3*c^2*f^2) + 2*a*b*d*(2*d^3*e^3 - 4*c*
d^2*e^2*f - 10*c^2*d*e*f^2 - 3*c^3*f^3) + b^2*c*(-8*d^3*e^3 + 20*c*d^2*e^2
*f + 3*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a
*d)*(-(b*e*(d*e - c*f)*(-(a*d*f*(2*d*e + 3*c*f)) + b*(2*d^2*e^2 + 3*c^2*f^
2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) - 3*c*(b*c - a*d)*f^2*
(a*f*(6*d*e - c*f) + b*e*(-7*d*e + 2*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(6*Sqrt[b/a]*c^2*(b*c - a*d)^2*e^2*(b*e -
 a*f)*(d*e - c*f)^3*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1279, normalized size of antiderivative = 1.68, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {426, 421, 25, 402, 25, 400, 313, 320, 413, 413, 412, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2} \left(c+dx^2\right)^{5/2} \left(e+fx^2\right)^2} \, dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)}dx}{de-cf}-\frac{f \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{421}$$

$$\frac{d\left(\frac{f^2 \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d \int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \; 402$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int \frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \; 400$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}-\frac{d\cdots}{3c(\cdots}\cdots\right.}{(de-cf)^2}\cdots}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \; 313$$

$$\frac{d\left(\frac{d\left(-\frac{\frac{b(ad(de-4cf)-3bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$\downarrow$ 320

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\ \text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2d}{\sqrt{c}\sqrt{c+dx^2}(bc-}}{3c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$\downarrow$ 413

$$\frac{d\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-\cdots}{\sqrt{c}\cdots}}{3c(bc-ad)}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf\cdots}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$\Big\downarrow$ 413

$$\frac{d\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}\cdots}{\cdots}}{3c(bc-ad)}\cdots\right)}{(de-cf)\cdots}\cdots\right)}{de-cf\cdots}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$\Big\downarrow$ 412

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right)}{de-cf}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}$$

$\downarrow$ 426

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)}\cdots\right.}{\cdots}\right)}{de-cf}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}$$

$\downarrow$ 421

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\cdots}{3c(bc-ad)}\cdots\right)}{\cdots}\right)}{de-cf}\cdots$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}$$

$\downarrow$ 25

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\cdots}{3c(bc-ad)}\cdots\right)}{\cdots}\right)}{de-cf}\cdots$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}$$

$$\Big\downarrow \ 400$$

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)\cdots}\cdots\right)}{\cdots}\right)}{de-cf}\cdots$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}$$

$$\Big\downarrow \ 313$$

$$\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\operatorname{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\cdots}{3c(bc-ad)\cdots}\right.}{\cdots}\right.}{de-cf}$$

$$\frac{f\left(\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}}{de-cf}\cdots\right.}{de-cf}$$

$\Big\downarrow$ 320

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{}\right.}{de-cf}$$

$$\frac{f\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\right.}{de-cf}$$

$$\Big\downarrow \; 414$$

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right)}{de-cf}$$

$$\frac{f\left(\frac{d\left(d\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}$$

$\downarrow$ 424

$$d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right.$$

$$de-cf$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$$de-$$

$\downarrow$ 406

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}\cdots}\cdots\right.}{\cdots}\cdots\right)}{de-cf}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}$$

↓ 320

$$d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right)$$

$$de-cf$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$\downarrow$ 388

$$d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right.$$

$$de-cf$$

$$f\left(\frac{d\left(d\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$\downarrow$ 313

$$d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right.$$

$$de-cf$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$\downarrow$ 413

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{}\right)}{de-cf}$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\right.}{a\left(d\right.}}}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$\downarrow$ 413

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}\cdots}\cdots\right.}{\cdots}\right.}{de-cf}\cdots$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{\cdots}$$

$\downarrow$ 412

$$\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{\cdots}\right.\cdots}{de-cf}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(\cdots\right)}{a\left(\cdots\right)}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.}{\cdots}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(d*((d*(-1/3*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^
(3/2)) - ((Sqrt[d]*(b*c*(4*d*e - 7*c*f) - a*d*(2*d*e - 5*c*f))*Sqrt[a + b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*
c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[
c]*(a*d*(d*e - 4*c*f) - 3*b*c*(d*e - 2*c*f))*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*c*(b*c - a*d))))/(d*e
- c*f)^2 + (Sqrt[-a]*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticP
i[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*(d*e
 - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(d*e - c*f) - (f*(-((f*((f^2*
x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2
)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e -
 a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqr
t[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])))/(d*e - c*f)) + (d*((d*(-((Sqrt[d]*(d*e...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3334 vs. $2(728) = 1456$.

Time = 20.63 (sec) , antiderivative size = 3335, normalized size of antiderivative = 4.39

method	result	size
elliptic	Expression too large to display	3335
default	Expression too large to display	8404

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-8/3/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*d^3*b/(a*d-b*c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*a*f*EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^4*b/(a*d-b*c)^2
/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*a*e*EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))*b*d^2/(c*f-d*e)^2/(a*d-b*c)/c-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^3/(c*f-d*e)
/(c^2*f^2-2*c*d*e*f+d^2*e^2)/e/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))*a*d^4/c/(a*d-b*c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2)/(c*f-d*e)*b*e+1/2*
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*b*f^3/(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2)/e/(a*f-b*e)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3*(b*d*x^2+a*d)/c^2*d^2/(a
*d-b*c)^2*x*(4*a*c*d*f-a*d^2*e-5*b*c^2*f+2*b*c*d*e)/(c^2*f^2-2*c*d*e*f+d^2
*e^2)/(c*f-d*e)/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)-3*f^3/(c*f-d*e)/(c^2*f^2-2
*c*d*e*f+d^2*e^2)/(a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{\frac{5}{2}}\left(f\,x^2+e\right)^2}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^2,x)
```

3.166 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 517

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{f^2x\sqrt{c+dx^2}}{2e(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)} - \frac{\sqrt{b}(abcf^2-a^2df^2-2b^2e(de-cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2\sqrt{a}(bc-ad)e(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}\sqrt{b}(a^2df^2-abf(6de+cf)+2b^2e(de+2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c(bc-ad)e(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}f^2(be(5de-4cf)-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/2*f^2*x*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e
)-1/2*b^(1/2)*(a*b*c*f^2-a^2*d*f^2-2*b^2*e*(-c*f+d*e))*(d*x^2+c)^(1/2)*Ell
ipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(-a*
d+b*c)/e/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)-1/2*a^(1/2)*b^(1/2)*(a^2*d*f^2-a*b*f*(c*f+6*d*e)+2*b^2*e*(2*c*f+d*e
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/c/(-a*d+b*c)/e/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)+1/2*a^(3/2)*f^2*(b*e*(-4*c*f+5*d*e)-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1
/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(
1/2))/b^(1/2)/c/e^2/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.13 (sec) , antiderivative size = 1659, normalized size of antiderivative = 3.21

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(2*b^3*Sqrt[b/a]*c*d*e^4*x - 2*b^3*Sqrt[b/a]*c^2*e^3*f*x - a^3*
(b/a)^(3/2)*c^2*e*f^3*x + a^3*Sqrt[b/a]*c*d*e*f^3*x + 2*b^3*Sqrt[b/a]*d^2*
e^4*x^3 - 2*b^3*Sqrt[b/a]*c^2*e^2*f^2*x^3 - a*b^2*Sqrt[b/a]*c^2*e*f^3*x^3
+ a^3*Sqrt[b/a]*d^2*e*f^3*x^3 + 2*b^3*Sqrt[b/a]*d^2*e^3*f*x^5 - 2*b^3*Sqrt
[b/a]*c*d*e^2*f^2*x^5 - a*b^2*Sqrt[b/a]*c*d*e*f^3*x^5 + a^3*(b/a)^(3/2)*d^
2*e*f^3*x^5 + I*b*c*e*(-(a*b*c*f^2) + a^2*d*f^2 + 2*b^2*e*(d*e - c*f))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] - I*b*(-(b*c) + a*d)*e*(2*b*e + a*f)*(-(d*e) + c*f)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + (5*I)*a*b^2*c*d*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] - (5*I)*a^2*b*d^2*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*a*b^2*c^2*e^2
*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*a^2*b*c*d*e^2*f^2*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + (2*I)*a^3*d^2*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a^2*b*c^
2*e*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a^3*c*d*e*f^3*Sqrt[1 + (b*x^2)/a...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 426$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(be-af)^2}\right)}{be-af}-$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\text{ EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\text{ Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 424

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\text{ EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\text{ Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{be-af}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}}{be-af}$$

$$\frac{f\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c}}{2e(e+fx^2)(be-af}\right.}{be-af}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}}{be-af}$$

$$\frac{f\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}\right.}{be-af}$$

$\Big\downarrow$ 388

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}\cdots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\cdots}\right.}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\cdots\right.}\cdots}{2e(be-af)(de-cf)}\cdots\right)}{be-af}$$

$\downarrow$ 313

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}\cdots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\cdots}\right.}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+\cdots}}\cdots\right.\right.}{2e(be-af)(de-cf)}\cdots\right)}{be-af}$$

$\downarrow$ 413

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}\cdots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}$$

$$\frac{f\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\cdots}{\cdots}\right.\cdots\right.}{2e(be-af)(de-cf)}\cdots\right.}{be-af}$$

$$\Big\downarrow \; 413$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\cdots}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+\cdots\right)}{a\left(dx^2+\cdots\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\cdots\right.}{be-af}$$

$$\Big\downarrow \; 412$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+c}{a(dx^2+c}}}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\frac{\sqrt{}}{}\right.}{be-af}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)
*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqr
t[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/
(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*
e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*
e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(b*e - a*f)) + (b*((b*((Sqrt[b
]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (
a*d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]) - (Sqrt[c]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(b*e - a*f)^2 + (
a^(3/2)*f^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)
/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(b*e - a*f)^2*Sqrt[a + b*x^2]*Sq
rt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/(b*e - a*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. 2(491) = 982.

Time = 11.27 (sec) , antiderivative size = 1339, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	1339
default	Expression too large to display	2813

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)*b^2/a/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/2*f^3/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))/a*b^2/(a*f-b*e)^2-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))*b*d*f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)+c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*b^3/(a*d-b*c)/a/(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(a*
f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(a*f-b*e)*EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/(a
*f-b*e)*f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a*c-1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*f^2/e/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.167 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 708

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx =$$
$$-\frac{b(abcf^2-a^2df^2-2b^2e(de-cf))\,x}{2a(bc-ad)e(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{f^2x}{2e(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{d}(2b^3ce(de-cf)^2+a^3d^2f^2(2de+cf)-2a^2bdf(2d^2e^2+c^2f^2)+ab^2(2d^3e^3+c^3f^3))\sqrt{a+bx^2}\,E\Big(\text{arcta}}{2a\sqrt{c}(bc-ad)^2e(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{c}\sqrt{d}(2abcdf^2(4de-cf)-a^2d^2f^2(4de-cf)+b^2(4d^3e^3-8cd^2e^2f+c^3f^3))\sqrt{a+bx^2}\,\text{EllipticF}\Big(\text{arctar}}{2a(bc-ad)^2e(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}f^3(be(7de-4cf)-af(4de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/2*b*(a*b*c*f^2-a^2*d*f^2-2*b^2*e*(-c*f+d*e))*x/a/(-a*d+b*c)/e/(-a*f+b*e
)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*f^2*x/e/(-a*f+b*e)/(-c*
f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*d^(1/2)*(2*b^3*c*e*(-
c*f+d*e)^2+a^3*d^2*f^2*(c*f+2*d*e)-2*a^2*b*d*f*(c^2*f^2+2*d^2*e^2)+a*b^2*(
c^3*f^3+2*d^3*e^3))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/e/(-a*f+b*e)^2/(-c*f+d*e
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(1/2)*d^(1/2)*(2
*a*b*c*d*f^2*(-c*f+4*d*e)-a^2*d^2*f^2*(-c*f+4*d*e)+b^2*(c^3*f^3-8*c*d^2*e^
2*f+4*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),
(1-b*c/a/d)^(1/2))/a/(-a*d+b*c)^2/e/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f^3*(b*e*(-4*c*f+7*d*e)-a*f*
(-c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/
2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)^2/(-c*f+d*e)^3/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.45 (sec) , antiderivative size = 3040, normalized size of antiderivative = 4.29

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*((b^4*x)/(a*(-(b*c) + a*d)^2*(-(b*e) + a*f
)^2*(a + b*x^2)) + (d^4*x)/(c*(b*c - a*d)^2*(-(d*e) + c*f)^2*(c + d*x^2))
+ (f^4*x)/(2*e*(b*e - a*f)^2*(d*e - c*f)^2*(e + f*x^2))) - (Sqrt[(a + b*x^
2)*(c + d*x^2)]*(((-2*I)*b^4*c^2*d^2*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*Sqrt[(a + b*x^2)*(c + d*x^2)])
- ((2*I)*a*b^3*c*d^3*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(Elliptic
E[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)]))/(Sqrt[b/a]*Sqrt[(a + b*x^2)*(c + d*x^2)]) + ((4*I)*b^4*c^3
*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/
(Sqrt[b/a]*Sqrt[(a + b*x^2)*(c + d*x^2)]) + ((4*I)*a^2*b^2*c*d^3*e^2*f*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*S
qrt[(a + b*x^2)*(c + d*x^2)]) - ((2*I)*b^4*c^4*e*f^2*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*Sqrt[(a + b*x^2)*(c
 + d*x^2)]) - ((2*I)*a^3*b*c*d^3*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*Sqrt[(a + b*x^2)*(c + d*x^2)])...
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 1366, normalized size of antiderivative = 1.93, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.781$, Rules used = {426, 421, 25, 402, 400, 313, 320, 416, 313, 414, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{a(bde+bcf-2adf)-bd(be-af)x^2}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(-2a^2df+abde+b^2ce\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2df+abde+b^2ce\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \cdots}{\cdots}\right.}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2 \cdots\right.}{\sqrt{c}\sqrt{\cdots}}}{a(bc-ad)}\right)}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 416

$$\frac{b\left(\frac{f^2\left(\frac{d\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots}{a(\cdots}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af\cdots}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+\cdots\right)}{a\left(c+\cdots\right)}}}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{be-af\cdots}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}}{\cdots}\right)}{\cdots}\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\big\downarrow$ 426

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}}{\cdots}\right)}{\cdots}\right)}{be-af}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{be-af}$$

$\big\downarrow$ 421

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}}{}\right.}{}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}\right)}{be-af}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}}{}\right.}{}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \, 400$$

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}\cdots}{\cdots}\cdots\right.}{\cdots}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \, 313$$

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c\cdots}}}{de-cf\cdots}\right.}{be-af}$$

$\downarrow$ 320

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\cdots\right.}{be-af}$$

$\downarrow$ 414

$$b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad be-af\qquad\qquad}$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.$$

$$\overline{\qquad\qquad be-af\qquad\qquad}$$

$$\downarrow 424$$

$$b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad be-af\cdots}$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\overline{\qquad\qquad be-\cdots}$$

$\downarrow$ 406

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc}{}\right.}{}}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(b}{a(d}}}\right.}{(de-cf)^2}\right.}{de-cf}}{}$$

$\downarrow$ 320

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\cdots}$$

$\downarrow$ 388

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{\cdots}$$

$\downarrow$ 313

$$b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad be-af\qquad\cdots}$$

$$f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right)}{a\left(d\cdots\right)}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.\cdots$$

↓ 413

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{be-af\cdots}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(b\cdots}{a(d\cdots}}\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}$$

$\downarrow$ 413

$$
\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{be-af}\cdots
$$

$$
f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\cdots\right.}{a\left(d\cdots\right.}}\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.
$$

$$\Big\downarrow \; 412$$

$$\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}$$

$$\frac{f\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(a\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\ldots\right.}{a\left(d\ldots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(b*((b*((b*(b*e - a*f)*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
- (-((Sqrt[d]*(b^2*c*e + a*b*d*e - 2*a^2*d*f)*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*(2*b*d*e + b*
c*f - 3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2]))/(a*(b*c - a*d))))/(b*e - a*f)^2 + (f^2*((Sqrt[d]*Sqrt[a +
 b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*
(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (a^(3
/2)*f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sqrt[
a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[(a*(c
 + d*x^2))/(c*(a + b*x^2))])))/(b*e - a*f)^2))/(b*e - a*f) - (f*(-((f*((f^
2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x
^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e
 - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3678 vs. 2(676) = 1352.

Time = 20.61 (sec) , antiderivative size = 3679, normalized size of antiderivative = 5.20

method	result	size
elliptic	Expression too large to display	3679
default	Expression too large to display	5035

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^4/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*
x^2+e)-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*d^2*b^4/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*c*f^2-a*d*e*f
-b*c*e*f+b*d*e^2)^2*e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*f^2
/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2+7/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*
e^2)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^
(1/2))*b*d+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^3/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))-2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*d^3*b^2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*c*f^2-a
*d*e*f-b*c*e*f+b*d*e^2)^2*e*f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))+2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*d^3*b^2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*c*f^2-a*d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{}{b^2d^2f^2x^{12}+2ab\,d^2f^2x^{10}+2b^2cd\,f^2x^{10}+2b^2d^2ef\,x^{10}+a^2d}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2*e**2 + 2*a**2*c**2*e*f*
x**2 + a**2*c**2*f**2*x**4 + 2*a**2*c*d*e**2*x**2 + 4*a**2*c*d*e*f*x**4 +
2*a**2*c*d*f**2*x**6 + a**2*d**2*e**2*x**4 + 2*a**2*d**2*e*f*x**6 + a**2*d
**2*f**2*x**8 + 2*a*b*c**2*e**2*x**2 + 4*a*b*c**2*e*f*x**4 + 2*a*b*c**2*f*
*2*x**6 + 4*a*b*c*d*e**2*x**4 + 8*a*b*c*d*e*f*x**6 + 4*a*b*c*d*f**2*x**8 +
 2*a*b*d**2*e**2*x**6 + 4*a*b*d**2*e*f*x**8 + 2*a*b*d**2*f**2*x**10 + b**2
*c**2*e**2*x**4 + 2*b**2*c**2*e*f*x**6 + b**2*c**2*f**2*x**8 + 2*b**2*c*d*
e**2*x**6 + 4*b**2*c*d*e*f*x**8 + 2*b**2*c*d*f**2*x**10 + b**2*d**2*e**2*x
**8 + 2*b**2*d**2*e*f*x**10 + b**2*d**2*f**2*x**12),x)
```

3.168 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 747

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx =$$

$$-\frac{b(3abcf^2-3a^2df^2-2b^2e(de-cf))\,x\sqrt{c+dx^2}}{6a(bc-ad)e(be-af)^2(de-cf)\,(a+bx^2)^{3/2}}$$

$$+\frac{f^2x\sqrt{c+dx^2}}{2e(be-af)(de-cf)\,(a+bx^2)^{3/2}\,(e+fx^2)}$$

$$-\frac{\sqrt{b}(6a^3bcdf^3-3a^4d^2f^3-4b^4ce^2(de-cf)+8ab^3e(d^2e^2+cdef-2c^2f^2)-a^2b^2f(20d^2e^2-20cdef+3c}{6a^{3/2}(bc-ad)^2e(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{\sqrt{b}(2b^4cde^3+3a^4d^2f^3-6a^3bdf^2(6de+cf)+a^2b^2f(24d^2e^2+50cdef+3c^2f^2)-2ab^3e(3d^2e^2+8cdef-}{6\sqrt{a}c(bc-ad)^2e(be-af)^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}f^3(be(7de-6cf)-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/6*b*(3*a*b*c*f^2-3*a^2*d*f^2-2*b^2*e*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/a/(-
a*d+b*c)/e/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(3/2)+1/2*f^2*x*(d*x^2+c)^(1/
2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(3/2)/(f*x^2+e)-1/6*b^(1/2)*(6*a^3*b*
c*d*f^3-3*a^4*d^2*f^3-4*b^4*c*e^2*(-c*f+d*e)+8*a*b^3*e*(-2*c^2*f^2+c*d*e*f
+d^2*e^2)-a^2*b^2*f*(3*c^2*f^2-20*c*d*e*f+20*d^2*e^2))*(d*x^2+c)^(1/2)*Ell
ipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/(-a*
d+b*c)^2/e/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-1/6*b^(1/2)*(2*b^4*c*d*e^3+3*a^4*d^2*f^3-6*a^3*b*d*f^2*(c*f+6*d*e
)+a^2*b^2*f*(3*c^2*f^2+50*c*d*e*f+24*d^2*e^2)-2*a*b^3*e*(9*c^2*f^2+8*c*d*e
*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(
1-a*d/b/c)^(1/2))/a^(1/2)/c/(-a*d+b*c)^2/e/(-a*f+b*e)^4/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*f^3*(b*e*(-6*c*f+7*d*e)-a*f*(-c*
f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1
-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-a*f+b*e)^4/(-c*f+d*e)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ex(c+dx^2)\left(3a^2(bc-ad)^2f^4(a+bx^2)^2+2ab^3(bc-ad)e(be\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2*f^4*(a + b*x^2)^2 + 2*a*b^
3*(b*c - a*d)*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2) + 4*b^3*e*(d*e - c*f)*
(b^2*c*e + 5*a^2*d*f - 2*a*b*(d*e + 2*c*f))*(a + b*x^2)*(e + f*x^2)) + I*(
a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(-(b*c*e*(6
*a^3*b*c*d*f^3 - 3*a^4*d^2*f^3 + 4*b^4*c*e^2*(-(d*e) + c*f) + a^2*b^2*f*(-
20*d^2*e^2 + 20*c*d*e*f - 3*c^2*f^2) + 8*a*b^3*e*(d^2*e^2 + c*d*e*f - 2*c^
2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + (b*c - a*d)*(b*e
*(-(d*e) + c*f)*(4*b^3*c*e^2 + 3*a^3*d*f^2 - 3*a^2*b*f*(-6*d*e + c*f) - 2*
a*b^2*e*(3*d*e + 8*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
3*a^2*(-(b*c) + a*d)*f^2*(b*e*(7*d*e - 6*c*f) + a*f*(-2*d*e + c*f))*Ellipt
icPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(6*a^2*Sqrt[b/a]
*(b*c - a*d)^2*e^2*(b*e - a*f)^3*(d*e - c*f)*(a + b*x^2)^(3/2)*Sqrt[c + d*
x^2]*(e + f*x^2))
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 1283, normalized size of antiderivative = 1.72, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {426, 421, 25, 402, 25, 400, 313, 320, 413, 413, 412, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{b\left(\frac{\frac{b\left(7a^2df-5abcf-4abde+2b^2ce\right)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{}{\sqrt{bx^2+}}}{}\right.}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{313}$$

$$b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}\right)}{(be-af)^2}\right.$$

$$\overline{\qquad\qquad\qquad\qquad be-af}$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{320}$$

$$b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2\right.}{a\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad\qquad\qquad\qquad be-af}$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf\cdots\right.}{a\sqrt{c+\cdots}}}{3a(bc-ad)}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af\cdots}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 413

$$\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a\cdots\right.}{\cdots}}{3a(bc-ad)}\cdots\right.}{(be-af\cdots}\cdots\right.}{be-af\cdots}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 412

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+ \cdots\right)}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\Big\downarrow$ 426

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+ \cdots\right)}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}$$

$\Big\downarrow$ 421

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int-\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \ 400$$

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \ 313$$

$$b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}+\cdots\right)}{(be-af)^2}\right)$$

$$\overline{\qquad be-af \qquad}$$

$$f\left(\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+\cdots}}}{be-a\cdots}\right)$$

$$\overline{\qquad be-af \qquad}$$

↓ 320

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad be-af\qquad\qquad}$$

$$f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(be-af)^2}\right)}{be-af}-\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad be-af\qquad\qquad}$$

$$\downarrow 414$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{a}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right)}{be-af}\right)}{be-af}$$

$\downarrow$ 424

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$\overline{\qquad be-af\cdots}$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(a\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.$$

$$\overline{\qquad be-\cdots}$$

↓ 406

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{}\right)}{}\right)$$

$$\overline{be-af}$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(a\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right)}{(be-af)^2}\right)}{be-af}\right)$$

$\downarrow$ 320

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+cx}}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad be-af}$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(a\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\downarrow 388$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}\cdots}\cdots\right.}{\cdots}\cdots\right.}{be-af\cdots}$$

$$\frac{f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right.}{\cdots}$$

↓ 313

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right.$$

$$be-af$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(a\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$\downarrow$ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right)}{be-af}\right.}{}$$

$\downarrow$ 413

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad be-af\qquad\qquad}$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$\downarrow$ 412

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{}\right.$$

$$be-af$$

$$f\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right)}{be-af}\right.$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(b*((b*((b*(b*e - a*f)*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/
2)) + ((Sqrt[b]*(2*b^2*c*e - 4*a*b*d*e - 5*a*b*c*f + 7*a^2*d*f)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*(b
*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c
]*Sqrt[d]*(b^2*c*e + 6*a^2*d*f - a*b*(3*d*e + 4*c*f))*Sqrt[a + b*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*a*(b*c - a*d))))/(b*e
 - a*f)^2 + (Sqrt[-a]*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*(b*
e - a*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(b*e - a*f) - (f*(-((f*((f^2
*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^
2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
 b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d
]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e
- a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sq
rt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])))/(b*e - a*f)) + (b*((b*((Sqrt[b]*(b*e ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. $2(715) = 1430$.

Time = 20.53 (sec) , antiderivative size = 2509, normalized size of antiderivative = 3.36

method	result	size
elliptic	Expression too large to display	2509
default	Expression too large to display	9310

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^4*b/(a*
f-b*e)/(a^2*f^2-2*a*b*e*f+b^2*e^2)/e/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/(b*f*x^2+b*e)-1/3*b/(a*f-b*e)^2/(a*d-b*c)/a*x*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-2/3*(b*d*x^2+b*c)*b^2/a^2/(a*d-b*c)^2*x*(5*
a^2*d*f-4*a*b*c*f-2*a*b*d*e+b^2*c*e)/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)
/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))*d*b^2/(a*f-b*e)^2/(a*d-b*c)/a-8/3*c^2/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
b^4/(a*d-b*c)^2/a/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*f*EllipticE(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+10/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^3/(a*d-b*c)^2/(a^
2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-4/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))*b^3/(a*d-b*c)/a/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*d*e+2/3/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^4/(a*d-b*c)/a^2/
(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*c*e-4/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^4/(a*d-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.169 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1047

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx =$$

$$-\frac{b(3abcf^2-3a^2df^2-2b^2e(de-cf))x}{6a(bc-ad)e(be-af)^2(de-cf)(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$

$$-\frac{b(6a^3bcdf^3-3a^4d^2f^3-4b^4ce^2(de-cf)+4ab^3e(3d^2e^2+cdef-4c^2f^2)-3a^2b^2f(8d^2e^2-8cdef+c^2f^2 \ldots}{6a^2(bc-ad)^2e(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$+\frac{f^2x}{2e(be-af)(de-cf)(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}$$

$$+\frac{\sqrt{d}(4b^5c^2e^2(de-cf)^2+3a^5d^3f^3(2de+cf)-2ab^4ce(de-cf)^2(7de+8cf)-9a^4bd^2f^2(2d^2e^2+c^2f^2)+ \ldots}{6a^2\sqrt{c}(bc-ad)^3e(be \ldots}$$

$$-\frac{\sqrt{c}\sqrt{d}(2b^4ce(de-cf)^3-3a^4d^3f^3(4de-cf)+3a^3bd^2f^2(2d^2e^2+10cdef-3c^2f^2)+3a^2b^2df(8d^3e^3-20 \ldots}{6a^2(bc-ad)^3e(be-af)^2(de- \ldots}$$

$$+\frac{c^{3/2}f^4(3be(3de-2cf)-af(4de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)^3(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/6*b*(3*a*b*c*f^2-3*a^2*d*f^2-2*b^2*e*(-c*f+d*e))*x/a/(-a*d+b*c)/e/(-a*f
+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)-1/6*b*(6*a^3*b*c*d*f^3-
3*a^4*d^2*f^3-4*b^4*c*e^2*(-c*f+d*e)+4*a*b^3*e*(-4*c^2*f^2+c*d*e*f+3*d^2*e
^2)-3*a^2*b^2*f*(c^2*f^2-8*c*d*e*f+8*d^2*e^2))*x/a^2/(-a*d+b*c)^2/e/(-a*f+
b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*f^2*x/e/(-a*f+b*e)/(
-c*f+d*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/6*d^(1/2)*(4*b^5*c^2
*e^2*(-c*f+d*e)^2+3*a^5*d^3*f^3*(c*f+2*d*e)-2*a*b^4*c*e*(-c*f+d*e)^2*(8*c*
f+7*d*e)-9*a^4*b*d^2*f^2*(c^2*f^2+2*d^2*e^2)+9*a^3*b^2*d*f*(c^3*f^3+2*d^3*
e^3)-a^2*b^3*(3*c^4*f^4-26*c^3*d*e*f^3+52*c^2*d^2*e^2*f^2-26*c*d^3*e^3*f+6
*d^4*e^4))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^3/e/(-a*f+b*e)^3/(-c*f+d*e)^2/(c*
(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/6*c^(1/2)*d^(1/2)*(2*b^4*c*
e*(-c*f+d*e)^3-3*a^4*d^3*f^3*(-c*f+4*d*e)+3*a^3*b*d^2*f^2*(-3*c^2*f^2+10*c
*d*e*f+2*d^2*e^2)+3*a^2*b^2*d*f*(3*c^3*f^3-20*c*d^2*e^2*f+8*d^3*e^3)-3*a*b
^3*(c^4*f^4-10*c*d^3*e^3*f+6*d^4*e^4))*(b*x^2+a)^(1/2)*InverseJacobiAM(arc
tan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/(-a*d+b*c)^3/e/(-a*f+b*e)^2/
(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/2*c^(3/2)*f
^4*(3*b*e*(-2*c*f+3*d*e)-a*f*(-c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1
/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2
/(-a*f+b*e)^3/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.69 (sec) , antiderivative size = 765, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}ex\Big(3a^2c(bc-ad)^3f^5(a+bx^2)^2(c+dx^2)+6a^2d^5e(be-$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*x*(3*a^2*c*(b*c - a*d)^3*f^5*(a + b*x^2)^2*(c + d*x^2) + 6*
a^2*d^5*e*(b*e - a*f)^3*(a + b*x^2)^2*(e + f*x^2) - 2*a*b^4*c*(-(b*c) + a*
d)*e*(-(b*e) + a*f)*(d*e - c*f)^2*(c + d*x^2)*(e + f*x^2) - 2*b^4*c*e*(d*e
 - c*f)^2*(2*b^2*c*e + 13*a^2*d*f - a*b*(7*d*e + 8*c*f))*(a + b*x^2)*(c +
d*x^2)*(e + f*x^2))) - I*c*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*(e + f*x^2)*(-(b*e*(4*b^5*c^2*e^2*(d*e - c*f)^2 + 3*a^5*d^3*f^3*(2*d*
e + c*f) - 2*a*b^4*c*e*(d*e - c*f)^2*(7*d*e + 8*c*f) - 9*a^4*b*d^2*f^2*(2*
d^2*e^2 + c^2*f^2) + 9*a^3*b^2*d*f*(2*d^3*e^3 + c^3*f^3) + a^2*b^3*(-6*d^4
*e^4 + 26*c*d^3*e^3*f - 52*c^2*d^2*e^2*f^2 + 26*c^3*d*e*f^3 - 3*c^4*f^4))*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + (b*c - a*d)*(b*e*(d*e -
c*f)*(-6*a^3*b*c*d*f^3 + 3*a^4*d^2*f^3 + 4*b^4*c*e^2*(d*e - c*f) - 4*a*b^3
*e*(3*d^2*e^2 + c*d*e*f - 4*c^2*f^2) + 3*a^2*b^2*f*(8*d^2*e^2 - 8*c*d*e*f
+ c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*a^2*(b*c -
a*d)^2*f^3*(3*b*e*(3*d*e - 2*c*f) + a*f*(-4*d*e + c*f))*EllipticPi[(a*f)/(
b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(6*a^2*Sqrt[b/a]*c*(b*c - a*
d)^3*e^2*(b*e - a*f)^3*(d*e - c*f)^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e
+ f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)^{3/2}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\left(\frac{\frac{d\int \frac{b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\right.}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\big\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{\frac{d\left(\frac{\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{bx\left(9a^2df-5ab\ldots\right.}{a\sqrt{a+bx^2}\sqrt{\ldots}}}{3a(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\big\downarrow$ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}}{3a(bc-ad)}}{(be-af)^2}\right.$$

$$\overline{\qquad be-af \qquad}$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 320

$$b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\text{arct}\ldots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\ldots\right.}{\ldots}\right)}{3a(bc-a\ldots}\ldots\right)}{be-af}$$

$$\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc}\right.}{}\right.}{}\right.$$

$$\overline{\qquad be}$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 400

$$\frac{b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\right.}{}\right.}{}$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 313

$$b\left(\frac{f^2\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf \cdots\right.}{a\sqrt{a+bx^2}\sqrt{c} \cdots}\right.}{\cdots}\right.$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$\big\downarrow$ 320

$$b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{(be-af)^2}+\right.$$

$$\frac{f\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}$$

$$\Big\downarrow \text{414}$$

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\right.}{a\left(dx^2\right.}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.$$

$$\frac{f\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \text{426}$$

$$b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\right.}{a\left(dx^2\right.}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 421

$$\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.}{f\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int-\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\right)}$$

$$be-af$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{(de-cf)^2}}{(be-af)^2}}{\frac{f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dxf^2}{(be-af)^2}+\frac{b\int\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\right)}{be-af}}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{(de-cf)^2}}{(be-af)^2}}{}$$

$$\frac{f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\int\frac{a(bde+bcf-2adf)-bd(be-af)x^2}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\right.}{be-af}$$

$$\downarrow \text{400}$$

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx^2\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{(be-af)^2}\cdots\right.$$

$$\frac{f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{ab(2bde+bcf-3adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(-2dfa^2+bdea+b^2ce\right)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx\cdots}{bc-ad}\cdots}{a(bc-ad)}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.}{be-af}\cdots$$

↓ 313

$$b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{ab(2bde+bcf-3adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\left(-2dfa^2+bdea+b^2ce\right)\sqrt{bx^2+a}E\left(\text{arct}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d}}}{a(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$be-af$$

↓ 320

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx^2\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{(be-af)^2}\cdots\right.$$

$$f\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}\left(-2dfa^2+bdea+\cdots\right.}{\sqrt{c}(bc\cdots}}{a(bc-ad)}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.$$

$$\overline{\qquad\qquad\qquad}$$
$$be-af$$

↓ 416

$$b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\left(\frac{d\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{a(bc}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$be-af$$

↓ 313

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\right.}{a\left(dx^2\right.}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx}}}{}\right.}{}\right.}{be-af}\right.$$

$$be-af$$

↓ 414

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx^2\ldots\right.}}}\ldots\right.}{(de-cf)^2}\ldots\right.}{(be-af)^2}\ldots\right.$$

$$\overline{f\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\ldots\right.}$$

$$be\ldots$$

↓ 426

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx^2\ldots\right.}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\right.$$

↓ 421

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\right.}{a\left(dx^2\right.}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\right.$$

↓ 25

$$b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{arc}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\right.}{a\left(dx^2\right.}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.$$

$$f\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\right.$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3093 vs. $2(1009) = 2018$.

Time = 21.74 (sec) , antiderivative size = 3094, normalized size of antiderivative = 2.96

method	result	size
elliptic	Expression too large to display	3094
default	Expression too large to display	14457

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(7/3*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*d*b^5/(a*d-b*c)^3/a/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*e*EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^6/(a*d-b*c)^
3/a^2/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*e*EllipticE(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))*b*f^3*d/(c*f-d*e)^2/(a*f-b*e)/(a^2*f^2-2*a*b*e*f+b^2*e^2)-1
3/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^3/(a*d
-b*c)^2/(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*f-b*e)*d*f+1/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^4/c/(a*c*d*f-a*d^2*e-b*c^2*f+b*c
*d*e)^2+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*
b^3*d/(a*d-b*c)^2/(a*f-b*e)^2/a-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))*a*d^5/c/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)^
2+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{}{b^3d^2f^2x^{14}+3a\,b^2d^2f^2x^{12}+2b^3cd\,f^2x^{12}+2b^3d^2ef\,x^{12}+3a}$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c**2*e**2 + 2*a**3*c**2*e*f*
x**2 + a**3*c**2*f**2*x**4 + 2*a**3*c*d*e**2*x**2 + 4*a**3*c*d*e*f*x**4 +
2*a**3*c*d*f**2*x**6 + a**3*d**2*e**2*x**4 + 2*a**3*d**2*e*f*x**6 + a**3*d
**2*f**2*x**8 + 3*a**2*b*c**2*e**2*x**2 + 6*a**2*b*c**2*e*f*x**4 + 3*a**2*
b*c**2*f**2*x**6 + 6*a**2*b*c*d*e**2*x**4 + 12*a**2*b*c*d*e*f*x**6 + 6*a**
2*b*c*d*f**2*x**8 + 3*a**2*b*d**2*e**2*x**6 + 6*a**2*b*d**2*e*f*x**8 + 3*a
**2*b*d**2*f**2*x**10 + 3*a*b**2*c**2*e**2*x**4 + 6*a*b**2*c**2*e*f*x**6 +
 3*a*b**2*c**2*f**2*x**8 + 6*a*b**2*c*d*e**2*x**6 + 12*a*b**2*c*d*e*f*x**8
 + 6*a*b**2*c*d*f**2*x**10 + 3*a*b**2*d**2*e**2*x**8 + 6*a*b**2*d**2*e*f*x
**10 + 3*a*b**2*d**2*f**2*x**12 + b**3*c**2*e**2*x**6 + 2*b**3*c**2*e*f*x*
*8 + b**3*c**2*f**2*x**10 + 2*b**3*c*d*e**2*x**8 + 4*b**3*c*d*e*f*x**10 +
2*b**3*c*d*f**2*x**12 + b**3*d**2*e**2*x**10 + 2*b**3*d**2*e*f*x**12 + b**
3*d**2*f**2*x**14),x)
```

3.170 $\int \frac{1}{(a+bx^2)^2 \sqrt{c-dx^2} \sqrt{e+fx^2}} dx$

Optimal result

Integrand size = 33, antiderivative size = 426

$$\int \frac{1}{(a+bx^2)^2 \sqrt{c-dx^2} \sqrt{e+fx^2}} dx = \frac{b^2 x \sqrt{c-dx^2} \sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\, E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\, \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a(bc+ad)\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}(b^2ce-3a^2df+ab(2de-2cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\, \text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

output
```
1/2*b^2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(b*x^2+a
)+1/2*b*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1/2)*(f*x^2+e)^(1/2)*EllipticE(d^(1/2
)*x/c^(1/2),(-c*f/d/e)^(1/2))/a/(a*d+b*c)/(-a*f+b*e)/(-d*x^2+c)^(1/2)/(1+f
*x^2/e)^(1/2)-1/2*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)*Elli
pticF(d^(1/2)*x/c^(1/2),(-c*f/d/e)^(1/2))/a/(a*d+b*c)/(-d*x^2+c)^(1/2)/(f*
x^2+e)^(1/2)+1/2*c^(1/2)*(b^2*c*e-3*a^2*d*f+a*b*(-2*c*f+2*d*e))*(1-d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2),-b*c/a/d,(-c*f/d/e)
^(1/2))/a^2/d^(1/2)/(a*d+b*c)/(-a*f+b*e)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.85 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+bx^2)^2 \sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{-\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}-\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}-ibc\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)+ic\sqrt{-\frac{d}{c}}(}{}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(-((b^2*c*e*x)/(a + b*x^2)) + (b^2*d*e*x^3)/(a + b*x^2) - (b^2*c*f*x^3)/(a
 + b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) - I*b*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d
*e))] + I*c*Sqrt[-(d/c)]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + (I*b^2*c*e*Sqrt[
1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sq
rt[-(d/c)]*x], -((c*f)/(d*e))])/(a*Sqrt[-(d/c)]) - (2*I)*b*c*Sqrt[-(d/c)]*
e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*Arc
Sinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + ((2*I)*b*d*f*Sqrt[1 - (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -
((c*f)/(d*e))])/(-(d/c))^(3/2) + (3*I)*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*
x], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[
e + f*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 \sqrt{c-dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{ 424}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\int \frac{bx^2+a}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{f}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{\frac{fx^2}{e}+1}(be-af)\int \frac{1}{\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\int\frac{\sqrt{fx^2+e}}{\sqrt{c-dx^2}}dx}{f}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{fx^2+e}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\int\frac{\sqrt{\frac{fx^2}{e}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int\frac{1}{(bx^2+a)\sqrt{c-dx^2}\sqrt{fx^2+e}}dx}{2a(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{fx^2+e}}dx}{2a\sqrt{c-dx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}}dx}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{df\left(\frac{b\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}-\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{\sqrt{d}f\sqrt{c-dx^2}\sqrt{e+fx^2}}\right)}{2a(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a +
b*x^2)) + (d*f*((b*Sqrt[c]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f*Sqrt[c - d*x^2]*Sq
rt[1 + (f*x^2)/e]) - (Sqrt[c]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(Sqrt[d]*f
*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])))/(2*a*(b*c + a*d)*(b*e - a*f)) + (Sqrt[
c]*(b^2*c*e - 3*a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*
f)/(d*e))])/(2*a^2*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e
+ f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(369) = 738$.

Time = 8.82 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.32

method	result
elliptic	$\frac{\sqrt{(-x^2d+c)(f\,x^2+e)}\left(-\frac{b^2x\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}{2a\left(a^2df+abcf-abde-ce\,b^2\right)\left(b\,x^2+a\right)}-\frac{df\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2\left(a^2df+abcf-abde-ce\,b^2\right)\sqrt{\frac{d}{c}}\,\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}+\frac{bde\sqrt{1-\frac{d}{}}}{2\left(a^2df+abc\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/2*b^2/a/
(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)*x*(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(
b*x^2+a)-1/2*d*f/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/(1/c*d)^(1/2)*(1-d*x^2/
c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(
x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f+a*b*c*f-a*b*d*e
-b^2*c*e)/a*e/(1/c*d)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+
c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1
/2))-1/2*b*d/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*e/(1/c*d)^(1/2)*(1-d*x^2/
c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticE(
x*(1/c*d)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+3/2/(a^2*d*f+a*b*c*f-a*b*d*e-b^2
*c*e)/(1/c*d)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-
d*e*x^2+c*e)^(1/2)*EllipticPi(x*(1/c*d)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(1/c*d
)^(1/2))*d*f+1/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*b/(1/c*d)^(1/2)*(1-d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Elliptic
Pi(x*(1/c*d)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(1/c*d)^(1/2))*c*f-1/(a^2*d*f+a*b
*c*f-a*b*d*e-b^2*c*e)/a*b/(1/c*d)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2
)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(1/c*d)^(1/2),-b*c/a/d
,(-f/e)^(1/2)/(1/c*d)^(1/2))*d*e-1/2/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a^2
*b^2/(1/c*d)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d
*e*x^2+c*e)^(1/2)*EllipticPi(x*(1/c*d)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(1/c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}}{-b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6-b^2de\,x^6-a^2df\,x^4+2abcf\,x^4-2abde\,x^4+b^2ce\,x^4+a^2cf\,x^2-a^2de}$$

input
```
int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output

```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2))/(a**2*c*e + a**2*c*f*x**2 - a**2*d
*e*x**2 - a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4
 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 - b**2*d*e*x**6 - b**2*d
*f*x**8),x)
```

3.171 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 424

$$
\begin{aligned}
&\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx \\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\,(a+bx^2)\,\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{e}\sqrt{f}(be-2af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2ac(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{be^{3/2}(b^2ce+3a^2df-2ab(de+cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2a^2c(bc-ad)\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)/(f*x^2+e)^(1/2)+1/2*b*e^(1/
2)*f^(1/2)*(d*x^2+c)^(1/2)*EllipticE(f^(1/2)*x/e^(1/2)/(1+f*x^2/e)^(1/2),(
1-d*e/c/f)^(1/2))/a/(-a*d+b*c)/(-a*f+b*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/
(f*x^2+e)^(1/2)-1/2*e^(1/2)*f^(1/2)*(-2*a*f+b*e)*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(f^(1/2)*x/e^(1/2)),(1-d*e/c/f)^(1/2))/a/c/(-a*f+b*e)^2/(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*b*e^(3/2)*(b^2*c*e+3*a^2*d*
f-2*a*b*(c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(f^(1/2)*x/e^(1/2)/(1+f*x^2/e
)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))/a^2/c/(-a*d+b*c)/f^(1/2)/(-a*f+b*e)^2
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}+\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}+ibc\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ic\sqrt{\frac{d}{c}}(be-af)\cdots}{}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x^3)/(a +
b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*c*
Sqrt[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1
 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
)/(a*Sqrt[d/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + ((2*I
)*b*c*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (3*I)*a*c*Sqrt[d/c]*f*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(2*a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{424}$$

$$\frac{(3a^2df-2ab(cf+de)+b^2ce)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{406}$$

$$\frac{(3a^2df-2ab(cf+de)+b^2ce)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(a\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{320}$$

$$\frac{(3a^2df-2ab(cf+de)+b^2ce)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(b\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx+\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{388}$$

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{e\int \frac{\sqrt{dx^2+c}}{\left(fx^2+e\right)^{3/2}}dx}{d}\right)+\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2a(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$$\Big\downarrow \textbf{413}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{2a\sqrt{c+dx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$$\Big\downarrow \textbf{413}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\int \frac{1}{(bx^2+a)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{2a\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

$\big\downarrow$ 412

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{df\left(\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+b\left(\frac{x\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}\right)\right)}{2a(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a +
b*x^2)) - (d*f*(b*((x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sq
rt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])) + (a*Sqrt[e]
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])))/(2*a*(
b*c - a*d)*(b*e - a*f)) + (Sqrt[-c]*(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*
f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin
[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*
f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(398) = 796$.

Time = 8.59 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.29

method	result
elliptic	$\frac{\sqrt{(x^2d+c)(f\,x^2+e)}\left(\frac{b^2x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{2a\left(a^2df-abcf-abde+ce\,b^2\right)\left(b\,x^2+a\right)}-\frac{df\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2\left(a^2df-abcf-abde+ce\,b^2\right)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{bde\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{}}{2\left(a^2df-abcf-a\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*b^2/a/(a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^
2+a)-1/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*d*f/(-1/c*d)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-
1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/a*e/(-1/c*d)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))-1/2*b*d/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*e/(-1/c*d)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*
(-1/c*d)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+3/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/(-1/c*d)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-1/c*d
)^(1/2))*d*f-1/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*b/(-1/c*d)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
Pi(x*(-1/c*d)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*c*f-1/(a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/a*b/(-1/c*d)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2),b*c/a/
d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*d*e+1/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a
^2*b^2/(-1/c*d)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+2abde\,x^4+b^2ce\,x^4+a^2cf\,x^2+a^2de\,x^2}$$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2))/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4
 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)
```

3.172 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}{\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 959

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx =$$

$$-\frac{(8a^2d^3e^2f^2+b^2e(105d^3e^3-140cd^2e^2f+21c^2def^2+6c^3f^3)-abf(110d^3e^3-131cd^2e^2f+12c^2def^2+9}{24e^2f^4(be-af)\sqrt{a+bx^2}}$$

$$+\frac{d^3x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^3}-\frac{(de-cf)^3x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^3\left(e+fx^2\right)^2}$$

$$+\frac{(de-cf)^2(be(11de+2cf)-af(10de+3cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^3(be-af)\left(e+fx^2\right)}$$

$$+\frac{\sqrt{a}(8a^2d^3e^2f^2+b^2e(105d^3e^3-140cd^2e^2f+21c^2def^2+6c^3f^3)-abf(110d^3e^3-131cd^2e^2f+12c^2def^2}{24\sqrt{b}e^2f^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(8a^2d^3e^2f^2(9de-11cf)+b^2e(105d^4e^4-175cd^3e^3f+63c^2d^2e^2f^2+3c^3def^3-12c^4f^4)-abf(180d^4}{24\sqrt{b}ce^2f^4(be-af)^2\sqrt{a}}$$

$$+\frac{a^{3/2}(de-cf)^2\left(35b^2d^2e^4-2abef(30d^2e^2+3cdef+2c^2f^2)+a^2f^2(24d^2e^2+8cdef+3c^2f^2)\right)\sqrt{c+dx^2}\,\mathrm{E}}{8\sqrt{b}ce^3f^4(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/24*(8*a^2*d^3*e^2*f^2+b^2*e*(6*c^3*f^3+21*c^2*d*e*f^2-140*c*d^2*e^2*f+1
05*d^3*e^3)-a*b*f*(9*c^3*f^3+12*c^2*d*e*f^2-131*c*d^2*e^2*f+110*d^3*e^3))*
x*(d*x^2+c)^(1/2)/e^2/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)+1/3*d^3*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/f^3-1/4*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/e/f^3/(f*x^2+e)^2+1/8*(-c*f+d*e)^2*(b*e*(2*c*f+11*d*e)-a*f*(3*c*f+10*d*e
))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^3/(-a*f+b*e)/(f*x^2+e)+1/24*a^(
1/2)*(8*a^2*d^3*e^2*f^2+b^2*e*(6*c^3*f^3+21*c^2*d*e*f^2-140*c*d^2*e^2*f+10
5*d^3*e^3)-a*b*f*(9*c^3*f^3+12*c^2*d*e*f^2-131*c*d^2*e^2*f+110*d^3*e^3))*(
d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(
1/2))/b^(1/2)/e^2/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)-1/24*a^(3/2)*(8*a^2*d^3*e^2*f^2*(-11*c*f+9*d*e)+b^2*e*(-12*c^4*f^4+
3*c^3*d*e*f^3+63*c^2*d^2*e^2*f^2-175*c*d^3*e^3*f+105*d^4*e^4)-a*b*f*(-9*c^
4*f^4-9*c^3*d*e*f^3+81*c^2*d^2*e^2*f^2-275*c*d^3*e^3*f+180*d^4*e^4))*(d*x^
2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^
(1/2)/c/e^2/f^4/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+1/8*a^(3/2)*(-c*f+d*e)^2*(35*b^2*d^2*e^4-2*a*b*e*f*(2*c^2*f^2+3*c*d*e*f
+30*d^2*e^2)+a^2*f^2*(3*c^2*f^2+8*c*d*e*f+24*d^2*e^2))*(d*x^2+c)^(1/2)*Ell
ipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b
^(1/2)/c/e^3/f^4/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.75 (sec) , antiderivative size = 636, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)(c+dx^2)\left(6e(be-af)(de-cf)^3-3(de-cf)^2(be(11d\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))/(e + f*x^2)^3,x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(6*e*(b*e - a*f)*(d*e - c*f)^
3 - 3*(d*e - c*f)^2*(b*e*(11*d*e + 2*c*f) - a*f*(10*d*e + 3*c*f))*(e + f*x
^2) - 8*d^3*e^2*(b*e - a*f)*(e + f*x^2)^2)) + I*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*(e + f*x^2)^2*(c*e*f*(8*a^2*d^3*e^2*f^2 + b^2*e*(105*d^3*e^3
 - 140*c*d^2*e^2*f + 21*c^2*d*e*f^2 + 6*c^3*f^3) - a*b*f*(110*d^3*e^3 - 13
1*c*d^2*e^2*f + 12*c^2*d*e*f^2 + 9*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - e*(8*a^2*d^3*e^2*f^2*(9*d*e - 10*c*f) + a*b*f*(-180*d^
4*e^4 + 197*c*d^3*e^3*f + 17*c^2*d^2*e^2*f^2 - 9*c^3*d*e*f^3 - 9*c^4*f^4)
+ b^2*e*(105*d^4*e^4 - 105*c*d^3*e^3*f - 35*c^2*d^2*e^2*f^2 + 21*c^3*d*e*f
^3 + 6*c^4*f^4))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*(d*e -
 c*f)^2*(35*b^2*d^2*e^4 - 2*a*b*e*f*(30*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) +
 a^2*f^2*(24*d^2*e^2 + 8*c*d*e*f + 3*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(24*Sqrt[b/a]*e^3*f^5*(b*e - a*f)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{7/2}}{\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^{7/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{7/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \ 425$$

$$\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\int \frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{406}$$

$$b\left(\frac{d\left(\frac{d\left(\frac{c(3bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int -}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 320$$

$$b\left(\frac{d\left(\frac{d\left(\frac{2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)}}{f}}{f}\right.$$

$$\frac{\ldots}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 388$$

$$b\left(\frac{d\left(\frac{d\left(\frac{2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{(\cdots}{\sqrt{bx\cdots}}}{\cdots}}{f}\cdots\right.$$

$$\frac{\cdots}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\downarrow 313$$

$$b\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}}{f}\right.$$

$$\frac{\quad}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\downarrow 420$$

$$b\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 324

$$b\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\cdots\right)}{f}\cdots\right)}{f}\cdots\right)$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\big\downarrow$ 388

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right)}{f}\right)}{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}$$

$$\frac{}{f}$$

$\Big\downarrow$ 313

$$b\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right.}{}\right.$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$b\left(\frac{d\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}\right.}{}\right.$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right)}{}\right)$$

$$(be-af)\left(\frac{d\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}}{f}\right.}{f}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 420

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\right.}{f}\right.}{f}$$

↓ 324

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\cdots \quad (d$$

$$f$$

↓ 320

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2\cdots\right.}{\sqrt{bx^2+\cdots}}}{f}\cdots}{f}\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-\cdots}{\cdots}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.}{\ldots}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+\ldots}}{\sqrt{bx^2+a}\left(fx\ldots\right.}}{f}\right.}{f}\right.}{f}\right.$$

↓ 414

$$b\left(d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}}{f}\right.$$

$\downarrow$ 425

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\cdots}\right.}{\cdots}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right.}{f}\right.}{f}\cdots\right.$$

↓ 414

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\ldots}}{\ldots}\right)$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}\ldots}{a\sqrt{d}ef\ldots}\right)}{f}}{\ldots}\right)$$

↓ 425

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}}{\cdots}\right.$$

↓ 413

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}\right.}{f}\right.}{}\right.$$

↓ 413

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{\quad}\right)}{\quad}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}}{\quad}\right.$$

↓ 412

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}}{f}}{}\right.$$

↓ 424

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\cdots}{\cdots}\right.$$

$$\left(d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}\cdots\right.\right.$$

↓ 406

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\cdots\right)}{\cdots}\right.\cdots$$

$$\cdots\left(d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\cdots\right)}{f}\cdots\right.\right.\cdots$$

↓ 320

$$\left(\left(d \left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b} + \frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + 2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}} - \frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b} \right) \right) \right)$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4512 vs. 2(915) = 1830.

Time = 26.80 (sec) , antiderivative size = 4513, normalized size of antiderivative = 4.71

method	result	size
elliptic	Expression too large to display	4513
risch	Expression too large to display	4595
default	Expression too large to display	8407

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*d^3/f^3*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-9/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))*d^3/f^4*b*c*e+13/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^4/f^5*e^3/(a*f-b*e)-3*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*d^3/f^4*b*e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-7/8*c^3/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*d*b^2/f^2/(a*f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))+7/8*c^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/f^2/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))+3/f^3*e/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/
2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d^4+35/8/f^5*e^3/(a*f-b*e)/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2
*d^4-5/f^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a^2*c*d^3-1/2/e^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(7/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{7/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{7/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{7}{2}}}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(7/2)/(f*x^2+e)^3,x)
```

3.173 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 795

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^3}\,dx = \\
&-\frac{b(af(14d^2e^2-3cdef-3c^2f^2)-be(15d^2e^2-5cdef-2c^2f^2))\,x\sqrt{c+dx^2}}{8e^2f^3(be-af)\sqrt{a+bx^2}} \\
&+\frac{(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^2\left(e+fx^2\right)^2} \\
&+\frac{(de-cf)(3af(2de+cf)-be(7de+2cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^2(be-af)\left(e+fx^2\right)} \\
&+\frac{\sqrt{a}\sqrt{b}(af(14d^2e^2-3cdef-3c^2f^2)-be(15d^2e^2-5cdef-2c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8e^2f^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}(8a^2d^3e^2f^2-abf(24d^3e^3-13cd^2e^2f+2c^2def^2+3c^3f^3)+b^2e(15d^3e^3-10cd^2e^2f-c^2def^2+4c^3f^3}{8\sqrt{b}ce^2f^3(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{a^{3/2}(de-cf)\left(15b^2d^2e^4-2abef(12d^2e^2+cdef+2c^2f^2)+a^2f^2(8d^2e^2+4cdef+3c^2f^2)\right)\sqrt{c+dx^2}\,\mathrm{Elli}}{8\sqrt{b}ce^3f^3(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/8*b*(a*f*(-3*c^2*f^2-3*c*d*e*f+14*d^2*e^2)-b*e*(-2*c^2*f^2-5*c*d*e*f+15
*d^2*e^2))*x*(d*x^2+c)^(1/2)/e^2/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)+1/4*(-c*f+
d*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^2+1/8*(-c*f+d*e)*
(3*a*f*(c*f+2*d*e)-b*e*(2*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^
2/f^2/(-a*f+b*e)/(f*x^2+e)+1/8*a^(1/2)*b^(1/2)*(a*f*(-3*c^2*f^2-3*c*d*e*f+
14*d^2*e^2)-b*e*(-2*c^2*f^2-5*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*Ellipti
cE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/f^3/(-a*f+b*
e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*(8*a^2*d^3*
e^2*f^2-a*b*f*(3*c^3*f^3+2*c^2*d*e*f^2-13*c*d^2*e^2*f+24*d^3*e^3)+b^2*e*(4
*c^3*f^3-c^2*d*e*f^2-10*c*d^2*e^2*f+15*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f^3/(-a*
f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*(-c*f
+d*e)*(15*b^2*d^2*e^4-2*a*b*e*f*(2*c^2*f^2+c*d*e*f+12*d^2*e^2)+a^2*f^2*(3*
c^2*f^2+4*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f^3/(-a*f+b*
e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2(de-cf)x(a+bx^2)\left(c+dx^2\right)\left(2e(be-af)(de-cf)-(-3af(2de+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
(Sqrt[b/a]*e*f^2*(d*e - c*f)*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d
*e - c*f) - (-3*a*f*(2*d*e + c*f) + b*e*(7*d*e + 2*c*f))*(e + f*x^2)) + I*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(a*f*(14*d^
2*e^2 - 3*c*d*e*f - 3*c^2*f^2) + b*e*(-15*d^2*e^2 + 5*c*d*e*f + 2*c^2*f^2)
)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + e*(8*a^2*d^3*e^2*f^2 +
b^2*e*(15*d^3*e^3 - 5*c^2*d*e*f^2 - 2*c^3*f^3) + a*b*f*(-24*d^3*e^3 + 3*c*
d^2*e^2*f + 2*c^2*d*e*f^2 + 3*c^3*f^3))*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - (d*e - c*f)*(15*b^2*d^2*e^4 - 2*a*b*e*f*(12*d^2*e^2 + c*d*e
*f + 2*c^2*f^2) + a^2*f^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2))*EllipticPi[
(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*f^4*(
b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 420$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{d\left(\frac{d\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{(d\cdots}{\sqrt{bx^2\cdots}}}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$
\frac{b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\right)}{f}
$$

$$
\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}
$$

$\downarrow$ 414

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a}{a(c}}}\right)}{f}}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c\left(a\right.}{a\left(c\right.}}}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\right.}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 414

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a\cdots}{a(c\cdots}}}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\cdots\right.}{f}\right.}{f}$$

$\downarrow$ 425

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2}}{f}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{bx^2+a}}-\frac{(de-cf)\int\frac{}{\sqrt{bx^2+a}\sqrt{d}}}{f}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{f}\right.}{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\int}{\sqrt{}}\right.}{f}\right.}{f}\right.}$$

↓ 412

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}}{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{f}\right.}{f}\right.}$$

$\downarrow$ 424

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}\right)-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{}\right.}{f}\right.$$

↓ 406

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{}\right.}{}\right.$$

↓ 320

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{f}\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

↓ 413

$$b\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

↓ 412

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

↓ 433

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}$$

↓ 2009

$$b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3241 vs. $2(757) = 1514$.

Time = 7.43 (sec) , antiderivative size = 3242, normalized size of antiderivative = 4.08

method	result	size
elliptic	Expression too large to display	3242
default	Expression too large to display	5943

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/f^2/(a*f-b
*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))*a^2*d^3-9/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))*b^2*d^3/f^4*e^2/(a*f-b*e)-5/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/f^2/(a*f-b*e)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/8*c^2/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/f^2
/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8*f/e^3/(a
*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)
^(1/2))*a^2*c^3+1/8/e^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2*d-1/2/e^2/(a*f-b*e)/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c^3-15/8
/f^4*e^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*b^2*d^3+1/4*(c^2*f^2-2*c*d*e*f+d^2*e^2)/e/f^2*x*(b*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d**2*f*x + 4*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*d**3*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d**3*f
*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*d*f*x + 4*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b*c*d**2*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
c*d**2*f*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**3*e*x**3 + 4*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e
**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**
2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a
**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2*x**2 + 6*a*b
*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a*b*c*d*e**3
*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6 + 4*a*b*c*d*f**
4*x**8 - 3*a*b*d**2*e**4*x**2 - 7*a*b*d**2*e**3*f*x**4 - 3*a*b*d**2*e**2*f
**2*x**6 + 3*a*b*d**2*e*f**3*x**8 + 2*a*b*d**2*f**4*x**10 + 2*b**2*c**2*e*
*3*f*x**2 + 6*b**2*c**2*e**2*f**2*x**4 + 6*b**2*c**2*e*f**3*x**6 + 2*b**2*
c**2*f**4*x**8 - 3*b**2*c*d*e**4*x**2 - 7*b**2*c*d*e**3*f*x**4 - 3*b**2*c*
d*e**2*f**2*x**6 + 3*b**2*c*d*e*f**3*x**8 + 2*b**2*c*d*f**4*x**10 - 3*b**2
*d**2*e**4*x**4 - 9*b**2*d**2*e**3*f*x**6 - 9*b**2*d**2*e**2*f**2*x**8 - 3
*b**2*d**2*e*f**3*x**10),x)*a**3*d**5*e**2*f**3 + 8*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6*
a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 +...
```

3.174 $\int \frac{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 566

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef(e+fx^2)^2}$$
$$-\frac{(de-cf)(be(3de+2cf)-af(2de+3cf))x\sqrt{a+bx^2}}{8e^2f^2(be-af)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{c}\sqrt{d}(be(3de+2cf)-af(2de+3cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2f^2(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{3c^{3/2}\sqrt{d}(bde^2-acf^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae^2f^2(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}(3b^2d^2e^4+3a^2c^2f^4-2abef(2d^2e^2-cdef+2c^2f^2))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{8a\sqrt{d}e^3f^2(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/4*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^2-1/8*(-c*
f+d*e)*(b*e*(2*c*f+3*d*e)-a*f*(3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(-a
*f+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/8*c^(1/2)*d^(1/2)*(b*e*(2*c*f+3*d*e)-a
*f*(3*c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))/e^2/f^2/(-a*f+b*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)-3/8*c^(3/2)*d^(1/2)*(-a*c*f^2+b*d*e^2)*(b*x^2+a)^(1/2)*I
nverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/f^2/(-c*
f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(3/2)*(3*b^2*
d^2*e^4+3*a^2*c^2*f^4-2*a*b*e*f*(2*c^2*f^2-c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(
1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^
(1/2))/a/d^(1/2)/e^3/f^2/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(af^2(5ce+2dex^2+3cfx^2)-be(2cf(2e+fx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(a*f^2*(5*c*e + 2*d*e*x^2 + 3
*c*f*x^2) - b*e*(2*c*f*(2*e + f*x^2) + d*e*(e + 3*f*x^2)))) + I*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(b*e*(3*d*e + 2*c*f)
- a*f*(2*d*e + 3*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*
e*(b*e*(3*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) - a*f*(4*d^2*e^2 + c*d*e*f + 3*
c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*b^2*d^2*e^4
+ 3*a^2*c^2*f^4 + 2*a*b*e*f*(-2*d^2*e^2 + c*d*e*f - 2*c^2*f^2))*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*f^3*
(b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 414}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-(de-\right.}{f}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(f}}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}}{f}\right.}{}\right.}{f}$$

$\downarrow$ 413

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{}{\sqrt{bx^2+a}\sqrt{\ldots}}}{\ldots}\right.}{f}\right.$$

$$\overline{}$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(\ldots}}{f}\right.}{\ldots}\right.$$

$$\overline{}$$

$$f$$

$$\Big\downarrow \text{ 412}$$

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{}{\sqrt{b\ldots}}}{\ldots}\right.}{f}\right.$$

$$\overline{}$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{}{\sqrt{bx^2+a}\sqrt{\ldots}}}{\ldots}\right.}{\ldots}\right.$$

$$\overline{}$$

$$f$$

$$\Big\downarrow \text{ 424}$$

$$
\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\right.}{\cdots}\right.}{\cdots}}{f}
$$

$$
\frac{}{(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{\cdots}{\sqrt{bx\cdots}}}{2e(be-af)(de-\cdots}\right.}{f}\right.}{f}\right.}
$$

↓ 406

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\quad}{2e(\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{\quad}{\sqrt{\ldots}}\right.}{2e(be-af)(de-c\ldots}\right.}{\ldots}\right.}{f}\right.$$

↓ 320

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\ldots}{2e(\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\ldots\right.\right.}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d\ldots}}\right.}{2e(be\ldots}\right.}{\ldots}\right.}{f}\right.$$

↓ 388

$$(de-cf)\left(\frac{}{2e(\cdots}\right.$$

$$(de-cf)\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.$$

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\cdots\right.$$

$$bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d\cdots}}\right.$$

$$(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\cdots\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right)}{f\cdots}\right.$$

↓ 313

$$(de-cf)\left(\frac{}{2e(\cdots}\right.$$

$$(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.$$

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\cdots\right.$$

$$bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E(\cdots}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.\right.$$

$$(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\cdots\right.$$

$$(be-af)\left(\left(d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.\right.\right.$$

↓ 413

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 413

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 412

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

$$(be-af)\left(d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.\right.$$

↓ 433

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E(\cdots}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2009

$$b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\cdots}{2e(\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{d\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E(\cdots}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. $2(534) = 1068$.

Time = 5.96 (sec) , antiderivative size = 1895, normalized size of antiderivative = 3.35

method	result	size
elliptic	Expression too large to display	1895
default	Expression too large to display	3631

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*(c*f-d*e)
/e/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*(3*a*c*f^2+2*a*
d*e*f-2*b*c*e*f-3*b*d*e^2)/f/e^2/(a*f-b*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/(f*x^2+e)+3/8*b/e^2/(a*f-b*e)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))*a-3/8*b^2/f^2/(a*f-b*e)*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d-3/8*b/e^2/(a*f-b*e)*c^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a+3/8*b^2/f^2/(a*
f-b*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d-1
/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/f^2
/(a*f-b*e)*a+5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))*b^2*d^2/f^3*e/(a*f-b*e)-1/4*b^2/f/e/(a*f-b*e)*c^2/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/4*b^2/f/e/(a*f-b*e)*c^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}\,(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d**2*x - 2*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b*c*d*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**
2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2
*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*
a**2*d**2*e*f**3*x**6 + 2*a**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*
c**2*e**2*f**2*x**2 + 6*a*b*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*
b*c*d*e**4 - 5*a*b*c*d*e**3*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*
e*f**3*x**6 + 4*a*b*c*d*f**4*x**8 - 3*a*b*d**2*e**4*x**2 - 7*a*b*d**2*e**3
*f*x**4 - 3*a*b*d**2*e**2*f**2*x**6 + 3*a*b*d**2*e*f**3*x**8 + 2*a*b*d**2*
f**4*x**10 + 2*b**2*c**2*e**3*f*x**2 + 6*b**2*c**2*e**2*f**2*x**4 + 6*b**2
*c**2*e*f**3*x**6 + 2*b**2*c**2*f**4*x**8 - 3*b**2*c*d*e**4*x**2 - 7*b**2*
c*d*e**3*f*x**4 - 3*b**2*c*d*e**2*f**2*x**6 + 3*b**2*c*d*e*f**3*x**8 + 2*b
**2*c*d*f**4*x**10 - 3*b**2*d**2*e**4*x**4 - 9*b**2*d**2*e**3*f*x**6 - 9*b
**2*d**2*e**2*f**2*x**8 - 3*b**2*d**2*e*f**3*x**10),x)*a**2*b*d**4*e**2*f*
*2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6
*a**2*c*d*e**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 +
 2*a**2*d**2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3
*x**6 + 2*a**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2*x
**2 + 6*a*b*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a
*b*c*d*e**3*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6 +...
```

3.175 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 555

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx \\
&= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4e\,(e+fx^2)^2} + \frac{(af(2de-3cf)-be(de-2cf))x\sqrt{a+bx^2}}{8e^2f(be-af)\sqrt{c+dx^2}\,(e+fx^2)} \\
&\quad - \frac{\sqrt{c}\sqrt{d}(af(2de-3cf)-be(de-2cf))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2f(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{c^{3/2}\sqrt{d}(bde^2-af(4de-3cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae^2f(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{c^{3/2}(b^2d^2e^4+a^2cf^3(4de-3cf)-2abcef^2(3de-2cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1\right.}{8a\sqrt{d}e^3f(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(f*x^2+e)^2+1/8*(a*f*(-3*c*f+2*d*e
)-b*e*(-2*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/f/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*
x^2+e)-1/8*c^(1/2)*d^(1/2)*(a*f*(-3*c*f+2*d*e)-b*e*(-2*c*f+d*e))*(b*x^2+a)
^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/e^
2/f/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
1/8*c^(3/2)*d^(1/2)*(b*d*e^2-a*f*(-3*c*f+4*d*e))*(b*x^2+a)^(1/2)*InverseJa
cobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/f/(-c*f+d*e)^2/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(3/2)*(b^2*d^2*e^4+a^
2*c*f^3*(-3*c*f+4*d*e)-2*a*b*c*e*f^2*(-2*c*f+3*d*e))*(b*x^2+a)^(1/2)*Ellip
ticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d
^(1/2)/e^3/f/(-a*f+b*e)/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)(c+dx^2)(2e(be-af)(de-cf)+(be(de-2cf)+af(-2de+3cf))(e+fx^2))-i\sqrt{}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f) +
(b*e*(d*e - 2*c*f) + a*f*(-2*d*e + 3*c*f))*(e + f*x^2)) - I*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(a*f*(2*d*e - 3*c*f) + b*
e*(-(d*e) + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*e*(
d*e - c*f)*(-3*a*c*f^2 + b*e*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - (b^2*d^2*e^4 + a^2*c*f^3*(4*d*e - 3*c*f) + 2*a*b*c*e*f^
2*(-3*d*e + 2*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)]))/(8*Sqrt[b/a]*e^3*f^2*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{b\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{b\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\downarrow \text{412}$$

$$\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\downarrow \text{424}$$

$$\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int\cdots}{2e(be-\cdots}\cdots\right)}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int\frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}\right)}{f}-\frac{(de-cf)\int\cdots}{\cdots}\right)}{f}$$

$$\downarrow \text{406}$$

$$\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)\cdots}{\cdots}\cdots\right)}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\cdots}{2e\left(e+fx^2\right)(be-\cdots}\right)}{f}\cdots\right)}{f}$$

$$\downarrow \text{320}$$

$$b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\cdots\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right)}{2e(be-af)(de-cf)}\cdots\right)}{\cdots}\cdots\right)$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{d\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\cdots\right)}\cdots}{2e(be-af)(de-cf)}\cdots\right)}{f}\cdots\right)$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$\downarrow$ 388

$$\frac{b\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\cdots\right.}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.}{2e(be-\cdots}\cdots\right.}{\cdots}\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)\cdots}{\cdots}\right.}{f}\cdots\right.}{f}$$

↓ 313

$$b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\cdots\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.}{2e(be-af)(de-cf)}\right.}{f}\cdots\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$\downarrow 413$$

$$b\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.\right.}{}\right.}{}\right.$$

$$f$$

$$(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.$$

$$f$$

↓ 413

$$b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.\right.}{}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.}{f}$$

↓ 412

$$b\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.\right.}{}\right.}{}\right.$$

$$\frac{}{(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.}$$

$$f$$

↓ 433

$$\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.\right.}{}\right.}{}\right.}{(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.}$$

$\downarrow$ 2009

$$b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.\right.}{}\right.}{}\right.$$

$$\frac{}{(be-af)\left(\frac{d\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right)}{f}\right.}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(523) = 1046$.

Time = 1.53 (sec) , antiderivative size = 1928, normalized size of antiderivative = 3.47

method	result	size
elliptic	Expression too large to display	1928
default	Expression too large to display	3504

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*x/e*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*(3*a*c*f^2-2*a*d*e*f-2*b*c*
e*f+b*d*e^2)/e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*x*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)/(f*x^2+e)+1/8*b^2*d^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*e/f
^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/4*b^2/e
/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-1/4*b^2/e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*c^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)/e^3*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2-1/8/(a*c*f^2-a*d*e*f-b*c*e*f+b*
d*e^2)*e/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(
-b/a)^(1/2))*b^2*d^2-3/8*b/e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*f*c^2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a-3/8*b*d/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f^3x^6+3e\,f^2x^4+3e^2f\,x^2+e^3}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e**3 + 3*e**2*f*x**2 + 3*e*f**2*x
**4 + f**3*x**6),x)
```

3.176 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 606

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = -\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{4e(de-cf)\,(e+fx^2)^2}+\frac{(be(5de-2cf)-3af(2de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} - \frac{\sqrt{c}\sqrt{d}(be(5de-2cf)-3af(2de-cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}(3bcde^2-a(8d^2e^2-8cdef+3c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(3b^2d^2e^4-2abef(6d^2e^2-5cdef+2c^2f^2)+a^2f^2(8d^2e^2-8cdef+3c^2f^2))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1\right.}{8a\sqrt{d}e^3(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^2+1/8*(b*e
*(-2*c*f+5*d*e)-3*a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c*f
+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*c^(1/2)*d^(1/2)*(b*e*(-2*c*f+5*d*e)-3*
a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(1/2)*d^(1/2)*(3*b*c*d*e^2-a*(3*c^2*f^2-8
*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1
/2)),(1-b*c/a/d)^(1/2))/a/e^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)+1/8*c^(3/2)*(3*b^2*d^2*e^4-2*a*b*e*f*(2*c^2*f^2-5*c*d*e*f
+6*d^2*e^2)+a^2*f^2*(3*c^2*f^2-8*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*Ellip
ticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d
^(1/2)/e^3/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+
c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.66 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+(be(5de-2cf)+3af(-2de+cf))\,(e+fx^2))+}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f)
+ (b*e*(5*d*e - 2*c*f) + 3*a*f*(-2*d*e + c*f))*(e + f*x^2))) + I*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(-3*a*f*(-2*d*e + c*
f) + b*e*(-5*d*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- b*e*(d*e - c*f)*(b*e*(3*d*e - 2*c*f) + a*f*(-4*d*e + 3*c*f))*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*b^2*d^2*e^4 - 2*a*b*e*f*(6*d^2*e^
2 - 5*c*d*e*f + 2*c^2*f^2) + a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f + 3*c^2*f^2))*
EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a
]*e^3*f*(b*e - a*f)*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x
^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \ 424$$

$$\frac{b\left(\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \ 406$$

$$\frac{b\left(-\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c}\cdots}{2e(e+fx^2)(be-af\cdots}\cdots\right.}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \ 320$$

$$\frac{b\left(-\frac{bd\left(f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\cdots\right.}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$\downarrow$ 388

$$b\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\right.}}{2e(be-af)(de-cf)}\right.$$

$$f$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$\downarrow$ 313

$$b\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\right.}{b\sqrt{d}\sqrt{c+}}\right.\right.}{2e(be-af)(de-cf)}\right.$$

$$f$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$\downarrow$ 413

$$b\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{2e(be-af)(de-cf)}\right.$$

$$f$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+\cdots}}{b\sqrt{c+\cdots}}\cdots\right.\right.}{2e(be-af)(de-\cdots}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}$$

$\Big\downarrow$ 412

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\cdots\right.}{2e(be-\cdots}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}$$

$\Big\downarrow$ 433

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\cdots\right.}{2e(be-\cdots}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\int \left(-\frac{f^{3/2}}{8(-e)^{3/2}\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{f^{3/2}}{8(-e)^{3/2}\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{3f}{16e^2\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{\cdots}}\cdots\right.}{f}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\frac{\sqrt{}}{}\right.}{f}$$

$$\frac{(be-af)\left(-\frac{\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{\int\frac{1}{(fx+\sqrt{-e}\sqrt{f})^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{3\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf}{16e^2}\right.}{f}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2504 vs. 2(574) = 1148.

Time = 9.19 (sec) , antiderivative size = 2505, normalized size of antiderivative = 4.13

method	result	size
elliptic	Expression too large to display	2505
default	Expression too large to display	4175

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f/(c*f-d*
e)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*f*(3*a*c*f^2-6*
a*d*e*f-2*b*c*e*f+5*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/(c*f-d*
e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-3/8*b^2*d^2/(a*c*f^2-a*
d*e*f-b*c*e*f+b*d*e^2)*e/(c*f-d*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))-1/4*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(c*f-
d*e)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*f+5
/8*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(c*f-d*e)*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d+1/4*b^2/(a*c*f^2-a*d*e*f-b*c*e*
f+b*d*e^2)/e/(c*f-d*e)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))*f-5/8*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(c*f-d*e)*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d-7/8*b*d/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(c*f-d*e)*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*c+3/4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^3x^8+c\,f^3x^6+3de\,f^2x^6+3ce\,f^2x^4+3d\,e^2f\,x^4+3c\,e^2f\,x^2+d\,e^3x^2+c\,e^3}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**3 + 3*c*e**2*f*x**2 + 3*c*e*
f**2*x**4 + c*f**3*x**6 + d*e**3*x**2 + 3*d*e**2*f*x**4 + 3*d*e*f**2*x**6
+ d*f**3*x**8),x)
```

3.177 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 636

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx=-\frac{fx\sqrt{a+bx^2}}{4e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^2}$$
$$+\frac{f(af(8de-3cf)-be(7de-2cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{d}(af(8d^2e^2+10cdef-3c^2f^2)-be(8d^2e^2+9cdef-2c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8\sqrt{c}e^2(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{3\sqrt{c}\sqrt{d}f(5bcde^2-a(8d^2e^2-4cdef+c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}f(15b^2d^2e^4+3a^2f^2(8d^2e^2-4cdef+c^2f^2)-2abef(20d^2e^2-7cdef+2c^2f^2))\sqrt{a+bx^2}\,\text{EllipticPi}\,(}{8a\sqrt{d}e^3(be-af)(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/4*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^2+1/8*f*(a
*f*(-3*c*f+8*d*e)-b*e*(-2*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c
*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*d^(1/2)*(a*f*(-3*c^2*f^2+10*c*d*e*
f+8*d^2*e^2)-b*e*(-2*c^2*f^2+9*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*Ellipti
cE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(1/2)/e^2/(-a*
f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+3/8*c^
(1/2)*d^(1/2)*f*(5*b*c*d*e^2-a*(c^2*f^2-4*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1
/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/(-c
*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(3/2)*f*(1
5*b^2*d^2*e^4+3*a^2*f^2*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(2*c^2*f^2
-7*c*d*e*f+20*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*
x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^3/(-a*f+b*e)/(-c*f+d
*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(2cef^2(be-af)(de-cf)\,(c+dx^2)+cf^2(be(9de-2cf)\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(2*c*e*f^2*(b*e - a*f)*(d*e - c*f)*(c + d*x^2)
+ c*f^2*(b*e*(9*d*e - 2*c*f) + a*f*(-10*d*e + 3*c*f))*(c + d*x^2)*(e + f*x
^2) + 8*d^3*e^2*(b*e - a*f)*(e + f*x^2)^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*e*(b*e*(8*d^2*e^2 + 9*c*d*e*f - 2*c^2*f^
2) + a*f*(-8*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)]) - b*e*(d*e - c*f)*(b*e*(7*d*e - 2*c*f) + a*f*(-8*d*e
 + 3*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*b^2*d^2*e^
4 + 3*a^2*f^2*(8*d^2*e^2 - 4*c*d*e*f + c^2*f^2) - 2*a*b*e*f*(20*d^2*e^2 -
7*c*d*e*f + 2*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(8*Sqrt[b/a]*c*e^3*(b*e - a*f)*(d*e - c*f)^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 426}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{de-cf}}{f}$$

$$\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\downarrow \text{320}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{de-cf}-}{f}$$

$$\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \ 414$$

$$b\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\Big(\text{a}}{\sqrt{c}\sqrt{c+dx^2}(bc-a}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \ 424$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\ldots\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a\ldots}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f\ldots}$$

$$\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\ldots\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a\ldots}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}$$

$$\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\text{ar}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a\cdots}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 388

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx^2\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx^2\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\cdots\right)}{(de-cf)^2}\right)}{de-cf}\right.$$

The last visible term, cut off at the right edge of the page: numerator "$\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\big($a…", denominator "$\sqrt{c}(bc-ad)\sqrt{\tfrac{c(bx\ldots}{a(dx\ldots}}$".

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{412}$$

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \; 426$$

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e}}{de-cf}\right.}{f}$$

$\downarrow$ 25

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3\cdots}}{de-cf}\right.}{f}$$

$\downarrow$ 400

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\right)}{de-cf}\cdots\right)$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+\cdots\right.}\cdots}{de-cf}\right)}{de-cf}\cdots\right)}{f}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\cdots}{\sqrt{bx^2\cdots}}\right.}{de-cf}\right.}{f}$$

↓ 320

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

↓ 414

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\mathrm{a}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{\cdots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}\right.}{f}$$

↓ 424

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}\cdots\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{\cdots}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right.$$

↓ 406

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\text{a}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

↓ 320

$$b\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.}{}\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{d\,\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\text{a}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{(de-cf)^2}}{de-cf}\right.\right.\ldots$$

$$(be-af)\left(\frac{d\left(\frac{d\,\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\ldots}\right.}{(de-cf)^2}}{de-cf}\right.\right.\ldots$$

↓ 313

$$b \Bigg(\frac{d \left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} - \frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}}{(de-cf)^2}}{de-cf}$$

$$(be-af) \Bigg(\frac{d \left(\frac{d \left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} - \frac{\sqrt{d}(de-c}{\sqrt{}}}{(de-cf)^2}}{de-cf}}{}$$

↓ 413

$$b\left(\frac{d\left(\frac{d\,\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{a}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots}{de-cf}\cdots\right)\cdots$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\,\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{\cdots}}\cdots\right)}{(de-cf)^2}\cdots}{de-cf}\cdots\right)}{\cdots}\cdots\right)\cdots$$

↓ 413

$$b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\mathrm{a}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)$$

$$\overline{\qquad\cdots\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{\cdots}}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}\cdots\right)$$

$$\overline{\qquad\cdots\qquad}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. $2(604) = 1208$.

Time = 19.16 (sec) , antiderivative size = 2670, normalized size of antiderivative = 4.20

method	result	size
elliptic	Expression too large to display	2670
default	Expression too large to display	4864

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f^2/(c*f-
d*e)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*f^2*(3*a*c*
f^2-10*a*d*e*f-2*b*c*e*f+9*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/
(c*f-d*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-(b*d*x^2+a*d)/
c*d^2*x/(c*f-d*e)^3/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)/(c*f-d*e)^2*a*f+9/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+
b*d*e^2)/(c*f-d*e)^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9/
8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*d*f*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(c*f-d*e)^2*Ellip
ticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8/(a*c*f^2-a*d*e*f-b*c*e*f
+b*d*e^2)/e^3/(c*f-d*e)^2*f^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e
,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2+3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/
(c*f-d*e)^2/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1
/2)/(-b/a)^(1/2))*a^2*d^2-5/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(c*f-d*e)^2*
f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{d^2f^3x^{10}+2cd\,f^3x^8+3d^2e\,f^2x^8+c^2f^3x^6+6cde\,f^2x^6+3d^2e^2f\,x^6+3c}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e**3 + 3*c**2*e**2*f*x**2 +
3*c**2*e*f**2*x**4 + c**2*f**3*x**6 + 2*c*d*e**3*x**2 + 6*c*d*e**2*f*x**4
+ 6*c*d*e*f**2*x**6 + 2*c*d*f**3*x**8 + d**2*e**3*x**4 + 3*d**2*e**2*f*x**
6 + 3*d**2*e*f**2*x**8 + d**2*f**3*x**10),x)
```

3.178 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 923

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx =$$

$$-\frac{d(af(8d^2e^2+36cdef-9c^2f^2)-be(8d^2e^2+33cdef-6c^2f^2))\,x\sqrt{a+bx^2}}{24ce^2(be-af)(de-cf)^3\,(c+dx^2)^{3/2}}$$

$$-\frac{fx\sqrt{a+bx^2}}{4e(de-cf)\,(c+dx^2)^{3/2}\,(e+fx^2)^2}+\frac{f(af(10de-3cf)-be(9de-2cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)^2\,(c+dx^2)^{3/2}\,(e+fx^2)}$$

$$+\frac{\sqrt{d}(b^2ce(8d^3e^3-80cd^2e^2f-39c^2def^2+6c^3f^3)+a^2df(16d^3e^3-88cd^2e^2f-42c^2def^2+9c^3f^3)-ab(16}{24c^{3/2}(bc-ad)e^2(be-af)(de-cf}$$

$$-\frac{\sqrt{d}(105b^2c^3de^2f^2+3a^2cdf^2(48d^2e^2-16cdef+3c^2f^2)-ab(8d^4e^4-16cd^3e^3f+257c^2d^2e^2f^2-48c^3def}{24a\sqrt{c}(bc-ad)e^2(de-cf)^5\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+\frac{c^{3/2}f^2(35b^2d^2e^4-2abef(42d^2e^2-9cdef+2c^2f^2)+a^2f^2(48d^2e^2-16cdef+3c^2f^2))\sqrt{a+bx^2}\,\mathrm{EllipticP}}{8a\sqrt{d}e^3(be-af)(de-cf)^5\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/24*d*(a*f*(-9*c^2*f^2+36*c*d*e*f+8*d^2*e^2)-b*e*(-6*c^2*f^2+33*c*d*e*f+
8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(d*x^2+c)^(3/2
)-1/4*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^2+1/8*f*(
a*f*(-3*c*f+10*d*e)-b*e*(-2*c*f+9*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(
-c*f+d*e)^2/(d*x^2+c)^(3/2)/(f*x^2+e)+1/24*d^(1/2)*(b^2*c*e*(6*c^3*f^3-39*
c^2*d*e*f^2-80*c*d^2*e^2*f+8*d^3*e^3)+a^2*d*f*(9*c^3*f^3-42*c^2*d*e*f^2-88
*c*d^2*e^2*f+16*d^3*e^3)-a*b*(9*c^4*f^4-36*c^3*d*e*f^3-119*c^2*d^2*e^2*f^2
-80*c*d^3*e^3*f+16*d^4*e^4))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/(-a*d+b*c)/e^2/(-a*f+b*e)/(-c*
f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/24*d^(1/2)*(105
*b^2*c^3*d*e^2*f^2+3*a^2*c*d*f^2*(3*c^2*f^2-16*c*d*e*f+48*d^2*e^2)-a*b*(9*
c^4*f^4-48*c^3*d*e*f^3+257*c^2*d^2*e^2*f^2-16*c*d^3*e^3*f+8*d^4*e^4))*(b*x
^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a
/c^(1/2)/(-a*d+b*c)/e^2/(-c*f+d*e)^5/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)+1/8*c^(3/2)*f^2*(35*b^2*d^2*e^4-2*a*b*e*f*(2*c^2*f^2-9*c*d*e*f+
42*d^2*e^2)+a^2*f^2*(3*c^2*f^2-16*c*d*e*f+48*d^2*e^2))*(b*x^2+a)^(1/2)*Ell
ipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a
/d^(1/2)/e^3/(-a*f+b*e)/(-c*f+d*e)^5/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.92 (sec) , antiderivative size = 731, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ex(a+bx^2)\left(6c^2(bc-ad)ef^3(be-af)(de-cf)\left(c+dx^2\right)^2+3c^2(bc\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*x*(a + b*x^2)*(6*c^2*(b*c - a*d)*e*f^3*(b*e - a*f)*(d*e - c
*f)*(c + d*x^2)^2 + 3*c^2*(b*c - a*d)*f^3*(b*e*(13*d*e - 2*c*f) + a*f*(-14
*d*e + 3*c*f))*(c + d*x^2)^2*(e + f*x^2) + 8*c*d^3*(b*c - a*d)*e^2*(b*e -
a*f)*(-(d*e) + c*f)*(e + f*x^2)^2 + 8*d^3*e^2*(b*e - a*f)*(a*d*(2*d*e - 11
*c*f) + b*c*(-(d*e) + 10*c*f))*(c + d*x^2)*(e + f*x^2)^2)) - I*c*Sqrt[1 +
(b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*e*(b^2*c*e*(
8*d^3*e^3 - 80*c*d^2*e^2*f - 39*c^2*d*e*f^2 + 6*c^3*f^3) + a^2*d*f*(16*d^3
*e^3 - 88*c*d^2*e^2*f - 42*c^2*d*e*f^2 + 9*c^3*f^3) + a*b*(-16*d^4*e^4 + 8
0*c*d^3*e^3*f + 119*c^2*d^2*e^2*f^2 + 36*c^3*d*e*f^3 - 9*c^4*f^4))*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + (b*c - a*d)*(b*e*(d*e - c*f)*(b
*e*(8*d^2*e^2 + 33*c*d*e*f - 6*c^2*f^2) + a*f*(-8*d^2*e^2 - 36*c*d*e*f + 9
*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*c*f*(35*b^2*
d^2*e^4 - 2*a*b*e*f*(42*d^2*e^2 - 9*c*d*e*f + 2*c^2*f^2) + a^2*f^2*(48*d^2
*e^2 - 16*c*d*e*f + 3*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)])))/(24*Sqrt[b/a]*c^2*(b*c - a*d)*e^3*(b*e - a*f)*(d*e -
 c*f)^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{5/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}(}}{de-cf}}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}(}}{de-cf}}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 400

$$\frac{b\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx\cdots}{3c\left(\cdots\right.}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\frac{b(ad(de-4cf)-3bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2de-5cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)}{(de-cf)^2}}{de-cf}}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7cf)-ad(2d\cdots}{\sqrt{c}\sqrt{c+dx^2}(bc-\cdots}}{3c(bc-ad)}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\Big\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(bc(4de-7c}{\sqrt{c}\sqrt{c}}}{3c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(}{}}{3c(bc-ad)}\right.}{(de-cf)^2}}{de-cf}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\downarrow$ 412

$$b\left(\frac{d\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}(de-cf)^2}+\frac{d\left(-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(ad(de-4cf)-3bc(de-2cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{\cdots}}{\cdots}}{3c(bc-ad)}\cdots\right)}{(de\cdots}\right)}{de-cf}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{f}$$

$\Big\downarrow$ 426

$$\frac{b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right)}{(de-\cdots}\right)}{de-cf}\cdots\right)}{}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{b\cdots}}}{\cdots}\right.}{de-cf}\cdots\right)}{f}$$

$$\Big\downarrow \; 421$$

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{(de\cdots}\right)}{de-cf}\right.$$

$$\overline{(be-af)\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{}{\sqrt{bx^2+a}(\cdots}}{\cdots}\right.}{\cdots}\right.}$$

$$f$$

↓ 25

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\ldots}\right)}{(de\ldots}\right)}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{\ldots}{\sqrt{bx^2+a}\left(dx\ldots\right.}}{\ldots}\right.}{\ldots}\right.}{f}$$

$\downarrow$ 400

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{dx}}{\sqrt{c}}\right.\right.}{\cdots}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right)}{(de\cdots}\right)}{de-cf}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{\cdots}{\sqrt{bx^2+a}\left(dx\cdots\right.}}{\cdots}\right.}{\cdots}\cdots\right)}{f}$$

↓ 313

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}$$

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\cdots\right)}{(de-\cdots}\right)}{de-cf}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{\cdots}{\sqrt{bx^2+a}\left(dx\cdots\right.}}{\cdots}\right.}{\cdots}\right.}{f}$$

↓ 320

$$
b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\cdots\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right)}{\cdots}\right)}{de-cf}\cdots\right)
$$

$$
\frac{\cdots}{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{\cdots}{\sqrt{bx^2+a}\left(\cdots\right.}}{\cdots}\cdots\right.}{\cdots}\right)}
$$

$$
f
$$

↓ 402

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right)}{(de-\cdots}\right)}{de-cf}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(\cdots\right.}}{\cdots}\right)}{de-cf}\right.$$

↓ 25

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{dx}}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right.}{(de\cdots}\right)}{de-cf}\right.}{\cdots}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(\cdots\right.}}{\cdots}\right)}{de-cf}\right.\cdots$$

↓ 400

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{dx}}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{(de\text{-}}\right)}{de-cf}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3b}{}}{3c(bc-ad)}\right.}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}\right.$$

↓ 313

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{dx}}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right)}{(de-}\right)}{de-cf}\right.$$

$$\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-}{3c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}\right.}{}$$

↓ 320

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{(de-}\right)}{de-cf}\right.$$

$$\left(\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{}\right.}{(de-cf)^2}\right.}{de-cf}\right.\right.$$

↓ 413

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right.}{(de\cdots}\right)}{de-cf}\right.$$

$$\left(\left(\frac{d\left(\frac{\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2\sqrt{bx^2+a}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right.}{(de-\cdots}\right)}{de-cf}\right.\right.$$

$\downarrow$ 413

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\ldots}\right)}{(de-\ldots}\right)}{de-cf}\right.$$

$$\left(d\left(\frac{d\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx f^2}{(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\text{arct}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2\ldots}}}{\ldots}\right)}{\ldots}\right)}{de-cf}\right.\right.$$

↓ 412

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots}{\cdots}\right)}{(de\cdots}\right)}{de-cf}\cdots\right)$$

$$\overline{\left(\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}}{\cdots}\right)}{\cdots}\right)}{de-cf}\cdots\right.\right.}$$

↓ 414

$$b \left(\frac{d \left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}} + \frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} - \frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right)}{(de\cdots}\right)}{de-cf}\right)}{\cdots}$$

$$\left(\left(d\,\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}} + \frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}} - \frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}}{\cdots}\right)}{\cdots}\right)}{de-cf}\cdots\right.\right.$$

↓ 424

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right)}{(de\cdots}\right)}{de-cf}\cdots$$

$$\frac{\cdots}{\left(\left(d\,\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}}{\cdots}\right)}{\cdots}\right)}{de-cf}\cdots\right.\right.}$$

↓ 406

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\ldots}\right)}{(de\ldots}\right)}{de-cf}\right.$$

$$\left(\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}}{\ldots}\right)}{\ldots}\right)}{de-cf}\right.\right.$$

↓ 320

$$b\left(\frac{d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{\cdots}\right)}{(de\cdots}\right)}{de-cf}\right.$$

$$\overline{\left(\left(d\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(de-cf)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}-\frac{\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}}{\cdots}\right)}{\cdots}\right)\right.\right.}$$

$$d\,\frac{\cdots}{de-cf}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4338 vs. $2(885) = 1770$.

Time = 21.47 (sec) , antiderivative size = 4339, normalized size of antiderivative = 4.70

method	result	size
elliptic	Expression too large to display	4339
default	Expression too large to display	14866

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/8*f^3*(3*a*
c*f^2-14*a*d*e*f-2*b*c*e*f+13*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e
^2/(c*f-d*e)^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/4*f^3/(c*
f-d*e)^3/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2-11/3/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^3/c/(c*f-d*e)^4*a*
f+2/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^4/c^
2/(c*f-d*e)^4*a*e-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))*d^3/c/(c*f-d*e)^4*b*e-15/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))*f^3*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(c*f-
d*e)^3*a*c+3/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/
(c*f-d*e)^3*a*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/8*c^2/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/(c*f-d*e)^3*a*Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+7/4*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^3*b/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}\,(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \int \frac{}{d^3f^3x^{12}+3c\,d^2f^3x^{10}+3d^3e\,f^2x^{10}+3c^2d\,f^3x^8+9c\,d^2e\,f^2x^8+3d^3e^2f\,x}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**3*e**3 + 3*c**3*e**2*f*x**2 +
3*c**3*e*f**2*x**4 + c**3*f**3*x**6 + 3*c**2*d*e**3*x**2 + 9*c**2*d*e**2*f
*x**4 + 9*c**2*d*e*f**2*x**6 + 3*c**2*d*f**3*x**8 + 3*c*d**2*e**3*x**4 + 9
*c*d**2*e**2*f*x**6 + 9*c*d**2*e*f**2*x**8 + 3*c*d**2*f**3*x**10 + d**3*e*
*3*x**6 + 3*d**3*e**2*f*x**8 + 3*d**3*e*f**2*x**10 + d**3*f**3*x**12),x)
```

3.179 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 792

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \frac{b(af(50d^2e^2-9cdef-9c^2f^2)-be(105d^2e^2-95cdef+6c^2f^2))\,x\sqrt{c+\ldots}}{24e^2f^4\sqrt{a+bx^2}}$$
$$+\frac{bd^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^3}-\frac{(be-af)(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^3(e+fx^2)^2}$$
$$+\frac{(de-cf)(be(11de-2cf)-3af(2de+cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^3(e+fx^2)}$$
$$-\frac{\sqrt{a}\sqrt{b}(af(50d^2e^2-9cdef-9c^2f^2)-be(105d^2e^2-95cdef+6c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\ldots\right.}{24e^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(24a^2d^3e^2f^2+b^2de^2(105d^2e^2-130cdef+33c^2f^2)-abf(120d^3e^3-103cd^2e^2f+6c^2def^2+9c^3f^3))\ldots}{24\sqrt{b}ce^2f^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(de-cf)\,(5b^2de^3(7de-4cf)-10abde^2f(4de-cf)+a^2f^2(8d^2e^2+4cdef+3c^2f^2))\,\sqrt{c+dx^2}\,\mathrm{Ellip}\ldots}{8\sqrt{b}ce^3f^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/24*b*(a*f*(-9*c^2*f^2-9*c*d*e*f+50*d^2*e^2)-b*e*(6*c^2*f^2-95*c*d*e*f+10
5*d^2*e^2))*x*(d*x^2+c)^(1/2)/e^2/f^4/(b*x^2+a)^(1/2)+1/3*b*d^2*x*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/f^3-1/4*(-a*f+b*e)*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/e/f^3/(f*x^2+e)^2+1/8*(-c*f+d*e)*(b*e*(-2*c*f+11*d*e)-3*a*
f*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^3/(f*x^2+e)-1/24*a^
(1/2)*b^(1/2)*(a*f*(-9*c^2*f^2-9*c*d*e*f+50*d^2*e^2)-b*e*(6*c^2*f^2-95*c*d
*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),(1-a*d/b/c)^(1/2))/e^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)-1/24*a^(3/2)*(24*a^2*d^3*e^2*f^2+b^2*d*e^2*(33*c^2*f^2-130*c*d*e*f
+105*d^2*e^2)-a*b*f*(9*c^3*f^3+6*c^2*d*e*f^2-103*c*d^2*e^2*f+120*d^3*e^3))
*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/
2))/b^(1/2)/c/e^2/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)+1/8*a^(3/2)*(-c*f+d*e)*(5*b^2*d*e^3*(-4*c*f+7*d*e)-10*a*b*d*e^2*f*(
-c*f+4*d*e)+a^2*f^2*(3*c^2*f^2+4*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*Ellip
ticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(
1/2)/c/e^3/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.09 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \frac{-\frac{ef^2x(a+bx^2)(c+dx^2)\left(6e(be-af)(de-cf)^2-3(de-cf)(be(11de-2cf)-3af(2de+cf))(e+fx^2)-\right.}{(e+fx^2)^2}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
(-((e*f^2*x*(a + b*x^2)*(c + d*x^2)*(6*e*(b*e - a*f)*(d*e - c*f)^2 - 3*(d*
e - c*f)*(b*e*(11*d*e - 2*c*f) - 3*a*f*(2*d*e + c*f))*(e + f*x^2) - 8*b*d^
2*e^2*(e + f*x^2)^2))/(e + f*x^2)^2) - (I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*(-(b*c*e*f*(b*e*(105*d^2*e^2 - 95*c*d*e*f + 6*c^2*f^2) + a*f*(-50*
d^2*e^2 + 9*c*d*e*f + 9*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)]) + e*(24*a^2*d^3*e^2*f^2 + b^2*e*(105*d^3*e^3 - 60*c*d^2*e^2*f - 35
*c^2*d*e*f^2 + 6*c^3*f^3) + a*b*f*(-120*d^3*e^3 + 65*c*d^2*e^2*f + 6*c^2*d
*e*f^2 + 9*c^3*f^3))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*(d
*e - c*f)*(5*b^2*d*e^3*(7*d*e - 4*c*f) + 10*a*b*d*e^2*f*(-4*d*e + c*f) + a
^2*f^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(24*e^3*f^5*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{5/2}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\int\frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{c(3bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int ...}{...}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}\cdots}{f}\cdots\right)}{f}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{d\left(\frac{2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}-\frac{(de-cf)\int \frac{(\cdots}{\sqrt{bx\cdots}}}{\cdots}}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right)}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\big\downarrow$ 324

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 320

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{5/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right)}{f}\right)$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right.}{f}\right.}{}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 414}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\right)}{f}\right)}{}\right)$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\ldots}{f}\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 420$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\cdots\right.}{f}\right.}{f}$$

↓ 324

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\ldots\right.$$

$$\overline{\qquad f \qquad}$$

(be

$$\downarrow \text{320}$$

$$b\left(b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2\cdots\right.}{\sqrt{bx^2+\cdots}}}{f}\cdots\right)}{f}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-\ldots}{\ldots}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+\ldots}}{\sqrt{bx^2+a}\left(fx\ldots\right.}}{f}\right.}{f}\right.}{f}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{}\right)}{}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}\right)}{f}\right)}{f}\right.$$

↓ 425

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\ldots}{a\sqrt{d}ef\ldots}\right.}{f}\right.}{f}\ldots\right.$$

$\Big\downarrow$ 414

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\cdots\right)}{\cdots}\cdots\right)$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\cdots\right)}{f}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 425

$$b\left(b\left(d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.\right.$$

$$(be-af)\left(b\left(d\,\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right.\right.$$

↓ 413

$$
b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{}\right.
$$

$$
(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}\right.}{}\right.
$$

$$
-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}\ldots}{a\sqrt{d}ef\ldots}
$$

↓ 413

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\ldots}\right.}{\ldots}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}\ldots}{a\sqrt{d}ef\ldots}\right.}{f}\right.}{\ldots}\right.$$

↓ 412

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{\;}\right.}{\;}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}\right.}{f}\right.}{\;}\right.$$

↓ 424

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}}{}\right.}{}$$

$$\left(\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}}{f}\right.\right.$$

↓ 406

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{3b}\right)}{f}\right.}{}\right.$$

$$\left(\left(\frac{d\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}}{a\sqrt{d}ef}\right)}{f}\right.\right.$$

↓ 320

$$\left(\left(d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{3b}\right)\right)\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2812 vs. $2(748) = 1496$.

Time = 26.99 (sec) , antiderivative size = 2813, normalized size of antiderivative = 3.55

method	result	size
elliptic	Expression too large to display	2813
risch	Expression too large to display	4810
default	Expression too large to display	6114

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)*((b*x^2+a)*(
d*x^2+c))^(1/2)*(-35/24/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))*d/f^3*b^2*c^2+35/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))*d^3/f^5*b^2*e^2+95/24*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d/f^3*b^2*Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-5/2/f^3/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c^2*d-35/8*e^2/f^5/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
b^2*d^3+65/24/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)*b*d^2/f^3*a*c-5/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))*d^3/f^4*a*b*e-5/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))*d^2/f^4*b^2*c*e+25/12*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2/f^3*b*a*EllipticE(x*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2} (c+dx^2)^{5/2}}{(e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

3.180 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 619

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3} dx = \frac{b(be(15de-2cf)-af(2de+3cf))x\sqrt{c+dx^2}}{8e^2f^3\sqrt{a+bx^2}}$$
$$+\frac{(be-af)(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^2(e+fx^2)^2}$$
$$-\frac{(be(7de-2cf)-af(2de+3cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^2(e+fx^2)}$$
$$-\frac{\sqrt{a}\sqrt{b}(be(15de-2cf)-af(2de+3cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8e^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}\sqrt{b}(bde^2(15de-7cf)-af(12d^2e^2-cdef-3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2f^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{3a^{3/2}(a^2c^2f^4+b^2de^3(5de-4cf)-2abde^2f(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{8\sqrt{b}ce^3f^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/8*b*(b*e*(-2*c*f+15*d*e)-a*f*(3*c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^2/f^3/(b
*x^2+a)^(1/2)+1/4*(-a*f+b*e)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
e/f^2/(f*x^2+e)^2-1/8*(b*e*(-2*c*f+7*d*e)-a*f*(3*c*f+2*d*e))*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/e^2/f^2/(f*x^2+e)-1/8*a^(1/2)*b^(1/2)*(b*e*(-2*c*f+15
*d*e)-a*f*(3*c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)+1/8*a^(3/2)*b^(1/2)*(b*d*e^2*(-7*c*f+15*d*e)-a*f*(-3*c^2*f^2
-c*d*e*f+12*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(
1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)-3/8*a^(3/2)*(a^2*c^2*f^4+b^2*d*e^3*(-4*c*f+5*d*e)-2*a*
b*d*e^2*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f^3/(-a*f+b*e)/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.55 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \frac{\frac{ef^2x(a+bx^2)(c+dx^2)\left(af^2(5ce+2dex^2+3cfx^2)+be(2cf^2x^2-de(5e+7fx^2))\right)}{(e+fx^2)^2} - \frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{d}{}}}{}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
((e*f^2*x*(a + b*x^2)*(c + d*x^2)*(a*f^2*(5*c*e + 2*d*e*x^2 + 3*c*f*x^2) +
 b*e*(2*c*f^2*x^2 - d*e*(5*e + 7*f*x^2))))/(e + f*x^2)^2 - (I*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(-(b*c*e*f*(b*e*(-15*d*e + 2*c*f) + a*f*(2*d*e
 + 3*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) - b*e*(b*e*(15*
d^2*e^2 + 3*c*d*e*f - 2*c^2*f^2) - a*f*(12*d^2*e^2 + c*d*e*f + 3*c^2*f^2))
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*(a^2*c^2*f^4 + b^2*d*e
^3*(5*d*e - 4*c*f) + 2*a*b*d*e^2*f*(-2*d*e + c*f))*EllipticPi[(a*f)/(b*e),
 I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(8*e^3*f^4*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 320$$

$$b\left(\frac{b\left(\frac{d\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}}{f}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{b\left(\frac{d\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx}{\sqrt{bx^2-}}}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a-}{a(c-}}}\right)}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 425$$

$$
\frac{b\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{de}f\sqrt{c+dx^2}\sqrt{\frac{c\left(a-\cdots\right)}{a\left(c-\cdots\right)}}}\cdots}{f}\cdots}{f}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}
$$

$$\downarrow 414$$

$$b\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a-}{a(c-}}}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right.}{f}\right.}{f}$$

$$\downarrow \text{425}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2}}{f}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{bx^2+a}}-\frac{(de-cf)\int\frac{}{\sqrt{bx^2+a}\sqrt{dx}}}{f}\right.}{f}\right.}{f}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\int}{\sqrt{}}\right.}{f}\right.}{f}\right.$$

↓ 412

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{f}}{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{f}\right.}{f}\right.}$$

$\downarrow$ 424

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}\right)-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{}\right.}{f}\right.$$

↓ 406

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-c}{}\right.}{}\right.}{}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{f}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

↓ 412

$$b \left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f} \right.$$

↓ 433

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}}{f}\right.$$

↓ 2009

$$b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. 2(581) = 1162.

Time = 7.12 (sec) , antiderivative size = 1688, normalized size of antiderivative = 2.73

method	result	size
elliptic	Expression too large to display	1688
default	Expression too large to display	3576

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)*((b*x^2+a)*(
d*x^2+c))^(1/2)*(1/4*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/f^2*x*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*(3*a*c*f^2+2*a*d*e*f+2*b*c*e*f-7*
b*d*e^2)/f^2/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+3/2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/f^3*a-3/8/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d/f^3*c-1
5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2/
f^4*e+15/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/f^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-3/2/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*a*b*d^2-3/2/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c*d+15/8*e/f^4/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d^2+1/8/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.181 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 551

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = -\frac{(be-af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef(e+fx^2)^2} - \frac{(af(2de-3cf)+be(3de-2cf))x\sqrt{a+bx^2}}{8e^2f^2\sqrt{c+dx^2}(e+fx^2)} + \frac{\sqrt{c}\sqrt{d}(af(2de-3cf)+be(3de-2cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2f^2(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{d}(be-af)(be(3de-4cf)+af(4de-3cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae^2f^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(2abcde^2f^2+b^2de^3(3de-4cf)-a^2cf^3(4de-3cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{8a\sqrt{d}e^3f^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/4*(-a*f+b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^2-1/8*(a*f
*(-3*c*f+2*d*e)+b*e*(-2*c*f+3*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(d*x^2+c)^(1
/2)/(f*x^2+e)+1/8*c^(1/2)*d^(1/2)*(a*f*(-3*c*f+2*d*e)+b*e*(-2*c*f+3*d*e))*
(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^
(1/2))/e^2/f^2/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
1/8*c^(3/2)*d^(1/2)*(-a*f+b*e)*(b*e*(-4*c*f+3*d*e)+a*f*(-3*c*f+4*d*e))*(b*
x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/
a/e^2/f^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8
*c^(3/2)*(2*a*b*c*d*e^2*f^2+b^2*d*e^3*(-4*c*f+3*d*e)-a^2*c*f^3*(-3*c*f+4*d
*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/
d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^3/f^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+(be(-3de+2cf)+c$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f)
+ (b*e*(-3*d*e + 2*c*f) + a*f*(-2*d*e + 3*c*f))*(e + f*x^2))) + I*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*c*e*f*(b*e*(-3*d*e + 2*
c*f) + a*f*(-2*d*e + 3*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
]) - b*e*(d*e - c*f)*(3*a*c*f^2 + b*e*(3*d*e + 2*c*f))*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*a*b*c*d*e^2*f^2 + b^2*d*e^3*(3*d*e - 4*c*
f) + a^2*c*f^3*(-4*d*e + 3*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*f^3*(d*e - c*f)*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 423}$$

$$\frac{b\left(\frac{bd\int \frac{e-fx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{b\left(\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{b\left(\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+\ldots}{a(c+\ldots}}}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 413$$

$$\frac{b\left(\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2\sqrt{a+bx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\ldots\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a\ldots}{a(c\ldots}}}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 413$$

$$\frac{b\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E}{b\sqrt{d}\sqrt{c}}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$\Big\downarrow$ 412

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}}{}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$\Big\downarrow$ 425

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}}{}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 425

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(de-}{}\right.}{f}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{2ef^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}d}{f}\right.}{}\right.}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{be}\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}\cdots}{}\right.\right.}{2ef^2}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(\cdots}}{f}\cdots\right.}{}\cdots\right)}{f}$$

$\downarrow$ 412

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{be}\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}\cdots}{}\right.\right.}{2ef^2}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{be}f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{c}\cdots}}{}\cdots\right.}{}\cdots\right)}{f}$$

$\downarrow$ 424

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+cx}}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{\cdots}{\sqrt{bx^2\cdots}}}{2e(be-af)(de-\cdots}\right.}{f}\right.}{f}\cdots}{\cdots}$$

$\downarrow$ 406

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+cx}}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{\cdots}{\sqrt{\cdots}}\right.}{2e(be-af)(de-c\cdots}\right.}{\cdots}\right.}{f}\cdots}{\cdots}$$

$\downarrow$ 320

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d}}\right.}{2e(be}\right.}{}\right.}{f}\right.$$

↓ 388

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d}}\right.}{}\right.}{}\right.}{f}\right.$$

↓ 313

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c(\cdots)}{a(\cdots)}}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{f\cdots}\right.$$

↓ 413

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c(\cdots)}{a(\cdots)}}}\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 413

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}\,e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\,d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}\,x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}\,ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}\,f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 412

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\right.}{}\cdots}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\operatorname{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

↓ 433

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\cdots\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\cdots\right.}{b\sqrt{d}\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2009

$$\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\left(\frac{ac}{e}-\right.}{}\right.}{f}$$

$$\frac{}{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E(\ldots}{b\sqrt{d}\sqrt{\frac{c(\ldots}{a(\ldots}}}\right.\right.}{}\right.}{}\right.}{}\right.}$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. 2(519) = 1038.

Time = 5.98 (sec) , antiderivative size = 1790, normalized size of antiderivative = 3.25

method	result	size
elliptic	Expression too large to display	1790
default	Expression too large to display	3444

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*(a*f-b*e)
/e/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*(3*a*c*f^2-2*a*
d*e*f+2*b*c*e*f-3*b*d*e^2)/f/e^2/(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*d*b^2/f^3-3/8*b/e^2/(c*f-d*e)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))*a+3/8*b^2/f^2/(c*f-d*e)*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d+3/8*b/e^2/(c*f-d*e)*c^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a-7/8/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d/f^2/(c*f-d*e)*c+5
/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2/f
^3*e/(c*f-d*e)+1/4*b^2/f/e/(c*f-d*e)*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-1/4*b^2/f/e/(c*f-d*e)*c^2/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^3}\,dx = \int \frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{\left(e+fx^2\right)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^3}\,dx = \int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*x - sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b**2*c*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**
2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2
*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*
a**2*d**2*e*f**3*x**6 + 2*a**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*
c**2*e**2*f**2*x**2 + 6*a*b*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*
b*c*d*e**4 - 5*a*b*c*d*e**3*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*
e*f**3*x**6 + 4*a*b*c*d*f**4*x**8 - 3*a*b*d**2*e**4*x**2 - 7*a*b*d**2*e**3
*f*x**4 - 3*a*b*d**2*e**2*f**2*x**6 + 3*a*b*d**2*e*f**3*x**8 + 2*a*b*d**2*
f**4*x**10 + 2*b**2*c**2*e**3*f*x**2 + 6*b**2*c**2*e**2*f**2*x**4 + 6*b**2
*c**2*e*f**3*x**6 + 2*b**2*c**2*f**4*x**8 - 3*b**2*c*d*e**4*x**2 - 7*b**2*
c*d*e**3*f*x**4 - 3*b**2*c*d*e**2*f**2*x**6 + 3*b**2*c*d*e*f**3*x**8 + 2*b
**2*c*d*f**4*x**10 - 3*b**2*d**2*e**4*x**4 - 9*b**2*d**2*e**3*f*x**6 - 9*b
**2*d**2*e**2*f**2*x**8 - 3*b**2*d**2*e*f**3*x**10),x)*a*b**3*c*d**2*e**2*
f**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f +
 6*a**2*c*d*e**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6
 + 2*a**2*d**2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f*
*3*x**6 + 2*a**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2
*x**2 + 6*a*b*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5
*a*b*c*d*e**3*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6...
```

3.182 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)^3} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 730

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{b(3af(2de-cf)-be(de+2cf))x\sqrt{c+dx^2}}{8e^2f(de-cf)^2\sqrt{a+bx^2}}$$
$$+\frac{3b(bde^2-af(2de-cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2}-\frac{fx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4e(de-cf)\,(e+fx^2)^2}$$
$$-\frac{3f(bde^2-af(2de-cf))\,x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2\,(e+fx^2)}$$
$$-\frac{\sqrt{a}\sqrt{b}(3af(2de-cf)-be(de+2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8e^2f(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}\sqrt{b}(bde^2-af(4de-3cf))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2f(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(b^2de^3(de-4cf)+2abde^2f(2de+cf)-a^2f^2(8d^2e^2-8cdef+3c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\right.}{8\sqrt{b}ce^3f(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/8*b*(3*a*f*(-c*f+2*d*e)-b*e*(2*c*f+d*e))*x*(d*x^2+c)^(1/2)/e^2/f/(-c*f+d
*e)^2/(b*x^2+a)^(1/2)+3/8*b*(b*d*e^2-a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2-1/4*f*x*(b*x^2+a)^(3/2)*(d*x^2+
c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^2-3/8*f*(b*d*e^2-a*f*(-c*f+2*d*e))*x*(b*x^
2+a)^(3/2)*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2+e)-1/8*a^(1/
2)*b^(1/2)*(3*a*f*(-c*f+2*d*e)-b*e*(2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/f/(-c*f+d*e)^2/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*b^(1/2)*(b*d*e
^2-a*f*(-3*c*f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*(b^2*d*e^3*(-4*c*f+d*e)+2*a*b*d*
e^2*f*(c*f+2*d*e)-a^2*f^2*(3*c^2*f^2-8*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)
*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2
))/b^(1/2)/c/e^3/f/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.84 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+(3af(-2de+cf)+be(}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f) +
(3*a*f*(-2*d*e + c*f) + b*e*(d*e + 2*c*f))*(e + f*x^2)) - I*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*c*e*f*(3*a*f*(-2*d*e + c*f) +
 b*e*(d*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*
(d*e - c*f)*(a*f*(4*d*e - 3*c*f) + b*e*(d*e - 2*c*f))*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] - (b^2*d*e^3*(d*e - 4*c*f) + 2*a*b*d*e^2*f*(2*d
*e + c*f) + a^2*f^2*(-8*d^2*e^2 + 8*c*d*e*f - 3*c^2*f^2))*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*f^2*(d*e -
 c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 412$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 424$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \cdots}{2e(be \cdots}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int\frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}\right)}{f}-\frac{(be-af)\int \cdots}{\cdots}\cdots\right.}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2c\cdots}{\cdots}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\cdots}{2e\left(e+fx^2\right)(be-\cdots}\cdots\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{2e(be-af)(de-cf)}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right.}}{2e(be-af)(de-cf)}\right.}{f}\right.}{f}$$

↓ 388

$$
\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.\cdots}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}\cdots}\cdots}{2e(b\cdots}\cdots\right)}{\cdots}\cdots\right)}{f\cdots}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)\cdots}{\cdots}\cdots\right)}{f}\cdots\right)}{f\cdots}
$$

↓ 313

$$
\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\text{arctan}\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\cdots\right.}{a\left(dx^2+\cdots\right.}}}\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{f\cdots}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right)}{2e(be-af)(de-cf)}\cdots\right)}{f\cdots}\cdots\right)}{f\cdots}
$$

↓ 413

$$b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\text{arctan}\ldots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\ldots\right.}{a\left(dx^2+\ldots\right.}}}\right.\right.}{\ldots}\right.}{\ldots}\right.$$

$$\frac{\ldots}{f}$$

$$(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\ldots\right.}{2e(be-af)(de-cf)}\right.}{f}\right.$$

$$\frac{\ldots}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\cdots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\cdots\right.}{a\left(dx^2+\cdots\right.}}}\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\cdots\right.}{2e(be-af)(de-cf)\cdots}\cdots\right.}{f\cdots}\cdots\right.}{f\cdots}$$

↓ 412

$$b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\ldots\right.}{a\left(dx^2+\ldots\right.}}}\ldots\right.\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\ldots\right.}{2e(be-af)(de-cf)}\ldots\right.}{f}\ldots\right.}{f}$$

↓ 433

$$b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\right.}{a\left(dx^2+\right.}}}\right.\right.}{}\right.}{}\right.$$

$$\frac{}{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.}$$

$\downarrow$ 2009

$$b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+\right.}{a\left(dx^2+\right.}}}\right.\right.}{}\right.}{}\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{2e(be-af)(de-cf)}\right.}{f}\right.}$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. 2(686) = 1372.

Time = 9.17 (sec) , antiderivative size = 2021, normalized size of antiderivative = 2.77

method	result	size
elliptic	Expression too large to display	2021
default	Expression too large to display	4097

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*(a*f-b*e)
/(c*f-d*e)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*(3*a*c*
f^2-6*a*d*e*f+2*b*c*e*f+b*d*e^2)/e^2/(c*f-d*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/(f*x^2+e)+3/4*b/e/(c*f-d*e)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*d-7/8*b*d/e/(c*f-d*e)^2/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a*c+1/2*b*d^2/f/(c*f-d*e)^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a-3/8*b^2*d
/f/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))*c+3/8*b/e^2/(c*f-d*e)^2*f*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))*a-3/8*b/e^2/(c*f-d*e)^2*f*c^2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a-1/8*b^2/(c*f-d*e)^2/f*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d-1/(c*f-d*e)^2*f/e^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d\,f^3x^8+c\,f^3x^6+3de\,f^2x^6+3ce\,f^2x^4+3d\,e^2f\,x^4+3c\,e^2f\,x^2+d\,e^3x^2 -}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^3x^8+c\,f^3x^6+3de\,f^2x^6+3ce\,f^2x^4+3d\,e^2f\,x^4+3c\,e^2f\,x^2+d\,e^3x^2+c\,e^3}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e**3 + 3*c*e**2*f*x**2 + 3
*c*e*f**2*x**4 + c*f**3*x**6 + d*e**3*x**2 + 3*d*e**2*f*x**4 + 3*d*e*f**2*
x**6 + d*f**3*x**8),x)*b + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**3
 + 3*c*e**2*f*x**2 + 3*c*e*f**2*x**4 + c*f**3*x**6 + d*e**3*x**2 + 3*d*e**
2*f*x**4 + 3*d*e*f**2*x**6 + d*f**3*x**8),x)*a
```

3.183 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 787

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3} dx = -\frac{(bc-ad)\left(bde^2(8de+7cf)-af(8d^2e^2+10cdef-3c^2f^2)\right)x\sqrt{a+bx^2}}{8ce^2(be-af)(de-cf)^3\sqrt{c+dx^2}} + \frac{b\left(bce(13de+2cf)-a(8d^2e^2+10cdef-3c^2f^2)\right)x\sqrt{c+dx^2}}{8ce^2(de-cf)^3\sqrt{a+bx^2}} - \frac{fx(a+bx^2)^{3/2}}{4e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^2} - \frac{f(5bde^2-af(8de-3cf))\,x(a+bx^2)^{3/2}}{8e^2(be-af)(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}\left(bce(13de+2cf)-a(8d^2e^2+10cdef-3c^2f^2)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8ce^2(de-cf)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{b}(5bde^2-af(8de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{3a^{3/2}\left(2abde^2f(4de+cf)-b^2de^3(de+4cf)-a^2f^2(8d^2e^2-4cdef+c^2f^2)\right)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be}\right.}{8\sqrt{b}ce^3(be-af)(de-cf)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/8*(-a*d+b*c)*(b*d*e^2*(7*c*f+8*d*e)-a*f*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^
2))*x*(b*x^2+a)^(1/2)/c/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)+1/8*b*
(b*c*e*(2*c*f+13*d*e)-a*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c)^(1/
2)/c/e^2/(-c*f+d*e)^3/(b*x^2+a)^(1/2)-1/4*f*x*(b*x^2+a)^(3/2)/e/(-c*f+d*e)
/(d*x^2+c)^(1/2)/(f*x^2+e)^2-1/8*f*(5*b*d*e^2-a*f*(-3*c*f+8*d*e))*x*(b*x^2
+a)^(3/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*a^(1/2
)*b^(1/2)*(b*c*e*(2*c*f+13*d*e)-a*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2)
)/c/e^2/(-c*f+d*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a
^(3/2)*b^(1/2)*(5*b*d*e^2-a*f*(-3*c*f+8*d*e))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/(-a*f+b*e)/(-c*f+d*
e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-3/8*a^(3/2)*(2*a*b*d*
e^2*f*(c*f+4*d*e)-b^2*d*e^3*(4*c*f+d*e)-a^2*f^2*(c^2*f^2-4*c*d*e*f+8*d^2*e
^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/
b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(-a*f+b*e)/(-c*f+d*e)^3/(b*x^2+a)^(1/
2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.83 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}efx(a+bx^2)\Big(2cef(be-af)(de-cf)(c+dx^2)+cf(be(5de+2cf)}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f*x*(a + b*x^2)*(2*c*e*f*(b*e - a*f)*(d*e - c*f)*(c + d*x^2
) + c*f*(b*e*(5*d*e + 2*c*f) + a*f*(-10*d*e + 3*c*f))*(c + d*x^2)*(e + f*x
^2) + 8*d^2*(b*c - a*d)*e^2*(e + f*x^2)^2)) + I*c*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*e*f*(b*c*e*(13*d*e + 2*c*f) + a*(-8*d^2
*e^2 - 10*c*d*e*f + 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]) - b*e*(d*e - c*f)*(b*e*(3*d*e + 2*c*f) + a*f*(-8*d*e + 3*c*f))*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*(-2*a*b*d*e^2*f*(4*d*e + c*f
) + b^2*d*e^3*(d*e + 4*c*f) + a^2*f^2*(8*d^2*e^2 - 4*c*d*e*f + c^2*f^2))*E
llipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]
*c*e^3*f*(d*e - c*f)^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+\right.}}{f}\right.}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{}{\sqrt{bx^2+a}(}}{}\right.$$

$$\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

(the fraction above is the denominator f of the large expression)

$\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{f}-\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 414$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\operatorname{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\text{ar}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ac}\right.}{(de-cf)^2}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 426

$$b\left(\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\text{ar}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad}\right.}{(de-cf)^2}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int}{}\right.}{f}\right.}{f}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\cdots}{\sqrt{bx^2+a}\left(dx^2+\cdots\right.}}{de\cdots}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\Big\downarrow\ 25$$

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right.}}{de-c}\right.}{}\right.}{f}$$

$$\downarrow 400$$

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+}}{de-cf}}{f}\right.}{f}$$

↓ 313

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx f^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int}{\sqrt{bx^2}}\right.}{f}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}\right.$$

↓ 424

$$\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.\right.}$$

↓ 406

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right)}{(de-cf)^2}\ldots\right)}{f}\ldots\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\ldots\right)}{(de-cf)^2}\ldots\right)}{de-cf}\ldots\right)}{\ldots}\ldots\right.$$

↓ 320

$$
\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{c}\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}\cdots\right)}
$$

↓ 388

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right)}{(de-cf)^2}\ldots\right)}{f}\ldots\right)$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\ldots\right)}{(de-cf)^2}\ldots\right)}{de-cf}\ldots\right)}{\ldots}\ldots\right)$$

↓ 413

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\mathrm{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

↓ 412

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

↓ 426

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.}{}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{}\right.$$

$$\Big\downarrow \text{421}$$

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{(de-cf)^2}\ldots\right.}{f}\ldots\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\ldots\right.}{(de-cf)^2}\ldots\right.}{de-cf}\ldots\right.}\ldots\right.}$$

↓ 25

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\right.}{(de-cf)^2}\right.}{de-cf}\right.\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$b\left(d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\right.}{\sqrt{c}}\right.}{(de-cf)^2}\right.\right.$$

$$\overline{\qquad de-cf \qquad}$$

↓ 313

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(dx}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\left(\left(d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c}{\sqrt{c}}\right.}{(de-cf)^2}\right.\right.\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\left(\left(d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\right.}{\sqrt{c}}\right.}{(de-cf)^2}\right.\right.\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\text{ar}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{f}\cdots\right.$$

$$\left(\left(d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{c}\cdots}\cdots\right.}{(de-cf)^2}\cdots\right.\right.\right.$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. $2(743) = 1486$.

Time = 10.54 (sec) , antiderivative size = 2179, normalized size of antiderivative = 2.77

method	result	size
elliptic	Expression too large to display	2179
default	Expression too large to display	4562

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f*(a*f-b*
e)/(c*f-d*e)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/8*f*(
3*a*c*f^2-10*a*d*e*f+2*b*c*e*f+5*b*d*e^2)/e^2/(c*f-d*e)^3*x*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-(b*d*x^2+a*d)*(a*d-b*c)*d/(c*f-d*e)^3/c*x/
((x^2+c/d)*(b*d*x^2+a*d))^(1/2)-11/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))*b*d/e/(c*f-d*e)^3*f*a*c+1/4*c^2/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/e/
(c*f-d*e)^3*f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/4*c^2/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*b^2/e/(c*f-d*e)^3*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-3/2/(c*f-d*e)^3*f^2/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c*d+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/(c*f-d*e)^3*a+1/8/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d/(c*f-d*e)^3*c-13/8*c/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*d*b^2/(c*f-d*e)^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^{\cdots}}}{d^2f^3x^{10}+2cd\,f^3x^8+3d^2e\,f^2x^8+c^2f^3x^6+6cde\,f^2x^6+3d^2e^2f\,x^6+3\cdots}\right.$$
$$+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f^3x^{10}+2cd\,f^3x^8+3d^2e\,f^2x^8+c^2f^3x^6+6cde\,f^2x^6+3d^2e^2f\,x^6+3c^2e\,f^2x^4+6cd\,e^2f\,x^4+d^2e^3x\cdots}\right.$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e**3 + 3*c**2*e**2*f*x*
*2 + 3*c**2*e*f**2*x**4 + c**2*f**3*x**6 + 2*c*d*e**3*x**2 + 6*c*d*e**2*f*
x**4 + 6*c*d*e*f**2*x**6 + 2*c*d*f**3*x**8 + d**2*e**3*x**4 + 3*d**2*e**2*
f*x**6 + 3*d**2*e*f**2*x**8 + d**2*f**3*x**10),x)*b + int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(c**2*e**3 + 3*c**2*e**2*f*x**2 + 3*c**2*e*f**2*x**4 +
 c**2*f**3*x**6 + 2*c*d*e**3*x**2 + 6*c*d*e**2*f*x**4 + 6*c*d*e*f**2*x**6
+ 2*c*d*f**3*x**8 + d**2*e**3*x**4 + 3*d**2*e**2*f*x**6 + 3*d**2*e*f**2*x*
*8 + d**2*f**3*x**10),x)*a
```

3.184 $\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 785

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{(ac-bd)\left(bce^2(8ce+27df)-af(8c^2e^2+36cdef-9d^2f^2)\right)x\sqrt{a+bx^2}}{24de^2(be-af)(ce-df)^3(d+cx^2)^{3/2}}$$
$$-\frac{fx(a+bx^2)^{3/2}}{4e(ce-df)(d+cx^2)^{3/2}(e+fx^2)^2}-\frac{f(7bce^2-af(10ce-3df))x(a+bx^2)^{3/2}}{8e^2(be-af)(ce-df)^2(d+cx^2)^{3/2}(e+fx^2)}$$
$$+\frac{\sqrt{c}\left(bde(16c^2e^2+83cdef+6d^2f^2)+a(16c^3e^3-88c^2de^2f-42cd^2ef^2+9d^3f^3)\right)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{c}\cdots}{\sqrt{a}\cdots}\right)\cdots\right.}{24d^{3/2}e^2(ce-df)^4\sqrt{\frac{d(a+bx^2)}{a(d+cx^2)}}\sqrt{d+cx^2}}$$
$$+\frac{\sqrt{c}\left(15b^2d^2e^2f(3ce+4df)+3a^2df^2(48c^2e^2-16cdef+3d^2f^2)-abe(8c^3e^3+56c^2de^2f+149cd^2ef^2-3d\cdots\right.}{24a\sqrt{d}e^2(ce-df)^5\sqrt{\frac{d(a+bx^2)}{a(d+cx^2)}}\sqrt{d+cx^2}}$$
$$+\frac{d^{3/2}f\left(10abce^2f(6ce+df)-5b^2ce^3(3ce+4df)-a^2f^2(48c^2e^2-16cdef+3d^2f^2)\right)\sqrt{a+bx^2}\,\text{EllipticPi}\left(1\cdots\right.}{8a\sqrt{c}e^3(ce-df)^5\sqrt{\frac{d(a+bx^2)}{a(d+cx^2)}}\sqrt{d+cx^2}}$$

output
```
1/24*(a*c-b*d)*(b*c*e^2*(8*c*e+27*d*f)-a*f*(8*c^2*e^2+36*c*d*e*f-9*d^2*f^2
))*x*(b*x^2+a)^(1/2)/d/e^2/(-a*f+b*e)/(c*e-d*f)^3/(c*x^2+d)^(3/2)-1/4*f*x*
(b*x^2+a)^(3/2)/e/(c*e-d*f)/(c*x^2+d)^(3/2)/(f*x^2+e)^2-1/8*f*(7*b*c*e^2-a
*f*(10*c*e-3*d*f))*x*(b*x^2+a)^(3/2)/e^2/(-a*f+b*e)/(c*e-d*f)^2/(c*x^2+d)^
(3/2)/(f*x^2+e)+1/24*c^(1/2)*(b*d*e*(16*c^2*e^2+83*c*d*e*f+6*d^2*f^2)+a*(1
6*c^3*e^3-88*c^2*d*e^2*f-42*c*d^2*e*f^2+9*d^3*f^3))*(b*x^2+a)^(1/2)*Ellipt
icE(c^(1/2)*x/d^(1/2)/(1+c*x^2/d)^(1/2),(1-b*d/a/c)^(1/2))/d^(3/2)/e^2/(c*
e-d*f)^4/(d*(b*x^2+a)/a/(c*x^2+d))^(1/2)/(c*x^2+d)^(1/2)+1/24*c^(1/2)*(15*
b^2*d^2*e^2*f*(3*c*e+4*d*f)+3*a^2*d*f^2*(48*c^2*e^2-16*c*d*e*f+3*d^2*f^2)-
a*b*e*(8*c^3*e^3+56*c^2*d*e^2*f+149*c*d^2*e*f^2-3*d^3*f^3))*(b*x^2+a)^(1/2
)*InverseJacobiAM(arctan(c^(1/2)*x/d^(1/2)),(1-b*d/a/c)^(1/2))/a/d^(1/2)/e
^2/(c*e-d*f)^5/(d*(b*x^2+a)/a/(c*x^2+d))^(1/2)/(c*x^2+d)^(1/2)+1/8*d^(3/2)
*f*(10*a*b*c*e^2*f*(6*c*e+d*f)-5*b^2*c*e^3*(3*c*e+4*d*f)-a^2*f^2*(48*c^2*e
^2-16*c*d*e*f+3*d^2*f^2))*(b*x^2+a)^(1/2)*EllipticPi(c^(1/2)*x/d^(1/2)/(1+
c*x^2/d)^(1/2),1-d*f/c/e,(1-b*d/a/c)^(1/2))/a/c^(1/2)/e^3/(c*e-d*f)^5/(d*(
b*x^2+a)/a/(c*x^2+d))^(1/2)/(c*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.28 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(6d^2ef^2(be-af)(ce-df)\left(d+cx^2\right)^2+3d^2f^2(be(9ce+2c\right.}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((d + c*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(6*d^2*e*f^2*(b*e - a*f)*(c*e - d*f)*(d + c*x^2
)^2 + 3*d^2*f^2*(b*e*(9*c*e + 2*d*f) + a*f*(-14*c*e + 3*d*f))*(d + c*x^2)^
2*(e + f*x^2) + 8*c^2*d*(a*c - b*d)*e^2*(c*e - d*f)*(e + f*x^2)^2 + 8*c^2*
e^2*(a*c*(2*c*e - 11*d*f) + b*d*(2*c*e + 7*d*f))*(d + c*x^2)*(e + f*x^2)^2
) - I*d*Sqrt[1 + (b*x^2)/a]*(d + c*x^2)*Sqrt[1 + (c*x^2)/d]*(e + f*x^2)^2*
(-(b*e*(b*d*e*(16*c^2*e^2 + 83*c*d*e*f + 6*d^2*f^2) + a*(16*c^3*e^3 - 88*c
^2*d*e^2*f - 42*c*d^2*e*f^2 + 9*d^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*c)/(b*d)]) + b*e*(c*e - d*f)*(-(b*d*e*(29*c*e + 6*d*f)) + a*(8*c^2*e^
2 + 36*c*d*e*f - 9*d^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*c)/(b*d)
] + 3*d*(-10*a*b*c*e^2*f*(6*c*e + d*f) + 5*b^2*c*e^3*(3*c*e + 4*d*f) + a^2
*f^2*(48*c^2*e^2 - 16*c*d*e*f + 3*d^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcS
inh[Sqrt[b/a]*x], (a*c)/(b*d)]))/(24*Sqrt[b/a]*d^2*e^3*(c*e - d*f)^4*Sqrt[
a + b*x^2]*(d + c*x^2)^(3/2)*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx^2+d)^{5/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(cx^2+d)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(cx^2+d)^{5/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}(fx^2+e)}dx}{(ce-df)^2}-\frac{c\int-\frac{-cfx^2+ce-2df}{\sqrt{bx^2+a}(cx^2+d)^{5/2}}dx}{(ce-df)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}(fx^2+e)}dx}{(ce-df)^2}+\frac{c\int\frac{-cfx^2+ce-2df}{\sqrt{bx^2+a}(cx^2+d)^{5/2}}dx}{(ce-df)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(cx^2+d)^{5/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{(ce-df)^2}+\frac{c\left(\frac{cx\sqrt{a+bx^2}(ce-df)}{3d\left(cx^2+d\right)^{3/2}(ac-bd)}-\frac{\int -\frac{bc(ce-df)x^2+ac(2ce-5df)-3bd(ce-2df)}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}}dx}{3d(ac-bd)}\right)}{(ce-df)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\cdots\right.}}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{(ce-df)^2}+\frac{c\left(\frac{\int \frac{bc(ce-df)x^2+ac(2ce-5df)-3bd(ce-2df)}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}}dx}{3d(ac-bd)}+\frac{cx\sqrt{a+bx^2}(ce-df)}{3d\left(cx^2+d\right)^{3/2}(ac-bd)}\right)}{(ce-df)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{\cdots}}}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\big\downarrow$ 400

$$b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{(ce-df)^2}+\frac{c\left(\frac{-\frac{c(bd(4ce-7df)-ac(2ce-5df))\int\frac{\sqrt{bx^2+a}}{\left(cx^2+d\right)^{3/2}}dx}{ac-bd}-\frac{b(ac(ce-4df)-3bd(ce-2df))\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}}dx}{ac-bd}}{3d(ac-bd)}+\frac{cx\sqrt{\cdots}}{3d\left(cx\cdots\right.}\cdots\right)}{(ce-df)^2}\cdots\right)}{f}\cdots\right.$$

$$\frac{\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 313$$

$$b\left(\frac{b\left(\frac{b\left(\frac{c\left(\frac{-\frac{b(ac(ce-4df)-3bd(ce-2df))\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}}dx}{ac-bd}-\frac{\sqrt{c}\sqrt{a+bx^2}(bd(4ce-7df)-ac(2ce-5df))E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bd}{ac}\right)}{\sqrt{d}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d(a+bx^2)}{a(cx^2+d)}}}}{3d(ac-bd)}+\frac{cx\sqrt{a+bx^2}(ce-df)}{3d\left(cx^2+d\right)^{3/2}(ac-bd)}\right)}{(ce-df)^2}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{(ce-df)^2}+\frac{c\left(\frac{-\frac{b\sqrt{d}\sqrt{a+bx^2}(ac(ce-4df)-3bd(ce-2df))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bd}{ac}\right)}{a\sqrt{c}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d\left(a+bx^2\right)}{a\left(cx^2+d\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(bd(4ce-7df)-ac(2ce-}{\sqrt{d}\sqrt{cx^2+d}(ac-b}}{3d(ac-bd)}}{(ce-df)^2}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(ce-df)^2}+\frac{c\left(\frac{-\frac{b\sqrt{d}\sqrt{a+bx^2}(ac(ce-4df)-3bd(ce-2df))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bd}{ac}\right)}{a\sqrt{c}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d\left(a+bx^2\right)}{a\left(cx^2+d\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(bd(4ce-7df}{\sqrt{d}\sqrt{cx}}}{3d(ac-bd)}\right.}{(ce-df)^2}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{cx^2+d}(ce-df)^2}+\frac{c\left(\frac{-\frac{b\sqrt{d}\sqrt{a+bx^2}(ac(ce-4df)-3bd(ce-2df))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bd}{ac}\right)}{a\sqrt{c}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d\left(a+bx^2\right)}{a\left(cx^2+d\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(b\cdots}{\cdots}}{3d(ac-bd)}\cdots\right)}{(ce-df)^2}\cdots\right)}{f}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow\ 412$$

$$b\left(b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{cx^2+d}(ce-df)^2}+\frac{c\left(-\frac{b\sqrt{d}\sqrt{a+bx^2}(ac(ce-4df)-3bd(ce-2df))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bd}{ac}\right)}{a\sqrt{c}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d(a+bx^2)}{a(cx^2+d)}}}-\frac{\sqrt{c}\sqrt{a-\cdots}}{\cdots}\right.}{3d(ac-bd)\cdots}}{(ce-\cdots}\right)}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 426$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{-a}f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{cx^2+d}(ce-df)^2}+\frac{c\left(\frac{-\frac{b\sqrt{d}\sqrt{a+bx^2}(ac(ce-4df)-3bd(ce-2df))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bd}{ac}\right)}{a\sqrt{c}\sqrt{cx^2+d}(ac-bd)\sqrt{\frac{d(a+bx^2)}{a(cx^2+d)}}}-\frac{\sqrt{c}\sqrt{a-\cdots}}{\cdots}}{3d(ac-bd)}\cdots\right)}{(ce-\cdots}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{c\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)}dx}{ce-df}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}\left(fx^2+e\right)^2}dx}{ce-df}\right)}{f}-\frac{(be-af)\left(\frac{c\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}\left(fx^2+e\right)^2}dx}{ce-df}-\cdots\right)}{f}\cdots\right)}{f}$$

$$\Big\downarrow \; 421$$

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\cdots\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}\cdots}{\cdots}\right)}{(ce-\cdots}\right)}{f}\cdots\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{c\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx}{(ce-df)^2}-\frac{c\int-\frac{-cfx^2+ce-2df}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}}dx}{(ce-df)^2}\right)}{ce-df}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}\left(fx^2+e\right)^2}dx}{ce-df}\right)}{f}-\frac{(be-af)\left(\frac{c\int\frac{\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right.}{\cdots}\right.}$$

$$f$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dxf^2}{(ce-df)^2}+\frac{c\int\frac{-cfx^2+ce-2df}{\sqrt{bx^2+a}\left(cx^2+d\right)^{5/2}}dx}{(ce-df)^2}\right)}{ce-df}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}\left(fx^2+e\right)^2}dx}{ce-df}\right)}{f}-\frac{(be-af)\left(\frac{c\int}{\sqrt{bx^2}}\right.}{}\right.$$

$$f$$

↓ 402

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{c\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dxf^2}{(ce-df)^2}+\frac{c\left(\frac{c(ce-df)x\sqrt{bx^2+a}}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}-\frac{\int-\frac{bc(ce-df)x^2+ac(2ce-5df)-3bd(ce-2df)}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}}dx}{3d(ac-bd)}\right)}{(ce-df)^2}\right)}{ce-df}-\frac{f\int\frac{}{\sqrt{bx^2+a}\left(cx\right.}}{}\right)}{f}\right.$$

↓ 25

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right)}{(ce-}\right)}{f}\right.}{}$$

$$(be-af)\left(\frac{b\left(\frac{c\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dxf^2}{(ce-df)^2}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{\int\frac{bc(ce-df)x^2+ac(2ce-5df)-3bd(ce-2df)}{\sqrt{bx^2+a}\left(cx^2+d\right)^{3/2}}dx}{3d(ac-bd)}\right)}{(ce-df)^2}\right)}{ce-df}-\frac{f\int\frac{}{\sqrt{bx^2+a}\left(cx^2\right.}}{c}\right)}{f}\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\ldots\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}\ldots}{\ldots}\right)}{(ce-\ldots}\right)}{f}\ldots\right)$$

$$(be-af)\left(\frac{b\left(\frac{c\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{cx^2+d}\left(fx^2+e\right)}dx\,f^2}{(ce-df)^2}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{c(bd(4ce-7df)-ac(2ce-5df))\int\frac{\sqrt{bx^2+a}}{\left(cx^2+d\right)^{3/2}}dx}{ac-bd}-\frac{b(ac(ce-4df)-3bd\ldots}{\ldots}}{3d(ac-bd)}\right)}{(ce-df)^2}\ldots\right)}{ce-df}\ldots\right)}{f}\ldots\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}\,x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right)}{f}\right.}{}$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bd}{ac}\right)-}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\cdots}}\sqrt{cx^2+d}}$$

↓ 320

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\ldots\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}\ldots}{\ldots}\right)}{(ce-\ldots}\right)}{f}\ldots$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bd}{ac}\right)}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\ldots}}\sqrt{cx^2+d}}-\ldots$$

↓ 413

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}\,x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right|}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}$$

↓ 413

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+ax}}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{cx}}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right)}{(ce-}\right)}{f}\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\text{arcta}\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{}}\sqrt{cx^2+}}$$

$$\Big\downarrow \text{412}$$

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\left(\ldots\right)}}}$$

↓ 426

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+ax}}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\cdots\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}\cdots}{\cdots}\right)}{\cdots}\right)}{f}\cdots\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\cdots}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\cdots}}\cdots}$$

↓ 421

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right)}{(ce-}\right)}{f}\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\ldots}}}$$

$$\Big\downarrow \; 25$$

$$b \left(\frac{b \left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be}, \arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right), \frac{ac}{bd}\right) f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}} + \frac{c \left(\frac{c(ce-df)\sqrt{bx^2+a}\,x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}} + \frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{} \right.}{(ce-} \right.}{f} \right.$$

$$- \frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{\left(\ldots\right)}}}$$

↓ 400

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}\,x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

$$-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}\,E}{\sqrt{\frac{d\left(bx^2+a\right)}{}}}$$

↓ 313

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right)}{f}\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{cx^2}{d}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ac}{bd}\right)f^2}{\sqrt{b}e(ce-df)^2\sqrt{bx^2+a}\sqrt{cx^2+d}}+\frac{c\left(\frac{c(ce-df)\sqrt{bx^2+a}x}{3d(ac-bd)\left(cx^2+d\right)^{3/2}}+\frac{-\frac{\sqrt{c}(bd(4ce-7df)-ac(2ce-5df))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right.}{\sqrt{d}(ac-bd)\sqrt{\frac{d\left(bx^2+a\right)}{a\left(cx^2+d\right)}}\sqrt{cx^2+d}}}{}\right.}{(ce-}\right.}{f}\right.$$

input
```
Int[(a + b*x^2)^(3/2)/((d + c*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2690 vs. 2(747) = 1494.

Time = 21.80 (sec) , antiderivative size = 2691, normalized size of antiderivative = 3.43

method	result	size
elliptic	Expression too large to display	2691
default	Expression too large to display	8333

input
```
int((b*x^2+a)^(3/2)/(c*x^2+d)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(c*x^2+d))^(1/2)/(b*x^2+a)^(1/2)/(c*x^2+d)^(1/2)*(1/3*(a*c-b*d)
/(c*e-d*f)^3/d*x*(b*c*x^4+a*c*x^2+b*d*x^2+a*d)^(1/2)/(x^2+1/c*d)^2+3/2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(b*c*x^4+a*c*x^2+b*d*x^2+a*d
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*c+b*d)/d/b)^(1/2))*b*c^2/(c*e-d*f)
^4*a*f+5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(b*c*x^4+a*c*x
^2+b*d*x^2+a*d)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*c+b*d)/d/b)^(1/2))*b
^2*c/(c*e-d*f)^4*d*f+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/
(b*c*x^4+a*c*x^2+b*d*x^2+a*d)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*c+b*d)
/d/b)^(1/2))*b*c^2/(c*e-d*f)^3/d*a+1/3*(b*c*x^2+a*c)*c*(2*a*c^2*e-11*a*c*d
*f+2*b*c*d*e+7*b*d^2*f)/(c*e-d*f)^4/d^2*x/((x^2+1/c*d)*(b*c*x^2+a*c))^(1/2
)-5/4/(c*e-d*f)^4/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(
b*c*x^4+a*c*x^2+b*d*x^2+a*d)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-c/d
)^(1/2)/(-b/a)^(1/2))*a*b*c*d-7/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^
2)^(1/2)/(b*c*x^4+a*c*x^2+b*d*x^2+a*d)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*c+b*d)/d/b)^(1/2))*b^2*c^2/(c*e-d*f)^4*e+15/8/(c*e-d*f)^4*e/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(b*c*x^4+a*c*x^2+b*d*x^2+a*d)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-c/d)^(1/2)/(-b/a)^(1/2))*b^2*c^2-15/8/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(b*c*x^4+a*c*x^2+b*d*x^2+
a*d)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*c+b*d)/d/b)^(1/2))*b*c/(c*e-d*f
)^4/e*a*d*f^2+7/4*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+c/d*x^2)^(1/2)/(b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x^2+d)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x**2+d)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx^2+d)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x^2+d)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(3/2)/((c*x^2 + d)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx^2+d)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x^2+d)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((c*x^2 + d)^(5/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x^2+d)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((d + c*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((d + c*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(d+cx^2)^{5/2}(e+fx^2)^3}\,dx = \left(\int \frac{}{c^3f^3x^{12}+3c^3e\,f^2x^{10}+3c^2d\,f^3x^{10}+3c^3e^2f\,x^8+9c^2de\,f^2x^8+3c\,d^2f^3}\right.$$
$$+\left(\int \frac{\sqrt{b\,x^2+a}\,\sqrt{c\,x^2}}{c^3f^3x^{12}+3c^3e\,f^2x^{10}+3c^2d\,f^3x^{10}+3c^3e^2f\,x^8+9c^2de\,f^2x^8+3c\,d^2f^3x^8+c^3e^3x^6+9c^2d\,e^2f\,x^6+9c}\right.$$

input
```
int((b*x^2+a)^(3/2)/(c*x^2+d)^(5/2)/(f*x^2+e)^3,x)
```

output

```
int((sqrt(a + b*x**2)*sqrt(c*x**2 + d)*x**2)/(c**3*e**3*x**6 + 3*c**3*e**2
*f*x**8 + 3*c**3*e*f**2*x**10 + c**3*f**3*x**12 + 3*c**2*d*e**3*x**4 + 9*c
**2*d*e**2*f*x**6 + 9*c**2*d*e*f**2*x**8 + 3*c**2*d*f**3*x**10 + 3*c*d**2*
e**3*x**2 + 9*c*d**2*e**2*f*x**4 + 9*c*d**2*e*f**2*x**6 + 3*c*d**2*f**3*x*
*8 + d**3*e**3 + 3*d**3*e**2*f*x**2 + 3*d**3*e*f**2*x**4 + d**3*f**3*x**6)
,x)*b + int((sqrt(a + b*x**2)*sqrt(c*x**2 + d))/(c**3*e**3*x**6 + 3*c**3*e
**2*f*x**8 + 3*c**3*e*f**2*x**10 + c**3*f**3*x**12 + 3*c**2*d*e**3*x**4 +
9*c**2*d*e**2*f*x**6 + 9*c**2*d*e*f**2*x**8 + 3*c**2*d*f**3*x**10 + 3*c*d*
*2*e**3*x**2 + 9*c*d**2*e**2*f*x**4 + 9*c*d**2*e*f**2*x**6 + 3*c*d**2*f**3
*x**8 + d**3*e**3 + 3*d**3*e**2*f*x**2 + 3*d**3*e*f**2*x**4 + d**3*f**3*x*
*6),x)*a
```

3.185 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 942

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \frac{b(a^2f^2(274d^2e^2-45cdef-45c^2f^2)-abef(1155d^2e^2-1016cdef+45c^2}{120e^2f^5\sqrt{a+bx^2}}$$

$$-\frac{bd(15bde-11bcf-11adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15f^4}$$

$$+\frac{b^2d^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5f^3}+\frac{(be-af)^2(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^4(e+fx^2)^2}$$

$$-\frac{3(be-af)(de-cf)(be(5de-2cf)-af(2de+cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^4(e+fx^2)}$$

$$-\frac{\sqrt{a}\sqrt{b}(a^2f^2(274d^2e^2-45cdef-45c^2f^2)-abef(1155d^2e^2-1016cdef+45c^2f^2)+b^2e^2(945d^2e^2-115}{120e^2f^5\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}(120a^2d^3e^2f^2+b^2e(945d^3e^3-1470cd^2e^2f+617c^2def^2-60c^3f^3)-abf(840d^3e^3-827cd^2e^2f+30c}{120\sqrt{b}ce^2f^5\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}(de-cf)\left(2abef(28d^2e^2-11cdef-2c^2f^2)-a^2f^2(8d^2e^2+4cdef+3c^2f^2)-b^2e^2(63d^2e^2-56cdef}{8\sqrt{b}ce^3f^5\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/120*b*(a^2*f^2*(-45*c^2*f^2-45*c*d*e*f+274*d^2*e^2)-a*b*e*f*(45*c^2*f^2-
1016*c*d*e*f+1155*d^2*e^2)+b^2*e^2*(274*c^2*f^2-1155*c*d*e*f+945*d^2*e^2))
*x*(d*x^2+c)^(1/2)/e^2/f^5/(b*x^2+a)^(1/2)-1/15*b*d*(-11*a*d*f-11*b*c*f+15
*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^4+1/5*b^2*d^2*x^3*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/f^3+1/4*(-a*f+b*e)^2*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/e/f^4/(f*x^2+e)^2-3/8*(-a*f+b*e)*(-c*f+d*e)*(b*e*(-2*c*f+5*d
*e)-a*f*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^4/(f*x^2+e)-1
/120*a^(1/2)*b^(1/2)*(a^2*f^2*(-45*c^2*f^2-45*c*d*e*f+274*d^2*e^2)-a*b*e*f
*(45*c^2*f^2-1016*c*d*e*f+1155*d^2*e^2)+b^2*e^2*(274*c^2*f^2-1155*c*d*e*f+
945*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2
),(1-a*d/b/c)^(1/2))/e^2/f^5/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+1/120*a^(3/2)*(120*a^2*d^3*e^2*f^2+b^2*e*(-60*c^3*f^3+617*c^2*d*e*f^2-1
470*c*d^2*e^2*f+945*d^3*e^3)-a*b*f*(45*c^3*f^3+30*c^2*d*e*f^2-827*c*d^2*e^
2*f+840*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)
),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f^5/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)+1/8*a^(3/2)*(-c*f+d*e)*(2*a*b*e*f*(-2*c^2*f^2-11*c*d*e*f+28*d
^2*e^2)-a^2*f^2*(3*c^2*f^2+4*c*d*e*f+8*d^2*e^2)-b^2*e^2*(8*c^2*f^2-56*c*d*
e*f+63*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f^5/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.76 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \frac{\frac{ef^2x(a+bx^2)(c+dx^2)\left(30e(be-af)^2(de-cf)^2-45(be-af)(de-cf)(be(5de-2cf)-af(2de+cf))(e+\right.}{(e+fx^2)^2}}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
((e*f^2*x*(a + b*x^2)*(c + d*x^2)*(30*e*(b*e - a*f)^2*(d*e - c*f)^2 - 45*(
b*e - a*f)*(d*e - c*f)*(b*e*(5*d*e - 2*c*f) - a*f*(2*d*e + c*f))*(e + f*x^
2) - 8*b*d*e^2*(15*b*d*e - 11*b*c*f - 11*a*d*f)*(e + f*x^2)^2 + 24*b^2*d^2
*e^2*f*x^2*(e + f*x^2)^2))/(e + f*x^2)^2 - (I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(b*c*e*f*(a^2*f^2*(274*d^2*e^2 - 45*c*d*e*f - 45*c^2*f^2) + a*
b*e*f*(-1155*d^2*e^2 + 1016*c*d*e*f - 45*c^2*f^2) + b^2*e^2*(945*d^2*e^2 -
 1155*c*d*e*f + 274*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] + e*(120*a^3*d^3*e^2*f^3 + b^3*e^2*(-945*d^3*e^3 + 840*c*d^2*e^2*f + 19
5*c^2*d*e*f^2 - 154*c^3*f^3) + 3*a*b^2*e*f*(595*d^3*e^3 - 495*c*d^2*e^2*f
- 43*c^2*d*e*f^2 + 15*c^3*f^3) + a^2*b*f^2*(-960*d^3*e^3 + 613*c*d^2*e^2*f
 + 30*c^2*d*e*f^2 + 45*c^3*f^3))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] + 15*(b*e - a*f)*(d*e - c*f)*(2*a*b*e*f*(-28*d^2*e^2 + 11*c*d*e*f +
2*c^2*f^2) + a^2*f^2*(8*d^2*e^2 + 4*c*d*e*f + 3*c^2*f^2) + b^2*e^2*(63*d^2
*e^2 - 56*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(120*e^3*f^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{5/2}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \ 425$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\int \frac{\sqrt{bx^2+a}\left(2d(3bc-ad)x^2+c(5bc-ad)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/}}{\left(fx^2+e\right)^2}}{f}\right.}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\int \frac{d\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\int \frac{\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{fx^2+e}dx}{f}\right)}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{f}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{5b}+\frac{dx}{\cdots}\right.}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{388}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(}{5b}\right.}{f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{313}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}}{f}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 418

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}}{f}}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\downarrow 25$$

$$b\left(b\left(d\left(\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\cdots\right.\right.\right.$$

5b

f

f

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 403

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\cdots}{5b}}{f}}{}\right)}{}\right)$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x\cdots}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 406$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}\right.}{f}\right.}{}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$b\left(b\left(d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}\right.\right.\right.$$

$$f$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 388$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{1}{3}\left(\left(-2a^2d^2+7abcd+3b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2}{3}x}{5b}}{f}}{}\right)}{}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\right.\right.}{5b}\right.}{f}\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.\right.}{5b}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 425

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.\right.}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right.}{}\right.$$

$$f$$

↓ 318

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.\right.}{5b}\cdots\right)}{f}\cdots\right)}{\cdots}\cdots\right.}{\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{3b}+\frac{\int\frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{5/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\cdots\right.}{f}$$

↓ 406

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3b}+\frac{c(3bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\right.}{f}\right.}{}\right.$$

$$f$$

↓ 320

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.}{}\right.$$

$$\left(\left(\left(\left(\frac{(3bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+2d(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx\right)\right.\right.\right.\right.$$

↓ 388

$$b \left(b \left(\frac{d \left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b} + \frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x + \frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.\right.$$

↓ 313

$$b\left(b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.\right.$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.}{}\right.$$

↓ 324

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{}}{b\sqrt{}}\right.\right.}{5b}\right.}{f}\right.}{}\right.$$

↓ 320

$$b \Bigg(b \Bigg(d \Bigg(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b} + \frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.\right.}{5b} \cdots$$

$$\cdots \Bigg/ f \cdots$$

↓ 388

$$b \left(\frac{b \left(\frac{d \left(\frac{dx\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b} + \frac{\frac{2}{3}(3bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x + \frac{1}{3}\left(\frac{(9bc-ad)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), 1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}} + \left(3b^2c^2+7abdc-2a^2d^2\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.\right.}{5b}\right.\right.}{f}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4348 vs. 2(892) = 1784.

Time = 30.44 (sec) , antiderivative size = 4349, normalized size of antiderivative = 4.62

method	result	size
elliptic	Expression too large to display	4349
risch	Expression too large to display	4719
default	Expression too large to display	9510

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)*((b*x^2+a)*(
d*x^2+c))^(1/2)*(3/8/e^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/
c*d)^(1/2)/(-b/a)^(1/2))*a^3*c^3-1/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2
),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*d^3-1/f^3/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^3*c^3-63/8/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^6*b^3*d^3*e^3+137/
60*c^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*b^3/f^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))+63/8*e^3/f^6/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2
)/(-b/a)^(1/2))*b^3*d^3+127/15*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/f^3*a*EllipticE(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-77/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^3/f^4*e*Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+63/8*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*b^3/f...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

3.186 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 728

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = -\frac{b(5b^2e^2(21de-10cf)-abef(95de-9cf)+3a^2f^2(2de+3cf))\,x\sqrt{c+dx^2}}{24e^2f^4\sqrt{a+bx^2}} + \frac{b^2dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^3} - \frac{(be-af)^2(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^3(e+fx^2)^2} + \frac{(be-af)(be(11de-6cf)-af(2de+3cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^3(e+fx^2)} + \frac{\sqrt{a}\sqrt{b}(5b^2e^2(21de-10cf)-abef(95de-9cf)+3a^2f^2(2de+3cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{24e^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{b}(3af(20d^2e^2-cdef-3c^2f^2)-be(105d^2e^2-85cdef+12c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{24ce^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(3a^2c^2f^4-2abef(10d^2e^2-5cdef-2c^2f^2)+b^2e^2(35d^2e^2-40cdef+8c^2f^2))\sqrt{c+dx^2}\,\text{EllipticPi}\left(\right.}{8\sqrt{b}ce^3f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/24*b*(5*b^2*e^2*(-10*c*f+21*d*e)-a*b*e*f*(-9*c*f+95*d*e)+3*a^2*f^2*(3*c
*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^2/f^4/(b*x^2+a)^(1/2)+1/3*b^2*d*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)/f^3-1/4*(-a*f+b*e)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/e/f^3/(f*x^2+e)^2+1/8*(-a*f+b*e)*(b*e*(-6*c*f+11*d*e)-a*f*(
3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^3/(f*x^2+e)+1/24*a^(
1/2)*b^(1/2)*(5*b^2*e^2*(-10*c*f+21*d*e)-a*b*e*f*(-9*c*f+95*d*e)+3*a^2*f^2
*(3*c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),(1-a*d/b/c)^(1/2))/e^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)+1/24*a^(3/2)*b^(1/2)*(3*a*f*(-3*c^2*f^2-c*d*e*f+20*d^2*e^2)-b*e*(12*c
^2*f^2-85*c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(
1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*(3*a^2*c^2*f^4-2*a*b*e*f*(-2*c^2*f^2-5*c*d*
e*f+10*d^2*e^2)+b^2*e^2*(8*c^2*f^2-40*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)
*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2
))/b^(1/2)/c/e^3/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.08 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \frac{-\frac{ef^2x(a+bx^2)(c+dx^2)\left(6e(be-af)^2(de-cf)-3(be-af)(be(11de-6cf)-af(2de+3cf))(e+fx^2)-\right.}{(e+fx^2)^2}}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
(-((e*f^2*x*(a + b*x^2)*(c + d*x^2)*(6*e*(b*e - a*f)^2*(d*e - c*f) - 3*(b*
e - a*f)*(b*e*(11*d*e - 6*c*f) - a*f*(2*d*e + 3*c*f))*(e + f*x^2) - 8*b^2*
d*e^2*(e + f*x^2)^2))/(e + f*x^2)^2) - (I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*(b*c*e*f*(a*b*e*f*(95*d*e - 9*c*f) - 3*a^2*f^2*(2*d*e + 3*c*f) + 5
*b^2*e^2*(-21*d*e + 10*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + b*e*(b^2*e^2*(105*d^2*e^2 - 15*c*d*e*f - 26*c^2*f^2) + 3*a^2*f^2*(20*d
^2*e^2 + c*d*e*f + 3*c^2*f^2) + a*b*e*f*(-165*d^2*e^2 + 20*c*d*e*f + 9*c^2
*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*(b*e - a*f)*(3*a
^2*c^2*f^4 + 2*a*b*e*f*(-10*d^2*e^2 + 5*c*d*e*f + 2*c^2*f^2) + b^2*e^2*(35
*d^2*e^2 - 40*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(24*e^3*f^5*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^{3/2}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 418}$$

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}+\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\sqrt{dx^2+c}}dx}{f^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{\int \frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int \frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 406

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}\right)}{f}-\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\frac{1}{3}fx\sqrt{\ldots}\right.}{f^2}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{f^2}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{(de-cf)^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\ldots\right.}{b\sqrt{d}\sqrt{c+dx^2}\ldots}\right.\right.\right.}{f^2}\right.}{f}\right.$$

$$\frac{\ldots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 414$$

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-adf-4bcf+3bde)\right.\right.}{f^2}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-adf-4bcf+3bde)\right.\right.}{f^2}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\,d\right.}{f}\right.}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(\ldots}{\ldots}\right.\right.}{f^2}\ldots}{f}\ldots\right.}{\ldots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\ldots\right.}{f}\ldots\right.}{f}$$

$$\Big\downarrow \text{324}$$

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(}{}\right.\right.}{f^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be}{}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(\cdots}{\cdots}\right.\right.}{f^2}\cdots}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2\cdots}{\sqrt{bx^2+a}\cdots}}{f}\cdots}{f}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3}{}\right.\right.}{f^2}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-}{}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{\cdots}\right.\right.}{f^2}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c\cdots}}{\sqrt{bx^2+a}(fx\cdots}}{f}\right)}{f}\cdots}{f}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3}{}\right.\right.}{f^2}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\mathrm{E}}{a\sqrt{d}ef}\right.}{f}\right.}{f}\right.$$

↓ 425

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{\cdots}\right.\right.}{f^2}\cdots}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}}{f}\cdots}{f}\cdots\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{}\right.\right.}{f^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right.}{f}\right.\right.$$

↓ 425

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{\cdots}\right.\right.}{f^2}\cdots\right)}{f}\cdots\right)$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\cdots\right)}{f}\cdots\right)}{\cdots}\cdots\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{\cdots}\right.\right.}{f^2\cdots}\right)}{f}\cdots\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{b\left(d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}\cdots\right.}$$

↓ 413

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3\cdots}{\cdots}\right.\right.}{f^2}\cdots\right)}{f}\cdots\right)$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}\cdots\right)}{\cdots}\right.$$

↓ 412

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(3}{}\right.\right.}{f^2}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}}{a\sqrt{d}ef}}{f}}{}\right.\right.$$

↓ 424

$$b\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(\ldots}{\ldots}\right.\right.}{f^2}\ldots\right)}{f}\ldots}{\left(b\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\ldots}{a\sqrt{d}ef\ldots}\ldots\right)}{f}\ldots\right.}$$

↓ 406

$$b\left(\frac{b\left(\frac{a^{3/2}(de-cf)^2\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{d\left(\frac{1}{3}\left((3bde-4bcf-adf)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}(\cdots}{\cdots}\right.\right.}{f^2}\right)}{f}\right.$$

$$\left(b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{E}\cdots}{a\sqrt{d}ef\cdots}\right)}{f}\right.\right.$$

↓ 320

↓ 388

↓ 313

↓ 413

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2816 vs. $2(684) = 1368$.

Time = 27.79 (sec) , antiderivative size = 2817, normalized size of antiderivative = 3.87

method	result	size
elliptic	Expression too large to display	2817
risch	Expression too large to display	4810
default	Expression too large to display	6123

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)*((b*x^2+a)*(
d*x^2+c))^(1/2)*(1/3*d*b^2/f^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/8*(
3*a^2*c*f^3+2*a^2*d*e*f^2+3*a*b*c*e*f^2-13*a*b*d*e^2*f-6*b^2*c*e^2*f+11*b^
2*d*e^3)/e^2/f^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+5/6/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d*b^2/f^3*a*c-55/
8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2/f^4*a*
d^2*e-5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^
3/f^4*c*d*e+95/24*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d/f^3*b^2*a*EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))-35/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d/f^4*b^3*e*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+1/8/e^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),
a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*b*c^2+1/2/f^2/e/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b^2*c^2-25/4/f^
3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.187 $\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 684

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \frac{b(b^2e^2(15de-14cf)-a^2f^2(2de-3cf)-abef(5de-3cf))x\sqrt{c+dx^2}}{8e^2f^3(de-cf)\sqrt{a+bx^2}}$$
$$+\frac{(be-af)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ef^2(e+fx^2)^2}$$
$$-\frac{(be-af)(be(7de-6cf)+af(2de-3cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^2(de-cf)(e+fx^2)}$$
$$-\frac{\sqrt{a}\sqrt{b}(b^2e^2(15de-14cf)-a^2f^2(2de-3cf)-abef(5de-3cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8e^2f^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}\sqrt{b}(3acf^2-be(15de-4cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(a^2cf^3(4de-3cf)+2abcef^2(de-2cf)-b^2e^2(15d^2e^2-24cdef+8c^2f^2))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\right.}{8\sqrt{b}ce^3f^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/8*b*(b^2*e^2*(-14*c*f+15*d*e)-a^2*f^2*(-3*c*f+2*d*e)-a*b*e*f*(-3*c*f+5*d
*e))*x*(d*x^2+c)^(1/2)/e^2/f^3/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/4*(-a*f+b*e)^2
*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^2-1/8*(-a*f+b*e)*(b*e*(
-6*c*f+7*d*e)+a*f*(-3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^
2/(-c*f+d*e)/(f*x^2+e)-1/8*a^(1/2)*b^(1/2)*(b^2*e^2*(-14*c*f+15*d*e)-a^2*f
^2*(-3*c*f+2*d*e)-a*b*e*f*(-3*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/f^3/(-c*f+d*e)/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*b^(1/2)*(3*a*c*f^2-b
*e*(-4*c*f+15*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/
2)),(1-a*d/b/c)^(1/2))/c/e^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)+1/8*a^(3/2)*(a^2*c*f^3*(-3*c*f+4*d*e)+2*a*b*c*e*f^2*(-2*c*f+d*e)-b^
2*e^2*(8*c^2*f^2-24*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f
^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.84 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2(be-af)x(a+bx^2)(c+dx^2)(2e(be-af)(de-cf)-(be(7de-6c$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
(Sqrt[b/a]*e*f^2*(b*e - a*f)*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d
*e - c*f) - (b*e*(7*d*e - 6*c*f) + a*f*(2*d*e - 3*c*f))*(e + f*x^2)) + I*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(a^2*f^2*(2*
d*e - 3*c*f) + a*b*e*f*(5*d*e - 3*c*f) + b^2*e^2*(-15*d*e + 14*c*f))*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*e - a*f)*(3*b*e*(d*e - c*f)
*(a*c*f^2 + b*e*(5*d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] + (a^2*c*f^3*(4*d*e - 3*c*f) + 2*a*b*c*e*f^2*(d*e - 2*c*f) + b^2*e^2
*(-15*d^2*e^2 + 24*c*d*e*f - 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])))/(8*Sqrt[b/a]*e^3*f^4*(d*e - c*f)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{410}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(\frac{b\left(\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{b\left(\frac{b\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{b\left(\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2\cdots}}{(f\cdots}}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\cdots\right)}{a\left(c+\cdots\right)}}}\cdots\right)}{f}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 423

$$b\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a+}{a(c+}}}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{bd\int\frac{e-fx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 406

$$b\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a+}{a(c+}}}}{f}\right.}{}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}\right)}{f}-\frac{(be-a}{}\right.}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.$$

$$\overline{\qquad(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(f\cdots\right.}\cdots\right)}{f}\cdots\right.\qquad}$$

$$f$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}cx}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int\frac{\cdots}{\sqrt{bx^2\cdots}}\cdots\right)}{f}\cdots\right.}{f}$$

$$\downarrow 313$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{a}{e}\right.\right.}{f}\right.}$$

$$f$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\left(\frac{ac}{e}\cdots\right.}{\cdots}\right.}{f}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\left(\frac{ac}{e}\cdots\right.}{\cdots}\right)}{f}\cdots\right)}{f}$$

↓ 412

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,cx}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.}{f}$$

↓ 425

$$
\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{f}\cdots\right)}{\cdots}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}cx}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a\cdots}}{\cdots}\cdots\right)}{f}\cdots\right)}{f\cdots}
$$

↓ 425

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{\cdots}}\right.}{f}\right.}{\left(be-af\right)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,cx}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\cdots}{\cdots}\right.}{f}\right.}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 413

$$b\left(\frac{\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}\,e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 412

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{\cdot}}}{f}}{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\cdots}{}\right)}{f}}$$

$\downarrow$ 424

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 406

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}\cdots}{\cdots}\right)}{f}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{}}\right)}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e\left(fx^2+e\right)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{\cdots}}\cdots\right)}{f}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a\cdots}}{\cdots}\cdots\right)}{f}\cdots\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{\cdot}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 412

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

↓ 433

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(be-af)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de}\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x}{2e(fx^2+e)}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{2ef^2}+\frac{\sqrt{-a}}{}\right.}{f}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3150 vs. 2(646) = 1292.

Time = 7.49 (sec) , antiderivative size = 3151, normalized size of antiderivative = 4.61

method	result	size
elliptic	Expression too large to display	3151
default	Expression too large to display	5950

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-9/8/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^3*d^2/f^4*e^2/(c*f
-d*e)-3/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*b^3/f^2/(c*f-d*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))+3/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3/f^2/(c*f-d*e)*EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8*f/e^3/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*c^2-1/2/e^2/(c*f
-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(
1/2))*a^3*c*d+3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))*b^2/f^3*a*d-3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))*b^3*d/f^4*e+c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3/f^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-1/f^2/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d*f*x - 4*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b**2*c*f*x + 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2
*d*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d*f*x**3 + 4*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**3*c*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c*f*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d*e*x**3 + 6*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e
**2*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**
2*e**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a
**2*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2*x**2 + 6*a*b
*c**2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a*b*c*d*e**3
*f*x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6 + 4*a*b*c*d*f**
4*x**8 - 3*a*b*d**2*e**4*x**2 - 7*a*b*d**2*e**3*f*x**4 - 3*a*b*d**2*e**2*f
**2*x**6 + 3*a*b*d**2*e*f**3*x**8 + 2*a*b*d**2*f**4*x**10 + 2*b**2*c**2*e*
*3*f*x**2 + 6*b**2*c**2*e**2*f**2*x**4 + 6*b**2*c**2*e*f**3*x**6 + 2*b**2*
c**2*f**4*x**8 - 3*b**2*c*d*e**4*x**2 - 7*b**2*c*d*e**3*f*x**4 - 3*b**2*c*
d*e**2*f**2*x**6 + 3*b**2*c*d*e*f**3*x**8 + 2*b**2*c*d*f**4*x**10 - 3*b**2
*d**2*e**4*x**4 - 9*b**2*d**2*e**3*f*x**6 - 9*b**2*d**2*e**2*f**2*x**8 - 3
*b**2*d**2*e*f**3*x**10),x)*a**3*b**2*d**3*e**2*f**3 + 12*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**
2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*...
```

3.188 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 833

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = &-\frac{3b(be-af)(be(de-2cf)+af(2de-cf))x\sqrt{c+dx^2}}{8e^2f^2(de-cf)^2\sqrt{a+bx^2}} \\
&+\frac{b(be(de-4cf)+3af(2de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f(de-cf)^2} \\
&-\frac{b(3af(2de-cf)-be(de+2cf))x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2} \\
&-\frac{fx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{4e(de-cf)(e+fx^2)^2}+\frac{f(3af(2de-cf)-be(de+2cf))x(a+bx^2)^{5/2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2(e+fx^2)} \\
&+\frac{3\sqrt{a}\sqrt{b}(be-af)(be(de-2cf)+af(2de-cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{8e^2f^2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{a^{3/2}\sqrt{b}(be(3de-4cf)+af(4de-3cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2f^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}(2abef(2d^2e^2-7cdef+2c^2f^2)+a^2f^2(8d^2e^2-8cdef+3c^2f^2)+b^2e^2(3d^2e^2-8cdef+8c^2f^2))\sqrt{c}}{8\sqrt{b}ce^3f^2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-3/8*b*(-a*f+b*e)*(b*e*(-2*c*f+d*e)+a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^
2/f^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)+1/8*b*(b*e*(-4*c*f+d*e)+3*a*f*(-c*f+2*d
*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f/(-c*f+d*e)^2-1/8*b*(3*a*f*(-c
*f+2*d*e)-b*e*(2*c*f+d*e))*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e
)/(-c*f+d*e)^2-1/4*f*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2
+e)^2+1/8*f*(3*a*f*(-c*f+2*d*e)-b*e*(2*c*f+d*e))*x*(b*x^2+a)^(5/2)*(d*x^2+
c)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2+e)+3/8*a^(1/2)*b^(1/2)*(-a*f+b
*e)*(b*e*(-2*c*f+d*e)+a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/f^2/(-c*f+d*e)^2/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*b^(1/2)*(b*e*(-4*c*f
+3*d*e)+a*f*(-3*c*f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*(2*a*b*e*f*(2*c^2*f^2-7*c*d*e*f+2*d^
2*e^2)+a^2*f^2*(3*c^2*f^2-8*c*d*e*f+8*d^2*e^2)+b^2*e^2*(8*c^2*f^2-8*c*d*e*
f+3*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f^2/(-c*f+d*e)^2/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.12 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ef^2(be-af)x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)-3(be(de-2\ldots}{\ldots}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f^2*(b*e - a*f)*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*
(d*e - c*f) - 3*(b*e*(d*e - 2*c*f) + a*f*(2*d*e - c*f))*(e + f*x^2))) - I*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(3*b*c*e*f*(b*e - a*
f)*(a*f*(-2*d*e + c*f) + b*e*(-(d*e) + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] + b*e*(d*e - c*f)*(a*b*e*f*(d*e - 3*c*f) + a^2*f^2*(4*
d*e - 3*c*f) + b^2*e^2*(3*d*e - 2*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - (b*e - a*f)*(2*a*b*e*f*(2*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2)
+ a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f + 3*c^2*f^2) + b^2*e^2*(3*d^2*e^2 - 8*c*d
*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)]))/(8*Sqrt[b/a]*e^3*f^3*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\cdots\right)}{f}\cdots\right)}{f}$$
$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-\cdots}{\cdots}\right)}{f}\cdots\right)}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(f\cdots\right.}}{f}\cdots\right)}{f}\cdots\right)}{f}$$
$$\frac{(be-af)\left(\frac{b\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}\cdots}{f}\cdots\right)}{\cdots}\right)}{f}$$

$\downarrow$ 413

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{}{\sqrt{bx^2+a}\sqrt{}}}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\cdots\right.}}{f}\cdots\right.}{}\cdots\right.}{f}$$

$\Big\downarrow$ 412

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\cdots}{\cdots}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{}{\sqrt{bx^2+a}\sqrt{}}}{}\cdots\right.}{}\cdots\right.}{f}$$

$\Big\downarrow$ 424

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{\cdots}{2e\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\cdots}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{\cdots}{\sqrt{b\cdots}}}{2e(be-af)(de\cdots}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 406

$$\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{}{2e\cdots}\right.}{}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\cdots\right.}{2e(be-af)(de-\cdots}\right.}{}\right.}{f}\right.}{}$$

↓ 320

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{}{2e}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\right.}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{2e(b}\right.}{}\right.}{f}\right.$$

↓ 388

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\ldots\right.}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\ldots}}\right.}{\ldots}\right.}{\ldots}\right.}{f\ldots}\right.$$

↓ 313

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{\cdots}{2e\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\cdots}{b\sqrt{d}\sqrt{\frac{c\cdots}{a\cdots}}}\right.\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 413

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{}{2e}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E}{b\sqrt{d}\sqrt{\frac{c}{a}}}\right.\right.\right.}{}\right.}{}\right.}{}\right.$$

↓ 413

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{}{2e}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E}{b\sqrt{d}\sqrt{\frac{c}{a}}}\right.\right.}{}\right.}{}\right.}{}\right.$$

↓ 412

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{\ldots}{2e\ldots}\right.}{\ldots}\right.$$

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\ldots}{\ldots}\right.$$

$$(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\ldots}{b\sqrt{d}\sqrt{\frac{c\ldots}{a\ldots}}}\right.\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\ldots}{\ldots}\right.}{\ldots}\right.$$

↓ 433

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{}{2e}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E}{b\sqrt{d}\sqrt{\frac{c}{a}}}\right.\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$(be-af)\Bigg(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-$$

$$(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.$$

$$b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\right.$$

$$bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E}{b\sqrt{d}\sqrt{\frac{c}{a}}}\right.\right.$$

$$(be-af)\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)\left(fx^2+e\right)}-\right.$$

$$(be-af)\left(b\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.\right.$$

input
```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3287 vs. 2(783) = 1566.

Time = 9.43 (sec) , antiderivative size = 3288, normalized size of antiderivative = 3.95

method	result	size
elliptic	Expression too large to display	3288
default	Expression too large to display	6616

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^3/f^3+3/4*b/(c*f-d*e)
^2/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a^2*d
-3/8/f*b^2/(c*f-d*e)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))*a*d+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))*b*d^2/f/(c*f-d*e)^2*a^2-5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))*b^3*d^2*e^2/f^3/(c*f-d*e)^2-3/4/f*b^3/(c*f-d*e)^2
*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/4/f*b
^3/(c*f-d*e)^2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))+3/8/(c*f-d*e)^2*f^2/e^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^3*c^2-1/2/(c*f-d*e)^2/f/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 2*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6
*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 + 6
*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a**2*d**2*f**4*x**
8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2*x**2 + 6*a*b*c**2*e*f**3*x**4
 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a*b*c*d*e**3*f*x**2 + 3*a*b*c
*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6 + 4*a*b*c*d*f**4*x**8 - 3*a*b*d*
*2*e**4*x**2 - 7*a*b*d**2*e**3*f*x**4 - 3*a*b*d**2*e**2*f**2*x**6 + 3*a*b*
d**2*e*f**3*x**8 + 2*a*b*d**2*f**4*x**10 + 2*b**2*c**2*e**3*f*x**2 + 6*b**
2*c**2*e**2*f**2*x**4 + 6*b**2*c**2*e*f**3*x**6 + 2*b**2*c**2*f**4*x**8 -
3*b**2*c*d*e**4*x**2 - 7*b**2*c*d*e**3*f*x**4 - 3*b**2*c*d*e**2*f**2*x**6
+ 3*b**2*c*d*e*f**3*x**8 + 2*b**2*c*d*f**4*x**10 - 3*b**2*d**2*e**4*x**4 -
 9*b**2*d**2*e**3*f*x**6 - 9*b**2*d**2*e**2*f**2*x**8 - 3*b**2*d**2*e*f**3
*x**10),x)*a**2*b**3*d**2*e**2*f**2 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6*a**2*c*d*e*f
**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 + 6*a**2*d**2*e*
*2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a**2*d**2*f**4*x**8 + 2*a*b*c**
2*e**3*f + 6*a*b*c**2*e**2*f**2*x**2 + 6*a*b*c**2*e*f**3*x**4 + 2*a*b*c**2
*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a*b*c*d*e**3*f*x**2 + 3*a*b*c*d*e**2*f**2*
x**4 + 9*a*b*c*d*e*f**3*x**6 + 4*a*b*c*d*f**4*x**8 - 3*a*b*d**2*e**4*x*...
```

3.189 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 954

$$
\begin{aligned}
&\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \\
&-\frac{(bc-ad)^2\left(af(8d^2e^2+10cdef-3c^2f^2)-be(8d^2e^2+5cdef+2c^2f^2)\right)x\sqrt{a+bx^2}}{8cde^2(be-af)(de-cf)^3\sqrt{c+dx^2}} \\
&+\frac{b\left(3abcef(9de+cf)-b^2ce^2(de+14cf)-a^2f(8d^2e^2+10cdef-3c^2f^2)\right)x\sqrt{c+dx^2}}{8ce^2f(de-cf)^3\sqrt{a+bx^2}} \\
&-\frac{b^2(af(8de-3cf)-be(3de+2cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8de^2(be-af)(de-cf)^2} \\
&-\frac{fx(a+bx^2)^{5/2}}{4e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^2}+\frac{f(af(8de-3cf)-be(3de+2cf))x(a+bx^2)^{5/2}}{8e^2(be-af)(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)} \\
&-\frac{\sqrt{a}\sqrt{b}\left(3abcef(9de+cf)-b^2ce^2(de+14cf)-a^2f(8d^2e^2+10cdef-3c^2f^2)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{8ce^2f(de-cf)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}\sqrt{b}(af(8de-3cf)-be(de+4cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{8ce^2f(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}\left(b^2e^2(d^2e^2-8cdef-8c^2f^2)+2abef(4d^2e^2+13cdef-2c^2f^2)-3a^2f^2(8d^2e^2-4cdef+c^2f^2)\right)\sqrt{c-}}{8\sqrt{b}ce^3f(de-cf)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/8*(-a*d+b*c)^2*(a*f*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2)-b*e*(2*c^2*f^2+5*
c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(d*x
^2+c)^(1/2)+1/8*b*(3*a*b*c*e*f*(c*f+9*d*e)-b^2*c*e^2*(14*c*f+d*e)-a^2*f*(-
3*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c)^(1/2)/c/e^2/f/(-c*f+d*e)^3/(b
*x^2+a)^(1/2)-1/8*b^2*(a*f*(-3*c*f+8*d*e)-b*e*(2*c*f+3*d*e))*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d/e^2/(-a*f+b*e)/(-c*f+d*e)^2-1/4*f*x*(b*x^2+a)^(5/2)
/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^2+1/8*f*(a*f*(-3*c*f+8*d*e)-b*e*(2
*c*f+3*d*e))*x*(b*x^2+a)^(5/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)
/(f*x^2+e)-1/8*a^(1/2)*b^(1/2)*(3*a*b*c*e*f*(c*f+9*d*e)-b^2*c*e^2*(14*c*f+
d*e)-a^2*f*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/e^2/f/(-c*f+d*e)^3/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*b^(1/2)*(a*f*(
-3*c*f+8*d*e)-b*e*(4*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1
/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*(b^2*e^2*(-8*c^2*f^2-8*c*d*e*f+d^
2*e^2)+2*a*b*e*f*(-2*c^2*f^2+13*c*d*e*f+4*d^2*e^2)-3*a^2*f^2*(c^2*f^2-4*c*
d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f/(-c*f+d*e)^3/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.12 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\left(2ce(be-af)^2(de-cf)(c+dx^2)+c(be-af)(af(-1\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(2*c*e*(b*e - a*f)^2*(d*e - c*f)*(c + d*x^2
) + c*(b*e - a*f)*(a*f*(-10*d*e + 3*c*f) + b*e*(d*e + 6*c*f))*(c + d*x^2)*
(e + f*x^2) + 8*d*(b*c - a*d)^2*e^2*(e + f*x^2)^2) + I*c*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*e*f*(-3*a*b*c*e*f*(9*d*e + c*f) +
b^2*c*e^2*(d*e + 14*c*f) + a^2*f*(8*d^2*e^2 + 10*c*d*e*f - 3*c^2*f^2))*Ell
ipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*e - a*f)*(b*e*(d*e - c*f)
*(b*e*(d*e - 6*c*f) + a*f*(8*d*e - 3*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] + (-2*a*b*e*f*(4*d^2*e^2 + 13*c*d*e*f - 2*c^2*f^2) + 3*a^2
*f^2*(8*d^2*e^2 - 4*c*d*e*f + c^2*f^2) + b^2*e^2*(-(d^2*e^2) + 8*c*d*e*f +
 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
))/(8*Sqrt[b/a]*c*e^3*f^2*(d*e - c*f)^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e
 + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{416}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{414}$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{425}$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}d}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}}{f}\right.}{}\right.}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int -\right.}{}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{}{\sqrt{bx}}}{}\right.}{}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int \cdots}{\cdots}\right.}{f}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\cdots}{\sqrt{bx^2\cdots}}}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad\qquad}$$

$$\Big\downarrow \; 400$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right.\cdots}}{f}\cdots\right.}{f}\cdots\right.}{f}$$

↓ 313

$$(be-af)\left(\frac{b\left(\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx f^2}{(de-cf)^2}+\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{\cdots}{\cdots}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx f^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\cdots}{\cdots}\right)}{f}\right.$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{}\ldots}{}\right.}{}\right.}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.}{(de-cf)^2}\right.}{f}\right.}{f}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{ar}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

$$(be-af)\left(\frac{b\left(\frac{b\,c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\ldots\right.}{(de-cf)^2}\ldots\right)}{f}\ldots}{f}\ldots\right.$$

$\downarrow$ 426

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{ar}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\right.}{(de-cf)^2}\right.}{f}\ldots\right.}{f}\ldots\right.}$$

↓ 421

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{ar}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\right.}{\ldots}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\ldots}{\sqrt{c}\ldots}\right.}{(de-cf)^2}\right.}{f}\right.}{\ldots}\right.$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},a\cdots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-c\cdots}{\sqrt{c}\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 400

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots}{\ldots}\ldots\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\right.$$

↓ 320

$$
b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\mathrm{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.
$$

↓ 414

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 424

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 406

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\mathrm{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx^2\ldots}}}\ldots\right)}{\ldots}\ldots\right)}{\ldots}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^{2}\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2}{a(dx^2}}}\right.}{}\right.}{}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{ar}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

↓ 412

$$(be-af)\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},a\cdots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\cdots\right.}{a\left(dx^2\cdots\right.}}}\cdots\right.\right.$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{\cdots}{\cdots}\right.$$

↓ 426

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\text{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

$$\Big\downarrow \; 421$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\cdots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\cdots}{a(dx^2\cdots}}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow 25$$

$$b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\mathrm{a}\ldots\right.}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2\ldots}{a(dx\ldots}}}\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3357 vs. $2(904) = 1808$.

Time = 19.21 (sec) , antiderivative size = 3358, normalized size of antiderivative = 3.52

method	result	size
elliptic	Expression too large to display	3358
default	Expression too large to display	7114

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-3/4/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/(c*f-d*e)^3*c^2*b^3-
1/(c*f-d*e)^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/
(-b/a)^(1/2))*b^3*c^2+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))*b^2*d/(c*f-d*e)^3*a*c-1/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))*b^3*d^2/(c*f-d*e)^3*e^2/f^2-27/8*c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
d*b^2/(c*f-d*e)^3*a*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*d^2*b/(c*f-d*e)^3*a^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))+1/8*(3*a^2*c*f^3-10*a^2*d*e*f^2+3*a*b*c*e*f^2+11*a*b*d*e^2*f-6*b^
2*c*e^2*f-b^2*d*e^3)/(c*f-d*e)^3/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/(f*x^2+e)-11/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))*b*d/(c*f-d*e)^3/e*f*a^2*c+3/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/(c*f-d*e)^3/e^2*f^2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 20*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(4*a**2*c**2*d*e**3*f + 12*a**2*c**2*d*e**2*f**2*
x**2 + 12*a**2*c**2*d*e*f**3*x**4 + 4*a**2*c**2*d*f**4*x**6 + 8*a**2*c*d**
2*e**3*f*x**2 + 24*a**2*c*d**2*e**2*f**2*x**4 + 24*a**2*c*d**2*e*f**3*x**6
 + 8*a**2*c*d**2*f**4*x**8 + 4*a**2*d**3*e**3*f*x**4 + 12*a**2*d**3*e**2*f
**2*x**6 + 12*a**2*d**3*e*f**3*x**8 + 4*a**2*d**3*f**4*x**10 + 2*a*b*c**3*
e**3*f + 6*a*b*c**3*e**2*f**2*x**2 + 6*a*b*c**3*e*f**3*x**4 + 2*a*b*c**3*f
**4*x**6 - a*b*c**2*d*e**4 + 5*a*b*c**2*d*e**3*f*x**2 + 21*a*b*c**2*d*e**2
*f**2*x**4 + 23*a*b*c**2*d*e*f**3*x**6 + 8*a*b*c**2*d*f**4*x**8 - 2*a*b*c*
d**2*e**4*x**2 + 4*a*b*c*d**2*e**3*f*x**4 + 24*a*b*c*d**2*e**2*f**2*x**6 +
 28*a*b*c*d**2*e*f**3*x**8 + 10*a*b*c*d**2*f**4*x**10 - a*b*d**3*e**4*x**4
 + a*b*d**3*e**3*f*x**6 + 9*a*b*d**3*e**2*f**2*x**8 + 11*a*b*d**3*e*f**3*x
**10 + 4*a*b*d**3*f**4*x**12 + 2*b**2*c**3*e**3*f*x**2 + 6*b**2*c**3*e**2*
f**2*x**4 + 6*b**2*c**3*e*f**3*x**6 + 2*b**2*c**3*f**4*x**8 - b**2*c**2*d*
e**4*x**2 + b**2*c**2*d*e**3*f*x**4 + 9*b**2*c**2*d*e**2*f**2*x**6 + 11*b*
*2*c**2*d*e*f**3*x**8 + 4*b**2*c**2*d*f**4*x**10 - 2*b**2*c*d**2*e**4*x**4
 - 4*b**2*c*d**2*e**3*f*x**6 + 4*b**2*c*d**2*e*f**3*x**10 + 2*b**2*c*d**2*
f**4*x**12 - b**2*d**3*e**4*x**6 - 3*b**2*d**3*e**3*f*x**8 - 3*b**2*d**3*e
**2*f**2*x**10 - b**2*d**3*e*f**3*x**12),x)*a**2*b**3*c*d**2*e**2*f**2 - 4
0*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(4*a**2*c**2*d*e**3*f + ...
```

3.190 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 995

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = -\frac{(bc-ad)^2\left(af(8d^2e^2+36cdef-9c^2f^2)-be(8d^2e^2+21cdef+6c^2f^2)\right)x\sqrt{a+bx^2}}{24cde^2(be-af)(de-cf)^3\left(c+dx^2\right)^{3/2}}$$
$$-\frac{b^2(af(10de-3cf)-be(5de+2cf))x\sqrt{a+bx^2}}{8de^2(be-af)(de-cf)^2\sqrt{c+dx^2}}-\frac{fx(a+bx^2)^{5/2}}{4e(de-cf)\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}$$
$$+\frac{f(af(10de-3cf)-be(5de+2cf))x(a+bx^2)^{5/2}}{8e^2(be-af)(de-cf)^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}$$
$$-\frac{\sqrt{d}(5b^2c^2e^2(11de+10cf)-3abce(8d^2e^2+59cdef+3c^2f^2)-a^2(16d^3e^3-88cd^2e^2f-42c^2def^2+9c^3f}{24c^{3/2}e^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{(3a^3cdf^2(48d^2e^2-16cdef+3c^2f^2)-3b^3c^2e^2(3d^2e^2+24cdef+8c^2f^2)+ab^2cde^2(32d^2e^2+146cdef+1}{24a\sqrt{c}\sqrt{d}e^2(de-cf)}$$
$$-\frac{c^{3/2}(be-af)\left(2abef(18d^2e^2+19cdef-2c^2f^2)-a^2f^2(48d^2e^2-16cdef+3c^2f^2)-b^2e^2(3d^2e^2+24cde\right.}{8a\sqrt{d}e^3(de-cf)^5\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/24*(-a*d+b*c)^2*(a*f*(-9*c^2*f^2+36*c*d*e*f+8*d^2*e^2)-b*e*(6*c^2*f^2+2
1*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(d
*x^2+c)^(3/2)-1/8*b^2*(a*f*(-3*c*f+10*d*e)-b*e*(2*c*f+5*d*e))*x*(b*x^2+a)^
(1/2)/d/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)-1/4*f*x*(b*x^2+a)^(5/2
)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^2+1/8*f*(a*f*(-3*c*f+10*d*e)-b*e*
(2*c*f+5*d*e))*x*(b*x^2+a)^(5/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(3/
2)/(f*x^2+e)-1/24*d^(1/2)*(5*b^2*c^2*e^2*(10*c*f+11*d*e)-3*a*b*c*e*(3*c^2*
f^2+59*c*d*e*f+8*d^2*e^2)-a^2*(9*c^3*f^3-42*c^2*d*e*f^2-88*c*d^2*e^2*f+16*
d^3*e^3))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/c^(3/2)/e^2/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)+1/24*(3*a^3*c*d*f^2*(3*c^2*f^2-16*c*d*e*f+48*d^2*e^2)-3*b^
3*c^2*e^2*(8*c^2*f^2+24*c*d*e*f+3*d^2*e^2)+a*b^2*c*d*e^2*(137*c^2*f^2+146*
c*d*e*f+32*d^2*e^2)-a^2*b*d*e*(-6*c^3*f^3+185*c^2*d*e*f^2+128*c*d^2*e^2*f+
8*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b
*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/e^2/(-c*f+d*e)^5/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(3/2)*(-a*f+b*e)*(2*a*b*e*f*(-2*c^2*f^2+19*c
*d*e*f+18*d^2*e^2)-a^2*f^2*(3*c^2*f^2-16*c*d*e*f+48*d^2*e^2)-b^2*e^2*(8*c^
2*f^2+24*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/
(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^3/(-c*f+d*e)^5/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}efx(a+bx^2)\left(6c^2ef(be-af)^2(de-cf)\left(c+dx^2\right)^2+3c^2f(be-af)\right.}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f*x*(a + b*x^2)*(6*c^2*e*f*(b*e - a*f)^2*(d*e - c*f)*(c + d
*x^2)^2 + 3*c^2*f*(b*e - a*f)*(a*f*(-14*d*e + 3*c*f) + b*e*(5*d*e + 6*c*f)
)*(c + d*x^2)^2*(e + f*x^2) + 8*c*d*(b*c - a*d)^2*e^2*(-(d*e) + c*f)*(e +
f*x^2)^2 + 8*d*(b*c - a*d)*e^2*(a*d*(2*d*e - 11*c*f) + b*c*(5*d*e + 4*c*f)
)*(c + d*x^2)*(e + f*x^2)^2)) - I*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1
 + (d*x^2)/c]*(e + f*x^2)^2*(b*e*f*(5*b^2*c^2*e^2*(11*d*e + 10*c*f) - 3*a*
b*c*e*(8*d^2*e^2 + 59*c*d*e*f + 3*c^2*f^2) + a^2*(-16*d^3*e^3 + 88*c*d^2*e
^2*f + 42*c^2*d*e*f^2 - 9*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] + b*e*(d*e - c*f)*(-(a*b*c*e*f*(61*d*e + 9*c*f)) + b^2*c*e^2*(9*d
*e + 26*c*f) + a^2*f*(8*d^2*e^2 + 36*c*d*e*f - 9*c^2*f^2))*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*c*(b*e - a*f)*(-2*a*b*e*f*(18*d^2*e^2
+ 19*c*d*e*f - 2*c^2*f^2) + a^2*f^2*(48*d^2*e^2 - 16*c*d*e*f + 3*c^2*f^2)
+ b^2*e^2*(3*d^2*e^2 + 24*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(24*Sqrt[b/a]*c^2*e^3*f*(d*e - c*f)^4
*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{5/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-a}{}\right.}{f}-\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}+\frac{x\sqrt{a+bx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}}{(de-cf)^2}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-4cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}}{(de-cf)^2}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-\cdots}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-\cdots}}{3c}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)$$
$$\overline{\qquad\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad\qquad}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 425$$

$$b\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{d\left(\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{a+bx^2}(ad(2de-5cf)-bc(de-}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-}}{3c}}{(de-cf)^2}}{f}}{}}{}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}}{f}\right.}{}\right.}{f}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\right.}{}\right.$$

$$f$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\cdots}{\cdots}}{3c}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\cdots}{\cdots}-\cdots\right.}{\cdots}\right.}{f}$$

↓ 402

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{\cdots}}{3c}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{3c(bc-ad)\left(dx^2+c\right)^{3/2}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\cdots}{\sqrt{bx\cdots}}}{\cdots}\right.}{f}\right.$$

$$\Big\downarrow \text{ 25}$$

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\int\frac{bd(de-cf)x^2+ad(2de-5cf)-3bc(de-2cf)}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3c(bc-ad)}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{}{\sqrt{bx}}}{}\right.}{f}\right.$$

$\downarrow$ 400

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5\cdots}{\cdots}}{3c}\cdots\right.}{(de-cf)^2}\cdots\right)}{f}\cdots\right.$$

$$(be-af)\left(b\left(\frac{b\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{3c(bc-ad)(dx^2+c)^{3/2}}-\frac{\frac{d(bc(4de-7cf)-ad(2de-5cf))\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}+\frac{b(ad(de-4cf)-3b\cdots}{\cdots}}{3c(bc-ad)}\cdots\right.}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{f}\cdots\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\cdots}{\cdots}}{3c}\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}+\cdots$$

↓ 320

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5\cdots}{\cdots}}{3c}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots\right)$$

$$\cdots\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}+\cdots$$

↓ 413

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right)}{(de-cf)^2}\right)}{f}\right)$$

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{\ldots}}\sqrt{dx^2+c}}$$

↓ 413

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\cdots}{\cdots}}{3c\cdots}\right)}{(de-cf)^2\cdots}\cdots\right)}{f\cdots}\cdots\right.$$

$$\cdots\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}E\left(\text{arct}\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2\cdots}}\cdots$$

↓ 412

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\ldots}{\ldots}}{3c\ldots}\right)}{(de-cf)^2\ldots}\right)}{f\ldots}\right.$$

$$\left(\left(\left(\ldots\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}\ldots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\ldots}}}\right.\right.\right.$$

↓ 426

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right.}{(de-cf)^2}\right.}{f}\right.$$

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\left(\ldots\right)}}}$$

↓ 421

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c \ldots}{\ldots}}{3c \ldots}\ldots\right)}{(de-cf)^2 \ldots}\ldots\right)}{f}\ldots\right)}{\ldots}$$

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a \ldots}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\ldots}}}$$

↓ 25

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right)}{(de-cf)^2}\right)}{f}\right)$$

$$\frac{\sqrt{d}(bc(4de-7cf)-ad(2de-5cf))\sqrt{bx^2+a}}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\ldots}}}$$

↓ 402

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c}{}}{3c}\right.}{(de-cf)^2}\right.}{f}\right.$$

↓ 25

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\cdots}{\cdots}}{3c\cdots}\right.}{(de-cf)^2\cdots}\right.}{f\cdots}\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5\cdots}{\cdots}}{3c}\right)}{(de-cf)^2}\right)}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5c\cdots}{\cdots}}{3c}\right)}{(de-cf)^2}\right)}{f}\right.$$

↓ 320

$$b\,\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(de-cf)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(ad(2de-5}{}}{3c}}{(de-cf)^2}}{f}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4003 vs. $2(951) = 1902$.

Time = 20.40 (sec) , antiderivative size = 4004, normalized size of antiderivative = 4.02

method	result	size
elliptic	Expression too large to display	4004
default	Expression too large to display	12398

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/8*f*(3*a^2*
c*f^3-14*a^2*d*e*f^2+3*a*b*c*e*f^2+19*a*b*d*e^2*f-6*b^2*c*e^2*f-5*b^2*d*e^
3)/(c*f-d*e)^4/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/3*(a^
2*d^2-2*a*b*c*d+b^2*c^2)/d/(c*f-d*e)^3/c*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/(x^2+c/d)^2+1/4*(a^2*f^2-2*a*b*e*f+b^2*e^2)*f/(c*f-d*e)^3/e*x*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2-1/3*(b*d*x^2+a*d)*(11*a^2*c*d^2*f
-2*a^2*d^3*e-15*a*b*c^2*d*f-3*a*b*c*d^2*e+4*b^2*c^3*f+5*b^2*c^2*d*e)/(c*f-
d*e)^4/c^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+2/3/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2/(c*f-d*e)^3*a*d-3/4/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/(c*f-d*e)^4*c^2*b^3*f-15/
8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d/(c*f-d
*e)^4/e*f^2*a^2*c-1/(c*f-d*e)^4*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^3*c^2-1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b/(c*f-d*e)^3/c*a^2*d^2+3/2/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 54*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**6)/(6*a**2*c**3*d*e**3*f + 18*a**2*c**3*d*e**2*f**2*
x**2 + 18*a**2*c**3*d*e*f**3*x**4 + 6*a**2*c**3*d*f**4*x**6 + 18*a**2*c**2
*d**2*e**3*f*x**2 + 54*a**2*c**2*d**2*e**2*f**2*x**4 + 54*a**2*c**2*d**2*e
*f**3*x**6 + 18*a**2*c**2*d**2*f**4*x**8 + 18*a**2*c*d**3*e**3*f*x**4 + 54
*a**2*c*d**3*e**2*f**2*x**6 + 54*a**2*c*d**3*e*f**3*x**8 + 18*a**2*c*d**3*
f**4*x**10 + 6*a**2*d**4*e**3*f*x**6 + 18*a**2*d**4*e**2*f**2*x**8 + 18*a*
*2*d**4*e*f**3*x**10 + 6*a**2*d**4*f**4*x**12 + 2*a*b*c**4*e**3*f + 6*a*b*
c**4*e**2*f**2*x**2 + 6*a*b*c**4*e*f**3*x**4 + 2*a*b*c**4*f**4*x**6 + a*b*
c**3*d*e**4 + 15*a*b*c**3*d*e**3*f*x**2 + 39*a*b*c**3*d*e**2*f**2*x**4 + 3
7*a*b*c**3*d*e*f**3*x**6 + 12*a*b*c**3*d*f**4*x**8 + 3*a*b*c**2*d**2*e**4*
x**2 + 33*a*b*c**2*d**2*e**3*f*x**4 + 81*a*b*c**2*d**2*e**2*f**2*x**6 + 75
*a*b*c**2*d**2*e*f**3*x**8 + 24*a*b*c**2*d**2*f**4*x**10 + 3*a*b*c*d**3*e*
*4*x**4 + 29*a*b*c*d**3*e**3*f*x**6 + 69*a*b*c*d**3*e**2*f**2*x**8 + 63*a*
b*c*d**3*e*f**3*x**10 + 20*a*b*c*d**3*f**4*x**12 + a*b*d**4*e**4*x**6 + 9*
a*b*d**4*e**3*f*x**8 + 21*a*b*d**4*e**2*f**2*x**10 + 19*a*b*d**4*e*f**3*x*
*12 + 6*a*b*d**4*f**4*x**14 + 2*b**2*c**4*e**3*f*x**2 + 6*b**2*c**4*e**2*f
**2*x**4 + 6*b**2*c**4*e*f**3*x**6 + 2*b**2*c**4*f**4*x**8 + b**2*c**3*d*e
**4*x**2 + 9*b**2*c**3*d*e**3*f*x**4 + 21*b**2*c**3*d*e**2*f**2*x**6 + 19*
b**2*c**3*d*e*f**3*x**8 + 6*b**2*c**3*d*f**4*x**10 + 3*b**2*c**2*d**2*e...
```

3.191 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 1059

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx =$$

$$-\frac{b(ab^2de^2f(8de-17cf)-3a^3df^3(2de-cf)+a^2bdef^2(5de+4cf)-b^3e^2(15d^2e^2-26cdef+8c^2f^2))\,x\sqrt{}}{8de^2f^3(de-cf)^2\sqrt{a+bx^2}}$$

$$-\frac{b(be-af)(be(5de-8cf)+3af(2de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8e^2f^2(de-cf)^2}$$

$$+\frac{3b(be(de-2cf)+af(2de-cf))x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{8e^2f(de-cf)^2}$$

$$-\frac{b(be(de-4cf)+3af(2de-cf))x(a+bx^2)^{5/2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2}-\frac{fx(a+bx^2)^{7/2}\sqrt{c+dx^2}}{4e(de-cf)\,(e+fx^2)^2}$$

$$+\frac{f(be(de-4cf)+3af(2de-cf))x(a+bx^2)^{7/2}\sqrt{c+dx^2}}{8e^2(be-af)(de-cf)^2\,(e+fx^2)}$$

$$+\frac{\sqrt{a}\sqrt{b}(ab^2de^2f(8de-17cf)-3a^3df^3(2de-cf)+a^2bdef^2(5de+4cf)-b^3e^2(15d^2e^2-26cdef+8c^2f^2)}{8de^2f^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}\sqrt{b}(b^2e^2(15de-16cf)-abef(3de-5cf)-a^2f^2(4de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{8ce^2f^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(be-af)\,(a^2f^2(8d^2e^2-8cdef+3c^2f^2)+2abef(6d^2e^2-13cdef+4c^2f^2)+3b^2e^2(5d^2e^2-12cdef}{8\sqrt{b}ce^3f^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/8*b*(a*b^2*d*e^2*f*(-17*c*f+8*d*e)-3*a^3*d*f^3*(-c*f+2*d*e)+a^2*b*d*e*f
^2*(4*c*f+5*d*e)-b^3*e^2*(8*c^2*f^2-26*c*d*e*f+15*d^2*e^2))*x*(d*x^2+c)^(1
/2)/d/e^2/f^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)-1/8*b*(-a*f+b*e)*(b*e*(-8*c*f+5
*d*e)+3*a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/f^2/(-c*f+
d*e)^2+3/8*b*(b*e*(-2*c*f+d*e)+a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(3/2)*(d*x^2+
c)^(1/2)/e^2/f/(-c*f+d*e)^2-1/8*b*(b*e*(-4*c*f+d*e)+3*a*f*(-c*f+2*d*e))*x*
(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2-1/4*f*x*(b*x^2
+a)^(7/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^2+1/8*f*(b*e*(-4*c*f+d*e)
+3*a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(7/2)*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e)/(-c*
f+d*e)^2/(f*x^2+e)+1/8*a^(1/2)*b^(1/2)*(a*b^2*d*e^2*f*(-17*c*f+8*d*e)-3*a^
3*d*f^3*(-c*f+2*d*e)+a^2*b*d*e*f^2*(4*c*f+5*d*e)-b^3*e^2*(8*c^2*f^2-26*c*d
*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))/d/e^2/f^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*b^(1/2)*(b^2*e^2*(-16*c*f+15*d*e)-a*b*e*
f*(-5*c*f+3*d*e)-a^2*f^2*(-3*c*f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(a
rctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f^3/(-c*f+d*e)/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*(-a*f+b*e)*(a^2*f^2*(3*
c^2*f^2-8*c*d*e*f+8*d^2*e^2)+2*a*b*e*f*(4*c^2*f^2-13*c*d*e*f+6*d^2*e^2)+3*
b^2*e^2*(8*c^2*f^2-12*c*d*e*f+5*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/
2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.98 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}def^2(be-af)^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)-(be(7de-1$$

input
```
Integrate[(a + b*x^2)^(7/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*d*e*f^2*(b*e - a*f)^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f
)*(d*e - c*f) - (b*e*(7*d*e - 10*c*f) - 3*a*f*(-2*d*e + c*f))*(e + f*x^2))
 - I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(3*a^3
*d*f^3*(2*d*e - c*f) - a^2*b*d*e*f^2*(5*d*e + 4*c*f) + a*b^2*d*e^2*f*(-8*d
*e + 17*c*f) + b^3*e^2*(15*d^2*e^2 - 26*c*d*e*f + 8*c^2*f^2))*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e*(d*e - c*f)*(-3*a*b^2*d*e^2*f*(6*
d*e - 5*c*f) + a^3*d*f^3*(-4*d*e + 3*c*f) + a^2*b*d*e*f^2*(-(d*e) + 4*c*f)
 + b^3*e^2*(15*d^2*e^2 - 6*c*d*e*f - 8*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] + d*(b*e - a*f)^2*(a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f +
3*c^2*f^2) + 2*a*b*e*f*(6*d^2*e^2 - 13*c*d*e*f + 4*c^2*f^2) + 3*b^2*e^2*(5
*d^2*e^2 - 12*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*d*e^3*f^4*(d*e - c*f)^2*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{(bx^2+a)^{5/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{5/2}}{\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\left(\frac{b\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 324

$$\frac{b\left(\frac{b\left(\frac{b\left(a\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{b\left(b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(bx^2\cdots\right.}{\sqrt{dx^2+c\cdots}}}{f}\cdots\right.$$

$$\frac{\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}\right)}{f}-\frac{(b\cdots}{\cdots}\right)$$

$$\frac{\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+d}{c(a+b}}}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{425}$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+d}{c(a+b}}}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^3}d}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\text{a}\right.}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+d}{c(a+b}}}\right)}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\operatorname{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}\right.}{f}\right.}{(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2\right.}}{f}\right.}{f}\right.}{f}\right.}$$

$\downarrow$ 413

$$\frac{b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\text{a}\cdots\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\cdots\right.}{c\left(bx^2\cdots\right.}}}\cdots\right)}{f}\cdots\right)}{(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{bx^2+a}}-\frac{(be-af)\int\frac{\cdots}{\sqrt{bx^2+a}\sqrt{d\cdots}}}{f}\cdots\right)}{f}\cdots\right)}{f}\cdots\right)}$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\ldots\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\ldots\right.}{c\left(bx^2\ldots\right.}}}\ldots\right)}{f}\ldots\right.$$

$$\overline{(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-af)\int\ldots}{\sqrt{\ldots}}\ldots\right)}{f}\ldots\right)}{f}\ldots\right.}$$

$\downarrow$ 412

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-}{}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 424

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-}{}\right.}{}\right.}{f}\right.$$

↓ 406

$$
b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}}{f}\right.}{(be-af)\left(\frac{b\left(\frac{a^{3/2}\sqrt{b}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{(be-}{}\right.}{}\right.}{}\right.}
$$

↓ 320

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}}{f}\right.$$

↓ 388

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}\right)}{f}$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}}{f}\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}\right)}{f}$$

↓ 412

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}}{f}\right.$$

↓ 433

$$b\,\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a(dx^2}{c(bx^2}}}\right.}{f}$$

↓ 2009

$$b\left(\frac{b\left(\frac{b\left(b\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},a\right.}{\sqrt{b}cef\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2\right.}{c\left(bx^2\right.}}}}{f}\right.$$

input
```
Int[(a + b*x^2)^(7/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4536 vs. 2(1003) = 2006.

Time = 9.78 (sec) , antiderivative size = 4537, normalized size of antiderivative = 4.28

method	result	size
elliptic	Expression too large to display	4537
default	Expression too large to display	9180

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/8/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*e/f^2/(c*f-d*
e)^2*a^2-5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
*b^2*d/f/(c*f-d*e)^2*a^2*c-7/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))*b*d/e/(c*f-d*e)^2*a^3*c-19/8/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^4*d*e^2/f^3/(c*f-d*e)^2*c-7/4/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^3*d^2*e^2/f^3/(c
*f-d*e)^2*a+1/4*(a^3*f^3-3*a^2*b*e*f^2+3*a*b^2*e^2*f-b^3*e^3)/(c*f-d*e)/e/
f^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2-c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^3/f
^2/(c*f-d*e)^2*e*a*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+23/4
/(c*f-d*e)^2/f^2*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(
1/2)/(-b/a)^(1/2))*a*b^3*c*d-5/4/(c*f-d*e)^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(7/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{7}{2}}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.192 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1199

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

output
```
1/8*(-a*d+b*c)^3*(a*f*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2)-b*e*(4*c^2*f^2+3*c
*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d^2/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(d*
x^2+c)^(1/2)+1/8*b*(a^2*b*c*d*e*f^2*(4*c*f+41*d*e)-a*b^2*c*d*e^2*f*(41*c*f
+4*d*e)-b^3*c*e^2*(-8*c^2*f^2-10*c*d*e*f+3*d^2*e^2)-a^3*d*f^2*(-3*c^2*f^2+
10*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c)^(1/2)/c/d/e^2/f^2/(-c*f+d*e)^3/(b*x^2+a
)^(1/2)-1/8*b^2*(3*a^2*d*f^2*(-3*c*f+8*d*e)-b^2*e*(-4*c^2*f^2-7*c*d*e*f+d^
2*e^2)-a*b*f*(-3*c^2*f^2+19*c*d*e*f+9*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/d^2/e^2/f/(-a*f+b*e)/(-c*f+d*e)^2-1/8*b^3*(a*f*(-3*c*f+8*d*e)-b*e*
(4*c*f+d*e))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/e^2/(-a*f+b*e)/(-c*f+d*
e)^2-1/4*f*x*(b*x^2+a)^(7/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^2+1/8*
f*(a*f*(-3*c*f+8*d*e)-b*e*(4*c*f+d*e))*x*(b*x^2+a)^(7/2)/e^2/(-a*f+b*e)/(-
c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*a^(1/2)*b^(1/2)*(a^2*b*c*d*e*f^2*
(4*c*f+41*d*e)-a*b^2*c*d*e^2*f*(41*c*f+4*d*e)-b^3*c*e^2*(-8*c^2*f^2-10*c*d
*e*f+3*d^2*e^2)-a^3*d*f^2*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/d/e^
2/f^2/(-c*f+d*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(
3/2)*b^(1/2)*(-a*f+b*e)*(b*e*(-8*c*f+3*d*e)+a*f*(-3*c*f+8*d*e))*(d*x^2+c)^
(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f
^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2
)*(-a*f+b*e)*(3*a^2*f^2*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)+2*a*b*e*f*(4*c^2*...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 635, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}def^2x(a+bx^2)\left(2ce(be-af)^3(de-cf)(c+dx^2)+c(be-af)^2(af\right.}{}$$

input
```
Integrate[(a + b*x^2)^(7/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*d*e*f^2*x*(a + b*x^2)*(2*c*e*(b*e - a*f)^3*(d*e - c*f)*(c + d
*x^2) + c*(b*e - a*f)^2*(a*f*(-10*d*e + 3*c*f) + b*e*(-3*d*e + 10*c*f))*(c
 + d*x^2)*(e + f*x^2) + 8*(b*c - a*d)^3*e^2*f*(e + f*x^2)^2)) + I*c*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*e*f*(-(a^2*b*c*d*e*f^2*
(41*d*e + 4*c*f)) + a*b^2*c*d*e^2*f*(4*d*e + 41*c*f) + b^3*c*e^2*(3*d^2*e^
2 - 10*c*d*e*f - 8*c^2*f^2) + a^3*d*f^2*(8*d^2*e^2 + 10*c*d*e*f - 3*c^2*f^
2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e*(d*e - c*f)*(a*b^
2*d*e^2*f*(2*d*e - 17*c*f) + a^3*d*f^3*(-8*d*e + 3*c*f) + a^2*b*d*e*f^2*(1
1*d*e + 4*c*f) + b^3*e^2*(3*d^2*e^2 - 6*c*d*e*f + 8*c^2*f^2))*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(b*e - a*f)^2*(3*a^2*f^2*(8*d^2*e^2
 - 4*c*d*e*f + c^2*f^2) + 2*a*b*e*f*(4*d^2*e^2 - 23*c*d*e*f + 4*c^2*f^2) +
 3*b^2*e^2*(d^2*e^2 - 4*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*c*d*e^3*f^3*(d*e - c*f)^3*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^{5/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{5/2}}{(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{417}$$

$$\frac{b\left(\frac{b\left(\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{b\left(\frac{(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e}}{f}\right.}{}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{}}{(dx^2+c}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 416

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}-\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{d\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}d\cdots}{f}\cdots\right.}{\cdots}\right)}{f}$$

$\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{c+dx^2}(be-af)\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a+bx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\ldots\right.}{\sqrt{c}\sqrt{c+dx^2}(de-cf\ldots}\right.\right.}{\ldots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\ldots}{(dx^2+\ldots}}{\ldots}\right.}{\ldots}\right.$$

$$\overline{\hspace{20em}}$$

$$f$$

↓ 414

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\cdots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right.\right.}{\cdots}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad f\qquad\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}\cdots}{f}\cdots\right.}{f}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad f\qquad\cdots}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\right.\right.\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}}{f}\right.}{}\right.}{f}\right.$$

↓ 421

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx\ldots}{a(dx\ldots}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+\ldots}}{(de-cf)^2}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{f}\ldots\right.$$

↓ 25

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{\ldots}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}d\ldots}{(de-cf)^2}\right.}{\ldots}\right.}{\ldots}\right.}{f}\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.\right.}{\ldots}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.\right.}{}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx}{a(dx}}}\right.}{}\right.}{}\right.$$

↓ 426

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx\ldots}{a(dx\ldots}}}\right.}{\ldots}\right.}{\ldots}\right.$$

$$\Big\downarrow \text{421}$$

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{(bc-ad)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx}{a(dx}}}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+aE}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+aE}\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.\right.}{\ldots}\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{a^{3/2}(be-af)\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bc-ad)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\right.$$

↓ 313

$$(be-af)\ \frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\text{arctan}\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{\ldots}$$

↓ 320

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{}\right.$$

↓ 414

$$(be-af)\;\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan \cdots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\cdots\right.}{a\left(dx\cdots\right.}}}\cdots\right.}{\cdots}$$

↓ 424

$$\left(\quad (be-af) \left(\frac{b \left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan \cdots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx \cdots\right.}{a\left(dx \cdots\right.}}} \cdots \right.}{\cdots} \right. \right.$$

↓ 406

$$(be-af)\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}$$

↓ 320

$$(be-af)\ \frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.}{\ldots}$$

↓ 388

$$(be-af)\ b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.$$

↓ 313

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\ldots\right.}{\ldots}\right.$$

↓ 413

$$(be-af)\left(b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\ldots\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\ldots\right.}{a\left(dx\ldots\right.}}}\right.\right.$$

↓ 413

$$(be-af)\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\right.}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{}\right.$$

input
```
Int[(a + b*x^2)^(7/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 417
```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(
(a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[S
qrt[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 PosQ[d/c] && PosQ[f/e]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4538 vs. 2(1143) = 2286.

Time = 11.61 (sec) , antiderivative size = 4539, normalized size of antiderivative = 3.79

method	result	size
elliptic	Expression too large to display	4539
default	Expression too large to display	9740

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-11/8/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*e/f/(c*f-d*
e)^3*a^2-1/4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
*b^3*d^2*e^2/f^2/(c*f-d*e)^3*a-15/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))*b^4*d*e^2/f^2/(c*f-d*e)^3*c+5/4*c^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^4
*e/(c*f-d*e)^3/f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-5/4*c^
2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*b^4*e/(c*f-d*e)^3/f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))+3/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^4*e^2/(c*f-d*e)^3/f^2*EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/2/(c*f-d*e)^3*f^2/e^2/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^4*c*d+1/4/(c*f-d*e
)^3*f^2/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a^3*b*c^2+11/8/(c*f-d*e)^3*f/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(7/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{7}{2}}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.193 $\int \frac{\left(a+bx^2\right)^{7/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1221

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

output
```
1/24*(-a*d+b*c)^3*(a*f*(-9*c^2*f^2+36*c*d*e*f+8*d^2*e^2)-b*e*(12*c^2*f^2+1
5*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c/d^2/e^2/(-a*f+b*e)/(-c*f+d*e)^3/
(d*x^2+c)^(3/2)-1/8*b^2*(3*a^2*d*f^2*(-3*c*f+10*d*e)-a*b*f*(-3*c^2*f^2+21*
c*d*e*f+17*d^2*e^2)+b^2*e*(4*c^2*f^2+9*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)
/d^2/e^2/f/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)-1/8*b^3*(a*f*(-3*c*f+10
*d*e)-b*e*(4*c*f+3*d*e))*x^3*(b*x^2+a)^(1/2)/d/e^2/(-a*f+b*e)/(-c*f+d*e)^2
/(d*x^2+c)^(1/2)-1/4*f*x*(b*x^2+a)^(7/2)/e/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x
^2+e)^2+1/8*f*(a*f*(-3*c*f+10*d*e)-b*e*(4*c*f+3*d*e))*x*(b*x^2+a)^(7/2)/e^
2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(3/2)/(f*x^2+e)-1/24*(5*a*b^2*c^2*d*e^
2*f*(31*c*f+32*d*e)-b^3*c^2*e^2*(8*c^2*f^2+94*c*d*e*f+3*d^2*e^2)-a^2*b*c*d
*e*f*(12*c^2*f^2+271*c*d*e*f+32*d^2*e^2)-a^3*d*f*(9*c^3*f^3-42*c^2*d*e*f^2
-88*c*d^2*e^2*f+16*d^3*e^3))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(1/2)/e^2/f/(-c*f+d*e)^4/(c*
(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/24*(3*b^4*c^2*e^3*(-24*c^2*
f^2-12*c*d*e*f+d^2*e^2)-3*a^4*c*d*f^3*(3*c^2*f^2-16*c*d*e*f+48*d^2*e^2)+5*
a*b^3*c^2*e^2*f*(16*c^2*f^2+49*c*d*e*f+19*d^2*e^2)-a^2*b^2*c*d*e^2*f*(223*
c^2*f^2+319*c*d*e*f+88*d^2*e^2)+a^3*b*d*e*f*(-9*c^3*f^3+221*c^2*d*e*f^2+20
0*c*d^2*e^2*f+8*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x
/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/e^2/f/(-c*f+d*e)^5/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(3/2)*(-a*f+b*e)^2*(b^2*e...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.78 (sec) , antiderivative size = 761, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}def^2x(a+bx^2)\left(6c^2e(be-af)^3(de-cf)\left(c+dx^2\right)^2+3c^2(be-af)^2(c\right.}{}$$

input
```
Integrate[(a + b*x^2)^(7/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*d*e*f^2*x*(a + b*x^2)*(6*c^2*e*(b*e - a*f)^3*(d*e - c*f)*(c + d
*x^2)^2 + 3*c^2*(b*e - a*f)^2*(a*f*(-14*d*e + 3*c*f) + b*e*(d*e + 10*c*f))
*(c + d*x^2)^2*(e + f*x^2) + 8*c*(b*c - a*d)^3*e^2*(-(d*e) + c*f)*(e + f*x
^2)^2 + 8*(b*c - a*d)^2*e^2*(a*d*(2*d*e - 11*c*f) + b*c*(8*d*e + c*f))*(c
+ d*x^2)*(e + f*x^2)^2) - I*c*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*
x^2)/c]*(e + f*x^2)^2*(-(b*e*f*(-5*a*b^2*c^2*d*e^2*f*(32*d*e + 31*c*f) + b
^3*c^2*e^2*(3*d^2*e^2 + 94*c*d*e*f + 8*c^2*f^2) + a^2*b*c*d*e*f*(32*d^2*e^
2 + 271*c*d*e*f + 12*c^2*f^2) + a^3*d*f*(16*d^3*e^3 - 88*c*d^2*e^2*f - 42*
c^2*d*e*f^2 + 9*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
+ b*e*(d*e - c*f)*(-3*a^2*b*c*d*e*f^2*(31*d*e + 4*c*f) + 15*a*b^2*c*d*e^2*
f*(2*d*e + 5*c*f) + a^3*d*f^2*(8*d^2*e^2 + 36*c*d*e*f - 9*c^2*f^2) + b^3*c
*e^2*(3*d^2*e^2 - 30*c*d*e*f - 8*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] + 3*c*d*(b*e - a*f)^2*(-2*a*b*e*f*(6*d^2*e^2 + 33*c*d*e*f
- 4*c^2*f^2) + a^2*f^2*(48*d^2*e^2 - 16*c*d*e*f + 3*c^2*f^2) + b^2*e^2*(-(
d^2*e^2) + 12*c*d*e*f + 24*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)]))/(24*Sqrt[b/a]*c^2*d*e^3*f^2*(d*e - c*f)^4*Sqrt[a
 + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{7/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^{5/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{5/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{419}$$

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{\sqrt{bx^2+a}\left(bfc^2+d^2(be-af)x^2+ad(de-2cf)\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{401}$$

$$b\left(\frac{b\left(\frac{-\frac{\int -\frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-\ldots}{\ldots}\right)$$

$$\frac{\ldots}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\frac{\int \frac{d\left(b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\ldots}{\ldots}\right)$$

$$\frac{\ldots}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{b(ad(de-4cf)+bc(2de+cf))x^2+a(ad(2de-5cf)+bc(de+2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \cdots}{\cdots}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(cf+2de))\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx-ab(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{f}-\cdots\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{313}$$

$$b\left(\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\frac{f(be-af)\int\frac{\sqrt{bx^2}}{\sqrt{dx^2+c}(}}{(de-cf)^2}\right.}{f}\right.$$

$$f$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\downarrow 320$$

$$b\left(\frac{b\left(\frac{\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{f}{}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 414$$

$$b\left(\frac{b\left(\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}\cdots\right)$$

$$\frac{\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)^{3/2}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{425}$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{a+bx^2}(ad(2de-5cf)+bc(cf+2de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3c}-\frac{x\sqrt{a+bx^2}(bc-ad)(de-cf)}{3c\left(c+dx^2\right)^{3/2}}-\cdots\right)}{(de-cf)^2}}{f}\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\sqrt{\cdots}}{\left(dx^2+c\right)\cdots}}{f}\cdots\right)}{f}\cdots\right.}{f}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(b\int\frac{\cdots}{\left(dx^2+\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{f}$$

$$\downarrow 25$$

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^{5/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\sqrt{\cdots}}{\left(dx^2+c\right)^{\cdots}}\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{f}$$

↓ 401

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}-\frac{\int-\frac{d(b(de-4cf)x^2+a(2de-5cf))}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{5/2}(fx^2+e)^2}dx}{f}\right)}{f}\right.$$

↓ 25

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}}{(de-cf)^2}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}-\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\int\frac{d\left(b(de-4cf)x^2+a(2de-5cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3cd}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\right.$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}-}{(de-cf)^2}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{3c\left(dx^2+c\right)^{3/2}}+\frac{\int\frac{b(de-4cf)x^2+a(2de-5cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{3c}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\right.\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{3c(dx^2+c)^{3/2}}+\frac{\frac{ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}}{3c}\right)}{(de-cf)^2}\right.}{f}\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{ab(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{(ad(2de-5cf)-bc(de-4cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\right.\right.}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{\ldots}}\sqrt{dx^2+\ldots}}$$

↓ 320

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\cdots\right)}{f}\right.$$

$$\left(\left(\left(\left(\quad\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}-\frac{(ad(2de-5cf)-bc(de-4cf)\cdots}{\sqrt{c}\sqrt{d}(bc-ad\cdots}\right.\right.\right.\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\ldots}}\sqrt{dx^2+c}}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

↓ 421

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

↓ 25

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c(dx^2+c)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$$\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{\ldots}}\sqrt{dx^2+c}}$$

↓ 402

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}\right.$$

↓ 413

$$b\frac{b\left(\frac{\frac{(ad(2de-5cf)+bc(2de+cf))\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(de-cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}}{3c}-\frac{(bc-ad)(de-cf)x\sqrt{bx^2+a}}{3c\left(dx^2+c\right)^{3/2}}}{(de-cf)^2}-\right.}{f}$$

input
```
Int[(a + b*x^2)^(7/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5212 vs. 2(1171) = 2342.

Time = 19.57 (sec) , antiderivative size = 5213, normalized size of antiderivative = 4.27

method	result	size
elliptic	Expression too large to display	5213
default	Expression too large to display	16403

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima
")
```

output
```
integrate((b*x^2 + a)^(7/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(7/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{7}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(7/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^3,x)
```

3.194 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 669

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx \\
&= \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4e(be-af)(de-cf)\,(e+fx^2)^2} - \frac{3f(be(3de-2cf)-af(2de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\
&\quad + \frac{3\sqrt{c}\sqrt{d}f(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}(af(8d^2e^2-8cdef+3c^2f^2)-be(8d^2e^2-9cdef+4c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{8ae^2(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{c^{3/2}f(a^2f^2(8d^2e^2-8cdef+3c^2f^2)-2abef(10d^2e^2-11cdef+4c^2f^2)+b^2e^2(15d^2e^2-20cdef+8c^2}{8a\sqrt{d}e^3(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx}}
\end{aligned}$$

output
```
1/4*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e
)^2-3/8*f*(b*e*(-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*
f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+3/8*c^(1/2)*d^(1/2)*f*(b*e*(
-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2
)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(1/2)*d^(1/2)*(a*f*(3*c^2
*f^2-8*c*d*e*f+8*d^2*e^2)-b*e*(4*c^2*f^2-9*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(
1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/(-
a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*
c^(3/2)*f*(a^2*f^2*(3*c^2*f^2-8*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(4*c^2*f^2-11
*c*d*e*f+10*d^2*e^2)+b^2*e^2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^
(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)
^(1/2))/a/d^(1/2)/e^3/(-a*f+b*e)^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.19 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+3(be(3de-2cf)+af(-2de+cf))\,(e+fx^2))-i}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f) +
3*(b*e*(3*d*e - 2*c*f) + a*f*(-2*d*e + c*f))*(e + f*x^2)) - I*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(3*b*c*e*f*(a*f*(2*d*e - c*f) +
b*e*(-3*d*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e
*(d*e - c*f)*(b*e*(7*d*e - 6*c*f) + a*f*(-4*d*e + 3*c*f))*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f + 3*c^2*f
^2) - 2*a*b*e*f*(10*d^2*e^2 - 11*c*d*e*f + 4*c^2*f^2) + b^2*e^2*(15*d^2*e^
2 - 20*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(-\frac{3f}{8e^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-ef-f^2x^2)}-\frac{3f}{16e^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,\left(\sqrt{-e}\sqrt{f}-fx\right)^2}-\frac{3f}{16e^2\sqrt{a+bx^2}\sqrt{c+d}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3f\int \frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{16e^2}-\frac{3f\int \frac{1}{\left(fx+\sqrt{-e}\sqrt{f}\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{16e^2}-\frac{f^{3/2}\int \frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{8(-e)^{3/2}}-\frac{f^{3/2}\int \frac{1}{\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{8(-e)^{3/2}}+\frac{3\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{8\sqrt{b}e^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. 2(637) = 1274.

Time = 11.13 (sec) , antiderivative size = 2616, normalized size of antiderivative = 3.91

method	result	size
elliptic	Expression too large to display	2616
default	Expression too large to display	4650

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)^2+3/8*f^2*(a*c*f^2-2*a*d*e*f-2*b*c*e*f+3*b*d*e^2)/(a*c*f^2-a*d*e*f-b*
c*e*f+b*d*e^2)^2/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+11/4/
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d-3/4*f^2*b^2/(a*c*f^2-a*d
*e*f-b*c*e*f+b*d*e^2)^2/e*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))+3/4*f^2*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*c^2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8/(a*c*f^2-a
*d*e*f-b*c*e*f+b*d*e^2)^2/e^3*f^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2+1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e
^2)^2/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a^2*d^2-7/8*b^2*d^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(a*c*f^2-a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,f^3x^{10}+ad\,f^3x^8+bc\,f^3x^8+3bde\,f^2x^8+ac\,f^3x^6+3ade\,f^2x^6+3bce\,f^2x^6+3bd\,e^2f\,x^6+3ace\,f^2x}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**3 + 3*a*c*e**2*f*x**2 + 3*
a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 + 3*a*
d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3*b*c*
e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b*d*e*
f**2*x**8 + b*d*f**3*x**10),x)
```

3.195 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 835

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \frac{f^2x\sqrt{a+bx^2}}{4e(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^2}$$
$$+\frac{f^2(be(11de-6cf)-af(8de-3cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{d}(a^2df^2(8d^2e^2+10cdef-3c^2f^2)+b^2e(8d^3e^3+13c^2def^2-6c^3f^3)-abf(16d^3e^3+13cd^2e^2f+4c^2de}{8\sqrt{c}(bc-ad)e^2(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{c}\sqrt{d}(3a^2df^2(8d^2e^2-4cdef+c^2f^2)-b^2e(8d^3e^3-32cd^2e^2f+13c^2def^2-4c^3f^3)-abf(16d^3e^3+19c}{8a(bc-ad)e^2(be-af)(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{c^{3/2}f^2(3a^2f^2(8d^2e^2-4cdef+c^2f^2)-2abef(28d^2e^2-17cdef+4c^2f^2)+b^2e^2(35d^2e^2-28cdef+8c^2f^2}{8a\sqrt{d}e^3(be-af)^2(de-cf)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/4*f^2*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e
)^2+1/8*f^2*(b*e*(-6*c*f+11*d*e)-a*f*(-3*c*f+8*d*e))*x*(b*x^2+a)^(1/2)/e^2
/(-a*f+b*e)^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*d^(1/2)*(a^2*d*f^
2*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2)+b^2*e*(-6*c^3*f^3+13*c^2*d*e*f^2+8*d^3
*e^3)-a*b*f*(-3*c^3*f^3+4*c^2*d*e*f^2+13*c*d^2*e^2*f+16*d^3*e^3))*(b*x^2+a
)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c
^(1/2)/(-a*d+b*c)/e^2/(-a*f+b*e)^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(d*x^2+c)^(1/2)-1/8*c^(1/2)*d^(1/2)*(3*a^2*d*f^2*(c^2*f^2-4*c*d*e*f+
8*d^2*e^2)-b^2*e*(-4*c^3*f^3+13*c^2*d*e*f^2-32*c*d^2*e^2*f+8*d^3*e^3)-a*b*
f*(3*c^3*f^3-8*c^2*d*e*f^2+19*c*d^2*e^2*f+16*d^3*e^3))*(b*x^2+a)^(1/2)*Inv
erseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/(-a*d+b*c)/e^2
/(-a*f+b*e)/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1
/8*c^(3/2)*f^2*(3*a^2*f^2*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(4*c^2*f
^2-17*c*d*e*f+28*d^2*e^2)+b^2*e^2*(8*c^2*f^2-28*c*d*e*f+35*d^2*e^2))*(b*x^
2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c
/a/d)^(1/2))/a/d^(1/2)/e^3/(-a*f+b*e)^2/(-c*f+d*e)^4/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.08 (sec) , antiderivative size = 595, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}ex(a+bx^2)\left(2c(bc-ad)ef^3(be-af)(de-cf)\,(c+dx^2)\right.}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*x*(a + b*x^2)*(2*c*(b*c - a*d)*e*f^3*(b*e - a*f)*(d*e - c*f
)*(c + d*x^2) + c*(b*c - a*d)*f^3*(b*e*(13*d*e - 6*c*f) + a*f*(-10*d*e + 3
*c*f))*(c + d*x^2)*(e + f*x^2) + 8*d^4*e^2*(b*e - a*f)^2*(e + f*x^2)^2)) -
 I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*e*(a^2*d*f^2
*(8*d^2*e^2 + 10*c*d*e*f - 3*c^2*f^2) + b^2*e*(8*d^3*e^3 + 13*c^2*d*e*f^2
- 6*c^3*f^3) + a*b*f*(-16*d^3*e^3 - 13*c*d^2*e^2*f - 4*c^2*d*e*f^2 + 3*c^3
*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*f*(b*e
*(d*e - c*f)*(b*e*(11*d*e - 6*c*f) + a*f*(-8*d*e + 3*c*f))*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b^2*e^2*(-35*d^2*e^2 + 28*c*d*e*f - 8*c
^2*f^2) - 3*a^2*f^2*(8*d^2*e^2 - 4*c*d*e*f + c^2*f^2) + 2*a*b*e*f*(28*d^2*
e^2 - 17*c*d*e*f + 4*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])))/(8*Sqrt[b/a]*c*(b*c - a*d)*e^3*(b*e - a*f)^2*(d*e - c
*f)^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{426}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{421}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{400}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{313}$$

$$\frac{d\left(\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{de-cf}}{de-cf}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 320

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}\right)}{de-cf}-}{de-cf}$$

$$\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 414

$$d\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\ldots\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a\ldots}\right.}{(de-cf)^2}}{de-cf}\right.}{de-cf}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 424

$$d\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\ldots\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a\ldots}\right.}{(de-cf)^2}}{de-cf}\right.}{de-\ldots}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 406

$$d\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 320

$$d\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$\Big\downarrow$ 388

$$d\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(a\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-a}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \; 313$$

$$d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx}{a(d}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \; 413$$

$$d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}$$

$\Big\downarrow$ 413

$$d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}$$

$\Big\downarrow$ 412

$$d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}$$

$\downarrow$ 433

$$d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx\right.}{a\left(dx\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\frac{f\int\left(-\frac{f^{3/2}}{8(-e)^{3/2}\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{f^{3/2}}{8(-e)^{3/2}\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{3f}{16e^2\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{de-cf}$$

$\downarrow$ 2009

$$\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(a\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(b\ldots\right.}{a\left(d\ldots\right.}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}$$

$$\frac{f\left(-\frac{\int\frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{\int\frac{1}{\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{3\int\frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf}{16e^2}-\frac{3\int\frac{\ldots}{(fx+\ldots}\ldots\right.}{de-cf}$$

input

```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 400

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3336 vs. 2(803) = 1606.

Time = 20.31 (sec) , antiderivative size = 3337, normalized size of antiderivative = 4.00

method	result	size
elliptic	Expression too large to display	3337
default	Expression too large to display	8681

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f^3/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(c*f-d*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/(f*x^2+e)^2+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*a*d^4/c/(a*d-b*c)/(c*f-d*e)^3-11/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/
e/(c*f-d*e)*a*c+3/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)
^2/e^2/(c*f-d*e)*a*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/8*
c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(c*f-d*e)*a*E
llipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/4*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^3*b
/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(c*f-d*e)*a*EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))-3/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*
c*e*f+b*d*e^2)^2/e/(c*f-d*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))+3/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.196 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 810

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3} dx = \frac{f^2x\sqrt{c+dx^2}}{4e(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)^2}$$
$$+ \frac{f^2(be(11de-8cf)-3af(2de-cf))x\sqrt{c+dx^2}}{8e^2(be-af)^2(de-cf)^2\sqrt{a+bx^2}(e+fx^2)}$$
$$- \frac{\sqrt{b}(ab^2cef^2(13de-10cf)-8b^3e^2(de-cf)^2+3a^3df^3(2de-cf)-a^2bf^2(13d^2e^2-4cdef-3c^2f^2))\sqrt{c}}{8\sqrt{a}(bc-ad)e^2(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+ \frac{\sqrt{a}\sqrt{b}(a^3df^3(4de-3cf)+ab^2ef(32d^2e^2-19cdef-12c^2f^2)-a^2bf^2(13d^2e^2-8cdef-3c^2f^2)-8b^3e^2(}{8c(bc-ad)e^2(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{}}$$
$$+ \frac{a^{3/2}f^2(a^2f^2(8d^2e^2-8cdef+3c^2f^2)-2abef(14d^2e^2-17cdef+6c^2f^2)+b^2e^2(35d^2e^2-56cdef+24c^2}{8\sqrt{b}ce^3(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/4*f^2*x*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e
)^2+1/8*f^2*(b*e*(-8*c*f+11*d*e)-3*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^2
/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)-1/8*b^(1/2)*(a*b^2*c*
e*f^2*(-10*c*f+13*d*e)-8*b^3*e^2*(-c*f+d*e)^2+3*a^3*d*f^3*(-c*f+2*d*e)-a^2
*b*f^2*(-3*c^2*f^2-4*c*d*e*f+13*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(-a*d+b*c)/e^2/(-
a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/
8*a^(1/2)*b^(1/2)*(a^3*d*f^3*(-3*c*f+4*d*e)+a*b^2*e*f*(-12*c^2*f^2-19*c*d*
e*f+32*d^2*e^2)-a^2*b*f^2*(-3*c^2*f^2-8*c*d*e*f+13*d^2*e^2)-8*b^3*e^2*(-3*
c^2*f^2+2*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/e^2/(-a*f+b*e)^4/(-c*f+d*e)/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*f^2*(a^2*f^2*(3*
c^2*f^2-8*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(6*c^2*f^2-17*c*d*e*f+14*d^2*e^2)+b
^2*e^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1
/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3
/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.51 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}ex(c+dx^2)\left(2a(bc-ad)ef^3(be-af)(de-cf)(a+\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*x*(c + d*x^2)*(2*a*(b*c - a*d)*e*f^3*(b*e - a*f)
*(d*e - c*f)*(a + b*x^2) + a*(b*c - a*d)*f^3*(b*e*(13*d*e - 10*c*f) + 3*a*
f*(-2*d*e + c*f))*(a + b*x^2)*(e + f*x^2) - 8*b^4*e^2*(d*e - c*f)^2*(e + f
*x^2)^2)) + I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e
*(8*b^3*e^2*(d*e - c*f)^2 + 3*a^3*d*f^3*(-2*d*e + c*f) + a*b^2*c*e*f^2*(-1
3*d*e + 10*c*f) + a^2*b*f^2*(13*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2))*Elliptic
E[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(d*e - c*f)*(a*b
*e*f*(11*d*e - 10*c*f) + 8*b^2*e^2*(d*e - c*f) + a^2*f^2*(-4*d*e + 3*c*f))
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + a*f*(b^2*e^2*(-35*d^2*e^
2 + 56*c*d*e*f - 24*c^2*f^2) + a^2*f^2*(-8*d^2*e^2 + 8*c*d*e*f - 3*c^2*f^2
) + 2*a*b*e*f*(14*d^2*e^2 - 17*c*d*e*f + 6*c^2*f^2))*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))))/(8*b*(b*c - a*d)*e^3*(b*e - a*
f)^3*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int-\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\ldots}}{be-af}\ldots\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(be-af)^2}\right)}{be-af}-\ldots\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}(\ldots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(\ldots}{a(\ldots}}}\ldots\right)}{(be-af)^2}\ldots\right)}{be-af}\ldots\right)$$

$$\overline{\qquad\qquad be-af\qquad\qquad}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 424

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}(\ldots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(\ldots}{a(\ldots}}}\ldots\right)}{(be-af)^2}\ldots\right)}{be-af}\ldots\right)$$

$$\overline{\qquad\qquad be-\ldots}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 406

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(\right.}{a\left(\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\Big\downarrow$ 320

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(\right.}{a\left(\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\Big\downarrow$ 388

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(}{a(}}}\right.}{(be-af)^2}}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{(be-af)^2}}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.}{\cdots}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 413

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.}{\cdots}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 412

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\big\downarrow$ 433

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\left(-\frac{f^{3/2}}{8(-e)^{3/2}\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{f^{3/2}}{8(-e)^{3/2}\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{3f}{16e^2\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{be-af}$$

$\big\downarrow$ 2009

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\left(-\frac{\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{\int\frac{1}{(fx+\sqrt{-e}\sqrt{f})^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{3\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf}{16e^2}-\frac{3\int\frac{}{(fx+}}{}\right.}{be-af}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3116 vs. 2(778) = 1556.

Time = 20.79 (sec) , antiderivative size = 3117, normalized size of antiderivative = 3.85

method	result	size
elliptic	Expression too large to display	3117
default	Expression too large to display	9458

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(3/8*c^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*e)*a*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(b*d*x^2+b*c)*b^3/a/(a*d-b*c)*x/(
a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+17/4/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)^2/(a*f-b*e)*f^3/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1
/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))*b^3/a/(a*f-b*e)^3-13/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^2*b^2/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-5/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d
*e^2)^2/e/(a*f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/4
*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)*El
lipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/8*f^3*(3*a*c*f^2-6*a*d*
e*f-10*b*c*e*f+13*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*
e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/4*f^3/(a*c*f^2-a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.197 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1130

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/8*b*(a^3*d*f^3*(-3*c*f+8*d*e)+5*a*b^2*c*e*f^2*(-2*c*f+3*d*e)-8*b^3*e^2*
(-c*f+d*e)^2-a^2*b*f^2*(-3*c^2*f^2-2*c*d*e*f+15*d^2*e^2))*x/a/(-a*d+b*c)/e
^2/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/4*f^2*x/e/(
-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2+1/8*f^2*(
b*e*(-8*c*f+13*d*e)-a*f*(-3*c*f+8*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(b
*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/8*d^(1/2)*(8*b^4*c*e^2*(-c*f+d*e
)^3-a^4*d^2*f^3*(-3*c^2*f^2+10*c*d*e*f+8*d^2*e^2)+a^3*b*d*f^2*(-6*c^3*f^3+
10*c^2*d*e*f^2+17*c*d^2*e^2*f+24*d^3*e^3)+a*b^3*e*(-10*c^4*f^4+17*c^3*d*e*
f^3+8*d^4*e^4)-a^2*b^2*f*(-3*c^4*f^4-10*c^3*d*e*f^3+34*c^2*d^2*e^2*f^2+24*
d^4*e^4))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/a/c^(1/2)/(-a*d+b*c)^2/e^2/(-a*f+b*e)^3/(-c*f+d*e)^3/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(1/2)*d^(1/2)*(3*a^3*d^2
*f^3*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)-a^2*b*d*f^2*(6*c^3*f^3-16*c^2*d*e*f^2+2
3*c*d^2*e^2*f+32*d^3*e^3)+b^3*e*(-8*c^4*f^4+17*c^3*d*e*f^3-40*c*d^3*e^3*f+
16*d^4*e^4)-a*b^2*f*(-3*c^4*f^4-4*c^3*d*e*f^3+34*c^2*d^2*e^2*f^2-80*c*d^3*
e^3*f+8*d^4*e^4))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)
),(1-b*c/a/d)^(1/2))/a/(-a*d+b*c)^2/e^2/(-a*f+b*e)^2/(-c*f+d*e)^4/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-3/8*c^(3/2)*f^3*(a^2*f^2*(c^2*f^2-
4*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(2*c^2*f^2-9*c*d*e*f+12*d^2*e^2)+b^2*e^2*(8
*c^2*f^2-24*c*d*e*f+21*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.37 (sec) , antiderivative size = 805, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}ex\left(2ac(bc-ad)^2ef^4(-be+af)(-de+cf)\,(a+bx^2\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*e*x*(2*a*c*(b*c - a*d)^2*e*f^4*(-(b*e) + a*f)*(-(d*e
) + c*f)*(a + b*x^2)*(c + d*x^2) + a*c*(b*c - a*d)^2*f^4*(b*e*(17*d*e - 10
*c*f) + a*f*(-10*d*e + 3*c*f))*(a + b*x^2)*(c + d*x^2)*(e + f*x^2) + 8*a*d
^5*e^2*(b*e - a*f)^3*(a + b*x^2)*(e + f*x^2)^2 + 8*b^5*c*e^2*(d*e - c*f)^3
*(c + d*x^2)*(e + f*x^2)^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
(e + f*x^2)^2*(-(b*e*(-8*b^4*c*e^2*(-(d*e) + c*f)^3 + a^4*d^2*f^3*(-8*d^2*
e^2 - 10*c*d*e*f + 3*c^2*f^2) + a^3*b*d*f^2*(24*d^3*e^3 + 17*c*d^2*e^2*f +
 10*c^2*d*e*f^2 - 6*c^3*f^3) + a*b^3*e*(8*d^4*e^4 + 17*c^3*d*e*f^3 - 10*c^
4*f^4) + a^2*b^2*f*(-24*d^4*e^4 - 34*c^2*d^2*e^2*f^2 + 10*c^3*d*e*f^3 + 3*
c^4*f^4))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) - (b*c - a*d)*(-
(b*e*(d*e - c*f)*(8*b^3*e^2*(d*e - c*f)^2 + 5*a*b^2*c*e*f^2*(-3*d*e + 2*c*
f) + a^3*d*f^3*(-8*d*e + 3*c*f) + a^2*b*f^2*(15*d^2*e^2 - 2*c*d*e*f - 3*c^
2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + 3*a*(-(b*c) + a*
d)*f^2*(a^2*f^2*(8*d^2*e^2 - 4*c*d*e*f + c^2*f^2) - 2*a*b*e*f*(12*d^2*e^2
- 9*c*d*e*f + 2*c^2*f^2) + b^2*e^2*(21*d^2*e^2 - 24*c*d*e*f + 8*c^2*f^2))*
EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))))/(8*b*c*(b
*c - a*d)^2*e^3*(b*e - a*f)^3*(d*e - c*f)^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 426$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \; 426$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int\frac{a(bde+bcf-2adf)-bd(be-af)x^2}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}}{be-af}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}\right)}{be-af}$$

$\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(-2a^2df+abde+b^2ce)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{bc-ad}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}}{be-af}\right)}{be-af}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}\right)}{be-af}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{ab(-3adf+bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2df+abde+b^2ce\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int -}{(}}{be-af}\right.$$

$$\frac{}{be-af}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$\downarrow$ 320

$$
b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}\left(-2a^2\cdots\right.}{\sqrt{c}\sqrt{\cdots}}}{a(bc-ad)}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.
$$

$$
\frac{\cdots}{be-af}\cdots
$$

$$
\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}
$$

$\downarrow$ 416

$$b\left(\frac{b\left(\frac{f^2\left(\frac{d\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{de-cf}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{a(b\cdots}\right.}{(be-af)^2}}{be-af}\right.}{be-af}$$

$$\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}(-3adf+bcf+2bde)\,\text{EllipticF}\left(\text{ar}\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+b\cdots}{a(c+c\cdots}}}\cdots\right.}{\cdots}\cdots\right)}{be-af}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{de-cf}\right)}{be-af}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{b\left(\frac{f^2\left(\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\right)}{(be-af)^2}+\frac{b\left(\frac{bx(be-af)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{b\sqrt{c}\sqrt{a+bx^2}(\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}\right.}{b\cdots}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$\big\downarrow$ 426

$$\frac{b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right)}{\ldots}\right)}{be-af}\ldots\right)}{\ldots}$$

$$\frac{f\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow \text{ 421}$$

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{be-af}\cdots\right)$$

$$\frac{f\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$$\Big\downarrow\ 25$$

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}}{\cdots}\right)}{\cdots}\cdots\right)}{be-af}\cdots\right)$$

$$\frac{f\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{de-cf}\right)}{be-af}$$

$\downarrow$ 400

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\cdots\right)}{\cdots}\cdots}{be-af}\cdots\right)$$

$$\frac{\cdots}{\cdots}$$

$$\frac{f\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\cdots\right)}{be-af}$$

↓ 313

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}\cdots\right)$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx\,f^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c\cdots}}}{de-cf}\cdots\right)}{de-cf}\cdots\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}\cdots$$

$$\left(f\frac{d\left(d\frac{d\left(\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)}{de-cf}-\cdots\right)}{de-cf}\cdots\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\cdots}{\sqrt{c}(bc-ad)\sqrt{\cdots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}\right.$$

↓ 424

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\cdots}{\sqrt{c}(bc-ad)\sqrt{\cdots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de\cdots}\right.$$

↓ 406

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\ldots\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right.}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\cdots}{\sqrt{c}(bc-ad)\sqrt{\cdots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\cdots}\right.$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right)}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\cdots}{\sqrt{c}(bc-ad)\sqrt{\cdots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\cdots}{\cdots}\right.}{\cdots}\right)}{be-af}\cdots$$

$$f\left(d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\mathrm{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\cdots}{c\cdots}}}\right.}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)\cdots$$

↓ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}}{be-af}\ldots\right.$$

$$f\left(d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\right.}{de-cf}\ldots\right.\right.$$

↓ 413

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right)}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{\ldots}\right.$$

$\downarrow$ 412

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}\right.}{be-af}\right.$$

$$f\left(d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.\right.$$

↓ 433

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}\right.}{\ldots}}{be-af}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\right.}{de-cf}\right.\right.$$

↓ 2009

$$b\left(\frac{b\left(\frac{\left(\frac{\sqrt{d}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{a^{3/2}f\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{b\sqrt{c}(2bde+bc\ldots}{\ldots}}{\ldots}\right.}{\ldots}\right)}{be-af}\ldots\right.$$

$$\overline{f\left(\frac{d\left(\frac{d\,c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\right.}{(de-cf)^2}\ldots\right)}{de-cf}\ldots\right.}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6154 vs. $2(1092) = 2184$.

Time = 22.86 (sec) , antiderivative size = 6155, normalized size of antiderivative = 5.45

method	result	size
elliptic	Expression too large to display	6155
default	Expression too large to display	14829

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.198 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)^3} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 1161

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3} \, dx = \text{Too large to display}$$

output
```
-1/24*b*(9*a*b^2*c*e*f^2*(-4*c*f+5*d*e)-8*b^3*e^2*(-c*f+d*e)^2+9*a^3*d*f^3
*(-c*f+2*d*e)-9*a^2*b*f^2*(-c^2*f^2-2*c*d*e*f+5*d^2*e^2))*x*(d*x^2+c)^(1/2
)/a/(-a*d+b*c)/e^2/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(3/2)+1/4*f^2*x*(d*
x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(3/2)/(f*x^2+e)^2+1/8*f^2*(
b*e*(-10*c*f+13*d*e)-3*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e)^
2/(-c*f+d*e)^2/(b*x^2+a)^(3/2)/(f*x^2+e)+1/24*b^(1/2)*(16*b^5*c*e^3*(-c*f+
d*e)^2-9*a^5*d^2*f^4*(-c*f+2*d*e)-8*a*b^4*e^2*(-c*f+d*e)^2*(11*c*f+4*d*e)+
3*a^4*b*d*f^3*(-6*c^2*f^2-2*c*d*e*f+17*d^2*e^2)-3*a^3*b^2*c*f^3*(-3*c^2*f^
2-22*c*d*e*f+34*d^2*e^2)+a^2*b^3*e*f*(-42*c^3*f^3+155*c^2*d*e*f^2-208*c*d^
2*e^2*f+104*d^3*e^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/(-a*d+b*c)^2/e^2/(-a*f+b*e)^4/(-c*f+d
*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/24*b^(1/2)*(3*a^5*
d^2*f^4*(-3*c*f+4*d*e)-8*b^5*c*d*e^4*(-c*f+d*e)-3*a^4*b*d*f^3*(-6*c^2*f^2-
8*c*d*e*f+17*d^2*e^2)-a^2*b^3*e*f*(-48*c^3*f^3-317*c^2*d*e*f^2+248*c*d^2*e
^2*f+120*d^3*e^3)+8*a*b^4*e^2*(-18*c^3*f^3+7*c^2*d*e*f^2+8*c*d^2*e^2*f+3*d
^3*e^3)+3*a^3*b^2*f^2*(-3*c^3*f^3-28*c^2*d*e*f^2-46*c*d^2*e^2*f+80*d^3*e^3
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/a^(1/2)/c/(-a*d+b*c)^2/e^2/(-a*f+b*e)^5/(-c*f+d*e)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*f^3*(a^2*f^2*(3*c^2*f^2-8*c*d*e
*f+8*d^2*e^2)-2*a*b*e*f*(8*c^2*f^2-23*c*d*e*f+18*d^2*e^2)+3*b^2*e^2*(16...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.27 (sec) , antiderivative size = 893, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ex(c+dx^2)\left(6a^2(bc-ad)^2ef^4(-be+af)(-de+cf)\,(a+bx\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*x*(c + d*x^2)*(6*a^2*(b*c - a*d)^2*e*f^4*(-(b*e) + a*f)*(-(d*
e) + c*f)*(a + b*x^2)^2 + 3*a^2*(b*c - a*d)^2*f^4*(b*e*(17*d*e - 14*c*f) +
 3*a*f*(-2*d*e + c*f))*(a + b*x^2)^2*(e + f*x^2) + 8*a*b^4*(-(b*c) + a*d)*
e^2*(-(b*e) + a*f)*(d*e - c*f)^2*(e + f*x^2)^2 + 8*b^4*e^2*(d*e - c*f)^2*(
2*b^2*c*e + 13*a^2*d*f - a*b*(4*d*e + 11*c*f))*(a + b*x^2)*(e + f*x^2)^2)
- I*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b
*c*e*(16*b^5*c*e^3*(d*e - c*f)^2 + 9*a^5*d^2*f^4*(-2*d*e + c*f) - 8*a*b^4*
e^2*(d*e - c*f)^2*(4*d*e + 11*c*f) + 3*a^4*b*d*f^3*(17*d^2*e^2 - 2*c*d*e*f
 - 6*c^2*f^2) + 3*a^3*b^2*c*f^3*(-34*d^2*e^2 + 22*c*d*e*f + 3*c^2*f^2) + a
^2*b^3*e*f*(104*d^3*e^3 - 208*c*d^2*e^2*f + 155*c^2*d*e*f^2 - 42*c^3*f^3))
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + (b*c - a*d)*(b*e*(-(d*e
) + c*f)*(3*a^4*d*f^3*(4*d*e - 3*c*f) + 16*b^4*c*e^3*(-(d*e) + c*f) + 8*a*
b^3*e^2*(3*d^2*e^2 + 8*c*d*e*f - 11*c^2*f^2) + 3*a^3*b*f^2*(-15*d^2*e^2 +
10*c*d*e*f + 3*c^2*f^2) - 3*a^2*b^2*e*f*(32*d^2*e^2 - 47*c*d*e*f + 14*c^2*
f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*a^2*(-(b*c) + a*d
)*f^2*(a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f + 3*c^2*f^2) - 2*a*b*e*f*(18*d^2*e^2
 - 23*c*d*e*f + 8*c^2*f^2) + 3*b^2*e^2*(21*d^2*e^2 - 36*c*d*e*f + 16*c^2*f
^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(24*a
^2*Sqrt[b/a]*(b*c - a*d)^2*e^3*(b*e - a*f)^4*(d*e - c*f)^2*(a + b*x^2)^(3/
2)*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 421

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(\cdots\right.}}{be-af}\cdots\right.}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(f\cdots\right.}}{be-af}\cdots\right.}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \ 400$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{b\left(7a^2df-5abcf-4abde+2b^2ce\right)\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int\frac{\cdots}{\sqrt{bx^2+\cdots}}\cdots}{\cdots}}{be-af}\cdots\right)}{be-af}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 313

$$b\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{d\left(6a^2df-ab(4cf+3de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx\sqrt{c+dx^2}(be-af)}{3a\left(a+bx^2\right)^{3/2}(bc-ad}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-af}\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2\right.}{a\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}}{(be-af)^2}}{be-af}}{be-af}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\Big\downarrow$ 413

$$b\left(\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf\cdots\right.}{a\sqrt{c+d\cdots}}}{3a(bc-ad)}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right)}{be-af}\cdots$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 413$$

$$\frac{b\left(\frac{b\left(\frac{f^2\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)^2}+\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2d\cdots\right.}{\cdots}}{3a(bc-ad)}\cdots\right)}{(be-af)^2\cdots}\right)}{be-af}\cdots\right)}{be-af\cdots}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 412$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-af}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 426$$

$$b\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}\left(7a^2df-5abcf-4abde+2b^2ce\right)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(6a^2df-ab(4cf+3de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3a(bc-ad)}}{(be-af)^2}}{be-af}}{}$$

$$\frac{f\left(\frac{b\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\cdots\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}\cdots}\cdots\right)}{(be\cdots}\cdots\right)}{be-af}\cdots\right)$$

$$\frac{f\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int-\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{(be}\right)}{be-af}$$

$$\frac{f\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(be-af)^2}+\frac{b\int\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 400

$$\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}$$

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\cdots\right.}{(be\cdots}\right.}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int\frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(bde+bcf-2adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\cdots\right.$$

$$be-af$$

↓ 313

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{}\right.}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{\left(b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx\,f^2}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{(bde+bcf-2adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}\right)-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\right.}}{be-af}\right.}{be-af}\right.$$

↓ 320

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{}\right)}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(be-af)^2}\right)}{be-af}\right)-}{be-af}\right.$$

↓ 414

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right.}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-af}\right.$$

$\downarrow$ 424

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right)}{be-af}$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}\right)}{be-af}\right)}{be}$$

↓ 406

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}\,x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\cdots\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\cdots\right.}{(be\cdots}\right.}{be-af}\cdots\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\cdots}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\cdots\right)}{a\left(dx^2+c\cdots\right)}}}\cdots\right.}{(be-af)^2}\right.}{be-af}\cdots\right.}{\cdots}\right.\cdots$$

↓ 320

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.}{}\right.$$

$$\downarrow 388$$

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{}\right)}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}}{be-af}\right.}{}\right.$$

↓ 313

$$b\left(b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right.\right.$$

$$\overline{\qquad be-af \qquad}$$

$$f\left(b\left(b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}\right.\right.\right.$$

$$\overline{\qquad be-af \qquad}$$

↓ 413

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.\right.$$

$$\Big\downarrow \text{413}$$

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\mathrm{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}}\right.}{(be-af)^2}\right)}{be-af}\right.\right.$$

↓ 412

$$b\left(\frac{b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}}{}\right.}{(be}\right.}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right.}{be-af}\right.}{}\right.$$

↓ 433

$$\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}$$

$$b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\right.$$

$$b\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{\cdots}{(be\cdots}\right.$$

$$b\,\frac{\cdots}{be-af}$$

$$b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\cdots}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\right.$$

$$b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{\cdots}{(be-af)^2}\right.$$

$$b\,\frac{\cdots}{be-af}$$

$$f\,\frac{\cdots}{\cdots}$$

↓ 2009

$$b\left(\frac{b\left(\frac{b\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)f^2}{\sqrt{b}e(be-af)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}+\frac{b\left(\frac{b(be-af)\sqrt{dx^2+c}x}{3a(bc-ad)\left(bx^2+a\right)^{3/2}}+\frac{\sqrt{b}\left(7dfa^2-4bdea-5bcfa+2b^2ce\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}\right.}{(be}\right)}{be-af}\right.$$

$$f\left(\frac{b\left(\frac{b\,a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}\right.}{(be-af)^2}\right)}{be-af}\right.$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4029 vs. 2(1123) = 2246.

Time = 22.76 (sec) , antiderivative size = 4030, normalized size of antiderivative = 3.47

method	result	size
elliptic	Expression too large to display	4030
default	Expression too large to display	23079

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-7/4*c^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*f^4*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)^2*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+7/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^4*b^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-f^5/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*e
)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))*a^2*c*d-2*f^5/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*e)^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*
c^2+1/8*f^4*(3*a*c*f^2-6*a*d*e*f-14*b*c*e*f+17*b*d*e^2)/(a*c*f^2-a*d*e*f-b
*c*e*f+b*d*e^2)^2/e^2/(a*f-b*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f
*x^2+e)+23/4*f^4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)^2/e/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d
+1/4*f^4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(a*f-b*e)^2*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)^2+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^3}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.199 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1572

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/24*b*(3*a*b^2*c*e*f^2*(-12*c*f+17*d*e)+3*a^3*d*f^3*(-3*c*f+8*d*e)-8*b^3
*e^2*(-c*f+d*e)^2-3*a^2*b*f^2*(-3*c^2*f^2-4*c*d*e*f+17*d^2*e^2))*x/a/(-a*d
+b*c)/e^2/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)-1/24*b
*(3*a^5*d^2*f^4*(-3*c*f+8*d*e)-16*b^5*c*e^3*(-c*f+d*e)^2+8*a*b^4*e^2*(-c*f
+d*e)^2*(11*c*f+6*d*e)-3*a^4*b*d*f^3*(-6*c^2*f^2+2*c*d*e*f+19*d^2*e^2)+3*a
^3*b^2*c*f^3*(-3*c^2*f^2-20*c*d*e*f+38*d^2*e^2)-3*a^2*b^3*e*f*(-14*c^3*f^3
+59*c^2*d*e*f^2-80*c*d^2*e^2*f+40*d^3*e^3))*x/a^2/(-a*d+b*c)^2/e^2/(-a*f+b
*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/4*f^2*x/e/(-a*f+b*e)/
(-c*f+d*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2-1/8*f^2*(a*f*(-3*c*
f+8*d*e)-5*b*e*(-2*c*f+3*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(
3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/24*d^(1/2)*(16*b^6*c^2*e^3*(-c*f+d*e)^3-8
*a*b^5*c*e^2*(-c*f+d*e)^3*(11*c*f+7*d*e)-3*a^6*d^3*f^4*(-3*c^2*f^2+10*c*d*
e*f+8*d^2*e^2)+3*a^5*b*d^2*f^3*(-9*c^3*f^3+16*c^2*d*e*f^2+21*c*d^2*e^2*f+3
2*d^3*e^3)-9*a^4*b^2*d*f^2*(-3*c^4*f^4-4*c^3*d*e*f^3+21*c^2*d^2*e^2*f^2+16
*d^4*e^4)-a^2*b^4*e*(-42*c^5*f^5+191*c^4*d*e*f^4-384*c^3*d^2*e^2*f^3+384*c
^2*d^3*e^3*f^2-128*c*d^4*e^4*f+24*d^5*e^5)+3*a^3*b^3*f*(-3*c^5*f^5-32*c^4*
d*e*f^4+63*c^3*d^2*e^2*f^3+32*d^5*e^5))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*
x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^3/e^
2/(-a*f+b*e)^4/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2
)-1/24*c^(1/2)*d^(1/2)*(8*b^5*c*e^2*(-c*f+d*e)^4+9*a^5*d^3*f^4*(c^2*f^2...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.74 (sec) , antiderivative size = 9241, normalized size of antiderivative = 5.88

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{5/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{5/2}\left(dx^2+c\right)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{6dfa^2-b(3de+5cf)a+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{}{\left(bx^2+a\right.}}{}\right.}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-3bdea-5bcfa+3bd(be-af)x^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}dx}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{}{\left(bx^2+a\right)^{3}}}{}\right)}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\Big\downarrow$ 402

$$b\left(\frac{b\left(\frac{b\left(\frac{\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{\int -\frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-af}\right.$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{d\left(b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}}{be-af}}{}\right.$$

$$be-af$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\frac{d\int \frac{b\left(9dfa^2-6bdea-5bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(3de+2cf)a+b^2ce\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{a(bc-ad)}+\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}}{3a(bc-ad)}+\frac{bx(be-af)}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}+\cdots}{be-af}\cdots}{be-af}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\big\downarrow$ 400

$$b\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{a(bc-ad)}+\frac{bx\left(9a^2df-5\cdots\right.}{a\sqrt{a+bx\cdots}}}{3a(bc-ad)}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right)}{be-a\cdots}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 313$$

$$b\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{ab\left(15a^2df-7abcf-9abde+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right.}{a(bc-ad)}}{3a(bc-ad)}}{(be-af)^2}}{be-af}\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5cf+7de)+2b^3c^2e\right)E\left(\text{arct}\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{}}{3a(bc-a}\right.}{}\right.}{be-af}\right.}{}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(\cdots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(\cdots}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 25

$$b\left(b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\frac{d\left(\frac{\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(3de-7cf)-ab^2c(5c\cdots\right.}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 400

$$b\left(\frac{b\left(\frac{f^2\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{(be-af)^2}+\frac{b\left(\frac{bx\left(9a^2df-5abcf-6abde+2b^2ce\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}+\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\downarrow 313$$

$$b\left(\frac{b\left(\frac{\left(\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{(bde-2bcf+adf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-a\cdots}{3a(bc-ad)\left(bx^2+\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{f\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 320

$$b\left(b\left(\frac{\frac{\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{(de-cf)^2}\right)f^2}{(be-af)^2}\right.\right.$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$\downarrow$ 414

$$b\left(\frac{b\left(\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(\ldots\right.}{a\left(\ldots\right.}}}\ldots\right.}{(de-cf)^2}\ldots\right.}{(be-af)^2}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

$$\frac{f\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^3}dx}{be-af}\right)}{be-af}$$

$$\Big\downarrow \; 426$$

$$
\frac{b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\cdots\right)}{(de-cf)^2}\cdots}{(be-af)^2}\cdots\right)}{\cdots}\cdots
$$

$$
\frac{f\left(\frac{b\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{d\cdots}}\cdots}{de\cdots}\cdots\right.}{be-af}\cdots\right.}{be-af}
$$

↓ 421

$$b\left(b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E(}{\sqrt{c}(bc-ad)\sqrt{\frac{c(}{a(}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.\right.$$

$$\frac{f\left(\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int-\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(}}{de-cf}\right.}{}\right.}{be-af}$$

$\Big\downarrow$ 25

$$b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}\,e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{(be-af)^2}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{f\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dx\,f^2}{(be-af)^2}+\frac{b\int\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}(dx^2+c)^{3/2}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\right)}{be-af}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx\cdots}}{de-cf}\cdots\right.}{\cdots}\cdots\right.}{be-af}$$

↓ 402

$$b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E(\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right)}{(de-cf)^2}\cdots}{(be-af)^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$f\left(\frac{b\left(\frac{b\left(\frac{b\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}(fx^2+e)}dxf^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\int\frac{a(bde+bcf-2adf)-bd(be-af)x^2}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{a(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{be-af}\cdots\right)}{be-af}\cdots\right.$$

↓ 400

$$b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\right.}{(de-cf)^2}}{(be-af)^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\left(b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx\,f^2}{(be-af)^2}+\frac{b\left(\frac{b(be-af)x}{a(bc-ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{\frac{ab(2bde+bcf-3adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d\left(-2dfa^2+bdea+b^2ce\right)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}\cdots}{bc-ad}}{a(bc-ad)}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.\right.$$

↓ 313

$$b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\Big(}{\sqrt{c}(bc-ad)\sqrt{\frac{c(}{a(}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.}{}\right.$$

$$\frac{ab(2bde+bcf-3adf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\left(-2dfa^2+bdea+b^2ce\right)\sqrt{bx^2+a}E\Big(}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}}$$

↓ 320

$$b\left(b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\right.}{(de-cf)^2}}{(be-af)^2}\right.\right.$$

$$\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}-\frac{\sqrt{d}\left(-2dfa^2+bde\cdots\right.}{\sqrt{c}\cdots}$$

↓ 416

$$b\left(b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c\left(\cdots\right.}{a\left(\cdots\right.}}}\cdots\right.}{(de-cf)^2}\cdots\right.}{(be-af)^2}\cdots$$

$$\cdots\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\cdots\right.}{\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{\cdots}}\sqrt{dx^2+c}}\cdots$$

↓ 313

$$b\left(\frac{b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{(de-cf)^2}\cdots}{(be-af)^2}\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{b\sqrt{c}(2bde+bcf-3adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\cdots\right.}{\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{\cdots}}\cdots}\cdots$$

↓ 414

$$b\left(b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right)}{(de-cf)^2}\cdots}{(be-af)^2}\cdots\right.\right.$$

$$\left(\left(\left(\cdots\right.\right.\right.\quad\left(\cdots\right.\quad\frac{b\sqrt{c}(2bde-\cdots}{\cdots}$$

↓ 426

$$b\left(b\left(\frac{\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\cdots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\cdots}{a(\cdots}}}\cdots\right.}{(de-cf)^2}\cdots}{(be-af)^2}\cdots\right.}{\cdots}\cdots\right.$$

$$\left(\left(\left(\cdots\right.\right.\right.\qquad \frac{b\sqrt{c}(2bde-\cdots}{\cdots}$$

$$\Big\downarrow \text{421}$$

$$b\left(\frac{b\left(\frac{\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\left(\ldots\right.}{\sqrt{c}(bc-ad)\sqrt{\frac{c(\ldots}{a(\ldots}}}\right.}{(de-cf)^2}\right.}{(be-af)^2}\right.}{\ldots}\right.$$

$$\left(\left(\left(\ldots\right.\right.\right. \qquad \frac{b\sqrt{c}(2bde-\ldots}{\ldots}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4747 vs. 2(1528) = 3056.

Time = 25.18 (sec) , antiderivative size = 4748, normalized size of antiderivative = 3.02

method	result	size
elliptic	Expression too large to display	4748
default	Expression too large to display	33380

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2*f^4/(a*c*f^2-a*d*
e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)^2/(c*f-d*e)*a+33/8/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d*f^4/(a*c*f^2-a*d*e*f-b*c*e*f
+b*d*e^2)^2/(a*f-b*e)^2/(c*f-d*e)*c-19/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)
^2*e/(a*f-b*e)^2/(c*f-d*e)-11/3*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^6/(a*d-b*c)^3/a/(a*f-b*e)
^4*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-7/3*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d
*b^6/(a*d-b*c)^3/a/(a*f-b*e)^4*e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))+2/3*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^7/(a*d-b*c)^3/a^2/(a*f-b*e)^4*e*EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-21/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^4*b^2/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)^2/(c*f-d*e)*EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+1/8*f^5*(3*a*c*f^2-10*a*d*e*f-14*b*c*e*f+2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^3,x)
```

3.200 $\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx &= ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3\\&\quad+\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5\\&\quad+\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7\\&\quad+\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9\\&\quad+\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}\end{aligned}$$

output
```
a*c*e^4*x+1/3*e^3*(4*a*c*f+a*d*e+b*c*e)*x^3+1/5*e^2*(2*a*f*(3*c*f+2*d*e)+b
*e*(4*c*f+d*e))*x^5+2/7*e*f*(a*f*(2*c*f+3*d*e)+b*e*(3*c*f+2*d*e))*x^7+1/9*
f^2*(a*f*(c*f+4*d*e)+2*b*e*(2*c*f+3*d*e))*x^9+1/11*f^3*(a*d*f+b*c*f+4*b*d*
e)*x^11+1/13*b*d*f^4*x^13
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3 +\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5 +\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7 +\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9 +\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

output
```
a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3
*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d
*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9
)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx$$

$$\Big\downarrow \; 396$$

$$\int \left(e^3x^2(4acf+ade+bce)+e^2x^4(2af(3cf+2de)+be(4cf+de))+f^3x^{10}(adf+bcf+4bde)+f^2x^8(af(cf+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

input

```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

output

```
a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3
*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d
*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9
)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Defintions of rubi rules used

rule 396

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

method	result
default	$\frac{bd\,f^4x^{13}}{13}+\frac{((ad+bc)f^4+4bde\,f^3)x^{11}}{11}+\frac{(ac\,f^4+4(ad+bc)e\,f^3+6bd\,e^2f^2)x^9}{9}+\frac{(4ace\,f^3+6(ad+bc)e^2f^2+4bd\,e^3f)x^7}{7}+$
norman	$\frac{bd\,f^4x^{13}}{13}+\left(\frac{1}{11}ad\,f^4+\frac{1}{11}bc\,f^4+\frac{4}{11}bde\,f^3\right)x^{11}+\left(\frac{1}{9}ac\,f^4+\frac{4}{9}ade\,f^3+\frac{4}{9}bce\,f^3+\frac{2}{3}bd\,e^2f^2\right)x^9$
gosper	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$
risch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$
parallelrisch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+$
orering	$x(3465bd\,f^4x^{12}+4095ad\,f^4x^{10}+4095bc\,f^4x^{10}+16380bde\,f^3x^{10}+5005ac\,f^4x^8+20020ade\,f^3x^8+20020bce\,f^3x^8+30030bd\,e^2$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*f^4+4*(a*
d+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+b*c)*e^2*f^2+4*b*d
*e^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*f+b*d*e^4)*x^5+1/3*(4*a*c*e
^3*f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = \frac{1}{13}bdf^4x^{13} + \frac{1}{11}\left(4\,bdef^3 + (bc+ad)f^4\right)x^{11} + \frac{1}{9}\left(6\,bde^2f^2 + acf^4 + 4\,(bc+ad)ef^3\right)x^9 + \frac{2}{7}\left(2\,bde^3f + 2\,acef^3 + 3\,(bc+ad)e^2f^2\right)x^7 + ace^4x + \frac{1}{5}\left(bde^4 + 6\,ace^2f^2 + 4\,(bc+ad)e^3f\right)x^5 + \frac{1}{3}\left(4\,ace^3f + (bc+ad)e^4\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d
*e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e
*f^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f
^2 + 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.37

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx = ace^4x+\frac{bdf^4x^{13}}{13}+x^{11}\left(\frac{adf^4}{11}+\frac{bcf^4}{11}+\frac{4bdef^3}{11}\right)+x^9\left(\frac{acf^4}{9}+\frac{4adef^3}{9}+\frac{4bcef^3}{9}+\frac{2bde^2f^2}{3}\right)+x^7\cdot\left(\frac{4acef^3}{7}+\frac{6ade^2f^2}{7}+\frac{6bce^2f^2}{7}+\frac{4bde^3f}{7}\right)+x^5\cdot\left(\frac{6ace^2f^2}{5}+\frac{4ade^3f}{5}+\frac{4bce^3f}{5}+\frac{bde^4}{5}\right)+x^3\cdot\left(\frac{4ace^3f}{3}+\frac{ade^4}{3}+\frac{bce^4}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

output
```
a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11 + 4*b*d*
e*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e*f**3/9 + 2*b*d*e*
*2*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*f**2/7 + 6*b*c*e**2*f**2/7
+ 4*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*
f/5 + b*d*e**4/5) + x**3*(4*a*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx = \frac{1}{13}\,bdf^4x^{13}+\frac{1}{11}\left(4\,bdef^3+(bc+ad)f^4\right)x^{11}+\frac{1}{9}\left(6\,bde^2f^2+acf^4+4\,(bc+ad)ef^3\right)x^9+\frac{2}{7}\left(2\,bde^3f+2\,acef^3+3\,(bc+ad)e^2f^2\right)x^7+ace^4x+\frac{1}{5}\left(bde^4+6\,ace^2f^2+4\,(bc+ad)e^3f\right)x^5+\frac{1}{3}\left(4\,ace^3f+(bc+ad)e^4\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d
*e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e
*f^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f
^2 + 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = \frac{1}{13}bdf^4x^{13} + \frac{4}{11}bdef^3x^{11} + \frac{1}{11}bcf^4x^{11} + \frac{1}{11}adf^4x^{11} + \frac{2}{3}bde^2f^2x^9 + \frac{4}{9}bcef^3x^9 + \frac{4}{9}adef^3x^9 + \frac{1}{9}acf^4x^9 + \frac{4}{7}bde^3fx^7 + \frac{6}{7}bce^2f^2x^7 + \frac{6}{7}ade^2f^2x^7 + \frac{4}{7}acef^3x^7 + \frac{1}{5}bde^4x^5 + \frac{4}{5}bce^3fx^5 + \frac{4}{5}ade^3fx^5 + \frac{6}{5}ace^2f^2x^5 + \frac{1}{3}bce^4x^3 + \frac{1}{3}ade^4x^3 + \frac{4}{3}ace^3fx^3 + ace^4x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/13*b*d*f^4*x^13 + 4/11*b*d*e*f^3*x^11 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4
*x^11 + 2/3*b*d*e^2*f^2*x^9 + 4/9*b*c*e*f^3*x^9 + 4/9*a*d*e*f^3*x^9 + 1/9*
a*c*f^4*x^9 + 4/7*b*d*e^3*f*x^7 + 6/7*b*c*e^2*f^2*x^7 + 6/7*a*d*e^2*f^2*x^
7 + 4/7*a*c*e*f^3*x^7 + 1/5*b*d*e^4*x^5 + 4/5*b*c*e^3*f*x^5 + 4/5*a*d*e^3*
f*x^5 + 6/5*a*c*e^2*f^2*x^5 + 1/3*b*c*e^4*x^3 + 1/3*a*d*e^4*x^3 + 4/3*a*c*
e^3*f*x^3 + a*c*e^4*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = x^3\left(\frac{a\,d\,e^4}{3}+\frac{b\,c\,e^4}{3}+\frac{4\,a\,c\,e^3\,f}{3}\right)+x^{11}\left(\frac{a\,d\,f^4}{11}+\frac{b\,c\,f^4}{11}+\frac{4\,b\,d\,e\,f^3}{11}\right)+x^5\left(\frac{b\,d\,e^4}{5}+\frac{4\,a\,d\,e^3\,f}{5}+\frac{4\,b\,c\,e^3\,f}{5}+\frac{6\,a\,c\,e^2\,f^2}{5}\right)+x^9\left(\frac{a\,c\,f^4}{9}+\frac{4\,a\,d\,e\,f^3}{9}+\frac{4\,b\,c\,e\,f^3}{9}+\frac{2\,b\,d\,e^2\,f^2}{3}\right)+\frac{2\,e\,f\,x^7\,(2\,a\,c\,f^2+2\,b\,d\,e^2+3\,a\,d\,e\,f+3\,b\,c\,e\,f)}{7}+a\,c\,e^4\,x+\frac{b\,d\,f^4\,x^{13}}{13}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x)
```

output
```
x^3*((a*d*e^4)/3 + (b*c*e^4)/3 + (4*a*c*e^3*f)/3) + x^11*((a*d*f^4)/11 + (
b*c*f^4)/11 + (4*b*d*e*f^3)/11) + x^5*((b*d*e^4)/5 + (4*a*d*e^3*f)/5 + (4*
b*c*e^3*f)/5 + (6*a*c*e^2*f^2)/5) + x^9*((a*c*f^4)/9 + (4*a*d*e*f^3)/9 + (
4*b*c*e*f^3)/9 + (2*b*d*e^2*f^2)/3) + (2*e*f*x^7*(2*a*c*f^2 + 2*b*d*e^2 +
3*a*d*e*f + 3*b*c*e*f))/7 + a*c*e^4*x + (b*d*f^4*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = \frac{x(3465bd\,f^4x^{12}+4095ad\,f^4x^{10}+4095bc\,f^4x^{10}+16380bde\,f^3x^{10}+5005ac\,f^4x^8+20020ade\,f^3x^8+200}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x)
```

output

```
(x*(45045*a*c*e**4 + 60060*a*c*e**3*f*x**2 + 54054*a*c*e**2*f**2*x**4 + 25
740*a*c*e*f**3*x**6 + 5005*a*c*f**4*x**8 + 15015*a*d*e**4*x**2 + 36036*a*d
*e**3*f*x**4 + 38610*a*d*e**2*f**2*x**6 + 20020*a*d*e*f**3*x**8 + 4095*a*d
*f**4*x**10 + 15015*b*c*e**4*x**2 + 36036*b*c*e**3*f*x**4 + 38610*b*c*e**2
*f**2*x**6 + 20020*b*c*e*f**3*x**8 + 4095*b*c*f**4*x**10 + 9009*b*d*e**4*x
**4 + 25740*b*d*e**3*f*x**6 + 30030*b*d*e**2*f**2*x**8 + 16380*b*d*e*f**3*
x**10 + 3465*b*d*f**4*x**12))/45045
```

3.201 $\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = {} & ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3 \\ & +\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5 \\ & +\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7 \\ & +\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}\end{aligned}$$

output
```
a*c*e^3*x+1/3*e^2*(3*a*c*f+a*d*e+b*c*e)*x^3+1/5*e*(3*a*f*(c*f+d*e)+b*e*(3*
c*f+d*e))*x^5+1/7*f*(3*b*e*(c*f+d*e)+a*f*(c*f+3*d*e))*x^7+1/9*f^2*(a*d*f+b
*c*f+3*b*d*e)*x^9+1/11*b*d*f^3*x^11
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3+\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5+\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7+\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f)
+ b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x
^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx$$

$$\Big\downarrow \text{396}$$

$$\int (e^2x^2(3acf+ade+bce)+f^2x^8(adf+bcf+3bde)+fx^6(af(cf+3de)+3be(cf+de))+ex^4(3af(cf+de)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

input

```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output

```
a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f)
+ b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x
^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Defintions of rubi rules used

rule 396

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

method	result
default	$\frac{bd\,f^3x^{11}}{11}+\frac{((ad+bc)f^3+3bde\,f^2)x^9}{9}+\frac{(ac\,f^3+3(ad+bc)e\,f^2+3bd\,e^2f)x^7}{7}+\frac{(3ace\,f^2+3(ad+bc)e^2f+bd\,e^3)x^5}{5}+$
norman	$\frac{bd\,f^3x^{11}}{11}+\left(\frac{1}{9}ad\,f^3+\frac{1}{9}bc\,f^3+\frac{1}{3}bde\,f^2\right)x^9+\left(\frac{1}{7}ac\,f^3+\frac{3}{7}ade\,f^2+\frac{3}{7}bce\,f^2+\frac{3}{7}bd\,e^2f\right)x^7+$
gosper	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7$
risch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7$
parallelrisch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7$
orering	$\frac{x(315bd\,f^3x^{10}+385ad\,f^3x^8+385bc\,f^3x^8+1155bde\,f^2x^8+495ac\,f^3x^6+1485ade\,f^2x^6+1485bce\,f^2x^6+1485bd\,e^2f\,x^6+2079a}{3465}$

input

```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^3+3*(a*d+
b*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)*e^2*f+b*d*e^3)*x^
5+1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx = \frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\left(3\,bdef^2+(bc+ad)f^3\right)x^9 + \frac{1}{7}\left(3\,bde^2f+acf^3+3\,(bc+ad)ef^2\right)x^7 + ace^3x+\frac{1}{5}\left(bde^3+3\,acef^2+3\,(bc+ad)e^2f\right)x^5 + \frac{1}{3}\left(3\,ace^2f+(bc+ad)e^3\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e
^2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a
*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*
x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx = ace^3x+\frac{bdf^3x^{11}}{11}+x^9\left(\frac{adf^3}{9}+\frac{bcf^3}{9}+\frac{bdef^2}{3}\right) + x^7\left(\frac{acf^3}{7}+\frac{3adef^2}{7}+\frac{3bcef^2}{7}+\frac{3bde^2f}{7}\right) + x^5\cdot\left(\frac{3acef^2}{5}+\frac{3ade^2f}{5}+\frac{3bce^2f}{5}+\frac{bde^3}{5}\right) + x^3\left(ace^2f+\frac{ade^3}{3}+\frac{bce^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

output
```
a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 + b*d*e*f**
2/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/7 + 3*b*d*e**2*f/7
) + x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b*c*e**2*f/5 + b*d*e**3/5) +
 x**3*(a*c*e**2*f + a*d*e**3/3 + b*c*e**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = \frac{1}{11}bdf^3x^{11} + \frac{1}{9}(3\,bdef^2 + (bc+ad)f^3)x^9 + \frac{1}{7}(3\,bde^2f + acf^3 + 3\,(bc+ad)ef^2)x^7 + ace^3x + \frac{1}{5}(bde^3 + 3\,acef^2 + 3\,(bc+ad)e^2f)x^5 + \frac{1}{3}(3\,ace^2f + (bc+ad)e^3)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e
^2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a
*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*
x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = \frac{1}{11}bdf^3x^{11} + \frac{1}{3}bdef^2x^9 + \frac{1}{9}bcf^3x^9 + \frac{1}{9}adf^3x^9 + \frac{3}{7}bde^2fx^7 + \frac{3}{7}bcef^2x^7 + \frac{3}{7}adef^2x^7 + \frac{1}{7}acf^3x^7 + \frac{1}{5}bde^3x^5 + \frac{3}{5}bce^2fx^5 + \frac{3}{5}ade^2fx^5 + \frac{3}{5}acef^2x^5 + \frac{1}{3}bce^3x^3 + \frac{1}{3}ade^3x^3 + ace^2fx^3 + ace^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/11*b*d*f^3*x^11 + 1/3*b*d*e*f^2*x^9 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9
+ 3/7*b*d*e^2*f*x^7 + 3/7*b*c*e*f^2*x^7 + 3/7*a*d*e*f^2*x^7 + 1/7*a*c*f^3*
x^7 + 1/5*b*d*e^3*x^5 + 3/5*b*c*e^2*f*x^5 + 3/5*a*d*e^2*f*x^5 + 3/5*a*c*e*
f^2*x^5 + 1/3*b*c*e^3*x^3 + 1/3*a*d*e^3*x^3 + a*c*e^2*f*x^3 + a*c*e^3*x
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = x^5\left(\frac{b\,d\,e^3}{5}+\frac{3\,a\,c\,e\,f^2}{5}+\frac{3\,a\,d\,e^2\,f}{5}+\frac{3\,b\,c\,e^2\,f}{5}\right) + x^7\left(\frac{a\,c\,f^3}{7}+\frac{3\,a\,d\,e\,f^2}{7}+\frac{3\,b\,c\,e\,f^2}{7}+\frac{3\,b\,d\,e^2\,f}{7}\right) + x^3\left(\frac{a\,d\,e^3}{3}+\frac{b\,c\,e^3}{3}+a\,c\,e^2\,f\right) + x^9\left(\frac{a\,d\,f^3}{9}+\frac{b\,c\,f^3}{9}+\frac{b\,d\,e\,f^2}{3}\right) + a\,c\,e^3\,x+\frac{b\,d\,f^3\,x^{11}}{11}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

output
```
x^5*((b*d*e^3)/5 + (3*a*c*e*f^2)/5 + (3*a*d*e^2*f)/5 + (3*b*c*e^2*f)/5) +
x^7*((a*c*f^3)/7 + (3*a*d*e*f^2)/7 + (3*b*c*e*f^2)/7 + (3*b*d*e^2*f)/7) +
x^3*((a*d*e^3)/3 + (b*c*e^3)/3 + a*c*e^2*f) + x^9*((a*d*f^3)/9 + (b*c*f^3)
/9 + (b*d*e*f^2)/3) + a*c*e^3*x + (b*d*f^3*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.30

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx$$
$$=\frac{x(315bdf^3x^{10}+385adf^3x^8+385bcf^3x^8+1155bdef^2x^8+495acf^3x^6+1485adef^2x^6+1485bcef^2x^6}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x)
```

output
```
(x*(3465*a*c*e**3 + 3465*a*c*e**2*f*x**2 + 2079*a*c*e*f**2*x**4 + 495*a*c*
f**3*x**6 + 1155*a*d*e**3*x**2 + 2079*a*d*e**2*f*x**4 + 1485*a*d*e*f**2*x*
*6 + 385*a*d*f**3*x**8 + 1155*b*c*e**3*x**2 + 2079*b*c*e**2*f*x**4 + 1485*
b*c*e*f**2*x**6 + 385*b*c*f**3*x**8 + 693*b*d*e**3*x**4 + 1485*b*d*e**2*f*
x**6 + 1155*b*d*e*f**2*x**8 + 315*b*d*f**3*x**10))/3465
```

3.202 $\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3 + \frac{1}{5}(af(2de+cf)+be(de+2cf))x^5 + \frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

output
```
a*c*e^2*x+1/3*e*(2*a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*f*(c*f+2*d*e)+b*e*(2*c*f+
d*e))*x^5+1/7*f*(a*d*f+b*c*f+2*b*d*e)*x^7+1/9*b*d*f^2*x^9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3 + \frac{1}{5}(bde^2+2bcef+2adef+acf^2)x^5 + \frac{1}{7}f(2bde+bcf+adf)x^7+\frac{1}{9}bdf^2x^9$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b*c*e*f +
2*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f
^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(fx^6(adf+bcf+2bde)+x^4(af(cf+2de)+be(2cf+de))+ex^2(2acf+ade+bce)+ace^2+bdf^2x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*
e*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x
^9)/9
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
default	$\frac{bd\,f^2x^9}{9}+\frac{((ad+bc)f^2+2dbef)x^7}{7}+\frac{(ac\,f^2+2(ad+bc)ef+bd\,e^2)x^5}{5}+\frac{(2acef+(ad+bc)e^2)x^3}{3}+ac\,e^2x$
norman	$\frac{bd\,f^2x^9}{9}+\left(\frac{1}{7}ad\,f^2+\frac{1}{7}bc\,f^2+\frac{2}{7}dbef\right)x^7+\left(\frac{1}{5}ac\,f^2+\frac{2}{5}adef+\frac{2}{5}bcef+\frac{1}{5}bd\,e^2\right)x^5+\left(\frac{2}{3}acef\right.$
gosper	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7dbef+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2$ -
risch	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7dbef+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2$ -
parallelrisch	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7dbef+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2$ -
orering	$\frac{x(35bd\,f^2x^8+45ad\,f^2x^6+45bc\,f^2x^6+90bdef\,x^6+63ac\,f^2x^4+126adef\,x^4+126bcef\,x^4+63bd\,e^2x^4+210acef\,x^2+105ad\,e^2x^2}{315}$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*d*b*e*f)*x^7+1/5*(a*c*f^2+2*(a*d+b*c)
*e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{1}{7}(2bdef + (bc+ad)f^2)x^7 + \frac{1}{5}(bde^2 + acf^2 + 2(bc+ad)ef)x^5 + ace^2x + \frac{1}{3}(2acef + (bc+ad)e^2)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a
*c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)
*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = ace^2x + \frac{bdf^2x^9}{9} + x^7\left(\frac{adf^2}{7} + \frac{bcf^2}{7} + \frac{2bdef}{7}\right) + x^5\left(\frac{acf^2}{5} + \frac{2adef}{5} + \frac{2bcef}{5} + \frac{bde^2}{5}\right) + x^3\cdot\left(\frac{2acef}{3} + \frac{ade^2}{3} + \frac{bce^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

output
```
a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*b*d*e*f/7
) + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5) + x**3*(2*a
*c*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{1}{7}(2bdef + (bc+ad)f^2)x^7 + \frac{1}{5}(bde^2 + acf^2 + 2(bc+ad)ef)x^5 + ace^2x + \frac{1}{3}(2acef + (bc+ad)e^2)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a
*c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)
*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{2}{7}bdefx^7 + \frac{1}{7}bcf^2x^7 + \frac{1}{7}adf^2x^7 + \frac{1}{5}bde^2x^5 + \frac{2}{5}bcefx^5 + \frac{2}{5}adefx^5 + \frac{1}{5}acf^2x^5 + \frac{1}{3}bce^2x^3 + \frac{1}{3}ade^2x^3 + \frac{2}{3}acefx^3 + ace^2x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/9*b*d*f^2*x^9 + 2/7*b*d*e*f*x^7 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 1/
5*b*d*e^2*x^5 + 2/5*b*c*e*f*x^5 + 2/5*a*d*e*f*x^5 + 1/5*a*c*f^2*x^5 + 1/3*
b*c*e^2*x^3 + 1/3*a*d*e^2*x^3 + 2/3*a*c*e*f*x^3 + a*c*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = x^5\left(\frac{a\,c\,f^2}{5}+\frac{b\,d\,e^2}{5}+\frac{2\,a\,d\,e\,f}{5}+\frac{2\,b\,c\,e\,f}{5}\right) + x^3\left(\frac{a\,d\,e^2}{3}+\frac{b\,c\,e^2}{3}+\frac{2\,a\,c\,e\,f}{3}\right) + x^7\left(\frac{a\,d\,f^2}{7}+\frac{b\,c\,f^2}{7}+\frac{2\,b\,d\,e\,f}{7}\right) + a\,c\,e^2\,x+\frac{b\,d\,f^2\,x^9}{9}$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

output
```
x^5*((a*c*f^2)/5 + (b*d*e^2)/5 + (2*a*d*e*f)/5 + (2*b*c*e*f)/5) + x^3*((a*
d*e^2)/3 + (b*c*e^2)/3 + (2*a*c*e*f)/3) + x^7*((a*d*f^2)/7 + (b*c*f^2)/7 +
 (2*b*d*e*f)/7) + a*c*e^2*x + (b*d*f^2*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{x(35bd\,f^2x^8+45ad\,f^2x^6+45bc\,f^2x^6+90bdef\,x^6+63ac\,f^2x^4+126adef\,x^4+126bcef\,x^4+63bd\,e^2x^4 \cdots}{315}$$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x)
```

output
```
(x*(315*a*c*e**2 + 210*a*c*e*f*x**2 + 63*a*c*f**2*x**4 + 105*a*d*e**2*x**2
 + 126*a*d*e*f*x**4 + 45*a*d*f**2*x**6 + 105*b*c*e**2*x**2 + 126*b*c*e*f*x
**4 + 45*b*c*f**2*x**6 + 63*b*d*e**2*x**4 + 90*b*d*e*f*x**6 + 35*b*d*f**2*
x**8))/315
```

3.203 $\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7$$

output
```
a*c*e*x+1/3*(a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*d*f+b*c*f+b*d*e)*x^5+1/7*b*d*f*
x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/
5 + (b*d*f*x^7)/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(x^4(adf+bcf+bde)+x^2(acf+ade+bce)+ace+bdfx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

input
```
Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/
5 + (b*d*f*x^7)/7
```


Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{bdf\,x^7}{7}+\frac{((ad+bc)f+bde)x^5}{5}+\frac{(acf+(ad+bc)e)x^3}{3}+acex$	53
norman	$\frac{bdf\,x^7}{7}+\left(\frac{1}{5}adf+\frac{1}{5}bcf+\frac{1}{5}bde\right)x^5+\left(\frac{1}{3}acf+\frac{1}{3}ade+\frac{1}{3}bce\right)x^3+acex$	55
gosper	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
risch	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
parallelrisch	$\frac{1}{7}bdf\,x^7+\frac{1}{5}x^5adf+\frac{1}{5}x^5bcf+\frac{1}{5}x^5bde+\frac{1}{3}x^3acf+\frac{1}{3}x^3ade+\frac{1}{3}x^3bce+acex$	63
orering	$\frac{x(15bdf\,x^6+21adf\,x^4+21bcf\,x^4+21bde\,x^4+35acf\,x^2+35ade\,x^2+35bce\,x^2+105ace)}{105}$	66

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)*x^3+a*c*
e*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx=\frac{1}{7}bdfx^7+\frac{1}{5}(bde+(bc+ad)f)x^5+acex+\frac{1}{3}(acf+(bc+ad)e)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

output
```
1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (
b*c + a*d)*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex + \frac{bdfx^7}{7} + x^5\left(\frac{adf}{5} + \frac{bcf}{5} + \frac{bde}{5}\right) + x^3\left(\frac{acf}{3} + \frac{ade}{3} + \frac{bce}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

output
```
a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**3*(a*c*f/
3 + a*d*e/3 + b*c*e/3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7 + \frac{1}{5}(bde + (bc+ad)f)x^5 + acex + \frac{1}{3}(acf + (bc+ad)e)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (
b*c + a*d)*e)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7+\frac{1}{5}bdex^5+\frac{1}{5}bcfx^5+\frac{1}{5}adfx^5+\frac{1}{3}bcex^3+\frac{1}{3}adex^3+\frac{1}{3}acfx^3+acex$$

input
```
integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

output
```
1/7*b*d*f*x^7 + 1/5*b*d*e*x^5 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/3*b*c*e*
x^3 + 1/3*a*d*e*x^3 + 1/3*a*c*f*x^3 + a*c*e*x
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{b\,d\,f\,x^7}{7}+\left(\frac{a\,d\,f}{5}+\frac{b\,c\,f}{5}+\frac{b\,d\,e}{5}\right)x^5+\left(\frac{a\,c\,f}{3}+\frac{a\,d\,e}{3}+\frac{b\,c\,e}{3}\right)x^3+a\,c\,e\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)*(e + f*x^2),x)
```

output
```
x^3*((a*c*f)/3 + (a*d*e)/3 + (b*c*e)/3) + x^5*((a*d*f)/5 + (b*c*f)/5 + (b*
d*e)/5) + a*c*e*x + (b*d*f*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$$
$$=\frac{x(15bdf\,x^6+21adf\,x^4+21bcf\,x^4+21bde\,x^4+35acf\,x^2+35ade\,x^2+35bce\,x^2+105ace)}{105}$$

input
```
int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x)
```

output
```
(x*(105*a*c*e + 35*a*c*f*x**2 + 35*a*d*e*x**2 + 21*a*d*f*x**4 + 35*b*c*e*x
**2 + 21*b*c*f*x**4 + 21*b*d*e*x**4 + 15*b*d*f*x**6))/105
```

3.204 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx = -\frac{(bde-bcf-adf)x}{f^2} + \frac{bdx^3}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

output
```
-(-a*d*f-b*c*f+b*d*e)*x/f^2+1/3*b*d*x^3/f+(-a*f+b*e)*(-c*f+d*e)*arctan(f^(
1/2)*x/e^(1/2))/e^(1/2)/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx = \frac{(-bde+bcf+adf)x}{f^2} + \frac{bdx^3}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*f)*(d*e -
 c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(3bde-3bcf-2adf)x^2+a(de-3cf)}{fx^2+e}dx}{3f}+\frac{dx(a+bx^2)}{3f}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\int \frac{(3bde-3bcf-2adf)x^2+a(de-3cf)}{fx^2+e}dx}{3f}$$

$$\Big\downarrow \text{299}$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\frac{x(-2adf-3bcf+3bde)}{f}-\frac{3(be-af)(de-cf)\int \frac{1}{fx^2+e}dx}{f}}{3f}$$

$$\Big\downarrow \text{218}$$

$$\frac{dx(a+bx^2)}{3f}-\frac{\frac{x(-2adf-3bcf+3bde)}{f}-\frac{3(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)}{\sqrt{e}f^{3/2}}}{3f}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(d*x*(a + b*x^2))/(3*f) - (((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/f - (3*(b*e -
 a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
default	$\frac{\frac{1}{3}bdf\,x^3+adfx+bcfx-bdex}{f^2}+\frac{(ac\,f^2-adef-bcef+bd\,e^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{f^2\sqrt{ef}}$
risch	$\frac{bd\,x^3}{3f}+\frac{adx}{f}+\frac{bcx}{f}-\frac{bdex}{f^2}-\frac{\ln(fx+\sqrt{-ef})ac}{2\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})ade}{2f\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})bce}{2f\sqrt{-ef}}-\frac{\ln(fx+\sqrt{-ef})bd\,e^2}{2f^2\sqrt{-ef}}+$ ln

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(1/3*b*d*f*x^3+a*d*f*x+b*c*f*x-b*d*e*x)+(a*c*f^2-a*d*e*f-b*c*e*f+b*d
*e^2)/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \left[\frac{2\,bdef^2x^3-3\left(bde^2+acf^2-(bc+ad)ef\right)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right)-6\left(bde^2f-(bc+ad)ef^2\right)x}{6\,ef^3}, \frac{bde\ldots}{}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/6*(2*b*d*e*f^2*x^3 - 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*sqrt(-e*f)
*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^2*f - (b*c + a*d
)*e*f^2)*x)/(e*f^3), 1/3*(b*d*e*f^2*x^3 + 3*(b*d*e^2 + a*c*f^2 - (b*c + a*
d)*e*f)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^2*f - (b*c + a*d)*e*f^2
)*x)/(e*f^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.78

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx &= \frac{bdx^3}{3f} + x\left(\frac{ad}{f}+\frac{bc}{f}-\frac{bde}{f^2}\right) \\ &- \frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(-\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2} \\ &+ \frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2}\end{aligned}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

output
```
b*d*x**3/(3*f) + x*(a*d/f + b*c/f - b*d*e/f**2) - sqrt(-1/(e*f**5))*(a*f -
 b*e)*(c*f - d*e)*log(-e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a
*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f -
b*e)*(c*f - d*e)*log(e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c
*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{(bde^2-bcef-adef+acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^2} + \frac{bdf^2x^3-3\,bdefx+3\,bcf^2x+3\,adf^2x}{3\,f^3}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
(b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f
^2) + 1/3*(b*d*f^2*x^3 - 3*b*d*e*f*x + 3*b*c*f^2*x + 3*a*d*f^2*x)/f^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = x\left(\frac{a\,d+b\,c}{f}-\frac{b\,d\,e}{f^2}\right)+\frac{b\,d\,x^3}{3\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,f^{5/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x)
```

output
```
x*((a*d + b*c)/f - (b*d*e)/f^2) + (b*d*x^3)/(3*f) + (atan((f^(1/2)*x*(a*f
- b*e)*(c*f - d*e))/(e^(1/2)*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)))*(a*
f - b*e)*(c*f - d*e))/(e^(1/2)*f^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ac\,f^2-3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)adef-3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)bcef+3\sqrt{f}\,\sqrt{e}\,atan\left(\right.}{3e\,f^3}$$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x)
```

output
```
(3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*f**2 - 3*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*d*e*f - 3*sqrt(f)*sqrt(e)*atan((f*x)/(
sqrt(f)*sqrt(e)))*b*c*e*f + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*b*d*e**2 + 3*a*d*e*f**2*x + 3*b*c*e*f**2*x - 3*b*d*e**2*f*x + b*d*e*f**2
*x**3)/(3*e*f**3)
```

3.205 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2}+\frac{(be-af)(de-cf)x}{2ef^2(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

output
```
b*d*x/f^2+1/2*(-a*f+b*e)*(-c*f+d*e)*x/e/f^2/(f*x^2+e)-1/2*(b*e*(-c*f+3*d*e
)-a*f*(c*f+d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2}+\frac{(be-af)(de-cf)x}{2ef^2(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
(b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) - ((b*e*(3
*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(
5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{b(3de-cf)x^2+a(de+cf)}{fx^2+e}dx}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(3de-cf)x^2+a(de+cf)}{fx^2+e}dx}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{bx(3de-cf)}{f}-\frac{(be(3de-cf)-af(cf+de))\int \frac{1}{fx^2+e}dx}{f}}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{bx(3de-cf)}{f}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{\sqrt{e}f^{3/2}}}{2ef}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
-1/2*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)) + ((b*(3*d*e - c*f)*x)/
f - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(S
qrt[e]*f^(3/2)))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

method	result
default	$\frac{bdx}{f^{2}}+\frac{\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)x}{2e\left(f\,x^{2}+e\right)}+\frac{\left(ac\,f^{2}+adef+bcef-3bd\,e^{2}\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}}{f^{2}}$
risch	$\frac{bdx}{f^{2}}+\frac{(ac\,f^{2}-adef-bcef+bd\,e^{2})x}{2e\,f^{2}(f\,x^{2}+e)}-\frac{\ln\left(fx+\sqrt{-ef}\right)ac}{4\sqrt{-ef}\,e}-\frac{\ln\left(fx+\sqrt{-ef}\right)ad}{4f\sqrt{-ef}}-\frac{\ln\left(fx+\sqrt{-ef}\right)bc}{4f\sqrt{-ef}}+\frac{3e\ln\left(fx+\sqrt{-ef}\right)bd}{4f^{2}\sqrt{-ef}}+$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
b*d*x/f^2+1/f^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(
a*c*f^2+a*d*e*f+b*c*e*f-3*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.35

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,bde^2f^2x^3 + (3\,bde^3 - acef^2 - (bc+ad)e^2f + (3\,bde^2f - acf^3 - (bc+ad)ef^2)x^2)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{\cdot}}{fx^2\cdot}\right.}{4\,(e^2f^4x^2 + e^3f^3)}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*b*d*e^2*f^2*x^3 + (3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*
b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sq
rt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^
2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3), 1/2*(2*b*d*e^2*f^2*x^3 - (3*b*d*e^3 - a
*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)
*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^3*f + a*c*e*f^3 - (b*c +
a*d)*e^2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(87) = 174$.

Time = 0.60 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$= \frac{bdx}{f^2} + \frac{x(acf^2-adef-bcef+bde^2)}{2e^2f^2+2ef^3x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2+adef+bcef-3bde^2)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2+adef+bcef-3bde^2)\log\left(e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*f**2 + 2*
e*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*
e**2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4 + sqrt(-1/(e**3*f**5))*(a
*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e**2)*log(e**2*f**2*sqrt(-1/(e**3*f**5
)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2} - \frac{(3\,bde^2-bcef-adef-acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\sqrt{ef}ef^2} + \frac{bde^2x-bcefx-adefx+acf^2x}{2\,(fx^2+e)ef^2}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b*d*x/f^2 - 1/2*(3*b*d*e^2 - b*c*e*f - a*d*e*f - a*c*f^2)*arctan(f*x/sqrt(
e*f))/(sqrt(e*f)*e*f^2) + 1/2*(b*d*e^2*x - b*c*e*f*x - a*d*e*f*x + a*c*f^2
*x)/((f*x^2 + e)*e*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{b\,d\,x}{f^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(a\,c\,f^2-3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f\right)}{2\,e^{3/2}\,f^{5/2}} + \frac{x\,\left(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f\right)}{2\,e\,\left(f^3\,x^2+e\,f^2\right)}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

output
```
(b*d*x)/f^2 + (atan((f^(1/2)*x)/e^(1/2))*(a*c*f^2 - 3*b*d*e^2 + a*d*e*f +
b*c*e*f))/(2*e^(3/2)*f^(5/2)) + (x*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)
)/(2*e*(e*f^2 + f^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.80

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$= \frac{\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ace\,f^2+\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ac\,f^3x^2+\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ad\,e^2f+\sqrt{f}\,\sqrt{e}\,atan\left(\right.}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
(sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e*f**2 + sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*f**3*x**2 + sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*d*e**2*f + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(
e)))*a*d*e*f**2*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e
**2*f + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e*f**2*x**2 - 3*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d*e**3 - 3*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d*e**2*f*x**2 + a*c*e*f**3*x - a*d*e**2*f
**2*x - b*c*e**2*f**2*x + 3*b*d*e**3*f*x + 2*b*d*e**2*f**2*x**3)/(2*e**2*f
**3*(e + f*x**2))
```

3.206 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} dx = \frac{(be-af)(de-cf)x}{4ef^2(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

output

```
1/4*(-a*f+b*e)*(-c*f+d*e)*x/e/f^2/(f*x^2+e)^2-1/8*(b*e*(-c*f+5*d*e)-a*f*(3
*c*f+d*e))*x/e^2/f^2/(f*x^2+e)+1/8*(b*e*(c*f+3*d*e)+a*f*(3*c*f+d*e))*arcta
n(f^(1/2)*x/e^(1/2))/e^(5/2)/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} dx = \frac{(be-af)(de-cf)x}{4ef^2(e+fx^2)^2} + \frac{(be(-5de+cf)+af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d*e + c*f)
 + a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a
*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{b(3de+cf)x^2+a(de+3cf)}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(3de+cf)x^2+a(de+3cf)}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(af(3cf+de)+be(cf+3de))\int \frac{1}{fx^2+e}dx}{2ef}-\frac{x(be(cf+3de)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{2e^{3/2}f^{3/2}}-\frac{x(be(cf+3de)-af(3cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
-1/4*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(3*d*e
+ c*f) - a*f*(d*e + 3*c*f))*x)/(e*f*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a
*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(3/2)))/(4*e*f
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

method	result
default	$\frac{\frac{(3ac\,f^2+adef+bcef-5bd\,e^2)x^3}{8e^2f}+\frac{(5ac\,f^2-adef-bcef-3bd\,e^2)x}{8e\,f^2}}{(f\,x^2+e)^2}+\frac{(3ac\,f^2+adef+bcef+3bd\,e^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8e^2f^2\sqrt{ef}}$
risch	$\frac{\frac{(3ac\,f^2+adef+bcef-5bd\,e^2)x^3}{8e^2f}+\frac{(5ac\,f^2-adef-bcef-3bd\,e^2)x}{8e\,f^2}}{(f\,x^2+e)^2}-\frac{3\ln(fx+\sqrt{-ef})ac}{16\sqrt{-ef}\,e^2}-\frac{\ln(fx+\sqrt{-ef})ad}{16\sqrt{-ef}\,fe}-\frac{\ln(fx+\sqrt{-ef})bc}{16\sqrt{-ef}\,fe}-$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f^2-a*d*e*
f-b*c*e*f-3*b*d*e^2)/e/f^2*x)/(f*x^2+e)^2+1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f+3
*b*d*e^2)/e^2/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.57

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$$

$$= \left[-\frac{2\,(5\,bde^3f^2-3\,acef^4-(bc+ad)e^2f^3)x^3+(3\,bde^4+3\,ace^2f^2+(bc+ad)e^3f+(3\,bde^2f^2+3\,acf^4+\right.$$

$$-\frac{(5\,bde^3f^2-3\,acef^4-(bc+ad)e^2f^3)x^3-(3\,bde^4+3\,ace^2f^2+(bc+ad)e^3f+(3\,bde^2f^2+3\,acf^4+(}{8\,(e^3}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 + (3*b*d
*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4 + (b
*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f^2)
*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*
e^4*f - 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x
^2 + e^5*f^3), -1/8*((5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x
^3 - (3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a
*c*f^4 + (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*
d)*e^2*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^4*f - 5*a*c*e^
2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 + e^5*f^3)]
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^2)\,(c+dx^2)}{(e+fx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2+adef+bcef+3bde^2)\log\left(-e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2+adef+bcef+3bde^2)\log\left(e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16} + \frac{x^3\cdot(3acf^3+adef^2+bcef^2-5bde^2f)+x(5acef^2-ade^2f-bce^2f-3bde^3)}{8e^4f^2+16e^3f^3x^2+8e^2f^4x^4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
-sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(-e
**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f**5))*(3*a*c*f**2 +
 a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/1
6 + (x**3*(3*a*c*f**3 + a*d*e*f**2 + b*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c
*e*f**2 - a*d*e**2*f - b*c*e**2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f*
*3*x**2 + 8*e**2*f**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{(3\,bde^2+bcef+adef+3\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\,\sqrt{ef}e^2f^2} - \frac{5\,bde^2fx^3-bcef^2x^3-adef^2x^3-3\,acf^3x^3+3\,bde^3x+bce^2fx+ade^2fx-5\,acef^2x}{8\,(fx^2+e)^2e^2f^2}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(3*b*d*e^2 + b*c*e*f + a*d*e*f + 3*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqr
t(e*f)*e^2*f^2) - 1/8*(5*b*d*e^2*f*x^3 - b*c*e*f^2*x^3 - a*d*e*f^2*x^3 - 3
*a*c*f^3*x^3 + 3*b*d*e^3*x + b*c*e^2*f*x + a*d*e^2*f*x - 5*a*c*e*f^2*x)/((
f*x^2 + e)^2*e^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(3\,a\,c\,f^2+3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^{5/2}\,f^{5/2}} - \frac{\frac{x\,(3\,b\,d\,e^2-5\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e\,f^2}-\frac{x^3\,(3\,a\,c\,f^2-5\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^2\,f}}{e^2+2\,e\,f\,x^2+f^2\,x^4}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

output
```
(atan((f^(1/2)*x)/e^(1/2))*(3*a*c*f^2 + 3*b*d*e^2 + a*d*e*f + b*c*e*f))/(8
*e^(5/2)*f^(5/2)) - ((x*(3*b*d*e^2 - 5*a*c*f^2 + a*d*e*f + b*c*e*f))/(8*e*
f^2) - (x^3*(3*a*c*f^2 - 5*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*e^2*f))/(e^2 +
 f^2*x^4 + 2*e*f*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 433, normalized size of antiderivative = 3.28

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$$
$$= \frac{3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ac\,e^2f^2+6\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ace\,f^3x^2+3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ac\,f^4x^4+\sqrt{f}\,\sqrt{e}}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x)
```

output

```
(3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e**2*f**2 + 6*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e*f**3*x**2 + 3*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*f**4*x**4 + sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*a*d*e**3*f + 2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*a*d*e**2*f**2*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a
*d*e*f**3*x**4 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e**3*f
+ 2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e**2*f**2*x**2 + sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e*f**3*x**4 + 3*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d*e**4 + 6*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*b*d*e**3*f*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f
)*sqrt(e)))*b*d*e**2*f**2*x**4 + 5*a*c*e**2*f**3*x + 3*a*c*e*f**4*x**3 - a
*d*e**3*f**2*x + a*d*e**2*f**3*x**3 - b*c*e**3*f**2*x + b*c*e**2*f**3*x**3
 - 3*b*d*e**4*f*x - 5*b*d*e**3*f**2*x**3)/(8*e**3*f**3*(e**2 + 2*e*f*x**2
+ f**2*x**4))
```

3.207 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(be-af)(de-cf)x}{6ef^2(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

output
```
1/6*(-a*f+b*e)*(-c*f+d*e)*x/e/f^2/(f*x^2+e)^3-1/24*(b*e*(-c*f+7*d*e)-a*f*(
5*c*f+d*e))*x/e^2/f^2/(f*x^2+e)^2+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*x/e
^3/f^2/(f*x^2+e)+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*arctan(f^(1/2)*x/e^(
1/2))/e^(7/2)/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(be-af)(de-cf)x}{6ef^2(e+fx^2)^3} + \frac{(be(-7de+cf)+af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d*e + c*f)
 + a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) +
a*f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(
d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {401, 25, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{3b(de+cf)x^2+a(de+5cf)}{(fx^2+e)^3}dx}{6ef} - \frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b(de+cf)x^2+a(de+5cf)}{(fx^2+e)^3}dx}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$\Big\downarrow$ 298

$$\frac{\frac{3(af(5cf+de)+be(cf+de))\int \frac{1}{(fx^2+e)^2}dx}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$\Big\downarrow$ 215

$$\frac{\frac{3(af(5cf+de)+be(cf+de))\left(\frac{\int \frac{1}{fx^2+e}dx}{2e}+\frac{x}{2e(e+fx^2)}\right)}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}\sqrt{f}}+\frac{x}{2e(e+fx^2)}\right)(af(5cf+de)+be(cf+de))}{4ef}-\frac{x(3be(cf+de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
-1/6*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^3) + (-1/4*((3*b*e*(d*e
+ c*f) - a*f*(d*e + 5*c*f))*x)/(e*f*(e + f*x^2)^2) + (3*(b*e*(d*e + c*f) +
 a*f*(d*e + 5*c*f))*(x/(2*e*(e + f*x^2)) + ArcTan[(Sqrt[f]*x)/Sqrt[e]]/(2*
e^(3/2)*Sqrt[f])))/(4*e*f))/(6*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)x^{5}}{16e^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef-bd\,e^{2}\right)x^{3}}{6e^{2}f}+\frac{\left(11ac\,f^{2}-adef-bcef-bd\,e^{2}\right)x}{16e\,f^{2}}}{\left(f\,x^{2}+e\right)^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)\arctan(}{16e^{3}f^{2}\sqrt{ef}}$
risch	$\frac{\frac{\left(5ac\,f^{2}+adef+bcef+bd\,e^{2}\right)x^{5}}{16e^{3}}+\frac{\left(5ac\,f^{2}+adef+bcef-bd\,e^{2}\right)x^{3}}{6e^{2}f}+\frac{\left(11ac\,f^{2}-adef-bcef-bd\,e^{2}\right)x}{16e\,f^{2}}}{\left(f\,x^{2}+e\right)^{3}}-\frac{5\ln\left(fx+\sqrt{-ef}\right)ac}{32\sqrt{-ef}\,e^{3}}-\frac{\ln\left(fx+\sqrt{-e}\right.}{32\sqrt{-ef}\,f}$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+a*d*e*f+b
*c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e*f-b*d*e^2)/e/f^2*
x)/(f*x^2+e)^3+1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3/f^2/(e*f)^(1/2
)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$$
$$= \left[\frac{6\,(bde^3f^3+5\,acef^5+(bc+ad)e^2f^4)x^5-16\,(bde^4f^2-5\,ace^2f^4-(bc+ad)e^3f^3)x^3-3\,(bde^5+5\,ace^3\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 16*(b*d*e
^4*f^2 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 - 3*(b*d*e^5 + 5*a*c*e^3
*f^2 + (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4
*f + 3*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*
f + 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sq
rt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)
*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3), 1/48
*(3*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 8*(b*d*e^4*f^2
 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 + 3*(b*d*e^5 + 5*a*c*e^3*f^2 +
 (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3
*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*
a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) -
3*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e
^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3)]
```

Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}$$
$$+\frac{x^5\cdot(15acf^4+3adef^3+3bcef^3+3bde^2f^2)+x^3\cdot(40acef^3+8ade^2f^2+8bce^2f^2-8bde^3f)+x(33ac}{48e^6f^2+144e^5f^3x^2+144e^4f^4x^4+48e^3f^5x^6}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
-sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(-e**
4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a
*d*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 +
(x**5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3
*(40*a*c*e*f**3 + 8*a*d*e**2*f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(3
3*a*c*e**2*f**2 - 3*a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2
 + 144*e**5*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(bde^2+bcef+adef+5\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\sqrt{ef}e^3f^2} + \frac{3\,bde^2f^2x^5+3\,bcef^3x^5+3\,adef^3x^5+15\,acf^4x^5-8\,bde^3fx^3+8\,bce^2f^2x^3+8\,ade^2f^2x^3+40\,acef^3x^3}{48\,(fx^2+e)^3e^3f^2}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(b*d*e^2 + b*c*e*f + a*d*e*f + 5*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt
(e*f)*e^3*f^2) + 1/48*(3*b*d*e^2*f^2*x^5 + 3*b*c*e*f^3*x^5 + 3*a*d*e*f^3*x
^5 + 15*a*c*f^4*x^5 - 8*b*d*e^3*f*x^3 + 8*b*c*e^2*f^2*x^3 + 8*a*d*e^2*f^2*
x^3 + 40*a*c*e*f^3*x^3 - 3*b*d*e^4*x - 3*b*c*e^3*f*x - 3*a*d*e^3*f*x + 33*
a*c*e^2*f^2*x)/((f*x^2 + e)^3*e^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{\frac{x^5\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^3}-\frac{x\,(b\,d\,e^2-11\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e\,f^2}+\frac{x^3\,(5\,a\,c\,f^2-b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{6\,e^2\,f}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^{7/2}\,f^{5/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

output
```
((x^5*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e^3) - (x*(b*d*e^2 -
11*a*c*f^2 + a*d*e*f + b*c*e*f))/(16*e*f^2) + (x^3*(5*a*c*f^2 - b*d*e^2 +
a*d*e*f + b*c*e*f))/(6*e^2*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4)
 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(
16*e^(7/2)*f^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.57

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e**3*f**2 + 45*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e**2*f**3*x**2 + 45*sqrt(f)*s
qrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*e*f**4*x**4 + 15*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*f**5*x**6 + 3*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*d*e**4*f + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqr
t(e)))*a*d*e**3*f**2*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*a*d*e**2*f**3*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*d
*e*f**4*x**6 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e**4*f
+ 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e**3*f**2*x**2 + 9*s
qrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e**2*f**3*x**4 + 3*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*e*f**4*x**6 + 3*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d*e**5 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(s
qrt(f)*sqrt(e)))*b*d*e**4*f*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*s
qrt(e)))*b*d*e**3*f**2*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*b*d*e**2*f**3*x**6 + 33*a*c*e**3*f**3*x + 40*a*c*e**2*f**4*x**3 + 15*a
*c*e*f**5*x**5 - 3*a*d*e**4*f**2*x + 8*a*d*e**3*f**3*x**3 + 3*a*d*e**2*f**
4*x**5 - 3*b*c*e**4*f**2*x + 8*b*c*e**3*f**3*x**3 + 3*b*c*e**2*f**4*x**5 -
 3*b*d*e**5*f*x - 8*b*d*e**4*f**2*x**3 + 3*b*d*e**3*f**3*x**5)/(48*e**4*f*
*3*(e**3 + 3*e**2*f*x**2 + 3*e*f**2*x**4 + f**3*x**6))
```

3.208 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 226

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$$
$$=ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\big(bce(2de+3cf)+a\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^5$$
$$+\frac{1}{7}\big(af\big(3d^2e^2+6cdef+c^2f^2\big)+be\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^7$$
$$+\frac{1}{9}f\big(adf(3de+2cf)+b\big(3d^2e^2+6cdef+c^2f^2\big)\big)x^9$$
$$+\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

output
```
a*c^2*e^3*x+1/3*c*e^2*(3*a*c*f+2*a*d*e+b*c*e)*x^3+1/5*e*(b*c*e*(3*c*f+2*d*
e)+a*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5+1/7*(a*f*(c^2*f^2+6*c*d*e*f+3*d^2*
e^2)+b*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*f*(a*d*f*(2*c*f+3*d*e)+b*(
c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^9+1/11*d*f^2*(a*d*f+2*b*c*f+3*b*d*e)*x^11+
1/13*b*d^2*f^3*x^13
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$$
$$=ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5$$
$$+\frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right)+be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7$$
$$+\frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9$$
$$+\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
 + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 +
(d*f^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$$
$$\Big\downarrow \text{396}$$
$$\int \left(fx^8\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+x^6\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
 + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2
 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 +
(f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 +
(d*f^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05

method	result
default	$\frac{b\,d^2f^3x^{13}}{13}+\frac{((a\,d^2+2bcd)\,f^3+3b\,d^2e\,f^2)\,x^{11}}{11}+\frac{((2acd+b\,c^2)\,f^3+3(a\,d^2+2bcd)\,e\,f^2+3b\,d^2e^2f)\,x^9}{9}+\frac{(a\,c^2f^3+3(2acd+b$
norman	$\frac{b\,d^2f^3x^{13}}{13}+\left(\frac{1}{11}a\,d^2f^3+\frac{2}{11}bcd\,f^3+\frac{3}{11}b\,d^2e\,f^2\right)x^{11}+\left(\frac{2}{9}acd\,f^3+\frac{1}{3}a\,d^2e\,f^2+\frac{1}{9}b\,c^2f^3+\frac{2}{3}bcde$
gosper	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$
risch	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$
parallelrisch	$\frac{1}{13}b\,d^2f^3x^{13}+\frac{1}{11}x^{11}a\,d^2f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2e\,f^2+\frac{2}{9}x^9acd\,f^3+\frac{1}{3}x^9a\,d^2e\,f^2+\frac{1}{9}x^9b$
orering	$x(3465b\,d^2f^3x^{12}+4095a\,d^2f^3x^{10}+8190bcd\,f^3x^{10}+12285b\,d^2e\,f^2x^{10}+10010acd\,f^3x^8+15015a\,d^2e\,f^2x^8+5005b\,c^2f^3x^8+300$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11+1/9*((2*
a*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*x^9+1/7*(a*c^2*f^3
+3*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*f+e^3*b*d^2)*x^7+1/5*(3*a*c
^2*e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^2+2*b*c*d)*e^3)*x^5+1/3*(3*a*c^2*e^2
*f+(2*a*c*d+b*c^2)*e^3)*x^3+a*c^2*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}\int (a+bx^2)\,(c+dx^2)^2\,(e+fx^2)^3\,dx &= \frac{1}{13}\,bd^2f^3x^{13}+\frac{1}{11}\,(3\,bd^2ef^2+(2\,bcd+ad^2)\,f^3)x^{11}\\ &\quad+\frac{1}{9}\,(3\,bd^2e^2f+3\,(2\,bcd+ad^2)\,ef^2+(bc^2+2\,acd)\,f^3)x^9\\ &\quad+\frac{1}{7}\,(bd^2e^3+ac^2f^3+3\,(2\,bcd+ad^2)\,e^2f+3\,(bc^2+2\,acd)\,ef^2)x^7\\ &\quad+ac^2e^3x+\frac{1}{5}\,(3\,ac^2ef^2+(2\,bcd+ad^2)\,e^3+3\,(bc^2+2\,acd)\,e^2f)x^5\\ &\quad+\frac{1}{3}\,(3\,ac^2e^2f+(bc^2+2\,acd)\,e^3)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 +
1/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^
9 + 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*
a*c*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e
^3 + 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*
d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx = ac^2e^3x + \frac{bd^2f^3x^{13}}{13} + x^{11}\left(\frac{ad^2f^3}{11} + \frac{2bcdf^3}{11} + \frac{3bd^2ef^2}{11}\right) + x^9 \cdot \left(\frac{2acdf^3}{9} + \frac{ad^2ef^2}{3} + \frac{bc^2f^3}{9} + \frac{2bcdef^2}{3} + \frac{bd^2e^2f}{3}\right) + x^7\left(\frac{ac^2f^3}{7} + \frac{6acdef^2}{7} + \frac{3ad^2e^2f}{7} + \frac{3bc^2ef^2}{7} + \frac{6bcde^2f}{7} + \frac{bd^2e^3}{7}\right) + x^5 \cdot \left(\frac{3ac^2ef^2}{5} + \frac{6acde^2f}{5} + \frac{ad^2e^3}{5} + \frac{3bc^2e^2f}{5} + \frac{2bcde^3}{5}\right) + x^3\left(ac^2e^2f + \frac{2acde^3}{3} + \frac{bc^2e^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

output
```
a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*b*c*d*f**
3/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d**2*e*f**2/3 + b*c*
*2*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3) + x**7*(a*c**2*f**3/7 + 6*
a*c*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7
+ b*d**2*e**3/7) + x**5*(3*a*c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**
3/5 + 3*b*c**2*e**2*f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*
e**3/3 + b*c**2*e**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}\int (a+bx^2)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx &= \frac{1}{13}bd^2f^3x^{13}+\frac{1}{11}\left(3\,bd^2ef^2+\left(2\,bcd+ad^2\right)f^3\right)x^{11}\\&\quad+\frac{1}{9}\left(3\,bd^2e^2f+3\left(2\,bcd+ad^2\right)ef^2+\left(bc^2+2\,acd\right)f^3\right)x^9\\&\quad+\frac{1}{7}\left(bd^2e^3+ac^2f^3+3\left(2\,bcd+ad^2\right)e^2f+3\left(bc^2+2\,acd\right)ef^2\right)x^7\\&\quad+ac^2e^3x+\frac{1}{5}\left(3\,ac^2ef^2+\left(2\,bcd+ad^2\right)e^3+3\left(bc^2+2\,acd\right)e^2f\right)x^5\\&\quad+\frac{1}{3}\left(3\,ac^2e^2f+\left(bc^2+2\,acd\right)e^3\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 +
1/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^
9 + 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*
a*c*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e
^3 + 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*
d)*e^3)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \frac{1}{13}bd^2f^3x^{13}+\frac{3}{11}bd^2ef^2x^{11}+\frac{2}{11}bcdf^3x^{11}+\frac{1}{11}ad^2f^3x^{11}+\frac{1}{3}bd^2e^2fx^9+\frac{2}{3}bcdef^2x^9+\frac{1}{3}ad^2ef^2x^9+\frac{1}{9}bc^2f^3x^9+\frac{2}{9}acdf^3x^9+\frac{1}{7}bd^2e^3x^7+\frac{6}{7}bcde^2fx^7+\frac{3}{7}ad^2e^2fx^7+\frac{3}{7}bc^2ef^2x^7+\frac{6}{7}acdef^2x^7+\frac{1}{7}ac^2f^3x^7+\frac{2}{5}bcde^3x^5+\frac{1}{5}ad^2e^3x^5+\frac{3}{5}bc^2e^2fx^5+\frac{6}{5}acde^2fx^5+\frac{3}{5}ac^2ef^2x^5+\frac{1}{3}bc^2e^3x^3+\frac{2}{3}acde^3x^3+ac^2e^2fx^3+ac^2e^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/13*b*d^2*f^3*x^13 + 3/11*b*d^2*e*f^2*x^11 + 2/11*b*c*d*f^3*x^11 + 1/11*a
*d^2*f^3*x^11 + 1/3*b*d^2*e^2*f*x^9 + 2/3*b*c*d*e*f^2*x^9 + 1/3*a*d^2*e*f^
2*x^9 + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 + 1/7*b*d^2*e^3*x^7 + 6/7*b*
c*d*e^2*f*x^7 + 3/7*a*d^2*e^2*f*x^7 + 3/7*b*c^2*e*f^2*x^7 + 6/7*a*c*d*e*f^
2*x^7 + 1/7*a*c^2*f^3*x^7 + 2/5*b*c*d*e^3*x^5 + 1/5*a*d^2*e^3*x^5 + 3/5*b*
c^2*e^2*f*x^5 + 6/5*a*c*d*e^2*f*x^5 + 3/5*a*c^2*e*f^2*x^5 + 1/3*b*c^2*e^3*
x^3 + 2/3*a*c*d*e^3*x^3 + a*c^2*e^2*f*x^3 + a*c^2*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx=x^5\left(\frac{3\,b\,c^2\,e^2\,f}{5}+\frac{3\,a\,c^2\,e\,f^2}{5}+\frac{2\,b\,c\,d\,e^3}{5}+\frac{6\,a\,c\,d\,e^2\,f}{5}+\frac{a\,d^2\,e^3}{5}\right)+x^9\left(\frac{b\,c^2\,f^3}{9}+\frac{2\,b\,c\,d\,e\,f^2}{3}+\frac{2\,a\,c\,d\,f^3}{9}+\frac{b\,d^2\,e^2\,f}{3}+\frac{a\,d^2\,e\,f^2}{3}\right)+x^7\left(\frac{3\,b\,c^2\,e\,f^2}{7}+\frac{a\,c^2\,f^3}{7}+\frac{6\,b\,c\,d\,e^2\,f}{7}+\frac{6\,a\,c\,d\,e\,f^2}{7}+\frac{b\,d^2\,e^3}{7}+\frac{3\,a\,d^2\,e^2\,f}{7}\right)+\frac{b\,d^2\,f^3\,x^{13}}{13}+\frac{c\,e^2\,x^3\,\left(3\,a\,c\,f+2\,a\,d\,e+b\,c\,e\right)}{3}+\frac{d\,f^2\,x^{11}\,\left(a\,d\,f+2\,b\,c\,f+3\,b\,d\,e\right)}{11}+a\,c^2\,e^3\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

output
```
x^5*((a*d^2*e^3)/5 + (2*b*c*d*e^3)/5 + (3*a*c^2*e*f^2)/5 + (3*b*c^2*e^2*f)
/5 + (6*a*c*d*e^2*f)/5) + x^9*((b*c^2*f^3)/9 + (2*a*c*d*f^3)/9 + (a*d^2*e*
f^2)/3 + (b*d^2*e^2*f)/3 + (2*b*c*d*e*f^2)/3) + x^7*((a*c^2*f^3)/7 + (b*d^
2*e^3)/7 + (3*a*d^2*e^2*f)/7 + (3*b*c^2*e*f^2)/7 + (6*a*c*d*e*f^2)/7 + (6*
b*c*d*e^2*f)/7) + (b*d^2*f^3*x^13)/13 + (c*e^2*x^3*(3*a*c*f + 2*a*d*e + b*
c*e))/3 + (d*f^2*x^11*(a*d*f + 2*b*c*f + 3*b*d*e))/11 + a*c^2*e^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \frac{x(3465b\,d^2f^3x^{12}+4095a\,d^2f^3x^{10}+8190bcd\,f^3x^{10}+12285b\,d^2e\,f^2x^{10}+10010acd\,f^3x^8+15015a\,d^2e\,f^2\cdots}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

output
```
(x*(45045*a*c**2*e**3 + 45045*a*c**2*e**2*f*x**2 + 27027*a*c**2*e*f**2*x**
4 + 6435*a*c**2*f**3*x**6 + 30030*a*c*d*e**3*x**2 + 54054*a*c*d*e**2*f*x**
4 + 38610*a*c*d*e*f**2*x**6 + 10010*a*c*d*f**3*x**8 + 9009*a*d**2*e**3*x**
4 + 19305*a*d**2*e**2*f*x**6 + 15015*a*d**2*e*f**2*x**8 + 4095*a*d**2*f**3
*x**10 + 15015*b*c**2*e**3*x**2 + 27027*b*c**2*e**2*f*x**4 + 19305*b*c**2*
e*f**2*x**6 + 5005*b*c**2*f**3*x**8 + 18018*b*c*d*e**3*x**4 + 38610*b*c*d*
e**2*f*x**6 + 30030*b*c*d*e*f**2*x**8 + 8190*b*c*d*f**3*x**10 + 6435*b*d**
2*e**3*x**6 + 15015*b*d**2*e**2*f*x**8 + 12285*b*d**2*e*f**2*x**10 + 3465*
b*d**2*f**3*x**12))/45045
```

3.209 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3 + \frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 + \frac{1}{9}df(adf+2b(de+cf))x^9+\frac{1}{11}bd^2f^2x^{11}$$

output

```
a*c^2*e^2*x+1/3*c*e*(b*c*e+2*a*(c*f+d*e))*x^3+1/5*(2*b*c*e*(c*f+d*e)+a*(c^
2*f^2+4*c*d*e*f+d^2*e^2))*x^5+1/7*(2*a*d*f*(c*f+d*e)+b*(c^2*f^2+4*c*d*e*f+
d^2*e^2))*x^7+1/9*d*f*(a*d*f+2*b*(c*f+d*e))*x^9+1/11*b*d^2*f^2*x^11
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right) \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx = ac^2e^2x + \frac{1}{3}ce(bce+2a(de+cf))x^3 + \frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 + \frac{1}{9}df(adf+2b(de+cf))x^9 + \frac{1}{11}bd^2f^2x^{11}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f
) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*
(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x
^9)/9 + (b*d^2*f^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx$$

$$\Big\downarrow \ 396$$

$$\int \left(x^6\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+x^4\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+dfx^8(adf+$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f
) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*
(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x
^9)/9 + (b*d^2*f^2*x^11)/11
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

method	result
default	$\frac{b\,d^2f^2x^{11}}{11}+\frac{\left(\left(a\,d^2+2bcd\right)f^2+2b\,d^2ef\right)x^9}{9}+\frac{\left(\left(2acd+b\,c^2\right)f^2+2\left(a\,d^2+2bcd\right)ef+b\,d^2e^2\right)x^7}{7}+\frac{\left(a\,c^2f^2+2\left(2acd+b\,c^2\right)ef\right.}{5}$
norman	$\frac{b\,d^2f^2x^{11}}{11}+\left(\frac{1}{9}a\,d^2f^2+\frac{2}{9}bcd\,f^2+\frac{2}{9}b\,d^2ef\right)x^9+\left(\frac{2}{7}acd\,f^2+\frac{2}{7}a\,d^2ef+\frac{1}{7}b\,c^2f^2+\frac{4}{7}bcdef+\frac{1}{7}\right.$
gosper	$\frac{1}{11}b\,d^2f^2x^{11}+\frac{1}{9}x^9a\,d^2f^2+\frac{2}{9}x^9bcd\,f^2+\frac{2}{9}x^9b\,d^2ef+\frac{2}{7}x^7acd\,f^2+\frac{2}{7}x^7a\,d^2ef+\frac{1}{7}x^7b\,c^2f^2+$
risch	$\frac{1}{11}b\,d^2f^2x^{11}+\frac{1}{9}x^9a\,d^2f^2+\frac{2}{9}x^9bcd\,f^2+\frac{2}{9}x^9b\,d^2ef+\frac{2}{7}x^7acd\,f^2+\frac{2}{7}x^7a\,d^2ef+\frac{1}{7}x^7b\,c^2f^2+$
parallelrisch	$\frac{1}{11}b\,d^2f^2x^{11}+\frac{1}{9}x^9a\,d^2f^2+\frac{2}{9}x^9bcd\,f^2+\frac{2}{9}x^9b\,d^2ef+\frac{2}{7}x^7acd\,f^2+\frac{2}{7}x^7a\,d^2ef+\frac{1}{7}x^7b\,c^2f^2+$
orering	$\frac{x\left(315b\,d^2f^2x^{10}+385a\,d^2f^2x^8+770bcd\,f^2x^8+770b\,d^2ef\,x^8+990acd\,f^2x^6+990a\,d^2ef\,x^6+495b\,c^2f^2x^6+1980bcdef\,x^6+495b\right.}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*e*f)*x^9+1/7*((2*a*c*
d+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*e*f+b*d^2*e^2)*x^7+1/5*(a*c^2*f^2+2*(2*a*c*
d+b*c^2)*e*f+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*a*c^2*e*f+(2*a*c*d+b*c^2)*e^2
)*x^3+a*c^2*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx &= \frac{1}{11}bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2ef+\left(2\,bcd+ad^2\right)f^2\right)x^9 \\ &\quad+\frac{1}{7}\left(bd^2e^2+2\left(2\,bcd+ad^2\right)ef+\left(bc^2+2\,acd\right)f^2\right)x^7 \\ &\quad+ac^2e^2x+\frac{1}{5}\left(ac^2f^2+\left(2\,bcd+ad^2\right)e^2+2\left(bc^2+2\,acd\right)ef\right)x^5 \\ &\quad+\frac{1}{3}\left(2\,ac^2ef+\left(bc^2+2\,acd\right)e^2\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*
(b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*
e^2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*
x^5 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx = ac^2e^2x+\frac{bd^2f^2x^{11}}{11}+x^9\left(\frac{ad^2f^2}{9}+\frac{2bcdf^2}{9}+\frac{2bd^2ef}{9}\right)+x^7\cdot\left(\frac{2acdf^2}{7}+\frac{2ad^2ef}{7}+\frac{bc^2f^2}{7}+\frac{4bcdef}{7}+\frac{bd^2e^2}{7}\right)+x^5\left(\frac{ac^2f^2}{5}+\frac{4acdef}{5}+\frac{ad^2e^2}{5}+\frac{2bc^2ef}{5}+\frac{2bcde^2}{5}\right)+x^3\cdot\left(\frac{2ac^2ef}{3}+\frac{2acde^2}{3}+\frac{bc^2e^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

output
```
a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*c*d*f**2/
9 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/
7 + 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 +
 a*d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 +
 2*a*c*d*e**2/3 + b*c**2*e**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx = \frac{1}{11}\,bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2ef+\left(2\,bcd+ad^2\right)f^2\right)x^9+\frac{1}{7}\left(bd^2e^2+2\left(2\,bcd+ad^2\right)ef+\left(bc^2+2\,acd\right)f^2\right)x^7+ac^2e^2x+\frac{1}{5}\left(ac^2f^2+\left(2\,bcd+ad^2\right)e^2+2\left(bc^2+2\,acd\right)ef\right)x^5+\frac{1}{3}\left(2\,ac^2ef+\left(bc^2+2\,acd\right)e^2\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*
(b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*
e^2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*
x^5 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx &= \frac{1}{11}bd^2f^2x^{11} + \frac{2}{9}bd^2efx^9 + \frac{2}{9}bcdf^2x^9 \\ &+ \frac{1}{9}ad^2f^2x^9 + \frac{1}{7}bd^2e^2x^7 + \frac{4}{7}bcdefx^7 \\ &+ \frac{2}{7}ad^2efx^7 + \frac{1}{7}bc^2f^2x^7 + \frac{2}{7}acdf^2x^7 \\ &+ \frac{2}{5}bcde^2x^5 + \frac{1}{5}ad^2e^2x^5 + \frac{2}{5}bc^2efx^5 \\ &+ \frac{4}{5}acdefx^5 + \frac{1}{5}ac^2f^2x^5 + \frac{1}{3}bc^2e^2x^3 \\ &+ \frac{2}{3}acde^2x^3 + \frac{2}{3}ac^2efx^3 + ac^2e^2x\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/11*b*d^2*f^2*x^11 + 2/9*b*d^2*e*f*x^9 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^
2*x^9 + 1/7*b*d^2*e^2*x^7 + 4/7*b*c*d*e*f*x^7 + 2/7*a*d^2*e*f*x^7 + 1/7*b*
c^2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 2/5*b*c*d*e^2*x^5 + 1/5*a*d^2*e^2*x^5 +
2/5*b*c^2*e*f*x^5 + 4/5*a*c*d*e*f*x^5 + 1/5*a*c^2*f^2*x^5 + 1/3*b*c^2*e^2*
x^3 + 2/3*a*c*d*e^2*x^3 + 2/3*a*c^2*e*f*x^3 + a*c^2*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = x^5\left(\frac{2\,b\,c^2\,e\,f}{5}+\frac{a\,c^2\,f^2}{5}+\frac{2\,b\,c\,d\,e^2}{5}+\frac{4\,a\,c\,d\,e\,f}{5}+\frac{a\,d^2\,e^2}{5}\right)+x^7\left(\frac{b\,c^2\,f^2}{7}+\frac{4\,b\,c\,d\,e\,f}{7}+\frac{2\,a\,c\,d\,f^2}{7}+\frac{b\,d^2\,e^2}{7}+\frac{2\,a\,d^2\,e\,f}{7}\right)+\frac{b\,d^2\,f^2\,x^{11}}{11}+a\,c^2\,e^2\,x+\frac{c\,e\,x^3\,(2\,a\,c\,f+2\,a\,d\,e+b\,c\,e)}{3}+\frac{d\,f\,x^9\,(a\,d\,f+2\,b\,c\,f+2\,b\,d\,e)}{9}$$

input
```
int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

output
```
x^5*((a*c^2*f^2)/5 + (a*d^2*e^2)/5 + (2*b*c*d*e^2)/5 + (2*b*c^2*e*f)/5 + (
4*a*c*d*e*f)/5) + x^7*((b*c^2*f^2)/7 + (b*d^2*e^2)/7 + (2*a*c*d*f^2)/7 + (
2*a*d^2*e*f)/7 + (4*b*c*d*e*f)/7) + (b*d^2*f^2*x^11)/11 + a*c^2*e^2*x + (c
*e*x^3*(2*a*c*f + 2*a*d*e + b*c*e))/3 + (d*f*x^9*(a*d*f + 2*b*c*f + 2*b*d*
e))/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.30

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{x(315b\,d^2f^2x^{10}+385a\,d^2f^2x^8+770bcd\,f^2x^8+770b\,d^2ef\,x^8+990acd\,f^2x^6+990a\,d^2ef\,x^6+495b\,c^2f^2x}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

output
```
(x*(3465*a*c**2*e**2 + 2310*a*c**2*e*f*x**2 + 693*a*c**2*f**2*x**4 + 2310*
a*c*d*e**2*x**2 + 2772*a*c*d*e*f*x**4 + 990*a*c*d*f**2*x**6 + 693*a*d**2*e
**2*x**4 + 990*a*d**2*e*f*x**6 + 385*a*d**2*f**2*x**8 + 1155*b*c**2*e**2*x
**2 + 1386*b*c**2*e*f*x**4 + 495*b*c**2*f**2*x**6 + 1386*b*c*d*e**2*x**4 +
 1980*b*c*d*e*f*x**6 + 770*b*c*d*f**2*x**8 + 495*b*d**2*e**2*x**6 + 770*b*
d**2*e*f*x**8 + 315*b*d**2*f**2*x**10))/3465
```

3.210 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = -\frac{(adf(de-2cf)-b(de-cf)^2)\,x}{f^3} - \frac{d(bde-2bcf-adf)x^3}{3f^2} + \frac{bd^2x^5}{5f} - \frac{(be-af)(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

output

```
-(a*d*f*(-2*c*f+d*e)-b*(-c*f+d*e)^2)*x/f^3-1/3*d*(-a*d*f-2*b*c*f+b*d*e)*x^
3/f^2+1/5*b*d^2*x^5/f-(-a*f+b*e)*(-c*f+d*e)^2*arctan(f^(1/2)*x/e^(1/2))/e^
(1/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{(b(de-cf)^2+adf(-de+2cf))\,x}{f^3} + \frac{d(-bde+2bcf+adf)x^3}{3f^2} + \frac{bd^2x^5}{5f} - \frac{(be-af)(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e) + 2*b*c*
f + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*A
rcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {403, 25, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int -\frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f} + \frac{bx(c+dx^2)^2}{5f}$$

$$\Big\downarrow \; 25$$

$$\frac{bx(c+dx^2)^2}{5f} - \frac{\int \frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}$$

$$\Big\downarrow \textbf{403}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{\int -\frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}+\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}}{5f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}}{5f}$$

$$\Big\downarrow \textbf{299}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)(de-cf)^2\int \frac{1}{fx^2+e}dx}{f}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{bx\left(c+dx^2\right)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
(b*x*(c + d*x^2)^2)/(5*f) - (((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))
/(3*f) - (-(((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2
*f^2))*x)/f) - (15*(b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(3/2)))/(3*f))/(5*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

method	result
default	$\frac{\frac{1}{5}b\,d^{2}x^{5}f^{2}+\frac{1}{3}a\,d^{2}f^{2}x^{3}+\frac{2}{3}bcd\,f^{2}x^{3}-\frac{1}{3}b\,d^{2}ef\,x^{3}+2acd\,f^{2}x-a\,d^{2}efx+b\,c^{2}f^{2}x-2bcdefx+b\,d^{2}e^{2}x}{f^{3}}+\frac{(a\,c^{2}f^{3}-2acde\,f^{2}+a\,d^{2}e^{2}f-$
risch	$\frac{b\,d^{2}x^{5}}{5f}+\frac{a\,d^{2}x^{3}}{3f}+\frac{2bcd\,x^{3}}{3f}-\frac{b\,d^{2}e\,x^{3}}{3f^{2}}+\frac{2acdx}{f}-\frac{a\,d^{2}ex}{f^{2}}+\frac{b\,c^{2}x}{f}-\frac{2bcdex}{f^{2}}+\frac{b\,d^{2}e^{2}x}{f^{3}}-\frac{\ln(fx+\sqrt{-ef})a\,c^{2}}{2\sqrt{-ef}}+\frac{\ln(f$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+2/3*b*c*d*f^2*x^3-1/3*b*d^2*e*f
*x^3+2*a*c*d*f^2*x-a*d^2*e*f*x+b*c^2*f^2*x-2*b*c*d*e*f*x+b*d^2*e^2*x)+(a*c
^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^3/
(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.16

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \left[\frac{6\,bd^2ef^3x^5-10\,(bd^2e^2f^2-(2\,bcd+ad^2)ef^3)x^3+15\,(bd^2e^3-ac^2f^3-(2\,bcd+ad^2)e^2f+(bc^2+2\,acd}{30\,ef^4}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/30*(6*b*d^2*e*f^3*x^5 - 10*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^
3 + 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d
)*e*f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(b*
d^2*e^3*f - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4
), 1/15*(3*b*d^2*e*f^3*x^5 - 5*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x
^3 - 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*
d)*e*f^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(b*d^2*e^3*f - (2*b*c*d + a
*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(105) = 210.

Time = 0.50 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.99

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{bd^2x^5}{5f}+x^3\left(\frac{ad^2}{3f}+\frac{2bcd}{3f}-\frac{bd^2e}{3f^2}\right)+x\left(\frac{2acd}{f}-\frac{ad^2e}{f^2}+\frac{bc^2}{f}-\frac{2bcde}{f^2}+\frac{bd^2e^2}{f^3}\right) - \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
b*d**2*x**5/(5*f) + x**3*(a*d**2/(3*f) + 2*b*c*d/(3*f) - b*d**2*e/(3*f**2)
) + x*(2*a*c*d/f - a*d**2*e/f**2 + b*c**2/f - 2*b*c*d*e/f**2 + b*d**2*e**2
/f**3) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(-e*f**3*sqrt(-1/
(e*f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**
2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + sqrt(-1/
(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(a*f - b
*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*
e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = -\frac{(bd^2e^3 - 2\,bcde^2f - ad^2e^2f + bc^2ef^2 + 2\,acdef^2 - ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^3} + \frac{3\,bd^2f^4x^5 - 5\,bd^2ef^3x^3 + 10\,bcdf^4x^3 + 5\,ad^2f^4x^3 + 15\,bd^2e^2f^2x - 30\,bcdef^3x - 15\,ad^2ef^3x + 15\,bc^2f}{15\,f^5}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
-(b*d^2*e^3 - 2*b*c*d*e^2*f - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 -
a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^3) + 1/15*(3*b*d^2*f^4*x^5 -
 5*b*d^2*e*f^3*x^3 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 + 15*b*d^2*e^2*f^2
*x - 30*b*c*d*e*f^3*x - 15*a*d^2*e*f^3*x + 15*b*c^2*f^4*x + 30*a*c*d*f^4*x
)/f^5
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = x^3\left(\frac{a\,d^2+2\,b\,c\,d}{3\,f}-\frac{b\,d^2\,e}{3\,f^2}\right)+x\left(\frac{b\,c^2+2\,a\,d\,c}{f}-\frac{e\left(\frac{a\,d^2+2\,b\,c\,d}{f}-\frac{b\,d^2\,e}{f^2}\right)}{f}\right)+\frac{b\,d^2\,x^5}{5\,f}+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,f^{7/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

output
```
x^3*((a*d^2 + 2*b*c*d)/(3*f) - (b*d^2*e)/(3*f^2)) + x*((b*c^2 + 2*a*c*d)/f
 - (e*((a*d^2 + 2*b*c*d)/f - (b*d^2*e)/f^2))/f) + (b*d^2*x^5)/(5*f) + (ata
n((f^(1/2)*x*(a*f - b*e)*(c*f - d*e)^2)/(e^(1/2)*(a*c^2*f^3 - b*d^2*e^3 +
a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f)))*(a*f - b*e)*(
c*f - d*e)^2)/(e^(1/2)*f^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.34

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= \frac{15\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,c^2f^3 - 30\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)acde\,f^2 + 15\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,d^2e^2f - 15\sqrt{f}}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*f**3 - 30*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e*f**2 + 15*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*a*d**2*e**2*f - 15*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*b*c**2*e*f**2 + 30*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*b*c*d*e**2*f - 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*b*d**2*e**3 + 30*a*c*d*e*f**3*x - 15*a*d**2*e**2*f**2*x + 5*a*d**2*e*f**3
*x**3 + 15*b*c**2*e*f**3*x - 30*b*c*d*e**2*f**2*x + 10*b*c*d*e*f**3*x**3 +
 15*b*d**2*e**3*f*x - 5*b*d**2*e**2*f**2*x**3 + 3*b*d**2*e*f**3*x**5)/(15*
e*f**4)
```

3.211 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx = -\frac{d(2bde-2bcf-adf)x}{f^3} + \frac{bd^2x^3}{3f^2} - \frac{(be-af)(de-cf)^2x}{2ef^3(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

output
```
-d*(-a*d*f-2*b*c*f+2*b*d*e)*x/f^3+1/3*b*d^2*x^3/f^2-1/2*(-a*f+b*e)*(-c*f+d
*e)^2*x/e/f^3/(f*x^2+e)+1/2*(-c*f+d*e)*(b*e*(-c*f+5*d*e)-a*f*(c*f+3*d*e))*
arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx = \frac{d(-2bde+2bcf+adf)x}{f^3} + \frac{bd^2x^3}{3f^2} - \frac{(be-af)(de-cf)^2x}{2ef^3(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
(d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((b*e - a*f
)*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f)
 - a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {401, 25, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\left(c+dx^2\right)^2}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}+\frac{dx(c+dx^2)(5be-3af)}{3f}}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\int \frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3(de-cf)(be(5de-cf)-af(cf+3de))\int \frac{1}{fx^2+e}dx}{f}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
-1/2*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) + ((d*(5*b*e - 3*a*f)
*x*(c + d*x^2))/(3*f) - ((d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*
x)/f - (3*(d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(Sqrt
[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

method	result
default	$\frac{d\left(\frac{1}{3}bdf\,x^3+adfx+2bcfx-2bdex\right)}{f^3}+\frac{\frac{\left(a\,c^2f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-e^3b\,d^2\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(a\,c^2f^3+2acde\,f^2-3a\,d^2e^2f+b\,c^2e\,f^2-6\right.}{2e\sqrt{ef}}}{f^3}$
risch	$\frac{b\,d^2x^3}{3f^2}+\frac{d^2ax}{f^2}+\frac{2dbcx}{f^2}-\frac{2d^2bex}{f^3}+\frac{(a\,c^2f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-e^3b\,d^2)x}{2e\,f^3(f\,x^2+e)}-\frac{\ln\left(fx+\sqrt{-ef}\right)a\,c^2}{4\sqrt{-ef}\,e}-$ ln

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
d/f^3*(1/3*b*d*f*x^3+a*d*f*x+2*b*c*f*x-2*b*d*e*x)+1/f^3*(1/2*(a*c^2*f^3-2*
a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/e*x/(f*x^2+e)
+1/2*(a*c^2*f^3+2*a*c*d*e*f^2-3*a*d^2*e^2*f+b*c^2*e*f^2-6*b*c*d*e^2*f+5*b*
d^2*e^3)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(122) = 244.

Time = 0.09 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,bd^2e^2f^3x^5 - 4\,(5\,bd^2e^3f^2 - 3\,(2\,bcd + ad^2)e^2f^3)x^3 - 3\,(5\,bd^2e^4 + ac^2ef^3 - 3\,(2\,bcd + ad^2)e^3f + (bc^2\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b*d^2*e^2*f^3*x^5 - 4*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*
f^3)*x^3 - 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c
^2 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e
^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f
)*x - e)/(f*x^2 + e)) - 6*(5*b*d^2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^
2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4), 1/6*(2
*b*d^2*e^2*f^3*x^5 - 2*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f^3)*x^3
 + 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a
*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*f^2 +
 (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(5*b*d^
2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^
2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(128) = 256.

Time = 1.20 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.55

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{bd^2x^3}{3f^2} + x\left(\frac{ad^2}{f^2}+\frac{2bcd}{f^2}-\frac{2bd^2e}{f^3}\right) + \frac{x(ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3)}{2e^2f^3+2ef^4x^2} - \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(-\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3}+x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3}+x\right)}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
b*d**2*x**3/(3*f**2) + x*(a*d**2/f**2 + 2*b*c*d/f**2 - 2*b*d**2*e/f**3) +
x*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*
e**2*f - b*d**2*e**3)/(2*e**2*f**3 + 2*e*f**4*x**2) - sqrt(-1/(e**3*f**7))
*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(-e**2*f**3*
sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e
**2)/(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b
*c*d*e**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*
c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f**3*sqrt(-1/(e**3*f**
7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/(a*c**2*f**3
 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b
*d**2*e**3) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx &= \frac{(5\,bd^2e^3-6\,bcde^2f-3\,ad^2e^2f+bc^2ef^2+2\,acdef^2+ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,\sqrt{ef}ef^3} \\ &\quad - \frac{bd^2e^3x-2\,bcde^2fx-ad^2e^2fx+bc^2ef^2x+2\,acdef^2x-ac^2f^3x}{2\,(fx^2+e)ef^3} \\ &\quad + \frac{bd^2f^4x^3-6\,bd^2ef^3x+6\,bcdf^4x+3\,ad^2f^4x}{3\,f^6}\end{aligned}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(5*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e
*f^2 + a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e*f^3) - 1/2*(b*d^2*e^3
*x - 2*b*c*d*e^2*f*x - a*d^2*e^2*f*x + b*c^2*e*f^2*x + 2*a*c*d*e*f^2*x - a
*c^2*f^3*x)/((f*x^2 + e)*e*f^3) + 1/3*(b*d^2*f^4*x^3 - 6*b*d^2*e*f^3*x + 6
*b*c*d*f^4*x + 3*a*d^2*f^4*x)/f^6
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = x\left(\frac{a\,d^2+2\,b\,c\,d}{f^2}-\frac{2\,b\,d^2\,e}{f^3}\right)+\frac{b\,d^2\,x^3}{3\,f^2} + \frac{x\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}{2\,e\,(f^4\,x^2+e\,f^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^2\,e\,f^2+a\,c^2\,f^3-6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3-3\,a\,d^2\,e^2\,f)}\right)\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,\ldots}{2\,e^{3/2}\,f^{7/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

output
```
x*((a*d^2 + 2*b*c*d)/f^2 - (2*b*d^2*e)/f^3) + (b*d^2*x^3)/(3*f^2) + (x*(a*
c^2*f^3 - b*d^2*e^3 + a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*
e^2*f))/(2*e*(e*f^3 + f^4*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)*(a*c*f^2 -
5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^2*f^3 + 5*b*d^2*e^3 - 3*a*
d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 - 6*b*c*d*e^2*f)))*(c*f - d*e)*(a*
c*f^2 - 5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.54

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,c^2e\,f^3+3\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,c^2f^4x^2+6\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)acd\,e^2f^2+6\sqrt{f}\ldots}{\ldots}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e*f**3 + 3*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*f**4*x**2 + 6*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e**2*f**2 + 6*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e*f**3*x**2 - 9*sqrt(f)*sqrt(e)*atan((f*x)/(
sqrt(f)*sqrt(e)))*a*d**2*e**3*f - 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sq
rt(e)))*a*d**2*e**2*f**2*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*b*c**2*e**2*f**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b
*c**2*e*f**3*x**2 - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d
*e**3*f - 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**2*f**2
*x**2 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**4 + 15*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**3*f*x**2 + 3*a*c**
2*e*f**4*x - 6*a*c*d*e**2*f**3*x + 9*a*d**2*e**3*f**2*x + 6*a*d**2*e**2*f*
*3*x**3 - 3*b*c**2*e**2*f**3*x + 18*b*c*d*e**3*f**2*x + 12*b*c*d*e**2*f**3
*x**3 - 15*b*d**2*e**4*f*x - 10*b*d**2*e**3*f**2*x**3 + 2*b*d**2*e**2*f**3
*x**5)/(6*e**2*f**4*(e + f*x**2))
```

3.212 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 183

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx &= \frac{bd^2x}{f^3} - \frac{(be-af)(de-cf)^2x}{4ef^3(e+fx^2)^2} + \frac{(de-cf)(be(9de-cf)-af(5de+3cf))x}{8e^2f^3(e+fx^2)} \\ &\quad - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}\end{aligned}$$

output

```
b*d^2*x/f^3-1/4*(-a*f+b*e)*(-c*f+d*e)^2*x/e/f^3/(f*x^2+e)^2+1/8*(-c*f+d*e)
*(b*e*(-c*f+9*d*e)-a*f*(3*c*f+5*d*e))*x/e^2/f^3/(f*x^2+e)-1/8*(b*e*(-c^2*f
^2-6*c*d*e*f+15*d^2*e^2)-a*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*arctan(f^(1/
2)*x/e^(1/2))/e^(5/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{bd^2x}{f^3} - \frac{(be-af)(de-cf)^2x}{4ef^3(e+fx^2)^2} + \frac{(de-cf)(be(9de-cf)-af(5de+3cf))x}{8e^2f^3(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2)^2) + ((
d*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^3*(e + f*
x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*
e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {401, 25, 401, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$\Big\downarrow \text{401}$$
$$-\frac{\int -\frac{(dx^2+c)(d(5be-af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)(d(5be-af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef} - \frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{-\frac{\frac{\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{f} - \frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef} - \frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}} - \frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef} - \frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{2ef(e+fx^2)}}{4ef} - \frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

input

```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output

```
-1/4*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(5*d*
e - c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b*
e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/f) + ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)
/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

method	result
default	$\frac{b\,d^{2} x}{f^{3}}+\frac{\frac{\frac{f \left(3 a \,c^{2} f^{3}+2 a c d e \,f^{2}-5 a \,d^{2} e^{2} f +b \,c^{2} e \,f^{2}-10 b c d \,e^{2} f +9 e^{3} b \,d^{2}\right) x^{3}}{8 e^{2}}+\frac{\left(5 a \,c^{2} f^{3}-2 a c d e \,f^{2}-3 a \,d^{2} e^{2} f -b \,c^{2} e \,f^{2}-6 b c d \,e^{2} f +7 e^{3} b \,d^{2}\right) x}{8 e}}{\left(f \,x^{2}+e \right)^{2}}+}{f^{3}}$
risch	$\frac{b\,d^{2} x}{f^{3}}+\frac{\frac{f \left(3 a \,c^{2} f^{3}+2 a c d e \,f^{2}-5 a \,d^{2} e^{2} f +b \,c^{2} e \,f^{2}-10 b c d \,e^{2} f +9 e^{3} b \,d^{2}\right) x^{3}}{8 e^{2}}+\frac{\left(5 a \,c^{2} f^{3}-2 a c d e \,f^{2}-3 a \,d^{2} e^{2} f -b \,c^{2} e \,f^{2}-6 b c d \,e^{2} f +7 e^{3} b \,d^{2}\right) x}{8 e}}{f^{3} \left(f \,x^{2}+e \right)^{2}}-$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
b*d^2*x/f^3+1/f^3*((1/8*f*(3*a*c^2*f^3+2*a*c*d*e*f^2-5*a*d^2*e^2*f+b*c^2*e
*f^2-10*b*c*d*e^2*f+9*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-2*a*c*d*e*f^2-3*
a*d^2*e^2*f-b*c^2*e*f^2-6*b*c*d*e^2*f+7*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3
*a*c^2*f^3+2*a*c*d*e*f^2+3*a*d^2*e^2*f+b*c^2*e*f^2+6*b*c*d*e^2*f-15*b*d^2*
e^3)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(169) = 338$.

Time = 0.09 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.25

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*b*d^2*e^3*f^3*x^5 + 2*(25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 - 5*(2*b
*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 + (15*b*d^2*e^5 - 3
*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2 + (
15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*
a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2
)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt
(-e*f)*x - e)/(f*x^2 + e)) + 2*(15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*
c*d + a*d^2)*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*
f^5*x^2 + e^5*f^4), 1/8*(8*b*d^2*e^3*f^3*x^5 + (25*b*d^2*e^4*f^2 + 3*a*c^2
*e*f^5 - 5*(2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 - (1
5*b*d^2*e^5 - 3*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c
*d)*e^3*f^2 + (15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^
3 - (b*c^2 + 2*a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(
2*b*c*d + a*d^2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(e*f)*arcta
n(sqrt(e*f)*x/e) + (15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a*d^2)
*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2 + e^
5*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(177) = 354$.

Time = 4.59 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{x^3\cdot(3ac^2f^4+2acdef^3-5ad^2e^2f^2+bc^2ef^3-10bcde^2f^2+9bd^2e^3f)+x(5ac^2ef^3-2acde^2f^2-3ad^2}{8e^4f^3+16e^3f^4x^2+8e^2f^5x^4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a
*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(-e**3*
f**3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 +
2*a*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d
**2*e**3)*log(e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**
4 + 2*a*c*d*e*f**3 - 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f*
*2 + 9*b*d**2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*
e**3*f - b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**4*f**3
+ 16*e**3*f**4*x**2 + 8*e**2*f**5*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{bd^2x}{f^3} - \frac{(15\,bd^2e^3 - 6\,bcde^2f - 3\,ad^2e^2f - bc^2ef^2 - 2\,acdef^2 - 3\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\sqrt{ef}e^2f^3}$$
$$+ \frac{9\,bd^2e^3fx^3 - 10\,bcde^2f^2x^3 - 5\,ad^2e^2f^2x^3 + bc^2ef^3x^3 + 2\,acdef^3x^3 + 3\,ac^2f^4x^3 + 7\,bd^2e^4x - 6\,bcde^3f}{8\,(fx^2+e)^2e^2f^3}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b*d^2*x/f^3 - 1/8*(15*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f - b*c^2*e*
f^2 - 2*a*c*d*e*f^2 - 3*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^
3) + 1/8*(9*b*d^2*e^3*f*x^3 - 10*b*c*d*e^2*f^2*x^3 - 5*a*d^2*e^2*f^2*x^3 +
 b*c^2*e*f^3*x^3 + 2*a*c*d*e*f^3*x^3 + 3*a*c^2*f^4*x^3 + 7*b*d^2*e^4*x - 6
*b*c*d*e^3*f*x - 3*a*d^2*e^3*f*x - b*c^2*e^2*f^2*x - 2*a*c*d*e^2*f^2*x + 5
*a*c^2*e*f^3*x)/((f*x^2 + e)^2*e^2*f^3)
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\left(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-15\,b\,d^2\,e^3+3\,a\,d^2\,e^2\,f\right)}{8\,e^{5/2}\,f^{7/2}}$$
$$-\frac{\frac{x\left(b\,c^2\,e\,f^2-5\,a\,c^2\,f^3+6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-7\,b\,d^2\,e^3+3\,a\,d^2\,e^2\,f\right)}{8\,e}-\frac{x^3\left(b\,c^2\,e\,f^3+3\,a\,c^2\,f^4-10\,b\,c\,d\,e^2\,f^2+2\,a\,c\,d\,e\,f^3+9\,b\,d^2\,e^3\,f-5\,a\,d^2}{8\,e^2}}{e^2\,f^3+2\,e\,f^4\,x^2+f^5\,x^4}$$
$$+\frac{b\,d^2\,x}{f^3}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

output
```
(atan((f^(1/2)*x)/e^(1/2))*(3*a*c^2*f^3 - 15*b*d^2*e^3 + 3*a*d^2*e^2*f + b
*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c*d*e^2*f))/(8*e^(5/2)*f^(7/2)) - ((x*(3*
a*d^2*e^2*f - 7*b*d^2*e^3 - 5*a*c^2*f^3 + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*
b*c*d*e^2*f))/(8*e) - (x^3*(3*a*c^2*f^4 - 5*a*d^2*e^2*f^2 + b*c^2*e*f^3 +
9*b*d^2*e^3*f - 10*b*c*d*e^2*f^2 + 2*a*c*d*e*f^3))/(8*e^2))/(e^2*f^3 + f^5
*x^4 + 2*e*f^4*x^2) + (b*d^2*x)/f^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.92

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output

```
(3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e**2*f**3 + 6*sqrt
(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e*f**4*x**2 + 3*sqrt(f)*s
qrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*f**5*x**4 + 2*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e**3*f**2 + 4*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*a*c*d*e**2*f**3*x**2 + 2*sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*a*c*d*e*f**4*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*a*d**2*e**4*f + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)
))*a*d**2*e**3*f**2*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a*d**2*e**2*f**3*x**4 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c
**2*e**3*f**2 + 2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e**
2*f**3*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e*f**4*
x**4 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**4*f + 12*s
qrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**3*f**2*x**2 + 6*sqrt
(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**2*f**3*x**4 - 15*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**5 - 30*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**4*f*x**2 - 15*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**3*f**2*x**4 + 5*a*c**2*e**2*f**4*x + 3*
a*c**2*e*f**5*x**3 - 2*a*c*d*e**3*f**3*x + 2*a*c*d*e**2*f**4*x**3 - 3*a*d*
*2*e**4*f**2*x - 5*a*d**2*e**3*f**3*x**3 - b*c**2*e**3*f**3*x + b*c**2*e**
2*f**4*x**3 - 6*b*c*d*e**4*f**2*x - 10*b*c*d*e**3*f**3*x**3 + 15*b*d**2...
```

3.213 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 244

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx &= -\frac{(be-af)(de-cf)^2x}{6ef^3(e+fx^2)^3}+\frac{(de-cf)(be(13de-cf)-af(7de+5cf))x}{24e^2f^3(e+fx^2)^2}\\&\quad-\frac{(be(11d^2e^2-2cdef-c^2f^2)-af(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^3(e+fx^2)}\\&\quad+\frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}\end{aligned}$$

output
```
-1/6*(-a*f+b*e)*(-c*f+d*e)^2*x/e/f^3/(f*x^2+e)^3+1/24*(-c*f+d*e)*(b*e*(-c*
f+13*d*e)-a*f*(5*c*f+7*d*e))*x/e^2/f^3/(f*x^2+e)^2-1/16*(b*e*(-c^2*f^2-2*c
*d*e*f+11*d^2*e^2)-a*f*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*x/e^3/f^3/(f*x^2+e)+
1/16*(b*e*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+a*f*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))
*arctan(f^(1/2)*x/e^(1/2))/e^(7/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$
$$= -\frac{(be-af)(de-cf)^2x}{6ef^3(e+fx^2)^3} + \frac{(de-cf)(be(13de-cf)-af(7de+5cf))x}{24e^2f^3(e+fx^2)^2} + \frac{(be(-11d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))x}{16e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
-1/6*((b*e - a*f)*(d*e - c*f)^2*x)/(e*f^3*(e + f*x^2)^3) + ((d*e - c*f)*(b
*e*(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e + f*x^2)^2) + (
(b*e*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^
2*f^2))*x)/(16*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(16*e^(7/2)*f^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {401, 25, 401, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$
$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(dx^2+c)\left(d(5be+af)x^2+c(be+5af)\right)}{(fx^2+e)^3}dx}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{(dx^2+c)\left(d(5be+af)x^2+c(be+5af)\right)}{(fx^2+e)^3}dx}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 401

$$\frac{-\frac{\int -\frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 298

$$\frac{\frac{\frac{3\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{2ef}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)(de(af+\cdots}{4ef(e\cdots}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{2e^{3/2}f^{3/2}}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)(de(a\cdots}{4ef\cdots}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
-1/6*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^3) + (-1/4*((d*e*(5*b*
e + a*f) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)^2) + (-1/2*(
(a*f*(3*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f -
3*c^2*f^2))*x)/(e*f*(e + f*x^2)) + (3*(b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^
2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(
2*e^(3/2)*f^(3/2)))/(4*e*f))/(6*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.18

method	result
default	$\frac{\frac{\left(5a\,c^2 f^3+2acde\,f^2+a\,d^2e^2f+b\,c^2e\,f^2+2bcd\,e^2f-11e^3b\,d^2\right)x^5}{16e^3f}+\frac{\left(5a\,c^2f^3+2acde\,f^2-a\,d^2e^2f+b\,c^2e\,f^2-2bcd\,e^2f-5e^3b\,d^2\right)x^3}{6e^2f^2}+\frac{\left(11a\,c^2f^3-2a\cdots\right.}{}}{\left(f\,x^2+e\right)^3}$
risch	$\frac{\frac{\left(5a\,c^2 f^3+2acde\,f^2+a\,d^2e^2f+b\,c^2e\,f^2+2bcd\,e^2f-11e^3b\,d^2\right)x^5}{16e^3f}+\frac{\left(5a\,c^2f^3+2acde\,f^2-a\,d^2e^2f+b\,c^2e\,f^2-2bcd\,e^2f-5e^3b\,d^2\right)x^3}{6e^2f^2}+\frac{\left(11a\,c^2f^3-2a\cdots\right.}{}}{\left(f\,x^2+e\right)^3}$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f-11*
b*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^2*e^2*f+b*c^2*e*f^
2-2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(11*a*c^2*f^3-2*a*c*d*e*f^2-
a*d^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+1/
16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f+5*b*d^
2*e^3)/e^3/f^3/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(228) = 456.

Time = 0.10 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.20

$$\int \frac{(a+bx^2)\,(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[-1/96*(6*(11*b*d^2*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 -
(b*c^2 + 2*a*c*d)*e^2*f^5)*x^5 + 16*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (
2*b*c*d + a*d^2)*e^4*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 + 3*(5*b*d^2*e^6
 + 5*a*c^2*e^3*f^3 + (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 +
 (5*b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a
*c*d)*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*
e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*
f^4 + (2*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(-e*
f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6*(5*b*d^2*e^6*f - 11*a
*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*f^3)*x)/(
e^4*f^7*x^6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4), -1/48*(3*(11*b*d^2
*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (b*c^2 + 2*a*c*d)*e
^2*f^5)*x^5 + 8*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*b*c*d + a*d^2)*e^4
*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 - 3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 +
 (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e^3*f^3 +
5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)*e*f^5)*x^6 + 3
*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2
*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (2*b*c*d + a*d
^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x
/e) + 3*(5*b*d^2*e^6*f - 11*a*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(240) = 480$.

Time = 18.07 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{e^7f^7}}\cdot(5ac^2f^3+2acdef^2+ad^2e^2f+bc^2ef^2+2bcde^2f+5bd^2e^3)\log\left(-e^4f^3\sqrt{-\frac{1}{e^7f^7}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{e^7f^7}}\cdot(5ac^2f^3+2acdef^2+ad^2e^2f+bc^2ef^2+2bcde^2f+5bd^2e^3)\log\left(e^4f^3\sqrt{-\frac{1}{e^7f^7}}+x\right)}{32} + \frac{x^5\cdot(15ac^2f^5+6acdef^4+3ad^2e^2f^3+3bc^2ef^4+6bcde^2f^3-33bd^2e^3f^2)+x^3\cdot(40ac^2ef^4+16acde^2f}{48e^6f^3+14}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

output
```
-sqrt(-1/(e**7*f**7))*(5*a*c**2*f**3 + 2*a*c*d*e*f**2 + a*d**2*e**2*f + b*
c**2*e*f**2 + 2*b*c*d*e**2*f + 5*b*d**2*e**3)*log(-e**4*f**3*sqrt(-1/(e**7
*f**7)) + x)/32 + sqrt(-1/(e**7*f**7))*(5*a*c**2*f**3 + 2*a*c*d*e*f**2 + a
*d**2*e**2*f + b*c**2*e*f**2 + 2*b*c*d*e**2*f + 5*b*d**2*e**3)*log(e**4*f*
*3*sqrt(-1/(e**7*f**7)) + x)/32 + (x**5*(15*a*c**2*f**5 + 6*a*c*d*e*f**4 +
 3*a*d**2*e**2*f**3 + 3*b*c**2*e*f**4 + 6*b*c*d*e**2*f**3 - 33*b*d**2*e**3
*f**2) + x**3*(40*a*c**2*e*f**4 + 16*a*c*d*e**2*f**3 - 8*a*d**2*e**3*f**2
+ 8*b*c**2*e**2*f**3 - 16*b*c*d*e**3*f**2 - 40*b*d**2*e**4*f) + x*(33*a*c*
*2*e**2*f**3 - 6*a*c*d*e**3*f**2 - 3*a*d**2*e**4*f - 3*b*c**2*e**3*f**2 -
6*b*c*d*e**4*f - 15*b*d**2*e**5))/(48*e**6*f**3 + 144*e**5*f**4*x**2 + 144
*e**4*f**5*x**4 + 48*e**3*f**6*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \frac{(5\,bd^2e^3+2\,bcde^2f+ad^2e^2f+bc^2ef^2+2\,acdef^2+5\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\,\sqrt{ef}e^3f^3} - \frac{33\,bd^2e^3f^2x^5-6\,bcde^2f^3x^5-3\,ad^2e^2f^3x^5-3\,bc^2ef^4x^5-6\,acdef^4x^5-15\,ac^2f^5x^5+40\,bd^2e^4fx^3+1}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(5*b*d^2*e^3 + 2*b*c*d*e^2*f + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*
f^2 + 5*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^3) - 1/48*(33*b*
d^2*e^3*f^2*x^5 - 6*b*c*d*e^2*f^3*x^5 - 3*a*d^2*e^2*f^3*x^5 - 3*b*c^2*e*f^
4*x^5 - 6*a*c*d*e*f^4*x^5 - 15*a*c^2*f^5*x^5 + 40*b*d^2*e^4*f*x^3 + 16*b*c
*d*e^3*f^2*x^3 + 8*a*d^2*e^3*f^2*x^3 - 8*b*c^2*e^2*f^3*x^3 - 16*a*c*d*e^2*
f^3*x^3 - 40*a*c^2*e*f^4*x^3 + 15*b*d^2*e^5*x + 6*b*c*d*e^4*f*x + 3*a*d^2*
e^4*f*x + 3*b*c^2*e^3*f^2*x + 6*a*c*d*e^3*f^2*x - 33*a*c^2*e^2*f^3*x)/((f*
x^2 + e)^3*e^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \frac{\frac{x^3\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3-2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-5\,b\,d^2\,e^3-a\,d^2\,e^2\,f)}{6\,e^2\,f^2}-\frac{x\,(b\,c^2\,e\,f^2-11\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e\,f^3}+}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e^{7/2}\,f^{7/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

output
```
((x^3*(5*a*c^2*f^3 - 5*b*d^2*e^3 - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f
^2 - 2*b*c*d*e^2*f))/(6*e^2*f^2) - (x*(5*b*d^2*e^3 - 11*a*c^2*f^3 + a*d^2*
e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e*f^3) + (x^5*(5
*a*c^2*f^3 - 11*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*
b*c*d*e^2*f))/(16*e^3*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) + (a
tan((f^(1/2)*x)/e^(1/2))*(5*a*c^2*f^3 + 5*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*
e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e^(7/2)*f^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 988, normalized size of antiderivative = 4.05

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e**3*f**3 + 45*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e**2*f**4*x**2 + 45*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*e*f**5*x**4 + 15*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*f**6*x**6 + 6*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e**4*f**2 + 18*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e**3*f**3*x**2 + 18*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*c*d*e**2*f**4*x**4 + 6*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*c*d*e*f**5*x**6 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt
(f)*sqrt(e)))*a*d**2*e**5*f + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*a*d**2*e**4*f**2*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*a*d**2*e**3*f**3*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
a*d**2*e**2*f**4*x**6 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*
c**2*e**4*f**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e*
*3*f**3*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e**2
*f**4*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e*f**5
*x**6 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**5*f + 18*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**4*f**2*x**2 + 18*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**3*f**3*x**4 + 6*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**2*f**4*x**6 + 15*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d**2*e**6 + 45*sqrt(f)*sqrt(e)...
```

3.214 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx &= ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3\\
&\quad+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5\\
&\quad+\frac{1}{7}\left(3bce\left(d^2e^2+3cdef+c^2f^2\right)\right.\\
&\qquad\left.+a\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7\\
&\quad+\frac{1}{9}\left(3adf\left(d^2e^2+3cdef+c^2f^2\right)\right.\\
&\qquad\left.+b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9\\
&\quad+\frac{3}{11}df\left(adf(de+cf)\right.\\
&\qquad\left.+b\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{11}\\
&\quad+\frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13}+\frac{1}{15}bd^3f^3x^{15}\end{aligned}$$

output
```
a*c^3*e^3*x+1/3*c^2*e^2*(b*c*e+3*a*(c*f+d*e))*x^3+3/5*c*e*(b*c*e*(c*f+d*e)
+a*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(3*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^
2)+a*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/9*(3*a*d*f*(c^2*
f^2+3*c*d*e*f+d^2*e^2)+b*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^
9+3/11*d*f*(a*d*f*(c*f+d*e)+b*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^11+1/13*d^2*f
^2*(a*d*f+3*b*(c*f+d*e))*x^13+1/15*b*d^3*f^3*x^15
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = ac^3e^3x + \frac{1}{3}c^2e^2(bce+3a(de+cf))x^3 + \frac{3}{5}ce\big(bce(de+cf)+a(d^2e^2+3cdef+c^2f^2)\big)x^5 + \frac{1}{7}\big(3bce(d^2e^2+3cdef+c^2f^2) + a(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3)\big)x^7 + \frac{1}{9}\big(3adf(d^2e^2+3cdef+c^2f^2) + b(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3)\big)x^9 + \frac{3}{11}df\big(adf(de+cf) + b(d^2e^2+3cdef+c^2f^2)\big)x^{11} + \frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13} + \frac{1}{15}bd^3f^3x^{15}$$

input
```
Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d
*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2
 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3
*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f
) + b*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d
*e + c*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx$$

$$\Big\downarrow \; 396$$

$$\int \left(3dfx^{10}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+3cex^4\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+c^2e^2x^2\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

input
```
Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d
*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2
 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3
*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f
) + b*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d
*e + c*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09

method	result
default	$\frac{b\,d^{3} f^{3} x^{15}}{15}+\frac{\left(\left(a\,d^{3}+3 b c\,d^{2}\right) f^{3}+3 b\,d^{3} e\,f^{2}\right) x^{13}}{13}+\frac{\left(\left(3 a c\,d^{2}+3 b\,c^{2} d\right) f^{3}+3 \left(a\,d^{3}+3 b c\,d^{2}\right) e\,f^{2}+3 b\,d^{3} e^{2} f\right) x^{11}}{11}+\frac{((3 a\,c^{2} d+c}{}$
norman	$a\,c^{3} e^{3} x+\left(c^{3} a\,e^{2} f+a\,c^{2} d\,e^{3}+\frac{1}{3} b\,c^{3} e^{3}\right) x^{3}+\left(\frac{3}{5} c^{3} a e\,f^{2}+\frac{9}{5} a\,c^{2} d\,e^{2} f+\frac{3}{5} a c\,d^{2} e^{3}+\frac{3}{5} b\,c^{3} e^{2} f\right.$
gosper	$\frac{9}{5} x^{5} a\,c^{2} d\,e^{2} f+x^{3} c^{3} a\,e^{2} f+x^{9} b c\,d^{2} e^{2} f+\frac{9}{7} x^{7} b\,c^{2} d\,e^{2} f+x^{9} a c\,d^{2} e\,f^{2}+x^{9} b\,c^{2} d e\,f^{2}+\frac{9}{11} x^{11} b$
risch	$\frac{9}{5} x^{5} a\,c^{2} d\,e^{2} f+x^{3} c^{3} a\,e^{2} f+x^{9} b c\,d^{2} e^{2} f+\frac{9}{7} x^{7} b\,c^{2} d\,e^{2} f+x^{9} a c\,d^{2} e\,f^{2}+x^{9} b\,c^{2} d e\,f^{2}+\frac{9}{11} x^{11} b$
parallelrisch	$\frac{9}{5} x^{5} a\,c^{2} d\,e^{2} f+x^{3} c^{3} a\,e^{2} f+x^{9} b c\,d^{2} e^{2} f+\frac{9}{7} x^{7} b\,c^{2} d\,e^{2} f+x^{9} a c\,d^{2} e\,f^{2}+x^{9} b\,c^{2} d e\,f^{2}+\frac{9}{11} x^{11} b$
orering	$\frac{x \left(3003 b\,d^{3} f^{3} x^{14}+3465 a\,d^{3} f^{3} x^{12}+10395 b c\,d^{2} f^{3} x^{12}+10395 b\,d^{3} e\,f^{2} x^{12}+12285 a c\,d^{2} f^{3} x^{10}+12285 a\,d^{3} e\,f^{2} x^{10}+12285 b\,c^{2} d\,f}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^13+1/11*(
(3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*d^3*e^2*f)*x^11+1/
9*((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d)*e*f^2+3*(a*d^3+3*b*c*d^2)
*e^2*f+b*d^3*e^3)*x^9+1/7*(c^3*a*f^3+3*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^
2+3*b*c^2*d)*e^2*f+(a*d^3+3*b*c*d^2)*e^3)*x^7+1/5*(3*c^3*a*e*f^2+3*(3*a*c^
2*d+b*c^3)*e^2*f+(3*a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*c^3*a*e^2*f+(3*a*c^
2*d+b*c^3)*e^3)*x^3+a*c^3*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$$
$$= \frac{1}{15}bd^3f^3x^{15} + \frac{1}{13}\left(3\,bd^3ef^2 + \left(3\,bcd^2 + ad^3\right)f^3\right)x^{13}$$
$$+ \frac{3}{11}\left(bd^3e^2f + \left(3\,bcd^2 + ad^3\right)ef^2 + \left(bc^2d + acd^2\right)f^3\right)x^{11}$$
$$+ \frac{1}{9}\left(bd^3e^3 + 3\left(3\,bcd^2 + ad^3\right)e^2f + 9\left(bc^2d + acd^2\right)ef^2 + \left(bc^3 + 3\,ac^2d\right)f^3\right)x^9$$
$$+ ac^3e^3x$$
$$+ \frac{1}{7}\left(ac^3f^3 + \left(3\,bcd^2 + ad^3\right)e^3 + 9\left(bc^2d + acd^2\right)e^2f + 3\left(bc^3 + 3\,ac^2d\right)ef^2\right)x^7$$
$$+ \frac{3}{5}\left(ac^3ef^2 + \left(bc^2d + acd^2\right)e^3 + \left(bc^3 + 3\,ac^2d\right)e^2f\right)x^5$$
$$+ \frac{1}{3}\left(3\,ac^3e^2f + \left(bc^3 + 3\,ac^2d\right)e^3\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13
+ 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)
*x^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^
2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (
3*b*c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*
d)*e*f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*
c^2*d)*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.36

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = ac^3e^3x + \frac{bd^3f^3x^{15}}{15} + x^{13}\left(\frac{ad^3f^3}{13} + \frac{3bcd^2f^3}{13} + \frac{3bd^3ef^2}{13}\right) + x^{11}\cdot\left(\frac{3acd^2f^3}{11} + \frac{3ad^3ef^2}{11} + \frac{3bc^2df^3}{11} + \frac{9bcd^2ef^2}{11} + \frac{3bd^3e^2f}{11}\right) + x^9\left(\frac{ac^2df^3}{3} + acd^2ef^2 + \frac{ad^3e^2f}{3} + \frac{bc^3f^3}{9} + bc^2def^2 + bcd^2e^2f + \frac{bd^3e^3}{9}\right) + x^7\left(\frac{ac^3f^3}{7} + \frac{9ac^2def^2}{7} + \frac{9acd^2e^2f}{7} + \frac{ad^3e^3}{7} + \frac{3bc^3ef^2}{7} + \frac{9bc^2de^2f}{7} + \frac{3bcd^2e^3}{7}\right) + x^5\cdot\left(\frac{3ac^3ef^2}{5} + \frac{9ac^2de^2f}{5} + \frac{3acd^2e^3}{5} + \frac{3bc^3e^2f}{5} + \frac{3bc^2de^3}{5}\right) + x^3\left(ac^3e^2f + ac^2de^3 + \frac{bc^3e^3}{3}\right)$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

output
```
a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*b*c*d**2*
f**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/11 + 3*a*d**3*e*f**
2/11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/11 + 3*b*d**3*e**2*f/11) + x
**9*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 +
 b*c**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7
+ 9*a*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f
**2/7 + 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5
 + 9*a*c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*b*c**2*
d*e**3/5) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx$$
$$= \frac{1}{15}bd^3f^3x^{15} + \frac{1}{13}\left(3\,bd^3ef^2 + \left(3\,bcd^2 + ad^3\right)f^3\right)x^{13}$$
$$+ \frac{3}{11}\left(bd^3e^2f + \left(3\,bcd^2 + ad^3\right)ef^2 + \left(bc^2d + acd^2\right)f^3\right)x^{11}$$
$$+ \frac{1}{9}\left(bd^3e^3 + 3\left(3\,bcd^2 + ad^3\right)e^2f + 9\left(bc^2d + acd^2\right)ef^2 + \left(bc^3 + 3\,ac^2d\right)f^3\right)x^9$$
$$+ ac^3e^3x$$
$$+ \frac{1}{7}\left(ac^3f^3 + \left(3\,bcd^2 + ad^3\right)e^3 + 9\left(bc^2d + acd^2\right)e^2f + 3\left(bc^3 + 3\,ac^2d\right)ef^2\right)x^7$$
$$+ \frac{3}{5}\left(ac^3ef^2 + \left(bc^2d + acd^2\right)e^3 + \left(bc^3 + 3\,ac^2d\right)e^2f\right)x^5$$
$$+ \frac{1}{3}\left(3\,ac^3e^2f + \left(bc^3 + 3\,ac^2d\right)e^3\right)x^3$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13
+ 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)
*x^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^
2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (
3*b*c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*
d)*e*f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*
c^2*d)*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.32

$$\int (a+bx^2) (c+dx^2)^3 (e+fx^2)^3 \, dx = \frac{1}{15} bd^3f^3x^{15} + \frac{3}{13} bd^3ef^2x^{13} + \frac{3}{13} bcd^2f^3x^{13} + \frac{1}{13} ad^3f^3x^{13} + \frac{3}{11} bd^3e^2fx^{11} + \frac{9}{11} bcd^2ef^2x^{11} + \frac{3}{11} ad^3ef^2x^{11} + \frac{3}{11} bc^2df^3x^{11} + \frac{3}{11} acd^2f^3x^{11} + \frac{1}{9} bd^3e^3x^9 + bcd^2e^2fx^9 + \frac{1}{3} ad^3e^2fx^9 + bc^2def^2x^9 + acd^2ef^2x^9 + \frac{1}{9} bc^3f^3x^9 + \frac{1}{3} ac^2df^3x^9 + \frac{3}{7} bcd^2e^3x^7 + \frac{1}{7} ad^3e^3x^7 + \frac{9}{7} bc^2de^2fx^7 + \frac{9}{7} acd^2e^2fx^7 + \frac{3}{7} bc^3ef^2x^7 + \frac{9}{7} ac^2def^2x^7 + \frac{1}{7} ac^3f^3x^7 + \frac{3}{5} bc^2de^3x^5 + \frac{3}{5} acd^2e^3x^5 + \frac{3}{5} bc^3e^2fx^5 + \frac{9}{5} ac^2de^2fx^5 + \frac{3}{5} ac^3ef^2x^5 + \frac{1}{3} bc^3e^3x^3 + ac^2de^3x^3 + ac^3e^2fx^3 + ac^3e^3x$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/15*b*d^3*f^3*x^15 + 3/13*b*d^3*e*f^2*x^13 + 3/13*b*c*d^2*f^3*x^13 + 1/13
*a*d^3*f^3*x^13 + 3/11*b*d^3*e^2*f*x^11 + 9/11*b*c*d^2*e*f^2*x^11 + 3/11*a
*d^3*e*f^2*x^11 + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f^3*x^11 + 1/9*b*d^
3*e^3*x^9 + b*c*d^2*e^2*f*x^9 + 1/3*a*d^3*e^2*f*x^9 + b*c^2*d*e*f^2*x^9 +
a*c*d^2*e*f^2*x^9 + 1/9*b*c^3*f^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + 3/7*b*c*d^2*
e^3*x^7 + 1/7*a*d^3*e^3*x^7 + 9/7*b*c^2*d*e^2*f*x^7 + 9/7*a*c*d^2*e^2*f*x^
7 + 3/7*b*c^3*e*f^2*x^7 + 9/7*a*c^2*d*e*f^2*x^7 + 1/7*a*c^3*f^3*x^7 + 3/5*
b*c^2*d*e^3*x^5 + 3/5*a*c*d^2*e^3*x^5 + 3/5*b*c^3*e^2*f*x^5 + 9/5*a*c^2*d*
e^2*f*x^5 + 3/5*a*c^3*e*f^2*x^5 + 1/3*b*c^3*e^3*x^3 + a*c^2*d*e^3*x^3 + a*
c^3*e^2*f*x^3 + a*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.08

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = x^5\left(\frac{3\,b\,c^3\,e^2\,f}{5}+\frac{3\,a\,c^3\,e\,f^2}{5}+\frac{3\,b\,c^2\,d\,e^3}{5}+\frac{9\,a\,c^2\,d\,e^2\,f}{5}+\frac{3\,a\,c\,d^2\,e^3}{5}\right)+x^{11}\left(\frac{3\,b\,c^2\,d\,f^3}{11}+\frac{9\,b\,c\,d^2\,e\,f^2}{11}+\frac{3\,a\,c\,d^2\,f^3}{11}+\frac{3\,b\,d^3\,e^2\,f}{11}+\frac{3\,a\,d^3\,e\,f^2}{11}\right)+x^7\left(\frac{3\,b\,c^3\,e\,f^2}{7}+\frac{a\,c^3\,f^3}{7}+\frac{9\,b\,c^2\,d\,e^2\,f}{7}+\frac{9\,a\,c^2\,d\,e\,f^2}{7}+\frac{3\,b\,c\,d^2\,e^3}{7}+\frac{9\,a\,c\,d^2\,e^2\,f}{7}+\frac{a\,d^3\,e^3}{7}\right)+x^9\left(\frac{b\,c^3\,f^3}{9}+b\,c^2\,d\,e\,f^2+\frac{a\,c^2\,d\,f^3}{3}+b\,c\,d^2\,e^2\,f+a\,c\,d^2\,e\,f^2+\frac{b\,d^3\,e^3}{9}+\frac{a\,d^3\,e^2\,f}{3}\right)+\frac{b\,d^3\,f^3\,x^{15}}{15}+\frac{c^2\,e^2\,x^3\,(3\,a\,c\,f+3\,a\,d\,e+b\,c\,e)}{3}+\frac{d^2\,f^2\,x^{13}\,(a\,d\,f+3\,b\,c\,f+3\,b\,d\,e)}{13}+a\,c^3\,e^3\,x$$

input
```
int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

output
```
x^5*((3*a*c*d^2*e^3)/5 + (3*b*c^2*d*e^3)/5 + (3*a*c^3*e*f^2)/5 + (3*b*c^3*
e^2*f)/5 + (9*a*c^2*d*e^2*f)/5) + x^11*((3*a*c*d^2*f^3)/11 + (3*b*c^2*d*f^
3)/11 + (3*a*d^3*e*f^2)/11 + (3*b*d^3*e^2*f)/11 + (9*b*c*d^2*e*f^2)/11) +
x^7*((a*c^3*f^3)/7 + (a*d^3*e^3)/7 + (3*b*c*d^2*e^3)/7 + (3*b*c^3*e*f^2)/7
 + (9*a*c*d^2*e^2*f)/7 + (9*a*c^2*d*e*f^2)/7 + (9*b*c^2*d*e^2*f)/7) + x^9*
((b*c^3*f^3)/9 + (b*d^3*e^3)/9 + (a*c^2*d*f^3)/3 + (a*d^3*e^2*f)/3 + a*c*d
^2*e*f^2 + b*c*d^2*e^2*f + b*c^2*d*e*f^2) + (b*d^3*f^3*x^15)/15 + (c^2*e^2
*x^3*(3*a*c*f + 3*a*d*e + b*c*e))/3 + (d^2*f^2*x^13*(a*d*f + 3*b*c*f + 3*b
*d*e))/13 + a*c^3*e^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.35

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$$
$$= \frac{x(3003b\,d^3f^3x^{14}+3465a\,d^3f^3x^{12}+10395bc\,d^2f^3x^{12}+10395b\,d^3e\,f^2x^{12}+12285ac\,d^2f^3x^{10}+12285a\,d^3e}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x)
```

output
```
(x*(45045*a*c**3*e**3 + 45045*a*c**3*e**2*f*x**2 + 27027*a*c**3*e*f**2*x**
4 + 6435*a*c**3*f**3*x**6 + 45045*a*c**2*d*e**3*x**2 + 81081*a*c**2*d*e**2
*f*x**4 + 57915*a*c**2*d*e*f**2*x**6 + 15015*a*c**2*d*f**3*x**8 + 27027*a*
c*d**2*e**3*x**4 + 57915*a*c*d**2*e**2*f*x**6 + 45045*a*c*d**2*e*f**2*x**8
 + 12285*a*c*d**2*f**3*x**10 + 6435*a*d**3*e**3*x**6 + 15015*a*d**3*e**2*f
*x**8 + 12285*a*d**3*e*f**2*x**10 + 3465*a*d**3*f**3*x**12 + 15015*b*c**3*
e**3*x**2 + 27027*b*c**3*e**2*f*x**4 + 19305*b*c**3*e*f**2*x**6 + 5005*b*c
**3*f**3*x**8 + 27027*b*c**2*d*e**3*x**4 + 57915*b*c**2*d*e**2*f*x**6 + 45
045*b*c**2*d*e*f**2*x**8 + 12285*b*c**2*d*f**3*x**10 + 19305*b*c*d**2*e**3
*x**6 + 45045*b*c*d**2*e**2*f*x**8 + 36855*b*c*d**2*e*f**2*x**10 + 10395*b
*c*d**2*f**3*x**12 + 5005*b*d**3*e**3*x**8 + 12285*b*d**3*e**2*f*x**10 + 1
0395*b*d**3*e*f**2*x**12 + 3003*b*d**3*f**3*x**14))/45045
```

3.215 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 180

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = -\frac{(b(de-cf)^3-adf(d^2e^2-3cdef+3c^2f^2))\,x}{f^4} - \frac{d(adf(de-3cf)-b(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^3} - \frac{d^2(bde-3bcf-adf)x^5}{5f^2} + \frac{bd^3x^7}{7f} + \frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

output
```
-(b*(-c*f+d*e)^3-a*d*f*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x/f^4-1/3*d*(a*d*f*(
-3*c*f+d*e)-b*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x^3/f^3-1/5*d^2*(-a*d*f-3*b*c
*f+b*d*e)*x^5/f^2+1/7*b*d^3*x^7/f+(-a*f+b*e)*(-c*f+d*e)^3*arctan(f^(1/2)*x
/e^(1/2))/e^(1/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{(-b(de-cf)^3+adf(d^2e^2-3cdef+3c^2f^2))\,x}{f^4} + \frac{d(adf(-de+3cf)+b(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^3} + \frac{d^2(-bde+3bcf+adf)x^5}{5f^2} + \frac{bd^3x^7}{7f} + \frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^4 + (
d*(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f
^3) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5*f^2) + (b*d^3*x^7)/(7*f) +
 ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {403, 25, 403, 25, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int -\frac{(dx^2+c)^2((7bde-6bcf-7adf)x^2+c(be-7af))}{fx^2+e}dx}{7f} + \frac{bx(c+dx^2)^3}{7f}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}$$

$$\Big\downarrow \ 403$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}+\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}}{7f}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}$$

$$\Big\downarrow \ 403$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\int \frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3\right)\right)x^2+c\left(7af\left(5d^2e^2-12cdfe+15c^2f^2\right)\cdots\right.}{fx^2+e}}{3f}\cdots}{5f}}{7f}$$

$$\Big\downarrow \ 299$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)(de-cf)^3\int \frac{1}{fx^2+e}dx}{f}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3\right)\cdots\right.}{f}}{3f}\cdots}{5f}}{7f}$$

$$\Big\downarrow \ 218$$

$$\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3\cdots\right.\right.}{f}}{3f}\cdots}{5f}}{7f}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
(b*x*(c + d*x^2)^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^
2)/(5*f) - (-1/3*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f +
24*c^2*f^2))*x*(c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c
^2*f^2) - b*(105*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3)
)*x)/f + (105*(b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt
[e]*f^(3/2)))/(3*f))/(5*f))/(7*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.67

method	result
default	$\frac{\frac{1}{7}b\,d^3x^7f^3+\frac{1}{5}a\,d^3f^3x^5+\frac{3}{5}bc\,d^2f^3x^5-\frac{1}{5}b\,d^3e\,f^2x^5+ac\,d^2f^3x^3-\frac{1}{3}a\,d^3e\,f^2x^3+b\,c^2d\,f^3x^3-bc\,d^2e\,f^2x^3+\frac{1}{3}b\,d^3e^2f\,x^3+3a\,c^2d\,f^3x-3}{f^4}$
risch	$\frac{3bc\,d^2x^5}{5f}-\frac{b\,d^3e\,x^5}{5f^2}+\frac{a\,d^3x^5}{5f}+\frac{b\,c^3x}{f}-\frac{\ln\left(fx+\sqrt{-ef}\right)c^3a}{2\sqrt{-ef}}+\frac{\ln\left(-fx+\sqrt{-ef}\right)c^3a}{2\sqrt{-ef}}-\frac{3\ln\left(-fx+\sqrt{-ef}\right)bc\,d^2e^3}{2f^3\sqrt{-ef}}-\frac{a\,d^3e}{3f^2}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/7*b*d^3*x^7*f^3+1/5*a*d^3*f^3*x^5+3/5*b*c*d^2*f^3*x^5-1/5*b*d^3*e
*f^2*x^5+a*c*d^2*f^3*x^3-1/3*a*d^3*e*f^2*x^3+b*c^2*d*f^3*x^3-b*c*d^2*e*f^2
*x^3+1/3*b*d^3*e^2*f*x^3+3*a*c^2*d*f^3*x-3*a*c*d^2*e*f^2*x+a*d^3*e^2*f*x+b
*c^3*f^3*x-3*b*c^2*d*e*f^2*x+3*b*c*d^2*e^2*f*x-b*d^3*e^3*x)+(a*c^3*f^4-3*a
*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-3
*b*c*d^2*e^3*f+b*d^3*e^4)/f^4/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.26

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{e+fx^2}\,dx$$

$$= \left[\frac{30\,bd^3ef^4x^7-42\,(bd^3e^2f^3-(3\,bcd^2+ad^3)ef^4)x^5+70\,(bd^3e^3f^2-(3\,bcd^2+ad^3)e^2f^3+3\,(bc^2d+acd^2}{}\right.$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/210*(30*b*d^3*e*f^4*x^7 - 42*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*d^3)*e*f^4
)*x^5 + 70*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^2*d + a*c
*d^2)*e*f^4)*x^3 - 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3*f
+ 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(-e*f)*lo
g((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 210*(b*d^3*e^4*f - (3*b*c*d^
2 + a*d^3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e
*f^4)*x)/(e*f^5), 1/105*(15*b*d^3*e*f^4*x^7 - 21*(b*d^3*e^2*f^3 - (3*b*c*d
^2 + a*d^3)*e*f^4)*x^5 + 35*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 +
 3*(b*c^2*d + a*c*d^2)*e*f^4)*x^3 + 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^
2 + a*d^3)*e^3*f + 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f
^3)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 105*(b*d^3*e^4*f - (3*b*c*d^2 + a*d^
3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x)
/(e*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(172) = 344$.

Time = 0.73 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{bd^3x^7}{7f} + x^5\left(\frac{ad^3}{5f} + \frac{3bcd^2}{5f} - \frac{bd^3e}{5f^2}\right) + x^3\left(\frac{acd^2}{f} - \frac{ad^3e}{3f^2} + \frac{bc^2d}{f} - \frac{bcd^2e}{f^2} + \frac{bd^3e^2}{3f^3}\right)$$
$$+ x\left(\frac{3ac^2d}{f} - \frac{3acd^2e}{f^2} + \frac{ad^3e^2}{f^3} + \frac{bc^3}{f} - \frac{3bc^2de}{f^2} + \frac{3bcd^2e^2}{f^3} - \frac{bd^3e^3}{f^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4} + x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4} + x\right)}{2}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
b*d**3*x**7/(7*f) + x**5*(a*d**3/(5*f) + 3*b*c*d**2/(5*f) - b*d**3*e/(5*f*
*2)) + x**3*(a*c*d**2/f - a*d**3*e/(3*f**2) + b*c**2*d/f - b*c*d**2*e/f**2
 + b*d**3*e**2/(3*f**3)) + x*(3*a*c**2*d/f - 3*a*c*d**2*e/f**2 + a*d**3*e*
*2/f**3 + b*c**3/f - 3*b*c**2*d*e/f**2 + 3*b*c*d**2*e**2/f**3 - b*d**3*e**
3/f**4) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log(-e*f**4*sqrt(-1
/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3
*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2
 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)
*(c*f - d*e)**3*log(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a
*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*
c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)
/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{(bd^3e^4 - 3\,bcd^2e^3f - ad^3e^3f + 3\,bc^2de^2f^2 + 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 + ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^4} + \frac{15\,bd^3f^6x^7 - 21\,bd^3ef^5x^5 + 63\,bcd^2f^6x^5 + 21\,ad^3f^6x^5 + 35\,bd^3e^2f^4x^3 - 105\,bcd^2ef^5x^3 - 35\,ad^3ef^5x^3}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
(b*d^3*e^4 - 3*b*c*d^2*e^3*f - a*d^3*e^3*f + 3*b*c^2*d*e^2*f^2 + 3*a*c*d^2
*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 + a*c^3*f^4)*arctan(f*x/sqrt(e*f)
)/(sqrt(e*f)*f^4) + 1/105*(15*b*d^3*f^6*x^7 - 21*b*d^3*e*f^5*x^5 + 63*b*c*
d^2*f^6*x^5 + 21*a*d^3*f^6*x^5 + 35*b*d^3*e^2*f^4*x^3 - 105*b*c*d^2*e*f^5*
x^3 - 35*a*d^3*e*f^5*x^3 + 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x^3 - 105
*b*d^3*e^3*f^3*x + 315*b*c*d^2*e^2*f^4*x + 105*a*d^3*e^2*f^4*x - 315*b*c^2
*d*e*f^5*x - 315*a*c*d^2*e*f^5*x + 105*b*c^3*f^6*x + 315*a*c^2*d*f^6*x)/f^
7
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = x\left(\frac{b\,c^3+3\,a\,d\,c^2}{f}+\frac{e\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{f}-\frac{3\,c\,d\,(a\,d+b\,c)}{f}\right)}{f}\right) + x^5\left(\frac{a\,d^3+3\,b\,c\,d^2}{5\,f}-\frac{b\,d^3\,e}{5\,f^2}\right) - x^3\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{3\,f}-\frac{c\,d\,(a\,d+b\,c)}{f}\right)+\frac{b\,d^3\,x^7}{7\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
x*((b*c^3 + 3*a*c^2*d)/f + (e*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))
/f - (3*c*d*(a*d + b*c))/f))/f) + x^5*((a*d^3 + 3*b*c*d^2)/(5*f) - (b*d^3*
e)/(5*f^2)) - x^3*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/(3*f) - (c*
d*(a*d + b*c))/f) + (b*d^3*x^7)/(7*f) + (atan((f^(1/2)*x*(a*f - b*e)*(c*f
- d*e)^3)/(e^(1/2)*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b*c^3*e*f^3 - 3*
a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2)))
*(a*f - b*e)*(c*f - d*e)^3)/(e^(1/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.45

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{105\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,c^3f^4-315\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)a\,c^2de\,f^3+315\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)ac\,d^2e^2f^2 -}{}$$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x)
```

output
```
(105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*f**4 - 315*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e*f**3 + 315*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**2*f**2 - 105*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*d**3*e**3*f - 105*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*b*c**3*e*f**3 + 315*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*b*c**2*d*e**2*f**2 - 315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*b*c*d**2*e**3*f + 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqr
t(e)))*b*d**3*e**4 + 315*a*c**2*d*e*f**4*x - 315*a*c*d**2*e**2*f**3*x + 10
5*a*c*d**2*e*f**4*x**3 + 105*a*d**3*e**3*f**2*x - 35*a*d**3*e**2*f**3*x**3
 + 21*a*d**3*e*f**4*x**5 + 105*b*c**3*e*f**4*x - 315*b*c**2*d*e**2*f**3*x
+ 105*b*c**2*d*e*f**4*x**3 + 315*b*c*d**2*e**3*f**2*x - 105*b*c*d**2*e**2*
f**3*x**3 + 63*b*c*d**2*e*f**4*x**5 - 105*b*d**3*e**4*f*x + 35*b*d**3*e**3
*f**2*x**3 - 21*b*d**3*e**2*f**3*x**5 + 15*b*d**3*e*f**4*x**7)/(105*e*f**5
)
```

3.216 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = -\frac{d(adf(2de-3cf)-3b(de-cf)^2)x}{f^4} - \frac{d^2(2bde-3bcf-adf)x^3}{3f^3} + \frac{bd^3x^5}{5f^2} + \frac{(be-af)(de-cf)^3x}{2ef^4(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

output
```
-d*(a*d*f*(-3*c*f+2*d*e)-3*b*(-c*f+d*e)^2)*x/f^4-1/3*d^2*(-a*d*f-3*b*c*f+2
*b*d*e)*x^3/f^3+1/5*b*d^3*x^5/f^2+1/2*(-a*f+b*e)*(-c*f+d*e)^3*x/e/f^4/(f*x
^2+e)-1/2*(-c*f+d*e)^2*(b*e*(-c*f+7*d*e)-a*f*(c*f+5*d*e))*arctan(f^(1/2)*x
/e^(1/2))/e^(3/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{d(3b(de-cf)^2+adf(-2de+3cf))\,x}{f^4} + \frac{d^2(-2bde+3bcf+adf)x^3}{3f^3} + \frac{bd^3x^5}{5f^2} + \frac{(be-af)(de-cf)^3x}{2ef^4(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
(d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2*b*d*e +
3*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((b*e - a*f)*(d*e -
c*f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*
(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.43, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {401, 25, 403, 25, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 401$$

$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef} - \frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}+\frac{dx(c+dx^2)^2(7be-5af)}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\int \frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\left(5d^2e^2-6cdfe-3c^2f^2\right)-be\left(35d^2e^2-54cdfe+15c^2\right.\right.}{fx^2+e}}{3f}}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15(de-cf)^2(be(7de-cf)-af(cf+5de))\int\frac{1}{fx^2+e}dx}{f}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+\frac{dx\left(\right.}{}}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+d}{5f}}{2ef}$$

$$\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
-1/2*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*a*f)
*x*(c + d*x^2)^2)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e - 3*c*
f))*x*(c + d*x^2))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2
) - b*e*(105*d^2*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*f)^2
*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqr
t[e]*f^(3/2)))/(3*f))/(5*f))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.73

method	result
default	$\frac{d\left(\frac{1}{5}b\,d^{2}x^{5}f^{2}+\frac{1}{3}a\,d^{2}f^{2}x^{3}+bcd\,f^{2}x^{3}-\frac{2}{3}b\,d^{2}ef\,x^{3}+3acd\,f^{2}x-2a\,d^{2}efx+3b\,c^{2}f^{2}x-6bcdefx+3b\,d^{2}e^{2}x\right)}{f^{4}}+\frac{\left(c^{3}a\,f^{4}-3a\,c^{2}de\,f^{3}+3ac\right.}{}$
risch	$-\frac{9e\ln\left(-fx+\sqrt{-ef}\right)ac\,d^{2}}{4f^{2}\sqrt{-ef}}-\frac{9e\ln\left(-fx+\sqrt{-ef}\right)b\,c^{2}d}{4f^{2}\sqrt{-ef}}+\frac{15e^{2}\ln\left(-fx+\sqrt{-ef}\right)bc\,d^{2}}{4f^{3}\sqrt{-ef}}+\frac{d^{2}bc\,x^{3}}{f^{2}}-\frac{2d^{3}be\,x^{3}}{3f^{3}}+\frac{\ln\left(-fx+\sqrt{-\right.}{4f\sqrt{-e}}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
d/f^4*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+b*c*d*f^2*x^3-2/3*b*d^2*e*f*x^3
+3*a*c*d*f^2*x-2*a*d^2*e*f*x+3*b*c^2*f^2*x-6*b*c*d*e*f*x+3*b*d^2*e^2*x)+1/
f^4*(1/2*(a*c^3*f^4-3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*
f^3+3*b*c^2*d*e^2*f^2-3*b*c*d^2*e^3*f+b*d^3*e^4)/e*x/(f*x^2+e)+1/2*(a*c^3*
f^4+3*a*c^2*d*e*f^3-9*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3-9*b*c^2*d*
e^2*f^2+15*b*c*d^2*e^3*f-7*b*d^3*e^4)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(162) = 324.

Time = 0.10 (sec) , antiderivative size = 834, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b*d^3*e^2*f^4*x^7 - 4*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e
^2*f^4)*x^5 + 20*(7*b*d^3*e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c
^2*d + a*c*d^2)*e^2*f^4)*x^3 + 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^
2 + a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2
*f^3 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c
^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(-e*f)*log((
f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(7*b*d^3*e^5*f + a*c^3*e*f^5
 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 - (b*c^3
+ 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5), 1/30*(6*b*d^3*e^2*f^4*x^
7 - 2*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^2*f^4)*x^5 + 10*(7*b*d^3*
e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4)*x
^3 - 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4*f + 9*(b*c^
2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3 + (7*b*d^3*e^4*f - a*
c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d + a*c*d^2)*e^2*f^3 -
(b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(7*b*
d^3*e^5*f + a*c^3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c
*d^2)*e^3*f^3 - (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(167) = 334$.

Time = 1.94 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$

$$= \frac{bd^3x^5}{5f^2} + x^3\left(\frac{ad^3}{3f^2} + \frac{bcd^2}{f^2} - \frac{2bd^3e}{3f^3}\right) + x\left(\frac{3acd^2}{f^2} - \frac{2ad^3e}{f^3} + \frac{3bc^2d}{f^2} - \frac{6bcd^2e}{f^3} + \frac{3bd^3e^2}{f^4}\right)$$
$$+ \frac{x(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4)}{2e^2f^4 + 2ef^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f}\right.}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bde^2)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7b}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+}\right.}{4}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
b*d**3*x**5/(5*f**2) + x**3*(a*d**3/(3*f**2) + b*c*d**2/f**2 - 2*b*d**3*e/
(3*f**3)) + x*(3*a*c*d**2/f**2 - 2*a*d**3*e/f**3 + 3*b*c**2*d/f**2 - 6*b*c
*d**2*e/f**3 + 3*b*d**3*e**2/f**4) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 +
3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**
2 - 3*b*c*d**2*e**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(
-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**
2)*log(-e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*
f + b*c*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e*
*2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*
d**2*e**3*f - 7*b*d**3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*
(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*
f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**
3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f + b*c*
*3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**3*e**4) + x
)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx =$$
$$-\frac{(7\,bd^3e^4 - 15\,bcd^2e^3f - 5\,ad^3e^3f + 9\,bc^2de^2f^2 + 9\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 - ac^3f^4)\arctan\left(\frac{f}{\sqrt{\cdot}}\right.}{2\sqrt{ef}ef^4}$$
$$+\frac{bd^3e^4x - 3\,bcd^2e^3fx - ad^3e^3fx + 3\,bc^2de^2f^2x + 3\,acd^2e^2f^2x - bc^3ef^3x - 3\,ac^2def^3x + ac^3f^4x}{2\,(fx^2+e)ef^4}$$
$$+\frac{3\,bd^3f^8x^5 - 10\,bd^3ef^7x^3 + 15\,bcd^2f^8x^3 + 5\,ad^3f^8x^3 + 45\,bd^3e^2f^6x - 90\,bcd^2ef^7x - 30\,ad^3ef^7x + 45\,b}{15\,f^{10}}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(7*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2 +
 9*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - a*c^3*f^4)*arctan(f*x
/sqrt(e*f))/(sqrt(e*f)*e*f^4) + 1/2*(b*d^3*e^4*x - 3*b*c*d^2*e^3*f*x - a*d
^3*e^3*f*x + 3*b*c^2*d*e^2*f^2*x + 3*a*c*d^2*e^2*f^2*x - b*c^3*e*f^3*x - 3
*a*c^2*d*e*f^3*x + a*c^3*f^4*x)/((f*x^2 + e)*e*f^4) + 1/15*(3*b*d^3*f^8*x^
5 - 10*b*d^3*e*f^7*x^3 + 15*b*c*d^2*f^8*x^3 + 5*a*d^3*f^8*x^3 + 45*b*d^3*e
^2*f^6*x - 90*b*c*d^2*e*f^7*x - 30*a*d^3*e*f^7*x + 45*b*c^2*d*f^8*x + 45*a
*c*d^2*f^8*x)/f^10
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = x^3\left(\frac{a\,d^3+3\,b\,c\,d^2}{3\,f^2}-\frac{2\,b\,d^3\,e}{3\,f^3}\right) - x\left(\frac{2\,e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^2}-\frac{2\,b\,d^3\,e}{f^3}\right)}{f}+\frac{b\,d^3\,e^2}{f^4}-\frac{3\,c\,d\,(a\,d+b\,c)}{f^2}\right)+\frac{b\,d^3\,x^5}{5\,f^2} + \frac{x\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}{2\,e\,(f^5\,x^2+e\,f^4)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)^2\,(a\,c\,f^2-7\,b\,d\,e^2+5\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+a\,c^3\,f^4-9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f-9\,a\,c\,d^2\,e^2\,f^2-7\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f)}\right)\,(c\,f-d\,e)^2\,(a\,c\,f^2-}{2\,e^{3/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
x^3*((a*d^3 + 3*b*c*d^2)/(3*f^2) - (2*b*d^3*e)/(3*f^3)) - x*((2*e*((a*d^3
+ 3*b*c*d^2)/f^2 - (2*b*d^3*e)/f^3))/f + (b*d^3*e^2)/f^4 - (3*c*d*(a*d + b
*c))/f^2) + (b*d^3*x^5)/(5*f^2) + (x*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f
- b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*
b*c^2*d*e^2*f^2))/(2*e*(e*f^4 + f^5*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)^2
*(a*c*f^2 - 7*b*d*e^2 + 5*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^3*f^4 - 7*b*d^
3*e^4 + 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f -
 9*a*c*d^2*e^2*f^2 - 9*b*c^2*d*e^2*f^2)))*(c*f - d*e)^2*(a*c*f^2 - 7*b*d*e
^2 + 5*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 723, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output

```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e*f**4 + 15*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*f**5*x**2 + 45*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**2*f**3 + 45*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e*f**4*x**2 - 135*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**3*f**2 - 135*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**2*f**3*x**2 + 75*sqrt(f)*sqrt(e)*at
an((f*x)/(sqrt(f)*sqrt(e)))*a*d**3*e**4*f + 75*sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*a*d**3*e**3*f**2*x**2 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(
sqrt(f)*sqrt(e)))*b*c**3*e**2*f**3 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f
)*sqrt(e)))*b*c**3*e*f**4*x**2 - 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*s
qrt(e)))*b*c**2*d*e**3*f**2 - 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*b*c**2*d*e**2*f**3*x**2 + 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sq
rt(e)))*b*c*d**2*e**4*f + 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*b*c*d**2*e**3*f**2*x**2 - 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*b*d**3*e**5 - 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*d**3
*e**4*f*x**2 + 15*a*c**3*e*f**5*x - 45*a*c**2*d*e**2*f**4*x + 135*a*c*d**2
*e**3*f**3*x + 90*a*c*d**2*e**2*f**4*x**3 - 75*a*d**3*e**4*f**2*x - 50*a*d
**3*e**3*f**3*x**3 + 10*a*d**3*e**2*f**4*x**5 - 15*b*c**3*e**2*f**4*x + 13
5*b*c**2*d*e**3*f**3*x + 90*b*c**2*d*e**2*f**4*x**3 - 225*b*c*d**2*e**4*f*
*2*x - 150*b*c*d**2*e**3*f**3*x**3 + 30*b*c*d**2*e**2*f**4*x**5 + 105*b...
```

3.217 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 221

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = -\frac{d^2(3bde-3bcf-adf)x}{f^4}+\frac{bd^3x^3}{3f^3}+\frac{(be-af)(de-cf)^3x}{4ef^4(e+fx^2)^2}-\frac{(de-cf)^2(be(13de-cf)-3af(3de+cf))x}{8e^2f^4(e+fx^2)}+\frac{(de-cf)\left(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}$$

output

```
-d^2*(-a*d*f-3*b*c*f+3*b*d*e)*x/f^4+1/3*b*d^3*x^3/f^3+1/4*(-a*f+b*e)*(-c*f
+d*e)^3*x/e/f^4/(f*x^2+e)^2-1/8*(-c*f+d*e)^2*(b*e*(-c*f+13*d*e)-3*a*f*(c*f
+3*d*e))*x/e^2/f^4/(f*x^2+e)+1/8*(-c*f+d*e)*(b*e*(-c^2*f^2-10*c*d*e*f+35*d
^2*e^2)-3*a*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(
5/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{d^2(-3bde+3bcf+adf)x}{f^4}+\frac{bd^3x^3}{3f^3} + \frac{(be-af)(de-cf)^3x}{4ef^4(e+fx^2)^2} - \frac{(de-cf)^2(be(13de-cf)-3af(3de+cf))x}{8e^2f^4(e+fx^2)} + \frac{(de-cf)\left(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) + ((b*e - a
*f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c*f)^2*(b*e*(13*d*e
 - c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e + f*x^2)) + ((d*e - c*f)*(
b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c
^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {401, 25, 401, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-3af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef} - \frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx^2+c)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{(fx^2+e)^2}dx}{4ef} - \frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 401

$$\frac{-\frac{\int \frac{(dx^2+c)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef} - \frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef} - \frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 403

$$\frac{-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f} - \frac{dx\left(c+dx^2\right)(be(35de-3cf)-\cdots}{3f}}{2ef}\cdots}{4ef} \cdots - \frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 299

$$\frac{-\frac{\frac{-\frac{3(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{f} - \frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{f}}{3f} - \cdots}{2ef}\cdots}{4ef} \cdots - \frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}} - \frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{f}\cdots}{3f}\cdots}{2ef}\cdots}{4ef} \cdots - \frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

input

```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
-1/4*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*
e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(
d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-((d*
(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d*e
*f + 3*c^2*f^2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^
2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.56

method	result
default	$\frac{d^{2}\left(\frac{1}{3} b d f\, x^{3}+a d f x+3 b c f x-3 b d e x\right)}{f^{4}}+\frac{\frac{f \left(3 c^{3} a\, f^{4}+3 a\, c^{2} d e\, f^{3}-15 a c\, d^{2} e^{2} f^{2}+9 a\, d^{3} e^{3} f+b\, c^{3} e\, f^{3}-15 b\, c^{2} d\, e^{2} f^{2}+27 b c\, d^{2} e^{3} f-13 e^{4} b\, d^{3}\right) x^{3}}{8 e^{2}}+}{\left(f\, x^{2}+e\right)}$
risch	$\frac{b\, d^{3} x^{3}}{3 f^{3}}+\frac{d^{3} a x}{f^{3}}+\frac{3 d^{2} b c x}{f^{3}}-\frac{3 d^{3} b e x}{f^{4}}+\frac{\frac{f \left(3 c^{3} a\, f^{4}+3 a\, c^{2} d e\, f^{3}-15 a c\, d^{2} e^{2} f^{2}+9 a\, d^{3} e^{3} f+b\, c^{3} e\, f^{3}-15 b\, c^{2} d\, e^{2} f^{2}+27 b c\, d^{2} e^{3} f-13 e^{4} b\, d^{3}\right)}{8 e^{2}}}{f^{4} (f}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
d^2/f^4*(1/3*b*d*f*x^3+a*d*f*x+3*b*c*f*x-3*b*d*e*x)+1/f^4*((1/8*f*(3*a*c^3
*f^4+3*a*c^2*d*e*f^3-15*a*c*d^2*e^2*f^2+9*a*d^3*e^3*f+b*c^3*e*f^3-15*b*c^2
*d*e^2*f^2+27*b*c*d^2*e^3*f-13*b*d^3*e^4)/e^2*x^3+1/8*(5*a*c^3*f^4-3*a*c^2
*d*e*f^3-9*a*c*d^2*e^2*f^2+7*a*d^3*e^3*f-b*c^3*e*f^3-9*b*c^2*d*e^2*f^2+21*
b*c*d^2*e^3*f-11*b*d^3*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a*c^3*f^4+3*a*c^2*d*e*
f^3+9*a*c*d^2*e^2*f^2-15*a*d^3*e^3*f+b*c^3*e*f^3+9*b*c^2*d*e^2*f^2-45*b*c*
d^2*e^3*f+35*b*d^3*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(205) = 410$.

Time = 0.10 (sec) , antiderivative size = 1102, normalized size of antiderivative = 4.99

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*b*d^3*e^3*f^4*x^7 - 16*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*
e^3*f^4)*x^5 - 2*(175*b*d^3*e^5*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^
3)*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^
5)*x^3 - 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f
+ 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*
e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*
d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^
3*e*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 +
 (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x
- e)/(f*x^2 + e)) - 6*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 +
a*d^3)*e^5*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f
^4)*x)/(e^3*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5), 1/24*(8*b*d^3*e^3*f^4*x^7
- 8*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e^3*f^4)*x^5 - (175*b*d^3*e^5
*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)*e^4*f^3 + 45*(b*c^2*d + a*c*
d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^3 + 3*(35*b*d^3*e^6 + 3*a*
c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2
 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b
*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2
*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5 - 15*(3*b*c*d^2 + a*d^3
)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(214) = 428$.

Time = 10.49 (sec) , antiderivative size = 865, normalized size of antiderivative = 3.91

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{bd^3x^3}{3f^3} + x\left(\frac{ad^3}{f^3} + \frac{3bcd^2}{f^3} - \frac{3bd^3e}{f^4}\right) - \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)\left(3ac^2f^3 + 6acdef^2 + 15ad^2e^2f + bc^2ef^2 + 10bcde^2f - 35bd^2e^3\right)\log\left(-\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(\cdots}{3ac^3f^4+3ac^2de\cdots}\right.}{16} + \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)\left(3ac^2f^3 + 6acdef^2 + 15ad^2e^2f + bc^2ef^2 + 10bcde^2f - 35bd^2e^3\right)\log\left(\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf\cdots}{3ac^3f^4+3ac^2def^3\cdots}\right.}{16} + \frac{x^3\cdot\left(3ac^3f^5 + 3ac^2def^4 - 15acd^2e^2f^3 + 9ad^3e^3f^2 + bc^3ef^4 - 15bc^2de^2f^3 + 27bcd^2e^3f^2 - 13bd^3e^4f\right) \cdots}{8e^4f^4 + 16e^3f^5x^{\cdots}}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
b*d**3*x**3/(3*f**3) + x*(a*d**3/f**3 + 3*b*c*d**2/f**3 - 3*b*d**3*e/f**4)
 - sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(-e**3
*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 1
5*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c
**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b
*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4)
 + x)/16 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**
2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*l
og(e**3*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f
**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)
/(3*a*c**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**
3*f + b*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**
3*e**4) + x)/16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c*d**2*e
**2*f**3 + 9*a*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*d*e**2*f**3 + 27
*b*c*d**2*e**3*f**2 - 13*b*d**3*e**4*f) + x*(5*a*c**3*e*f**4 - 3*a*c**2*d*
e**2*f**3 - 9*a*c*d**2*e**3*f**2 + 7*a*d**3*e**4*f - b*c**3*e**2*f**3 - 9*
b*c**2*d*e**3*f**2 + 21*b*c*d**2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4 +
16*e**3*f**5*x**2 + 8*e**2*f**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{(35\,bd^3e^4 - 45\,bcd^2e^3f - 15\,ad^3e^3f + 9\,bc^2de^2f^2 + 9\,acd^2e^2f^2 + bc^3ef^3 + 3\,ac^2def^3 + 3\,ac^3f^4)\arctan\Big(}{8\sqrt{ef}e^2f^4}$$
$$- \frac{13\,bd^3e^4fx^3 - 27\,bcd^2e^3f^2x^3 - 9\,ad^3e^3f^2x^3 + 15\,bc^2de^2f^3x^3 + 15\,acd^2e^2f^3x^3 - bc^3ef^4x^3 - 3\,ac^2def^4x}{}$$
$$+ \frac{bd^3f^6x^3 - 9\,bd^3ef^5x + 9\,bcd^2f^6x + 3\,ad^3f^6x}{3\,f^9}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(35*b*d^3*e^4 - 45*b*c*d^2*e^3*f - 15*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2
+ 9*a*c*d^2*e^2*f^2 + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 3*a*c^3*f^4)*arctan(
f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^4) - 1/8*(13*b*d^3*e^4*f*x^3 - 27*b*c*d^2*
e^3*f^2*x^3 - 9*a*d^3*e^3*f^2*x^3 + 15*b*c^2*d*e^2*f^3*x^3 + 15*a*c*d^2*e^
2*f^3*x^3 - b*c^3*e*f^4*x^3 - 3*a*c^2*d*e*f^4*x^3 - 3*a*c^3*f^5*x^3 + 11*b
*d^3*e^5*x - 21*b*c*d^2*e^4*f*x - 7*a*d^3*e^4*f*x + 9*b*c^2*d*e^3*f^2*x +
9*a*c*d^2*e^3*f^2*x + b*c^3*e^2*f^3*x + 3*a*c^2*d*e^2*f^3*x - 5*a*c^3*e*f^
4*x)/((f*x^2 + e)^2*e^2*f^4) + 1/3*(b*d^3*f^6*x^3 - 9*b*d^3*e*f^5*x + 9*b*
c*d^2*f^6*x + 3*a*d^3*f^6*x)/f^9
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^3\,e\,f^4+3\,a\,c^3\,f^5-15\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4+27\,b\,c\,d^2\,e^3\,f^2-15\,a\,c\,d^2\,e^2\,f^3-13\,b\,d^3\,e^4\,f+9\,a\,d^3\,e^3\,f^2)}{8\,e^2} - \frac{x\,(b\,c^3\,e\,f^3-5\,a\,c^3\,f^4+9\,b\,c^2\,d}{}}{e^2\,f^4+2\,e\,f^5\,x^2+f^6\,x^4}$$
$$+ x\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^3} - \frac{3\,b\,d^3\,e}{f^4}\right) + \frac{b\,d^3\,x^3}{3\,f^3}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+10\,b\,c\,d\,e^2\,f+6\,a\,c\,d\,e\,f^2-35\,b\,d^2\,e^3+15\,a\,d^2\,e^2\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+3\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3-45\,b\,c\,d^2\,e^3\,f+9\,a\,c\,d^2\,e^2\,f^2+35\,b\,d^3\,e^4-15\,a\,d^3\,e^3\,f)}\right)(c\,f-d\,e)\,(b\,c^2\,e}{8\,e^{5/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
((x^3*(3*a*c^3*f^5 + 9*a*d^3*e^3*f^2 + b*c^3*e*f^4 - 13*b*d^3*e^4*f + 3*a*
c^2*d*e*f^4 - 15*a*c*d^2*e^2*f^3 + 27*b*c*d^2*e^3*f^2 - 15*b*c^2*d*e^2*f^3
))/(8*e^2) - (x*(11*b*d^3*e^4 - 5*a*c^3*f^4 - 7*a*d^3*e^3*f + b*c^3*e*f^3
+ 3*a*c^2*d*e*f^3 - 21*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e^2*f
^2))/(8*e))/(e^2*f^4 + f^6*x^4 + 2*e*f^5*x^2) + x*((a*d^3 + 3*b*c*d^2)/f^3
 - (3*b*d^3*e)/f^4) + (b*d^3*x^3)/(3*f^3) + (atan((f^(1/2)*x*(c*f - d*e)*(
3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*e*f^2 + 6*a*c*d*e*f^2
+ 10*b*c*d*e^2*f))/(e^(1/2)*(3*a*c^3*f^4 + 35*b*d^3*e^4 - 15*a*d^3*e^3*f +
 b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 - 45*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*
b*c^2*d*e^2*f^2)))*(c*f - d*e)*(3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*
f + b*c^2*e*f^2 + 6*a*c*d*e*f^2 + 10*b*c*d*e^2*f))/(8*e^(5/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1032, normalized size of antiderivative = 4.67

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output

```
(9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e**2*f**4 + 18*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e*f**5*x**2 + 9*sqrt(f)*
sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*f**6*x**4 + 9*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**3*f**3 + 18*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**2*f**4*x**2 + 9*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e*f**5*x**4 + 27*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**4*f**2 + 54*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**3*f**3*x**2 + 27*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**2*f**4*x**4 - 45*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*d**3*e**5*f - 90*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt
(f)*sqrt(e)))*a*d**3*e**4*f**2*x**2 - 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*a*d**3*e**3*f**3*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*b*c**3*e**3*f**3 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*b*c**3*e**2*f**4*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*b*c**3*e*f**5*x**4 + 27*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*
c**2*d*e**4*f**2 + 54*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2
*d*e**3*f**3*x**2 + 27*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**
2*d*e**2*f**4*x**4 - 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c
*d**2*e**5*f - 270*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*d**2*
e**4*f**2*x**2 - 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*...
```

3.218 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 295

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4} dx = \frac{bd^3x}{f^4} + \frac{(be-af)(de-cf)^3x}{6ef^4(e+fx^2)^3} - \frac{(de-cf)^2(be(19de-cf)-af(13de+5cf))x}{24e^2f^4(e+fx^2)^2} + \frac{(de-cf)(be(29d^2e^2-4cdef-c^2f^2)-af(11d^2e^2+8cdef+5c^2f^2))x}{16e^3f^4(e+fx^2)} - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}$$

output
```
b*d^3*x/f^4+1/6*(-a*f+b*e)*(-c*f+d*e)^3*x/e/f^4/(f*x^2+e)^3-1/24*(-c*f+d*e
)^2*(b*e*(-c*f+19*d*e)-a*f*(5*c*f+13*d*e))*x/e^2/f^4/(f*x^2+e)^2+1/16*(-c*
f+d*e)*(b*e*(-c^2*f^2-4*c*d*e*f+29*d^2*e^2)-a*f*(5*c^2*f^2+8*c*d*e*f+11*d^
2*e^2))*x/e^3/f^4/(f*x^2+e)-1/16*(b*e*(-c^3*f^3-3*c^2*d*e*f^2-15*c*d^2*e^2
*f+35*d^3*e^3)-a*f*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arct
an(f^(1/2)*x/e^(1/2))/e^(7/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{bd^3x}{f^4} + \frac{(be-af)(de-cf)^3x}{6ef^4(e+fx^2)^3} - \frac{(de-cf)^2(be(19de-cf)-af(13de+5cf))x}{24e^2f^4(e+fx^2)^2} + \frac{(de-cf)(be(29d^2e^2-4cdef-c^2f^2)-af(11d^2e^2+8cdef+5c^2f^2))x}{16e^3f^4(e+fx^2)} - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2)^3) - ((
d*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))*x)/(24*e^2*f^4*(e
 + f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 - 4*c*d*e*f - c^2*f^2) - a*f*
(11*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*
(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 +
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(16*e^(7/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {401, 25, 401, 25, 401, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{(dx^2+c)^2(d(7be-af)x^2+c(be+5af))}{(fx^2+e)^3}dx}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$\downarrow$ 25

$$\frac{\int \frac{(dx^2+c)^2(d(7be-af)x^2+c(be+5af))}{(fx^2+e)^3}dx}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$\downarrow$ 401

$$-\frac{-\frac{\int -\frac{(dx^2+c)(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af)))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{(dx^2+c)(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af)))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$\downarrow$ 401

$$\frac{-\frac{-\frac{\int \frac{c(af(5d^2e^2+6cdfe-15c^2f^2)-be(35d^2e^2+6cdfe+3c^2f^2))-d(be(105d^2e^2-10cdfe-3c^2f^2)-af(15d^2e^2+14cdfe+15c^2f^2))x^2}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)(be(-3c^2f^2}{}}{4ef}}{6ef}$$

$$\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$\downarrow$ 299

$$-\frac{\frac{3(be(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3)-af(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3))\int\frac{1}{fx^2+e}dx}{f}-\frac{dx(be(-3c^2f^2-10cdef+105d^2e^2)-af(15c^2f^2+14cdef}{f}}{2ef}$$

$$4ef$$

$$6ef$$

$$\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

$$\Big\downarrow 218$$

$$\frac{-\frac{-\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-af\left(15c^2f^2+14cd\right.\right.}{f}}{2ef}}{4ef}}{6ef}$$

$$\frac{x\left(c+dx^2\right)^3(be-af)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
-1/6*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^3) + (-1/4*((b*e*(7*d*
e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2
*((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f*(5*d^2*e^2 + 4*c*d*e*f +
 15*c^2*f^2))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b*e*(105*d^2*e^2 -
 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))*x)/
f) + (3*(b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f
*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*
x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f))/(6*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.41

method	result
default	$\frac{b\,d^3 x}{f^4}+\frac{\frac{f^2\left(5c^3 a\,f^4+3a\,c^2 de\,f^3+3ac\,d^2 e^2 f^2-11a\,d^3 e^3 f+b\,c^3 e\,f^3+3b\,c^2 d\,e^2 f^2-33bc\,d^2 e^3 f+29e^4 b\,d^3\right)x^5}{16e^3}+\frac{f\left(5c^3 a\,f^4+3a\,c^2 de\,f^3-3ac\,d^2 e^2\right.}{}}{}$
risch	$\frac{b\,d^3 x}{f^4}+\frac{\frac{f^2\left(5c^3 a\,f^4+3a\,c^2 de\,f^3+3ac\,d^2 e^2 f^2-11a\,d^3 e^3 f+b\,c^3 e\,f^3+3b\,c^2 d\,e^2 f^2-33bc\,d^2 e^3 f+29e^4 b\,d^3\right)x^5}{16e^3}+\frac{f\left(5c^3 a\,f^4+3a\,c^2 de\,f^3-3ac\,d^2 e^2\right.}{}}{}$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
b*d^3*x/f^4+1/f^4*((1/16*f^2*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^
2-11*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-33*b*c*d^2*e^3*f+29*b*d^3*e
^4)/e^3*x^5+1/6*f*(5*a*c^3*f^4+3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e
^3*f+b*c^3*e*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+17*b*d^3*e^4)/e^2*x^3+
1/16*(11*a*c^3*f^4-3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*f-b*c^3*e
*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+19*b*d^3*e^4)/e*x)/(f*x^2+e)^3+1/1
6*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3
+3*b*c^2*d*e^2*f^2+15*b*c*d^2*e^3*f-35*b*d^3*e^4)/e^3/(e*f)^(1/2)*arctan(f
*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(279) = 558$.

Time = 0.11 (sec) , antiderivative size = 1422, normalized size of antiderivative = 4.82

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(96*b*d^3*e^4*f^4*x^7 + 6*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*
b*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^
2*d)*e^2*f^6)*x^5 + 16*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2
 + a*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3
*f^5)*x^3 + 3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*
f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^
3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c
*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a
*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4
 - (b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5
- 5*(3*b*c*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 +
 3*a*c^2*d)*e^3*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x
^2 + e)) + 6*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^
6*f^2 - 3*(b*c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e
^4*f^8*x^6 + 3*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5), 1/48*(48*b*d^3*e^4*
f^4*x^7 + 3*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b*c*d^2 + a*d^3)*e^4
*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*d)*e^2*f^6)*x^5 +
8*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^3 - 3*
(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5)*x^3 - 3*(35*b*d
^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3*(b*c^2*d + a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \frac{bd^3x}{f^4} - \frac{(35\,bd^3e^4 - 15\,bcd^2e^3f - 5\,ad^3e^3f - 3\,bc^2de^2f^2 - 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 - 5\,ac^3f^4)\arctan\left(\right.}{16\sqrt{ef}e^3f^4} + \frac{87\,bd^3e^4f^2x^5 - 99\,bcd^2e^3f^3x^5 - 33\,ad^3e^3f^3x^5 + 9\,bc^2de^2f^4x^5 + 9\,acd^2e^2f^4x^5 + 3\,bc^3ef^5x^5 + 9\,ac^2def^5}{}$$

input
```
integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
b*d^3*x/f^4 - 1/16*(35*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f - 3*b*
c^2*d*e^2*f^2 - 3*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 5*a*c^
3*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^4) + 1/48*(87*b*d^3*e^4*f^2*
x^5 - 99*b*c*d^2*e^3*f^3*x^5 - 33*a*d^3*e^3*f^3*x^5 + 9*b*c^2*d*e^2*f^4*x^
5 + 9*a*c*d^2*e^2*f^4*x^5 + 3*b*c^3*e*f^5*x^5 + 9*a*c^2*d*e*f^5*x^5 + 15*a
*c^3*f^6*x^5 + 136*b*d^3*e^5*f*x^3 - 120*b*c*d^2*e^4*f^2*x^3 - 40*a*d^3*e^
4*f^2*x^3 - 24*b*c^2*d*e^3*f^3*x^3 - 24*a*c*d^2*e^3*f^3*x^3 + 8*b*c^3*e^2*
f^4*x^3 + 24*a*c^2*d*e^2*f^4*x^3 + 40*a*c^3*e*f^5*x^3 + 57*b*d^3*e^6*x - 4
5*b*c*d^2*e^5*f*x - 15*a*d^3*e^5*f*x - 9*b*c^2*d*e^4*f^2*x - 9*a*c*d^2*e^4
*f^2*x - 3*b*c^3*e^3*f^3*x - 9*a*c^2*d*e^3*f^3*x + 33*a*c^3*e^2*f^4*x)/((f
*x^2 + e)^3*e^3*f^4)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{\frac{x^3\,\left(b\,c^3\,e\,f^4+5\,a\,c^3\,f^5-3\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4-15\,b\,c\,d^2\,e^3\,f^2-3\,a\,c\,d^2\,e^2\,f^3+17\,b\,d^3\,e^4\,f-5\,a\,d^3\,e^3\,f^2\right)}{6\,e^2}+\frac{x^5\,\left(b\,c^3\,e\,f^5+5\,a\,c^3\,f^6+3\,b\,c^2\,d\,e\right.}{}}{e^3\,f}$$
$$+\frac{b\,d^3\,x}{f^4}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(b\,c^3\,e\,f^3+5\,a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2-35\,b\,d^3\,e\right.}{16\,e^{7/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
((x^3*(5*a*c^3*f^5 - 5*a*d^3*e^3*f^2 + b*c^3*e*f^4 + 17*b*d^3*e^4*f + 3*a*
c^2*d*e*f^4 - 3*a*c*d^2*e^2*f^3 - 15*b*c*d^2*e^3*f^2 - 3*b*c^2*d*e^2*f^3))
/(6*e^2) + (x^5*(5*a*c^3*f^6 - 11*a*d^3*e^3*f^3 + 29*b*d^3*e^4*f^2 + b*c^3
*e*f^5 + 3*a*c^2*d*e*f^5 + 3*a*c*d^2*e^2*f^4 - 33*b*c*d^2*e^3*f^3 + 3*b*c^
2*d*e^2*f^4))/(16*e^3) - (x*(5*a*d^3*e^3*f - 19*b*d^3*e^4 - 11*a*c^3*f^4 +
 b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*
b*c^2*d*e^2*f^2))/(16*e))/(e^3*f^4 + f^7*x^6 + 3*e*f^6*x^4 + 3*e^2*f^5*x^2
) + (b*d^3*x)/f^4 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c^3*f^4 - 35*b*d^3*e^4
 + 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*
c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2))/(16*e^(7/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1373, normalized size of antiderivative = 4.65

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e**3*f**4 + 45*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e**2*f**5*x**2 + 45*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*e*f**6*x**4 + 15*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**3*f**7*x**6 + 9*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**4*f**3 + 27*sqrt(f)*sqrt(e)*at
an((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**3*f**4*x**2 + 27*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e**2*f**5*x**4 + 9*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*a*c**2*d*e*f**6*x**6 + 9*sqrt(f)*sqrt(e)*atan
((f*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**5*f**2 + 27*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**4*f**3*x**2 + 27*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**3*f**4*x**4 + 9*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*c*d**2*e**2*f**5*x**6 + 15*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*a*d**3*e**6*f + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqr
t(f)*sqrt(e)))*a*d**3*e**5*f**2*x**2 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt
(f)*sqrt(e)))*a*d**3*e**4*f**3*x**4 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*a*d**3*e**3*f**4*x**6 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*b*c**3*e**4*f**3 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*b*c**3*e**3*f**4*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*b*c**3*e**2*f**5*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
b*c**3*e*f**6*x**6 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*...
```

3.219 $\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)} dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)} dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)} + \frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}$$

output
```
-(-a*d+b*c)*arctan(d^(1/2)*x/c^(1/2))/c^(1/2)/d^(1/2)/(-c*f+d*e)+(-a*f+b*e
)*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)/f^(1/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)} dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(-de+cf)} + \frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]*(-(d*e) + c*f))
 + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{ 397}$$

$$\frac{(be-af)\int \frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)))
 + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

method	result
default	$\frac{(af-be)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(cf-de)\sqrt{ef}}+\frac{(-ad+bc)\arctan\left(\frac{xd}{\sqrt{cd}}\right)}{(cf-de)\sqrt{cd}}$
risch	$-\frac{\ln\left(e\,f^2x-(-ef)^{\frac{3}{2}}\right)af}{2(cf-de)\sqrt{-ef}}+\frac{\ln\left(e\,f^2x-(-ef)^{\frac{3}{2}}\right)be}{2(cf-de)\sqrt{-ef}}+\frac{\ln\left(-e\,f^2x-(-ef)^{\frac{3}{2}}\right)af}{2(cf-de)\sqrt{-ef}}-\frac{\ln\left(-e\,f^2x-(-ef)^{\frac{3}{2}}\right)be}{2(cf-de)\sqrt{-ef}}-\frac{\ln\left(-c\,d^2x-(-c\right.}{2\sqrt{-cd}\,(cf-}$

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(a*f-b*e)/(c*f-d*e)/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))+(-a*d+b*c)/(c*f-d*
e)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.70

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = \left[-\frac{(bc-ad)\sqrt{-cd}ef\log\left(\frac{dx^2+2\sqrt{-cd}x-c}{dx^2+c}\right)+(bcde-acdf)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right)}{2(cd^2e^2f-c^2def^2)},\right.$$
$$-\frac{(bc-ad)\sqrt{-cd}ef\log\left(\frac{dx^2+2\sqrt{-cd}x-c}{dx^2+c}\right)-2(bcde-acdf)\sqrt{ef}\arctan\left(\frac{\sqrt{ef}x}{e}\right)}{2(cd^2e^2f-c^2def^2)},$$
$$-\frac{2(bc-ad)\sqrt{cd}ef\arctan\left(\frac{\sqrt{cd}x}{c}\right)+(bcde-acdf)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right)}{2(cd^2e^2f-c^2def^2)},$$
$$\left.-\frac{(bc-ad)\sqrt{cd}ef\arctan\left(\frac{\sqrt{cd}x}{c}\right)-(bcde-acdf)\sqrt{ef}\arctan\left(\frac{\sqrt{ef}x}{e}\right)}{cd^2e^2f-c^2def^2}\right]$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/2*((b*c - a*d)*sqrt(-c*d)*e*f*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2
+ c)) + (b*c*d*e - a*c*d*f)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f
*x^2 + e)))/(c*d^2*e^2*f - c^2*d*e*f^2), -1/2*((b*c - a*d)*sqrt(-c*d)*e*f*
log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b*c*d*e - a*c*d*f)*sqrt
(e*f)*arctan(sqrt(e*f)*x/e))/(c*d^2*e^2*f - c^2*d*e*f^2), -1/2*(2*(b*c - a
*d)*sqrt(c*d)*e*f*arctan(sqrt(c*d)*x/c) + (b*c*d*e - a*c*d*f)*sqrt(-e*f)*l
og((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)))/(c*d^2*e^2*f - c^2*d*e*f^2),
 -((b*c - a*d)*sqrt(c*d)*e*f*arctan(sqrt(c*d)*x/c) - (b*c*d*e - a*c*d*f)*s
qrt(e*f)*arctan(sqrt(e*f)*x/e))/(c*d^2*e^2*f - c^2*d*e*f^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}(de-cf)} + \frac{(be-af)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(de-cf)\sqrt{ef}}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
-(b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*(d*e - c*f)) + (b*e - a*f)*a
rctan(f*x/sqrt(e*f))/((d*e - c*f)*sqrt(e*f))
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 1245, normalized size of antiderivative = 14.48

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
(a*d*e*f*atan((b^2*c^3*f^2*x*(-c*d)^(3/2)*1i + a^2*c*d^2*f^2*x*(-c*d)^(3/2
)*1i + b^2*c^4*d*f^2*x*(-c*d)^(1/2)*1i + a^2*c^2*d^3*f^2*x*(-c*d)^(1/2)*2i
 + b^2*c^2*d^3*e^2*x*(-c*d)^(1/2)*1i + a^2*d^3*e*f*x*(-c*d)^(3/2)*1i - a*b
*c^2*d*f^2*x*(-c*d)^(3/2)*2i + b^2*c^2*d*e*f*x*(-c*d)^(3/2)*1i - a*b*c^3*d
^2*f^2*x*(-c*d)^(1/2)*2i - a*b*c*d^2*e*f*x*(-c*d)^(3/2)*2i - a*b*c^2*d^3*e
*f*x*(-c*d)^(1/2)*2i)/(a^2*c^3*d^3*f^2 + b^2*c^3*d^3*e^2 - a^2*c^2*d^4*e*f
 - b^2*c^4*d^2*e*f))*(-c*d)^(1/2)*1i)/(c*d^2*e^2*f - c^2*d*e*f^2) - (b*c*e
*f*atan((b^2*c^3*f^2*x*(-c*d)^(3/2)*1i + a^2*c*d^2*f^2*x*(-c*d)^(3/2)*1i +
 b^2*c^4*d*f^2*x*(-c*d)^(1/2)*1i + a^2*c^2*d^3*f^2*x*(-c*d)^(1/2)*2i + b^2
*c^2*d^3*e^2*x*(-c*d)^(1/2)*1i + a^2*d^3*e*f*x*(-c*d)^(3/2)*1i - a*b*c^2*d
*f^2*x*(-c*d)^(3/2)*2i + b^2*c^2*d*e*f*x*(-c*d)^(3/2)*1i - a*b*c^3*d^2*f^2
*x*(-c*d)^(1/2)*2i - a*b*c*d^2*e*f*x*(-c*d)^(3/2)*2i - a*b*c^2*d^3*e*f*x*(
-c*d)^(1/2)*2i)/(a^2*c^3*d^3*f^2 + b^2*c^3*d^3*e^2 - a^2*c^2*d^4*e*f - b^2
*c^4*d^2*e*f))*(-c*d)^(1/2)*1i)/(c*d^2*e^2*f - c^2*d*e*f^2) - (a*c*d*f*ata
n((b^2*d^2*e^3*x*(-e*f)^(3/2)*1i + a^2*d^2*e*f^2*x*(-e*f)^(3/2)*1i + b^2*d
^2*e^4*f*x*(-e*f)^(1/2)*1i + a^2*d^2*e^2*f^3*x*(-e*f)^(1/2)*2i + b^2*c^2*e
^2*f^3*x*(-e*f)^(1/2)*1i + a^2*c*d*f^3*x*(-e*f)^(3/2)*1i - a*b*d^2*e^2*f*x
*(-e*f)^(3/2)*2i + b^2*c*d*e^2*f*x*(-e*f)^(3/2)*1i - a*b*d^2*e^3*f^2*x*(-e
*f)^(1/2)*2i - a*b*c*d*e*f^2*x*(-e*f)^(3/2)*2i - a*b*c*d*e^2*f^3*x*(-e*f)^
(1/2)*2i)/(a^2*d^2*e^3*f^3 + b^2*c^2*e^3*f^3 - a^2*c*d*e^2*f^4 - b^2*c*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)}\,dx = \frac{-\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)adef+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)bcef+\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)acdf-\sqrt{f}\,\sqrt{e}\,atan\left(\frac{fx}{\sqrt{f}\,\sqrt{e}}\right)\dots}{cdef\,(cf-de)}$$

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
( - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d*e*f + sqrt(d)*sqrt(c
)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*e*f + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt
(f)*sqrt(e)))*a*c*d*f - sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c*
d*e)/(c*d*e*f*(c*f - d*e))
```

3.220 $\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 140

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{(be-af)x}{2e(de-cf)(e+fx^2)} - \frac{\sqrt{d}(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)^2} - \frac{(af(3de-cf)-be(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}\sqrt{f}(de-cf)^2}$$

output
```
1/2*(-a*f+b*e)*x/e/(-c*f+d*e)/(f*x^2+e)-d^(1/2)*(-a*d+b*c)*arctan(d^(1/2)*
x/c^(1/2))/c^(1/2)/(-c*f+d*e)^2-1/2*(a*f*(-c*f+3*d*e)-b*e*(c*f+d*e))*arcta
n(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(1/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{\frac{(be-af)(de-cf)x}{e(e+fx^2)} + \frac{2\sqrt{d}(-bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{(af(-3de+cf)+be(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}\sqrt{f}}}{2(de-cf)^2}$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(((b*e - a*f)*(d*e - c*f)*x)/(e*(e + f*x^2)) + (2*Sqrt[d]*(-(b*c) + a*d)*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c] + ((a*f*(-3*d*e + c*f) + b*e*(d*e + c*
f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*Sqrt[f]))/(2*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {402, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{\int -\frac{-d(be-af)x^2+bce-2ade+acf}{(dx^2+c)(fx^2+e)}dx}{2e(de-cf)}+\frac{x(be-af)}{2e(e+fx^2)(de-cf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\int \frac{-d(be-af)x^2+bce-2ade+acf}{(dx^2+c)(fx^2+e)}dx}{2e(de-cf)}$$

$$\Big\downarrow \text{397}$$

$$\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2de(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}+\frac{(af(3de-cf)-be(cf+de))\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
((b*e - a*f)*x)/(2*e*(d*e - c*f)*(e + f*x^2)) - ((2*Sqrt[d]*(b*c - a*d)*e*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + ((a*f*(3*d*e - c*f) -
 b*e*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f
)))/(2*e*(d*e - c*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)x}{2e\left(f\,x^{2}+e\right)}+\frac{\left(ac\,f^{2}-3adef+bcef+bd\,e^{2}\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}}{\left(cf-de\right)^{2}}+\frac{\left(ad-bc\right)d\arctan\left(\frac{xd}{\sqrt{cd}}\right)}{\left(cf-de\right)^{2}\sqrt{cd}}$	131
risch	Expression too large to display	2213

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(a*
c*f^2-3*a*d*e*f+b*c*e*f+b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))+(a
*d-b*c)*d/(c*f-d*e)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 997, normalized size of antiderivative = 7.12

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*((b*c - a*d)*e^2*f^2*x^2 + (b*c - a*d)*e^3*f)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b*d*e^3 + a*c*e*f^2 + (b*c - 3*
a*d)*e^2*f + (b*d*e^2*f + a*c*f^3 + (b*c - 3*a*d)*e*f^2)*x^2)*sqrt(-e*f)*l
og((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 2*(b*d*e^3*f + a*c*e*f^3 -
(b*c + a*d)*e^2*f^2)*x)/(d^2*e^5*f - 2*c*d*e^4*f^2 + c^2*e^3*f^3 + (d^2*e^
4*f^2 - 2*c*d*e^3*f^3 + c^2*e^2*f^4)*x^2), 1/2*((b*d*e^3 + a*c*e*f^2 + (b*
c - 3*a*d)*e^2*f + (b*d*e^2*f + a*c*f^3 + (b*c - 3*a*d)*e*f^2)*x^2)*sqrt(e
*f)*arctan(sqrt(e*f)*x/e) - ((b*c - a*d)*e^2*f^2*x^2 + (b*c - a*d)*e^3*f)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b*d*e^3*f +
a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x)/(d^2*e^5*f - 2*c*d*e^4*f^2 + c^2*e^3*f
^3 + (d^2*e^4*f^2 - 2*c*d*e^3*f^3 + c^2*e^2*f^4)*x^2), -1/4*(4*((b*c - a*d
)*e^2*f^2*x^2 + (b*c - a*d)*e^3*f)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b*d*e^
3 + a*c*e*f^2 + (b*c - 3*a*d)*e^2*f + (b*d*e^2*f + a*c*f^3 + (b*c - 3*a*d)
*e*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 2*
(b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x)/(d^2*e^5*f - 2*c*d*e^4*f^
2 + c^2*e^3*f^3 + (d^2*e^4*f^2 - 2*c*d*e^3*f^3 + c^2*e^2*f^4)*x^2), -1/2*(
2*((b*c - a*d)*e^2*f^2*x^2 + (b*c - a*d)*e^3*f)*sqrt(d/c)*arctan(x*sqrt(d/
c)) - (b*d*e^3 + a*c*e*f^2 + (b*c - 3*a*d)*e^2*f + (b*d*e^2*f + a*c*f^3 +
(b*c - 3*a*d)*e*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - (b*d*e^3*f + a
*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x)/(d^2*e^5*f - 2*c*d*e^4*f^2 + c^2*e^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2}\,dx = -\frac{(bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^2e^2-2\,cdef+c^2f^2)\sqrt{cd}} + \frac{(bde^2+bcef-3\,adef+acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,(d^2e^3-2\,cde^2f+c^2ef^2)\sqrt{ef}} + \frac{bex-afx}{2\,(de^2-cef)(fx^2+e)}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-(b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sq
rt(c*d)) + 1/2*(b*d*e^2 + b*c*e*f - 3*a*d*e*f + a*c*f^2)*arctan(f*x/sqrt(e
*f))/((d^2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*sqrt(e*f)) + 1/2*(b*e*x - a*f*x)
/((d*e^2 - c*e*f)*(f*x^2 + e))
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 5567, normalized size of antiderivative = 39.76

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
(x*(a*f - b*e))/(2*e*(e + f*x^2)*(c*f - d*e)) - (atan(((((x*(b^2*d^5*e^4*f
 + a^2*c^2*d^3*f^5 + 13*a^2*d^5*e^2*f^3 + 5*b^2*c^2*d^3*e^2*f^3 - 6*a*b*d^
5*e^3*f^2 - 6*a^2*c*d^4*e*f^4 + 2*b^2*c*d^4*e^3*f^2 - 12*a*b*c*d^4*e^2*f^3
 + 2*a*b*c^2*d^3*e*f^4))/(2*(d^2*e^4 + c^2*e^2*f^2 - 2*c*d*e^3*f)) - ((-e^
3*f)^(1/2)*((18*a*c*d^6*e^5*f^3 - 4*a*d^7*e^6*f^2 + 2*a*c^5*d^2*e*f^7 + 2*
b*c*d^6*e^6*f^2 - 32*a*c^2*d^5*e^4*f^4 + 28*a*c^3*d^4*e^3*f^5 - 12*a*c^4*d
^3*e^2*f^6 - 8*b*c^2*d^5*e^5*f^3 + 12*b*c^3*d^4*e^4*f^4 - 8*b*c^4*d^3*e^3*
f^5 + 2*b*c^5*d^2*e^2*f^6)/(d^3*e^5 - c^3*e^2*f^3 + 3*c^2*d*e^3*f^2 - 3*c*
d^2*e^4*f) - (x*(-e^3*f)^(1/2)*(a*c*f^2 + b*d*e^2 - 3*a*d*e*f + b*c*e*f)*(
16*d^7*e^7*f^2 - 48*c*d^6*e^6*f^3 + 32*c^2*d^5*e^5*f^4 + 32*c^3*d^4*e^4*f^
5 - 48*c^4*d^3*e^3*f^6 + 16*c^5*d^2*e^2*f^7))/(8*(d^2*e^5*f + c^2*e^3*f^3
- 2*c*d*e^4*f^2)*(d^2*e^4 + c^2*e^2*f^2 - 2*c*d*e^3*f)))*(a*c*f^2 + b*d*e^
2 - 3*a*d*e*f + b*c*e*f))/(4*(d^2*e^5*f + c^2*e^3*f^3 - 2*c*d*e^4*f^2)))*(
-e^3*f)^(1/2)*(a*c*f^2 + b*d*e^2 - 3*a*d*e*f + b*c*e*f)*1i)/(4*(d^2*e^5*f
+ c^2*e^3*f^3 - 2*c*d*e^4*f^2)) + (((x*(b^2*d^5*e^4*f + a^2*c^2*d^3*f^5 +
13*a^2*d^5*e^2*f^3 + 5*b^2*c^2*d^3*e^2*f^3 - 6*a*b*d^5*e^3*f^2 - 6*a^2*c*d
^4*e*f^4 + 2*b^2*c*d^4*e^3*f^2 - 12*a*b*c*d^4*e^2*f^3 + 2*a*b*c^2*d^3*e*f^
4))/(2*(d^2*e^4 + c^2*e^2*f^2 - 2*c*d*e^3*f)) + ((-e^3*f)^(1/2)*((18*a*c*d
^6*e^5*f^3 - 4*a*d^7*e^6*f^2 + 2*a*c^5*d^2*e*f^7 + 2*b*c*d^6*e^6*f^2 - 32*
a*c^2*d^5*e^4*f^4 + 28*a*c^3*d^4*e^3*f^5 - 12*a*c^4*d^3*e^2*f^6 - 8*b*c...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.06

$$\int \frac{a + bx^2}{(c + dx^2)(e + fx^2)^2} dx$$
$$= \frac{2\sqrt{d}\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\sqrt{c}}\right) ad\,e^3 f + 2\sqrt{d}\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\sqrt{c}}\right) ad\,e^2 f^2 x^2 - 2\sqrt{d}\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\sqrt{c}}\right) bc\,e^3 f - 2\sqrt{d}\sqrt{c}\,\ldots}{}$$

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
(2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d*e**3*f + 2*sqrt(d)*sq
rt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d*e**2*f**2*x**2 - 2*sqrt(d)*sqrt(c)
*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*e**3*f - 2*sqrt(d)*sqrt(c)*atan((d*x)/(
sqrt(d)*sqrt(c)))*b*c*e**2*f**2*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*a*c**2*e*f**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a
*c**2*f**3*x**2 - 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e*
*2*f - 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*e*f**2*x**2 +
 sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e**2*f + sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*e*f**2*x**2 + sqrt(f)*sqrt(e)*at
an((f*x)/(sqrt(f)*sqrt(e)))*b*c*d*e**3 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*b*c*d*e**2*f*x**2 + a*c**2*e*f**3*x - a*c*d*e**2*f**2*x - b*c
**2*e**2*f**2*x + b*c*d*e**3*f*x)/(2*c*e**2*f*(c**2*e*f**2 + c**2*f**3*x**
2 - 2*c*d*e**2*f - 2*c*d*e*f**2*x**2 + d**2*e**3 + d**2*e**2*f*x**2))
```

3.221 $\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 222

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{(be-af)x}{4e(de-cf)(e+fx^2)^2}-\frac{(af(7de-3cf)-be(3de+cf))x}{8e^2(de-cf)^2(e+fx^2)} - \frac{d^{3/2}(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)^3} + \frac{(be(3d^2e^2+6cdef-c^2f^2)-af(15d^2e^2-10cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}\sqrt{f}(de-cf)^3}$$

output
```
1/4*(-a*f+b*e)*x/e/(-c*f+d*e)/(f*x^2+e)^2-1/8*(a*f*(-3*c*f+7*d*e)-b*e*(c*f
+3*d*e))*x/e^2/(-c*f+d*e)^2/(f*x^2+e)-d^(3/2)*(-a*d+b*c)*arctan(d^(1/2)*x/
c^(1/2))/c^(1/2)/(-c*f+d*e)^3+1/8*(b*e*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2)-a*f*
(3*c^2*f^2-10*c*d*e*f+15*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/f^(1/
2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2}{(c+dx^2)\,(e+fx^2)^3}\,dx = \frac{1}{8}\left(\frac{2(be-af)x}{e(de-cf)\,(e+fx^2)^2}+\frac{(be(3de+cf)+af(-7de+3cf))x}{e^2(de-cf)^2\,(e+fx^2)} + \frac{8d^{3/2}(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(-de+cf)^3} + \frac{(af(-15d^2e^2+10cdef-3c^2f^2)+be(3d^2e^2+6cdef-c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{5/2}\sqrt{f}(de-cf)^3}\right)$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((2*(b*e - a*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + ((b*e*(3*d*e + c*f) + a
*f*(-7*d*e + 3*c*f))*x)/(e^2*(d*e - c*f)^2*(e + f*x^2)) + (8*d^(3/2)*(b*c
- a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(-(d*e) + c*f)^3) + ((a*f*(-1
5*d^2*e^2 + 10*c*d*e*f - 3*c^2*f^2) + b*e*(3*d^2*e^2 + 6*c*d*e*f - c^2*f^2
))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*Sqrt[f]*(d*e - c*f)^3))/8
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {402, 25, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{\int -\frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}+\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}$$

$\Big\downarrow$ 25

$$\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\int \frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}$$

$\Big\downarrow$ 402

$$\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\int \frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e(e+fx^2)(de-cf)}}{4e(de-cf)}$$

$\Big\downarrow$ 397

$$\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\frac{8d^2e^2(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e(e+fx^2)(de-cf)}}{4e(de-cf)}$$

$\Big\downarrow$ 218

$$\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\frac{8d^{3/2}e^2(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e(e+fx^2)(de-cf)}}{4e(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((b*e - a*f)*x)/(4*e*(d*e - c*f)*(e + f*x^2)^2) - (((a*f*(7*d*e - 3*c*f) -
 b*e*(3*d*e + c*f))*x)/(2*e*(d*e - c*f)*(e + f*x^2)) + ((8*d^(3/2)*(b*c -
a*d)*e^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - ((b*e*(3*d^2
*e^2 + 6*c*d*e*f - c^2*f^2) - a*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2))*A
rcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)))/(2*e*(d*e - c*f
)))/(4*e*(d*e - c*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{\frac{f\left(3a\,c^{2}f^{3}-10acde\,f^{2}+7a\,d^{2}e^{2}f+b\,c^{2}e\,f^{2}+2bcd\,e^{2}f-3e^{3}b\,d^{2}\right)x^{3}}{8e^{2}}+\frac{\left(5a\,c^{2}f^{3}-14acde\,f^{2}+9a\,d^{2}e^{2}f-b\,c^{2}e\,f^{2}+6bcd\,e^{2}f-5e^{3}b\,d^{2}\right)x}{8e}}{\left(f\,x^{2}+e\right)^{2}}+\frac{\left(3a\,c^{2}f^{3}-\cdots\right.}{\cdots}}{(cf-de)^{3}}$
risch	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^3*((1/8*f*(3*a*c^2*f^3-10*a*c*d*e*f^2+7*a*d^2*e^2*f+b*c^2*e*f^
2+2*b*c*d*e^2*f-3*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-14*a*c*d*e*f^2+9*a*d
^2*e^2*f-b*c^2*e*f^2+6*b*c*d*e^2*f-5*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3*a*
c^2*f^3-10*a*c*d*e*f^2+15*a*d^2*e^2*f+b*c^2*e*f^2-6*b*c*d*e^2*f-3*b*d^2*e^
3)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-(a*d-b*c)*d^2/(c*f-d*e)^3/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(200) = 400$.

Time = 3.56 (sec) , antiderivative size = 2339, normalized size of antiderivative = 10.54

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(3*b*d^2*e^4*f^2 - 3*a*c^2*e*f^5 - (2*b*c*d + 7*a*d^2)*e^3*f^3 -
(b*c^2 - 10*a*c*d)*e^2*f^4)*x^3 + 8*((b*c*d - a*d^2)*e^3*f^3*x^4 + 2*(b*c*
d - a*d^2)*e^4*f^2*x^2 + (b*c*d - a*d^2)*e^5*f)*sqrt(-d/c)*log((d*x^2 - 2*
c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - (3*b*d^2*e^5 - 3*a*c^2*e^2*f^3 + 3*(2*b
*c*d - 5*a*d^2)*e^4*f - (b*c^2 - 10*a*c*d)*e^3*f^2 + (3*b*d^2*e^3*f^2 - 3*
a*c^2*f^5 + 3*(2*b*c*d - 5*a*d^2)*e^2*f^3 - (b*c^2 - 10*a*c*d)*e*f^4)*x^4
+ 2*(3*b*d^2*e^4*f - 3*a*c^2*e*f^4 + 3*(2*b*c*d - 5*a*d^2)*e^3*f^2 - (b*c^
2 - 10*a*c*d)*e^2*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f
*x^2 + e)) + 2*(5*b*d^2*e^5*f - 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + 3*a*d^2)*e^
4*f^2 + (b*c^2 + 14*a*c*d)*e^3*f^3)*x)/(d^3*e^8*f - 3*c*d^2*e^7*f^2 + 3*c^
2*d*e^6*f^3 - c^3*e^5*f^4 + (d^3*e^6*f^3 - 3*c*d^2*e^5*f^4 + 3*c^2*d*e^4*f
^5 - c^3*e^3*f^6)*x^4 + 2*(d^3*e^7*f^2 - 3*c*d^2*e^6*f^3 + 3*c^2*d*e^5*f^4
 - c^3*e^4*f^5)*x^2), 1/8*((3*b*d^2*e^4*f^2 - 3*a*c^2*e*f^5 - (2*b*c*d + 7
*a*d^2)*e^3*f^3 - (b*c^2 - 10*a*c*d)*e^2*f^4)*x^3 + (3*b*d^2*e^5 - 3*a*c^2
*e^2*f^3 + 3*(2*b*c*d - 5*a*d^2)*e^4*f - (b*c^2 - 10*a*c*d)*e^3*f^2 + (3*b
*d^2*e^3*f^2 - 3*a*c^2*f^5 + 3*(2*b*c*d - 5*a*d^2)*e^2*f^3 - (b*c^2 - 10*a
*c*d)*e*f^4)*x^4 + 2*(3*b*d^2*e^4*f - 3*a*c^2*e*f^4 + 3*(2*b*c*d - 5*a*d^2
)*e^3*f^2 - (b*c^2 - 10*a*c*d)*e^2*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/
e) + 4*((b*c*d - a*d^2)*e^3*f^3*x^4 + 2*(b*c*d - a*d^2)*e^4*f^2*x^2 + (b*c
*d - a*d^2)*e^5*f)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = &-\frac{(bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3)\sqrt{cd}}\\&+\frac{(3\,bd^2e^3+6\,bcde^2f-15\,ad^2e^2f-bc^2ef^2+10\,acdef^2-3\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\,(d^3e^5-3\,cd^2e^4f+3\,c^2de^3f^2-c^3e^2f^3)\sqrt{ef}}\\&+\frac{3\,bde^2fx^3+bcef^2x^3-7\,adef^2x^3+3\,acf^3x^3+5\,bde^3x-bce^2fx-9\,ade^2fx+5\,acef^2x}{8\,(d^2e^4-2\,cde^3f+c^2e^2f^2)(fx^2+e)^2}\end{aligned}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-(b*c*d^2 - a*d^3)*arctan(d*x/sqrt(c*d))/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2
*d*e*f^2 - c^3*f^3)*sqrt(c*d)) + 1/8*(3*b*d^2*e^3 + 6*b*c*d*e^2*f - 15*a*d
^2*e^2*f - b*c^2*e*f^2 + 10*a*c*d*e*f^2 - 3*a*c^2*f^3)*arctan(f*x/sqrt(e*f
))/((d^3*e^5 - 3*c*d^2*e^4*f + 3*c^2*d*e^3*f^2 - c^3*e^2*f^3)*sqrt(e*f)) +
 1/8*(3*b*d*e^2*f*x^3 + b*c*e*f^2*x^3 - 7*a*d*e*f^2*x^3 + 3*a*c*f^3*x^3 +
5*b*d*e^3*x - b*c*e^2*f*x - 9*a*d*e^2*f*x + 5*a*c*e*f^2*x)/((d^2*e^4 - 2*c
*d*e^3*f + c^2*e^2*f^2)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 12640, normalized size of antiderivative = 56.94

$$\int \frac{a+bx^2}{(c+dx^2)\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
((x*(5*a*c*f^2 + 5*b*d*e^2 - 9*a*d*e*f - b*c*e*f))/(8*e*(c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) + (x^3*(3*a*c*f^3 - 7*a*d*e*f^2 + b*c*e*f^2 + 3*b*d*e^2*f))
/(8*e^2*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)))/(e^2 + f^2*x^4 + 2*e*f*x^2) - at
an(((((256*a*d^10*e^10*f^2 - 1760*a*c*d^9*e^9*f^3 - 160*b*c*d^9*e^10*f^2 +
 5280*a*c^2*d^8*e^8*f^4 - 9056*a*c^3*d^7*e^7*f^5 + 9760*a*c^4*d^6*e^6*f^6
- 6816*a*c^5*d^5*e^5*f^7 + 3040*a*c^6*d^4*e^4*f^8 - 800*a*c^7*d^3*e^3*f^9
+ 96*a*c^8*d^2*e^2*f^10 + 992*b*c^2*d^8*e^9*f^3 - 2592*b*c^3*d^7*e^8*f^4 +
 3680*b*c^4*d^6*e^7*f^5 - 3040*b*c^5*d^5*e^6*f^6 + 1440*b*c^6*d^4*e^5*f^7
- 352*b*c^7*d^3*e^4*f^8 + 32*b*c^8*d^2*e^3*f^9)/(64*(d^6*e^10 + c^6*e^4*f^
6 - 6*c^5*d*e^5*f^5 + 15*c^2*d^4*e^8*f^2 - 20*c^3*d^3*e^7*f^3 + 15*c^4*d^2
*e^6*f^4 - 6*c*d^5*e^9*f)) - (x*(-(9*a^2*c^4*f^6 + 9*b^2*d^4*e^6 + 225*a^2
*d^4*e^4*f^2 + b^2*c^4*e^2*f^4 + 6*a*b*c^4*e*f^5 - 90*a*b*d^4*e^5*f + 190*
a^2*c^2*d^2*e^2*f^4 + 30*b^2*c^2*d^2*e^4*f^2 - 60*a^2*c^3*d*e*f^5 + 36*b^2
*c*d^3*e^5*f - 300*a^2*c*d^3*e^3*f^3 - 12*b^2*c^3*d*e^3*f^3 + 132*a*b*c^2*
d^2*e^3*f^3 - 120*a*b*c*d^3*e^4*f^2 - 56*a*b*c^3*d*e^2*f^4)/(256*(d^6*e^11
*f + c^6*e^5*f^7 - 6*c*d^5*e^10*f^2 - 6*c^5*d*e^6*f^6 + 15*c^2*d^4*e^9*f^3
 - 20*c^3*d^3*e^8*f^4 + 15*c^4*d^2*e^7*f^5)))^(1/2)*(256*d^9*e^11*f^2 - 12
80*c*d^8*e^10*f^3 + 2304*c^2*d^7*e^9*f^4 - 1280*c^3*d^6*e^8*f^5 - 1280*c^4
*d^5*e^7*f^6 + 2304*c^5*d^4*e^6*f^7 - 1280*c^6*d^3*e^5*f^8 + 256*c^7*d^2*e
^4*f^9))/(32*(d^4*e^8 + c^4*e^4*f^4 - 4*c^3*d*e^5*f^3 + 6*c^2*d^2*e^6*f...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1037, normalized size of antiderivative = 4.67

$$\int \frac{a+bx^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
( - 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**2*e**5*f - 16*sqr
t(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**2*e**4*f**2*x**2 - 8*sqrt(
d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**2*e**3*f**3*x**4 + 8*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d*e**5*f + 16*sqrt(d)*sqrt(c)*a
tan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d*e**4*f**2*x**2 + 8*sqrt(d)*sqrt(c)*atan
((d*x)/(sqrt(d)*sqrt(c)))*b*c*d*e**3*f**3*x**4 + 3*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*c**3*e**2*f**3 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*a*c**3*e*f**4*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*a*c**3*f**5*x**4 - 10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(
e)))*a*c**2*d*e**3*f**2 - 20*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a*c**2*d*e**2*f**3*x**2 - 10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*a*c**2*d*e*f**4*x**4 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
a*c*d**2*e**4*f + 30*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d**
2*e**3*f**2*x**2 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c*d*
*2*e**2*f**3*x**4 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**3*e
**3*f**2 + 2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**3*e**2*f**
3*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**3*e*f**4*x**4
- 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*d*e**4*f - 12*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*d*e**3*f**2*x**2 - 6*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b*c**2*d*e**2*f**3*x**4 - 3*...
```

3.222 $\int \frac{a+bx^2}{\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = -\frac{f(2bce-ade-acf)x}{2ce(de-cf)^2(e+fx^2)} - \frac{(bc-ad)x}{2c(de-cf)(c+dx^2)(e+fx^2)} + \frac{\sqrt{d}(ad(de-5cf)+bc(de+3cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(de-cf)^3} + \frac{\sqrt{f}(af(5de-cf)-be(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}(de-cf)^3}$$

output
```
-1/2*f*(-a*c*f-a*d*e+2*b*c*e)*x/c/e/(-c*f+d*e)^2/(f*x^2+e)-1/2*(-a*d+b*c)*
x/c/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)+1/2*d^(1/2)*(a*d*(-5*c*f+d*e)+b*c*(3*c*
f+d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/(-c*f+d*e)^3+1/2*f^(1/2)*(a*f*(-
c*f+5*d*e)-b*e*(c*f+3*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{(c+dx^2)^2 (e+fx^2)^2} \, dx = \frac{1}{2} \left(\frac{d(-bc+ad)x}{c(de-cf)^2 (c+dx^2)} + \frac{f(-be+af)x}{e(de-cf)^2 (e+fx^2)} - \frac{\sqrt{d}(ad(de-5cf)+bc(de+3cf)) \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(-de+cf)^3} - \frac{\sqrt{f}(af(-5de+cf)+be(3de+cf)) \arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}(de-cf)^3} \right)$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
((d*(-(b*c) + a*d)*x)/(c*(d*e - c*f)^2*(c + d*x^2)) + (f*(-(b*e) + a*f)*x)
/(e*(d*e - c*f)^2*(e + f*x^2)) - (Sqrt[d]*(a*d*(d*e - 5*c*f) + b*c*(d*e +
3*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(-(d*e) + c*f)^3) - (Sqrt[f]
*(a*f*(-5*d*e + c*f) + b*e*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^
(3/2)*(d*e - c*f)^3))/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {402, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^2 (e+fx^2)^2} \, dx$$

$$\Big\downarrow \text{402}$$

$$-\frac{\int -\frac{-3(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^2}dx}{2c(de-cf)} - \frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{2\left(-df(2bce-ade-acf)x^2+bce(de+cf)+a\left(d^2e^2-4cdfe+c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e(e+fx^2)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-df(2bce-ade-acf)x^2+bce(de+cf)+a\left(d^2e^2-4cdfe+c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e(e+fx^2)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{de(ad(de-5cf)+bc(3cf+de))\int \frac{1}{dx^2+c}dx}{de-cf}+\frac{cf(af(5de-cf)-be(cf+3de))\int \frac{1}{fx^2+e}dx}{de-cf}}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e(e+fx^2)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\sqrt{d}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-5cf)+bc(3cf+de))}{\sqrt{c}(de-cf)}+\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5de-cf)-be(cf+3de))}{\sqrt{e}(de-cf)}}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e(e+fx^2)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)) + (-((f*(2*b*
c*e - a*d*e - a*c*f)*x)/(e*(d*e - c*f)*(e + f*x^2))) + ((Sqrt[d]*e*(a*d*(d
*e - 5*c*f) + b*c*(d*e + 3*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*
e - c*f)) + (c*Sqrt[f]*(a*f*(5*d*e - c*f) - b*e*(3*d*e + c*f))*ArcTan[(Sqr
t[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(e*(d*e - c*f)))/(2*c*(d*e - c*f)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
default	$\frac{f\left(\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)x}{2e\left(f\,x^{2}+e\right)}+\frac{\left(ac\,f^{2}-5adef+bcef+3bd\,e^{2}\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}\right)}{\left(cf-de\right)^{3}}+\frac{d\left(\frac{\left(acdf-a\,d^{2}e-b\,c^{2}f+bcde\right)x}{2c\left(x^{2}d+c\right)}+\frac{\left(5acdf-a\,d^{2}e-3b\,c^{2}f \cdots\right.}{2c \cdots}\right.}{\left(cf-de\right)^{3}}$
risch	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
f/(c*f-d*e)^3*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(a*
c*f^2-5*a*d*e*f+b*c*e*f+3*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))+
d/(c*f-d*e)^3*(1/2*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c*x/(d*x^2+c)+1/2*(5*
a*c*d*f-a*d^2*e-3*b*c^2*f-b*c*d*e)/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(190) = 380.

Time = 4.35 (sec) , antiderivative size = 2381, normalized size of antiderivative = 11.13

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(2*b*c^2*d*e*f^2 - a*c^2*d*f^3 - (2*b*c*d^2 - a*d^3)*e^2*f)*x^3 +
(((b*c*d^2 + a*d^3)*e^2*f + (3*b*c^2*d - 5*a*c*d^2)*e*f^2)*x^4 + (b*c^2*d
+ a*c*d^2)*e^3 + (3*b*c^3 - 5*a*c^2*d)*e^2*f + ((b*c*d^2 + a*d^3)*e^3 + 4*
(b*c^2*d - a*c*d^2)*e^2*f + (3*b*c^3 - 5*a*c^2*d)*e*f^2)*x^2)*sqrt(-d/c)*l
og((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - (3*b*c^2*d*e^3 + a*c^3*e*
f^2 + (3*b*c*d^2*e^2*f + a*c^2*d*f^3 + (b*c^2*d - 5*a*c*d^2)*e*f^2)*x^4 +
(b*c^3 - 5*a*c^2*d)*e^2*f + (3*b*c*d^2*e^3 + a*c^3*f^3 + (4*b*c^2*d - 5*a*
c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*d)*e*f^2)*x^2)*sqrt(-f/e)*log((f*x^2 + 2*e
*x*sqrt(-f/e) - e)/(f*x^2 + e)) - 2*(a*c*d^2*e^2*f + a*c^3*f^3 + (b*c*d^2
- a*d^3)*e^3 - (b*c^3 + a*c^2*d)*e*f^2)*x)/(c^2*d^3*e^5 - 3*c^3*d^2*e^4*f
+ 3*c^4*d*e^3*f^2 - c^5*e^2*f^3 + (c*d^4*e^4*f - 3*c^2*d^3*e^3*f^2 + 3*c^3
*d^2*e^2*f^3 - c^4*d*e*f^4)*x^4 + (c*d^4*e^5 - 2*c^2*d^3*e^4*f + 2*c^4*d*e
^2*f^3 - c^5*e*f^4)*x^2), 1/4*(2*(2*b*c^2*d*e*f^2 - a*c^2*d*f^3 - (2*b*c*d
^2 - a*d^3)*e^2*f)*x^3 - 2*(3*b*c^2*d*e^3 + a*c^3*e*f^2 + (3*b*c*d^2*e^2*f
 + a*c^2*d*f^3 + (b*c^2*d - 5*a*c*d^2)*e*f^2)*x^4 + (b*c^3 - 5*a*c^2*d)*e^
2*f + (3*b*c*d^2*e^3 + a*c^3*f^3 + (4*b*c^2*d - 5*a*c*d^2)*e^2*f + (b*c^3
- 4*a*c^2*d)*e*f^2)*x^2)*sqrt(f/e)*arctan(x*sqrt(f/e)) + (((b*c*d^2 + a*d^
3)*e^2*f + (3*b*c^2*d - 5*a*c*d^2)*e*f^2)*x^4 + (b*c^2*d + a*c*d^2)*e^3 +
(3*b*c^3 - 5*a*c^2*d)*e^2*f + ((b*c*d^2 + a*d^3)*e^3 + 4*(b*c^2*d - a*c*d^
2)*e^2*f + (3*b*c^3 - 5*a*c^2*d)*e*f^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx &= \frac{(bcd^2e+ad^3e+3\,bc^2df-5\,acd^2f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(cd^3e^3-3\,c^2d^2e^2f+3\,c^3def^2-c^4f^3)\sqrt{cd}}\\ &\quad-\frac{(3\,bde^2f+bcef^2-5\,adef^2+acf^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,(d^3e^4-3\,cd^2e^3f+3\,c^2de^2f^2-c^3ef^3)\sqrt{ef}}\\ &\quad-\frac{2\,bcdefx^3-ad^2efx^3-acdf^2x^3+bcde^2x-ad^2e^2x+bc^2efx-ac^2f^2x}{2\,(cd^2e^3-2\,c^2de^2f+c^3ef^2)(dfx^4+dex^2+cfx^2+ce)}\end{aligned}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d^2*e + a*d^3*e + 3*b*c^2*d*f - 5*a*c*d^2*f)*arctan(d*x/sqrt(c*d)
)/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*sqrt(c*d)) - 1/
2*(3*b*d*e^2*f + b*c*e*f^2 - 5*a*d*e*f^2 + a*c*f^3)*arctan(f*x/sqrt(e*f))/
((d^3*e^4 - 3*c*d^2*e^3*f + 3*c^2*d*e^2*f^2 - c^3*e*f^3)*sqrt(e*f)) - 1/2*
(2*b*c*d*e*f*x^3 - a*d^2*e*f*x^3 - a*c*d*f^2*x^3 + b*c*d*e^2*x - a*d^2*e^2
*x + b*c^2*e*f*x - a*c^2*f^2*x)/((c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*(
d*f*x^4 + d*e*x^2 + c*f*x^2 + c*e))
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 9047, normalized size of antiderivative = 42.28

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
((x*(a*c^2*f^2 + a*d^2*e^2 - b*c*d*e^2 - b*c^2*e*f))/(2*c*e*(c^2*f^2 + d^2
*e^2 - 2*c*d*e*f)) + (d*f*x^3*(a*c*f + a*d*e - 2*b*c*e))/(2*c*e*(c^2*f^2 +
 d^2*e^2 - 2*c*d*e*f)))/(c*e + x^2*(c*f + d*e) + d*f*x^4) + (atan(((((x*(a
^2*c^4*d^3*f^7 + a^2*d^7*e^4*f^3 + 50*a^2*c^2*d^5*e^2*f^5 + 10*b^2*c^2*d^5
*e^4*f^3 + 12*b^2*c^3*d^4*e^3*f^4 + 10*b^2*c^4*d^3*e^2*f^5 - 10*a^2*c*d^6*
e^3*f^4 - 10*a^2*c^3*d^4*e*f^6 - 34*a*b*c^2*d^5*e^3*f^4 - 34*a*b*c^3*d^4*e
^2*f^5 + 2*a*b*c*d^6*e^4*f^3 + 2*a*b*c^4*d^3*e*f^6))/(2*(c^2*d^4*e^6 + c^6
*e^2*f^4 - 4*c^3*d^3*e^5*f - 4*c^5*d*e^3*f^3 + 6*c^4*d^2*e^4*f^2)) - (((2*
a*c*d^10*e^9*f^2 + 2*a*c^9*d^2*e*f^10 - 20*a*c^2*d^9*e^8*f^3 + 80*a*c^3*d^
8*e^7*f^4 - 172*a*c^4*d^7*e^6*f^5 + 220*a*c^5*d^6*e^5*f^6 - 172*a*c^6*d^5*
e^4*f^7 + 80*a*c^7*d^4*e^3*f^8 - 20*a*c^8*d^3*e^2*f^9 + 2*b*c^2*d^9*e^9*f^
2 - 10*b*c^3*d^8*e^8*f^3 + 18*b*c^4*d^7*e^7*f^4 - 10*b*c^5*d^6*e^6*f^5 - 1
0*b*c^6*d^5*e^5*f^6 + 18*b*c^7*d^4*e^4*f^7 - 10*b*c^8*d^3*e^3*f^8 + 2*b*c^
9*d^2*e^2*f^9)/(c^2*d^6*e^8 + c^8*e^2*f^6 - 6*c^3*d^5*e^7*f - 6*c^7*d*e^3*
f^5 + 15*c^4*d^4*e^6*f^2 - 20*c^5*d^3*e^5*f^3 + 15*c^6*d^2*e^4*f^4) - (x*(
-e^3*f)^(1/2)*(a*c*f^2 + 3*b*d*e^2 - 5*a*d*e*f + b*c*e*f)*(16*c^2*d^9*e^9*
f^2 - 80*c^3*d^8*e^8*f^3 + 144*c^4*d^7*e^7*f^4 - 80*c^5*d^6*e^6*f^5 - 80*c
^6*d^5*e^5*f^6 + 144*c^7*d^4*e^4*f^7 - 80*c^8*d^3*e^3*f^8 + 16*c^9*d^2*e^2
*f^9))/(8*(d^3*e^6 - c^3*e^3*f^3 + 3*c^2*d*e^4*f^2 - 3*c*d^2*e^5*f)*(c^2*d
^4*e^6 + c^6*e^2*f^4 - 4*c^3*d^3*e^5*f - 4*c^5*d*e^3*f^3 + 6*c^4*d^2*e^...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.12

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d*e**3*f + 5*sqrt(
d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d*e**2*f**2*x**2 - sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**2*e**4 + 4*sqrt(d)*sqrt(c)*a
tan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**2*e**3*f*x**2 + 5*sqrt(d)*sqrt(c)*atan
((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**2*e**2*f**2*x**4 - sqrt(d)*sqrt(c)*atan((
d*x)/(sqrt(d)*sqrt(c)))*a*d**3*e**4*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqr
t(d)*sqrt(c)))*a*d**3*e**3*f*x**4 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*
sqrt(c)))*b*c**3*e**3*f - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*
b*c**3*e**2*f**2*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*
*2*d*e**4 - 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c**2*d*e**3*
f*x**2 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c**2*d*e**2*f**
2*x**4 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d**2*e**4*x**2
- sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d**2*e**3*f*x**4 + sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**4*e*f**2 + sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*a*c**4*f**3*x**2 - 5*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*a*c**3*d*e**2*f - 4*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*a*c**3*d*e*f**2*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*a*c**3*d*f**3*x**4 - 5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*a*c**2*d**2*e**2*f*x**2 - 5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*a*c**2*d**2*e*f**2*x**4 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqr...
```

3.223 $\int \frac{a+bx^2}{\left(c+dx^2\right)^2 \left(e+fx^2\right)^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 313

$$\begin{aligned}
&\int \frac{a+bx^2}{\left(c+dx^2\right)^2 \left(e+fx^2\right)^3} \, dx \\
&= -\frac{f(3bce-2ade-acf)x}{4ce(de-cf)^2\left(e+fx^2\right)^2} - \frac{(bc-ad)x}{2c(de-cf)\left(c+dx^2\right)\left(e+fx^2\right)^2} \\
&\quad - \frac{f\left(bce(11de+cf)-a\left(4d^2e^2+11cdef-3c^2f^2\right)\right)x}{8ce^2(de-cf)^3\left(e+fx^2\right)} \\
&\quad + \frac{d^{3/2}(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(de-cf)^4} \\
&\quad - \frac{\sqrt{f}\left(be\left(15d^2e^2+10cdef-c^2f^2\right)-af\left(35d^2e^2-14cdef+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}(de-cf)^4}
\end{aligned}$$

output

```
-1/4*f*(-a*c*f-2*a*d*e+3*b*c*e)*x/c/e/(-c*f+d*e)^2/(f*x^2+e)^2-1/2*(-a*d+b
*c)*x/c/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)^2-1/8*f*(b*c*e*(c*f+11*d*e)-a*(-3*c
^2*f^2+11*c*d*e*f+4*d^2*e^2))*x/c/e^2/(-c*f+d*e)^3/(f*x^2+e)+1/2*d^(3/2)*(
a*d*(-7*c*f+d*e)+b*c*(5*c*f+d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/(-c*f+
d*e)^4-1/8*f^(1/2)*(b*e*(-c^2*f^2+10*c*d*e*f+15*d^2*e^2)-a*f*(3*c^2*f^2-14
*c*d*e*f+35*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(\frac{4d^2(bc-ad)x}{c(-de+cf)^3(c+dx^2)}+\frac{2f(-be+af)x}{e(de-cf)^2(e+fx^2)^2} - \frac{f(be(7de+cf)+af(-11de+3cf))x}{e^2(de-cf)^3(e+fx^2)} + \frac{4d^{3/2}(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(de-cf)^4} + \frac{\sqrt{f}(be(-15d^2e^2-10cdef+c^2f^2)+af(35d^2e^2-14cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{5/2}(de-cf)^4}\right)$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
((4*d^2*(b*c - a*d)*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)) + (2*f*(-(b*e) + a
*f)*x)/(e*(d*e - c*f)^2*(e + f*x^2)^2) - (f*(b*e*(7*d*e + c*f) + a*f*(-11*
d*e + 3*c*f))*x)/(e^2*(d*e - c*f)^3*(e + f*x^2)) + (4*d^(3/2)*(a*d*(d*e -
7*c*f) + b*c*(d*e + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(d*e - c
*f)^4) + (Sqrt[f]*(b*e*(-15*d^2*e^2 - 10*c*d*e*f + c^2*f^2) + a*f*(35*d^2*
e^2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*(d*e
- c*f)^4))/8
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {402, 25, 402, 27, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^3}\,dx$$

$\Big\downarrow$ 402

$$-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e(e+fx^2)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e(e+fx^2)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-3c^2f^2+11\right.\right.}{2e\left(e+fx^2\right)(de-cf}}{2e(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{4d^2e^2(ad(de-7cf)+bc(5cf+de))\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{cf\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-3c^2f^2+11\right.\right.}{2e\left(e+fx^2\right)(de-cf}}{2e(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{4d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-7cf)+bc(5cf+de))}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-\cdots\right.\right.}{2e\left(e+fx\cdots\right.}}{2e(de-cf)}\cdots}{2c(de-cf)}$$

$$\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)^2) + (-1/2*(f*
(3*b*c*e - 2*a*d*e - a*c*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(f*(b
*c*e*(11*d*e + c*f) - a*(4*d^2*e^2 + 11*c*d*e*f - 3*c^2*f^2))*x)/(e*(d*e -
 c*f)*(e + f*x^2)) + ((4*d^(3/2)*e^2*(a*d*(d*e - 7*c*f) + b*c*(d*e + 5*c*f
))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (c*Sqrt[f]*(b*e*(1
5*d^2*e^2 + 10*c*d*e*f - c^2*f^2) - a*f*(35*d^2*e^2 - 14*c*d*e*f + 3*c^2*f
^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f))
)/(2*e*(d*e - c*f)))/(2*c*(d*e - c*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.06

method	result
default	$\frac{f\left(\frac{\frac{f\left(3a\,c^2 f^3-14acde\,f^2+11a\,d^2e^2f+b\,c^2e\,f^2+6bcd\,e^2f-7e^3b\,d^2\right)x^3}{8e^2}+\frac{\left(5a\,c^2f^3-18acde\,f^2+13a\,d^2e^2f-b\,c^2e\,f^2+10bcd\,e^2f-9e^3b\,d^2\right)x}{8e}}{\left(f\,x^2+e\right)^2}+\frac{(3a}{}\right.}{(cf-de)^4}$
risch	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
f/(c*f-d*e)^4*((1/8*f*(3*a*c^2*f^3-14*a*c*d*e*f^2+11*a*d^2*e^2*f+b*c^2*e*f
^2+6*b*c*d*e^2*f-7*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-18*a*c*d*e*f^2+13*a
*d^2*e^2*f-b*c^2*e*f^2+10*b*c*d*e^2*f-9*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3
*a*c^2*f^3-14*a*c*d*e*f^2+35*a*d^2*e^2*f+b*c^2*e*f^2-10*b*c*d*e^2*f-15*b*d
^2*e^3)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d^2/(c*f-d*e)^4*(1/2*(a*c
*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c*x/(d*x^2+c)+1/2*(7*a*c*d*f-a*d^2*e-5*b*c^2
*f-b*c*d*e)/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(287) = 574.

Time = 17.64 (sec) , antiderivative size = 4763, normalized size of antiderivative = 15.22

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.42

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{(bcd^3e+ad^4e+5\,bc^2d^2f-7\,acd^3f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(cd^4e^4-4\,c^2d^3e^3f+6\,c^3d^2e^2f^2-4\,c^4def^3+c^5f^4)\sqrt{cd}}$$
$$-\frac{(15\,bd^2e^3f+10\,bcde^2f^2-35\,ad^2e^2f^2-bc^2ef^3+14\,acdef^3-3\,ac^2f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\,(d^4e^6-4\,cd^3e^5f+6\,c^2d^2e^4f^2-4\,c^3de^3f^3+c^4e^2f^4)\sqrt{ef}}$$
$$-\frac{bcd^2x-ad^3x}{2\,(cd^3e^3-3\,c^2d^2e^2f+3\,c^3def^2-c^4f^3)(dx^2+c)}$$
$$-\frac{7\,bde^2f^2x^3+bcef^3x^3-11\,adef^3x^3+3\,acf^4x^3+9\,bde^3fx-bce^2f^2x-13\,ade^2f^2x+5\,acef^3x}{8\,(d^3e^5-3\,cd^2e^4f+3\,c^2de^3f^2-c^3e^2f^3)(fx^2+e)^2}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/2*(b*c*d^3*e + a*d^4*e + 5*b*c^2*d^2*f - 7*a*c*d^3*f)*arctan(d*x/sqrt(c*
d))/((c*d^4*e^4 - 4*c^2*d^3*e^3*f + 6*c^3*d^2*e^2*f^2 - 4*c^4*d*e*f^3 + c^
5*f^4)*sqrt(c*d)) - 1/8*(15*b*d^2*e^3*f + 10*b*c*d*e^2*f^2 - 35*a*d^2*e^2*
f^2 - b*c^2*e*f^3 + 14*a*c*d*e*f^3 - 3*a*c^2*f^4)*arctan(f*x/sqrt(e*f))/((
d^4*e^6 - 4*c*d^3*e^5*f + 6*c^2*d^2*e^4*f^2 - 4*c^3*d*e^3*f^3 + c^4*e^2*f^
4)*sqrt(e*f)) - 1/2*(b*c*d^2*x - a*d^3*x)/((c*d^3*e^3 - 3*c^2*d^2*e^2*f +
3*c^3*d*e*f^2 - c^4*f^3)*(d*x^2 + c)) - 1/8*(7*b*d*e^2*f^2*x^3 + b*c*e*f^3
*x^3 - 11*a*d*e*f^3*x^3 + 3*a*c*f^4*x^3 + 9*b*d*e^3*f*x - b*c*e^2*f^2*x -
13*a*d*e^2*f^2*x + 5*a*c*e*f^3*x)/((d^3*e^5 - 3*c*d^2*e^4*f + 3*c^2*d*e^3*
f^2 - c^3*e^2*f^3)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 17499, normalized size of antiderivative = 55.91

$$\int \frac{a+bx^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
((x^5*(3*a*c^2*d*f^4 - 4*a*d^3*e^2*f^2 - 11*a*c*d^2*e*f^3 + b*c^2*d*e*f^3
+ 11*b*c*d^2*e^2*f^2))/(8*c*e^2*(c^3*f^3 - d^3*e^3 + 3*c*d^2*e^2*f - 3*c^2
*d*e*f^2)) + (x*(5*a*c^3*f^3 - 4*a*d^3*e^3 + 4*b*c*d^2*e^3 - b*c^3*e*f^2 -
 13*a*c^2*d*e*f^2 + 9*b*c^2*d*e^2*f))/(8*c*e*(c*f - d*e)*(c^2*f^2 + d^2*e^
2 - 2*c*d*e*f)) + (f*x^3*(3*a*c^3*f^3 - 8*a*d^3*e^3 + 17*b*c*d^2*e^3 + b*c
^3*e*f^2 - 13*a*c*d^2*e^2*f - 6*a*c^2*d*e*f^2 + 6*b*c^2*d*e^2*f))/(8*c*e^2
*(c*f - d*e)*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)))/(c*e^2 + x^2*(d*e^2 + 2*c*e
*f) + x^4*(c*f^2 + 2*d*e*f) + d*f^2*x^6) - atan(((((256*a*c*d^13*e^13*f^2
- 3584*a*c^2*d^12*e^12*f^3 + 20160*a*c^3*d^11*e^11*f^4 - 63168*a*c^4*d^10*
e^10*f^5 + 125184*a*c^5*d^9*e^9*f^6 - 166656*a*c^6*d^8*e^8*f^7 + 153216*a*
c^7*d^7*e^7*f^8 - 97920*a*c^8*d^6*e^6*f^9 + 43008*a*c^9*d^5*e^5*f^10 - 125
44*a*c^10*d^4*e^4*f^11 + 2240*a*c^11*d^3*e^3*f^12 - 192*a*c^12*d^2*e^2*f^1
3 + 256*b*c^2*d^12*e^13*f^2 - 1472*b*c^3*d^11*e^12*f^3 + 2496*b*c^4*d^10*e
^11*f^4 + 2304*b*c^5*d^9*e^10*f^5 - 16128*b*c^6*d^8*e^9*f^6 + 29568*b*c^7*
d^7*e^8*f^7 - 29568*b*c^8*d^6*e^7*f^8 + 17664*b*c^9*d^5*e^6*f^9 - 6144*b*c
^10*d^4*e^5*f^10 + 1088*b*c^11*d^3*e^4*f^11 - 64*b*c^12*d^2*e^3*f^12)/(128
*(c^2*d^9*e^13 - c^11*e^4*f^9 - 9*c^3*d^8*e^12*f + 9*c^10*d*e^5*f^8 + 36*c
^4*d^7*e^11*f^2 - 84*c^5*d^6*e^10*f^3 + 126*c^6*d^5*e^9*f^4 - 126*c^7*d^4*
e^8*f^5 + 84*c^8*d^3*e^7*f^6 - 36*c^9*d^2*e^6*f^7)) - (x*(-(9*a^2*c^4*f^7
+ 225*b^2*d^4*e^6*f + 1225*a^2*d^4*e^4*f^3 + b^2*c^4*e^2*f^5 + 6*a*b*c^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2169, normalized size of antiderivative = 6.93

$$\int \frac{a+bx^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
( - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d**2*e**5*f -
56*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d**2*e**4*f**2*x**
2 - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d**2*e**3*f**3
*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**3*e**6 - 20
*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**3*e**5*f*x**2 - 52*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**3*e**4*f**2*x**4 - 28*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d**3*e**3*f**3*x**6 + 4*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**4*e**6*x**2 + 8*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**4*e**5*f*x**4 + 4*sqrt(d)*sqrt
(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**4*e**4*f**2*x**6 + 20*sqrt(d)*sqrt(
c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c**3*d*e**5*f + 40*sqrt(d)*sqrt(c)*atan
((d*x)/(sqrt(d)*sqrt(c)))*b*c**3*d*e**4*f**2*x**2 + 20*sqrt(d)*sqrt(c)*ata
n((d*x)/(sqrt(d)*sqrt(c)))*b*c**3*d*e**3*f**3*x**4 + 4*sqrt(d)*sqrt(c)*ata
n((d*x)/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**6 + 28*sqrt(d)*sqrt(c)*atan((d*x
)/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**5*f*x**2 + 44*sqrt(d)*sqrt(c)*atan((d*
x)/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**4*f**2*x**4 + 20*sqrt(d)*sqrt(c)*atan
((d*x)/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**3*f**3*x**6 + 4*sqrt(d)*sqrt(c)*a
tan((d*x)/(sqrt(d)*sqrt(c)))*b*c*d**3*e**6*x**2 + 8*sqrt(d)*sqrt(c)*atan((
d*x)/(sqrt(d)*sqrt(c)))*b*c*d**3*e**5*f*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x
)/(sqrt(d)*sqrt(c)))*b*c*d**3*e**4*f**2*x**6 + 3*sqrt(f)*sqrt(e)*atan((...
```

3.224 $\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 448

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx &= \frac{f(bce(de+11cf)+a(3d^2e^2-13cdef-2c^2f^2))\,x}{8c^2e(de-cf)^3(e+fx^2)^2} \\ &\quad - \frac{(bc-ad)x}{4c(de-cf)(c+dx^2)^2(e+fx^2)^2} + \frac{(ad(3de-11cf)+bc(de+7cf))x}{8c^2(de-cf)^2(c+dx^2)(e+fx^2)^2} \\ &\quad + \frac{f(bce(d^2e^2+22cdef+c^2f^2)+3a(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3))\,x}{8c^2e^2(de-cf)^4(e+fx^2)} \\ &\quad + \frac{d^{3/2}(bc(d^2e^2-14cdef-35c^2f^2)+3ad(d^2e^2-6cdef+21c^2f^2))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(de-cf)^5} \\ &\quad + \frac{f^{3/2}(be(35d^2e^2+14cdef-c^2f^2)-3af(21d^2e^2-6cdef+c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}(de-cf)^5}\end{aligned}$$

output
```
1/8*f*(b*c*e*(11*c*f+d*e)+a*(-2*c^2*f^2-13*c*d*e*f+3*d^2*e^2))*x/c^2/e/(-c
*f+d*e)^3/(f*x^2+e)^2-1/4*(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^2/(f*x^2+e)^
2+1/8*(a*d*(-11*c*f+3*d*e)+b*c*(7*c*f+d*e))*x/c^2/(-c*f+d*e)^2/(d*x^2+c)/(
f*x^2+e)^2+1/8*f*(b*c*e*(c^2*f^2+22*c*d*e*f+d^2*e^2)+3*a*(c^3*f^3-5*c^2*d*
e*f^2-5*c*d^2*e^2*f+d^3*e^3))*x/c^2/e^2/(-c*f+d*e)^4/(f*x^2+e)+1/8*d^(3/2)
*(b*c*(-35*c^2*f^2-14*c*d*e*f+d^2*e^2)+3*a*d*(21*c^2*f^2-6*c*d*e*f+d^2*e^2
))*arctan(d^(1/2)*x/c^(1/2))/c^(5/2)/(-c*f+d*e)^5+1/8*f^(3/2)*(b*e*(-c^2*f
^2+14*c*d*e*f+35*d^2*e^2)-3*a*f*(c^2*f^2-6*c*d*e*f+21*d^2*e^2))*arctan(f^(
1/2)*x/e^(1/2))/e^(5/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx$$
$$= \frac{1}{8}\left(\frac{2d^2(bc-ad)x}{c(-de+cf)^3(c+dx^2)^2}+\frac{d^2(3ad(de-5cf)+bc(de+11cf))x}{c^2(de-cf)^4(c+dx^2)} + \frac{2f^2(be-af)x}{e(de-cf)^3(e+fx^2)^2}+\frac{f^2(3af(-5de+cf)+be(11de+cf))x}{e^2(de-cf)^4(e+fx^2)} + \frac{d^{3/2}(-3ad(d^2e^2-6cdef+21c^2f^2)+bc(-d^2e^2+14cdef+35c^2f^2))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(-de+cf)^5} + \frac{f^{3/2}(be(35d^2e^2+14cdef-c^2f^2)-3af(21d^2e^2-6cdef+c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{5/2}(de-cf)^5}\right)$$

input
```
Integrate[(a + b*x^2)/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
((2*d^2*(b*c - a*d)*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)^2) + (d^2*(3*a*d*(d
*e - 5*c*f) + b*c*(d*e + 11*c*f))*x)/(c^2*(d*e - c*f)^4*(c + d*x^2)) + (2*
f^2*(b*e - a*f)*x)/(e*(d*e - c*f)^3*(e + f*x^2)^2) + (f^2*(3*a*f*(-5*d*e +
 c*f) + b*e*(11*d*e + c*f))*x)/(e^2*(d*e - c*f)^4*(e + f*x^2)) + (d^(3/2)*
(-3*a*d*(d^2*e^2 - 6*c*d*e*f + 21*c^2*f^2) + b*c*(-(d^2*e^2) + 14*c*d*e*f
+ 35*c^2*f^2))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(-(d*e) + c*f)^5) + (
f^(3/2)*(b*e*(35*d^2*e^2 + 14*c*d*e*f - c^2*f^2) - 3*a*f*(21*d^2*e^2 - 6*c
*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*(d*e - c*f)^5))/8
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {402, 25, 402, 25, 402, 27, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{402}$$

$$-\frac{\int -\frac{-7(bc-ad)fx^2+bce+3ade-4acf}{\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{4c(de-cf)}-\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-7(bc-ad)fx^2+bce+3ade-4acf}{\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{4c(de-cf)}-\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x(ad(3de-11cf)+bc(7cf+de))}{2c(c+dx^2)(e+fx^2)^2(de-cf)}-\frac{\int -\frac{5f(ad(3de-11cf)+bc(de+7cf))x^2+bce(de-9cf)+a\left(3d^2e^2-3cdfe+8c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}}{4c(de-cf)}-\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5f(ad(3de-11cf)+bc(de+7cf))x^2+bce(de-9cf)+a\left(3d^2e^2-3cdfe+8c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}+\frac{x(ad(3de-11cf)+bc(7cf+de))}{2c(c+dx^2)(e+fx^2)^2(de-cf)}}{4c(de-cf)}-\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{4\left(3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2-13cdef+3d^2e^2\right)\right.}{e\left(e+fx^2\right)^2(de-}}{2c(de-cf)}}{4c(de-cf)}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2-13cdef+3d^2e^2\right)+b\right.}{e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}}{4c(de-cf)}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{2\left(df\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+c^3f^3\right)\right)x^2+bce\left(d^3e^3-13cd^2fe^2-13c^2df^2e+c^3f^3\right)+3a\left(d^4e^4-5cd^3fe^3+16c^2d^2f^2e^2-5c^3df^3\right.\right.}{\left(dx^2+c\right)\left(fx^2+e\right)}}{2e(de-cf)}}{e(de-cf)}}{2c(de-cf)}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{df\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+c^3f^3\right)\right)x^2+bce\left(d^3e^3-13cd^2fe^2-13c^2df^2e+c^3f^3\right)+3a\left(d^4e^4-5cd^3fe^3+16c^2d^2f^2e^2-5c^3df^3e+\right.}{\left(dx^2+c\right)\left(fx^2+e\right)}}{e(de-cf)}}{e(de-cf)}}{2c(de-cf)}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\frac{d^2e^2\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)+bc\left(-35c^2f^2-14cdef+d^2e^2\right)\right)\int\frac{1}{dx^2+c}dx}{de-cf}+\frac{c^2f^2\left(be\left(-c^2f^2+14cdef+35d^2e^2\right)-3af\left(c^2f^2-6cdef+21d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{e(de-cf)}}{e(de-cf)}}{2c(de-cf)}}{4c(de}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\frac{c^2f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+14cdef+35d^2e^2\right)-3af\left(c^2f^2-6cdef+21d^2e^2\right)\right)}{\sqrt{e}(de-cf)}+\frac{d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)+bc\left(-35c^2f^2-14cdef\right.\right.}{\sqrt{c}(de-cf)}}{e(de-cf)}}{e(de-cf)}}{2c(de-cf)}}{4c}$$

$$\frac{x(bc-ad)}{4c\left(c+dx^2\right)^2\left(e+fx^2\right)^2(de-cf)}$$

input
```
Int[(a + b*x^2)/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
-1/4*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)^2*(e + f*x^2)^2) + (((a*d*
(3*d*e - 11*c*f) + b*c*(d*e + 7*c*f))*x)/(2*c*(d*e - c*f)*(c + d*x^2)*(e +
 f*x^2)^2) + ((f*(b*c*e*(d*e + 11*c*f) + a*(3*d^2*e^2 - 13*c*d*e*f - 2*c^2
*f^2))*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + ((f*(b*c*e*(d^2*e^2 + 22*c*d*e*f
 + c^2*f^2) + 3*a*(d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + c^3*f^3))*x)/
(e*(d*e - c*f)*(e + f*x^2)) + ((d^(3/2)*e^2*(b*c*(d^2*e^2 - 14*c*d*e*f - 3
5*c^2*f^2) + 3*a*d*(d^2*e^2 - 6*c*d*e*f + 21*c^2*f^2))*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + (c^2*f^(3/2)*(b*e*(35*d^2*e^2 + 14*c*d*e
*f - c^2*f^2) - 3*a*f*(21*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*
x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(e*(d*e - c*f)))/(e*(d*e - c*f)))/(2*c
*(d*e - c*f)))/(4*c*(d*e - c*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.03

method	result
default	$\frac{f^2\left(\frac{\frac{f\left(3a\,c^2f^3-18acde\,f^2+15a\,d^2e^2f+b\,c^2e\,f^2+10bcd\,e^2f-11e^3b\,d^2\right)x^3}{8e^2}+\frac{\left(5a\,c^2f^3-22acde\,f^2+17a\,d^2e^2f-b\,c^2e\,f^2+14bcd\,e^2f-13e^3b\,d^2\right)x}{8e}}{\left(f\,x^2+e\right)^2}\right.}{(cf-de)^5}$
risch	Expression too large to display

input
```
int((b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
f^2/(c*f-d*e)^5*((1/8*f*(3*a*c^2*f^3-18*a*c*d*e*f^2+15*a*d^2*e^2*f+b*c^2*e
*f^2+10*b*c*d*e^2*f-11*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-22*a*c*d*e*f^2+
17*a*d^2*e^2*f-b*c^2*e*f^2+14*b*c*d*e^2*f-13*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1
/8*(3*a*c^2*f^3-18*a*c*d*e*f^2+63*a*d^2*e^2*f+b*c^2*e*f^2-14*b*c*d*e^2*f-3
5*b*d^2*e^3)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d^2/(c*f-d*e)^5*((1/
8*d*(15*a*c^2*d*f^2-18*a*c*d^2*e*f+3*a*d^3*e^2-11*b*c^3*f^2+10*b*c^2*d*e*f
+b*c*d^2*e^2)/c^2*x^3+1/8*(17*a*c^2*d*f^2-22*a*c*d^2*e*f+5*a*d^3*e^2-13*b*
c^3*f^2+14*b*c^2*d*e*f-b*c*d^2*e^2)/c*x)/(d*x^2+c)^2+1/8*(63*a*c^2*d*f^2-1
8*a*c*d^2*e*f+3*a*d^3*e^2-35*b*c^3*f^2-14*b*c^2*d*e*f+b*c*d^2*e^2)/c^2/(c*
d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(420) = 840$.

Time = 56.84 (sec) , antiderivative size = 7784, normalized size of antiderivative = 17.38

$$\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{(c+dx^2)^3 (e+fx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(420) = 840$.

Time = 0.14 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.02

$$\int \frac{a+bx^2}{(c+dx^2)^3 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(b*c*d^4*e^2 + 3*a*d^5*e^2 - 14*b*c^2*d^3*e*f - 18*a*c*d^4*e*f - 35*b*
c^3*d^2*f^2 + 63*a*c^2*d^3*f^2)*arctan(d*x/sqrt(c*d))/((c^2*d^5*e^5 - 5*c^
3*d^4*e^4*f + 10*c^4*d^3*e^3*f^2 - 10*c^5*d^2*e^2*f^3 + 5*c^6*d*e*f^4 - c^
7*f^5)*sqrt(c*d)) + 1/8*(35*b*d^2*e^3*f^2 + 14*b*c*d*e^2*f^3 - 63*a*d^2*e^
2*f^3 - b*c^2*e*f^4 + 18*a*c*d*e*f^4 - 3*a*c^2*f^5)*arctan(f*x/sqrt(e*f))/
((d^5*e^7 - 5*c*d^4*e^6*f + 10*c^2*d^3*e^5*f^2 - 10*c^3*d^2*e^4*f^3 + 5*c^
4*d*e^3*f^4 - c^5*e^2*f^5)*sqrt(e*f)) + 1/8*(b*c*d^4*e^3*f^2*x^7 + 3*a*d^5
*e^3*f^2*x^7 + 22*b*c^2*d^3*e^2*f^3*x^7 - 15*a*c*d^4*e^2*f^3*x^7 + b*c^3*d
^2*e*f^4*x^7 - 15*a*c^2*d^3*e*f^4*x^7 + 3*a*c^3*d^2*f^5*x^7 + 2*b*c*d^4*e^
4*f*x^5 + 6*a*d^5*e^4*f*x^5 + 34*b*c^2*d^3*e^3*f^2*x^5 - 25*a*c*d^4*e^3*f^
2*x^5 + 34*b*c^3*d^2*e^2*f^3*x^5 - 34*a*c^2*d^3*e^2*f^3*x^5 + 2*b*c^4*d*e*
f^4*x^5 - 25*a*c^3*d^2*e*f^4*x^5 + 6*a*c^4*d*f^5*x^5 + b*c*d^4*e^5*x^3 + 3
*a*d^5*e^5*x^3 + 9*b*c^2*d^3*e^4*f*x^3 - 5*a*c*d^4*e^4*f*x^3 + 52*b*c^3*d^
2*e^3*f^2*x^3 - 34*a*c^2*d^3*e^3*f^2*x^3 + 9*b*c^4*d*e^2*f^3*x^3 - 34*a*c^
3*d^2*e^2*f^3*x^3 + b*c^5*e*f^4*x^3 - 5*a*c^4*d*e*f^4*x^3 + 3*a*c^5*f^5*x^
3 - b*c^2*d^3*e^5*x + 5*a*c*d^4*e^5*x + 13*b*c^3*d^2*e^4*f*x - 17*a*c^2*d^
3*e^4*f*x + 13*b*c^4*d*e^3*f^2*x - b*c^5*e^2*f^3*x - 17*a*c^4*d*e^2*f^3*x
+ 5*a*c^5*e*f^4*x)/((c^2*d^4*e^6 - 4*c^3*d^3*e^5*f + 6*c^4*d^2*e^4*f^2 - 4
*c^5*d*e^3*f^3 + c^6*e^2*f^4)*(d*f*x^4 + d*e*x^2 + c*f*x^2 + c*e)^2)
```

Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 98059, normalized size of antiderivative = 218.88

$$\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/((c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
((x*(5*a*c^4*f^4 + 5*a*d^4*e^4 - b*c*d^3*e^4 - b*c^4*e*f^3 - 17*a*c*d^3*e^
3*f - 17*a*c^3*d*e*f^3 + 13*b*c^2*d^2*e^3*f + 13*b*c^3*d*e^2*f^2))/(8*c*e*
(c^4*f^4 + d^4*e^4 + 6*c^2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)) +
 (x^5*(6*a*c^4*d*f^5 + 6*a*d^5*e^4*f + 2*b*c*d^4*e^4*f + 2*b*c^4*d*e*f^4 -
 25*a*c*d^4*e^3*f^2 - 25*a*c^3*d^2*e*f^4 - 34*a*c^2*d^3*e^2*f^3 + 34*b*c^2
*d^3*e^3*f^2 + 34*b*c^3*d^2*e^2*f^3))/(8*c^2*e^2*(c^4*f^4 + d^4*e^4 + 6*c^
2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)) + (x^3*(3*a*c^5*f^5 + 3*a*
d^5*e^5 + b*c*d^4*e^5 + b*c^5*e*f^4 - 5*a*c*d^4*e^4*f - 5*a*c^4*d*e*f^4 +
9*b*c^2*d^3*e^4*f + 9*b*c^4*d*e^2*f^3 - 34*a*c^2*d^3*e^3*f^2 - 34*a*c^3*d^
2*e^2*f^3 + 52*b*c^3*d^2*e^3*f^2))/(8*c^2*e^2*(c^4*f^4 + d^4*e^4 + 6*c^2*d
^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)) + (d*f*x^7*(3*a*c^3*d*f^4 + 3
*a*d^4*e^3*f + b*c*d^3*e^3*f + b*c^3*d*e*f^3 - 15*a*c*d^3*e^2*f^2 - 15*a*c
^2*d^2*e*f^3 + 22*b*c^2*d^2*e^2*f^2))/(8*c^2*e^2*(c^4*f^4 + d^4*e^4 + 6*c^
2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)))/(x^4*(c^2*f^2 + d^2*e^2 +
 4*c*d*e*f) + x^2*(2*c*d*e^2 + 2*c^2*e*f) + x^6*(2*c*d*f^2 + 2*d^2*e*f) +
c^2*e^2 + d^2*f^2*x^8) - atan(((((768*a*c^2*d^16*e^16*f^2 - 11520*a*c^3*d^
15*e^15*f^3 + 85248*a*c^4*d^14*e^14*f^4 - 391680*a*c^5*d^13*e^13*f^5 + 121
4208*a*c^6*d^12*e^12*f^6 - 2654976*a*c^7*d^11*e^11*f^7 + 4204800*a*c^8*d^1
0*e^10*f^8 - 4893696*a*c^9*d^9*e^9*f^9 + 4204800*a*c^10*d^8*e^8*f^10 - 265
4976*a*c^11*d^7*e^7*f^11 + 1214208*a*c^12*d^6*e^6*f^12 - 391680*a*c^13*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3522, normalized size of antiderivative = 7.86

$$\int \frac{a+bx^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
( - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**4*d**2*e**5*f**2
 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**4*d**2*e**4*f**3
*x**2 - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**4*d**2*e**3*
f**4*x**4 + 18*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**3*d**3*e
**6*f - 90*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**3*d**3*e**5*
f**2*x**2 - 234*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**3*d**3*
e**4*f**3*x**4 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**3*
d**3*e**3*f**4*x**6 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*
*2*d**4*e**7 + 30*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d**
4*e**6*f*x**2 + 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2*d**
4*e**5*f**2*x**4 - 90*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c**2
*d**4*e**4*f**3*x**6 - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*
c**2*d**4*e**3*f**4*x**8 - 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))
*a*c*d**5*e**7*x**2 + 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*
d**5*e**6*f*x**4 + 30*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d*
*5*e**5*f**2*x**6 + 18*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*c*d
**5*e**4*f**3*x**8 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**
6*e**7*x**4 - 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**6*e**6*
f*x**6 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*d**6*e**5*f**2*
x**8 + 35*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b*c**5*d*e**5*f...
```

3.225 $\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 362

$$\begin{aligned}
&\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^3 \, dx \\
&= a^2c^2e^3x + \frac{1}{3}ace^2(2bce+2ade+3acf)x^3 \\
&+ \frac{1}{5}e\left(b^2c^2e^2+2abce(2de+3cf)+a^2\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5 \\
&+ \frac{1}{7}\left(b^2ce^2(2de+3cf)+a^2f\left(3d^2e^2+6cdef+c^2f^2\right)+2abe\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 \\
&+ \frac{1}{9}\left(a^2df^2(3de+2cf)+2abf\left(3d^2e^2+6cdef+c^2f^2\right)+b^2e\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9 \\
&+ \frac{1}{11}f\left(a^2d^2f^2+2abdf(3de+2cf)+b^2\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^{11} \\
&+ \frac{1}{13}bdf^2(3bde+2bcf+2adf)x^{13} + \frac{1}{15}b^2d^2f^3x^{15}
\end{aligned}$$

output

```
a^2*c^2*e^3*x+1/3*a*c*e^2*(3*a*c*f+2*a*d*e+2*b*c*e)*x^3+1/5*e*(b^2*c^2*e^2
+2*a*b*c*e*(3*c*f+2*d*e)+a^2*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5+1/7*(b^2*c
*e^2*(3*c*f+2*d*e)+a^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+2*a*b*e*(3*c^2*f^2+
6*c*d*e*f+d^2*e^2))*x^7+1/9*(a^2*d*f^2*(2*c*f+3*d*e)+2*a*b*f*(c^2*f^2+6*c*
d*e*f+3*d^2*e^2)+b^2*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^9+1/11*f*(a^2*d^2*
f^2+2*a*b*d*f*(2*c*f+3*d*e)+b^2*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^11+1/13*b
*d*f^2*(2*a*d*f+2*b*c*f+3*b*d*e)*x^13+1/15*b^2*d^2*f^3*x^15
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \, dx$$
$$= a^2c^2e^3x+\frac{1}{3}ace^2(2bce+2ade+3acf)x^3$$
$$+\frac{1}{5}e\left(b^2c^2e^2+2abce(2de+3cf)+a^2\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5$$
$$+\frac{1}{7}\left(b^2ce^2(2de+3cf)+a^2f\left(3d^2e^2+6cdef+c^2f^2\right)+2abe\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7$$
$$+\frac{1}{9}\left(a^2df^2(3de+2cf)+2abf\left(3d^2e^2+6cdef+c^2f^2\right)+b^2e\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9$$
$$+\frac{1}{11}f\left(a^2d^2f^2+2abdf(3de+2cf)+b^2\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^{11}$$
$$+\frac{1}{13}bdf^2(3bde+2bcf+2adf)x^{13}+\frac{1}{15}b^2d^2f^3x^{15}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a^2*c^2*e^3*x + (a*c*e^2*(2*b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b^2*c^
2*e^2 + 2*a*b*c*e*(2*d*e + 3*c*f) + a^2*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))
*x^5)/5 + ((b^2*c*e^2*(2*d*e + 3*c*f) + a^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2
*f^2) + 2*a*b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + ((a^2*d*f^2*(3
*d*e + 2*c*f) + 2*a*b*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b^2*e*(d^2*e^2
 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (f*(a^2*d^2*f^2 + 2*a*b*d*f*(3*d*e + 2
*c*f) + b^2*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^11)/11 + (b*d*f^2*(3*b*d*
e + 2*b*c*f + 2*a*d*f)*x^13)/13 + (b^2*d^2*f^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(fx^{10}\left(a^2d^2f^2+2abdf(2cf+3de)+b^2\left(c^2f^2+6cdef+3d^2e^2\right)\right)+x^8\left(a^2df^2(2cf+3de)+2abf\left(c^2f^2+6cdef\right.\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{11}fx^{11}\left(a^2d^2f^2+2abdf(2cf+3de)+b^2\left(c^2f^2+6cdef+3d^2e^2\right)\right)+ \\ \frac{1}{9}x^9\left(a^2df^2(2cf+3de)+2abf\left(c^2f^2+6cdef+3d^2e^2\right)+b^2e\left(3c^2f^2+6cdef+d^2e^2\right)\right)+ \\ \frac{1}{7}x^7\left(a^2f\left(c^2f^2+6cdef+3d^2e^2\right)+2abe\left(3c^2f^2+6cdef+d^2e^2\right)+b^2ce^2(3cf+2de)\right)+ \\ \frac{1}{5}ex^5\left(a^2\left(3c^2f^2+6cdef+d^2e^2\right)+2abce(3cf+2de)+b^2c^2e^2\right)+a^2c^2e^3x+\frac{1}{3}ace^2x^3(3acf+ \\ 2ade+2bce)+\frac{1}{13}bdf^2x^{13}(2adf+2bcf+3bde)+\frac{1}{15}b^2d^2f^3x^{15}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
a^2*c^2*e^3*x + (a*c*e^2*(2*b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b^2*c^
2*e^2 + 2*a*b*c*e*(2*d*e + 3*c*f) + a^2*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))
*x^5)/5 + ((b^2*c*e^2*(2*d*e + 3*c*f) + a^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2
*f^2) + 2*a*b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + ((a^2*d*f^2*(3
*d*e + 2*c*f) + 2*a*b*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b^2*e*(d^2*e^2
 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (f*(a^2*d^2*f^2 + 2*a*b*d*f*(3*d*e + 2
*c*f) + b^2*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^11)/11 + (b*d*f^2*(3*b*d*
e + 2*b*c*f + 2*a*d*f)*x^13)/13 + (b^2*d^2*f^3*x^15)/15
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.09

method	result
default	$\frac{b^2d^2f^3x^{15}}{15}+\frac{((2ab\,d^2+2b^2cd)f^3+3b^2d^2e\,f^2)x^{13}}{13}+\frac{((a^2d^2+4abcd+b^2c^2)f^3+3(2ab\,d^2+2b^2cd)e\,f^2+3b^2d^2e^2f)x^{11}}{11}+$
norman	$a^2c^2e^3x+\left(a^2c^2e^2f+\frac{2}{3}a^2cd\,e^3+\frac{2}{3}ab\,c^2e^3\right)x^3+\left(\frac{3}{5}a^2c^2e\,f^2+\frac{6}{5}a^2cd\,e^2f+\frac{1}{5}a^2d^2e^3+\frac{6}{5}ab\,c^2\right.$
gosper	$\frac{6}{5}x^5a^2cd\,e^2f+x^3a^2c^2e^2f+\frac{2}{3}x^3a^2cd\,e^3+\frac{1}{5}x^5a^2d^2e^3+\frac{3}{5}x^5a^2c^2e\,f^2+\frac{3}{7}x^7a^2d^2e^2f+\frac{1}{5}x^5b^2c^2$
risch	$\frac{6}{5}x^5a^2cd\,e^2f+x^3a^2c^2e^2f+\frac{2}{3}x^3a^2cd\,e^3+\frac{1}{5}x^5a^2d^2e^3+\frac{3}{5}x^5a^2c^2e\,f^2+\frac{3}{7}x^7a^2d^2e^2f+\frac{1}{5}x^5b^2c^2$
parallelrisch	$\frac{6}{5}x^5a^2cd\,e^2f+x^3a^2c^2e^2f+\frac{2}{3}x^3a^2cd\,e^3+\frac{1}{5}x^5a^2d^2e^3+\frac{3}{5}x^5a^2c^2e\,f^2+\frac{3}{7}x^7a^2d^2e^2f+\frac{1}{5}x^5b^2c^2$
orering	$x(3003b^2d^2f^3x^{14}+6930ab\,d^2f^3x^{12}+6930b^2cd\,f^3x^{12}+10395b^2d^2e\,f^2x^{12}+4095a^2d^2f^3x^{10}+16380abcd\,f^3x^{10}+24570ab\,d^2e\,f$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*b^2*d^2*f^3*x^15+1/13*((2*a*b*d^2+2*b^2*c*d)*f^3+3*b^2*d^2*e*f^2)*x^1
3+1/11*((a^2*d^2+4*a*b*c*d+b^2*c^2)*f^3+3*(2*a*b*d^2+2*b^2*c*d)*e*f^2+3*b^
2*d^2*e^2*f)*x^11+1/9*((2*a^2*c*d+2*a*b*c^2)*f^3+3*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*e*f^2+3*(2*a*b*d^2+2*b^2*c*d)*e^2*f+b^2*d^2*e^3)*x^9+1/7*(a^2*c^2*f^3
+3*(2*a^2*c*d+2*a*b*c^2)*e*f^2+3*(a^2*d^2+4*a*b*c*d+b^2*c^2)*e^2*f+(2*a*b*
d^2+2*b^2*c*d)*e^3)*x^7+1/5*(3*a^2*c^2*e*f^2+3*(2*a^2*c*d+2*a*b*c^2)*e^2*f
+(a^2*d^2+4*a*b*c*d+b^2*c^2)*e^3)*x^5+1/3*(3*a^2*c^2*e^2*f+(2*a^2*c*d+2*a*
b*c^2)*e^3)*x^3+a^2*c^2*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \, dx$$
$$= \frac{1}{15} b^2 d^2 f^3 x^{15} + \frac{1}{13} \left(3 b^2 d^2 e f^2 + 2 \left(b^2 c d + a b d^2\right) f^3\right) x^{13}$$
$$+ \frac{1}{11} \left(3 b^2 d^2 e^2 f + 6 \left(b^2 c d + a b d^2\right) e f^2 + \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) f^3\right) x^{11}$$
$$+ \frac{1}{9} \left(b^2 d^2 e^3 + 6 \left(b^2 c d + a b d^2\right) e^2 f + 3 \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) e f^2 + 2 \left(a b c^2 + a^2 c d\right) f^3\right) x^9$$
$$+ a^2 c^2 e^3 x$$
$$+ \frac{1}{7} \left(a^2 c^2 f^3 + 2 \left(b^2 c d + a b d^2\right) e^3 + 3 \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) e^2 f + 6 \left(a b c^2 + a^2 c d\right) e f^2\right) x^7$$
$$+ \frac{1}{5} \left(3 a^2 c^2 e f^2 + \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) e^3 + 6 \left(a b c^2 + a^2 c d\right) e^2 f\right) x^5$$
$$+ \frac{1}{3} \left(3 a^2 c^2 e^2 f + 2 \left(a b c^2 + a^2 c d\right) e^3\right) x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/15*b^2*d^2*f^3*x^15 + 1/13*(3*b^2*d^2*e*f^2 + 2*(b^2*c*d + a*b*d^2)*f^3)
*x^13 + 1/11*(3*b^2*d^2*e^2*f + 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*f^3)*x^11 + 1/9*(b^2*d^2*e^3 + 6*(b^2*c*d + a*b*d^2)*e
^2*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e*f^2 + 2*(a*b*c^2 + a^2*c*d)*f^3
)*x^9 + a^2*c^2*e^3*x + 1/7*(a^2*c^2*f^3 + 2*(b^2*c*d + a*b*d^2)*e^3 + 3*(
b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f + 6*(a*b*c^2 + a^2*c*d)*e*f^2)*x^7 +
1/5*(3*a^2*c^2*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3 + 6*(a*b*c^2 +
a^2*c*d)*e^2*f)*x^5 + 1/3*(3*a^2*c^2*e^2*f + 2*(a*b*c^2 + a^2*c*d)*e^3)*x^
3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.44

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \,dx = a^2c^2e^3x + \frac{b^2d^2f^3x^{15}}{15} + x^{13} \cdot \left(\frac{2abd^2f^3}{13} + \frac{2b^2cdf^3}{13} + \frac{3b^2d^2ef^2}{13}\right) + x^{11}\left(\frac{a^2d^2f^3}{11} + \frac{4abcdf^3}{11} + \frac{6abd^2ef^2}{11} + \frac{b^2c^2f^3}{11} + \frac{6b^2cdef^2}{11} + \frac{3b^2d^2e^2f}{11}\right) + x^9 \cdot \left(\frac{2a^2cdf^3}{9} + \frac{a^2d^2ef^2}{3} + \frac{2abc^2f^3}{9} + \frac{4abcdef^2}{3} + \frac{2abd^2e^2f}{3} + \frac{b^2c^2ef^2}{3} + \frac{2b^2cde^2f}{3} + \frac{b^2d^2e^3}{9}\right) + x^7\left(\frac{a^2c^2f^3}{7} + \frac{6a^2cdef^2}{7} + \frac{3a^2d^2e^2f}{7} + \frac{6abc^2ef^2}{7} + \frac{12abcde^2f}{7} + \frac{2abd^2e^3}{7} + \frac{3b^2c^2e^2f}{7} + \frac{2b^2cde^3}{7}\right) + x^5 \cdot \left(\frac{3a^2c^2ef^2}{5} + \frac{6a^2cde^2f}{5} + \frac{a^2d^2e^3}{5} + \frac{6abc^2e^2f}{5} + \frac{4abcde^3}{5} + \frac{b^2c^2e^3}{5}\right) + x^3\left(a^2c^2e^2f + \frac{2a^2cde^3}{3} + \frac{2abc^2e^3}{3}\right)$$

input

```
integrate((b*x**2+a)**2*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

output
```
a**2*c**2*e**3*x + b**2*d**2*f**3*x**15/15 + x**13*(2*a*b*d**2*f**3/13 + 2
*b**2*c*d*f**3/13 + 3*b**2*d**2*e*f**2/13) + x**11*(a**2*d**2*f**3/11 + 4*
a*b*c*d*f**3/11 + 6*a*b*d**2*e*f**2/11 + b**2*c**2*f**3/11 + 6*b**2*c*d*e*
f**2/11 + 3*b**2*d**2*e**2*f/11) + x**9*(2*a**2*c*d*f**3/9 + a**2*d**2*e*f
**2/3 + 2*a*b*c**2*f**3/9 + 4*a*b*c*d*e*f**2/3 + 2*a*b*d**2*e**2*f/3 + b**
2*c**2*e*f**2/3 + 2*b**2*c*d*e**2*f/3 + b**2*d**2*e**3/9) + x**7*(a**2*c**
2*f**3/7 + 6*a**2*c*d*e*f**2/7 + 3*a**2*d**2*e**2*f/7 + 6*a*b*c**2*e*f**2/
7 + 12*a*b*c*d*e**2*f/7 + 2*a*b*d**2*e**3/7 + 3*b**2*c**2*e**2*f/7 + 2*b**
2*c*d*e**3/7) + x**5*(3*a**2*c**2*e*f**2/5 + 6*a**2*c*d*e**2*f/5 + a**2*d*
*2*e**3/5 + 6*a*b*c**2*e**2*f/5 + 4*a*b*c*d*e**3/5 + b**2*c**2*e**3/5) + x
**3*(a**2*c**2*e**2*f + 2*a**2*c*d*e**3/3 + 2*a*b*c**2*e**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \,dx \\
&= \frac{1}{15}\, b^2d^2f^3x^{15} + \frac{1}{13}\left(3\, b^2d^2ef^2 + 2\left(b^2cd+abd^2\right)f^3\right)x^{13} \\
&\quad + \frac{1}{11}\left(3\, b^2d^2e^2f + 6\left(b^2cd+abd^2\right)ef^2 + \left(b^2c^2+4\, abcd+a^2d^2\right)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(b^2d^2e^3 + 6\left(b^2cd+abd^2\right)e^2f + 3\left(b^2c^2+4\, abcd+a^2d^2\right)ef^2 + 2\left(abc^2+a^2cd\right)f^3\right)x^9 \\
&\quad + a^2c^2e^3x \\
&\quad + \frac{1}{7}\left(a^2c^2f^3 + 2\left(b^2cd+abd^2\right)e^3 + 3\left(b^2c^2+4\, abcd+a^2d^2\right)e^2f + 6\left(abc^2+a^2cd\right)ef^2\right)x^7 \\
&\quad + \frac{1}{5}\left(3\, a^2c^2ef^2 + \left(b^2c^2+4\, abcd+a^2d^2\right)e^3 + 6\left(abc^2+a^2cd\right)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\, a^2c^2e^2f + 2\left(abc^2+a^2cd\right)e^3\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/15*b^2*d^2*f^3*x^15 + 1/13*(3*b^2*d^2*e*f^2 + 2*(b^2*c*d + a*b*d^2)*f^3)
*x^13 + 1/11*(3*b^2*d^2*e^2*f + 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*f^3)*x^11 + 1/9*(b^2*d^2*e^3 + 6*(b^2*c*d + a*b*d^2)*e
^2*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e*f^2 + 2*(a*b*c^2 + a^2*c*d)*f^3
)*x^9 + a^2*c^2*e^3*x + 1/7*(a^2*c^2*f^3 + 2*(b^2*c*d + a*b*d^2)*e^3 + 3*(
b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f + 6*(a*b*c^2 + a^2*c*d)*e*f^2)*x^7 +
1/5*(3*a^2*c^2*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3 + 6*(a*b*c^2 +
a^2*c*d)*e^2*f)*x^5 + 1/3*(3*a^2*c^2*e^2*f + 2*(a*b*c^2 + a^2*c*d)*e^3)*x^
3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.37

$$
\begin{aligned}
\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \,dx &= \frac{1}{15}\,b^2d^2f^3x^{15}+\frac{3}{13}\,b^2d^2ef^2x^{13}+\frac{2}{13}\,b^2cdf^3x^{13} \\
&+\frac{2}{13}\,abd^2f^3x^{13}+\frac{3}{11}\,b^2d^2e^2fx^{11} \\
&+\frac{6}{11}\,b^2cdef^2x^{11}+\frac{6}{11}\,abd^2ef^2x^{11} \\
&+\frac{1}{11}\,b^2c^2f^3x^{11}+\frac{4}{11}\,abcdf^3x^{11} \\
&+\frac{1}{11}\,a^2d^2f^3x^{11}+\frac{1}{9}\,b^2d^2e^3x^9+\frac{2}{3}\,b^2cde^2fx^9 \\
&+\frac{2}{3}\,abd^2e^2fx^9+\frac{1}{3}\,b^2c^2ef^2x^9+\frac{4}{3}\,abcdef^2x^9 \\
&+\frac{1}{3}\,a^2d^2ef^2x^9+\frac{2}{9}\,abc^2f^3x^9+\frac{2}{9}\,a^2cdf^3x^9 \\
&+\frac{2}{7}\,b^2cde^3x^7+\frac{2}{7}\,abd^2e^3x^7+\frac{3}{7}\,b^2c^2e^2fx^7 \\
&+\frac{12}{7}\,abcde^2fx^7+\frac{3}{7}\,a^2d^2e^2fx^7+\frac{6}{7}\,abc^2ef^2x^7 \\
&+\frac{6}{7}\,a^2cdef^2x^7+\frac{1}{7}\,a^2c^2f^3x^7+\frac{1}{5}\,b^2c^2e^3x^5 \\
&+\frac{4}{5}\,abcde^3x^5+\frac{1}{5}\,a^2d^2e^3x^5+\frac{6}{5}\,abc^2e^2fx^5 \\
&+\frac{6}{5}\,a^2cde^2fx^5+\frac{3}{5}\,a^2c^2ef^2x^5+\frac{2}{3}\,abc^2e^3x^3 \\
&+\frac{2}{3}\,a^2cde^3x^3+a^2c^2e^2fx^3+a^2c^2e^3x
\end{aligned}
$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/15*b^2*d^2*f^3*x^15 + 3/13*b^2*d^2*e*f^2*x^13 + 2/13*b^2*c*d*f^3*x^13 +
2/13*a*b*d^2*f^3*x^13 + 3/11*b^2*d^2*e^2*f*x^11 + 6/11*b^2*c*d*e*f^2*x^11
+ 6/11*a*b*d^2*e*f^2*x^11 + 1/11*b^2*c^2*f^3*x^11 + 4/11*a*b*c*d*f^3*x^11
+ 1/11*a^2*d^2*f^3*x^11 + 1/9*b^2*d^2*e^3*x^9 + 2/3*b^2*c*d*e^2*f*x^9 + 2/
3*a*b*d^2*e^2*f*x^9 + 1/3*b^2*c^2*e*f^2*x^9 + 4/3*a*b*c*d*e*f^2*x^9 + 1/3*
a^2*d^2*e*f^2*x^9 + 2/9*a*b*c^2*f^3*x^9 + 2/9*a^2*c*d*f^3*x^9 + 2/7*b^2*c*
d*e^3*x^7 + 2/7*a*b*d^2*e^3*x^7 + 3/7*b^2*c^2*e^2*f*x^7 + 12/7*a*b*c*d*e^2
*f*x^7 + 3/7*a^2*d^2*e^2*f*x^7 + 6/7*a*b*c^2*e*f^2*x^7 + 6/7*a^2*c*d*e*f^2
*x^7 + 1/7*a^2*c^2*f^3*x^7 + 1/5*b^2*c^2*e^3*x^5 + 4/5*a*b*c*d*e^3*x^5 + 1
/5*a^2*d^2*e^3*x^5 + 6/5*a*b*c^2*e^2*f*x^5 + 6/5*a^2*c*d*e^2*f*x^5 + 3/5*a
^2*c^2*e*f^2*x^5 + 2/3*a*b*c^2*e^3*x^3 + 2/3*a^2*c*d*e^3*x^3 + a^2*c^2*e^2
*f*x^3 + a^2*c^2*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^3 \, dx = x^5 \left(\frac{3a^2c^2ef^2}{5} + \frac{6a^2cde^2f}{5} + \frac{a^2d^2e^3}{5} + \frac{6abc^2e^2f}{5} + \frac{4abcde^3}{5} + \frac{b^2c^2e^3}{5}\right) + x^{11}\left(\frac{a^2d^2f^3}{11} + \frac{4abcdf^3}{11} + \frac{6abd^2ef^2}{11} + \frac{b^2c^2f^3}{11} + \frac{6b^2cdef^2}{11} + \frac{3b^2d^2e^2f}{11}\right) + x^7\left(\frac{a^2c^2f^3}{7} + \frac{6a^2cdef^2}{7} + \frac{3a^2d^2e^2f}{7} + \frac{6abc^2ef^2}{7} + \frac{12abcde^2f}{7} + \frac{2abd^2e^3}{7} + \frac{3b^2c^2e^2f}{7} + \frac{2b^2cde^3}{7}\right) + x^9\left(\frac{2a^2cdf^3}{9} + \frac{a^2d^2ef^2}{3} + \frac{2abc^2f^3}{9} + \frac{4abcdef^2}{3} + \frac{2abd^2e^2f}{3} + \frac{b^2c^2ef^2}{3} + \frac{2b^2cde^2f}{3} + \frac{b^2d^2e^3}{9}\right) + a^2c^2e^3x + \frac{b^2d^2f^3x^{15}}{15} + \frac{ace^2x^3(3acf + 2ade + 2bce)}{3} + \frac{bdf^2x^{13}(2adf + 2bcf + 3bde)}{13}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

output
```
x^5*((a^2*d^2*e^3)/5 + (b^2*c^2*e^3)/5 + (3*a^2*c^2*e*f^2)/5 + (6*a*b*c^2*
e^2*f)/5 + (6*a^2*c*d*e^2*f)/5 + (4*a*b*c*d*e^3)/5) + x^11*((a^2*d^2*f^3)/
11 + (b^2*c^2*f^3)/11 + (3*b^2*d^2*e^2*f)/11 + (6*a*b*d^2*e*f^2)/11 + (6*b
^2*c*d*e*f^2)/11 + (4*a*b*c*d*f^3)/11) + x^7*((a^2*c^2*f^3)/7 + (3*a^2*d^2
*e^2*f)/7 + (3*b^2*c^2*e^2*f)/7 + (2*a*b*d^2*e^3)/7 + (2*b^2*c*d*e^3)/7 +
(6*a*b*c^2*e*f^2)/7 + (6*a^2*c*d*e*f^2)/7 + (12*a*b*c*d*e^2*f)/7) + x^9*((
b^2*d^2*e^3)/9 + (a^2*d^2*e*f^2)/3 + (b^2*c^2*e*f^2)/3 + (2*a*b*c^2*f^3)/9
 + (2*a^2*c*d*f^3)/9 + (2*a*b*d^2*e^2*f)/3 + (2*b^2*c*d*e^2*f)/3 + (4*a*b*
c*d*e*f^2)/3) + a^2*c^2*e^3*x + (b^2*d^2*f^3*x^15)/15 + (a*c*e^2*x^3*(3*a*
c*f + 2*a*d*e + 2*b*c*e))/3 + (b*d*f^2*x^13*(2*a*d*f + 2*b*c*f + 3*b*d*e))
/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.38

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \,dx = \frac{x(3003b^2d^2f^3x^{14}+6930ab\,d^2f^3x^{12}+6930b^2cd\,f^3x^{12}+10395b^2d^2e\,f^2x^{12}+4095a^2d^2f^3x^{10}+16380abcd}{}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

output
```
(x*(45045*a**2*c**2*e**3 + 45045*a**2*c**2*e**2*f*x**2 + 27027*a**2*c**2*e
*f**2*x**4 + 6435*a**2*c**2*f**3*x**6 + 30030*a**2*c*d*e**3*x**2 + 54054*a
**2*c*d*e**2*f*x**4 + 38610*a**2*c*d*e*f**2*x**6 + 10010*a**2*c*d*f**3*x**
8 + 9009*a**2*d**2*e**3*x**4 + 19305*a**2*d**2*e**2*f*x**6 + 15015*a**2*d*
*2*e*f**2*x**8 + 4095*a**2*d**2*f**3*x**10 + 30030*a*b*c**2*e**3*x**2 + 54
054*a*b*c**2*e**2*f*x**4 + 38610*a*b*c**2*e*f**2*x**6 + 10010*a*b*c**2*f**
3*x**8 + 36036*a*b*c*d*e**3*x**4 + 77220*a*b*c*d*e**2*f*x**6 + 60060*a*b*c
*d*e*f**2*x**8 + 16380*a*b*c*d*f**3*x**10 + 12870*a*b*d**2*e**3*x**6 + 300
30*a*b*d**2*e**2*f*x**8 + 24570*a*b*d**2*e*f**2*x**10 + 6930*a*b*d**2*f**3
*x**12 + 9009*b**2*c**2*e**3*x**4 + 19305*b**2*c**2*e**2*f*x**6 + 15015*b*
*2*c**2*e*f**2*x**8 + 4095*b**2*c**2*f**3*x**10 + 12870*b**2*c*d*e**3*x**6
 + 30030*b**2*c*d*e**2*f*x**8 + 24570*b**2*c*d*e*f**2*x**10 + 6930*b**2*c*
d*f**3*x**12 + 5005*b**2*d**2*e**3*x**8 + 12285*b**2*d**2*e**2*f*x**10 + 1
0395*b**2*d**2*e*f**2*x**12 + 3003*b**2*d**2*f**3*x**14))/45045
```

3.226 $\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 244

$$\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx = a^2c^2e^2x + \frac{2}{3}ace(bce+ade+acf)x^3 + \frac{1}{5}\left(b^2c^2e^2 + 4abce(de+cf) + a^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 + \frac{2}{7}\left(b^2ce(de+cf) + a^2df(de+cf) + ab\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 + \frac{1}{9}\left(a^2d^2f^2 + 4abdf(de+cf) + b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^9 + \frac{2}{11}bdf(bde+bcf+adf)x^{11} + \frac{1}{13}b^2d^2f^2x^{13}$$

output
```
a^2*c^2*e^2*x+2/3*a*c*e*(a*c*f+a*d*e+b*c*e)*x^3+1/5*(b^2*c^2*e^2+4*a*b*c*e
*(c*f+d*e)+a^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^5+2/7*(b^2*c*e*(c*f+d*e)+a^2
*d*f*(c*f+d*e)+a*b*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^7+1/9*(a^2*d^2*f^2+4*a*b
*d*f*(c*f+d*e)+b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^9+2/11*b*d*f*(a*d*f+b*c*
f+b*d*e)*x^11+1/13*b^2*d^2*f^2*x^13
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx = a^2c^2e^2x + \frac{2}{3}ace(bce+ade+acf)x^3 + \frac{1}{5}(b^2c^2e^2 + 4abce(de+cf) + a^2(d^2e^2+4cdef+c^2f^2))x^5 + \frac{2}{7}(b^2ce(de+cf) + a^2df(de+cf) + ab(d^2e^2+4cdef+c^2f^2))x^7 + \frac{1}{9}(a^2d^2f^2 + 4abdf(de+cf) + b^2(d^2e^2+4cdef+c^2f^2))x^9 + \frac{2}{11}bdf(bde+bcf+adf)x^{11} + \frac{1}{13}b^2d^2f^2x^{13}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a^2*c^2*e^2*x + (2*a*c*e*(b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b^2*c^2*e^2 +
4*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + (2*(
b^2*c*e*(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^7)/7 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c
*d*e*f + c^2*f^2))*x^9)/9 + (2*b*d*f*(b*d*e + b*c*f + a*d*f)*x^11)/11 + (b
^2*d^2*f^2*x^13)/13
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(x^8\left(a^2d^2f^2+4abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)+2x^6\left(a^2df(cf+de)+ab\left(c^2f^2+4cdef+d^2e^2\right)+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}x^9\left(a^2d^2f^2+4abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{2}{7}x^7\left(a^2df(cf+de)+ab\left(c^2f^2+4cdef+d^2e^2\right)+b^2ce(cf+de)\right)+\frac{1}{5}x^5\left(a^2\left(c^2f^2+4cdef+d^2e^2\right)+4abce(cf+de)+b^2c^2e^2\right)+a^2c^2e^2x+\frac{2}{11}bdfx^{11}(adf+bcf+bde)+\frac{2}{3}acex^3(acf+ade+bce)+\frac{1}{13}b^2d^2f^2x^{13}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
a^2*c^2*e^2*x + (2*a*c*e*(b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b^2*c^2*e^2 +
4*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + (2*(
b^2*c*e*(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^7)/7 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c
*d*e*f + c^2*f^2))*x^9)/9 + (2*b*d*f*(b*d*e + b*c*f + a*d*f)*x^11)/11 + (b
^2*d^2*f^2*x^13)/13
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.17

method	result
default	$\frac{b^2d^2f^2x^{13}}{13}+\frac{((2ab\,d^2+2b^2cd)f^2+2b^2d^2ef)x^{11}}{11}+\frac{((a^2d^2+4abcd+b^2c^2)f^2+2(2ab\,d^2+2b^2cd)ef+b^2d^2e^2)x^9}{9}+ ((2a^2c$
norman	$\frac{b^2d^2f^2x^{13}}{13}+\left(\frac{2}{11}a\,d^2f^2b+\frac{2}{11}b^2cd\,f^2+\frac{2}{11}b^2d^2ef\right)x^{11}+\left(\frac{1}{9}a^2d^2f^2+\frac{4}{9}abcd\,f^2+\frac{4}{9}ab\,d^2ef+\frac{1}{9}\right.$
gosper	$\frac{1}{5}x^5a^2c^2f^2+\frac{2}{3}x^3a^2c^2ef+\frac{2}{7}x^7ab\,c^2f^2+\frac{2}{7}x^7ab\,d^2e^2+\frac{2}{7}x^7b^2c^2ef+\frac{1}{5}x^5a^2d^2e^2+\frac{2}{11}x^{11}a\,d^2f$
risch	$\frac{1}{5}x^5a^2c^2f^2+\frac{2}{3}x^3a^2c^2ef+\frac{2}{7}x^7ab\,c^2f^2+\frac{2}{7}x^7ab\,d^2e^2+\frac{2}{7}x^7b^2c^2ef+\frac{1}{5}x^5a^2d^2e^2+\frac{2}{11}x^{11}a\,d^2f$
parallelrisch	$\frac{1}{5}x^5a^2c^2f^2+\frac{2}{3}x^3a^2c^2ef+\frac{2}{7}x^7ab\,c^2f^2+\frac{2}{7}x^7ab\,d^2e^2+\frac{2}{7}x^7b^2c^2ef+\frac{1}{5}x^5a^2d^2e^2+\frac{2}{11}x^{11}a\,d^2f$
orering	$x(3465b^2d^2f^2x^{12}+8190ab\,d^2f^2x^{10}+8190b^2cd\,f^2x^{10}+8190b^2d^2ef\,x^{10}+5005a^2d^2f^2x^8+20020abcd\,f^2x^8+20020ab\,d^2ef\,x^8+$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d^2*f^2*x^13+1/11*((2*a*b*d^2+2*b^2*c*d)*f^2+2*b^2*d^2*e*f)*x^11+
1/9*((a^2*d^2+4*a*b*c*d+b^2*c^2)*f^2+2*(2*a*b*d^2+2*b^2*c*d)*e*f+b^2*d^2*e
^2)*x^9+1/7*((2*a^2*c*d+2*a*b*c^2)*f^2+2*(a^2*d^2+4*a*b*c*d+b^2*c^2)*e*f+(
2*a*b*d^2+2*b^2*c*d)*e^2)*x^7+1/5*(a^2*c^2*f^2+2*(2*a^2*c*d+2*a*b*c^2)*e*f
+(a^2*d^2+4*a*b*c*d+b^2*c^2)*e^2)*x^5+1/3*(2*a^2*c^2*e*f+(2*a^2*c*d+2*a*b*
c^2)*e^2)*x^3+a^2*c^2*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx \\
&= \frac{1}{13}b^2d^2f^2x^{13} + \frac{2}{11}\left(b^2d^2ef + \left(b^2cd + abd^2\right)f^2\right)x^{11} \\
&\quad + \frac{1}{9}\left(b^2d^2e^2 + 4\left(b^2cd + abd^2\right)ef + \left(b^2c^2 + 4\,abcd + a^2d^2\right)f^2\right)x^9 \\
&\quad + \frac{2}{7}\left(\left(b^2cd + abd^2\right)e^2 + \left(b^2c^2 + 4\,abcd + a^2d^2\right)ef + \left(abc^2 + a^2cd\right)f^2\right)x^7 \\
&\quad + a^2c^2e^2x + \frac{1}{5}\left(a^2c^2f^2 + \left(b^2c^2 + 4\,abcd + a^2d^2\right)e^2 + 4\left(abc^2 + a^2cd\right)ef\right)x^5 \\
&\quad + \frac{2}{3}\left(a^2c^2ef + \left(abc^2 + a^2cd\right)e^2\right)x^3
\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
1/13*b^2*d^2*f^2*x^13 + 2/11*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^11
+ 1/9*(b^2*d^2*e^2 + 4*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*f^2)*x^9 + 2/7*((b^2*c*d + a*b*d^2)*e^2 + (b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*e*f + (a*b*c^2 + a^2*c*d)*f^2)*x^7 + a^2*c^2*e^2*x + 1/5*(a^2*c^2*f
^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^5
+ 2/3*(a^2*c^2*e*f + (a*b*c^2 + a^2*c*d)*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.52

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx = a^2c^2e^2x + \frac{b^2d^2f^2x^{13}}{13} + x^{11}\cdot\left(\frac{2abd^2f^2}{11} + \frac{2b^2cdf^2}{11} + \frac{2b^2d^2ef}{11}\right) + x^9\left(\frac{a^2d^2f^2}{9} + \frac{4abcdf^2}{9} + \frac{4abd^2ef}{9} + \frac{b^2c^2f^2}{9} + \frac{4b^2cdef}{9} + \frac{b^2d^2e^2}{9}\right) + x^7\cdot\left(\frac{2a^2cdf^2}{7} + \frac{2a^2d^2ef}{7} + \frac{2abc^2f^2}{7} + \frac{8abcdef}{7} + \frac{2abd^2e^2}{7} + \frac{2b^2c^2ef}{7} + \frac{2b^2cde^2}{7}\right) + x^5\left(\frac{a^2c^2f^2}{5} + \frac{4a^2cdef}{5} + \frac{a^2d^2e^2}{5} + \frac{4abc^2ef}{5} + \frac{4abcde^2}{5} + \frac{b^2c^2e^2}{5}\right) + x^3\cdot\left(\frac{2a^2c^2ef}{3} + \frac{2a^2cde^2}{3} + \frac{2abc^2e^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

output
```
a**2*c**2*e**2*x + b**2*d**2*f**2*x**13/13 + x**11*(2*a*b*d**2*f**2/11 + 2
*b**2*c*d*f**2/11 + 2*b**2*d**2*e*f/11) + x**9*(a**2*d**2*f**2/9 + 4*a*b*c
*d*f**2/9 + 4*a*b*d**2*e*f/9 + b**2*c**2*f**2/9 + 4*b**2*c*d*e*f/9 + b**2*
d**2*e**2/9) + x**7*(2*a**2*c*d*f**2/7 + 2*a**2*d**2*e*f/7 + 2*a*b*c**2*f*
*2/7 + 8*a*b*c*d*e*f/7 + 2*a*b*d**2*e**2/7 + 2*b**2*c**2*e*f/7 + 2*b**2*c*
d*e**2/7) + x**5*(a**2*c**2*f**2/5 + 4*a**2*c*d*e*f/5 + a**2*d**2*e**2/5 +
 4*a*b*c**2*e*f/5 + 4*a*b*c*d*e**2/5 + b**2*c**2*e**2/5) + x**3*(2*a**2*c*
*2*e*f/3 + 2*a**2*c*d*e**2/3 + 2*a*b*c**2*e**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.11

$$\begin{aligned}\int (a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2 \, dx &= \frac{1}{13} b^2 d^2 f^2 x^{13} + \frac{2}{11} \left(b^2 d^2 e f + \left(b^2 c d + a b d^2\right) f^2\right) x^{11} \\ &+ \frac{1}{9} \left(b^2 d^2 e^2 + 4 \left(b^2 c d + a b d^2\right) e f + \left(b^2 c^2 + 4 \, a b c d + a^2 d^2\right) f^2\right) x^9 \\ &+ \frac{2}{7} \left(\left(b^2 c d + a b d^2\right) e^2 + \left(b^2 c^2 + 4 \, a b c d + a^2 d^2\right) e f + \left(a b c^2 + a^2 c d\right) f^2\right) x^7 \\ &+ a^2 c^2 e^2 x + \frac{1}{5} \left(a^2 c^2 f^2 + \left(b^2 c^2 + 4 \, a b c d + a^2 d^2\right) e^2 + 4 \left(a b c^2 + a^2 c d\right) e f\right) x^5 \\ &+ \frac{2}{3} \left(a^2 c^2 e f + \left(a b c^2 + a^2 c d\right) e^2\right) x^3\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/13*b^2*d^2*f^2*x^13 + 2/11*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^11
+ 1/9*(b^2*d^2*e^2 + 4*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*f^2)*x^9 + 2/7*((b^2*c*d + a*b*d^2)*e^2 + (b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*e*f + (a*b*c^2 + a^2*c*d)*f^2)*x^7 + a^2*c^2*e^2*x + 1/5*(a^2*c^2*f
^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^5
+ 2/3*(a^2*c^2*e*f + (a*b*c^2 + a^2*c*d)*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.43

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx = \frac{1}{13} b^2d^2f^2x^{13} + \frac{2}{11} b^2d^2efx^{11} + \frac{2}{11} b^2cdf^2x^{11} + \frac{2}{11} abd^2f^2x^{11} + \frac{1}{9} b^2d^2e^2x^9 + \frac{4}{9} b^2cdefx^9 + \frac{4}{9} abd^2efx^9 + \frac{1}{9} b^2c^2f^2x^9 + \frac{4}{9} abcdf^2x^9 + \frac{1}{9} a^2d^2f^2x^9 + \frac{2}{7} b^2cde^2x^7 + \frac{2}{7} abd^2e^2x^7 + \frac{2}{7} b^2c^2efx^7 + \frac{8}{7} abcdefx^7 + \frac{2}{7} a^2d^2efx^7 + \frac{2}{7} abc^2f^2x^7 + \frac{2}{7} a^2cdf^2x^7 + \frac{1}{5} b^2c^2e^2x^5 + \frac{4}{5} abcde^2x^5 + \frac{1}{5} a^2d^2e^2x^5 + \frac{4}{5} abc^2efx^5 + \frac{4}{5} a^2cdefx^5 + \frac{1}{5} a^2c^2f^2x^5 + \frac{2}{3} abc^2e^2x^3 + \frac{2}{3} a^2cde^2x^3 + \frac{2}{3} a^2c^2efx^3 + a^2c^2e^2x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/13*b^2*d^2*f^2*x^13 + 2/11*b^2*d^2*e*f*x^11 + 2/11*b^2*c*d*f^2*x^11 + 2/
11*a*b*d^2*f^2*x^11 + 1/9*b^2*d^2*e^2*x^9 + 4/9*b^2*c*d*e*f*x^9 + 4/9*a*b*
d^2*e*f*x^9 + 1/9*b^2*c^2*f^2*x^9 + 4/9*a*b*c*d*f^2*x^9 + 1/9*a^2*d^2*f^2*
x^9 + 2/7*b^2*c*d*e^2*x^7 + 2/7*a*b*d^2*e^2*x^7 + 2/7*b^2*c^2*e*f*x^7 + 8/
7*a*b*c*d*e*f*x^7 + 2/7*a^2*d^2*e*f*x^7 + 2/7*a*b*c^2*f^2*x^7 + 2/7*a^2*c*
d*f^2*x^7 + 1/5*b^2*c^2*e^2*x^5 + 4/5*a*b*c*d*e^2*x^5 + 1/5*a^2*d^2*e^2*x^
5 + 4/5*a*b*c^2*e*f*x^5 + 4/5*a^2*c*d*e*f*x^5 + 1/5*a^2*c^2*f^2*x^5 + 2/3*
a*b*c^2*e^2*x^3 + 2/3*a^2*c*d*e^2*x^3 + 2/3*a^2*c^2*e*f*x^3 + a^2*c^2*e^2*
x
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \,dx = x^7 \left(\frac{2\,a^2\,c\,d\,f^2}{7}+\frac{2\,a^2\,d^2\,e\,f}{7}+\frac{2\,a\,b\,c^2\,f^2}{7}+\frac{8\,a\,b\,c\,d\,e\,f}{7}+\frac{2\,a\,b\,d^2\,e^2}{7}+\frac{2\,b^2\,c^2\,e\,f}{7}+\frac{2\,b^2\,c\,d\,e^2}{7}\right)+x^5\left(\frac{a^2\,c^2\,f^2}{5}+\frac{4\,a^2\,c\,d\,e\,f}{5}+\frac{a^2\,d^2\,e^2}{5}+\frac{4\,a\,b\,c^2\,e\,f}{5}+\frac{4\,a\,b\,c\,d\,e^2}{5}+\frac{b^2\,c^2\,e^2}{5}\right)+x^9\left(\frac{a^2\,d^2\,f^2}{9}+\frac{4\,a\,b\,c\,d\,f^2}{9}+\frac{4\,a\,b\,d^2\,e\,f}{9}+\frac{b^2\,c^2\,f^2}{9}+\frac{4\,b^2\,c\,d\,e\,f}{9}+\frac{b^2\,d^2\,e^2}{9}\right)+a^2\,c^2\,e^2\,x+\frac{b^2\,d^2\,f^2\,x^{13}}{13}+\frac{2\,a\,c\,e\,x^3\,(a\,c\,f+a\,d\,e+b\,c\,e)}{3}+\frac{2\,b\,d\,f\,x^{11}\,(a\,d\,f+b\,c\,f+b\,d\,e)}{11}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

output
```
x^7*((2*a*b*c^2*f^2)/7 + (2*a*b*d^2*e^2)/7 + (2*a^2*c*d*f^2)/7 + (2*b^2*c*
d*e^2)/7 + (2*a^2*d^2*e*f)/7 + (2*b^2*c^2*e*f)/7 + (8*a*b*c*d*e*f)/7) + x^
5*((a^2*c^2*f^2)/5 + (a^2*d^2*e^2)/5 + (b^2*c^2*e^2)/5 + (4*a*b*c*d*e^2)/5
 + (4*a*b*c^2*e*f)/5 + (4*a^2*c*d*e*f)/5) + x^9*((a^2*d^2*f^2)/9 + (b^2*c^
2*f^2)/9 + (b^2*d^2*e^2)/9 + (4*a*b*c*d*f^2)/9 + (4*a*b*d^2*e*f)/9 + (4*b^
2*c*d*e*f)/9) + a^2*c^2*e^2*x + (b^2*d^2*f^2*x^13)/13 + (2*a*c*e*x^3*(a*c*
f + a*d*e + b*c*e))/3 + (2*b*d*f*x^11*(a*d*f + b*c*f + b*d*e))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.44

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx$$
$$= \frac{x(3465b^2d^2f^2x^{12}+8190ab\,d^2f^2x^{10}+8190b^2cd\,f^2x^{10}+8190b^2d^2ef\,x^{10}+5005a^2d^2f^2x^8+20020abcd\,f^2x}{}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

output
```
(x*(45045*a**2*c**2*e**2 + 30030*a**2*c**2*e*f*x**2 + 9009*a**2*c**2*f**2*
x**4 + 30030*a**2*c*d*e**2*x**2 + 36036*a**2*c*d*e*f*x**4 + 12870*a**2*c*d
*f**2*x**6 + 9009*a**2*d**2*e**2*x**4 + 12870*a**2*d**2*e*f*x**6 + 5005*a*
*2*d**2*f**2*x**8 + 30030*a*b*c**2*e**2*x**2 + 36036*a*b*c**2*e*f*x**4 + 1
2870*a*b*c**2*f**2*x**6 + 36036*a*b*c*d*e**2*x**4 + 51480*a*b*c*d*e*f*x**6
 + 20020*a*b*c*d*f**2*x**8 + 12870*a*b*d**2*e**2*x**6 + 20020*a*b*d**2*e*f
*x**8 + 8190*a*b*d**2*f**2*x**10 + 9009*b**2*c**2*e**2*x**4 + 12870*b**2*c
**2*e*f*x**6 + 5005*b**2*c**2*f**2*x**8 + 12870*b**2*c*d*e**2*x**6 + 20020
*b**2*c*d*e*f*x**8 + 8190*b**2*c*d*f**2*x**10 + 5005*b**2*d**2*e**2*x**8 +
 8190*b**2*d**2*e*f*x**10 + 3465*b**2*d**2*f**2*x**12))/45045
```

3.227 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{e+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 181

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{e+fx^2}\,dx &= \frac{(bde-bcf-adf)(af(de-2cf)-be(de-cf))x}{f^4} \\ &+ \frac{(a^2d^2f^2-2abdf(de-2cf)+b^2(de-cf)^2)x^3}{3f^3} \\ &- \frac{bd(bde-2bcf-2adf)x^5}{5f^2}+\frac{b^2d^2x^7}{7f} \\ &+ \frac{(be-af)^2(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}\end{aligned}$$

output

```
(-a*d*f-b*c*f+b*d*e)*(a*f*(-2*c*f+d*e)-b*e*(-c*f+d*e))*x/f^4+1/3*(a^2*d^2*
f^2-2*a*b*d*f*(-2*c*f+d*e)+b^2*(-c*f+d*e)^2)*x^3/f^3-1/5*b*d*(-2*a*d*f-2*b
*c*f+b*d*e)*x^5/f^2+1/7*b^2*d^2*x^7/f+(-a*f+b*e)^2*(-c*f+d*e)^2*arctan(f^(
1/2)*x/e^(1/2))/e^(1/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{e+fx^2}\,dx = -\frac{(a^2df^2(de-2cf)+b^2e(de-cf)^2-2abf(de-cf)^2)\,x}{f^4} + \frac{(a^2d^2f^2-2abdf(de-2cf)+b^2(de-cf)^2)\,x^3}{3f^3} - \frac{bd(bde-2bcf-2adf)x^5}{5f^2} + \frac{b^2d^2x^7}{7f} + \frac{(be-af)^2(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
-(((a^2*d*f^2*(d*e - 2*c*f) + b^2*e*(d*e - c*f)^2 - 2*a*b*f*(d*e - c*f)^2)
*x)/f^4) + ((a^2*d^2*f^2 - 2*a*b*d*f*(d*e - 2*c*f) + b^2*(d*e - c*f)^2)*x^
3)/(3*f^3) - (b*d*(b*d*e - 2*b*c*f - 2*a*d*f)*x^5)/(5*f^2) + (b^2*d^2*x^7)
/(7*f) + ((b*e - a*f)^2*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e
]*f^(9/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {420, 290, 403, 25, 403, 25, 299, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{e+fx^2}\,dx$$

$$\Big\downarrow \; 420$$

$$\frac{b\int (bx^2+a)\,(dx^2+c)^2\,dx}{f} - \frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{290}$$

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{\int -\frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}+\frac{bx(c+dx^2)^2}{5f}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\int \frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}\right)}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{\int -\frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}+\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}}{5f}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}}{5f}\right)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)(de-cf)^2\int \frac{1}{fx^2+e}dx}{f}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}\right)}{f}}{}$$

$\Big\downarrow$ 218

$$\frac{b\int \left(bd^2x^6+d(2bc+ad)x^4+c(bc+2ad)x^2+ac^2\right)dx}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}\right)}{f}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7\right)}{f}-\frac{(be-af)\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}\right)}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
(b*(a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x
^7)/7))/f - ((b*e - a*f)*((b*x*(c + d*x^2)^2)/(5*f) - (((5*b*d*e - 4*b*c*f
 - 5*a*d*f)*x*(c + d*x^2))/(3*f) - (-(((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^
2*e^2 - 25*c*d*e*f + 8*c^2*f^2))*x)/f) - (15*(b*e - a*f)*(d*e - c*f)^2*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.61

method	result
default	$\frac{\frac{b^2d^2x^7f^3}{7}+\frac{\left((adf+bcf-bde)bd\,f^2+bdf\left(ad\,f^2+bc\,f^2\right)\right)x^5}{5}+\frac{\left((adf+bcf-bde)\left(ad\,f^2+bc\,f^2\right)+bdf\left(2ac\,f^2-adef-bcef+bd\,e^2\right)\right)x^3}{3}+(adf+bcf-}{f^4}$
risch	$\frac{2\ln(-fx+\sqrt{-ef})abcd\,e^2}{f^2\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})a^2cde}{f\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})ab\,c^2e}{f\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})ab\,d^2e^3}{f^3\sqrt{-ef}}+\frac{\ln(fx+\sqrt{-ef})b^2cd\,e^3}{f^3\sqrt{-ef}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/7*b^2*d^2*x^7*f^3+1/5*((a*d*f+b*c*f-b*d*e)*b*d*f^2+b*d*f*(a*d*f^2
+b*c*f^2))*x^5+1/3*((a*d*f+b*c*f-b*d*e)*(a*d*f^2+b*c*f^2)+b*d*f*(2*a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2))*x^3+(a*d*f+b*c*f-b*d*e)*(2*a*c*f^2-a*d*e*f-b*c*
e*f+b*d*e^2)*x)+(a^2*c^2*f^4-2*a^2*c*d*e*f^3+a^2*d^2*e^2*f^2-2*a*b*c^2*e*f
^3+4*a*b*c*d*e^2*f^2-2*a*b*d^2*e^3*f+b^2*c^2*e^2*f^2-2*b^2*c*d*e^3*f+b^2*d
^2*e^4)/f^4/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.60

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{e+fx^2}\,dx$$

$$= \left[\frac{30\,b^2d^2ef^4x^7-42\,(b^2d^2e^2f^3-2\,(b^2cd+abd^2)ef^4)x^5+70\,(b^2d^2e^3f^2-2\,(b^2cd+abd^2)e^2f^3+(b^2c^2+4}{}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/210*(30*b^2*d^2*e*f^4*x^7 - 42*(b^2*d^2*e^2*f^3 - 2*(b^2*c*d + a*b*d^2)
*e*f^4)*x^5 + 70*(b^2*d^2*e^3*f^2 - 2*(b^2*c*d + a*b*d^2)*e^2*f^3 + (b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*e*f^4)*x^3 - 105*(b^2*d^2*e^4 + a^2*c^2*f^4 - 2*
(b^2*c*d + a*b*d^2)*e^3*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^2 - 2*(a
*b*c^2 + a^2*c*d)*e*f^3)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^
2 + e)) - 210*(b^2*d^2*e^4*f - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 +
4*a*b*c*d + a^2*d^2)*e^2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*x)/(e*f^5), 1/
105*(15*b^2*d^2*e*f^4*x^7 - 21*(b^2*d^2*e^2*f^3 - 2*(b^2*c*d + a*b*d^2)*e*
f^4)*x^5 + 35*(b^2*d^2*e^3*f^2 - 2*(b^2*c*d + a*b*d^2)*e^2*f^3 + (b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*e*f^4)*x^3 + 105*(b^2*d^2*e^4 + a^2*c^2*f^4 - 2*(b^
2*c*d + a*b*d^2)*e^3*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^2 - 2*(a*b*
c^2 + a^2*c*d)*e*f^3)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 105*(b^2*d^2*e^4*f
 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^3
 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*x)/(e*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(177) = 354$.

Time = 1.00 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.36

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{e+fx^2}\,dx = \frac{b^2d^2x^7}{7f} + x^5\cdot\left(\frac{2abd^2}{5f}+\frac{2b^2cd}{5f}-\frac{b^2d^2e}{5f^2}\right) + x^3\left(\frac{a^2d^2}{3f}+\frac{4abcd}{3f}-\frac{2abd^2e}{3f^2}+\frac{b^2c^2}{3f}-\frac{2b^2cde}{3f^2}+\frac{b^2d^2e^2}{3f^3}\right) + x\left(\frac{2a^2cd}{f}-\frac{a^2d^2e}{f^2}+\frac{2abc^2}{f}-\frac{4abcde}{f^2}+\frac{2abd^2e^2}{f^3}-\frac{b^2c^2e}{f^2}+\frac{2b^2cde^2}{f^3}-\frac{b^2d^2e^3}{f^4}\right) - \frac{\sqrt{-\frac{1}{ef^9}}(af-be)^2(cf-de)^2\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)^2(cf-de)^2}{a^2c^2f^4-2a^2cdef^3+a^2d^2e^2f^2-2abc^2ef^3+4abcde^2f^2-2abd^2e^3f+b^2c^2e^2f^2-2b^2cde^3f+b\cdots}\right.}{2} + \frac{\sqrt{-\frac{1}{ef^9}}(af-be)^2(cf-de)^2\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)^2(cf-de)^2}{a^2c^2f^4-2a^2cdef^3+a^2d^2e^2f^2-2abc^2ef^3+4abcde^2f^2-2abd^2e^3f+b^2c^2e^2f^2-2b^2cde^3f+b^2d^2\cdots}\right.}{2}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
b**2*d**2*x**7/(7*f) + x**5*(2*a*b*d**2/(5*f) + 2*b**2*c*d/(5*f) - b**2*d*
*2*e/(5*f**2)) + x**3*(a**2*d**2/(3*f) + 4*a*b*c*d/(3*f) - 2*a*b*d**2*e/(3
*f**2) + b**2*c**2/(3*f) - 2*b**2*c*d*e/(3*f**2) + b**2*d**2*e**2/(3*f**3)
) + x*(2*a**2*c*d/f - a**2*d**2*e/f**2 + 2*a*b*c**2/f - 4*a*b*c*d*e/f**2 +
 2*a*b*d**2*e**2/f**3 - b**2*c**2*e/f**2 + 2*b**2*c*d*e**2/f**3 - b**2*d**
2*e**3/f**4) - sqrt(-1/(e*f**9))*(a*f - b*e)**2*(c*f - d*e)**2*log(-e*f**4
*sqrt(-1/(e*f**9))*(a*f - b*e)**2*(c*f - d*e)**2/(a**2*c**2*f**4 - 2*a**2*
c*d*e*f**3 + a**2*d**2*e**2*f**2 - 2*a*b*c**2*e*f**3 + 4*a*b*c*d*e**2*f**2
 - 2*a*b*d**2*e**3*f + b**2*c**2*e**2*f**2 - 2*b**2*c*d*e**3*f + b**2*d**2
*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)**2*(c*f - d*e)**2*log(e*f**4
*sqrt(-1/(e*f**9))*(a*f - b*e)**2*(c*f - d*e)**2/(a**2*c**2*f**4 - 2*a**2*
c*d*e*f**3 + a**2*d**2*e**2*f**2 - 2*a*b*c**2*e*f**3 + 4*a*b*c*d*e**2*f**2
 - 2*a*b*d**2*e**3*f + b**2*c**2*e**2*f**2 - 2*b**2*c*d*e**3*f + b**2*d**2
*e**4) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{e+fx^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(168) = 336$.

Time = 0.12 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.04

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{e+fx^2} \, dx$$

$$= \frac{(b^2d^2e^4 - 2\,b^2cde^3f - 2\,abd^2e^3f + b^2c^2e^2f^2 + 4\,abcde^2f^2 + a^2d^2e^2f^2 - 2\,abc^2ef^3 - 2\,a^2cdef^3 + a^2c^2f^4)}{\sqrt{ef}\,f^4}$$

$$+ \frac{15\,b^2d^2f^6x^7 - 21\,b^2d^2ef^5x^5 + 42\,b^2cdf^6x^5 + 42\,abd^2f^6x^5 + 35\,b^2d^2e^2f^4x^3 - 70\,b^2cdef^5x^3 - 70\,abd^2ef^5}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
(b^2*d^2*e^4 - 2*b^2*c*d*e^3*f - 2*a*b*d^2*e^3*f + b^2*c^2*e^2*f^2 + 4*a*b
*c*d*e^2*f^2 + a^2*d^2*e^2*f^2 - 2*a*b*c^2*e*f^3 - 2*a^2*c*d*e*f^3 + a^2*c
^2*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^4) + 1/105*(15*b^2*d^2*f^6*x^7
- 21*b^2*d^2*e*f^5*x^5 + 42*b^2*c*d*f^6*x^5 + 42*a*b*d^2*f^6*x^5 + 35*b^2*
d^2*e^2*f^4*x^3 - 70*b^2*c*d*e*f^5*x^3 - 70*a*b*d^2*e*f^5*x^3 + 35*b^2*c^2
*f^6*x^3 + 140*a*b*c*d*f^6*x^3 + 35*a^2*d^2*f^6*x^3 - 105*b^2*d^2*e^3*f^3*
x + 210*b^2*c*d*e^2*f^4*x + 210*a*b*d^2*e^2*f^4*x - 105*b^2*c^2*e*f^5*x -
420*a*b*c*d*e*f^5*x - 105*a^2*d^2*e*f^5*x + 210*a*b*c^2*f^6*x + 210*a^2*c*
d*f^6*x)/f^7
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{e+fx^2}\,dx = x^3\left(\frac{a^2d^2+4abcd+b^2c^2}{3f}+\frac{e\left(\frac{b^2d^2e}{f^2}-\frac{2bd(ad+bc)}{f}\right)}{3f}\right) - x^5\left(\frac{b^2d^2e}{5f^2}-\frac{2bd(ad+bc)}{5f}\right) - x\left(\frac{e\left(\frac{a^2d^2+4abcd+b^2c^2}{f}+\frac{e\left(\frac{b^2d^2e}{f^2}-\frac{2bd(ad+bc)}{f}\right)}{f}\right)}{f}-\frac{2ac(ad+bc)}{f}\right)+\frac{b^2d^2x^7}{7f} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x(af-be)^2(cf-de)^2}{\sqrt{e}\,(a^2c^2f^4-2a^2cdef^3+a^2d^2e^2f^2-2abc^2ef^3+4abcde^2f^2-2abd^2e^3f+b^2c^2e^2f^2-2b^2cde^3f+b^2d^2e^4)}\right)(af-be}{\sqrt{e}\,f^{9/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2),x)
```

output
```
x^3*((a^2*d^2 + b^2*c^2 + 4*a*b*c*d)/(3*f) + (e*((b^2*d^2*e)/f^2 - (2*b*d*
(a*d + b*c))/f))/(3*f)) - x^5*((b^2*d^2*e)/(5*f^2) - (2*b*d*(a*d + b*c))/(
5*f)) - x*((e*((a^2*d^2 + b^2*c^2 + 4*a*b*c*d)/f + (e*((b^2*d^2*e)/f^2 - (
2*b*d*(a*d + b*c))/f))/f))/f - (2*a*c*(a*d + b*c))/f) + (b^2*d^2*x^7)/(7*f
) + (atan((f^(1/2)*x*(a*f - b*e)^2*(c*f - d*e)^2)/(e^(1/2)*(a^2*c^2*f^4 +
b^2*d^2*e^4 + a^2*d^2*e^2*f^2 + b^2*c^2*e^2*f^2 - 2*a*b*c^2*e*f^3 - 2*a*b*
d^2*e^3*f - 2*a^2*c*d*e*f^3 - 2*b^2*c*d*e^3*f + 4*a*b*c*d*e^2*f^2))*(a*f
- b*e)^2*(c*f - d*e)^2)/(e^(1/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{e+fx^2}\,dx$$
$$= \frac{-105a^2d^2e^2f^3x+35a^2d^2ef^4x^3-105b^2c^2e^2f^3x+35b^2c^2ef^4x^3-105b^2d^2e^4fx+35b^2d^2e^3f^2x^3-21b^2d^2}{}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e),x)
```

output
```
(105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*f**4 - 210*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e*f**3 + 105*sqrt(f)*
sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**2*f**2 - 210*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e*f**3 + 420*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**2*f**2 - 210*sqrt(f)*sqrt(e)*atan
((f*x)/(sqrt(f)*sqrt(e)))*a*b*d**2*e**3*f + 105*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*b**2*c**2*e**2*f**2 - 210*sqrt(f)*sqrt(e)*atan((f*x)/(
sqrt(f)*sqrt(e)))*b**2*c*d*e**3*f + 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f
)*sqrt(e)))*b**2*d**2*e**4 + 210*a**2*c*d*e*f**4*x - 105*a**2*d**2*e**2*f*
*3*x + 35*a**2*d**2*e*f**4*x**3 + 210*a*b*c**2*e*f**4*x - 420*a*b*c*d*e**2
*f**3*x + 140*a*b*c*d*e*f**4*x**3 + 210*a*b*d**2*e**3*f**2*x - 70*a*b*d**2
*e**2*f**3*x**3 + 42*a*b*d**2*e*f**4*x**5 - 105*b**2*c**2*e**2*f**3*x + 35
*b**2*c**2*e*f**4*x**3 + 210*b**2*c*d*e**3*f**2*x - 70*b**2*c*d*e**2*f**3*
x**3 + 42*b**2*c*d*e*f**4*x**5 - 105*b**2*d**2*e**4*f*x + 35*b**2*d**2*e**
3*f**2*x**3 - 21*b**2*d**2*e**2*f**3*x**5 + 15*b**2*d**2*e*f**4*x**7)/(105
*e*f**5)
```

3.228 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx &= \frac{(a^2d^2f^2-4abdf(de-cf)+b^2(3d^2e^2-4cdef+c^2f^2))\,x}{f^4} \\ &\quad -\frac{2bd(bde-bcf-adf)x^3}{3f^3}+\frac{b^2d^2x^5}{5f^2}+\frac{(be-af)^2(de-cf)^2x}{2ef^4\,(e+fx^2)} \\ &\quad -\frac{(be-af)(de-cf)(be(7de-3cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}\end{aligned}$$

output
```
(a^2*d^2*f^2-4*a*b*d*f*(-c*f+d*e)+b^2*(c^2*f^2-4*c*d*e*f+3*d^2*e^2))*x/f^4
-2/3*b*d*(-a*d*f-b*c*f+b*d*e)*x^3/f^3+1/5*b^2*d^2*x^5/f^2+1/2*(-a*f+b*e)^2
*(-c*f+d*e)^2*x/e/f^4/(f*x^2+e)-1/2*(-a*f+b*e)*(-c*f+d*e)*(b*e*(-3*c*f+7*d
*e)-a*f*(c*f+3*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{(a^2d^2f^2+4abdf(-de+cf)+b^2(3d^2e^2-4cdef+c^2f^2))\,x}{f^4} - \frac{2bd(bde-bcf-adf)x^3}{3f^3} + \frac{b^2d^2x^5}{5f^2} + \frac{(be-af)^2(de-cf)^2x}{2ef^4(e+fx^2)} - \frac{(be-af)(de-cf)(be(7de-3cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
((a^2*d^2*f^2 + 4*a*b*d*f*(-(d*e) + c*f) + b^2*(3*d^2*e^2 - 4*c*d*e*f + c^
2*f^2))*x)/f^4 - (2*b*d*(b*d*e - b*c*f - a*d*f)*x^3)/(3*f^3) + (b^2*d^2*x^
5)/(5*f^2) + ((b*e - a*f)^2*(d*e - c*f)^2*x)/(2*e*f^4*(e + f*x^2)) - ((b*e
 - a*f)*(d*e - c*f)*(b*e*(7*d*e - 3*c*f) - a*f*(3*d*e + c*f))*ArcTan[(Sqrt
[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.67, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {425, 401, 25, 403, 25, 299, 218, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^2}{fx^2+e}dx}{f} - \frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^2}{fx^2+e}dx}{f}-\frac{(be-af)\left(-\frac{\int -\frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^2}{fx^2+e}dx}{f}-\frac{(be-af)\left(\frac{\int \frac{(dx^2+c)(d(5be-3af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{\int -\frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}+\frac{bx(c+dx^2)^2}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{\int -\frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}+\frac{dx(c+dx^2)(5be-3af)}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\int \frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\int \frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\int \frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3(de-cf)(be(5de-cf)-af(cf+3de))\int \frac{1}{fx^2+e}dx}{f}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\Big\downarrow$ 218

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\int\frac{(dx^2+c)((5bde-4bcf-5adf)x^2+c(be-5af))}{fx^2+e}dx}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{\int-\frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}+\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{\int\frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdfe+8c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\Big\downarrow$ 299

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)(de-cf)^2\int\frac{1}{fx^2+e}dx}{f}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{bx(c+dx^2)^2}{5f}-\frac{\frac{x(c+dx^2)(-5adf-4bcf+5bde)}{3f}-\frac{-\frac{15(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{f}}{3f}}{5f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
(b*((b*x*(c + d*x^2)^2)/(5*f) - (((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x
^2))/(3*f) - (-(((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8
*c^2*f^2))*x)/f) - (15*(b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f)))/f - ((b*e - a*f)*(-1/2*((b*e - a*f)*
x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) + ((d*(5*b*e - 3*a*f)*x*(c + d*x^2))/(3
*f) - ((d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/f - (3*(d*e - c
*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(
Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.79

method	result
default	$\frac{\frac{1}{5}f^2x^5b^2d^2+\frac{2}{3}ab\,d^2f^2x^3+\frac{2}{3}b^2cd\,f^2x^3-\frac{2}{3}b^2d^2ef\,x^3+a^2d^2f^2x+4abcd\,f^2x-4ab\,d^2efx+b^2c^2f^2x-4b^2cdefx+3b^2d^2e^2x}{f^4}+\frac{\left(a^2c^2f^4\right.}{}$
risch	$\frac{2ab\,d^2x^3}{3f^2}-\frac{\ln\left(fx+\sqrt{-ef}\right)ab\,c^2}{2f\sqrt{-ef}}+\frac{3e\ln\left(fx+\sqrt{-ef}\right)b^2c^2}{4f^2\sqrt{-ef}}+\frac{7e^3\ln\left(fx+\sqrt{-ef}\right)b^2d^2}{4f^4\sqrt{-ef}}+\frac{\ln\left(-fx+\sqrt{-ef}\right)a^2cd}{2f\sqrt{-ef}}-\frac{3e\ln\left(-fx+\right.}{4f^2\sqrt{}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/5*f^2*x^5*b^2*d^2+2/3*a*b*d^2*f^2*x^3+2/3*b^2*c*d*f^2*x^3-2/3*b^2
*d^2*e*f*x^3+a^2*d^2*f^2*x+4*a*b*c*d*f^2*x-4*a*b*d^2*e*f*x+b^2*c^2*f^2*x-4
*b^2*c*d*e*f*x+3*b^2*d^2*e^2*x)+1/f^4*(1/2*(a^2*c^2*f^4-2*a^2*c*d*e*f^3+a^
2*d^2*e^2*f^2-2*a*b*c^2*e*f^3+4*a*b*c*d*e^2*f^2-2*a*b*d^2*e^3*f+b^2*c^2*e^
2*f^2-2*b^2*c*d*e^3*f+b^2*d^2*e^4)/e*x/(f*x^2+e)+1/2*(a^2*c^2*f^4+2*a^2*c*
d*e*f^3-3*a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-12*a*b*c*d*e^2*f^2+10*a*b*d^2*e^
3*f-3*b^2*c^2*e^2*f^2+10*b^2*c*d*e^3*f-7*b^2*d^2*e^4)/e/(e*f)^(1/2)*arctan
(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(192) = 384.

Time = 0.10 (sec) , antiderivative size = 934, normalized size of antiderivative = 4.49

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b^2*d^2*e^2*f^4*x^7 - 4*(7*b^2*d^2*e^3*f^3 - 10*(b^2*c*d + a*b*d
^2)*e^2*f^4)*x^5 + 20*(7*b^2*d^2*e^4*f^2 - 10*(b^2*c*d + a*b*d^2)*e^3*f^3
+ 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^4)*x^3 + 15*(7*b^2*d^2*e^5 - a^2
*c^2*e*f^4 - 10*(b^2*c*d + a*b*d^2)*e^4*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*e^3*f^2 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^3 + (7*b^2*d^2*e^4*f - a^2*c^2*f
^5 - 10*(b^2*c*d + a*b*d^2)*e^3*f^2 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^
2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-
e*f)*x - e)/(f*x^2 + e)) + 30*(7*b^2*d^2*e^5*f + a^2*c^2*e*f^5 - 10*(b^2*c
*d + a*b*d^2)*e^4*f^2 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^3 - 2*(a*b
*c^2 + a^2*c*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5), 1/30*(6*b^2*d^2*e^2*f
^4*x^7 - 2*(7*b^2*d^2*e^3*f^3 - 10*(b^2*c*d + a*b*d^2)*e^2*f^4)*x^5 + 10*(
7*b^2*d^2*e^4*f^2 - 10*(b^2*c*d + a*b*d^2)*e^3*f^3 + 3*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*e^2*f^4)*x^3 - 15*(7*b^2*d^2*e^5 - a^2*c^2*e*f^4 - 10*(b^2*c*
d + a*b*d^2)*e^4*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^2 - 2*(a*b*c^
2 + a^2*c*d)*e^2*f^3 + (7*b^2*d^2*e^4*f - a^2*c^2*f^5 - 10*(b^2*c*d + a*b*
d^2)*e^3*f^2 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^3 - 2*(a*b*c^2 + a^
2*c*d)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(7*b^2*d^2*e^5*f +
 a^2*c^2*e*f^5 - 10*(b^2*c*d + a*b*d^2)*e^4*f^2 + 3*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*e^3*f^3 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f
^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(199) = 398$.

Time = 2.87 (sec) , antiderivative size = 780, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{b^2d^2x^5}{5f^2} + x^3\cdot\left(\frac{2abd^2}{3f^2} + \frac{2b^2cd}{3f^2} - \frac{2b^2d^2e}{3f^3}\right) + x\left(\frac{a^2d^2}{f^2} + \frac{4abcd}{f^2} - \frac{4abd^2e}{f^3} + \frac{b^2c^2}{f^2} - \frac{4b^2cde}{f^3} + \frac{3b^2d^2e^2}{f^4}\right) + \frac{x(a^2c^2f^4 - 2a^2cdef^3 + a^2d^2e^2f^2 - 2abc^2ef^3 + 4abcde^2f^2 - 2abd^2e^3f + b^2c^2e^2f^2 - 2b^2cde^3f + b^2d^2e^4}{2e^2f^4 + 2ef^5x^2} - \frac{\sqrt{-\frac{1}{e^3f^9}}(af-be)(cf-de)(acf^2+3adef+3bcef-7bde^2)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(af-be)(c}{a^2c^2f^4+2a^2cdef^3-3a^2d^2e^2f^2+2abc^2ef^3-1}\right.}{4} + \frac{\sqrt{-\frac{1}{e^3f^9}}(af-be)(cf-de)(acf^2+3adef+3bcef-7bde^2)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(af-be)(cf-}{a^2c^2f^4+2a^2cdef^3-3a^2d^2e^2f^2+2abc^2ef^3-12ab}\right.}{4}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
b**2*d**2*x**5/(5*f**2) + x**3*(2*a*b*d**2/(3*f**2) + 2*b**2*c*d/(3*f**2)
- 2*b**2*d**2*e/(3*f**3)) + x*(a**2*d**2/f**2 + 4*a*b*c*d/f**2 - 4*a*b*d**
2*e/f**3 + b**2*c**2/f**2 - 4*b**2*c*d*e/f**3 + 3*b**2*d**2*e**2/f**4) + x
*(a**2*c**2*f**4 - 2*a**2*c*d*e*f**3 + a**2*d**2*e**2*f**2 - 2*a*b*c**2*e*
f**3 + 4*a*b*c*d*e**2*f**2 - 2*a*b*d**2*e**3*f + b**2*c**2*e**2*f**2 - 2*b
**2*c*d*e**3*f + b**2*d**2*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(-1/(
e**3*f**9))*(a*f - b*e)*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + 3*b*c*e*f - 7*
b*d*e**2)*log(-e**2*f**4*sqrt(-1/(e**3*f**9))*(a*f - b*e)*(c*f - d*e)*(a*c
*f**2 + 3*a*d*e*f + 3*b*c*e*f - 7*b*d*e**2)/(a**2*c**2*f**4 + 2*a**2*c*d*e
*f**3 - 3*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f**3 - 12*a*b*c*d*e**2*f**2 +
 10*a*b*d**2*e**3*f - 3*b**2*c**2*e**2*f**2 + 10*b**2*c*d*e**3*f - 7*b**2*
d**2*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(a*f - b*e)*(c*f - d*e)*(a*c*f**2
 + 3*a*d*e*f + 3*b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*f**9))*
(a*f - b*e)*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + 3*b*c*e*f - 7*b*d*e**2)/(a
**2*c**2*f**4 + 2*a**2*c*d*e*f**3 - 3*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f
**3 - 12*a*b*c*d*e**2*f**2 + 10*a*b*d**2*e**3*f - 3*b**2*c**2*e**2*f**2 +
10*b**2*c*d*e**3*f - 7*b**2*d**2*e**4) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(192) = 384$.

Time = 0.13 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^2} \, dx = -\frac{(7\,b^2d^2e^4 - 10\,b^2cde^3f - 10\,abd^2e^3f + 3\,b^2c^2e^2f^2 + 12\,abcde^2f^2 + 3\,a^2d^2e^2f^2 - 2\,abc^2ef^3 - 2\,a^2cdef^3}{2\sqrt{ef}ef^4} + \frac{b^2d^2e^4x - 2\,b^2cde^3fx - 2\,abd^2e^3fx + b^2c^2e^2f^2x + 4\,abcde^2f^2x + a^2d^2e^2f^2x - 2\,abc^2ef^3x - 2\,a^2cdef^3}{2\,(fx^2+e)ef^4} + \frac{3\,b^2d^2f^8x^5 - 10\,b^2d^2ef^7x^3 + 10\,b^2cdf^8x^3 + 10\,abd^2f^8x^3 + 45\,b^2d^2e^2f^6x - 60\,b^2cdef^7x - 60\,abd^2ef^7x -}{15\,f^{10}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(7*b^2*d^2*e^4 - 10*b^2*c*d*e^3*f - 10*a*b*d^2*e^3*f + 3*b^2*c^2*e^2*
f^2 + 12*a*b*c*d*e^2*f^2 + 3*a^2*d^2*e^2*f^2 - 2*a*b*c^2*e*f^3 - 2*a^2*c*d
*e*f^3 - a^2*c^2*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e*f^4) + 1/2*(b^2*d
^2*e^4*x - 2*b^2*c*d*e^3*f*x - 2*a*b*d^2*e^3*f*x + b^2*c^2*e^2*f^2*x + 4*a
*b*c*d*e^2*f^2*x + a^2*d^2*e^2*f^2*x - 2*a*b*c^2*e*f^3*x - 2*a^2*c*d*e*f^3
*x + a^2*c^2*f^4*x)/((f*x^2 + e)*e*f^4) + 1/15*(3*b^2*d^2*f^8*x^5 - 10*b^2
*d^2*e*f^7*x^3 + 10*b^2*c*d*f^8*x^3 + 10*a*b*d^2*f^8*x^3 + 45*b^2*d^2*e^2*
f^6*x - 60*b^2*c*d*e*f^7*x - 60*a*b*d^2*e*f^7*x + 15*b^2*c^2*f^8*x + 60*a*
b*c*d*f^8*x + 15*a^2*d^2*f^8*x)/f^10
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^2}\,dx$$
$$= x\left(\frac{a^2d^2+4abcd+b^2c^2}{f^2}+\frac{2e\left(\frac{2b^2d^2e}{f^3}-\frac{2bd(ad+bc)}{f^2}\right)}{f}-\frac{b^2d^2e^2}{f^4}\right)$$
$$-x^3\left(\frac{2b^2d^2e}{3f^3}-\frac{2bd(ad+bc)}{3f^2}\right)$$
$$+\frac{x(a^2c^2f^4-2a^2cdef^3+a^2d^2e^2f^2-2abc^2ef^3+4abcde^2f^2-2abd^2e^3f+b^2c^2e^2f^2-2b^2c\cdots}{2e(f^5x^2+ef^4)}$$
$$+\frac{b^2d^2x^5}{5f^2}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x(af-be)(cf-de)(acf^2-7bde^2+3adef+3bcef)}{\sqrt{e}(a^2c^2f^4+2a^2cdef^3-3a^2d^2e^2f^2+2abc^2ef^3-12abcde^2f^2+10abd^2e^3f-3b^2c^2e^2f^2+10b^2cde^3f-7b^2d^2e^4)}\right)(af\cdots}{2e^{3/2}f^{9/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

output
```
x*((a^2*d^2 + b^2*c^2 + 4*a*b*c*d)/f^2 + (2*e*((2*b^2*d^2*e)/f^3 - (2*b*d*
(a*d + b*c))/f^2))/f - (b^2*d^2*e^2)/f^4) - x^3*((2*b^2*d^2*e)/(3*f^3) - (
2*b*d*(a*d + b*c))/(3*f^2)) + (x*(a^2*c^2*f^4 + b^2*d^2*e^4 + a^2*d^2*e^2*
f^2 + b^2*c^2*e^2*f^2 - 2*a*b*c^2*e*f^3 - 2*a*b*d^2*e^3*f - 2*a^2*c*d*e*f^
3 - 2*b^2*c*d*e^3*f + 4*a*b*c*d*e^2*f^2))/(2*e*(e*f^4 + f^5*x^2)) + (b^2*d
^2*x^5)/(5*f^2) + (atan((f^(1/2)*x*(a*f - b*e)*(c*f - d*e)*(a*c*f^2 - 7*b*
d*e^2 + 3*a*d*e*f + 3*b*c*e*f))/(e^(1/2)*(a^2*c^2*f^4 - 7*b^2*d^2*e^4 - 3*
a^2*d^2*e^2*f^2 - 3*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 + 10*a*b*d^2*e^3*f +
 2*a^2*c*d*e*f^3 + 10*b^2*c*d*e^3*f - 12*a*b*c*d*e^2*f^2))))*(a*f - b*e)*(c
*f - d*e)*(a*c*f^2 - 7*b*d*e^2 + 3*a*d*e*f + 3*b*c*e*f))/(2*e^(3/2)*f^(9/2
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 844, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e*f**4 + 15*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*f**5*x**2 + 30*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**2*f**3 + 30*sqrt(f)*s
qrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e*f**4*x**2 - 45*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**3*f**2 - 45*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**2*f**3*x**2 + 30*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**2*f**3 + 30*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e*f**4*x**2 - 180*sqrt(f)*sqrt(e)*a
tan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**3*f**2 - 180*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**2*f**3*x**2 + 150*sqrt(f)*sqrt(e)*atan
((f*x)/(sqrt(f)*sqrt(e)))*a*b*d**2*e**4*f + 150*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*b*d**2*e**3*f**2*x**2 - 45*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*b**2*c**2*e**3*f**2 - 45*sqrt(f)*sqrt(e)*atan((f*x)/(
sqrt(f)*sqrt(e)))*b**2*c**2*e**2*f**3*x**2 + 150*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*b**2*c*d*e**4*f + 150*sqrt(f)*sqrt(e)*atan((f*x)/(sqr
t(f)*sqrt(e)))*b**2*c*d*e**3*f**2*x**2 - 105*sqrt(f)*sqrt(e)*atan((f*x)/(s
qrt(f)*sqrt(e)))*b**2*d**2*e**5 - 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*
sqrt(e)))*b**2*d**2*e**4*f*x**2 + 15*a**2*c**2*e*f**5*x - 30*a**2*c*d*e**2
*f**4*x + 45*a**2*d**2*e**3*f**3*x + 30*a**2*d**2*e**2*f**4*x**3 - 30*a*b*
c**2*e**2*f**4*x + 180*a*b*c*d*e**3*f**3*x + 120*a*b*c*d*e**2*f**4*x**3...
```

3.229 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 262

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^3}\,dx = -\frac{bd(3bde-2bcf-2adf)x}{f^4}+\frac{b^2d^2x^3}{3f^3}$$
$$+\frac{(be-af)^2(de-cf)^2x}{4ef^4(e+fx^2)^2}-\frac{(be-af)(de-cf)(be(13de-5cf)-af(5de+3cf))x}{8e^2f^4(e+fx^2)}$$
$$-\frac{(2abef(15d^2e^2-6cdef-c^2f^2)-b^2e^2(35d^2e^2-30cdef+3c^2f^2)-a^2f^2(3d^2e^2+2cdef+3c^2f^2))\,\text{arc}}{8e^{5/2}f^{9/2}}$$

output
```
-b*d*(-2*a*d*f-2*b*c*f+3*b*d*e)*x/f^4+1/3*b^2*d^2*x^3/f^3+1/4*(-a*f+b*e)^2
*(-c*f+d*e)^2*x/e/f^4/(f*x^2+e)^2-1/8*(-a*f+b*e)*(-c*f+d*e)*(b*e*(-5*c*f+1
3*d*e)-a*f*(3*c*f+5*d*e))*x/e^2/f^4/(f*x^2+e)-1/8*(2*a*b*e*f*(-c^2*f^2-6*c
*d*e*f+15*d^2*e^2)-b^2*e^2*(3*c^2*f^2-30*c*d*e*f+35*d^2*e^2)-a^2*f^2*(3*c^
2*f^2+2*c*d*e*f+3*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^3}\,dx = -\frac{bd(3bde-2bcf-2adf)x}{f^4}+\frac{b^2d^2x^3}{3f^3} + \frac{(be-af)^2(de-cf)^2x}{4ef^4(e+fx^2)^2} - \frac{(be-af)(de-cf)(be(13de-5cf)-af(5de+3cf))x}{8e^2f^4(e+fx^2)} + \frac{(2abef(-15d^2e^2+6cdef+c^2f^2)+b^2e^2(35d^2e^2-30cdef+3c^2f^2)+a^2f^2(3d^2e^2+2cdef+3c^2f^2))\, a}{8e^{5/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
-((b*d*(3*b*d*e - 2*b*c*f - 2*a*d*f)*x)/f^4) + (b^2*d^2*x^3)/(3*f^3) + ((b
*e - a*f)^2*(d*e - c*f)^2*x)/(4*e*f^4*(e + f*x^2)^2) - ((b*e - a*f)*(d*e -
 c*f)*(b*e*(13*d*e - 5*c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^4*(e + f*x^
2)) + ((2*a*b*e*f*(-15*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b^2*e^2*(35*d^2*e^
2 - 30*c*d*e*f + 3*c^2*f^2) + a^2*f^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.57, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {425, 401, 25, 401, 299, 218, 403, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^2}{(fx^2+e)^2}dx}{f} - \frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^2}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)\left(d(5be-3af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)\left(d(5be-af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-3af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-3af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-3af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\frac{\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{f}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+}{2ef\left(e+fx^2\right)}}{4ef}\right.}{f}$$

$\Big\downarrow$ 218

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-3af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf}{2ef\left(e+fx^2\right)}}{4ef}\right.}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{\frac{\int -\frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}+\frac{dx\left(c+dx^2\right)(5be-3af)}{3f}}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf}{2ef\left(e+fx^2\right)}}{4ef}\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)(5be-3af)}{3f}-\frac{\int \frac{d(be(15de-13cf)-3af(3de-cf))x^2+c(be(5de-3cf)-3af(de+cf))}{fx^2+e}dx}{3f}}{2ef}-\frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf}{2ef\left(e+fx^2\right)}}{4ef}\right.}{f}$$

$\Big\downarrow$ 299

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3(de-cf)(be(5de-cf)-af(cf+3de))\int\frac{1}{fx^2+e}dx}{f}}{3f}}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3c\cdots}{2ef\left(e+fx^2\right)}}{4ef}\cdots\right)}{f}$$

$\Big\downarrow$ 218

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)(5be-3af)}{3f}-\frac{\frac{dx(be(15de-13cf)-3af(3de-cf))}{f}-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{\sqrt{e}f^{3/2}}}{3f}}{2ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3c\cdots}{2ef\left(e+fx^2\right)}}{4ef}\cdots\right)}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(b*(-1/2*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) + ((d*(5*b*e - 3*
a*f)*x*(c + d*x^2))/(3*f) - ((d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*
f))*x)/f - (3*(d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(
Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f)))/f - ((b*e - a*f)*
(-1/4*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(5*d
*e - c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b
*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/f) + ((b*e*(15*d^2*e^2 - 6*c*d*e
*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x
)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.55

method	result
default	$\frac{bd\left(\frac{1}{3}bdf\,x^3+2adfx+2bcfx-3bdex\right)}{f^4}+\frac{\frac{f\left(3a^2c^2f^4+2a^2cde\,f^3-5a^2d^2e^2f^2+2ab\,c^2e\,f^3-20abcd\,e^2f^2+18ab\,d^2e^3f-5b^2c^2e^2f^2+18b^2cd\,e^3f\right.}{8e^2}}{}$
risch	$\frac{b^2d^2x^3}{3f^3}+\frac{2b\,d^2ax}{f^3}+\frac{2b^2dcx}{f^3}-\frac{3b^2d^2ex}{f^4}+\frac{\frac{f\left(3a^2c^2f^4+2a^2cde\,f^3-5a^2d^2e^2f^2+2ab\,c^2e\,f^3-20abcd\,e^2f^2+18ab\,d^2e^3f-5b^2c^2e^2f^2+18\right.}{8e^2}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
b*d/f^4*(1/3*b*d*f*x^3+2*a*d*f*x+2*b*c*f*x-3*b*d*e*x)+1/f^4*((1/8*f*(3*a^2
*c^2*f^4+2*a^2*c*d*e*f^3-5*a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-20*a*b*c*d*e^2*
f^2+18*a*b*d^2*e^3*f-5*b^2*c^2*e^2*f^2+18*b^2*c*d*e^3*f-13*b^2*d^2*e^4)/e^
2*x^3+1/8*(5*a^2*c^2*f^4-2*a^2*c*d*e*f^3-3*a^2*d^2*e^2*f^2-2*a*b*c^2*e*f^3
-12*a*b*c*d*e^2*f^2+14*a*b*d^2*e^3*f-3*b^2*c^2*e^2*f^2+14*b^2*c*d*e^3*f-11
*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a^2*c^2*f^4+2*a^2*c*d*e*f^3+3*a^2*d^
2*e^2*f^2+2*a*b*c^2*e*f^3+12*a*b*c*d*e^2*f^2-30*a*b*d^2*e^3*f+3*b^2*c^2*e^
2*f^2-30*b^2*c*d*e^3*f+35*b^2*d^2*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(244) = 488.

Time = 0.11 (sec) , antiderivative size = 1238, normalized size of antiderivative = 4.73

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*b^2*d^2*e^3*f^4*x^7 - 16*(7*b^2*d^2*e^4*f^3 - 6*(b^2*c*d + a*b*d
^2)*e^3*f^4)*x^5 - 2*(175*b^2*d^2*e^5*f^2 - 9*a^2*c^2*e*f^6 - 150*(b^2*c*d
 + a*b*d^2)*e^4*f^3 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^4 - 6*(a*b*
c^2 + a^2*c*d)*e^2*f^5)*x^3 - 3*(35*b^2*d^2*e^6 + 3*a^2*c^2*e^2*f^4 - 30*(
b^2*c*d + a*b*d^2)*e^5*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^4*f^2 + 2*(
a*b*c^2 + a^2*c*d)*e^3*f^3 + (35*b^2*d^2*e^4*f^2 + 3*a^2*c^2*f^6 - 30*(b^2
*c*d + a*b*d^2)*e^3*f^3 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^4 + 2*(a
*b*c^2 + a^2*c*d)*e*f^5)*x^4 + 2*(35*b^2*d^2*e^5*f + 3*a^2*c^2*e*f^5 - 30*
(b^2*c*d + a*b*d^2)*e^4*f^2 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^3 +
2*(a*b*c^2 + a^2*c*d)*e^2*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x
 - e)/(f*x^2 + e)) - 6*(35*b^2*d^2*e^6*f - 5*a^2*c^2*e^2*f^5 - 30*(b^2*c*d
 + a*b*d^2)*e^5*f^2 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^4*f^3 + 2*(a*b*c
^2 + a^2*c*d)*e^3*f^4)*x)/(e^3*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5), 1/24*(8
*b^2*d^2*e^3*f^4*x^7 - 8*(7*b^2*d^2*e^4*f^3 - 6*(b^2*c*d + a*b*d^2)*e^3*f^
4)*x^5 - (175*b^2*d^2*e^5*f^2 - 9*a^2*c^2*e*f^6 - 150*(b^2*c*d + a*b*d^2)*
e^4*f^3 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^4 - 6*(a*b*c^2 + a^2*c*
d)*e^2*f^5)*x^3 + 3*(35*b^2*d^2*e^6 + 3*a^2*c^2*e^2*f^4 - 30*(b^2*c*d + a*
b*d^2)*e^5*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^4*f^2 + 2*(a*b*c^2 + a^
2*c*d)*e^3*f^3 + (35*b^2*d^2*e^4*f^2 + 3*a^2*c^2*f^6 - 30*(b^2*c*d + a*b*d
^2)*e^3*f^3 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^4 + 2*(a*b*c^2 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(260) = 520$.

Time = 19.17 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{b^2d^2x^3}{3f^3} + x\left(\frac{2abd^2}{f^3} + \frac{2b^2cd}{f^3} - \frac{3b^2d^2e}{f^4}\right) - \frac{\sqrt{-\frac{1}{e^5f^9}}\cdot(3a^2c^2f^4 + 2a^2cdef^3 + 3a^2d^2e^2f^2 + 2abc^2ef^3 + 12abcde^2f^2 - 30abd^2e^3f + 3b^2c^2e^2f^2 - 30b}{16} + \frac{\sqrt{-\frac{1}{e^5f^9}}\cdot(3a^2c^2f^4 + 2a^2cdef^3 + 3a^2d^2e^2f^2 + 2abc^2ef^3 + 12abcde^2f^2 - 30abd^2e^3f + 3b^2c^2e^2f^2 - 30b}{16} + \frac{x^3\cdot(3a^2c^2f^5 + 2a^2cdef^4 - 5a^2d^2e^2f^3 + 2abc^2ef^4 - 20abcde^2f^3 + 18abd^2e^3f^2 - 5b^2c^2e^2f^3 + 18b^2cde^3}{}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
b**2*d**2*x**3/(3*f**3) + x*(2*a*b*d**2/f**3 + 2*b**2*c*d/f**3 - 3*b**2*d*
*2*e/f**4) - sqrt(-1/(e**5*f**9))*(3*a**2*c**2*f**4 + 2*a**2*c*d*e*f**3 +
3*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f**3 + 12*a*b*c*d*e**2*f**2 - 30*a*b*
d**2*e**3*f + 3*b**2*c**2*e**2*f**2 - 30*b**2*c*d*e**3*f + 35*b**2*d**2*e*
*4)*log(-e**3*f**4*sqrt(-1/(e**5*f**9)) + x)/16 + sqrt(-1/(e**5*f**9))*(3*
a**2*c**2*f**4 + 2*a**2*c*d*e*f**3 + 3*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*
f**3 + 12*a*b*c*d*e**2*f**2 - 30*a*b*d**2*e**3*f + 3*b**2*c**2*e**2*f**2 -
 30*b**2*c*d*e**3*f + 35*b**2*d**2*e**4)*log(e**3*f**4*sqrt(-1/(e**5*f**9)
) + x)/16 + (x**3*(3*a**2*c**2*f**5 + 2*a**2*c*d*e*f**4 - 5*a**2*d**2*e**2
*f**3 + 2*a*b*c**2*e*f**4 - 20*a*b*c*d*e**2*f**3 + 18*a*b*d**2*e**3*f**2 -
 5*b**2*c**2*e**2*f**3 + 18*b**2*c*d*e**3*f**2 - 13*b**2*d**2*e**4*f) + x*
(5*a**2*c**2*e*f**4 - 2*a**2*c*d*e**2*f**3 - 3*a**2*d**2*e**3*f**2 - 2*a*b
*c**2*e**2*f**3 - 12*a*b*c*d*e**3*f**2 + 14*a*b*d**2*e**4*f - 3*b**2*c**2*
e**3*f**2 + 14*b**2*c*d*e**4*f - 11*b**2*d**2*e**5))/(8*e**4*f**4 + 16*e**
3*f**5*x**2 + 8*e**2*f**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^3}\,dx$$

$$= \frac{(35\,b^2d^2e^4 - 30\,b^2cde^3f - 30\,abd^2e^3f + 3\,b^2c^2e^2f^2 + 12\,abcde^2f^2 + 3\,a^2d^2e^2f^2 + 2\,abc^2ef^3 + 2\,a^2cdef^3 +}{8\,\sqrt{ef}e^2f^4}$$

$$- \frac{13\,b^2d^2e^4fx^3 - 18\,b^2cde^3f^2x^3 - 18\,abd^2e^3f^2x^3 + 5\,b^2c^2e^2f^3x^3 + 20\,abcde^2f^3x^3 + 5\,a^2d^2e^2f^3x^3 - 2\,abc}{}$$

$$+ \frac{b^2d^2f^6x^3 - 9\,b^2d^2ef^5x + 6\,b^2cdf^6x + 6\,abd^2f^6x}{3\,f^9}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(35*b^2*d^2*e^4 - 30*b^2*c*d*e^3*f - 30*a*b*d^2*e^3*f + 3*b^2*c^2*e^2*
f^2 + 12*a*b*c*d*e^2*f^2 + 3*a^2*d^2*e^2*f^2 + 2*a*b*c^2*e*f^3 + 2*a^2*c*d
*e*f^3 + 3*a^2*c^2*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^4) - 1/8*(1
3*b^2*d^2*e^4*f*x^3 - 18*b^2*c*d*e^3*f^2*x^3 - 18*a*b*d^2*e^3*f^2*x^3 + 5*
b^2*c^2*e^2*f^3*x^3 + 20*a*b*c*d*e^2*f^3*x^3 + 5*a^2*d^2*e^2*f^3*x^3 - 2*a
*b*c^2*e*f^4*x^3 - 2*a^2*c*d*e*f^4*x^3 - 3*a^2*c^2*f^5*x^3 + 11*b^2*d^2*e^
5*x - 14*b^2*c*d*e^4*f*x - 14*a*b*d^2*e^4*f*x + 3*b^2*c^2*e^3*f^2*x + 12*a
*b*c*d*e^3*f^2*x + 3*a^2*d^2*e^3*f^2*x + 2*a*b*c^2*e^2*f^3*x + 2*a^2*c*d*e
^2*f^3*x - 5*a^2*c^2*e*f^4*x)/((f*x^2 + e)^2*e^2*f^4) + 1/3*(b^2*d^2*f^6*x
^3 - 9*b^2*d^2*e*f^5*x + 6*b^2*c*d*f^6*x + 6*a*b*d^2*f^6*x)/f^9
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{\frac{x^3\,(3\,a^2\,c^2\,f^5+2\,a^2\,c\,d\,e\,f^4-5\,a^2\,d^2\,e^2\,f^3+2\,a\,b\,c^2\,e\,f^4-20\,a\,b\,c\,d\,e^2\,f^3+18\,a\,b\,d^2\,e^3\,f^2-5\,b^2\,c^2\,e^2\,f^3+18\,b^2\,c\,d\,e^3\,f^2-13\,b^2\,d^2\,e^4\,f)}{8\,e^2} - \frac{x\,(-5\,a^2}{}}{e^2\,f^4+2\,e\,f^5\,x^2+f^6} - x\left(\frac{3\,b^2\,d^2\,e}{f^4}-\frac{2\,b\,d\,(a\,d+b\,c)}{f^3}\right)+\frac{b^2\,d^2\,x^3}{3\,f^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(3\,a^2\,c^2\,f^4+2\,a^2\,c\,d\,e\,f^3+3\,a^2\,d^2\,e^2\,f^2+2\,a\,b\,c^2\,e\,f^3+12\,a\,b\,c\,d\,e^2\,f^2-30\,a\,b\,d^2\,e^3\,f+3\,b}{8\,e^{5/2}\,f^{9/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

output
```
((x^3*(3*a^2*c^2*f^5 - 13*b^2*d^2*e^4*f - 5*a^2*d^2*e^2*f^3 - 5*b^2*c^2*e^
2*f^3 + 2*a*b*c^2*e*f^4 + 2*a^2*c*d*e*f^4 + 18*a*b*d^2*e^3*f^2 + 18*b^2*c*
d*e^3*f^2 - 20*a*b*c*d*e^2*f^3))/(8*e^2) - (x*(11*b^2*d^2*e^4 - 5*a^2*c^2*
f^4 + 3*a^2*d^2*e^2*f^2 + 3*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 - 14*a*b*d^2
*e^3*f + 2*a^2*c*d*e*f^3 - 14*b^2*c*d*e^3*f + 12*a*b*c*d*e^2*f^2))/(8*e))/
(e^2*f^4 + f^6*x^4 + 2*e*f^5*x^2) - x*((3*b^2*d^2*e)/f^4 - (2*b*d*(a*d + b
*c))/f^3) + (b^2*d^2*x^3)/(3*f^3) + (atan((f^(1/2)*x)/e^(1/2))*(3*a^2*c^2*
f^4 + 35*b^2*d^2*e^4 + 3*a^2*d^2*e^2*f^2 + 3*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e
*f^3 - 30*a*b*d^2*e^3*f + 2*a^2*c*d*e*f^3 - 30*b^2*c*d*e^3*f + 12*a*b*c*d*
e^2*f^2))/(8*e^(5/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1195, normalized size of antiderivative = 4.56

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output

```
(9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e**2*f**4 + 18*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e*f**5*x**2 + 9*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*f**6*x**4 + 6*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**3*f**3 + 12*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**2*f**4*x**2 + 6*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e*f**5*x**4 + 9*sqrt(f)*sqrt(
e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**4*f**2 + 18*sqrt(f)*sqrt(e)*
atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**3*f**3*x**2 + 9*sqrt(f)*sqrt(e)
*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**2*f**4*x**4 + 6*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**3*f**3 + 12*sqrt(f)*sqrt(e)*at
an((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**2*f**4*x**2 + 6*sqrt(f)*sqrt(e)*at
an((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e*f**5*x**4 + 36*sqrt(f)*sqrt(e)*atan
((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**4*f**2 + 72*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**3*f**3*x**2 + 36*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**2*f**4*x**4 - 90*sqrt(f)*sqrt(e)*atan((f*x
)/(sqrt(f)*sqrt(e)))*a*b*d**2*e**5*f - 180*sqrt(f)*sqrt(e)*atan((f*x)/(sqr
t(f)*sqrt(e)))*a*b*d**2*e**4*f**2*x**2 - 90*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*a*b*d**2*e**3*f**3*x**4 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*b**2*c**2*e**4*f**2 + 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*b**2*c**2*e**3*f**3*x**2 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(s...
```

3.230 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 340

$$
\begin{aligned}
\int \frac{(a+bx^2)^2(c+dx^2)^2}{(e+fx^2)^4}\,dx &= \frac{b^2d^2x}{f^4}+\frac{(be-af)^2(de-cf)^2x}{6ef^4(e+fx^2)^3} \\
&- \frac{(be-af)(de-cf)(be(19de-7cf)-af(7de+5cf))x}{24e^2f^4(e+fx^2)^2} \\
&- \frac{(2abef(11d^2e^2-2cdef-c^2f^2)-b^2e^2(29d^2e^2-22cdef+c^2f^2)-a^2f^2(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^4(e+fx^2)} \\
&- \frac{(b^2e^2(35d^2e^2-10cdef-c^2f^2)-2abef(5d^2e^2+2cdef+c^2f^2)-a^2f^2(d^2e^2+2cdef+5c^2f^2))\arctan}{16e^{7/2}f^{9/2}}
\end{aligned}
$$

output
```
b^2*d^2*x/f^4+1/6*(-a*f+b*e)^2*(-c*f+d*e)^2*x/e/f^4/(f*x^2+e)^3-1/24*(-a*f
+b*e)*(-c*f+d*e)*(b*e*(-7*c*f+19*d*e)-a*f*(5*c*f+7*d*e))*x/e^2/f^4/(f*x^2+
e)^2-1/16*(2*a*b*e*f*(-c^2*f^2-2*c*d*e*f+11*d^2*e^2)-b^2*e^2*(c^2*f^2-22*c
*d*e*f+29*d^2*e^2)-a^2*f^2*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*x/e^3/f^4/(f*x^2
+e)-1/16*(b^2*e^2*(-c^2*f^2-10*c*d*e*f+35*d^2*e^2)-2*a*b*e*f*(c^2*f^2+2*c*
d*e*f+5*d^2*e^2)-a^2*f^2*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*arctan(f^(1/2)*x/e
^(1/2))/e^(7/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4} dx = \frac{b^2d^2x}{f^4} + \frac{(be-af)^2(de-cf)^2x}{6ef^4(e+fx^2)^3}$$
$$- \frac{(be-af)(de-cf)(be(19de-7cf)-af(7de+5cf))x}{24e^2f^4(e+fx^2)^2}$$
$$+ \frac{(b^2e^2(29d^2e^2-22cdef+c^2f^2)+2abef(-11d^2e^2+2cdef+c^2f^2)+a^2f^2(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^4(e+fx^2)}$$
$$- \frac{(b^2e^2(35d^2e^2-10cdef-c^2f^2)-2abef(5d^2e^2+2cdef+c^2f^2)-a^2f^2(d^2e^2+2cdef+5c^2f^2))\arctan}{16e^{7/2}f^{9/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
(b^2*d^2*x)/f^4 + ((b*e - a*f)^2*(d*e - c*f)^2*x)/(6*e*f^4*(e + f*x^2)^3)
- ((b*e - a*f)*(d*e - c*f)*(b*e*(19*d*e - 7*c*f) - a*f*(7*d*e + 5*c*f))*x)
/(24*e^2*f^4*(e + f*x^2)^2) + ((b^2*e^2*(29*d^2*e^2 - 22*c*d*e*f + c^2*f^2
) + 2*a*b*e*f*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a^2*f^2*(d^2*e^2 + 2*c
*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b^2*e^2*(35*d^2*e^2 -
 10*c*d*e*f - c^2*f^2) - 2*a*b*e*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - a^2
*f^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e
^(7/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.48, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {425, 401, 25, 401, 25, 298, 218, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4} dx$$
$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^2}{(fx^2+e)^3}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^2}{(fx^2+e)^4}dx}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)\left(d(5be-af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)\left(d(5be+af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be-af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(\frac{\int \frac{\left(dx^2+c\right)\left(d(5be+af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int -\frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)(de(af+5be)-cf(5af+be))}{4ef\left(e+fx^2\right)^2}}{6ef}-\frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{\frac{\int \frac{d(be(15de+cf)+af(3de+5cf))x^2+c(de(5be+af)+3cf(be+5af))}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)(de(af+5be)-cf(5af+be))}{4ef\left(e+fx^2\right)^2}}{6ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$\downarrow$ 298

$$\frac{b\left(\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{\frac{\frac{3\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{2ef}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+\cdots}{\cdots}}{6ef}\cdots\right)}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{-\frac{\int \frac{c(af(de-3cf)-be(5de+cf))-d(be(15de-cf)-3af(de+cf))x^2}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef(e+fx^2)^2}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{2e^{3/2}f^{3/2}}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(\cdots}{\cdots}}{6ef}\cdots\right)}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{-\frac{\frac{\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{f}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{2e^{3/2}f^{3/2}}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(}{}}{6ef}\right.}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx(be(15de-cf)-3af(cf+de))}{f}}{2ef}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{2e^{3/2}f^{3/2}}-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(}{}}{6ef}\right.}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
(b*(-1/4*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(
5*d*e - c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d
*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/f) + ((b*e*(15*d^2*e^2 - 6*c*
d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f
]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f)))/f - ((b*e - a*f)*(-1/
6*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^3) + (-1/4*((d*e*(5*b*e +
 a*f) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(e*f*(e + f*x^2)^2) + (-1/2*((a*
f*(3*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f - 3*c
^2*f^2))*x)/(e*f*(e + f*x^2)) + (3*(b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
+ a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e
^(3/2)*f^(3/2)))/(4*e*f))/(6*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.46

method	result
default	$\frac{b^2d^2x}{f^4}+\frac{\frac{f^2\left(5a^2c^2f^4+2a^2cde\,f^3+a^2d^2e^2f^2+2ab\,c^2e\,f^3+4abcd\,e^2f^2-22ab\,d^2e^3f+b^2c^2e^2f^2-22b^2cd\,e^3f+29b^2d^2e^4\right)x^5}{16e^3}+\frac{f\left(5a^2c^2f^4+2a^2c\right.}{}}{}$
risch	$\frac{b^2d^2x}{f^4}+\frac{\frac{f^2\left(5a^2c^2f^4+2a^2cde\,f^3+a^2d^2e^2f^2+2ab\,c^2e\,f^3+4abcd\,e^2f^2-22ab\,d^2e^3f+b^2c^2e^2f^2-22b^2cd\,e^3f+29b^2d^2e^4\right)x^5}{16e^3}+\frac{f\left(5a^2c^2f^4+2a^2c\right.}{}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
b^2*d^2*x/f^4+1/f^4*((1/16*f^2*(5*a^2*c^2*f^4+2*a^2*c*d*e*f^3+a^2*d^2*e^2*
f^2+2*a*b*c^2*e*f^3+4*a*b*c*d*e^2*f^2-22*a*b*d^2*e^3*f+b^2*c^2*e^2*f^2-22*
b^2*c*d*e^3*f+29*b^2*d^2*e^4)/e^3*x^5+1/6*f*(5*a^2*c^2*f^4+2*a^2*c*d*e*f^3
-a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-4*a*b*c*d*e^2*f^2-10*a*b*d^2*e^3*f-b^2*c^
2*e^2*f^2-10*b^2*c*d*e^3*f+17*b^2*d^2*e^4)/e^2*x^3+1/16*(11*a^2*c^2*f^4-2*
a^2*c*d*e*f^3-a^2*d^2*e^2*f^2-2*a*b*c^2*e*f^3-4*a*b*c*d*e^2*f^2-10*a*b*d^2
*e^3*f-b^2*c^2*e^2*f^2-10*b^2*c*d*e^3*f+19*b^2*d^2*e^4)/e*x)/(f*x^2+e)^3+1
/16*(5*a^2*c^2*f^4+2*a^2*c*d*e*f^3+a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3+4*a*b*c
*d*e^2*f^2+10*a*b*d^2*e^3*f+b^2*c^2*e^2*f^2+10*b^2*c*d*e^3*f-35*b^2*d^2*e^
4)/e^3/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(322) = 644$.

Time = 0.14 (sec) , antiderivative size = 1596, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(96*b^2*d^2*e^4*f^4*x^7 + 6*(77*b^2*d^2*e^5*f^3 + 5*a^2*c^2*e*f^7 -
22*(b^2*c*d + a*b*d^2)*e^4*f^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^5 +
 2*(a*b*c^2 + a^2*c*d)*e^2*f^6)*x^5 + 16*(35*b^2*d^2*e^6*f^2 + 5*a^2*c^2*e
^2*f^6 - 10*(b^2*c*d + a*b*d^2)*e^5*f^3 - (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*
e^4*f^4 + 2*(a*b*c^2 + a^2*c*d)*e^3*f^5)*x^3 + 3*(35*b^2*d^2*e^7 - 5*a^2*c
^2*e^3*f^4 - 10*(b^2*c*d + a*b*d^2)*e^6*f - (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*e^5*f^2 - 2*(a*b*c^2 + a^2*c*d)*e^4*f^3 + (35*b^2*d^2*e^4*f^3 - 5*a^2*c^
2*f^7 - 10*(b^2*c*d + a*b*d^2)*e^3*f^4 - (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e
^2*f^5 - 2*(a*b*c^2 + a^2*c*d)*e*f^6)*x^6 + 3*(35*b^2*d^2*e^5*f^2 - 5*a^2*
c^2*e*f^6 - 10*(b^2*c*d + a*b*d^2)*e^4*f^3 - (b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*e^3*f^4 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^5)*x^4 + 3*(35*b^2*d^2*e^6*f - 5*
a^2*c^2*e^2*f^5 - 10*(b^2*c*d + a*b*d^2)*e^5*f^2 - (b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*e^4*f^3 - 2*(a*b*c^2 + a^2*c*d)*e^3*f^4)*x^2)*sqrt(-e*f)*log((f*x
^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6*(35*b^2*d^2*e^7*f + 11*a^2*c^2*e
^3*f^5 - 10*(b^2*c*d + a*b*d^2)*e^6*f^2 - (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*
e^5*f^3 - 2*(a*b*c^2 + a^2*c*d)*e^4*f^4)*x)/(e^4*f^8*x^6 + 3*e^5*f^7*x^4 +
 3*e^6*f^6*x^2 + e^7*f^5), 1/48*(48*b^2*d^2*e^4*f^4*x^7 + 3*(77*b^2*d^2*e^
5*f^3 + 5*a^2*c^2*e*f^7 - 22*(b^2*c*d + a*b*d^2)*e^4*f^4 + (b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*e^3*f^5 + 2*(a*b*c^2 + a^2*c*d)*e^2*f^6)*x^5 + 8*(35*b^2*
d^2*e^6*f^2 + 5*a^2*c^2*e^2*f^6 - 10*(b^2*c*d + a*b*d^2)*e^5*f^3 - (b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4}\,dx = \frac{b^2d^2x}{f^4}$$
$$-\frac{(35\,b^2d^2e^4 - 10\,b^2cde^3f - 10\,abd^2e^3f - b^2c^2e^2f^2 - 4\,abcde^2f^2 - a^2d^2e^2f^2 - 2\,abc^2ef^3 - 2\,a^2cdef^3 - \cdots}{16\,\sqrt{ef}e^3f^4}$$
$$+\frac{87\,b^2d^2e^4f^2x^5 - 66\,b^2cde^3f^3x^5 - 66\,abd^2e^3f^3x^5 + 3\,b^2c^2e^2f^4x^5 + 12\,abcde^2f^4x^5 + 3\,a^2d^2e^2f^4x^5 + 6\,ab\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
b^2*d^2*x/f^4 - 1/16*(35*b^2*d^2*e^4 - 10*b^2*c*d*e^3*f - 10*a*b*d^2*e^3*f
 - b^2*c^2*e^2*f^2 - 4*a*b*c*d*e^2*f^2 - a^2*d^2*e^2*f^2 - 2*a*b*c^2*e*f^3
 - 2*a^2*c*d*e*f^3 - 5*a^2*c^2*f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f
^4) + 1/48*(87*b^2*d^2*e^4*f^2*x^5 - 66*b^2*c*d*e^3*f^3*x^5 - 66*a*b*d^2*e
^3*f^3*x^5 + 3*b^2*c^2*e^2*f^4*x^5 + 12*a*b*c*d*e^2*f^4*x^5 + 3*a^2*d^2*e^
2*f^4*x^5 + 6*a*b*c^2*e*f^5*x^5 + 6*a^2*c*d*e*f^5*x^5 + 15*a^2*c^2*f^6*x^5
 + 136*b^2*d^2*e^5*f*x^3 - 80*b^2*c*d*e^4*f^2*x^3 - 80*a*b*d^2*e^4*f^2*x^3
 - 8*b^2*c^2*e^3*f^3*x^3 - 32*a*b*c*d*e^3*f^3*x^3 - 8*a^2*d^2*e^3*f^3*x^3
+ 16*a*b*c^2*e^2*f^4*x^3 + 16*a^2*c*d*e^2*f^4*x^3 + 40*a^2*c^2*e*f^5*x^3 +
 57*b^2*d^2*e^6*x - 30*b^2*c*d*e^5*f*x - 30*a*b*d^2*e^5*f*x - 3*b^2*c^2*e^
4*f^2*x - 12*a*b*c*d*e^4*f^2*x - 3*a^2*d^2*e^4*f^2*x - 6*a*b*c^2*e^3*f^3*x
 - 6*a^2*c*d*e^3*f^3*x + 33*a^2*c^2*e^2*f^4*x)/((f*x^2 + e)^3*e^3*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4} dx = \frac{b^2 d^2 x}{f^4}$$
$$- \frac{\frac{x^3 (-5 a^2 c^2 f^5 - 2 a^2 c d e f^4 + a^2 d^2 e^2 f^3 - 2 a b c^2 e f^4 + 4 a b c d e^2 f^3 + 10 a b d^2 e^3 f^2 + b^2 c^2 e^2 f^3 + 10 b^2 c d e^3 f^2 - 17 b^2 d^2 e^4 f)}{6 e^2} - \frac{x^5 (5 a^2}{}}{}$$
$$+ \frac{\operatorname{atan}\left(\frac{\sqrt{f}\, x}{\sqrt{e}}\right) (5 a^2 c^2 f^4 + 2 a^2 c d e f^3 + a^2 d^2 e^2 f^2 + 2 a b c^2 e f^3 + 4 a b c d e^2 f^2 + 10 a b d^2 e^3 f + b^2 c^2}{16 e^{7/2} f^{9/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

output
```
(b^2*d^2*x)/f^4 - ((x^3*(a^2*d^2*e^2*f^3 - 17*b^2*d^2*e^4*f - 5*a^2*c^2*f^
5 + b^2*c^2*e^2*f^3 - 2*a*b*c^2*e*f^4 - 2*a^2*c*d*e*f^4 + 10*a*b*d^2*e^3*f
^2 + 10*b^2*c*d*e^3*f^2 + 4*a*b*c*d*e^2*f^3))/(6*e^2) - (x^5*(5*a^2*c^2*f^
6 + a^2*d^2*e^2*f^4 + b^2*c^2*e^2*f^4 + 29*b^2*d^2*e^4*f^2 + 2*a*b*c^2*e*f
^5 + 2*a^2*c*d*e*f^5 - 22*a*b*d^2*e^3*f^3 - 22*b^2*c*d*e^3*f^3 + 4*a*b*c*d
*e^2*f^4))/(16*e^3) + (x*(a^2*d^2*e^2*f^2 - 19*b^2*d^2*e^4 - 11*a^2*c^2*f^
4 + b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 + 10*a*b*d^2*e^3*f + 2*a^2*c*d*e*f^3
 + 10*b^2*c*d*e^3*f + 4*a*b*c*d*e^2*f^2))/(16*e))/(e^3*f^4 + f^7*x^6 + 3*e
*f^6*x^4 + 3*e^2*f^5*x^2) + (atan((f^(1/2)*x)/e^(1/2))*(5*a^2*c^2*f^4 - 35
*b^2*d^2*e^4 + a^2*d^2*e^2*f^2 + b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 + 10*a*
b*d^2*e^3*f + 2*a^2*c*d*e*f^3 + 10*b^2*c*d*e^3*f + 4*a*b*c*d*e^2*f^2))/(16
*e^(7/2)*f^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1595, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e**3*f**4 + 45
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e**2*f**5*x**2 +
45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*e*f**6*x**4 + 1
5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*f**7*x**6 + 6*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**4*f**3 + 18*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**3*f**4*x**2 + 18*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e**2*f**5*x**4 + 6*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*e*f**6*x**6 + 3*sqrt(f)*
sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**5*f**2 + 9*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**4*f**3*x**2 + 9*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**3*f**4*x**4 + 3*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**2*e**2*f**5*x**6 + 6*sqrt(f)*s
qrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**4*f**3 + 18*sqrt(f)*sqrt(
e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**3*f**4*x**2 + 18*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e**2*f**5*x**4 + 6*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*e*f**6*x**6 + 12*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**5*f**2 + 36*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**4*f**3*x**2 + 36*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**3*f**4*x**4 + 12*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d*e**2*f**5*x**6 + 30*sqrt(f)*sqrt(e)*...
```

3.231 $\int (a+bx^2)^2 (c+dx^2)^3 (e+fx^2)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 493

$$
\begin{aligned}
&\int (a+bx^2)^2 (c+dx^2)^3 (e+fx^2)^3 \, dx \\
&= a^2c^3e^3x + \frac{1}{3}ac^2e^2(2bce+3a(de+cf))x^3 \\
&\quad + \frac{1}{5}ce\left(b^2c^2e^2+6abce(de+cf)+3a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(3b^2c^2e^2(de+cf)\right. \\
&\qquad \left. + 6abce\left(d^2e^2+3cdef+c^2f^2\right)+a^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7 \\
&\quad + \frac{1}{9}\left(3b^2ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2df\left(d^2e^2+3cdef+c^2f^2\right)\right. \\
&\qquad \left. + 2ab\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9 + \frac{1}{11}\left(3a^2d^2f^2(de+cf)\right. \\
&\qquad \left. + 6abdf\left(d^2e^2+3cdef+c^2f^2\right)+b^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{11} \\
&\quad + \frac{1}{13}df\left(a^2d^2f^2+6abdf(de+cf)+3b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{13} \\
&\quad + \frac{1}{15}bd^2f^2(2adf+3b(de+cf))x^{15} + \frac{1}{17}b^2d^3f^3x^{17}
\end{aligned}
$$

output
```
a^2*c^3*e^3*x+1/3*a*c^2*e^2*(2*b*c*e+3*a*(c*f+d*e))*x^3+1/5*c*e*(b^2*c^2*e
^2+6*a*b*c*e*(c*f+d*e)+3*a^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(3*b^2*c
^2*e^2*(c*f+d*e)+6*a*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)+a^2*(c^3*f^3+9*c^2*
d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/9*(3*b^2*c*e*(c^2*f^2+3*c*d*e*f+d^2*
e^2)+3*a^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)+2*a*b*(c^3*f^3+9*c^2*d*e*f^2+9*
c*d^2*e^2*f+d^3*e^3))*x^9+1/11*(3*a^2*d^2*f^2*(c*f+d*e)+6*a*b*d*f*(c^2*f^2
+3*c*d*e*f+d^2*e^2)+b^2*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^1
1+1/13*d*f*(a^2*d^2*f^2+6*a*b*d*f*(c*f+d*e)+3*b^2*(c^2*f^2+3*c*d*e*f+d^2*e
^2))*x^13+1/15*b*d^2*f^2*(2*a*d*f+3*b*(c*f+d*e))*x^15+1/17*b^2*d^3*f^3*x^1
7
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx$$
$$= a^2c^3e^3x+\frac{1}{3}ac^2e^2(2bce+3a(de+cf))x^3 + \frac{1}{5}ce\left(b^2c^2e^2+6abce(de+cf)+3a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(3b^2c^2e^2(de+cf) + 6abce\left(d^2e^2+3cdef+c^2f^2\right)+a^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7 + \frac{1}{9}\left(3b^2ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2df\left(d^2e^2+3cdef+c^2f^2\right) + 2ab\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9+\frac{1}{11}\left(3a^2d^2f^2(de+cf) + 6abdf\left(d^2e^2+3cdef+c^2f^2\right)+b^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{11} + \frac{1}{13}df\left(a^2d^2f^2+6abdf(de+cf)+3b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{13} + \frac{1}{15}bd^2f^2(2adf+3b(de+cf))x^{15}+\frac{1}{17}b^2d^3f^3x^{17}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a^2*c^3*e^3*x + (a*c^2*e^2*(2*b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (c*e*(b^2*
c^2*e^2 + 6*a*b*c*e*(d*e + c*f) + 3*a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x
^5)/5 + ((3*b^2*c^2*e^2*(d*e + c*f) + 6*a*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2
*f^2) + a^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 +
((3*b^2*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*d*f*(d^2*e^2 + 3*c*d*e
*f + c^2*f^2) + 2*a*b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^9)/9 + ((3*a^2*d^2*f^2*(d*e + c*f) + 6*a*b*d*f*(d^2*e^2 + 3*c*d*e*f + c
^2*f^2) + b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^11)/1
1 + (d*f*(a^2*d^2*f^2 + 6*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2))*x^13)/13 + (b*d^2*f^2*(2*a*d*f + 3*b*(d*e + c*f))*x^15)/15 +
(b^2*d^3*f^3*x^17)/17
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(dfx^{12}\left(a^2d^2f^2+6abdf(cf+de)+3b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)+cex^4\left(3a^2\left(c^2f^2+3cdef+d^2e^2\right)+6abce(cf\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{13}dfx^{13}\left(a^2d^2f^2+6abdf(cf+de)+3b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{5}cex^5\left(3a^2\left(c^2f^2+3cdef+d^2e^2\right)+6abce(cf+de)+b^2c^2e^2\right)+\frac{1}{11}x^{11}\left(3a^2d^2f^2(cf+de)+6abdf\left(c^2f^2+3cdef+d^2e^2\right)+b^2\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{9}x^9\left(3a^2df\left(c^2f^2+3cdef+d^2e^2\right)+2ab\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3b^2ce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(a^2\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+6abce\left(c^2f^2+3cdef+d^2e^2\right)+3b^2c^2e^2(cf+de)\right)+a^2c^3e^3x+\frac{1}{3}ac^2e^2x^3(3a(cf+de)+2bce)+\frac{1}{15}bd^2f^2x^{15}(2adf+3b(cf+de))+\frac{1}{17}b^2d^3f^3x^{17}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a^2*c^3*e^3*x + (a*c^2*e^2*(2*b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (c*e*(b^2*
c^2*e^2 + 6*a*b*c*e*(d*e + c*f) + 3*a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x
^5)/5 + ((3*b^2*c^2*e^2*(d*e + c*f) + 6*a*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2
*f^2) + a^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 +
((3*b^2*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*d*f*(d^2*e^2 + 3*c*d*e
*f + c^2*f^2) + 2*a*b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^9)/9 + ((3*a^2*d^2*f^2*(d*e + c*f) + 6*a*b*d*f*(d^2*e^2 + 3*c*d*e*f + c
^2*f^2) + b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^11)/1
1 + (d*f*(a^2*d^2*f^2 + 6*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2))*x^13)/13 + (b*d^2*f^2*(2*a*d*f + 3*b*(d*e + c*f))*x^15)/15 +
(b^2*d^3*f^3*x^17)/17
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^2d^3f^3x^{17}}{17}+\frac{\left(\left(2a\,d^3b+3b^2c\,d^2\right)f^3+3b^2d^3e\,f^2\right)x^{15}}{15}+\frac{\left(\left(a^2d^3+6ac\,d^2b+3b^2c^2d\right)f^3+3\left(2a\,d^3b+3b^2c\,d^2\right)e\,f^2+3b^2d^3e^2f\right)x^{1}}{13}$
norman	$a^2c^3e^3x+\left(a^2c^3e^2f+a^2c^2d\,e^3+\frac{2}{3}ab\,c^3e^3\right)x^3+\left(\frac{3}{5}a^2c^3e\,f^2+\frac{9}{5}a^2c^2d\,e^2f+\frac{3}{5}a^2c\,d^2e^3+\frac{6}{5}ab\right.$
gosper	$\frac{9}{5}x^5a^2c^2d\,e^2f+\frac{6}{5}x^5ab\,c^3e^2f+\frac{6}{5}x^5ab\,c^2d\,e^3+\frac{9}{7}x^7a^2c^2de\,f^2+\frac{9}{7}x^7a^2c\,d^2e^2f+\frac{1}{3}x^9b^2c^3e\,f^2+$
risch	$\frac{9}{5}x^5a^2c^2d\,e^2f+\frac{6}{5}x^5ab\,c^3e^2f+\frac{6}{5}x^5ab\,c^2d\,e^3+\frac{9}{7}x^7a^2c^2de\,f^2+\frac{9}{7}x^7a^2c\,d^2e^2f+\frac{1}{3}x^9b^2c^3e\,f^2+$
parallelrisch	$\frac{9}{5}x^5a^2c^2d\,e^2f+\frac{6}{5}x^5ab\,c^3e^2f+\frac{6}{5}x^5ab\,c^2d\,e^3+\frac{9}{7}x^7a^2c^2de\,f^2+\frac{9}{7}x^7a^2c\,d^2e^2f+\frac{1}{3}x^9b^2c^3e\,f^2+$
orering	$\frac{x\left(45045b^2d^3f^3x^{16}+102102ab\,d^3f^3x^{14}+153153b^2c\,d^2f^3x^{14}+153153b^2d^3e\,f^2x^{14}+58905a^2d^3f^3x^{12}+353430abc\,d^2f^3x^{12}+35\right.}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/17*b^2*d^3*f^3*x^17+1/15*((2*a*b*d^3+3*b^2*c*d^2)*f^3+3*b^2*d^3*e*f^2)*x
^15+1/13*((a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*d)*f^3+3*(2*a*b*d^3+3*b^2*c*d^2)*
e*f^2+3*b^2*d^3*e^2*f)*x^13+1/11*((3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*f^3+3*
(a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*d)*e*f^2+3*(2*a*b*d^3+3*b^2*c*d^2)*e^2*f+b^
2*d^3*e^3)*x^11+1/9*((3*a^2*c^2*d+2*a*b*c^3)*f^3+3*(3*a^2*c*d^2+6*a*b*c^2*
d+b^2*c^3)*e*f^2+3*(a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*d)*e^2*f+(2*a*b*d^3+3*b^
2*c*d^2)*e^3)*x^9+1/7*(a^2*c^3*f^3+3*(3*a^2*c^2*d+2*a*b*c^3)*e*f^2+3*(3*a^
2*c*d^2+6*a*b*c^2*d+b^2*c^3)*e^2*f+(a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*d)*e^3)*
x^7+1/5*(3*a^2*c^3*e*f^2+3*(3*a^2*c^2*d+2*a*b*c^3)*e^2*f+(3*a^2*c*d^2+6*a*
b*c^2*d+b^2*c^3)*e^3)*x^5+1/3*(3*a^2*c^3*e^2*f+(3*a^2*c^2*d+2*a*b*c^3)*e^3
)*x^3+a^2*c^3*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^2 (c+dx^2)^3 (e+fx^2)^3 \, dx$$
$$= \frac{1}{17} b^2 d^3 f^3 x^{17} + \frac{1}{15} \left(3 b^2 d^3 e f^2 + \left(3 b^2 c d^2 + 2 abd^3\right) f^3\right) x^{15}$$
$$+ \frac{1}{13} \left(3 b^2 d^3 e^2 f + 3 \left(3 b^2 c d^2 + 2 abd^3\right) e f^2 + \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) f^3\right) x^{13}$$
$$+ \frac{1}{11} \left(b^2 d^3 e^3 + 3 \left(3 b^2 c d^2 + 2 abd^3\right) e^2 f + 3 \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e f^2 + \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) f^3$$
$$+ \frac{1}{9} \left(\left(3 b^2 c d^2 + 2 abd^3\right) e^3 + 3 \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e^2 f + 3 \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e f^2 + \left(2 abc^3\right.\right.$$
$$+ a^2 c^3 e^3 x$$
$$+ \frac{1}{7} \left(a^2 c^3 f^3 + \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e^3 + 3 \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e^2 f + 3 \left(2 abc^3 + 3 a^2 c^2 d\right) e f^2\right)$$
$$+ \frac{1}{5} \left(3 a^2 c^3 e f^2 + \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e^3 + 3 \left(2 abc^3 + 3 a^2 c^2 d\right) e^2 f\right) x^5$$
$$+ \frac{1}{3} \left(3 a^2 c^3 e^2 f + \left(2 abc^3 + 3 a^2 c^2 d\right) e^3\right) x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/17*b^2*d^3*f^3*x^17 + 1/15*(3*b^2*d^3*e*f^2 + (3*b^2*c*d^2 + 2*a*b*d^3)*
f^3)*x^15 + 1/13*(3*b^2*d^3*e^2*f + 3*(3*b^2*c*d^2 + 2*a*b*d^3)*e*f^2 + (3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*f^3)*x^13 + 1/11*(b^2*d^3*e^3 + 3*(3*b
^2*c*d^2 + 2*a*b*d^3)*e^2*f + 3*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e*f^
2 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*f^3)*x^11 + 1/9*((3*b^2*c*d^2 +
2*a*b*d^3)*e^3 + 3*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^2*f + 3*(b^2*c^
3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e*f^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*f^3)*x^9
+ a^2*c^3*e^3*x + 1/7*(a^2*c^3*f^3 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)
*e^3 + 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f + 3*(2*a*b*c^3 + 3*a^
2*c^2*d)*e*f^2)*x^7 + 1/5*(3*a^2*c^3*e*f^2 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^
2*c*d^2)*e^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f)*x^5 + 1/3*(3*a^2*c^3*e^2
*f + (2*a*b*c^3 + 3*a^2*c^2*d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.46

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \,dx = a^2c^3e^3x + \frac{b^2d^3f^3x^{17}}{17} + x^{15} \cdot \left(\frac{2abd^3f^3}{15} + \frac{b^2cd^2f^3}{5} + \frac{b^2d^3ef^2}{5}\right) + x^{13}\left(\frac{a^2d^3f^3}{13} + \frac{6abcd^2f^3}{13} + \frac{6abd^3ef^2}{13} + \frac{3b^2c^2df^3}{13} + \frac{9b^2cd^2ef^2}{13} + \frac{3b^2d^3e^2f}{13}\right) + x^{11} \cdot \left(\frac{3a^2cd^2f^3}{11} + \frac{3a^2d^3ef^2}{11} + \frac{6abc^2df^3}{11} + \frac{18abcd^2ef^2}{11} + \frac{6abd^3e^2f}{11} + \frac{b^2c^3f^3}{11} + \frac{9b^2c^2def^2}{11} + \frac{9b^2cd^2e^2f}{11} + \frac{b^2d^3e^3}{11}\right) + x^9\left(\frac{a^2c^2df^3}{3} + a^2cd^2ef^2 + \frac{a^2d^3e^2f}{3} + \frac{2abc^3f^3}{9} + 2abc^2def^2 + 2abcd^2e^2f + \frac{2abd^3e^3}{9} + \frac{b^2c^3ef^2}{3} + b^2c^2de^2f + \frac{b^2cd^2e^3}{3}\right) + x^7\left(\frac{a^2c^3f^3}{7} + \frac{9a^2c^2def^2}{7} + \frac{9a^2cd^2e^2f}{7} + \frac{a^2d^3e^3}{7} + \frac{6abc^3ef^2}{7} + \frac{18abc^2de^2f}{7} + \frac{6abcd^2e^3}{7} + \frac{3b^2c^3e^2f}{7} + \frac{3b^2c^2de^3}{7}\right) + x^5 \cdot \left(\frac{3a^2c^3ef^2}{5} + \frac{9a^2c^2de^2f}{5} + \frac{3a^2cd^2e^3}{5} + \frac{6abc^3e^2f}{5} + \frac{6abc^2de^3}{5} + \frac{b^2c^3e^3}{5}\right) + x^3\left(a^2c^3e^2f + a^2c^2de^3 + \frac{2abc^3e^3}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

output
```
a**2*c**3*e**3*x + b**2*d**3*f**3*x**17/17 + x**15*(2*a*b*d**3*f**3/15 + b
**2*c*d**2*f**3/5 + b**2*d**3*e*f**2/5) + x**13*(a**2*d**3*f**3/13 + 6*a*b
*c*d**2*f**3/13 + 6*a*b*d**3*e*f**2/13 + 3*b**2*c**2*d*f**3/13 + 9*b**2*c*
d**2*e*f**2/13 + 3*b**2*d**3*e**2*f/13) + x**11*(3*a**2*c*d**2*f**3/11 + 3
*a**2*d**3*e*f**2/11 + 6*a*b*c**2*d*f**3/11 + 18*a*b*c*d**2*e*f**2/11 + 6*
a*b*d**3*e**2*f/11 + b**2*c**3*f**3/11 + 9*b**2*c**2*d*e*f**2/11 + 9*b**2*
c*d**2*e**2*f/11 + b**2*d**3*e**3/11) + x**9*(a**2*c**2*d*f**3/3 + a**2*c*
d**2*e*f**2 + a**2*d**3*e**2*f/3 + 2*a*b*c**3*f**3/9 + 2*a*b*c**2*d*e*f**2
 + 2*a*b*c*d**2*e**2*f + 2*a*b*d**3*e**3/9 + b**2*c**3*e*f**2/3 + b**2*c**
2*d*e**2*f + b**2*c*d**2*e**3/3) + x**7*(a**2*c**3*f**3/7 + 9*a**2*c**2*d*
e*f**2/7 + 9*a**2*c*d**2*e**2*f/7 + a**2*d**3*e**3/7 + 6*a*b*c**3*e*f**2/7
 + 18*a*b*c**2*d*e**2*f/7 + 6*a*b*c*d**2*e**3/7 + 3*b**2*c**3*e**2*f/7 + 3
*b**2*c**2*d*e**3/7) + x**5*(3*a**2*c**3*e*f**2/5 + 9*a**2*c**2*d*e**2*f/5
 + 3*a**2*c*d**2*e**3/5 + 6*a*b*c**3*e**2*f/5 + 6*a*b*c**2*d*e**3/5 + b**2
*c**3*e**3/5) + x**3*(a**2*c**3*e**2*f + a**2*c**2*d*e**3 + 2*a*b*c**3*e**
3/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \, dx$$

$$= \frac{1}{17} b^2 d^3 f^3 x^{17} + \frac{1}{15} \left(3 b^2 d^3 e f^2 + \left(3 b^2 c d^2 + 2 abd^3\right) f^3\right) x^{15}$$

$$+ \frac{1}{13} \left(3 b^2 d^3 e^2 f + 3 \left(3 b^2 c d^2 + 2 abd^3\right) e f^2 + \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) f^3\right) x^{13}$$

$$+ \frac{1}{11} \left(b^2 d^3 e^3 + 3 \left(3 b^2 c d^2 + 2 abd^3\right) e^2 f + 3 \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e f^2 + \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) f^3$$

$$+ \frac{1}{9} \left(\left(3 b^2 c d^2 + 2 abd^3\right) e^3 + 3 \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e^2 f + 3 \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e f^2 + \left(2 abc^3\right.$$

$$+ a^2 c^3 e^3 x$$

$$+ \frac{1}{7} \left(a^2 c^3 f^3 + \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) e^3 + 3 \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e^2 f + 3 \left(2 abc^3 + 3 a^2 c^2 d\right) e f^2\right)$$

$$+ \frac{1}{5} \left(3 a^2 c^3 e f^2 + \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) e^3 + 3 \left(2 abc^3 + 3 a^2 c^2 d\right) e^2 f\right) x^5$$

$$+ \frac{1}{3} \left(3 a^2 c^3 e^2 f + \left(2 abc^3 + 3 a^2 c^2 d\right) e^3\right) x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/17*b^2*d^3*f^3*x^17 + 1/15*(3*b^2*d^3*e*f^2 + (3*b^2*c*d^2 + 2*a*b*d^3)*
f^3)*x^15 + 1/13*(3*b^2*d^3*e^2*f + 3*(3*b^2*c*d^2 + 2*a*b*d^3)*e*f^2 + (3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*f^3)*x^13 + 1/11*(b^2*d^3*e^3 + 3*(3*b
^2*c*d^2 + 2*a*b*d^3)*e^2*f + 3*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e*f^
2 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*f^3)*x^11 + 1/9*((3*b^2*c*d^2 +
2*a*b*d^3)*e^3 + 3*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^2*f + 3*(b^2*c^
3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e*f^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*f^3)*x^9
+ a^2*c^3*e^3*x + 1/7*(a^2*c^3*f^3 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)
*e^3 + 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f + 3*(2*a*b*c^3 + 3*a^
2*c^2*d)*e*f^2)*x^7 + 1/5*(3*a^2*c^3*e*f^2 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^
2*c*d^2)*e^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f)*x^5 + 1/3*(3*a^2*c^3*e^2
*f + (2*a*b*c^3 + 3*a^2*c^2*d)*e^3)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.42

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \,dx = \frac{1}{17}\,b^2d^3f^3x^{17} + \frac{1}{5}\,b^2d^3ef^2x^{15} + \frac{1}{5}\,b^2cd^2f^3x^{15} + \frac{2}{15}\,abd^3f^3x^{15} + \frac{3}{13}\,b^2d^3e^2fx^{13} + \frac{9}{13}\,b^2cd^2ef^2x^{13} + \frac{6}{13}\,abd^3ef^2x^{13} + \frac{3}{13}\,b^2c^2df^3x^{13} + \frac{6}{13}\,abcd^2f^3x^{13} + \frac{1}{13}\,a^2d^3f^3x^{13} + \frac{1}{11}\,b^2d^3e^3x^{11} + \frac{9}{11}\,b^2cd^2e^2fx^{11} + \frac{6}{11}\,abd^3e^2fx^{11} + \frac{9}{11}\,b^2c^2def^2x^{11} + \frac{18}{11}\,abcd^2ef^2x^{11} + \frac{3}{11}\,a^2d^3ef^2x^{11} + \frac{1}{11}\,b^2c^3f^3x^{11} + \frac{6}{11}\,abc^2df^3x^{11} + \frac{3}{11}\,a^2cd^2f^3x^{11} + \frac{1}{3}\,b^2cd^2e^3x^9 + \frac{2}{9}\,abd^3e^3x^9 + b^2c^2de^2fx^9 + 2\,abcd^2e^2fx^9 + \frac{1}{3}\,a^2d^3e^2fx^9 + \frac{1}{3}\,b^2c^3ef^2x^9 + 2\,abc^2def^2x^9 + a^2cd^2ef^2x^9 + \frac{2}{9}\,abc^3f^3x^9 + \frac{1}{3}\,a^2c^2df^3x^9 + \frac{3}{7}\,b^2c^2de^3x^7 + \frac{6}{7}\,abcd^2e^3x^7 + \frac{1}{7}\,a^2d^3e^3x^7 + \frac{3}{7}\,b^2c^3e^2fx^7 + \frac{18}{7}\,abc^2de^2fx^7 + \frac{9}{7}\,a^2cd^2e^2fx^7 + \frac{6}{7}\,abc^3ef^2x^7 + \frac{9}{7}\,a^2c^2def^2x^7 + \frac{1}{7}\,a^2c^3f^3x^7 + \frac{1}{5}\,b^2c^3e^3x^5 + \frac{6}{5}\,abc^2de^3x^5 + \frac{3}{5}\,a^2cd^2e^3x^5 + \frac{6}{5}\,abc^3e^2fx^5 + \frac{9}{5}\,a^2c^2de^2fx^5 + \frac{3}{5}\,a^2c^3ef^2x^5 + \frac{2}{3}\,abc^3e^3x^3 + a^2c^2de^3x^3 + a^2c^3e^2fx^3 + a^2c^3e^3x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

output

```
1/17*b^2*d^3*f^3*x^17 + 1/5*b^2*d^3*e*f^2*x^15 + 1/5*b^2*c*d^2*f^3*x^15 +
2/15*a*b*d^3*f^3*x^15 + 3/13*b^2*d^3*e^2*f*x^13 + 9/13*b^2*c*d^2*e*f^2*x^1
3 + 6/13*a*b*d^3*e*f^2*x^13 + 3/13*b^2*c^2*d*f^3*x^13 + 6/13*a*b*c*d^2*f^3
*x^13 + 1/13*a^2*d^3*f^3*x^13 + 1/11*b^2*d^3*e^3*x^11 + 9/11*b^2*c*d^2*e^2
*f*x^11 + 6/11*a*b*d^3*e^2*f*x^11 + 9/11*b^2*c^2*d*e*f^2*x^11 + 18/11*a*b*
c*d^2*e*f^2*x^11 + 3/11*a^2*d^3*e*f^2*x^11 + 1/11*b^2*c^3*f^3*x^11 + 6/11*
a*b*c^2*d*f^3*x^11 + 3/11*a^2*c*d^2*f^3*x^11 + 1/3*b^2*c*d^2*e^3*x^9 + 2/9
*a*b*d^3*e^3*x^9 + b^2*c^2*d*e^2*f*x^9 + 2*a*b*c*d^2*e^2*f*x^9 + 1/3*a^2*d
^3*e^2*f*x^9 + 1/3*b^2*c^3*e*f^2*x^9 + 2*a*b*c^2*d*e*f^2*x^9 + a^2*c*d^2*e
*f^2*x^9 + 2/9*a*b*c^3*f^3*x^9 + 1/3*a^2*c^2*d*f^3*x^9 + 3/7*b^2*c^2*d*e^3
*x^7 + 6/7*a*b*c*d^2*e^3*x^7 + 1/7*a^2*d^3*e^3*x^7 + 3/7*b^2*c^3*e^2*f*x^7
 + 18/7*a*b*c^2*d*e^2*f*x^7 + 9/7*a^2*c*d^2*e^2*f*x^7 + 6/7*a*b*c^3*e*f^2*
x^7 + 9/7*a^2*c^2*d*e*f^2*x^7 + 1/7*a^2*c^3*f^3*x^7 + 1/5*b^2*c^3*e^3*x^5
+ 6/5*a*b*c^2*d*e^3*x^5 + 3/5*a^2*c*d^2*e^3*x^5 + 6/5*a*b*c^3*e^2*f*x^5 +
9/5*a^2*c^2*d*e^2*f*x^5 + 3/5*a^2*c^3*e*f^2*x^5 + 2/3*a*b*c^3*e^3*x^3 + a^
2*c^2*d*e^3*x^3 + a^2*c^3*e^2*f*x^3 + a^2*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.17

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \, dx = x^7 \left(\frac{a^2 c^3 f^3}{7}+\frac{9 a^2 c^2 d e f^2}{7}+\frac{9 a^2 c d^2 e^2 f}{7}+\frac{a^2 d^3 e^3}{7}+\frac{6 a b c^3 e f^2}{7}+\frac{18 a b c^2 d e^2 f}{7}+\frac{6 a b c d^2 e^3}{7}+\frac{3 b^2 c^3 e^2 f}{7}+\frac{3 b^2 c^2 d e^3}{7}\right)+x^{11} \left(\frac{3 a^2 c d^2 f^3}{11}+\frac{3 a^2 d^3 e f^2}{11}+\frac{6 a b c^2 d f^3}{11}+\frac{18 a b c d^2 e f^2}{11}+\frac{6 a b d^3 e^2 f}{11}+\frac{b^2 c^3 f^3}{11}+\frac{9 b^2 c^2 d e f^2}{11}+\frac{9 b^2 c d^2 e^2 f}{11}+\frac{b^2 d^3 e^3}{11}\right)+x^5 \left(\frac{3 a^2 c^3 e f^2}{5}+\frac{9 a^2 c^2 d e^2 f}{5}+\frac{3 a^2 c d^2 e^3}{5}+\frac{6 a b c^3 e^2 f}{5}+\frac{6 a b c^2 d e^3}{5}+\frac{b^2 c^3 e^3}{5}\right)+x^9 \left(\frac{a^2 c^2 d f^3}{3}+a^2 c d^2 e f^2+\frac{a^2 d^3 e^2 f}{3}+\frac{2 a b c^3 f^3}{9}+2 a b c^2 d e f^2+2 a b c d^2 e^2 f+\frac{2 a b d^3 e^3}{9}+\frac{b^2 c^3 e f^2}{3}+b^2 c^2 d e^2 f+\frac{b^2 c d^2 e^3}{3}\right)+x^{13} \left(\frac{a^2 d^3 f^3}{13}+\frac{6 a b c d^2 f^3}{13}+\frac{6 a b d^3 e f^2}{13}+\frac{3 b^2 c^2 d f^3}{13}+\frac{9 b^2 c d^2 e f^2}{13}+\frac{3 b^2 d^3 e^2 f}{13}\right)+a^2 c^3 e^3 x+\frac{b^2 d^3 f^3 x^{17}}{17}+\frac{a c^2 e^2 x^3 (3 a c f+3 a d e+2 b c e)}{3}+\frac{b d^2 f^2 x^{15} (2 a d f+3 b c f+3 b d e)}{15}$$

input

```
int((a + b*x^2)^2*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

output
```
x^7*((a^2*c^3*f^3)/7 + (a^2*d^3*e^3)/7 + (3*b^2*c^2*d*e^3)/7 + (3*b^2*c^3*
e^2*f)/7 + (6*a*b*c*d^2*e^3)/7 + (6*a*b*c^3*e*f^2)/7 + (9*a^2*c*d^2*e^2*f)
/7 + (9*a^2*c^2*d*e*f^2)/7 + (18*a*b*c^2*d*e^2*f)/7) + x^11*((b^2*c^3*f^3)
/11 + (b^2*d^3*e^3)/11 + (3*a^2*c*d^2*f^3)/11 + (3*a^2*d^3*e*f^2)/11 + (6*
a*b*c^2*d*f^3)/11 + (6*a*b*d^3*e^2*f)/11 + (9*b^2*c*d^2*e^2*f)/11 + (9*b^2
*c^2*d*e*f^2)/11 + (18*a*b*c*d^2*e*f^2)/11) + x^5*((b^2*c^3*e^3)/5 + (3*a^
2*c*d^2*e^3)/5 + (3*a^2*c^3*e*f^2)/5 + (6*a*b*c^2*d*e^3)/5 + (6*a*b*c^3*e^
2*f)/5 + (9*a^2*c^2*d*e^2*f)/5) + x^9*((a^2*c^2*d*f^3)/3 + (b^2*c*d^2*e^3)
/3 + (a^2*d^3*e^2*f)/3 + (b^2*c^3*e*f^2)/3 + (2*a*b*c^3*f^3)/9 + (2*a*b*d^
3*e^3)/9 + a^2*c*d^2*e*f^2 + b^2*c^2*d*e^2*f + 2*a*b*c*d^2*e^2*f + 2*a*b*c
^2*d*e*f^2) + x^13*((a^2*d^3*f^3)/13 + (3*b^2*c^2*d*f^3)/13 + (3*b^2*d^3*e
^2*f)/13 + (6*a*b*c*d^2*f^3)/13 + (6*a*b*d^3*e*f^2)/13 + (9*b^2*c*d^2*e*f^
2)/13) + a^2*c^3*e^3*x + (b^2*d^3*f^3*x^17)/17 + (a*c^2*e^2*x^3*(3*a*c*f +
 3*a*d*e + 2*b*c*e))/3 + (b*d^2*f^2*x^15*(2*a*d*f + 3*b*c*f + 3*b*d*e))/15
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.43

$$\int (a+bx^2)^2 (c+dx^2)^3 (e+fx^2)^3 \, dx$$
$$= \frac{x(45045b^2d^3f^3x^{16} + 102102ab\,d^3f^3x^{14} + 153153b^2c\,d^2f^3x^{14} + 153153b^2d^3e\,f^2x^{14} + 58905a^2d^3f^3x^{12} + 35}{}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3*(f*x^2+e)^3,x)
```

output
```
(x*(765765*a**2*c**3*e**3 + 765765*a**2*c**3*e**2*f*x**2 + 459459*a**2*c**
3*e*f**2*x**4 + 109395*a**2*c**3*f**3*x**6 + 765765*a**2*c**2*d*e**3*x**2
+ 1378377*a**2*c**2*d*e**2*f*x**4 + 984555*a**2*c**2*d*e*f**2*x**6 + 25525
5*a**2*c**2*d*f**3*x**8 + 459459*a**2*c*d**2*e**3*x**4 + 984555*a**2*c*d**
2*e**2*f*x**6 + 765765*a**2*c*d**2*e*f**2*x**8 + 208845*a**2*c*d**2*f**3*x
**10 + 109395*a**2*d**3*e**3*x**6 + 255255*a**2*d**3*e**2*f*x**8 + 208845*
a**2*d**3*e*f**2*x**10 + 58905*a**2*d**3*f**3*x**12 + 510510*a*b*c**3*e**3
*x**2 + 918918*a*b*c**3*e**2*f*x**4 + 656370*a*b*c**3*e*f**2*x**6 + 170170
*a*b*c**3*f**3*x**8 + 918918*a*b*c**2*d*e**3*x**4 + 1969110*a*b*c**2*d*e**
2*f*x**6 + 1531530*a*b*c**2*d*e*f**2*x**8 + 417690*a*b*c**2*d*f**3*x**10 +
 656370*a*b*c*d**2*e**3*x**6 + 1531530*a*b*c*d**2*e**2*f*x**8 + 1253070*a*
b*c*d**2*e*f**2*x**10 + 353430*a*b*c*d**2*f**3*x**12 + 170170*a*b*d**3*e**
3*x**8 + 417690*a*b*d**3*e**2*f*x**10 + 353430*a*b*d**3*e*f**2*x**12 + 102
102*a*b*d**3*f**3*x**14 + 153153*b**2*c**3*e**3*x**4 + 328185*b**2*c**3*e*
*2*f*x**6 + 255255*b**2*c**3*e*f**2*x**8 + 69615*b**2*c**3*f**3*x**10 + 32
8185*b**2*c**2*d*e**3*x**6 + 765765*b**2*c**2*d*e**2*f*x**8 + 626535*b**2*
c**2*d*e*f**2*x**10 + 176715*b**2*c**2*d*f**3*x**12 + 255255*b**2*c*d**2*e
**3*x**8 + 626535*b**2*c*d**2*e**2*f*x**10 + 530145*b**2*c*d**2*e*f**2*x**
12 + 153153*b**2*c*d**2*f**3*x**14 + 69615*b**2*d**3*e**3*x**10 + 176715*b
**2*d**3*e**2*f*x**12 + 153153*b**2*d**3*e*f**2*x**14 + 45045*b**2*d**3...
```

3.232 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{e+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 290

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{e+fx^2}\,dx &= \frac{(b^2e(de-cf)^3-2abf(de-cf)^3+a^2df^2(d^2e^2-3cdef+3c^2f^2))\,x}{f^5} \\ &\quad-\frac{(a^2d^2f^2(de-3cf)+b^2(de-cf)^3-2abdf(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^4} \\ &\quad+\frac{d(a^2d^2f^2-2abdf(de-3cf)+b^2(d^2e^2-3cdef+3c^2f^2))\,x^5}{5f^3} \\ &\quad-\frac{bd^2(bde-3bcf-2adf)x^7}{7f^2}+\frac{b^2d^3x^9}{9f}-\frac{(be-af)^2(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{11/2}}\end{aligned}$$

output
```
(b^2*e*(-c*f+d*e)^3-2*a*b*f*(-c*f+d*e)^3+a^2*d*f^2*(3*c^2*f^2-3*c*d*e*f+d^
2*e^2))*x/f^5-1/3*(a^2*d^2*f^2*(-3*c*f+d*e)+b^2*(-c*f+d*e)^3-2*a*b*d*f*(3*
c^2*f^2-3*c*d*e*f+d^2*e^2))*x^3/f^4+1/5*d*(a^2*d^2*f^2-2*a*b*d*f*(-3*c*f+d
*e)+b^2*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x^5/f^3-1/7*b*d^2*(-2*a*d*f-3*b*c*f
+b*d*e)*x^7/f^2+1/9*b^2*d^3*x^9/f-(-a*f+b*e)^2*(-c*f+d*e)^3*arctan(f^(1/2)
*x/e^(1/2))/e^(1/2)/f^(11/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2} dx$$
$$= \frac{(b^2e(de-cf)^3 + 2abf(-de+cf)^3 + a^2df^2(d^2e^2-3cdef+3c^2f^2))\,x}{f^5} + \frac{(-b^2(de-cf)^3 + a^2d^2f^2(-de+3cf) + 2abdf(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^4} + \frac{d(a^2d^2f^2 - 2abdf(de-3cf) + b^2(d^2e^2-3cdef+3c^2f^2))\,x^5}{5f^3} - \frac{bd^2(bde-3bcf-2adf)x^7}{7f^2} + \frac{b^2d^3x^9}{9f} - \frac{(be-af)^2(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{11/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((b^2*e*(d*e - c*f)^3 + 2*a*b*f*(-(d*e) + c*f)^3 + a^2*d*f^2*(d^2*e^2 - 3*
c*d*e*f + 3*c^2*f^2))*x)/f^5 + ((-(b^2*(d*e - c*f)^3) + a^2*d^2*f^2*(-(d*e
) + 3*c*f) + 2*a*b*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f^4) + (
d*(a^2*d^2*f^2 - 2*a*b*d*f*(d*e - 3*c*f) + b^2*(d^2*e^2 - 3*c*d*e*f + 3*c^
2*f^2))*x^5)/(5*f^3) - (b*d^2*(b*d*e - 3*b*c*f - 2*a*d*f)*x^7)/(7*f^2) + (
b^2*d^3*x^9)/(9*f) - ((b*e - a*f)^2*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(Sqrt[e]*f^(11/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {420, 290, 403, 25, 403, 25, 403, 299, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2} dx$$

$$\Big\downarrow \ 420$$

$$\frac{b\int \left(bx^2+a\right)\left(dx^2+c\right)^3dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \ 290$$

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \ 403$$

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{\int -\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}+\frac{bx\left(c+dx^2\right)^3}{7f}\right)}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}\right)}{f}$$

$$\Big\downarrow \ 403$$

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}+\frac{x\left(c+dx^2\right)^2(-7adf-}{5f}}{7f}\right.}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-$$

$$\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f \cdots\right.\right.\right.}{fx^2+e}}{5f}}{7f}\cdots\right.}{f}$$

$\downarrow$ 403

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-$$

$$\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3\right)\right)x^2+c \cdots}{fx^2+e}}{3f}}{\cdots}\right.}{f}$$

$\downarrow$ 299

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-$$

$$\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)(de-cf)^3\int \frac{1}{fx^2+e}dx}{f}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231 \cdots\right.\right.}{f}}{3f}}{7f}\right.}{f}$$

$\downarrow$ 218

$$\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-$$

$$\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+2 \cdots\right.\right.}{f}}{3f}}{7f}\right.}{f}$$

$\downarrow$ 2009

$$\frac{b\left(\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9\right)}{f}-$$

$$\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+2\right)\right)}{f}}{3f}}{7f}\right)}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
(b*(a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2
*(3*b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9))/f - ((b*e - a*f)*((b*x*(c + d*x^2)
^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(5*f) - (-1/3
*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(
c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105
*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/f + (105*(b
*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3
*f))/(5*f))/(7*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeBe[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(274) = 548$.

Time = 0.56 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.10

method	result
default	$\frac{a^2 c d^2 f^4 x^3+\frac{1}{9} b^2 d^3 x^9 f^4+2ab c^3 f^4 x-b^2 c^3 e f^3 x+\frac{2}{7} ab d^3 f^4 x^7+3b^2 c^2 d e^2 f^2 x-2ab d^3 e^3 f x+\frac{3}{7} b^2 c d^2 f^4 x^7-\frac{1}{7} b^2 d^3 e f^3 x^7+\frac{3}{5} b^2 c^2 d f^4}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^5*(a^2*c*d^2*f^4*x^3+1/9*b^2*d^3*x^9*f^4+2*a*b*c^3*f^4*x-b^2*c^3*e*f^3
*x+2/7*a*b*d^3*f^4*x^7+3*b^2*c^2*d*e^2*f^2*x-2*a*b*d^3*e^3*f*x+3/7*b^2*c*d
^2*f^4*x^7-1/7*b^2*d^3*e*f^3*x^7+3/5*b^2*c^2*d*f^4*x^5+b^2*c*d^2*e^2*f^2*x
^3+b^2*d^3*e^4*x+1/5*a^2*d^3*f^4*x^5+6/5*a*b*c*d^2*f^4*x^5-1/3*a^2*d^3*e*f
^3*x^3-1/3*b^2*d^3*e^3*f*x^3+3*a^2*c^2*d*f^4*x+a^2*d^3*e^2*f^2*x-2/5*a*b*d
^3*e*f^3*x^5-b^2*c^2*d*e*f^3*x^3-3*a^2*c*d^2*e*f^3*x-3/5*b^2*c*d^2*e*f^3*x
^5-2*a*b*c*d^2*e*f^3*x^3+2*a*b*c^2*d*f^4*x^3+2/3*a*b*d^3*e^2*f^2*x^3+1/5*b
^2*d^3*e^2*f^2*x^5-6*a*b*c^2*d*e*f^3*x+6*a*b*c*d^2*e^2*f^2*x-3*b^2*c*d^2*e
^3*f*x+1/3*b^2*c^3*f^4*x^3)+(a^2*c^3*f^5-3*a^2*c^2*d*e*f^4+3*a^2*c*d^2*e^2
*f^3-a^2*d^3*e^3*f^2-2*a*b*c^3*e*f^4+6*a*b*c^2*d*e^2*f^3-6*a*b*c*d^2*e^3*f
^2+2*a*b*d^3*e^4*f+b^2*c^3*e^2*f^3-3*b^2*c^2*d*e^3*f^2+3*b^2*c*d^2*e^4*f-b
^2*d^3*e^5)/f^5/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 1068, normalized size of antiderivative = 3.68

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/630*(70*b^2*d^3*e*f^5*x^9 - 90*(b^2*d^3*e^2*f^4 - (3*b^2*c*d^2 + 2*a*b*
d^3)*e*f^5)*x^7 + 126*(b^2*d^3*e^3*f^3 - (3*b^2*c*d^2 + 2*a*b*d^3)*e^2*f^4
 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e*f^5)*x^5 - 210*(b^2*d^3*e^4*f^2
 - (3*b^2*c*d^2 + 2*a*b*d^3)*e^3*f^3 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^
3)*e^2*f^4 - (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e*f^5)*x^3 + 315*(b^2*d
^3*e^5 - a^2*c^3*f^5 - (3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f + (3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*e^3*f^2 - (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f
^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^4)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)
*x - e)/(f*x^2 + e)) + 630*(b^2*d^3*e^5*f - (3*b^2*c*d^2 + 2*a*b*d^3)*e^4*
f^2 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^3*f^3 - (b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*e^2*f^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^5)*x)/(e*f^6), 1
/315*(35*b^2*d^3*e*f^5*x^9 - 45*(b^2*d^3*e^2*f^4 - (3*b^2*c*d^2 + 2*a*b*d^
3)*e*f^5)*x^7 + 63*(b^2*d^3*e^3*f^3 - (3*b^2*c*d^2 + 2*a*b*d^3)*e^2*f^4 +
(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e*f^5)*x^5 - 105*(b^2*d^3*e^4*f^2 -
(3*b^2*c*d^2 + 2*a*b*d^3)*e^3*f^3 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*
e^2*f^4 - (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e*f^5)*x^3 - 315*(b^2*d^3*
e^5 - a^2*c^3*f^5 - (3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f + (3*b^2*c^2*d + 6*a*b
*c*d^2 + a^2*d^3)*e^3*f^2 - (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f^3
+ (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^4)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 315*(
b^2*d^3*e^5*f - (3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f^2 + (3*b^2*c^2*d + 6*a*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(291) = 582$.

Time = 1.66 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.14

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
b**2*d**3*x**9/(9*f) + x**7*(2*a*b*d**3/(7*f) + 3*b**2*c*d**2/(7*f) - b**2
*d**3*e/(7*f**2)) + x**5*(a**2*d**3/(5*f) + 6*a*b*c*d**2/(5*f) - 2*a*b*d**
3*e/(5*f**2) + 3*b**2*c**2*d/(5*f) - 3*b**2*c*d**2*e/(5*f**2) + b**2*d**3*
e**2/(5*f**3)) + x**3*(a**2*c*d**2/f - a**2*d**3*e/(3*f**2) + 2*a*b*c**2*d
/f - 2*a*b*c*d**2*e/f**2 + 2*a*b*d**3*e**2/(3*f**3) + b**2*c**3/(3*f) - b*
*2*c**2*d*e/f**2 + b**2*c*d**2*e**2/f**3 - b**2*d**3*e**3/(3*f**4)) + x*(3
*a**2*c**2*d/f - 3*a**2*c*d**2*e/f**2 + a**2*d**3*e**2/f**3 + 2*a*b*c**3/f
 - 6*a*b*c**2*d*e/f**2 + 6*a*b*c*d**2*e**2/f**3 - 2*a*b*d**3*e**3/f**4 - b
**2*c**3*e/f**2 + 3*b**2*c**2*d*e**2/f**3 - 3*b**2*c*d**2*e**3/f**4 + b**2
*d**3*e**4/f**5) - sqrt(-1/(e*f**11))*(a*f - b*e)**2*(c*f - d*e)**3*log(-e
*f**5*sqrt(-1/(e*f**11))*(a*f - b*e)**2*(c*f - d*e)**3/(a**2*c**3*f**5 - 3
*a**2*c**2*d*e*f**4 + 3*a**2*c*d**2*e**2*f**3 - a**2*d**3*e**3*f**2 - 2*a*
b*c**3*e*f**4 + 6*a*b*c**2*d*e**2*f**3 - 6*a*b*c*d**2*e**3*f**2 + 2*a*b*d*
*3*e**4*f + b**2*c**3*e**2*f**3 - 3*b**2*c**2*d*e**3*f**2 + 3*b**2*c*d**2*
e**4*f - b**2*d**3*e**5) + x)/2 + sqrt(-1/(e*f**11))*(a*f - b*e)**2*(c*f -
 d*e)**3*log(e*f**5*sqrt(-1/(e*f**11))*(a*f - b*e)**2*(c*f - d*e)**3/(a**2
*c**3*f**5 - 3*a**2*c**2*d*e*f**4 + 3*a**2*c*d**2*e**2*f**3 - a**2*d**3*e*
*3*f**2 - 2*a*b*c**3*e*f**4 + 6*a*b*c**2*d*e**2*f**3 - 6*a*b*c*d**2*e**3*f
**2 + 2*a*b*d**3*e**4*f + b**2*c**3*e**2*f**3 - 3*b**2*c**2*d*e**3*f**2 +
3*b**2*c*d**2*e**4*f - b**2*d**3*e**5) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(274) = 548$.

Time = 0.12 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{e+fx^2}\,dx = -\frac{(b^2d^3e^5 - 3\,b^2cd^2e^4f - 2\,abd^3e^4f + 3\,b^2c^2de^3f^2 + 6\,abcd^2e^3f^2 + a^2d^3e^3f^2 - b^2c^3e^2f^3 - 6\,abc^2de^2f^3 -}{\sqrt{ef}\,f^5} + \frac{35\,b^2d^3f^8x^9 - 45\,b^2d^3ef^7x^7 + 135\,b^2cd^2f^8x^7 + 90\,abd^3f^8x^7 + 63\,b^2d^3e^2f^6x^5 - 189\,b^2cd^2ef^7x^5 - 126\,a}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
-(b^2*d^3*e^5 - 3*b^2*c*d^2*e^4*f - 2*a*b*d^3*e^4*f + 3*b^2*c^2*d*e^3*f^2
+ 6*a*b*c*d^2*e^3*f^2 + a^2*d^3*e^3*f^2 - b^2*c^3*e^2*f^3 - 6*a*b*c^2*d*e^
2*f^3 - 3*a^2*c*d^2*e^2*f^3 + 2*a*b*c^3*e*f^4 + 3*a^2*c^2*d*e*f^4 - a^2*c^
3*f^5)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^5) + 1/315*(35*b^2*d^3*f^8*x^9 -
 45*b^2*d^3*e*f^7*x^7 + 135*b^2*c*d^2*f^8*x^7 + 90*a*b*d^3*f^8*x^7 + 63*b^
2*d^3*e^2*f^6*x^5 - 189*b^2*c*d^2*e*f^7*x^5 - 126*a*b*d^3*e*f^7*x^5 + 189*
b^2*c^2*d*f^8*x^5 + 378*a*b*c*d^2*f^8*x^5 + 63*a^2*d^3*f^8*x^5 - 105*b^2*d
^3*e^3*f^5*x^3 + 315*b^2*c*d^2*e^2*f^6*x^3 + 210*a*b*d^3*e^2*f^6*x^3 - 315
*b^2*c^2*d*e*f^7*x^3 - 630*a*b*c*d^2*e*f^7*x^3 - 105*a^2*d^3*e*f^7*x^3 + 1
05*b^2*c^3*f^8*x^3 + 630*a*b*c^2*d*f^8*x^3 + 315*a^2*c*d^2*f^8*x^3 + 315*b
^2*d^3*e^4*f^4*x - 945*b^2*c*d^2*e^3*f^5*x - 630*a*b*d^3*e^3*f^5*x + 945*b
^2*c^2*d*e^2*f^6*x + 1890*a*b*c*d^2*e^2*f^6*x + 315*a^2*d^3*e^2*f^6*x - 31
5*b^2*c^3*e*f^7*x - 1890*a*b*c^2*d*e*f^7*x - 945*a^2*c*d^2*e*f^7*x + 630*a
*b*c^3*f^8*x + 945*a^2*c^2*d*f^8*x)/f^9
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{e+fx^2}\,dx$$

$$= x^3 \left(\frac{3\,a^2\,c\,d^2+6\,a\,b\,c^2\,d+b^2\,c^3}{3\,f} - \frac{e\left(\frac{a^2\,d^3+6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d}{f}+\frac{e\left(\frac{b^2\,d^3\,e}{f^2}-\frac{b\,d^2\,(2\,a\,d+3\,b\,c)}{f}\right)}{f}\right)}{3\,f}\right)$$

$$+ x^5\left(\frac{a^2\,d^3+6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d}{5\,f}+\frac{e\left(\frac{b^2\,d^3\,e}{f^2}-\frac{b\,d^2\,(2\,a\,d+3\,b\,c)}{f}\right)}{5\,f}\right)$$

$$- x^7\left(\frac{b^2\,d^3\,e}{7\,f^2}-\frac{b\,d^2\,(2\,a\,d+3\,b\,c)}{7\,f}\right)$$

$$- x\left(\frac{e\left(\frac{3\,a^2\,c\,d^2+6\,a\,b\,c^2\,d+b^2\,c^3}{f}-\frac{e\left(\frac{a^2\,d^3+6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d}{f}+\frac{e\left(\frac{b^2\,d^3\,e}{f^2}-\frac{b\,d^2\,(2\,a\,d+3\,b\,c)}{f}\right)}{f}\right)}{f}\right)}{f} - \frac{a\,c^2\,(3\,a\,d+2\,b\,c)}{f}\right) + \frac{b^2\,d^3\,x^9}{9\,f}$$

$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)^2\,(c\,f-d\,e)^3}{\sqrt{e}\,(a^2\,c^3\,f^5-3\,a^2\,c^2\,d\,e\,f^4+3\,a^2\,c\,d^2\,e^2\,f^3-a^2\,d^3\,e^3\,f^2-2\,a\,b\,c^3\,e\,f^4+6\,a\,b\,c^2\,d\,e^2\,f^3-6\,a\,b\,c\,d^2\,e^3\,f^2+2\,a\,b\,d^3\,e^4\,f+b^2\,c^3\,e^2\,f^3-3\,b^2 \cdots}\right.}{\sqrt{e}\,f^{11/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
x^3*((b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^2*d)/(3*f) - (e*((a^2*d^3 + 3*b^2*c^
2*d + 6*a*b*c*d^2)/f + (e*((b^2*d^3*e)/f^2 - (b*d^2*(2*a*d + 3*b*c))/f))/f
))/(3*f)) + x^5*((a^2*d^3 + 3*b^2*c^2*d + 6*a*b*c*d^2)/(5*f) + (e*((b^2*d^
3*e)/f^2 - (b*d^2*(2*a*d + 3*b*c))/f))/(5*f)) - x^7*((b^2*d^3*e)/(7*f^2) -
 (b*d^2*(2*a*d + 3*b*c))/(7*f)) - x*((e*((b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^
2*d)/f - (e*((a^2*d^3 + 3*b^2*c^2*d + 6*a*b*c*d^2)/f + (e*((b^2*d^3*e)/f^2
 - (b*d^2*(2*a*d + 3*b*c))/f))/f))/f))/f - (a*c^2*(3*a*d + 2*b*c))/f) + (b
^2*d^3*x^9)/(9*f) + (atan((f^(1/2)*x*(a*f - b*e)^2*(c*f - d*e)^3)/(e^(1/2)
*(a^2*c^3*f^5 - b^2*d^3*e^5 - a^2*d^3*e^3*f^2 + b^2*c^3*e^2*f^3 - 2*a*b*c^
3*e*f^4 + 2*a*b*d^3*e^4*f - 3*a^2*c^2*d*e*f^4 + 3*b^2*c*d^2*e^4*f + 3*a^2*
c*d^2*e^2*f^3 - 3*b^2*c^2*d*e^3*f^2 - 6*a*b*c*d^2*e^3*f^2 + 6*a*b*c^2*d*e^
2*f^3)))*(a*f - b*e)^2*(c*f - d*e)^3)/(e^(1/2)*f^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.85

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e),x)
```

output

```
(315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*f**5 - 945*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**4 + 945*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f**3 - 315*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**3*f**2 - 630*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e*f**4 + 1890*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**2*f**3 - 1890*sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d**2*e**3*f**2 + 630*sqrt(f)*sqrt(
e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*d**3*e**4*f + 315*sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*b**2*c**3*e**2*f**3 - 945*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*b**2*c**2*d*e**3*f**2 + 945*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*b**2*c*d**2*e**4*f - 315*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*b**2*d**3*e**5 + 945*a**2*c**2*d*e*f**5*x - 945*a**2
*c*d**2*e**2*f**4*x + 315*a**2*c*d**2*e*f**5*x**3 + 315*a**2*d**3*e**3*f**
3*x - 105*a**2*d**3*e**2*f**4*x**3 + 63*a**2*d**3*e*f**5*x**5 + 630*a*b*c*
*3*e*f**5*x - 1890*a*b*c**2*d*e**2*f**4*x + 630*a*b*c**2*d*e*f**5*x**3 + 1
890*a*b*c*d**2*e**3*f**3*x - 630*a*b*c*d**2*e**2*f**4*x**3 + 378*a*b*c*d**
2*e*f**5*x**5 - 630*a*b*d**3*e**4*f**2*x + 210*a*b*d**3*e**3*f**3*x**3 - 1
26*a*b*d**3*e**2*f**4*x**5 + 90*a*b*d**3*e*f**5*x**7 - 315*b**2*c**3*e**2*
f**4*x + 105*b**2*c**3*e*f**5*x**3 + 945*b**2*c**2*d*e**3*f**3*x - 315*b**
2*c**2*d*e**2*f**4*x**3 + 189*b**2*c**2*d*e*f**5*x**5 - 945*b**2*c*d**2...
```

3.233 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 275

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^2}\,dx &= -\frac{(a^2d^2f^2(2de-3cf)-6abdf(de-cf)^2+b^2(de-cf)^2(4de-cf))\,x}{f^5} \\ &+\frac{d(a^2d^2f^2-2abdf(2de-3cf)+3b^2(de-cf)^2)\,x^3}{3f^4} \\ &-\frac{bd^2(2bde-3bcf-2adf)x^5}{5f^3}+\frac{b^2d^3x^7}{7f^2}-\frac{(be-af)^2(de-cf)^3x}{2ef^5\,(e+fx^2)} \\ &+\frac{(be-af)(de-cf)^2(3be(3de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{11/2}}\end{aligned}$$

output

```
-(a^2*d^2*f^2*(-3*c*f+2*d*e)-6*a*b*d*f*(-c*f+d*e)^2+b^2*(-c*f+d*e)^2*(-c*f
+4*d*e))*x/f^5+1/3*d*(a^2*d^2*f^2-2*a*b*d*f*(-3*c*f+2*d*e)+3*b^2*(-c*f+d*e
)^2)*x^3/f^4-1/5*b*d^2*(-2*a*d*f-3*b*c*f+2*b*d*e)*x^5/f^3+1/7*b^2*d^3*x^7/
f^2-1/2*(-a*f+b*e)^2*(-c*f+d*e)^3*x/e/f^5/(f*x^2+e)+1/2*(-a*f+b*e)*(-c*f+d
*e)^2*(3*b*e*(-c*f+3*d*e)-a*f*(c*f+5*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/
2)/f^(11/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} dx$$
$$= \frac{(6abdf(de-cf)^2 - b^2(de-cf)^2(4de-cf) + a^2d^2f^2(-2de+3cf))\,x}{f^5}$$
$$+ \frac{d(a^2d^2f^2 + 3b^2(de-cf)^2 + 2abdf(-2de+3cf))\,x^3}{3f^4}$$
$$- \frac{bd^2(2bde-3bcf-2adf)x^5}{5f^3} + \frac{b^2d^3x^7}{7f^2} - \frac{(be-af)^2(de-cf)^3x}{2ef^5(e+fx^2)}$$
$$+ \frac{(be-af)(de-cf)^2(3be(3de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{11/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
((6*a*b*d*f*(d*e - c*f)^2 - b^2*(d*e - c*f)^2*(4*d*e - c*f) + a^2*d^2*f^2*
(-2*d*e + 3*c*f))*x)/f^5 + (d*(a^2*d^2*f^2 + 3*b^2*(d*e - c*f)^2 + 2*a*b*d
*f*(-2*d*e + 3*c*f))*x^3)/(3*f^4) - (b*d^2*(2*b*d*e - 3*b*c*f - 2*a*d*f)*x
^5)/(5*f^3) + (b^2*d^3*x^7)/(7*f^2) - ((b*e - a*f)^2*(d*e - c*f)^3*x)/(2*e
*f^5*(e + f*x^2)) + ((b*e - a*f)*(d*e - c*f)^2*(3*b*e*(3*d*e - c*f) - a*f*
(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(11/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 401, 25, 403, 25, 403, 25, 299, 218, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} dx$$
$$\Big\downarrow \; 425$$

$$\frac{b\int\frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\int\frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 401

$$\frac{b\int\frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(-\frac{\int-\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\int\frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(\frac{\int\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\downarrow$ 403

$$\frac{b\left(\frac{\int-\frac{(dx^2+c)^2((7bde-6bcf-7adf)x^2+c(be-7af))}{fx^2+e}dx}{7f}+\frac{bx(c+dx^2)^3}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{\int-\frac{(dx^2+c)(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf)))}{fx^2+e}dx}{5f}+\frac{dx(c+dx^2)^2(7be-5af)}{5f}}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\int\frac{(dx^2+c)^2((7bde-6bcf-7adf)x^2+c(be-7af))}{fx^2+e}dx}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\int\frac{(dx^2+c)(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf)))}{fx^2+e}dx}{5f}}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}$$

$\downarrow$ 403

$$b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{\int -\frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}+\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}}{7f}\right.$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\int \frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\left(5d^2e^2-6cdfe-3c^2f^2\right)-be\left(35d^2e^2-\right.\right.}{fx^2+e}}{3f}}{5f}}{2ef}\right.$$

$$\frac{}{f}$$

$\downarrow$ 25

$$b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\int \frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\left(5d^2e^2-6cdfe-3c^2f^2\right)-be\left(35d^2e^2-\right.\right.}{fx^2+e}}{3f}}{5f}}{2ef}\right.$$

$$\frac{}{f}$$

$\downarrow$ 299

$$b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int \frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15(de-cf)^2(be(7de-cf)-af(cf+5de))\int \frac{1}{fx^2+e}dx}{f}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2\right.\right.}{f}}{3f}}{5f}}{2ef}\right.$$

$$\frac{}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{(dx^2+c)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+10\cdots\right.\right.}{f}}{3f}}{5f}}{2ef}\cdots\right.}{f}$$

$\downarrow$ 403

$$\frac{b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\int\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3\right)\right)x^2+c\left(7af\left(5d^2e^2-\cdots\right.\right.}{fx^2+e}}{3f}\cdots}{7f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+10\cdots\right.\right.}{f}}{3f}}{5f}}{2ef}\cdots\right.}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)(de-cf)^3\int\frac{1}{fx^2+e}dx}{f}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280\cdots\right.\right.}{f}}{3f}}{5f}}{7f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+10\cdots\right.\right.}{f}}{3f}}{5f}}{2ef}\cdots\right.}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{bx(c+dx^2)^3}{7f}-\frac{\frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-2\right.\right.}{f}}{3f}}{5f}}{7f}}{f}$$

$$\frac{(be-af)\left(\frac{\frac{dx(c+dx^2)^2(7be-5af)}{5f}-\frac{\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+10\right.\right.}{f}}{3f}}{5f}}{2ef}\right.}{f}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
(b*((b*x*(c + d*x^2)^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x
^2)^2)/(5*f) - (-1/3*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*
f + 24*c^2*f^2))*x*(c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f +
33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*
f^3))*x)/f + (105*(b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(
Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/(7*f)))/f - ((b*e - a*f)*(-1/2*((b*e - a*f
)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*a*f)*x*(c + d*x^2)^2
)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d*x^2
))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2
*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*f)^2*(b*e*(7*d*e - c
*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(
3*f))/(5*f))/(2*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(257) = 514$.

Time = 0.68 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.23

method	result
default	$\frac{\frac{1}{7}b^2d^3x^7f^3+\frac{2}{5}ab\,d^3f^3x^5+\frac{3}{5}b^2c\,d^2f^3x^5-\frac{2}{5}b^2d^3e\,f^2x^5+\frac{1}{3}a^2d^3f^3x^3+2abc\,d^2f^3x^3-\frac{4}{3}ab\,d^3e\,f^2x^3+b^2c^2d\,f^3x^3-2b^2c\,d^2e\,f^2x^3+b^2d^2}{f^5}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^5*(1/7*b^2*d^3*x^7*f^3+2/5*a*b*d^3*f^3*x^5+3/5*b^2*c*d^2*f^3*x^5-2/5*b
^2*d^3*e*f^2*x^5+1/3*a^2*d^3*f^3*x^3+2*a*b*c*d^2*f^3*x^3-4/3*a*b*d^3*e*f^2
*x^3+b^2*c^2*d*f^3*x^3-2*b^2*c*d^2*e*f^2*x^3+b^2*d^3*e^2*f*x^3+3*a^2*c*d^2
*f^3*x-2*a^2*d^3*e*f^2*x+6*a*b*c^2*d*f^3*x-12*a*b*c*d^2*e*f^2*x+6*a*b*d^3*
e^2*f*x+b^2*c^3*f^3*x-6*b^2*c^2*d*e*f^2*x+9*b^2*c*d^2*e^2*f*x-4*b^2*d^3*e^
3*x)+1/f^5*(1/2*(a^2*c^3*f^5-3*a^2*c^2*d*e*f^4+3*a^2*c*d^2*e^2*f^3-a^2*d^3
*e^3*f^2-2*a*b*c^3*e*f^4+6*a*b*c^2*d*e^2*f^3-6*a*b*c*d^2*e^3*f^2+2*a*b*d^3
*e^4*f+b^2*c^3*e^2*f^3-3*b^2*c^2*d*e^3*f^2+3*b^2*c*d^2*e^4*f-b^2*d^3*e^5)/
e*x/(f*x^2+e)+1/2*(a^2*c^3*f^5+3*a^2*c^2*d*e*f^4-9*a^2*c*d^2*e^2*f^3+5*a^2
*d^3*e^3*f^2+2*a*b*c^3*e*f^4-18*a*b*c^2*d*e^2*f^3+30*a*b*c*d^2*e^3*f^2-14*
a*b*d^3*e^4*f-3*b^2*c^3*e^2*f^3+15*b^2*c^2*d*e^3*f^2-21*b^2*c*d^2*e^4*f+9*
b^2*d^3*e^5)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(257) = 514$.

Time = 0.10 (sec) , antiderivative size = 1442, normalized size of antiderivative = 5.24

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*b^2*d^3*e^2*f^5*x^9 - 12*(9*b^2*d^3*e^3*f^4 - 7*(3*b^2*c*d^2 +
2*a*b*d^3)*e^2*f^5)*x^7 + 28*(9*b^2*d^3*e^4*f^3 - 7*(3*b^2*c*d^2 + 2*a*b*d
^3)*e^3*f^4 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^2*f^5)*x^5 - 140*(
9*b^2*d^3*e^5*f^2 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f^3 + 5*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*e^3*f^4 - 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
e^2*f^5)*x^3 - 105*(9*b^2*d^3*e^6 + a^2*c^3*e*f^5 - 7*(3*b^2*c*d^2 + 2*a*b
*d^3)*e^5*f + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^4*f^2 - 3*(b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^3*f^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f^4
+ (9*b^2*d^3*e^5*f + a^2*c^3*f^6 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f^2 + 5
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^3*f^3 - 3*(b^2*c^3 + 6*a*b*c^2*d
+ 3*a^2*c*d^2)*e^2*f^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^5)*x^2)*sqrt(-e*f)*
log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 210*(9*b^2*d^3*e^6*f - a^2
*c^3*e*f^6 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)*e^5*f^2 + 5*(3*b^2*c^2*d + 6*a*b*
c*d^2 + a^2*d^3)*e^4*f^3 - 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^3*f^4
 + (2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f^5)*x)/(e^2*f^7*x^2 + e^3*f^6), 1/210*(3
0*b^2*d^3*e^2*f^5*x^9 - 6*(9*b^2*d^3*e^3*f^4 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)
*e^2*f^5)*x^7 + 14*(9*b^2*d^3*e^4*f^3 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)*e^3*f^
4 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^2*f^5)*x^5 - 70*(9*b^2*d^3*e
^5*f^2 - 7*(3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f^3 + 5*(3*b^2*c^2*d + 6*a*b*c*d^
2 + a^2*d^3)*e^3*f^4 - 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f^5)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(264) = 528$.

Time = 5.69 (sec) , antiderivative size = 1096, normalized size of antiderivative = 3.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
b**2*d**3*x**7/(7*f**2) + x**5*(2*a*b*d**3/(5*f**2) + 3*b**2*c*d**2/(5*f**
2) - 2*b**2*d**3*e/(5*f**3)) + x**3*(a**2*d**3/(3*f**2) + 2*a*b*c*d**2/f**
2 - 4*a*b*d**3*e/(3*f**3) + b**2*c**2*d/f**2 - 2*b**2*c*d**2*e/f**3 + b**2
*d**3*e**2/f**4) + x*(3*a**2*c*d**2/f**2 - 2*a**2*d**3*e/f**3 + 6*a*b*c**2
*d/f**2 - 12*a*b*c*d**2*e/f**3 + 6*a*b*d**3*e**2/f**4 + b**2*c**3/f**2 - 6
*b**2*c**2*d*e/f**3 + 9*b**2*c*d**2*e**2/f**4 - 4*b**2*d**3*e**3/f**5) + x
*(a**2*c**3*f**5 - 3*a**2*c**2*d*e*f**4 + 3*a**2*c*d**2*e**2*f**3 - a**2*d
**3*e**3*f**2 - 2*a*b*c**3*e*f**4 + 6*a*b*c**2*d*e**2*f**3 - 6*a*b*c*d**2*
e**3*f**2 + 2*a*b*d**3*e**4*f + b**2*c**3*e**2*f**3 - 3*b**2*c**2*d*e**3*f
**2 + 3*b**2*c*d**2*e**4*f - b**2*d**3*e**5)/(2*e**2*f**5 + 2*e*f**6*x**2)
 - sqrt(-1/(e**3*f**11))*(a*f - b*e)*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f
+ 3*b*c*e*f - 9*b*d*e**2)*log(-e**2*f**5*sqrt(-1/(e**3*f**11))*(a*f - b*e)
*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + 3*b*c*e*f - 9*b*d*e**2)/(a**2*c**3
*f**5 + 3*a**2*c**2*d*e*f**4 - 9*a**2*c*d**2*e**2*f**3 + 5*a**2*d**3*e**3*
f**2 + 2*a*b*c**3*e*f**4 - 18*a*b*c**2*d*e**2*f**3 + 30*a*b*c*d**2*e**3*f*
*2 - 14*a*b*d**3*e**4*f - 3*b**2*c**3*e**2*f**3 + 15*b**2*c**2*d*e**3*f**2
 - 21*b**2*c*d**2*e**4*f + 9*b**2*d**3*e**5) + x)/4 + sqrt(-1/(e**3*f**11)
)*(a*f - b*e)*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + 3*b*c*e*f - 9*b*d*e**
2)*log(e**2*f**5*sqrt(-1/(e**3*f**11))*(a*f - b*e)*(c*f - d*e)**2*(a*c*f**
2 + 5*a*d*e*f + 3*b*c*e*f - 9*b*d*e**2)/(a**2*c**3*f**5 + 3*a**2*c**2*d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(257) = 514$.

Time = 0.12 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2}\,dx$$
$$= \frac{(9\,b^2d^3e^5 - 21\,b^2cd^2e^4f - 14\,abd^3e^4f + 15\,b^2c^2de^3f^2 + 30\,abcd^2e^3f^2 + 5\,a^2d^3e^3f^2 - 3\,b^2c^3e^2f^3 - 18\,abc^2}{2\,\sqrt{ef}ef^5}$$
$$- \frac{b^2d^3e^5x - 3\,b^2cd^2e^4fx - 2\,abd^3e^4fx + 3\,b^2c^2de^3f^2x + 6\,abcd^2e^3f^2x + a^2d^3e^3f^2x - b^2c^3e^2f^3x - 6\,abc^2}{2\,(fx^2+e)ef^5}$$
$$+ \frac{15\,b^2d^3f^{12}x^7 - 42\,b^2d^3ef^{11}x^5 + 63\,b^2cd^2f^{12}x^5 + 42\,abd^3f^{12}x^5 + 105\,b^2d^3e^2f^{10}x^3 - 210\,b^2cd^2ef^{11}x^3 -}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(9*b^2*d^3*e^5 - 21*b^2*c*d^2*e^4*f - 14*a*b*d^3*e^4*f + 15*b^2*c^2*d*
e^3*f^2 + 30*a*b*c*d^2*e^3*f^2 + 5*a^2*d^3*e^3*f^2 - 3*b^2*c^3*e^2*f^3 - 1
8*a*b*c^2*d*e^2*f^3 - 9*a^2*c*d^2*e^2*f^3 + 2*a*b*c^3*e*f^4 + 3*a^2*c^2*d*
e*f^4 + a^2*c^3*f^5)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e*f^5) - 1/2*(b^2*d^
3*e^5*x - 3*b^2*c*d^2*e^4*f*x - 2*a*b*d^3*e^4*f*x + 3*b^2*c^2*d*e^3*f^2*x
+ 6*a*b*c*d^2*e^3*f^2*x + a^2*d^3*e^3*f^2*x - b^2*c^3*e^2*f^3*x - 6*a*b*c^
2*d*e^2*f^3*x - 3*a^2*c*d^2*e^2*f^3*x + 2*a*b*c^3*e*f^4*x + 3*a^2*c^2*d*e*
f^4*x - a^2*c^3*f^5*x)/((f*x^2 + e)*e*f^5) + 1/105*(15*b^2*d^3*f^12*x^7 -
42*b^2*d^3*e*f^11*x^5 + 63*b^2*c*d^2*f^12*x^5 + 42*a*b*d^3*f^12*x^5 + 105*
b^2*d^3*e^2*f^10*x^3 - 210*b^2*c*d^2*e*f^11*x^3 - 140*a*b*d^3*e*f^11*x^3 +
 105*b^2*c^2*d*f^12*x^3 + 210*a*b*c*d^2*f^12*x^3 + 35*a^2*d^3*f^12*x^3 - 4
20*b^2*d^3*e^3*f^9*x + 945*b^2*c*d^2*e^2*f^10*x + 630*a*b*d^3*e^2*f^10*x -
 630*b^2*c^2*d*e*f^11*x - 1260*a*b*c*d^2*e*f^11*x - 210*a^2*d^3*e*f^11*x +
 105*b^2*c^3*f^12*x + 630*a*b*c^2*d*f^12*x + 315*a^2*c*d^2*f^12*x)/f^14
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.67

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} dx = x \left(\frac{3a^2 c d^2 + 6abc^2 d + b^2 c^3}{f^2} - \frac{2e\left(\frac{a^2 d^3+6abcd^2+3b^2c^2d}{f^2} + \frac{2e\left(\frac{2b^2d^3e}{f^3} - \frac{bd^2(2ad+3bc)}{f^2}\right)}{f} - \frac{b^2d^3e^2}{f^4}\right)}{f} + \frac{e^2\left(\frac{2b^2d^3e}{f^3} - \frac{bd^2(2ad+3bc)}{f^2}\right)}{f^2} \right) - x^5\left(\frac{2b^2d^3e}{5f^3} - \frac{bd^2(2ad+3bc)}{5f^2}\right) + x^3\left(\frac{a^2d^3+6abcd^2+3b^2c^2d}{3f^2} + \frac{2e\left(\frac{2b^2d^3e}{f^3} - \frac{bd^2(2ad+3bc)}{f^2}\right)}{3f} - \frac{b^2d^3e^2}{3f^4}\right) + \frac{b^2d^3x^7}{7f^2} + \frac{x(a^2c^3f^5 - 3a^2c^2def^4 + 3a^2cd^2e^2f^3 - a^2d^3e^3f^2 - 2abc^3ef^4 + 6abc^2de^2f^3 - 6abcd^2e^3f^2 +}{2e(f^6x^2+ef^5)} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x(af-be)(cf-de)^2(acf^2-9bde^2+5adef+3bcef)}{\sqrt{e}(a^2c^3f^5+3a^2c^2def^4-9a^2cd^2e^2f^3+5a^2d^3e^3f^2+2abc^3ef^4-18abc^2de^2f^3+30abcd^2e^3f^2-14abd^3e^4f-3b^2c^3e^2f^3}\right.}{2e^{3/2}f^{11}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
x*((b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^2*d)/f^2 - (2*e*((a^2*d^3 + 3*b^2*c^2*
d + 6*a*b*c*d^2)/f^2 + (2*e*((2*b^2*d^3*e)/f^3 - (b*d^2*(2*a*d + 3*b*c))/f
^2))/f - (b^2*d^3*e^2)/f^4))/f + (e^2*((2*b^2*d^3*e)/f^3 - (b*d^2*(2*a*d +
 3*b*c))/f^2))/f^2) - x^5*((2*b^2*d^3*e)/(5*f^3) - (b*d^2*(2*a*d + 3*b*c))
/(5*f^2)) + x^3*((a^2*d^3 + 3*b^2*c^2*d + 6*a*b*c*d^2)/(3*f^2) + (2*e*((2*
b^2*d^3*e)/f^3 - (b*d^2*(2*a*d + 3*b*c))/f^2))/(3*f) - (b^2*d^3*e^2)/(3*f^
4)) + (b^2*d^3*x^7)/(7*f^2) + (x*(a^2*c^3*f^5 - b^2*d^3*e^5 - a^2*d^3*e^3*
f^2 + b^2*c^3*e^2*f^3 - 2*a*b*c^3*e*f^4 + 2*a*b*d^3*e^4*f - 3*a^2*c^2*d*e*
f^4 + 3*b^2*c*d^2*e^4*f + 3*a^2*c*d^2*e^2*f^3 - 3*b^2*c^2*d*e^3*f^2 - 6*a*
b*c*d^2*e^3*f^2 + 6*a*b*c^2*d*e^2*f^3))/(2*e*(e*f^5 + f^6*x^2)) + (atan((f
^(1/2)*x*(a*f - b*e)*(c*f - d*e)^2*(a*c*f^2 - 9*b*d*e^2 + 5*a*d*e*f + 3*b*
c*e*f))/(e^(1/2)*(a^2*c^3*f^5 + 9*b^2*d^3*e^5 + 5*a^2*d^3*e^3*f^2 - 3*b^2*
c^3*e^2*f^3 + 2*a*b*c^3*e*f^4 - 14*a*b*d^3*e^4*f + 3*a^2*c^2*d*e*f^4 - 21*
b^2*c*d^2*e^4*f - 9*a^2*c*d^2*e^2*f^3 + 15*b^2*c^2*d*e^3*f^2 + 30*a*b*c*d^
2*e^3*f^2 - 18*a*b*c^2*d*e^2*f^3)))*(a*f - b*e)*(c*f - d*e)^2*(a*c*f^2 - 9
*b*d*e^2 + 5*a*d*e*f + 3*b*c*e*f))/(2*e^(3/2)*f^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1264, normalized size of antiderivative = 4.60

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output

```
(105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e*f**5 + 105*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*f**6*x**2 + 315*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**2*f**4 + 315*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**5*x**2 - 945*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**3*f**3 - 945*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f**4*x**2 +
 525*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**4*f**2 + 5
25*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**3*f**3*x**2
+ 210*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**2*f**4 + 2
10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e*f**5*x**2 - 18
90*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**3*f**3 - 18
90*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**2*f**4*x**2
 + 3150*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d**2*e**4*f**2
 + 3150*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d**2*e**3*f**3
*x**2 - 1470*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*d**3*e**5*f
 - 1470*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*d**3*e**4*f**2*x
**2 - 315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**2*c**3*e**3*f**
3 - 315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**2*c**3*e**2*f**4*
x**2 + 1575*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**2*c**2*d*e**4
*f**2 + 1575*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**2*c**2*d*...
```

3.234 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 333

$$\begin{aligned}
&\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^3}\,dx \\
&= \frac{d(a^2d^2f^2-6abdf(de-cf)+3b^2(2d^2e^2-3cdef+c^2f^2))\,x}{f^5} \\
&\quad - \frac{bd^2(3bde-3bcf-2adf)x^3}{3f^4} + \frac{b^2d^3x^5}{5f^3} - \frac{(be-af)^2(de-cf)^3x}{4ef^5\,(e+fx^2)^2} \\
&\quad + \frac{(be-af)(de-cf)^2(be(17de-5cf)-3af(3de+cf))x}{8e^2f^5\,(e+fx^2)} \\
&\quad + \frac{(de-cf)\,(2abef(35d^2e^2-10cdef-c^2f^2)-3b^2e^2(21d^2e^2-14cdef+c^2f^2)-3a^2f^2(5d^2e^2+2cdef}{8e^{5/2}f^{11/2}}
\end{aligned}$$

output

```
d*(a^2*d^2*f^2-6*a*b*d*f*(-c*f+d*e)+3*b^2*(c^2*f^2-3*c*d*e*f+2*d^2*e^2))*x
/f^5-1/3*b*d^2*(-2*a*d*f-3*b*c*f+3*b*d*e)*x^3/f^4+1/5*b^2*d^3*x^5/f^3-1/4*
(-a*f+b*e)^2*(-c*f+d*e)^3*x/e/f^5/(f*x^2+e)^2+1/8*(-a*f+b*e)*(-c*f+d*e)^2*
(b*e*(-5*c*f+17*d*e)-3*a*f*(c*f+3*d*e))*x/e^2/f^5/(f*x^2+e)+1/8*(-c*f+d*e)
*(2*a*b*e*f*(-c^2*f^2-10*c*d*e*f+35*d^2*e^2)-3*b^2*e^2*(c^2*f^2-14*c*d*e*f
+21*d^2*e^2)-3*a^2*f^2*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*arctan(f^(1/2)*x/e^(
1/2))/e^(5/2)/f^(11/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^3} \, dx$$
$$= \frac{d(a^2d^2f^2+6abdf(-de+cf)+3b^2(2d^2e^2-3cdef+c^2f^2))\,x}{f^5}$$
$$- \frac{bd^2(3bde-3bcf-2adf)x^3}{3f^4} + \frac{b^2d^3x^5}{5f^3} - \frac{(be-af)^2(de-cf)^3x}{4ef^5\,(e+fx^2)^2}$$
$$+ \frac{(be-af)(de-cf)^2(be(17de-5cf)-3af(3de+cf))x}{8e^2f^5\,(e+fx^2)}$$
$$- \frac{(de-cf)\,(3b^2e^2(21d^2e^2-14cdef+c^2f^2)+3a^2f^2(5d^2e^2+2cdef+c^2f^2)+2abef(-35d^2e^2+10cde}{8e^{5/2}f^{11/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(d*(a^2*d^2*f^2 + 6*a*b*d*f*(-(d*e) + c*f) + 3*b^2*(2*d^2*e^2 - 3*c*d*e*f
+ c^2*f^2))*x)/f^5 - (b*d^2*(3*b*d*e - 3*b*c*f - 2*a*d*f)*x^3)/(3*f^4) + (
b^2*d^3*x^5)/(5*f^3) - ((b*e - a*f)^2*(d*e - c*f)^3*x)/(4*e*f^5*(e + f*x^2
)^2) + ((b*e - a*f)*(d*e - c*f)^2*(b*e*(17*d*e - 5*c*f) - 3*a*f*(3*d*e + c
*f))*x)/(8*e^2*f^5*(e + f*x^2)) - ((d*e - c*f)*(3*b^2*e^2*(21*d^2*e^2 - 14
*c*d*e*f + c^2*f^2) + 3*a^2*f^2*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*a*b*
e*f*(-35*d^2*e^2 + 10*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*
e^(5/2)*f^(11/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.76, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {425, 401, 25, 401, 403, 25, 299, 218, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)^3}{\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x(c+dx^2)^3}{4ef(e+}\right.}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{b\left(\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}+\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}-\frac{dx\left(c+dx^2\right)\cdots}{\cdots}}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}-\frac{dx\left(c+dx^2\right)\cdots}{\cdots}}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{-\frac{\frac{-\frac{3(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int \frac{1}{fx^2+e}dx}{f}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cde\cdots\right)\right)}{f}}{3f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$\downarrow$ 218

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-$$

$$\frac{(be-af)\left(-\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100c\cdots\right.\right.}{f}}{3f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$\downarrow$ 403

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\frac{\int\frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\left(5d^2e^2-6cdfe-3c^2f^2\right)-be\left(35d^2e^2-54cdfe+15c\cdots\right.\right.}{fx^2+e}}{3f}\cdots}{5f}}{2ef}\cdots\right)}{f}$$

$$\frac{(be-af)\left(-\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100c\cdots\right.\right.}{f}}{3f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\frac{\frac{15(de-cf)^2(be(7de-cf)-af(cf+5de))\int\frac{1}{fx^2+e}dx}{f}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+\frac{dx\left(\cdots\right.}{\cdots}}{5f}}{2ef}\cdots\right)}{f}$$

$$\frac{(be-af)\left(-\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100c\cdots\right.\right.}{f}}{3f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\frac{\frac{15\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{\sqrt{e}f^{3/2}}+\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{f}}{3f}+\frac{d}{}}{5f}}{2ef}\right.}{f}$$

$$\frac{(be-af)\left(\frac{-\frac{-\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100c\right.\right.}{f}}{3f}}{2ef}}{4ef}\right.}{f}$$

input

```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output

```
(b*(-1/2*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*
a*f)*x*(c + d*x^2)^2)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e -
3*c*f))*x*(c + d*x^2))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2
*f^2) - b*e*(105*d^2*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*
f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/(2*e*f)))/f - ((b*e - a*f)*(-1/4*((b*e -
a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*e - c*f) - 3*
a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(d*(b*e*(35*d*
e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-((d*(3*a*f*(15*d^
2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d*e*f + 3*c^2*f^
2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*
f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]
*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(315) = 630$.

Time = 0.69 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.94

method	result
default	$\frac{d\left(\frac{1}{5}f^2x^5b^2d^2+\frac{2}{3}ab\,d^2f^2x^3+b^2cd\,f^2x^3-b^2d^2ef\,x^3+a^2d^2f^2x+6abcd\,f^2x-6ab\,d^2efx+3b^2c^2f^2x-9b^2cdefx+6b^2d^2e^2x\right)}{f^5}+\frac{f\left(3a^2\right.}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
d/f^5*(1/5*f^2*x^5*b^2*d^2+2/3*a*b*d^2*f^2*x^3+b^2*c*d*f^2*x^3-b^2*d^2*e*f
*x^3+a^2*d^2*f^2*x+6*a*b*c*d*f^2*x-6*a*b*d^2*e*f*x+3*b^2*c^2*f^2*x-9*b^2*c
*d*e*f*x+6*b^2*d^2*e^2*x)+1/f^5*((1/8*f*(3*a^2*c^3*f^5+3*a^2*c^2*d*e*f^4-1
5*a^2*c*d^2*e^2*f^3+9*a^2*d^3*e^3*f^2+2*a*b*c^3*e*f^4-30*a*b*c^2*d*e^2*f^3
+54*a*b*c*d^2*e^3*f^2-26*a*b*d^3*e^4*f-5*b^2*c^3*e^2*f^3+27*b^2*c^2*d*e^3*
f^2-39*b^2*c*d^2*e^4*f+17*b^2*d^3*e^5)/e^2*x^3+1/8*(5*a^2*c^3*f^5-3*a^2*c^
2*d*e*f^4-9*a^2*c*d^2*e^2*f^3+7*a^2*d^3*e^3*f^2-2*a*b*c^3*e*f^4-18*a*b*c^2
*d*e^2*f^3+42*a*b*c*d^2*e^3*f^2-22*a*b*d^3*e^4*f-3*b^2*c^3*e^2*f^3+21*b^2*
c^2*d*e^3*f^2-33*b^2*c*d^2*e^4*f+15*b^2*d^3*e^5)/e*x)/(f*x^2+e)^2+1/8*(3*a
^2*c^3*f^5+3*a^2*c^2*d*e*f^4+9*a^2*c*d^2*e^2*f^3-15*a^2*d^3*e^3*f^2+2*a*b*
c^3*e*f^4+18*a*b*c^2*d*e^2*f^3-90*a*b*c*d^2*e^3*f^2+70*a*b*d^3*e^4*f+3*b^2
*c^3*e^2*f^3-45*b^2*c^2*d*e^3*f^2+105*b^2*c*d^2*e^4*f-63*b^2*d^3*e^5)/e^2/
(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(315) = 630$.

Time = 0.13 (sec) , antiderivative size = 1852, normalized size of antiderivative = 5.56

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*b^2*d^3*e^3*f^5*x^9 - 16*(9*b^2*d^3*e^4*f^4 - 5*(3*b^2*c*d^2 +
2*a*b*d^3)*e^3*f^5)*x^7 + 16*(63*b^2*d^3*e^5*f^3 - 35*(3*b^2*c*d^2 + 2*a*b
*d^3)*e^4*f^4 + 15*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^3*f^5)*x^5 + 10
*(315*b^2*d^3*e^6*f^2 + 9*a^2*c^3*e*f^7 - 175*(3*b^2*c*d^2 + 2*a*b*d^3)*e^
5*f^3 + 75*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^4*f^4 - 15*(b^2*c^3 + 6
*a*b*c^2*d + 3*a^2*c*d^2)*e^3*f^5 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f^6)*x
^3 + 15*(63*b^2*d^3*e^7 - 3*a^2*c^3*e^2*f^5 - 35*(3*b^2*c*d^2 + 2*a*b*d^3)
*e^6*f + 15*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^5*f^2 - 3*(b^2*c^3 + 6
*a*b*c^2*d + 3*a^2*c*d^2)*e^4*f^3 - (2*a*b*c^3 + 3*a^2*c^2*d)*e^3*f^4 + (6
3*b^2*d^3*e^5*f^2 - 3*a^2*c^3*f^7 - 35*(3*b^2*c*d^2 + 2*a*b*d^3)*e^4*f^3 +
 15*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^3*f^4 - 3*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*e^2*f^5 - (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^6)*x^4 + 2*(63*b
^2*d^3*e^6*f - 3*a^2*c^3*e*f^6 - 35*(3*b^2*c*d^2 + 2*a*b*d^3)*e^5*f^2 + 15
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^4*f^3 - 3*(b^2*c^3 + 6*a*b*c^2*d
+ 3*a^2*c*d^2)*e^3*f^4 - (2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f^5)*x^2)*sqrt(-e*f
)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(63*b^2*d^3*e^7*f + 5
*a^2*c^3*e^2*f^6 - 35*(3*b^2*c*d^2 + 2*a*b*d^3)*e^6*f^2 + 15*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*e^5*f^3 - 3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)
*e^4*f^4 - (2*a*b*c^3 + 3*a^2*c^2*d)*e^3*f^5)*x)/(e^3*f^8*x^4 + 2*e^4*f^7*
x^2 + e^5*f^6), 1/120*(24*b^2*d^3*e^3*f^5*x^9 - 8*(9*b^2*d^3*e^4*f^4 - ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. 2(335) = 670.

Time = 82.32 (sec) , antiderivative size = 1508, normalized size of antiderivative = 4.53

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
b**2*d**3*x**5/(5*f**3) + x**3*(2*a*b*d**3/(3*f**3) + b**2*c*d**2/f**3 - b
**2*d**3*e/f**4) + x*(a**2*d**3/f**3 + 6*a*b*c*d**2/f**3 - 6*a*b*d**3*e/f*
*4 + 3*b**2*c**2*d/f**3 - 9*b**2*c*d**2*e/f**4 + 6*b**2*d**3*e**2/f**5) -
sqrt(-1/(e**5*f**11))*(c*f - d*e)*(3*a**2*c**2*f**4 + 6*a**2*c*d*e*f**3 +
15*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f**3 + 20*a*b*c*d*e**2*f**2 - 70*a*b
*d**2*e**3*f + 3*b**2*c**2*e**2*f**2 - 42*b**2*c*d*e**3*f + 63*b**2*d**2*e
**4)*log(-e**3*f**5*sqrt(-1/(e**5*f**11))*(c*f - d*e)*(3*a**2*c**2*f**4 +
6*a**2*c*d*e*f**3 + 15*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f**3 + 20*a*b*c*
d*e**2*f**2 - 70*a*b*d**2*e**3*f + 3*b**2*c**2*e**2*f**2 - 42*b**2*c*d*e**
3*f + 63*b**2*d**2*e**4)/(3*a**2*c**3*f**5 + 3*a**2*c**2*d*e*f**4 + 9*a**2
*c*d**2*e**2*f**3 - 15*a**2*d**3*e**3*f**2 + 2*a*b*c**3*e*f**4 + 18*a*b*c*
*2*d*e**2*f**3 - 90*a*b*c*d**2*e**3*f**2 + 70*a*b*d**3*e**4*f + 3*b**2*c**
3*e**2*f**3 - 45*b**2*c**2*d*e**3*f**2 + 105*b**2*c*d**2*e**4*f - 63*b**2*
d**3*e**5) + x)/16 + sqrt(-1/(e**5*f**11))*(c*f - d*e)*(3*a**2*c**2*f**4 +
 6*a**2*c*d*e*f**3 + 15*a**2*d**2*e**2*f**2 + 2*a*b*c**2*e*f**3 + 20*a*b*c
*d*e**2*f**2 - 70*a*b*d**2*e**3*f + 3*b**2*c**2*e**2*f**2 - 42*b**2*c*d*e*
*3*f + 63*b**2*d**2*e**4)*log(e**3*f**5*sqrt(-1/(e**5*f**11))*(c*f - d*e)*
(3*a**2*c**2*f**4 + 6*a**2*c*d*e*f**3 + 15*a**2*d**2*e**2*f**2 + 2*a*b*c**
2*e*f**3 + 20*a*b*c*d*e**2*f**2 - 70*a*b*d**2*e**3*f + 3*b**2*c**2*e**2*f*
*2 - 42*b**2*c*d*e**3*f + 63*b**2*d**2*e**4)/(3*a**2*c**3*f**5 + 3*a**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(315) = 630$.

Time = 0.13 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^3}\,dx =$$

$$-\frac{(63\,b^2d^3e^5 - 105\,b^2cd^2e^4f - 70\,abd^3e^4f + 45\,b^2c^2de^3f^2 + 90\,abcd^2e^3f^2 + 15\,a^2d^3e^3f^2 - 3\,b^2c^3e^2f^3 - 1}{8\,\sqrt{ef}e^2f^5}$$

$$+\frac{17\,b^2d^3e^5fx^3 - 39\,b^2cd^2e^4f^2x^3 - 26\,abd^3e^4f^2x^3 + 27\,b^2c^2de^3f^3x^3 + 54\,abcd^2e^3f^3x^3 + 9\,a^2d^3e^3f^3x^3 - 5}{}$$

$$+\frac{3\,b^2d^3f^{12}x^5 - 15\,b^2d^3ef^{11}x^3 + 15\,b^2cd^2f^{12}x^3 + 10\,abd^3f^{12}x^3 + 90\,b^2d^3e^2f^{10}x - 135\,b^2cd^2ef^{11}x - 90\,ab}{15\,f^{15}}$$

input

```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output

```
-1/8*(63*b^2*d^3*e^5 - 105*b^2*c*d^2*e^4*f - 70*a*b*d^3*e^4*f + 45*b^2*c^2
*d*e^3*f^2 + 90*a*b*c*d^2*e^3*f^2 + 15*a^2*d^3*e^3*f^2 - 3*b^2*c^3*e^2*f^3
 - 18*a*b*c^2*d*e^2*f^3 - 9*a^2*c*d^2*e^2*f^3 - 2*a*b*c^3*e*f^4 - 3*a^2*c^
2*d*e*f^4 - 3*a^2*c^3*f^5)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^5) + 1/8
*(17*b^2*d^3*e^5*f*x^3 - 39*b^2*c*d^2*e^4*f^2*x^3 - 26*a*b*d^3*e^4*f^2*x^3
 + 27*b^2*c^2*d*e^3*f^3*x^3 + 54*a*b*c*d^2*e^3*f^3*x^3 + 9*a^2*d^3*e^3*f^3
*x^3 - 5*b^2*c^3*e^2*f^4*x^3 - 30*a*b*c^2*d*e^2*f^4*x^3 - 15*a^2*c*d^2*e^2
*f^4*x^3 + 2*a*b*c^3*e*f^5*x^3 + 3*a^2*c^2*d*e*f^5*x^3 + 3*a^2*c^3*f^6*x^3
 + 15*b^2*d^3*e^6*x - 33*b^2*c*d^2*e^5*f*x - 22*a*b*d^3*e^5*f*x + 21*b^2*c
^2*d*e^4*f^2*x + 42*a*b*c*d^2*e^4*f^2*x + 7*a^2*d^3*e^4*f^2*x - 3*b^2*c^3*
e^3*f^3*x - 18*a*b*c^2*d*e^3*f^3*x - 9*a^2*c*d^2*e^3*f^3*x - 2*a*b*c^3*e^2
*f^4*x - 3*a^2*c^2*d*e^2*f^4*x + 5*a^2*c^3*e*f^5*x)/((f*x^2 + e)^2*e^2*f^5
) + 1/15*(3*b^2*d^3*f^12*x^5 - 15*b^2*d^3*e*f^11*x^3 + 15*b^2*c*d^2*f^12*x
^3 + 10*a*b*d^3*f^12*x^3 + 90*b^2*d^3*e^2*f^10*x - 135*b^2*c*d^2*e*f^11*x
- 90*a*b*d^3*e*f^11*x + 45*b^2*c^2*d*f^12*x + 90*a*b*c*d^2*f^12*x + 15*a^2
*d^3*f^12*x)/f^15
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 901, normalized size of antiderivative = 2.71

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^3} \, dx$$

$$= x\left(\frac{a^2 d^3 + 6 a b c d^2 + 3 b^2 c^2 d}{f^3} + \frac{3 e \left(\frac{3 b^2 d^3 e}{f^4} - \frac{b d^2 (2 a d + 3 b c)}{f^3}\right)}{f} - \frac{3 b^2 d^3 e^2}{f^5}\right) - \frac{\frac{x (-5 a^2 c^3 f^5 + 3 a^2 c^2 d e f^4 + 9 a^2 c d^2 e^2 f^3 - 7 a^2 d^3 e^3 f^2 + 2 a b c^3 e f^4 + 18 a b c^2 d e^2 f^3 - 42 a b c d^2 e^3 f^2 + 22 a b d^3 e^4 f + 3 b^2 c^3 e^2 f^3 - 21 b^2}{8 e}}{}$$

$$- x^3 \left(\frac{b^2 d^3 e}{f^4} - \frac{b d^2 (2 a d + 3 b c)}{3 f^3}\right) + \frac{b^2 d^3 x^5}{5 f^3} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\, x (c f - d e) (3 a^2 c^2 f^4 + 6 a^2 c d e f^3 + 15 a^2 d^2 e^2 f^2 + 2 a b c^2 e f^3 + 20 a b c d e^2 f^2 - 70 a b d^2 e^3 f + 3 b^2 c^2 e^2 f}{\sqrt{e} (3 a^2 c^3 f^5 + 3 a^2 c^2 d e f^4 + 9 a^2 c d^2 e^2 f^3 - 15 a^2 d^3 e^3 f^2 + 2 a b c^3 e f^4 + 18 a b c^2 d e^2 f^3 - 90 a b c d^2 e^3 f^2 + 70 a b d^3 e^4 f + 3 b^2 c^3 e^2}\right.}{}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
x*((a^2*d^3 + 3*b^2*c^2*d + 6*a*b*c*d^2)/f^3 + (3*e*((3*b^2*d^3*e)/f^4 - (
b*d^2*(2*a*d + 3*b*c))/f^3))/f - (3*b^2*d^3*e^2)/f^5) - ((x*(3*b^2*c^3*e^2
*f^3 - 15*b^2*d^3*e^5 - 7*a^2*d^3*e^3*f^2 - 5*a^2*c^3*f^5 + 2*a*b*c^3*e*f^
4 + 22*a*b*d^3*e^4*f + 3*a^2*c^2*d*e*f^4 + 33*b^2*c*d^2*e^4*f + 9*a^2*c*d^
2*e^2*f^3 - 21*b^2*c^2*d*e^3*f^2 - 42*a*b*c*d^2*e^3*f^2 + 18*a*b*c^2*d*e^2
*f^3))/(8*e) - (x^3*(3*a^2*c^3*f^6 + 17*b^2*d^3*e^5*f + 9*a^2*d^3*e^3*f^3
- 5*b^2*c^3*e^2*f^4 + 2*a*b*c^3*e*f^5 - 26*a*b*d^3*e^4*f^2 + 3*a^2*c^2*d*e
*f^5 - 15*a^2*c*d^2*e^2*f^4 - 39*b^2*c*d^2*e^4*f^2 + 27*b^2*c^2*d*e^3*f^3
+ 54*a*b*c*d^2*e^3*f^3 - 30*a*b*c^2*d*e^2*f^4))/(8*e^2))/(e^2*f^5 + f^7*x^
4 + 2*e*f^6*x^2) - x^3*((b^2*d^3*e)/f^4 - (b*d^2*(2*a*d + 3*b*c))/(3*f^3))
 + (b^2*d^3*x^5)/(5*f^3) + (atan((f^(1/2)*x*(c*f - d*e)*(3*a^2*c^2*f^4 + 6
3*b^2*d^2*e^4 + 15*a^2*d^2*e^2*f^2 + 3*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 -
 70*a*b*d^2*e^3*f + 6*a^2*c*d*e*f^3 - 42*b^2*c*d*e^3*f + 20*a*b*c*d*e^2*f^
2))/(e^(1/2)*(3*a^2*c^3*f^5 - 63*b^2*d^3*e^5 - 15*a^2*d^3*e^3*f^2 + 3*b^2*
c^3*e^2*f^3 + 2*a*b*c^3*e*f^4 + 70*a*b*d^3*e^4*f + 3*a^2*c^2*d*e*f^4 + 105
*b^2*c*d^2*e^4*f + 9*a^2*c*d^2*e^2*f^3 - 45*b^2*c^2*d*e^3*f^2 - 90*a*b*c*d
^2*e^3*f^2 + 18*a*b*c^2*d*e^2*f^3)))*(c*f - d*e)*(3*a^2*c^2*f^4 + 63*b^2*d
^2*e^4 + 15*a^2*d^2*e^2*f^2 + 3*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 - 70*a*b
*d^2*e^3*f + 6*a^2*c*d*e*f^3 - 42*b^2*c*d*e^3*f + 20*a*b*c*d*e^2*f^2))/(8*
e^(5/2)*f^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1724, normalized size of antiderivative = 5.18

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^3} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
(45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e**2*f**5 + 90
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e*f**6*x**2 + 45*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*f**7*x**4 + 45*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**3*f**4 + 90*sqrt
(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**2*f**5*x**2 + 45*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**6*x**4 + 13
5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**4*f**3 + 27
0*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**3*f**4*x**2
 + 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f**5
*x**4 - 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**5*f
**2 - 450*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**4*f**
3*x**2 - 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**3*
f**4*x**4 + 30*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**3
*f**4 + 60*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**2*f**
5*x**2 + 30*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e*f**6*
x**4 + 270*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**4*f
**3 + 540*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**3*f*
*4*x**2 + 270*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**
2*f**5*x**4 - 1350*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*d**
2*e**5*f**2 - 2700*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c*...
```

3.235 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 426

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{(e+fx^2)^4}\,dx = -\frac{bd^2(4bde-3bcf-2adf)x}{f^5}+\frac{b^2d^3x^3}{3f^4} -\frac{(be-af)^2(de-cf)^3x}{6ef^5(e+fx^2)^3}+\frac{(be-af)(de-cf)^2(be(25de-7cf)-af(13de+5cf))x}{24e^2f^5(e+fx^2)^2} +\frac{(de-cf)\left(2abef(29d^2e^2-4cdef-c^2f^2)-b^2e^2(55d^2e^2-32cdef+c^2f^2)-a^2f^2(11d^2e^2+8cdef+5\right.}{16e^3f^5(e+fx^2)} -\frac{(2abef(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-b^2e^2(105d^3e^3-105cd^2e^2f+15c^2def^2+c^3f^3)-a^2f^2}{16e^{7/2}f^{11/2}}$$

output
```
-b*d^2*(-2*a*d*f-3*b*c*f+4*b*d*e)*x/f^5+1/3*b^2*d^3*x^3/f^4-1/6*(-a*f+b*e)
^2*(-c*f+d*e)^3*x/e/f^5/(f*x^2+e)^3+1/24*(-a*f+b*e)*(-c*f+d*e)^2*(b*e*(-7*
c*f+25*d*e)-a*f*(5*c*f+13*d*e))*x/e^2/f^5/(f*x^2+e)^2+1/16*(-c*f+d*e)*(2*a
*b*e*f*(-c^2*f^2-4*c*d*e*f+29*d^2*e^2)-b^2*e^2*(c^2*f^2-32*c*d*e*f+55*d^2*
e^2)-a^2*f^2*(5*c^2*f^2+8*c*d*e*f+11*d^2*e^2))*x/e^3/f^5/(f*x^2+e)-1/16*(2
*a*b*e*f*(-c^3*f^3-3*c^2*d*e*f^2-15*c*d^2*e^2*f+35*d^3*e^3)-b^2*e^2*(c^3*f
^3+15*c^2*d*e*f^2-105*c*d^2*e^2*f+105*d^3*e^3)-a^2*f^2*(5*c^3*f^3+3*c^2*d*
e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arctan(f^(1/2)*x/e^(1/2))/e^(7/2)/f^(11/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4} \, dx$$

$$= \frac{-48bd^2\sqrt{f}(4bde-3bcf-2adf)x+16b^2d^3f^{3/2}x^3+\frac{8\sqrt{f}(be-af)^2(-de+cf)^3x}{e(e+fx^2)^3}+\frac{2\sqrt{f}(-be+af)(de-cf)^2(af(13de+5cf}{e^2(e+fx^2)^2}}{}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(-48*b*d^2*Sqrt[f]*(4*b*d*e - 3*b*c*f - 2*a*d*f)*x + 16*b^2*d^3*f^(3/2)*x^
3 + (8*Sqrt[f]*(b*e - a*f)^2*(-(d*e) + c*f)^3*x)/(e*(e + f*x^2)^3) + (2*Sq
rt[f]*(-(b*e) + a*f)*(d*e - c*f)^2*(a*f*(13*d*e + 5*c*f) + b*e*(-25*d*e +
7*c*f))*x)/(e^2*(e + f*x^2)^2) + (3*Sqrt[f]*(-(d*e) + c*f)*(b^2*e^2*(55*d^
2*e^2 - 32*c*d*e*f + c^2*f^2) + 2*a*b*e*f*(-29*d^2*e^2 + 4*c*d*e*f + c^2*f
^2) + a^2*f^2*(11*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(e^3*(e + f*x^2)) +
 (3*(2*a*b*e*f*(-35*d^3*e^3 + 15*c*d^2*e^2*f + 3*c^2*d*e*f^2 + c^3*f^3) +
b^2*e^2*(105*d^3*e^3 - 105*c*d^2*e^2*f + 15*c^2*d*e*f^2 + c^3*f^3) + a^2*f
^2*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f
]*x)/Sqrt[e]])/e^(7/2))/(48*f^(11/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.65, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {425, 401, 25, 401, 25, 401, 299, 218, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4} \, dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^3}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^4}dx}{f}$$

$$\Big\downarrow \; 401$$

$$\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{6ef(e+fx^2)^3}\right)}{f}$$

$$\Big\downarrow \; 401$$

$$\frac{b\left(\frac{-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af))\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-af(5cf+de))}{4ef\left(e+fx^2\right)^2}}{6ef}-\frac{x\left(c+dx^2\right)^3(\ldots}{6ef(e+f\ldots}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{-\frac{\int \frac{(dx^2+c)(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{\frac{\int \frac{(dx^2+c)(d(be(35de-cf)-5af(de+cf))x^2+c(de(7be-af)+3cf(be+5af)))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x(c+dx^2)^3(be-}{6ef(e+fx^2)}\right.}{f}$$

$\downarrow$ 401

$$\frac{b\left(\frac{-\frac{\int \frac{(dx^2+c)(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$\frac{(be-af)\left(\frac{\frac{-\frac{\int \frac{c(af(5d^2e^2+6cdfe-15c^2f^2)-be(35d^2e^2+6cdfe+3c^2f^2))-d(be(105d^2e^2-10cdfe-3c^2f^2)-af(15d^2e^2+14cdfe+15c^2f^2))x^2}{fx^2+e}dx}{2ef}-\frac{x(c+}{}}{4ef}}{6ef}\right.}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{-\frac{\int \frac{(dx^2+c)(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}\right)}{f}$$

$$(be-af)\left(\frac{\frac{-\frac{\frac{3(be(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3)-af(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3))\int \frac{1}{fx^2+e}dx}{f}-\frac{dx(be(-3c^2f^2-10cdef+105d^2e^2)-af}{f}}{2ef}}{4ef}}{6}\right.$$

$\downarrow$ 218

$$b\left(\frac{-\frac{\int \frac{(dx^2+c)(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2)}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{2ef(e+fx^2)}}{4ef}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}\right)$$
$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{-\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\right.}{f}}{2ef}}{4ef}\right.$$

$\downarrow$ 403

$$b\left(\frac{-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x^2}{fx^2+e}dx}{3f}-\frac{dx(c+dx^2)(be(35de-3c}{3f}}{2ef}}{4ef}\right.$$
$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{-\frac{\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\right.}{f}}{2ef}}{4ef}\right.$$

$\downarrow$ 299

$$\frac{b\left(-\frac{-\frac{-\frac{3(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{f}}{3f}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\cdots\right)}{f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}\cdots$$

$$(be-af)\left(\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}}{2ef}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\cdots\right)}{f}\cdots}{4ef}\cdots\right)\cdots$$

$\downarrow$ 218

$$\frac{b\left(-\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{\sqrt{e}f^{3/2}}}{3f}-\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\cdots\right)\right)}{f}\cdots}{2ef}\cdots}{4ef}\cdots\right)}{f}\cdots$$

$$(be-af)\left(\frac{-\frac{-\frac{3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}f^{3/2}}}{2ef}-\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\cdots\right)}{f}\cdots}{4ef}\cdots\right)\cdots$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*(-1/4*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(
7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1
/3*(d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-
((d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c
*d*e*f + 3*c^2*f^2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f
- c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/S
qrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)))/f - ((b*e - a*f)*(-1
/6*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^3) + (-1/4*((b*e*(7*d*e
- c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^2) + (-1/2*(
(b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f*(5*d^2*e^2 + 4*c*d*e*f + 1
5*c^2*f^2))*x*(c + d*x^2))/(e*f*(e + f*x^2)) - (-((d*(b*e*(105*d^2*e^2 - 1
0*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))*x)/f)
 + (3*(b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(
5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)
/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(2*e*f))/(4*e*f))/(6*e*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.70

method	result
default	$\frac{b\,d^2\left(\frac{1}{3}bdf\,x^3+2adfx+3bcfx-4bdex\right)}{f^5}+\frac{\frac{f^2\left(5a^2c^3f^5+3a^2c^2de\,f^4+3a^2c\,d^2e^2f^3-11a^2d^3e^3f^2+2ab\,c^3e\,f^4+6ab\,c^2d\,e^2f^3-66abc\,d^2e^3f^2+5\right.}{16e^3}}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
b*d^2/f^5*(1/3*b*d*f*x^3+2*a*d*f*x+3*b*c*f*x-4*b*d*e*x)+1/f^5*((1/16*f^2*(
5*a^2*c^3*f^5+3*a^2*c^2*d*e*f^4+3*a^2*c*d^2*e^2*f^3-11*a^2*d^3*e^3*f^2+2*a
*b*c^3*e*f^4+6*a*b*c^2*d*e^2*f^3-66*a*b*c*d^2*e^3*f^2+58*a*b*d^3*e^4*f+b^2
*c^3*e^2*f^3-33*b^2*c^2*d*e^3*f^2+87*b^2*c*d^2*e^4*f-55*b^2*d^3*e^5)/e^3*x
^5+1/6*f*(5*a^2*c^3*f^5+3*a^2*c^2*d*e*f^4-3*a^2*c*d^2*e^2*f^3-5*a^2*d^3*e^
3*f^2+2*a*b*c^3*e*f^4-6*a*b*c^2*d*e^2*f^3-30*a*b*c*d^2*e^3*f^2+34*a*b*d^3*
e^4*f-b^2*c^3*e^2*f^3-15*b^2*c^2*d*e^3*f^2+51*b^2*c*d^2*e^4*f-35*b^2*d^3*e
^5)/e^2*x^3+1/16*(11*a^2*c^3*f^5-3*a^2*c^2*d*e*f^4-3*a^2*c*d^2*e^2*f^3-5*a
^2*d^3*e^3*f^2-2*a*b*c^3*e*f^4-6*a*b*c^2*d*e^2*f^3-30*a*b*c*d^2*e^3*f^2+38
*a*b*d^3*e^4*f-b^2*c^3*e^2*f^3-15*b^2*c^2*d*e^3*f^2+57*b^2*c*d^2*e^4*f-41*
b^2*d^3*e^5)/e*x)/(f*x^2+e)^3+1/16*(5*a^2*c^3*f^5+3*a^2*c^2*d*e*f^4+3*a^2*
c*d^2*e^2*f^3+5*a^2*d^3*e^3*f^2+2*a*b*c^3*e*f^4+6*a*b*c^2*d*e^2*f^3+30*a*b
*c*d^2*e^3*f^2-70*a*b*d^3*e^4*f+b^2*c^3*e^2*f^3+15*b^2*c^2*d*e^3*f^2-105*b
^2*c*d^2*e^4*f+105*b^2*d^3*e^5)/e^3/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1138 vs. 2(404) = 808.

Time = 0.18 (sec) , antiderivative size = 2296, normalized size of antiderivative = 5.39

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/96*(32*b^2*d^3*e^4*f^5*x^9 - 96*(3*b^2*d^3*e^5*f^4 - (3*b^2*c*d^2 + 2*a
*b*d^3)*e^4*f^5)*x^7 - 6*(231*b^2*d^3*e^6*f^3 - 5*a^2*c^3*e*f^8 - 77*(3*b^
2*c*d^2 + 2*a*b*d^3)*e^5*f^4 + 11*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^
4*f^5 - (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^3*f^6 - (2*a*b*c^3 + 3*a^2
*c^2*d)*e^2*f^7)*x^5 - 16*(105*b^2*d^3*e^7*f^2 - 5*a^2*c^3*e^2*f^7 - 35*(3
*b^2*c*d^2 + 2*a*b*d^3)*e^6*f^3 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*
e^5*f^4 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^4*f^5 - (2*a*b*c^3 + 3*a
^2*c^2*d)*e^3*f^6)*x^3 - 3*(105*b^2*d^3*e^8 + 5*a^2*c^3*e^3*f^5 - 35*(3*b^
2*c*d^2 + 2*a*b*d^3)*e^7*f + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^6*f
^2 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^5*f^3 + (2*a*b*c^3 + 3*a^2*c^
2*d)*e^4*f^4 + (105*b^2*d^3*e^5*f^3 + 5*a^2*c^3*f^8 - 35*(3*b^2*c*d^2 + 2*
a*b*d^3)*e^4*f^4 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^3*f^5 + (b^2*
c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2*f^6 + (2*a*b*c^3 + 3*a^2*c^2*d)*e*f^7
)*x^6 + 3*(105*b^2*d^3*e^6*f^2 + 5*a^2*c^3*e*f^7 - 35*(3*b^2*c*d^2 + 2*a*b
*d^3)*e^5*f^3 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^4*f^4 + (b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^3*f^5 + (2*a*b*c^3 + 3*a^2*c^2*d)*e^2*f^6)
*x^4 + 3*(105*b^2*d^3*e^7*f + 5*a^2*c^3*e^2*f^6 - 35*(3*b^2*c*d^2 + 2*a*b*
d^3)*e^6*f^2 + 5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^5*f^3 + (b^2*c^3
+ 6*a*b*c^2*d + 3*a^2*c*d^2)*e^4*f^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*e^3*f^5)*
x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(105*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4} dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(404) = 808.

Time = 0.13 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{(105\,b^2d^3e^5 - 105\,b^2cd^2e^4f - 70\,abd^3e^4f + 15\,b^2c^2de^3f^2 + 30\,abcd^2e^3f^2 + 5\,a^2d^3e^3f^2 + b^2c^3e^2f^3 + 6\,abc}{16\sqrt{ef}e^3f^5}$$
$$- \frac{165\,b^2d^3e^5f^2x^5 - 261\,b^2cd^2e^4f^3x^5 - 174\,abd^3e^4f^3x^5 + 99\,b^2c^2de^3f^4x^5 + 198\,abcd^2e^3f^4x^5 + 33\,a^2d^3e^3f}{}$$
$$+ \frac{b^2d^3f^8x^3 - 12\,b^2d^3ef^7x + 9\,b^2cd^2f^8x + 6\,abd^3f^8x}{3\,f^{12}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(105*b^2*d^3*e^5 - 105*b^2*c*d^2*e^4*f - 70*a*b*d^3*e^4*f + 15*b^2*c^
2*d*e^3*f^2 + 30*a*b*c*d^2*e^3*f^2 + 5*a^2*d^3*e^3*f^2 + b^2*c^3*e^2*f^3 +
 6*a*b*c^2*d*e^2*f^3 + 3*a^2*c*d^2*e^2*f^3 + 2*a*b*c^3*e*f^4 + 3*a^2*c^2*d
*e*f^4 + 5*a^2*c^3*f^5)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^5) - 1/48*(
165*b^2*d^3*e^5*f^2*x^5 - 261*b^2*c*d^2*e^4*f^3*x^5 - 174*a*b*d^3*e^4*f^3*
x^5 + 99*b^2*c^2*d*e^3*f^4*x^5 + 198*a*b*c*d^2*e^3*f^4*x^5 + 33*a^2*d^3*e^
3*f^4*x^5 - 3*b^2*c^3*e^2*f^5*x^5 - 18*a*b*c^2*d*e^2*f^5*x^5 - 9*a^2*c*d^2
*e^2*f^5*x^5 - 6*a*b*c^3*e*f^6*x^5 - 9*a^2*c^2*d*e*f^6*x^5 - 15*a^2*c^3*f^
7*x^5 + 280*b^2*d^3*e^6*f*x^3 - 408*b^2*c*d^2*e^5*f^2*x^3 - 272*a*b*d^3*e^
5*f^2*x^3 + 120*b^2*c^2*d*e^4*f^3*x^3 + 240*a*b*c*d^2*e^4*f^3*x^3 + 40*a^2
*d^3*e^4*f^3*x^3 + 8*b^2*c^3*e^3*f^4*x^3 + 48*a*b*c^2*d*e^3*f^4*x^3 + 24*a
^2*c*d^2*e^3*f^4*x^3 - 16*a*b*c^3*e^2*f^5*x^3 - 24*a^2*c^2*d*e^2*f^5*x^3 -
 40*a^2*c^3*e*f^6*x^3 + 123*b^2*d^3*e^7*x - 171*b^2*c*d^2*e^6*f*x - 114*a*
b*d^3*e^6*f*x + 45*b^2*c^2*d*e^5*f^2*x + 90*a*b*c*d^2*e^5*f^2*x + 15*a^2*d
^3*e^5*f^2*x + 3*b^2*c^3*e^4*f^3*x + 18*a*b*c^2*d*e^4*f^3*x + 9*a^2*c*d^2*
e^4*f^3*x + 6*a*b*c^3*e^3*f^4*x + 9*a^2*c^2*d*e^3*f^4*x - 33*a^2*c^3*e^2*f
^5*x)/((f*x^2 + e)^3*e^3*f^5) + 1/3*(b^2*d^3*f^8*x^3 - 12*b^2*d^3*e*f^7*x
+ 9*b^2*c*d^2*f^8*x + 6*a*b*d^3*f^8*x)/f^12
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4}\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(5\,a^2\,c^3\,f^5+3\,a^2\,c^2\,d\,e\,f^4+3\,a^2\,c\,d^2\,e^2\,f^3+5\,a^2\,d^3\,e^3\,f^2+2\,a\,b\,c^3\,e\,f^4+6\,a\,b\,c^2\,d\,e^2\,f^3+30\ldots}{16\,e^{7/2}\,f^{11/2}}$$

$$-\frac{\frac{x\,\left(-11\,a^2\,c^3\,f^5+3\,a^2\,c^2\,d\,e\,f^4+3\,a^2\,c\,d^2\,e^2\,f^3+5\,a^2\,d^3\,e^3\,f^2+2\,a\,b\,c^3\,e\,f^4+6\,a\,b\,c^2\,d\,e^2\,f^3+30\,a\,b\,c\,d^2\,e^3\,f^2-38\,a\,b\,d^3\,e^4\,f+b^2\,c^3\,e^2\,f^3+15\,b^2\ldots}{16\,e}}{\ldots}$$

$$-x\left(\frac{4\,b^2\,d^3\,e}{f^5}-\frac{b\,d^2\,(2\,a\,d+3\,b\,c)}{f^4}\right)+\frac{b^2\,d^3\,x^3}{3\,f^4}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
(atan((f^(1/2)*x)/e^(1/2))*(5*a^2*c^3*f^5 + 105*b^2*d^3*e^5 + 5*a^2*d^3*e^
3*f^2 + b^2*c^3*e^2*f^3 + 2*a*b*c^3*e*f^4 - 70*a*b*d^3*e^4*f + 3*a^2*c^2*d
*e*f^4 - 105*b^2*c*d^2*e^4*f + 3*a^2*c*d^2*e^2*f^3 + 15*b^2*c^2*d*e^3*f^2
+ 30*a*b*c*d^2*e^3*f^2 + 6*a*b*c^2*d*e^2*f^3))/(16*e^(7/2)*f^(11/2)) - ((x
*(41*b^2*d^3*e^5 - 11*a^2*c^3*f^5 + 5*a^2*d^3*e^3*f^2 + b^2*c^3*e^2*f^3 +
2*a*b*c^3*e*f^4 - 38*a*b*d^3*e^4*f + 3*a^2*c^2*d*e*f^4 - 57*b^2*c*d^2*e^4*
f + 3*a^2*c*d^2*e^2*f^3 + 15*b^2*c^2*d*e^3*f^2 + 30*a*b*c*d^2*e^3*f^2 + 6*
a*b*c^2*d*e^2*f^3))/(16*e) - (x^5*(5*a^2*c^3*f^7 - 11*a^2*d^3*e^3*f^4 + b^
2*c^3*e^2*f^5 - 55*b^2*d^3*e^5*f^2 + 2*a*b*c^3*e*f^6 + 58*a*b*d^3*e^4*f^3
+ 3*a^2*c^2*d*e*f^6 + 3*a^2*c*d^2*e^2*f^5 + 87*b^2*c*d^2*e^4*f^3 - 33*b^2*
c^2*d*e^3*f^4 - 66*a*b*c*d^2*e^3*f^4 + 6*a*b*c^2*d*e^2*f^5))/(16*e^3) + (x
^3*(35*b^2*d^3*e^5*f - 5*a^2*c^3*f^6 + 5*a^2*d^3*e^3*f^3 + b^2*c^3*e^2*f^4
 - 2*a*b*c^3*e*f^5 - 34*a*b*d^3*e^4*f^2 - 3*a^2*c^2*d*e*f^5 + 3*a^2*c*d^2*
e^2*f^4 - 51*b^2*c*d^2*e^4*f^2 + 15*b^2*c^2*d*e^3*f^3 + 30*a*b*c*d^2*e^3*f
^3 + 6*a*b*c^2*d*e^2*f^4))/(6*e^2))/(e^3*f^5 + f^8*x^6 + 3*e*f^7*x^4 + 3*e
^2*f^6*x^2) - x*((4*b^2*d^3*e)/f^5 - (b*d^2*(2*a*d + 3*b*c))/f^4) + (b^2*d
^3*x^3)/(3*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2236, normalized size of antiderivative = 5.25

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

output
```
(15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e**3*f**5 + 45
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e**2*f**6*x**2 +
45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e*f**7*x**4 + 1
5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*f**8*x**6 + 9*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**4*f**4 + 27*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**3*f**5*x**2 + 27
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**2*f**6*x**4
+ 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**7*x**6
+ 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**5*f**3 +
27*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**4*f**4*x**
2 + 27*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**3*f**5
*x**4 + 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f
**6*x**6 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**6
*f**2 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**5*f*
*3*x**2 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**4*
f**4*x**4 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*d**3*e**
3*f**5*x**6 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**
4*f**4 + 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**3*f*
*5*x**2 + 18*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**2*f
**6*x**4 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e*f...
```

3.236 $\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx=\frac{b^2x}{df}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(de-cf)}-\frac{(be-af)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}(de-cf)}$$

output
```
b^2*x/d/f+(-a*d+b*c)^2*arctan(d^(1/2)*x/c^(1/2))/c^(1/2)/d^(3/2)/(-c*f+d*e
)-(-a*f+b*e)^2*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)/f^(3/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx=\frac{b^2x}{df}-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(-de+cf)}-\frac{(be-af)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}(de-cf)}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b^2*x)/(d*f) - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/
2)*(-(d*e) + c*f)) - ((b*e - a*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*
f^(3/2)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {420, 299, 218, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \frac{bx^2+a}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b\left(\frac{bx}{f}-\frac{(be-af)\int \frac{1}{fx^2+e}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\int \frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{d}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*((b*x)/f - ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))
)/d - ((b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*S
qrt[d]*(d*e - c*f))) + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*
Sqrt[f]*(d*e - c*f))))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^2 x}{df}+\frac{(a^2 f^2-2abfe+b^2 e^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{f(cf-de)\sqrt{ef}}+\frac{(-a^2 d^2+2abcd-b^2 c^2)\arctan\left(\frac{xd}{\sqrt{cd}}\right)}{(cf-de)d\sqrt{cd}}$
risch	$\frac{b^2 x}{df}-\frac{f\ln\left(e f^2 x-(-ef)^{\frac{3}{2}}\right)a^2}{2\sqrt{-ef}\,(cf-de)}+\frac{\ln\left(e f^2 x-(-ef)^{\frac{3}{2}}\right)abe}{\sqrt{-ef}\,(cf-de)}-\frac{\ln\left(e f^2 x-(-ef)^{\frac{3}{2}}\right)b^2 e^2}{2f\sqrt{-ef}\,(cf-de)}+\frac{f\ln\left(-e f^2 x-(-ef)^{\frac{3}{2}}\right)a^2}{2\sqrt{-ef}\,(cf-de)}-\frac{\ln(-}{}$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d/f+1/f*(a^2*f^2-2*a*b*e*f+b^2*e^2)/(c*f-d*e)/(e*f)^(1/2)*arctan(f*x
/(e*f)^(1/2))+(-a^2*d^2+2*a*b*c*d-b^2*c^2)/(c*f-d*e)/d/(c*d)^(1/2)*arctan(
x*d/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 678, normalized size of antiderivative = 6.71

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx$$
$$=\left[\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cd}ef^2\log\left(\frac{dx^2+2\sqrt{-cd}x-c}{dx^2+c}\right)+(b^2cd^2e^2-2\,abcd^2ef+a^2cd^2f^2)\sqrt{-ef}\log\left(\frac{fx^2-}{}\right.}{2\,(cd^3e^2f^2-c^2d^2ef^3)}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*e*f^2*log((d*x^2 + 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + (b^2*c*d^2*e^2 - 2*a*b*c*d^2*e*f + a^2*c*d^2*
f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(b^2*c*d
^2*e^2*f - b^2*c^2*d*e*f^2)*x)/(c*d^3*e^2*f^2 - c^2*d^2*e*f^3), 1/2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*e*f^2*log((d*x^2 + 2*sqrt(-c*d)*x -
c)/(d*x^2 + c)) - 2*(b^2*c*d^2*e^2 - 2*a*b*c*d^2*e*f + a^2*c*d^2*f^2)*sqrt
(e*f)*arctan(sqrt(e*f)*x/e) + 2*(b^2*c*d^2*e^2*f - b^2*c^2*d*e*f^2)*x)/(c*
d^3*e^2*f^2 - c^2*d^2*e*f^3), 1/2*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
c*d)*e*f^2*arctan(sqrt(c*d)*x/c) + (b^2*c*d^2*e^2 - 2*a*b*c*d^2*e*f + a^2*
c*d^2*f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(b
^2*c*d^2*e^2*f - b^2*c^2*d*e*f^2)*x)/(c*d^3*e^2*f^2 - c^2*d^2*e*f^3), ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*e*f^2*arctan(sqrt(c*d)*x/c) - (b^2*
c*d^2*e^2 - 2*a*b*c*d^2*e*f + a^2*c*d^2*f^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/
e) + (b^2*c*d^2*e^2*f - b^2*c^2*d*e*f^2)*x)/(c*d^3*e^2*f^2 - c^2*d^2*e*f^3
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x}{df} + \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^2e-cdf)\sqrt{cd}} - \frac{(b^2e^2-2\,abef+a^2f^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(def-cf^2)\sqrt{ef}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
b^2*x/(d*f) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((d^2*
e - c*d*f)*sqrt(c*d)) - (b^2*e^2 - 2*a*b*e*f + a^2*f^2)*arctan(f*x/sqrt(e*
f))/((d*e*f - c*f^2)*sqrt(e*f))
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 3684, normalized size of antiderivative = 36.48

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
(atan((((((-c*d^3)^(1/2)*(a*d - b*c)^2*((4*a^2*c^2*d^3*f^5 + 4*a^2*d^5*e^2
*f^3 + 8*b^2*c^2*d^3*e^2*f^3 - 8*a^2*c*d^4*e*f^4 - 4*b^2*c*d^4*e^3*f^2 - 4
*b^2*c^3*d^2*e*f^4)/(d*f) + (x*(-c*d^3)^(1/2)*(a*d - b*c)^2*(4*c^3*d^3*f^6
 + 4*d^6*e^3*f^3 - 4*c*d^5*e^2*f^4 - 4*c^2*d^4*e*f^5))/(d*f*(c^2*d^3*f - c
*d^4*e))))/(2*(c^2*d^3*f - c*d^4*e)) + (2*x*(2*a^4*d^4*f^4 + b^4*c^4*f^4 +
 b^4*d^4*e^4 + 6*a^2*b^2*c^2*d^2*f^4 + 6*a^2*b^2*d^4*e^2*f^2 - 4*a*b^3*c^3
*d*f^4 - 4*a^3*b*c*d^3*f^4 - 4*a*b^3*d^4*e^3*f - 4*a^3*b*d^4*e*f^3))/(d*f)
)*(-c*d^3)^(1/2)*(a*d - b*c)^2*1i)/(2*(c^2*d^3*f - c*d^4*e)) - ((((-c*d^3)
^(1/2)*(a*d - b*c)^2*((4*a^2*c^2*d^3*f^5 + 4*a^2*d^5*e^2*f^3 + 8*b^2*c^2*d
^3*e^2*f^3 - 8*a^2*c*d^4*e*f^4 - 4*b^2*c*d^4*e^3*f^2 - 4*b^2*c^3*d^2*e*f^4
)/(d*f) - (x*(-c*d^3)^(1/2)*(a*d - b*c)^2*(4*c^3*d^3*f^6 + 4*d^6*e^3*f^3 -
 4*c*d^5*e^2*f^4 - 4*c^2*d^4*e*f^5))/(d*f*(c^2*d^3*f - c*d^4*e))))/(2*(c^2
*d^3*f - c*d^4*e)) - (2*x*(2*a^4*d^4*f^4 + b^4*c^4*f^4 + b^4*d^4*e^4 + 6*a
^2*b^2*c^2*d^2*f^4 + 6*a^2*b^2*d^4*e^2*f^2 - 4*a*b^3*c^3*d*f^4 - 4*a^3*b*c
*d^3*f^4 - 4*a*b^3*d^4*e^3*f - 4*a^3*b*d^4*e*f^3))/(d*f))*(-c*d^3)^(1/2)*(
a*d - b*c)^2*1i)/(2*(c^2*d^3*f - c*d^4*e)))/((2*(b^6*c^2*d*e^3 - 2*a^5*b*d
^3*f^3 + b^6*c^3*e^2*f + a^2*b^4*c^3*f^3 + a^2*b^4*d^3*e^3 - 2*a*b^5*c*d^2
*e^3 - 2*a*b^5*c^3*e*f^2 - 4*a^3*b^3*c^2*d*f^3 + 5*a^4*b^2*c*d^2*f^3 - 4*a
^3*b^3*d^3*e^2*f + 5*a^4*b^2*d^3*e*f^2 + 9*a^2*b^4*c*d^2*e^2*f + 9*a^2*b^4
*c^2*d*e*f^2 - 12*a^3*b^3*c*d^2*e*f^2 - 6*a*b^5*c^2*d*e^2*f))/(d*f) + (...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)a^2d^2e\,f^2+2\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)abcde\,f^2-\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)b^2c^2e\,f^2+\sqrt{f}\,\sqrt{e}}{c\,d^2}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e),x)
```

output
```
( - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e*f**2 + 2*sqr
t(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e*f**2 - sqrt(d)*sqrt(c
)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e*f**2 + sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*f**2 - 2*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*a*b*c*d**2*e*f + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(
e)))*b**2*c*d**2*e**2 + b**2*c**2*d*e*f**2*x - b**2*c*d**2*e**2*f*x)/(c*d*
*2*e*f**2*(c*f - d*e))
```

3.237 $\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx = -\frac{(be-af)^2x}{2ef(de-cf)(e+fx^2)} + \frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)^2} + \frac{(be-af)(be(de-3cf)+af(3de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}(de-cf)^2}$$

output
```
-1/2*(-a*f+b*e)^2*x/e/f/(-c*f+d*e)/(f*x^2+e)+(-a*d+b*c)^2*arctan(d^(1/2)*x
/c^(1/2))/c^(1/2)/d^(1/2)/(-c*f+d*e)^2+1/2*(-a*f+b*e)*(b*e*(-3*c*f+d*e)+a*
f*(-c*f+3*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(3/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{-\frac{(be-af)^2(de-cf)x}{ef(e+fx^2)} + \frac{2(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}} + \frac{(be-af)(be(de-3cf)+af(3de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}f^{3/2}}}{2(de-cf)^2}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(-(((b*e - a*f)^2*(d*e - c*f)*x)/(e*f*(e + f*x^2))) + (2*(b*c - a*d)^2*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]) + ((b*e - a*f)*(b*e*(d*e - 3*c
*f) + a*f*(3*d*e - c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*f^(3/2)))/(
2*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {419, 25, 299, 218, 401, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(bx^2+a)(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{dx^2+c}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{dx^2+c}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\int \frac{(bx^2+a)(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \frac{(bx^2+a)(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{f\left(b(be(de-3cf)+af(de+cf))x^2+a(af(3de-cf)-be(de+cf))\right)}{fx^2+e}dx}{2ef}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{b(be(de-3cf)+af(de+cf))x^2+a(af(3de-cf)-be(de+cf))}{fx^2+e}dx}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)(af(3de-cf)+be(de-3cf))\int \frac{1}{fx^2+e}dx}{f}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
-((d*(b*c - a*d)*((b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqr
t[c]*d^(3/2))))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*f)*x*(a + b*x^2))/
(2*e*(e + f*x^2)) - ((b*(b*e*(d*e - 3*c*f) + a*f*(d*e + c*f))*x)/f - ((b*e
 - a*f)*(b*e*(d*e - 3*c*f) + a*f*(3*d*e - c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]
])/(Sqrt[e]*f^(3/2)))/(2*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.36

method	result
default	$\frac{\frac{\left(a^{2} c f^{3}-a^{2} d e f^{2}-2 a b c e f^{2}+2 a b d e^{2} f+b^{2} c e^{2} f-b^{2} d e^{3}\right) x}{2 e f \left(f x^{2}+e\right)}+\frac{\left(a^{2} c f^{3}-3 a^{2} d e f^{2}+2 a b c e f^{2}+2 a b d e^{2} f-3 b^{2} c e^{2} f+b^{2} d e^{3}\right) \arctan\left(\frac{f x}{\sqrt{e f}}\right)}{2 e f \sqrt{e f}}}{(c f-d e)^{2}}+\cdots$
risch	$\frac{(a^{2} f^{2}-2 a b f e+b^{2} e^{2}) x}{2 e f (c f-d e)(f x^{2}+e)}-\frac{f^{2} \ln\left(e f^{2} x-(-e f)^{\frac{3}{2}}\right) a^{2} c}{4 \sqrt{-e f}\,(c f-d e)^{2} e}+\frac{3 f \ln\left(e f^{2} x-(-e f)^{\frac{3}{2}}\right) a^{2} d}{4 \sqrt{-e f}\,(c f-d e)^{2}}-\frac{f \ln\left(e f^{2} x-(-e f)^{\frac{3}{2}}\right) a b c}{2 \sqrt{-e f}\,(c f-d e)^{2}}-\cdots$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^2*(1/2*(a^2*c*f^3-a^2*d*e*f^2-2*a*b*c*e*f^2+2*a*b*d*e^2*f+b^2*
c*e^2*f-b^2*d*e^3)/e/f*x/(f*x^2+e)+1/2*(a^2*c*f^3-3*a^2*d*e*f^2+2*a*b*c*e*
f^2+2*a*b*d*e^2*f-3*b^2*c*e^2*f+b^2*d*e^3)/e/f/(e*f)^(1/2)*arctan(f*x/(e*f
)^(1/2)))+(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c*f-d*e)^2/(c*d)^(1/2)*arctan(x*d/(
c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(134) = 268.

Time = 5.25 (sec) , antiderivative size = 1702, normalized size of antiderivative = 11.05

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2*f^3*x^2 + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*e^3*f^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2
+ c)) + (b^2*c*d^2*e^4 + a^2*c^2*d*e*f^3 - (3*b^2*c^2*d - 2*a*b*c*d^2)*e^3
*f + (2*a*b*c^2*d - 3*a^2*c*d^2)*e^2*f^2 + (b^2*c*d^2*e^3*f + a^2*c^2*d*f^
4 - (3*b^2*c^2*d - 2*a*b*c*d^2)*e^2*f^2 + (2*a*b*c^2*d - 3*a^2*c*d^2)*e*f^
3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(b^2*
c*d^2*e^4*f - a^2*c^2*d*e*f^4 - (b^2*c^2*d + 2*a*b*c*d^2)*e^3*f^2 + (2*a*b
*c^2*d + a^2*c*d^2)*e^2*f^3)*x)/(c*d^3*e^5*f^2 - 2*c^2*d^2*e^4*f^3 + c^3*d
*e^3*f^4 + (c*d^3*e^4*f^3 - 2*c^2*d^2*e^3*f^4 + c^3*d*e^2*f^5)*x^2), 1/2*(
(b^2*c*d^2*e^4 + a^2*c^2*d*e*f^3 - (3*b^2*c^2*d - 2*a*b*c*d^2)*e^3*f + (2*
a*b*c^2*d - 3*a^2*c*d^2)*e^2*f^2 + (b^2*c*d^2*e^3*f + a^2*c^2*d*f^4 - (3*b
^2*c^2*d - 2*a*b*c*d^2)*e^2*f^2 + (2*a*b*c^2*d - 3*a^2*c*d^2)*e*f^3)*x^2)*
sqrt(e*f)*arctan(sqrt(e*f)*x/e) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2*f^3
*x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^3*f^2)*sqrt(-c*d)*log((d*x^2 - 2*
sqrt(-c*d)*x - c)/(d*x^2 + c)) - (b^2*c*d^2*e^4*f - a^2*c^2*d*e*f^4 - (b^2
*c^2*d + 2*a*b*c*d^2)*e^3*f^2 + (2*a*b*c^2*d + a^2*c*d^2)*e^2*f^3)*x)/(c*d
^3*e^5*f^2 - 2*c^2*d^2*e^4*f^3 + c^3*d*e^3*f^4 + (c*d^3*e^4*f^3 - 2*c^2*d^
2*e^3*f^4 + c^3*d*e^2*f^5)*x^2), 1/4*(4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e
^2*f^3*x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^3*f^2)*sqrt(c*d)*arctan(sqr
t(c*d)*x/c) - (b^2*c*d^2*e^4 + a^2*c^2*d*e*f^3 - (3*b^2*c^2*d - 2*a*b*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.37

$$\begin{aligned}&\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2}\,dx\\&=\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^2e^2-2\,cdef+c^2f^2)\sqrt{cd}}\\&\quad+\frac{(b^2de^3-3\,b^2ce^2f+2\,abde^2f+2\,abcef^2-3\,a^2def^2+a^2cf^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,(d^2e^3f-2\,cde^2f^2+c^2ef^3)\sqrt{ef}}\\&\quad-\frac{b^2e^2x-2\,abefx+a^2f^2x}{2\,(de^2f-cef^2)(fx^2+e)}\end{aligned}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((d^2*e^2 - 2*c*d*e*
f + c^2*f^2)*sqrt(c*d)) + 1/2*(b^2*d*e^3 - 3*b^2*c*e^2*f + 2*a*b*d*e^2*f +
 2*a*b*c*e*f^2 - 3*a^2*d*e*f^2 + a^2*c*f^3)*arctan(f*x/sqrt(e*f))/((d^2*e^
3*f - 2*c*d*e^2*f^2 + c^2*e*f^3)*sqrt(e*f)) - 1/2*(b^2*e^2*x - 2*a*b*e*f*x
 + a^2*f^2*x)/((d*e^2*f - c*e*f^2)*(f*x^2 + e))
```

Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 8320, normalized size of antiderivative = 54.03

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
(x*(a^2*f^2 + b^2*e^2 - 2*a*b*e*f))/(2*e*f*(e + f*x^2)*(c*f - d*e)) - (ata
n((((a*f - b*e)*((x*(b^4*d^5*e^6 + a^4*c^2*d^3*f^6 + 13*a^4*d^5*e^2*f^4 -
2*a^2*b^2*d^5*e^4*f^2 + 9*b^4*c^2*d^3*e^4*f^2 + 4*a*b^3*d^5*e^5*f - 6*a^4*
c*d^4*e*f^5 - 6*b^4*c*d^4*e^5*f - 12*a^3*b*d^5*e^3*f^3 + 4*b^4*c^4*d*e^2*f
^4 - 8*a*b^3*c*d^4*e^4*f^2 - 24*a^3*b*c*d^4*e^2*f^4 + 4*a^3*b*c^2*d^3*e*f^
5 - 12*a*b^3*c^2*d^3*e^3*f^3 - 16*a*b^3*c^3*d^2*e^2*f^4 + 28*a^2*b^2*c*d^4
*e^3*f^3 + 22*a^2*b^2*c^2*d^3*e^2*f^4))/(2*(d^2*e^4*f + c^2*e^2*f^3 - 2*c*
d*e^3*f^2)) - (((4*a^2*d^7*e^6*f^3 + 32*a^2*c^2*d^5*e^4*f^5 - 28*a^2*c^3*d
^4*e^3*f^6 + 12*a^2*c^4*d^3*e^2*f^7 - 8*b^2*c^2*d^5*e^6*f^3 + 12*b^2*c^3*d
^4*e^5*f^4 - 8*b^2*c^4*d^3*e^4*f^5 + 2*b^2*c^5*d^2*e^3*f^6 - 18*a^2*c*d^6*
e^5*f^4 - 2*a^2*c^5*d^2*e*f^8 + 2*b^2*c*d^6*e^7*f^2 + 16*a*b*c^2*d^5*e^5*f
^4 - 24*a*b*c^3*d^4*e^4*f^5 + 16*a*b*c^4*d^3*e^3*f^6 - 4*a*b*c^5*d^2*e^2*f
^7 - 4*a*b*c*d^6*e^6*f^3)/(d^3*e^5*f - c^3*e^2*f^4 - 3*c*d^2*e^4*f^2 + 3*c
^2*d*e^3*f^3) - (x*(a*f - b*e)*(-e^3*f^3)^(1/2)*(a*c*f^2 - b*d*e^2 - 3*a*d
*e*f + 3*b*c*e*f)*(16*d^7*e^7*f^3 - 48*c*d^6*e^6*f^4 + 32*c^2*d^5*e^5*f^5
+ 32*c^3*d^4*e^4*f^6 - 48*c^4*d^3*e^3*f^7 + 16*c^5*d^2*e^2*f^8))/(8*(d^2*e
^4*f + c^2*e^2*f^3 - 2*c*d*e^3*f^2)*(c^2*e^3*f^5 + d^2*e^5*f^3 - 2*c*d*e^4
*f^4)))*(a*f - b*e)*(-e^3*f^3)^(1/2)*(a*c*f^2 - b*d*e^2 - 3*a*d*e*f + 3*b*
c*e*f))/(4*(c^2*e^3*f^5 + d^2*e^5*f^3 - 2*c*d*e^4*f^4)))*(-e^3*f^3)^(1/2)*
(a*c*f^2 - b*d*e^2 - 3*a*d*e*f + 3*b*c*e*f)*1i)/(4*(c^2*e^3*f^5 + d^2*e...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 710, normalized size of antiderivative = 4.61

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
(2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e**3*f**2 + 2*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e**2*f**3*x**2 - 4*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e**3*f**2 - 4*sqrt(d
)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e**2*f**3*x**2 + 2*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e**3*f**2 + 2*sqrt(d)*sqr
t(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e**2*f**3*x**2 + sqrt(f)*sqrt
(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**3 + sqrt(f)*sqrt(e)*ata
n((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*f**4*x**2 - 3*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f**2 - 3*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e*f**3*x**2 + 2*sqrt(f)*sqrt(e)*atan((f
*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e**2*f**2 + 2*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a*b*c**2*d*e*f**3*x**2 + 2*sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*a*b*c*d**2*e**3*f + 2*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*a*b*c*d**2*e**2*f**2*x**2 - 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqr
t(f)*sqrt(e)))*b**2*c**2*d*e**3*f - 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*
sqrt(e)))*b**2*c**2*d*e**2*f**2*x**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)
*sqrt(e)))*b**2*c*d**2*e**4 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*b**2*c*d**2*e**3*f*x**2 + a**2*c**2*d*e*f**4*x - a**2*c*d**2*e**2*f**3*x
 - 2*a*b*c**2*d*e**2*f**3*x + 2*a*b*c*d**2*e**3*f**2*x + b**2*c**2*d*e**3*
f**2*x - b**2*c*d**2*e**4*f*x)/(2*c*d*e**2*f**2*(c**2*e*f**2 + c**2*f**...
```

3.238 $\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 274

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = -\frac{(be-af)^2x}{4ef(de-cf)(e+fx^2)^2} + \frac{(be-af)(be(de-5cf)+af(7de-3cf))x}{8e^2f(de-cf)^2(e+fx^2)} + \frac{\sqrt{d}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)^3} + \frac{(b^2e^2(d^2e^2-6cdef-3c^2f^2)+2abef(3d^2e^2+6cdef-c^2f^2)-a^2f^2(15d^2e^2-10cdef+3c^2f^2))\,\text{arcta}}{8e^{5/2}f^{3/2}(de-cf)^3}$$

output
```
-1/4*(-a*f+b*e)^2*x/e/f/(-c*f+d*e)/(f*x^2+e)^2+1/8*(-a*f+b*e)*(b*e*(-5*c*f
+d*e)+a*f*(-3*c*f+7*d*e))*x/e^2/f/(-c*f+d*e)^2/(f*x^2+e)+d^(1/2)*(-a*d+b*c
)^2*arctan(d^(1/2)*x/c^(1/2))/c^(1/2)/(-c*f+d*e)^3+1/8*(b^2*e^2*(-3*c^2*f^
2-6*c*d*e*f+d^2*e^2)+2*a*b*e*f*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2)-a^2*f^2*(3*c
^2*f^2-10*c*d*e*f+15*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/f^(3/2)/(
-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{1}{8}\left(-\frac{2(be-af)^2x}{ef(de-cf)(e+fx^2)^2}\right.$$
$$+\frac{(be-af)(be(de-5cf)+af(7de-3cf))x}{e^2f(de-cf)^2(e+fx^2)}-\frac{8\sqrt{d}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(-de+cf)^3}$$
$$+\frac{(b^2e^2(d^2e^2-6cdef-3c^2f^2)+a^2f^2(-15d^2e^2+10cdef-3c^2f^2)+2abef(3d^2e^2+6cdef-c^2f^2))\,\mathrm{arc}}{e^{5/2}f^{3/2}(de-cf)^3}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((-2*(b*e - a*f)^2*x)/(e*f*(d*e - c*f)*(e + f*x^2)^2) + ((b*e - a*f)*(b*e*
(d*e - 5*c*f) + a*f*(7*d*e - 3*c*f))*x)/(e^2*f*(d*e - c*f)^2*(e + f*x^2))
- (8*Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(-(d*e) +
 c*f)^3) + ((b^2*e^2*(d^2*e^2 - 6*c*d*e*f - 3*c^2*f^2) + a^2*f^2*(-15*d^2*
e^2 + 10*c*d*e*f - 3*c^2*f^2) + 2*a*b*e*f*(3*d^2*e^2 + 6*c*d*e*f - c^2*f^2
))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*f^(3/2)*(d*e - c*f)^3))/8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {419, 25, 397, 218, 401, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx$$
$$\Big\downarrow \; 419$$

$$-\frac{\int -\frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 397$$

$$\frac{\int \frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\int \frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\int \frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{f\left(b(af(5de-cf)-be(de+3cf))x^2+a(af(7de-3cf)-be(3de+cf))\right)}{(fx^2+e)^2}dx}{4ef}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{b(af(5de-cf)-be(de+3cf))x^2+a(af(7de-3cf)-be(3de+cf))}{(fx^2+e)^2}dx}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)\int\frac{1}{fx^2+e}dx}{2ef}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
-((d*(b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqr
t[d]*(d*e - c*f))) + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sq
rt[f]*(d*e - c*f))))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*f)*x*(a + b*x
^2))/(4*e*(e + f*x^2)^2) - (-1/2*((b*e - a*f)*(a*f*(7*d*e - 3*c*f) - b*e*(
d*e + 3*c*f))*x)/(e*f*(e + f*x^2)) + ((a^2*f^2*(7*d*e - 3*c*f) + 2*a*b*e*f
*(d*e - c*f) - b^2*e^2*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3
/2)*f^(3/2)))/(4*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.58

method	result
default	$\frac{\frac{\frac{\left(3a^2c^2f^4-10a^2cde\,f^3+7a^2d^2e^2f^2+2ab\,c^2e\,f^3+4abcd\,e^2f^2-6ab\,d^2e^3f-5b^2c^2e^2f^2+6b^2cd\,e^3f-b^2d^2e^4\right)x^3}{8e^2}+\frac{\left(5a^2c^2f^4-14a^2cde\,f^3+9a^2d^2e^2 \right.}{}}{\left(f\,x^2+e\right)^2}}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^3*((1/8*(3*a^2*c^2*f^4-10*a^2*c*d*e*f^3+7*a^2*d^2*e^2*f^2+2*a*
b*c^2*e*f^3+4*a*b*c*d*e^2*f^2-6*a*b*d^2*e^3*f-5*b^2*c^2*e^2*f^2+6*b^2*c*d*
e^3*f-b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-14*a^2*c*d*e*f^3+9*a^2*d^2*e
^2*f^2-2*a*b*c^2*e*f^3+12*a*b*c*d*e^2*f^2-10*a*b*d^2*e^3*f-3*b^2*c^2*e^2*f
^2+2*b^2*c*d*e^3*f+b^2*d^2*e^4)/e/f*x)/(f*x^2+e)^2+1/8*(3*a^2*c^2*f^4-10*a
^2*c*d*e*f^3+15*a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-12*a*b*c*d*e^2*f^2-6*a*b*d
^2*e^3*f+3*b^2*c^2*e^2*f^2+6*b^2*c*d*e^3*f-b^2*d^2*e^4)/e^2/f/(e*f)^(1/2)*
arctan(f*x/(e*f)^(1/2)))-d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c*f-d*e)^3/(c*d)^(
1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(252) = 504.

Time = 18.60 (sec) , antiderivative size = 3241, normalized size of antiderivative = 11.83

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)\left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to display
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{(b^2c^2d - 2\,abcd^2 + a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^3e^3 - 3\,cd^2e^2f + 3\,c^2def^2 - c^3f^3)\sqrt{cd}} + \frac{(b^2d^2e^4 - 6\,b^2cde^3f + 6\,abd^2e^3f - 3\,b^2c^2e^2f^2 + 12\,abcde^2f^2 - 15\,a^2d^2e^2f^2 - 2\,abc^2ef^3 + 10\,a^2cdef^3 -}{8\,(d^3e^5f - 3\,cd^2e^4f^2 + 3\,c^2de^3f^3 - c^3e^2f^4)\sqrt{ef}} + \frac{b^2de^3fx^3 - 5\,b^2ce^2f^2x^3 + 6\,abde^2f^2x^3 + 2\,abcef^3x^3 - 7\,a^2def^3x^3 + 3\,a^2cf^4x^3 - b^2de^4x - 3\,b^2ce^3fx +}{8\,(d^2e^4f - 2\,cde^3f^2 + c^2e^2f^3)(fx^2+e)^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/((d^3*e^3 - 3*c*
d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sqrt(c*d)) + 1/8*(b^2*d^2*e^4 - 6*b^2
*c*d*e^3*f + 6*a*b*d^2*e^3*f - 3*b^2*c^2*e^2*f^2 + 12*a*b*c*d*e^2*f^2 - 15
*a^2*d^2*e^2*f^2 - 2*a*b*c^2*e*f^3 + 10*a^2*c*d*e*f^3 - 3*a^2*c^2*f^4)*arc
tan(f*x/sqrt(e*f))/((d^3*e^5*f - 3*c*d^2*e^4*f^2 + 3*c^2*d*e^3*f^3 - c^3*e
^2*f^4)*sqrt(e*f)) + 1/8*(b^2*d*e^3*f*x^3 - 5*b^2*c*e^2*f^2*x^3 + 6*a*b*d*
e^2*f^2*x^3 + 2*a*b*c*e*f^3*x^3 - 7*a^2*d*e*f^3*x^3 + 3*a^2*c*f^4*x^3 - b^
2*d*e^4*x - 3*b^2*c*e^3*f*x + 10*a*b*d*e^3*f*x - 2*a*b*c*e^2*f^2*x - 9*a^2
*d*e^2*f^2*x + 5*a^2*c*e*f^3*x)/((d^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)
*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 13717, normalized size of antiderivative = 50.06

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
((x^3*(3*a^2*c*f^3 + b^2*d*e^3 - 7*a^2*d*e*f^2 - 5*b^2*c*e^2*f + 2*a*b*c*e
*f^2 + 6*a*b*d*e^2*f))/(8*e^2*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (x*(b^2*d
*e^3 - 5*a^2*c*f^3 + 9*a^2*d*e*f^2 + 3*b^2*c*e^2*f + 2*a*b*c*e*f^2 - 10*a*
b*d*e^2*f))/(8*e*(c^2*f^3 + d^2*e^2*f - 2*c*d*e*f^2)))/(e^2 + f^2*x^4 + 2*
e*f*x^2) + (atan(((((x*(b^4*d^7*e^8 + 9*a^4*c^4*d^3*f^8 + 289*a^4*d^7*e^4*
f^4 + 6*a^2*b^2*d^7*e^6*f^2 + 190*a^4*c^2*d^5*e^2*f^6 + 30*b^4*c^2*d^5*e^6
*f^2 + 36*b^4*c^3*d^4*e^5*f^3 + 73*b^4*c^4*d^3*e^4*f^4 + 12*a*b^3*d^7*e^7*
f - 12*b^4*c*d^6*e^7*f - 180*a^3*b*d^7*e^5*f^3 - 300*a^4*c*d^6*e^3*f^5 - 6
0*a^4*c^3*d^4*e*f^7 - 48*a*b^3*c*d^6*e^6*f^2 - 496*a^3*b*c*d^6*e^4*f^4 + 1
2*a^3*b*c^4*d^3*e*f^7 - 184*a*b^3*c^2*d^5*e^5*f^3 - 304*a*b^3*c^3*d^4*e^4*
f^4 + 12*a*b^3*c^4*d^3*e^3*f^5 + 344*a^2*b^2*c*d^6*e^5*f^3 + 264*a^3*b*c^2
*d^5*e^3*f^5 - 112*a^3*b*c^3*d^4*e^2*f^6 + 468*a^2*b^2*c^2*d^5*e^4*f^4 - 7
2*a^2*b^2*c^3*d^4*e^3*f^5 + 22*a^2*b^2*c^4*d^3*e^2*f^6))/(32*(d^4*e^8*f +
c^4*e^4*f^5 - 4*c*d^3*e^7*f^2 - 4*c^3*d*e^5*f^4 + 6*c^2*d^2*e^6*f^3)) - ((
(256*a^2*d^10*e^10*f^3 + 5280*a^2*c^2*d^8*e^8*f^5 - 9056*a^2*c^3*d^7*e^7*f
^6 + 9760*a^2*c^4*d^6*e^6*f^7 - 6816*a^2*c^5*d^5*e^5*f^8 + 3040*a^2*c^6*d^
4*e^4*f^9 - 800*a^2*c^7*d^3*e^3*f^10 + 96*a^2*c^8*d^2*e^2*f^11 - 96*b^2*c^
2*d^8*e^10*f^3 - 96*b^2*c^3*d^7*e^9*f^4 + 800*b^2*c^4*d^6*e^8*f^5 - 1440*b
^2*c^5*d^5*e^7*f^6 + 1248*b^2*c^6*d^4*e^6*f^7 - 544*b^2*c^7*d^3*e^5*f^8 +
96*b^2*c^8*d^2*e^4*f^9 - 1760*a^2*c*d^9*e^9*f^4 + 32*b^2*c*d^9*e^11*f^2...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1587, normalized size of antiderivative = 5.79

$$\int \frac{(a+bx^2)^2}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
( - 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e**5*f**2 -
16*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e**4*f**3*x**2
- 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**2*e**3*f**4*x**4
 + 16*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e**5*f**2 + 32
*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e**4*f**3*x**2 + 16
*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d*e**3*f**4*x**4 - 8*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e**5*f**2 - 16*sqr
t(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e**4*f**3*x**2 - 8*sq
rt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*e**3*f**4*x**4 + 3*s
qrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e**2*f**4 + 6*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*e*f**5*x**2 + 3*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**3*f**6*x**4 - 10*sqrt(f)*sq
rt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**3*f**3 - 20*sqrt(f)*sqr
t(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e**2*f**4*x**2 - 10*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c**2*d*e*f**5*x**4 + 15*sqrt(f
)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**4*f**2 + 30*sqrt(f)
*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**3*f**3*x**2 + 15*sqr
t(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d**2*e**2*f**4*x**4 + 2*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**3*f**3 + 4*sqrt(
f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**3*e**2*f**4*x**2 + 2*sq...
```

3.239 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = -\frac{(4abcdef-b^2ce(de+cf)-a^2df(de+cf))\,x}{2cde(de-cf)^2\,(e+fx^2)} + \frac{(bc-ad)^2x}{2cd(de-cf)\,(c+dx^2)\,(e+fx^2)} - \frac{(bc-ad)(ad(de-5cf)+bc(3de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}(de-cf)^3} - \frac{(be-af)(af(5de-cf)-be(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}\sqrt{f}(de-cf)^3}$$

output
```
-1/2*(4*a*b*c*d*e*f-b^2*c*e*(c*f+d*e)-a^2*d*f*(c*f+d*e))*x/c/d/e/(-c*f+d*e
)^2/(f*x^2+e)+1/2*(-a*d+b*c)^2*x/c/d/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)-1/2*(-
a*d+b*c)*(a*d*(-5*c*f+d*e)+b*c*(c*f+3*d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3
/2)/d^(1/2)/(-c*f+d*e)^3-1/2*(-a*f+b*e)*(a*f*(-c*f+5*d*e)-b*e*(3*c*f+d*e))
*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(1/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{1}{2}\left(\frac{(bc-ad)^2x}{c(de-cf)^2(c+dx^2)}+\frac{(be-af)^2x}{e(de-cf)^2(e+fx^2)} + \frac{(bc-ad)(ad(de-5cf)+bc(3de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}(-de+cf)^3} + \frac{(be-af)(af(-5de+cf)+be(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}\sqrt{f}(de-cf)^3}\right)$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(((b*c - a*d)^2*x)/(c*(d*e - c*f)^2*(c + d*x^2)) + ((b*e - a*f)^2*x)/(e*(d
*e - c*f)^2*(e + f*x^2)) + ((b*c - a*d)*(a*d*(d*e - 5*c*f) + b*c*(3*d*e +
c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]*(-(d*e) + c*f)^3) + ((
b*e - a*f)*(a*f*(-5*d*e + c*f) + b*e*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqr
t[e]])/(e^(3/2)*Sqrt[f]*(d*e - c*f)^3))/2
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {425, 402, 25, 397, 218, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)^2}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{\int -\frac{-d(be-af)x^2+bce-2ade+acf}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(be-af)}{2e(e+fx^2)(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\int \frac{-d(be-af)x^2+bce-2ade+acf}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$\Big\downarrow$ 397

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2de(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}+\frac{(af(3de-cf)-be(cf+de))\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$\Big\downarrow$ 218

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\int\frac{2\left(-df(2bce-ade-acf)x^2+bce(de+cf)+a\left(d^2e^2-4cdfe+c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e\left(e+fx^2\right)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\int\frac{-df(2bce-ade-acf)x^2+bce(de+cf)+a\left(d^2e^2-4cdfe+c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e\left(e+fx^2\right)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\frac{de(ad(de-5cf)+bc(3cf+de))\int\frac{1}{dx^2+c}dx}{de-cf}+\frac{cf(af(5de-cf)-be(cf+3de))\int\frac{1}{fx^2+e}dx}{de-cf}}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e\left(e+fx^2\right)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{b\left(\frac{x(be-af)}{2e(e+fx^2)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{\frac{\frac{\sqrt{d}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-5cf)+bc(3cf+de))}{\sqrt{c}(de-cf)}+\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5de-cf)-be(cf+3de))}{\sqrt{e}(de-cf)}}{e(de-cf)}-\frac{fx(-acf-ade+2bce)}{e\left(e+fx^2\right)(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{d}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(b*(((b*e - a*f)*x)/(2*e*(d*e - c*f)*(e + f*x^2)) - ((2*Sqrt[d]*(b*c - a*d
)*e*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + ((a*f*(3*d*e - c*
f) - b*e*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e -
 c*f)))/(2*e*(d*e - c*f))))/d - ((b*c - a*d)*(-1/2*((b*c - a*d)*x)/(c*(d*e
 - c*f)*(c + d*x^2)*(e + f*x^2)) + (-((f*(2*b*c*e - a*d*e - a*c*f)*x)/(e*(
d*e - c*f)*(e + f*x^2))) + ((Sqrt[d]*e*(a*d*(d*e - 5*c*f) + b*c*(d*e + 3*c
*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + (c*Sqrt[f]*(a*f*
(5*d*e - c*f) - b*e*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(
d*e - c*f)))/(e*(d*e - c*f)))/(2*c*(d*e - c*f))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.22

method	result
default	$\frac{\frac{\left(a^{2} c\,f^{3}-a^{2} d e\,f^{2}-2 a b c e\,f^{2}+2 a b d\,e^{2} f+b^{2} c\,e^{2} f-b^{2} d\,e^{3}\right) x}{2 e \left(f\,x^{2}+e\right)}+\frac{\left(a^{2} c\,f^{3}-5 a^{2} d e\,f^{2}+2 a b c e\,f^{2}+6 a b d\,e^{2} f-3 b^{2} c\,e^{2} f-b^{2} d\,e^{3}\right) \arctan\left(\frac{f x}{\sqrt{e f}}\right)}{2 e \sqrt{e f}}}{\left(c f-d e\right)^{3}}+\frac{\left(a^{2}\right.}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^3*(1/2*(a^2*c*f^3-a^2*d*e*f^2-2*a*b*c*e*f^2+2*a*b*d*e^2*f+b^2*
c*e^2*f-b^2*d*e^3)/e*x/(f*x^2+e)+1/2*(a^2*c*f^3-5*a^2*d*e*f^2+2*a*b*c*e*f^
2+6*a*b*d*e^2*f-3*b^2*c*e^2*f-b^2*d*e^3)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1
/2)))+1/(c*f-d*e)^3*(1/2*(a^2*c*d^2*f-a^2*d^3*e-2*a*b*c^2*d*f+2*a*b*c*d^2*
e+b^2*c^3*f-b^2*c^2*d*e)/c*x/(d*x^2+c)+1/2*(5*a^2*c*d^2*f-a^2*d^3*e-6*a*b*
c^2*d*f-2*a*b*c*d^2*e+b^2*c^3*f+3*b^2*c^2*d*e)/c/(c*d)^(1/2)*arctan(x*d/(c
*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(234) = 468$.

Time = 19.59 (sec) , antiderivative size = 3861, normalized size of antiderivative = 14.97

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx$$
$$= -\frac{(3\,b^2c^2de - 2\,abcd^2e - a^2d^3e + b^2c^3f - 6\,abc^2df + 5\,a^2cd^2f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(cd^3e^3 - 3\,c^2d^2e^2f + 3\,c^3def^2 - c^4f^3)\sqrt{cd}}$$
$$+ \frac{(b^2de^3 + 3\,b^2ce^2f - 6\,abde^2f - 2\,abcef^2 + 5\,a^2def^2 - a^2cf^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,(d^3e^4 - 3\,cd^2e^3f + 3\,c^2de^2f^2 - c^3ef^3)\sqrt{ef}}$$
$$+ \frac{b^2cde^2x^3 + b^2c^2efx^3 - 4\,abcdefx^3 + a^2d^2efx^3 + a^2cdf^2x^3 + 2\,b^2c^2e^2x - 2\,abcde^2x + a^2d^2e^2x - 2\,abc^2}{2\,(cd^2e^3 - 2\,c^2de^2f + c^3ef^2)(dfx^4 + dex^2 + cfx^2 + ce)}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c^2*d*e - 2*a*b*c*d^2*e - a^2*d^3*e + b^2*c^3*f - 6*a*b*c^2*d*
f + 5*a^2*c*d^2*f)*arctan(d*x/sqrt(c*d))/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3
*c^3*d*e*f^2 - c^4*f^3)*sqrt(c*d)) + 1/2*(b^2*d*e^3 + 3*b^2*c*e^2*f - 6*a*
b*d*e^2*f - 2*a*b*c*e*f^2 + 5*a^2*d*e*f^2 - a^2*c*f^3)*arctan(f*x/sqrt(e*f
))/((d^3*e^4 - 3*c*d^2*e^3*f + 3*c^2*d*e^2*f^2 - c^3*e*f^3)*sqrt(e*f)) + 1
/2*(b^2*c*d*e^2*x^3 + b^2*c^2*e*f*x^3 - 4*a*b*c*d*e*f*x^3 + a^2*d^2*e*f*x^
3 + a^2*c*d*f^2*x^3 + 2*b^2*c^2*e^2*x - 2*a*b*c*d*e^2*x + a^2*d^2*e^2*x -
2*a*b*c^2*e*f*x + a^2*c^2*f^2*x)/((c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*
(d*f*x^4 + d*e*x^2 + c*f*x^2 + c*e))
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 12581, normalized size of antiderivative = 48.76

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
((x^3*(a^2*c*d*f^2 + b^2*c*d*e^2 + a^2*d^2*e*f + b^2*c^2*e*f - 4*a*b*c*d*e
*f))/(2*c*e*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (x*(a^2*c^2*f^2 + a^2*d^2*e
^2 + 2*b^2*c^2*e^2 - 2*a*b*c*d*e^2 - 2*a*b*c^2*e*f))/(2*c*e*(c^2*f^2 + d^2
*e^2 - 2*c*d*e*f)))/(c*e + x^2*(c*f + d*e) + d*f*x^4) + (atan(((((-e^3*f)^(
1/2)*(a*f - b*e)*((x*(a^4*c^4*d^3*f^7 + a^4*d^7*e^4*f^3 + 50*a^4*c^2*d^5*e
^2*f^5 + 6*b^4*c^3*d^4*e^5*f^2 + 18*b^4*c^4*d^3*e^4*f^3 + 6*b^4*c^5*d^2*e^
3*f^4 - 10*a^4*c*d^6*e^3*f^4 - 10*a^4*c^3*d^4*e*f^6 + b^4*c^2*d^5*e^6*f +
b^4*c^6*d*e^2*f^5 + 4*a^3*b*c*d^6*e^4*f^3 + 4*a^3*b*c^4*d^3*e*f^6 - 12*a*b
^3*c^2*d^5*e^5*f^2 - 52*a*b^3*c^3*d^4*e^4*f^3 - 52*a*b^3*c^4*d^3*e^3*f^4 -
 12*a*b^3*c^5*d^2*e^2*f^5 - 68*a^3*b*c^2*d^5*e^3*f^4 - 68*a^3*b*c^3*d^4*e^
2*f^5 + 44*a^2*b^2*c^2*d^5*e^4*f^3 + 104*a^2*b^2*c^3*d^4*e^3*f^4 + 44*a^2*
b^2*c^4*d^3*e^2*f^5))/(2*(c^2*d^4*e^6 + c^6*e^2*f^4 - 4*c^3*d^3*e^5*f - 4*
c^5*d*e^3*f^3 + 6*c^4*d^2*e^4*f^2)) - (((20*a^2*c^2*d^9*e^8*f^3 - 80*a^2*c
^3*d^8*e^7*f^4 + 172*a^2*c^4*d^7*e^6*f^5 - 220*a^2*c^5*d^6*e^5*f^6 + 172*a
^2*c^6*d^5*e^4*f^7 - 80*a^2*c^7*d^4*e^3*f^8 + 20*a^2*c^8*d^3*e^2*f^9 + 4*b
^2*c^3*d^8*e^9*f^2 - 24*b^2*c^4*d^7*e^8*f^3 + 60*b^2*c^5*d^6*e^7*f^4 - 80*
b^2*c^6*d^5*e^6*f^5 + 60*b^2*c^7*d^4*e^5*f^6 - 24*b^2*c^8*d^3*e^4*f^7 + 4*
b^2*c^9*d^2*e^3*f^8 - 2*a^2*c*d^10*e^9*f^2 - 2*a^2*c^9*d^2*e*f^10 - 4*a*b*
c^2*d^9*e^9*f^2 + 20*a*b*c^3*d^8*e^8*f^3 - 36*a*b*c^4*d^7*e^7*f^4 + 20*a*b
*c^5*d^6*e^6*f^5 + 20*a*b*c^6*d^5*e^5*f^6 - 36*a*b*c^7*d^4*e^4*f^7 + 20...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1759, normalized size of antiderivative = 6.82

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2 (e+fx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**2*e**3*f**2
+ 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**2*e**2*f**3
*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**3*e**4*f +
 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**3*e**3*f**2*x**
2 + 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**3*e**2*f**3*
x**4 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**4*e**4*f*x**2
 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**4*e**3*f**2*x**4
- 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c**3*d*e**3*f**2 - 6
*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c**3*d*e**2*f**3*x**2 -
 2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c**2*d**2*e**4*f - 8*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c**2*d**2*e**3*f**2*x**2
 - 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c**2*d**2*e**2*f**3
*x**4 - 2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d**3*e**4*f*
x**2 - 2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b*c*d**3*e**3*f**
2*x**4 + sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**4*e**3*f**2
 + sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**4*e**2*f**3*x**2
+ 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**3*d*e**4*f + 4*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**3*d*e**3*f**2*x**2 +
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**3*d*e**2*f**3*x**4 +
 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**2*c**2*d**2*e**4*f*...
```

3.240 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 409

$$\begin{aligned}
&\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx \\
&= -\frac{(6abcdef-a^2df(2de+cf)-b^2ce(de+2cf))\,x}{4cde(de-cf)^2(e+fx^2)^2}+\frac{(bc-ad)^2x}{2cd(de-cf)(c+dx^2)(e+fx^2)^2} \\
&\quad-\frac{(2abcef(11de+cf)-3b^2ce^2(de+3cf)-a^2f(4d^2e^2+11cdef-3c^2f^2))\,x}{8ce^2(de-cf)^3(e+fx^2)} \\
&\quad-\frac{\sqrt{d}(bc-ad)(ad(de-7cf)+3bc(de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(de-cf)^4} \\
&\quad-\frac{(2abef(15d^2e^2+10cdef-c^2f^2)-3b^2e^2(d^2e^2+6cdef+c^2f^2)-a^2f^2(35d^2e^2-14cdef+3c^2f^2))\,\mathrm{arc}}{8e^{5/2}\sqrt{f}(de-cf)^4}
\end{aligned}$$

output
```
-1/4*(6*a*b*c*d*e*f-a^2*d*f*(c*f+2*d*e)-b^2*c*e*(2*c*f+d*e))*x/c/d/e/(-c*f
+d*e)^2/(f*x^2+e)^2+1/2*(-a*d+b*c)^2*x/c/d/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)^
2-1/8*(2*a*b*c*e*f*(c*f+11*d*e)-3*b^2*c*e^2*(3*c*f+d*e)-a^2*f*(-3*c^2*f^2+
11*c*d*e*f+4*d^2*e^2))*x/c/e^2/(-c*f+d*e)^3/(f*x^2+e)-1/2*d^(1/2)*(-a*d+b*
c)*(a*d*(-7*c*f+d*e)+3*b*c*(c*f+d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/(-
c*f+d*e)^4-1/8*(2*a*b*e*f*(-c^2*f^2+10*c*d*e*f+15*d^2*e^2)-3*b^2*e^2*(c^2*
f^2+6*c*d*e*f+d^2*e^2)-a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2))*arctan(f
^(1/2)*x/e^(1/2))/e^(5/2)/f^(1/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(-\frac{4d(bc-ad)^2x}{c(-de+cf)^3(c+dx^2)}+\frac{2(be-af)^2x}{e(de-cf)^2(e+fx^2)^2}\right.$$
$$+\frac{(be-af)(af(-11de+3cf)+be(3de+5cf))x}{e^2(de-cf)^3(e+fx^2)}$$
$$+\frac{4\sqrt{d}(-bc+ad)(ad(de-7cf)+3bc(de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(de-cf)^4}$$
$$+\frac{(2abef(-15d^2e^2-10cdef+c^2f^2)+3b^2e^2(d^2e^2+6cdef+c^2f^2)+a^2f^2(35d^2e^2-14cdef+3c^2f^2))\,a}{e^{5/2}\sqrt{f}(de-cf)^4}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
((-4*d*(b*c - a*d)^2*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)) + (2*(b*e - a*f)^
2*x)/(e*(d*e - c*f)^2*(e + f*x^2)^2) + ((b*e - a*f)*(a*f*(-11*d*e + 3*c*f)
 + b*e*(3*d*e + 5*c*f))*x)/(e^2*(d*e - c*f)^3*(e + f*x^2)) + (4*Sqrt[d]*(-
(b*c) + a*d)*(a*d*(d*e - 7*c*f) + 3*b*c*(d*e + c*f))*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(c^(3/2)*(d*e - c*f)^4) + ((2*a*b*e*f*(-15*d^2*e^2 - 10*c*d*e*f +
c^2*f^2) + 3*b^2*e^2*(d^2*e^2 + 6*c*d*e*f + c^2*f^2) + a^2*f^2*(35*d^2*e^2
 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*Sqrt[f]*
(d*e - c*f)^4))/8
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.56, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {425, 402, 25, 402, 27, 397, 218, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{\int -\frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}+\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\int \frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\int \frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\int \frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\frac{8d^2e^2(bc-ad)\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+\cdots}{2e\left(e+fx^2\right)(de-cf\cdots}}{4e(de-cf)}\cdots}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\frac{8d^{3/2}e^2(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}+\frac{x(af(7de-3cf)-}{2e\left(e+fx^2\right)}}{4e(de-cf)}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\int\frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\frac{8d^{3/2}e^2(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}+\frac{x(af(7de-3cf)-}{2e\left(e+fx^2\right)}}{4e(de-cf)}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\frac{\int\frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx\left(bce(cf+11de\right.}{2e}}{2e(de-cf)}}{2c(de-cf)}\right.}{d}$$

$\downarrow$ 397

$$b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\frac{8d^{3/2}e^2(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}+\frac{x(af(7de-3cf)-}{2e\left(e+fx^2\right)}}{4e(de-cf)}\right.$$

$$\overline{d}$$

$$(bc-ad)\left(\frac{\frac{\frac{4d^2e^2(ad(de-7cf)+bc(5cf+de))\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{cf\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{fx\left(bce(cf+11de\right.}{2e}}{2e(de-cf)}}{2c(de-cf)}\right.$$

$$\overline{d}$$

$\downarrow$ 218

$$b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\frac{\frac{8d^{3/2}e^2(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}+\frac{x(af(7de-3cf)-}{2e\left(e+fx^2\right)}}{4e(de-cf)}\right.$$

$$\overline{d}$$

$$(bc-ad)\left(\frac{\frac{\frac{4d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-7cf)+bc(5cf+de))}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx\left(bc\right.}{}}{2e(de-cf)}}{2c(de-cf)}\right.$$

$$\overline{d}$$

input
```
Int[(a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(b*(((b*e - a*f)*x)/(4*e*(d*e - c*f)*(e + f*x^2)^2) - (((a*f*(7*d*e - 3*c*
f) - b*e*(3*d*e + c*f))*x)/(2*e*(d*e - c*f)*(e + f*x^2)) + ((8*d^(3/2)*(b*
c - a*d)*e^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - ((b*e*(3
*d^2*e^2 + 6*c*d*e*f - c^2*f^2) - a*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2
))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)))/(2*e*(d*e -
 c*f)))/(4*e*(d*e - c*f))))/d - ((b*c - a*d)*(-1/2*((b*c - a*d)*x)/(c*(d*e
 - c*f)*(c + d*x^2)*(e + f*x^2)^2) + (-1/2*(f*(3*b*c*e - 2*a*d*e - a*c*f)*
x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(f*(b*c*e*(11*d*e + c*f) - a*(4*d
^2*e^2 + 11*c*d*e*f - 3*c^2*f^2))*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((4*d^(
3/2)*e^2*(a*d*(d*e - 7*c*f) + b*c*(d*e + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(Sqrt[c]*(d*e - c*f)) - (c*Sqrt[f]*(b*e*(15*d^2*e^2 + 10*c*d*e*f - c^2
*f^2) - a*f*(35*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*c*(d
*e - c*f))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.32

method	result
default	$\frac{\frac{f\left(3a^2c^2f^4-14a^2cde\,f^3+11a^2d^2e^2f^2+2ab\,c^2e\,f^3+12abcd\,e^2f^2-14ab\,d^2e^3f-5b^2c^2e^2f^2+2b^2cd\,e^3f+3b^2d^2e^4\right)x^3}{8e^2}+\frac{\left(5a^2c^2f^4-18a^2cde\,f^3+13\right.}{}}{\left(f\,x^2+e\right)^2}$
risch	Expression too large to display

input
```
int((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^4*((1/8*f*(3*a^2*c^2*f^4-14*a^2*c*d*e*f^3+11*a^2*d^2*e^2*f^2+2
*a*b*c^2*e*f^3+12*a*b*c*d*e^2*f^2-14*a*b*d^2*e^3*f-5*b^2*c^2*e^2*f^2+2*b^2
*c*d*e^3*f+3*b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-18*a^2*c*d*e*f^3+13*a
^2*d^2*e^2*f^2-2*a*b*c^2*e*f^3+20*a*b*c*d*e^2*f^2-18*a*b*d^2*e^3*f-3*b^2*c
^2*e^2*f^2-2*b^2*c*d*e^3*f+5*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a^2*c^2*
f^4-14*a^2*c*d*e*f^3+35*a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-20*a*b*c*d*e^2*f^2
-30*a*b*d^2*e^3*f+3*b^2*c^2*e^2*f^2+18*b^2*c*d*e^3*f+3*b^2*d^2*e^4)/e^2/(e
*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d/(c*f-d*e)^4*(1/2*(a^2*c*d^2*f-a^2*d^3
*e-2*a*b*c^2*d*f+2*a*b*c*d^2*e+b^2*c^3*f-b^2*c^2*d*e)/c*x/(d*x^2+c)+1/2*(7
*a^2*c*d^2*f-a^2*d^3*e-10*a*b*c^2*d*f-2*a*b*c*d^2*e+3*b^2*c^3*f+3*b^2*c^2*
d*e)/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(381) = 762$.

Time = 64.86 (sec) , antiderivative size = 6809, normalized size of antiderivative = 16.65

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx$$
$$= -\frac{(3\,b^2c^2d^2e - 2\,abcd^3e - a^2d^4e + 3\,b^2c^3df - 10\,abc^2d^2f + 7\,a^2cd^3f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(cd^4e^4 - 4\,c^2d^3e^3f + 6\,c^3d^2e^2f^2 - 4\,c^4def^3 + c^5f^4)\sqrt{cd}}$$
$$+ \frac{(3\,b^2d^2e^4 + 18\,b^2cde^3f - 30\,abd^2e^3f + 3\,b^2c^2e^2f^2 - 20\,abcde^2f^2 + 35\,a^2d^2e^2f^2 + 2\,abc^2ef^3 - 14\,a^2cde}{8\,(d^4e^6 - 4\,cd^3e^5f + 6\,c^2d^2e^4f^2 - 4\,c^3de^3f^3 + c^4e^2f^4)\sqrt{ef}}$$
$$+ \frac{b^2c^2dx - 2\,abcd^2x + a^2d^3x}{2\,(cd^3e^3 - 3\,c^2d^2e^2f + 3\,c^3def^2 - c^4f^3)(dx^2 + c)}$$
$$+ \frac{3\,b^2de^3fx^3 + 5\,b^2ce^2f^2x^3 - 14\,abde^2f^2x^3 - 2\,abcef^3x^3 + 11\,a^2def^3x^3 - 3\,a^2cf^4x^3 + 5\,b^2de^4x + 3\,b^2ce}{8\,(d^3e^5 - 3\,cd^2e^4f + 3\,c^2de^3f^2 - c^3e^2f^3)(fx^2 +}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c^2*d^2*e - 2*a*b*c*d^3*e - a^2*d^4*e + 3*b^2*c^3*d*f - 10*a*b
*c^2*d^2*f + 7*a^2*c*d^3*f)*arctan(d*x/sqrt(c*d))/((c*d^4*e^4 - 4*c^2*d^3*
e^3*f + 6*c^3*d^2*e^2*f^2 - 4*c^4*d*e*f^3 + c^5*f^4)*sqrt(c*d)) + 1/8*(3*b
^2*d^2*e^4 + 18*b^2*c*d*e^3*f - 30*a*b*d^2*e^3*f + 3*b^2*c^2*e^2*f^2 - 20*
a*b*c*d*e^2*f^2 + 35*a^2*d^2*e^2*f^2 + 2*a*b*c^2*e*f^3 - 14*a^2*c*d*e*f^3
+ 3*a^2*c^2*f^4)*arctan(f*x/sqrt(e*f))/((d^4*e^6 - 4*c*d^3*e^5*f + 6*c^2*d
^2*e^4*f^2 - 4*c^3*d*e^3*f^3 + c^4*e^2*f^4)*sqrt(e*f)) + 1/2*(b^2*c^2*d*x
- 2*a*b*c*d^2*x + a^2*d^3*x)/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2
 - c^4*f^3)*(d*x^2 + c)) + 1/8*(3*b^2*d*e^3*f*x^3 + 5*b^2*c*e^2*f^2*x^3 -
14*a*b*d*e^2*f^2*x^3 - 2*a*b*c*e*f^3*x^3 + 11*a^2*d*e*f^3*x^3 - 3*a^2*c*f^
4*x^3 + 5*b^2*d*e^4*x + 3*b^2*c*e^3*f*x - 18*a*b*d*e^3*f*x + 2*a*b*c*e^2*f
^2*x + 13*a^2*d*e^2*f^2*x - 5*a^2*c*e*f^3*x)/((d^3*e^5 - 3*c*d^2*e^4*f + 3
*c^2*d*e^3*f^2 - c^3*e^2*f^3)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 127501, normalized size of antiderivative = 311.74

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
atan(((((3584*a^2*c^2*d^12*e^12*f^3 - 20160*a^2*c^3*d^11*e^11*f^4 + 63168*
a^2*c^4*d^10*e^10*f^5 - 125184*a^2*c^5*d^9*e^9*f^6 + 166656*a^2*c^6*d^8*e^
8*f^7 - 153216*a^2*c^7*d^7*e^7*f^8 + 97920*a^2*c^8*d^6*e^6*f^9 - 43008*a^2
*c^9*d^5*e^5*f^10 + 12544*a^2*c^10*d^4*e^4*f^11 - 2240*a^2*c^11*d^3*e^3*f^
12 + 192*a^2*c^12*d^2*e^2*f^13 + 576*b^2*c^3*d^11*e^13*f^2 - 4416*b^2*c^4*
d^10*e^12*f^3 + 14592*b^2*c^5*d^9*e^11*f^4 - 26880*b^2*c^6*d^8*e^10*f^5 +
29568*b^2*c^7*d^7*e^9*f^6 - 18816*b^2*c^8*d^6*e^8*f^7 + 5376*b^2*c^9*d^5*e
^7*f^8 + 768*b^2*c^10*d^4*e^6*f^9 - 960*b^2*c^11*d^3*e^5*f^10 + 192*b^2*c^
12*d^2*e^4*f^11 - 256*a^2*c*d^13*e^13*f^2 - 512*a*b*c^2*d^12*e^13*f^2 + 29
44*a*b*c^3*d^11*e^12*f^3 - 4992*a*b*c^4*d^10*e^11*f^4 - 4608*a*b*c^5*d^9*e
^10*f^5 + 32256*a*b*c^6*d^8*e^9*f^6 - 59136*a*b*c^7*d^7*e^8*f^7 + 59136*a*
b*c^8*d^6*e^7*f^8 - 35328*a*b*c^9*d^5*e^6*f^9 + 12288*a*b*c^10*d^4*e^5*f^1
0 - 2176*a*b*c^11*d^3*e^4*f^11 + 128*a*b*c^12*d^2*e^3*f^12)/(128*(c^2*d^9*
e^13 - c^11*e^4*f^9 - 9*c^3*d^8*e^12*f + 9*c^10*d*e^5*f^8 + 36*c^4*d^7*e^1
1*f^2 - 84*c^5*d^6*e^10*f^3 + 126*c^6*d^5*e^9*f^4 - 126*c^7*d^4*e^8*f^5 +
84*c^8*d^3*e^7*f^6 - 36*c^9*d^2*e^6*f^7)) - (x*(-(72*a^4*c^15*f^16 - ((144
*a^4*c^15*f^16 + 256*a^4*d^15*e^15*f + 144*b^4*c^3*d^12*e^16 + 144*b^4*c^1
5*e^4*f^12 + 352*a^2*b^2*c^15*e^2*f^14 + 48384*a^4*c^2*d^13*e^13*f^3 - 195
440*a^4*c^3*d^12*e^12*f^4 + 397376*a^4*c^4*d^11*e^11*f^5 - 286944*a^4*c^5*
d^10*e^10*f^6 - 504000*a^4*c^6*d^9*e^9*f^7 + 1638000*a^4*c^7*d^8*e^8*f^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3303, normalized size of antiderivative = 8.08

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output

```
( - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**2*e**5*f
**2 - 56*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**2*e**4
*f**3*x**2 - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d*
*2*e**3*f**4*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c
*d**3*e**6*f - 20*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**
3*e**5*f**2*x**2 - 52*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c
*d**3*e**4*f**3*x**4 - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*
*2*c*d**3*e**3*f**4*x**6 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))
*a**2*d**4*e**6*f*x**2 + 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a
**2*d**4*e**5*f**2*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*
a**2*d**4*e**4*f**3*x**6 + 40*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c))
)*a*b*c**3*d*e**5*f**2 + 80*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*
a*b*c**3*d*e**4*f**3*x**2 + 40*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)
))*a*b*c**3*d*e**3*f**4*x**4 + 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(
c)))*a*b*c**2*d**2*e**6*f + 56*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)
))*a*b*c**2*d**2*e**5*f**2*x**2 + 88*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*s
qrt(c)))*a*b*c**2*d**2*e**4*f**3*x**4 + 40*sqrt(d)*sqrt(c)*atan((d*x)/(sqr
t(d)*sqrt(c)))*a*b*c**2*d**2*e**3*f**4*x**6 + 8*sqrt(d)*sqrt(c)*atan((d*x)
/(sqrt(d)*sqrt(c)))*a*b*c*d**3*e**6*f*x**2 + 16*sqrt(d)*sqrt(c)*atan((d*x)
/(sqrt(d)*sqrt(c)))*a*b*c*d**3*e**5*f**2*x**4 + 8*sqrt(d)*sqrt(c)*atan(...
```

3.241 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 588

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx$$

$$= -\frac{f(b^2c^2e(7de+5cf)-2abcde(de+11cf)-a^2d(3d^2e^2-13cdef-2c^2f^2))\,x}{8c^2de(de-cf)^3(e+fx^2)^2}$$

$$+\frac{(bc-ad)^2x}{4cd(de-cf)(c+dx^2)^2(e+fx^2)^2}-\frac{(bc-ad)(ad(3de-11cf)+bc(5de+3cf))x}{8c^2d(de-cf)^2(c+dx^2)(e+fx^2)^2}$$

$$-\frac{f(12b^2c^2e^2(de+cf)-2abce(d^2e^2+22cdef+c^2f^2)-3a^2(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3))\,x}{8c^2e^2(de-cf)^4(e+fx^2)}$$

$$+\frac{\sqrt{d}(2abcd(d^2e^2-14cdef-35c^2f^2)+3b^2c^2(d^2e^2+10cdef+5c^2f^2)+3a^2d^2(d^2e^2-6cdef+21c^2f^2)}{8c^{5/2}(de-cf)^5}$$

$$+\frac{\sqrt{f}(2abef(35d^2e^2+14cdef-c^2f^2)-3a^2f^2(21d^2e^2-6cdef+c^2f^2)-3b^2e^2(5d^2e^2+10cdef+c^2f^2}{8e^{5/2}(de-cf)^5}$$

output
```
-1/8*f*(b^2*c^2*e*(5*c*f+7*d*e)-2*a*b*c*d*e*(11*c*f+d*e)-a^2*d*(-2*c^2*f^2
-13*c*d*e*f+3*d^2*e^2))*x/c^2/d/e/(-c*f+d*e)^3/(f*x^2+e)^2+1/4*(-a*d+b*c)^
2*x/c/d/(-c*f+d*e)/(d*x^2+c)^2/(f*x^2+e)^2-1/8*(-a*d+b*c)*(a*d*(-11*c*f+3*
d*e)+b*c*(3*c*f+5*d*e))*x/c^2/d/(-c*f+d*e)^2/(d*x^2+c)/(f*x^2+e)^2-1/8*f*(
12*b^2*c^2*e^2*(c*f+d*e)-2*a*b*c*e*(c^2*f^2+22*c*d*e*f+d^2*e^2)-3*a^2*(c^3
*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3))*x/c^2/e^2/(-c*f+d*e)^4/(f*x^2+e
)+1/8*d^(1/2)*(2*a*b*c*d*(-35*c^2*f^2-14*c*d*e*f+d^2*e^2)+3*b^2*c^2*(5*c^2
*f^2+10*c*d*e*f+d^2*e^2)+3*a^2*d^2*(21*c^2*f^2-6*c*d*e*f+d^2*e^2))*arctan(
d^(1/2)*x/c^(1/2))/c^(5/2)/(-c*f+d*e)^5+1/8*f^(1/2)*(2*a*b*e*f*(-c^2*f^2+1
4*c*d*e*f+35*d^2*e^2)-3*a^2*f^2*(c^2*f^2-6*c*d*e*f+21*d^2*e^2)-3*b^2*e^2*(
c^2*f^2+10*c*d*e*f+5*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/(-c*f+d*e
)^5
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx$$
$$= \frac{1}{8}\Bigg(-\frac{2d(bc-ad)^2x}{c(-de+cf)^3(c+dx^2)^2} + \frac{d(-bc+ad)(3ad(de-5cf)+bc(5de+7cf))x}{c^2(de-cf)^4(c+dx^2)} - \frac{2f(be-af)^2x}{e(de-cf)^3(e+fx^2)^2} + \frac{f(-be+af)(3af(-5de+cf)+be(7de+5cf))x}{e^2(de-cf)^4(e+fx^2)}$$
$$-\frac{\sqrt{d}(2abcd(d^2e^2-14cdef-35c^2f^2)+3b^2c^2(d^2e^2+10cdef+5c^2f^2)+3a^2d^2(d^2e^2-6cdef+21c^2f^2)}{c^{5/2}(-de+cf)^5}$$
$$-\frac{\sqrt{f}(2abef(-35d^2e^2-14cdef+c^2f^2)+3a^2f^2(21d^2e^2-6cdef+c^2f^2)+3b^2e^2(5d^2e^2+10cdef+c^2}{e^{5/2}(de-cf)^5}$$

input
```
Integrate[(a + b*x^2)^2/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
((-2*d*(b*c - a*d)^2*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)^2) + (d*(-(b*c) +
a*d)*(3*a*d*(d*e - 5*c*f) + b*c*(5*d*e + 7*c*f))*x)/(c^2*(d*e - c*f)^4*(c
+ d*x^2)) - (2*f*(b*e - a*f)^2*x)/(e*(d*e - c*f)^3*(e + f*x^2)^2) + (f*(-(
b*e) + a*f)*(3*a*f*(-5*d*e + c*f) + b*e*(7*d*e + 5*c*f))*x)/(e^2*(d*e - c*
f)^4*(e + f*x^2)) - (Sqrt[d]*(2*a*b*c*d*(d^2*e^2 - 14*c*d*e*f - 35*c^2*f^2
) + 3*b^2*c^2*(d^2*e^2 + 10*c*d*e*f + 5*c^2*f^2) + 3*a^2*d^2*(d^2*e^2 - 6*
c*d*e*f + 21*c^2*f^2))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(-(d*e) + c*f
)^5) - (Sqrt[f]*(2*a*b*e*f*(-35*d^2*e^2 - 14*c*d*e*f + c^2*f^2) + 3*a^2*f^
2*(21*d^2*e^2 - 6*c*d*e*f + c^2*f^2) + 3*b^2*e^2*(5*d^2*e^2 + 10*c*d*e*f +
 c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*(d*e - c*f)^5))/8
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.50, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {425, 402, 25, 402, 25, 27, 402, 27, 397, 218, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3 (e+fx^2)^3} \, dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{bx^2+a}{(dx^2+c)^2(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)^3(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-7(bc-ad)fx^2+bce+3ade-4acf}{(dx^2+c)^2(fx^2+e)^3}dx}{4c(de-cf)}-\frac{x(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-7(bc-ad)fx^2+bce+3ade-4acf}{\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{4c(de-cf)}-\frac{x(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{x(ad(3de-11cf)+bc(7cf+de))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{\int -\frac{5f(ad(3de-11cf)+bc(de+7cf))x^2+bce(de-9cf)+a\left(3d^2e^2-3cdfe+8c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}}{4c(de-cf)}-\frac{x(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}\right)}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{5f(ad(3de-11cf)+bc(de+7cf))x^2+bce(de-9cf)+a\left(3d^2e^2-3cdfe+8c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}+\frac{x(ad(3de-11cf)+bc(7cf+de))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}}{4c(de-cf)}-\frac{x(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}\right)}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{\int \frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(-acf-2ade+3bce)}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{5f(ad(3de-11cf)+bc(de+7cf))x^2+bce(de-9cf)+a\left(3d^2e^2-3cdfe+8c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}+\frac{x(ad(3de-11cf)+bc(7cf+de))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}}{4c(de-cf)}-\frac{x(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(\cdots}\cdots\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\frac{\frac{\int \frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-3c^2f\cdots\right.\right.}{2e\left(e+fx^2\right)(de\cdots}}{2e(de-cf)}\cdots}{2c(de-cf)}\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\frac{\int \frac{4\left(3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2\cdots\right.\right.}{\cdots}}{2c(de-cf)}\cdots}{4c(de-cf)}\cdots\right)}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{\int \frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-3c^2f\cdots\right.\right.}{2e\left(e+fx^2\right)(d\cdots}}{2e(de-cf)}}{2c(de-cf)}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2-13\cdots\right.\right.}{e\left(e\cdots\right.}}{2c(de-cf)}}{4c(de-cf)}\right.}{d}$$

$\downarrow$ 397

$$\frac{b\left(\frac{\frac{\frac{4d^2e^2(ad(de-7cf)+bc(5cf+de))\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{cf\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-a\left(-3c^2f\cdots\right.\right.}{2e\left(e+fx^2\right)(d\cdots}}{2e(de-cf)}}{2c(de-cf)}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2-13\cdots\right.\right.}{e\left(e\cdots\right.}}{2c(de-cf)}}{4c(de-cf)}\right.}{d}$$

$\downarrow$ 218

$$\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-7cf)+bc(5cf+de))}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2+10cdef+15d^2e^2\right)-af\left(3c^2f^2-14cdef+35d^2e^2\right)\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx\left(bce(cf+11de)-\cdots\right.}{2e\left(\cdots\right.}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{\frac{\frac{\int\frac{3df\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bce\left(d^2e^2-11cdfe-2c^2f^2\right)+3a\left(d^3e^3-3cd^2fe^2+8c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{e(de-cf)}+\frac{fx\left(a\left(-2c^2f^2-13\cdots\right.\right.}{e\left(e\cdots\right.}}{2c(de-cf)}}{4c(de-cf)}\cdots\right.}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bce(11de+cf)-a\left(\cdots\right.\right.}{2e(de-\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{(ad(3de-11cf)+bc(de+7cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}+\frac{\frac{f\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}+\frac{f\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+\cdots\right.\right.}{e(de-cf)\left(fx^2+e\right)}\cdots}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bce(11de+cf)-a\left(\cdots\right.\right.}{2e(de-\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}$$

$$(bc-ad)\left(\frac{(ad(3de-11cf)+bc(de+7cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}+\frac{\frac{f\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}+\frac{f\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+\cdots\right.\right.}{e(de-cf)\left(fx^2+e\right)}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 397

$$\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bce(11de+cf)-a\left(\cdots\right.\right.}{2e(de-\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}$$

$$(bc-ad)\left(\frac{(ad(3de-11cf)+bc(de+7cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}+\frac{\frac{f\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}+\frac{f\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+\cdots\right.\right.}{e(de-cf)\left(fx^2+e\right)}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 218

$$\frac{b\left(\frac{\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bce(11de+cf)-a\left(\cdots\right.\right.}{2e(de-\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}$$

$$(bc-ad)\left(\frac{\frac{(ad(3de-11cf)+bc(de+7cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}+\frac{\frac{f\left(bce(de+11cf)+a\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}+\frac{\frac{f\left(bce\left(d^2e^2+22cdfe+c^2f^2\right)+3a\left(d^3e^3-5cd^2fe^2-5c^2df^2e+\cdots\right.\right.}{e(de-cf)\left(fx^2+e\right)}\cdots}{\cdots}}{\cdots}}{\cdots}\cdots\right.$$

input

```
Int[(a + b*x^2)^2/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output

```
(b*(-1/2*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)^2) + (-1/2
*(f*(3*b*c*e - 2*a*d*e - a*c*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(
f*(b*c*e*(11*d*e + c*f) - a*(4*d^2*e^2 + 11*c*d*e*f - 3*c^2*f^2))*x)/(e*(d
*e - c*f)*(e + f*x^2)) + ((4*d^(3/2)*e^2*(a*d*(d*e - 7*c*f) + b*c*(d*e + 5
*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (c*Sqrt[f]*(b*
e*(15*d^2*e^2 + 10*c*d*e*f - c^2*f^2) - a*f*(35*d^2*e^2 - 14*c*d*e*f + 3*c
^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c
*f)))/(2*e*(d*e - c*f)))/(2*c*(d*e - c*f))))/d - ((b*c - a*d)*(-1/4*((b*c
- a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)^2*(e + f*x^2)^2) + (((a*d*(3*d*e - 11
*c*f) + b*c*(d*e + 7*c*f))*x)/(2*c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)^2)
+ ((f*(b*c*e*(d*e + 11*c*f) + a*(3*d^2*e^2 - 13*c*d*e*f - 2*c^2*f^2))*x)/(
e*(d*e - c*f)*(e + f*x^2)^2) + ((f*(b*c*e*(d^2*e^2 + 22*c*d*e*f + c^2*f^2)
 + 3*a*(d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + c^3*f^3))*x)/(e*(d*e - c
*f)*(e + f*x^2)) + ((d^(3/2)*e^2*(b*c*(d^2*e^2 - 14*c*d*e*f - 35*c^2*f^2)
+ 3*a*d*(d^2*e^2 - 6*c*d*e*f + 21*c^2*f^2))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
Sqrt[c]*(d*e - c*f)) + (c^2*f^(3/2)*(b*e*(35*d^2*e^2 + 14*c*d*e*f - c^2*f^
2) - 3*a*f*(21*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
)/(Sqrt[e]*(d*e - c*f)))/(e*(d*e - c*f)))/(e*(d*e - c*f)))/(2*c*(d*e - c*f
)))/(4*c*(d*e - c*f))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.29

method	result
default	$f\left(\frac{\frac{f\left(3a^{2}c^{2}f^{4}-18a^{2}cde\,f^{3}+15a^{2}d^{2}e^{2}f^{2}+2ab\,c^{2}e\,f^{3}+20abcd\,e^{2}f^{2}-22ab\,d^{2}e^{3}f-5b^{2}c^{2}e^{2}f^{2}-2b^{2}cd\,e^{3}f+7b^{2}d^{2}e^{4}\right)x^{3}}{8e^{2}}+\frac{\left(5a^{2}c^{2}f^{4}-22a^{2}cde\,f^{3}\right.}{}}{\left(f\,x^{2}+e\right)^{2}}\right.$
risch	Expression too large to display

input
```
int((b*x^2+a)^2/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
f/(c*f-d*e)^5*((1/8*f*(3*a^2*c^2*f^4-18*a^2*c*d*e*f^3+15*a^2*d^2*e^2*f^2+2
*a*b*c^2*e*f^3+20*a*b*c*d*e^2*f^2-22*a*b*d^2*e^3*f-5*b^2*c^2*e^2*f^2-2*b^2
*c*d*e^3*f+7*b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-22*a^2*c*d*e*f^3+17*a
^2*d^2*e^2*f^2-2*a*b*c^2*e*f^3+28*a*b*c*d*e^2*f^2-26*a*b*d^2*e^3*f-3*b^2*c
^2*e^2*f^2-6*b^2*c*d*e^3*f+9*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a^2*c^2*
f^4-18*a^2*c*d*e*f^3+63*a^2*d^2*e^2*f^2+2*a*b*c^2*e*f^3-28*a*b*c*d*e^2*f^2
-70*a*b*d^2*e^3*f+3*b^2*c^2*e^2*f^2+30*b^2*c*d*e^3*f+15*b^2*d^2*e^4)/e^2/(
e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d/(c*f-d*e)^5*((1/8*d*(15*a^2*c^2*d^2*
f^2-18*a^2*c*d^3*e*f+3*a^2*d^4*e^2-22*a*b*c^3*d*f^2+20*a*b*c^2*d^2*e*f+2*a
*b*c*d^3*e^2+7*b^2*c^4*f^2-2*b^2*c^3*d*e*f-5*b^2*c^2*d^2*e^2)/c^2*x^3+1/8*
(17*a^2*c^2*d^2*f^2-22*a^2*c*d^3*e*f+5*a^2*d^4*e^2-26*a*b*c^3*d*f^2+28*a*b
*c^2*d^2*e*f-2*a*b*c*d^3*e^2+9*b^2*c^4*f^2-6*b^2*c^3*d*e*f-3*b^2*c^2*d^2*e
^2)/c*x)/(d*x^2+c)^2+1/8*(63*a^2*c^2*d^2*f^2-18*a^2*c*d^3*e*f+3*a^2*d^4*e^
2-70*a*b*c^3*d*f^2-28*a*b*c^2*d^2*e*f+2*a*b*c*d^3*e^2+15*b^2*c^4*f^2+30*b^
2*c^3*d*e*f+3*b^2*c^2*d^2*e^2)/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^2}{(c + dx^2)^3 (e + fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^2)^2}{(c + dx^2)^3 (e + fx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1271 vs. $2(560) = 1120$.

Time = 0.13 (sec) , antiderivative size = 1271, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2*d^3*e^2 + 2*a*b*c*d^4*e^2 + 3*a^2*d^5*e^2 + 30*b^2*c^3*d^2*
e*f - 28*a*b*c^2*d^3*e*f - 18*a^2*c*d^4*e*f + 15*b^2*c^4*d*f^2 - 70*a*b*c^
3*d^2*f^2 + 63*a^2*c^2*d^3*f^2)*arctan(d*x/sqrt(c*d))/((c^2*d^5*e^5 - 5*c^
3*d^4*e^4*f + 10*c^4*d^3*e^3*f^2 - 10*c^5*d^2*e^2*f^3 + 5*c^6*d*e*f^4 - c^
7*f^5)*sqrt(c*d)) - 1/8*(15*b^2*d^2*e^4*f + 30*b^2*c*d*e^3*f^2 - 70*a*b*d^
2*e^3*f^2 + 3*b^2*c^2*e^2*f^3 - 28*a*b*c*d*e^2*f^3 + 63*a^2*d^2*e^2*f^3 +
2*a*b*c^2*e*f^4 - 18*a^2*c*d*e*f^4 + 3*a^2*c^2*f^5)*arctan(f*x/sqrt(e*f))/
((d^5*e^7 - 5*c*d^4*e^6*f + 10*c^2*d^3*e^5*f^2 - 10*c^3*d^2*e^4*f^3 + 5*c^
4*d*e^3*f^4 - c^5*e^2*f^5)*sqrt(e*f)) - 1/8*(12*b^2*c^2*d^3*e^3*f^2*x^7 -
2*a*b*c*d^4*e^3*f^2*x^7 - 3*a^2*d^5*e^3*f^2*x^7 + 12*b^2*c^3*d^2*e^2*f^3*x
^7 - 44*a*b*c^2*d^3*e^2*f^3*x^7 + 15*a^2*c*d^4*e^2*f^3*x^7 - 2*a*b*c^3*d^2
*e*f^4*x^7 + 15*a^2*c^2*d^3*e*f^4*x^7 - 3*a^2*c^3*d^2*f^5*x^7 + 19*b^2*c^2
*d^3*e^4*f*x^5 - 4*a*b*c*d^4*e^4*f*x^5 - 6*a^2*d^5*e^4*f*x^5 + 34*b^2*c^3*
d^2*e^3*f^2*x^5 - 68*a*b*c^2*d^3*e^3*f^2*x^5 + 25*a^2*c*d^4*e^3*f^2*x^5 +
19*b^2*c^4*d*e^2*f^3*x^5 - 68*a*b*c^3*d^2*e^2*f^3*x^5 + 34*a^2*c^2*d^3*e^2
*f^3*x^5 - 4*a*b*c^4*d*e*f^4*x^5 + 25*a^2*c^3*d^2*e*f^4*x^5 - 6*a^2*c^4*d*
f^5*x^5 + 5*b^2*c^2*d^3*e^5*x^3 - 2*a*b*c*d^4*e^5*x^3 - 3*a^2*d^5*e^5*x^3
+ 31*b^2*c^3*d^2*e^4*f*x^3 - 18*a*b*c^2*d^3*e^4*f*x^3 + 5*a^2*c*d^4*e^4*f*
x^3 + 31*b^2*c^4*d*e^3*f^2*x^3 - 104*a*b*c^3*d^2*e^3*f^2*x^3 + 34*a^2*c^2*
d^3*e^3*f^2*x^3 + 5*b^2*c^5*e^2*f^3*x^3 - 18*a*b*c^4*d*e^2*f^3*x^3 + 34...
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 161006, normalized size of antiderivative = 273.82

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/((c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
- atan(((((768*a^2*c^2*d^16*e^16*f^2 - 11520*a^2*c^3*d^15*e^15*f^3 + 85248
*a^2*c^4*d^14*e^14*f^4 - 391680*a^2*c^5*d^13*e^13*f^5 + 1214208*a^2*c^6*d^
12*e^12*f^6 - 2654976*a^2*c^7*d^11*e^11*f^7 + 4204800*a^2*c^8*d^10*e^10*f^
8 - 4893696*a^2*c^9*d^9*e^9*f^9 + 4204800*a^2*c^10*d^8*e^8*f^10 - 2654976*
a^2*c^11*d^7*e^7*f^11 + 1214208*a^2*c^12*d^6*e^6*f^12 - 391680*a^2*c^13*d^
5*e^5*f^13 + 85248*a^2*c^14*d^4*e^4*f^14 - 11520*a^2*c^15*d^3*e^3*f^15 + 7
68*a^2*c^16*d^2*e^2*f^16 + 768*b^2*c^4*d^14*e^16*f^2 - 3072*b^2*c^5*d^13*e
^15*f^3 - 10752*b^2*c^6*d^12*e^14*f^4 + 107520*b^2*c^7*d^11*e^13*f^5 - 357
120*b^2*c^8*d^10*e^12*f^6 + 681984*b^2*c^9*d^9*e^11*f^7 - 838656*b^2*c^10*
d^8*e^10*f^8 + 681984*b^2*c^11*d^7*e^9*f^9 - 357120*b^2*c^12*d^6*e^8*f^10
+ 107520*b^2*c^13*d^5*e^7*f^11 - 10752*b^2*c^14*d^4*e^6*f^12 - 3072*b^2*c^
15*d^3*e^5*f^13 + 768*b^2*c^16*d^2*e^4*f^14 + 512*a*b*c^3*d^15*e^16*f^2 -
11776*a*b*c^4*d^14*e^15*f^3 + 82944*a*b*c^5*d^13*e^14*f^4 - 293888*a*b*c^6
*d^12*e^13*f^5 + 601600*a*b*c^7*d^11*e^12*f^6 - 705024*a*b*c^8*d^10*e^11*f
^7 + 325632*a*b*c^9*d^9*e^10*f^8 + 325632*a*b*c^10*d^8*e^9*f^9 - 705024*a*
b*c^11*d^7*e^8*f^10 + 601600*a*b*c^12*d^6*e^7*f^11 - 293888*a*b*c^13*d^5*e
^6*f^12 + 82944*a*b*c^14*d^4*e^5*f^13 - 11776*a*b*c^15*d^3*e^4*f^14 + 512*
a*b*c^16*d^2*e^3*f^15)/(512*(c^4*d^12*e^16 + c^16*e^4*f^12 - 12*c^5*d^11*e
^15*f - 12*c^15*d*e^5*f^11 + 66*c^6*d^10*e^14*f^2 - 220*c^7*d^9*e^13*f^3 +
 495*c^8*d^8*e^12*f^4 - 792*c^9*d^7*e^11*f^5 + 924*c^10*d^6*e^10*f^6 - ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5234, normalized size of antiderivative = 8.90

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
( - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**4*d**2*e**5*f
**2 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**4*d**2*e**
4*f**3*x**2 - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**4*d
**2*e**3*f**4*x**4 + 18*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2
*c**3*d**3*e**6*f - 90*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*
c**3*d**3*e**5*f**2*x**2 - 234*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)
))*a**2*c**3*d**3*e**4*f**3*x**4 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)
*sqrt(c)))*a**2*c**3*d**3*e**3*f**4*x**6 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(s
qrt(d)*sqrt(c)))*a**2*c**2*d**4*e**7 + 30*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt
(d)*sqrt(c)))*a**2*c**2*d**4*e**6*f*x**2 + 6*sqrt(d)*sqrt(c)*atan((d*x)/(s
qrt(d)*sqrt(c)))*a**2*c**2*d**4*e**5*f**2*x**4 - 90*sqrt(d)*sqrt(c)*atan((
d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**4*e**4*f**3*x**6 - 63*sqrt(d)*sqrt(c)
*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c**2*d**4*e**3*f**4*x**8 - 6*sqrt(d)*s
qrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**5*e**7*x**2 + 6*sqrt(d)*sqr
t(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**5*e**6*f*x**4 + 30*sqrt(d)*sq
rt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**5*e**5*f**2*x**6 + 18*sqrt(d
)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*c*d**5*e**4*f**3*x**8 - 3*sqr
t(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**6*e**7*x**4 - 6*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**6*e**6*f*x**6 - 3*sqrt(d)*s
qrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*d**6*e**5*f**2*x**8 + 70*sqrt...
```

3.242 $\int (a+bx^2)^3 (c+dx^2)^3 (e+fx^2)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 653

$$\int (a+bx^2)^3 (c+dx^2)^3 (e+fx^2)^3 \, dx = a^3c^3e^3x+a^2c^2e^2(bce+ade+acf)x^3+\frac{3}{5}ace\left(b^2c^2e^2+3abce(de+cf)+a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(b^3c^3e^3+9ab^2c^2e^2(de+cf)+9a^2bce\left(d^2e^2+3cdef+c^2f^2\right)+a^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7 + \frac{1}{3}\left(b^3c^2e^2(de+cf)+3ab^2ce\left(d^2e^2+3cdef+c^2f^2\right)+a^3df\left(d^2e^2+3cdef+c^2f^2\right)+a^2b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9 + \frac{3}{11}\left(a^3d^2f^2(de+cf)+b^3ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2bdf\left(d^2e^2+3cdef+c^2f^2\right)+ab^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{11} + \frac{1}{13}\left(a^3d^3f^3+9a^2bd^2f^2(de+cf)+9ab^2df\left(d^2e^2+3cdef+c^2f^2\right)+b^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{13} + \frac{1}{5}bdf\left(a^2d^2f^2+3abdf(de+cf)+b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{15} + \frac{3}{17}b^2d^2f^2(bde+bcf+adf)x^{17}+\frac{1}{19}b^3d^3f^3x^{19}$$

output
```
a^3*c^3*e^3*x+a^2*c^2*e^2*(a*c*f+a*d*e+b*c*e)*x^3+3/5*a*c*e*(b^2*c^2*e^2+3
*a*b*c*e*(c*f+d*e)+a^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(b^3*c^3*e^3+9
*a*b^2*c^2*e^2*(c*f+d*e)+9*a^2*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)+a^3*(c^3*
f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/3*(b^3*c^2*e^2*(c*f+d*e)+3
*a*b^2*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)+a^3*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)
+a^2*b*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^9+3/11*(a^3*d^2*f^
2*(c*f+d*e)+b^3*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)+3*a^2*b*d*f*(c^2*f^2+3*c*d
*e*f+d^2*e^2)+a*b^2*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^11+1/
13*(a^3*d^3*f^3+9*a^2*b*d^2*f^2*(c*f+d*e)+9*a*b^2*d*f*(c^2*f^2+3*c*d*e*f+d
^2*e^2)+b^3*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^13+1/5*b*d*f*
(a^2*d^2*f^2+3*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^15+3/1
7*b^2*d^2*f^2*(a*d*f+b*c*f+b*d*e)*x^17+1/19*b^3*d^3*f^3*x^19
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (c+dx^2)^3 (e+fx^2)^3 \, dx = a^3c^3e^3x+a^2c^2e^2(bce+ade+acf)x^3+\frac{3}{5}ace\left(b^2c^2e^2+3abce(de+cf)+a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5+\frac{1}{7}\left(b^3c^3e^3+9ab^2c^2e^2(de+cf)+9a^2bce\left(d^2e^2+3cdef+c^2f^2\right)+a^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7+\frac{1}{3}\left(b^3c^2e^2(de+cf)+3ab^2ce\left(d^2e^2+3cdef+c^2f^2\right)+a^3df\left(d^2e^2+3cdef+c^2f^2\right)+a^2b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9+\frac{3}{11}\left(a^3d^2f^2(de+cf)+b^3ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2bdf\left(d^2e^2+3cdef+c^2f^2\right)+ab^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{11}+\frac{1}{13}\left(a^3d^3f^3+9a^2bd^2f^2(de+cf)+9ab^2df\left(d^2e^2+3cdef+c^2f^2\right)+b^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^{13}+\frac{1}{5}bdf\left(a^2d^2f^2+3abdf(de+cf)+b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{15}+\frac{3}{17}b^2d^2f^2(bde+bcf+adf)x^{17}+\frac{1}{19}b^3d^3f^3x^{19}$$

input
```
Integrate[(a + b*x^2)^3*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a^3*c^3*e^3*x + a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^3 + (3*a*c*e*(b^2*c^
2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/
5 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^7)/7 + ((b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/3 + (3*(a^3*d^2*f^2*(d*e + c
*f) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^11)/11 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*
(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e
*f^2 + c^3*f^3))*x^13)/13 + (b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) +
b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^15)/5 + (3*b^2*d^2*f^2*(b*d*e + b*c
*f + a*d*f)*x^17)/17 + (b^3*d^3*f^3*x^19)/19
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \, dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(a^3c^3e^3+3bdfx^{14}\left(a^2d^2f^2+3abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)+3acex^4\left(a^2\left(c^2f^2+3cdef+d^2e^2\right)\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$
\begin{aligned}
&a^3c^3e^3x + \frac{1}{5}bdfx^{15}\left(a^2d^2f^2 + 3abdf(cf+de) + b^2\left(c^2f^2 + 3cdef + d^2e^2\right)\right) + \\
&\frac{3}{5}acex^5\left(a^2\left(c^2f^2 + 3cdef + d^2e^2\right) + 3abce(cf+de) + b^2c^2e^2\right) + a^2c^2e^2x^3(acf + ade + bce) + \\
&\frac{1}{13}x^{13}\left(a^3d^3f^3 + 9a^2bd^2f^2(cf+de) + 9ab^2df\left(c^2f^2 + 3cdef + d^2e^2\right) + b^3\left(c^3f^3 + 9c^2def^2 + 9cd^2e^2f + d^3e^3\right)\right) + \\
&\frac{3}{11}x^{11}\left(a^3d^2f^2(cf+de) + 3a^2bdf\left(c^2f^2 + 3cdef + d^2e^2\right) + ab^2\left(c^3f^3 + 9c^2def^2 + 9cd^2e^2f + d^3e^3\right) + b^3ce\left(c^2f^2 + \right.\right. \\
&\frac{1}{3}x^9\left(a^3df\left(c^2f^2 + 3cdef + d^2e^2\right) + a^2b\left(c^3f^3 + 9c^2def^2 + 9cd^2e^2f + d^3e^3\right) + 3ab^2ce\left(c^2f^2 + 3cdef + d^2e^2\right) + b^3c\right. \\
&\frac{1}{7}x^7\left(a^3\left(c^3f^3 + 9c^2def^2 + 9cd^2e^2f + d^3e^3\right) + 9a^2bce\left(c^2f^2 + 3cdef + d^2e^2\right) + 9ab^2c^2e^2(cf+de) + b^3c^3e^3\right) + \\
&\frac{3}{17}b^2d^2f^2x^{17}(adf + bcf + bde) + \frac{1}{19}b^3d^3f^3x^{19}
\end{aligned}
$$

input
```
Int[(a + b*x^2)^3*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

output
```
a^3*c^3*e^3*x + a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^3 + (3*a*c*e*(b^2*c^
2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/
5 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^7)/7 + ((b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/3 + (3*(a^3*d^2*f^2*(d*e + c
*f) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^11)/11 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*
(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e
*f^2 + c^3*f^3))*x^13)/13 + (b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) +
b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^15)/5 + (3*b^2*d^2*f^2*(b*d*e + b*c
*f + a*d*f)*x^17)/17 + (b^3*d^3*f^3*x^19)/19
```

Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.17

method	result
default	$\frac{b^3d^3f^3x^{19}}{19}+\frac{((3ab^2d^3+3b^3cd^2)f^3+3b^3d^3ef^2)x^{17}}{17}+\frac{((3a^2bd^3+9ab^2cd^2+3b^3c^2d)f^3+3(3ab^2d^3+3b^3cd^2)ef^2+3b^3d^3$
norman	$c^3a^3e^3x+(c^3a^3e^2f+a^3c^2de^3+a^2bc^3e^3)x^3+\left(\frac{3}{5}c^3a^3ef^2+\frac{9}{5}a^3c^2de^2f+\frac{3}{5}a^3cd^2e^3+\frac{9}{5}a^2b$
gosper	$\frac{9}{11}x^{11}a^2bd^3e^2f+\frac{3}{17}x^{17}ab^2d^3f^3+\frac{3}{17}x^{17}b^3cd^2f^3+\frac{3}{17}x^{17}b^3d^3ef^2+a^3c^3e^2fx^3+a^3c^2de^3x^3$
risch	$\frac{9}{11}x^{11}a^2bd^3e^2f+\frac{3}{17}x^{17}ab^2d^3f^3+\frac{3}{17}x^{17}b^3cd^2f^3+\frac{3}{17}x^{17}b^3d^3ef^2+a^3c^3e^2fx^3+a^3c^2de^3x^3$
parallelrisch	$\frac{9}{11}x^{11}a^2bd^3e^2f+\frac{3}{17}x^{17}ab^2d^3f^3+\frac{3}{17}x^{17}b^3cd^2f^3+\frac{3}{17}x^{17}b^3d^3ef^2+a^3c^3e^2fx^3+a^3c^2de^3x^3$
orering	$x(255255b^3d^3f^3x^{18}+855855ab^2d^3f^3x^{16}+855855b^3cd^2f^3x^{16}+855855b^3d^3ef^2x^{16}+969969a^2bd^3f^3x^{14}+2909907ab^2cd^2f^3$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/19*b^3*d^3*f^3*x^19+1/17*((3*a*b^2*d^3+3*b^3*c*d^2)*f^3+3*b^3*d^3*e*f^2)
*x^17+1/15*((3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2*d)*f^3+3*(3*a*b^2*d^3+3*b
^3*c*d^2)*e*f^2+3*b^3*d^3*e^2*f)*x^15+1/13*((a^3*d^3+9*a^2*b*c*d^2+9*a*b^2
*c^2*d+b^3*c^3)*f^3+3*(3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2*d)*e*f^2+3*(3*a
*b^2*d^3+3*b^3*c*d^2)*e^2*f+b^3*d^3*e^3)*x^13+1/11*((3*a^3*c*d^2+9*a^2*b*c
^2*d+3*a*b^2*c^3)*f^3+3*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*e*f^
2+3*(3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2*d)*e^2*f+(3*a*b^2*d^3+3*b^3*c*d^2
)*e^3)*x^11+1/9*((3*a^3*c^2*d+3*a^2*b*c^3)*f^3+3*(3*a^3*c*d^2+9*a^2*b*c^2*
d+3*a*b^2*c^3)*e*f^2+3*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*e^2*f
+(3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2*d)*e^3)*x^9+1/7*(c^3*a^3*f^3+3*(3*a^
3*c^2*d+3*a^2*b*c^3)*e*f^2+3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*e^2*f
+(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*e^3)*x^7+1/5*(3*c^3*a^3*e*f
^2+3*(3*a^3*c^2*d+3*a^2*b*c^3)*e^2*f+(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^
3)*e^3)*x^5+1/3*(3*c^3*a^3*e^2*f+(3*a^3*c^2*d+3*a^2*b*c^3)*e^3)*x^3+c^3*a^
3*e^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^3 (c+dx^2)^3 (e+fx^2)^3 \, dx$$
$$= \frac{1}{19} b^3d^3f^3x^{19} + \frac{3}{17} (b^3d^3ef^2 + (b^3cd^2 + ab^2d^3)f^3)x^{17}$$
$$+ \frac{1}{5} (b^3d^3e^2f + 3 (b^3cd^2 + ab^2d^3)ef^2 + (b^3c^2d + 3\,ab^2cd^2 + a^2bd^3)f^3)x^{15}$$
$$+ \frac{1}{13} (b^3d^3e^3 + 9 (b^3cd^2 + ab^2d^3)e^2f + 9 (b^3c^2d + 3\,ab^2cd^2 + a^2bd^3)ef^2 + (b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + \cdots$$
$$+ \frac{3}{11} ((b^3cd^2 + ab^2d^3)e^3 + 3 (b^3c^2d + 3\,ab^2cd^2 + a^2bd^3)e^2f + (b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3)ef^2 + \cdots$$
$$+ a^3c^3e^3x$$
$$+ \frac{1}{3} ((b^3c^2d + 3\,ab^2cd^2 + a^2bd^3)e^3 + (b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3)e^2f + 3 (ab^2c^3 + 3\,a^2bc^2d + a^3c\cdots$$
$$+ \frac{1}{7} (a^3c^3f^3 + (b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3)e^3 + 9 (ab^2c^3 + 3\,a^2bc^2d + a^3cd^2)e^2f + 9 (a^2bc^3 + a^3c\cdots$$
$$+ \frac{3}{5} (a^3c^3ef^2 + (ab^2c^3 + 3\,a^2bc^2d + a^3cd^2)e^3 + 3 (a^2bc^3 + a^3c^2d)e^2f)x^5$$
$$+ (a^3c^3e^2f + (a^2bc^3 + a^3c^2d)e^3)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
1/19*b^3*d^3*f^3*x^19 + 3/17*(b^3*d^3*e*f^2 + (b^3*c*d^2 + a*b^2*d^3)*f^3)
*x^17 + 1/5*(b^3*d^3*e^2*f + 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d
+ 3*a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^15 + 1/13*(b^3*d^3*e^3 + 9*(b^3*c*d^2
+ a*b^2*d^3)*e^2*f + 9*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (b^
3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3)*x^13 + 3/11*((b^3*c*
d^2 + a*b^2*d^3)*e^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^2*f + (
b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*f^2 + (a*b^2*c^3 + 3*
a^2*b*c^2*d + a^3*c*d^2)*f^3)*x^11 + a^3*c^3*e^3*x + 1/3*((b^3*c^2*d + 3*a
*b^2*c*d^2 + a^2*b*d^3)*e^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a
^3*d^3)*e^2*f + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e*f^2 + (a^2*b*c
^3 + a^3*c^2*d)*f^3)*x^9 + 1/7*(a^3*c^3*f^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9
*a^2*b*c*d^2 + a^3*d^3)*e^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^
2*f + 9*(a^2*b*c^3 + a^3*c^2*d)*e*f^2)*x^7 + 3/5*(a^3*c^3*e*f^2 + (a*b^2*c
^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^3 + 3*(a^2*b*c^3 + a^3*c^2*d)*e^2*f)*x^5
 + (a^3*c^3*e^2*f + (a^2*b*c^3 + a^3*c^2*d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.54

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

output
```
a**3*c**3*e**3*x + b**3*d**3*f**3*x**19/19 + x**17*(3*a*b**2*d**3*f**3/17
+ 3*b**3*c*d**2*f**3/17 + 3*b**3*d**3*e*f**2/17) + x**15*(a**2*b*d**3*f**3
/5 + 3*a*b**2*c*d**2*f**3/5 + 3*a*b**2*d**3*e*f**2/5 + b**3*c**2*d*f**3/5
+ 3*b**3*c*d**2*e*f**2/5 + b**3*d**3*e**2*f/5) + x**13*(a**3*d**3*f**3/13
+ 9*a**2*b*c*d**2*f**3/13 + 9*a**2*b*d**3*e*f**2/13 + 9*a*b**2*c**2*d*f**3
/13 + 27*a*b**2*c*d**2*e*f**2/13 + 9*a*b**2*d**3*e**2*f/13 + b**3*c**3*f**
3/13 + 9*b**3*c**2*d*e*f**2/13 + 9*b**3*c*d**2*e**2*f/13 + b**3*d**3*e**3/
13) + x**11*(3*a**3*c*d**2*f**3/11 + 3*a**3*d**3*e*f**2/11 + 9*a**2*b*c**2
*d*f**3/11 + 27*a**2*b*c*d**2*e*f**2/11 + 9*a**2*b*d**3*e**2*f/11 + 3*a*b*
*2*c**3*f**3/11 + 27*a*b**2*c**2*d*e*f**2/11 + 27*a*b**2*c*d**2*e**2*f/11
+ 3*a*b**2*d**3*e**3/11 + 3*b**3*c**3*e*f**2/11 + 9*b**3*c**2*d*e**2*f/11
+ 3*b**3*c*d**2*e**3/11) + x**9*(a**3*c**2*d*f**3/3 + a**3*c*d**2*e*f**2 +
 a**3*d**3*e**2*f/3 + a**2*b*c**3*f**3/3 + 3*a**2*b*c**2*d*e*f**2 + 3*a**2
*b*c*d**2*e**2*f + a**2*b*d**3*e**3/3 + a*b**2*c**3*e*f**2 + 3*a*b**2*c**2
*d*e**2*f + a*b**2*c*d**2*e**3 + b**3*c**3*e**2*f/3 + b**3*c**2*d*e**3/3)
+ x**7*(a**3*c**3*f**3/7 + 9*a**3*c**2*d*e*f**2/7 + 9*a**3*c*d**2*e**2*f/7
 + a**3*d**3*e**3/7 + 9*a**2*b*c**3*e*f**2/7 + 27*a**2*b*c**2*d*e**2*f/7 +
 9*a**2*b*c*d**2*e**3/7 + 9*a*b**2*c**3*e**2*f/7 + 9*a*b**2*c**2*d*e**3/7
+ b**3*c**3*e**3/7) + x**5*(3*a**3*c**3*e*f**2/5 + 9*a**3*c**2*d*e**2*f/5
+ 3*a**3*c*d**2*e**3/5 + 9*a**2*b*c**3*e**2*f/5 + 9*a**2*b*c**2*d*e**3/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \, dx$$
$$= \frac{1}{19}b^3d^3f^3x^{19} + \frac{3}{17}\left(b^3d^3ef^2 + \left(b^3cd^2+ab^2d^3\right)f^3\right)x^{17}$$
$$+ \frac{1}{5}\left(b^3d^3e^2f + 3\left(b^3cd^2+ab^2d^3\right)ef^2 + \left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)f^3\right)x^{15}$$
$$+ \frac{1}{13}\left(b^3d^3e^3 + 9\left(b^3cd^2+ab^2d^3\right)e^2f + 9\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)ef^2 + \left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2 + \right.\right.$$
$$+ \frac{3}{11}\left(\left(b^3cd^2+ab^2d^3\right)e^3 + 3\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)e^2f + \left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)ef^2 + \right.$$
$$+ a^3c^3e^3x$$
$$+ \frac{1}{3}\left(\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)e^3 + \left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e^2f + 3\left(ab^2c^3+3\,a^2bc^2d+a^3c\right.\right.$$
$$+ \frac{1}{7}\left(a^3c^3f^3 + \left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e^3 + 9\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)e^2f + 9\left(a^2bc^3+a^3c\right.\right.$$
$$+ \frac{3}{5}\left(a^3c^3ef^2 + \left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)e^3 + 3\left(a^2bc^3+a^3c^2d\right)e^2f\right)x^5$$
$$+ \left(a^3c^3e^2f + \left(a^2bc^3+a^3c^2d\right)e^3\right)x^3$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/19*b^3*d^3*f^3*x^19 + 3/17*(b^3*d^3*e*f^2 + (b^3*c*d^2 + a*b^2*d^3)*f^3)
*x^17 + 1/5*(b^3*d^3*e^2*f + 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d
+ 3*a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^15 + 1/13*(b^3*d^3*e^3 + 9*(b^3*c*d^2
+ a*b^2*d^3)*e^2*f + 9*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (b^
3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3)*x^13 + 3/11*((b^3*c*
d^2 + a*b^2*d^3)*e^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^2*f + (
b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*f^2 + (a*b^2*c^3 + 3*
a^2*b*c^2*d + a^3*c*d^2)*f^3)*x^11 + a^3*c^3*e^3*x + 1/3*((b^3*c^2*d + 3*a
*b^2*c*d^2 + a^2*b*d^3)*e^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a
^3*d^3)*e^2*f + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e*f^2 + (a^2*b*c
^3 + a^3*c^2*d)*f^3)*x^9 + 1/7*(a^3*c^3*f^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9
*a^2*b*c*d^2 + a^3*d^3)*e^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^
2*f + 9*(a^2*b*c^3 + a^3*c^2*d)*e*f^2)*x^7 + 3/5*(a^3*c^3*e*f^2 + (a*b^2*c
^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^3 + 3*(a^2*b*c^3 + a^3*c^2*d)*e^2*f)*x^5
 + (a^3*c^3*e^2*f + (a^2*b*c^3 + a^3*c^2*d)*e^3)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 984, normalized size of antiderivative = 1.51

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \left(e+fx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/19*b^3*d^3*f^3*x^19 + 3/17*b^3*d^3*e*f^2*x^17 + 3/17*b^3*c*d^2*f^3*x^17
+ 3/17*a*b^2*d^3*f^3*x^17 + 1/5*b^3*d^3*e^2*f*x^15 + 3/5*b^3*c*d^2*e*f^2*x
^15 + 3/5*a*b^2*d^3*e*f^2*x^15 + 1/5*b^3*c^2*d*f^3*x^15 + 3/5*a*b^2*c*d^2*
f^3*x^15 + 1/5*a^2*b*d^3*f^3*x^15 + 1/13*b^3*d^3*e^3*x^13 + 9/13*b^3*c*d^2
*e^2*f*x^13 + 9/13*a*b^2*d^3*e^2*f*x^13 + 9/13*b^3*c^2*d*e*f^2*x^13 + 27/1
3*a*b^2*c*d^2*e*f^2*x^13 + 9/13*a^2*b*d^3*e*f^2*x^13 + 1/13*b^3*c^3*f^3*x^
13 + 9/13*a*b^2*c^2*d*f^3*x^13 + 9/13*a^2*b*c*d^2*f^3*x^13 + 1/13*a^3*d^3*
f^3*x^13 + 3/11*b^3*c*d^2*e^3*x^11 + 3/11*a*b^2*d^3*e^3*x^11 + 9/11*b^3*c^
2*d*e^2*f*x^11 + 27/11*a*b^2*c*d^2*e^2*f*x^11 + 9/11*a^2*b*d^3*e^2*f*x^11
+ 3/11*b^3*c^3*e*f^2*x^11 + 27/11*a*b^2*c^2*d*e*f^2*x^11 + 27/11*a^2*b*c*d
^2*e*f^2*x^11 + 3/11*a^3*d^3*e*f^2*x^11 + 3/11*a*b^2*c^3*f^3*x^11 + 9/11*a
^2*b*c^2*d*f^3*x^11 + 3/11*a^3*c*d^2*f^3*x^11 + 1/3*b^3*c^2*d*e^3*x^9 + a*
b^2*c*d^2*e^3*x^9 + 1/3*a^2*b*d^3*e^3*x^9 + 1/3*b^3*c^3*e^2*f*x^9 + 3*a*b^
2*c^2*d*e^2*f*x^9 + 3*a^2*b*c*d^2*e^2*f*x^9 + 1/3*a^3*d^3*e^2*f*x^9 + a*b^
2*c^3*e*f^2*x^9 + 3*a^2*b*c^2*d*e*f^2*x^9 + a^3*c*d^2*e*f^2*x^9 + 1/3*a^2*
b*c^3*f^3*x^9 + 1/3*a^3*c^2*d*f^3*x^9 + 1/7*b^3*c^3*e^3*x^7 + 9/7*a*b^2*c^
2*d*e^3*x^7 + 9/7*a^2*b*c*d^2*e^3*x^7 + 1/7*a^3*d^3*e^3*x^7 + 9/7*a*b^2*c^
3*e^2*f*x^7 + 27/7*a^2*b*c^2*d*e^2*f*x^7 + 9/7*a^3*c*d^2*e^2*f*x^7 + 9/7*a
^2*b*c^3*e*f^2*x^7 + 9/7*a^3*c^2*d*e*f^2*x^7 + 1/7*a^3*c^3*f^3*x^7 + 3/5*a
*b^2*c^3*e^3*x^5 + 9/5*a^2*b*c^2*d*e^3*x^5 + 3/5*a^3*c*d^2*e^3*x^5 + 9/...
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.20

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx$$
$$= x^{13}\left(\frac{a^3\,d^3\,f^3}{13}+\frac{9\,a^2\,b\,c\,d^2\,f^3}{13}+\frac{9\,a^2\,b\,d^3\,e\,f^2}{13}+\frac{9\,a\,b^2\,c^2\,d\,f^3}{13}+\frac{27\,a\,b^2\,c\,d^2\,e\,f^2}{13}+\frac{9\,a\,b^2\,d^3\,e^2\,f}{13}+\frac{b^3\,c^3\,f^3}{13}+\frac{9\,b^3\,c^2\,d\,e\,f^2}{13}+\frac{9\,b^3\,c\,d^2\,e^2\,f}{13}+\frac{b^3\,d^3\,e^3}{13}\right)$$
$$+x^9\left(\frac{a^3\,c^2\,d\,f^3}{3}+a^3\,c\,d^2\,e\,f^2+\frac{a^3\,d^3\,e^2\,f}{3}+\frac{a^2\,b\,c^3\,f^3}{3}+3\,a^2\,b\,c^2\,d\,e\,f^2+3\,a^2\,b\,c\,d^2\,e^2\,f+\frac{a^2\,b\,d^3\,e^3}{3}+a\,b^2\,c^3\,e\,f^2+3\,a\,b^2\,c^2\,d\,e^2\,f+a\,b^2\,c\,d^2\,e^3+\frac{b^3\,c^3\,e^2\,f}{3}+\frac{b^3\,c^2\,d\,e^3}{3}\right)$$
$$+x^{11}\left(\frac{3\,a^3\,c\,d^2\,f^3}{11}+\frac{3\,a^3\,d^3\,e\,f^2}{11}+\frac{9\,a^2\,b\,c^2\,d\,f^3}{11}+\frac{27\,a^2\,b\,c\,d^2\,e\,f^2}{11}+\frac{9\,a^2\,b\,d^3\,e^2\,f}{11}+\frac{3\,a\,b^2\,c^3\,f^3}{11}+\frac{27\,a\,b^2\,c^2\,d\,e\,f^2}{11}+\frac{27\,a\,b^2\,c\,d^2\,e^2\,f}{11}+\frac{3\,a\,b^2\,d^3\,e^3}{11}+\frac{3\,b^3\,c^3\,e\,f^2}{11}+\frac{9\,b^3\,c^2\,d\,e^2\,f}{11}+\frac{3\,b^3\,c\,d^2\,e^3}{11}\right)$$
$$+x^7\left(\frac{a^3\,c^3\,f^3}{7}+\frac{9\,a^3\,c^2\,d\,e\,f^2}{7}+\frac{9\,a^3\,c\,d^2\,e^2\,f}{7}+\frac{a^3\,d^3\,e^3}{7}+\frac{9\,a^2\,b\,c^3\,e\,f^2}{7}+\frac{27\,a^2\,b\,c^2\,d\,e^2\,f}{7}+\frac{9\,a^2\,b\,c\,d^2\,e^3}{7}+\frac{9\,a\,b^2\,c^3\,e^2\,f}{7}+\frac{9\,a\,b^2\,c^2\,d\,e^3}{7}+\frac{b^3\,c^3\,e^3}{7}\right)+a^3\,c^3\,e^3\,x$$
$$+\frac{b^3\,d^3\,f^3\,x^{19}}{19}+a^2\,c^2\,e^2\,x^3\,(a\,c\,f+a\,d\,e+b\,c\,e)+\frac{3\,b^2\,d^2\,f^2\,x^{17}\,(a\,d\,f+b\,c\,f+b\,d\,e)}{17}$$
$$+\frac{3\,a\,c\,e\,x^5\,(a^2\,c^2\,f^2+3\,a^2\,c\,d\,e\,f+a^2\,d^2\,e^2+3\,a\,b\,c^2\,e\,f+3\,a\,b\,c\,d\,e^2+b^2\,c^2\,e^2)}{5}$$
$$+\frac{b\,d\,f\,x^{15}\,(a^2\,d^2\,f^2+3\,a\,b\,c\,d\,f^2+3\,a\,b\,d^2\,e\,f+b^2\,c^2\,f^2+3\,b^2\,c\,d\,e\,f+b^2\,d^2\,e^2)}{5}$$

input
```
int((a + b*x^2)^3*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

output
```
x^13*((a^3*d^3*f^3)/13 + (b^3*c^3*f^3)/13 + (b^3*d^3*e^3)/13 + (9*a*b^2*c^
2*d*f^3)/13 + (9*a^2*b*c*d^2*f^3)/13 + (9*a*b^2*d^3*e^2*f)/13 + (9*a^2*b*d
^3*e*f^2)/13 + (9*b^3*c*d^2*e^2*f)/13 + (9*b^3*c^2*d*e*f^2)/13 + (27*a*b^2
*c*d^2*e*f^2)/13) + x^9*((a^2*b*c^3*f^3)/3 + (a^2*b*d^3*e^3)/3 + (a^3*c^2*
d*f^3)/3 + (b^3*c^2*d*e^3)/3 + (a^3*d^3*e^2*f)/3 + (b^3*c^3*e^2*f)/3 + a*b
^2*c*d^2*e^3 + a*b^2*c^3*e*f^2 + a^3*c*d^2*e*f^2 + 3*a*b^2*c^2*d*e^2*f + 3
*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2) + x^11*((3*a*b^2*c^3*f^3)/11 + (
3*a*b^2*d^3*e^3)/11 + (3*a^3*c*d^2*f^3)/11 + (3*b^3*c*d^2*e^3)/11 + (3*a^3
*d^3*e*f^2)/11 + (3*b^3*c^3*e*f^2)/11 + (9*a^2*b*c^2*d*f^3)/11 + (9*a^2*b*
d^3*e^2*f)/11 + (9*b^3*c^2*d*e^2*f)/11 + (27*a*b^2*c*d^2*e^2*f)/11 + (27*a
*b^2*c^2*d*e*f^2)/11 + (27*a^2*b*c*d^2*e*f^2)/11) + x^7*((a^3*c^3*f^3)/7 +
 (a^3*d^3*e^3)/7 + (b^3*c^3*e^3)/7 + (9*a*b^2*c^2*d*e^3)/7 + (9*a^2*b*c*d^
2*e^3)/7 + (9*a*b^2*c^3*e^2*f)/7 + (9*a^2*b*c^3*e*f^2)/7 + (9*a^3*c*d^2*e^
2*f)/7 + (9*a^3*c^2*d*e*f^2)/7 + (27*a^2*b*c^2*d*e^2*f)/7) + a^3*c^3*e^3*x
 + (b^3*d^3*f^3*x^19)/19 + a^2*c^2*e^2*x^3*(a*c*f + a*d*e + b*c*e) + (3*b^
2*d^2*f^2*x^17*(a*d*f + b*c*f + b*d*e))/17 + (3*a*c*e*x^5*(a^2*c^2*f^2 + a
^2*d^2*e^2 + b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f))
/5 + (b*d*f*x^15*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2
+ 3*a*b*d^2*e*f + 3*b^2*c*d*e*f))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 993, normalized size of antiderivative = 1.52

$$\int (a+bx^2)^3 (c+dx^2)^3 (e+fx^2)^3 \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3*(f*x^2+e)^3,x)
```

output
```
(x*(4849845*a**3*c**3*e**3 + 4849845*a**3*c**3*e**2*f*x**2 + 2909907*a**3*
c**3*e*f**2*x**4 + 692835*a**3*c**3*f**3*x**6 + 4849845*a**3*c**2*d*e**3*x
**2 + 8729721*a**3*c**2*d*e**2*f*x**4 + 6235515*a**3*c**2*d*e*f**2*x**6 +
1616615*a**3*c**2*d*f**3*x**8 + 2909907*a**3*c*d**2*e**3*x**4 + 6235515*a*
*3*c*d**2*e**2*f*x**6 + 4849845*a**3*c*d**2*e*f**2*x**8 + 1322685*a**3*c*d
**2*f**3*x**10 + 692835*a**3*d**3*e**3*x**6 + 1616615*a**3*d**3*e**2*f*x**
8 + 1322685*a**3*d**3*e*f**2*x**10 + 373065*a**3*d**3*f**3*x**12 + 4849845
*a**2*b*c**3*e**3*x**2 + 8729721*a**2*b*c**3*e**2*f*x**4 + 6235515*a**2*b*
c**3*e*f**2*x**6 + 1616615*a**2*b*c**3*f**3*x**8 + 8729721*a**2*b*c**2*d*e
**3*x**4 + 18706545*a**2*b*c**2*d*e**2*f*x**6 + 14549535*a**2*b*c**2*d*e*f
**2*x**8 + 3968055*a**2*b*c**2*d*f**3*x**10 + 6235515*a**2*b*c*d**2*e**3*x
**6 + 14549535*a**2*b*c*d**2*e**2*f*x**8 + 11904165*a**2*b*c*d**2*e*f**2*x
**10 + 3357585*a**2*b*c*d**2*f**3*x**12 + 1616615*a**2*b*d**3*e**3*x**8 +
3968055*a**2*b*d**3*e**2*f*x**10 + 3357585*a**2*b*d**3*e*f**2*x**12 + 9699
69*a**2*b*d**3*f**3*x**14 + 2909907*a*b**2*c**3*e**3*x**4 + 6235515*a*b**2
*c**3*e**2*f*x**6 + 4849845*a*b**2*c**3*e*f**2*x**8 + 1322685*a*b**2*c**3*
f**3*x**10 + 6235515*a*b**2*c**2*d*e**3*x**6 + 14549535*a*b**2*c**2*d*e**2
*f*x**8 + 11904165*a*b**2*c**2*d*e*f**2*x**10 + 3357585*a*b**2*c**2*d*f**3
*x**12 + 4849845*a*b**2*c*d**2*e**3*x**8 + 11904165*a*b**2*c*d**2*e**2*f*x
**10 + 10072755*a*b**2*c*d**2*e*f**2*x**12 + 2909907*a*b**2*c*d**2*f**3...
```

3.243 $\int \frac{(a+bx^2)^3(c+dx^2)^3}{e+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 428

$$\begin{aligned}\int \frac{(a+bx^2)^3(c+dx^2)^3}{e+fx^2}\,dx = &-\frac{(b^3e^2(de-cf)^3-3ab^2ef(de-cf)^3+3a^2bf^2(de-cf)^3-a^3df^3(d^2e^2-3cdef+3c^2f^2))\,x}{f^6}\\ &-\frac{(a^3d^2f^3(de-3cf)-b^3e(de-cf)^3+3ab^2f(de-cf)^3-3a^2bdf^2(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^5}\\ &+\frac{(a^3d^3f^3-3a^2bd^2f^2(de-3cf)-b^3(de-cf)^3+3ab^2df(d^2e^2-3cdef+3c^2f^2))\,x^5}{5f^4}\\ &+\frac{bd(3a^2d^2f^2-3abdf(de-3cf)+b^2(d^2e^2-3cdef+3c^2f^2))\,x^7}{7f^3}\\ &-\frac{b^2d^2(bde-3bcf-3adf)x^9}{9f^2}+\frac{b^3d^3x^{11}}{11f}+\frac{(be-af)^3(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{13/2}}\end{aligned}$$

output
```
-(b^3*e^2*(-c*f+d*e)^3-3*a*b^2*e*f*(-c*f+d*e)^3+3*a^2*b*f^2*(-c*f+d*e)^3-a
^3*d*f^3*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x/f^6-1/3*(a^3*d^2*f^3*(-3*c*f+d*e
)-b^3*e*(-c*f+d*e)^3+3*a*b^2*f*(-c*f+d*e)^3-3*a^2*b*d*f^2*(3*c^2*f^2-3*c*d
*e*f+d^2*e^2))*x^3/f^5+1/5*(a^3*d^3*f^3-3*a^2*b*d^2*f^2*(-3*c*f+d*e)-b^3*(
-c*f+d*e)^3+3*a*b^2*d*f*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x^5/f^4+1/7*b*d*(3*
a^2*d^2*f^2-3*a*b*d*f*(-3*c*f+d*e)+b^2*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x^7/
f^3-1/9*b^2*d^2*(-3*a*d*f-3*b*c*f+b*d*e)*x^9/f^2+1/11*b^3*d^3*x^11/f+(-a*f
+b*e)^3*(-c*f+d*e)^3*arctan(f^(1/2)*x/e^(1/2))/e^(1/2)/f^(13/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx$$
$$= \frac{(-b^3e^2(de-cf)^3 + 3ab^2ef(de-cf)^3 + 3a^2bf^2(-de+cf)^3 + a^3df^3(d^2e^2-3cdef+3c^2f^2))\,x}{f^6}$$
$$+ \frac{(b^3e(de-cf)^3 + 3ab^2f(-de+cf)^3 + a^3d^2f^3(-de+3cf) + 3a^2bdf^2(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^5}$$
$$+ \frac{(a^3d^3f^3 - 3a^2bd^2f^2(de-3cf) - b^3(de-cf)^3 + 3ab^2df(d^2e^2-3cdef+3c^2f^2))\,x^5}{5f^4}$$
$$+ \frac{bd(3a^2d^2f^2 - 3abdf(de-3cf) + b^2(d^2e^2-3cdef+3c^2f^2))\,x^7}{7f^3}$$
$$- \frac{b^2d^2(bde-3bcf-3adf)x^9}{9f^2} + \frac{b^3d^3x^{11}}{11f} + \frac{(be-af)^3(de-cf)^3 \arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{13/2}}$$

input
```
Integrate[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((-(b^3*e^2*(d*e - c*f)^3) + 3*a*b^2*e*f*(d*e - c*f)^3 + 3*a^2*b*f^2*(-(d*
e) + c*f)^3 + a^3*d*f^3*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^6 + ((b^3*
e*(d*e - c*f)^3 + 3*a*b^2*f*(-(d*e) + c*f)^3 + a^3*d^2*f^3*(-(d*e) + 3*c*f
) + 3*a^2*b*d*f^2*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f^5) + ((a^3*
d^3*f^3 - 3*a^2*b*d^2*f^2*(d*e - 3*c*f) - b^3*(d*e - c*f)^3 + 3*a*b^2*d*f*
(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^5)/(5*f^4) + (b*d*(3*a^2*d^2*f^2 - 3*
a*b*d*f*(d*e - 3*c*f) + b^2*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^7)/(7*f^3
) - (b^2*d^2*(b*d*e - 3*b*c*f - 3*a*d*f)*x^9)/(9*f^2) + (b^3*d^3*x^11)/(11
*f) + ((b*e - a*f)^3*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f
^(13/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {420, 290, 420, 290, 403, 25, 403, 25, 403, 299, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int (bx^2+a)^2 (dx^2+c)^3 dx}{f} - \frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{290}$$

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2\right.}{f}$$

$$\frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$\frac{(be-af)\left(\frac{b\int \left(bx^2+a\right)\left(dx^2+c\right)^3dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}\right)}{f}$$

$\Big\downarrow$ 290

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$\frac{(be-af)\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}\right)}{f}$$

$\Big\downarrow$ 403

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$\frac{(be-af)\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{\int -\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}d}{7f}\right.}{f}\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$\frac{(be-af)\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+\right.}{fx^2+e}}{7f}\right.}{f}\right.}{f}$$

$\Big\downarrow$ 403

$$\frac{b\int\left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int-\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(\right.}{}}{}\right.}{}\right.}{f}$$

$\downarrow$ 25

$$\frac{b\int\left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int}{}\right.}{}\right.}{f}$$

$\downarrow$ 403

$$\frac{b\int\left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2\right.}{f}$$

$$(be-af)\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int}{}\right.}{}\right.$$

$\downarrow$ 299

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$(be-af)\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\right.}{}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{b\int \left(b^2d^3x^{10}+bd^2(3bc+2ad)x^8+d\left(3b^2c^2+6abdc+a^2d^2\right)x^6+c\left(b^2c^2+6abdc+3a^2d^2\right)x^4+ac^2(2bc+3ad)x^2}{f}$$

$$(be-af)\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9\right)}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\right.}{}\right.$$

input
```
Int[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
(b*(a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d +
3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(
3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11))/f - ((b*e - a*f)*((b*(a*c^3*x
+ (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b*c + a*
d)*x^7)/7 + (b*d^3*x^9)/9))/f - ((b*e - a*f)*((b*x*(c + d*x^2)^3)/(7*f) -
(((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(5*f) - (-1/3*((7*a*d*f*(
5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/
f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3 - 2
80*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/f + (105*(b*e - a*f)*(d
*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/
(7*f)))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(410) = 820$.

Time = 0.68 (sec) , antiderivative size = 927, normalized size of antiderivative = 2.17

method	result
default	$\frac{\frac{b^3 d^3 x^{11} f^5}{11}+\frac{\left((adf+bcf-bde)b^2 d^2 f^4+bdf\left(2ab\,d^2 f^4+2b^2 cd\,f^4\right)\right)x^9}{9}+\frac{\left((adf+bcf-bde)\left(2ab\,d^2 f^4+2b^2 cd\,f^4\right)+bdf\left(a^2 d^2 f^4+5abcd\,f^4-ab\,d^2 e\,f^3 \right.\right.}{7}}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f^6*(1/11*b^3*d^3*x^11*f^5+1/9*((a*d*f+b*c*f-b*d*e)*b^2*d^2*f^4+b*d*f*(2
*a*b*d^2*f^4+2*b^2*c*d*f^4))*x^9+1/7*((a*d*f+b*c*f-b*d*e)*(2*a*b*d^2*f^4+2
*b^2*c*d*f^4)+b*d*f*(a^2*d^2*f^4+5*a*b*c*d*f^4-a*b*d^2*e*f^3+b^2*c^2*f^4-b
^2*c*d*e*f^3+b^2*d^2*e^2*f^2))*x^7+1/5*((a*d*f+b*c*f-b*d*e)*(a^2*d^2*f^4+5
*a*b*c*d*f^4-a*b*d^2*e*f^3+b^2*c^2*f^4-b^2*c*d*e*f^3+b^2*d^2*e^2*f^2)+b*d*
f*(3*a^2*c*d*f^4-a^2*d^2*e*f^3+3*a*b*c^2*f^4-2*a*b*c*d*e*f^3+a*b*d^2*e^2*f
^2-b^2*c^2*e*f^3+b^2*c*d*e^2*f^2))*x^5+1/3*((a*d*f+b*c*f-b*d*e)*(3*a^2*c*d
*f^4-a^2*d^2*e*f^3+3*a*b*c^2*f^4-2*a*b*c*d*e*f^3+a*b*d^2*e^2*f^2-b^2*c^2*e
*f^3+b^2*c*d*e^2*f^2)+b*d*f*(3*a^2*c^2*f^4-3*a^2*c*d*e*f^3+a^2*d^2*e^2*f^2
-3*a*b*c^2*e*f^3+5*a*b*c*d*e^2*f^2-2*a*b*d^2*e^3*f+b^2*c^2*e^2*f^2-2*b^2*c
*d*e^3*f+b^2*d^2*e^4))*x^3+(a*d*f+b*c*f-b*d*e)*(3*a^2*c^2*f^4-3*a^2*c*d*e*
f^3+a^2*d^2*e^2*f^2-3*a*b*c^2*e*f^3+5*a*b*c*d*e^2*f^2-2*a*b*d^2*e^3*f+b^2*
c^2*e^2*f^2-2*b^2*c*d*e^3*f+b^2*d^2*e^4)*x)+(a^3*c^3*f^6-3*a^3*c^2*d*e*f^5
+3*a^3*c*d^2*e^2*f^4-a^3*d^3*e^3*f^3-3*a^2*b*c^3*e*f^5+9*a^2*b*c^2*d*e^2*f
^4-9*a^2*b*c*d^2*e^3*f^3+3*a^2*b*d^3*e^4*f^2+3*a*b^2*c^3*e^2*f^4-9*a*b^2*c
^2*d*e^3*f^3+9*a*b^2*c*d^2*e^4*f^2-3*a*b^2*d^3*e^5*f-b^3*c^3*e^3*f^3+3*b^3
*c^2*d*e^4*f^2-3*b^3*c*d^2*e^5*f+b^3*d^3*e^6)/f^6/(e*f)^(1/2)*arctan(f*x/(
e*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1604, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/6930*(630*b^3*d^3*e*f^6*x^11 - 770*(b^3*d^3*e^2*f^5 - 3*(b^3*c*d^2 + a*
b^2*d^3)*e*f^6)*x^9 + 990*(b^3*d^3*e^3*f^4 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2
*f^5 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e*f^6)*x^7 - 1386*(b^3*d^
3*e^4*f^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^3*f^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d
^2 + a^2*b*d^3)*e^2*f^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d
^3)*e*f^6)*x^5 + 2310*(b^3*d^3*e^5*f^2 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^4*f^3
 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^3*f^4 - (b^3*c^3 + 9*a*b^2*
c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^2*f^5 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d +
a^3*c*d^2)*e*f^6)*x^3 - 3465*(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a
*b^2*d^3)*e^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3
*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3
*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5)*sqrt(
-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6930*(b^3*d^3*e^6*f
- 3*(b^3*c*d^2 + a*b^2*d^3)*e^5*f^2 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b
*d^3)*e^4*f^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^
4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^5 - 3*(a^2*b*c^3 + a^3
*c^2*d)*e*f^6)*x)/(e*f^7), 1/3465*(315*b^3*d^3*e*f^6*x^11 - 385*(b^3*d^3*e
^2*f^5 - 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^6)*x^9 + 495*(b^3*d^3*e^3*f^4 - 3*(
b^3*c*d^2 + a*b^2*d^3)*e^2*f^5 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*e*f^6)*x^7 - 693*(b^3*d^3*e^4*f^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^3*f^4 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1379 vs. $2(428) = 856$.

Time = 3.79 (sec) , antiderivative size = 1379, normalized size of antiderivative = 3.22

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
b**3*d**3*x**11/(11*f) + x**9*(a*b**2*d**3/(3*f) + b**3*c*d**2/(3*f) - b**
3*d**3*e/(9*f**2)) + x**7*(3*a**2*b*d**3/(7*f) + 9*a*b**2*c*d**2/(7*f) - 3
*a*b**2*d**3*e/(7*f**2) + 3*b**3*c**2*d/(7*f) - 3*b**3*c*d**2*e/(7*f**2) +
 b**3*d**3*e**2/(7*f**3)) + x**5*(a**3*d**3/(5*f) + 9*a**2*b*c*d**2/(5*f)
- 3*a**2*b*d**3*e/(5*f**2) + 9*a*b**2*c**2*d/(5*f) - 9*a*b**2*c*d**2*e/(5*
f**2) + 3*a*b**2*d**3*e**2/(5*f**3) + b**3*c**3/(5*f) - 3*b**3*c**2*d*e/(5
*f**2) + 3*b**3*c*d**2*e**2/(5*f**3) - b**3*d**3*e**3/(5*f**4)) + x**3*(a*
*3*c*d**2/f - a**3*d**3*e/(3*f**2) + 3*a**2*b*c**2*d/f - 3*a**2*b*c*d**2*e
/f**2 + a**2*b*d**3*e**2/f**3 + a*b**2*c**3/f - 3*a*b**2*c**2*d*e/f**2 + 3
*a*b**2*c*d**2*e**2/f**3 - a*b**2*d**3*e**3/f**4 - b**3*c**3*e/(3*f**2) +
b**3*c**2*d*e**2/f**3 - b**3*c*d**2*e**3/f**4 + b**3*d**3*e**4/(3*f**5)) +
 x*(3*a**3*c**2*d/f - 3*a**3*c*d**2*e/f**2 + a**3*d**3*e**2/f**3 + 3*a**2*
b*c**3/f - 9*a**2*b*c**2*d*e/f**2 + 9*a**2*b*c*d**2*e**2/f**3 - 3*a**2*b*d
**3*e**3/f**4 - 3*a*b**2*c**3*e/f**2 + 9*a*b**2*c**2*d*e**2/f**3 - 9*a*b**
2*c*d**2*e**3/f**4 + 3*a*b**2*d**3*e**4/f**5 + b**3*c**3*e**2/f**3 - 3*b**
3*c**2*d*e**3/f**4 + 3*b**3*c*d**2*e**4/f**5 - b**3*d**3*e**5/f**6) - sqrt
(-1/(e*f**13))*(a*f - b*e)**3*(c*f - d*e)**3*log(-e*f**6*sqrt(-1/(e*f**13)
)*(a*f - b*e)**3*(c*f - d*e)**3/(a**3*c**3*f**6 - 3*a**3*c**2*d*e*f**5 + 3
*a**3*c*d**2*e**2*f**4 - a**3*d**3*e**3*f**3 - 3*a**2*b*c**3*e*f**5 + 9*a*
*2*b*c**2*d*e**2*f**4 - 9*a**2*b*c*d**2*e**3*f**3 + 3*a**2*b*d**3*e**4*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(410) = 820$.

Time = 0.14 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.37

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
(b^3*d^3*e^6 - 3*b^3*c*d^2*e^5*f - 3*a*b^2*d^3*e^5*f + 3*b^3*c^2*d*e^4*f^2
 + 9*a*b^2*c*d^2*e^4*f^2 + 3*a^2*b*d^3*e^4*f^2 - b^3*c^3*e^3*f^3 - 9*a*b^2
*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 - a^3*d^3*e^3*f^3 + 3*a*b^2*c^3*e^2
*f^4 + 9*a^2*b*c^2*d*e^2*f^4 + 3*a^3*c*d^2*e^2*f^4 - 3*a^2*b*c^3*e*f^5 - 3
*a^3*c^2*d*e*f^5 + a^3*c^3*f^6)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^6) + 1/
3465*(315*b^3*d^3*f^10*x^11 - 385*b^3*d^3*e*f^9*x^9 + 1155*b^3*c*d^2*f^10*
x^9 + 1155*a*b^2*d^3*f^10*x^9 + 495*b^3*d^3*e^2*f^8*x^7 - 1485*b^3*c*d^2*e
*f^9*x^7 - 1485*a*b^2*d^3*e*f^9*x^7 + 1485*b^3*c^2*d*f^10*x^7 + 4455*a*b^2
*c*d^2*f^10*x^7 + 1485*a^2*b*d^3*f^10*x^7 - 693*b^3*d^3*e^3*f^7*x^5 + 2079
*b^3*c*d^2*e^2*f^8*x^5 + 2079*a*b^2*d^3*e^2*f^8*x^5 - 2079*b^3*c^2*d*e*f^9
*x^5 - 6237*a*b^2*c*d^2*e*f^9*x^5 - 2079*a^2*b*d^3*e*f^9*x^5 + 693*b^3*c^3
*f^10*x^5 + 6237*a*b^2*c^2*d*f^10*x^5 + 6237*a^2*b*c*d^2*f^10*x^5 + 693*a^
3*d^3*f^10*x^5 + 1155*b^3*d^3*e^4*f^6*x^3 - 3465*b^3*c*d^2*e^3*f^7*x^3 - 3
465*a*b^2*d^3*e^3*f^7*x^3 + 3465*b^3*c^2*d*e^2*f^8*x^3 + 10395*a*b^2*c*d^2
*e^2*f^8*x^3 + 3465*a^2*b*d^3*e^2*f^8*x^3 - 1155*b^3*c^3*e*f^9*x^3 - 10395
*a*b^2*c^2*d*e*f^9*x^3 - 10395*a^2*b*c*d^2*e*f^9*x^3 - 1155*a^3*d^3*e*f^9*
x^3 + 3465*a*b^2*c^3*f^10*x^3 + 10395*a^2*b*c^2*d*f^10*x^3 + 3465*a^3*c*d^
2*f^10*x^3 - 3465*b^3*d^3*e^5*f^5*x + 10395*b^3*c*d^2*e^4*f^6*x + 10395*a*
b^2*d^3*e^4*f^6*x - 10395*b^3*c^2*d*e^3*f^7*x - 31185*a*b^2*c*d^2*e^3*f^7*
x - 10395*a^2*b*d^3*e^3*f^7*x + 3465*b^3*c^3*e^2*f^8*x + 31185*a*b^2*c^...
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.96

$$\int \frac{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}{e+fx^2}\,dx$$

$$= x\left(\frac{e\left(\frac{e\left(\frac{a^3\,d^3+9\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d+b^3\,c^3}{f}-\frac{e\left(\frac{e\left(\frac{b^3\,d^3\,e}{f^2}-\frac{3\,b^2\,d^2\,(a\,d+b\,c)}{f}\right)}{f}+\frac{3\,b\,d\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{f}\right)}{f}\right)}{f}-\frac{3\,a\,c\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{f}\cdots\right)}{f}\cdots\right.$$

$$+\frac{3\,a^2\,c^2\,(a\,d+b\,c)}{f}\Bigg)$$

$$\left(\left(\cdots e\left(\frac{e\left(\frac{b^3\,d^3\,e}{f^2}-\frac{3\,b^2\,d^2\,(a\,d+b\,c)}{f}\right)}{f}+\frac{3\,b\,d\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{f}\right)\cdots\right.\right.$$

input
```
int(((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
x*((e*((e*((a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2)/f - (e*((e*
((b^3*d^3*e)/f^2 - (3*b^2*d^2*(a*d + b*c))/f))/f + (3*b*d*(a^2*d^2 + b^2*c
^2 + 3*a*b*c*d))/f))/f))/f - (3*a*c*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f))/f
 + (3*a^2*c^2*(a*d + b*c))/f) - x^3*((e*((a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*
d + 9*a^2*b*c*d^2)/f - (e*((e*((b^3*d^3*e)/f^2 - (3*b^2*d^2*(a*d + b*c))/f
))/f + (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f))/f))/(3*f) - (a*c*(a^2*d
^2 + b^2*c^2 + 3*a*b*c*d))/f) + x^5*((a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d +
9*a^2*b*c*d^2)/(5*f) - (e*((e*((b^3*d^3*e)/f^2 - (3*b^2*d^2*(a*d + b*c))/f
))/f + (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f))/(5*f)) + x^7*((e*((b^3*
d^3*e)/f^2 - (3*b^2*d^2*(a*d + b*c))/f))/(7*f) + (3*b*d*(a^2*d^2 + b^2*c^2
 + 3*a*b*c*d))/(7*f)) - x^9*((b^3*d^3*e)/(9*f^2) - (b^2*d^2*(a*d + b*c))/(
3*f)) + (b^3*d^3*x^11)/(11*f) + (atan((f^(1/2)*x*(a*f - b*e)^3*(c*f - d*e)
^3)/(e^(1/2)*(a^3*c^3*f^6 + b^3*d^3*e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^
3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^
2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4
+ 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*
a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*f^4)))*(a*f - b*e)^3*(c*f - d*e)^3
)/(e^(1/2)*f^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.02

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{e+fx^2} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e),x)
```

output
```
(3465*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*f**6 - 10395
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e*f**5 + 10395*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**2*f**4 - 3465
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**3*e**3*f**3 - 10395
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**3*e*f**5 + 31185*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**2*d*e**2*f**4 - 31
185*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c*d**2*e**3*f**3
+ 10395*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*d**3*e**4*f**
2 + 10395*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c**3*e**2*f
**4 - 31185*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c**2*d*e*
*3*f**3 + 31185*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c*d**
2*e**4*f**2 - 10395*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*d
**3*e**5*f - 3465*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**3*c**3*
e**3*f**3 + 10395*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**3*c**2*
d*e**4*f**2 - 10395*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**3*c*d
**2*e**5*f + 3465*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*b**3*d**3*
e**6 + 10395*a**3*c**2*d*e*f**6*x - 10395*a**3*c*d**2*e**2*f**5*x + 3465*a
**3*c*d**2*e*f**6*x**3 + 3465*a**3*d**3*e**3*f**4*x - 1155*a**3*d**3*e**2*
f**5*x**3 + 693*a**3*d**3*e*f**6*x**5 + 10395*a**2*b*c**3*e*f**6*x - 31185
*a**2*b*c**2*d*e**2*f**5*x + 10395*a**2*b*c**2*d*e*f**6*x**3 + 31185*a*...
```

3.244 $\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 387

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^2}\,dx = -\frac{(a^3d^2f^3(2de-3cf)-9a^2bdf^2(de-cf)^2-b^3e(5de-2cf)(de-cf)^2+3ab^2f(de-cf)^2(4de-cf))\,x}{f^6} - \frac{(bde-bcf-adf)\,(a^2d^2f^2-abdf(5de-8cf)+b^2(4d^2e^2-5cdef+c^2f^2))\,x^3}{3f^5} + \frac{3bd(a^2d^2f^2-abdf(2de-3cf)+b^2(de-cf)^2)\,x^5}{5f^4} - \frac{b^2d^2(2bde-3bcf-3adf)x^7}{7f^3} + \frac{b^3d^3x^9}{9f^2} + \frac{(be-af)^3(de-cf)^3x}{2ef^6\,(e+fx^2)} - \frac{(be-af)^2(de-cf)^2(be(11de-5cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{13/2}}$$

output
```
-(a^3*d^2*f^3*(-3*c*f+2*d*e)-9*a^2*b*d*f^2*(-c*f+d*e)^2-b^3*e*(-2*c*f+5*d*
e)*(-c*f+d*e)^2+3*a*b^2*f*(-c*f+d*e)^2*(-c*f+4*d*e))*x/f^6-1/3*(-a*d*f-b*c
*f+b*d*e)*(a^2*d^2*f^2-a*b*d*f*(-8*c*f+5*d*e)+b^2*(c^2*f^2-5*c*d*e*f+4*d^2
*e^2))*x^3/f^5+3/5*b*d*(a^2*d^2*f^2-a*b*d*f*(-3*c*f+2*d*e)+b^2*(-c*f+d*e)^
2)*x^5/f^4-1/7*b^2*d^2*(-3*a*d*f-3*b*c*f+2*b*d*e)*x^7/f^3+1/9*b^3*d^3*x^9/
f^2+1/2*(-a*f+b*e)^3*(-c*f+d*e)^3*x/e/f^6/(f*x^2+e)-1/2*(-a*f+b*e)^2*(-c*f
+d*e)^2*(b*e*(-5*c*f+11*d*e)-a*f*(c*f+5*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^
(3/2)/f^(13/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^2}\,dx$$
$$= \frac{(9a^2bdf^2(de-cf)^2+b^3e(5de-2cf)(de-cf)^2+3ab^2f(de-cf)^2(-4de+cf)+a^3d^2f^3(-2de+3cf))}{f^6}$$
$$+\frac{(a^3d^3f^3+9ab^2df(de-cf)^2-b^3(de-cf)^2(4de-cf)+3a^2bd^2f^2(-2de+3cf))\,x^3}{3f^5}$$
$$+\frac{3bd(a^2d^2f^2+b^2(de-cf)^2+abdf(-2de+3cf))\,x^5}{5f^4}$$
$$-\frac{b^2d^2(2bde-3bcf-3adf)x^7}{7f^3}+\frac{b^3d^3x^9}{9f^2}+\frac{(be-af)^3(de-cf)^3x}{2ef^6(e+fx^2)}$$
$$-\frac{(be-af)^2(de-cf)^2(be(11de-5cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{13/2}}$$

input
```
Integrate[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
((9*a^2*b*d*f^2*(d*e - c*f)^2 + b^3*e*(5*d*e - 2*c*f)*(d*e - c*f)^2 + 3*a*
b^2*f*(d*e - c*f)^2*(-4*d*e + c*f) + a^3*d^2*f^3*(-2*d*e + 3*c*f))*x)/f^6
+ ((a^3*d^3*f^3 + 9*a*b^2*d*f*(d*e - c*f)^2 - b^3*(d*e - c*f)^2*(4*d*e - c
*f) + 3*a^2*b*d^2*f^2*(-2*d*e + 3*c*f))*x^3)/(3*f^5) + (3*b*d*(a^2*d^2*f^2
 + b^2*(d*e - c*f)^2 + a*b*d*f*(-2*d*e + 3*c*f))*x^5)/(5*f^4) - (b^2*d^2*(
2*b*d*e - 3*b*c*f - 3*a*d*f)*x^7)/(7*f^3) + (b^3*d^3*x^9)/(9*f^2) + ((b*e
- a*f)^3*(d*e - c*f)^3*x)/(2*e*f^6*(e + f*x^2)) - ((b*e - a*f)^2*(d*e - c*
f)^2*(b*e*(11*d*e - 5*c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]
])/(2*e^(3/2)*f^(13/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 880 vs. 2(387) = 774.

Time = 1.64 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.27, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.821$, Rules used = {425, 420, 290, 403, 25, 403, 25, 403, 299, 218, 425, 401, 25, 403, 25, 403, 25, 299, 218, 403, 299, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3 \left(c+dx^2\right)^3}{\left(e+fx^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b \int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af) \int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b \left(\frac{b \int \left(bx^2+a\right) \left(dx^2+c\right)^3dx}{f}-\frac{(be-af) \int \frac{\left(bx^2+a\right) \left(dx^2+c\right)^3}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af) \int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{b \left(\frac{b \int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af) \int \frac{\left(bx^2+a\right) \left(dx^2+c\right)^3}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af) \int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{b\left(\frac{b\int (bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3)dx}{f}-\frac{(be-af)\left(\frac{\int -\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}+\frac{bx\left(c+dx\right.}{7f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\int (bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}}{7f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 403

$$\frac{b\left(\frac{b\int (bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int -\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-(\right.}{f}}{}\right.}{}\right.}{f}$$

$$\frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right)}{f}$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 403

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{\left(7adf\left(15d^2\cdots\right.\right.}{\cdots}}{\cdots}\right)}{\cdots}\right)$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 299

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)(d\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right)$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 218

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\text{ ar}}{}}{}}{}\right)}{}\right)$$

$$\frac{(be-af)\int\frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 425

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\text{ ar}}{}}{}}{}\right)}{}\right)$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af)\int\frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 401

$$\frac{b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{105(be-af)\,\mathrm{ar}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(-\frac{\int-\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{105(be-af)\,\mathrm{ar}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\text{ar}\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.}{(be-af)\left(\frac{b\left(\frac{\int -\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}+\frac{bx\left(c+dx^2\right)^3}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2\cdots\right.}{fx^2+e}\cdots}{5f}\cdots\right.}{\cdots}\right)}{f}}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\text{ar}\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.}{(be-af)\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}\right)}{f}-\frac{(be-af)\left(\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right)}{f}}$$

$$\Big\downarrow \; 403$$

$$b\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\text{ar}}{}}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}+\frac{x\left(c+dx^2\right)^2(-7adf-}{5f}\right.}{f}\right.$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\mathrm{ar}\cdots}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f\cdots\right.\right.\right.}{fx^2+e}}{5f}}{7f}\cdots\right)}{f}\cdots\right.}{\cdots}$$

↓ 299

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\mathrm{ar}}{}}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f\right.\right.\right.}{fx^2+e}}{5f}}{7f}\right.}{f}\right.$$

$\downarrow$ 218

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\frac{105(be-af)\,\text{a}\ldots}{\ldots}}{\ldots}\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f\ldots\right.\right.\right.}{fx^2+e}}{5f}}{7f}\right.}{f}\right.$$

$\downarrow$ 403

$$b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{105(be-af)\,\mathrm{arc}}{\sqrt{\ }}}{}\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{\int\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3\right)\right)x^2+c\left(\right.}{fx^2+e}}{3f}}{7}\right)}{f}\right.$$

$\downarrow$ 299

$$\frac{b\left(\frac{b\int\left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{105(be-af)\,\text{arc}\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{\frac{105(be-af)\int\frac{1}{fx^2+e}dx(de-cf)^3}{f}+\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2\cdots\right.\right.}{f}}{3f}}{5f\cdots}}{7f}\cdots\right)}{f}\cdots\right)}{\cdots}$$

$\downarrow$ 218

$$b\left(\frac{b\int \left(bd^3x^8+d^2(3bc+ad)x^6+3cd(bc+ad)x^4+c^2(bc+3ad)x^2+ac^3\right)dx}{f}-\frac{(be-af)\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{105(be-af)\operatorname{arc}\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280c\cdots\right.\right.}{f}\cdots}{3f}\cdots}{7f}\cdots\right.}{f}\cdots\right.}{\cdots}$$

$\downarrow$ 2009

$$b\left(\frac{b\left(\frac{1}{9}bd^3x^9+\frac{1}{7}d^2(3bc+ad)x^7+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{3}c^2(bc+3ad)x^3+ac^3x\right)}{f}-\frac{(be-af)\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{105(be-af)\,a\cdots}{\cdots}}{\cdots}\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280c\cdots\right.\right.}{f}}{3f}}{7f}\cdots\right.}{f}\cdots\right.$$

input
```
Int[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
(b*((b*(a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 +
(d^2*(3*b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9))/f - ((b*e - a*f)*((b*x*(c + d*
x^2)^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(5*f) - (
-1/3*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))
*x*(c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*
(105*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/f + (10
5*(b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2))
)/(3*f))/(5*f))/(7*f)))/f))/f - ((b*e - a*f)*((b*((b*x*(c + d*x^2)^3)/(7*f
) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(5*f) - (-1/3*((7*a*d
*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^
2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3
 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/f + (105*(b*e - a*f
)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*
f))/(7*f)))/f - ((b*e - a*f)*(-1/2*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e +
 f*x^2)) + ((d*(7*b*e - 5*a*f)*x*(c + d*x^2)^2)/(5*f) - ((d*(b*e*(35*d*e -
 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d*x^2))/(3*f) + ((d*(5*a*f*(15*d^
2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2 - 190*c*d*e*f + 81*c^2*
f^2))*x)/f + (15*(d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/(2*e*f)))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(367) = 734$.

Time = 0.62 (sec) , antiderivative size = 996, normalized size of antiderivative = 2.57

method	result
default	$-\frac{6}{5}a\,b^2d^3e\,f^3x^5+\frac{1}{3}b^3c^3f^4x^3+3a\,b^2c^3f^4x+\frac{9}{5}a\,b^2c\,d^2f^4x^5+9a^2b\,c^2d\,f^4x+9a^2b\,d^3e^2f^2x+\frac{3}{5}a^2b\,d^3f^4x^5+\frac{3}{5}b^3c^2d\,f^4x^5+\frac{3}{5}b^3d^3e^2f^2$
risch	Expression too large to display

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^6*(-6/5*a*b^2*d^3*e*f^3*x^5+1/3*b^3*c^3*f^4*x^3+3*a*b^2*c^3*f^4*x+9/5*
a*b^2*c*d^2*f^4*x^5+9*a^2*b*c^2*d*f^4*x+9*a^2*b*d^3*e^2*f^2*x+3/5*a^2*b*d^
3*f^4*x^5+3/5*b^3*c^2*d*f^4*x^5+3/5*b^3*d^3*e^2*f^2*x^5-4/3*b^3*d^3*e^3*f*
x^3+3*a^3*c*d^2*f^4*x-2*a^3*d^3*e*f^3*x+1/3*a^3*d^3*f^4*x^3-6*a*b^2*c*d^2*
e*f^3*x^3-18*a^2*b*c*d^2*e*f^3*x-18*a*b^2*c^2*d*e*f^3*x+27*a*b^2*c*d^2*e^2
*f^2*x-6/5*b^3*c*d^2*e*f^3*x^5+3*a^2*b*c*d^2*f^4*x^3-2*a^2*b*d^3*e*f^3*x^3
-2*b^3*c^2*d*e*f^3*x^3+3*b^3*c*d^2*e^2*f^2*x^3+5*b^3*d^3*e^4*x-12*a*b^2*d^
3*e^3*f*x+9*b^3*c^2*d*e^2*f^2*x-12*b^3*c*d^2*e^3*f*x+1/9*d^3*x^9*b^3*f^4+3
/7*a*b^2*d^3*f^4*x^7+3/7*b^3*c*d^2*f^4*x^7-2/7*b^3*d^3*e*f^3*x^7+3*a*b^2*c
^2*d*f^4*x^3+3*a*b^2*d^3*e^2*f^2*x^3-2*b^3*c^3*e*f^3*x)+1/f^6*(1/2*(a^3*c^
3*f^6-3*a^3*c^2*d*e*f^5+3*a^3*c*d^2*e^2*f^4-a^3*d^3*e^3*f^3-3*a^2*b*c^3*e*
f^5+9*a^2*b*c^2*d*e^2*f^4-9*a^2*b*c*d^2*e^3*f^3+3*a^2*b*d^3*e^4*f^2+3*a*b^
2*c^3*e^2*f^4-9*a*b^2*c^2*d*e^3*f^3+9*a*b^2*c*d^2*e^4*f^2-3*a*b^2*d^3*e^5*
f-b^3*c^3*e^3*f^3+3*b^3*c^2*d*e^4*f^2-3*b^3*c*d^2*e^5*f+b^3*d^3*e^6)/e*x/(
f*x^2+e)+1/2*(a^3*c^3*f^6+3*a^3*c^2*d*e*f^5-9*a^3*c*d^2*e^2*f^4+5*a^3*d^3*
e^3*f^3+3*a^2*b*c^3*e*f^5-27*a^2*b*c^2*d*e^2*f^4+45*a^2*b*c*d^2*e^3*f^3-21
*a^2*b*d^3*e^4*f^2-9*a*b^2*c^3*e^2*f^4+45*a*b^2*c^2*d*e^3*f^3-63*a*b^2*c*d
^2*e^4*f^2+27*a*b^2*d^3*e^5*f+5*b^3*c^3*e^3*f^3-21*b^3*c^2*d*e^4*f^2+27*b^
3*c*d^2*e^5*f-11*b^3*d^3*e^6)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(367) = 734$.

Time = 0.12 (sec) , antiderivative size = 2076, normalized size of antiderivative = 5.36

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/1260*(140*b^3*d^3*e^2*f^6*x^11 - 20*(11*b^3*d^3*e^3*f^5 - 27*(b^3*c*d^2
 + a*b^2*d^3)*e^2*f^6)*x^9 + 36*(11*b^3*d^3*e^4*f^4 - 27*(b^3*c*d^2 + a*b^
2*d^3)*e^3*f^5 + 21*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^2*f^6)*x^7 -
 84*(11*b^3*d^3*e^5*f^3 - 27*(b^3*c*d^2 + a*b^2*d^3)*e^4*f^4 + 21*(b^3*c^2
*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^3*f^5 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*e^2*f^6)*x^5 + 420*(11*b^3*d^3*e^6*f^2 - 27*(b^3*c*d
^2 + a*b^2*d^3)*e^5*f^3 + 21*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f
^4 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^5 + 9*(a*
b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^6)*x^3 + 315*(11*b^3*d^3*e^7 -
a^3*c^3*e*f^6 - 27*(b^3*c*d^2 + a*b^2*d^3)*e^6*f + 21*(b^3*c^2*d + 3*a*b^2
*c*d^2 + a^2*b*d^3)*e^5*f^2 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 +
 a^3*d^3)*e^4*f^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^3*f^4 - 3*
(a^2*b*c^3 + a^3*c^2*d)*e^2*f^5 + (11*b^3*d^3*e^6*f - a^3*c^3*f^7 - 27*(b^
3*c*d^2 + a*b^2*d^3)*e^5*f^2 + 21*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*
e^4*f^3 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^4 +
9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^5 - 3*(a^2*b*c^3 + a^3*c^2
*d)*e*f^6)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) +
 630*(11*b^3*d^3*e^7*f + a^3*c^3*e*f^7 - 27*(b^3*c*d^2 + a*b^2*d^3)*e^6*f^
2 + 21*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^5*f^3 - 5*(b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^4*f^4 + 9*(a*b^2*c^3 + 3*a^2*b*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(379) = 758$.

Time = 13.48 (sec) , antiderivative size = 1590, normalized size of antiderivative = 4.11

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^2}\, dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
b**3*d**3*x**9/(9*f**2) + x**7*(3*a*b**2*d**3/(7*f**2) + 3*b**3*c*d**2/(7*
f**2) - 2*b**3*d**3*e/(7*f**3)) + x**5*(3*a**2*b*d**3/(5*f**2) + 9*a*b**2*
c*d**2/(5*f**2) - 6*a*b**2*d**3*e/(5*f**3) + 3*b**3*c**2*d/(5*f**2) - 6*b*
*3*c*d**2*e/(5*f**3) + 3*b**3*d**3*e**2/(5*f**4)) + x**3*(a**3*d**3/(3*f**
2) + 3*a**2*b*c*d**2/f**2 - 2*a**2*b*d**3*e/f**3 + 3*a*b**2*c**2*d/f**2 -
6*a*b**2*c*d**2*e/f**3 + 3*a*b**2*d**3*e**2/f**4 + b**3*c**3/(3*f**2) - 2*
b**3*c**2*d*e/f**3 + 3*b**3*c*d**2*e**2/f**4 - 4*b**3*d**3*e**3/(3*f**5))
+ x*(3*a**3*c*d**2/f**2 - 2*a**3*d**3*e/f**3 + 9*a**2*b*c**2*d/f**2 - 18*a
**2*b*c*d**2*e/f**3 + 9*a**2*b*d**3*e**2/f**4 + 3*a*b**2*c**3/f**2 - 18*a*
b**2*c**2*d*e/f**3 + 27*a*b**2*c*d**2*e**2/f**4 - 12*a*b**2*d**3*e**3/f**5
 - 2*b**3*c**3*e/f**3 + 9*b**3*c**2*d*e**2/f**4 - 12*b**3*c*d**2*e**3/f**5
 + 5*b**3*d**3*e**4/f**6) + x*(a**3*c**3*f**6 - 3*a**3*c**2*d*e*f**5 + 3*a
**3*c*d**2*e**2*f**4 - a**3*d**3*e**3*f**3 - 3*a**2*b*c**3*e*f**5 + 9*a**2
*b*c**2*d*e**2*f**4 - 9*a**2*b*c*d**2*e**3*f**3 + 3*a**2*b*d**3*e**4*f**2
+ 3*a*b**2*c**3*e**2*f**4 - 9*a*b**2*c**2*d*e**3*f**3 + 9*a*b**2*c*d**2*e*
*4*f**2 - 3*a*b**2*d**3*e**5*f - b**3*c**3*e**3*f**3 + 3*b**3*c**2*d*e**4*
f**2 - 3*b**3*c*d**2*e**5*f + b**3*d**3*e**6)/(2*e**2*f**6 + 2*e*f**7*x**2
) - sqrt(-1/(e**3*f**13))*(a*f - b*e)**2*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*
e*f + 5*b*c*e*f - 11*b*d*e**2)*log(-e**2*f**6*sqrt(-1/(e**3*f**13))*(a*f -
 b*e)**2*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + 5*b*c*e*f - 11*b*d*e**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(367) = 734$.

Time = 0.13 (sec) , antiderivative size = 1022, normalized size of antiderivative = 2.64

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(11*b^3*d^3*e^6 - 27*b^3*c*d^2*e^5*f - 27*a*b^2*d^3*e^5*f + 21*b^3*c^
2*d*e^4*f^2 + 63*a*b^2*c*d^2*e^4*f^2 + 21*a^2*b*d^3*e^4*f^2 - 5*b^3*c^3*e^
3*f^3 - 45*a*b^2*c^2*d*e^3*f^3 - 45*a^2*b*c*d^2*e^3*f^3 - 5*a^3*d^3*e^3*f^
3 + 9*a*b^2*c^3*e^2*f^4 + 27*a^2*b*c^2*d*e^2*f^4 + 9*a^3*c*d^2*e^2*f^4 - 3
*a^2*b*c^3*e*f^5 - 3*a^3*c^2*d*e*f^5 - a^3*c^3*f^6)*arctan(f*x/sqrt(e*f))/
(sqrt(e*f)*e*f^6) + 1/2*(b^3*d^3*e^6*x - 3*b^3*c*d^2*e^5*f*x - 3*a*b^2*d^3
*e^5*f*x + 3*b^3*c^2*d*e^4*f^2*x + 9*a*b^2*c*d^2*e^4*f^2*x + 3*a^2*b*d^3*e
^4*f^2*x - b^3*c^3*e^3*f^3*x - 9*a*b^2*c^2*d*e^3*f^3*x - 9*a^2*b*c*d^2*e^3
*f^3*x - a^3*d^3*e^3*f^3*x + 3*a*b^2*c^3*e^2*f^4*x + 9*a^2*b*c^2*d*e^2*f^4
*x + 3*a^3*c*d^2*e^2*f^4*x - 3*a^2*b*c^3*e*f^5*x - 3*a^3*c^2*d*e*f^5*x + a
^3*c^3*f^6*x)/((f*x^2 + e)*e*f^6) + 1/315*(35*b^3*d^3*f^16*x^9 - 90*b^3*d^
3*e*f^15*x^7 + 135*b^3*c*d^2*f^16*x^7 + 135*a*b^2*d^3*f^16*x^7 + 189*b^3*d
^3*e^2*f^14*x^5 - 378*b^3*c*d^2*e*f^15*x^5 - 378*a*b^2*d^3*e*f^15*x^5 + 18
9*b^3*c^2*d*f^16*x^5 + 567*a*b^2*c*d^2*f^16*x^5 + 189*a^2*b*d^3*f^16*x^5 -
 420*b^3*d^3*e^3*f^13*x^3 + 945*b^3*c*d^2*e^2*f^14*x^3 + 945*a*b^2*d^3*e^2
*f^14*x^3 - 630*b^3*c^2*d*e*f^15*x^3 - 1890*a*b^2*c*d^2*e*f^15*x^3 - 630*a
^2*b*d^3*e*f^15*x^3 + 105*b^3*c^3*f^16*x^3 + 945*a*b^2*c^2*d*f^16*x^3 + 94
5*a^2*b*c*d^2*f^16*x^3 + 105*a^3*d^3*f^16*x^3 + 1575*b^3*d^3*e^4*f^12*x -
3780*b^3*c*d^2*e^3*f^13*x - 3780*a*b^2*d^3*e^3*f^13*x + 2835*b^3*c^2*d*e^2
*f^14*x + 8505*a*b^2*c*d^2*e^2*f^14*x + 2835*a^2*b*d^3*e^2*f^14*x - 630...
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1177, normalized size of antiderivative = 3.04

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int(((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
x^5*((2*e*((2*b^3*d^3*e)/f^3 - (3*b^2*d^2*(a*d + b*c))/f^2))/(5*f) - (b^3*
d^3*e^2)/(5*f^4) + (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/(5*f^2)) + x^3*
((a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(3*f^2) - (2*e*((2*e*
((2*b^3*d^3*e)/f^3 - (3*b^2*d^2*(a*d + b*c))/f^2))/f - (b^3*d^3*e^2)/f^4 +
 (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f^2))/(3*f) + (e^2*((2*b^3*d^3*e)
/f^3 - (3*b^2*d^2*(a*d + b*c))/f^2))/(3*f^2)) - x*((2*e*((a^3*d^3 + b^3*c^
3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2)/f^2 - (2*e*((2*e*((2*b^3*d^3*e)/f^3 - (
3*b^2*d^2*(a*d + b*c))/f^2))/f - (b^3*d^3*e^2)/f^4 + (3*b*d*(a^2*d^2 + b^2
*c^2 + 3*a*b*c*d))/f^2))/f + (e^2*((2*b^3*d^3*e)/f^3 - (3*b^2*d^2*(a*d + b
*c))/f^2))/f^2))/f + (e^2*((2*e*((2*b^3*d^3*e)/f^3 - (3*b^2*d^2*(a*d + b*c
))/f^2))/f - (b^3*d^3*e^2)/f^4 + (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f
^2))/f^2 - (3*a*c*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f^2) - x^7*((2*b^3*d^3*
e)/(7*f^3) - (3*b^2*d^2*(a*d + b*c))/(7*f^2)) + (x*(a^3*c^3*f^6 + b^3*d^3*
e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*
e^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^
2*b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^
2*e^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*
e^2*f^4))/(2*e*(e*f^6 + f^7*x^2)) + (b^3*d^3*x^9)/(9*f^2) + (atan((f^(1/2)
*x*(a*f - b*e)^2*(c*f - d*e)^2*(a*c*f^2 - 11*b*d*e^2 + 5*a*d*e*f + 5*b*c*e
*f))/(e^(1/2)*(a^3*c^3*f^6 - 11*b^3*d^3*e^6 + 5*a^3*d^3*e^3*f^3 + 5*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1885, normalized size of antiderivative = 4.87

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output

```
(315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e*f**6 + 315*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*f**7*x**2 + 945*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**2*f**5 + 945*sq
rt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e*f**6*x**2 - 2835
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**3*f**4 - 283
5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**2*f**5*x**2
 + 1575*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**3*e**4*f**3
+ 1575*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**3*e**3*f**4*x
**2 + 945*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**3*e**2*f
**5 + 945*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**3*e*f**6
*x**2 - 8505*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**2*d*e
**3*f**4 - 8505*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**2*
d*e**2*f**5*x**2 + 14175*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**
2*b*c*d**2*e**4*f**3 + 14175*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a**2*b*c*d**2*e**3*f**4*x**2 - 6615*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*s
qrt(e)))*a**2*b*d**3*e**5*f**2 - 6615*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*
sqrt(e)))*a**2*b*d**3*e**4*f**3*x**2 - 2835*sqrt(f)*sqrt(e)*atan((f*x)/(sq
rt(f)*sqrt(e)))*a*b**2*c**3*e**3*f**4 - 2835*sqrt(f)*sqrt(e)*atan((f*x)/(s
qrt(f)*sqrt(e)))*a*b**2*c**3*e**2*f**5*x**2 + 14175*sqrt(f)*sqrt(e)*atan((
f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c**2*d*e**4*f**3 + 14175*sqrt(f)*sqrt(e)...
```

3.245 $\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 455

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{(a^3d^3f^3 - 9a^2bd^2f^2(de-cf) + 9ab^2df(2d^2e^2 - 3cdef + c^2f^2) - b^3(10d^3e^3 - 18cd^2e^2f + 9c^2def^2 - c^3f}{f^6} + \frac{bd(a^2d^2f^2 - 3abdf(de-cf) + b^2(2d^2e^2 - 3cdef + c^2f^2))x^3}{f^5} - \frac{3b^2d^2(bde - bcf - adf)x^5}{5f^4} + \frac{b^3d^3x^7}{7f^3} + \frac{(be-af)^3(de-cf)^3x}{4ef^6(e+fx^2)^2} - \frac{3(be-af)^2(de-cf)^2(be(7de-3cf) - af(3de+cf))x}{8e^2f^6(e+fx^2)} - \frac{3(be-af)(de-cf)\,(2abef(15d^2e^2 - 6cdef - c^2f^2) - a^2f^2(5d^2e^2 + 2cdef + c^2f^2) - b^2e^2(33d^2e^2 - 3}{8e^{5/2}f^{13/2}}$$

output
```
(a^3*d^3*f^3-9*a^2*b*d^2*f^2*(-c*f+d*e)+9*a*b^2*d*f*(c^2*f^2-3*c*d*e*f+2*d
^2*e^2)-b^3*(-c^3*f^3+9*c^2*d*e*f^2-18*c*d^2*e^2*f+10*d^3*e^3))*x/f^6+b*d*
(a^2*d^2*f^2-3*a*b*d*f*(-c*f+d*e)+b^2*(c^2*f^2-3*c*d*e*f+2*d^2*e^2))*x^3/f
^5-3/5*b^2*d^2*(-a*d*f-b*c*f+b*d*e)*x^5/f^4+1/7*b^3*d^3*x^7/f^3+1/4*(-a*f+
b*e)^3*(-c*f+d*e)^3*x/e/f^6/(f*x^2+e)^2-3/8*(-a*f+b*e)^2*(-c*f+d*e)^2*(b*e
*(-3*c*f+7*d*e)-a*f*(c*f+3*d*e))*x/e^2/f^6/(f*x^2+e)-3/8*(-a*f+b*e)*(-c*f+
d*e)*(2*a*b*e*f*(-c^2*f^2-6*c*d*e*f+15*d^2*e^2)-a^2*f^2*(c^2*f^2+2*c*d*e*f
+5*d^2*e^2)-b^2*e^2*(5*c^2*f^2-30*c*d*e*f+33*d^2*e^2))*arctan(f^(1/2)*x/e^
(1/2))/e^(5/2)/f^(13/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{(a^3d^3f^3+9a^2bd^2f^2(-de+cf)+9ab^2df(2d^2e^2-3cdef+c^2f^2)+b^3(-10d^3e^3+18cd^2e^2f-9c^2def^2+}{f^6}$$
$$+\frac{bd(a^2d^2f^2+3abdf(-de+cf)+b^2(2d^2e^2-3cdef+c^2f^2))x^3}{f^5}$$
$$-\frac{3b^2d^2(bde-bcf-adf)x^5}{5f^4}+\frac{b^3d^3x^7}{7f^3}+\frac{(be-af)^3(de-cf)^3x}{4ef^6(e+fx^2)^2}$$
$$-\frac{3(be-af)^2(de-cf)^2(be(7de-3cf)-af(3de+cf))x}{8e^2f^6(e+fx^2)}$$
$$+\frac{3(be-af)(de-cf)(a^2f^2(5d^2e^2+2cdef+c^2f^2)+2abef(-15d^2e^2+6cdef+c^2f^2)+b^2e^2(33d^2e^2-}{8e^{5/2}f^{13/2}}$$

input
```
Integrate[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(-(d*e) + c*f) + 9*a*b^2*d*f*(2*d^2*e^2 -
3*c*d*e*f + c^2*f^2) + b^3*(-10*d^3*e^3 + 18*c*d^2*e^2*f - 9*c^2*d*e*f^2 +
 c^3*f^3))*x)/f^6 + (b*d*(a^2*d^2*f^2 + 3*a*b*d*f*(-(d*e) + c*f) + b^2*(2*
d^2*e^2 - 3*c*d*e*f + c^2*f^2))*x^3)/f^5 - (3*b^2*d^2*(b*d*e - b*c*f - a*d
*f)*x^5)/(5*f^4) + (b^3*d^3*x^7)/(7*f^3) + ((b*e - a*f)^3*(d*e - c*f)^3*x)
/(4*e*f^6*(e + f*x^2)^2) - (3*(b*e - a*f)^2*(d*e - c*f)^2*(b*e*(7*d*e - 3*
c*f) - a*f*(3*d*e + c*f))*x)/(8*e^2*f^6*(e + f*x^2)) + (3*(b*e - a*f)*(d*e
 - c*f)*(a^2*f^2*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*a*b*e*f*(-15*d^2*e^
2 + 6*c*d*e*f + c^2*f^2) + b^2*e^2*(33*d^2*e^2 - 30*c*d*e*f + 5*c^2*f^2))*
ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(13/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1127 vs. $2(455) = 910$.

Time = 1.75 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.48, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {425, 425, 401, 25, 401, 403, 25, 299, 218, 403, 25, 299, 218, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^3} dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(bx^2+a)^2(dx^2+c)^3}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)^2(dx^2+c)^3}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{b\int \frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{(bx^2+a)(dx^2+c)^3}{(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 401

$$\frac{b\left(\frac{b\int\frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(-\frac{\int-\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\left(-\frac{\int-\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(-\frac{\int-\frac{(dx^2+c)^2(d(7be-3af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef}-\cdots\right.}{f}\right.}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\int\frac{(bx^2+a)(dx^2+c)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(\frac{\int\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{\int\frac{(dx^2+c)^2(d(7be-5af)x^2+c(be+af))}{fx^2+e}dx}{2ef}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}\right)}{f}-\frac{(be-af)\left(\frac{\int\frac{(dx^2+c)^2(d(7be-3af)x^2+c(be+3af))}{(fx^2+e)^2}dx}{4ef}-\frac{x(c+d\cdots}{4ef\cdots}\right.}{f}\right.}{f}$$

$\downarrow$ 401

$$\frac{b\left(\frac{b\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af)\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de\cdots\right.}{fx^2+e}}{2ef}\cdots\right.}{\cdots}\cdots\right.}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{b\left(\frac{b\left(\frac{\int -\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}+\frac{bx\left(c+dx^2\right)^3}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5\cdots\right.}{fx^2+e}}{5f}\cdots}{2ef}\cdots\right.}{f}\cdots\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}+\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\cdots\right.$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de\right.}{f}}{2ef}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\right.}{}$$

$\downarrow$ 299

$$\frac{b\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int \frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de\right.}{f}}{2ef}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\right.}{}$$

$\big\downarrow$ 218

$$b\left(\frac{b\left(\frac{bx\left(c+dx^2\right)^3}{7f}-\frac{\int\frac{\left(dx^2+c\right)^2\left((7bde-6bcf-7adf)x^2+c(be-7af)\right)}{fx^2+e}dx}{7f}\right)}{f}-\frac{(be-af)\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de\right.}{f}}{2ef}\right.}{f}\right.$$

$$\overline{f}$$

$$(be-af)\left(\frac{b\left(\frac{\frac{dx\left(c+dx^2\right)^2(7be-5af)}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\right.$$

$\big\downarrow$ 403

$$b\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}+\frac{\int -\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\int \frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right.\right.}{f}}{5f}}{2ef}\right)}{f}\right.$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\int\frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right.\right.}{f}}{5f}}{2ef}\right.}{f}\right.$$

↓ 299

$$b\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\int\frac{1}{fx^2+e}dx(de-cf)^2}{f}+\frac{d\left(5af\left(15d^2\ldots\right.\right.}{3f}}{5f}}{2ef}\right.}{f}\right.$$

↓ 218

$$b\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\int \frac{\left(dx^2+c\right)\left(c(be(7de-11cf)-7af(de-5cf))-\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdfe+24c^2f^2\right)\right)x^2\right)}{fx^2+e}dx}{5f}}{7f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}+\frac{d\left(5af\left(15\right.\right.}{3f}}{5f}}{2ef}\right.}{f}\right.$$

↓ 403

$$b\left(\frac{b\left(\frac{bx(dx^2+c)^3}{7f}-\frac{(7bde-6bcf-7adf)x(dx^2+c)^2}{5f}-\frac{\frac{\int\frac{(7adf(15d^2e^2-40cdfe+33c^2f^2)-b(105d^3e^3-280cd^2fe^2+231c^2df^2e-48c^3f^3))x^2+c(7af(5d^2e^2-}{fx^2+e}}{3f}}{7f}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x(dx^2+c)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x(dx^2+c)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}+\frac{d(5af(15}{3f}}{5f}}{2ef}\right.}{f}\right.$$

↓ 299

$$b\left(\frac{b\left(\frac{bx(dx^2+c)^3}{7f}-\frac{(7bde-6bcf-7adf)x(dx^2+c)^2}{5f}-\frac{\frac{\frac{105(be-af)\int\frac{1}{fx^2+e}dx(de-cf)^3}{f}+\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c^2df\cdots\right.\right.}{f}}{3f}\cdots}{5f}\cdots}{7f}\cdots\right)}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x(dx^2+c)^2}{5f}-\frac{d(be(35de-33cf)-5af(5de-3cf))x(dx^2+c)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}+\frac{d\left(5af\left(15\cdots\right.\right.}{3f}\cdots}{5f}\cdots}{2ef}\cdots\right)}{f}\cdots\right.$$

$\downarrow$ 218

$$b\left(\frac{b\left(\frac{bx\left(dx^2+c\right)^3}{7f}-\frac{\frac{(7bde-6bcf-7adf)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{\frac{105(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{3/2}}+\frac{\left(7adf\left(15d^2e^2-40cdfe+33c^2f^2\right)-b\left(105d^3e^3-280cd^2fe^2+231c\cdots\right.\right.}{f}}{3f}\cdots}{5f}\cdots}{7f}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}+\frac{d\left(5af\left(15\cdots\right.\right.}{3f}\cdots}{5f}\cdots}{2ef}\cdots\right.}{f}\cdots\right.$$

input
```
Int[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(b*((b*((b*x*(c + d*x^2)^3)/(7*f) - (((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c +
 d*x^2)^2)/(5*f) - (-1/3*((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*
d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/f + (((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*
f + 33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*
c^3*f^3))*x)/f + (105*(b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]
])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/(7*f)))/f - ((b*e - a*f)*(-1/2*((b*e -
 a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*a*f)*x*(c + d*x^
2)^2)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d
*x^2))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105
*d^2*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*f)^2*(b*e*(7*d*e
 - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)
))/(3*f))/(5*f))/(2*e*f)))/f))/f - ((b*e - a*f)*((b*(-1/2*((b*e - a*f)*x*(
c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e - 5*a*f)*x*(c + d*x^2)^2)/(5*
f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d*x^2))/(3
*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2
- 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e - c*f)^2*(b*e*(7*d*e - c*f) -
 a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))
/(5*f))/(2*e*f)))/f - ((b*e - a*f)*(-1/4*((b*e - a*f)*x*(c + d*x^2)^3)/(e*
f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d
*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(437) = 874$.

Time = 0.66 (sec) , antiderivative size = 1015, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1015
risch	Expression too large to display	1774

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f^6*(1/7*b^3*d^3*x^7*f^3+3/5*a*b^2*d^3*f^3*x^5+3/5*b^3*c*d^2*f^3*x^5-3/5
*b^3*d^3*e*f^2*x^5+a^2*b*d^3*f^3*x^3+3*a*b^2*c*d^2*f^3*x^3-3*a*b^2*d^3*e*f
^2*x^3+b^3*c^2*d*f^3*x^3-3*b^3*c*d^2*e*f^2*x^3+2*b^3*d^3*e^2*f*x^3+a^3*d^3
*f^3*x+9*a^2*b*c*d^2*f^3*x-9*a^2*b*d^3*e*f^2*x+9*a*b^2*c^2*d*f^3*x-27*a*b^
2*c*d^2*e*f^2*x+18*a*b^2*d^3*e^2*f*x+b^3*c^3*f^3*x-9*b^3*c^2*d*e*f^2*x+18*
b^3*c*d^2*e^2*f*x-10*b^3*d^3*e^3*x)+1/f^6*((3/8*f*(a^3*c^3*f^6+a^3*c^2*d*e
*f^5-5*a^3*c*d^2*e^2*f^4+3*a^3*d^3*e^3*f^3+a^2*b*c^3*e*f^5-15*a^2*b*c^2*d*
e^2*f^4+27*a^2*b*c*d^2*e^3*f^3-13*a^2*b*d^3*e^4*f^2-5*a*b^2*c^3*e^2*f^4+27
*a*b^2*c^2*d*e^3*f^3-39*a*b^2*c*d^2*e^4*f^2+17*a*b^2*d^3*e^5*f+3*b^3*c^3*e
^3*f^3-13*b^3*c^2*d*e^4*f^2+17*b^3*c*d^2*e^5*f-7*b^3*d^3*e^6)/e^2*x^3+1/8*
(5*a^3*c^3*f^6-3*a^3*c^2*d*e*f^5-9*a^3*c*d^2*e^2*f^4+7*a^3*d^3*e^3*f^3-3*a
^2*b*c^3*e*f^5-27*a^2*b*c^2*d*e^2*f^4+63*a^2*b*c*d^2*e^3*f^3-33*a^2*b*d^3*
e^4*f^2-9*a*b^2*c^3*e^2*f^4+63*a*b^2*c^2*d*e^3*f^3-99*a*b^2*c*d^2*e^4*f^2+
45*a*b^2*d^3*e^5*f+7*b^3*c^3*e^3*f^3-33*b^3*c^2*d*e^4*f^2+45*b^3*c*d^2*e^5
*f-19*b^3*d^3*e^6)/e*x)/(f*x^2+e)^2+3/8*(a^3*c^3*f^6+a^3*c^2*d*e*f^5+3*a^3
*c*d^2*e^2*f^4-5*a^3*d^3*e^3*f^3+a^2*b*c^3*e*f^5+9*a^2*b*c^2*d*e^2*f^4-45*
a^2*b*c*d^2*e^3*f^3+35*a^2*b*d^3*e^4*f^2+3*a*b^2*c^3*e^2*f^4-45*a*b^2*c^2*
d*e^3*f^3+105*a*b^2*c*d^2*e^4*f^2-63*a*b^2*d^3*e^5*f-5*b^3*c^3*e^3*f^3+35*
b^3*c^2*d*e^4*f^2-63*b^3*c*d^2*e^5*f+33*b^3*d^3*e^6)/e^2/(e*f)^(1/2)*arcta
n(f*x/(e*f)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1270 vs. $2(435) = 870$.

Time = 0.20 (sec) , antiderivative size = 2560, normalized size of antiderivative = 5.63

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/560*(80*b^3*d^3*e^3*f^6*x^11 - 16*(11*b^3*d^3*e^4*f^5 - 21*(b^3*c*d^2 +
 a*b^2*d^3)*e^3*f^6)*x^9 + 16*(33*b^3*d^3*e^5*f^4 - 63*(b^3*c*d^2 + a*b^2*
d^3)*e^4*f^5 + 35*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^3*f^6)*x^7 - 1
12*(33*b^3*d^3*e^6*f^3 - 63*(b^3*c*d^2 + a*b^2*d^3)*e^5*f^4 + 35*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^5 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^
2*b*c*d^2 + a^3*d^3)*e^3*f^6)*x^5 - 70*(165*b^3*d^3*e^7*f^2 - 3*a^3*c^3*e*
f^8 - 315*(b^3*c*d^2 + a*b^2*d^3)*e^6*f^3 + 175*(b^3*c^2*d + 3*a*b^2*c*d^2
 + a^2*b*d^3)*e^5*f^4 - 25*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*
d^3)*e^4*f^5 + 15*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^3*f^6 - 3*(a^2
*b*c^3 + a^3*c^2*d)*e^2*f^7)*x^3 - 105*(33*b^3*d^3*e^8 + a^3*c^3*e^2*f^6 -
 63*(b^3*c*d^2 + a*b^2*d^3)*e^7*f + 35*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*
d^3)*e^6*f^2 - 5*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^5*f
^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^4*f^4 + (a^2*b*c^3 + a^3*
c^2*d)*e^3*f^5 + (33*b^3*d^3*e^6*f^2 + a^3*c^3*f^8 - 63*(b^3*c*d^2 + a*b^2
*d^3)*e^5*f^3 + 35*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^4 - 5*(b^
3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^5 + 3*(a*b^2*c^3 +
3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^6 + (a^2*b*c^3 + a^3*c^2*d)*e*f^7)*x^4 +
2*(33*b^3*d^3*e^7*f + a^3*c^3*e*f^7 - 63*(b^3*c*d^2 + a*b^2*d^3)*e^6*f^2 +
 35*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^5*f^3 - 5*(b^3*c^3 + 9*a*b^2
*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^4*f^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3\,(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(435) = 870.

Time = 0.13 (sec) , antiderivative size = 1093, normalized size of antiderivative = 2.40

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
3/8*(33*b^3*d^3*e^6 - 63*b^3*c*d^2*e^5*f - 63*a*b^2*d^3*e^5*f + 35*b^3*c^2
*d*e^4*f^2 + 105*a*b^2*c*d^2*e^4*f^2 + 35*a^2*b*d^3*e^4*f^2 - 5*b^3*c^3*e^
3*f^3 - 45*a*b^2*c^2*d*e^3*f^3 - 45*a^2*b*c*d^2*e^3*f^3 - 5*a^3*d^3*e^3*f^
3 + 3*a*b^2*c^3*e^2*f^4 + 9*a^2*b*c^2*d*e^2*f^4 + 3*a^3*c*d^2*e^2*f^4 + a^
2*b*c^3*e*f^5 + a^3*c^2*d*e*f^5 + a^3*c^3*f^6)*arctan(f*x/sqrt(e*f))/(sqrt
(e*f)*e^2*f^6) - 1/8*(21*b^3*d^3*e^6*f*x^3 - 51*b^3*c*d^2*e^5*f^2*x^3 - 51
*a*b^2*d^3*e^5*f^2*x^3 + 39*b^3*c^2*d*e^4*f^3*x^3 + 117*a*b^2*c*d^2*e^4*f^
3*x^3 + 39*a^2*b*d^3*e^4*f^3*x^3 - 9*b^3*c^3*e^3*f^4*x^3 - 81*a*b^2*c^2*d*
e^3*f^4*x^3 - 81*a^2*b*c*d^2*e^3*f^4*x^3 - 9*a^3*d^3*e^3*f^4*x^3 + 15*a*b^
2*c^3*e^2*f^5*x^3 + 45*a^2*b*c^2*d*e^2*f^5*x^3 + 15*a^3*c*d^2*e^2*f^5*x^3
- 3*a^2*b*c^3*e*f^6*x^3 - 3*a^3*c^2*d*e*f^6*x^3 - 3*a^3*c^3*f^7*x^3 + 19*b
^3*d^3*e^7*x - 45*b^3*c*d^2*e^6*f*x - 45*a*b^2*d^3*e^6*f*x + 33*b^3*c^2*d*
e^5*f^2*x + 99*a*b^2*c*d^2*e^5*f^2*x + 33*a^2*b*d^3*e^5*f^2*x - 7*b^3*c^3*
e^4*f^3*x - 63*a*b^2*c^2*d*e^4*f^3*x - 63*a^2*b*c*d^2*e^4*f^3*x - 7*a^3*d^
3*e^4*f^3*x + 9*a*b^2*c^3*e^3*f^4*x + 27*a^2*b*c^2*d*e^3*f^4*x + 9*a^3*c*d
^2*e^3*f^4*x + 3*a^2*b*c^3*e^2*f^5*x + 3*a^3*c^2*d*e^2*f^5*x - 5*a^3*c^3*e
*f^6*x)/((f*x^2 + e)^2*e^2*f^6) + 1/35*(5*b^3*d^3*f^18*x^7 - 21*b^3*d^3*e*
f^17*x^5 + 21*b^3*c*d^2*f^18*x^5 + 21*a*b^2*d^3*f^18*x^5 + 70*b^3*d^3*e^2*
f^16*x^3 - 105*b^3*c*d^2*e*f^17*x^3 - 105*a*b^2*d^3*e*f^17*x^3 + 35*b^3*c^
2*d*f^18*x^3 + 105*a*b^2*c*d^2*f^18*x^3 + 35*a^2*b*d^3*f^18*x^3 - 350*b...
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 1299, normalized size of antiderivative = 2.85

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^3} dx = \text{Too large to display}$$

input
```
int(((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
((3*x^3*(a^3*c^3*f^7 - 7*b^3*d^3*e^6*f + 3*a^3*d^3*e^3*f^4 + 3*b^3*c^3*e^3
*f^4 + a^2*b*c^3*e*f^6 + a^3*c^2*d*e*f^6 - 5*a*b^2*c^3*e^2*f^5 + 17*a*b^2*
d^3*e^5*f^2 - 13*a^2*b*d^3*e^4*f^3 - 5*a^3*c*d^2*e^2*f^5 + 17*b^3*c*d^2*e^
5*f^2 - 13*b^3*c^2*d*e^4*f^3 - 39*a*b^2*c*d^2*e^4*f^3 + 27*a*b^2*c^2*d*e^3
*f^4 + 27*a^2*b*c*d^2*e^3*f^4 - 15*a^2*b*c^2*d*e^2*f^5))/(8*e^2) - (x*(19*
b^3*d^3*e^6 - 5*a^3*c^3*f^6 - 7*a^3*d^3*e^3*f^3 - 7*b^3*c^3*e^3*f^3 + 3*a^
2*b*c^3*e*f^5 - 45*a*b^2*d^3*e^5*f + 3*a^3*c^2*d*e*f^5 - 45*b^3*c*d^2*e^5*
f + 9*a*b^2*c^3*e^2*f^4 + 33*a^2*b*d^3*e^4*f^2 + 9*a^3*c*d^2*e^2*f^4 + 33*
b^3*c^2*d*e^4*f^2 + 99*a*b^2*c*d^2*e^4*f^2 - 63*a*b^2*c^2*d*e^3*f^3 - 63*a
^2*b*c*d^2*e^3*f^3 + 27*a^2*b*c^2*d*e^2*f^4))/(8*e))/(e^2*f^6 + f^8*x^4 +
2*e*f^7*x^2) + x^3*((e*((3*b^3*d^3*e)/f^4 - (3*b^2*d^2*(a*d + b*c))/f^3))/
f - (b^3*d^3*e^2)/f^5 + (b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f^3) + x*((a
^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2)/f^3 - (3*e*((3*e*((3*b^3
*d^3*e)/f^4 - (3*b^2*d^2*(a*d + b*c))/f^3))/f - (3*b^3*d^3*e^2)/f^5 + (3*b
*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f^3))/f + (3*e^2*((3*b^3*d^3*e)/f^4 -
(3*b^2*d^2*(a*d + b*c))/f^3))/f^2 - (b^3*d^3*e^3)/f^6) - x^5*((3*b^3*d^3*e
)/(5*f^4) - (3*b^2*d^2*(a*d + b*c))/(5*f^3)) + (b^3*d^3*x^7)/(7*f^3) + (3*
atan((f^(1/2)*x*(a*f - b*e)*(c*f - d*e)*(a^2*c^2*f^4 + 33*b^2*d^2*e^4 + 5*
a^2*d^2*e^2*f^2 + 5*b^2*c^2*e^2*f^2 + 2*a*b*c^2*e*f^3 - 30*a*b*d^2*e^3*f +
 2*a^2*c*d*e*f^3 - 30*b^2*c*d*e^3*f + 12*a*b*c*d*e^2*f^2))/(e^(1/2)*(a^...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 2493, normalized size of antiderivative = 5.48

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output

```
(105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e**2*f**6 + 2
10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e*f**7*x**2 + 1
05*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*f**8*x**4 + 105
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**3*f**5 + 210
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**2*f**6*x**2
+ 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e*f**7*x**
4 + 315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**4*f**
4 + 630*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**3*f**
5*x**2 + 315*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**
2*f**6*x**4 - 525*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**3*
e**5*f**3 - 1050*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**3*e
**4*f**4*x**2 - 525*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d**
3*e**3*f**5*x**4 + 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*
b*c**3*e**3*f**5 + 210*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*
b*c**3*e**2*f**6*x**2 + 105*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
a**2*b*c**3*e*f**7*x**4 + 945*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*a**2*b*c**2*d*e**4*f**4 + 1890*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(
e)))*a**2*b*c**2*d*e**3*f**5*x**2 + 945*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f
)*sqrt(e)))*a**2*b*c**2*d*e**2*f**6*x**4 - 4725*sqrt(f)*sqrt(e)*atan((f*x)
/(sqrt(f)*sqrt(e)))*a**2*b*c*d**2*e**5*f**3 - 9450*sqrt(f)*sqrt(e)*atan...
```

3.246 $\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 554

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^4} dx$$
$$= \frac{bd(3a^2d^2f^2-3abdf(4de-3cf)+b^2(10d^2e^2-12cdef+3c^2f^2))x}{f^6}$$
$$-\frac{b^2d^2(4bde-3bcf-3adf)x^3}{3f^5}+\frac{b^3d^3x^5}{5f^4}+\frac{(be-af)^3(de-cf)^3x}{6ef^6(e+fx^2)^3}$$
$$-\frac{(be-af)^2(de-cf)^2(be(31de-13cf)-af(13de+5cf))x}{24e^2f^6(e+fx^2)^2}$$
$$-\frac{(be-af)(de-cf)(4abef(19d^2e^2-5cdef-2c^2f^2)-a^2f^2(11d^2e^2+8cdef+5c^2f^2)-b^2e^2(89d^2e^2 -}{16e^3f^6(e+fx^2)}$$
$$-\frac{(b^3e^3(231d^3e^3-315cd^2e^2f+105c^2def^2-5c^3f^3)+3a^2bef^2(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3) -}{1}$$

output
```
b*d*(3*a^2*d^2*f^2-3*a*b*d*f*(-3*c*f+4*d*e)+b^2*(3*c^2*f^2-12*c*d*e*f+10*d
^2*e^2))*x/f^6-1/3*b^2*d^2*(-3*a*d*f-3*b*c*f+4*b*d*e)*x^3/f^5+1/5*b^3*d^3*
x^5/f^4+1/6*(-a*f+b*e)^3*(-c*f+d*e)^3*x/e/f^6/(f*x^2+e)^3-1/24*(-a*f+b*e)^
2*(-c*f+d*e)^2*(b*e*(-13*c*f+31*d*e)-a*f*(5*c*f+13*d*e))*x/e^2/f^6/(f*x^2+
e)^2-1/16*(-a*f+b*e)*(-c*f+d*e)*(4*a*b*e*f*(-2*c^2*f^2-5*c*d*e*f+19*d^2*e^
2)-a^2*f^2*(5*c^2*f^2+8*c*d*e*f+11*d^2*e^2)-b^2*e^2*(11*c^2*f^2-76*c*d*e*f
+89*d^2*e^2))*x/e^3/f^6/(f*x^2+e)-1/16*(b^3*e^3*(-5*c^3*f^3+105*c^2*d*e*f^
2-315*c*d^2*e^2*f+231*d^3*e^3)+3*a^2*b*e*f^2*(-c^3*f^3-3*c^2*d*e*f^2-15*c*
d^2*e^2*f+35*d^3*e^3)-3*a*b^2*e^2*f*(c^3*f^3+15*c^2*d*e*f^2-105*c*d^2*e^2*
f+105*d^3*e^3)-a^3*f^3*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*
arctan(f^(1/2)*x/e^(1/2))/e^(7/2)/f^(13/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{bd(3a^2d^2f^2+3abdf(-4de+3cf)+b^2(10d^2e^2-12cdef+3c^2f^2))x}{f^6} - \frac{b^2d^2(4bde-3bcf-3adf)x^3}{3f^5} + \frac{b^3d^3x^5}{5f^4} + \frac{(be-af)^3(de-cf)^3x}{6ef^6(e+fx^2)^3} - \frac{(be-af)^2(de-cf)^2(be(31de-13cf)-af(13de+5cf))x}{24e^2f^6(e+fx^2)^2} + \frac{(be-af)(de-cf)(4abef(-19d^2e^2+5cdef+2c^2f^2)+a^2f^2(11d^2e^2+8cdef+5c^2f^2)+b^2e^2(89d^2e^2}{16e^3f^6(e+fx^2)} - \frac{(b^3e^3(231d^3e^3-315cd^2e^2f+105c^2def^2-5c^3f^3)+3a^2bef^2(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-}{1}$$

input
```
Integrate[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*d*(3*a^2*d^2*f^2 + 3*a*b*d*f*(-4*d*e + 3*c*f) + b^2*(10*d^2*e^2 - 12*c*
d*e*f + 3*c^2*f^2))*x)/f^6 - (b^2*d^2*(4*b*d*e - 3*b*c*f - 3*a*d*f)*x^3)/(
3*f^5) + (b^3*d^3*x^5)/(5*f^4) + ((b*e - a*f)^3*(d*e - c*f)^3*x)/(6*e*f^6*
(e + f*x^2)^3) - ((b*e - a*f)^2*(d*e - c*f)^2*(b*e*(31*d*e - 13*c*f) - a*f
*(13*d*e + 5*c*f))*x)/(24*e^2*f^6*(e + f*x^2)^2) + ((b*e - a*f)*(d*e - c*f
)*(4*a*b*e*f*(-19*d^2*e^2 + 5*c*d*e*f + 2*c^2*f^2) + a^2*f^2*(11*d^2*e^2 +
 8*c*d*e*f + 5*c^2*f^2) + b^2*e^2*(89*d^2*e^2 - 76*c*d*e*f + 11*c^2*f^2))*
x)/(16*e^3*f^6*(e + f*x^2)) - ((b^3*e^3*(231*d^3*e^3 - 315*c*d^2*e^2*f + 1
05*c^2*d*e*f^2 - 5*c^3*f^3) + 3*a^2*b*e*f^2*(35*d^3*e^3 - 15*c*d^2*e^2*f -
 3*c^2*d*e*f^2 - c^3*f^3) - 3*a*b^2*e^2*f*(105*d^3*e^3 - 105*c*d^2*e^2*f +
 15*c^2*d*e*f^2 + c^3*f^3) - a^3*f^3*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*
e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(13/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1308 vs. $2(554) = 1108$.

Time = 1.91 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.36, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {425, 425, 401, 25, 401, 25, 401, 299, 218, 403, 25, 299, 218, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3 \left(c+dx^2\right)^3}{\left(e+fx^2\right)^4} \, dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}{\left(fx^2+e\right)^4}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^3}{\left(fx^2+e\right)^4}dx}{f}\right)}{f}$$

$\downarrow$ 401

$$\frac{b\left(\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3}{4ef\left(e+f\right.}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}\right)}{f}-\frac{(be-af)\left(-\frac{\int -\frac{\left(dx^2+c\right)^2\left(d(7be-af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\right.}{f}\right.}{f}$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\frac{(be-af)\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}\right.}{f}\right.$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{b\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-3af)x^2+c(be+3af)\right)}{\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}\right)}{f}-\frac{(be-af)\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-af)x^2+c(be+5af)\right)}{\left(fx^2+e\right)^3}dx}{6ef}-\frac{x\left(c+d\right.}{6ef}\right.}{f}\right.$$

$$\frac{}{f}$$

$$\downarrow 401$$

$$b\left(\frac{b\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int\frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(\right.}{fx^2+e}}{2ef}}{4ef}\right.}{}\right.$$

$$\frac{}{f}$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{\int\frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^3}{4ef\left(e+f\right.}\right.}{f}\right.$$

$$\downarrow 25$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(\cdots\right.\right.}{fx^2+e}}{2ef}\cdots}{4ef}\cdots\right.}{\cdots}}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{2ef\left(e+fx^2\right)}}{4ef}-\frac{x\left(c+dx^2\right)^3\cdots}{4ef\left(e+f\cdots\right.}\right.}{f}\cdots\right.}{\cdots}$$

↓ 401

$$b\left(\frac{b\left(\frac{\int\frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\int\frac{\left(dx^2+c\right)\left(c(3af(de-\right.}{}}{}}{4ef}\right.}{}\right.$$
$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\int\frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}}{4ef}-\frac{(be-af)x\left(\right.}{4ef\left(fx^2\right.}\right)}{f}\right.$$

$\downarrow$ 299

$$b\left(\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\int\frac{\left(dx^2+c\right)\left(c(3af(de-\cdots\right.}{\cdots}}{4ef}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\int\frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}}{4ef}-\frac{(be-af)x\left(\cdots\right.}{4ef\left(fx^2\cdots\right.}\cdots\right.}{f}\cdots\right.}{\cdots}$$

$$\downarrow \; 218$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(d(7be-5af)x^2+c(be+af)\right)}{fx^2+e}dx}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-\right.}{}}{}}{4ef}\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\int \frac{\left(dx^2+c\right)\left(c(3af(de-cf)-be(7de+cf))-d(be(35de-3cf)-3af(5de+3cf))x^2\right)}{fx^2+e}dx}{2ef}}{4ef}-\frac{(be-af)x\left(\right.}{4ef\left(fx^2\right.}\right.}{f}\right.$$

$\downarrow$ 403

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}+\frac{\int -\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\ldots}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\frac{\int \frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(10\ldots\right.\right.}{fx^2+e}}{3f}}{2ef}}{4ef}\right.}{f}\right.$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\cdots\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\left(-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{\frac{\int\frac{c\left(be\left(35d^2e^2-24cdfe-3c^2f^2\right)-3af\left(5d^2e^2+3c^2f^2\right)\right)-d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(10\cdots\right.\right.}{fx^2+e}}{3f}}{2ef}\cdots}{4ef}\right.}{f}\cdots\right.}{\cdots}$$

$\downarrow$ 299

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\cdots\right.}{\cdots}\right.$$

$$(be-af)\left(\frac{b\left(-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{-\frac{-\frac{d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x}{f}-\frac{3(de-cf)\left(be\left(35d^2e^2-10c\cdots\right.\right.}{\cdots}}{3f}\cdots}{4ef}\cdots\right.}{f}\cdots\right.$$

$\downarrow$ 218

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\int\frac{\left(dx^2+c\right)\left(d(be(35de-33cf)-5af(5de-3cf))x^2+c(be(7de-5cf)-5af(de+cf))\right)}{fx^2+e}dx}{5f}}{2ef}-\frac{(be-af)x\left(dx^2+c\right)^3}{2ef\left(fx^2+e\right)}\right)}{f}-\frac{(be-af)\left(\right.}{}\right.$$

$$(be-af)\left(\frac{b\left(-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{-\frac{\frac{d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x}{f}-\frac{3(de-cf)\left(be\left(35d^2e^2-10c\right.\right.}{}}{3f}}{4ef}\right)}{f}\right.$$

↓ 403

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\int\frac{d\left(5af\left(15d^2e^2-22cdfe+3c^2f^2\right)-be\left(105d^2e^2-190cdfe+81c^2f^2\right)\right)x^2+c\left(5af\right.}{fx^2+e}}{3f}}{5f}}{2ef}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{-\frac{\frac{d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x}{f}-\frac{3(de-cf)\left(be\left(35d^2e^2-10c\right.\right.}{}}{3f}}{4ef}}{}\right.}{f}\right.$$

↓ 299

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\int\frac{1}{fx^2+e}dx(de-cf)^2}{f}+\frac{d\left(5af\left(15d^2e^2-22cdfe+3\right.\right.}{3f}}{5f}}{2ef}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{-\frac{d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x}{f}-\frac{3(de-cf)\left(be\left(35d^2e^2-10\right.\right.}{}}{3f}}{4ef}\right.}{f}\right.$$

↓ 218

$$b\left(\frac{b\left(\frac{\frac{d(7be-5af)x\left(dx^2+c\right)^2}{5f}-\frac{\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(dx^2+c\right)}{3f}+\frac{\frac{15(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{3/2}}+\frac{d\left(5af\left(15d^2e^2-22cdf\cdots\right.\right.}{3f}}{5f}}{2ef}\cdots}{f}\cdots\right.\right.$$

$$(be-af)\left(\frac{b\left(-\frac{(be(7de-cf)-3af(de+cf))x\left(dx^2+c\right)^2}{2ef\left(fx^2+e\right)}-\frac{-\frac{\frac{d\left(3af\left(15d^2e^2-4cdfe-3c^2f^2\right)-be\left(105d^2e^2-100cdfe+3c^2f^2\right)\right)x}{f}-\frac{3(de-cf)\left(be\left(35d^2e^2-10c\cdots\right.\right.}{\cdots}}{3f}}{4ef}\cdots\right.}{f}\cdots\right.$$

input

```
Int[((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*((b*(-1/2*((b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)) + ((d*(7*b*e
- 5*a*f)*x*(c + d*x^2)^2)/(5*f) - ((d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*d*
e - 3*c*f))*x*(c + d*x^2))/(3*f) + ((d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3
*c^2*f^2) - b*e*(105*d^2*e^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/f + (15*(d*e
- c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(Sqrt[e]*f^(3/2)))/(3*f))/(5*f))/(2*e*f)))/f - ((b*e - a*f)*(-1/4*((b*
e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*e - c*f)
- 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(d*(b*e*(3
5*d*e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-((d*(3*a*f*(1
5*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d*e*f + 3*c^
2*f^2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) -
3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqr
t[e]*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)))/f))/f - ((b*e - a*f)*((b*(-1/4*((
b*e - a*f)*x*(c + d*x^2)^3)/(e*f*(e + f*x^2)^2) + (-1/2*((b*e*(7*d*e - c*f
) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)) - (-1/3*(d*(b*e*
(35*d*e - 3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/f + (-((d*(3*a*f*
(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d*e*f + 3*
c^2*f^2))*x)/f) - (3*(d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2)
- 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(S
qrt[e]*f^(3/2)))/(3*f))/(2*e*f))/(4*e*f)))/f - ((b*e - a*f)*(-1/6*((b*e...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(534) = 1068$.

Time = 0.62 (sec) , antiderivative size = 1095, normalized size of antiderivative = 1.98

method	result	size
default	Expression too large to display	1095
risch	Expression too large to display	1861

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
b*d/f^6*(1/5*f^2*x^5*b^2*d^2+a*b*d^2*f^2*x^3+b^2*c*d*f^2*x^3-4/3*b^2*d^2*e
*f*x^3+3*a^2*d^2*f^2*x+9*a*b*c*d*f^2*x-12*a*b*d^2*e*f*x+3*b^2*c^2*f^2*x-12
*b^2*c*d*e*f*x+10*b^2*d^2*e^2*x)+1/f^6*((1/16*f^2*(5*a^3*c^3*f^6+3*a^3*c^2
*d*e*f^5+3*a^3*c*d^2*e^2*f^4-11*a^3*d^3*e^3*f^3+3*a^2*b*c^3*e*f^5+9*a^2*b*
c^2*d*e^2*f^4-99*a^2*b*c*d^2*e^3*f^3+87*a^2*b*d^3*e^4*f^2+3*a*b^2*c^3*e^2*
f^4-99*a*b^2*c^2*d*e^3*f^3+261*a*b^2*c*d^2*e^4*f^2-165*a*b^2*d^3*e^5*f-11*
b^3*c^3*e^3*f^3+87*b^3*c^2*d*e^4*f^2-165*b^3*c*d^2*e^5*f+89*b^3*d^3*e^6)/e
^3*x^5+1/6*f*(5*a^3*c^3*f^6+3*a^3*c^2*d*e*f^5-3*a^3*c*d^2*e^2*f^4-5*a^3*d^
3*e^3*f^3+3*a^2*b*c^3*e*f^5-9*a^2*b*c^2*d*e^2*f^4-45*a^2*b*c*d^2*e^3*f^3+5
1*a^2*b*d^3*e^4*f^2-3*a*b^2*c^3*e^2*f^4-45*a*b^2*c^2*d*e^3*f^3+153*a*b^2*c
*d^2*e^4*f^2-105*a*b^2*d^3*e^5*f-5*b^3*c^3*e^3*f^3+51*b^3*c^2*d*e^4*f^2-10
5*b^3*c*d^2*e^5*f+59*b^3*d^3*e^6)/e^2*x^3+1/16*(11*a^3*c^3*f^6-3*a^3*c^2*d
*e*f^5-3*a^3*c*d^2*e^2*f^4-5*a^3*d^3*e^3*f^3-3*a^2*b*c^3*e*f^5-9*a^2*b*c^2
*d*e^2*f^4-45*a^2*b*c*d^2*e^3*f^3+57*a^2*b*d^3*e^4*f^2-3*a*b^2*c^3*e^2*f^4
-45*a*b^2*c^2*d*e^3*f^3+171*a*b^2*c*d^2*e^4*f^2-123*a*b^2*d^3*e^5*f-5*b^3*
c^3*e^3*f^3+57*b^3*c^2*d*e^4*f^2-123*b^3*c*d^2*e^5*f+71*b^3*d^3*e^6)/e*x)/
(f*x^2+e)^3+1/16*(5*a^3*c^3*f^6+3*a^3*c^2*d*e*f^5+3*a^3*c*d^2*e^2*f^4+5*a^
3*d^3*e^3*f^3+3*a^2*b*c^3*e*f^5+9*a^2*b*c^2*d*e^2*f^4+45*a^2*b*c*d^2*e^3*f
^3-105*a^2*b*d^3*e^4*f^2+3*a*b^2*c^3*e^2*f^4+45*a*b^2*c^2*d*e^3*f^3-315*a*
b^2*c*d^2*e^4*f^2+315*a*b^2*d^3*e^5*f+5*b^3*c^3*e^3*f^3-105*b^3*c^2*d*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(532) = 1064$.

Time = 0.35 (sec) , antiderivative size = 3136, normalized size of antiderivative = 5.66

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(532) = 1064$.

Time = 0.12 (sec) , antiderivative size = 1230, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-1/16*(231*b^3*d^3*e^6 - 315*b^3*c*d^2*e^5*f - 315*a*b^2*d^3*e^5*f + 105*b
^3*c^2*d*e^4*f^2 + 315*a*b^2*c*d^2*e^4*f^2 + 105*a^2*b*d^3*e^4*f^2 - 5*b^3
*c^3*e^3*f^3 - 45*a*b^2*c^2*d*e^3*f^3 - 45*a^2*b*c*d^2*e^3*f^3 - 5*a^3*d^3
*e^3*f^3 - 3*a*b^2*c^3*e^2*f^4 - 9*a^2*b*c^2*d*e^2*f^4 - 3*a^3*c*d^2*e^2*f
^4 - 3*a^2*b*c^3*e*f^5 - 3*a^3*c^2*d*e*f^5 - 5*a^3*c^3*f^6)*arctan(f*x/sqr
t(e*f))/(sqrt(e*f)*e^3*f^6) + 1/48*(267*b^3*d^3*e^6*f^2*x^5 - 495*b^3*c*d^
2*e^5*f^3*x^5 - 495*a*b^2*d^3*e^5*f^3*x^5 + 261*b^3*c^2*d*e^4*f^4*x^5 + 78
3*a*b^2*c*d^2*e^4*f^4*x^5 + 261*a^2*b*d^3*e^4*f^4*x^5 - 33*b^3*c^3*e^3*f^5
*x^5 - 297*a*b^2*c^2*d*e^3*f^5*x^5 - 297*a^2*b*c*d^2*e^3*f^5*x^5 - 33*a^3*
d^3*e^3*f^5*x^5 + 9*a*b^2*c^3*e^2*f^6*x^5 + 27*a^2*b*c^2*d*e^2*f^6*x^5 + 9
*a^3*c*d^2*e^2*f^6*x^5 + 9*a^2*b*c^3*e*f^7*x^5 + 9*a^3*c^2*d*e*f^7*x^5 + 1
5*a^3*c^3*f^8*x^5 + 472*b^3*d^3*e^7*f*x^3 - 840*b^3*c*d^2*e^6*f^2*x^3 - 84
0*a*b^2*d^3*e^6*f^2*x^3 + 408*b^3*c^2*d*e^5*f^3*x^3 + 1224*a*b^2*c*d^2*e^5
*f^3*x^3 + 408*a^2*b*d^3*e^5*f^3*x^3 - 40*b^3*c^3*e^4*f^4*x^3 - 360*a*b^2*
c^2*d*e^4*f^4*x^3 - 360*a^2*b*c*d^2*e^4*f^4*x^3 - 40*a^3*d^3*e^4*f^4*x^3 -
 24*a*b^2*c^3*e^3*f^5*x^3 - 72*a^2*b*c^2*d*e^3*f^5*x^3 - 24*a^3*c*d^2*e^3*
f^5*x^3 + 24*a^2*b*c^3*e^2*f^6*x^3 + 24*a^3*c^2*d*e^2*f^6*x^3 + 40*a^3*c^3
*e*f^7*x^3 + 213*b^3*d^3*e^8*x - 369*b^3*c*d^2*e^7*f*x - 369*a*b^2*d^3*e^7
*f*x + 171*b^3*c^2*d*e^6*f^2*x + 513*a*b^2*c*d^2*e^6*f^2*x + 171*a^2*b*d^3
*e^6*f^2*x - 15*b^3*c^3*e^5*f^3*x - 135*a*b^2*c^2*d*e^5*f^3*x - 135*a^2...
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 1132, normalized size of antiderivative = 2.04

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} \, dx = \text{Too large to display}$$

input
```
int(((a + b*x^2)^3*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
x*((4*e*((4*b^3*d^3*e)/f^5 - (3*b^2*d^2*(a*d + b*c))/f^4))/f - (6*b^3*d^3*
e^2)/f^6 + (3*b*d*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/f^4) - ((x^3*(5*a^3*d^3
*e^3*f^4 - 59*b^3*d^3*e^6*f - 5*a^3*c^3*f^7 + 5*b^3*c^3*e^3*f^4 - 3*a^2*b*
c^3*e*f^6 - 3*a^3*c^2*d*e*f^6 + 3*a*b^2*c^3*e^2*f^5 + 105*a*b^2*d^3*e^5*f^
2 - 51*a^2*b*d^3*e^4*f^3 + 3*a^3*c*d^2*e^2*f^5 + 105*b^3*c*d^2*e^5*f^2 - 5
1*b^3*c^2*d*e^4*f^3 - 153*a*b^2*c*d^2*e^4*f^3 + 45*a*b^2*c^2*d*e^3*f^4 + 4
5*a^2*b*c*d^2*e^3*f^4 + 9*a^2*b*c^2*d*e^2*f^5))/(6*e^2) - (x^5*(5*a^3*c^3*
f^8 - 11*a^3*d^3*e^3*f^5 - 11*b^3*c^3*e^3*f^5 + 89*b^3*d^3*e^6*f^2 + 3*a^2
*b*c^3*e*f^7 + 3*a^3*c^2*d*e*f^7 + 3*a*b^2*c^3*e^2*f^6 - 165*a*b^2*d^3*e^5
*f^3 + 87*a^2*b*d^3*e^4*f^4 + 3*a^3*c*d^2*e^2*f^6 - 165*b^3*c*d^2*e^5*f^3
+ 87*b^3*c^2*d*e^4*f^4 + 261*a*b^2*c*d^2*e^4*f^4 - 99*a*b^2*c^2*d*e^3*f^5
- 99*a^2*b*c*d^2*e^3*f^5 + 9*a^2*b*c^2*d*e^2*f^6))/(16*e^3) + (x*(5*a^3*d^
3*e^3*f^3 - 71*b^3*d^3*e^6 - 11*a^3*c^3*f^6 + 5*b^3*c^3*e^3*f^3 + 3*a^2*b*
c^3*e*f^5 + 123*a*b^2*d^3*e^5*f + 3*a^3*c^2*d*e*f^5 + 123*b^3*c*d^2*e^5*f
+ 3*a*b^2*c^3*e^2*f^4 - 57*a^2*b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 - 57*b^
3*c^2*d*e^4*f^2 - 171*a*b^2*c*d^2*e^4*f^2 + 45*a*b^2*c^2*d*e^3*f^3 + 45*a^
2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*f^4))/(16*e))/(e^3*f^6 + f^9*x^6 + 3
*e*f^8*x^4 + 3*e^2*f^7*x^2) - x^3*((4*b^3*d^3*e)/(3*f^5) - (b^2*d^2*(a*d +
 b*c))/f^4) + (b^3*d^3*x^5)/(5*f^4) + (atan((f^(1/2)*x)/e^(1/2))*(5*a^3*c^
3*f^6 - 231*b^3*d^3*e^6 + 5*a^3*d^3*e^3*f^3 + 5*b^3*c^3*e^3*f^3 + 3*a^2...
```

Reduce [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 3156, normalized size of antiderivative = 5.70

$$\int \frac{(a+bx^2)^3 (c+dx^2)^3}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

output

```
(75*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e**3*f**6 + 22
5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e**2*f**7*x**2 +
 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*e*f**8*x**4 +
 75*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**3*f**9*x**6 + 45
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**4*f**5 + 135
*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**3*f**6*x**2
+ 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e**2*f**7*
x**4 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e*f**8
*x**6 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**5*
f**4 + 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*e**4*
f**5*x**2 + 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**2*
e**3*f**6*x**4 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d
**2*e**2*f**7*x**6 + 75*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3
*d**3*e**6*f**3 + 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*d
**3*e**5*f**4*x**2 + 225*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**
3*d**3*e**4*f**5*x**4 + 75*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a
**3*d**3*e**3*f**6*x**6 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a**2*b*c**3*e**4*f**5 + 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a**2*b*c**3*e**3*f**6*x**2 + 135*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*a**2*b*c**3*e**2*f**7*x**4 + 45*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(...
```

3.247 $\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = -\frac{b^2(bde+bcf-3adf)x}{d^2f^2}+\frac{b^3x^3}{3df} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}(de-cf)}+\frac{(be-af)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}(de-cf)}$$

output
```
-b^2*(-3*a*d*f+b*c*f+b*d*e)*x/d^2/f^2+1/3*b^3*x^3/d/f-(-a*d+b*c)^3*arctan(
d^(1/2)*x/c^(1/2))/c^(1/2)/d^(5/2)/(-c*f+d*e)+(-a*f+b*e)^3*arctan(f^(1/2)*
x/e^(1/2))/e^(1/2)/f^(5/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = -\frac{b^2(bde+bcf-3adf)x}{d^2f^2}+\frac{b^3x^3}{3df} + \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}(-de+cf)}+\frac{(be-af)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}(de-cf)}$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
-((b^2*(b*d*e + b*c*f - 3*a*d*f)*x)/(d^2*f^2)) + (b^3*x^3)/(3*d*f) + ((b*c
 - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2)*(-(d*e) + c*f)) +
((b*e - a*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.72, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {420, 300, 420, 299, 218, 397, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \frac{(bx^2+a)^2}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{bx^2+a}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\int \frac{1}{fx^2+e}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\int \frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}\right)}{d}\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\int \left(\frac{b^2x^2}{f}-\frac{b(be-2af)}{f^2}+\frac{b^2e^2-2abfe+a^2f^2}{f^2(fx^2+e)}\right)dx}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{d}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(be-af)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}-\frac{bx(be-2af)}{f^2}+\frac{b^2x^3}{3f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{d}\right)}{d}$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*(-((b*(b*e - 2*a*f)*x)/f^2) + (b^2*x^3)/(3*f) + ((b*e - a*f)^2*ArcTan[(
Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))))/d - ((b*c - a*d)*((b*((b*x)/f - (
(b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2))))/d - ((b*c - a
*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]*(d*e - c
*f))) + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e -
c*f))))/d))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

method	result
default	$\frac{b^2\left(\frac{1}{3}bdf\,x^3+3adfx-bcfx-bdex\right)}{d^2f^2}+\frac{\left(a^3f^3-3a^2be\,f^2+3a\,b^2e^2f-b^3e^3\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{f^2(cf-de)\sqrt{ef}}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\text{arct}}{d^2(cf-de)\sqrt{cd}}$
risch	$\frac{b^3x^3}{3df}+\frac{3b^2ax}{df}-\frac{b^3cx}{d^2f}-\frac{b^3ex}{d\,f^2}-\frac{f\ln\left(e\,f^2x-(-ef)^{\frac{3}{2}}\right)a^3}{2\sqrt{-ef}\,(cf-de)}+\frac{3\ln\left(e\,f^2x-(-ef)^{\frac{3}{2}}\right)a^2be}{2\sqrt{-ef}\,(cf-de)}-\frac{3\ln\left(e\,f^2x-(-ef)^{\frac{3}{2}}\right)a\,b^2e^2}{2f\sqrt{-ef}\,(cf-de)}+$

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
b^2/d^2/f^2*(1/3*b*d*f*x^3+3*a*d*f*x-b*c*f*x-b*d*e*x)+1/f^2*(a^3*f^3-3*a^2
*b*e*f^2+3*a*b^2*e^2*f-b^3*e^3)/(c*f-d*e)/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/
2))+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/d^2/(c*f-d*e)/(c*d)^(1/
2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(114) = 228$.

Time = 10.63 (sec) , antiderivative size = 1081, normalized size of antiderivative = 8.19

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*e*
f^3*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^3*c*d^3*e^2*f^2 -
 b^3*c^2*d^2*e*f^3)*x^3 + 3*(b^3*c*d^3*e^3 - 3*a*b^2*c*d^3*e^2*f + 3*a^2*b
*c*d^3*e*f^2 - a^3*c*d^3*f^3)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/
(f*x^2 + e)) + 6*(b^3*c*d^3*e^3*f - 3*a*b^2*c*d^3*e^2*f^2 - (b^3*c^3*d - 3
*a*b^2*c^2*d^2)*e*f^3)*x)/(c*d^4*e^2*f^3 - c^2*d^3*e*f^4), -1/6*(3*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*e*f^3*log((d*x^2 +
 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^3*c*d^3*e^2*f^2 - b^3*c^2*d^2*e*f
^3)*x^3 - 6*(b^3*c*d^3*e^3 - 3*a*b^2*c*d^3*e^2*f + 3*a^2*b*c*d^3*e*f^2 - a
^3*c*d^3*f^3)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 6*(b^3*c*d^3*e^3*f - 3*a*b
^2*c*d^3*e^2*f^2 - (b^3*c^3*d - 3*a*b^2*c^2*d^2)*e*f^3)*x)/(c*d^4*e^2*f^3
- c^2*d^3*e*f^4), -1/6*(6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(c*d)*e*f^3*arctan(sqrt(c*d)*x/c) - 2*(b^3*c*d^3*e^2*f^2 - b^3*c^2
*d^2*e*f^3)*x^3 + 3*(b^3*c*d^3*e^3 - 3*a*b^2*c*d^3*e^2*f + 3*a^2*b*c*d^3*e
*f^2 - a^3*c*d^3*f^3)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 +
 e)) + 6*(b^3*c*d^3*e^3*f - 3*a*b^2*c*d^3*e^2*f^2 - (b^3*c^3*d - 3*a*b^2*c
^2*d^2)*e*f^3)*x)/(c*d^4*e^2*f^3 - c^2*d^3*e*f^4), -1/3*(3*(b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*e*f^3*arctan(sqrt(c*d)*x/c)
 - (b^3*c*d^3*e^2*f^2 - b^3*c^2*d^2*e*f^3)*x^3 - 3*(b^3*c*d^3*e^3 - 3*a*b^
2*c*d^3*e^2*f + 3*a^2*b*c*d^3*e*f^2 - a^3*c*d^3*f^3)*sqrt(e*f)*arctan(s...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^3e-cd^2f)\sqrt{cd}} + \frac{(b^3e^3-3\,ab^2e^2f+3\,a^2bef^2-a^3f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(def^2-cf^3)\sqrt{ef}} + \frac{b^3d^2f^2x^3-3\,b^3d^2efx-3\,b^3cdf^2x+9\,ab^2d^2f^2x}{3\,d^3f^3}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))
/((d^3*e - c*d^2*f)*sqrt(c*d)) + (b^3*e^3 - 3*a*b^2*e^2*f + 3*a^2*b*e*f^2
- a^3*f^3)*arctan(f*x/sqrt(e*f))/((d*e*f^2 - c*f^3)*sqrt(e*f)) + 1/3*(b^3*
d^2*f^2*x^3 - 3*b^3*d^2*e*f*x - 3*b^3*c*d*f^2*x + 9*a*b^2*d^2*f^2*x)/(d^3*
f^3)
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 5412, normalized size of antiderivative = 41.00

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
x*((3*a*b^2)/(d*f) - (b^3*(c*f + d*e))/(d^2*f^2)) + (b^3*x^3)/(3*d*f) + (a
tan((((((4*a^3*c^2*d^5*f^7 + 4*a^3*d^7*e^2*f^5 - 4*b^3*c^2*d^5*e^3*f^4 -
4*b^3*c^3*d^4*e^2*f^5 - 8*a^3*c*d^6*e*f^6 + 4*b^3*c*d^6*e^4*f^3 + 4*b^3*c^
4*d^3*e*f^6 - 12*a*b^2*c*d^6*e^3*f^4 - 12*a*b^2*c^3*d^4*e*f^6 + 24*a*b^2*c
^2*d^5*e^2*f^5)/(d^3*f^3) + (x*(-c*d^5)^(1/2)*(a*d - b*c)^3*(4*c^3*d^5*f^8
 + 4*d^8*e^3*f^5 - 4*c*d^7*e^2*f^6 - 4*c^2*d^6*e*f^7))/(d^3*f^3*(c^2*d^5*f
 - c*d^6*e)))*(-c*d^5)^(1/2)*(a*d - b*c)^3)/(2*(c^2*d^5*f - c*d^6*e)) + (2
*x*(2*a^6*d^6*f^6 + b^6*c^6*f^6 + b^6*d^6*e^6 + 15*a^2*b^4*c^4*d^2*f^6 - 2
0*a^3*b^3*c^3*d^3*f^6 + 15*a^4*b^2*c^2*d^4*f^6 + 15*a^2*b^4*d^6*e^4*f^2 -
20*a^3*b^3*d^6*e^3*f^3 + 15*a^4*b^2*d^6*e^2*f^4 - 6*a*b^5*c^5*d*f^6 - 6*a^
5*b*c*d^5*f^6 - 6*a*b^5*d^6*e^5*f - 6*a^5*b*d^6*e*f^5))/(d^3*f^3))*(-c*d^5
)^(1/2)*(a*d - b*c)^3*1i)/(2*(c^2*d^5*f - c*d^6*e)) - (((((4*a^3*c^2*d^5*f
^7 + 4*a^3*d^7*e^2*f^5 - 4*b^3*c^2*d^5*e^3*f^4 - 4*b^3*c^3*d^4*e^2*f^5 - 8
*a^3*c*d^6*e*f^6 + 4*b^3*c*d^6*e^4*f^3 + 4*b^3*c^4*d^3*e*f^6 - 12*a*b^2*c*
d^6*e^3*f^4 - 12*a*b^2*c^3*d^4*e*f^6 + 24*a*b^2*c^2*d^5*e^2*f^5)/(d^3*f^3)
 - (x*(-c*d^5)^(1/2)*(a*d - b*c)^3*(4*c^3*d^5*f^8 + 4*d^8*e^3*f^5 - 4*c*d^
7*e^2*f^6 - 4*c^2*d^6*e*f^7))/(d^3*f^3*(c^2*d^5*f - c*d^6*e)))*(-c*d^5)^(1
/2)*(a*d - b*c)^3)/(2*(c^2*d^5*f - c*d^6*e)) - (2*x*(2*a^6*d^6*f^6 + b^6*c
^6*f^6 + b^6*d^6*e^6 + 15*a^2*b^4*c^4*d^2*f^6 - 20*a^3*b^3*c^3*d^3*f^6 + 1
5*a^4*b^2*c^2*d^4*f^6 + 15*a^2*b^4*d^6*e^4*f^2 - 20*a^3*b^3*d^6*e^3*f^3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.68

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)}\,dx$$
$$= \frac{-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)a^3d^3e\,f^3+9\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)a^2bc\,d^2e\,f^3-9\sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)a\,b^2c^2de\,f^3+}{}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e*f**3 + 9*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e*f**3 - 9*sqrt
(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**2*d*e*f**3 + 3*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**3*c**3*e*f**3 + 3*sqrt(f)*sqrt(e
)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**3*f**3 - 9*sqrt(f)*sqrt(e)*atan(
(f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c*d**3*e*f**2 + 9*sqrt(f)*sqrt(e)*atan((f*
x)/(sqrt(f)*sqrt(e)))*a*b**2*c*d**3*e**2*f - 3*sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*b**3*c*d**3*e**3 + 9*a*b**2*c**2*d**2*e*f**3*x - 9*a*b*
*2*c*d**3*e**2*f**2*x - 3*b**3*c**3*d*e*f**3*x + b**3*c**2*d**2*e*f**3*x**
3 + 3*b**3*c*d**3*e**3*f*x - b**3*c*d**3*e**2*f**2*x**3)/(3*c*d**3*e*f**3*
(c*f - d*e))
```

3.248 $\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 169

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{b^3x}{df^2} + \frac{(be-af)^3x}{2ef^2(de-cf)(e+fx^2)} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(de-cf)^2} - \frac{(be-af)^2(be(3de-5cf)+af(3de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}(de-cf)^2}$$

output
```
b^3*x/d/f^2+1/2*(-a*f+b*e)^3*x/e/f^2/(-c*f+d*e)/(f*x^2+e)-(-a*d+b*c)^3*arc
tan(d^(1/2)*x/c^(1/2))/c^(1/2)/d^(3/2)/(-c*f+d*e)^2-1/2*(-a*f+b*e)^2*(b*e*
(-5*c*f+3*d*e)+a*f*(-c*f+3*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/f^(5/2)
/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{b^3x}{df^2} + \frac{(be-af)^3x}{2ef^2(de-cf)(e+fx^2)} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}(de-cf)^2} - \frac{(be-af)^2(be(3de-5cf)+af(3de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}(de-cf)^2}$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(b^3*x)/(d*f^2) + ((b*e - a*f)^3*x)/(2*e*f^2*(d*e - c*f)*(e + f*x^2)) - ((
b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2)*(d*e - c*f)^2)
- ((b*e - a*f)^2*(b*e*(3*d*e - 5*c*f) + a*f*(3*d*e - c*f))*ArcTan[(Sqrt[f]
*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/2)*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.85, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {419, 25, 300, 401, 27, 403, 25, 299, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \; 419$$

$$-\frac{\int -\frac{(bx^2+a)^2(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2} - \frac{d(bc-ad)\int \frac{(bx^2+a)^2}{dx^2+c}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^2(bde^2+(bc-ad)f^2x^2-af(2de-cf))}{(fx^2+e)^2}dx}{(de-cf)^2} - \frac{d(bc-ad)\int \frac{(bx^2+a)^2}{dx^2+c}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \frac{(bx^2+a)^2\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \left(\frac{b^2x^2}{d}-\frac{b(bc-2ad)}{d^2}+\frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{f\left(bx^2+a\right)\left(b(be(3de-5cf)-af(de-3cf))x^2+a(af(3de-cf)-be(de+cf))\right)}{fx^2+e}dx}{2ef}}{(de-cf)^2}-\frac{d(bc-ad)\int \left(\frac{b^2x^2}{d}-\frac{b(bc-2ad)}{d^2}+\frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{\left(bx^2+a\right)\left(b(be(3de-5cf)-af(de-3cf))x^2+a(af(3de-cf)-be(de+cf))\right)}{fx^2+e}dx}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\int \left(\frac{b^2x^2}{d}-\frac{b(bc-2ad)}{d^2}+\frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\frac{\int -\frac{a\left(b^2(3de-5cf)e^2+2abf(de+3cf)e-3a^2f^2(3de-cf)\right)-b\left(-3b^2(3de-5cf)e^2+2abf(3de-11cf)e+a^2f^2(7de+3cf)\right)x^2}{fx^2+e}dx}{3f}+\cdots}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\int \left(\frac{b^2x^2}{d}-\frac{b(bc-2ad)}{d^2}+\frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\frac{bx\left(a+bx^2\right)(be(3de-5cf)-af(de-3cf))}{3f}-\frac{\int \frac{a\left(b^2(3de-5cf)e^2+2abf(de+3cf)e-3a^2f^2(3de-cf)\right)-b\left(-3b^2(3de-5cf)e^2+2a\cdots\right.}{fx^2+e}\cdots}{3f}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\int \left(\frac{b^2x^2}{d}-\frac{b(bc-2ad)}{d^2}+\frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)} - \frac{\frac{bx(a+bx^2)(be(3de-5cf)-af(de-3cf))}{3f} - \frac{-\frac{3(be-af)^2(af(3de-cf)+be(3de-5cf))\int\frac{1}{fx^2+e}dx}{f} - \frac{bx(a^2f^2(3cf+7de)+2abef}{3f}}{2e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\left(\frac{b^2x^2}{d} - \frac{b(bc-2ad)}{d^2} + \frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)} - \frac{\frac{bx(a+bx^2)(be(3de-5cf)-af(de-3cf))}{3f} - \frac{-\frac{bx(a^2f^2(3cf+7de)+2abef(3de-11cf)-3b^2e^2(3de-5cf))}{f} - \frac{3(be-af)^2\arctan(}{3f}}{2e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\left(\frac{b^2x^2}{d} - \frac{b(bc-2ad)}{d^2} + \frac{b^2c^2-2abdc+a^2d^2}{d^2(dx^2+c)}\right)dx}{(de-cf)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{2e(e+fx^2)} - \frac{\frac{bx(a+bx^2)(be(3de-5cf)-af(de-3cf))}{3f} - \frac{-\frac{bx(a^2f^2(3cf+7de)+2abef(3de-11cf)-3b^2e^2(3de-5cf))}{f} - \frac{3(be-af)^2\arctan(}{3f}}{2e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^3}{3d}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
-((d*(b*c - a*d)*(-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a
*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))))/(d*e - c*f)^2) + ((
(b*e - a*f)*(d*e - c*f)*x*(a + b*x^2)^2)/(2*e*(e + f*x^2)) - ((b*(b*e*(3*d
*e - 5*c*f) - a*f*(d*e - 3*c*f))*x*(a + b*x^2))/(3*f) - (-((b*(2*a*b*e*f*(
3*d*e - 11*c*f) - 3*b^2*e^2*(3*d*e - 5*c*f) + a^2*f^2*(7*d*e + 3*c*f))*x)/
f) - (3*(b*e - a*f)^2*(b*e*(3*d*e - 5*c*f) + a*f*(3*d*e - c*f))*ArcTan[(Sq
rt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(3/2)))/(3*f))/(2*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.76

method	result
default	$\frac{b^3 x}{d f^2}+\frac{\frac{\left(a^3 c f^4-a^3 d e f^3-3a^2 b c e f^3+3a^2 b d e^2 f^2+3a b^2 c e^2 f^2-3a b^2 d e^3 f-b^3 c e^3 f+e^4 b^3 d\right) x}{2e\left(f x^2+e\right)}+\frac{\left(a^3 c f^4-3a^3 d e f^3+3a^2 b c e f^3+3a^2 b d e^2 f^2-\right.}{}}{f^2 (cf-de)^2}$
risch	Expression too large to display

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d/f^2+1/f^2/(c*f-d*e)^2*(1/2*(a^3*c*f^4-a^3*d*e*f^3-3*a^2*b*c*e*f^3+
3*a^2*b*d*e^2*f^2+3*a*b^2*c*e^2*f^2-3*a*b^2*d*e^3*f-b^3*c*e^3*f+b^3*d*e^4)
/e*x/(f*x^2+e)+1/2*(a^3*c*f^4-3*a^3*d*e*f^3+3*a^2*b*c*e*f^3+3*a^2*b*d*e^2*
f^2-9*a*b^2*c*e^2*f^2+3*a*b^2*d*e^3*f+5*b^3*c*e^3*f-3*b^3*d*e^4)/e/(e*f)^(
1/2)*arctan(f*x/(e*f)^(1/2)))+1/d*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)/(c*f-d*e)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(149) = 298$.

Time = 64.52 (sec) , antiderivative size = 2588, normalized size of antiderivative = 15.31

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(b^3*c*d^3*e^4*f^2 - 2*b^3*c^2*d^2*e^3*f^3 + b^3*c^3*d*e^2*f^4)*x^
3 + 2*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e^2*f^4*x^2 + (
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e^3*f^3)*sqrt(-c*d)*log
((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + (3*b^3*c*d^3*e^5 - a^3*c^2*d^
2*e*f^4 - (5*b^3*c^2*d^2 + 3*a*b^2*c*d^3)*e^4*f + 3*(3*a*b^2*c^2*d^2 - a^2
*b*c*d^3)*e^3*f^2 - 3*(a^2*b*c^2*d^2 - a^3*c*d^3)*e^2*f^3 + (3*b^3*c*d^3*e
^4*f - a^3*c^2*d^2*f^5 - (5*b^3*c^2*d^2 + 3*a*b^2*c*d^3)*e^3*f^2 + 3*(3*a*
b^2*c^2*d^2 - a^2*b*c*d^3)*e^2*f^3 - 3*(a^2*b*c^2*d^2 - a^3*c*d^3)*e*f^4)*
x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b^3*c
*d^3*e^5*f + a^3*c^2*d^2*e*f^5 - (5*b^3*c^2*d^2 + 3*a*b^2*c*d^3)*e^4*f^2 +
 (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*e^3*f^3 - (3*a^2*b*c^2*d^
2 + a^3*c*d^3)*e^2*f^4)*x)/(c*d^4*e^5*f^3 - 2*c^2*d^3*e^4*f^4 + c^3*d^2*e^
3*f^5 + (c*d^4*e^4*f^4 - 2*c^2*d^3*e^3*f^5 + c^3*d^2*e^2*f^6)*x^2), 1/2*(2
*(b^3*c*d^3*e^4*f^2 - 2*b^3*c^2*d^2*e^3*f^3 + b^3*c^3*d*e^2*f^4)*x^3 - (3*
b^3*c*d^3*e^5 - a^3*c^2*d^2*e*f^4 - (5*b^3*c^2*d^2 + 3*a*b^2*c*d^3)*e^4*f
+ 3*(3*a*b^2*c^2*d^2 - a^2*b*c*d^3)*e^3*f^2 - 3*(a^2*b*c^2*d^2 - a^3*c*d^3
)*e^2*f^3 + (3*b^3*c*d^3*e^4*f - a^3*c^2*d^2*f^5 - (5*b^3*c^2*d^2 + 3*a*b^
2*c*d^3)*e^3*f^2 + 3*(3*a*b^2*c^2*d^2 - a^2*b*c*d^3)*e^2*f^3 - 3*(a^2*b*c^
2*d^2 - a^3*c*d^3)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + ((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e^2*f^4*x^2 + (b^3*c^3 - 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{b^3x}{df^2} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^3e^2 - 2\,cd^2ef + c^2df^2)\sqrt{cd}} - \frac{(3\,b^3de^4 - 5\,b^3ce^3f - 3\,ab^2de^3f + 9\,ab^2ce^2f^2 - 3\,a^2bde^2f^2 - 3\,a^2bcef^3 + 3\,a^3def^3 - a^3cf^4)\arctan\left(\frac{f}{\sqrt{}}\right.}{2\,(d^2e^3f^2 - 2\,cde^2f^3 + c^2ef^4)\sqrt{ef}} + \frac{b^3e^3x - 3\,ab^2e^2fx + 3\,a^2bef^2x - a^3f^3x}{2\,(de^2f^2 - cef^3)(fx^2 + e)}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b^3*x/(d*f^2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan
(d*x/sqrt(c*d))/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt(c*d)) - 1/2*(3*b
^3*d*e^4 - 5*b^3*c*e^3*f - 3*a*b^2*d*e^3*f + 9*a*b^2*c*e^2*f^2 - 3*a^2*b*d
*e^2*f^2 - 3*a^2*b*c*e*f^3 + 3*a^3*d*e*f^3 - a^3*c*f^4)*arctan(f*x/sqrt(e*
f))/((d^2*e^3*f^2 - 2*c*d*e^2*f^3 + c^2*e*f^4)*sqrt(e*f)) + 1/2*(b^3*e^3*x
 - 3*a*b^2*e^2*f*x + 3*a^2*b*e*f^2*x - a^3*f^3*x)/((d*e^2*f^2 - c*e*f^3)*(
f*x^2 + e))
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 11519, normalized size of antiderivative = 68.16

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
(b^3*x)/(d*f^2) + (atan(((((x*(9*a^2*b^4*d^6*e^6*f^2 - a^6*c^2*d^4*f^8 - 1
3*a^6*d^6*e^2*f^6 - 4*b^6*c^6*e^2*f^6 - 9*b^6*d^6*e^8 - 36*a^3*b^3*d^6*e^5
*f^3 + 9*a^4*b^2*d^6*e^4*f^4 - 25*b^6*c^2*d^4*e^6*f^2 + 18*a*b^5*d^6*e^7*f
 + 6*a^6*c*d^5*e*f^7 + 30*b^6*c*d^5*e^7*f + 18*a^5*b*d^6*e^3*f^5 - 84*a*b^
5*c*d^5*e^6*f^2 + 24*a*b^5*c^5*d*e^2*f^6 + 36*a^5*b*c*d^5*e^2*f^6 - 6*a^5*
b*c^2*d^4*e*f^7 + 90*a*b^5*c^2*d^4*e^5*f^3 + 42*a^2*b^4*c*d^5*e^5*f^3 + 72
*a^3*b^3*c*d^5*e^4*f^4 - 78*a^4*b^2*c*d^5*e^3*f^5 - 111*a^2*b^4*c^2*d^4*e^
4*f^4 - 60*a^2*b^4*c^4*d^2*e^2*f^6 + 44*a^3*b^3*c^2*d^4*e^3*f^5 + 80*a^3*b
^3*c^3*d^3*e^2*f^6 - 51*a^4*b^2*c^2*d^4*e^2*f^6))/(2*(d^3*e^4*f^3 - 2*c*d^
2*e^3*f^4 + c^2*d*e^2*f^5)) + ((-c*d^3)^(1/2)*(a*d - b*c)^3*((16*a^3*d^8*e
^6*f^5 + 128*a^3*c^2*d^6*e^4*f^7 - 112*a^3*c^3*d^5*e^3*f^8 + 48*a^3*c^4*d^
4*e^2*f^9 + 112*b^3*c^2*d^6*e^7*f^4 - 208*b^3*c^3*d^5*e^6*f^5 + 192*b^3*c^
4*d^4*e^5*f^6 - 88*b^3*c^5*d^3*e^4*f^7 + 16*b^3*c^6*d^2*e^3*f^8 - 72*a^3*c
*d^7*e^5*f^6 - 8*a^3*c^5*d^3*e*f^10 - 24*b^3*c*d^7*e^8*f^3 + 24*a*b^2*c*d^
7*e^7*f^4 - 24*a^2*b*c*d^7*e^6*f^5 - 96*a*b^2*c^2*d^6*e^6*f^5 + 144*a*b^2*
c^3*d^5*e^5*f^6 - 96*a*b^2*c^4*d^4*e^4*f^7 + 24*a*b^2*c^5*d^3*e^3*f^8 + 96
*a^2*b*c^2*d^6*e^5*f^6 - 144*a^2*b*c^3*d^5*e^4*f^7 + 96*a^2*b*c^4*d^4*e^3*
f^8 - 24*a^2*b*c^5*d^3*e^2*f^9)/(4*(d^4*e^5*f^3 - 3*c*d^3*e^4*f^4 - c^3*d*
e^2*f^6 + 3*c^2*d^2*e^3*f^5)) - (x*(-c*d^3)^(1/2)*(a*d - b*c)^3*(16*d^8*e^
7*f^5 - 48*c*d^7*e^6*f^6 + 32*c^2*d^6*e^5*f^7 + 32*c^3*d^5*e^4*f^8 - 48...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1064, normalized size of antiderivative = 6.30

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^2,x)
```

output

```
(2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e**3*f**3 + 2*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e**2*f**4*x**2 - 6*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e**3*f**3 - 6*
sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e**2*f**4*x**2
 + 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**2*d*e**3*f**3
 + 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**2*d*e**2*f**4
*x**2 - 2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**3*c**3*e**3*f**
3 - 2*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*b**3*c**3*e**2*f**4*x*
*2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d**2*e*f**4 +
 sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d**2*f**5*x**2 -
3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**3*e**2*f**3 - 3*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**3*c*d**3*e*f**4*x**2 + 3*
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**2*d**2*e**2*f**3 +
 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c**2*d**2*e*f**4*x
**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c*d**3*e**3*f
**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*b*c*d**3*e**2*f
**3*x**2 - 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c**2*d**
2*e**3*f**2 - 9*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b**2*c**2*
d**2*e**2*f**3*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*
*2*c*d**3*e**4*f + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*...
```

3.249 $\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 286

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{(be-af)^3x}{4ef^2(de-cf)(e+fx^2)^2}$$
$$-\frac{(be-af)^2(be(5de-9cf)+af(7de-3cf))x}{8e^2f^2(de-cf)^2(e+fx^2)} - \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)^3}$$
$$+\frac{(be-af)\left(2abef(3d^2e^2-14cdef+3c^2f^2)+a^2f^2(15d^2e^2-10cdef+3c^2f^2)+b^2e^2(3d^2e^2-10cdef\right.}{8e^{5/2}f^{5/2}(de-cf)^3}$$

output
```
1/4*(-a*f+b*e)^3*x/e/f^2/(-c*f+d*e)/(f*x^2+e)^2-1/8*(-a*f+b*e)^2*(b*e*(-9*
c*f+5*d*e)+a*f*(-3*c*f+7*d*e))*x/e^2/f^2/(-c*f+d*e)^2/(f*x^2+e)-(-a*d+b*c)
^3*arctan(d^(1/2)*x/c^(1/2))/c^(1/2)/d^(1/2)/(-c*f+d*e)^3+1/8*(-a*f+b*e)*(
2*a*b*e*f*(3*c^2*f^2-14*c*d*e*f+3*d^2*e^2)+a^2*f^2*(3*c^2*f^2-10*c*d*e*f+1
5*d^2*e^2)+b^2*e^2*(15*c^2*f^2-10*c*d*e*f+3*d^2*e^2))*arctan(f^(1/2)*x/e^(
1/2))/e^(5/2)/f^(5/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{1}{8}\left(\frac{2(be-af)^3x}{ef^2(de-cf)(e+fx^2)^2}\right.$$

$$-\frac{(be-af)^2(be(5de-9cf)+af(7de-3cf))x}{e^2f^2(de-cf)^2(e+fx^2)}+\frac{8(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(-de+cf)^3}$$

$$+\frac{(be-af)\left(2abef(3d^2e^2-14cdef+3c^2f^2)+a^2f^2(15d^2e^2-10cdef+3c^2f^2)+b^2e^2(3d^2e^2-10cdef\right.}{e^{5/2}f^{5/2}(de-cf)^3}$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((2*(b*e - a*f)^3*x)/(e*f^2*(d*e - c*f)*(e + f*x^2)^2) - ((b*e - a*f)^2*(b
*e*(5*d*e - 9*c*f) + a*f*(7*d*e - 3*c*f))*x)/(e^2*f^2*(d*e - c*f)^2*(e + f
*x^2)) + (8*(b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]*(-
(d*e) + c*f)^3) + ((b*e - a*f)*(2*a*b*e*f*(3*d^2*e^2 - 14*c*d*e*f + 3*c^2*
f^2) + a^2*f^2*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2) + b^2*e^2*(3*d^2*e^2
- 10*c*d*e*f + 15*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*f^(5/2)*
(d*e - c*f)^3))/8
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.52, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {419, 25, 401, 27, 401, 25, 299, 218, 420, 299, 218, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(bx^2+a)^2\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^2\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{f\left(bx^2+a\right)\left(b(be(de-5cf)+af(3de+cf))x^2+a(af(7de-3cf)-be(3de+cf))\right)}{\left(fx^2+e\right)^2}dx}{4ef}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{\left(bx^2+a\right)\left(b(be(de-5cf)+af(3de+cf))x^2+a(af(7de-3cf)-be(3de+cf))\right)}{\left(fx^2+e\right)^2}dx}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{-\frac{\int -\frac{b\left(3b^2(de-5cf)e^2+4abf(3de+cf)e-a^2f^2(7de-3cf)\right)x^2+a\left(b^2(de-5cf)e^2+a^2f^2(7de-3cf)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(a+bx^2\right)(be-af)(\ldots}{2e\ldots}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\int \frac{b\left(3b^2(de-5cf)e^2+4abf(3de+cf)e-a^2f^2(7de-3cf)\right)x^2+a\left(b^2(de-5cf)e^2+a^2f^2(7de-3cf)\right)}{fx^2+e}dx}{2ef}-\frac{x\left(a+bx^2\right)(be-af)(af(\ldots}{2ef\left(e\ldots\right.}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 299

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2(de-5cf)\right)\int \cdots}{f}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 420

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{b\int \frac{bx^2+a}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 299

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\int \frac{1}{fx^2+e}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 397

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\int\frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int\frac{1}{dx^2+c}dx}{de-cf}\right)}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 218

$$\frac{\frac{x(a+bx^2)^2(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\frac{bx\left(-a^2f^2(7de-3cf)+4abef(cf+3de)+3b^2e^2(de-5cf)\right)}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(7de-3cf)+3b^2e^2\cdots\right.}{\sqrt{e}f^{3/2}}}{2ef}}{4e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{b\left(\frac{bx}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{3/2}}\right)}{d}-\frac{(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{d}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
-((d*(b*c - a*d)*((b*((b*x)/f - ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(3/2))))/d - ((b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(Sqrt[c]*Sqrt[d]*(d*e - c*f))) + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/S
qrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f))))/d))/(d*e - c*f)^2) + (((b*e - a*f
)*(d*e - c*f)*x*(a + b*x^2)^2)/(4*e*(e + f*x^2)^2) - (-1/2*((b*e - a*f)*(b
*e*(d*e - 5*c*f) + a*f*(7*d*e - 3*c*f))*x*(a + b*x^2))/(e*f*(e + f*x^2)) +
 ((b*(3*b^2*e^2*(d*e - 5*c*f) - a^2*f^2*(7*d*e - 3*c*f) + 4*a*b*e*f*(3*d*e
 + c*f))*x)/f - ((b*e - a*f)*(3*b^2*e^2*(d*e - 5*c*f) + 2*a*b*e*f*(7*d*e -
 3*c*f) + a^2*f^2*(7*d*e - 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f
^(3/2)))/(2*e*f))/(4*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(264) = 528$.

Time = 0.90 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.09

method	result
default	$\frac{\frac{\left(3a^3c^2f^5-10a^3cde\,f^4+7a^3d^2e^2f^3+3a^2b\,c^2e\,f^4+6a^2bcd\,e^2f^3-9a^2b\,d^2e^3f^2-15a\,b^2c^2e^2f^3+18a\,b^2cd\,e^3f^2-3a\,b^2d^2e^4f+9b^3c^2e^3f^2-14b^3cd\,e\cdots}{8e^2f}}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^3*((1/8*(3*a^3*c^2*f^5-10*a^3*c*d*e*f^4+7*a^3*d^2*e^2*f^3+3*a^
2*b*c^2*e*f^4+6*a^2*b*c*d*e^2*f^3-9*a^2*b*d^2*e^3*f^2-15*a*b^2*c^2*e^2*f^3
+18*a*b^2*c*d*e^3*f^2-3*a*b^2*d^2*e^4*f+9*b^3*c^2*e^3*f^2-14*b^3*c*d*e^4*f
+5*b^3*d^2*e^5)/e^2/f*x^3+1/8*(5*a^3*c^2*f^5-14*a^3*c*d*e*f^4+9*a^3*d^2*e^
2*f^3-3*a^2*b*c^2*e*f^4+18*a^2*b*c*d*e^2*f^3-15*a^2*b*d^2*e^3*f^2-9*a*b^2*
c^2*e^2*f^3+6*a*b^2*c*d*e^3*f^2+3*a*b^2*d^2*e^4*f+7*b^3*c^2*e^3*f^2-10*b^3
*c*d*e^4*f+3*b^3*d^2*e^5)/e/f^2*x)/(f*x^2+e)^2+1/8*(3*a^3*c^2*f^5-10*a^3*c
*d*e*f^4+15*a^3*d^2*e^2*f^3+3*a^2*b*c^2*e*f^4-18*a^2*b*c*d*e^2*f^3-9*a^2*b
*d^2*e^3*f^2+9*a*b^2*c^2*e^2*f^3+18*a*b^2*c*d*e^3*f^2-3*a*b^2*d^2*e^4*f-15
*b^3*c^2*e^3*f^2+10*b^3*c*d*e^4*f-3*b^3*d^2*e^5)/e^2/f^2/(e*f)^(1/2)*arcta
n(f*x/(e*f)^(1/2)))+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(c*f-d*
e)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(264) = 528$.

Time = 112.85 (sec) , antiderivative size = 4843, normalized size of antiderivative = 16.93

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(264) = 528$.

Time = 0.13 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = -\frac{(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(d^3e^3 - 3cd^2e^2f + 3c^2def^2 - c^3f^3)\sqrt{cd}}$$
$$+ \frac{(3b^3d^2e^5 - 10b^3cde^4f + 3ab^2d^2e^4f + 15b^3c^2e^3f^2 - 18ab^2cde^3f^2 + 9a^2bd^2e^3f^2 - 9ab^2c^2e^2f^3 + 18a^2}{8(d^3e^5f^2 - 3cd^2e^4f^3 + 3c^2de^3f^4 - c}$$
$$- \frac{5b^3de^4fx^3 - 9b^3ce^3f^2x^3 - 3ab^2de^3f^2x^3 + 15ab^2ce^2f^3x^3 - 9a^2bde^2f^3x^3 - 3a^2bcef^4x^3 + 7a^3def^4x^3}{8(d^2e^4f^2 -}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))
/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sqrt(c*d)) + 1/8*(3*
b^3*d^2*e^5 - 10*b^3*c*d*e^4*f + 3*a*b^2*d^2*e^4*f + 15*b^3*c^2*e^3*f^2 -
18*a*b^2*c*d*e^3*f^2 + 9*a^2*b*d^2*e^3*f^2 - 9*a*b^2*c^2*e^2*f^3 + 18*a^2*
b*c*d*e^2*f^3 - 15*a^3*d^2*e^2*f^3 - 3*a^2*b*c^2*e*f^4 + 10*a^3*c*d*e*f^4
- 3*a^3*c^2*f^5)*arctan(f*x/sqrt(e*f))/((d^3*e^5*f^2 - 3*c*d^2*e^4*f^3 + 3
*c^2*d*e^3*f^4 - c^3*e^2*f^5)*sqrt(e*f)) - 1/8*(5*b^3*d*e^4*f*x^3 - 9*b^3*
c*e^3*f^2*x^3 - 3*a*b^2*d*e^3*f^2*x^3 + 15*a*b^2*c*e^2*f^3*x^3 - 9*a^2*b*d
*e^2*f^3*x^3 - 3*a^2*b*c*e*f^4*x^3 + 7*a^3*d*e*f^4*x^3 - 3*a^3*c*f^5*x^3 +
 3*b^3*d*e^5*x - 7*b^3*c*e^4*f*x + 3*a*b^2*d*e^4*f*x + 9*a*b^2*c*e^3*f^2*x
 - 15*a^2*b*d*e^3*f^2*x + 3*a^2*b*c*e^2*f^3*x + 9*a^3*d*e^2*f^3*x - 5*a^3*
c*e*f^4*x)/((d^2*e^4*f^2 - 2*c*d*e^3*f^3 + c^2*e^2*f^4)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 18014, normalized size of antiderivative = 62.99

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
((x^3*(3*a^3*c*f^4 - 5*b^3*d*e^4 - 7*a^3*d*e*f^3 + 9*b^3*c*e^3*f + 3*a^2*b
*c*e*f^3 + 3*a*b^2*d*e^3*f - 15*a*b^2*c*e^2*f^2 + 9*a^2*b*d*e^2*f^2))/(8*e
^2*(c^2*f^3 + d^2*e^2*f - 2*c*d*e*f^2)) - (x*(3*b^3*d*e^4 - 5*a^3*c*f^4 +
9*a^3*d*e*f^3 - 7*b^3*c*e^3*f + 3*a^2*b*c*e*f^3 + 3*a*b^2*d*e^3*f + 9*a*b^
2*c*e^2*f^2 - 15*a^2*b*d*e^2*f^2))/(8*e*f*(c^2*f^3 + d^2*e^2*f - 2*c*d*e*f
^2)))/(e^2 + f^2*x^4 + 2*e*f*x^2) + (atan((((-c*d)^(1/2)*((x*(9*b^6*d^7*e^
10 + 9*a^6*c^4*d^3*f^10 + 289*a^6*d^7*e^4*f^6 + 63*a^2*b^4*d^7*e^8*f^2 - 3
6*a^3*b^3*d^7*e^7*f^3 - 9*a^4*b^2*d^7*e^6*f^4 + 190*a^6*c^2*d^5*e^2*f^8 +
190*b^6*c^2*d^5*e^8*f^2 - 300*b^6*c^3*d^4*e^7*f^3 + 225*b^6*c^4*d^3*e^6*f^
4 + 18*a*b^5*d^7*e^9*f - 60*b^6*c*d^6*e^9*f - 270*a^5*b*d^7*e^5*f^5 - 300*
a^6*c*d^6*e^3*f^7 - 60*a^6*c^3*d^4*e*f^9 + 64*b^6*c^6*d*e^4*f^6 - 168*a*b^
5*c*d^6*e^8*f^2 - 744*a^5*b*c*d^6*e^4*f^6 + 18*a^5*b*c^4*d^3*e*f^9 + 396*a
*b^5*c^2*d^5*e^7*f^3 - 360*a*b^5*c^3*d^4*e^6*f^4 - 270*a*b^5*c^4*d^3*e^5*f
^5 - 384*a*b^5*c^5*d^2*e^4*f^6 - 180*a^2*b^4*c*d^6*e^7*f^3 + 144*a^3*b^3*c
*d^6*e^6*f^4 + 924*a^4*b^2*c*d^6*e^5*f^5 + 396*a^5*b*c^2*d^5*e^3*f^7 - 168
*a^5*b*c^3*d^4*e^2*f^8 + 162*a^2*b^4*c^2*d^5*e^6*f^4 + 924*a^2*b^4*c^3*d^4
*e^5*f^5 + 951*a^2*b^4*c^4*d^3*e^4*f^6 - 1496*a^3*b^3*c^2*d^5*e^5*f^5 - 11
36*a^3*b^3*c^3*d^4*e^4*f^6 - 36*a^3*b^3*c^4*d^3*e^3*f^7 + 1122*a^4*b^2*c^2
*d^5*e^4*f^6 - 180*a^4*b^2*c^3*d^4*e^3*f^7 + 63*a^4*b^2*c^4*d^3*e^2*f^8))/
(32*(c^4*e^4*f^7 + d^4*e^8*f^3 - 4*c*d^3*e^7*f^4 - 4*c^3*d*e^5*f^6 + 6*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2204, normalized size of antiderivative = 7.71

$$\int \frac{(a+bx^2)^3}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
( - 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e**5*f**3 -
16*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e**4*f**4*x**2
- 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**3*e**3*f**5*x**4
 + 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e**5*f**
3 + 48*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e**4*f*
*4*x**2 + 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d**2*e
**3*f**5*x**4 - 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c*
*2*d*e**5*f**3 - 48*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c
**2*d*e**4*f**4*x**2 - 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*
b**2*c**2*d*e**3*f**5*x**4 + 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)
))*b**3*c**3*e**5*f**3 + 16*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*
b**3*c**3*e**4*f**4*x**2 + 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))
*b**3*c**3*e**3*f**5*x**4 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e))
)*a**3*c**3*d*e**2*f**5 + 6*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
a**3*c**3*d*e*f**6*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*
a**3*c**3*d*f**7*x**4 - 10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a
**3*c**2*d**2*e**3*f**4 - 20*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a**3*c**2*d**2*e**2*f**5*x**2 - 10*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sq
rt(e)))*a**3*c**2*d**2*e*f**6*x**4 + 15*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f
)*sqrt(e)))*a**3*c*d**3*e**4*f**3 + 30*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt...
```

3.250 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 299

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2} dx = -\frac{(6a^2bcd^2ef^2-3ab^2cdef(de+cf)-a^3d^2f^2(de+cf)+b^3(cd^2e^3+c^3ef^2))x}{2cd^2ef(de-cf)^2(e+fx^2)} - \frac{(bc-ad)^3x}{2cd^2(de-cf)(c+dx^2)(e+fx^2)} + \frac{(bc-ad)^2(ad(de-5cf)+bc(5de-cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}(de-cf)^3} + \frac{(be-af)^2(be(de-5cf)+af(5de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}(de-cf)^3}$$

output
```
-1/2*(6*a^2*b*c*d^2*e*f^2-3*a*b^2*c*d*e*f*(c*f+d*e)-a^3*d^2*f^2*(c*f+d*e)+
b^3*(c^3*e*f^2+c*d^2*e^3))*x/c/d^2/e/f/(-c*f+d*e)^2/(f*x^2+e)-1/2*(-a*d+b*
c)^3*x/c/d^2/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)+1/2*(-a*d+b*c)^2*(a*d*(-5*c*f+
d*e)+b*c*(-c*f+5*d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/d^(3/2)/(-c*f+d*e
)^3+1/2*(-a*f+b*e)^2*(b*e*(-5*c*f+d*e)+a*f*(-c*f+5*d*e))*arctan(f^(1/2)*x/
e^(1/2))/e^(3/2)/f^(3/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{1}{2}\left(-\frac{(bc-ad)^3x}{cd(de-cf)^2(c+dx^2)}-\frac{(be-af)^3x}{ef(de-cf)^2(e+fx^2)} + \frac{(bc-ad)^2(bc(-5de+cf)+ad(-de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}(-de+cf)^3} + \frac{(be-af)^2(be(de-5cf)+af(5de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}f^{3/2}(de-cf)^3}\right)$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(-(((b*c - a*d)^3*x)/(c*d*(d*e - c*f)^2*(c + d*x^2))) - ((b*e - a*f)^3*x)/
(e*f*(d*e - c*f)^2*(e + f*x^2)) + ((b*c - a*d)^2*(b*c*(-5*d*e + c*f) + a*d
*(-(d*e) + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*d^(3/2)*(-(d*e) +
 c*f)^3) + ((b*e - a*f)^2*(b*e*(d*e - 5*c*f) + a*f*(5*d*e - c*f))*ArcTan[(
Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*f^(3/2)*(d*e - c*f)^3))/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 639 vs. 2(299) = 598.

Time = 1.04 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.14, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {425, 419, 25, 299, 218, 401, 27, 299, 218, 425, 402, 25, 397, 218, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2 \left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)\left(fx^2+e\right)^2}dx}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \text{419}$$

$$\frac{b\left(-\frac{\int -\frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{dx^2+c}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{dx^2+c}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{\int \frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{d}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\int \frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int \frac{f\left(b(be(de-3cf)+af(de+cf))x^2+a(af(3de-cf)-be(de+cf))\right)}{fx^2+e}dx}{2ef}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int \frac{b(be(de-3cf)+af(de+cf))x^2+a(af(3de-cf)-be(de+cf))}{fx^2+e}dx}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \ 299$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)(af(3de-cf)+be(de-3cf))\int \frac{1}{fx^2+e}dx}{f}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}$$

$$\Big\downarrow \ 425$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{bx^2+a}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{\int-\frac{-d(be-af)x^2+bce-2ade+acf}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int-\frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-a\ldots}{2c\left(c+dx^2\right)\left(e+f\ldots\right.}\right.}{d}\right.}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\int\frac{-d(be-af)x^2+bce-2ade+acf}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(d\ldots}\right.}{d}\right.}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2de(bc-ad)\int\frac{1}{dx^2+c}dx}{de-cf}+\frac{(af(3de-cf)-be(cf+de))\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{-3(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}}{2c(de-cf)}\right.}{d}\right.}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{-3(bc-ad)fx^2+bce+ad}{\left(dx^2+c\right)\left(fx^2+e\right)}}{2c(de-cf)}\right.}{}\right.}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{2\left(-df(2bce-ade-acf)\right.}{(}}{}\right.}{}\right.}{d}$$

$$\Big\downarrow\ 27$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int\frac{-df(2bce-ade-acf)x^2}{\left(d\right.}}{}\right.}{}\right.}{d}$$

$$\Big\downarrow\ 397$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{de(ad(de-5cf)+bc(3cf+}{de-cf}}{}\right.}{}\right.}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{2e\left(e+fx^2\right)}-\frac{\frac{bx(af(cf+de)+be(de-3cf))}{f}-\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)+be(de-3cf))}{\sqrt{e}f^{3/2}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{\frac{2\sqrt{d}e(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3de-cf)-be(cf+de))}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{d}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de}{\sqrt{c}(de-}}{}\right.}{}\right.}{d}$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(b*(-((d*(b*c - a*d)*((b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/
(Sqrt[c]*d^(3/2))))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*f)*x*(a + b*x^
2))/(2*e*(e + f*x^2)) - ((b*(b*e*(d*e - 3*c*f) + a*f*(d*e + c*f))*x)/f - (
(b*e - a*f)*(b*e*(d*e - 3*c*f) + a*f*(3*d*e - c*f))*ArcTan[(Sqrt[f]*x)/Sqr
t[e]])/(Sqrt[e]*f^(3/2)))/(2*e))/(d*e - c*f)^2))/d - ((b*c - a*d)*((b*(((b
*e - a*f)*x)/(2*e*(d*e - c*f)*(e + f*x^2)) - ((2*Sqrt[d]*(b*c - a*d)*e*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + ((a*f*(3*d*e - c*f) - b*
e*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)))
/(2*e*(d*e - c*f))))/d - ((b*c - a*d)*(-1/2*((b*c - a*d)*x)/(c*(d*e - c*f)
*(c + d*x^2)*(e + f*x^2)) + (-((f*(2*b*c*e - a*d*e - a*c*f)*x)/(e*(d*e - c
*f)*(e + f*x^2))) + ((Sqrt[d]*e*(a*d*(d*e - 5*c*f) + b*c*(d*e + 3*c*f))*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + (c*Sqrt[f]*(a*f*(5*d*e
- c*f) - b*e*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c
*f)))/(e*(d*e - c*f)))/(2*c*(d*e - c*f))))/d))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.50

method	result
default	$\frac{\frac{\left(a^{3} c f^{4}-a^{3} d e f^{3}-3 a^{2} b c e f^{3}+3 a^{2} b d e^{2} f^{2}+3 a b^{2} c e^{2} f^{2}-3 a b^{2} d e^{3} f-b^{3} c e^{3} f+e^{4} b^{3} d\right) x}{2 e f \left(f x^{2}+e\right)}+\frac{\left(a^{3} c f^{4}-5 a^{3} d e f^{3}+3 a^{2} b c e f^{3}+9 a^{2} b d e^{2} f^{2}-9 a b^{2} c e^{2} f \cdots}{2 e f \sqrt{e f} \cdots}}{\left(c f-d e\right)^{3}}$
risch	Expression too large to display

input
```
int((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^3*(1/2*(a^3*c*f^4-a^3*d*e*f^3-3*a^2*b*c*e*f^3+3*a^2*b*d*e^2*f^
2+3*a*b^2*c*e^2*f^2-3*a*b^2*d*e^3*f-b^3*c*e^3*f+b^3*d*e^4)/e/f*x/(f*x^2+e)
+1/2*(a^3*c*f^4-5*a^3*d*e*f^3+3*a^2*b*c*e*f^3+9*a^2*b*d*e^2*f^2-9*a*b^2*c*
e^2*f^2-3*a*b^2*d*e^3*f+5*b^3*c*e^3*f-b^3*d*e^4)/e/f/(e*f)^(1/2)*arctan(f*
x/(e*f)^(1/2)))+1/(c*f-d*e)^3*(1/2*(a^3*c*d^3*f-a^3*d^4*e-3*a^2*b*c^2*d^2*
f+3*a^2*b*c*d^3*e+3*a*b^2*c^3*d*f-3*a*b^2*c^2*d^2*e-b^3*c^4*f+b^3*c^3*d*e)
/c/d*x/(d*x^2+c)+1/2*(5*a^3*c*d^3*f-a^3*d^4*e-9*a^2*b*c^2*d^2*f-3*a^2*b*c*
d^3*e+3*a*b^2*c^3*d*f+9*a*b^2*c^2*d^2*e+b^3*c^4*f-5*b^3*c^3*d*e)/c/d/(c*d)
^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(275) = 550$.

Time = 146.92 (sec) , antiderivative size = 5227, normalized size of antiderivative = 17.48

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(275) = 550.

Time = 0.13 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx$$
$$= \frac{(5\,b^3c^3de - 9\,ab^2c^2d^2e + 3\,a^2bcd^3e + a^3d^4e - b^3c^4f - 3\,ab^2c^3df + 9\,a^2bc^2d^2f - 5\,a^3cd^3f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(cd^4e^3 - 3\,c^2d^3e^2f + 3\,c^3d^2ef^2 - c^4df^3)\sqrt{cd}}$$
$$+ \frac{(b^3de^4 - 5\,b^3ce^3f + 3\,ab^2de^3f + 9\,ab^2ce^2f^2 - 9\,a^2bde^2f^2 - 3\,a^2bcef^3 + 5\,a^3def^3 - a^3cf^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right.}{2\,(d^3e^4f - 3\,cd^2e^3f^2 + 3\,c^2de^2f^3 - c^3ef^4)\sqrt{ef}}$$
$$- \frac{b^3cd^2e^3x^3 - 3\,ab^2cd^2e^2fx^3 + b^3c^3ef^2x^3 - 3\,ab^2c^2def^2x^3 + 6\,a^2bcd^2ef^2x^3 - a^3d^3ef^2x^3 - a^3cd^2f^3x^3 +}{2\,(cd^3e^3f - 2\,c^2d^2e^2f^2 + c^3def^3)(d}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^3*c^3*d*e - 9*a*b^2*c^2*d^2*e + 3*a^2*b*c*d^3*e + a^3*d^4*e - b^3
*c^4*f - 3*a*b^2*c^3*d*f + 9*a^2*b*c^2*d^2*f - 5*a^3*c*d^3*f)*arctan(d*x/s
qrt(c*d))/((c*d^4*e^3 - 3*c^2*d^3*e^2*f + 3*c^3*d^2*e*f^2 - c^4*d*f^3)*sqr
t(c*d)) + 1/2*(b^3*d*e^4 - 5*b^3*c*e^3*f + 3*a*b^2*d*e^3*f + 9*a*b^2*c*e^2
*f^2 - 9*a^2*b*d*e^2*f^2 - 3*a^2*b*c*e*f^3 + 5*a^3*d*e*f^3 - a^3*c*f^4)*ar
ctan(f*x/sqrt(e*f))/((d^3*e^4*f - 3*c*d^2*e^3*f^2 + 3*c^2*d*e^2*f^3 - c^3*
e*f^4)*sqrt(e*f)) - 1/2*(b^3*c*d^2*e^3*x^3 - 3*a*b^2*c*d^2*e^2*f*x^3 + b^3
*c^3*e*f^2*x^3 - 3*a*b^2*c^2*d*e*f^2*x^3 + 6*a^2*b*c*d^2*e*f^2*x^3 - a^3*d
^3*e*f^2*x^3 - a^3*c*d^2*f^3*x^3 + b^3*c^2*d*e^3*x + b^3*c^3*e^2*f*x - 6*a
*b^2*c^2*d*e^2*f*x + 3*a^2*b*c*d^2*e^2*f*x - a^3*d^3*e^2*f*x + 3*a^2*b*c^2
*d*e*f^2*x - a^3*c^2*d*f^3*x)/((c*d^3*e^3*f - 2*c^2*d^2*e^2*f^2 + c^3*d*e*
f^3)*(d*f*x^4 + d*e*x^2 + c*f*x^2 + c*e))
```

Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 142283, normalized size of antiderivative = 475.86

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
atan(((((160*a^3*c^2*d^10*e^8*f^4 - 640*a^3*c^3*d^9*e^7*f^5 + 1376*a^3*c^4
*d^8*e^6*f^6 - 1760*a^3*c^5*d^7*e^5*f^7 + 1376*a^3*c^6*d^6*e^4*f^8 - 640*a
^3*c^7*d^5*e^3*f^9 + 160*a^3*c^8*d^4*e^2*f^10 - 16*b^3*c^3*d^9*e^10*f^2 +
80*b^3*c^4*d^8*e^9*f^3 - 144*b^3*c^5*d^7*e^8*f^4 + 80*b^3*c^6*d^6*e^7*f^5
+ 80*b^3*c^7*d^5*e^6*f^6 - 144*b^3*c^8*d^4*e^5*f^7 + 80*b^3*c^9*d^3*e^4*f^
8 - 16*b^3*c^10*d^2*e^3*f^9 - 16*a^3*c*d^11*e^9*f^3 - 16*a^3*c^9*d^3*e*f^1
1 + 96*a*b^2*c^3*d^9*e^9*f^3 - 576*a*b^2*c^4*d^8*e^8*f^4 + 1440*a*b^2*c^5*
d^7*e^7*f^5 - 1920*a*b^2*c^6*d^6*e^6*f^6 + 1440*a*b^2*c^7*d^5*e^5*f^7 - 57
6*a*b^2*c^8*d^4*e^4*f^8 + 96*a*b^2*c^9*d^3*e^3*f^9 - 48*a^2*b*c^2*d^10*e^9
*f^3 + 240*a^2*b*c^3*d^9*e^8*f^4 - 432*a^2*b*c^4*d^8*e^7*f^5 + 240*a^2*b*c
^5*d^7*e^6*f^6 + 240*a^2*b*c^6*d^6*e^5*f^7 - 432*a^2*b*c^7*d^5*e^4*f^8 + 2
40*a^2*b*c^8*d^4*e^3*f^9 - 48*a^2*b*c^9*d^3*e^2*f^10)/(8*(c^2*d^7*e^8*f +
c^8*d*e^2*f^7 - 6*c^3*d^6*e^7*f^2 + 15*c^4*d^5*e^6*f^3 - 20*c^5*d^4*e^5*f^
4 + 15*c^6*d^3*e^4*f^5 - 6*c^7*d^2*e^3*f^6)) - (x*((2360*a^6*c^3*d^11*e^8*
f^6 - 8*a^6*c^11*d^3*f^14 - 8*b^6*c^3*d^11*e^14 - 8*a^6*d^14*e^11*f^3 - 8*
b^6*c^14*e^3*f^11 - 800*a^6*c^2*d^12*e^9*f^5 - ((4720*a^6*c^3*d^11*e^8*f^6
 - 16*b^6*c^3*d^11*e^14 - 16*a^6*d^14*e^11*f^3 - 16*b^6*c^14*e^3*f^11 - 16
00*a^6*c^2*d^12*e^9*f^5 - 16*a^6*c^11*d^3*f^14 - 6880*a^6*c^4*d^10*e^7*f^7
 + 3520*a^6*c^5*d^9*e^6*f^8 + 3520*a^6*c^6*d^8*e^5*f^9 - 6880*a^6*c^7*d^7*
e^4*f^10 + 4720*a^6*c^8*d^6*e^3*f^11 - 1600*a^6*c^9*d^5*e^2*f^12 - 1600...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2451, normalized size of antiderivative = 8.20

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**3*e**3*f**3
+ 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**3*e**2*f**4
*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**4*e**4*f**
2 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**4*e**3*f**3*
x**2 + 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**4*e**2*f*
*4*x**4 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**5*e**4*f**
2*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**5*e**3*f**3
*x**4 - 9*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**3*d**2*e
**3*f**3 - 9*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**3*d**
2*e**2*f**4*x**2 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*
c**2*d**3*e**4*f**2 - 12*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**
2*b*c**2*d**3*e**3*f**3*x**2 - 9*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(
c)))*a**2*b*c**2*d**3*e**2*f**4*x**4 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(
d)*sqrt(c)))*a**2*b*c*d**4*e**4*f**2*x**2 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(
sqrt(d)*sqrt(c)))*a**2*b*c*d**4*e**3*f**3*x**4 + 3*sqrt(d)*sqrt(c)*atan((d
*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**4*d*e**3*f**3 + 3*sqrt(d)*sqrt(c)*atan((d
*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**4*d*e**2*f**4*x**2 + 9*sqrt(d)*sqrt(c)*at
an((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**3*d**2*e**4*f**2 + 12*sqrt(d)*sqrt(c
)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**3*d**2*e**3*f**3*x**2 + 3*sqrt(d
)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*c**3*d**2*e**2*f**4*x**4...
```

3.251 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 505

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^3}\,dx$$
$$= -\frac{(9a^2bcd^2ef^2 - a^3d^2f^2(2de+cf) - 3ab^2cdef(de+2cf) + b^3(cd^2e^3+2c^3ef^2))\,x}{4cd^2ef(de-cf)^2\,(e+fx^2)^2}$$
$$-\frac{(bc-ad)^3x}{2cd^2(de-cf)\,(c+dx^2)\,(e+fx^2)^2}$$
$$-\frac{(3a^2bcdef^2(11de+cf) - 9ab^2cde^2f(de+3cf) - b^3ce^2(d^2e^2-9cdef-4c^2f^2) - a^3df^2(4d^2e^2+11cde}{8cde^2f(de-cf)^3\,(e+fx^2)}$$
$$+\frac{(bc-ad)^2(ad(de-7cf)+bc(5de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}(de-cf)^4}$$
$$+\frac{(be-af)\,(b^2e^2(d^2e^2-10cdef-15c^2f^2)+2abef(5d^2e^2+22cdef-3c^2f^2)-a^2f^2(35d^2e^2-14cdef}{8e^{5/2}f^{3/2}(de-cf)^4}$$

output
```
-1/4*(9*a^2*b*c*d^2*e*f^2-a^3*d^2*f^2*(c*f+2*d*e)-3*a*b^2*c*d*e*f*(2*c*f+d
*e)+b^3*(2*c^3*e*f^2+c*d^2*e^3))*x/c/d^2/e/f/(-c*f+d*e)^2/(f*x^2+e)^2-1/2*
(-a*d+b*c)^3*x/c/d^2/(-c*f+d*e)/(d*x^2+c)/(f*x^2+e)^2-1/8*(3*a^2*b*c*d*e*f
^2*(c*f+11*d*e)-9*a*b^2*c*d*e^2*f*(3*c*f+d*e)-b^3*c*e^2*(-4*c^2*f^2-9*c*d*
e*f+d^2*e^2)-a^3*d*f^2*(-3*c^2*f^2+11*c*d*e*f+4*d^2*e^2))*x/c/d/e^2/f/(-c*
f+d*e)^3/(f*x^2+e)+1/2*(-a*d+b*c)^2*(a*d*(-7*c*f+d*e)+b*c*(c*f+5*d*e))*arc
tan(d^(1/2)*x/c^(1/2))/c^(3/2)/d^(1/2)/(-c*f+d*e)^4+1/8*(-a*f+b*e)*(b^2*e^
2*(-15*c^2*f^2-10*c*d*e*f+d^2*e^2)+2*a*b*e*f*(-3*c^2*f^2+22*c*d*e*f+5*d^2*
e^2)-a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/
e^(5/2)/f^(3/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(\frac{4(bc-ad)^3x}{c(-de+cf)^3(c+dx^2)} - \frac{2(be-af)^3x}{ef(de-cf)^2(e+fx^2)^2} + \frac{(be-af)^2(be(de-9cf)+af(11de-3cf))x}{e^2f(de-cf)^3(e+fx^2)} + \frac{4(bc-ad)^2(ad(de-7cf)+bc(5de+cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}(de-cf)^4} + \frac{(be-af)\left(b^2e^2(d^2e^2-10cdef-15c^2f^2)+a^2f^2(-35d^2e^2+14cdef-3c^2f^2)+2abef(5d^2e^2+22cde\ldots\right.}{e^{5/2}f^{3/2}(de-cf)^4}\right.$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
((4*(b*c - a*d)^3*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)) - (2*(b*e - a*f)^3*x
)/(e*f*(d*e - c*f)^2*(e + f*x^2)^2) + ((b*e - a*f)^2*(b*e*(d*e - 9*c*f) +
a*f*(11*d*e - 3*c*f))*x)/(e^2*f*(d*e - c*f)^3*(e + f*x^2)) + (4*(b*c - a*d
)^2*(a*d*(d*e - 7*c*f) + b*c*(5*d*e + c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
c^(3/2)*Sqrt[d]*(d*e - c*f)^4) + ((b*e - a*f)*(b^2*e^2*(d^2*e^2 - 10*c*d*e
*f - 15*c^2*f^2) + a^2*f^2*(-35*d^2*e^2 + 14*c*d*e*f - 3*c^2*f^2) + 2*a*b*
e*f*(5*d^2*e^2 + 22*c*d*e*f - 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^
(5/2)*f^(3/2)*(d*e - c*f)^4))/8
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.89, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {425, 419, 25, 397, 218, 401, 27, 298, 218, 425, 402, 25, 402, 27, 397, 218, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 419$$

$$\frac{b\left(-\frac{\int -\frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\int \frac{\left(bx^2+a\right)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{bx^2+a}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\left(bx^2+a\right)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{b\left(\frac{\int \frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\int \frac{1}{fx^2+e}dx}{de-cf}-\frac{(bc-ad)\int \frac{1}{dx^2+c}dx}{de-cf}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\left(\frac{\int \frac{(bx^2+a)\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(de-cf)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{401}$$

$$\frac{b\left(\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{f\left(b(af(5de-cf)-be(de+3cf))x^2+a(af(7de-3cf)-be(3de+cf))\right)}{(fx^2+e)^2}dx}{4ef}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\,\text{arc}}{\sqrt{c}\sqrt{d}(}\right.}{(de-cf)^2}}{d}$$

$$\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\frac{x(a+bx^2)(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{b(af(5de-cf)-be(de+3cf))x^2+a(af(7de-3cf)-be(3de+cf))}{(fx^2+e)^2}dx}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}\sqrt{f}(de-cf)}-\frac{(bc-ad)\arctan}{\sqrt{c}\sqrt{d}(de-}\right.}{(de-cf)^2}}{d}$$

$$\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{298}$$

$$b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)\int\frac{1}{fx^2+e}dx}{2ef}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be-\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\int\frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 218

$$b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\int\frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 425

$$b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(be\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{bx^2+a}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{bx^2+a}{\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(bc}{}\right.}{}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{\int -\frac{-3d(be-af)x^2+bce-4ade+3acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}+\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}-\frac{x(bc}{2c\left(c+dx^2\right)\left(e\right.}\right.}{d}\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(bc}{}\right.}{}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\int \frac{-3d(be-af)x^2+bce-4ade+3acf}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right.}\right.}{d}\right.}{d}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(b\cdots}{\cdots}\right.}{\cdots}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\int\frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}\right)}{d}-\cdots\right)}{d}$$

$\big\downarrow$ 27

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(b\cdots}{\cdots}\right.}{\cdots}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\int\frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{x(af(7de-3cf)-be(cf+3de))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}\right)}{d}-\cdots\right)}{d}$$

$\big\downarrow$ 397

$$\frac{b\left(\frac{\frac{x\left(a+bx^2\right)(be-af)(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(a^2f^2(7de-3cf)+2abef(de-cf)-b^2e^2(3cf+de)\right)}{2e^{3/2}f^{3/2}}-\frac{x(be-af)(af(7de-3cf)-be(3cf+de))}{2ef\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{(b\ldots}{}\right.}{}\right)}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{x(be-af)}{4e\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\frac{8d^2e^2(bc-ad)\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{\left(be\left(-c^2f^2+6cdef+3d^2e^2\right)-af\left(3c^2f^2-10cdef+15d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}+\frac{x(af(7de-3\ldots}{2e\left(e+f\ldots\right.}}{4e(de-cf)}\right.}{d}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{b\left(\frac{\frac{(be-af)(de-cf)x\left(bx^2+a\right)}{4e\left(fx^2+e\right)^2}-\frac{\frac{\left(-b^2(de+3cf)e^2+2abf(de-cf)e+a^2f^2(7de-3cf)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}}-\frac{(be-af)(af(7de-3cf)-be(de+3cf))x}{2ef\left(fx^2+e\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\ldots\right.}{}\right)}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3\ldots\right.\right.}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{\frac{(be-af)(de-cf)x\left(bx^2+a\right)}{4e\left(fx^2+e\right)^2}-\frac{\frac{\left(-b^2(de+3cf)e^2+2abf(de-cf)e+a^2f^2(7de-3cf)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}}-\frac{(be-af)(af(7de-3cf)-be(de+3cf))x}{2ef\left(fx^2+e\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\right.}{}\right.}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3\right.\right.}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

↓ 397

$$\frac{b\left(\frac{\frac{(be-af)(de-cf)x\left(bx^2+a\right)}{4e\left(fx^2+e\right)^2}-\frac{\frac{\left(-b^2(de+3cf)e^2+2abf(de-cf)e+a^2f^2(7de-3cf)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}}-\frac{(be-af)(af(7de-3cf)-be(de+3cf))x}{2ef\left(fx^2+e\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\cdots\right.}{\cdots}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3\cdots\right.\right.}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.}{}$$

$\downarrow$ 218

$$b\left(\frac{\frac{(be-af)(de-cf)x\left(bx^2+a\right)}{4e\left(fx^2+e\right)^2}-\frac{\frac{\left(-b^2(de+3cf)e^2+2abf(de-cf)e+a^2f^2(7de-3cf)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{3/2}}-\frac{(be-af)(af(7de-3cf)-be(de+3cf))x}{2ef\left(fx^2+e\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\cdots\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$$(bc-ad)\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3\cdots\right.\right.}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(b*(-((d*(b*c - a*d)*(-(((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*Sqrt[d]*(d*e - c*f))) + ((b*e - a*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e
]*Sqrt[f]*(d*e - c*f))))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*f)*x*(a +
 b*x^2))/(4*e*(e + f*x^2)^2) - (-1/2*((b*e - a*f)*(a*f*(7*d*e - 3*c*f) - b
*e*(d*e + 3*c*f))*x)/(e*f*(e + f*x^2)) + ((a^2*f^2*(7*d*e - 3*c*f) + 2*a*b
*e*f*(d*e - c*f) - b^2*e^2*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*
e^(3/2)*f^(3/2)))/(4*e))/(d*e - c*f)^2))/d - ((b*c - a*d)*((b*(((b*e - a*f
)*x)/(4*e*(d*e - c*f)*(e + f*x^2)^2) - (((a*f*(7*d*e - 3*c*f) - b*e*(3*d*e
 + c*f))*x)/(2*e*(d*e - c*f)*(e + f*x^2)) + ((8*d^(3/2)*(b*c - a*d)*e^2*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - ((b*e*(3*d^2*e^2 + 6*c*
d*e*f - c^2*f^2) - a*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt
[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)))/(2*e*(d*e - c*f))))/(4*e*(d
*e - c*f))))/d - ((b*c - a*d)*(-1/2*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*
x^2)*(e + f*x^2)^2) + (-1/2*(f*(3*b*c*e - 2*a*d*e - a*c*f)*x)/(e*(d*e - c*
f)*(e + f*x^2)^2) + (-1/2*(f*(b*c*e*(11*d*e + c*f) - a*(4*d^2*e^2 + 11*c*d
*e*f - 3*c^2*f^2))*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((4*d^(3/2)*e^2*(a*d*(
d*e - 7*c*f) + b*c*(d*e + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d
*e - c*f)) - (c*Sqrt[f]*(b*e*(15*d^2*e^2 + 10*c*d*e*f - c^2*f^2) - a*f*(35
*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*
(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*c*(d*e - c*f))))...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.49

method	result
default	$\frac{\frac{\left(3a^3c^2f^5-14a^3cde\,f^4+11a^3d^2e^2f^3+3a^2b\,c^2e\,f^4+18a^2bcd\,e^2f^3-21a^2b\,d^2e^3f^2-15a\,b^2c^2e^2f^3+6a\,b^2cd\,e^3f^2+9a\,b^2d^2e^4f+9b^3c^2e^3f^2-10b^3cd}{8e^2}}{}$
risch	Expression too large to display

input
```
int((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^4*((1/8*(3*a^3*c^2*f^5-14*a^3*c*d*e*f^4+11*a^3*d^2*e^2*f^3+3*a
^2*b*c^2*e*f^4+18*a^2*b*c*d*e^2*f^3-21*a^2*b*d^2*e^3*f^2-15*a*b^2*c^2*e^2*
f^3+6*a*b^2*c*d*e^3*f^2+9*a*b^2*d^2*e^4*f+9*b^3*c^2*e^3*f^2-10*b^3*c*d*e^4
*f+b^3*d^2*e^5)/e^2*x^3+1/8*(5*a^3*c^2*f^5-18*a^3*c*d*e*f^4+13*a^3*d^2*e^2
*f^3-3*a^2*b*c^2*e*f^4+30*a^2*b*c*d*e^2*f^3-27*a^2*b*d^2*e^3*f^2-9*a*b^2*c
^2*e^2*f^3-6*a*b^2*c*d*e^3*f^2+15*a*b^2*d^2*e^4*f+7*b^3*c^2*e^3*f^2-6*b^3*
c*d*e^4*f-b^3*d^2*e^5)/e/f*x)/(f*x^2+e)^2+1/8*(3*a^3*c^2*f^5-14*a^3*c*d*e*
f^4+35*a^3*d^2*e^2*f^3+3*a^2*b*c^2*e*f^4-30*a^2*b*c*d*e^2*f^3-45*a^2*b*d^2
*e^3*f^2+9*a*b^2*c^2*e^2*f^3+54*a*b^2*c*d*e^3*f^2+9*a*b^2*d^2*e^4*f-15*b^3
*c^2*e^3*f^2-10*b^3*c*d*e^4*f+b^3*d^2*e^5)/e^2/f/(e*f)^(1/2)*arctan(f*x/(e
*f)^(1/2)))-1/(c*f-d*e)^4*(1/2*(a^3*c*d^3*f-a^3*d^4*e-3*a^2*b*c^2*d^2*f+3*
a^2*b*c*d^3*e+3*a*b^2*c^3*d*f-3*a*b^2*c^2*d^2*e-b^3*c^4*f+b^3*c^3*d*e)/c*x
/(d*x^2+c)+1/2*(7*a^3*c*d^3*f-a^3*d^4*e-15*a^2*b*c^2*d^2*f-3*a^2*b*c*d^3*e
+9*a*b^2*c^3*d*f+9*a*b^2*c^2*d^2*e-b^3*c^4*f-5*b^3*c^3*d*e)/c/(c*d)^(1/2)*
arctan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2 (e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2 (e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2(e+fx^2)^3}\,dx$$
$$= \frac{(5b^3c^3de-9ab^2c^2d^2e+3a^2bcd^3e+a^3d^4e+b^3c^4f-9ab^2c^3df+15a^2bc^2d^2f-7a^3cd^3f)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2(cd^4e^4-4c^2d^3e^3f+6c^3d^2e^2f^2-4c^4def^3+c^5f^4)\sqrt{cd}}$$
$$+\frac{(b^3d^2e^5-10b^3cde^4f+9ab^2d^2e^4f-15b^3c^2e^3f^2+54ab^2cde^3f^2-45a^2bd^2e^3f^2+9ab^2c^2e^2f^3-30a^2b}{8(d^4e^6f-4cd^3e^5f^2+6c^2d^2e^4f^3-4c^3de^3}$$
$$-\frac{b^3c^3x-3ab^2c^2dx+3a^2bcd^2x-a^3d^3x}{2(cd^3e^3-3c^2d^2e^2f+3c^3def^2-c^4f^3)(dx^2+c)}$$
$$+\frac{b^3de^4fx^3-9b^3ce^3f^2x^3+9ab^2de^3f^2x^3+15ab^2ce^2f^3x^3-21a^2bde^2f^3x^3-3a^2bcef^4x^3+11a^3def^4x^3}{8(d^3e^5f-3cd^2e}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/2*(5*b^3*c^3*d*e - 9*a*b^2*c^2*d^2*e + 3*a^2*b*c*d^3*e + a^3*d^4*e + b^3
*c^4*f - 9*a*b^2*c^3*d*f + 15*a^2*b*c^2*d^2*f - 7*a^3*c*d^3*f)*arctan(d*x/
sqrt(c*d))/((c*d^4*e^4 - 4*c^2*d^3*e^3*f + 6*c^3*d^2*e^2*f^2 - 4*c^4*d*e*f
^3 + c^5*f^4)*sqrt(c*d)) + 1/8*(b^3*d^2*e^5 - 10*b^3*c*d*e^4*f + 9*a*b^2*d
^2*e^4*f - 15*b^3*c^2*e^3*f^2 + 54*a*b^2*c*d*e^3*f^2 - 45*a^2*b*d^2*e^3*f^
2 + 9*a*b^2*c^2*e^2*f^3 - 30*a^2*b*c*d*e^2*f^3 + 35*a^3*d^2*e^2*f^3 + 3*a^
2*b*c^2*e*f^4 - 14*a^3*c*d*e*f^4 + 3*a^3*c^2*f^5)*arctan(f*x/sqrt(e*f))/((
d^4*e^6*f - 4*c*d^3*e^5*f^2 + 6*c^2*d^2*e^4*f^3 - 4*c^3*d*e^3*f^4 + c^4*e^
2*f^5)*sqrt(e*f)) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a
^3*d^3*x)/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*(d*x^2
+ c)) + 1/8*(b^3*d*e^4*f*x^3 - 9*b^3*c*e^3*f^2*x^3 + 9*a*b^2*d*e^3*f^2*x^3
 + 15*a*b^2*c*e^2*f^3*x^3 - 21*a^2*b*d*e^2*f^3*x^3 - 3*a^2*b*c*e*f^4*x^3 +
 11*a^3*d*e*f^4*x^3 - 3*a^3*c*f^5*x^3 - b^3*d*e^5*x - 7*b^3*c*e^4*f*x + 15
*a*b^2*d*e^4*f*x + 9*a*b^2*c*e^3*f^2*x - 27*a^2*b*d*e^3*f^2*x + 3*a^2*b*c*
e^2*f^3*x + 13*a^3*d*e^2*f^3*x - 5*a^3*c*e*f^4*x)/((d^3*e^5*f - 3*c*d^2*e^
4*f^2 + 3*c^2*d*e^3*f^3 - c^3*e^2*f^4)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 184741, normalized size of antiderivative = 365.82

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
atan(((((20160*a^3*c^3*d^11*e^11*f^5 - 3584*a^3*c^2*d^12*e^12*f^4 - 63168*
a^3*c^4*d^10*e^10*f^6 + 125184*a^3*c^5*d^9*e^9*f^7 - 166656*a^3*c^6*d^8*e^
8*f^8 + 153216*a^3*c^7*d^7*e^7*f^9 - 97920*a^3*c^8*d^6*e^6*f^10 + 43008*a^
3*c^9*d^5*e^5*f^11 - 12544*a^3*c^10*d^4*e^4*f^12 + 2240*a^3*c^11*d^3*e^3*f
^13 - 192*a^3*c^12*d^2*e^2*f^14 + 64*b^3*c^3*d^11*e^14*f^2 + 192*b^3*c^4*d
^10*e^13*f^3 - 3840*b^3*c^5*d^9*e^12*f^4 + 16128*b^3*c^6*d^8*e^11*f^5 - 34
944*b^3*c^7*d^7*e^10*f^6 + 45696*b^3*c^8*d^6*e^9*f^7 - 37632*b^3*c^9*d^5*e
^8*f^8 + 19200*b^3*c^10*d^4*e^7*f^9 - 5568*b^3*c^11*d^3*e^6*f^10 + 704*b^3
*c^12*d^2*e^5*f^11 + 256*a^3*c*d^13*e^13*f^3 - 1728*a*b^2*c^3*d^11*e^13*f^
3 + 13248*a*b^2*c^4*d^10*e^12*f^4 - 43776*a*b^2*c^5*d^9*e^11*f^5 + 80640*a
*b^2*c^6*d^8*e^10*f^6 - 88704*a*b^2*c^7*d^7*e^9*f^7 + 56448*a*b^2*c^8*d^6*
e^8*f^8 - 16128*a*b^2*c^9*d^5*e^7*f^9 - 2304*a*b^2*c^10*d^4*e^6*f^10 + 288
0*a*b^2*c^11*d^3*e^5*f^11 - 576*a*b^2*c^12*d^2*e^4*f^12 + 768*a^2*b*c^2*d^
12*e^13*f^3 - 4416*a^2*b*c^3*d^11*e^12*f^4 + 7488*a^2*b*c^4*d^10*e^11*f^5
+ 6912*a^2*b*c^5*d^9*e^10*f^6 - 48384*a^2*b*c^6*d^8*e^9*f^7 + 88704*a^2*b*
c^7*d^7*e^8*f^8 - 88704*a^2*b*c^8*d^6*e^7*f^9 + 52992*a^2*b*c^9*d^5*e^6*f^
10 - 18432*a^2*b*c^10*d^4*e^5*f^11 + 3264*a^2*b*c^11*d^3*e^4*f^12 - 192*a^
2*b*c^12*d^2*e^3*f^13)/(128*(c^11*e^4*f^10 - c^2*d^9*e^13*f - 9*c^10*d*e^5
*f^9 + 9*c^3*d^8*e^12*f^2 - 36*c^4*d^7*e^11*f^3 + 84*c^5*d^6*e^10*f^4 - 12
6*c^6*d^5*e^9*f^5 + 126*c^7*d^4*e^8*f^6 - 84*c^8*d^3*e^7*f^7 + 36*c^9*d...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 4530, normalized size of antiderivative = 8.97

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
( - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**3*e**5*f
**3 - 56*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**3*e**4
*f**4*x**2 - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d*
*3*e**3*f**5*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c
*d**4*e**6*f**2 - 20*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*
d**4*e**5*f**3*x**2 - 52*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**
3*c*d**4*e**4*f**4*x**4 - 28*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))
*a**3*c*d**4*e**3*f**5*x**6 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c
)))*a**3*d**5*e**6*f**2*x**2 + 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(
c)))*a**3*d**5*e**5*f**3*x**4 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt
(c)))*a**3*d**5*e**4*f**4*x**6 + 60*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sq
rt(c)))*a**2*b*c**3*d**2*e**5*f**3 + 120*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(
d)*sqrt(c)))*a**2*b*c**3*d**2*e**4*f**4*x**2 + 60*sqrt(d)*sqrt(c)*atan((d*
x)/(sqrt(d)*sqrt(c)))*a**2*b*c**3*d**2*e**3*f**5*x**4 + 12*sqrt(d)*sqrt(c)
*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**2*d**3*e**6*f**2 + 84*sqrt(d)*sqr
t(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**2*d**3*e**5*f**3*x**2 + 132*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**2*d**3*e**4*f**4*x*
*4 + 60*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c**2*d**3*e**
3*f**5*x**6 + 12*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*c*d*
*4*e**6*f**2*x**2 + 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*...
```

3.252 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 621

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}\,dx = \frac{(bc-ad)^2(ad(3de-11cf)+bc(9de-cf))x}{8c^2(de-cf)^4(c+dx^2)}$$
$$+\frac{(6a^2bcd^2ef^2-3ab^2cdef(de+cf)-a^3d^2f^2(de+cf)+b^3(cd^2e^3+c^3ef^2))\,x}{4cd^2e(de-cf)^3(e+fx^2)^2}$$
$$-\frac{(bc-ad)^3x}{4cd^2(de-cf)(c+dx^2)^2(e+fx^2)^2}$$
$$+\frac{(3a^2bcdef^2(15de+cf)-3ab^2cde^2f(7de+9cf)-a^3df^2(4d^2e^2+15cdef-3c^2f^2)+b^3ce^2(3d^2e^2+9c}{8cde^2(de-cf)^4(e+fx^2)}$$
$$-\frac{3(bc-ad)(2abcd(d^2e^2-10cdef-7c^2f^2)+b^2c^2(5d^2e^2+10cdef+c^2f^2)+a^2d^2(d^2e^2-6cdef+21c}{8c^{5/2}\sqrt{d}(de-cf)^5}$$
$$-\frac{3(be-af)(2abef(7d^2e^2+10cdef-c^2f^2)-a^2f^2(21d^2e^2-6cdef+c^2f^2)-b^2e^2(d^2e^2+10cdef+5c}{8e^{5/2}\sqrt{f}(de-cf)^5}$$

output
```
1/8*(-a*d+b*c)^2*(a*d*(-11*c*f+3*d*e)+b*c*(-c*f+9*d*e))*x/c^2/(-c*f+d*e)^4
/(d*x^2+c)+1/4*(6*a^2*b*c*d^2*e*f^2-3*a*b^2*c*d*e*f*(c*f+d*e)-a^3*d^2*f^2*
(c*f+d*e)+b^3*(c^3*e*f^2+c*d^2*e^3))*x/c/d^2/e/(-c*f+d*e)^3/(f*x^2+e)^2-1/
4*(-a*d+b*c)^3*x/c/d^2/(-c*f+d*e)/(d*x^2+c)^2/(f*x^2+e)^2+1/8*(3*a^2*b*c*d
*e*f^2*(c*f+15*d*e)-3*a*b^2*c*d*e^2*f*(9*c*f+7*d*e)-a^3*d*f^2*(-3*c^2*f^2+
15*c*d*e*f+4*d^2*e^2)+b^3*c*e^2*(4*c^2*f^2+9*c*d*e*f+3*d^2*e^2))*x/c/d/e^2
/(-c*f+d*e)^4/(f*x^2+e)-3/8*(-a*d+b*c)*(2*a*b*c*d*(-7*c^2*f^2-10*c*d*e*f+d
^2*e^2)+b^2*c^2*(c^2*f^2+10*c*d*e*f+5*d^2*e^2)+a^2*d^2*(21*c^2*f^2-6*c*d*e
*f+d^2*e^2))*arctan(d^(1/2)*x/c^(1/2))/c^(5/2)/d^(1/2)/(-c*f+d*e)^5-3/8*(-
a*f+b*e)*(2*a*b*e*f*(-c^2*f^2+10*c*d*e*f+7*d^2*e^2)-a^2*f^2*(c^2*f^2-6*c*d
*e*f+21*d^2*e^2)-b^2*e^2*(5*c^2*f^2+10*c*d*e*f+d^2*e^2))*arctan(f^(1/2)*x/
e^(1/2))/e^(5/2)/f^(1/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}\,dx$$
$$= \frac{1}{8}\left(\frac{2(bc-ad)^3x}{c(-de+cf)^3(c+dx^2)^2} + \frac{3(bc-ad)^2(ad(de-5cf)+bc(3de+cf))x}{c^2(de-cf)^4(c+dx^2)} + \frac{2(be-af)^3x}{e(de-cf)^3(e+fx^2)^2} + \frac{3(be-af)^2(af(-5de+cf)+be(de+3cf))x}{e^2(de-cf)^4(e+fx^2)}\right.$$
$$+ \frac{3(bc-ad)\left(b^2c^2(5d^2e^2+10cdef+c^2f^2)-2abcd(-d^2e^2+10cdef+7c^2f^2)+a^2d^2(d^2e^2-6cdef+2\ldots\right.}{c^{5/2}\sqrt{d}(-de+cf)^5}$$
$$+ \frac{3(be-af)\left(-2abef(7d^2e^2+10cdef-c^2f^2)+a^2f^2(21d^2e^2-6cdef+c^2f^2)+b^2e^2(d^2e^2+10cdef+\ldots\right.}{e^{5/2}\sqrt{f}(de-cf)^5}$$

input
```
Integrate[(a + b*x^2)^3/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
((2*(b*c - a*d)^3*x)/(c*(-(d*e) + c*f)^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2
*(a*d*(d*e - 5*c*f) + b*c*(3*d*e + c*f))*x)/(c^2*(d*e - c*f)^4*(c + d*x^2)
) + (2*(b*e - a*f)^3*x)/(e*(d*e - c*f)^3*(e + f*x^2)^2) + (3*(b*e - a*f)^2
*(a*f*(-5*d*e + c*f) + b*e*(d*e + 3*c*f))*x)/(e^2*(d*e - c*f)^4*(e + f*x^2
)) + (3*(b*c - a*d)*(b^2*c^2*(5*d^2*e^2 + 10*c*d*e*f + c^2*f^2) - 2*a*b*c*
d*(-(d^2*e^2) + 10*c*d*e*f + 7*c^2*f^2) + a^2*d^2*(d^2*e^2 - 6*c*d*e*f + 2
1*c^2*f^2))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]*(-(d*e) + c*f)^5
) + (3*(b*e - a*f)*(-2*a*b*e*f*(7*d^2*e^2 + 10*c*d*e*f - c^2*f^2) + a^2*f^
2*(21*d^2*e^2 - 6*c*d*e*f + c^2*f^2) + b^2*e^2*(d^2*e^2 + 10*c*d*e*f + 5*c
^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*Sqrt[f]*(d*e - c*f)^5))/8
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1540 vs. $2(621) = 1242$.

Time = 2.20 (sec) , antiderivative size = 1540, normalized size of antiderivative = 2.48, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {425, 425, 402, 25, 402, 25, 27, 397, 218, 402, 27, 397, 218, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{(bx^2+a)^2}{(dx^2+c)^2(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^2}{(dx^2+c)^3(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{bx^2+a}{(dx^2+c)(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)^2(fx^2+e)^3}dx}{d}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{bx^2+a}{(dx^2+c)^2(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{bx^2+a}{(dx^2+c)^3(fx^2+e)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{\int -\frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}+\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-}\right.}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(-\frac{\int -\frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(-\frac{\int -\frac{-7(bc-ad)fx^2+bce+3ade-4acf}{(dx^2+c)^2(fx^2+e)^3}dx}{4c(de-cf)}-\frac{}{4c(}\right.}{d}\right.}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{x(be-af)}{4e(e+fx^2)^2(de-cf)}-\frac{\int \frac{-3d(be-af)x^2+bce-4ade+3acf}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{\int \frac{-5(bc-ad)fx^2+bce+ade-2acf}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}-\frac{x(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{-7(bc-ad)fx^2+bce+3ade-4acf}{(dx^2+c)^2(fx^2+e)^3}dx}{4c(de-cf)}-\frac{}{4c(c+dx^2)}\right.}{d}\right.}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\int \frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\cdots\right.}{\cdots}\right.}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{f(3bce-2ade-acf)x}{2e(de-cf)\left(fx^2+e\right)^2}}{2c(de-cf)}-\frac{(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}\right)}{d}-\cdots\right.$$

$$\downarrow 25$$

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\int \frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\right.}{}\right.$$

$$\frac{\qquad}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\int \frac{2\left(-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{f(3bce-2ade-acf)x}{2e(de-cf)\left(fx^2+e\right)^2}}{2c(de-cf)}-\frac{(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}\right)}{d}-\right.$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\int\frac{d(af(7de-3cf)-be(3de+cf))x^2+bce(5de-cf)-a\left(8d^2e^2-7cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}}{4e(de-cf)}\right)}{d}-\frac{(bc-ad)\left(\cdots\right.}{\cdots}\right.$$

$$\frac{}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\int\frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{f(3bce-2ade-acf)x}{2e(de-cf)\left(fx^2+e\right)^2}}{2c(de-cf)}-\frac{(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}\right)}{d}-\frac{(bc\cdots}{\cdots}\right.$$

$$\frac{}{d}$$

$\downarrow$ 397

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^2(bc-ad)e^2\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\int\frac{1}{fx^2+e}}{de-cf}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

$$\overline{\qquad d}$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\int\frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{f(3bce-2ade-acf)x}{2e(de-cf)\left(fx^2+e\right)^2}}{2c(de-cf)}-\frac{(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}\right)}{d}-\frac{(bc\cdots}{\cdots}\right.$$

↓ 218

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\int\frac{-3df(3bce-2ade-acf)x^2+bce(2de+cf)+a\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{f(3bce-2ade-acf)x}{2e(de-cf)\left(fx^2+e\right)^2}}{2c(de-cf)}-\frac{(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}\right)}{d}-\frac{(bc\cdots}{\cdots}\right.$$

$\downarrow$ 402

$$
\frac{b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\arctan}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}}{d}\right.}{(bc-ad)\left(\frac{b\left(\frac{\frac{\int\frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{f\left(bce(11de+\right.}{}}{2e(de-cf)}}{2c(de-cf)}\right.}{d}\right.}
$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}\cdots}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\cdots\right)}{d}\cdots\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{\int\frac{-df\left(bce(11de+cf)-a\left(4d^2e^2+11cdfe-3c^2f^2\right)\right)x^2+bce\left(4d^2e^2+9cdfe-c^2f^2\right)+a\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{f\left(bce(11de+\cdots\right.}{\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right)}{d}\cdots\right.$$

↓ 397

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\right.}{d}\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^2e^2(ad(de-7cf)+bc(de+5cf))\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{cf\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{f\left(bce(11de+c\right.}{2}}{2e(de-cf)}}{2c(de-cf)}\right.}{d}\right.$$

$\downarrow$ 218

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}}{d}\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bc\right.}{}}{2e(de-cf)}}{2c(de-cf)}}{d}\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}}{d}\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bc\right.}{}}{2e(de-cf)}}{2c(de-cf)}}{d}\right.$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\operatorname{arcta}\cdots}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\cdots\right.}{d}\cdots\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bc\cdots\right.}{\cdots}}{2e(de-cf)}}{2c(de-cf)}\cdots\right.}{d}\cdots\right.$$

↓ 397

$$
b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}\cdots}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}\cdots\right)}{d}\cdots\right.
$$

$$
(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bc\cdots\right.}{\cdots}}{2e(de-cf)}\cdots}{2c(de-cf)}\cdots\right)}{d}\cdots\right.
$$

$\downarrow$ 218

$$b\left(\frac{b\left(\frac{(be-af)x}{4e(de-cf)\left(fx^2+e\right)^2}-\frac{\frac{(af(7de-3cf)-be(3de+cf))x}{2e(de-cf)\left(fx^2+e\right)}+\frac{\frac{8d^{3/2}(bc-ad)e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\left(be\left(3d^2e^2+6cdfe-c^2f^2\right)-af\left(15d^2e^2-10cdfe+3c^2f^2\right)\right)\text{arcta}}{\sqrt{e}\sqrt{f}(de-cf)}}{2e(de-cf)}}{4e(de-cf)}}{d}\right.$$

$$(bc-ad)\left(\frac{b\left(\frac{\frac{\frac{4d^{3/2}e^2(ad(de-7cf)+bc(de+5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{c\sqrt{f}\left(be\left(15d^2e^2+10cdfe-c^2f^2\right)-af\left(35d^2e^2-14cdfe+3c^2f^2\right)\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{f\left(bc\right.}{}}{2e(de-cf)}}{2c(de-cf)}}{d}\right.$$

input
```
Int[(a + b*x^2)^3/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(b*((b*(((b*e - a*f)*x)/(4*e*(d*e - c*f)*(e + f*x^2)^2) - (((a*f*(7*d*e -
3*c*f) - b*e*(3*d*e + c*f))*x)/(2*e*(d*e - c*f)*(e + f*x^2)) + ((8*d^(3/2)
*(b*c - a*d)*e^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - ((b*
e*(3*d^2*e^2 + 6*c*d*e*f - c^2*f^2) - a*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2
*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)))/(2*e*(d
*e - c*f)))/(4*e*(d*e - c*f))))/d - ((b*c - a*d)*(-1/2*((b*c - a*d)*x)/(c*
(d*e - c*f)*(c + d*x^2)*(e + f*x^2)^2) + (-1/2*(f*(3*b*c*e - 2*a*d*e - a*c
*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(f*(b*c*e*(11*d*e + c*f) - a*
(4*d^2*e^2 + 11*c*d*e*f - 3*c^2*f^2))*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((4
*d^(3/2)*e^2*(a*d*(d*e - 7*c*f) + b*c*(d*e + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(Sqrt[c]*(d*e - c*f)) - (c*Sqrt[f]*(b*e*(15*d^2*e^2 + 10*c*d*e*f -
 c^2*f^2) - a*f*(35*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*
c*(d*e - c*f))))/d))/d - ((b*c - a*d)*((b*(-1/2*((b*c - a*d)*x)/(c*(d*e -
c*f)*(c + d*x^2)*(e + f*x^2)^2) + (-1/2*(f*(3*b*c*e - 2*a*d*e - a*c*f)*x)/
(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(f*(b*c*e*(11*d*e + c*f) - a*(4*d^2*
e^2 + 11*c*d*e*f - 3*c^2*f^2))*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((4*d^(3/2
)*e^2*(a*d*(d*e - 7*c*f) + b*c*(d*e + 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(Sqrt[c]*(d*e - c*f)) - (c*Sqrt[f]*(b*e*(15*d^2*e^2 + 10*c*d*e*f - c^2*f^
2) - a*f*(35*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 1051, normalized size of antiderivative = 1.69

method	result	size
default	Expression too large to display	1051
risch	Expression too large to display	3271

input
```
int((b*x^2+a)^3/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)^5*((3/8*f*(a^3*c^2*f^5-6*a^3*c*d*e*f^4+5*a^3*d^2*e^2*f^3+a^2*b
*c^2*e*f^4+10*a^2*b*c*d*e^2*f^3-11*a^2*b*d^2*e^3*f^2-5*a*b^2*c^2*e^2*f^3-2
*a*b^2*c*d*e^3*f^2+7*a*b^2*d^2*e^4*f+3*b^3*c^2*e^3*f^2-2*b^3*c*d*e^4*f-b^3
*d^2*e^5)/e^2*x^3+1/8*(5*a^3*c^2*f^5-22*a^3*c*d*e*f^4+17*a^3*d^2*e^2*f^3-3
*a^2*b*c^2*e*f^4+42*a^2*b*c*d*e^2*f^3-39*a^2*b*d^2*e^3*f^2-9*a*b^2*c^2*e^2
*f^3-18*a*b^2*c*d*e^3*f^2+27*a*b^2*d^2*e^4*f+7*b^3*c^2*e^3*f^2-2*b^3*c*d*e
^4*f-5*b^3*d^2*e^5)/e*x)/(f*x^2+e)^2+3/8*(a^3*c^2*f^5-6*a^3*c*d*e*f^4+21*a
^3*d^2*e^2*f^3+a^2*b*c^2*e*f^4-14*a^2*b*c*d*e^2*f^3-35*a^2*b*d^2*e^3*f^2+3
*a*b^2*c^2*e^2*f^3+30*a*b^2*c*d*e^3*f^2+15*a*b^2*d^2*e^4*f-5*b^3*c^2*e^3*f
^2-10*b^3*c*d*e^4*f-b^3*d^2*e^5)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-
1/(c*f-d*e)^5*((3/8*d*(5*a^3*c^2*d^3*f^2-6*a^3*c*d^4*e*f+a^3*d^5*e^2-11*a^
2*b*c^3*d^2*f^2+10*a^2*b*c^2*d^3*e*f+a^2*b*c*d^4*e^2+7*a*b^2*c^4*d*f^2-2*a
*b^2*c^3*d^2*e*f-5*a*b^2*c^2*d^3*e^2-b^3*c^5*f^2-2*b^3*c^4*d*e*f+3*b^3*c^3
*d^2*e^2)/c^2*x^3+1/8*(17*a^3*c^2*d^3*f^2-22*a^3*c*d^4*e*f+5*a^3*d^5*e^2-3
9*a^2*b*c^3*d^2*f^2+42*a^2*b*c^2*d^3*e*f-3*a^2*b*c*d^4*e^2+27*a*b^2*c^4*d*
f^2-18*a*b^2*c^3*d^2*e*f-9*a*b^2*c^2*d^3*e^2-5*b^3*c^5*f^2-2*b^3*c^4*d*e*f
+7*b^3*c^3*d^2*e^2)/c*x)/(d*x^2+c)^2+3/8*(21*a^3*c^2*d^3*f^2-6*a^3*c*d^4*e
*f+a^3*d^5*e^2-35*a^2*b*c^3*d^2*f^2-14*a^2*b*c^2*d^3*e*f+a^2*b*c*d^4*e^2+1
5*a*b^2*c^4*d*f^2+30*a*b^2*c^3*d^2*e*f+3*a*b^2*c^2*d^3*e^2-b^3*c^5*f^2-10*
b^3*c^4*d*e*f-5*b^3*c^3*d^2*e^2)/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**3/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1605 vs. $2(591) = 1182$.

Time = 0.13 (sec) , antiderivative size = 1605, normalized size of antiderivative = 2.58

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^3/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-3/8*(5*b^3*c^3*d^2*e^2 - 3*a*b^2*c^2*d^3*e^2 - a^2*b*c*d^4*e^2 - a^3*d^5*
e^2 + 10*b^3*c^4*d*e*f - 30*a*b^2*c^3*d^2*e*f + 14*a^2*b*c^2*d^3*e*f + 6*a
^3*c*d^4*e*f + b^3*c^5*f^2 - 15*a*b^2*c^4*d*f^2 + 35*a^2*b*c^3*d^2*f^2 - 2
1*a^3*c^2*d^3*f^2)*arctan(d*x/sqrt(c*d))/((c^2*d^5*e^5 - 5*c^3*d^4*e^4*f +
 10*c^4*d^3*e^3*f^2 - 10*c^5*d^2*e^2*f^3 + 5*c^6*d*e*f^4 - c^7*f^5)*sqrt(c
*d)) + 3/8*(b^3*d^2*e^5 + 10*b^3*c*d*e^4*f - 15*a*b^2*d^2*e^4*f + 5*b^3*c^
2*e^3*f^2 - 30*a*b^2*c*d*e^3*f^2 + 35*a^2*b*d^2*e^3*f^2 - 3*a*b^2*c^2*e^2*
f^3 + 14*a^2*b*c*d*e^2*f^3 - 21*a^3*d^2*e^2*f^3 - a^2*b*c^2*e*f^4 + 6*a^3*
c*d*e*f^4 - a^3*c^2*f^5)*arctan(f*x/sqrt(e*f))/((d^5*e^7 - 5*c*d^4*e^6*f +
 10*c^2*d^3*e^5*f^2 - 10*c^3*d^2*e^4*f^3 + 5*c^4*d*e^3*f^4 - c^5*e^2*f^5)*
sqrt(e*f)) + 1/8*(3*b^3*c^2*d^3*e^4*f*x^7 + 18*b^3*c^3*d^2*e^3*f^2*x^7 - 3
6*a*b^2*c^2*d^3*e^3*f^2*x^7 + 3*a^2*b*c*d^4*e^3*f^2*x^7 + 3*a^3*d^5*e^3*f^
2*x^7 + 3*b^3*c^4*d*e^2*f^3*x^7 - 36*a*b^2*c^3*d^2*e^2*f^3*x^7 + 66*a^2*b*
c^2*d^3*e^2*f^3*x^7 - 15*a^3*c*d^4*e^2*f^3*x^7 + 3*a^2*b*c^3*d^2*e*f^4*x^7
 - 15*a^3*c^2*d^3*e*f^4*x^7 + 3*a^3*c^3*d^2*f^5*x^7 + 5*b^3*c^2*d^3*e^5*x^
5 + 31*b^3*c^3*d^2*e^4*f*x^5 - 57*a*b^2*c^2*d^3*e^4*f*x^5 + 6*a^2*b*c*d^4*
e^4*f*x^5 + 6*a^3*d^5*e^4*f*x^5 + 31*b^3*c^4*d*e^3*f^2*x^5 - 102*a*b^2*c^3
*d^2*e^3*f^2*x^5 + 102*a^2*b*c^2*d^3*e^3*f^2*x^5 - 25*a^3*c*d^4*e^3*f^2*x^
5 + 5*b^3*c^5*e^2*f^3*x^5 - 57*a*b^2*c^4*d*e^2*f^3*x^5 + 102*a^2*b*c^3*d^2
*e^2*f^3*x^5 - 34*a^3*c^2*d^3*e^2*f^3*x^5 + 6*a^2*b*c^4*d*e*f^4*x^5 - 2...
```

Mupad [B] (verification not implemented)

Time = 24.53 (sec) , antiderivative size = 227222, normalized size of antiderivative = 365.90

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3/((c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
((x^3*(3*a^3*c^5*f^5 + 3*a^3*d^5*e^5 + 19*b^3*c^3*d^2*e^5 + 19*b^3*c^5*e^3
*f^2 - 34*a^3*c^2*d^3*e^3*f^2 - 34*a^3*c^3*d^2*e^2*f^3 + 3*a^2*b*c*d^4*e^5
 + 3*a^2*b*c^5*e*f^4 - 5*a^3*c*d^4*e^4*f - 5*a^3*c^4*d*e*f^4 + 34*b^3*c^4*
d*e^4*f - 15*a*b^2*c^2*d^3*e^5 - 15*a*b^2*c^5*e^2*f^3 - 93*a*b^2*c^3*d^2*e
^4*f - 93*a*b^2*c^4*d*e^3*f^2 + 27*a^2*b*c^2*d^3*e^4*f + 27*a^2*b*c^4*d*e^
2*f^3 + 156*a^2*b*c^3*d^2*e^3*f^2))/(8*c^2*e^2*(c^4*f^4 + d^4*e^4 + 6*c^2*
d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)) - (x*(3*a^2*b*c*d^3*e^4 - 5*
a^3*d^4*e^4 - 12*b^3*c^3*d*e^4 - 12*b^3*c^4*e^3*f - 5*a^3*c^4*f^4 + 3*a^2*
b*c^4*e*f^3 + 17*a^3*c*d^3*e^3*f + 17*a^3*c^3*d*e*f^3 + 9*a*b^2*c^2*d^2*e^
4 + 9*a*b^2*c^4*e^2*f^2 - 39*a^2*b*c^2*d^2*e^3*f - 39*a^2*b*c^3*d*e^2*f^2
+ 54*a*b^2*c^3*d*e^3*f))/(8*c*e*(c^4*f^4 + d^4*e^4 + 6*c^2*d^2*e^2*f^2 - 4
*c*d^3*e^3*f - 4*c^3*d*e*f^3)) + (x^5*(6*a^3*c^4*d*f^5 + 6*a^3*d^5*e^4*f +
 5*b^3*c^2*d^3*e^5 + 5*b^3*c^5*e^2*f^3 - 34*a^3*c^2*d^3*e^2*f^3 - 25*a^3*c
*d^4*e^3*f^2 - 25*a^3*c^3*d^2*e*f^4 + 31*b^3*c^3*d^2*e^4*f + 31*b^3*c^4*d*
e^3*f^2 - 57*a*b^2*c^2*d^3*e^4*f - 57*a*b^2*c^4*d*e^2*f^3 - 102*a*b^2*c^3*
d^2*e^3*f^2 + 102*a^2*b*c^2*d^3*e^3*f^2 + 102*a^2*b*c^3*d^2*e^2*f^3 + 6*a^
2*b*c*d^4*e^4*f + 6*a^2*b*c^4*d*e*f^4))/(8*c^2*e^2*(c^4*f^4 + d^4*e^4 + 6*
c^2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3)) + (3*d*f*x^7*(a^3*c^3*d*
f^4 + a^3*d^4*e^3*f + b^3*c^2*d^2*e^4 + b^3*c^4*e^2*f^2 + 6*b^3*c^3*d*e^3*
f - 5*a^3*c*d^3*e^2*f^2 - 5*a^3*c^2*d^2*e*f^3 - 12*a*b^2*c^2*d^2*e^3*f ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7120, normalized size of antiderivative = 11.47

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^3/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
( - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**4*d**3*e**5*f
**3 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**4*d**3*e**
4*f**4*x**2 - 63*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**4*d
**3*e**3*f**5*x**4 + 18*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3
*c**3*d**4*e**6*f**2 - 90*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*
*3*c**3*d**4*e**5*f**3*x**2 - 234*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt
(c)))*a**3*c**3*d**4*e**4*f**4*x**4 - 126*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt
(d)*sqrt(c)))*a**3*c**3*d**4*e**3*f**5*x**6 - 3*sqrt(d)*sqrt(c)*atan((d*x)
/(sqrt(d)*sqrt(c)))*a**3*c**2*d**5*e**7*f + 30*sqrt(d)*sqrt(c)*atan((d*x)/
(sqrt(d)*sqrt(c)))*a**3*c**2*d**5*e**6*f**2*x**2 + 6*sqrt(d)*sqrt(c)*atan(
(d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**5*e**5*f**3*x**4 - 90*sqrt(d)*sqrt(c
)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**5*e**4*f**4*x**6 - 63*sqrt(d)
*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c**2*d**5*e**3*f**5*x**8 - 6*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**6*e**7*f*x**2 + 6*s
qrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**6*e**6*f**2*x**4 +
30*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**6*e**5*f**3*x**
6 + 18*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*c*d**6*e**4*f**4
*x**8 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**7*e**7*f*x
**4 - 6*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**7*e**6*f**2*
x**6 - 3*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**7*e**5*f...
```

3.253 $\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 134

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)(be-af)} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)(de-cf)} + \frac{f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)(de-cf)}$$

output
```
b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/(-a*d+b*c)/(-a*f+b*e)-d^(3/2)*ar
ctan(d^(1/2)*x/c^(1/2))/c^(1/2)/(-a*d+b*c)/(-c*f+d*e)+f^(3/2)*arctan(f^(1/
2)*x/e^(1/2))/e^(1/2)/(-a*f+b*e)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)(-be+af)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)(-de+cf)} + \frac{f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)(de-cf)}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d)*(-(b*e) + a*
f)) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)*(-(d*e) +
 c*f)) + (f^(3/2)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f)*(d*e -
 c*f))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {422, 303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx$$

$$\Big\downarrow \text{422}$$

$$\frac{b\int \frac{1}{(bx^2+a)(fx^2+e)}dx}{bc-ad} - \frac{d\int \frac{1}{(dx^2+c)(fx^2+e)}dx}{bc-ad}$$

$$\Big\downarrow \text{303}$$

$$\frac{b\left(\frac{b\int \frac{1}{bx^2+a}dx}{be-af}-\frac{f\int \frac{1}{fx^2+e}dx}{be-af}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{f\int \frac{1}{fx^2+e}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \; 218$$

$$\frac{b\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}\right)}{bc-ad}-\frac{d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*e - a*f)) - (Sqrt[f]
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f))))/(b*c - a*d) - (d*((S
qrt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (Sqrt[f]*ArcTa
n[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(af-be)(ad-bc)\sqrt{ab}}+\frac{f^{2}\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(cf-de)(af-be)\sqrt{ef}}-\frac{d^{2}\arctan\left(\frac{xd}{\sqrt{cd}}\right)}{(ad-bc)(cf-de)\sqrt{cd}}$	117
risch	Expression too large to display	4033

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*f-b*e)/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^2/(c*f-d*e)/
(a*f-b*e)/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))-d^2/(a*d-b*c)/(c*f-d*e)/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2ce-abde-abcf+a^2df)\sqrt{ab}} - \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcde-ad^2e-bc^2f+acdf)\sqrt{cd}} + \frac{f^2\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{(bde^2-bcef-adef+acf^2)\sqrt{ef}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/((b^2*c*e - a*b*d*e - a*b*c*f + a^2*d*f)*sqrt(a*
b)) - d^2*arctan(d*x/sqrt(c*d))/((b*c*d*e - a*d^2*e - b*c^2*f + a*c*d*f)*s
qrt(c*d)) + f^2*arctan(f*x/sqrt(e*f))/((b*d*e^2 - b*c*e*f - a*d*e*f + a*c*
f^2)*sqrt(e*f))
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 26278, normalized size of antiderivative = 196.10

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)),x)
```

output
```
(atan((((((-c*d^3)^(1/2)*((((x*(16*a^5*b^2*d^7*f^7 + 16*b^7*c^5*d^2*f^7 +
16*b^7*d^7*e^5*f^2 + 16*a^2*b^5*c^3*d^4*f^7 + 16*a^3*b^4*c^2*d^5*f^7 + 16*
a^2*b^5*d^7*e^3*f^4 + 16*a^3*b^4*d^7*e^2*f^5 + 16*b^7*c^2*d^5*e^3*f^4 + 16
*b^7*c^3*d^4*e^2*f^5 - 32*a*b^6*c^4*d^3*f^7 - 32*a^4*b^3*c*d^6*f^7 - 32*a*
b^6*d^7*e^4*f^3 - 32*a^4*b^3*d^7*e*f^6 - 32*b^7*c*d^6*e^4*f^3 - 32*b^7*c^4
*d^3*e*f^6 + 64*a*b^6*c*d^6*e^3*f^4 + 64*a*b^6*c^3*d^4*e*f^6 + 64*a^3*b^4*
c*d^6*e*f^6 - 48*a*b^6*c^2*d^5*e^2*f^5 - 48*a^2*b^5*c*d^6*e^2*f^5 - 48*a^2
*b^5*c^2*d^5*e*f^6))/2 - ((-c*d^3)^(1/2)*((x*(-c*d^3)^(1/2)*(128*a^4*b^5*c
^6*d^3*f^9 - 32*a^3*b^6*c^7*d^2*f^9 - 192*a^5*b^4*c^5*d^4*f^9 + 128*a^6*b^
3*c^4*d^5*f^9 - 32*a^7*b^2*c^3*d^6*f^9 - 32*a^3*b^6*d^9*e^7*f^2 + 128*a^4*
b^5*d^9*e^6*f^3 - 192*a^5*b^4*d^9*e^5*f^4 + 128*a^6*b^3*d^9*e^4*f^5 - 32*a
^7*b^2*d^9*e^3*f^6 - 32*b^9*c^3*d^6*e^7*f^2 + 128*b^9*c^4*d^5*e^6*f^3 - 19
2*b^9*c^5*d^4*e^5*f^4 + 128*b^9*c^6*d^3*e^4*f^5 - 32*b^9*c^7*d^2*e^3*f^6 +
 32*a*b^8*c^2*d^7*e^7*f^2 - 128*a*b^8*c^3*d^6*e^6*f^3 + 96*a*b^8*c^4*d^5*e
^5*f^4 + 96*a*b^8*c^5*d^4*e^4*f^5 - 128*a*b^8*c^6*d^3*e^3*f^6 + 32*a*b^8*c
^7*d^2*e^2*f^7 + 32*a^2*b^7*c*d^8*e^7*f^2 + 32*a^2*b^7*c^7*d^2*e*f^8 - 128
*a^3*b^6*c*d^8*e^6*f^3 - 128*a^3*b^6*c^6*d^3*e*f^8 + 96*a^4*b^5*c*d^8*e^5*
f^4 + 96*a^4*b^5*c^5*d^4*e*f^8 + 96*a^5*b^4*c*d^8*e^4*f^5 + 96*a^5*b^4*c^4
*d^5*e*f^8 - 128*a^6*b^3*c*d^8*e^3*f^6 - 128*a^6*b^3*c^3*d^6*e*f^8 + 32*a^
7*b^2*c*d^8*e^2*f^7 + 32*a^7*b^2*c^2*d^7*e*f^8 + 96*a^2*b^7*c^3*d^6*e^5...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,c^2ef - \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bcd\,e^2 - \sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)a^2def + \sqrt{d}\,\sqrt{c}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{c}}\right)}{ace\left(a^2cd\,f^2 - a^2d^2ef - ab\,c^2f^2 + ab\,d^2e^2 + b^2c^2\right.}$$

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c**2*e*f - sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*d*e**2 - sqrt(d)*sqrt(c)*atan((d*x)/(s
qrt(d)*sqrt(c)))*a**2*d*e*f + sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c))
)*a*b*d*e**2 + sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a**2*c*d*f -
sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))*a*b*c**2*f)/(a*c*e*(a**2*c*d
*f**2 - a**2*d**2*e*f - a*b*c**2*f**2 + a*b*d**2*e**2 + b**2*c**2*e*f - b*
*2*c*d*e**2))
```

3.254 $\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2} dx = \frac{f^2x}{2e(be-af)(de-cf)(e+fx^2)} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)(be-af)^2} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)(de-cf)^2} + \frac{f^{3/2}(be(5de-3cf)-af(3de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}(be-af)^2(de-cf)^2}$$

output
```
1/2*f^2*x/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+b^(5/2)*arctan(b^(1/2)*x/a^(1/
2))/a^(1/2)/(-a*d+b*c)/(-a*f+b*e)^2-d^(5/2)*arctan(d^(1/2)*x/c^(1/2))/c^(1
/2)/(-a*d+b*c)/(-c*f+d*e)^2+1/2*f^(3/2)*(b*e*(-3*c*f+5*d*e)-a*f*(-c*f+3*d*
e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/(-a*f+b*e)^2/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx$$
$$= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)(be-af)^2} + \frac{\frac{f^2(de-cf)x}{e(be-af)(e+fx^2)} - \frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)} + \frac{f^{3/2}(be(5de-3cf)+af(-3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}(be-af)^2}}{2(de-cf)^2}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
-((b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d)*(b*e - a*f
)^2)) + ((f^2*(d*e - c*f)*x)/(e*(b*e - a*f)*(e + f*x^2)) - (2*d^(5/2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)) + (f^(3/2)*(b*e*(5*d*e - 3*
c*f) + a*f*(-3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*(b*e - a*
f)^2))/(2*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {421, 303, 218, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{421}$$
$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)}dx}{(be-af)^2} - \frac{f\int \frac{bfx^2+2be-af}{(dx^2+c)(fx^2+e)^2}dx}{(be-af)^2}$$
$$\Big\downarrow \text{303}$$

$$\frac{b^2\left(\frac{b\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^2+c}dx}{bc-ad}\right)}{(be-af)^2}-\frac{f\int \frac{bfx^2+2be-af}{(dx^2+c)(fx^2+e)^2}dx}{(be-af)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b^2\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)}{(be-af)^2}-\frac{f\int \frac{bfx^2+2be-af}{(dx^2+c)(fx^2+e)^2}dx}{(be-af)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)}{(be-af)^2}-\frac{f\left(\frac{\int \frac{-df(be-af)x^2+be(4de-3cf)-af(2de-cf)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx(be-af)}{2e(e+fx^2)(de-cf)}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{b^2\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)}{(be-af)^2}-\frac{f\left(\frac{\frac{2de(-adf-bcf+2bde)\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{f(be(5de-3cf)-af(3de-cf))\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{fx(be-af)}{2e(e+fx^2)(de-cf)}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b^2\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)}{(be-af)^2}-\frac{f\left(\frac{\frac{2\sqrt{d}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(-adf-bcf+2bde)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-3cf)-af(3de-cf))}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx(be-af)}{2e(e+fx^2)(de-cf)}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(b^2*((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[
d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))))/(b*e - a*f)^2 - (f
*(-1/2*(f*(b*e - a*f)*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((2*Sqrt[d]*e*(2*b*
d*e - b*c*f - a*d*f)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) -
(Sqrt[f]*(b*e*(5*d*e - 3*c*f) - a*f*(3*d*e - c*f))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f))))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

method	result
default	$-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(af-be)^{2}(ad-bc)\sqrt{ab}}+\frac{f^{2}\left(\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)x}{2e\left(f\,x^{2}+e\right)}+\frac{\left(ac\,f^{2}-3adef-3bcef+5bd\,e^{2}\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}\right)}{(cf-de)^{2}(af-be)^{2}}+\frac{d^{3}\arctan\left(\frac{xd}{\sqrt{cd}}\right)}{(ad-bc)(cf-de)^{2}\sqrt{}}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*f-b*e)^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^2/(c*f-d*
e)^2/(a*f-b*e)^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*
(a*c*f^2-3*a*d*e*f-3*b*c*e*f+5*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/
2)))+d^3/(a*d-b*c)/(c*f-d*e)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(175) = 350$.

Time = 0.13 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx$$
$$= \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3ce^2-ab^2de^2-2\,ab^2cef+2\,a^2bdef+a^2bcf^2-a^3df^2)\sqrt{ab}}$$
$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd^2e^2-ad^3e^2-2\,bc^2def+2\,acd^2ef+bc^3f^2-ac^2df^2)\sqrt{cd}}$$
$$+\frac{f^2x}{2\,(bde^3-bce^2f-ade^2f+acef^2)(fx^2+e)}$$
$$+\frac{(5\,bde^2f^2-3\,bcef^3-3\,adef^3+acf^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,(b^2d^2e^5-2\,b^2cde^4f-2\,abd^2e^4f+b^2c^2e^3f^2+4\,abcde^3f^2+a^2d^2e^3f^2-2\,abc^2e^2f^3-2\,a^2cde^2f^3+a^2}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((b^3*c*e^2 - a*b^2*d*e^2 - 2*a*b^2*c*e*f + 2*a^
2*b*d*e*f + a^2*b*c*f^2 - a^3*d*f^2)*sqrt(a*b)) - d^3*arctan(d*x/sqrt(c*d)
)/((b*c*d^2*e^2 - a*d^3*e^2 - 2*b*c^2*d*e*f + 2*a*c*d^2*e*f + b*c^3*f^2 -
a*c^2*d*f^2)*sqrt(c*d)) + 1/2*f^2*x/((b*d*e^3 - b*c*e^2*f - a*d*e^2*f + a*
c*e*f^2)*(f*x^2 + e)) + 1/2*(5*b*d*e^2*f^2 - 3*b*c*e*f^3 - 3*a*d*e*f^3 + a
*c*f^4)*arctan(f*x/sqrt(e*f))/((b^2*d^2*e^5 - 2*b^2*c*d*e^4*f - 2*a*b*d^2*
e^4*f + b^2*c^2*e^3*f^2 + 4*a*b*c*d*e^3*f^2 + a^2*d^2*e^3*f^2 - 2*a*b*c^2*
e^2*f^3 - 2*a^2*c*d*e^2*f^3 + a^2*c^2*e*f^4)*sqrt(e*f))
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 120009, normalized size of antiderivative = 591.18

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
(atan(((((x*(54*b^9*d^9*e^6*f^5 + a^2*b^7*c^4*d^5*f^11 + a^4*b^5*c^2*d^7*f
^11 + 107*a^2*b^7*d^9*e^4*f^7 - 48*a^3*b^6*d^9*e^3*f^8 + 9*a^4*b^5*d^9*e^2
*f^9 + 107*b^9*c^2*d^7*e^4*f^7 - 48*b^9*c^3*d^6*e^3*f^8 + 9*b^9*c^4*d^5*e^
2*f^9 - 118*a*b^8*d^9*e^5*f^6 - 118*b^9*c*d^8*e^5*f^6 + 192*a*b^8*c*d^8*e^
4*f^7 - 6*a*b^8*c^4*d^5*e*f^10 - 6*a^4*b^5*c*d^8*e*f^10 - 124*a*b^8*c^2*d^
7*e^3*f^8 + 40*a*b^8*c^3*d^6*e^2*f^9 - 124*a^2*b^7*c*d^8*e^3*f^8 - 8*a^2*b
^7*c^3*d^6*e*f^10 + 40*a^3*b^6*c*d^8*e^2*f^9 - 8*a^3*b^6*c^2*d^7*e*f^10 +
48*a^2*b^7*c^2*d^7*e^2*f^9))/(2*(b^4*d^4*e^10 + a^4*c^4*e^2*f^8 + a^4*d^4*
e^6*f^4 + b^4*c^4*e^6*f^4 + 6*a^2*b^2*c^4*e^4*f^6 + 6*a^2*b^2*d^4*e^8*f^2
+ 6*a^4*c^2*d^2*e^4*f^6 + 6*b^4*c^2*d^2*e^8*f^2 - 4*a*b^3*d^4*e^9*f - 4*b^
4*c*d^3*e^9*f - 4*a*b^3*c^4*e^5*f^5 - 4*a^3*b*c^4*e^3*f^7 - 4*a^3*b*d^4*e^
7*f^3 - 4*a^4*c*d^3*e^5*f^5 - 4*a^4*c^3*d*e^3*f^7 - 4*b^4*c^3*d*e^7*f^3 +
16*a*b^3*c*d^3*e^8*f^2 + 16*a*b^3*c^3*d*e^6*f^4 + 16*a^3*b*c*d^3*e^6*f^4 +
 16*a^3*b*c^3*d*e^4*f^6 - 24*a*b^3*c^2*d^2*e^7*f^3 - 24*a^2*b^2*c*d^3*e^7*
f^3 - 24*a^2*b^2*c^3*d*e^5*f^5 - 24*a^3*b*c^2*d^2*e^5*f^5 + 36*a^2*b^2*c^2
*d^2*e^6*f^4)) - (((2*a^2*b^8*c^8*d^2*f^13 - 2*a^3*b^7*c^7*d^3*f^13 - 2*a^
7*b^3*c^3*d^7*f^13 + 2*a^8*b^2*c^2*d^8*f^13 + 50*a^2*b^8*d^10*e^8*f^5 - 24
0*a^3*b^7*d^10*e^7*f^6 + 466*a^4*b^6*d^10*e^6*f^7 - 464*a^5*b^5*d^10*e^5*f
^8 + 246*a^6*b^4*d^10*e^4*f^9 - 64*a^7*b^3*d^10*e^3*f^10 + 6*a^8*b^2*d^10*
e^2*f^11 + 50*b^10*c^2*d^8*e^8*f^5 - 240*b^10*c^3*d^7*e^7*f^6 + 466*b^1...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1214, normalized size of antiderivative = 5.98

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**3*e**3*f**2 -
2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**3*e**2*f**3*x**2 +
 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**2*d*e**4*f + 4*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**2*d*e**3*f**2*x**2 - 2
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*d**2*e**5 - 2*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*d**2*e**4*f*x**2 + 2*sqrt(d
)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**2*e**3*f**2 + 2*sqrt(d)*sq
rt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*d**2*e**2*f**3*x**2 - 4*sqrt(d)*s
qrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*d**2*e**4*f - 4*sqrt(d)*sqrt(c
)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**2*b*d**2*e**3*f**2*x**2 + 2*sqrt(d)*sqr
t(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*d**2*e**5 + 2*sqrt(d)*sqrt(c)*at
an((d*x)/(sqrt(d)*sqrt(c)))*a*b**2*d**2*e**4*f*x**2 + sqrt(f)*sqrt(e)*atan
((f*x)/(sqrt(f)*sqrt(e)))*a**3*c**2*d*e*f**3 + sqrt(f)*sqrt(e)*atan((f*x)/
(sqrt(f)*sqrt(e)))*a**3*c**2*d*f**4*x**2 - 3*sqrt(f)*sqrt(e)*atan((f*x)/(s
qrt(f)*sqrt(e)))*a**3*c*d**2*e**2*f**2 - 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqr
t(f)*sqrt(e)))*a**3*c*d**2*e*f**3*x**2 - sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(
f)*sqrt(e)))*a**2*b*c**3*e*f**3 - sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt
(e)))*a**2*b*c**3*f**4*x**2 + 5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e
)))*a**2*b*c*d**2*e**3*f + 5*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*sqrt(e)))
*a**2*b*c*d**2*e**2*f**2*x**2 + 3*sqrt(f)*sqrt(e)*atan((f*x)/(sqrt(f)*s...
```

3.255 $\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 333

$$\begin{aligned}
&\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3} \, dx \\
&= \frac{f^2x}{4e(be-af)(de-cf)(e+fx^2)^2} + \frac{f^2(be(11de-7cf)-af(7de-3cf))x}{8e^2(be-af)^2(de-cf)^2(e+fx^2)} \\
&\quad + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)(be-af)^3} - \frac{d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)(de-cf)^3} \\
&\quad + \frac{f^{3/2}(a^2f^2(15d^2e^2-10cdef+3c^2f^2)-2abef(21d^2e^2-18cdef+5c^2f^2)+b^2e^2(35d^2e^2-42cdef+15}{8e^{5/2}(be-af)^3(de-cf)^3}
\end{aligned}$$

output

```
1/4*f^2*x/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2+1/8*f^2*(b*e*(-7*c*f+11*d*e)
-a*f*(-3*c*f+7*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x^2+e)+b^(7/2)*arc
tan(b^(1/2)*x/a^(1/2))/a^(1/2)/(-a*d+b*c)/(-a*f+b*e)^3-d^(7/2)*arctan(d^(1
/2)*x/c^(1/2))/c^(1/2)/(-a*d+b*c)/(-c*f+d*e)^3+1/8*f^(3/2)*(a^2*f^2*(3*c^2
*f^2-10*c*d*e*f+15*d^2*e^2)-2*a*b*e*f*(5*c^2*f^2-18*c*d*e*f+21*d^2*e^2)+b^
2*e^2*(15*c^2*f^2-42*c*d*e*f+35*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2
)/(-a*f+b*e)^3/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx$$
$$= \frac{1}{8}\left(\frac{2f^2x}{e(be-af)(de-cf)(e+fx^2)^2} + \frac{f^2(be(11de-7cf)+af(-7de+3cf))x}{e^2(be-af)^2(de-cf)^2(e+fx^2)} + \frac{8b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)(-be+af)^3} + \frac{8d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)(-de+cf)^3} + \frac{f^{3/2}(a^2f^2(15d^2e^2-10cdef+3c^2f^2)-2abef(21d^2e^2-18cdef+5c^2f^2)+b^2e^2(35d^2e^2-42cdef+1}{e^{5/2}(be-af)^3(de-cf)^3}\right.$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((2*f^2*x)/(e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2)^2) + (f^2*(b*e*(11*d*e -
 7*c*f) + a*f*(-7*d*e + 3*c*f))*x)/(e^2*(b*e - a*f)^2*(d*e - c*f)^2*(e + f
*x^2)) + (8*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d)*(
-(b*e) + a*f)^3) + (8*d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c -
 a*d)*(-(d*e) + c*f)^3) + (f^(3/2)*(a^2*f^2*(15*d^2*e^2 - 10*c*d*e*f + 3*c
^2*f^2) - 2*a*b*e*f*(21*d^2*e^2 - 18*c*d*e*f + 5*c^2*f^2) + b^2*e^2*(35*d^
2*e^2 - 42*c*d*e*f + 15*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(5/2)*(b
*e - a*f)^3*(d*e - c*f)^3))/8
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {421, 402, 402, 25, 397, 218, 422, 303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx$$

$$\Big\downarrow \ 421$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\int \frac{bfx^2+2be-af}{(dx^2+c)(fx^2+e)^3}dx}{(be-af)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{\int \frac{-3df(be-af)x^2+be(8de-7cf)-af(4de-3cf)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(be-af)}{4e(e+fx^2)^2(de-cf)}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 402$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{\frac{\int -\frac{df(be(11de-7cf)-af(7de-3cf))x^2+af\left(8d^2e^2-7cdfe+3c^2f^2\right)-be\left(16d^2e^2-19cdfe+7c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}-\frac{fx(be-a}{4e(e+fx^2)^2(}\right.}{(be-af)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{\int \frac{df(be(11de-7cf)-af(7de-3cf))x^2+af\left(8d^2e^2-7cdfe+3c^2f^2\right)-be\left(16d^2e^2-19cdfe+7c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}-\frac{fx(be-c}{4e(e+fx^2)^2(}\right.}{(be-af)^2}$$

$$\Big\downarrow \ 397$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{-\frac{f\left(af\left(3c^2f^2-10cdef+15d^2e^2\right)-be\left(7c^2f^2-26cdef+27d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}-\frac{8d^2e^2(-adf-bcf+2bde)\int\frac{1}{dx^2+c}dx}{de-cf}}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(de-cf)}}{4e(de-cf)}\right.}{(be-af)^2}$$

$$\Big\downarrow 218$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(3c^2f^2-10cdef+15d^2e^2\right)-be\left(7c^2f^2-26cdef+27d^2e^2\right)\right)}{\sqrt{e}(de-cf)}-\frac{8d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(-adf-bcf+2bde)}{\sqrt{c}(de-cf)}}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7c\cdots}{2e\left(e+fx^2\right)(de-c\cdots}}{4e(de-cf)}\cdots\right)}{(be-af)^2}$$

$$\Big\downarrow 422$$

$$\frac{b^2\left(\frac{b\int \frac{1}{\left(bx^2+a\right)\left(fx^2+e\right)}dx}{bc-ad}-\frac{d\int \frac{1}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{bc-ad}\right)}{(be-af)^2}-\frac{f\left(\frac{-\frac{-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(3c^2f^2-10cdef+15d^2e^2\right)-be\left(7c^2f^2-26cdef+27d^2e^2\right)\right)}{\sqrt{e}(de-cf)}-\frac{8d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(-adf-bcf+2bde)}{\sqrt{c}(de-cf)}}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7c\cdots}{2e\left(e+fx^2\right)(de-c\cdots}}{4e(de-cf)}\cdots\right)}{(be-af)^2}$$

$$\Big\downarrow 303$$

$$\frac{b^2\left(\frac{b\left(\frac{b\int \frac{1}{bx^2+a}dx}{be-af}-\frac{f\int \frac{1}{fx^2+e}dx}{be-af}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{f\int \frac{1}{fx^2+e}dx}{de-cf}\right)}{bc-ad}\right)}{(be-af)^2}-\frac{f\left(\frac{-\frac{-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(3c^2f^2-10cdef+15d^2e^2\right)-be\left(7c^2f^2-26cdef+27d^2e^2\right)\right)}{\sqrt{e}(de-cf)}-\frac{8d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(-adf-bcf+2bde)}{\sqrt{c}(de-cf)}}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7c\cdots}{2e\left(e+fx^2\right)(de-c\cdots}}{4e(de-cf)}\cdots\right)}{(be-af)^2}$$

$$\Big\downarrow 218$$

$$\frac{b^2\left(\frac{b\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}\right)}{bc-ad}-\frac{d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}\right)}{bc-ad}\right)}{(be-af)^2}-$$

$$\frac{f\left(\frac{-\frac{-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(3c^2f^2-10cdef+15d^2e^2\right)-be\left(7c^2f^2-26cdef+27d^2e^2\right)\right)}{\sqrt{e}(de-cf)}-\frac{8d^{3/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(-adf-bcf+2bde)}{\sqrt{c}(de-cf)}}{2e(de-cf)}-\frac{fx(be(11de-7cf)-af(7\ldots}{2e\left(e+fx^2\right)(de-c\ldots}}{4e(de-cf)}\right.}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(b^2*((b*((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*e - a*f)) - (S
qrt[f]*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f))))/(b*c - a*d) -
(d*((Sqrt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (Sqrt[f]
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f))))/(b*c - a*d)))/(b*e -
 a*f)^2 - (f*(-1/4*(f*(b*e - a*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2
*(f*(b*e*(11*d*e - 7*c*f) - a*f*(7*d*e - 3*c*f))*x)/(e*(d*e - c*f)*(e + f*
x^2)) - ((-8*d^(3/2)*e^2*(2*b*d*e - b*c*f - a*d*f)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(d*e - c*f)) - (Sqrt[f]*(a*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c
^2*f^2) - b*e*(27*d^2*e^2 - 26*c*d*e*f + 7*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sq
rt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f)))/(4*e*(d*e - c*f))))/(b*e
 - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.41

method	result
default	$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)(af-be)^3\sqrt{ab}} + \frac{f^2\left(\frac{f\left(3a^2c^2f^4-10a^2cde\,f^3+7a^2d^2e^2f^2-10ab\,c^2e\,f^3+28abcd\,e^2f^2-18ab\,d^2e^3f+7b^2c^2e^2f^2-18b^2cd\,e^3f+11b^2d\right.}{8e^2}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
b^4/(a*d-b*c)/(a*f-b*e)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^2/(c*f-d*e
)^3/(a*f-b*e)^3*((1/8*f*(3*a^2*c^2*f^4-10*a^2*c*d*e*f^3+7*a^2*d^2*e^2*f^2-
10*a*b*c^2*e*f^3+28*a*b*c*d*e^2*f^2-18*a*b*d^2*e^3*f+7*b^2*c^2*e^2*f^2-18*
b^2*c*d*e^3*f+11*b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-14*a^2*c*d*e*f^3+
9*a^2*d^2*e^2*f^2-14*a*b*c^2*e*f^3+36*a*b*c*d*e^2*f^2-22*a*b*d^2*e^3*f+9*b
^2*c^2*e^2*f^2-22*b^2*c*d*e^3*f+13*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a^
2*c^2*f^4-10*a^2*c*d*e*f^3+15*a^2*d^2*e^2*f^2-10*a*b*c^2*e*f^3+36*a*b*c*d*
e^2*f^2-42*a*b*d^2*e^3*f+15*b^2*c^2*e^2*f^2-42*b^2*c*d*e^3*f+35*b^2*d^2*e^
4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d^4/(a*d-b*c)/(c*f-d*e)^3/(c*d
)^(1/2)*arctan(x*d/(c*d)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(303) = 606$.

Time = 0.13 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.36

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx$$
$$= \frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^4ce^3 - ab^3de^3 - 3\,ab^3ce^2f + 3\,a^2b^2de^2f + 3\,a^2b^2cef^2 - 3\,a^3bdef^2 - a^3bcf^3 + a^4df^3)\sqrt{ab}}$$
$$- \frac{d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd^3e^3 - ad^4e^3 - 3\,bc^2d^2e^2f + 3\,acd^3e^2f + 3\,bc^3def^2 - 3\,ac^2d^2ef^2 - bc^4f^3 + ac^3df^3)\sqrt{cd}}$$
$$+ \frac{(35\,b^2d^2e^4f^2 - 42\,b^2cde^3f^3 - 42\,abd^2e^3f^3 + 15\,b^2c^2e^2f^4 + 3}{8\,(b^3d^3e^8 - 3\,b^3cd^2e^7f - 3\,ab^2d^3e^7f + 3\,b^3c^2de^6f^2 + 9\,ab^2cd^2e^6f^2 + 3\,a^2bd^3e^6f^2 - b^3c^3e^5f^3 - 9\,ab^2c^2d}$$
$$+ \frac{11\,bde^2f^3x^3 - 7\,bcef^4x^3 - 7\,adef^4x^3 + 3\,acf^5x^3 + 13\,bde^3f^2x - 9\,bce^2f^3x - 9\,ade^2f^3x + 5}{8\,(b^2d^2e^6 - 2\,b^2cde^5f - 2\,abd^2e^5f + b^2c^2e^4f^2 + 4\,abcde^4f^2 + a^2d^2e^4f^2 - 2\,abc^2e^3f^3 - 2\,a^2cde^3f^3 + a^2}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/((b^4*c*e^3 - a*b^3*d*e^3 - 3*a*b^3*c*e^2*f + 3*
a^2*b^2*d*e^2*f + 3*a^2*b^2*c*e*f^2 - 3*a^3*b*d*e*f^2 - a^3*b*c*f^3 + a^4*
d*f^3)*sqrt(a*b)) - d^4*arctan(d*x/sqrt(c*d))/((b*c*d^3*e^3 - a*d^4*e^3 -
3*b*c^2*d^2*e^2*f + 3*a*c*d^3*e^2*f + 3*b*c^3*d*e*f^2 - 3*a*c^2*d^2*e*f^2
- b*c^4*f^3 + a*c^3*d*f^3)*sqrt(c*d)) + 1/8*(35*b^2*d^2*e^4*f^2 - 42*b^2*c
*d*e^3*f^3 - 42*a*b*d^2*e^3*f^3 + 15*b^2*c^2*e^2*f^4 + 36*a*b*c*d*e^2*f^4
+ 15*a^2*d^2*e^2*f^4 - 10*a*b*c^2*e*f^5 - 10*a^2*c*d*e*f^5 + 3*a^2*c^2*f^6
)*arctan(f*x/sqrt(e*f))/((b^3*d^3*e^8 - 3*b^3*c*d^2*e^7*f - 3*a*b^2*d^3*e^
7*f + 3*b^3*c^2*d*e^6*f^2 + 9*a*b^2*c*d^2*e^6*f^2 + 3*a^2*b*d^3*e^6*f^2 -
b^3*c^3*e^5*f^3 - 9*a*b^2*c^2*d*e^5*f^3 - 9*a^2*b*c*d^2*e^5*f^3 - a^3*d^3*
e^5*f^3 + 3*a*b^2*c^3*e^4*f^4 + 9*a^2*b*c^2*d*e^4*f^4 + 3*a^3*c*d^2*e^4*f^
4 - 3*a^2*b*c^3*e^3*f^5 - 3*a^3*c^2*d*e^3*f^5 + a^3*c^3*e^2*f^6)*sqrt(e*f)
) + 1/8*(11*b*d*e^2*f^3*x^3 - 7*b*c*e*f^4*x^3 - 7*a*d*e*f^4*x^3 + 3*a*c*f^
5*x^3 + 13*b*d*e^3*f^2*x - 9*b*c*e^2*f^3*x - 9*a*d*e^2*f^3*x + 5*a*c*e*f^4
*x)/((b^2*d^2*e^6 - 2*b^2*c*d*e^5*f - 2*a*b*d^2*e^5*f + b^2*c^2*e^4*f^2 +
4*a*b*c*d*e^4*f^2 + a^2*d^2*e^4*f^2 - 2*a*b*c^2*e^3*f^3 - 2*a^2*c*d*e^3*f^
3 + a^2*c^2*e^2*f^4)*(f*x^2 + e)^2)
```

Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 113015, normalized size of antiderivative = 339.38

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
((x^3*(3*a*c*f^5 - 7*a*d*e*f^4 - 7*b*c*e*f^4 + 11*b*d*e^2*f^3))/(8*e^2*(a^
2*c^2*f^4 + b^2*d^2*e^4 + a^2*d^2*e^2*f^2 + b^2*c^2*e^2*f^2 - 2*a*b*c^2*e*
f^3 - 2*a*b*d^2*e^3*f - 2*a^2*c*d*e*f^3 - 2*b^2*c*d*e^3*f + 4*a*b*c*d*e^2*
f^2)) + (x*(5*a*c*f^4 - 9*a*d*e*f^3 - 9*b*c*e*f^3 + 13*b*d*e^2*f^2))/(8*e*
(a^2*c^2*f^4 + b^2*d^2*e^4 + a^2*d^2*e^2*f^2 + b^2*c^2*e^2*f^2 - 2*a*b*c^2
*e*f^3 - 2*a*b*d^2*e^3*f - 2*a^2*c*d*e*f^3 - 2*b^2*c*d*e^3*f + 4*a*b*c*d*e
^2*f^2)))/(e^2 + f^2*x^4 + 2*e*f*x^2) + symsum(log((1505*b^12*d^12*e^8*f^6
 + 9*a^4*b^8*c^4*d^8*f^14 + 3270*a^2*b^10*d^12*e^6*f^8 - 1380*a^3*b^9*d^12
*e^5*f^9 + 225*a^4*b^8*d^12*e^4*f^10 + 3270*b^12*c^2*d^10*e^6*f^8 - 1380*b
^12*c^3*d^9*e^5*f^9 + 225*b^12*c^4*d^8*e^4*f^10 - 3556*a*b^11*d^12*e^7*f^7
 - 3556*b^12*c*d^11*e^7*f^7 + 7288*a*b^11*c*d^11*e^6*f^8 - 5808*a*b^11*c^2
*d^10*e^5*f^9 + 2120*a*b^11*c^3*d^9*e^4*f^10 - 300*a*b^11*c^4*d^8*e^3*f^11
 - 5808*a^2*b^10*c*d^11*e^5*f^9 + 2120*a^3*b^9*c*d^11*e^4*f^10 - 60*a^3*b^
9*c^4*d^8*e*f^13 - 300*a^4*b^8*c*d^11*e^3*f^11 - 60*a^4*b^8*c^3*d^9*e*f^13
 + 4108*a^2*b^10*c^2*d^10*e^4*f^10 - 1376*a^2*b^10*c^3*d^9*e^3*f^11 + 190*
a^2*b^10*c^4*d^8*e^2*f^12 - 1376*a^3*b^9*c^2*d^10*e^3*f^11 + 440*a^3*b^9*c
^3*d^9*e^2*f^12 + 190*a^4*b^8*c^2*d^10*e^2*f^12)/(64*(b^8*d^8*e^20 + a^8*c
^8*e^4*f^16 + a^8*d^8*e^12*f^8 + b^8*c^8*e^12*f^8 + 28*a^2*b^6*c^8*e^10*f^
10 - 56*a^3*b^5*c^8*e^9*f^11 + 70*a^4*b^4*c^8*e^8*f^12 - 56*a^5*b^3*c^8*e^
7*f^13 + 28*a^6*b^2*c^8*e^6*f^14 + 28*a^2*b^6*d^8*e^18*f^2 - 56*a^3*b^5...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3509, normalized size of antiderivative = 10.54

$$\int \frac{1}{(a+bx^2)(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output

```
(8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*e**5*f**3 + 16*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*e**4*f**4*x**2 + 8
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*e**3*f**5*x**4 -
24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d*e**6*f**2 - 4
8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d*e**5*f**3*x**2
 - 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d*e**4*f**4*
x**4 + 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*d**2*e**
7*f + 48*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*d**2*e**6
*f**2*x**2 + 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*d*
*2*e**5*f**3*x**4 - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c
*d**3*e**8 - 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*d**3*
e**7*f*x**2 - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*d**3*
e**6*f**2*x**4 - 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*d**3
*e**5*f**3 - 16*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*d**3*e*
*4*f**4*x**2 - 8*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*d**3*e
**3*f**5*x**4 + 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*b*d*
*3*e**6*f**2 + 48*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*b*d**
3*e**5*f**3*x**2 + 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**3*b
*d**3*e**4*f**4*x**4 - 24*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a*
*2*b**2*d**3*e**7*f - 48*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*...
```

3.256 $\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 270

$$
\begin{aligned}
&\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx \\
&= -\frac{d^3x}{2c(bc-ad)(de-cf)^2(c+dx^2)} - \frac{f^3x}{2e(be-af)(de-cf)^2(e+fx^2)} \\
&\quad + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2(be-af)^2} - \frac{d^{5/2}(bc(3de-7cf)-ad(de-5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2(de-cf)^3} \\
&\quad - \frac{f^{5/2}(be(7de-3cf)-af(5de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}(be-af)^2(de-cf)^3}
\end{aligned}
$$

output
```
-1/2*d^3*x/c/(-a*d+b*c)/(-c*f+d*e)^2/(d*x^2+c)-1/2*f^3*x/e/(-a*f+b*e)/(-c*
f+d*e)^2/(f*x^2+e)+b^(7/2)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/(-a*d+b*c)^2/
(-a*f+b*e)^2-1/2*d^(5/2)*(b*c*(-7*c*f+3*d*e)-a*d*(-5*c*f+d*e))*arctan(d^(1
/2)*x/c^(1/2))/c^(3/2)/(-a*d+b*c)^2/(-c*f+d*e)^3-1/2*f^(5/2)*(b*e*(-3*c*f+
7*d*e)-a*f*(-c*f+5*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2)/(-a*f+b*e)^2/(-
c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx$$
$$= \frac{1}{2}\left(-\frac{d^3x}{c(bc-ad)(de-cf)^2(c+dx^2)} - \frac{f^3x}{e(be-af)(de-cf)^2(e+fx^2)} + \frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2(be-af)^2} - \frac{d^{5/2}(ad(de-5cf)+bc(-3de+7cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2(-de+cf)^3} - \frac{f^{5/2}(be(7de-3cf)+af(-5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}(be-af)^2(de-cf)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(-((d^3*x)/(c*(b*c - a*d)*(d*e - c*f)^2*(c + d*x^2))) - (f^3*x)/(e*(b*e -
a*f)*(d*e - c*f)^2*(e + f*x^2)) + (2*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(Sqrt[a]*(b*c - a*d)^2*(b*e - a*f)^2) - (d^(5/2)*(a*d*(d*e - 5*c*f) + b*c*
(-3*d*e + 7*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2*(-(d
*e) + c*f)^3) - (f^(5/2)*(b*e*(7*d*e - 3*c*f) + a*f*(-5*d*e + c*f))*ArcTan
[(Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*(b*e - a*f)^2*(d*e - c*f)^3))/2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {421, 316, 397, 218, 402, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b^2\int\frac{1}{(bx^2+a)(fx^2+e)^2}dx}{(bc-ad)^2}-\frac{d\int\frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{316}$$

$$\frac{b^2\left(\frac{\int\frac{-bfx^2+2be-af}{(bx^2+a)(fx^2+e)}dx}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int\frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{b^2\left(\frac{\frac{2b^2e\int\frac{1}{bx^2+a}dx}{be-af}-\frac{f(3be-af)\int\frac{1}{fx^2+e}dx}{be-af}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int\frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int\frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}-\frac{\int-\frac{3d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{(dx^2+c)(fx^2+e)^2}dx}{2c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\int\frac{3d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{(dx^2+c)(fx^2+e)^2}dx}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\int -\frac{2\left(-df\left(2bc^2f-ad(de+cf)\right)x^2+ad\left(d^2e^2-4cdfe+c^2f^2\right)-bc\left(3d^2e^2-7cdfe+2c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}+\frac{fx\left(2bc^2f-ad(cf+de)\right)}{e\left(e+fx^2\right)(de-cf)}}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 27

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{fx\left(2bc^2f-ad(cf+de)\right)}{e\left(e+fx^2\right)(de-cf)}-\frac{\int \frac{-df\left(2bc^2f-ad(de+cf)\right)x^2+ad\left(d^2e^2-4cdfe+c^2f^2\right)-bc\left(3d^2e^2-7cdfe+2c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{e(de-cf)}}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 397

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{fx\left(2bc^2f-ad(cf+de)\right)}{e\left(e+fx^2\right)(de-cf)}-\frac{\frac{cf\left(adf(5de-cf)+b\left(2c^2f^2-9cdef+3d^2e^2\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}-\frac{d^2e(bc(3de-7cf)-ad(de-5cf))\int\frac{1}{dx^2+c}dx}{de-cf}}{e(de-cf)}}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)(de-cf)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 218

$$\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(e+fx^2)(be-af)}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{fx\left(2bc^2f-ad(cf+de)\right)}{e\left(e+fx^2\right)(de-cf)}-\frac{\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(adf(5de-cf)+b\left(2c^2f^2-9cdef+3d^2e^2\right)\right)}{\sqrt{e}(de-cf)}-\frac{d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(bc(3de-7cf)-ad(de-5cf))}{\sqrt{c}(de-cf)}}{e(de-cf)}}{2c(de-cf)}+\frac{d}{2c(c+dx^2}\right.}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(b^2*(-1/2*(f*x)/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b^(3/2)*e*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(Sqrt[a]*(b*e - a*f)) - (Sqrt[f]*(3*b*e - a*f)*ArcTan[(Sqr
t[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f)))/(2*e*(b*e - a*f))))/(b*c - a*d)^2
 - (d*((d*(b*c - a*d)*x)/(2*c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)) + ((f*(
2*b*c^2*f - a*d*(d*e + c*f))*x)/(e*(d*e - c*f)*(e + f*x^2)) - (-((d^(3/2)*
e*(b*c*(3*d*e - 7*c*f) - a*d*(d*e - 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
Sqrt[c]*(d*e - c*f))) + (c*Sqrt[f]*(a*d*f*(5*d*e - c*f) + b*(3*d^2*e^2 - 9
*c*d*e*f + 2*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))
/(e*(d*e - c*f)))/(2*c*(d*e - c*f))))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2(af-be)^2\sqrt{ab}}+\frac{f^3\left(\frac{\left(ac f^2-adef-bcef+bd e^2\right)x}{2e\left(f x^2+e\right)}+\frac{\left(ac f^2-5adef-3bcef+7bd e^2\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}\right)}{(af-be)^2(cf-de)^3}+\frac{d^3\left(\frac{\left(acdf-a d^2e-b c^2\right.}{2c\left(x^2d+\right.}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
b^4/(a*d-b*c)^2/(a*f-b*e)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^3/(a*f-b
*e)^2/(c*f-d*e)^3*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2
*(a*c*f^2-5*a*d*e*f-3*b*c*e*f+7*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1
/2)))+d^3/(a*d-b*c)^2/(c*f-d*e)^3*(1/2*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c
*x/(d*x^2+c)+1/2*(5*a*c*d*f-a*d^2*e-7*b*c^2*f+3*b*c*d*e)/c/(c*d)^(1/2)*arc
tan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(238) = 476$.

Time = 0.14 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.94

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2}\,dx$$
$$= \frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^4c^2e^2-2\,ab^3cde^2+a^2b^2d^2e^2-2\,ab^3c^2ef+4\,a^2b^2cdef-2\,a^3bd^2ef+a^2b^2c^2f^2-2\,a^3bcdf^2+a^4d^2f^2)\sqrt{}}$$
$$-\frac{(3\,bcd^4e-ad^5e-7\,bc^2d^3f+5\,acd^4f)\arctan\left(\frac{dx}{\sqrt{c}}\right)}{2\,(b^2c^3d^3e^3-2\,abc^2d^4e^3+a^2cd^5e^3-3\,b^2c^4d^2e^2f+6\,abc^3d^3e^2f-3\,a^2c^2d^4e^2f+3\,b^2c^5def^2-6\,abc^4d^2}$$
$$-\frac{(7\,bde^2f^3-3\,bcef^4-5\,adef^4+acf^5)\arctan\left(\frac{}{\sqrt{}}\right)}{2\,(b^2d^3e^6-3\,b^2cd^2e^5f-2\,abd^3e^5f+3\,b^2c^2de^4f^2+6\,abcd^2e^4f^2+a^2d^3e^4f^2-b^2c^3e^3f^3-6\,abc^2de^3f^3}$$
$$-\frac{bd^3e^2fx^3-ad^3ef^2x^3+bc^2df^3x^3-acd^2f^3x^3+bd^3e^3x-ad^3e^2fx+bc}{2\,(b^2c^2d^2e^4-abcd^3e^4-2\,b^2c^3de^3f+abc^2d^2e^3f+a^2cd^3e^3f+b^2c^4e^2f^2+abc^3de^2f^2-2\,a^2c^2d^2e^2f^2-}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/((b^4*c^2*e^2 - 2*a*b^3*c*d*e^2 + a^2*b^2*d^2*e^
2 - 2*a*b^3*c^2*e*f + 4*a^2*b^2*c*d*e*f - 2*a^3*b*d^2*e*f + a^2*b^2*c^2*f^
2 - 2*a^3*b*c*d*f^2 + a^4*d^2*f^2)*sqrt(a*b)) - 1/2*(3*b*c*d^4*e - a*d^5*e
 - 7*b*c^2*d^3*f + 5*a*c*d^4*f)*arctan(d*x/sqrt(c*d))/((b^2*c^3*d^3*e^3 -
2*a*b*c^2*d^4*e^3 + a^2*c*d^5*e^3 - 3*b^2*c^4*d^2*e^2*f + 6*a*b*c^3*d^3*e^
2*f - 3*a^2*c^2*d^4*e^2*f + 3*b^2*c^5*d*e*f^2 - 6*a*b*c^4*d^2*e*f^2 + 3*a^
2*c^3*d^3*e*f^2 - b^2*c^6*f^3 + 2*a*b*c^5*d*f^3 - a^2*c^4*d^2*f^3)*sqrt(c*
d)) - 1/2*(7*b*d*e^2*f^3 - 3*b*c*e*f^4 - 5*a*d*e*f^4 + a*c*f^5)*arctan(f*x
/sqrt(e*f))/((b^2*d^3*e^6 - 3*b^2*c*d^2*e^5*f - 2*a*b*d^3*e^5*f + 3*b^2*c^
2*d*e^4*f^2 + 6*a*b*c*d^2*e^4*f^2 + a^2*d^3*e^4*f^2 - b^2*c^3*e^3*f^3 - 6*
a*b*c^2*d*e^3*f^3 - 3*a^2*c*d^2*e^3*f^3 + 2*a*b*c^3*e^2*f^4 + 3*a^2*c^2*d*
e^2*f^4 - a^2*c^3*e*f^5)*sqrt(e*f)) - 1/2*(b*d^3*e^2*f*x^3 - a*d^3*e*f^2*x
^3 + b*c^2*d*f^3*x^3 - a*c*d^2*f^3*x^3 + b*d^3*e^3*x - a*d^3*e^2*f*x + b*c
^3*f^3*x - a*c^2*d*f^3*x)/((b^2*c^2*d^2*e^4 - a*b*c*d^3*e^4 - 2*b^2*c^3*d*
e^3*f + a*b*c^2*d^2*e^3*f + a^2*c*d^3*e^3*f + b^2*c^4*e^2*f^2 + a*b*c^3*d*
e^2*f^2 - 2*a^2*c^2*d^2*e^2*f^2 - a*b*c^4*e*f^3 + a^2*c^3*d*e*f^3)*(d*f*x^
4 + d*e*x^2 + c*f*x^2 + c*e))
```

Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 123740, normalized size of antiderivative = 458.30

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^2} dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2),x)
```

output

```
symsum(log(root(315707392*a^9*b^8*c^13*d^10*e^13*f^10*z^6 + 229605376*a^11
*b^6*c^12*d^11*e^12*f^11*z^6 + 229605376*a^7*b^10*c^14*d^9*e^14*f^9*z^6 +
224387072*a^10*b^7*c^14*d^9*e^11*f^12*z^6 + 224387072*a^10*b^7*c^11*d^12*e
^14*f^9*z^6 + 224387072*a^8*b^9*c^15*d^8*e^12*f^11*z^6 + 224387072*a^8*b^9
*c^12*d^11*e^15*f^8*z^6 - 193077248*a^9*b^8*c^14*d^9*e^12*f^11*z^6 - 19307
7248*a^9*b^8*c^12*d^11*e^14*f^9*z^6 - 143503360*a^11*b^6*c^13*d^10*e^11*f^
12*z^6 - 143503360*a^11*b^6*c^11*d^12*e^13*f^10*z^6 - 143503360*a^7*b^10*c
^15*d^8*e^13*f^10*z^6 - 143503360*a^7*b^10*c^13*d^10*e^15*f^8*z^6 - 140894
208*a^10*b^7*c^15*d^8*e^10*f^13*z^6 - 140894208*a^10*b^7*c^10*d^13*e^15*f^
8*z^6 - 140894208*a^8*b^9*c^16*d^7*e^11*f^12*z^6 - 140894208*a^8*b^9*c^11*
d^12*e^16*f^7*z^6 + 114802688*a^12*b^5*c^13*d^10*e^10*f^13*z^6 + 114802688
*a^12*b^5*c^10*d^13*e^13*f^10*z^6 - 114802688*a^10*b^7*c^13*d^10*e^12*f^11
*z^6 - 114802688*a^10*b^7*c^12*d^11*e^13*f^10*z^6 - 114802688*a^8*b^9*c^14
*d^9*e^13*f^10*z^6 - 114802688*a^8*b^9*c^13*d^10*e^14*f^9*z^6 + 114802688*
a^6*b^11*c^16*d^7*e^13*f^10*z^6 + 114802688*a^6*b^11*c^13*d^10*e^16*f^7*z^
6 + 111992832*a^9*b^8*c^16*d^7*e^10*f^13*z^6 + 111992832*a^9*b^8*c^10*d^13
*e^16*f^7*z^6 + 86102016*a^13*b^4*c^11*d^12*e^11*f^12*z^6 + 86102016*a^5*b
^12*c^15*d^8*e^15*f^8*z^6 - 77529088*a^12*b^5*c^14*d^9*e^9*f^14*z^6 - 7752
9088*a^12*b^5*c^9*d^14*e^14*f^9*z^6 - 77529088*a^6*b^11*c^17*d^6*e^12*f^11
*z^6 - 77529088*a^6*b^11*c^12*d^11*e^17*f^6*z^6 + 77156352*a^11*b^6*c^1...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4534, normalized size of antiderivative = 16.79

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input

```
int(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**6*e**3*f**3 + 2*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**6*e**2*f**4*x**2 - 6*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**5*d*e**4*f**2 - 4*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**5*d*e**3*f**3*x**2 + 2
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**5*d*e**2*f**4*x**4
+ 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*d**2*e**5*f -
6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*d**2*e**3*f**3*x
**4 - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d**3*e**6
+ 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d**3*e**5*f*x*
*2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d**3*e**4*f
**2*x**4 - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*d**4*
e**6*x**2 - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*d**4
*e**5*f*x**4 + 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*c**2*d
**3*e**3*f**3 + 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*c**2*
d**3*e**2*f**4*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*c
*d**4*e**4*f**2 + 4*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*c*d
**4*e**3*f**3*x**2 + 5*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*
c*d**4*e**2*f**4*x**4 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4
*d**5*e**4*f**2*x**2 - sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a**4*
d**5*e**3*f**3*x**4 - 7*sqrt(d)*sqrt(c)*atan((d*x)/(sqrt(d)*sqrt(c)))*a...
```

3.257 $\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 400

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = -\frac{d^4x}{2c(bc-ad)(de-cf)^3(c+dx^2)} - \frac{f^3x}{4e(be-af)(de-cf)^2(e+fx^2)^2} - \frac{f^3(be(15de-7cf)-af(11de-3cf))x}{8e^2(be-af)^2(de-cf)^3(e+fx^2)} + \frac{b^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2(be-af)^3} + \frac{d^{7/2}(ad(de-7cf)-3bc(de-3cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2(de-cf)^4} - \frac{f^{5/2}(a^2f^2(35d^2e^2-14cdef+3c^2f^2)-2abef(45d^2e^2-26cdef+5c^2f^2)+3b^2e^2(21d^2e^2-18cdef+}{8e^{5/2}(be-af)^3(de-cf)^4}$$

output
```
-1/2*d^4*x/c/(-a*d+b*c)/(-c*f+d*e)^3/(d*x^2+c)-1/4*f^3*x/e/(-a*f+b*e)/(-c*
f+d*e)^2/(f*x^2+e)^2-1/8*f^3*(b*e*(-7*c*f+15*d*e)-a*f*(-3*c*f+11*d*e))*x/e
^2/(-a*f+b*e)^2/(-c*f+d*e)^3/(f*x^2+e)+b^(9/2)*arctan(b^(1/2)*x/a^(1/2))/a
^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^3+1/2*d^(7/2)*(a*d*(-7*c*f+d*e)-3*b*c*(-3*c
*f+d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/(-a*d+b*c)^2/(-c*f+d*e)^4-1/8*f
^(5/2)*(a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2)-2*a*b*e*f*(5*c^2*f^2-26*
c*d*e*f+45*d^2*e^2)+3*b^2*e^2*(5*c^2*f^2-18*c*d*e*f+21*d^2*e^2))*arctan(f^
(1/2)*x/e^(1/2))/e^(5/2)/(-a*f+b*e)^3/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(\frac{4d^4x}{c(bc-ad)(-de+cf)^3(c+dx^2)} - \frac{2f^3x}{e(be-af)(de-cf)^2(e+fx^2)^2} - \frac{f^3(be(15de-7cf)+af(-11de+3cf))x}{e^2(be-af)^2(de-cf)^3(e+fx^2)} - \frac{8b^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2(-be+af)^3} + \frac{4d^{7/2}(ad(de-7cf)+3bc(-de+3cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2(de-cf)^4} - \frac{f^{5/2}(a^2f^2(35d^2e^2-14cdef+3c^2f^2)-2abef(45d^2e^2-26cdef+5c^2f^2)+3b^2e^2(21d^2e^2-18cdef+\dots}{e^{5/2}(be-af)^3(de-cf)^4}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
((4*d^4*x)/(c*(b*c - a*d)*(-(d*e) + c*f)^3*(c + d*x^2)) - (2*f^3*x)/(e*(b*
e - a*f)*(d*e - c*f)^2*(e + f*x^2)^2) - (f^3*(b*e*(15*d*e - 7*c*f) + a*f*(
-11*d*e + 3*c*f))*x)/(e^2*(b*e - a*f)^2*(d*e - c*f)^3*(e + f*x^2)) - (8*b^
(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)^2*(-(b*e) + a*f)^3
) + (4*d^(7/2)*(a*d*(d*e - 7*c*f) + 3*b*c*(-(d*e) + 3*c*f))*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2*(d*e - c*f)^4) - (f^(5/2)*(a^2*f^2*(
35*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2) - 2*a*b*e*f*(45*d^2*e^2 - 26*c*d*e*f
+ 5*c^2*f^2) + 3*b^2*e^2*(21*d^2*e^2 - 18*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sq
rt[f]*x)/Sqrt[e]])/(e^(5/2)*(b*e - a*f)^3*(d*e - c*f)^4))/8
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {421, 316, 402, 25, 397, 218, 402, 27, 402, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 421$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \; 316$$

$$\frac{b^2\left(\frac{\int \frac{-3bfx^2+4be-3af}{(bx^2+a)(fx^2+e)^2}dx}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b^2\left(\frac{\frac{\int \frac{8b^2e^2-7abfe+3a^2f^2-bf(7be-3af)x^2}{(bx^2+a)(fx^2+e)}dx}{2e(be-af)}-\frac{fx(7be-3af)}{2e(e+fx^2)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}-\frac{\int -\frac{5d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\left(\frac{\frac{\int \frac{8b^2e^2-7abfe+3a^2f^2-bf(7be-3af)x^2}{(bx^2+a)(fx^2+e)}dx}{2e(be-af)}-\frac{fx(7be-3af)}{2e(e+fx^2)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{5d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{(dx^2+c)(fx^2+e)^3}dx}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 397$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^3e^2\int \frac{1}{bx^2+a}dx}{be-af}-\frac{f\left(3a^2f^2-10abef+15b^2e^2\right)\int \frac{1}{fx^2+e}dx}{be-af}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{5d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{5d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)^2(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\int \frac{2\left(-3df(ad(2de+cf)-bc(de+2cf))x^2+3bc\left(2d^2e^2-5cdfe+2c^2f^2\right)-ad\left(2d^2e^2-8cdfe+3c^2f^2\right)\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{4e(de-cf)}-\frac{fx(ad(cf+2de)-bc(2cf+de))}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}+\frac{dx(bc-\cdots}{2c(c+dx^2)(e+fx\cdots}\right.}{(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\int \frac{-3df(ad(2de+cf)-bc(de+2cf))x^2+3bc\left(2d^2e^2-5cdfe+2c^2f^2\right)-ad\left(2d^2e^2-8cdfe+3c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2e(de-cf)}-\frac{fx(ad(cf+2de)-bc(2cf+de))}{2e\left(e+fx^2\right)^2(de-cf)}}{2c(de-cf)}+\frac{dx(bc-ad)}{2c(c+dx^2)(e+fx^2)^2}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\frac{\int -\frac{df\left(ad\left(4d^2e^2+11cdfe-3c^2f^2\right)+3bc\left(d^2e^2-7cdfe+2c^2f^2\right)\right)x^2+ad\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)-3bc\left(4d^3e^3-13cd^2fe^2+7c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\cdots}{2e(de-cf)}}{2c(de-cf)}\cdots\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\int \frac{df\left(ad\left(4d^2e^2+11cdfe-3c^2f^2\right)+3bc\left(d^2e^2-7cdfe+2c^2f^2\right)\right)x^2+ad\left(4d^3e^3-24cd^2fe^2+11c^2df^2e-3c^3f^3\right)-3bc\left(4d^3e^3-13cd^2fe^2+7c^2df^2e-2c^3f^3\right)}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\cdots}{2e(de-cf)}}{2c(de-cf)}\cdots\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 397$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\frac{cf\left(adf\left(3c^2f^2-14cdef+35d^2e^2\right)+3b\left(-2c^3f^3+9c^2def^2-20cd^2e^2f+5d^3e^3\right)\right)\int\frac{1}{fx^2+e}dx}{de-cf}+\frac{4d^3e^2(ad(de-7cf)-3bc(de-3cf))\int\frac{1}{dx^2+c}dx}{de-cf}}{2e(de-cf)}-\frac{fx\left(ad\left(-3c\right.\right.}{}}{2e(de-cf)}}{2c(de-cf)}\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{b^2\left(\frac{\frac{\frac{8b^{5/2}e^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\left(3a^2f^2-10abef+15b^2e^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx(7be-3af)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{-\frac{\frac{c\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(adf\left(3c^2f^2-14cdef+35d^2e^2\right)+3b\left(-2c^3f^3+9c^2def^2-20cd^2e^2f+5d^3e^3\right)\right)}{\sqrt{e}(de-cf)}+\frac{4d^{5/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)(ad(de-7cf)-3bc(de-3cf))}{\sqrt{c}(de-cf)}}{2e(de-cf)}-\frac{f\cdots}{}}{2e(de-cf)}}{2c(de-cf)}\right.}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(b^2*(-1/4*(f*x)/(e*(b*e - a*f)*(e + f*x^2)^2) + (-1/2*(f*(7*b*e - 3*a*f)*
x)/(e*(b*e - a*f)*(e + f*x^2)) + ((8*b^(5/2)*e^2*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[a]*(b*e - a*f)) - (Sqrt[f]*(15*b^2*e^2 - 10*a*b*e*f + 3*a^2*f^2)
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f)))/(2*e*(b*e - a*f)))/(4
*e*(b*e - a*f))))/(b*c - a*d)^2 - (d*((d*(b*c - a*d)*x)/(2*c*(d*e - c*f)*(
c + d*x^2)*(e + f*x^2)^2) + (-1/2*(f*(a*d*(2*d*e + c*f) - b*c*(d*e + 2*c*f
))*x)/(e*(d*e - c*f)*(e + f*x^2)^2) + (-1/2*(f*(a*d*(4*d^2*e^2 + 11*c*d*e*
f - 3*c^2*f^2) + 3*b*c*(d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*x)/(e*(d*e - c*f
)*(e + f*x^2)) - ((4*d^(5/2)*e^2*(a*d*(d*e - 7*c*f) - 3*b*c*(d*e - 3*c*f))
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) + (c*Sqrt[f]*(a*d*f*(3
5*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2) + 3*b*(5*d^3*e^3 - 20*c*d^2*e^2*f + 9*
c^2*d*e*f^2 - 2*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f
)))/(2*e*(d*e - c*f)))/(2*e*(d*e - c*f)))/(2*c*(d*e - c*f))))/(b*c - a*d)^
2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.36

method	result
default	$-\frac{b^{5} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{(a d-b c)^{2} (a f-b e)^{3} \sqrt{a b}}+\frac{f^{3} \left(\frac{f \left(3 a^{2} c^{2} f^{4}-14 a^{2} c d e f^{3}+11 a^{2} d^{2} e^{2} f^{2}-10 a b c^{2} e f^{3}+36 a b c d e^{2} f^{2}-26 a b d^{2} e^{3} f+7 b^{2} c^{2} e^{2} f^{2}-22 b^{2} c d e^{3} f+\right.}{8 e^{2}}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^5/(a*d-b*c)^2/(a*f-b*e)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^3/(c*f-
d*e)^4/(a*f-b*e)^3*((1/8*f*(3*a^2*c^2*f^4-14*a^2*c*d*e*f^3+11*a^2*d^2*e^2*
f^2-10*a*b*c^2*e*f^3+36*a*b*c*d*e^2*f^2-26*a*b*d^2*e^3*f+7*b^2*c^2*e^2*f^2
-22*b^2*c*d*e^3*f+15*b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-18*a^2*c*d*e*
f^3+13*a^2*d^2*e^2*f^2-14*a*b*c^2*e*f^3+44*a*b*c*d*e^2*f^2-30*a*b*d^2*e^3*
f+9*b^2*c^2*e^2*f^2-26*b^2*c*d*e^3*f+17*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*
(3*a^2*c^2*f^4-14*a^2*c*d*e*f^3+35*a^2*d^2*e^2*f^2-10*a*b*c^2*e*f^3+52*a*b
*c*d*e^2*f^2-90*a*b*d^2*e^3*f+15*b^2*c^2*e^2*f^2-54*b^2*c*d*e^3*f+63*b^2*d
^2*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d^4/(a*d-b*c)^2/(c*f-d*e)
^4*(1/2*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c*x/(d*x^2+c)+1/2*(7*a*c*d*f-a*d
^2*e-9*b*c^2*f+3*b*c*d*e)/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(367) = 734$.

Time = 0.15 (sec) , antiderivative size = 1240, normalized size of antiderivative = 3.10

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b^5*arctan(b*x/sqrt(a*b))/((b^5*c^2*e^3 - 2*a*b^4*c*d*e^3 + a^2*b^3*d^2*e^
3 - 3*a*b^4*c^2*e^2*f + 6*a^2*b^3*c*d*e^2*f - 3*a^3*b^2*d^2*e^2*f + 3*a^2*
b^3*c^2*e*f^2 - 6*a^3*b^2*c*d*e*f^2 + 3*a^4*b*d^2*e*f^2 - a^3*b^2*c^2*f^3
+ 2*a^4*b*c*d*f^3 - a^5*d^2*f^3)*sqrt(a*b)) - 1/2*d^4*x/((b*c^2*d^3*e^3 -
a*c*d^4*e^3 - 3*b*c^3*d^2*e^2*f + 3*a*c^2*d^3*e^2*f + 3*b*c^4*d*e*f^2 - 3*
a*c^3*d^2*e*f^2 - b*c^5*f^3 + a*c^4*d*f^3)*(d*x^2 + c)) - 1/2*(3*b*c*d^5*e
 - a*d^6*e - 9*b*c^2*d^4*f + 7*a*c*d^5*f)*arctan(d*x/sqrt(c*d))/((b^2*c^3*
d^4*e^4 - 2*a*b*c^2*d^5*e^4 + a^2*c*d^6*e^4 - 4*b^2*c^4*d^3*e^3*f + 8*a*b*
c^3*d^4*e^3*f - 4*a^2*c^2*d^5*e^3*f + 6*b^2*c^5*d^2*e^2*f^2 - 12*a*b*c^4*d
^3*e^2*f^2 + 6*a^2*c^3*d^4*e^2*f^2 - 4*b^2*c^6*d*e*f^3 + 8*a*b*c^5*d^2*e*f
^3 - 4*a^2*c^4*d^3*e*f^3 + b^2*c^7*f^4 - 2*a*b*c^6*d*f^4 + a^2*c^5*d^2*f^4
)*sqrt(c*d)) - 1/8*(63*b^2*d^2*e^4*f^3 - 54*b^2*c*d*e^3*f^4 - 90*a*b*d^2*e
^3*f^4 + 15*b^2*c^2*e^2*f^5 + 52*a*b*c*d*e^2*f^5 + 35*a^2*d^2*e^2*f^5 - 10
*a*b*c^2*e*f^6 - 14*a^2*c*d*e*f^6 + 3*a^2*c^2*f^7)*arctan(f*x/sqrt(e*f))/(
(b^3*d^4*e^9 - 4*b^3*c*d^3*e^8*f - 3*a*b^2*d^4*e^8*f + 6*b^3*c^2*d^2*e^7*f
^2 + 12*a*b^2*c*d^3*e^7*f^2 + 3*a^2*b*d^4*e^7*f^2 - 4*b^3*c^3*d*e^6*f^3 -
18*a*b^2*c^2*d^2*e^6*f^3 - 12*a^2*b*c*d^3*e^6*f^3 - a^3*d^4*e^6*f^3 + b^3*
c^4*e^5*f^4 + 12*a*b^2*c^3*d*e^5*f^4 + 18*a^2*b*c^2*d^2*e^5*f^4 + 4*a^3*c*
d^3*e^5*f^4 - 3*a*b^2*c^4*e^4*f^5 - 12*a^2*b*c^3*d*e^4*f^5 - 6*a^3*c^2*d^2
*e^4*f^5 + 3*a^2*b*c^4*e^3*f^6 + 4*a^3*c^3*d*e^3*f^6 - a^3*c^4*e^2*f^7)...
```

Mupad [B] (verification not implemented)

Time = 33.86 (sec) , antiderivative size = 212642, normalized size of antiderivative = 531.60

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
symsum(log(- root(61018521600*a^13*b^8*c^11*d^16*e^21*f^12*z^6 + 610185216
00*a^9*b^12*c^19*d^8*e^17*f^16*z^6 - 839516160*a^14*b^7*c^22*d^5*e^9*f^24*
z^6 - 839516160*a^8*b^13*c^8*d^19*e^29*f^4*z^6 - 22304522240*a^17*b^4*c^14
*d^13*e^14*f^19*z^6 - 22304522240*a^16*b^5*c^15*d^12*e^14*f^19*z^6 - 22304
522240*a^6*b^15*c^15*d^12*e^24*f^9*z^6 - 22304522240*a^5*b^16*c^16*d^11*e^
24*f^9*z^6 + 35188113408*a^14*b^7*c^13*d^14*e^18*f^15*z^6 + 35188113408*a^
8*b^13*c^17*d^10*e^20*f^13*z^6 + 314353385472*a^12*b^9*c^16*d^11*e^17*f^16
*z^6 + 314353385472*a^10*b^11*c^14*d^13*e^21*f^12*z^6 + 818413568*a^12*b^9
*c^22*d^5*e^11*f^22*z^6 + 818413568*a^10*b^11*c^8*d^19*e^27*f^6*z^6 - 8016
36352*a^12*b^9*c^6*d^21*e^27*f^6*z^6 - 801636352*a^10*b^11*c^24*d^3*e^11*f
^22*z^6 + 750059520*a^15*b^6*c^21*d^6*e^9*f^24*z^6 + 750059520*a^7*b^14*c^
9*d^18*e^29*f^4*z^6 + 39332085760*a^11*b^10*c^20*d^7*e^14*f^19*z^6 + 39332
085760*a^11*b^10*c^10*d^17*e^24*f^9*z^6 - 9264955392*a^17*b^4*c^16*d^11*e^
12*f^21*z^6 - 9264955392*a^16*b^5*c^16*d^11*e^13*f^20*z^6 - 9264955392*a^6
*b^15*c^14*d^13*e^25*f^8*z^6 - 9264955392*a^5*b^16*c^14*d^13*e^26*f^7*z^6
+ 620232704*a^2*b^19*c^22*d^5*e^21*f^12*z^6 - 619970560*a^16*b^5*c^5*d^22*
e^24*f^9*z^6 - 619970560*a^6*b^15*c^25*d^2*e^14*f^19*z^6 + 13473546240*a^1
2*b^9*c^8*d^19*e^25*f^8*z^6 + 13473546240*a^10*b^11*c^22*d^5*e^13*f^20*z^6
 - 583532544*a^18*b^3*c^9*d^18*e^18*f^15*z^6 - 583532544*a^4*b^17*c^21*d^6
*e^20*f^13*z^6 - 129415249920*a^15*b^6*c^12*d^15*e^18*f^15*z^6 - 129415...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10641, normalized size of antiderivative = 26.60

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
( - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**7*e**5*f**4 -
16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**7*e**4*f**5*x**2
- 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**7*e**3*f**6*x**4
 + 32*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**6*d*e**6*f**3
+ 56*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**6*d*e**5*f**4*x
**2 + 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**6*d*e**4*f*
*5*x**4 - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**6*d*e**3
*f**6*x**6 - 48*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**5*d*
*2*e**7*f**2 - 64*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**5*
d**2*e**6*f**3*x**2 + 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**
4*c**5*d**2*e**5*f**4*x**4 + 32*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**4*c**5*d**2*e**4*f**5*x**6 + 32*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*b**4*c**4*d**3*e**8*f + 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**4*c**4*d**3*e**7*f**2*x**2 - 64*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*b**4*c**4*d**3*e**6*f**3*x**4 - 48*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*b**4*c**4*d**3*e**5*f**4*x**6 - 8*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**3*d**4*e**9 + 16*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**3*d**4*e**8*f*x**2 + 56*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**3*d**4*e**7*f**2*x**4 + 32*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**3*d**4*e**6*f**3*x**6 - 8...
```

3.258 $\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 532

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = -\frac{d^4x}{4c(bc-ad)(de-cf)^3(c+dx^2)^2} - \frac{d^4(bc(7de-19cf)-3ad(de-5cf))x}{8c^2(bc-ad)^2(de-cf)^4(c+dx^2)} + \frac{f^4x}{4e(be-af)(de-cf)^3(e+fx^2)^2} + \frac{f^4(be(19de-7cf)-3af(5de-cf))x}{8e^2(be-af)^2(de-cf)^4(e+fx^2)} + \frac{b^{11/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3(be-af)^3} - \frac{d^{7/2}(3a^2d^2(d^2e^2-6cdef+21c^2f^2)+3b^2c^2(5d^2e^2-22cdef+33c^2f^2)-2abcd(5d^2e^2-34cdef+77c}{8c^{5/2}(bc-ad)^3(de-cf)^5} + \frac{f^{7/2}(3a^2f^2(21d^2e^2-6cdef+c^2f^2)-2abef(77d^2e^2-34cdef+5c^2f^2)+3b^2e^2(33d^2e^2-22cdef+5c}{8e^{5/2}(be-af)^3(de-cf)^5}$$

output
```
-1/4*d^4*x/c/(-a*d+b*c)/(-c*f+d*e)^3/(d*x^2+c)^2-1/8*d^4*(b*c*(-19*c*f+7*d
*e)-3*a*d*(-5*c*f+d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*e)^4/(d*x^2+c)+1/4*f^4*
x/e/(-a*f+b*e)/(-c*f+d*e)^3/(f*x^2+e)^2+1/8*f^4*(b*e*(-7*c*f+19*d*e)-3*a*f
*(-c*f+5*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^4/(f*x^2+e)+b^(11/2)*arctan(b
^(1/2)*x/a^(1/2))/a^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^3-1/8*d^(7/2)*(3*a^2*d^2
*(21*c^2*f^2-6*c*d*e*f+d^2*e^2)+3*b^2*c^2*(33*c^2*f^2-22*c*d*e*f+5*d^2*e^2
)-2*a*b*c*d*(77*c^2*f^2-34*c*d*e*f+5*d^2*e^2))*arctan(d^(1/2)*x/c^(1/2))/c
^(5/2)/(-a*d+b*c)^3/(-c*f+d*e)^5+1/8*f^(7/2)*(3*a^2*f^2*(c^2*f^2-6*c*d*e*f
+21*d^2*e^2)-2*a*b*e*f*(5*c^2*f^2-34*c*d*e*f+77*d^2*e^2)+3*b^2*e^2*(5*c^2*
f^2-22*c*d*e*f+33*d^2*e^2))*arctan(f^(1/2)*x/e^(1/2))/e^(5/2)/(-a*f+b*e)^3
/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \frac{1}{8}\Bigg(\frac{2d^4x}{c(bc-ad)(-de+cf)^3(c+dx^2)^2} + \frac{d^4(3ad(de-5cf)+bc(-7de+19cf))x}{c^2(bc-ad)^2(de-cf)^4(c+dx^2)} + \frac{2f^4x}{e(be-af)(de-cf)^3(e+fx^2)^2} + \frac{f^4(be(19de-7cf)+3af(-5de+cf))x}{e^2(be-af)^2(de-cf)^4(e+fx^2)} + \frac{8b^{11/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3(be-af)^3} + \frac{d^{7/2}(3a^2d^2(d^2e^2-6cdef+21c^2f^2)+3b^2c^2(5d^2e^2-22cdef+33c^2f^2)-2abcd(5d^2e^2-34cdef+77c}{c^{5/2}(bc-ad)^3(-de+cf)^5} + \frac{f^{7/2}(3a^2f^2(21d^2e^2-6cdef+c^2f^2)-2abef(77d^2e^2-34cdef+5c^2f^2)+3b^2e^2(33d^2e^2-22cdef+5}{e^{5/2}(be-af)^3(de-cf)^5}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
((2*d^4*x)/(c*(b*c - a*d)*(-(d*e) + c*f)^3*(c + d*x^2)^2) + (d^4*(3*a*d*(d
*e - 5*c*f) + b*c*(-7*d*e + 19*c*f))*x)/(c^2*(b*c - a*d)^2*(d*e - c*f)^4*(
c + d*x^2)) + (2*f^4*x)/(e*(b*e - a*f)*(d*e - c*f)^3*(e + f*x^2)^2) + (f^4
*(b*e*(19*d*e - 7*c*f) + 3*a*f*(-5*d*e + c*f))*x)/(e^2*(b*e - a*f)^2*(d*e
- c*f)^4*(e + f*x^2)) + (8*b^(11/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*
(b*c - a*d)^3*(b*e - a*f)^3) + (d^(7/2)*(3*a^2*d^2*(d^2*e^2 - 6*c*d*e*f +
21*c^2*f^2) + 3*b^2*c^2*(5*d^2*e^2 - 22*c*d*e*f + 33*c^2*f^2) - 2*a*b*c*d*
(5*d^2*e^2 - 34*c*d*e*f + 77*c^2*f^2))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/
2)*(b*c - a*d)^3*(-(d*e) + c*f)^5) + (f^(7/2)*(3*a^2*f^2*(21*d^2*e^2 - 6*c
*d*e*f + c^2*f^2) - 2*a*b*e*f*(77*d^2*e^2 - 34*c*d*e*f + 5*c^2*f^2) + 3*b^
2*e^2*(33*d^2*e^2 - 22*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(e^(5/2)*(b*e - a*f)^3*(d*e - c*f)^5))/8
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 1052, normalized size of antiderivative = 1.98, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.679, Rules used = {421, 402, 25, 402, 402, 27, 402, 27, 397, 218, 421, 402, 402, 25, 397, 218, 422, 303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^3(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}-\frac{\int -\frac{7d(bc-ad)fx^2+bc(7de-8cf)-ad(3de-4cf)}{(dx^2+c)^2(fx^2+e)^3}dx}{4c(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{7d(bc-ad)fx^2+bc(7de-8cf)-ad(3de-4cf)}{\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{4c(de-cf)}+\frac{dx(bc-ad)}{4c(c+dx^2)^2(e+fx^2)^2(de-cf)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{\int \frac{-5df(bc(7de-15cf)-ad(3de-11cf))x^2+ad\left(3d^2e^2-3cdfe+8c^2f^2\right)-bc\left(7d^2e^2-15cdfe+16c^2f^2\right)}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{2c(de-cf)}}{4c(de-cf)}+\frac{d \cdots}{4c(c+dx^2) \cdots}}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{\frac{\int -\frac{4\left(3df\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bc\left(7d^3e^3-29cd^2fe^2+46c^2df^2e-12c^3f^3\right)-3a \cdots}{\left(dx^2+c\right)\left(fx^2+e\right)^2}}{4e(de-cf)} \cdots}{2c(de-cf)}}{4c(de-cf)} \cdots}{(bc-ad) \cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{\int \frac{3df\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x^2+bc\left(7d^3e^3-29cd^2fe^2+46c^2df^2e-12c^3f^3\right)-3ad \cdots}{\left(dx^2+c\right)\left(fx^2+e\right)^2}}{e(de-cf)} \cdots}{2c(de-cf)}}{4c(de-cf)} \cdots}{(bc-ad)^2}$$

$\Big\downarrow$ 402

$$d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{\int -\frac{2\left(df\left(3ad\left(d^3e^3-5cd^2fe^2-5c^2df^2e+c^3f^3\right)-bc\left(7d^3e^3-8cd^2fe^2-29c^2df^2e+6c^3f^3\right)\right)x^2+3ad\left(d^4e^4-5cd^3fe^3+\right.}{\left(dx^2+c\right)\left(fx^2+e\right)}}{2e(de-cf)}}{}}{}\right.$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$\Big\downarrow$ 27

$$d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{-\frac{\int \frac{df\left(3ad\left(d^3e^3-5cd^2fe^2-5c^2df^2e+c^3f^3\right)-bc\left(7d^3e^3-8cd^2fe^2-29c^2df^2e+6c^3f^3\right)\right)x^2+3ad\left(d^4e^4-5cd^3fe^3+16c\right.}{\left(dx^2+c\right)\left(fx^2+e\right)}}{e(de-cf)}}{}}{}}{}\right.$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$\Big\downarrow$ 397

$$d\left(\frac{\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{-\frac{\frac{d^3e^2\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)-bc\left(91c^2f^2-50cdef+7d^2e^2\right)\right)\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{c^2f^2\left(3adf\left(c^2f^2-6cdef+21d^2e^2\right)\right.}{}}{e(de-cf)}}{}}{}}{}\right.$$

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$\Big\downarrow$ 218

$$\frac{b^2\int \frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-$$

$$d\left(\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{-\frac{d^{5/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)-bc\left(91c^2f^2-50cdef+7d^2e^2\right)\right)}{\sqrt{c}(de-cf)}-\frac{c^2f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(3adf\left(\right.\right.}{e(de-cf)}}{}}{}\right.$$

$\downarrow$ 421

$$\frac{b^2\left(\frac{b^2\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{f\int \frac{bfx^2+2be-af}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$d\left(\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c\left(c+dx^2\right)\left(e+fx^2\right)^2(de-cf)}-\frac{-\frac{-\frac{d^{5/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)-bc\left(91c^2f^2-50cdef+7d^2e^2\right)\right)}{\sqrt{c}(de-cf)}-\frac{c^2f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(3adf\left(\right.\right.}{e(de-cf)}}{}}{}\right.$$

$\downarrow$ 402

$$\frac{b^2\left(\frac{b^2\int\frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{\int\frac{-3df(be-af)x^2+be(8de-7cf)-af(4de-3cf)}{(dx^2+c)(fx^2+e)^2}dx}{4e(de-cf)}-\frac{fx(be-af)}{4e(e+fx^2)^2(de-cf)}\right)}{(be-af)^2}\right)}{(bc-ad)^2}-$$

$$\frac{d\left(\frac{dx(bc(7de-15cf)-ad(3de-11cf))}{2c(c+dx^2)(e+fx^2)^2(de-cf)}-\frac{-\frac{-\frac{d^{5/2}e^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(3ad\left(21c^2f^2-6cdef+d^2e^2\right)-bc\left(91c^2f^2-50cdef+7d^2e^2\right)\right)}{\sqrt{c}(de-cf)}-\frac{c^2f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(3adf(\right.}{e(de-cf)}}{\ldots}}{\ldots}\right.}{\ldots}$$

$\downarrow$ 402

$$\frac{b^2\left(\frac{b^2\int\frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{\frac{\int-\frac{df(be(11de-7cf)-af(7de-3cf))x^2+af\left(8d^2e^2-7cdfe+3c^2f^2\right)-be\left(16d^2e^2-19cdfe+7c^2f^2\right)}{(dx^2+c)(fx^2+e)}dx}{2e(de-cf)}-\frac{f(be(11\ldots}{\ldots}}{4e(de-cf)}\ldots\right.}{(be-af)^2}\ldots\right.}{(bc-ad)^2}$$

$$\frac{d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)(dx^2+c)(fx^2+e)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)(fx^2+e)^2}-\frac{-\frac{f\left(3ad\left(d^3e^3\ldots\right.\right.}{\ldots}}{\ldots}}{\ldots}}{\ldots}\right.}{\ldots}$$

$\downarrow$ 25

$$b^2\left(\frac{b^2\int\frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)(fx^2+e)}-\frac{\int\frac{df(be(11de-7cf)-af(7de-3cf))x^2+af\left(8d^2e^2-7cdfe+3c^2f^2\right)-be\left(1\cdots\right.}{(dx^2+c)(fx^2+e)}}{2e(de-cf)}}{4e(de-cf)}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad (bc-ad)^2 \qquad}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)(dx^2+c)(fx^2+e)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)(fx^2+e)^2}-\frac{f\left(3ad\left(d^3e^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 397$$

$$b^2\left(\frac{b^2\int\frac{1}{(bx^2+a)(dx^2+c)(fx^2+e)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)(fx^2+e)}-\frac{-\frac{8d^2(2bde-bcf-adf)\int\frac{1}{dx^2+c}dxe^2}{de-cf}-\frac{f\left(af\left(15d^2e^2-10cdfe+3c^2f^2\right)-\cdots\right.}{\cdots}}{2e(de-cf)}}{4e(de-cf)}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad (bc-ad)^2 \qquad}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)(dx^2+c)(fx^2+e)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)(fx^2+e)^2}-\frac{f\left(3ad\left(d^3e^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 218

$$b^2\left(\frac{b^2\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{f\left(\frac{-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)\left(fx^2+e\right)}-\frac{-\frac{8d^{3/2}(2bde-bcf-adf)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)e^2}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\left(af\left(15d^2e^2-10cdfe+3c\right.\right.}{2e(de-cf)}}{4e(de-cf)}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad\qquad (bc-ad)^2 \qquad\qquad}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}-\frac{-\frac{f\left(3ad\left(d^3e^3\right.\right.}{}}{}}{}}{}\right.$$

$\downarrow$ 422

$$b^2\left(\frac{b^2\left(\frac{b\int\frac{1}{(bx^2+a)(fx^2+e)}dx}{bc-ad}-\frac{d\int\frac{1}{(dx^2+c)(fx^2+e)}dx}{bc-ad}\right)}{(be-af)^2}-\frac{f\left(\frac{-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)(fx^2+e)}-\frac{-\frac{8d^{3/2}(2bde-bcf-adf)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)e^2}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\cdots}{\cdots}}{4e(de-cf\cdots}}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad (bc-ad)^2 \qquad}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)(dx^2+c)(fx^2+e)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)(fx^2+e)^2}-\frac{-\frac{f\left(3ad\left(d^3e^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 303

$$b^2\left(\frac{b^2\left(\frac{b\left(\frac{b\int\frac{1}{bx^2+a}dx}{be-af}-\frac{f\int\frac{1}{fx^2+e}dx}{be-af}\right)}{bc-ad}-\frac{d\left(\frac{d\int\frac{1}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{1}{fx^2+e}dx}{de-cf}\right)}{bc-ad}\right)}{(be-af)^2}-\frac{f\left(\frac{-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)(fx^2+e)}-\frac{-\frac{8d^{3/2}(2bde-bcf-adf)\,\mathrm{arc}\cdots}{\sqrt{c}(de-cf)}}{\cdots}}{\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad (bc-ad)^2 \qquad}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)(dx^2+c)(fx^2+e)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)(fx^2+e)^2}-\frac{-\frac{f\left(3ad\left(d^3e^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 218$$

$$b^2\left(\frac{b^2\left(\frac{b\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}\right)}{bc-ad}-\frac{d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}\right)}{bc-ad}\right)}{(be-af)^2}-\frac{f\left(-\frac{f(be(11de-7cf)-af(7de-3cf))x}{2e(de-cf)\left(fx^2+e\right)}-\frac{-\frac{8d^{3/2}(\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad (bc-ad)^2}$$

$$d\left(\frac{d(bc-ad)x}{4c(de-cf)(dx^2+c)^2(fx^2+e)^2}+\frac{\frac{d(bc(7de-15cf)-ad(3de-11cf))x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)^2}-\frac{-\frac{f\left(bc\left(7d^2e^2-15cdfe-4c^2f^2\right)-ad\left(3d^2e^2-13cdfe-2c^2f^2\right)\right)x}{e(de-cf)\left(fx^2+e\right)^2}-\frac{-\frac{f\left(3ad\left(d^3e^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(b^2*((b^2*((b*((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*e - a*f)
) - (Sqrt[f]*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f))))/(b*c - a
*d) - (d*((Sqrt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (S
qrt[f]*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f))))/(b*c - a*d)))/
(b*e - a*f)^2 - (f*(-1/4*(f*(b*e - a*f)*x)/(e*(d*e - c*f)*(e + f*x^2)^2) +
 (-1/2*(f*(b*e*(11*d*e - 7*c*f) - a*f*(7*d*e - 3*c*f))*x)/(e*(d*e - c*f)*(
e + f*x^2)) - ((-8*d^(3/2)*e^2*(2*b*d*e - b*c*f - a*d*f)*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (Sqrt[f]*(a*f*(15*d^2*e^2 - 10*c*d*e*f
 + 3*c^2*f^2) - b*e*(27*d^2*e^2 - 26*c*d*e*f + 7*c^2*f^2))*ArcTan[(Sqrt[f]
*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f)))/(4*e*(d*e - c*f)))
)/(b*e - a*f)^2))/(b*c - a*d)^2 - (d*((d*(b*c - a*d)*x)/(4*c*(d*e - c*f)*(
c + d*x^2)^2*(e + f*x^2)^2) + ((d*(b*c*(7*d*e - 15*c*f) - a*d*(3*d*e - 11*
c*f))*x)/(2*c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)^2) - (-((f*(b*c*(7*d^2*e
^2 - 15*c*d*e*f - 4*c^2*f^2) - a*d*(3*d^2*e^2 - 13*c*d*e*f - 2*c^2*f^2))*x
)/(e*(d*e - c*f)*(e + f*x^2)^2)) - (-((f*(3*a*d*(d^3*e^3 - 5*c*d^2*e^2*f -
 5*c^2*d*e*f^2 + c^3*f^3) - b*c*(7*d^3*e^3 - 8*c*d^2*e^2*f - 29*c^2*d*e*f^
2 + 6*c^3*f^3))*x)/(e*(d*e - c*f)*(e + f*x^2))) - ((d^(5/2)*e^2*(3*a*d*(d^
2*e^2 - 6*c*d*e*f + 21*c^2*f^2) - b*c*(7*d^2*e^2 - 50*c*d*e*f + 91*c^2*f^2
))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (c^2*f^(3/2)*(3*a*
d*f*(21*d^2*e^2 - 6*c*d*e*f + c^2*f^2) + b*(35*d^3*e^3 - 112*c*d^2*e^2*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.55

method	result
default	$\frac{b^6 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^3(af-be)^3\sqrt{ab}}+\frac{f^4\left(\frac{f\left(3a^2c^2f^4-18a^2cde\,f^3+15a^2d^2e^2f^2-10ab\,c^2e\,f^3+44abcd\,e^2f^2-34ab\,d^2e^3f+7b^2c^2e^2f^2-26b^2cd\,e^3f+19b\right.}{8e^2}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
b^6/(a*d-b*c)^3/(a*f-b*e)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+f^4/(a*f-b
*e)^3/(c*f-d*e)^5*((1/8*f*(3*a^2*c^2*f^4-18*a^2*c*d*e*f^3+15*a^2*d^2*e^2*f
^2-10*a*b*c^2*e*f^3+44*a*b*c*d*e^2*f^2-34*a*b*d^2*e^3*f+7*b^2*c^2*e^2*f^2-
26*b^2*c*d*e^3*f+19*b^2*d^2*e^4)/e^2*x^3+1/8*(5*a^2*c^2*f^4-22*a^2*c*d*e*f
^3+17*a^2*d^2*e^2*f^2-14*a*b*c^2*e*f^3+52*a*b*c*d*e^2*f^2-38*a*b*d^2*e^3*f
+9*b^2*c^2*e^2*f^2-30*b^2*c*d*e^3*f+21*b^2*d^2*e^4)/e*x)/(f*x^2+e)^2+1/8*(
3*a^2*c^2*f^4-18*a^2*c*d*e*f^3+63*a^2*d^2*e^2*f^2-10*a*b*c^2*e*f^3+68*a*b*
c*d*e^2*f^2-154*a*b*d^2*e^3*f+15*b^2*c^2*e^2*f^2-66*b^2*c*d*e^3*f+99*b^2*d
^2*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))-d^4/(c*f-d*e)^5/(a*d-b*c)
^3*((1/8*d*(15*a^2*c^2*d^2*f^2-18*a^2*c*d^3*e*f+3*a^2*d^4*e^2-34*a*b*c^3*d
*f^2+44*a*b*c^2*d^2*e*f-10*a*b*c*d^3*e^2+19*b^2*c^4*f^2-26*b^2*c^3*d*e*f+7
*b^2*c^2*d^2*e^2)/c^2*x^3+1/8*(17*a^2*c^2*d^2*f^2-22*a^2*c*d^3*e*f+5*a^2*d
^4*e^2-38*a*b*c^3*d*f^2+52*a*b*c^2*d^2*e*f-14*a*b*c*d^3*e^2+21*b^2*c^4*f^2
-30*b^2*c^3*d*e*f+9*b^2*c^2*d^2*e^2)/c*x)/(d*x^2+c)^2+1/8*(63*a^2*c^2*d^2*
f^2-18*a^2*c*d^3*e*f+3*a^2*d^4*e^2-154*a*b*c^3*d*f^2+68*a*b*c^2*d^2*e*f-10
*a*b*c*d^3*e^2+99*b^2*c^4*f^2-66*b^2*c^3*d*e*f+15*b^2*c^2*d^2*e^2)/c^2/(c*
d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3373 vs. 2(496) = 992.

Time = 0.20 (sec) , antiderivative size = 3373, normalized size of antiderivative = 6.34

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b^6*arctan(b*x/sqrt(a*b))/((b^6*c^3*e^3 - 3*a*b^5*c^2*d*e^3 + 3*a^2*b^4*c*
d^2*e^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*
a^3*b^3*c*d^2*e^2*f + 3*a^4*b^2*d^3*e^2*f + 3*a^2*b^4*c^3*e*f^2 - 9*a^3*b^
3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2 - 3*a^5*b*d^3*e*f^2 - a^3*b^3*c^3*f^
3 + 3*a^4*b^2*c^2*d*f^3 - 3*a^5*b*c*d^2*f^3 + a^6*d^3*f^3)*sqrt(a*b)) - 1/
8*(15*b^2*c^2*d^6*e^2 - 10*a*b*c*d^7*e^2 + 3*a^2*d^8*e^2 - 66*b^2*c^3*d^5*
e*f + 68*a*b*c^2*d^6*e*f - 18*a^2*c*d^7*e*f + 99*b^2*c^4*d^4*f^2 - 154*a*b
*c^3*d^5*f^2 + 63*a^2*c^2*d^6*f^2)*arctan(d*x/sqrt(c*d))/((b^3*c^5*d^5*e^5
 - 3*a*b^2*c^4*d^6*e^5 + 3*a^2*b*c^3*d^7*e^5 - a^3*c^2*d^8*e^5 - 5*b^3*c^6
*d^4*e^4*f + 15*a*b^2*c^5*d^5*e^4*f - 15*a^2*b*c^4*d^6*e^4*f + 5*a^3*c^3*d
^7*e^4*f + 10*b^3*c^7*d^3*e^3*f^2 - 30*a*b^2*c^6*d^4*e^3*f^2 + 30*a^2*b*c^
5*d^5*e^3*f^2 - 10*a^3*c^4*d^6*e^3*f^2 - 10*b^3*c^8*d^2*e^2*f^3 + 30*a*b^2
*c^7*d^3*e^2*f^3 - 30*a^2*b*c^6*d^4*e^2*f^3 + 10*a^3*c^5*d^5*e^2*f^3 + 5*b
^3*c^9*d*e*f^4 - 15*a*b^2*c^8*d^2*e*f^4 + 15*a^2*b*c^7*d^3*e*f^4 - 5*a^3*c
^6*d^4*e*f^4 - b^3*c^10*f^5 + 3*a*b^2*c^9*d*f^5 - 3*a^2*b*c^8*d^2*f^5 + a^
3*c^7*d^3*f^5)*sqrt(c*d)) + 1/8*(99*b^2*d^2*e^4*f^4 - 66*b^2*c*d*e^3*f^5 -
 154*a*b*d^2*e^3*f^5 + 15*b^2*c^2*e^2*f^6 + 68*a*b*c*d*e^2*f^6 + 63*a^2*d^
2*e^2*f^6 - 10*a*b*c^2*e*f^7 - 18*a^2*c*d*e*f^7 + 3*a^2*c^2*f^8)*arctan(f*
x/sqrt(e*f))/((b^3*d^5*e^10 - 5*b^3*c*d^4*e^9*f - 3*a*b^2*d^5*e^9*f + 10*b
^3*c^2*d^3*e^8*f^2 + 15*a*b^2*c*d^4*e^8*f^2 + 3*a^2*b*d^5*e^8*f^2 - 10*...
```

Mupad [B] (verification not implemented)

Time = 69.16 (sec) , antiderivative size = 332957, normalized size of antiderivative = 625.86

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
symsum(log(root(12589439385600*a^18*b^7*c^24*d^13*e^13*f^24*z^6 + 12589439
385600*a^18*b^7*c^13*d^24*e^24*f^13*z^6 + 12589439385600*a^8*b^17*c^29*d^8
*e^18*f^19*z^6 + 12589439385600*a^8*b^17*c^18*d^19*e^29*f^8*z^6 - 62419639
468032*a^17*b^8*c^22*d^15*e^16*f^21*z^6 - 62419639468032*a^17*b^8*c^16*d^2
1*e^22*f^15*z^6 - 62419639468032*a^9*b^16*c^26*d^11*e^20*f^17*z^6 - 624196
39468032*a^9*b^16*c^20*d^17*e^26*f^11*z^6 + 1272179589120*a^20*b^5*c^22*d^
15*e^13*f^24*z^6 + 1272179589120*a^20*b^5*c^13*d^24*e^22*f^15*z^6 + 127217
9589120*a^6*b^19*c^29*d^8*e^20*f^17*z^6 + 1272179589120*a^6*b^19*c^20*d^17
*e^29*f^8*z^6 + 2607913369600*a^22*b^3*c^19*d^18*e^14*f^23*z^6 + 260791336
9600*a^22*b^3*c^14*d^23*e^19*f^18*z^6 + 2607913369600*a^4*b^21*c^28*d^9*e^
23*f^14*z^6 + 2607913369600*a^4*b^21*c^23*d^14*e^28*f^9*z^6 - 168343633920
*a^21*b^4*c^24*d^13*e^10*f^27*z^6 - 168343633920*a^21*b^4*c^10*d^27*e^24*f
^13*z^6 - 168343633920*a^5*b^20*c^32*d^5*e^18*f^19*z^6 - 168343633920*a^5*
b^20*c^18*d^19*e^32*f^5*z^6 + 93437810442240*a^16*b^9*c^20*d^17*e^19*f^18*
z^6 + 93437810442240*a^16*b^9*c^19*d^18*e^20*f^17*z^6 + 93437810442240*a^1
0*b^15*c^23*d^14*e^22*f^15*z^6 + 93437810442240*a^10*b^15*c^22*d^15*e^23*f
^14*z^6 + 9342812160*a^16*b^9*c^31*d^6*e^8*f^29*z^6 + 9342812160*a^16*b^9*
c^8*d^29*e^31*f^6*z^6 + 9342812160*a^10*b^15*c^34*d^3*e^11*f^26*z^6 + 9342
812160*a^10*b^15*c^11*d^26*e^34*f^3*z^6 + 2353004544*a^7*b^18*c^36*d*e^12*
f^25*z^6 + 2353004544*a^7*b^18*c^12*d^25*e^36*f*z^6 + 18374612484096*a^...
```

Reduce [B] (verification not implemented)

Time = 28.18 (sec) , antiderivative size = 20240, normalized size of antiderivative = 38.05

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output

```
(8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**10*e**5*f**5 + 16
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**10*e**4*f**6*x**2 +
 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**10*e**3*f**7*x**4
 - 40*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**9*d*e**6*f**4
- 64*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**9*d*e**5*f**5*x
**2 - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**9*d*e**4*f**
6*x**4 + 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**9*d*e**3
*f**7*x**6 + 80*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**8*d*
*2*e**7*f**3 + 80*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**8*
d**2*e**6*f**4*x**2 - 72*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**
5*c**8*d**2*e**5*f**5*x**4 - 64*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**5*c**8*d**2*e**4*f**6*x**6 + 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**5*c**8*d**2*e**3*f**7*x**8 - 80*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*b**5*c**7*d**3*e**8*f**2 + 200*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*b**5*c**7*d**3*e**6*f**4*x**4 + 80*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**7*d**3*e**5*f**5*x**6 - 40*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**7*d**3*e**4*f**6*x**8 + 40*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**6*d**4*e**9*f - 80*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**6*d**4*e**8*f**2*x**2 - 20
0*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*c**6*d**4*e**7*f**...
```

3.259 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 338

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx
&= \frac{b^4x}{2a(bc-ad)^2(be-af)^2(a+bx^2)} + \frac{d^4x}{2c(bc-ad)^2(de-cf)^2(c+dx^2)} \\
&+ \frac{f^4x}{2e(be-af)^2(de-cf)^2(e+fx^2)} \\
&+ \frac{b^{7/2}(b^2ce+9a^2df-5ab(de+cf))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3(be-af)^3} \\
&+ \frac{d^{7/2}(bc(5de-9cf)-ad(de-5cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3(de-cf)^3} \\
&+ \frac{f^{7/2}(be(9de-5cf)-af(5de-cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}(be-af)^3(de-cf)^3}
\end{aligned}$$

output
```
1/2*b^4*x/a/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)+1/2*d^4*x/c/(-a*d+b*c)^2/(
-c*f+d*e)^2/(d*x^2+c)+1/2*f^4*x/e/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x^2+e)+1/2*
b^(7/2)*(b^2*c*e+9*a^2*d*f-5*a*b*(c*f+d*e))*arctan(b^(1/2)*x/a^(1/2))/a^(3
/2)/(-a*d+b*c)^3/(-a*f+b*e)^3+1/2*d^(7/2)*(b*c*(-9*c*f+5*d*e)-a*d*(-5*c*f+
d*e))*arctan(d^(1/2)*x/c^(1/2))/c^(3/2)/(-a*d+b*c)^3/(-c*f+d*e)^3+1/2*f^(7
/2)*(b*e*(-5*c*f+9*d*e)-a*f*(-c*f+5*d*e))*arctan(f^(1/2)*x/e^(1/2))/e^(3/2
)/(-a*f+b*e)^3/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{1}{2}\left(\frac{b^4x}{a(bc-ad)^2(be-af)^2(a+bx^2)}+\frac{d^4x}{c(bc-ad)^2(de-cf)^2(c+dx^2)}+\frac{f^4x}{e(be-af)^2(de-cf)^2(e+fx^2)}+\frac{b^{7/2}(b^2ce+9a^2df-5ab(de+cf))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3(be-af)^3}+\frac{d^{7/2}(ad(de-5cf)+bc(-5de+9cf))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^3(-de+cf)^3}+\frac{f^{7/2}(be(9de-5cf)+af(-5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{e^{3/2}(be-af)^3(de-cf)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
((b^4*x)/(a*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x^2)) + (d^4*x)/(c*(b*c - a
*d)^2*(d*e - c*f)^2*(c + d*x^2)) + (f^4*x)/(e*(b*e - a*f)^2*(d*e - c*f)^2*
(e + f*x^2)) + (b^(7/2)*(b^2*c*e + 9*a^2*d*f - 5*a*b*(d*e + c*f))*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^3*(b*e - a*f)^3) + (d^(7/2)*(a*d
*(d*e - 5*c*f) + b*c*(-5*d*e + 9*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/
2)*(b*c - a*d)^3*(-(d*e) + c*f)^3) + (f^(7/2)*(b*e*(9*d*e - 5*c*f) + a*f*(
-5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(e^(3/2)*(b*e - a*f)^3*(d*e -
c*f)^3))/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 850 vs. 2(338) = 676.

Time = 1.19 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.51, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {426, 421, 25, 316, 397, 218, 402, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2} \, dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^2(dx^2+c)(fx^2+e)^2}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\int \frac{1}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^2(fx^2+e)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\left(\frac{b^2\int \frac{1}{(bx^2+a)(fx^2+e)^2}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{(dx^2+c)^2(fx^2+e)^2}dx}{(bc-ad)^2}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\left(\frac{b^2\int \frac{1}{\left(bx^2+a\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}$$

$\downarrow$ 316

$$\frac{b\left(\frac{d^2\left(\frac{\int \frac{-dfx^2+2de-cf}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{2e(de-cf)}-\frac{fx}{2e\left(e+fx^2\right)(de-cf)}\right)}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\left(\frac{b^2\left(\frac{\int \frac{-bfx^2+2be-af}{\left(bx^2+a\right)\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}$$

$\downarrow$ 397

$$\frac{b\left(\frac{d^2\left(\frac{\frac{2d^2e\int \frac{1}{dx^2+c}dx}{de-cf}-\frac{f(3de-cf)\int \frac{1}{fx^2+e}dx}{de-cf}}{2e(de-cf)}-\frac{fx}{2e\left(e+fx^2\right)(de-cf)}\right)}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\left(\frac{b^2\left(\frac{\frac{2b^2e\int \frac{1}{bx^2+a}dx}{be-af}-\frac{f(3be-af)\int \frac{1}{fx^2+e}dx}{be-af}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}$$

$\downarrow$ 218

$$\frac{b\left(\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{d^2\left(\frac{\frac{2d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(3de-cf)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx}{2e\left(e+fx^2\right)(de-cf)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\int \frac{bdx^2+2bc-ad}{\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{2a\left(a+bx^2\right)\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{4dfa^2-b(3de+2cf)a+3b(bc-ad)fx^2+b^2ce}{\left(bx^2+a\right)\left(fx^2+e\right)^2}dx}{2a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\left(\frac{\frac{2d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(3de-cf)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{}{2e(}\right.}{(bc-ad)^2}\right)}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{dx(bc-ad)}{2c\left(c+dx^2\right)\left(e+fx^2\right)(de-cf)}-\frac{\int -\frac{3d(bc-ad)fx^2+bc(3de-4cf)}{\left(dx^2+c\right)\left(fx^2+e\right.}}{2c(de-cf)}\right.}{(bc-ad)^2}\right.}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{\int \frac{4dfa^2-3bdea-2bcfa+3b(bc-ad)fx^2+b^2ce}{\left(bx^2+a\right)\left(fx^2+e\right)^2}dx}{2a(be-af)}+\frac{bx(bc-ad)}{2a\left(a+bx^2\right)\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\left(\frac{\frac{2d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(3de-cf)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{\cdots}{2e\left(\cdots\right.}\cdots\right)}{(bc-ad)^2}\cdots\right)}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\int \frac{3d(bc-ad)fx^2+bc(3de-4cf)-ad(de-2cf)}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{2c(de-cf)}+\frac{dx(bc\cdots}{2c\left(c+dx^2\right)\left(e\cdots\right.}\cdots\right)}{(bc-ad)^2}\cdots\right)}{bc-ad}$$

$\big\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{2\left(-2df^2a^3+bf(7de+cf)a^2-b^2e(3de+4cf)a+b^3ce^2+bf\left(-2dfa^2+bcfa+b^2ce\right)x^2\right)}{\left(bx^2+a\right)\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\left(-2a^2df+abcf+b^2ce\right)}{e\left(e+fx^2\right)(be-af)}}{2a(be-af)}+\frac{bx(bc-ad)}{2a\left(a+bx^2\right)\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}\cdots\right)}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{\int -\frac{2\left(-df\left(2bc^2f-ad(de+cf)\right)x^2+ad\left(d^2e^2-4cdfe+c^2f^2\right)-bc\left(\cdots\right.\right.}{\left(dx^2+c\right)\left(fx^2+e\right)}\cdots}{2e(de-cf)}\cdots}{2c(de-cf)}\cdots\right)}{\cdots}\cdots\right)}{bc-ad}$$

$\big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{-2df^2a^3+bf(7de+cf)a^2-b^2e(3de+4cf)a+b^3ce^2+bf\left(-2dfa^2+bcfa+b^2ce\right)x^2}{\left(bx^2+a\right)\left(fx^2+e\right)}dx}{e(be-af)}+\frac{fx\left(-2a^2df+abcf+b^2ce\right)}{e\left(e+fx^2\right)(be-af)}}{2a(be-af)}+\frac{bx(bc-ad)}{2a\left(a+bx^2\right)\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}+ \cdots\right)}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{\frac{fx\left(2bc^2f-ad(cf+de)\right)}{e\left(e+fx^2\right)(de-cf)}-\frac{\int \frac{-df\left(2bc^2f-ad(de+cf)\right)x^2+ad\left(d^2 \cdots\right.}{\left(dx^2 \cdots\right.}}{\cdots}}{2c(de-cf)} \cdots\right)}{\cdots}\right)}{bc-ad}$$

$$\downarrow 397$$

$$\frac{b\left(\frac{\left(\frac{\frac{2d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}(3de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx}{2e(de-cf)\left(fx^2+e\right)}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{2a(be-af)\left(bx^2+a\right)\left(fx^2+e\right)}+\frac{\frac{f\left(-2dfa^2+bcfa+b^2ce\right)x}{e(be-af)\left(fx^2+e\right)}+\frac{e\left(7\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(be-af)\left(fx^2+e\right)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{d(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)}+\frac{\frac{f\left(2bc^2f-ad(de+cf)\right)x}{e(de-cf)\left(fx^2+e\right)}-\frac{cf\left(ad\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

$$\Big\downarrow \; 218$$

$$\frac{b\left(\frac{\left(\frac{\frac{2d^{3/2}e\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(de-cf)}-\frac{\sqrt{f}(3de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(de-cf)}}{2e(de-cf)}-\frac{fx}{2e(de-cf)\left(fx^2+e\right)}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{2a(be-af)\left(bx^2+a\right)\left(fx^2+e\right)}+\frac{\frac{f\left(-2dfa^2+bcfa+b^2ce\right)x}{e(be-af)\left(fx^2+e\right)}+\frac{e\left(7\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

$$\frac{d\left(\frac{b^2\left(\frac{\frac{2b^{3/2}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(be-af)}-\frac{\sqrt{f}(3be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}(be-af)}}{2e(be-af)}-\frac{fx}{2e(be-af)\left(fx^2+e\right)}\right)}{(bc-ad)^2}-\frac{d\left(\frac{d(bc-ad)x}{2c(de-cf)\left(dx^2+c\right)\left(fx^2+e\right)}+\frac{\frac{f\left(2bc^2f-ad(de+cf)\right)x}{e(de-cf)\left(fx^2+e\right)}-\frac{c\sqrt{f}\left(\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(b*((d^2*(-1/2*(f*x)/(e*(d*e - c*f)*(e + f*x^2)) + ((2*d^(3/2)*e*ArcTan[(S
qrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(d*e - c*f)) - (Sqrt[f]*(3*d*e - c*f)*ArcTan[
(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f)))/(2*e*(d*e - c*f))))/(b*c - a*
d)^2 + (b*((b*(b*c - a*d)*x)/(2*a*(b*e - a*f)*(a + b*x^2)*(e + f*x^2)) + (
(f*(b^2*c*e + a*b*c*f - 2*a^2*d*f)*x)/(e*(b*e - a*f)*(e + f*x^2)) + ((b^(3
/2)*e*(b^2*c*e - 3*a*b*d*e - 5*a*b*c*f + 7*a^2*d*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*(b*e - a*f)) + (a*Sqrt[f]*(2*a^2*d*f^2 - a*b*f*(9*d*e + c*
f) + b^2*e*(3*d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a
*f)))/(e*(b*e - a*f)))/(2*a*(b*e - a*f))))/(b*c - a*d)^2))/(b*c - a*d) - (
d*((b^2*(-1/2*(f*x)/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b^(3/2)*e*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*e - a*f)) - (Sqrt[f]*(3*b*e - a*f)*ArcTan[(
Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(b*e - a*f)))/(2*e*(b*e - a*f))))/(b*c - a*d
)^2 - (d*((d*(b*c - a*d)*x)/(2*c*(d*e - c*f)*(c + d*x^2)*(e + f*x^2)) + ((
f*(2*b*c^2*f - a*d*(d*e + c*f))*x)/(e*(d*e - c*f)*(e + f*x^2)) - (-((d^(3/
2)*e*(b*c*(3*d*e - 7*c*f) - a*d*(d*e - 5*c*f))*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(Sqrt[c]*(d*e - c*f))) + (c*Sqrt[f]*(a*d*f*(5*d*e - c*f) + b*(3*d^2*e^2
- 9*c*d*e*f + 2*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*(d*e - c*f
)))/(e*(d*e - c*f)))/(2*c*(d*e - c*f))))/(b*c - a*d)^2))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^4\left(\frac{\left(a^2df-abcf-abde+ce\,b^2\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(9a^2df-5abcf-5abde+ce\,b^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^3(af-be)^3}+\frac{f^4\left(\frac{\left(ac\,f^2-adef-bcef+bd\,e^2\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(ac\,f^2-5adef-5bc\right.}{}\right.}{(af-be)^3(cf-de)^3}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
b^4/(a*d-b*c)^3/(a*f-b*e)^3*(1/2*(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*x/(b*
x^2+a)+1/2*(9*a^2*d*f-5*a*b*c*f-5*a*b*d*e+b^2*c*e)/a/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))+f^4/(a*f-b*e)^3/(c*f-d*e)^3*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+
b*d*e^2)/e*x/(f*x^2+e)+1/2*(a*c*f^2-5*a*d*e*f-5*b*c*e*f+9*b*d*e^2)/e/(e*f)
^(1/2)*arctan(f*x/(e*f)^(1/2)))+d^4/(a*d-b*c)^3/(c*f-d*e)^3*(1/2*(a*c*d*f-
a*d^2*e-b*c^2*f+b*c*d*e)/c*x/(d*x^2+c)+1/2*(5*a*c*d*f-a*d^2*e-9*b*c^2*f+5*
b*c*d*e)/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(302) = 604$.

Time = 0.19 (sec) , antiderivative size = 1680, normalized size of antiderivative = 4.97

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(b^6*c*e - 5*a*b^5*d*e - 5*a*b^5*c*f + 9*a^2*b^4*d*f)*arctan(b*x/sqrt(
a*b))/((a*b^6*c^3*e^3 - 3*a^2*b^5*c^2*d*e^3 + 3*a^3*b^4*c*d^2*e^3 - a^4*b^
3*d^3*e^3 - 3*a^2*b^5*c^3*e^2*f + 9*a^3*b^4*c^2*d*e^2*f - 9*a^4*b^3*c*d^2*
e^2*f + 3*a^5*b^2*d^3*e^2*f + 3*a^3*b^4*c^3*e*f^2 - 9*a^4*b^3*c^2*d*e*f^2
+ 9*a^5*b^2*c*d^2*e*f^2 - 3*a^6*b*d^3*e*f^2 - a^4*b^3*c^3*f^3 + 3*a^5*b^2*
c^2*d*f^3 - 3*a^6*b*c*d^2*f^3 + a^7*d^3*f^3)*sqrt(a*b)) + 1/2*(5*b*c*d^5*e
 - a*d^6*e - 9*b*c^2*d^4*f + 5*a*c*d^5*f)*arctan(d*x/sqrt(c*d))/((b^3*c^4*
d^3*e^3 - 3*a*b^2*c^3*d^4*e^3 + 3*a^2*b*c^2*d^5*e^3 - a^3*c*d^6*e^3 - 3*b^
3*c^5*d^2*e^2*f + 9*a*b^2*c^4*d^3*e^2*f - 9*a^2*b*c^3*d^4*e^2*f + 3*a^3*c^
2*d^5*e^2*f + 3*b^3*c^6*d*e*f^2 - 9*a*b^2*c^5*d^2*e*f^2 + 9*a^2*b*c^4*d^3*
e*f^2 - 3*a^3*c^3*d^4*e*f^2 - b^3*c^7*f^3 + 3*a*b^2*c^6*d*f^3 - 3*a^2*b*c^
5*d^2*f^3 + a^3*c^4*d^3*f^3)*sqrt(c*d)) + 1/2*(9*b*d*e^2*f^4 - 5*b*c*e*f^5
 - 5*a*d*e*f^5 + a*c*f^6)*arctan(f*x/sqrt(e*f))/((b^3*d^3*e^7 - 3*b^3*c*d^
2*e^6*f - 3*a*b^2*d^3*e^6*f + 3*b^3*c^2*d*e^5*f^2 + 9*a*b^2*c*d^2*e^5*f^2
+ 3*a^2*b*d^3*e^5*f^2 - b^3*c^3*e^4*f^3 - 9*a*b^2*c^2*d*e^4*f^3 - 9*a^2*b*
c*d^2*e^4*f^3 - a^3*d^3*e^4*f^3 + 3*a*b^2*c^3*e^3*f^4 + 9*a^2*b*c^2*d*e^3*
f^4 + 3*a^3*c*d^2*e^3*f^4 - 3*a^2*b*c^3*e^2*f^5 - 3*a^3*c^2*d*e^2*f^5 + a^
3*c^3*e*f^6)*sqrt(e*f)) + 1/2*(b^4*c*d^3*e^3*f*x^5 + a*b^3*d^4*e^3*f*x^5 -
 2*b^4*c^2*d^2*e^2*f^2*x^5 - 2*a^2*b^2*d^4*e^2*f^2*x^5 + b^4*c^3*d*e*f^3*x
^5 + a^3*b*d^4*e*f^3*x^5 + a*b^3*c^3*d*f^4*x^5 - 2*a^2*b^2*c^2*d^2*f^4*...
```

Mupad [B] (verification not implemented)

Time = 39.11 (sec) , antiderivative size = 221098, normalized size of antiderivative = 654.14

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
symsum(log(- root(838582272*a^23*b^4*c^12*d^15*e^10*f^17*z^6 + 838582272*a
^23*b^4*c^10*d^17*e^12*f^15*z^6 + 838582272*a^20*b^7*c^18*d^9*e^7*f^20*z^6
 + 838582272*a^20*b^7*c^7*d^20*e^18*f^9*z^6 + 838582272*a^18*b^9*c^20*d^7*
e^7*f^20*z^6 + 838582272*a^18*b^9*c^7*d^20*e^20*f^7*z^6 + 838582272*a^12*b
^15*c^23*d^4*e^10*f^17*z^6 + 838582272*a^12*b^15*c^10*d^17*e^23*f^4*z^6 +
838582272*a^10*b^17*c^23*d^4*e^12*f^15*z^6 + 838582272*a^10*b^17*c^12*d^15
*e^23*f^4*z^6 + 838582272*a^7*b^20*c^20*d^7*e^18*f^9*z^6 + 838582272*a^7*b
^20*c^18*d^9*e^20*f^7*z^6 + 13566541824*a^19*b^8*c^14*d^13*e^12*f^15*z^6 +
 13566541824*a^19*b^8*c^12*d^15*e^14*f^13*z^6 + 13566541824*a^18*b^9*c^16*
d^11*e^11*f^16*z^6 + 13566541824*a^18*b^9*c^11*d^16*e^16*f^11*z^6 + 135665
41824*a^16*b^11*c^18*d^9*e^11*f^16*z^6 + 13566541824*a^16*b^11*c^11*d^16*e
^18*f^9*z^6 + 13566541824*a^14*b^13*c^19*d^8*e^12*f^15*z^6 + 13566541824*a
^14*b^13*c^12*d^15*e^19*f^8*z^6 + 13566541824*a^12*b^15*c^19*d^8*e^14*f^13
*z^6 + 13566541824*a^12*b^15*c^14*d^13*e^19*f^8*z^6 + 13566541824*a^11*b^1
6*c^18*d^9*e^16*f^11*z^6 + 13566541824*a^11*b^16*c^16*d^11*e^18*f^9*z^6 -
510935040*a^23*b^4*c^14*d^13*e^8*f^19*z^6 - 510935040*a^23*b^4*c^8*d^19*e^
14*f^13*z^6 - 510935040*a^22*b^5*c^16*d^11*e^7*f^20*z^6 - 510935040*a^22*b
^5*c^7*d^20*e^16*f^11*z^6 - 510935040*a^16*b^11*c^22*d^5*e^7*f^20*z^6 - 51
0935040*a^16*b^11*c^7*d^20*e^22*f^5*z^6 - 510935040*a^14*b^13*c^23*d^4*e^8
*f^19*z^6 - 510935040*a^14*b^13*c^8*d^19*e^23*f^4*z^6 - 510935040*a^8*b...
```

Reduce [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 10200, normalized size of antiderivative = 30.18

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2 (e+fx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
(9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**6*d*e**3*f**
4 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**6*d*e**2*
f**5*x**2 - 27*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**
5*d**2*e**4*f**3 - 18*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b
**3*c**5*d**2*e**3*f**4*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*a**3*b**3*c**5*d**2*e**2*f**5*x**4 + 27*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**3*b**3*c**4*d**3*e**5*f**2 - 27*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**4*d**3*e**3*f**4*x**4 - 9*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**3*d**4*e**6*f + 18*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**3*d**4*e**5*f**2*x**2
+ 27*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c**3*d**4*e**
4*f**3*x**4 - 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*c*
*2*d**5*e**6*f*x**2 - 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3
*b**3*c**2*d**5*e**5*f**2*x**4 - 5*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a**2*b**4*c**7*e**3*f**4 - 5*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**2*b**4*c**7*e**2*f**5*x**2 + 10*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**2*b**4*c**6*d*e**4*f**3 + 14*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**2*b**4*c**6*d*e**3*f**4*x**2 + 4*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**6*d*e**2*f**5*x**4 - 17*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**5*d**2*e**4*f**3*x**2 ...
```

3.260 $\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 390

$$\begin{aligned}
&\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3 \, dx \\
&= \frac{(128b^4ce^3+7a^4df^3+48a^2b^2ef(de+cf)-10a^3bf^2(3de+cf)-32ab^3e^2(de+3cf))\,x\sqrt{a+bx^2}}{256b^4} \\
&\quad - \frac{(7a^3df^3+48ab^2ef(de+cf)-10a^2bf^2(3de+cf)-32b^3e^2(de+3cf))\,x(a+bx^2)^{3/2}}{128b^4} \\
&\quad + \frac{f(7a^2df^2+48b^2e(de+cf)-10abf(3de+cf))\,x^3(a+bx^2)^{3/2}}{96b^3} \\
&\quad - \frac{f^2(7adf-10b(3de+cf))x^5(a+bx^2)^{3/2}}{80b^2} + \frac{df^3x^7(a+bx^2)^{3/2}}{10b} \\
&\quad + \frac{a(128b^4ce^3+7a^4df^3+48a^2b^2ef(de+cf)-10a^3bf^2(3de+cf)-32ab^3e^2(de+3cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right.}{256b^{9/2}}
\end{aligned}$$

output
```
1/256*(128*b^4*c*e^3+7*a^4*d*f^3+48*a^2*b^2*e*f*(c*f+d*e)-10*a^3*b*f^2*(c*
f+3*d*e)-32*a*b^3*e^2*(3*c*f+d*e))*x*(b*x^2+a)^(1/2)/b^4-1/128*(7*a^3*d*f^
3+48*a*b^2*e*f*(c*f+d*e)-10*a^2*b*f^2*(c*f+3*d*e)-32*b^3*e^2*(3*c*f+d*e))*
x*(b*x^2+a)^(3/2)/b^4+1/96*f*(7*a^2*d*f^2+48*b^2*e*(c*f+d*e)-10*a*b*f*(c*f
+3*d*e))*x^3*(b*x^2+a)^(3/2)/b^3-1/80*f^2*(7*a*d*f-10*b*(c*f+3*d*e))*x^5*(
b*x^2+a)^(3/2)/b^2+1/10*d*f^3*x^7*(b*x^2+a)^(3/2)/b+1/256*a*(128*b^4*c*e^3
+7*a^4*d*f^3+48*a^2*b^2*e*f*(c*f+d*e)-10*a^3*b*f^2*(c*f+3*d*e)-32*a*b^3*e^
2*(3*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^4df^3+10a^3bf^2(45de+15cf+7dfx^2)-4a^2b^2f(5cf(36e+5fx^2)+d(180e^2+75e}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^4*d*f^3 + 10*a^3*b*f^2*(45*d*e + 15*c*f
 + 7*d*f*x^2) - 4*a^2*b^2*f*(5*c*f*(36*e + 5*f*x^2) + d*(180*e^2 + 75*e*f*
x^2 + 14*f^2*x^4)) + 16*a*b^3*(5*c*f*(18*e^2 + 6*e*f*x^2 + f^2*x^4) + 3*d*
(10*e^3 + 10*e^2*f*x^2 + 5*e*f^2*x^4 + f^3*x^6)) + 96*b^4*(5*c*(4*e^3 + 6*
e^2*f*x^2 + 4*e*f^2*x^4 + f^3*x^6) + d*x^2*(10*e^3 + 20*e^2*f*x^2 + 15*e*f
^2*x^4 + 4*f^3*x^6))) - 15*a*(128*b^4*c*e^3 + 7*a^4*d*f^3 + 48*a^2*b^2*e*f
*(d*e + c*f) - 10*a^3*b*f^2*(3*d*e + c*f) - 32*a*b^3*e^2*(d*e + 3*c*f))*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(3840*b^(9/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {403, 403, 403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3\,dx$$
$$\Big\downarrow \text{403}$$
$$\frac{\int \sqrt{bx^2+a}\left(fx^2+e\right)^2\left((6bde+10bcf-7adf)x^2+(10bc-ad)e\right)dx}{10b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 403

$$\frac{\frac{\int \sqrt{bx^2+a}(fx^2+e)\left(\left(24e(de+5cf)b^2-2af(33de+25cf)b+35a^2df^2\right)x^2+e\left(7dfa^2-14bdea-10bcfa+80b^2ce\right)\right)dx}{8b}+\frac{x\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2\left(-7ac\right.}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 403

$$\frac{\frac{\frac{\int \sqrt{bx^2+a}\left(e\left(-35df^2a^3+2bf(54de+25cf)a^2-36b^2e(3de+5cf)a+480b^3ce^2\right)-\left(-48e^2(de+15cf)b^3+8aef(36de+65cf)b^2-10a^2f^2(31de+15cf)b+105a^3df^3\right)x^2\right)d}{6b}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{\frac{15\left(7a^4df^3-10a^3bf^2(cf+3de)+48a^2b^2ef(cf+de)-32ab^3e^2(3cf+de)+128b^4ce^3\right)\int \sqrt{bx^2+a}dx}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3df^3-10a^2bf^2(15cf+31de)+8ab^2ef(65cf-\right.}{4b}}{6b}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{15\left(7a^4df^3-10a^3bf^2(cf+3de)+48a^2b^2ef(cf+de)-32ab^3e^2(3cf+de)+128b^4ce^3\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3df^3-10a^2bf^2(15cf+\right.}{}}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{15\left(7a^4df^3-10a^3bf^2(cf+3de)+48a^2b^2ef(cf+de)-32ab^3e^2(3cf+de)+128b^4ce^3\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3df^3-10a^2b\right.}{}}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

$\downarrow$ 219

$$\frac{\frac{x\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)\left(35a^2df^2-2abf(25cf+33de)+24b^2e(5cf+de)\right)}{6b}+\frac{15\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(7a^4df^3-10a^3bf^2(cf+3de)+48a^2b^2ef(c\cdots\right.}{4b}\cdots}{8b}\cdots$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}{10b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
(d*x*(a + b*x^2)^(3/2)*(e + f*x^2)^3)/(10*b) + (((6*b*d*e + 10*b*c*f - 7*a
*d*f)*x*(a + b*x^2)^(3/2)*(e + f*x^2)^2)/(8*b) + (((35*a^2*d*f^2 + 24*b^2*
e*(d*e + 5*c*f) - 2*a*b*f*(33*d*e + 25*c*f))*x*(a + b*x^2)^(3/2)*(e + f*x^
2))/(6*b) + (-1/4*((105*a^3*d*f^3 - 48*b^3*e^2*(d*e + 15*c*f) - 10*a^2*b*f
^2*(31*d*e + 15*c*f) + 8*a*b^2*e*f*(36*d*e + 65*c*f))*x*(a + b*x^2)^(3/2))
/b + (15*(128*b^4*c*e^3 + 7*a^4*d*f^3 + 48*a^2*b^2*e*f*(d*e + c*f) - 10*a^
3*b*f^2*(3*d*e + c*f) - 32*a*b^3*e^2*(d*e + 3*c*f))*((x*Sqrt[a + b*x^2])/2
 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b))/(8
*b))/(10*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{7a\left(a^3\left(ad-\frac{10bc}{7}\right)f^3-\frac{30a^2\left(ad-\frac{8bc}{5}\right)be\,f^2}{7}+\frac{48a\,b^2e^2(ad-2bc)f}{7}-\frac{32b^3e^3(ad-4bc)}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{256}-\frac{7\left(\left(\left(-\frac{32\left(\frac{4x^2d}{5}+c\right)x^6 f}{7}\right.\right.\right.}{}}{}$
risch	$-\frac{x(-384b^4d\,f^3x^8-48a\,b^3x^6d\,f^3-480b^4c\,f^3x^6-1440b^4de\,f^2x^6+56a^2b^2x^4d\,f^3-80a\,b^3c\,f^3x^4-240a\,b^3de\,f^2x^4-1920b^4ce}{}$
default	$c\,e^3\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+f^2(cf+3de)\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+\right.}{4b}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
7/256*(a*(a^3*(a*d-10/7*b*c)*f^3-30/7*a^2*(a*d-8/5*b*c)*b*e*f^2+48/7*a*b^2
*e^2*(a*d-2*b*c)*f-32/7*b^3*e^3*(a*d-4*b*c))*arctanh((b*x^2+a)^(1/2)/x/b^(
1/2))-((-32/7*(4/5*x^2*d+c)*x^6*f^3-128/7*(3/4*x^2*d+c)*x^4*e*f^2-192/7*(2
/3*x^2*d+c)*x^2*e^2*f-128/7*(1/2*x^2*d+c)*e^3)*b^(9/2)+((-16/21*(3/5*x^2*d
+c)*x^4*f^3-32/7*(1/2*x^2*d+c)*x^2*e*f^2-96/7*(1/3*x^2*d+c)*e^2*f-32/7*d*e
^3)*b^(7/2)+a*f*(((8/15*d*x^4+20/21*c*x^2)*f^2+48/7*(5/12*x^2*d+c)*e*f+48/
7*d*e^2)*b^(5/2)+a*f*(((-2/3*x^2*d-10/7*c)*f-30/7*d*e)*b^(3/2)+a*d*f*b^(1/
2))))*a)*(b*x^2+a)^(1/2)*x)/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.10

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/7680*(15*(32*(4*a*b^4*c - a^2*b^3*d)*e^3 - 48*(2*a^2*b^3*c - a^3*b^2*d)
*e^2*f + 6*(8*a^3*b^2*c - 5*a^4*b*d)*e*f^2 - (10*a^4*b*c - 7*a^5*d)*f^3)*s
qrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d*f^3*
x^9 + 48*(30*b^5*d*e*f^2 + (10*b^5*c + a*b^4*d)*f^3)*x^7 + 8*(240*b^5*d*e^
2*f + 30*(8*b^5*c + a*b^4*d)*e*f^2 + (10*a*b^4*c - 7*a^2*b^3*d)*f^3)*x^5 +
 10*(96*b^5*d*e^3 + 48*(6*b^5*c + a*b^4*d)*e^2*f + 6*(8*a*b^4*c - 5*a^2*b^
3*d)*e*f^2 - (10*a^2*b^3*c - 7*a^3*b^2*d)*f^3)*x^3 + 15*(32*(4*b^5*c + a*b
^4*d)*e^3 + 48*(2*a*b^4*c - a^2*b^3*d)*e^2*f - 6*(8*a^2*b^3*c - 5*a^3*b^2*
d)*e*f^2 + (10*a^3*b^2*c - 7*a^4*b*d)*f^3)*x)*sqrt(b*x^2 + a))/b^5, -1/384
0*(15*(32*(4*a*b^4*c - a^2*b^3*d)*e^3 - 48*(2*a^2*b^3*c - a^3*b^2*d)*e^2*f
 + 6*(8*a^3*b^2*c - 5*a^4*b*d)*e*f^2 - (10*a^4*b*c - 7*a^5*d)*f^3)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*d*f^3*x^9 + 48*(30*b^5*d*e
*f^2 + (10*b^5*c + a*b^4*d)*f^3)*x^7 + 8*(240*b^5*d*e^2*f + 30*(8*b^5*c +
a*b^4*d)*e*f^2 + (10*a*b^4*c - 7*a^2*b^3*d)*f^3)*x^5 + 10*(96*b^5*d*e^3 +
48*(6*b^5*c + a*b^4*d)*e^2*f + 6*(8*a*b^4*c - 5*a^2*b^3*d)*e*f^2 - (10*a^2
*b^3*c - 7*a^3*b^2*d)*f^3)*x^3 + 15*(32*(4*b^5*c + a*b^4*d)*e^3 + 48*(2*a*
b^4*c - a^2*b^3*d)*e^2*f - 6*(8*a^2*b^3*c - 5*a^3*b^2*d)*e*f^2 + (10*a^3*b
^2*c - 7*a^4*b*d)*f^3)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.74

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{df^3x^9}{10}+\frac{x^7\left(\frac{adf^3}{10}+bcf^3+3bdef^2\right)}{8b}+\frac{x^5\left(acf^3+3adef^2-\frac{7a\left(\frac{adf^3}{10}+bcf^3+3bdef^2\right)}{8b}+3bcef^2+3bde^2f\right)}{6b}+\frac{x^3\cdot\left(3acef^2+\right.}{} \right. \\ \sqrt{a}\left(ce^3x+\frac{df^3x^9}{9}+\frac{x^7\left(cf^3+3def^2\right)}{7}+\frac{x^5\cdot\left(3cef^2+3de^2f\right)}{5}+\frac{x^3\cdot\left(3ce^2f+de^3\right)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)*(f*x**2+e)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f**3*x**9/10 + x**7*(a*d*f**3/10 + b*c*f**3
 + 3*b*d*e*f**2)/(8*b) + x**5*(a*c*f**3 + 3*a*d*e*f**2 - 7*a*(a*d*f**3/10
+ b*c*f**3 + 3*b*d*e*f**2)/(8*b) + 3*b*c*e*f**2 + 3*b*d*e**2*f)/(6*b) + x*
*3*(3*a*c*e*f**2 + 3*a*d*e**2*f - 5*a*(a*c*f**3 + 3*a*d*e*f**2 - 7*a*(a*d*
f**3/10 + b*c*f**3 + 3*b*d*e*f**2)/(8*b) + 3*b*c*e*f**2 + 3*b*d*e**2*f)/(6
*b) + 3*b*c*e**2*f + b*d*e**3)/(4*b) + x*(3*a*c*e**2*f + a*d*e**3 - 3*a*(3
*a*c*e*f**2 + 3*a*d*e**2*f - 5*a*(a*c*f**3 + 3*a*d*e*f**2 - 7*a*(a*d*f**3/
10 + b*c*f**3 + 3*b*d*e*f**2)/(8*b) + 3*b*c*e*f**2 + 3*b*d*e**2*f)/(6*b) +
 3*b*c*e**2*f + b*d*e**3)/(4*b) + b*c*e**3)/(2*b)) + (a*c*e**3 - a*(3*a*c*
e**2*f + a*d*e**3 - 3*a*(3*a*c*e*f**2 + 3*a*d*e**2*f - 5*a*(a*c*f**3 + 3*a
*d*e*f**2 - 7*a*(a*d*f**3/10 + b*c*f**3 + 3*b*d*e*f**2)/(8*b) + 3*b*c*e*f*
*2 + 3*b*d*e**2*f)/(6*b) + 3*b*c*e**2*f + b*d*e**3)/(4*b) + b*c*e**3)/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
(x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*e**3*x + d*f**3*x**
9/9 + x**7*(c*f**3 + 3*d*e*f**2)/7 + x**5*(3*c*e*f**2 + 3*d*e**2*f)/5 + x*
*3*(3*c*e**2*f + d*e**3)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.35

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx = \frac{(bx^2+a)^{\frac{3}{2}}df^3x^7}{10\,b} - \frac{7\,(bx^2+a)^{\frac{3}{2}}adf^3x^5}{80\,b^2} + \frac{7\,(bx^2+a)^{\frac{3}{2}}a^2df^3x^3}{96\,b^3} + \frac{(3\,def^2+cf^3)(bx^2+a)^{\frac{3}{2}}x^5}{8\,b} + \frac{1}{2}\sqrt{bx^2+a}ce^3x - \frac{7\,(bx^2+a)^{\frac{3}{2}}a^3df^3x}{128\,b^4} + \frac{7\sqrt{bx^2+a}a^4df^3x}{256\,b^4} + \frac{ace^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{7\,a^5df^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{9}{2}}} - \frac{5\,(3\,def^2+cf^3)(bx^2+a)^{\frac{3}{2}}ax^3}{48\,b^2} + \frac{(de^2f+cef^2)(bx^2+a)^{\frac{3}{2}}x^3}{2\,b} + \frac{5\,(3\,def^2+cf^3)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^3} - \frac{5\,(3\,def^2+cf^3)\sqrt{bx^2+a}a^3x}{128\,b^3} - \frac{3\,(de^2f+cef^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} + \frac{3\,(de^2f+cef^2)\sqrt{bx^2+a}a^2x}{16\,b^2} + \frac{(de^3+3\,ce^2f)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(de^3+3\,ce^2f)\sqrt{bx^2+a}ax}{8\,b} - \frac{5\,(3\,def^2+cf^3)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{3\,(de^2f+cef^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{(de^3+3\,ce^2f)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(3/2)*d*f^3*x^7/b - 7/80*(b*x^2 + a)^(3/2)*a*d*f^3*x^5/b^
2 + 7/96*(b*x^2 + a)^(3/2)*a^2*d*f^3*x^3/b^3 + 1/8*(3*d*e*f^2 + c*f^3)*(b*
x^2 + a)^(3/2)*x^5/b + 1/2*sqrt(b*x^2 + a)*c*e^3*x - 7/128*(b*x^2 + a)^(3/
2)*a^3*d*f^3*x/b^4 + 7/256*sqrt(b*x^2 + a)*a^4*d*f^3*x/b^4 + 1/2*a*c*e^3*a
rcsinh(b*x/sqrt(a*b))/sqrt(b) + 7/256*a^5*d*f^3*arcsinh(b*x/sqrt(a*b))/b^(
9/2) - 5/48*(3*d*e*f^2 + c*f^3)*(b*x^2 + a)^(3/2)*a*x^3/b^2 + 1/2*(d*e^2*f
 + c*e*f^2)*(b*x^2 + a)^(3/2)*x^3/b + 5/64*(3*d*e*f^2 + c*f^3)*(b*x^2 + a)
^(3/2)*a^2*x/b^3 - 5/128*(3*d*e*f^2 + c*f^3)*sqrt(b*x^2 + a)*a^3*x/b^3 - 3
/8*(d*e^2*f + c*e*f^2)*(b*x^2 + a)^(3/2)*a*x/b^2 + 3/16*(d*e^2*f + c*e*f^2
)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(d*e^3 + 3*c*e^2*f)*(b*x^2 + a)^(3/2)*x/
b - 1/8*(d*e^3 + 3*c*e^2*f)*sqrt(b*x^2 + a)*a*x/b - 5/128*(3*d*e*f^2 + c*f
^3)*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/16*(d*e^2*f + c*e*f^2)*a^3*arcs
inh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(d*e^3 + 3*c*e^2*f)*a^2*arcsinh(b*x/sqrt(
a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.10

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3\,dx$$
$$= \frac{1}{3840}\left(2\left(4\left(6\left(8\,df^3x^2+\frac{30\,b^8def^2+10\,b^8cf^3+ab^7df^3}{b^8}\right)x^2+\frac{240\,b^8de^2f+240\,b^8cef^2+30\,ab^7def^2+}{b^8}\right.\right.\right.$$
$$-\frac{(128\,ab^4ce^3-32\,a^2b^3de^3-96\,a^2b^3ce^2f+48\,a^3b^2de^2f+48\,a^3b^2cef^2-30\,a^4bdef^2-10\,a^4bcf^3+7\,a^5d}{256\,b^{\frac{9}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*d*f^3*x^2 + (30*b^8*d*e*f^2 + 10*b^8*c*f^3 + a*b^7*d*f^
3)/b^8)*x^2 + (240*b^8*d*e^2*f + 240*b^8*c*e*f^2 + 30*a*b^7*d*e*f^2 + 10*a
*b^7*c*f^3 - 7*a^2*b^6*d*f^3)/b^8)*x^2 + 5*(96*b^8*d*e^3 + 288*b^8*c*e^2*f
 + 48*a*b^7*d*e^2*f + 48*a*b^7*c*e*f^2 - 30*a^2*b^6*d*e*f^2 - 10*a^2*b^6*c
*f^3 + 7*a^3*b^5*d*f^3)/b^8)*x^2 + 15*(128*b^8*c*e^3 + 32*a*b^7*d*e^3 + 96
*a*b^7*c*e^2*f - 48*a^2*b^6*d*e^2*f - 48*a^2*b^6*c*e*f^2 + 30*a^3*b^5*d*e*
f^2 + 10*a^3*b^5*c*f^3 - 7*a^4*b^4*d*f^3)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(
128*a*b^4*c*e^3 - 32*a^2*b^3*d*e^3 - 96*a^2*b^3*c*e^2*f + 48*a^3*b^2*d*e^2
*f + 48*a^3*b^2*c*e*f^2 - 30*a^4*b*d*e*f^2 - 10*a^4*b*c*f^3 + 7*a^5*d*f^3)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^3\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\left(f\,x^2+e\right)^3dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^3,x)
```

3.261 $\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^2 dx$

Optimal result

Integrand size = 28, antiderivative size = 267

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^2 dx = \frac{(64b^3ce^2-5a^3df^2+8a^2bf(2de+cf)-16ab^2e(de+2cf))\,x\sqrt{a+bx^2}}{128b^3} + \frac{(5a^2df^2-8abf(2de+cf)+16b^2e(de+2cf))\,x(a+bx^2)^{3/2}}{64b^3} - \frac{f(5adf-8b(2de+cf))x^3(a+bx^2)^{3/2}}{48b^2} + \frac{df^2x^5(a+bx^2)^{3/2}}{8b} + \frac{a(64b^3ce^2-5a^3df^2+8a^2bf(2de+cf)-16ab^2e(de+2cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}$$

output

```
1/128*(64*b^3*c*e^2-5*a^3*d*f^2+8*a^2*b*f*(c*f+2*d*e)-16*a*b^2*e*(2*c*f+d*
e))*x*(b*x^2+a)^(1/2)/b^3+1/64*(5*a^2*d*f^2-8*a*b*f*(c*f+2*d*e)+16*b^2*e*(
2*c*f+d*e))*x*(b*x^2+a)^(3/2)/b^3-1/48*f*(5*a*d*f-8*b*(c*f+2*d*e))*x^3*(b*
x^2+a)^(3/2)/b^2+1/8*d*f^2*x^5*(b*x^2+a)^(3/2)/b+1/128*a*(64*b^3*c*e^2-5*a
^3*d*f^2+8*a^2*b*f*(c*f+2*d*e)-16*a*b^2*e*(2*c*f+d*e))*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2 \, dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(15a^3df^2-2a^2bf(24de+12cf+5dfx^2)+8ab^2(2cf(6e+fx^2)+d(6e^2+4efx^2+f^2x^4))}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d*f^2 - 2*a^2*b*f*(24*d*e + 12*c*f + 5*
d*f*x^2) + 8*a*b^2*(2*c*f*(6*e + f*x^2) + d*(6*e^2 + 4*e*f*x^2 + f^2*x^4))
 + 16*b^3*(4*c*(3*e^2 + 3*e*f*x^2 + f^2*x^4) + d*x^2*(6*e^2 + 8*e*f*x^2 +
3*f^2*x^4))) + 3*a*(-64*b^3*c*e^2 + 5*a^3*d*f^2 - 8*a^2*b*f*(2*d*e + c*f)
+ 16*a*b^2*e*(d*e + 2*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(7
/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {403, 403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2 \, dx$$
$$\Big\downarrow \text{ 403}$$
$$\frac{\int \sqrt{bx^2+a}(fx^2+e)\left((4bde+8bcf-5adf)x^2+(8bc-ad)e\right)dx}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$
$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int \sqrt{bx^2+a}\left(\left(8e(de+8cf)b^2-4af(7de+6cf)b+15a^2df^2\right)x^2+e\left(5dfa^2-10bdea-8bcfa+48b^2ce\right)\right)dx}{6b}+\frac{x\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)\left(-5adf+8bcf+4bde\right)}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{3\left(-5a^3df^2+8a^2bf(cf+2de)-16ab^2e(2cf+de)+64b^3ce^2\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2df^2-4abf(6cf+7de)+8b^2e(8cf+de)\right)}{4b}}{6b}+\frac{x\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)}{6}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{3\left(-5a^3df^2+8a^2bf(cf+2de)-16ab^2e(2cf+de)+64b^3ce^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2df^2-4abf(6cf+7de)+8b^2e(8cf+de)\right)}{4b}}{6b}+\frac{x(}{}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{3\left(-5a^3df^2+8a^2bf(cf+2de)-16ab^2e(2cf+de)+64b^3ce^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2df^2-4abf(6cf+7de)+8b^2e(8cf+de)\right)}{4b}}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2df^2-4abf(6cf+7de)+8b^2e(8cf+de)\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3df^2+8a^2bf(cf+2de)-16ab^2e(2cf+de)+64b^3ce^2\right)}{4b}}{6b}}{8b}$$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}{8b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
(d*x*(a + b*x^2)^(3/2)*(e + f*x^2)^2)/(8*b) + (((4*b*d*e + 8*b*c*f - 5*a*d
*f)*x*(a + b*x^2)^(3/2)*(e + f*x^2))/(6*b) + (((15*a^2*d*f^2 - 4*a*b*f*(7*
d*e + 6*c*f) + 8*b^2*e*(d*e + 8*c*f))*x*(a + b*x^2)^(3/2))/(4*b) + (3*(64*
b^3*c*e^2 - 5*a^3*d*f^2 + 8*a^2*b*f*(2*d*e + c*f) - 16*a*b^2*e*(d*e + 2*c*
f))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*S
qrt[b])))/(4*b))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^2\left(ad-\frac{8bc}{5}\right)f^2-\frac{16abe(ad-2bc)f}{5}+\frac{16b^2e^2(ad-4bc)}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\sqrt{b\,x^2+a}\,x\left(\frac{64\left(\frac{\left(\frac{3x^2d}{4}+c\right)x^4f^2}{3}+\left(\frac{2x^2}{3}\right.\right.}{}\right.\right.}{}$
risch	$\frac{x\left(48b^3d\,f^2x^6+8a\,b^2x^4d\,f^2+64b^3c\,f^2x^4+128b^3def\,x^4-10a^2b\,x^2d\,f^2+16ac\,f^2b^2x^2+32a\,b^2def\,x^2+192b^3cef\,x^2+96b^3d\,e^2\right.}{384b^3}$
default	$c\,e^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+f(cf+2de)\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\right.}{}\right)}{2b}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/128/b^(7/2)*(a*(a^2*(a*d-8/5*b*c)*f^2-16/5*a*b*e*(a*d-2*b*c)*f+16/5*b^2
*e^2*(a*d-4*b*c))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*x*(64
/5*(1/3*(3/4*x^2*d+c)*x^4*f^2+(2/3*x^2*d+c)*x^2*e*f+(1/2*x^2*d+c)*e^2)*b^(
7/2)+a*(16/5*(1/3*(1/2*x^2*d+c)*x^2*f^2+2*(1/3*x^2*d+c)*e*f+d*e^2)*b^(5/2)
+a*(2*((-1/3*x^2*d-4/5*c)*f-8/5*d*e)*b^(3/2)+a*d*f*b^(1/2))*f)))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.01

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^2\,dx$$

$$= \left[-\frac{3\left(16\left(4\,ab^3c-a^2b^2d\right)e^2-16\left(2\,a^2b^2c-a^3bd\right)ef+\left(8\,a^3bc-5\,a^4d\right)f^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{\right.}}{}\right.$$

$$-\frac{3\left(16\left(4\,ab^3c-a^2b^2d\right)e^2-16\left(2\,a^2b^2c-a^3bd\right)ef+\left(8\,a^3bc-5\,a^4d\right)f^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,b^4d}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/768*(3*(16*(4*a*b^3*c - a^2*b^2*d)*e^2 - 16*(2*a^2*b^2*c - a^3*b*d)*e*
f + (8*a^3*b*c - 5*a^4*d)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*b^4*d*f^2*x^7 + 8*(16*b^4*d*e*f + (8*b^4*c + a*b^3*d)
*f^2)*x^5 + 2*(48*b^4*d*e^2 + 16*(6*b^4*c + a*b^3*d)*e*f + (8*a*b^3*c - 5*
a^2*b^2*d)*f^2)*x^3 + 3*(16*(4*b^4*c + a*b^3*d)*e^2 + 16*(2*a*b^3*c - a^2*
b^2*d)*e*f - (8*a^2*b^2*c - 5*a^3*b*d)*f^2)*x)*sqrt(b*x^2 + a))/b^4, -1/38
4*(3*(16*(4*a*b^3*c - a^2*b^2*d)*e^2 - 16*(2*a^2*b^2*c - a^3*b*d)*e*f + (8
*a^3*b*c - 5*a^4*d)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48
*b^4*d*f^2*x^7 + 8*(16*b^4*d*e*f + (8*b^4*c + a*b^3*d)*f^2)*x^5 + 2*(48*b^
4*d*e^2 + 16*(6*b^4*c + a*b^3*d)*e*f + (8*a*b^3*c - 5*a^2*b^2*d)*f^2)*x^3
+ 3*(16*(4*b^4*c + a*b^3*d)*e^2 + 16*(2*a*b^3*c - a^2*b^2*d)*e*f - (8*a^2*
b^2*c - 5*a^3*b*d)*f^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.55

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{df^2x^7}{8}+\frac{x^5\left(\frac{adf^2}{8}+bcf^2+2bdef\right)}{6b}+\frac{x^3\left(acf^2+2adef-\frac{5a\left(\frac{adf^2}{8}+bcf^2+2bdef\right)}{6b}+2bcef+bde^2\right)}{4b}+\frac{x\left(2acef+ade^2-\frac{3a\left(\right.}{}\right.}{} \right. \\ \sqrt{a}\left(ce^2x+\frac{df^2x^7}{7}+\frac{x^5(cf^2+2def)}{5}+\frac{x^3\cdot(2cef+de^2)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)*(f*x**2+e)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f**2*x**7/8 + x**5*(a*d*f**2/8 + b*c*f**2 +
 2*b*d*e*f)/(6*b) + x**3*(a*c*f**2 + 2*a*d*e*f - 5*a*(a*d*f**2/8 + b*c*f**
2 + 2*b*d*e*f)/(6*b) + 2*b*c*e*f + b*d*e**2)/(4*b) + x*(2*a*c*e*f + a*d*e*
*2 - 3*a*(a*c*f**2 + 2*a*d*e*f - 5*a*(a*d*f**2/8 + b*c*f**2 + 2*b*d*e*f)/(
6*b) + 2*b*c*e*f + b*d*e**2)/(4*b) + b*c*e**2)/(2*b)) + (a*c*e**2 - a*(2*a
*c*e*f + a*d*e**2 - 3*a*(a*c*f**2 + 2*a*d*e*f - 5*a*(a*d*f**2/8 + b*c*f**2
 + 2*b*d*e*f)/(6*b) + 2*b*c*e*f + b*d*e**2)/(4*b) + b*c*e**2)/(2*b))*Piece
wise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x
)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*e**2*x + d*f**2*x**7/7 + x*
*5*(c*f**2 + 2*d*e*f)/5 + x**3*(2*c*e*f + d*e**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.26

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx = \frac{(bx^2+a)^{\frac{3}{2}}df^2x^5}{8\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}adf^2x^3}{48\,b^2} + \frac{1}{2}\,\sqrt{bx^2+a}ce^2x + \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2df^2x}{64\,b^3} - \frac{5\,\sqrt{bx^2+a}a^3df^2x}{128\,b^3} + \frac{(2\,def+cf^2)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b} + \frac{ace^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} - \frac{5\,a^4df^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} - \frac{(2\,def+cf^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} + \frac{(2\,def+cf^2)\sqrt{bx^2+a}a^2x}{16\,b^2} + \frac{(de^2+2\,cef)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(de^2+2\,cef)\sqrt{bx^2+a}ax}{8\,b} + \frac{(2\,def+cf^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{(de^2+2\,cef)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*d*f^2*x^5/b - 5/48*(b*x^2 + a)^(3/2)*a*d*f^2*x^3/b^2
 + 1/2*sqrt(b*x^2 + a)*c*e^2*x + 5/64*(b*x^2 + a)^(3/2)*a^2*d*f^2*x/b^3 -
5/128*sqrt(b*x^2 + a)*a^3*d*f^2*x/b^3 + 1/6*(2*d*e*f + c*f^2)*(b*x^2 + a)^
(3/2)*x^3/b + 1/2*a*c*e^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d*f^2
*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/8*(2*d*e*f + c*f^2)*(b*x^2 + a)^(3/2)*
a*x/b^2 + 1/16*(2*d*e*f + c*f^2)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(d*e^2 +
2*c*e*f)*(b*x^2 + a)^(3/2)*x/b - 1/8*(d*e^2 + 2*c*e*f)*sqrt(b*x^2 + a)*a*x
/b + 1/16*(2*d*e*f + c*f^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(d*e^
2 + 2*c*e*f)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)^2\,dx$$
$$=\frac{1}{384}\left(2\left(4\left(6\,df^2x^2+\frac{16\,b^6def+8\,b^6cf^2+ab^5df^2}{b^6}\right)x^2+\frac{48\,b^6de^2+96\,b^6cef+16\,ab^5def+8\,ab^5cf^2-}{b^6}\right.\right.$$
$$-\frac{(64\,ab^3ce^2-16\,a^2b^2de^2-32\,a^2b^2cef+16\,a^3bdef+8\,a^3bcf^2-5\,a^4df^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*d*f^2*x^2 + (16*b^6*d*e*f + 8*b^6*c*f^2 + a*b^5*d*f^2)/b^6)
*x^2 + (48*b^6*d*e^2 + 96*b^6*c*e*f + 16*a*b^5*d*e*f + 8*a*b^5*c*f^2 - 5*a
^2*b^4*d*f^2)/b^6)*x^2 + 3*(64*b^6*c*e^2 + 16*a*b^5*d*e^2 + 32*a*b^5*c*e*f
 - 16*a^2*b^4*d*e*f - 8*a^2*b^4*c*f^2 + 5*a^3*b^3*d*f^2)/b^6)*sqrt(b*x^2 +
 a)*x - 1/128*(64*a*b^3*c*e^2 - 16*a^2*b^2*d*e^2 - 32*a^2*b^2*c*e*f + 16*a
^3*b*d*e*f + 8*a^3*b*c*f^2 - 5*a^4*d*f^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.84

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx$$
$$=\frac{15\sqrt{b\,x^2+a}\,a^3bd\,f^2x-24\sqrt{b\,x^2+a}\,a^2b^2c\,f^2x-48\sqrt{b\,x^2+a}\,a^2b^2defx-10\sqrt{b\,x^2+a}\,a^2b^2d\,f^2x^3+96\sqrt{}}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e)^2,x)
```

output
```
(15*sqrt(a + b*x**2)*a**3*b*d*f**2*x - 24*sqrt(a + b*x**2)*a**2*b**2*c*f**
2*x - 48*sqrt(a + b*x**2)*a**2*b**2*d*e*f*x - 10*sqrt(a + b*x**2)*a**2*b**
2*d*f**2*x**3 + 96*sqrt(a + b*x**2)*a*b**3*c*e*f*x + 16*sqrt(a + b*x**2)*a
*b**3*c*f**2*x**3 + 48*sqrt(a + b*x**2)*a*b**3*d*e**2*x + 32*sqrt(a + b*x*
*2)*a*b**3*d*e*f*x**3 + 8*sqrt(a + b*x**2)*a*b**3*d*f**2*x**5 + 192*sqrt(a
 + b*x**2)*b**4*c*e**2*x + 192*sqrt(a + b*x**2)*b**4*c*e*f*x**3 + 64*sqrt(
a + b*x**2)*b**4*c*f**2*x**5 + 96*sqrt(a + b*x**2)*b**4*d*e**2*x**3 + 128*
sqrt(a + b*x**2)*b**4*d*e*f*x**5 + 48*sqrt(a + b*x**2)*b**4*d*f**2*x**7 -
15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d*f**2 + 24*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*f**2 + 48*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*e*f - 96*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*e*f - 48*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*e**2 + 192*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c*e**2)/(384*b**4)
```

3.262 $\int \sqrt{a+bx^2}(c+dx^2)(e+fx^2)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 156

$$\int \sqrt{a+bx^2}(c+dx^2)(e+fx^2)\,dx = \frac{(8b^2ce+a^2df-2ab(de+cf))\,x\sqrt{a+bx^2}}{16b^2} - \frac{(adf-2b(de+cf))x(a+bx^2)^{3/2}}{8b^2} + \frac{dfx^3(a+bx^2)^{3/2}}{6b} + \frac{a(8b^2ce+a^2df-2ab(de+cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*(8*b^2*c*e+a^2*d*f-2*a*b*(c*f+d*e))*x*(b*x^2+a)^(1/2)/b^2-1/8*(a*d*f-
2*b*(c*f+d*e))*x*(b*x^2+a)^(3/2)/b^2+1/6*d*f*x^3*(b*x^2+a)^(3/2)/b+1/16*a*
(8*b^2*c*e+a^2*d*f-2*a*b*(c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(
5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-3a^2df+2ab(3de+3cf+dfx^2)+4b^2(6ce+3dex^2+3cfx^2+2dfx^4))-3a(8b^2ce+a^2d}{48b^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a^2*d*f + 2*a*b*(3*d*e + 3*c*f + d*f*x^2) +
 4*b^2*(6*c*e + 3*d*e*x^2 + 3*c*f*x^2 + 2*d*f*x^4)) - 3*a*(8*b^2*c*e + a^2
*d*f - 2*a*b*(d*e + c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(5/2)
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \ 403$$

$$\frac{\int \sqrt{bx^2+a}\left((6bde+2bcf-3adf)x^2+c(6be-af)\right)dx}{6b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{3\left(a^2df-2ab(cf+de)+8b^2ce\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(-3adf+2bcf+6bde)}{4b}}{6b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{3\left(a^2df-2ab(cf+de)+8b^2ce\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(-3adf+2bcf+6bde)}{4b}}{6b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{3\left(a^2df-2ab(cf+de)+8b^2ce\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(-3adf+2bcf+6bde)}{4b}}{6b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2df-2ab(cf+de)+8b^2ce\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(-3adf+2bcf+6bde)}{4b}}{6b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + (((6*b*d*e + 2*b*c*f - 3*a*d*f
)*x*(a + b*x^2)^(3/2))/(4*b) + (3*(8*b^2*c*e + a^2*d*f - 2*a*b*(d*e + c*f)
)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b])))/(4*b))/(6*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{a\left(8ce\,b^2+a^2df-2ab(cf+de)\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\left(\left(\left(-\frac{8df\,x^4}{3}+(-4cf-4de)x^2-8ce\right)b^{\frac{5}{2}}+a\left(\left(-\frac{2}{3}df\,x^2-2cf-2de\right)b^{\frac{3}{2}}+a\right.\right.\right.}{16b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-8b^2df\,x^4-2ab\,x^2df-12b^2cf\,x^2-12b^2de\,x^2+3a^2df-6abcf-6abde-24ce\,b^2\right)\sqrt{b\,x^2+a}}{48b^2}+\frac{a\left(a^2df-2abcf-2abde+8c\right.}{16b}$
default	$ce\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+(cf+de)\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/16*(a*(8*c*e*b^2+a^2*d*f-2*a*b*(c*f+d*e))*arctanh((b*x^2+a)^(1/2)/x/b^(1
/2))-((-8/3*d*f*x^4+(-4*c*f-4*d*e)*x^2-8*c*e)*b^(5/2)+a*((-2/3*d*f*x^2-2*c
*f-2*d*e)*b^(3/2)+a*d*f*b^(1/2)))*(b*x^2+a)^(1/2)*x)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$
$$= \left[\frac{3\,(2\,(4\,ab^2c-a^2bd)e-(2\,a^2bc-a^3d)f)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3dfx^5+2\,(6\,b^3d}{96\,b^3}\right.$$
$$-\frac{3\,(2\,(4\,ab^2c-a^2bd)e-(2\,a^2bc-a^3d)f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3dfx^5+2\,(6\,b^3de+(6\,b^3c+ab^2d)}{48\,b^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/96*(3*(2*(4*a*b^2*c - a^2*b*d)*e - (2*a^2*b*c - a^3*d)*f)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d*f*x^5 + 2*(6*b^3*d
*e + (6*b^3*c + a*b^2*d)*f)*x^3 + 3*(2*(4*b^3*c + a*b^2*d)*e + (2*a*b^2*c
- a^2*b*d)*f)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(2*(4*a*b^2*c - a^2*b*d)*e
 - (2*a^2*b*c - a^3*d)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8
*b^3*d*f*x^5 + 2*(6*b^3*d*e + (6*b^3*c + a*b^2*d)*f)*x^3 + 3*(2*(4*b^3*c +
 a*b^2*d)*e + (2*a*b^2*c - a^2*b*d)*f)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.33

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{dfx^5}{6}+\frac{x^3\left(\frac{adf}{6}+bcf+bde\right)}{4b}+\frac{x\left(acf+ade-\frac{3a\left(\frac{adf}{6}+bcf+bde\right)}{4b}+bce\right)}{2b}\right)+\left(ace-\frac{a\left(acf+ade-\frac{3a\left(\frac{adf}{6}+bcf+bde\right)}{4b}\cdots\right.}{2b}\right. \\ \sqrt{a}\left(cex+\frac{dfx^5}{5}+\frac{x^3(cf+de)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)*(f*x**2+e),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f*x**5/6 + x**3*(a*d*f/6 + b*c*f + b*d*e)/(
4*b) + x*(a*c*f + a*d*e - 3*a*(a*d*f/6 + b*c*f + b*d*e)/(4*b) + b*c*e)/(2*
b)) + (a*c*e - a*(a*c*f + a*d*e - 3*a*(a*d*f/6 + b*c*f + b*d*e)/(4*b) + b*
c*e)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne
(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*e*x + d*f*
x**5/5 + x**3*(c*f + d*e)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}dfx^3}{6\,b}+\frac{1}{2}\sqrt{bx^2+a}\,acex \\ &\quad -\frac{(bx^2+a)^{\frac{3}{2}}adfx}{8\,b^2}+\frac{\sqrt{bx^2+a}\,a^2dfx}{16\,b^2} \\ &\quad +\frac{ace\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}+\frac{a^3df\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} \\ &\quad +\frac{(bx^2+a)^{\frac{3}{2}}(de+cf)x}{4\,b}-\frac{\sqrt{bx^2+a}(de+cf)ax}{8\,b} \\ &\quad -\frac{(de+cf)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*d*f*x^3/b + 1/2*sqrt(b*x^2 + a)*c*e*x - 1/8*(b*x^2 +
 a)^(3/2)*a*d*f*x/b^2 + 1/16*sqrt(b*x^2 + a)*a^2*d*f*x/b^2 + 1/2*a*c*e*arc
sinh(b*x/sqrt(a*b))/sqrt(b) + 1/16*a^3*d*f*arcsinh(b*x/sqrt(a*b))/b^(5/2)
+ 1/4*(b*x^2 + a)^(3/2)*(d*e + c*f)*x/b - 1/8*sqrt(b*x^2 + a)*(d*e + c*f)*
a*x/b - 1/8*(d*e + c*f)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$
$$=\frac{1}{48}\left(2\left(4\,dfx^2+\frac{6\,b^4de+6\,b^4cf+ab^3df}{b^4}\right)x^2+\frac{3\,(8\,b^4ce+2\,ab^3de+2\,ab^3cf-a^2b^2df)}{b^4}\right)\sqrt{bx^2+a}\,x$$
$$-\frac{(8\,ab^2ce-2\,a^2bde-2\,a^2bcf+a^3df)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

output
```
1/48*(2*(4*d*f*x^2 + (6*b^4*d*e + 6*b^4*c*f + a*b^3*d*f)/b^4)*x^2 + 3*(8*b
^4*c*e + 2*a*b^3*d*e + 2*a*b^3*c*f - a^2*b^2*d*f)/b^4)*sqrt(b*x^2 + a)*x -
 1/16*(8*a*b^2*c*e - 2*a^2*b*d*e - 2*a^2*b*c*f + a^3*d*f)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.67

$$\int \sqrt{a+bx^2}(c+dx^2)\left(e+fx^2\right)\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,a^2bdfx+6\sqrt{b\,x^2+a}\,a\,b^2cfx+6\sqrt{b\,x^2+a}\,a\,b^2dex+2\sqrt{b\,x^2+a}\,a\,b^2df\,x^3+24\sqrt{b\,x^2+a}\,b^3}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)*(f*x^2+e),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2*b*d*f*x + 6*sqrt(a + b*x**2)*a*b**2*c*f*x + 6*
sqrt(a + b*x**2)*a*b**2*d*e*x + 2*sqrt(a + b*x**2)*a*b**2*d*f*x**3 + 24*sq
rt(a + b*x**2)*b**3*c*e*x + 12*sqrt(a + b*x**2)*b**3*c*f*x**3 + 12*sqrt(a
+ b*x**2)*b**3*d*e*x**3 + 8*sqrt(a + b*x**2)*b**3*d*f*x**5 + 3*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d*f - 6*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*f - 6*sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*e + 24*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a*b**2*c*e)/(48*b**3)
```

3.263 $\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \frac{dx\sqrt{a+bx^2}}{2f}-\frac{(2bde-2bcf-adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}f^2}+\frac{\sqrt{be-af}(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^2}$$

output
```
1/2*d*x*(b*x^2+a)^(1/2)/f-1/2*(-a*d*f-2*b*c*f+2*b*d*e)*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(1/2)/f^2+(-a*f+b*e)^(1/2)*(-c*f+d*e)*arctanh((-a*f+b*e)
^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \frac{dfx\sqrt{a+bx^2}+\frac{2\sqrt{-be+af}(de-cf)\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}}+\frac{(2bde-2bcf-adf)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{2f^2}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(d*f*x*Sqrt[a + b*x^2] + (2*Sqrt[-(b*e) + a*f]*(d*e - c*f)*ArcTan[(-(f*x*S
qrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/Sqrt
[e] + ((2*b*d*e - 2*b*c*f - a*d*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sq
rt[b])/(2*f^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2f}+\frac{dx\sqrt{a+bx^2}}{2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int \frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2f}$$

$$\Big\downarrow \text{398}$$

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}$$

$$\Big\downarrow \text{224}$$

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}$$

$$\Big\downarrow \text{219}$$

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}$$

$\Big\downarrow$ 291

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2f}$$

$\Big\downarrow$ 221

$$\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{be-af}(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTa
nh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))/(2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\sqrt{b\,x^{2}+a}\,d f x-\frac{\left(a d f+2 b c f-2 b d e\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)}{\sqrt{b}}+\frac{2 \left(a f-b e\right) \left(c f-d e\right) \arctan\left(\frac{e \sqrt{b\,x^{2}+a}}{x \sqrt{\left(a f-b e\right) e}}\right)}{\sqrt{\left(a f-b e\right) e}}}{2 f^{2}}$
risch	$\frac{d x \sqrt{b\,x^{2}+a}}{2 f}+\frac{\frac{\left(a d f+2 b c f-2 b d e\right) \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{f \sqrt{b}}+\frac{\left(a c\,f^{2}-a d e f-b c e f+b d\,e^{2}\right) \ln\left(\frac{\frac{2 a f-2 b e}{f}-\frac{2 b \sqrt{-e f}\left(x+\frac{\sqrt{-e f}}{f}\right)}{f}+2 \sqrt{\frac{a f-b e}{f}} \cdots}{\cdots}\right)}{\sqrt{-e f}\, f \sqrt{\frac{a f-b e}{f}} \cdots}}{\cdots}$
default	$\frac{d\left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{f}-\frac{\left(c f-d e\right)\left(\sqrt{\left(x+\frac{\sqrt{-e f}}{f}\right)^{2} b-\frac{2 b \sqrt{-e f}\left(x+\frac{\sqrt{-e f}}{f}\right)}{f}+\frac{a f-b e}{f}}-\frac{\sqrt{b}\,\sqrt{-e f}\,\ln\left(\frac{-\frac{b \sqrt{-e f}}{f} \cdots}{\cdots}\right)}{\cdots} \cdots\right)}{\cdots}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f^2*(-(b*x^2+a)^(1/2)*d*f*x-(a*d*f+2*b*c*f-2*b*d*e)/b^(1/2)*arctanh((
b*x^2+a)^(1/2)/x/b^(1/2))+2*(a*f-b*e)*(c*f-d*e)/((a*f-b*e)*e)^(1/2)*arctan
(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 777, normalized size of antiderivative = 6.07

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*d*f*x - (2*b*d*e - (2*b*c + a*d)*f)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (b*d*e - b*c*f)*sqrt((b*e -
a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^
2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*s
qrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(b*f^2), 1/4*(2*sqrt(b*x
^2 + a)*b*d*f*x + 2*(2*b*d*e - (2*b*c + a*d)*f)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) - (b*d*e - b*c*f)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 -
8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*
e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x
^4 + 2*e*f*x^2 + e^2)))/(b*f^2), 1/4*(2*sqrt(b*x^2 + a)*b*d*f*x - 2*(b*d*e
 - b*c*f)*sqrt(-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*f)*x^2 + a*e)*sqrt(b
*x^2 + a)*sqrt(-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 + (a*b*e - a^2*f)*x))
- (2*b*d*e - (2*b*c + a*d)*f)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a))/(b*f^2), 1/2*(sqrt(b*x^2 + a)*b*d*f*x + (2*b*d*e - (2*b*c + a
*d)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*d*e - b*c*f)*sqrt(
-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(
-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 + (a*b*e - a^2*f)*x)))/(b*f^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{e+fx^2}\,dx = \frac{-2\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)bcf+2\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)b}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e),x)
```

output
```
( - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*f + 2*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*b*d*e - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*f
 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x*
*2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*d*e + sqrt(a + b*x**2)*b*d*e
*f*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*d*e*f + 2*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*c*e*f - 2*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*d*e**2)/(2*b*e*f**2)
```

3.264 $\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}}{2ef(e+fx^2)} + \frac{\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f^2} - \frac{(2bde^2-af(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^2\sqrt{be-af}}$$

output
```
-1/2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/f/(f*x^2+e)+b^(1/2)*d*arctanh(b^(1/2)*
x/(b*x^2+a)^(1/2))/f^2-1/2*(2*b*d*e^2-a*f*(c*f+d*e))*arctanh((-a*f+b*e)^(1
/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^2/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{\frac{f(-de+cf)x\sqrt{a+bx^2}}{e(e+fx^2)} + \frac{(2bde^2-af(de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{e^{3/2}\sqrt{-be+af}} - 2\sqrt{b}d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2f^2}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
((f*(-(d*e) + c*f)*x*Sqrt[a + b*x^2])/(e*(e + f*x^2)) + ((2*b*d*e^2 - a*f*
(d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e
]*Sqrt[-(b*e) + a*f])])/(e^(3/2)*Sqrt[-(b*e) + a*f]) - 2*Sqrt[b]*d*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/(2*f^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {401, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{2bdex^2+a(de+cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2bdex^2+a(de+cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{2bde\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(2bde^2-af(cf+de))\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{2bde\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(2bde^2-af(cf+de))\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(2bde^2-af(cf+de))\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef\,(e+fx^2)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(2bde^2-af(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef\,(e+fx^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bde^2-af(cf+de))}{\sqrt{e}f\sqrt{be-af}}}{2ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{2ef\,(e+fx^2)}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
-1/2*((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*(e + f*x^2)) + ((2*Sqrt[b]*d*e*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((2*b*d*e^2 - a*f*(d*e + c*f))*Ar
cTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e
- a*f]))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-\left(f\,x^{2}+e\right)\left(ac\,f^{2}+adef-2bd\,e^{2}\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{\left(af-be\right)e}}\right)+\left(2de\sqrt{b}\,\left(f\,x^{2}+e\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^{2}+a}\,fx(cf-de)\right)\sqrt{}}{2\sqrt{\left(af-be\right)e}\,f^{2}e\left(f\,x^{2}+e\right)}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(f*x^2+e)*(a*c*f^2+a*d*e*f-2*b*d*e^2)*arctan(e*(b*x^2+a)^(1/2)/x/((a
*f-b*e)*e)^(1/2))+(2*d*e*b^(1/2)*(f*x^2+e)*arctanh((b*x^2+a)^(1/2)/x/b^(1/
2))+(b*x^2+a)^(1/2)*f*x*(c*f-d*e))*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2
)/f^2/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(121) = 242$.

Time = 1.03 (sec) , antiderivative size = 1267, normalized size of antiderivative = 8.86

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*sqrt(b*x^2 + a)*x -
 4*(b*d*e^4 - a*d*e^3*f + (b*d*e^3*f - a*d*e^2*f^2)*x^2)*sqrt(b)*log(-2*b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (2*b*d*e^3 - a*d*e^2*f - a*c*e*f^
2 + (2*b*d*e^2*f - a*d*e*f^2 - a*c*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b
^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^
2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^
2*x^4 + 2*e*f*x^2 + e^2)))/(b*e^4*f^2 - a*e^3*f^3 + (b*e^3*f^3 - a*e^2*f^4
)*x^2), -1/8*(4*(b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*sqrt(b*x^2 +
 a)*x + 8*(b*d*e^4 - a*d*e^3*f + (b*d*e^3*f - a*d*e^2*f^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*b*d*e^3 - a*d*e^2*f - a*c*e*f^2 + (
2*b*d*e^2*f - a*d*e*f^2 - a*c*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^
2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4
*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4
 + 2*e*f*x^2 + e^2)))/(b*e^4*f^2 - a*e^3*f^3 + (b*e^3*f^3 - a*e^2*f^4)*x^2
), -1/4*(2*(b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*sqrt(b*x^2 + a)*x
 - (2*b*d*e^3 - a*d*e^2*f - a*c*e*f^2 + (2*b*d*e^2*f - a*d*e*f^2 - a*c*f^3
)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)
*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)
*x)) - 2*(b*d*e^4 - a*d*e^3*f + (b*d*e^3*f - a*d*e^2*f^2)*x^2)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b*e^4*f^2 - a*e^3*f^3 + ...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(121) = 242$.

Time = 0.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = -\frac{\sqrt{b}d\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,f^2} + \frac{\left(2\,b^{\frac{3}{2}}de^2 - a\sqrt{b}def - a\sqrt{b}cf^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,\sqrt{-b^2e^2+abef}ef^2} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}de^2 - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}cef - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a\sqrt{b}def + \left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 f + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 be - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right.\right.}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b)*d*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^2 + 1/2*(2*b^(3/2)*d
*e^2 - a*sqrt(b)*d*e*f - a*sqrt(b)*c*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*
b*e*f)*e*f^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*d*e^2 - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*e*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2
*a*sqrt(b)*d*e*f + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*f^2 + a^2*s
qrt(b)*d*e*f - a^2*sqrt(b)*c*f^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*
f + a^2*f)*e*f^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 929, normalized size of antiderivative = 6.50

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*e*f**2 - sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a*c*f**3*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a*d*e**2*f - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqr
t(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*d*e*f**2*x**2 + 2*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*d*e**3 + 2*sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*b*d*e**2*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
- b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*
a*c*e*f**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(
a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*f**3*x**2 - sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*d*e**2*f - sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*a*d*e*f**2*x**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*
e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*d*
e**3 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a...
```

3.265 $\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 170

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3} dx = \frac{(de-cf)x(a+bx^2)^{3/2}}{4e(be-af)(e+fx^2)^2} + \frac{(4bce-ade-3acf)x\sqrt{a+bx^2}}{8e^2(be-af)(e+fx^2)} + \frac{a(4bce-ade-3acf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{3/2}}$$

output

```
1/4*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/e/(-a*f+b*e)/(f*x^2+e)^2+1/8*(-3*a*c*f-a*
d*e+4*b*c*e)*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(f*x^2+e)+1/8*a*(-3*a*c*f-a*
d*e+4*b*c*e)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(
-a*f+b*e)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 11.75 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx$$

$$= \frac{x\sqrt{a+bx^2}\left(\frac{4de\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{fx^2}{e}}} + \frac{4de\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{f}{e}\right)x^2}}{\sqrt{1+\frac{fx^2}{e}}}\right)}{\sqrt{\left(-\frac{b}{a}+\frac{f}{e}\right)x^2}\sqrt{1+\frac{bx^2}{a}}} - \frac{(de-cf)\left(e(2be(2e+fx^2)-af(5e+3fx^2))+\frac{a(4be-3af)(e+fx^2)^2\text{arc}}{\sqrt{\frac{(be-af)x}{e(a+bx^2}}}\right.}{(be-af)(e+fx^2)^2}}{8e^3f}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
(x*Sqrt[a + b*x^2]*((4*d*e*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])/(Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (f*x^2)/e]) + (4*d*e*ArcSin[Sqrt[(-(b/a) + f/e)*x^2]
/Sqrt[1 + (f*x^2)/e]])/(Sqrt[(-(b/a) + f/e)*x^2]*Sqrt[1 + (b*x^2)/a]) - ((
d*e - c*f)*(e*(2*b*e*(2*e + f*x^2) - a*f*(5*e + 3*f*x^2)) + (a*(4*b*e - 3*
a*f)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(Sqrt
[((b*e - a*f)*x^2)/(e*(a + b*x^2))]*(a + b*x^2))))/((b*e - a*f)*(e + f*x^2
)^2)))/(8*e^3*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {401, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 401$$

$$-\frac{\int -\frac{2b(de+cf)x^2+a(de+3cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{2b(de+cf)x^2+a(de+3cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 402

$$\frac{\frac{\int \frac{af(4bce-ade-3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(2be(cf+de)-af(3cf+de))}{2e(e+fx^2)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{af(-3acf-ade+4bce)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(2be(cf+de)-af(3cf+de))}{2e(e+fx^2)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 291

$$\frac{\frac{af(-3acf-ade+4bce)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(2be(cf+de)-af(3cf+de))}{2e(e+fx^2)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{af\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-3acf-ade+4bce)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(2be(cf+de)-af(3cf+de))}{2e(e+fx^2)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
-1/4*((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*(e + f*x^2)^2) + (((2*b*e*(d*e +
 c*f) - a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)
) + (a*f*(4*b*c*e - a*d*e - 3*a*c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*
Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{b\,x^{2}+a}\,\left(3ac\,f^{2}x^{2}+ad\,x^{2}ef-2bcef\,x^{2}-2bd\,e^{2}x^{2}+5acef-ad\,e^{2}-4bc\,e^{2}\right)x}{a\left(f\,x^{2}+e\right)^{2}}+\frac{(3acf+ade-4bce)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{\sqrt{(af-be)e}}\right)}{8(af-be)e^{2}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a/(a*f-b*e)/e^2*(-(b*x^2+a)^(1/2)/a*(3*a*c*f^2*x^2+a*d*e*f*x^2-2*b*c*
e*f*x^2-2*b*d*e^2*x^2+5*a*c*e*f-a*d*e^2-4*b*c*e^2)*x/(f*x^2+e)^2+(3*a*c*f+
a*d*e-4*b*c*e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e
)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(150) = 300.

Time = 0.90 (sec) , antiderivative size = 854, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/32*((3*a^2*c*e^2*f + (3*a^2*c*f^3 - (4*a*b*c - a^2*d)*e*f^2)*x^4 - (4*
a*b*c - a^2*d)*e^3 + 2*(3*a^2*c*e*f^2 - (4*a*b*c - a^2*d)*e^2*f)*x^2)*sqrt
(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(
4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*
e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((2*b^2*d*e^4 + 3*a
^2*c*e*f^3 + (2*b^2*c - 3*a*b*d)*e^3*f - (5*a*b*c - a^2*d)*e^2*f^2)*x^3 +
(5*a^2*c*e^2*f^2 + (4*b^2*c + a*b*d)*e^4 - (9*a*b*c + a^2*d)*e^3*f)*x)*sqr
t(b*x^2 + a))/(b^2*e^7 - 2*a*b*e^6*f + a^2*e^5*f^2 + (b^2*e^5*f^2 - 2*a*b*
e^4*f^3 + a^2*e^3*f^4)*x^4 + 2*(b^2*e^6*f - 2*a*b*e^5*f^2 + a^2*e^4*f^3)*x
^2), 1/16*((3*a^2*c*e^2*f + (3*a^2*c*f^3 - (4*a*b*c - a^2*d)*e*f^2)*x^4 -
(4*a*b*c - a^2*d)*e^3 + 2*(3*a^2*c*e*f^2 - (4*a*b*c - a^2*d)*e^2*f)*x^2)*s
qrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a
*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) + 2
*((2*b^2*d*e^4 + 3*a^2*c*e*f^3 + (2*b^2*c - 3*a*b*d)*e^3*f - (5*a*b*c - a^
2*d)*e^2*f^2)*x^3 + (5*a^2*c*e^2*f^2 + (4*b^2*c + a*b*d)*e^4 - (9*a*b*c +
a^2*d)*e^3*f)*x)*sqrt(b*x^2 + a))/(b^2*e^7 - 2*a*b*e^6*f + a^2*e^5*f^2 + (
b^2*e^5*f^2 - 2*a*b*e^4*f^3 + a^2*e^3*f^4)*x^4 + 2*(b^2*e^6*f - 2*a*b*e^5*
f^2 + a^2*e^4*f^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(150) = 300$.

Time = 0.35 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.05

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*(4*a*b^(3/2)*c*e - a^2*sqrt(b)*d*e - 3*a^2*sqrt(b)*c*f)*arctan(1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/
(sqrt(-b^2*e^2 + a*b*e*f)*(b*e^3 - a*e^2*f)) + 1/4*(8*(sqrt(b)*x - sqrt(b*
x^2 + a))^6*b^(5/2)*d*e^3*f - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*
d*e^2*f^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^3 + (sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*a^2*sqrt(b)*c*f^4 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4
 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 24*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a*b^(5/2)*d*e^3*f - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*
b^(5/2)*c*e^2*f^2 + 14*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*e^2*f
^2 + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 - 3*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d*e*f^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*a^3*sqrt(b)*c*f^4 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*d*e^
3*f + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 - 8*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*d*e^2*f^2 - 28*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^3*b^(3/2)*c*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt
(b)*d*e*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*c*f^4 + 2*a^4*
b^(3/2)*d*e^2*f^2 + 2*a^4*b^(3/2)*c*e*f^3 - a^5*sqrt(b)*d*e*f^3 - 3*a^5*sq
rt(b)*c*f^4)/((b*e^3*f^2 - a*e^2*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f +
 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2655, normalized size of antiderivative = 15.62

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e**2*f**3 - 12*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e*f**4*x**2 - 6*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**3*c*f**5*x**4 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**3*d*e**3*f**2 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e
**2*f**3*x**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e*f**4*x**
4 + 20*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**3*f**2 + 40*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**2*f**3*x**2 + 20*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e*f**4*x**4 + 4*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a**2*b*d*e**4*f + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqr...
```

3.266 $\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 284

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx &= -\frac{(de-cf)x\sqrt{a+bx^2}}{6ef\,(e+fx^2)^3} + \frac{(2be(de+2cf)-af(de+5cf))x\sqrt{a+bx^2}}{24e^2f(be-af)\,(e+fx^2)^2} \\ &\quad + \frac{(4b^2e^2(de+2cf)+3a^2f^2(de+5cf)-2abef(2de+13cf))\,x\sqrt{a+bx^2}}{48e^3f(be-af)^2\,(e+fx^2)} \\ &\quad + \frac{a(8b^2ce^2+a^2f(de+5cf)-2abe(de+6cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{16e^{7/2}(be-af)^{5/2}}\end{aligned}$$

output
```
-1/6*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/f/(f*x^2+e)^3+1/24*(2*b*e*(2*c*f+d*e)-
a*f*(5*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/f/(-a*f+b*e)/(f*x^2+e)^2+1/48*(4*b^
2*e^2*(2*c*f+d*e)+3*a^2*f^2*(5*c*f+d*e)-2*a*b*e*f*(13*c*f+2*d*e))*x*(b*x^2
+a)^(1/2)/e^3/f/(-a*f+b*e)^2/(f*x^2+e)+1/16*a*(8*b^2*c*e^2+a^2*f*(5*c*f+d*
e)-2*a*b*e*(6*c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2)
)/e^(7/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 11.55 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx$$

$$= \frac{x\sqrt{a+bx^2}\left(6d(be-af)^2(e+fx^2)\left(e(2be(2e+fx^2)-af(5e+3fx^2))+\frac{a(4be-3af)(e+fx^2)^2\operatorname{arctanh}\left(\sqrt{\cdots}\right)}{\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}(a+bx^2)}\right.\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
(x*Sqrt[a + b*x^2]*(6*d*(b*e - a*f)^2*(e + f*x^2)*(e*(2*b*e*(2*e + f*x^2)
- a*f*(5*e + 3*f*x^2)) + (a*(4*b*e - 3*a*f)*(e + f*x^2)^2*ArcTanh[Sqrt[((b
*e - a*f)*x^2)/(e*(a + b*x^2))]])/(Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]
*(a + b*x^2))) - (d*e - c*f)*((b*e - a*f)*(8*b^2*e^2*(3*e^2 + 3*e*f*x^2 +
f^2*x^4) - 2*a*b*e*f*(30*e^2 + 35*e*f*x^2 + 13*f^2*x^4) + a^2*f^2*(33*e^2
+ 40*e*f*x^2 + 15*f^2*x^4)) + (3*a*(8*b^2*e^2 - 12*a*b*e*f + 5*a^2*f^2)*Sq
rt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]*(e + f*x^2)^3*ArcTanh[Sqrt[((b*e - a
*f)*x^2)/(e*(a + b*x^2))]])/x^2)))/(48*e^3*f*(b*e - a*f)^3*(e + f*x^2)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {401, 25, 402, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{2b(de+2cf)x^2+a(de+5cf)}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2b(de+2cf)x^2+a(de+5cf)}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int -\frac{a(3af(de+5cf)-2be(de+8cf))-2b(2be(de+2cf)-af(de+5cf))x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}-\frac{\int \frac{a(3af(de+5cf)-2be(de+8cf))-2b(2be(de+2cf)-af(de+5cf))x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}-\frac{\frac{\int -\frac{3af\left(f(de+5cf)a^2-2be(de+6cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)-2abef(13cf+2de)+4b^2e^2(2cf+\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}-\frac{-\frac{3af\left(a^2f(5cf+de)-2abe(6cf+de)+8b^2ce^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)-2abef(13cf+\right.}{2e\left(e+fx^2\right)(be-a}}{4e(be-af)}}{6ef}-\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}-\frac{-\frac{3af\left(a^2f(5cf+de)-2abe(6cf+de)+8b^2ce^2\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)-2abef(1\cdots\right.}{2e\left(e+fx^2\right)(b\cdots}}{4e(be-af)}}{6ef}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow 221$$

$$\frac{\frac{x\sqrt{a+bx^2}(2be(2cf+de)-af(5cf+de))}{4e(e+fx^2)^2(be-af)}-\frac{-\frac{3af\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f(5cf+de)-2abe(6cf+de)+8b^2ce^2\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)-2abef(13\cdots\right.}{2e\left(e+fx^2\right)(be\cdots}}{4e(be-af)}}{6ef}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
-1/6*((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*(e + f*x^2)^3) + (((2*b*e*(d*e +
 2*c*f) - a*f*(d*e + 5*c*f))*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^
2)^2) - (-1/2*((4*b^2*e^2*(d*e + 2*c*f) + 3*a^2*f^2*(d*e + 5*c*f) - 2*a*b*
e*f*(2*d*e + 13*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) - (3*
a*f*(8*b^2*c*e^2 + a^2*f*(d*e + 5*c*f) - 2*a*b*e*(d*e + 6*c*f))*ArcTanh[(S
qrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)
))/(4*e*(b*e - a*f)))/(6*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{5\left(a\left(-\frac{2b(ad-4bc)e^2}{5}+\frac{af(ad-12bc)e}{5}+a^2cf^2\right)(fx^2+e)^3\arctan\left(\frac{e\sqrt{bx^2+a}}{x\sqrt{(af-be)e}}\right)-\frac{11\left(\frac{2\left(ad+4\left(\frac{x^2d}{2}+c\right)b\right)be^4}{11}-\frac{(da^2+20\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{1\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16/((a*f-b*e)*e)^(1/2)*(a*(-2/5*b*(a*d-4*b*c)*e^2+1/5*a*f*(a*d-12*b*c)*
e+a^2*c*f^2)*(f*x^2+e)^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-1
1/5*(2/11*(a*d+4*(1/2*x^2*d+c)*b)*b*e^4-1/11*(d*a^2+20*(7/30*x^2*d+c)*b*a-
8*(1/6*x^2*d+c)*b^2*x^2)*f*e^3+f^2*((8/33*x^2*d+c)*a^2-70/33*(2/35*x^2*d+c
)*b*x^2*a+8/33*b^2*c*x^4)*e^2+40/33*a*((3/40*x^2*d+c)*a-13/20*x^2*b*c)*x^2
*f^3*e+5/11*a^2*c*f^4*x^4)*((a*f-b*e)*e)^(1/2)*(b*x^2+a)^(1/2)*x)/(f*x^2+e
)^3/(a*f-b*e)^2/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(260) = 520$.

Time = 7.00 (sec) , antiderivative size = 1618, normalized size of antiderivative = 5.70

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/192*(3*(5*a^3*c*e^3*f^2 + (5*a^3*c*f^5 + 2*(4*a*b^2*c - a^2*b*d)*e^2*f^
3 - (12*a^2*b*c - a^3*d)*e*f^4)*x^6 + 2*(4*a*b^2*c - a^2*b*d)*e^5 - (12*a^
2*b*c - a^3*d)*e^4*f + 3*(5*a^3*c*e*f^4 + 2*(4*a*b^2*c - a^2*b*d)*e^3*f^2
- (12*a^2*b*c - a^3*d)*e^2*f^3)*x^4 + 3*(5*a^3*c*e^2*f^3 + 2*(4*a*b^2*c -
a^2*b*d)*e^4*f - (12*a^2*b*c - a^3*d)*e^3*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*lo
g(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*
e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 +
a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((4*b^3*d*e^5*f - 15*a^3*c*e*f^5 + 8*
(b^3*c - a*b^2*d)*e^4*f^2 - (34*a*b^2*c - 7*a^2*b*d)*e^3*f^3 + (41*a^2*b*c
 - 3*a^3*d)*e^2*f^4)*x^5 + 2*(6*b^3*d*e^6 - 20*a^3*c*e^2*f^4 + (12*b^3*c -
 13*a*b^2*d)*e^5*f - (47*a*b^2*c - 11*a^2*b*d)*e^4*f^2 + (55*a^2*b*c - 4*a
^3*d)*e^3*f^3)*x^3 - 3*(11*a^3*c*e^3*f^3 - 2*(4*b^3*c + a*b^2*d)*e^6 + (28
*a*b^2*c + 3*a^2*b*d)*e^5*f - (31*a^2*b*c + a^3*d)*e^4*f^2)*x)*sqrt(b*x^2
+ a))/(b^3*e^10 - 3*a*b^2*e^9*f + 3*a^2*b*e^8*f^2 - a^3*e^7*f^3 + (b^3*e^7
*f^3 - 3*a*b^2*e^6*f^4 + 3*a^2*b*e^5*f^5 - a^3*e^4*f^6)*x^6 + 3*(b^3*e^8*f
^2 - 3*a*b^2*e^7*f^3 + 3*a^2*b*e^6*f^4 - a^3*e^5*f^5)*x^4 + 3*(b^3*e^9*f -
 3*a*b^2*e^8*f^2 + 3*a^2*b*e^7*f^3 - a^3*e^6*f^4)*x^2), -1/96*(3*(5*a^3*c*
e^3*f^2 + (5*a^3*c*f^5 + 2*(4*a*b^2*c - a^2*b*d)*e^2*f^3 - (12*a^2*b*c - a
^3*d)*e*f^4)*x^6 + 2*(4*a*b^2*c - a^2*b*d)*e^5 - (12*a^2*b*c - a^3*d)*e^4*
f + 3*(5*a^3*c*e*f^4 + 2*(4*a*b^2*c - a^2*b*d)*e^3*f^2 - (12*a^2*b*c - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1777 vs. 2(260) = 520.

Time = 0.71 (sec) , antiderivative size = 1777, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-1/16*(8*a*b^(5/2)*c*e^2 - 2*a^2*b^(3/2)*d*e^2 - 12*a^2*b^(3/2)*c*e*f + a^
3*sqrt(b)*d*e*f + 5*a^3*sqrt(b)*c*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^5 - 2*a*b*e^4*
f + a^2*e^3*f^2)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/24*(24*(sqrt(b)*x - sqrt(b*
x^2 + a))^10*a*b^(5/2)*c*e^2*f^4 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*
b^(3/2)*d*e^2*f^4 - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*e*f^
5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d*e*f^5 + 15*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*f^6 - 96*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*b^(9/2)*d*e^5*f + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d*e^4*
f^2 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c*e^3*f^3 - 156*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*d*e^3*f^3 - 480*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a^2*b^(5/2)*c*e^2*f^4 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3
*b^(3/2)*d*e^2*f^4 + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*e*f
^5 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d*e*f^5 - 75*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*c*f^6 - 128*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*b^(11/2)*d*e^6 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c*e^5*f
 + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*d*e^5*f + 1216*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*a*b^(9/2)*c*e^4*f^2 - 432*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*a^2*b^(7/2)*d*e^4*f^2 - 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(
7/2)*c*e^3*f^3 + 328*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*d*e^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)}{(f\,x^2+e)^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2))/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 5233, normalized size of antiderivative = 18.43

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**3*f**5 - 45*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**2*f**6*x**2 - 45*sqrt(e)*sqrt(a
*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(
b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e*f**7*x**4 - 15*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a**4*c*f**8*x**6 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
- b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*
a**4*d*e**4*f**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**3*f*
*5*x**2 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**2*f**6*x**4 -
 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e*f**7*x**6 + 66*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**4*f**4 + 198*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**3*f**5*x**2 + 198*sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/...
```

3.267 $\int \sqrt{a+bx^2}(c+dx^2)^2\left(e+fx^2\right)^3\,dx$

Optimal result

Integrand size = 30, antiderivative size = 676

$$\int \sqrt{a+bx^2}(c+dx^2)^2\left(e+fx^2\right)^3\,dx$$
$$= \frac{(512b^5c^2e^3-21a^5d^2f^3+28a^4bdf^2(3de+2cf)-128ab^4ce^2(2de+3cf)-40a^3b^2f(3d^2e^2+6cdef+c^2f^2}{1024b^5}$$
$$+\frac{(21a^4d^2f^3-28a^3bdf^2(3de+2cf)+128b^4ce^2(2de+3cf)+40a^2b^2f(3d^2e^2+6cdef+c^2f^2)-64ab^3e(}{512b^5}$$
$$-\frac{(21a^3d^2f^3-28a^2bdf^2(3de+2cf)+40ab^2f(3d^2e^2+6cdef+c^2f^2)-64b^3e(d^2e^2+6cdef+3c^2f^2))x}{384b^4}$$
$$+\frac{f(21a^2d^2f^2-28abdf(3de+2cf)+40b^2(3d^2e^2+6cdef+c^2f^2))x^5(a+bx^2)^{3/2}}{320b^3}$$
$$+\frac{df^2(12bde+8bcf-3adf)x^7(a+bx^2)^{3/2}}{40b^2}+\frac{d^2f^3x^9(a+bx^2)^{3/2}}{12b}$$
$$+\frac{a(512b^5c^2e^3-21a^5d^2f^3+28a^4bdf^2(3de+2cf)-128ab^4ce^2(2de+3cf)-40a^3b^2f(3d^2e^2+6cdef+c}{1024b^{11/2}}$$

output
```
1/1024*(512*b^5*c^2*e^3-21*a^5*d^2*f^3+28*a^4*b*d*f^2*(2*c*f+3*d*e)-128*a*
b^4*c*e^2*(3*c*f+2*d*e)-40*a^3*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+64*a^2*
b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/512*(21*a^4*d
^2*f^3-28*a^3*b*d*f^2*(2*c*f+3*d*e)+128*b^4*c*e^2*(3*c*f+2*d*e)+40*a^2*b^2
*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-64*a*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))
*x*(b*x^2+a)^(3/2)/b^5-1/384*(21*a^3*d^2*f^3-28*a^2*b*d*f^2*(2*c*f+3*d*e)+
40*a*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-64*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2
*e^2))*x^3*(b*x^2+a)^(3/2)/b^4+1/320*f*(21*a^2*d^2*f^2-28*a*b*d*f*(2*c*f+3
*d*e)+40*b^2*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^5*(b*x^2+a)^(3/2)/b^3+1/40*d
*f^2*(-3*a*d*f+8*b*c*f+12*b*d*e)*x^7*(b*x^2+a)^(3/2)/b^2+1/12*d^2*f^3*x^9*
(b*x^2+a)^(3/2)/b+1/1024*a*(512*b^5*c^2*e^3-21*a^5*d^2*f^3+28*a^4*b*d*f^2*
(2*c*f+3*d*e)-128*a*b^4*c*e^2*(3*c*f+2*d*e)-40*a^3*b^2*f*(c^2*f^2+6*c*d*e*
f+3*d^2*e^2)+64*a^2*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 587, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx$$

$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(315a^5d^2f^3 - 210a^4bdf^2(6de+4cf+dfx^2) + 8a^3b^2f(75c^2f^2+10cdf(45e+7fx^2)+3d^2(\cdots}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5*d^2*f^3 - 210*a^4*b*d*f^2*(6*d*e + 4*c
*f + d*f*x^2) + 8*a^3*b^2*f*(75*c^2*f^2 + 10*c*d*f*(45*e + 7*f*x^2) + 3*d^
2*(75*e^2 + 35*e*f*x^2 + 7*f^2*x^4)) + 64*a*b^4*(5*c^2*f*(18*e^2 + 6*e*f*x
^2 + f^2*x^4) + 6*c*d*(10*e^3 + 10*e^2*f*x^2 + 5*e*f^2*x^4 + f^3*x^6) + d^
2*x^2*(10*e^3 + 15*e^2*f*x^2 + 9*e*f^2*x^4 + 2*f^3*x^6)) - 16*a^2*b^3*(5*c
^2*f^2*(36*e + 5*f*x^2) + 2*c*d*f*(180*e^2 + 75*e*f*x^2 + 14*f^2*x^4) + d^
2*(60*e^3 + 75*e^2*f*x^2 + 42*e*f^2*x^4 + 9*f^3*x^6)) + 128*b^5*(15*c^2*(4
*e^3 + 6*e^2*f*x^2 + 4*e*f^2*x^4 + f^3*x^6) + 6*c*d*x^2*(10*e^3 + 20*e^2*f
*x^2 + 15*e*f^2*x^4 + 4*f^3*x^6) + d^2*x^4*(20*e^3 + 45*e^2*f*x^2 + 36*e*f
^2*x^4 + 10*f^3*x^6))) + 15*a*(-512*b^5*c^2*e^3 + 21*a^5*d^2*f^3 - 28*a^4*
b*d*f^2*(3*d*e + 2*c*f) + 128*a*b^4*c*e^2*(2*d*e + 3*c*f) + 40*a^3*b^2*f*(
3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) - 64*a^2*b^3*e*(d^2*e^2 + 6*c*d*e*f + 3*c
^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(11/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 1009, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(fx^6\sqrt{a+bx^2}\left(c^2f^2+6cdef+3d^2e^2\right)+ex^4\sqrt{a+bx^2}\left(3c^2f^2+6cdef+d^2e^2\right)+c^2e^3\sqrt{a+bx^2}+ce^2x^2\sqrt{a+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}d^2f^3\sqrt{bx^2+a}x^{11}+\frac{ad^2f^3\sqrt{bx^2+a}x^9}{120b}+\frac{1}{10}df^2(3de+2cf)\sqrt{bx^2+a}x^9-\frac{3a^2d^2f^3\sqrt{bx^2+a}x^7}{320b^2}+\frac{adf^2(3de+2cf)\sqrt{bx^2+a}x^7}{80b}+\frac{1}{8}f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^7+\frac{7a^3d^2f^3\sqrt{bx^2+a}x^5}{640b^3}-\frac{7a^2df^2(3de+2cf)\sqrt{bx^2+a}x^5}{480b^2}+\frac{af\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^5}{48b}+\frac{1}{6}e\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^5-\frac{7a^4d^2f^3\sqrt{bx^2+a}x^3}{512b^4}+\frac{7a^3df^2(3de+2cf)\sqrt{bx^2+a}x^3}{384b^3}+\frac{1}{4}ce^2(2de+3cf)\sqrt{bx^2+a}x^3-\frac{5a^2f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{192b^2}+\frac{ae\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^3}{24b}+\frac{1}{2}c^2e^3\sqrt{bx^2+a}x+\frac{21a^5d^2f^3\sqrt{bx^2+a}x}{1024b^5}-\frac{7a^4df^2(3de+2cf)\sqrt{bx^2+a}x}{256b^4}+\frac{ace^2(2de+3cf)\sqrt{bx^2+a}x}{8b}+\frac{5a^3f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{128b^3}-\frac{a^2e\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x}{16b^2}+\frac{ac^2e^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{b}}-\frac{21a^6d^2f^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{1024b^{11/2}}+\frac{7a^5df^2(3de+2cf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{256b^{9/2}}-\frac{a^2ce^2(2de+3cf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{3/2}}-\frac{5a^4f\left(3d^2e^2+6cdfe+c^2f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{7/2}}+\frac{a^3e\left(d^2e^2+6cdfe+3c^2f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{5/2}}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
(c^2*e^3*x*Sqrt[a + b*x^2])/2 + (21*a^5*d^2*f^3*x*Sqrt[a + b*x^2])/(1024*b
^5) - (7*a^4*d*f^2*(3*d*e + 2*c*f)*x*Sqrt[a + b*x^2])/(256*b^4) + (a*c*e^2
*(2*d*e + 3*c*f)*x*Sqrt[a + b*x^2])/(8*b) + (5*a^3*f*(3*d^2*e^2 + 6*c*d*e*
f + c^2*f^2)*x*Sqrt[a + b*x^2])/(128*b^3) - (a^2*e*(d^2*e^2 + 6*c*d*e*f +
3*c^2*f^2)*x*Sqrt[a + b*x^2])/(16*b^2) - (7*a^4*d^2*f^3*x^3*Sqrt[a + b*x^2
])/(512*b^4) + (7*a^3*d*f^2*(3*d*e + 2*c*f)*x^3*Sqrt[a + b*x^2])/(384*b^3)
 + (c*e^2*(2*d*e + 3*c*f)*x^3*Sqrt[a + b*x^2])/4 - (5*a^2*f*(3*d^2*e^2 + 6
*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(192*b^2) + (a*e*(d^2*e^2 + 6*c*d
*e*f + 3*c^2*f^2)*x^3*Sqrt[a + b*x^2])/(24*b) + (7*a^3*d^2*f^3*x^5*Sqrt[a
+ b*x^2])/(640*b^3) - (7*a^2*d*f^2*(3*d*e + 2*c*f)*x^5*Sqrt[a + b*x^2])/(4
80*b^2) + (a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5*Sqrt[a + b*x^2])/(48*
b) + (e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^5*Sqrt[a + b*x^2])/6 - (3*a^2*
d^2*f^3*x^7*Sqrt[a + b*x^2])/(320*b^2) + (a*d*f^2*(3*d*e + 2*c*f)*x^7*Sqrt
[a + b*x^2])/(80*b) + (f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^7*Sqrt[a + b*
x^2])/8 + (a*d^2*f^3*x^9*Sqrt[a + b*x^2])/(120*b) + (d*f^2*(3*d*e + 2*c*f)
*x^9*Sqrt[a + b*x^2])/10 + (d^2*f^3*x^11*Sqrt[a + b*x^2])/12 + (a*c^2*e^3*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]) - (21*a^6*d^2*f^3*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(11/2)) + (7*a^5*d*f^2*(3*d*e + 2*
c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(9/2)) - (a^2*c*e^2*(2*d
*e + 3*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2)) - (5*a^4*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{21\left(a\left(a^3\left(-\frac{8}{3}abcd+a^2d^2+\frac{40}{21}b^2c^2\right)f^3-4a^2be\left(a^2d^2-\frac{20}{7}abcd+\frac{16}{7}b^2c^2\right)f^2+\frac{40a\left(a^2d^2-\frac{16}{5}abcd+\frac{16}{5}b^2c^2\right)b^2e^2f}{7}-\frac{64b^3e^3\left(a^2\right.}{}\right.\right.}{}$
default	$c^2e^3\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)+f^2d(2cf+3de)\left(\frac{x^7\left(bx^2+a\right)^{\frac{3}{2}}}{10b}-\frac{7a\left(\frac{x^5\left(bx^2+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^3}{}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-21/1024*(a*(a^3*(-8/3*a*b*c*d+a^2*d^2+40/21*b^2*c^2)*f^3-4*a^2*b*e*(a^2*d
^2-20/7*a*b*c*d+16/7*b^2*c^2)*f^2+40/7*a*(a^2*d^2-16/5*a*b*c*d+16/5*b^2*c^
2)*b^2*e^2*f-64/21*b^3*e^3*(a^2*d^2-4*a*b*c*d+8*b^2*c^2))*arctanh((b*x^2+a
)^(1/2)/x/b^(1/2))-((128/21*(2/3*d^2*x^4+8/5*c*d*x^2+c^2)*x^6*f^3+512/21*x
^4*(3/5*d^2*x^4+3/2*c*d*x^2+c^2)*e*f^2+256/7*(1/2*d^2*x^4+4/3*c*d*x^2+c^2)
*x^2*e^2*f+512/21*(1/3*d^2*x^4+c*d*x^2+c^2)*e^3)*b^(11/2)+a*((64/63*(2/5*d
^2*x^4+6/5*c*d*x^2+c^2)*x^4*f^3+128/21*(3/10*d^2*x^4+c*d*x^2+c^2)*x^2*e*f^
2+128/7*(1/6*d^2*x^4+2/3*c*d*x^2+c^2)*e^2*f+256/21*d*(1/6*x^2*d+c)*e^3)*b^
(9/2)+a*((-80/63*(9/25*d^2*x^4+28/25*c*d*x^2+c^2)*x^2*f^3-64/7*(7/30*d^2*x
^4+5/6*c*d*x^2+c^2)*e*f^2-128/7*(5/24*x^2*d+c)*d*e^2*f-64/21*d^2*e^3)*b^(7
/2)+a*(((16/9*c*d*x^2+40/21*c^2+8/15*d^2*x^4)*f^2+80/7*d*(7/30*x^2*d+c)*e*
f+40/7*d^2*e^2)*b^(5/2)+a*d*f*(((-8/3*c-2/3*x^2*d)*f-4*d*e)*b^(3/2)+a*d*f*
b^(1/2)))*f)))*(b*x^2+a)^(1/2)*x)/b^(11/2)
```

Fricas [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 1414, normalized size of antiderivative = 2.09

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(64*(8*a*b^5*c^2 - 4*a^2*b^4*c*d + a^3*b^3*d^2)*e^3 - 24*(16
*a^2*b^4*c^2 - 16*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e^2*f + 12*(16*a^3*b^3*c^2
- 20*a^4*b^2*c*d + 7*a^5*b*d^2)*e*f^2 - (40*a^4*b^2*c^2 - 56*a^5*b*c*d + 2
1*a^6*d^2)*f^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(1280*b^6*d^2*f^3*x^11 + 128*(36*b^6*d^2*e*f^2 + (24*b^6*c*d + a*b^5*d^2
)*f^3)*x^9 + 48*(120*b^6*d^2*e^2*f + 12*(20*b^6*c*d + a*b^5*d^2)*e*f^2 + (
40*b^6*c^2 + 8*a*b^5*c*d - 3*a^2*b^4*d^2)*f^3)*x^7 + 8*(320*b^6*d^2*e^3 +
120*(16*b^6*c*d + a*b^5*d^2)*e^2*f + 12*(80*b^6*c^2 + 20*a*b^5*c*d - 7*a^2
*b^4*d^2)*e*f^2 + (40*a*b^5*c^2 - 56*a^2*b^4*c*d + 21*a^3*b^3*d^2)*f^3)*x^
5 + 10*(64*(12*b^6*c*d + a*b^5*d^2)*e^3 + 24*(48*b^6*c^2 + 16*a*b^5*c*d -
5*a^2*b^4*d^2)*e^2*f + 12*(16*a*b^5*c^2 - 20*a^2*b^4*c*d + 7*a^3*b^3*d^2)*
e*f^2 - (40*a^2*b^4*c^2 - 56*a^3*b^3*c*d + 21*a^4*b^2*d^2)*f^3)*x^3 + 15*(
64*(8*b^6*c^2 + 4*a*b^5*c*d - a^2*b^4*d^2)*e^3 + 24*(16*a*b^5*c^2 - 16*a^2
*b^4*c*d + 5*a^3*b^3*d^2)*e^2*f - 12*(16*a^2*b^4*c^2 - 20*a^3*b^3*c*d + 7*
a^4*b^2*d^2)*e*f^2 + (40*a^3*b^3*c^2 - 56*a^4*b^2*c*d + 21*a^5*b*d^2)*f^3)
*x)*sqrt(b*x^2 + a))/b^6, -1/15360*(15*(64*(8*a*b^5*c^2 - 4*a^2*b^4*c*d +
a^3*b^3*d^2)*e^3 - 24*(16*a^2*b^4*c^2 - 16*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e^
2*f + 12*(16*a^3*b^3*c^2 - 20*a^4*b^2*c*d + 7*a^5*b*d^2)*e*f^2 - (40*a^4*b
^2*c^2 - 56*a^5*b*c*d + 21*a^6*d^2)*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (1280*b^6*d^2*f^3*x^11 + 128*(36*b^6*d^2*e*f^2 + (24*b^6*c...
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 1290, normalized size of antiderivative = 1.91

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*f**3*x**11/12 + x**9*(a*d**2*f**3/12 + 2
*b*c*d*f**3 + 3*b*d**2*e*f**2)/(10*b) + x**7*(2*a*c*d*f**3 + 3*a*d**2*e*f*
*2 - 9*a*(a*d**2*f**3/12 + 2*b*c*d*f**3 + 3*b*d**2*e*f**2)/(10*b) + b*c**2
*f**3 + 6*b*c*d*e*f**2 + 3*b*d**2*e**2*f)/(8*b) + x**5*(a*c**2*f**3 + 6*a*
c*d*e*f**2 + 3*a*d**2*e**2*f - 7*a*(2*a*c*d*f**3 + 3*a*d**2*e*f**2 - 9*a*(
a*d**2*f**3/12 + 2*b*c*d*f**3 + 3*b*d**2*e*f**2)/(10*b) + b*c**2*f**3 + 6*
b*c*d*e*f**2 + 3*b*d**2*e**2*f)/(8*b) + 3*b*c**2*e*f**2 + 6*b*c*d*e**2*f +
 b*d**2*e**3)/(6*b) + x**3*(3*a*c**2*e*f**2 + 6*a*c*d*e**2*f + a*d**2*e**3
 - 5*a*(a*c**2*f**3 + 6*a*c*d*e*f**2 + 3*a*d**2*e**2*f - 7*a*(2*a*c*d*f**3
 + 3*a*d**2*e*f**2 - 9*a*(a*d**2*f**3/12 + 2*b*c*d*f**3 + 3*b*d**2*e*f**2)
/(10*b) + b*c**2*f**3 + 6*b*c*d*e*f**2 + 3*b*d**2*e**2*f)/(8*b) + 3*b*c**2
*e*f**2 + 6*b*c*d*e**2*f + b*d**2*e**3)/(6*b) + 3*b*c**2*e**2*f + 2*b*c*d*
e**3)/(4*b) + x*(3*a*c**2*e**2*f + 2*a*c*d*e**3 - 3*a*(3*a*c**2*e*f**2 + 6
*a*c*d*e**2*f + a*d**2*e**3 - 5*a*(a*c**2*f**3 + 6*a*c*d*e*f**2 + 3*a*d**2
*e**2*f - 7*a*(2*a*c*d*f**3 + 3*a*d**2*e*f**2 - 9*a*(a*d**2*f**3/12 + 2*b*
c*d*f**3 + 3*b*d**2*e*f**2)/(10*b) + b*c**2*f**3 + 6*b*c*d*e*f**2 + 3*b*d*
*2*e**2*f)/(8*b) + 3*b*c**2*e*f**2 + 6*b*c*d*e**2*f + b*d**2*e**3)/(6*b) +
 3*b*c**2*e**2*f + 2*b*c*d*e**3)/(4*b) + b*c**2*e**3)/(2*b)) + (a*c**2*e**
3 - a*(3*a*c**2*e**2*f + 2*a*c*d*e**3 - 3*a*(3*a*c**2*e*f**2 + 6*a*c*d*e**
2*f + a*d**2*e**3 - 5*a*(a*c**2*f**3 + 6*a*c*d*e*f**2 + 3*a*d**2*e**2*f...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.33

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(3/2)*d^2*f^3*x^9/b - 3/40*(b*x^2 + a)^(3/2)*a*d^2*f^3*x^
7/b^2 + 21/320*(b*x^2 + a)^(3/2)*a^2*d^2*f^3*x^5/b^3 - 7/128*(b*x^2 + a)^(
3/2)*a^3*d^2*f^3*x^3/b^4 + 1/10*(3*d^2*e*f^2 + 2*c*d*f^3)*(b*x^2 + a)^(3/2
)*x^7/b + 1/2*sqrt(b*x^2 + a)*c^2*e^3*x + 21/512*(b*x^2 + a)^(3/2)*a^4*d^2
*f^3*x/b^5 - 21/1024*sqrt(b*x^2 + a)*a^5*d^2*f^3*x/b^5 - 7/80*(3*d^2*e*f^2
 + 2*c*d*f^3)*(b*x^2 + a)^(3/2)*a*x^5/b^2 + 1/8*(3*d^2*e^2*f + 6*c*d*e*f^2
 + c^2*f^3)*(b*x^2 + a)^(3/2)*x^5/b + 1/2*a*c^2*e^3*arcsinh(b*x/sqrt(a*b))
/sqrt(b) - 21/1024*a^6*d^2*f^3*arcsinh(b*x/sqrt(a*b))/b^(11/2) + 7/96*(3*d
^2*e*f^2 + 2*c*d*f^3)*(b*x^2 + a)^(3/2)*a^2*x^3/b^3 - 5/48*(3*d^2*e^2*f +
6*c*d*e*f^2 + c^2*f^3)*(b*x^2 + a)^(3/2)*a*x^3/b^2 + 1/6*(d^2*e^3 + 6*c*d*
e^2*f + 3*c^2*e*f^2)*(b*x^2 + a)^(3/2)*x^3/b - 7/128*(3*d^2*e*f^2 + 2*c*d*
f^3)*(b*x^2 + a)^(3/2)*a^3*x/b^4 + 7/256*(3*d^2*e*f^2 + 2*c*d*f^3)*sqrt(b*
x^2 + a)*a^4*x/b^4 + 5/64*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*(b*x^2 + a
)^(3/2)*a^2*x/b^3 - 5/128*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*sqrt(b*x^2
 + a)*a^3*x/b^3 - 1/8*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*(b*x^2 + a)^(3
/2)*a*x/b^2 + 1/16*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*sqrt(b*x^2 + a)*a
^2*x/b^2 + 1/4*(2*c*d*e^3 + 3*c^2*e^2*f)*(b*x^2 + a)^(3/2)*x/b - 1/8*(2*c*
d*e^3 + 3*c^2*e^2*f)*sqrt(b*x^2 + a)*a*x/b + 7/256*(3*d^2*e*f^2 + 2*c*d*f^
3)*a^5*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/128*(3*d^2*e^2*f + 6*c*d*e*f^2 +
 c^2*f^3)*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*(d^2*e^3 + 6*c*d*e^...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.16

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*d^2*f^3*x^2 + (36*b^10*d^2*e*f^2 + 24*b^10*c*d*f^3
 + a*b^9*d^2*f^3)/b^10)*x^2 + 3*(120*b^10*d^2*e^2*f + 240*b^10*c*d*e*f^2 +
 12*a*b^9*d^2*e*f^2 + 40*b^10*c^2*f^3 + 8*a*b^9*c*d*f^3 - 3*a^2*b^8*d^2*f^
3)/b^10)*x^2 + (320*b^10*d^2*e^3 + 1920*b^10*c*d*e^2*f + 120*a*b^9*d^2*e^2
*f + 960*b^10*c^2*e*f^2 + 240*a*b^9*c*d*e*f^2 - 84*a^2*b^8*d^2*e*f^2 + 40*
a*b^9*c^2*f^3 - 56*a^2*b^8*c*d*f^3 + 21*a^3*b^7*d^2*f^3)/b^10)*x^2 + 5*(76
8*b^10*c*d*e^3 + 64*a*b^9*d^2*e^3 + 1152*b^10*c^2*e^2*f + 384*a*b^9*c*d*e^
2*f - 120*a^2*b^8*d^2*e^2*f + 192*a*b^9*c^2*e*f^2 - 240*a^2*b^8*c*d*e*f^2
+ 84*a^3*b^7*d^2*e*f^2 - 40*a^2*b^8*c^2*f^3 + 56*a^3*b^7*c*d*f^3 - 21*a^4*
b^6*d^2*f^3)/b^10)*x^2 + 15*(512*b^10*c^2*e^3 + 256*a*b^9*c*d*e^3 - 64*a^2
*b^8*d^2*e^3 + 384*a*b^9*c^2*e^2*f - 384*a^2*b^8*c*d*e^2*f + 120*a^3*b^7*d
^2*e^2*f - 192*a^2*b^8*c^2*e*f^2 + 240*a^3*b^7*c*d*e*f^2 - 84*a^4*b^6*d^2*
e*f^2 + 40*a^3*b^7*c^2*f^3 - 56*a^4*b^6*c*d*f^3 + 21*a^5*b^5*d^2*f^3)/b^10
)*sqrt(b*x^2 + a)*x - 1/1024*(512*a*b^5*c^2*e^3 - 256*a^2*b^4*c*d*e^3 + 64
*a^3*b^3*d^2*e^3 - 384*a^2*b^4*c^2*e^2*f + 384*a^3*b^3*c*d*e^2*f - 120*a^4
*b^2*d^2*e^2*f + 192*a^3*b^3*c^2*e*f^2 - 240*a^4*b^2*c*d*e*f^2 + 84*a^5*b*
d^2*e*f^2 - 40*a^4*b^2*c^2*f^3 + 56*a^5*b*c*d*f^3 - 21*a^6*d^2*f^3)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,\left(f\,x^2+e\right)^3\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\left(f\,x^2+e\right)^3dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

3.268 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 438

$$\begin{aligned}&\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx\\&=\frac{\left(128b^4c^2e^2+7a^4d^2f^2-64ab^3ce(de+cf)-20a^3bdf(de+cf)+16a^2b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x\sqrt{a+bx^2}}{256b^4}\\&\quad-\frac{\left(7a^3d^2f^2-64b^3ce(de+cf)-20a^2bdf(de+cf)+16ab^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x\left(a+bx^2\right)^{3/2}}{128b^4}\\&\quad+\frac{\left(7a^2d^2f^2-20abdf(de+cf)+16b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^3\left(a+bx^2\right)^{3/2}}{96b^3}\\&\quad-\frac{df(7adf-20b(de+cf))x^5\left(a+bx^2\right)^{3/2}}{80b^2}+\frac{d^2f^2x^7\left(a+bx^2\right)^{3/2}}{10b}\\&\quad+\frac{a\left(128b^4c^2e^2+7a^4d^2f^2-64ab^3ce(de+cf)-20a^3bdf(de+cf)+16a^2b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)\operatorname{arcta}}{256b^{9/2}}\end{aligned}$$

output
```
1/256*(128*b^4*c^2*e^2+7*a^4*d^2*f^2-64*a*b^3*c*e*(c*f+d*e)-20*a^3*b*d*f*(
c*f+d*e)+16*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^4-1/1
28*(7*a^3*d^2*f^2-64*b^3*c*e*(c*f+d*e)-20*a^2*b*d*f*(c*f+d*e)+16*a*b^2*(c^
2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(3/2)/b^4+1/96*(7*a^2*d^2*f^2-20*a*b
*d*f*(c*f+d*e)+16*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(3/2)/b^3
-1/80*d*f*(7*a*d*f-20*b*(c*f+d*e))*x^5*(b*x^2+a)^(3/2)/b^2+1/10*d^2*f^2*x^
7*(b*x^2+a)^(3/2)/b+1/256*a*(128*b^4*c^2*e^2+7*a^4*d^2*f^2-64*a*b^3*c*e*(c
*f+d*e)-20*a^3*b*d*f*(c*f+d*e)+16*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$$

$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^4d^2f^2+10a^3bdf(30de+30cf+7dfx^2)+16ab^3(10c^2f(6e+fx^2)+10cd(6e^2+4e}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^4*d^2*f^2 + 10*a^3*b*d*f*(30*d*e + 30*c
*f + 7*d*f*x^2) + 16*a*b^3*(10*c^2*f*(6*e + f*x^2) + 10*c*d*(6*e^2 + 4*e*f
*x^2 + f^2*x^4) + d^2*x^2*(10*e^2 + 10*e*f*x^2 + 3*f^2*x^4)) + 64*b^4*(10*
c^2*(3*e^2 + 3*e*f*x^2 + f^2*x^4) + 5*c*d*x^2*(6*e^2 + 8*e*f*x^2 + 3*f^2*x
^4) + d^2*x^4*(10*e^2 + 15*e*f*x^2 + 6*f^2*x^4)) - 8*a^2*b^2*(30*c^2*f^2 +
 5*c*d*f*(24*e + 5*f*x^2) + d^2*(30*e^2 + 25*e*f*x^2 + 7*f^2*x^4))) - 15*a
*(128*b^4*c^2*e^2 + 7*a^4*d^2*f^2 - 64*a*b^3*c*e*(d*e + c*f) - 20*a^3*b*d*
f*(d*e + c*f) + 16*a^2*b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(3840*b^(9/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(x^4\sqrt{a+bx^2}\left(c^2f^2+4cdef+d^2e^2\right)+c^2e^2\sqrt{a+bx^2}+2cex^2\sqrt{a+bx^2}(cf+de)+2dfx^6\sqrt{a+bx^2}(cf+de)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{7a^5d^2f^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{9/2}}-\frac{5a^4df\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{64b^{7/2}}-\frac{7a^4d^2f^2x\sqrt{a+bx^2}}{256b^4}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(c^2f^2+4cdef+d^2e^2\right)}{16b^{5/2}}+\frac{5a^3dfx\sqrt{a+bx^2}(cf+de)}{64b^3}+\frac{7a^3d^2f^2x^3\sqrt{a+bx^2}}{384b^3}-\frac{a^2ce\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{4b^{3/2}}-\frac{a^2x\sqrt{a+bx^2}\left(c^2f^2+4cdef+d^2e^2\right)}{16b^2}-\frac{5a^2dfx^3\sqrt{a+bx^2}(cf+de)}{96b^2}-\frac{7a^2d^2f^2x^5\sqrt{a+bx^2}}{480b^2}+\frac{ac^2e^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{6}x^5\sqrt{a+bx^2}\left(c^2f^2+4cdef+d^2e^2\right)+\frac{ax^3\sqrt{a+bx^2}\left(c^2f^2+4cdef+d^2e^2\right)}{24b}+\frac{1}{2}c^2e^2x\sqrt{a+bx^2}+\frac{acex\sqrt{a+bx^2}(cf+de)}{4b}+\frac{1}{4}dfx^7\sqrt{a+bx^2}(cf+de)+\frac{adfx^5\sqrt{a+bx^2}(cf+de)}{24b}+\frac{1}{2}cex^3\sqrt{a+bx^2}(cf+de)+\frac{1}{10}d^2f^2x^9\sqrt{a+bx^2}+\frac{ad^2f^2x^7\sqrt{a+bx^2}}{80b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
(c^2*e^2*x*Sqrt[a + b*x^2])/2 - (7*a^4*d^2*f^2*x*Sqrt[a + b*x^2])/(256*b^4
) + (a*c*e*(d*e + c*f)*x*Sqrt[a + b*x^2])/(4*b) + (5*a^3*d*f*(d*e + c*f)*x
*Sqrt[a + b*x^2])/(64*b^3) - (a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x*Sqrt[a
 + b*x^2])/(16*b^2) + (7*a^3*d^2*f^2*x^3*Sqrt[a + b*x^2])/(384*b^3) + (c*e
*(d*e + c*f)*x^3*Sqrt[a + b*x^2])/2 - (5*a^2*d*f*(d*e + c*f)*x^3*Sqrt[a +
b*x^2])/(96*b^2) + (a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])
/(24*b) - (7*a^2*d^2*f^2*x^5*Sqrt[a + b*x^2])/(480*b^2) + (a*d*f*(d*e + c*
f)*x^5*Sqrt[a + b*x^2])/(24*b) + ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^5*Sqrt
[a + b*x^2])/6 + (a*d^2*f^2*x^7*Sqrt[a + b*x^2])/(80*b) + (d*f*(d*e + c*f)
*x^7*Sqrt[a + b*x^2])/4 + (d^2*f^2*x^9*Sqrt[a + b*x^2])/10 + (a*c^2*e^2*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]) + (7*a^5*d^2*f^2*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(9/2)) - (a^2*c*e*(d*e + c*f)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*b^(3/2)) - (5*a^4*d*f*(d*e + c*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(64*b^(7/2)) + (a^3*(d^2*e^2 + 4*c*d*e*f +
c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{7a\left(a^4d^2f^2-\frac{20a^3bdf(cf+de)}{7}+\frac{16a^2b^2\left(c^2f^2+4cdef+d^2e^2\right)}{7}-\frac{64a\,b^3ce(cf+de)}{7}+\frac{128b^4c^2e^2}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{256}-\frac{7\left(\left(-\frac{128\left(\frac{3}{5}f^2\right.}{}\right.\right.}{}$
default	$c^2e^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+2df(cf+de)\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+\right.}{4b}\right.}{}\right)}{}\right.$
risch	$-\frac{x\left(-384d^2f^2b^4x^8-48a\,b^3d^2f^2x^6-960b^4cd\,f^2x^6-960b^4d^2ef\,x^6+56a^2b^2d^2f^2x^4-160a\,b^3cd\,f^2x^4-160a\,b^3d^2ef\,x^4-640b\right.}{}$

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
7/256*(a*(a^4*d^2*f^2-20/7*a^3*b*d*f*(c*f+d*e)+16/7*a^2*b^2*(c^2*f^2+4*c*d
*e*f+d^2*e^2)-64/7*a*b^3*c*e*(c*f+d*e)+128/7*b^4*c^2*e^2)*arctanh((b*x^2+a
)^(1/2)/x/b^(1/2))-((-128/21*(3/5*f^2*x^4+3/2*e*f*x^2+e^2)*x^4*d^2-128/7*c
*(1/2*f^2*x^4+4/3*e*f*x^2+e^2)*x^2*d-128/7*c^2*(1/3*f^2*x^4+e*f*x^2+e^2))*
b^(9/2)+a*((-32/21*(3/10*f^2*x^4+e*f*x^2+e^2)*x^2*d^2-64/7*c*(1/6*f^2*x^4+
2/3*e*f*x^2+e^2)*d-64/7*c^2*(1/6*f*x^2+e)*f)*b^(7/2)+(((40/21*e*f*x^2+16/7
*e^2+8/15*f^2*x^4)*d^2+64/7*c*(5/24*f*x^2+e)*f*d+16/7*c^2*f^2)*b^(5/2)+a*d
*f*(((-20/7*e-2/3*f*x^2)*d-20/7*c*f)*b^(3/2)+a*d*f*b^(1/2)))*a))*(b*x^2+a)
^(1/2)*x)/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 940, normalized size of antiderivative = 2.15

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/7680*(15*(16*(8*a*b^4*c^2 - 4*a^2*b^3*c*d + a^3*b^2*d^2)*e^2 - 4*(16*a^
2*b^3*c^2 - 16*a^3*b^2*c*d + 5*a^4*b*d^2)*e*f + (16*a^3*b^2*c^2 - 20*a^4*b
*c*d + 7*a^5*d^2)*f^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(384*b^5*d^2*f^2*x^9 + 48*(20*b^5*d^2*e*f + (20*b^5*c*d + a*b^4*d
^2)*f^2)*x^7 + 8*(80*b^5*d^2*e^2 + 20*(16*b^5*c*d + a*b^4*d^2)*e*f + (80*b
^5*c^2 + 20*a*b^4*c*d - 7*a^2*b^3*d^2)*f^2)*x^5 + 10*(16*(12*b^5*c*d + a*b
^4*d^2)*e^2 + 4*(48*b^5*c^2 + 16*a*b^4*c*d - 5*a^2*b^3*d^2)*e*f + (16*a*b^
4*c^2 - 20*a^2*b^3*c*d + 7*a^3*b^2*d^2)*f^2)*x^3 + 15*(16*(8*b^5*c^2 + 4*a
*b^4*c*d - a^2*b^3*d^2)*e^2 + 4*(16*a*b^4*c^2 - 16*a^2*b^3*c*d + 5*a^3*b^2
*d^2)*e*f - (16*a^2*b^3*c^2 - 20*a^3*b^2*c*d + 7*a^4*b*d^2)*f^2)*x)*sqrt(b
*x^2 + a))/b^5, -1/3840*(15*(16*(8*a*b^4*c^2 - 4*a^2*b^3*c*d + a^3*b^2*d^2
)*e^2 - 4*(16*a^2*b^3*c^2 - 16*a^3*b^2*c*d + 5*a^4*b*d^2)*e*f + (16*a^3*b^
2*c^2 - 20*a^4*b*c*d + 7*a^5*d^2)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) - (384*b^5*d^2*f^2*x^9 + 48*(20*b^5*d^2*e*f + (20*b^5*c*d + a*b^4
*d^2)*f^2)*x^7 + 8*(80*b^5*d^2*e^2 + 20*(16*b^5*c*d + a*b^4*d^2)*e*f + (80
*b^5*c^2 + 20*a*b^4*c*d - 7*a^2*b^3*d^2)*f^2)*x^5 + 10*(16*(12*b^5*c*d + a
*b^4*d^2)*e^2 + 4*(48*b^5*c^2 + 16*a*b^4*c*d - 5*a^2*b^3*d^2)*e*f + (16*a*
b^4*c^2 - 20*a^2*b^3*c*d + 7*a^3*b^2*d^2)*f^2)*x^3 + 15*(16*(8*b^5*c^2 + 4
*a*b^4*c*d - a^2*b^3*d^2)*e^2 + 4*(16*a*b^4*c^2 - 16*a^2*b^3*c*d + 5*a^3*b
^2*d^2)*e*f - (16*a^2*b^3*c^2 - 20*a^3*b^2*c*d + 7*a^4*b*d^2)*f^2)*x)*s...
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 804, normalized size of antiderivative = 1.84

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2f^2x^9}{10}+\frac{x^7\left(\frac{ad^2f^2}{10}+2bcdf^2+2bd^2ef\right)}{8b}+\frac{x^5\cdot\left(2acdf^2+2ad^2ef-\frac{7a\left(\frac{ad^2f^2}{10}+2bcdf^2+2bd^2ef\right)}{8b}+bc^2f^2+4bcdef+bd^2e^2\right)}{6b}\cdot\right. \\ \sqrt{a}\left(c^2e^2x+\frac{d^2f^2x^9}{9}+\frac{x^7\cdot\left(2cdf^2+2d^2ef\right)}{7}+\frac{x^5\left(c^2f^2+4cdef+d^2e^2\right)}{5}+\frac{x^3\cdot\left(2c^2ef+2cde^2\right)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*f**2*x**9/10 + x**7*(a*d**2*f**2/10 + 2*
b*c*d*f**2 + 2*b*d**2*e*f)/(8*b) + x**5*(2*a*c*d*f**2 + 2*a*d**2*e*f - 7*a
*(a*d**2*f**2/10 + 2*b*c*d*f**2 + 2*b*d**2*e*f)/(8*b) + b*c**2*f**2 + 4*b*
c*d*e*f + b*d**2*e**2)/(6*b) + x**3*(a*c**2*f**2 + 4*a*c*d*e*f + a*d**2*e*
*2 - 5*a*(2*a*c*d*f**2 + 2*a*d**2*e*f - 7*a*(a*d**2*f**2/10 + 2*b*c*d*f**2
 + 2*b*d**2*e*f)/(8*b) + b*c**2*f**2 + 4*b*c*d*e*f + b*d**2*e**2)/(6*b) +
2*b*c**2*e*f + 2*b*c*d*e**2)/(4*b) + x*(2*a*c**2*e*f + 2*a*c*d*e**2 - 3*a*
(a*c**2*f**2 + 4*a*c*d*e*f + a*d**2*e**2 - 5*a*(2*a*c*d*f**2 + 2*a*d**2*e*
f - 7*a*(a*d**2*f**2/10 + 2*b*c*d*f**2 + 2*b*d**2*e*f)/(8*b) + b*c**2*f**2
 + 4*b*c*d*e*f + b*d**2*e**2)/(6*b) + 2*b*c**2*e*f + 2*b*c*d*e**2)/(4*b) +
 b*c**2*e**2)/(2*b)) + (a*c**2*e**2 - a*(2*a*c**2*e*f + 2*a*c*d*e**2 - 3*a
*(a*c**2*f**2 + 4*a*c*d*e*f + a*d**2*e**2 - 5*a*(2*a*c*d*f**2 + 2*a*d**2*e
*f - 7*a*(a*d**2*f**2/10 + 2*b*c*d*f**2 + 2*b*d**2*e*f)/(8*b) + b*c**2*f**
2 + 4*b*c*d*e*f + b*d**2*e**2)/(6*b) + 2*b*c**2*e*f + 2*b*c*d*e**2)/(4*b)
+ b*c**2*e**2)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/s
qrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*
*2*e**2*x + d**2*f**2*x**9/9 + x**7*(2*c*d*f**2 + 2*d**2*e*f)/7 + x**5*(c*
*2*f**2 + 4*c*d*e*f + d**2*e**2)/5 + x**3*(2*c**2*e*f + 2*c*d*e**2)/3), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.32

$$\int \sqrt{a+bx^2}(c+dx^2)^2\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(3/2)*d^2*f^2*x^7/b - 7/80*(b*x^2 + a)^(3/2)*a*d^2*f^2*x^
5/b^2 + 7/96*(b*x^2 + a)^(3/2)*a^2*d^2*f^2*x^3/b^3 + 1/4*(d^2*e*f + c*d*f^
2)*(b*x^2 + a)^(3/2)*x^5/b + 1/2*sqrt(b*x^2 + a)*c^2*e^2*x - 7/128*(b*x^2
+ a)^(3/2)*a^3*d^2*f^2*x/b^4 + 7/256*sqrt(b*x^2 + a)*a^4*d^2*f^2*x/b^4 + 1
/2*a*c^2*e^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 7/256*a^5*d^2*f^2*arcsinh(b*
x/sqrt(a*b))/b^(9/2) - 5/24*(d^2*e*f + c*d*f^2)*(b*x^2 + a)^(3/2)*a*x^3/b^
2 + 1/6*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*(b*x^2 + a)^(3/2)*x^3/b + 5/32*(d^
2*e*f + c*d*f^2)*(b*x^2 + a)^(3/2)*a^2*x/b^3 - 5/64*(d^2*e*f + c*d*f^2)*sq
rt(b*x^2 + a)*a^3*x/b^3 - 1/8*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*(b*x^2 + a)^
(3/2)*a*x/b^2 + 1/16*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*sqrt(b*x^2 + a)*a^2*x
/b^2 + 1/2*(c*d*e^2 + c^2*e*f)*(b*x^2 + a)^(3/2)*x/b - 1/4*(c*d*e^2 + c^2*
e*f)*sqrt(b*x^2 + a)*a*x/b - 5/64*(d^2*e*f + c*d*f^2)*a^4*arcsinh(b*x/sqrt
(a*b))/b^(7/2) + 1/16*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*a^3*arcsinh(b*x/sqrt
(a*b))/b^(5/2) - 1/4*(c*d*e^2 + c^2*e*f)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2
)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.15

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(8\,d^2f^2x^2+\frac{20\,b^8d^2ef+20\,b^8cdf^2+ab^7d^2f^2}{b^8}\right)x^2+\frac{80\,b^8d^2e^2+320\,b^8cdef+20\,ab^7d^2e\right.\right.\right.\right.$$
$$-\frac{(128\,ab^4c^2e^2-64\,a^2b^3cde^2+16\,a^3b^2d^2e^2-64\,a^2b^3c^2ef+64\,a^3b^2cdef-20\,a^4bd^2ef+16\,a^3b^2c^2f^2-2}{256\,b^{\frac{9}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*d^2*f^2*x^2 + (20*b^8*d^2*e*f + 20*b^8*c*d*f^2 + a*b^7*
d^2*f^2)/b^8)*x^2 + (80*b^8*d^2*e^2 + 320*b^8*c*d*e*f + 20*a*b^7*d^2*e*f +
 80*b^8*c^2*f^2 + 20*a*b^7*c*d*f^2 - 7*a^2*b^6*d^2*f^2)/b^8)*x^2 + 5*(192*
b^8*c*d*e^2 + 16*a*b^7*d^2*e^2 + 192*b^8*c^2*e*f + 64*a*b^7*c*d*e*f - 20*a
^2*b^6*d^2*e*f + 16*a*b^7*c^2*f^2 - 20*a^2*b^6*c*d*f^2 + 7*a^3*b^5*d^2*f^2
)/b^8)*x^2 + 15*(128*b^8*c^2*e^2 + 64*a*b^7*c*d*e^2 - 16*a^2*b^6*d^2*e^2 +
 64*a*b^7*c^2*e*f - 64*a^2*b^6*c*d*e*f + 20*a^3*b^5*d^2*e*f - 16*a^2*b^6*c
^2*f^2 + 20*a^3*b^5*c*d*f^2 - 7*a^4*b^4*d^2*f^2)/b^8)*sqrt(b*x^2 + a)*x -
1/256*(128*a*b^4*c^2*e^2 - 64*a^2*b^3*c*d*e^2 + 16*a^3*b^2*d^2*e^2 - 64*a^
2*b^3*c^2*e*f + 64*a^3*b^2*c*d*e*f - 20*a^4*b*d^2*e*f + 16*a^3*b^2*c^2*f^2
 - 20*a^4*b*c*d*f^2 + 7*a^5*d^2*f^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))
)/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}(c+dx^2)^2\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}(c+dx^2)^2\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

3.269 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 267

$$\begin{aligned}\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx &= \frac{(64b^3c^2e-5a^3d^2f-16ab^2c(2de+cf)+8a^2bd(de+2cf))\,x\sqrt{a+bx^2}}{128b^3}\\ &+\frac{(5a^2d^2f+16b^2c(2de+cf)-8abd(de+2cf))\,x\left(a+bx^2\right)^{3/2}}{64b^3}\\ &-\frac{d(5adf-8b(de+2cf))x^3\left(a+bx^2\right)^{3/2}}{48b^2}+\frac{d^2fx^5\left(a+bx^2\right)^{3/2}}{8b}\\ &+\frac{a(64b^3c^2e-5a^3d^2f-16ab^2c(2de+cf)+8a^2bd(de+2cf))\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output

```
1/128*(64*b^3*c^2*e-5*a^3*d^2*f-16*a*b^2*c*(c*f+2*d*e)+8*a^2*b*d*(2*c*f+d*
e))*x*(b*x^2+a)^(1/2)/b^3+1/64*(5*a^2*d^2*f+16*b^2*c*(c*f+2*d*e)-8*a*b*d*(
2*c*f+d*e))*x*(b*x^2+a)^(3/2)/b^3-1/48*d*(5*a*d*f-8*b*(2*c*f+d*e))*x^3*(b*
x^2+a)^(3/2)/b^2+1/8*d^2*f*x^5*(b*x^2+a)^(3/2)/b+1/128*a*(64*b^3*c^2*e-5*a
^3*d^2*f-16*a*b^2*c*(c*f+2*d*e)+8*a^2*b*d*(2*c*f+d*e))*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(15a^3d^2f-2a^2bd(12de+24cf+5dfx^2)+8ab^2(6c^2f+d^2x^2(2e+fx^2)+4cd(3e+fx^2))}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^2*f - 2*a^2*b*d*(12*d*e + 24*c*f + 5*
d*f*x^2) + 8*a*b^2*(6*c^2*f + d^2*x^2*(2*e + f*x^2) + 4*c*d*(3*e + f*x^2))
 + 16*b^3*(6*c^2*(2*e + f*x^2) + 4*c*d*x^2*(3*e + 2*f*x^2) + d^2*x^4*(4*e
+ 3*f*x^2))) + 3*a*(-64*b^3*c^2*e + 5*a^3*d^2*f + 16*a*b^2*c*(2*d*e + c*f)
 - 8*a^2*b*d*(d*e + 2*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(7
/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {403, 403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\int \sqrt{bx^2+a}\left(dx^2+c\right)\left((8bde+4bcf-5adf)x^2+c(8be-af)\right)dx}{8b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int \sqrt{bx^2+a}\left(\left(8c(8de+cf)b^2-4ad(6de+7cf)b+15a^2d^2f\right)x^2+c\left(5dfa^2-8bdea-10bcfa+48b^2ce\right)\right)dx}{6b}+\frac{x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(-5adf+4bcf+8bde\right)}{6b}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{3\left(-5a^3d^2f+8a^2bd(2cf+de)-16ab^2c(cf+2de)+64b^3c^2e\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2d^2f-4abd(7cf+6de)+8b^2c(cf+8de)\right)}{4b}}{6b}+\frac{x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\dots}{6\dots}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{3\left(-5a^3d^2f+8a^2bd(2cf+de)-16ab^2c(cf+2de)+64b^3c^2e\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2d^2f-4abd(7cf+6de)+8b^2c(cf+8de)\right)}{4b}}{6b}+\frac{x(\dots}{\dots}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{3\left(-5a^3d^2f+8a^2bd(2cf+de)-16ab^2c(cf+2de)+64b^3c^2e\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2d^2f-4abd(7cf+6de)+8b^2c(cf+8de\dots\right.}{4b}}{6b}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x\left(a+bx^2\right)^{3/2}\left(15a^2d^2f-4abd(7cf+6de)+8b^2c(cf+8de)\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^2f+8a^2bd(2cf+de)-16ab^2c(cf+2de)+64b^3c^2e\dots\right.}{4b}}{6b}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (((8*b*d*e + 4*b*c*f - 5*a*d
*f)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + (((15*a^2*d^2*f + 8*b^2*c*(8*
d*e + c*f) - 4*a*b*d*(6*d*e + 7*c*f))*x*(a + b*x^2)^(3/2))/(4*b) + (3*(64*
b^3*c^2*e - 5*a^3*d^2*f - 16*a*b^2*c*(2*d*e + c*f) + 8*a^2*b*d*(d*e + 2*c*
f))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*S
qrt[b])))/(4*b))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^2\left(af-\frac{8be}{5}\right)d^2-\frac{16cba(af-2be)d}{5}+\frac{16b^2c^2(af-4be)}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\sqrt{b\,x^2+a}\left(\frac{64\left(\frac{\left(\frac{3f\,x^2}{4}+e\right)x^4d^2}{3}+c\left(\frac{2f}{3}\right.\right.}{}\right.\right.}{}$
risch	$\frac{x\left(48b^3d^2f\,x^6+8a\,b^2x^4d^2f+128b^3cdf\,x^4+64b^3d^2e\,x^4-10a^2b\,x^2d^2f+32a\,b^2cdf\,x^2+16a\,b^2d^2e\,x^2+96b^3c^2f\,x^2+192b^3cde\,x^2\right.}{384b^3}$
default	$c^2e\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+d(2cf+de)\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\right.}{}\right.}{2b}\right.$

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output

```
-5/128/b^(7/2)*(a*(a^2*(a*f-8/5*b*e)*d^2-16/5*c*b*a*(a*f-2*b*e)*d+16/5*b^2
*c^2*(a*f-4*b*e))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*(64/5
*(1/3*(3/4*f*x^2+e)*x^4*d^2+c*(2/3*f*x^2+e)*x^2*d+c^2*(1/2*f*x^2+e))*b^(7/
2)+a*(16/5*(1/3*x^2*(1/2*f*x^2+e)*d^2+2*c*(1/3*f*x^2+e)*d+c^2*f)*b^(5/2)+a
*d*(2*((-1/3*f*x^2-4/5*e)*d-8/5*c*f)*b^(3/2)+a*d*f*b^(1/2))))*x)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.06

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$

$$= \left[-\frac{3\,(8\,(8\,ab^3c^2-4\,a^2b^2cd+a^3bd^2)e-(16\,a^2b^2c^2-16\,a^3bcd+5\,a^4d^2)f)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{}\right.}{}\right.$$

$$-\frac{3\,(8\,(8\,ab^3c^2-4\,a^2b^2cd+a^3bd^2)e-(16\,a^2b^2c^2-16\,a^3bcd+5\,a^4d^2)f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,b^4d}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/768*(3*(8*(8*a*b^3*c^2 - 4*a^2*b^2*c*d + a^3*b*d^2)*e - (16*a^2*b^2*c^
2 - 16*a^3*b*c*d + 5*a^4*d^2)*f)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(48*b^4*d^2*f*x^7 + 8*(8*b^4*d^2*e + (16*b^4*c*d + a*b^
3*d^2)*f)*x^5 + 2*(8*(12*b^4*c*d + a*b^3*d^2)*e + (48*b^4*c^2 + 16*a*b^3*c
*d - 5*a^2*b^2*d^2)*f)*x^3 + 3*(8*(8*b^4*c^2 + 4*a*b^3*c*d - a^2*b^2*d^2)*
e + (16*a*b^3*c^2 - 16*a^2*b^2*c*d + 5*a^3*b*d^2)*f)*x)*sqrt(b*x^2 + a))/b
^4, -1/384*(3*(8*(8*a*b^3*c^2 - 4*a^2*b^2*c*d + a^3*b*d^2)*e - (16*a^2*b^2
*c^2 - 16*a^3*b*c*d + 5*a^4*d^2)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) - (48*b^4*d^2*f*x^7 + 8*(8*b^4*d^2*e + (16*b^4*c*d + a*b^3*d^2)*f)*x
^5 + 2*(8*(12*b^4*c*d + a*b^3*d^2)*e + (48*b^4*c^2 + 16*a*b^3*c*d - 5*a^2*
b^2*d^2)*f)*x^3 + 3*(8*(8*b^4*c^2 + 4*a*b^3*c*d - a^2*b^2*d^2)*e + (16*a*b
^3*c^2 - 16*a^2*b^2*c*d + 5*a^3*b*d^2)*f)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.55

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2fx^7}{8}+\frac{x^5\left(\frac{ad^2f}{8}+2bcdf+bd^2e\right)}{6b}+\frac{x^3\cdot\left(2acdf+ad^2e-\frac{5a\left(\frac{ad^2f}{8}+2bcdf+bd^2e\right)}{6b}+bc^2f+2bcde\right)}{4b}+\frac{x\left(ac^2f+2acde-\frac{3a(\cdots}{\cdots}\right.}{\cdots}\right. \\ \sqrt{a}\left(c^2ex+\frac{d^2fx^7}{7}+\frac{x^5\cdot(2cdf+d^2e)}{5}+\frac{x^3(c^2f+2cde)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2*(f*x**2+e),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*f*x**7/8 + x**5*(a*d**2*f/8 + 2*b*c*d*f
+ b*d**2*e)/(6*b) + x**3*(2*a*c*d*f + a*d**2*e - 5*a*(a*d**2*f/8 + 2*b*c*d
*f + b*d**2*e)/(6*b) + b*c**2*f + 2*b*c*d*e)/(4*b) + x*(a*c**2*f + 2*a*c*d
*e - 3*a*(2*a*c*d*f + a*d**2*e - 5*a*(a*d**2*f/8 + 2*b*c*d*f + b*d**2*e)/(
6*b) + b*c**2*f + 2*b*c*d*e)/(4*b) + b*c**2*e)/(2*b)) + (a*c**2*e - a*(a*c
**2*f + 2*a*c*d*e - 3*a*(2*a*c*d*f + a*d**2*e - 5*a*(a*d**2*f/8 + 2*b*c*d*
f + b*d**2*e)/(6*b) + b*c**2*f + 2*b*c*d*e)/(4*b) + b*c**2*e)/(2*b))*Piece
wise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x
)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c**2*e*x + d**2*f*x**7/7 + x*
*5*(2*c*d*f + d**2*e)/5 + x**3*(c**2*f + 2*c*d*e)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.26

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}d^2fx^5}{8\,b} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}ad^2fx^3}{48\,b^2} + \frac{1}{2}\sqrt{bx^2+a}c^2ex + \frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2d^2fx}{64\,b^3} - \frac{5\sqrt{bx^2+a}a^3d^2fx}{128\,b^3} + \frac{(d^2e+2\,cdf)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b} + \frac{ac^2e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{5\,a^4d^2f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} - \frac{(d^2e+2\,cdf)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} + \frac{(d^2e+2\,cdf)\sqrt{bx^2+a}a^2x}{16\,b^2} + \frac{(2\,cde+c^2f)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(2\,cde+c^2f)\sqrt{bx^2+a}ax}{8\,b} + \frac{(d^2e+2\,cdf)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{(2\,cde+c^2f)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input

```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*d^2*f*x^5/b - 5/48*(b*x^2 + a)^(3/2)*a*d^2*f*x^3/b^2
 + 1/2*sqrt(b*x^2 + a)*c^2*e*x + 5/64*(b*x^2 + a)^(3/2)*a^2*d^2*f*x/b^3 -
5/128*sqrt(b*x^2 + a)*a^3*d^2*f*x/b^3 + 1/6*(d^2*e + 2*c*d*f)*(b*x^2 + a)^
(3/2)*x^3/b + 1/2*a*c^2*e*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d^2*f
*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/8*(d^2*e + 2*c*d*f)*(b*x^2 + a)^(3/2)*
a*x/b^2 + 1/16*(d^2*e + 2*c*d*f)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(2*c*d*e
+ c^2*f)*(b*x^2 + a)^(3/2)*x/b - 1/8*(2*c*d*e + c^2*f)*sqrt(b*x^2 + a)*a*x
/b + 1/16*(d^2*e + 2*c*d*f)*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(2*c*
d*e + c^2*f)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$
$$= \frac{1}{384}\left(2\left(4\left(6\,d^2fx^2+\frac{8\,b^6d^2e+16\,b^6cdf+ab^5d^2f}{b^6}\right)x^2+\frac{96\,b^6cde+8\,ab^5d^2e+48\,b^6c^2f+16\,ab^5cdf-}{b^6}\right.\right.$$
$$-\frac{(64\,ab^3c^2e-32\,a^2b^2cde+8\,a^3bd^2e-16\,a^2b^2c^2f+16\,a^3bcdf-5\,a^4d^2f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*d^2*f*x^2 + (8*b^6*d^2*e + 16*b^6*c*d*f + a*b^5*d^2*f)/b^6)
*x^2 + (96*b^6*c*d*e + 8*a*b^5*d^2*e + 48*b^6*c^2*f + 16*a*b^5*c*d*f - 5*a
^2*b^4*d^2*f)/b^6)*x^2 + 3*(64*b^6*c^2*e + 32*a*b^5*c*d*e - 8*a^2*b^4*d^2*
e + 16*a*b^5*c^2*f - 16*a^2*b^4*c*d*f + 5*a^3*b^3*d^2*f)/b^6)*sqrt(b*x^2 +
 a)*x - 1/128*(64*a*b^3*c^2*e - 32*a^2*b^2*c*d*e + 8*a^3*b*d^2*e - 16*a^2*
b^2*c^2*f + 16*a^3*b*c*d*f - 5*a^4*d^2*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^2\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.84

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$$
$$=\frac{15\sqrt{b\,x^2+a}\,a^3b\,d^2fx-48\sqrt{b\,x^2+a}\,a^2b^2cdfx-24\sqrt{b\,x^2+a}\,a^2b^2d^2ex-10\sqrt{b\,x^2+a}\,a^2b^2d^2f\,x^3+48\sqrt{b}}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2*(f*x^2+e),x)
```

output
```
(15*sqrt(a + b*x**2)*a**3*b*d**2*f*x - 48*sqrt(a + b*x**2)*a**2*b**2*c*d*f
*x - 24*sqrt(a + b*x**2)*a**2*b**2*d**2*e*x - 10*sqrt(a + b*x**2)*a**2*b**
2*d**2*f*x**3 + 48*sqrt(a + b*x**2)*a*b**3*c**2*f*x + 96*sqrt(a + b*x**2)*
a*b**3*c*d*e*x + 32*sqrt(a + b*x**2)*a*b**3*c*d*f*x**3 + 16*sqrt(a + b*x**
2)*a*b**3*d**2*e*x**3 + 8*sqrt(a + b*x**2)*a*b**3*d**2*f*x**5 + 192*sqrt(a
 + b*x**2)*b**4*c**2*e*x + 96*sqrt(a + b*x**2)*b**4*c**2*f*x**3 + 192*sqrt
(a + b*x**2)*b**4*c*d*e*x**3 + 128*sqrt(a + b*x**2)*b**4*c*d*f*x**5 + 64*s
qrt(a + b*x**2)*b**4*d**2*e*x**5 + 48*sqrt(a + b*x**2)*b**4*d**2*f*x**7 -
15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d**2*f + 48*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*d*f + 24*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d**2*e - 48*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c**2*f - 96*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*d*e + 192*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c**2*e)/(384*b**4)
```

3.270 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 198

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx &= -\frac{d(4bde-8bcf-adf)x\sqrt{a+bx^2}}{8bf^2}+\frac{d^2x^3\sqrt{a+bx^2}}{4f}\\ &\quad-\frac{(a^2d^2f^2+4abdf(de-2cf)-8b^2(de-cf)^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}f^3}\\ &\quad-\frac{\sqrt{be-af}(de-cf)^2\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^3}\end{aligned}$$

output
```
-1/8*d*(-a*d*f-8*b*c*f+4*b*d*e)*x*(b*x^2+a)^(1/2)/b/f^2+1/4*d^2*x^3*(b*x^2
+a)^(1/2)/f-1/8*(a^2*d^2*f^2+4*a*b*d*f*(-2*c*f+d*e)-8*b^2*(-c*f+d*e)^2)*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/f^3-(-a*f+b*e)^(1/2)*(-c*f+d*e)^2
*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{e+fx^2}\,dx = \frac{\frac{dfx\sqrt{a+bx^2}(adf+b(-4de+8cf+2dfx^2))}{b} - \frac{8\sqrt{-be+af}(de-cf)^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}\left(e+fx^2\right)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}} - \frac{(-a^2d^2f^2-4abdf(de-2cf)+8b^2(de}{b^{3/2}}}{8f^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
((d*f*x*Sqrt[a + b*x^2]*(a*d*f + b*(-4*d*e + 8*c*f + 2*d*f*x^2)))/b - (8*S
qrt[-(b*e) + a*f]*(d*e - c*f)^2*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(
e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/Sqrt[e] - ((-(a^2*d^2*f^2) - 4*
a*b*d*f*(d*e - 2*c*f) + 8*b^2*(d*e - c*f)^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/b^(3/2))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {420, 299, 211, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{e+fx^2}\,dx$$

$$\Big\downarrow \; 420$$

$$\frac{d\int \sqrt{bx^2+a}\left(dx^2+c\right)dx}{f} - \frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{d\left(\frac{(4bc-ad)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 211

$$\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 403

$$\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{\int-\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{a+bx^2}}{2f}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}\right)}{f}$$

$\Big\downarrow$ 398

$$\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{2(be-af)(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2f}\right)}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{2(be-af)(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2f}\right)}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx(a+bx^2)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx(a+bx^2)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2f}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx(a+bx^2)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{be-af}(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{f}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
(d*((d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*d)*((x*Sqrt[a + b*x^2])/2
+ (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)))/f - ((d*e
 - c*f)*((d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e - 2*b*c*f - a*d*f)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2*Sqrt[b*e - a*f]*(d*e - c
*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))/(
2*f)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{b^{\frac{5}{2}}(-af+be)(cf-de)^{2}\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\left(-\frac{\left(-8b^{2}d^{2}e^{2}+4bdf(ad+4bc)e+f^{2}\left(a^{2}d^{2}-8abcd-8b^{2}c^{2}\right)\right)b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{8}\right.}{\sqrt{(af-be)e}\,f^{3}b^{\frac{5}{2}}}$
risch	$\frac{xd(2bdf\,x^{2}+adf+8bcf-4bde)\sqrt{b\,x^{2}+a}}{8b\,f^{2}}-\frac{\frac{\left(a^{2}d^{2}f^{2}-8abcd\,f^{2}+4ab\,d^{2}ef-8b^{2}c^{2}f^{2}+16b^{2}cdef-8b^{2}d^{2}e^{2}\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{f\sqrt{b}}+}{}$
default	$\frac{d\left(df\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+2cf\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)-de\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\right.\right.}{f^{2}}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(b^(5/2)*(-a*f+b*e)*(c*f-d*e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(
1/2))+(-1/8*(-8*b^2*d^2*e^2+4*b*d*f*(a*d+4*b*c)*e+f^2*(a^2*d^2-8*a*b*c*d-8
*b^2*c^2))*b*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+b^(3/2)*d*(b*x^2+a)^(1/2)*
x*f*(-1/2*b*d*e+1/8*((2*d*x^2+8*c)*b+a*d)*f))*((a*f-b*e)*e)^(1/2))/((a*f-b
*e)*e)^(1/2)/f^3/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.02

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/16*((8*b^2*d^2*e^2 - 4*(4*b^2*c*d + a*b*d^2)*e*f + (8*b^2*c^2 + 8*a*b*
c*d - a^2*d^2)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) - 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((b*e - a*f)/e)*log(
((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*
f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a
*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 2*(2*b^2*d^2*f^2*x^3 - (4*b^2*d^2*e
*f - (8*b^2*c*d + a*b*d^2)*f^2)*x)*sqrt(b*x^2 + a))/(b^2*f^3), -1/8*((8*b^
2*d^2*e^2 - 4*(4*b^2*c*d + a*b*d^2)*e*f + (8*b^2*c^2 + 8*a*b*c*d - a^2*d^2
)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*d^2*e^2 - 2*b^
2*c*d*e*f + b^2*c^2*f^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f +
 a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*
b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*
x^2 + e^2)) - (2*b^2*d^2*f^2*x^3 - (4*b^2*d^2*e*f - (8*b^2*c*d + a*b*d^2)*
f^2)*x)*sqrt(b*x^2 + a))/(b^2*f^3), 1/16*(8*(b^2*d^2*e^2 - 2*b^2*c*d*e*f +
 b^2*c^2*f^2)*sqrt(-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*f)*x^2 + a*e)*sq
rt(b*x^2 + a)*sqrt(-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 + (a*b*e - a^2*f)*
x)) - (8*b^2*d^2*e^2 - 4*(4*b^2*c*d + a*b*d^2)*e*f + (8*b^2*c^2 + 8*a*b*c*
d - a^2*d^2)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(2*b^2*d^2*f^2*x^3 - (4*b^2*d^2*e*f - (8*b^2*c*d + a*b*d^2)*f^2)*x)*sq
rt(b*x^2 + a))/(b^2*f^3), -1/8*((8*b^2*d^2*e^2 - 4*(4*b^2*c*d + a*b*d^2...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**2/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^2}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{e+fx^2}\,dx$$
$$= \frac{-8\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)b^2c^2f^2+16\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\right.}{\sqrt{e}\,\sqrt{b}}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e),x)
```

output
```
( - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*f**2 + 16*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e*f - 8*sqrt(e)*sqrt(a*f - b*e)*ata
n((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e
)*sqrt(b)))*b**2*d**2*e**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*
e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2
*c**2*f**2 + 16*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sq
rt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e*f - 8*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + s
qrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*d**2*e**2 + sqrt(a + b*x**2)*a*b
*d**2*e*f**2*x + 8*sqrt(a + b*x**2)*b**2*c*d*e*f**2*x - 4*sqrt(a + b*x**2)
*b**2*d**2*e**2*f*x + 2*sqrt(a + b*x**2)*b**2*d**2*e*f**2*x**3 - sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d**2*e*f**2 + 8*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d*e*f**2 - 4*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d**2*e**2*f + 8*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c**2*e*f**2 - 16*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c*d*e**2*f + 8*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*d**2*e**3)/(8*b**2*e*f**3)
```

3.271 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{d^2x\sqrt{a+bx^2}}{2f^2}+\frac{(de-cf)^2x\sqrt{a+bx^2}}{2ef^2(e+fx^2)} - \frac{d(4bde-4bcf-adf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}f^3} + \frac{(de-cf)(4bde^2-af(3de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^3\sqrt{be-af}}$$

output
```
1/2*d^2*x*(b*x^2+a)^(1/2)/f^2+1/2*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f^2/(f*
x^2+e)-1/2*d*(-a*d*f-4*b*c*f+4*b*d*e)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b
^(1/2)/f^3+1/2*(-c*f+d*e)*(4*b*d*e^2-a*f*(c*f+3*d*e))*arctanh((-a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^3/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx$$
$$= \frac{\frac{fx\sqrt{a+bx^2}(-2cdef+c^2f^2+d^2e(2e+fx^2))}{e(e+fx^2)} - \frac{(de-cf)(4bde^2-af(3de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{e^{3/2}\sqrt{-be+af}} + \frac{d(4bde-4bcf-adf)\log(}{\sqrt{b}}}{2f^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(-2*c*d*e*f + c^2*f^2 + d^2*e*(2*e + f*x^2)))/(e*(e
+ f*x^2)) - ((d*e - c*f)*(4*b*d*e^2 - a*f*(3*d*e + c*f))*ArcTan[(-(f*x*Sqr
t[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(e^(3/
2)*Sqrt[-(b*e) + a*f]) + (d*(4*b*d*e - 4*b*c*f - a*d*f)*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/Sqrt[b])/(2*f^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 412 vs. 2(195) = 390.

Time = 0.73 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.11, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {425, 420, 299, 224, 219, 398, 224, 219, 291, 221, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx$$
$$\Big\downarrow \ 425$$
$$\frac{b\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f} - \frac{(be-af)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+b\cdots}}{2e\left(e+fx\cdots\right.}\cdots\right.}{f}\cdots\right.}{f}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx\cdots}}{2e\left(e+fx^2\cdots\right.}\cdots\right.}{f}\cdots\right.}{f}$$

input

```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
(b*((d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sq
rt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f - ((b*e - a*f
)*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)
*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b
*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e -
 a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x
)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{4\sqrt{b}\,(cf-de)\left(bd\,e^{2}-\frac{1}{4}ac\,f^{2}-\frac{3}{4}adef\right)\left(f\,x^{2}+e\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\,\left(d\left(f\,x^{2}+e\right)e(-4bde+f(ad+4bc))\,\mathrm{arct}\right.}{2\sqrt{(af-be)e}\,\sqrt{b}\,f^{3}e\left(f\,x^{2}+e\right)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*b^(1/2)*(c*f-d*e)*(b*d*e^2-1/4*a*c*f^2-3/4*a*d*e*f)*(f*x^2+e)*arcta
n(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(d*(f*x^2+e
)*e*(-4*b*d*e+f*(a*d+4*b*c))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^
(1/2)*b^(1/2)*x*f*(2*d^2*e^2-2*d*(-1/2*x^2*d+c)*f*e+c^2*f^2)))/((a*f-b*e)*
e)^(1/2)/b^(1/2)/f^3/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(167) = 334$.

Time = 2.75 (sec) , antiderivative size = 2125, normalized size of antiderivative = 10.90

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(4*b^2*d^2*e^5 - (4*b^2*c*d + 5*a*b*d^2)*e^4*f + (4*a*b*c*d + a^2
*d^2)*e^3*f^2 + (4*b^2*d^2*e^4*f - (4*b^2*c*d + 5*a*b*d^2)*e^3*f^2 + (4*a*
b*c*d + a^2*d^2)*e^2*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - (4*b^2*d^2*e^4 + 2*a*b*c*d*e^2*f^2 + a*b*c^2*e*f^3 - (4*b^2
*c*d + 3*a*b*d^2)*e^3*f + (4*b^2*d^2*e^3*f + 2*a*b*c*d*e*f^3 + a*b*c^2*f^4
 - (4*b^2*c*d + 3*a*b*d^2)*e^2*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e
^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 +
4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^
4 + 2*e*f*x^2 + e^2)) - 4*((b^2*d^2*e^3*f^2 - a*b*d^2*e^2*f^3)*x^3 + (2*b^
2*d^2*e^4*f - a*b*c^2*e*f^4 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 2
*a*b*c*d)*e^2*f^3)*x)*sqrt(b*x^2 + a))/(b^2*e^4*f^3 - a*b*e^3*f^4 + (b^2*e
^3*f^4 - a*b*e^2*f^5)*x^2), 1/8*(4*(4*b^2*d^2*e^5 - (4*b^2*c*d + 5*a*b*d^2
)*e^4*f + (4*a*b*c*d + a^2*d^2)*e^3*f^2 + (4*b^2*d^2*e^4*f - (4*b^2*c*d +
5*a*b*d^2)*e^3*f^2 + (4*a*b*c*d + a^2*d^2)*e^2*f^3)*x^2)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*d^2*e^4 + 2*a*b*c*d*e^2*f^2 + a*b*c^2*
e*f^3 - (4*b^2*c*d + 3*a*b*d^2)*e^3*f + (4*b^2*d^2*e^3*f + 2*a*b*c*d*e*f^3
 + a*b*c^2*f^4 - (4*b^2*c*d + 3*a*b*d^2)*e^2*f^2)*x^2)*sqrt(b*e^2 - a*e*f)
*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a
^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2
 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((b^2*d^2*e^3*f^2 - a*b*d^2*e^2...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**2/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(167) = 334$.

Time = 0.16 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx$$

$$= \frac{\sqrt{bx^2+a}d^2x}{2f^2} + \frac{(4bd^2e-4bcdf-ad^2f)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\sqrt{b}f^3}$$

$$- \frac{\left(4b^{\frac{3}{2}}d^2e^3-4b^{\frac{3}{2}}cde^2f-3a\sqrt{b}d^2e^2f+2a\sqrt{b}cdef^2+a\sqrt{b}c^2f^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2be-af}{2\sqrt{-b^2e^2+abef}}\right)}{2\sqrt{-b^2e^2+abef}ef^3}$$

$$+ \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}d^2e^3-4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cde^2f-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d^2e^2f+2\left(\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f+4\right.}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*d^2*x/f^2 + 1/4*(4*b*d^2*e - 4*b*c*d*f - a*d^2*f)*log(
(sqrt(b)*x - sqrt(b*x^2 + a))^2)/(sqrt(b)*f^3) - 1/2*(4*b^(3/2)*d^2*e^3 -
4*b^(3/2)*c*d*e^2*f - 3*a*sqrt(b)*d^2*e^2*f + 2*a*sqrt(b)*c*d*e*f^2 + a*sq
rt(b)*c^2*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f
)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*e*f^3) + (2*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*b^(3/2)*d^2*e^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*b^(3/2)*c*d*e^2*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2*e^2*f
+ 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c^2*e*f^2 + 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*sqrt(b)*c*d*e*f^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sq
rt(b)*c^2*f^3 + a^2*sqrt(b)*d^2*e^2*f - 2*a^2*sqrt(b)*c*d*e*f^2 + a^2*sqrt
(b)*c^2*f^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f)*e*f^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1801, normalized size of antiderivative = 9.24

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*e*f**3 - sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*f**4*x**2 - 2*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a*b*c*d*e**2*f**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a*b*c*d*e*f**3*x**2 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s
qrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d**2*e
**3*f + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
 b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d**2*e**2*f**2*x**2 +
 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e**3*f + 4*sqrt(e)*sqrt
(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqr
t(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e**2*f**2*x**2 - 4*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*b**2*d**2*e**4 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*b**2*d**2*e**3*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e)
+ sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*...
```

3.272 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 267

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{(de-cf)^2x\sqrt{a+bx^2}}{4ef^2\,(e+fx^2)^2} - \frac{(de-cf)(2be(3de+cf)-af(5de+3cf))x\sqrt{a+bx^2}}{8e^2f^2(be-af)\,(e+fx^2)} + \frac{\sqrt{b}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f^3} - \frac{(8b^2d^2e^4-4abef(3d^2e^2+c^2f^2)+a^2f^2(3d^2e^2+2cdef+3c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}f^3(be-af)^{3/2}}$$

output
```
1/4*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f^2/(f*x^2+e)^2-1/8*(-c*f+d*e)*(2*b*e
*(c*f+3*d*e)-a*f*(3*c*f+5*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(-a*f+b*e)/(f*x^
2+e)+b^(1/2)*d^2*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/f^3-1/8*(8*b^2*d^2*e^4
-4*a*b*e*f*(c^2*f^2+3*d^2*e^2)+a^2*f^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*ar
ctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/f^3/(-a*f+b*e)^(
3/2)
```

Mathematica [A] (verified)

Time = 10.85 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^3}\,dx$$
$$= \frac{\frac{f(de-cf)x\sqrt{a+bx^2}(-2be(cf(2e+fx^2)+de(2e+3fx^2))+af(cf(5e+3fx^2)+de(3e+5fx^2)))}{e^2(be-af)(e+fx^2)^2} + \frac{(8b^2d^2e^4-4abef(3d^2e^2+c^2f^2)+a^2f^2(3d^2e^2}{e^{5/2}(-be+a}}{8f^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
((f*(d*e - c*f)*x*Sqrt[a + b*x^2]*(-2*b*e*(c*f*(2*e + f*x^2) + d*e*(2*e +
3*f*x^2)) + a*f*(c*f*(5*e + 3*f*x^2) + d*e*(3*e + 5*f*x^2))))/(e^2*(b*e -
a*f)*(e + f*x^2)^2) + ((8*b^2*d^2*e^4 - 4*a*b*e*f*(3*d^2*e^2 + c^2*f^2) +
a^2*f^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)
/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*(-(b*e) + a*f)^(3/2)) + 8*Sqrt[b]*d^
2*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*f^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 593 vs. 2(267) = 534.

Time = 0.85 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.22, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {425, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{\int \frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{(2bce-a(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(d}{4e\left(e+fx^2\right)^2(}\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de}{4e\left(e+fx^2\right)^2(b}\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 402$$

$$b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.$$

$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+b}\cdots}{}}{4e(be-af)}\right.}{f}\right.$$

$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad}$$

$\downarrow$ 27

$$b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.$$

$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(\cdots\right.}}{2e(be-af)}}{4e(be-\cdots}\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int\frac{1}{e-\frac{(be-af}{bx^2+}}}{2e(be-af)}}{4e(b}\right.}{}\right.}{f}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f(3cf+de)-4abe(2cf\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(b}\right.}{}\right.}{f}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(b*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f
)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[
b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e
- a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*
x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f - ((b
*e - a*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2
)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*
f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*
c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2))
 + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(S
qrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)
))/(4*e*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\frac{3\left(\frac{8b^2d^2e^4}{3}-4ab\,d^2e^3f+a^2d^2e^2f^2+\frac{2ac\,f^3(ad-2bc)e}{3}+a^2c^2f^4\right)\left(f\,x^2+e\right)^2\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\sqrt{(af-be)e}\left(d^2\left(b^{\frac{3}{2}}e-af\right.\right.}{\sqrt{(af-be)e}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/((a*f-b*e)*e)^(1/2)*(3/8*(8/3*b^2*d^2*e^4-4*a*b*d^2*e^3*f+a^2*d^2*e^2*f
^2+2/3*a*c*f^3*(a*d-2*b*c)*e+a^2*c^2*f^4)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(
1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(d^2*(b^(3/2)*e-a*f*b^(1/2
))*(f*x^2+e)^2*e^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-5/8*(c*f-d*e)*(b*x^2
+a)^(1/2)*x*f*(-4/5*b*d*e^3+3/5*(a*d-2*b*d*x^2-4/3*b*c)*f*e^2+((a*d-2/5*b*
c)*x^2+a*c)*f^2*e+3/5*a*c*f^3*x^2)))/f^3/(f*x^2+e)^2/(a*f-b*e)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(241) = 482$.

Time = 9.95 (sec) , antiderivative size = 3262, normalized size of antiderivative = 12.22

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**2/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1368 vs. $2(241) = 482$.

Time = 0.18 (sec) , antiderivative size = 1368, normalized size of antiderivative = 5.12

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(8*b^(5/2)*d^2*e^4 - 12*a*b^(3/2)*d^2*e^3*f + 3*a^2*sqrt(b)*d^2*e^2*f^
2 - 4*a*b^(3/2)*c^2*e*f^3 + 2*a^2*sqrt(b)*c*d*e*f^3 + 3*a^2*sqrt(b)*c^2*f^
4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*
e^2 + a*b*e*f))/((b*e^3*f^3 - a*e^2*f^4)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/2*s
qrt(b)*d^2*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^3 - 1/4*(16*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*b^(5/2)*d^2*e^4*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*b^(5/2)*c*d*e^3*f^2 - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d^2*e^
3*f^2 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d*e^2*f^3 + 5*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^2*e^2*f^3 + 4*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a*b^(3/2)*c^2*e*f^4 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*s
qrt(b)*c*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^2*f^5 +
 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^2*e^5 - 32*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*b^(7/2)*c*d*e^4*f - 88*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^
(5/2)*d^2*e^4*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^2*e^3*f^2 +
 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d*e^3*f^2 + 58*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^2*e^3*f^2 + 40*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a*b^(5/2)*c^2*e^2*f^3 - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(
3/2)*c*d*e^2*f^3 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^2*e^2*
f^3 - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c^2*e*f^4 + 6*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*c*d*e*f^4 + 9*(sqrt(b)*x - sqrt(b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 5191, normalized size of antiderivative = 19.44

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output

```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e**2*f**5 - 12*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e*f**6*x**2 - 6*sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*f**7*x**4 - 4*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**3*c*d*e**3*f**4 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a**3*c*d*e**2*f**5*x**2 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*
e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3
*c*d*e*f**6*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**4
*f**3 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
+ b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**3*f**4*x**2
 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x*
*2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**2*f**5*x**4 + 20*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c**2*e**3*f**4 + 40*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt...
```

3.273 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 351

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}\,dx = \frac{(de-cf)^2x\sqrt{a+bx^2}}{6ef^2\left(e+fx^2\right)^3} - \frac{(de-cf)(4be(2de+cf)-af(7de+5cf))x\sqrt{a+bx^2}}{24e^2f^2(be-af)\left(e+fx^2\right)^2} + \frac{(8b^2e^2(d^2e^2+cdef+c^2f^2)+3a^2f^2(d^2e^2+2cdef+5c^2f^2)-2abef(7d^2e^2+4cdef+13c^2f^2))\,x\sqrt{a+}}{48e^3f^2(be-af)^2\left(e+fx^2\right)} + \frac{a(8b^2c^2e^2-4abce(de+3cf)+a^2(d^2e^2+2cdef+5c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{16e^{7/2}(be-af)^{5/2}}$$

output
```
1/6*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f^2/(f*x^2+e)^3-1/24*(-c*f+d*e)*(4*b*
e*(c*f+2*d*e)-a*f*(5*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(-a*f+b*e)/(f*x
^2+e)^2+1/48*(8*b^2*e^2*(c^2*f^2+c*d*e*f+d^2*e^2)+3*a^2*f^2*(5*c^2*f^2+2*c
*d*e*f+d^2*e^2)-2*a*b*e*f*(13*c^2*f^2+4*c*d*e*f+7*d^2*e^2))*x*(b*x^2+a)^(1
/2)/e^3/f^2/(-a*f+b*e)^2/(f*x^2+e)+1/16*a*(8*b^2*c^2*e^2-4*a*b*c*e*(3*c*f+
d*e)+a^2*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)
/(b*x^2+a)^(1/2))/e^(7/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 15.19 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

$$= \frac{x\sqrt{a+bx^2}\left(\frac{24d^2e\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{fx^2}{e}}}+\frac{24d^2e\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{f}{e}\right)x^2}}{\sqrt{1+\frac{fx^2}{e}}}\right)}{\sqrt{\left(-\frac{b}{a}+\frac{f}{e}\right)x^2}\sqrt{1+\frac{bx^2}{a}}}-\frac{12d(de-cf)\left(e(2be(2e+fx^2)-af(5e+3fx^2))+\frac{a(4be-3af)(e+fx^2 \cdots}{\sqrt{\frac{(be \cdots}{e(\cdots}}}\right.}{(be-af)(e+fx^2)^2}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
(x*Sqrt[a + b*x^2]*((24*d^2*e*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])/(Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (f*x^2)/e]) + (24*d^2*e*ArcSin[Sqrt[(-(b/a) + f/e
)*x^2]/Sqrt[1 + (f*x^2)/e]])/(Sqrt[(-(b/a) + f/e)*x^2]*Sqrt[1 + (b*x^2)/a]
) - (12*d*(d*e - c*f)*(e*(2*b*e*(2*e + f*x^2) - a*f*(5*e + 3*f*x^2)) + (a*
(4*b*e - 3*a*f)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2
))]])/(Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]*(a + b*x^2))))/((b*e - a*f)
*(e + f*x^2)^2) + ((d*e - c*f)^2*((b*e - a*f)*(8*b^2*e^2*(3*e^2 + 3*e*f*x^
2 + f^2*x^4) - 2*a*b*e*f*(30*e^2 + 35*e*f*x^2 + 13*f^2*x^4) + a^2*f^2*(33*
e^2 + 40*e*f*x^2 + 15*f^2*x^4)) + (3*a*(8*b^2*e^2 - 12*a*b*e*f + 5*a^2*f^2
)*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]*(e + f*x^2)^3*ArcTanh[Sqrt[((b*e
 - a*f)*x^2)/(e*(a + b*x^2))]])/x^2))/((b*e - a*f)^3*(e + f*x^2)^3)))/(48*
e^3*f^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 947 vs. 2(351) = 702.

Time = 1.22 (sec) , antiderivative size = 947, normalized size of antiderivative = 2.70, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {425, 425, 402, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}-\frac{(be-af)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^4}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d\left(\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{4b(de-cf)x^2+6bce-ade-5acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4b(de-cf)x^2+6bce-ade-5acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}\right.}{f}\right.}{f}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4b(de-cf)x^2+6bce-ade-5acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}\right.}{f}\right.}{f}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{4b(de-cf)x^2+6bce-ade-5acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}\right.}{f}\right.$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\int}{4e(be}\right.}{f}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{b}}{6e(be-af)(}\right.}{}\right.$$

$$f$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\cdots}{\cdots}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}\right)}{f}-\frac{(de\cdots}{\cdots}\right.}{\cdots}$$

$$\downarrow 291$$

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\cdots}{\cdots}\right)}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}}{4e(be-af)}\right)}{f}-\cdots\right)}{\cdots}$$

↓ 221

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\cdots}{\cdots}\right.}{f}\cdots\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right)}{f}-\cdots\right.$$

$\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(}{}}{}\right.}{f}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right)}{f}-\right.$$

$\downarrow$ 27

$$b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\cdots}{\cdots}\right.}{f}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right)}{f}-\cdots\right.$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\cdots}{\cdots}\right.}{f}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right)}{f}-\cdots\right.}{\cdots}$$

$\downarrow$ 221

$$b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(}{}}{}\right.}{f}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\left(f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right)}{f}-\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
(b*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((
2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x
^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sq
rt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*c*f) +
a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((8*
b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(Sqrt[b*e
 - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e
*(b*e - a*f))))/f))/f - ((b*e - a*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/
(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*
f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a
*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqr
t[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))
/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(6*e*(b*e - a*f)*(e + f
*x^2)^3) + (((2*b*e*(2*d*e - 5*c*f) + a*f*(d*e + 5*c*f))*x*Sqrt[a + b*x^2]
)/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((4*b^2*e^2*(2*d*e - 11*c*f) - 3*a^2*
f^2*(d*e + 5*c*f) + 2*a*b*e*f*(5*d*e + 22*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b
*e - a*f)*(e + f*x^2)) + (3*(16*b^3*c*e^3 - 8*a*b^2*e^2*(d*e + 3*c*f) - a^
3*f^2*(d*e + 5*c*f) + 2*a^2*b*e*f*(2*d*e + 9*c*f))*ArcTanh[(Sqrt[b*e - a*f
]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e
- a*f)))/(6*e*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{5\left(a\left(\frac{\left(a^{2}d^{2}-4abcd+8b^{2}c^{2}\right)e^{2}}{5}+\frac{2acf(ad-6bc)e}{5}+a^{2}c^{2}f^{2}\right)\left(f\,x^{2}+e\right)^{3}\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)-\frac{11\left(\frac{\left(-a^{2}d^{2}+4d\left(\frac{x^{2}d}{6}+c\right)ba+\right.}{}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16*(a*(1/5*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*e^2+2/5*a*c*f*(a*d-6*b*c)*e+a^
2*c^2*f^2)*(f*x^2+e)^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-11/
5*(1/11*(-a^2*d^2+4*d*(1/6*x^2*d+c)*b*a+8*(1/3*d^2*x^4+c*d*x^2+c^2)*b^2)*e
^4-2/11*f*(d*(4/3*x^2*d+c)*a^2+10*(7/30*d^2*x^4+7/15*c*d*x^2+c^2)*b*a-4*c*
(1/3*x^2*d+c)*b^2*x^2)*e^3+((1/11*d^2*x^4+16/33*c*d*x^2+c^2)*a^2-70/33*c*(
4/35*x^2*d+c)*b*x^2*a+8/33*b^2*c^2*x^4)*f^2*e^2+40/33*a*c*((3/20*x^2*d+c)*
a-13/20*x^2*b*c)*x^2*f^3*e+5/11*a^2*c^2*f^4*x^4)*((a*f-b*e)*e)^(1/2)*(b*x^
2+a)^(1/2)*x)/((a*f-b*e)*e)^(1/2)/(f*x^2+e)^3/(a*f-b*e)^2/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(327) = 654$.

Time = 7.46 (sec) , antiderivative size = 1976, normalized size of antiderivative = 5.63

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[1/192*(3*(5*a^3*c^2*e^3*f^2 + (5*a^3*c^2*f^5 + (8*a*b^2*c^2 - 4*a^2*b*c*d
 + a^3*d^2)*e^2*f^3 - 2*(6*a^2*b*c^2 - a^3*c*d)*e*f^4)*x^6 + (8*a*b^2*c^2
- 4*a^2*b*c*d + a^3*d^2)*e^5 - 2*(6*a^2*b*c^2 - a^3*c*d)*e^4*f + 3*(5*a^3*
c^2*e*f^4 + (8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*e^3*f^2 - 2*(6*a^2*b*c^2
 - a^3*c*d)*e^2*f^3)*x^4 + 3*(5*a^3*c^2*e^2*f^3 + (8*a*b^2*c^2 - 4*a^2*b*c
*d + a^3*d^2)*e^4*f - 2*(6*a^2*b*c^2 - a^3*c*d)*e^3*f^2)*x^2)*sqrt(b*e^2 -
 a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^
2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqr
t(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((8*b^3*d^2*e^6 - 15*a^3*c^
2*e*f^5 + 2*(4*b^3*c*d - 11*a*b^2*d^2)*e^5*f + (8*b^3*c^2 - 16*a*b^2*c*d +
 17*a^2*b*d^2)*e^4*f^2 - (34*a*b^2*c^2 - 14*a^2*b*c*d + 3*a^3*d^2)*e^3*f^3
 + (41*a^2*b*c^2 - 6*a^3*c*d)*e^2*f^4)*x^5 - 2*(20*a^3*c^2*e^2*f^4 - (12*b
^3*c*d + a*b^2*d^2)*e^6 - (12*b^3*c^2 - 26*a*b^2*c*d - 5*a^2*b*d^2)*e^5*f
+ (47*a*b^2*c^2 - 22*a^2*b*c*d - 4*a^3*d^2)*e^4*f^2 - (55*a^2*b*c^2 - 8*a^
3*c*d)*e^3*f^3)*x^3 - 3*(11*a^3*c^2*e^3*f^3 - (8*b^3*c^2 + 4*a*b^2*c*d - a
^2*b*d^2)*e^6 + (28*a*b^2*c^2 + 6*a^2*b*c*d - a^3*d^2)*e^5*f - (31*a^2*b*c
^2 + 2*a^3*c*d)*e^4*f^2)*x)*sqrt(b*x^2 + a))/(b^3*e^10 - 3*a*b^2*e^9*f + 3
*a^2*b*e^8*f^2 - a^3*e^7*f^3 + (b^3*e^7*f^3 - 3*a*b^2*e^6*f^4 + 3*a^2*b*e^
5*f^5 - a^3*e^4*f^6)*x^6 + 3*(b^3*e^8*f^2 - 3*a*b^2*e^7*f^3 + 3*a^2*b*e^6*
f^4 - a^3*e^5*f^5)*x^4 + 3*(b^3*e^9*f - 3*a*b^2*e^8*f^2 + 3*a^2*b*e^7*f...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**2/(e + f*x**2)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2774 vs. $2(327) = 654$.

Time = 0.75 (sec) , antiderivative size = 2774, normalized size of antiderivative = 7.90

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-1/16*(8*a*b^(5/2)*c^2*e^2 - 4*a^2*b^(3/2)*c*d*e^2 + a^3*sqrt(b)*d^2*e^2 -
 12*a^2*b^(3/2)*c^2*e*f + 2*a^3*sqrt(b)*c*d*e*f + 5*a^3*sqrt(b)*c^2*f^2)*a
rctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2
+ a*b*e*f))/((b^2*e^5 - 2*a*b*e^4*f + a^2*e^3*f^2)*sqrt(-b^2*e^2 + a*b*e*f
)) + 1/24*(48*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d^2*e^5*f^2 - 96*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*d^2*e^4*f^3 + 48*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*d^2*e^3*f^4 - 24*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*a*b^(5/2)*c^2*e^2*f^5 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/
2)*c*d*e^2*f^5 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^2*e^2*f^
5 + 36*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c^2*e*f^6 - 6*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*d*e*f^6 - 15*(sqrt(b)*x - sqrt(b*x^
2 + a))^10*a^3*sqrt(b)*c^2*f^7 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/
2)*d^2*e^6*f + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*d*e^5*f^2 - 5
28*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d^2*e^5*f^2 - 384*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*a*b^(7/2)*c*d*e^4*f^3 + 480*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*a^2*b^(5/2)*d^2*e^4*f^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(
7/2)*c^2*e^3*f^4 + 312*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c*d*e^3
*f^4 - 174*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*d^2*e^3*f^4 + 480*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*e^2*f^5 - 120*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d*e^2*f^5 + 15*(sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^2}{{\left(f\,x^2+e\right)}^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^2)/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 7090, normalized size of antiderivative = 20.20

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^2}{(e+fx^2)^4} dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e**3*f**5 - 45*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e**2*f**6*x**2 - 45*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e*f**7*x**4 - 15*sqrt(e)*sqrt(a*f
 - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)
*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*f**8*x**6 - 6*sqrt(e)*sqrt(a*f - b*e)*ata
n((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e
)*sqrt(b)))*a**4*c*d*e**4*f**4 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**4*c*d*e**3*f**5*x**2 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*
c*d*e**2*f**6*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqr
t(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d*e*f
**7*x**6 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(
a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d**2*e**5*f**3 -
9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2)
 - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d**2*e**4*f**4*x**2 - 9*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - ...
```

3.274 $\int \sqrt{a+bx^2}(c+dx^2)^3\,(e+fx^2)^2\;dx$

Optimal result

Integrand size = 30, antiderivative size = 676

$$\int \sqrt{a+bx^2}(c+dx^2)^3\,(e+fx^2)^2\;dx$$
$$= \frac{(512b^5c^3e^2 - 21a^5d^3f^2 - 128ab^4c^2e(3de+2cf) + 28a^4bd^2f(2de+3cf) + 64a^2b^3c(3d^2e^2+6cdef+c^2f^2}{1024b^5}$$
$$+ \frac{(21a^4d^3f^2 + 128b^4c^2e(3de+2cf) - 28a^3bd^2f(2de+3cf) - 64ab^3c(3d^2e^2+6cdef+c^2f^2) + 40a^2b^2d(}{512b^5}$$
$$- \frac{(21a^3d^3f^2 - 28a^2bd^2f(2de+3cf) - 64b^3c(3d^2e^2+6cdef+c^2f^2) + 40ab^2d(d^2e^2+6cdef+3c^2f^2))\,x}{384b^4}$$
$$+ \frac{d(21a^2d^2f^2 - 28abdf(2de+3cf) + 40b^2(d^2e^2+6cdef+3c^2f^2))\,x^5(a+bx^2)^{3/2}}{320b^3}$$
$$+ \frac{d^2f(8bde+12bcf-3adf)x^7(a+bx^2)^{3/2}}{40b^2} + \frac{d^3f^2x^9(a+bx^2)^{3/2}}{12b}$$
$$+ \frac{a(512b^5c^3e^2 - 21a^5d^3f^2 - 128ab^4c^2e(3de+2cf) + 28a^4bd^2f(2de+3cf) + 64a^2b^3c(3d^2e^2+6cdef+c}{1024b^{11/2}}$$

output
```
1/1024*(512*b^5*c^3*e^2-21*a^5*d^3*f^2-128*a*b^4*c^2*e*(2*c*f+3*d*e)+28*a^
4*b*d^2*f*(3*c*f+2*d*e)+64*a^2*b^3*c*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-40*a^3*
b^2*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/512*(21*a^4*d
^3*f^2+128*b^4*c^2*e*(2*c*f+3*d*e)-28*a^3*b*d^2*f*(3*c*f+2*d*e)-64*a*b^3*c
*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+40*a^2*b^2*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))
*x*(b*x^2+a)^(3/2)/b^5-1/384*(21*a^3*d^3*f^2-28*a^2*b*d^2*f*(3*c*f+2*d*e)-
64*b^3*c*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+40*a*b^2*d*(3*c^2*f^2+6*c*d*e*f+d^2
*e^2))*x^3*(b*x^2+a)^(3/2)/b^4+1/320*d*(21*a^2*d^2*f^2-28*a*b*d*f*(3*c*f+2
*d*e)+40*b^2*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5*(b*x^2+a)^(3/2)/b^3+1/40*d
^2*f*(-3*a*d*f+12*b*c*f+8*b*d*e)*x^7*(b*x^2+a)^(3/2)/b^2+1/12*d^3*f^2*x^9*
(b*x^2+a)^(3/2)/b+1/1024*a*(512*b^5*c^3*e^2-21*a^5*d^3*f^2-128*a*b^4*c^2*e
*(2*c*f+3*d*e)+28*a^4*b*d^2*f*(3*c*f+2*d*e)+64*a^2*b^3*c*(c^2*f^2+6*c*d*e*
f+3*d^2*e^2)-40*a^3*b^2*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.86

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx$$

$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(315a^5d^3f^2 - 210a^4bd^2f(4de+6cf+dfx^2) + 64ab^4(10c^3f(6e+fx^2) + 15c^2d(6e^2+4efx}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5*d^3*f^2 - 210*a^4*b*d^2*f*(4*d*e + 6*c
*f + d*f*x^2) + 64*a*b^4*(10*c^3*f*(6*e + f*x^2) + 15*c^2*d*(6*e^2 + 4*e*f
*x^2 + f^2*x^4) + d^3*x^4*(5*e^2 + 6*e*f*x^2 + 2*f^2*x^4) + 3*c*d^2*x^2*(1
0*e^2 + 10*e*f*x^2 + 3*f^2*x^4)) - 16*a^2*b^3*(60*c^3*f^2 + 15*c^2*d*f*(24
*e + 5*f*x^2) + 6*c*d^2*(30*e^2 + 25*e*f*x^2 + 7*f^2*x^4) + d^3*x^2*(25*e^
2 + 28*e*f*x^2 + 9*f^2*x^4)) + 128*b^5*(20*c^3*(3*e^2 + 3*e*f*x^2 + f^2*x^
4) + 15*c^2*d*x^2*(6*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + 6*c*d^2*x^4*(10*e^2 +
15*e*f*x^2 + 6*f^2*x^4) + d^3*x^6*(15*e^2 + 24*e*f*x^2 + 10*f^2*x^4)) + 8*
a^3*b^2*d*(225*c^2*f^2 + 15*c*d*f*(30*e + 7*f*x^2) + d^2*(75*e^2 + 70*e*f*
x^2 + 21*f^2*x^4))) + 15*a*(-512*b^5*c^3*e^2 + 21*a^5*d^3*f^2 + 128*a*b^4*
c^2*e*(3*d*e + 2*c*f) - 28*a^4*b*d^2*f*(2*d*e + 3*c*f) - 64*a^2*b^3*c*(3*d
^2*e^2 + 6*c*d*e*f + c^2*f^2) + 40*a^3*b^2*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*
f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(11/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 1009, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(c^3e^2\sqrt{a+bx^2}+dx^6\sqrt{a+bx^2}\left(3c^2f^2+6cdef+d^2e^2\right)+cx^4\sqrt{a+bx^2}\left(c^2f^2+6cdef+3d^2e^2\right)+c^2ex^2\sqrt{a+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}d^3f^2\sqrt{bx^2+a}x^{11}+\frac{ad^3f^2\sqrt{bx^2+a}x^9}{120b}+\frac{1}{10}d^2f(2de+3cf)\sqrt{bx^2+a}x^9-\frac{3a^2d^3f^2\sqrt{bx^2+a}x^7}{320b^2}+\frac{ad^2f(2de+3cf)\sqrt{bx^2+a}x^7}{80b}+\frac{1}{8}d\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^7+\frac{7a^3d^3f^2\sqrt{bx^2+a}x^5}{640b^3}-\frac{7a^2d^2f(2de+3cf)\sqrt{bx^2+a}x^5}{480b^2}+\frac{1}{6}c\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^5+\frac{ad\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^5}{48b}-\frac{7a^4d^3f^2\sqrt{bx^2+a}x^3}{512b^4}+\frac{1}{4}c^2e(3de+2cf)\sqrt{bx^2+a}x^3+\frac{7a^3d^2f(2de+3cf)\sqrt{bx^2+a}x^3}{384b^3}+\frac{ac\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{24b}-\frac{5a^2d\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^3}{192b^2}+\frac{1}{2}c^3e^2\sqrt{bx^2+a}x+\frac{21a^5d^3f^2\sqrt{bx^2+a}x}{1024b^5}+\frac{ac^2e(3de+2cf)\sqrt{bx^2+a}x}{8b}-\frac{7a^4d^2f(2de+3cf)\sqrt{bx^2+a}x}{256b^4}-\frac{a^2c\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{16b^2}+\frac{5a^3d\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x}{128b^3}+\frac{ac^3e^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{b}}-\frac{21a^6d^3f^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{1024b^{11/2}}-\frac{a^2c^2e(3de+2cf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{3/2}}+\frac{7a^5d^2f(2de+3cf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{256b^{9/2}}+\frac{a^3c\left(3d^2e^2+6cdfe+c^2f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{5/2}}-\frac{5a^4d\left(d^2e^2+6cdfe+3c^2f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{7/2}}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

output
```
(c^3*e^2*x*Sqrt[a + b*x^2])/2 + (21*a^5*d^3*f^2*x*Sqrt[a + b*x^2])/(1024*b
^5) + (a*c^2*e*(3*d*e + 2*c*f)*x*Sqrt[a + b*x^2])/(8*b) - (7*a^4*d^2*f*(2*
d*e + 3*c*f)*x*Sqrt[a + b*x^2])/(256*b^4) - (a^2*c*(3*d^2*e^2 + 6*c*d*e*f
+ c^2*f^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (5*a^3*d*(d^2*e^2 + 6*c*d*e*f + 3
*c^2*f^2)*x*Sqrt[a + b*x^2])/(128*b^3) - (7*a^4*d^3*f^2*x^3*Sqrt[a + b*x^2
])/(512*b^4) + (c^2*e*(3*d*e + 2*c*f)*x^3*Sqrt[a + b*x^2])/4 + (7*a^3*d^2*
f*(2*d*e + 3*c*f)*x^3*Sqrt[a + b*x^2])/(384*b^3) + (a*c*(3*d^2*e^2 + 6*c*d
*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(24*b) - (5*a^2*d*(d^2*e^2 + 6*c*d*e*
f + 3*c^2*f^2)*x^3*Sqrt[a + b*x^2])/(192*b^2) + (7*a^3*d^3*f^2*x^5*Sqrt[a
+ b*x^2])/(640*b^3) - (7*a^2*d^2*f*(2*d*e + 3*c*f)*x^5*Sqrt[a + b*x^2])/(4
80*b^2) + (c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5*Sqrt[a + b*x^2])/6 + (a
*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^5*Sqrt[a + b*x^2])/(48*b) - (3*a^2*
d^3*f^2*x^7*Sqrt[a + b*x^2])/(320*b^2) + (a*d^2*f*(2*d*e + 3*c*f)*x^7*Sqrt
[a + b*x^2])/(80*b) + (d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^7*Sqrt[a + b*
x^2])/8 + (a*d^3*f^2*x^9*Sqrt[a + b*x^2])/(120*b) + (d^2*f*(2*d*e + 3*c*f)
*x^9*Sqrt[a + b*x^2])/10 + (d^3*f^2*x^11*Sqrt[a + b*x^2])/12 + (a*c^3*e^2*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]) - (21*a^6*d^3*f^2*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(11/2)) - (a^2*c^2*e*(3*d*e + 2*c*
f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2)) + (7*a^5*d^2*f*(2*d*e
 + 3*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(9/2)) + (a^3*c*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{21\left(a\left(a^{3}\left(\frac{40}{21}b^{2}e^{2}+a^{2}f^{2}-\frac{8}{3}abfe\right)d^{3}-4a^{2}cb\left(a^{2}f^{2}-\frac{20}{7}abfe+\frac{16}{7}b^{2}e^{2}\right)d^{2}+\frac{40\left(a^{2}f^{2}-\frac{16}{5}abfe+\frac{16}{5}b^{2}e^{2}\right)ac^{2}b^{2}d}{7}-\frac{64b^{3}c^{3}\left(a^{2}\right.}{}\right.\right.}{}$
default	$e^{2}c^{3}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d^{2}f(3cf+2de)\left(\frac{x^{7}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{10b}-\frac{7a\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^{3}}{}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-21/1024/b^(11/2)*(a*(a^3*(40/21*b^2*e^2+a^2*f^2-8/3*a*b*f*e)*d^3-4*a^2*c*
b*(a^2*f^2-20/7*a*b*f*e+16/7*b^2*e^2)*d^2+40/7*(a^2*f^2-16/5*a*b*f*e+16/5*
b^2*e^2)*a*c^2*b^2*d-64/21*b^3*c^3*(a^2*f^2-4*a*b*e*f+8*b^2*e^2))*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*((128/21*(2/3*f^2*x^4+8/5*e*f*x
^2+e^2)*x^6*d^3+512/21*(3/5*f^2*x^4+3/2*e*f*x^2+e^2)*c*x^4*d^2+256/7*c^2*(
1/2*f^2*x^4+4/3*e*f*x^2+e^2)*x^2*d+512/21*c^3*(1/3*f^2*x^4+e*f*x^2+e^2))*b
^(11/2)+a*((64/63*(2/5*f^2*x^4+6/5*e*f*x^2+e^2)*x^4*d^3+128/21*c*(3/10*f^2
*x^4+e*f*x^2+e^2)*x^2*d^2+128/7*c^2*(1/6*f^2*x^4+2/3*e*f*x^2+e^2)*d+256/21
*c^3*(1/6*f*x^2+e)*f)*b^(9/2)+a*((-80/63*x^2*(9/25*f^2*x^4+28/25*e*f*x^2+e
^2)*d^3-64/7*c*(7/30*f^2*x^4+5/6*e*f*x^2+e^2)*d^2-128/7*c^2*(5/24*f*x^2+e)
*f*d-64/21*f^2*c^3)*b^(7/2)+a*d*(((40/21*e^2+8/15*f^2*x^4+16/9*e*f*x^2)*d^
2+80/7*c*(7/30*f*x^2+e)*f*d+40/7*c^2*f^2)*b^(5/2)+a*d*(((-2/3*f*x^2-8/3*e)
*d-4*c*f)*b^(3/2)+a*d*f*b^(1/2))*f))))*x)
```

Fricas [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 1432, normalized size of antiderivative = 2.12

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="fricas")
```

output

```
[-1/30720*(15*(8*(64*a*b^5*c^3 - 48*a^2*b^4*c^2*d + 24*a^3*b^3*c*d^2 - 5*a
^4*b^2*d^3)*e^2 - 8*(32*a^2*b^4*c^3 - 48*a^3*b^3*c^2*d + 30*a^4*b^2*c*d^2
- 7*a^5*b*d^3)*e*f + (64*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 84*a^5*b*c*d^2
- 21*a^6*d^3)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*(1280*b^6*d^3*f^2*x^11 + 128*(24*b^6*d^3*e*f + (36*b^6*c*d^2 + a*b^5*
d^3)*f^2)*x^9 + 48*(40*b^6*d^3*e^2 + 8*(30*b^6*c*d^2 + a*b^5*d^3)*e*f + 3*
(40*b^6*c^2*d + 4*a*b^5*c*d^2 - a^2*b^4*d^3)*f^2)*x^7 + 8*(40*(24*b^6*c*d^
2 + a*b^5*d^3)*e^2 + 8*(240*b^6*c^2*d + 30*a*b^5*c*d^2 - 7*a^2*b^4*d^3)*e*
f + (320*b^6*c^3 + 120*a*b^5*c^2*d - 84*a^2*b^4*c*d^2 + 21*a^3*b^3*d^3)*f^
2)*x^5 + 10*(8*(144*b^6*c^2*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)*e^2 + 8*(9
6*b^6*c^3 + 48*a*b^5*c^2*d - 30*a^2*b^4*c*d^2 + 7*a^3*b^3*d^3)*e*f + (64*a
*b^5*c^3 - 120*a^2*b^4*c^2*d + 84*a^3*b^3*c*d^2 - 21*a^4*b^2*d^3)*f^2)*x^3
 + 15*(8*(64*b^6*c^3 + 48*a*b^5*c^2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*
e^2 + 8*(32*a*b^5*c^3 - 48*a^2*b^4*c^2*d + 30*a^3*b^3*c*d^2 - 7*a^4*b^2*d^
3)*e*f - (64*a^2*b^4*c^3 - 120*a^3*b^3*c^2*d + 84*a^4*b^2*c*d^2 - 21*a^5*b
*d^3)*f^2)*x)*sqrt(b*x^2 + a))/b^6, -1/15360*(15*(8*(64*a*b^5*c^3 - 48*a^2
*b^4*c^2*d + 24*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*e^2 - 8*(32*a^2*b^4*c^3 - 4
8*a^3*b^3*c^2*d + 30*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*e*f + (64*a^3*b^3*c^3 -
120*a^4*b^2*c^2*d + 84*a^5*b*c*d^2 - 21*a^6*d^3)*f^2)*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*f^2*x^11 + 128*(24*b^6*d^3*e*f ...
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 1290, normalized size of antiderivative = 1.91

$$\int \sqrt{a+bx^2}(c+dx^2)^3\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input

```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**3*f**2*x**11/12 + x**9*(a*d**3*f**2/12 + 3
*b*c*d**2*f**2 + 2*b*d**3*e*f)/(10*b) + x**7*(3*a*c*d**2*f**2 + 2*a*d**3*e
*f - 9*a*(a*d**3*f**2/12 + 3*b*c*d**2*f**2 + 2*b*d**3*e*f)/(10*b) + 3*b*c*
*2*d*f**2 + 6*b*c*d**2*e*f + b*d**3*e**2)/(8*b) + x**5*(3*a*c**2*d*f**2 +
6*a*c*d**2*e*f + a*d**3*e**2 - 7*a*(3*a*c*d**2*f**2 + 2*a*d**3*e*f - 9*a*(
a*d**3*f**2/12 + 3*b*c*d**2*f**2 + 2*b*d**3*e*f)/(10*b) + 3*b*c**2*d*f**2
+ 6*b*c*d**2*e*f + b*d**3*e**2)/(8*b) + b*c**3*f**2 + 6*b*c**2*d*e*f + 3*b
*c*d**2*e**2)/(6*b) + x**3*(a*c**3*f**2 + 6*a*c**2*d*e*f + 3*a*c*d**2*e**2
 - 5*a*(3*a*c**2*d*f**2 + 6*a*c*d**2*e*f + a*d**3*e**2 - 7*a*(3*a*c*d**2*f
**2 + 2*a*d**3*e*f - 9*a*(a*d**3*f**2/12 + 3*b*c*d**2*f**2 + 2*b*d**3*e*f)
/(10*b) + 3*b*c**2*d*f**2 + 6*b*c*d**2*e*f + b*d**3*e**2)/(8*b) + b*c**3*f
**2 + 6*b*c**2*d*e*f + 3*b*c*d**2*e**2)/(6*b) + 2*b*c**3*e*f + 3*b*c**2*d*
e**2)/(4*b) + x*(2*a*c**3*e*f + 3*a*c**2*d*e**2 - 3*a*(a*c**3*f**2 + 6*a*c
**2*d*e*f + 3*a*c*d**2*e**2 - 5*a*(3*a*c**2*d*f**2 + 6*a*c*d**2*e*f + a*d*
*3*e**2 - 7*a*(3*a*c*d**2*f**2 + 2*a*d**3*e*f - 9*a*(a*d**3*f**2/12 + 3*b*
c*d**2*f**2 + 2*b*d**3*e*f)/(10*b) + 3*b*c**2*d*f**2 + 6*b*c*d**2*e*f + b*
d**3*e**2)/(8*b) + b*c**3*f**2 + 6*b*c**2*d*e*f + 3*b*c*d**2*e**2)/(6*b) +
 2*b*c**3*e*f + 3*b*c**2*d*e**2)/(4*b) + b*c**3*e**2)/(2*b)) + (a*c**3*e**
2 - a*(2*a*c**3*e*f + 3*a*c**2*d*e**2 - 3*a*(a*c**3*f**2 + 6*a*c**2*d*e*f
+ 3*a*c*d**2*e**2 - 5*a*(3*a*c**2*d*f**2 + 6*a*c*d**2*e*f + a*d**3*e**2...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.33

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(3/2)*d^3*f^2*x^9/b - 3/40*(b*x^2 + a)^(3/2)*a*d^3*f^2*x^
7/b^2 + 21/320*(b*x^2 + a)^(3/2)*a^2*d^3*f^2*x^5/b^3 - 7/128*(b*x^2 + a)^(
3/2)*a^3*d^3*f^2*x^3/b^4 + 1/10*(2*d^3*e*f + 3*c*d^2*f^2)*(b*x^2 + a)^(3/2
)*x^7/b + 1/2*sqrt(b*x^2 + a)*c^3*e^2*x + 21/512*(b*x^2 + a)^(3/2)*a^4*d^3
*f^2*x/b^5 - 21/1024*sqrt(b*x^2 + a)*a^5*d^3*f^2*x/b^5 - 7/80*(2*d^3*e*f +
 3*c*d^2*f^2)*(b*x^2 + a)^(3/2)*a*x^5/b^2 + 1/8*(d^3*e^2 + 6*c*d^2*e*f + 3
*c^2*d*f^2)*(b*x^2 + a)^(3/2)*x^5/b + 1/2*a*c^3*e^2*arcsinh(b*x/sqrt(a*b))
/sqrt(b) - 21/1024*a^6*d^3*f^2*arcsinh(b*x/sqrt(a*b))/b^(11/2) + 7/96*(2*d
^3*e*f + 3*c*d^2*f^2)*(b*x^2 + a)^(3/2)*a^2*x^3/b^3 - 5/48*(d^3*e^2 + 6*c*
d^2*e*f + 3*c^2*d*f^2)*(b*x^2 + a)^(3/2)*a*x^3/b^2 + 1/6*(3*c*d^2*e^2 + 6*
c^2*d*e*f + c^3*f^2)*(b*x^2 + a)^(3/2)*x^3/b - 7/128*(2*d^3*e*f + 3*c*d^2*
f^2)*(b*x^2 + a)^(3/2)*a^3*x/b^4 + 7/256*(2*d^3*e*f + 3*c*d^2*f^2)*sqrt(b*
x^2 + a)*a^4*x/b^4 + 5/64*(d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*(b*x^2 + a
)^(3/2)*a^2*x/b^3 - 5/128*(d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*sqrt(b*x^2
 + a)*a^3*x/b^3 - 1/8*(3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*(b*x^2 + a)^(3
/2)*a*x/b^2 + 1/16*(3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*sqrt(b*x^2 + a)*a
^2*x/b^2 + 1/4*(3*c^2*d*e^2 + 2*c^3*e*f)*(b*x^2 + a)^(3/2)*x/b - 1/8*(3*c^
2*d*e^2 + 2*c^3*e*f)*sqrt(b*x^2 + a)*a*x/b + 7/256*(2*d^3*e*f + 3*c*d^2*f^
2)*a^5*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/128*(d^3*e^2 + 6*c*d^2*e*f + 3*c
^2*d*f^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*(3*c*d^2*e^2 + 6*c^...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.16

$$\int \sqrt{a+bx^2}(c+dx^2)^3\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*d^3*f^2*x^2 + (24*b^10*d^3*e*f + 36*b^10*c*d^2*f^2
 + a*b^9*d^3*f^2)/b^10)*x^2 + 3*(40*b^10*d^3*e^2 + 240*b^10*c*d^2*e*f + 8*
a*b^9*d^3*e*f + 120*b^10*c^2*d*f^2 + 12*a*b^9*c*d^2*f^2 - 3*a^2*b^8*d^3*f^
2)/b^10)*x^2 + (960*b^10*c*d^2*e^2 + 40*a*b^9*d^3*e^2 + 1920*b^10*c^2*d*e*
f + 240*a*b^9*c*d^2*e*f - 56*a^2*b^8*d^3*e*f + 320*b^10*c^3*f^2 + 120*a*b^
9*c^2*d*f^2 - 84*a^2*b^8*c*d^2*f^2 + 21*a^3*b^7*d^3*f^2)/b^10)*x^2 + 5*(11
52*b^10*c^2*d*e^2 + 192*a*b^9*c*d^2*e^2 - 40*a^2*b^8*d^3*e^2 + 768*b^10*c^
3*e*f + 384*a*b^9*c^2*d*e*f - 240*a^2*b^8*c*d^2*e*f + 56*a^3*b^7*d^3*e*f +
 64*a*b^9*c^3*f^2 - 120*a^2*b^8*c^2*d*f^2 + 84*a^3*b^7*c*d^2*f^2 - 21*a^4*
b^6*d^3*f^2)/b^10)*x^2 + 15*(512*b^10*c^3*e^2 + 384*a*b^9*c^2*d*e^2 - 192*
a^2*b^8*c*d^2*e^2 + 40*a^3*b^7*d^3*e^2 + 256*a*b^9*c^3*e*f - 384*a^2*b^8*c
^2*d*e*f + 240*a^3*b^7*c*d^2*e*f - 56*a^4*b^6*d^3*e*f - 64*a^2*b^8*c^3*f^2
 + 120*a^3*b^7*c^2*d*f^2 - 84*a^4*b^6*c*d^2*f^2 + 21*a^5*b^5*d^3*f^2)/b^10
)*sqrt(b*x^2 + a)*x - 1/1024*(512*a*b^5*c^3*e^2 - 384*a^2*b^4*c^2*d*e^2 +
192*a^3*b^3*c*d^2*e^2 - 40*a^4*b^2*d^3*e^2 - 256*a^2*b^4*c^3*e*f + 384*a^3
*b^3*c^2*d*e*f - 240*a^4*b^2*c*d^2*e*f + 56*a^5*b*d^3*e*f + 64*a^3*b^3*c^3
*f^2 - 120*a^4*b^2*c^2*d*f^2 + 84*a^5*b*c*d^2*f^2 - 21*a^6*d^3*f^2)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\left(f\,x^2+e\right)^2dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e)^2,x)
```

3.275 $\int \sqrt{a+bx^2}(c+dx^2)^3(e+fx^2)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 390

$$\begin{aligned}
&\int \sqrt{a+bx^2}(c+dx^2)^3(e+fx^2)\,dx \\
&= \frac{(128b^4c^3e+7a^4d^3f+48a^2b^2cd(de+cf)-32ab^3c^2(3de+cf)-10a^3bd^2(de+3cf))\,x\sqrt{a+bx^2}}{256b^4} \\
&\quad - \frac{(7a^3d^3f+48ab^2cd(de+cf)-32b^3c^2(3de+cf)-10a^2bd^2(de+3cf))\,x(a+bx^2)^{3/2}}{128b^4} \\
&\quad + \frac{d(7a^2d^2f+48b^2c(de+cf)-10abd(de+3cf))\,x^3(a+bx^2)^{3/2}}{96b^3} \\
&\quad - \frac{d^2(7adf-10b(de+3cf))x^5(a+bx^2)^{3/2}}{80b^2} + \frac{d^3fx^7(a+bx^2)^{3/2}}{10b} \\
&\quad + \frac{a(128b^4c^3e+7a^4d^3f+48a^2b^2cd(de+cf)-32ab^3c^2(3de+cf)-10a^3bd^2(de+3cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{9/2}}
\end{aligned}$$

output
```
1/256*(128*b^4*c^3*e+7*a^4*d^3*f+48*a^2*b^2*c*d*(c*f+d*e)-32*a*b^3*c^2*(c*
f+3*d*e)-10*a^3*b*d^2*(3*c*f+d*e))*x*(b*x^2+a)^(1/2)/b^4-1/128*(7*a^3*d^3*
f+48*a*b^2*c*d*(c*f+d*e)-32*b^3*c^2*(c*f+3*d*e)-10*a^2*b*d^2*(3*c*f+d*e))*
x*(b*x^2+a)^(3/2)/b^4+1/96*d*(7*a^2*d^2*f+48*b^2*c*(c*f+d*e)-10*a*b*d*(3*c
*f+d*e))*x^3*(b*x^2+a)^(3/2)/b^3-1/80*d^2*(7*a*d*f-10*b*(3*c*f+d*e))*x^5*(
b*x^2+a)^(3/2)/b^2+1/10*d^3*f*x^7*(b*x^2+a)^(3/2)/b+1/256*a*(128*b^4*c^3*e
+7*a^4*d^3*f+48*a^2*b^2*c*d*(c*f+d*e)-32*a*b^3*c^2*(c*f+3*d*e)-10*a^3*b*d^
2*(3*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.88

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^4d^3f+10a^3bd^2(15de+45cf+7dfx^2)+16ab^3(30c^3f+15cd^2x^2(2e+fx^2)+30c^2$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^4*d^3*f + 10*a^3*b*d^2*(15*d*e + 45*c*f
 + 7*d*f*x^2) + 16*a*b^3*(30*c^3*f + 15*c*d^2*x^2*(2*e + f*x^2) + 30*c^2*d
*(3*e + f*x^2) + d^3*x^4*(5*e + 3*f*x^2)) + 96*b^4*(10*c^3*(2*e + f*x^2) +
 10*c^2*d*x^2*(3*e + 2*f*x^2) + 5*c*d^2*x^4*(4*e + 3*f*x^2) + d^3*x^6*(5*e
 + 4*f*x^2)) - 4*a^2*b^2*d*(180*c^2*f + 15*c*d*(12*e + 5*f*x^2) + d^2*x^2*
(25*e + 14*f*x^2))) - 15*a*(128*b^4*c^3*e + 7*a^4*d^3*f + 48*a^2*b^2*c*d*(
d*e + c*f) - 32*a*b^3*c^2*(3*d*e + c*f) - 10*a^3*b*d^2*(d*e + 3*c*f))*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(3840*b^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {403, 403, 403, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \sqrt{bx^2+a}\left(dx^2+c\right)^2\left((10bde+6bcf-7adf)x^2+c(10be-af)\right)dx}{10b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 403

$$\frac{\frac{\int \sqrt{bx^2+a}\left(dx^2+c\right)\left(\left(24c(5de+cf)b^2-2ad(25de+33cf)b+35a^2d^2f\right)x^2+c\left(7dfa^2-10bdea-14bcfa+80b^2ce\right)\right)dx}{8b}+\frac{x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(-7ad\right.}{8b}}{10b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 403

$$\frac{\frac{\frac{\int \sqrt{bx^2+a}\left(c\left(-35d^2fa^3+2bd(25de+54cf)a^2-36b^2c(5de+3cf)a+480b^3c^2e\right)-\left(-48c^2(15de+cf)b^3+8acd(65de+36cf)b^2-10a^2d^2(15de+31cf)b+105a^3d^3f\right)x^2\right)d}{6b}}{8b}}{10b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{\frac{15\left(7a^4d^3f-10a^3bd^2(3cf+de)+48a^2b^2cd(cf+de)-32ab^3c^2(cf+3de)+128b^4c^3e\right)\int \sqrt{bx^2+a}dx}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3d^3f-10a^2bd^2(31cf+15de)+8ab^2cd(36cf-\right.}{4b}}{6b}}{8b}}{10b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{15\left(7a^4d^3f-10a^3bd^2(3cf+de)+48a^2b^2cd(cf+de)-32ab^3c^2(cf+3de)+128b^4c^3e\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3d^3f-10a^2bd^2(31cf+\right.}{}}{6b}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{15\left(7a^4d^3f-10a^3bd^2(3cf+de)+48a^2b^2cd(cf+de)-32ab^3c^2(cf+3de)+128b^4c^3e\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{x\left(a+bx^2\right)^{3/2}\left(105a^3d^3f-10a^2\right.}{}}{6b}}{8b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(35a^2d^2f-2abd(33cf+25de)+24b^2c(cf+5de)\right)}{6b}+\frac{15\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(7a^4d^3f-10a^3bd^2(3cf+de)+48a^2b^2cd(c\ldots\right.}{4b}}{8b}}{}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{10b}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*(c + d*x^2)^3)/(10*b) + (((10*b*d*e + 6*b*c*f - 7*a
*d*f)*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (((35*a^2*d^2*f + 24*b^2*
c*(5*d*e + c*f) - 2*a*b*d*(25*d*e + 33*c*f))*x*(a + b*x^2)^(3/2)*(c + d*x^
2))/(6*b) + (-1/4*((105*a^3*d^3*f - 48*b^3*c^2*(15*d*e + c*f) - 10*a^2*b*d
^2*(15*d*e + 31*c*f) + 8*a*b^2*c*d*(65*d*e + 36*c*f))*x*(a + b*x^2)^(3/2))
/b + (15*(128*b^4*c^3*e + 7*a^4*d^3*f + 48*a^2*b^2*c*d*(d*e + c*f) - 32*a*
b^3*c^2*(3*d*e + c*f) - 10*a^3*b*d^2*(d*e + 3*c*f))*((x*Sqrt[a + b*x^2])/2
 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b))/(8
*b))/(10*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{7a\left(a^3\left(af-\frac{10be}{7}\right)d^3-\frac{30a^2cb\left(af-\frac{8be}{5}\right)d^2}{7}+\frac{48a\,b^2c^2(af-2be)d}{7}-\frac{32b^3c^3(af-4be)}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{256}-\frac{7\left(\left(-\frac{32\left(\frac{4f\,x^2}{5}+e\right)x^6c}{7}\right.\right.}{}$
risch	$-\frac{x(-384f\,d^3b^4x^8-48a\,b^3d^3f\,x^6-1440b^4c\,d^2f\,x^6-480b^4d^3e\,x^6+56a^2b^2d^3f\,x^4-240a\,b^3c\,d^2f\,x^4-80a\,b^3d^3e\,x^4-1920b^4c}{}$
default	$e\,c^3\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+d^2(3cf+de)\left(\frac{x^5(b\,x^2+a)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^3(b\,x^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+}{4b}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
7/256/b^(9/2)*(a*(a^3*(a*f-10/7*b*e)*d^3-30/7*a^2*c*b*(a*f-8/5*b*e)*d^2+48
/7*a*b^2*c^2*(a*f-2*b*e)*d-32/7*b^3*c^3*(a*f-4*b*e))*arctanh((b*x^2+a)^(1/
2)/x/b^(1/2))-((-32/7*(4/5*f*x^2+e)*x^6*d^3-128/7*c*(3/4*f*x^2+e)*x^4*d^2-
192/7*c^2*(2/3*f*x^2+e)*x^2*d-128/7*c^3*(1/2*f*x^2+e))*b^(9/2)+a*((-16/21*
(3/5*f*x^2+e)*x^4*d^3-32/7*c*x^2*(1/2*f*x^2+e)*d^2-96/7*c^2*(1/3*f*x^2+e)*
d-32/7*f*c^3)*b^(7/2)+a*d*(((8/15*f*x^4+20/21*e*x^2)*d^2+48/7*(5/12*f*x^2+
e)*c*d+48/7*c^2*f)*b^(5/2)+a*(((-2/3*f*x^2-10/7*e)*d-30/7*c*f)*b^(3/2)+a*d
*f*b^(1/2))*d)))*(b*x^2+a)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.17

$$\int \sqrt{a+bx^2}(c+dx^2)^3(e+fx^2)\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/7680*(15*(2*(64*a*b^4*c^3 - 48*a^2*b^3*c^2*d + 24*a^3*b^2*c*d^2 - 5*a^4
*b*d^3)*e - (32*a^2*b^3*c^3 - 48*a^3*b^2*c^2*d + 30*a^4*b*c*d^2 - 7*a^5*d^
3)*f)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5
*d^3*f*x^9 + 48*(10*b^5*d^3*e + (30*b^5*c*d^2 + a*b^4*d^3)*f)*x^7 + 8*(10*
(24*b^5*c*d^2 + a*b^4*d^3)*e + (240*b^5*c^2*d + 30*a*b^4*c*d^2 - 7*a^2*b^3
*d^3)*f)*x^5 + 10*(2*(144*b^5*c^2*d + 24*a*b^4*c*d^2 - 5*a^2*b^3*d^3)*e +
(96*b^5*c^3 + 48*a*b^4*c^2*d - 30*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*f)*x^3 +
15*(2*(64*b^5*c^3 + 48*a*b^4*c^2*d - 24*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*e +
 (32*a*b^4*c^3 - 48*a^2*b^3*c^2*d + 30*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*f)*x)*
sqrt(b*x^2 + a))/b^5, -1/3840*(15*(2*(64*a*b^4*c^3 - 48*a^2*b^3*c^2*d + 24
*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*e - (32*a^2*b^3*c^3 - 48*a^3*b^2*c^2*d + 30*
a^4*b*c*d^2 - 7*a^5*d^3)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(384*b^5*d^3*f*x^9 + 48*(10*b^5*d^3*e + (30*b^5*c*d^2 + a*b^4*d^3)*f)*x^7
+ 8*(10*(24*b^5*c*d^2 + a*b^4*d^3)*e + (240*b^5*c^2*d + 30*a*b^4*c*d^2 - 7
*a^2*b^3*d^3)*f)*x^5 + 10*(2*(144*b^5*c^2*d + 24*a*b^4*c*d^2 - 5*a^2*b^3*d
^3)*e + (96*b^5*c^3 + 48*a*b^4*c^2*d - 30*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*f
)*x^3 + 15*(2*(64*b^5*c^3 + 48*a*b^4*c^2*d - 24*a^2*b^3*c*d^2 + 5*a^3*b^2*
d^3)*e + (32*a*b^4*c^3 - 48*a^2*b^3*c^2*d + 30*a^3*b^2*c*d^2 - 7*a^4*b*d^3
)*f)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.74

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^3fx^9}{10} + \frac{x^7\left(\frac{ad^3f}{10}+3bcd^2f+bd^3e\right)}{8b} + \frac{x^5\cdot\left(3acd^2f+ad^3e-\frac{7a\left(\frac{ad^3f}{10}+3bcd^2f+bd^3e\right)}{8b}+3bc^2df+3bcd^2e\right)}{6b} + \frac{x^3\cdot\left(3ac^2df+\right.}{} \right. \\ \sqrt{a}\left(c^3ex + \frac{d^3fx^9}{9} + \frac{x^7\cdot(3cd^2f+d^3e)}{7} + \frac{x^5\cdot(3c^2df+3cd^2e)}{5} + \frac{x^3\left(c^3f+3c^2de\right)}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3*(f*x**2+e),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**3*f*x**9/10 + x**7*(a*d**3*f/10 + 3*b*c*d*
*2*f + b*d**3*e)/(8*b) + x**5*(3*a*c*d**2*f + a*d**3*e - 7*a*(a*d**3*f/10
+ 3*b*c*d**2*f + b*d**3*e)/(8*b) + 3*b*c**2*d*f + 3*b*c*d**2*e)/(6*b) + x*
*3*(3*a*c**2*d*f + 3*a*c*d**2*e - 5*a*(3*a*c*d**2*f + a*d**3*e - 7*a*(a*d*
*3*f/10 + 3*b*c*d**2*f + b*d**3*e)/(8*b) + 3*b*c**2*d*f + 3*b*c*d**2*e)/(6
*b) + b*c**3*f + 3*b*c**2*d*e)/(4*b) + x*(a*c**3*f + 3*a*c**2*d*e - 3*a*(3
*a*c**2*d*f + 3*a*c*d**2*e - 5*a*(3*a*c*d**2*f + a*d**3*e - 7*a*(a*d**3*f/
10 + 3*b*c*d**2*f + b*d**3*e)/(8*b) + 3*b*c**2*d*f + 3*b*c*d**2*e)/(6*b) +
 b*c**3*f + 3*b*c**2*d*e)/(4*b) + b*c**3*e)/(2*b)) + (a*c**3*e - a*(a*c**3
*f + 3*a*c**2*d*e - 3*a*(3*a*c**2*d*f + 3*a*c*d**2*e - 5*a*(3*a*c*d**2*f +
 a*d**3*e - 7*a*(a*d**3*f/10 + 3*b*c*d**2*f + b*d**3*e)/(8*b) + 3*b*c**2*d
*f + 3*b*c*d**2*e)/(6*b) + b*c**3*f + 3*b*c**2*d*e)/(4*b) + b*c**3*e)/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
(x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c**3*e*x + d**3*f*x**
9/9 + x**7*(3*c*d**2*f + d**3*e)/7 + x**5*(3*c**2*d*f + 3*c*d**2*e)/5 + x*
*3*(c**3*f + 3*c**2*d*e)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.35

$$
\begin{aligned}
\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}d^3fx^7}{10\,b} - \frac{7\,(bx^2+a)^{\frac{3}{2}}ad^3fx^5}{80\,b^2} \\
&+ \frac{7\,(bx^2+a)^{\frac{3}{2}}a^2d^3fx^3}{96\,b^3} \\
&+ \frac{(d^3e+3\,cd^2f)(bx^2+a)^{\frac{3}{2}}x^5}{8\,b} + \frac{1}{2}\,\sqrt{bx^2+a}c^3ex \\
&- \frac{7\,(bx^2+a)^{\frac{3}{2}}a^3d^3fx}{128\,b^4} + \frac{7\,\sqrt{bx^2+a}a^4d^3fx}{256\,b^4} \\
&+ \frac{ac^3e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + \frac{7\,a^5d^3f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{9}{2}}} \\
&- \frac{5\,(d^3e+3\,cd^2f)(bx^2+a)^{\frac{3}{2}}ax^3}{48\,b^2} \\
&+ \frac{(cd^2e+c^2df)(bx^2+a)^{\frac{3}{2}}x^3}{2\,b} \\
&+ \frac{5\,(d^3e+3\,cd^2f)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^3} \\
&- \frac{5\,(d^3e+3\,cd^2f)\sqrt{bx^2+a}a^3x}{128\,b^3} \\
&- \frac{3\,(cd^2e+c^2df)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} \\
&+ \frac{3\,(cd^2e+c^2df)\sqrt{bx^2+a}a^2x}{16\,b^2} \\
&+ \frac{(3\,c^2de+c^3f)(bx^2+a)^{\frac{3}{2}}x}{4\,b} \\
&- \frac{(3\,c^2de+c^3f)\sqrt{bx^2+a}ax}{8\,b} \\
&- \frac{5\,(d^3e+3\,cd^2f)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} \\
&+ \frac{3\,(cd^2e+c^2df)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} \\
&- \frac{(3\,c^2de+c^3f)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}
\end{aligned}
$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(3/2)*d^3*f*x^7/b - 7/80*(b*x^2 + a)^(3/2)*a*d^3*f*x^5/b^
2 + 7/96*(b*x^2 + a)^(3/2)*a^2*d^3*f*x^3/b^3 + 1/8*(d^3*e + 3*c*d^2*f)*(b*
x^2 + a)^(3/2)*x^5/b + 1/2*sqrt(b*x^2 + a)*c^3*e*x - 7/128*(b*x^2 + a)^(3/
2)*a^3*d^3*f*x/b^4 + 7/256*sqrt(b*x^2 + a)*a^4*d^3*f*x/b^4 + 1/2*a*c^3*e*a
rcsinh(b*x/sqrt(a*b))/sqrt(b) + 7/256*a^5*d^3*f*arcsinh(b*x/sqrt(a*b))/b^(
9/2) - 5/48*(d^3*e + 3*c*d^2*f)*(b*x^2 + a)^(3/2)*a*x^3/b^2 + 1/2*(c*d^2*e
 + c^2*d*f)*(b*x^2 + a)^(3/2)*x^3/b + 5/64*(d^3*e + 3*c*d^2*f)*(b*x^2 + a)
^(3/2)*a^2*x/b^3 - 5/128*(d^3*e + 3*c*d^2*f)*sqrt(b*x^2 + a)*a^3*x/b^3 - 3
/8*(c*d^2*e + c^2*d*f)*(b*x^2 + a)^(3/2)*a*x/b^2 + 3/16*(c*d^2*e + c^2*d*f
)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(3*c^2*d*e + c^3*f)*(b*x^2 + a)^(3/2)*x/
b - 1/8*(3*c^2*d*e + c^3*f)*sqrt(b*x^2 + a)*a*x/b - 5/128*(d^3*e + 3*c*d^2
*f)*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/16*(c*d^2*e + c^2*d*f)*a^3*arcs
inh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(3*c^2*d*e + c^3*f)*a^2*arcsinh(b*x/sqrt(
a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.10

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(8\,d^3fx^2+\frac{10\,b^8d^3e+30\,b^8cd^2f+ab^7d^3f}{b^8}\right)x^2+\frac{240\,b^8cd^2e+10\,ab^7d^3e+240\,b^8c^2df+}{b^8}\right.\right.\right.$$
$$-\frac{(128\,ab^4c^3e-96\,a^2b^3c^2de+48\,a^3b^2cd^2e-10\,a^4bd^3e-32\,a^2b^3c^3f+48\,a^3b^2c^2df-30\,a^4bcd^2f+7\,a^5d^3}{256\,b^{\frac{9}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*d^3*f*x^2 + (10*b^8*d^3*e + 30*b^8*c*d^2*f + a*b^7*d^3*
f)/b^8)*x^2 + (240*b^8*c*d^2*e + 10*a*b^7*d^3*e + 240*b^8*c^2*d*f + 30*a*b
^7*c*d^2*f - 7*a^2*b^6*d^3*f)/b^8)*x^2 + 5*(288*b^8*c^2*d*e + 48*a*b^7*c*d
^2*e - 10*a^2*b^6*d^3*e + 96*b^8*c^3*f + 48*a*b^7*c^2*d*f - 30*a^2*b^6*c*d
^2*f + 7*a^3*b^5*d^3*f)/b^8)*x^2 + 15*(128*b^8*c^3*e + 96*a*b^7*c^2*d*e -
48*a^2*b^6*c*d^2*e + 10*a^3*b^5*d^3*e + 32*a*b^7*c^3*f - 48*a^2*b^6*c^2*d*
f + 30*a^3*b^5*c*d^2*f - 7*a^4*b^4*d^3*f)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(
128*a*b^4*c^3*e - 96*a^2*b^3*c^2*d*e + 48*a^3*b^2*c*d^2*e - 10*a^4*b*d^3*e
 - 32*a^2*b^3*c^3*f + 48*a^3*b^2*c^2*d*f - 30*a^4*b*c*d^2*f + 7*a^5*d^3*f)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^3\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\left(f\,x^2+e\right)dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3*(f*x^2+e),x)
```

3.276 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 312

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2} dx &= -\frac{d(a^2d^2f^2+2abdf(de-3cf)-8b^2(d^2e^2-3cdef+3c^2f^2))x\sqrt{a+bx^2}}{16b^2f^3} \\ &\quad -\frac{d^2(6bde-18bcf-adf)x^3\sqrt{a+bx^2}}{24bf^2}+\frac{d^3x^5\sqrt{a+bx^2}}{6f} \\ &\quad +\frac{(a^3d^3f^3+2a^2bd^2f^2(de-3cf)-16b^3(de-cf)^3+8ab^2df(d^2e^2-3cdef+3c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}f^4} \\ &\quad +\frac{\sqrt{be-af}(de-cf)^3\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^4}\end{aligned}$$

output
```
-1/16*d*(a^2*d^2*f^2+2*a*b*d*f*(-3*c*f+d*e)-8*b^2*(3*c^2*f^2-3*c*d*e*f+d^2
*e^2))*x*(b*x^2+a)^(1/2)/b^2/f^3-1/24*d^2*(-a*d*f-18*b*c*f+6*b*d*e)*x^3*(b
*x^2+a)^(1/2)/b/f^2+1/6*d^3*x^5*(b*x^2+a)^(1/2)/f+1/16*(a^3*d^3*f^3+2*a^2*
b*d^2*f^2*(-3*c*f+d*e)-16*b^3*(-c*f+d*e)^3+8*a*b^2*d*f*(3*c^2*f^2-3*c*d*e*
f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)/f^4+(-a*f+b*e)^(1/2
)*(-c*f+d*e)^3*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)
/f^4
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx$$

$$= \frac{\frac{dfx\sqrt{a+bx^2}(-3a^2d^2f^2+2abdf(-3de+9cf+dfx^2)+4b^2(18c^2f^2+9cdf(-2e+fx^2)+d^2(6e^2-3efx^2+2f^2x^4)))}{b^2} + \frac{48\sqrt{-be+af}(de-cf)^3 \arctan}{\sqrt{}}}{48f^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((d*f*x*Sqrt[a + b*x^2]*(-3*a^2*d^2*f^2 + 2*a*b*d*f*(-3*d*e + 9*c*f + d*f*
x^2) + 4*b^2*(18*c^2*f^2 + 9*c*d*f*(-2*e + f*x^2) + d^2*(6*e^2 - 3*e*f*x^2
 + 2*f^2*x^4))))/b^2 + (48*Sqrt[-(b*e) + a*f]*(d*e - c*f)^3*ArcTan[(-(f*x*
Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/Sqr
t[e] + (3*(-(a^3*d^3*f^3) - 2*a^2*b*d^2*f^2*(d*e - 3*c*f) + 16*b^3*(d*e -
c*f)^3 - 8*a*b^2*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/b^(5/2))/(48*f^4)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.27, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {420, 318, 299, 211, 224, 219, 420, 299, 211, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx$$

$$\Big\downarrow \text{420}$$

$$\frac{d\int \sqrt{bx^2+a}(dx^2+c)^2dx}{f} - \frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{d\left(\frac{\int \sqrt{bx^2+a}(d(8bc-3ad)x^2+c(6bc-ad))dx}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 420

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\sqrt{bx^2+a}\left(dx^2+c\right)dx}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$\Big\downarrow$ 299

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$\Big\downarrow$ 211

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$\big\downarrow$ 224

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{\int-\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{a+}}{2f}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2f}\right.}{f}}{f}$$

$$\Big\downarrow \ 398$$

$$\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-}{}\right.}{f}}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{(-adf-2bcf+2bde)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\ldots}{f}\ldots\right.}{f}\ldots\right.}{f}$$

$\downarrow$ 219

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc\ldots}{\sqrt{b}f}\ldots}{\ldots}\right.}{f}\ldots\right.}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc}{\sqrt{b}f}}{}\right.}{f}\right.}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc}{\sqrt{b}f}}{}\right.}{f}\right.}{f}$$

input

```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
(d*((d*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(8*b*c - 3*a*d)*x*(a +
 b*x^2)^(3/2))/(4*b) + (3*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b
*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(
6*b)))/f - ((d*e - c*f)*((d*((d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*d
)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b])))/(4*b)))/f - ((d*e - c*f)*((d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e
 - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2
*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
 b*x^2])])/(Sqrt[e]*f))/(2*f)))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{2(-af+be)(cf-de)^3 b^{\frac{9}{2}} \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(\frac{\left(-16b^3d^3e^3+8b^2d^2f(ad+6bc)e^2+2bd\,f^2\left(a^2d^2-12abcd-24b^2c^2\right)\right.}{}\right.}{}$
risch	$-\frac{xd\left(-8f^2x^4b^2d^2-2ab\,d^2f^2x^2-36b^2cd\,f^2x^2+12b^2d^2ef\,x^2+3a^2d^2f^2-18abcd\,f^2+6ab\,d^2ef-72b^2c^2f^2+72b^2cdef-24b^2\right.}{48b^2f^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*(-a*f+b*e)*(c*f-d*e)^3*b^(9/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e
)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(1/8*(-16*b^3*d^3*e^3+8*b^2*d^2*f*(a*d+6*b
*c)*e^2+2*b*d*f^2*(a^2*d^2-12*a*b*c*d-24*b^2*c^2)*e+f^3*(a^3*d^3-6*a^2*b*c
*d^2+24*a*b^2*c^2*d+16*b^3*c^3))*b^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b
^2*d^2*e^2-1/4*d*b*f*(2*(d*x^2+6*c)*b+a*d)*e-1/8*(4*(-2/3*d^2*x^4-3*c*d*x^
2-6*c^2)*b^2-6*a*d*(1/9*x^2*d+c)*b+a^2*d^2)*f^2)*d*(b*x^2+a)^(1/2)*x*f*b^(
5/2)))/((a*f-b*e)*e)^(1/2)/b^(9/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 28.22 (sec) , antiderivative size = 1809, normalized size of antiderivative = 5.80

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*b^3*d^3*e^3 - 8*(6*b^3*c*d^2 + a*b^2*d^3)*e^2*f + 2*(24*b^3*
c^2*d + 12*a*b^2*c*d^2 - a^2*b*d^3)*e*f^2 - (16*b^3*c^3 + 24*a*b^2*c^2*d -
 6*a^2*b*c*d^2 + a^3*d^3)*f^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 24*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b
^3*c^3*f^3)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4
 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f
)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) -
 2*(8*b^3*d^3*f^3*x^5 - 2*(6*b^3*d^3*e*f^2 - (18*b^3*c*d^2 + a*b^2*d^3)*f^
3)*x^3 + 3*(8*b^3*d^3*e^2*f - 2*(12*b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (24*b^3
*c^2*d + 6*a*b^2*c*d^2 - a^2*b*d^3)*f^3)*x)*sqrt(b*x^2 + a))/(b^3*f^4), 1/
48*(3*(16*b^3*d^3*e^3 - 8*(6*b^3*c*d^2 + a*b^2*d^3)*e^2*f + 2*(24*b^3*c^2*
d + 12*a*b^2*c*d^2 - a^2*b*d^3)*e*f^2 - (16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a
^2*b*c*d^2 + a^3*d^3)*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 1
2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt
((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(
4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^
2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + (8*b^3*d^3*f^3*
x^5 - 2*(6*b^3*d^3*e*f^2 - (18*b^3*c*d^2 + a*b^2*d^3)*f^3)*x^3 + 3*(8*b^3*
d^3*e^2*f - 2*(12*b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (24*b^3*c^2*d + 6*a*b^2*c
*d^2 - a^2*b*d^3)*f^3)*x)*sqrt(b*x^2 + a))/(b^3*f^4), -1/96*(48*(b^3*d^...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**3/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^3}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e),x)
```

output
```
( - 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**3*f**3 + 144*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**2*d*e*f**2 - 144*sqrt(e)*sqrt(a*f
- b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*
x)/(sqrt(e)*sqrt(b)))*b**3*c*d**2*e**2*f + 48*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*b**3*d**3*e**3 - 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*
e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3
*c**3*f**3 + 144*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*s
qrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**2*d*e*f**2
 - 144*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*
x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c*d**2*e**2*f + 48*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*d**3*e**3 - 3*sqrt(a + b*x**2)*a**
2*b*d**3*e*f**3*x + 18*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f**3*x - 6*sqrt(a
+ b*x**2)*a*b**2*d**3*e**2*f**2*x + 2*sqrt(a + b*x**2)*a*b**2*d**3*e*f**3*
x**3 + 72*sqrt(a + b*x**2)*b**3*c**2*d*e*f**3*x - 72*sqrt(a + b*x**2)*b**3
*c*d**2*e**2*f**2*x + 36*sqrt(a + b*x**2)*b**3*c*d**2*e*f**3*x**3 + 24*sqr
t(a + b*x**2)*b**3*d**3*e**3*f*x - 12*sqrt(a + b*x**2)*b**3*d**3*e**2*f...
```

3.277 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 265

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx &= -\frac{d^2(8bde-12bcf-adf)x\sqrt{a+bx^2}}{8bf^3}+\frac{d^3x^3\sqrt{a+bx^2}}{4f^2}-\frac{(de-cf)^3x\sqrt{a+bx^2}}{2ef^3(e+fx^2)}\\ &\quad-\frac{d(a^2d^2f^2+4abdf(2de-3cf)-24b^2(de-cf)^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}f^4}\\ &\quad-\frac{(de-cf)^2\left(6bde^2-af(5de+cf)\right)\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^4\sqrt{be-af}}\end{aligned}$$

output
```
-1/8*d^2*(-a*d*f-12*b*c*f+8*b*d*e)*x*(b*x^2+a)^(1/2)/b/f^3+1/4*d^3*x^3*(b*
x^2+a)^(1/2)/f^2-1/2*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^3/(f*x^2+e)-1/8*d*
(a^2*d^2*f^2+4*a*b*d*f*(-3*c*f+2*d*e)-24*b^2*(-c*f+d*e)^2)*arctanh(b^(1/2)
*x/(b*x^2+a)^(1/2))/b^(3/2)/f^4-1/2*(-c*f+d*e)^2*(6*b*d*e^2-a*f*(c*f+5*d*e
))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^4/(-a*f+b
*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^2}\,dx$$

$$= \frac{\frac{fx\sqrt{a+bx^2}\left(ad^3ef\left(e+fx^2\right)-2b\left(6c^2def^2-2c^3f^3-6cd^2ef\left(2e+fx^2\right)+d^3e\left(6e^2+3efx^2-f^2x^4\right)\right)\right)}{be\left(e+fx^2\right)}+\frac{4(de-cf)^2\left(6bde^2-af(5de+cf)\right)\arctan\Big(\cdots}{e^{3/2}\sqrt{-be+af}\cdots}}{8f^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(a*d^3*e*f*(e + f*x^2) - 2*b*(6*c^2*d*e*f^2 - 2*c^3*
f^3 - 6*c*d^2*e*f*(2*e + f*x^2) + d^3*e*(6*e^2 + 3*e*f*x^2 - f^2*x^4))))/(
b*e*(e + f*x^2)) + (4*(d*e - c*f)^2*(6*b*d*e^2 - a*f*(5*d*e + c*f))*ArcTan
[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f
])])/(e^(3/2)*Sqrt[-(b*e) + a*f]) - (d*(-(a^2*d^2*f^2) + 24*b^2*(d*e - c*f
)^2 + 4*a*b*d*f*(-2*d*e + 3*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(
3/2))/(8*f^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 420$$

$$\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 318

$$\frac{b\left(\frac{d\left(\frac{\int\frac{3d(2bc-ad)x^2+c(4bc-ad)}{\sqrt{bx^2+a}}dx}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 299

$$\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx\cdots}{\sqrt{bx^2+a}\cdots}}{f}\cdots\right.}{f}\cdots}{f}\cdots$$

$$\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+b\cdots}}{2b}\cdots\right.}{f}\cdots\right.}{f}\cdots}{f}\cdots$$

$$\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\cdots\right.}{f}\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\int \frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\int \frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\int\frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 221$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\big\downarrow$ 420

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 299

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots}{}\right)}{f}\cdots\right)}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\right.}{}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 398

$$b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{}{\sqrt{}}}{}\right.$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdot\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-}{}\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-\cdots}{}\right.}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right)}{f}\cdots\right)}{\cdots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\cdots\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\right.}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\right.}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f\cdots}$$

$$\downarrow 291$$

$$b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{6b^{\frac{5}{2}}\left(bd\,e^{2}-\frac{1}{6}ac\,f^{2}-\frac{5}{6}adef\right)(cf-de)^{2}\left(f\,x^{2}+e\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\left(-\frac{db\left(f\,x^{2}+e\right)e\left(-24b^{2}d^{2}e^{2}+8bdf(ad+6bc)e+f^{2}\left(a^{2}d\right.\right.}{4}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*f-b*e)*e)^(1/2)*(6*b^(5/2)*(b*d*e^2-1/6*a*c*f^2-5/6*a*d*e*f)*(c*f-
d*e)^2*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+(-1/4*d*b
*(f*x^2+e)*e*(-24*b^2*d^2*e^2+8*b*d*f*(a*d+6*b*c)*e+f^2*(a^2*d^2-12*a*b*c*
d-24*b^2*c^2))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+b^(3/2)*(b*x^2+a)^(1/2)*
x*f*(-3*b*d^3*e^3+1/4*d^2*((-6*d*x^2+24*c)*b+a*d)*f*e^2+1/4*d*f^2*((2*d^2*
x^4+12*c*d*x^2-12*c^2)*b+a*d^2*x^2)*e+b*c^3*f^3))*((a*f-b*e)*e)^(1/2))/b^(
5/2)/f^4/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(233) = 466.

Time = 15.93 (sec) , antiderivative size = 3401, normalized size of antiderivative = 12.83

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**3/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(233) = 466$.

Time = 0.17 (sec) , antiderivative size = 720, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^3x^2}{f^2} - \frac{8\,b^2d^3ef^6 - 12\,b^2cd^2f^7 - abd^3f^7}{b^2f^9}\right)x$$
$$- \frac{\left(6\,b^{\frac{3}{2}}d^3e^4 - 12\,b^{\frac{3}{2}}cd^2e^3f - 5\,a\sqrt{b}d^3e^3f + 6\,b^{\frac{3}{2}}c^2de^2f^2 + 9\,a\sqrt{b}cd^2e^2f^2 - 3\,a\sqrt{b}c^2def^3 - a\sqrt{b}c^3f^4\right)\operatorname{arc}}{2\sqrt{-b^2e^2+abef}\,ef^4}$$
$$- \frac{(24\,b^2d^3e^2 - 48\,b^2cd^2ef - 8\,abd^3ef + 24\,b^2c^2df^2 + 12\,abcd^2f^2 - a^2d^3f^2)\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{16\,b^{\frac{3}{2}}f^4}$$
$$- \frac{2\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^2d^3e^4 - 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^2cd^2e^3f - \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 abd^3e^3f + 6\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*d^3*x^2/f^2 - (8*b^2*d^3*e*f^6 - 12*b^2*c*d^2*f^7 -
 a*b*d^3*f^7)/(b^2*f^9))*x - 1/2*(6*b^(3/2)*d^3*e^4 - 12*b^(3/2)*c*d^2*e^3
*f - 5*a*sqrt(b)*d^3*e^3*f + 6*b^(3/2)*c^2*d*e^2*f^2 + 9*a*sqrt(b)*c*d^2*e
^2*f^2 - 3*a*sqrt(b)*c^2*d*e*f^3 - a*sqrt(b)*c^3*f^4)*arctan(-1/2*((sqrt(b
)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(
-b^2*e^2 + a*b*e*f)*e*f^4) - 1/16*(24*b^2*d^3*e^2 - 48*b^2*c*d^2*e*f - 8*a
*b*d^3*e*f + 24*b^2*c^2*d*f^2 + 12*a*b*c*d^2*f^2 - a^2*d^3*f^2)*log((sqrt(
b)*x - sqrt(b*x^2 + a))^2)/(b^(3/2)*f^4) - (2*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*b^2*d^3*e^4 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*c*d^2*e^3*f - (sqr
t(b)*x - sqrt(b*x^2 + a))^2*a*b*d^3*e^3*f + 6*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*b^2*c^2*d*e^2*f^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*c*d^2*e^2*f^
2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*c^3*e*f^3 - 3*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a*b*c^2*d*e*f^3 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*c^3*f^
4 + a^2*b*d^3*e^3*f - 3*a^2*b*c*d^2*e^2*f^2 + 3*a^2*b*c^2*d*e*f^3 - a^2*b*
c^3*f^4)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f)*sqrt(b)*e*f^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^3}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2906, normalized size of antiderivative = 10.97

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output

```
( - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**3*e*f**4 - 4*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**3*f**5*x**2 - 12*sqrt(e)*sqrt(a
*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(
b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**2*d*e**2*f**3 - 12*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a*b**2*c**2*d*e*f**4*x**2 + 36*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a*b**2*c*d**2*e**3*f**2 + 36*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*s
qrt(b)))*a*b**2*c*d**2*e**2*f**3*x**2 - 20*sqrt(e)*sqrt(a*f - b*e)*atan((s
qrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sq
rt(b)))*a*b**2*d**3*e**4*f - 20*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b
**2*d**3*e**3*f**2*x**2 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e)
 - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c
**2*d*e**3*f**2 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**2*d*e*
*2*f**3*x**2 - 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(...
```

3.278 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 322

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx &= \frac{d^3x\sqrt{a+bx^2}}{2f^3} - \frac{(de-cf)^3x\sqrt{a+bx^2}}{4ef^3\,(e+fx^2)^2} \\ &\quad - \frac{(de-cf)^2(3af(3de+cf)-2be(5de+cf))x\sqrt{a+bx^2}}{8e^2f^3(be-af)\,(e+fx^2)} \\ &\quad - \frac{d^2(6bde-6bcf-adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}f^4} \\ &\quad + \frac{(de-cf)\,(24b^2d^2e^4-4abef(10d^2e^2+cdef+c^2f^2)+3a^2f^2(5d^2e^2+2cdef+c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}f^4(be-af)^{3/2}}\end{aligned}$$

output

```
1/2*d^3*x*(b*x^2+a)^(1/2)/f^3-1/4*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^3/(f*
x^2+e)^2-1/8*(-c*f+d*e)^2*(3*a*f*(c*f+3*d*e)-2*b*e*(c*f+5*d*e))*x*(b*x^2+a
)^(1/2)/e^2/f^3/(-a*f+b*e)/(f*x^2+e)-1/2*d^2*(-a*d*f-6*b*c*f+6*b*d*e)*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^4+1/8*(-c*f+d*e)*(24*b^2*d^2*e^4-
4*a*b*e*f*(c^2*f^2+c*d*e*f+10*d^2*e^2)+3*a^2*f^2*(c^2*f^2+2*c*d*e*f+5*d^2*
e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/f^4/(-a*
f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 11.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^3}\,dx$$
$$=\frac{fx\sqrt{a+bx^2}\left(4d^3-\frac{2(de-cf)^3}{e(e+fx^2)^2}+\frac{(de-cf)^2(-3af(3de+cf)+2be(5de+cf))}{e^2(be-af)(e+fx^2)}\right)-\frac{(de-cf)\left(24b^2d^2e^4-4abef\left(10d^2e^2+cdef+c^2f^2\right)+3\right.}{e^{5/2}(-be-}}{8f^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(4*d^3 - (2*(d*e - c*f)^3)/(e*(e + f*x^2)^2) + ((d*e
- c*f)^2*(-3*a*f*(3*d*e + c*f) + 2*b*e*(5*d*e + c*f)))/(e^2*(b*e - a*f)*(e
 + f*x^2))) - ((d*e - c*f)*(24*b^2*d^2*e^4 - 4*a*b*e*f*(10*d^2*e^2 + c*d*e
*f + c^2*f^2) + 3*a^2*f^2*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[
-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*(-(b*e) + a*f)^(3/2)
) - (4*d^2*(6*b*d*e - 6*b*c*f - a*d*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])
/Sqrt[b])/(8*f^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1024 vs. 2(322) = 644.

Time = 1.59 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.18, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {425, 425, 420, 299, 224, 219, 398, 224, 219, 291, 221, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^3}\,dx$$
$$\Big\downarrow \; 425$$

$$\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}$$

$$\Big\downarrow \; 425$$

$$\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 420$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+\right.}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{(}{\sqrt{bx^2}}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{(\cdots}{\sqrt{bx^2\cdots}}}{f\cdots}\cdots\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\cdots}{\cdots}\right.}{f\cdots}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\int \cdots}{\cdots}\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\cdots}{\cdots}\right.}{\cdots}\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}\cdots}{f}\right.}{f}\cdots\right.}{f}$$

$\Big\downarrow$ 398

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d(\cdots}{\cdots}\right.}{\cdots}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\cdots\right.}{f}}{f}$$

$\downarrow$ 224

$$b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}\right.}{\cdots}\right.}{f}$$

$$\downarrow \text{219}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}\cdots\right.}{\cdots}\right)}{f}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\ldots\right.}{\ldots}\right.}{\ldots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2 \ldots}{\sqrt{bx^2+a}\left(f\ldots\right.}}{f}\right.}{\ldots}\right.}{f}$$

$$\downarrow 221$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d}{\dots}\right.}{\dots}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+\dots}{\sqrt{bx^2+a}\left(f\dots\right.}}{f}\right.}{\dots}\right)}{f}$$

$\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\frac{(d\cdots}{\cdots}\right.}{f}$$

$$\downarrow 27$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\ \cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{2e(be-af)}\right.}{f}\right.}{f}\right.$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f\cdots}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}\,d\cdots}{2e(be-af)}\cdots\right.}{f}\cdots\right)}{f}\cdots\right.$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right)}{f}\right.$$

$\downarrow$ 402

$$b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right.}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\right.}{f}\cdots\right.$$

↓ 27

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\cdots\right.}{f}\cdots\right.}\cdots$$

$$\downarrow 291$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d(\cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\right)}{f}\cdots\right.$$

↓ 221

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\right)}{f}\cdots\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(b*((d*((d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)
/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f - ((d*e -
 c*f)*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e -
c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sq
rt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b
*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*
f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f))/
f - ((b*e - a*f)*((d*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
)/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a +
b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh
[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3
/2))))/f))/f - ((d*e - c*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e
 - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]
*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e
- c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) +
(((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e -
 a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{3\left((cf-de)(f x^2+e)^2\left(\frac{f^2a^2\left(c^2f^2+2cdef+5d^2e^2\right)\sqrt{b}}{8}+(e^3b\,d^2-\frac{1}{6}a\,c^2f^3-\frac{1}{6}acde\,f^2-\frac{5}{3}a\,d^2e^2f)b^{\frac{3}{2}}e\right)\arctan\left(\frac{e\sqrt{b\,x^2+c}}{x\sqrt{(af-be}}\right.\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*((c*f-d*e)*(f*x^2+e)^2*(1/8*f^2*a^2*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)*b^(1/
2)+(e^3*b*d^2-1/6*a*c^2*f^3-1/6*a*c*d*e*f^2-5/3*a*d^2*e^2*f)*b^(3/2)*e)*ar
ctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+1/6*((a*f-b*e)*e)^(1/2)*(-d^
2*(a*f-b*e)*(f*x^2+e)^2*e^2*(-6*b*d*e+f*(a*d+6*b*c))*arctanh((b*x^2+a)^(1/
2)/x/b^(1/2))+(-5/4*a*(11/5*e^4*d^3-9/5*d^2*f*(-17/9*x^2*d+c)*e^3-3/5*(-4/
3*d^2*x^4+5*c*d*x^2+c^2)*d*f^2*e^2+c^2*f^3*(3/5*x^2*d+c)*e+3/5*c^3*f^4*x^2
)*f*b^(1/2)+(3*e^4*d^3-3*d^2*(-3/2*x^2*d+c)*f*e^3-9/2*d^2*x^2*f^2*(-2/9*x^
2*d+c)*e^2+c^2*f^3*(3/2*x^2*d+c)*e+1/2*c^3*f^4*x^2)*b^(3/2)*e)*(b*x^2+a)^(
1/2)*x*f))/((a*f-b*e)*e)^(1/2)/b^(1/2)/f^4/(f*x^2+e)^2/(a*f-b*e)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(290) = 580.

Time = 46.91 (sec) , antiderivative size = 5166, normalized size of antiderivative = 16.04

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**3/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(290) = 580$.

Time = 0.19 (sec) , antiderivative size = 1875, normalized size of antiderivative = 5.82

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*d^3*x/f^3 - 1/8*(24*b^(5/2)*d^3*e^5 - 24*b^(5/2)*c*d^2
*e^4*f - 40*a*b^(3/2)*d^3*e^4*f + 36*a*b^(3/2)*c*d^2*e^3*f^2 + 15*a^2*sqrt
(b)*d^3*e^3*f^2 - 9*a^2*sqrt(b)*c*d^2*e^2*f^3 + 4*a*b^(3/2)*c^3*e*f^4 - 3*
a^2*sqrt(b)*c^2*d*e*f^4 - 3*a^2*sqrt(b)*c^3*f^5)*arctan(1/2*((sqrt(b)*x -
sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*e^3*f^4
- a*e^2*f^5)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*b^(5/2)*d^3*e^5*f - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c*d^
2*e^4*f^2 - 32*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d^3*e^4*f^2 + 24*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d*e^3*f^3 + 60*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a*b^(3/2)*c*d^2*e^3*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*a^2*sqrt(b)*d^3*e^3*f^3 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*
c^2*d*e^2*f^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*d^2*e^2*f
^4 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c^3*e*f^5 + 3*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^2*d*e*f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*sqrt(b)*c^3*f^6 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^
3*e^6 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*d^2*e^5*f - 152*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*d^3*e^5*f + 48*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*b^(7/2)*c^2*d*e^4*f^2 + 264*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b
^(5/2)*c*d^2*e^4*f^2 + 102*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^3
*e^4*f^2 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3*e^3*f^3 - 72*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3}{\left(f\,x^2+e\right)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 8152, normalized size of antiderivative = 25.32

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^3}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
( - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*e**2*f**6 - 24*s
qrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*e*f**7*x**2 - 12*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*f**8*x**4 - 12*sqrt(e)*sqrt(a*
f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b
)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**2*d*e**3*f**5 - 24*sqrt(e)*sqrt(a*f - b*
e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(
sqrt(e)*sqrt(b)))*a**3*b*c**2*d*e**2*f**6*x**2 - 12*sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a**3*b*c**2*d*e*f**7*x**4 - 36*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a**3*b*c*d**2*e**4*f**4 - 72*sqrt(e)*sqrt(a*f - b*e)*atan((sq
rt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqr
t(b)))*a**3*b*c*d**2*e**3*f**5*x**2 - 36*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**3*b*c*d**2*e**2*f**6*x**4 + 60*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt
(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(
b)))*a**3*b*d**3*e**5*f**3 + 120*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f...
```

3.279 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 466

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx$$

$$= -\frac{(de-cf)^3x\sqrt{a+bx^2}}{6ef^3(e+fx^2)^3} + \frac{(de-cf)^2(2be(7de+2cf)-af(13de+5cf))x\sqrt{a+bx^2}}{24e^2f^3(be-af)(e+fx^2)^2}$$

$$-\frac{(de-cf)\left(4b^2e^2(11d^2e^2+5cdef+2c^2f^2)+3a^2f^2(11d^2e^2+8cdef+5c^2f^2)-2abef(40d^2e^2+19cde\ldots\right.}{48e^3f^3(be-af)^2(e+fx^2)}$$

$$+\frac{\sqrt{b}d^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f^4}$$

$$-\frac{(16b^3d^3e^6-8ab^2e^2f(5d^3e^3+c^3f^3)+6a^2bef^2(5d^3e^3+c^2def^2+2c^3f^3)-a^3f^3(5d^3e^3+3cd^2e^2f+3c^2\ldots}{16e^{7/2}f^4(be-af)^{5/2}}$$

output
```
-1/6*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^3/(f*x^2+e)^3+1/24*(-c*f+d*e)^2*(2
*b*e*(2*c*f+7*d*e)-a*f*(5*c*f+13*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^3/(-a*f+b*e
)/(f*x^2+e)^2-1/48*(-c*f+d*e)*(4*b^2*e^2*(2*c^2*f^2+5*c*d*e*f+11*d^2*e^2)+
3*a^2*f^2*(5*c^2*f^2+8*c*d*e*f+11*d^2*e^2)-2*a*b*e*f*(13*c^2*f^2+19*c*d*e*
f+40*d^2*e^2))*x*(b*x^2+a)^(1/2)/e^3/f^3/(-a*f+b*e)^2/(f*x^2+e)+b^(1/2)*d^
3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/f^4-1/16*(16*b^3*d^3*e^6-8*a*b^2*e^2*
f*(c^3*f^3+5*d^3*e^3)+6*a^2*b*e*f^2*(2*c^3*f^3+c^2*d*e*f^2+5*d^3*e^3)-a^3*
f^3*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arctanh((-a*f+b*e)^
(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(7/2)/f^4/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 12.08 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx$$

$$= \frac{\frac{f(de-cf)x\sqrt{a+bx^2}\left(-8e^2(de-cf)^2+\frac{2e(de-cf)(2be(7de+2cf)-af(13de+5cf))\left(e+fx^2\right)}{be-af}-\frac{\left(4b^2e^2\left(11d^2e^2+5cdef+2c^2f^2\right)+3a^2f^2\left(11d^2e^2+8cdef+5c^2f\right.\right.}{(be-af)^2}\right.}{e^3(e+fx^2)^3}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
((f*(d*e - c*f)*x*Sqrt[a + b*x^2]*(-8*e^2*(d*e - c*f)^2 + (2*e*(d*e - c*f)
*(2*b*e*(7*d*e + 2*c*f) - a*f*(13*d*e + 5*c*f))*(e + f*x^2))/(b*e - a*f) -
 ((4*b^2*e^2*(11*d^2*e^2 + 5*c*d*e*f + 2*c^2*f^2) + 3*a^2*f^2*(11*d^2*e^2
+ 8*c*d*e*f + 5*c^2*f^2) - 2*a*b*e*f*(40*d^2*e^2 + 19*c*d*e*f + 13*c^2*f^2
))*(e + f*x^2)^2)/(b*e - a*f)^2))/(e^3*(e + f*x^2)^3) - (3*(16*b^3*d^3*e^6
 - 8*a*b^2*e^2*f*(5*d^3*e^3 + c^3*f^3) + 6*a^2*b*e*f^2*(5*d^3*e^3 + c^2*d*
e*f^2 + 2*c^3*f^3) - a^3*f^3*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 +
5*c^3*f^3))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(
7/2)*(-(b*e) + a*f)^(5/2)) + 48*Sqrt[b]*d^3*Log[b*x + Sqrt[b]*Sqrt[a + b*x
^2]])/(48*f^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. 2(466) = 932.

Time = 2.04 (sec) , antiderivative size = 1559, normalized size of antiderivative = 3.35, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {425, 425, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^4}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}\right)}{f}$$

$\downarrow$ 398

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}\right)}{f}$$

$\downarrow$ 224

$$b\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de\ldots}{}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\ldots}{f}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de\ldots}{}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\ldots}{f}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\cdots\right.}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\cdots\right.}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}\right)}{f}\right)}{f}-\frac{(de-\cdots}{\cdots}\right.}{f}\cdots$$

$$\Big\downarrow\ 27$$

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}d}{4e(be-af)}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 291

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{f}\right.}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}}{4e(be-af)}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 221

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3a}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}}{4e(be-af)}\right.}{f}}{f}\right.$$

$\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.}{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{f}\right.}{f}\right.}$$

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{f}\right.$$

↓ 291

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}}{}\right.}{}\right.}{f}\right.$$

↓ 221

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}}{}\right.}{}}{f}\right.$$

↓ 402

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}}{}\right.}{}\right.}{f}\right.$$

↓ 27

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right)}{f}\right.$$

↓ 291

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right)$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right)}{f}\right)$$

↓ 221

$$b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{d\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{\cdots}}{2e(be-af)\left(fx^2+e\right)}\cdots\right.}{\cdots}\right.}{f}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*((d*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e -
 c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*S
qrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(
b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a
*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f -
 ((d*e - c*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f
*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*S
qrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e
- c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e
- 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x
^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTan
h[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(
3/2)))/(4*e*(b*e - a*f))))/f))/f))/f - ((b*e - a*f)*((d*((d*(((d*e - c*f)*
x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*
f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*
e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(
b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*
Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(
d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*S
qrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/f)...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.21

method	result
	$\frac{5(af-be)\left(f\,x^2+e\right)^3\left(-\frac{16b^3d^3e^6}{5}+8a\,b^2d^3e^5f-6a^2b\,d^3e^4f^2+a^3d^3e^3f^3+\frac{3ac\left(a^2d^2-2abcd+\frac{8}{3}b^2c^2\right)f^4e^2}{5}+\frac{3a^2c^2f^5(ad-4bc)e}{5}+a\right.}{16}$
pseudoelliptic	$-\frac{}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-(5/16*(a*f-b*e)*(f*x^2+e)^3*(-16/5*b^3*d^3*e^6+8*a*b^2*d^3*e^5*f-6*a^2*b*
d^3*e^4*f^2+a^3*d^3*e^3*f^3+3/5*a*c*(a^2*d^2-2*a*b*c*d+8/3*b^2*c^2)*f^4*e^
2+3/5*a^2*c^2*f^5*(a*d-4*b*c)*e+a^3*c^3*f^6)*arctan(e*(b*x^2+a)^(1/2)/x/((
a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(d^3*(f*x^2+e)^3*(-a^3*f^3*b^(1/2)+
b^(3/2)*(3*a^2*f^2-3*a*b*e*f+b^2*e^2)*e)*e^3*arctanh((b*x^2+a)^(1/2)/x/b^(
1/2))-11/16*(8/11*b^2*d^2*e^6-14/11*d*b*f*(a*d-4/7*(5/2*x^2*d+c)*b)*e^5+5/
11*(a^2*d^2-14/5*d*(53/21*x^2*d+c)*b*a+8/5*(11/6*d^2*x^4+5/2*c*d*x^2+c^2)*
b^2)*f^2*e^4+8/11*(d*(5/3*x^2*d+c)*a^2-5/2*(4/3*d^2*x^4+28/15*c*d*x^2+c^2)
*b*a+b^2*c*x^2*(5/6*x^2*d+c))*f^3*e^3+((64/33*c*d*x^2+d^2*x^4+c^2)*a^2-70/
33*c*(19/35*x^2*d+c)*b*x^2*a+8/33*b^2*c^2*x^4)*f^4*e^2+40/33*a*(a*(3/5*x^2
*d+c)-13/20*x^2*b*c)*c*x^2*f^5*e+5/11*a^2*c^2*f^6*x^4)*(a*f-b*e)*(c*f-d*e)
*(b*x^2+a)^(1/2)*x*f))/((a*f-b*e)*e)^(1/2)/e^3/(a*f-b*e)^3/(f*x^2+e)^3/f^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. $2(436) = 872$.

Time = 132.96 (sec) , antiderivative size = 6833, normalized size of antiderivative = 14.66

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**3/(e + f*x**2)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3864 vs. 2(436) = 872.

Time = 0.24 (sec) , antiderivative size = 3864, normalized size of antiderivative = 8.29

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(16*b^(7/2)*d^3*e^6 - 40*a*b^(5/2)*d^3*e^5*f + 30*a^2*b^(3/2)*d^3*e^4
*f^2 - 5*a^3*sqrt(b)*d^3*e^3*f^3 - 8*a*b^(5/2)*c^3*e^2*f^4 + 6*a^2*b^(3/2)
*c^2*d*e^2*f^4 - 3*a^3*sqrt(b)*c*d^2*e^2*f^4 + 12*a^2*b^(3/2)*c^3*e*f^5 -
3*a^3*sqrt(b)*c^2*d*e*f^5 - 5*a^3*sqrt(b)*c^3*f^6)*arctan(1/2*((sqrt(b)*x
- sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^5*
f^4 - 2*a*b*e^4*f^5 + a^2*e^3*f^6)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/2*sqrt(b)
*d^3*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^4 - 1/24*(144*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*b^(7/2)*d^3*e^6*f^2 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*b^(7/2)*c*d^2*e^5*f^3 - 312*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*d
^3*e^5*f^3 + 288*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c*d^2*e^4*f^4
+ 198*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*d^3*e^4*f^4 - 144*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*d^2*e^3*f^5 - 33*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a^3*sqrt(b)*d^3*e^3*f^5 + 24*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*a*b^(5/2)*c^3*e^2*f^6 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3
/2)*c^2*d*e^2*f^6 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*d^2*e
^2*f^6 - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c^3*e*f^7 + 9*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c^2*d*e*f^7 + 15*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a^3*sqrt(b)*c^3*f^8 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^
8*b^(9/2)*d^3*e^7*f - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*d^2*e^
6*f^2 - 2400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d^3*e^6*f^2 - 28...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^3}{(f\,x^2+e)^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^3)/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 11416, normalized size of antiderivative = 24.50

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e**3*f**7 - 45*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e**2*f**8*x**2 - 45*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e*f**9*x**4 - 15*sqrt(e)*sqrt(a*f
 - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)
*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*f**10*x**6 - 9*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a**4*c**2*d*e**4*f**6 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt
(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(
b)))*a**4*c**2*d*e**3*f**7*x**2 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**4*c**2*d*e**2*f**8*x**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*
*4*c**2*d*e*f**9*x**6 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d*
*2*e**5*f**5 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d**2*e**4*
f**6*x**2 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*...
```

3.280 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 120

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx = -\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)}+\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(de-cf)}$$

output
```
-(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1
/2)/(-c*f+d*e)+(-a*f+b*e)^(1/2)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+
a)^(1/2))/e^(1/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx = \frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}}-\frac{\sqrt{-be+af}\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}}}{-de+cf}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
((Sqrt[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))
/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] - (Sqrt[-(b*e) + a*f]*ArcTan[(-(f*
x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/S
qrt[e])/(-(d*e) + c*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{422}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}$$

$$\Big\downarrow \text{301}$$

$$\frac{d\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{de-cf}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{de-cf}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/(d*e -
 c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt[b*e -
 a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))
)/(d*e - c*f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{-\frac{(ad-bc)\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}+\frac{(af-be)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{\sqrt{(af-be)e}}}{cf-de}$	111
default	Expression too large to display	1460

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/(c*f-d*e)*(-(a*d-b*c)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))/(
(a*d-b*c)*c)^(1/2)+(a*f-b*e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/
x/((a*f-b*e)*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 977, normalized size of antiderivative = 8.14

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^
2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3
)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + sqrt
((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(
4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^
2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d*e - c*f), -1/
4*(2*sqrt(-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2
+ a)*sqrt(-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 + (a*b*e - a^2*f)*x)) + sqr
t((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*
(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x
^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(d*e - c*f), 1/
4*(2*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2
+ a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - sqr
t((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*
(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x
^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d*e - c*f), 1/
2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 +
a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - sqrt(
-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(
-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 + (a*b*e - a^2*f)*x)))/(d*e - c*f)...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)}\,dx = \frac{\left(b^{\frac{3}{2}}c-a\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(de-cf)} - \frac{\left(b^{\frac{3}{2}}e-a\sqrt{b}f\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\sqrt{-b^2e^2+abef}}\right)}{\sqrt{-b^2e^2+abef}(de-cf)}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
(b^(3/2)*c - a*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d +
2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(d*e - c*
f)) - (b^(3/2)*e - a*sqrt(b)*f)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^
2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*(d*
e - c*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)}\,dx$$
$$=\frac{\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{b\,x^2+a}-\sqrt{d}\,\sqrt{b}\,x}{\sqrt{c}\,\sqrt{b}}\right)e+\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}+\sqrt{d}\,\sqrt{b\,x^2+a}+\sqrt{d}\,\sqrt{b}\,x}{\sqrt{c}\,\sqrt{b}}\right)e-\sqrt{e}\,\sqrt{\cdots}}{ce\,(cf-de)}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2)
- sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*e + sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*s
qrt(b)))*e - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(
a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*c - sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*c)/(c*e*(c*f - d*e))
```

3.281 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 184

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = -\frac{fx\sqrt{a+bx^2}}{2e(de-cf)(e+fx^2)} - \frac{d\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^2} + \frac{(2bde^2-af(3de-cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}\sqrt{be-af}(de-cf)^2}$$

output

```
-1/2*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)-d*(-a*d+b*c)^(1/2)*arctanh
((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-c*f+d*e)^2+1/2*(2*b
*d*e^2-a*f*(-c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2
))/e^(3/2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{\frac{f(-de+cf)x\sqrt{a+bx^2}}{e(e+fx^2)} - \frac{2d\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} - \frac{(2bde^2+af(-3de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{e^{3/2}\sqrt{-be+af}}}{2(de-cf)^2}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
((f*(-(d*e) + c*f)*x*Sqrt[a + b*x^2])/(e*(e + f*x^2)) - (2*d*Sqrt[-(b*c) +
 a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[
-(b*c) + a*d])])/Sqrt[c] - ((2*b*d*e^2 + a*f*(-3*d*e + c*f))*ArcTan[(-(f*x
*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(e
^(3/2)*Sqrt[-(b*e) + a*f]))/(2*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {421, 301, 224, 219, 291, 221, 401, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{d^2\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^2\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}-\frac{\int-\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{398}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3de-cf)\right)}{\sqrt{e}f\sqrt{be-af}}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)}\right)}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(d^2*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/(d*e
 - c*f)^2 - (f*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(e + f*x^2)) + ((2*Sq
rt[b]*d*e*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((2*b*d*e^2 - a*f*(3*d
*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e
]*f*Sqrt[b*e - a*f]))/(2*e)))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{-\sqrt{(ad-bc)c}\,(f x^2+e)\,(ac f^2-3adef+2bd e^2)\arctan\left(\frac{e\sqrt{b x^2+a}}{x\sqrt{(af-be)e}}\right)+\left(-2de(f x^2+e)(ad-bc)\arctan\left(\frac{c\sqrt{b x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\sqrt{\quad}\right.}{2\sqrt{(ad-bc)c}\,\sqrt{(af-be)e}\,(cf-de)^2 e(f x^2+e)}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(1/2)*(-((a*d-b*c)*c)^(1/2)*(f*x^2+e
)*(a*c*f^2-3*a*d*e*f+2*b*d*e^2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(
1/2))+(-2*d*e*(f*x^2+e)*(a*d-b*c)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)
^(1/2))+((a*d-b*c)*c)^(1/2)*(c*f-d*e)*(b*x^2+a)^(1/2)*x*f)*((a*f-b*e)*e)^(
1/2))/(c*f-d*e)^2/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(158) = 316$.

Time $= 0.87$ (sec) , antiderivative size $= 377$, normalized size of antiderivative $= 2.05$

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{\left(b^{\frac{3}{2}}cd - a\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(d^2e^2-2\,cdef+c^2f^2)} - \frac{\left(2\,b^{\frac{3}{2}}de^2 - 3\,a\sqrt{b}def + a\sqrt{b}cf^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,(d^2e^3-2\,cde^2f+c^2ef^2)\sqrt{-b^2e^2+abef}} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}e - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a\sqrt{b}f + a^2\sqrt{b}f}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 f + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 be - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 af + a^2f\right)(de^2-cef)}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
(b^(3/2)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*
d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(d^2*
e^2 - 2*c*d*e*f + c^2*f^2)) - 1/2*(2*b^(3/2)*d*e^2 - 3*a*sqrt(b)*d*e*f + a
*sqrt(b)*c*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*
f)/sqrt(-b^2*e^2 + a*b*e*f))/((d^2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*sqrt(-b^
2*e^2 + a*b*e*f)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*e - (sqrt(b
)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*f + a^2*sqrt(b)*f)/(((sqrt(b)*x - sqrt(
b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*a*f + a^2*f)*(d*e^2 - c*e*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 1408, normalized size of antiderivative = 7.65

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**3*f - 2*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(
b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**2*f**2*x**2 + 2*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(
c)*sqrt(b)))*b*d*e**4 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) -
sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*d*e**3*
f*x**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a
+ b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**3*f - 2*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*
sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**2*f**2*x**2 + 2*sqrt(c)*sqrt(a*d - b*
c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(
sqrt(c)*sqrt(b)))*b*d*e**4 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*
c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*d*
e**3*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt
(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c**2*e*f**2 - sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt
(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c**2*f**3*x**2 + 3*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a*c*d*e**2*f + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt...
```

3.282 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 310

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^3}\,dx = -\frac{fx\sqrt{a+bx^2}}{4e(de-cf)(e+fx^2)^2} + \frac{f(af(7de-3cf)-2be(3de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)^2(e+fx^2)} - \frac{d^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^3} + \frac{(8b^2d^2e^4-4abef(6d^2e^2-3cdef+c^2f^2)+a^2f^2(15d^2e^2-10cdef+3c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{3/2}(de-cf)^3}$$

output
```
-1/4*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^2+1/8*f*(a*f*(-3*c*f+7*d*e
)-2*b*e*(-c*f+3*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2
+e)-d^2*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
))/c^(1/2)/(-c*f+d*e)^3+1/8*(8*b^2*d^2*e^4-4*a*b*e*f*(c^2*f^2-3*c*d*e*f+6*
d^2*e^2)+a^2*f^2*(3*c^2*f^2-10*c*d*e*f+15*d^2*e^2))*arctanh((-a*f+b*e)^(1/
2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 11.01 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{\frac{f(de-cf)x\sqrt{a+bx^2}\left(-2e(de-cf)-\frac{(2be(3de-cf)+af(-7de+3cf))(e+fx^2)}{be-af}\right)}{e^2(e+fx^2)^2}+\frac{8d^2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}-\frac{(8b^2d^2e^4-4abef(6d^2e^2}{}}{8(de-cf)^3}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((f*(d*e - c*f)*x*Sqrt[a + b*x^2]*(-2*e*(d*e - c*f) - ((2*b*e*(3*d*e - c*f
) + a*f*(-7*d*e + 3*c*f))*(e + f*x^2))/(b*e - a*f)))/(e^2*(e + f*x^2)^2) +
 (8*d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a +
 b*x^2]])])/Sqrt[c] - ((8*b^2*d^2*e^4 - 4*a*b*e*f*(6*d^2*e^2 - 3*c*d*e*f +
c^2*f^2) + a^2*f^2*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[-(b
*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*(-(b*e) + a*f)^(3/2)))/
(8*(d*e - c*f)^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.31, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {421, 401, 25, 27, 402, 25, 27, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^3}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}-\frac{\int -\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{2b(3de-cf)x^2+a(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 402

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\frac{\int -\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{\int \frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 291$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 422$$

$$\frac{d^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 301$$

$$\frac{d^2\left(\frac{d\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{d^2\left(\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 221

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(d^2*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/
(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt
[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[
e]*f)))/(d*e - c*f)))/(d*e - c*f)^2 - (f*((((d*e - c*f)*x*Sqrt[a + b*x^2])/
(4*e*(e + f*x^2)^2) + (-1/2*((a*f*(7*d*e - 3*c*f) - 2*b*e*(3*d*e - c*f))*x
*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) - (a*(a*f*(7*d*e - 3*c*f) -
4*b*e*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e)))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{3\left(\left(\frac{8b^2d^2e^4}{3}-8ab\,d^2e^3f+f^2\left(5a^2d^2+4abcd\right)e^2-\frac{10ac\left(ad+\frac{2bc}{5}\right)f^3e}{3}+a^2c^2f^4\right)\sqrt{(ad-bc)c}\,\left(f\,x^2+e\right)^2\arctan\left(\frac{e\sqrt{b\,x^2+c}}{x\sqrt{(af-be}}\right.\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)*((8/3*b^2*d^2*e^4-8*a*b*d^2*e^3*f+f^2*(5*a^2*d^2+
4*a*b*c*d)*e^2-10/3*a*c*(a*d+2/5*b*c)*f^3*e+a^2*c^2*f^4)*((a*d-b*c)*c)^(1/
2)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-8/3*(d^2*e^
2*(f*x^2+e)^2*(a*f-b*e)*(a*d-b*c)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)
^(1/2))+5/8*(8/5*b*d*e^3-9/5*f*(4/9*b*c+d*(-2/3*b*x^2+a))*e^2+(c*(-2/5*b*x
^2+a)-7/5*a*d*x^2)*f^2*e+3/5*a*c*f^3*x^2)*((a*d-b*c)*c)^(1/2)*(c*f-d*e)*(b
*x^2+a)^(1/2)*x*f)*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)/(c*f-d*e)^3/(f
*x^2+e)^2/(a*f-b*e)/e^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(281) = 562$.

Time = 2.00 (sec) , antiderivative size = 1173, normalized size of antiderivative = 3.78

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
(b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^
2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((d^3*e^3 - 3*c*d^2*e^2*f + 3
*c^2*d*e*f^2 - c^3*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/8*(8*b^(5/2)*d^2*e^4
 - 24*a*b^(3/2)*d^2*e^3*f + 12*a*b^(3/2)*c*d*e^2*f^2 + 15*a^2*sqrt(b)*d^2*
e^2*f^2 - 4*a*b^(3/2)*c^2*e*f^3 - 10*a^2*sqrt(b)*c*d*e*f^3 + 3*a^2*sqrt(b)
*c^2*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqr
t(-b^2*e^2 + a*b*e*f))/((b*d^3*e^6 - 3*b*c*d^2*e^5*f - a*d^3*e^5*f + 3*b*c
^2*d*e^4*f^2 + 3*a*c*d^2*e^4*f^2 - b*c^3*e^3*f^3 - 3*a*c^2*d*e^3*f^3 + a*c
^3*e^2*f^4)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*b^(5/2)*d*e^3*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*
f^2 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^3 + 7*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*a^2*sqrt(b)*c*f^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4 - 1
6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 104*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a*b^(5/2)*d*e^3*f + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(
5/2)*c*e^2*f^2 + 74*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*e^2*f^2
- 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 - 21*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d*e*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^3*sqrt(b)*c*f^4 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*d*e^3
*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 - 64*(sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)\,\left(f\,x^2+e\right)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 7591, normalized size of antiderivative = 24.49

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
(16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**
2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**5*f**3 + 32*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(
d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**4*f**4*x**2 + 16*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**3*f**5*x**4 - 64*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a**2*b*d**2*e**6*f**2 - 128*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sq
rt(c)*sqrt(b)))*a**2*b*d**2*e**5*f**3*x**2 - 64*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(
c)*sqrt(b)))*a**2*b*d**2*e**4*f**4*x**4 + 80*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*
sqrt(b)))*a*b**2*d**2*e**7*f + 160*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*
a*b**2*d**2*e**6*f**2*x**2 + 80*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*b
**2*d**2*e**5*f**3*x**4 - 32*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c)
 - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b**3*d
**2*e**8 - 64*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*s...
```

3.283 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 532

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx$$
$$= -\frac{fx\sqrt{a+bx^2}}{6e(de-cf)(e+fx^2)^3} + \frac{f(af(11de-5cf)-2be(5de-2cf))x\sqrt{a+bx^2}}{24e^2(be-af)(de-cf)^2(e+fx^2)^2}$$
$$-\frac{f(4b^2e^2(11d^2e^2-7cdef+2c^2f^2)+3a^2f^2(19d^2e^2-16cdef+5c^2f^2)-2abef(52d^2e^2-41cdef+13c}{48e^3(be-af)^2(de-cf)^3(e+fx^2)}$$
$$-\frac{d^3\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^4}$$
$$+\frac{(16b^3d^3e^6+2a^2bef^2(45d^3e^3-40cd^2e^2f+25c^2def^2-6c^3f^3)-a^3f^3(35d^3e^3-35cd^2e^2f+21c^2def^2 -}{16e^{7/2}(be-af)^{5/2}(de-cf}$$

output
```
-1/6*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^3+1/24*f*(a*f*(-5*c*f+11*d
*e)-2*b*e*(-2*c*f+5*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^2/(f
*x^2+e)^2-1/48*f*(4*b^2*e^2*(2*c^2*f^2-7*c*d*e*f+11*d^2*e^2)+3*a^2*f^2*(5*
c^2*f^2-16*c*d*e*f+19*d^2*e^2)-2*a*b*e*f*(13*c^2*f^2-41*c*d*e*f+52*d^2*e^2
))*x*(b*x^2+a)^(1/2)/e^3/(-a*f+b*e)^2/(-c*f+d*e)^3/(f*x^2+e)-d^3*(-a*d+b*c
)^(1/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-c*f+
d*e)^4+1/16*(16*b^3*d^3*e^6+2*a^2*b*e*f^2*(-6*c^3*f^3+25*c^2*d*e*f^2-40*c*
d^2*e^2*f+45*d^3*e^3)-a^3*f^3*(-5*c^3*f^3+21*c^2*d*e*f^2-35*c*d^2*e^2*f+35
*d^3*e^3)-8*a*b^2*e^2*f*(-c^3*f^3+4*c^2*d*e*f^2-6*c*d^2*e^2*f+9*d^3*e^3))*
arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(7/2)/(-a*f+b*e)^(5/
2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 12.20 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx = \frac{\frac{f(de-cf)x\sqrt{a+bx^2}\left(-8e^2(de-cf)^2-\frac{2e(de-cf)(2be(5de-2cf)+af(-11de+5cf))\left(e+fx^2\right)}{be-af}-\frac{\left(4b^2e^2\left(11d^2e^2-7cdef+2c^2f^2\right)+3a^2f^2\left(19d^2e^2-16cdef+5\right.\right.}{(be-af)^2}\right.}{e^3(e+fx^2)^3}}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^4),x]
```

output
```
((f*(d*e - c*f)*x*Sqrt[a + b*x^2]*(-8*e^2*(d*e - c*f)^2 - (2*e*(d*e - c*f)
*(2*b*e*(5*d*e - 2*c*f) + a*f*(-11*d*e + 5*c*f))*(e + f*x^2))/(b*e - a*f)
- ((4*b^2*e^2*(11*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2) + 3*a^2*f^2*(19*d^2*e^2
 - 16*c*d*e*f + 5*c^2*f^2) - 2*a*b*e*f*(52*d^2*e^2 - 41*c*d*e*f + 13*c^2*f
^2))*(e + f*x^2)^2)/(b*e - a*f)^2))/(e^3*(e + f*x^2)^3) + (48*d^3*Sqrt[-(b
*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[
c] + (3*(16*b^3*d^3*e^6 + 2*a^2*b*e*f^2*(45*d^3*e^3 - 40*c*d^2*e^2*f + 25*
c^2*d*e*f^2 - 6*c^3*f^3) - 8*a*b^2*e^2*f*(9*d^3*e^3 - 6*c*d^2*e^2*f + 4*c^
2*d*e*f^2 - c^3*f^3) + a^3*f^3*(-35*d^3*e^3 + 35*c*d^2*e^2*f - 21*c^2*d*e*
f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
)/(e^(7/2)*(-(b*e) + a*f)^(5/2)))/(48*(d*e - c*f)^4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.14, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.767, Rules used = {421, 401, 25, 27, 402, 402, 27, 291, 221, 421, 301, 224, 219, 291, 221, 401, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^4}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{6e(e+fx^2)^3}-\frac{\int -\frac{f(2b(5de-2cf)x^2+a(11de-5cf))}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6ef}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f(2b(5de-2cf)x^2+a(11de-5cf))}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e(e+fx^2)^3}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{2b(5de-2cf)x^2+a(11de-5cf)}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6e}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e(e+fx^2)^3}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2} - \frac{f\left(\frac{\frac{\int \frac{a(2be(17de-8cf)-3af(11de-5cf))-2b(af(11de-5cf)-2be(5de-2cf))x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)} - \frac{x\sqrt{a+bx^2}(af(11de-5cf)-2be(5de-2cf))}{4e\left(e+fx^2\right)^2(be-af)}}{6e} + \frac{x\sqrt{a+bx^2}(de-cf)}{6e(e+fx^2)^3}\right)}{(de-cf)^2}$$

$$\downarrow 402$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2} - \frac{f\left(\frac{\frac{\frac{\int -\frac{3a\left(-8b^2(2de-cf)e^2+2abf(13de-6cf)e-a^2f^2(11de-5cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)} - \frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-2cf)\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)} - \frac{x\sqrt{a+bx^2}(a}{4e}}{6e}\right.}{(de-cf)^2}$$

$$\downarrow 27$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2} - \frac{f\left(\frac{\frac{-\frac{3a\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)} - \frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-2c\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$$\downarrow 291$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 421

$$\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 301

$$\frac{d^2\left(\frac{d^2\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{-\frac{\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

↓ 224

$$\frac{d^2\left(\frac{d^2\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{-\frac{\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

↓ 219

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{-\frac{\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{-\frac{\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\cdots\right.}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int-\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\cdots\right.}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 27

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(de-cf)^2}-$$

$$\frac{f\left(\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 398

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{}}{}\right.}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\frac{f\left(\frac{-\frac{\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 224

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right.}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\frac{f\left(\frac{-\frac{\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 219

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+\right.}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\frac{f\left(\frac{\frac{-\frac{3a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\frac{f\left(\frac{\frac{-\frac{3a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\right.}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3\cdots\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(de-cf)^2}$$

$$\frac{f\left(\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(11de-5cf)+2abef(13de-6cf)-8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(11de-5cf)+2abef(28de-13cf)-4b^2e^2(5de\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e}\cdots\right.}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)*(e + f*x^2)^4),x]
```

output
```
-((f*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(6*e*(e + f*x^2)^3) + (-1/4*((a*f*(1
1*d*e - 5*c*f) - 2*b*e*(5*d*e - 2*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*
(e + f*x^2)^2) + (-1/2*((2*a*b*e*f*(28*d*e - 13*c*f) - 3*a^2*f^2*(11*d*e -
 5*c*f) - 4*b^2*e^2*(5*d*e - 2*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e
+ f*x^2)) - (3*a*(2*a*b*e*f*(13*d*e - 6*c*f) - a^2*f^2*(11*d*e - 5*c*f) -
8*b^2*e^2*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x
^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f)))/(6*e)))/(d*e - c*
f)^2) + (d^2*((d^2*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqr
t[c]*d)))/(d*e - c*f)^2 - (f*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(e + f*
x^2)) + ((2*Sqrt[b]*d*e*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((2*b*d*
e^2 - a*f*(3*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x
^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f]))/(2*e)))/(d*e - c*f)^2))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$-\frac{5\left(\left(\frac{16b^3d^3e^6}{5}-\frac{72a\,b^2d^3e^5f}{5}+18a\,d^2\left(ad+\frac{8bc}{15}\right)b\,f^2e^4-7ad\left(a^2d^2+\frac{16}{7}abcd+\frac{32}{35}b^2c^2\right)f^3e^3+7ac\left(a^2d^2+\frac{10}{7}abcd+\frac{8}{35}b^2c^2\right)\right.\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16/((a*d-b*c)*c)^(1/2)*((16/5*b^3*d^3*e^6-72/5*a*b^2*d^3*e^5*f+18*a*d^2
*(a*d+8/15*b*c)*b*f^2*e^4-7*a*d*(a^2*d^2+16/7*a*b*c*d+32/35*b^2*c^2)*f^3*e
^3+7*a*c*(a^2*d^2+10/7*a*b*c*d+8/35*b^2*c^2)*f^4*e^2-21/5*(a*d+4/7*b*c)*a^
2*c^2*f^5*e+a^3*c^3*f^6)*((a*d-b*c)*c)^(1/2)*(f*x^2+e)^3*arctan(e*(b*x^2+a
)^(1/2)/x/((a*f-b*e)*e)^(1/2))+16/5*((a*f-b*e)*e)^(1/2)*(d^3*e^3*(f*x^2+e)
^3*(a*f-b*e)^2*(a*d-b*c)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-1
1/16*(24/11*b^2*d^2*e^6-54/11*d*b*f*(-2/3*b*d*x^2+a*d+4/9*b*c)*e^5+29/11*(
44/87*b^2*d^2*x^4+(-250/87*a*b*d^2-28/29*b^2*c*d)*x^2+a^2*d^2+2*a*b*c*d+8/
29*b^2*c^2)*f^2*e^4-32/11*((13/12*a*b*d^2+7/24*b^2*c*d)*x^4+(-17/12*a^2*d^
2-7/3*a*b*c*d-1/4*b^2*c^2)*x^2+a*c*(a*d+5/8*b*c))*f^3*e^3+((19/11*a^2*d^2+
8/33*b^2*c^2+82/33*a*b*c*d)*x^4+(-128/33*a^2*c*d-70/33*b*c^2*a)*x^2+a^2*c^
2)*f^4*e^2+40/33*a*c*((-6/5*a*d-13/20*b*c)*x^2+a*c)*x^2*f^5*e+5/11*a^2*c^2
*f^6*x^4)*(c*f-d*e)*((a*d-b*c)*c)^(1/2)*(b*x^2+a)^(1/2)*x*f))/((a*f-b*e)*e
)^(1/2)/(a*f-b*e)^2/(c*f-d*e)^4/e^3/(f*x^2+e)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\,(e+fx^2)^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)*(f*x^2 + e)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3384 vs. $2(499) = 998$.

Time $= 6.36$ (sec) , antiderivative size $= 3384$, normalized size of antiderivative $= 6.36$

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
(b^(3/2)*c*d^3 - a*sqrt(b)*d^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^
2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((d^4*e^4 - 4*c*d^3*e^3*f + 6
*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + c^4*f^4)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/
16*(16*b^(7/2)*d^3*e^6 - 72*a*b^(5/2)*d^3*e^5*f + 48*a*b^(5/2)*c*d^2*e^4*f
^2 + 90*a^2*b^(3/2)*d^3*e^4*f^2 - 32*a*b^(5/2)*c^2*d*e^3*f^3 - 80*a^2*b^(3
/2)*c*d^2*e^3*f^3 - 35*a^3*sqrt(b)*d^3*e^3*f^3 + 8*a*b^(5/2)*c^3*e^2*f^4 +
 50*a^2*b^(3/2)*c^2*d*e^2*f^4 + 35*a^3*sqrt(b)*c*d^2*e^2*f^4 - 12*a^2*b^(3
/2)*c^3*e*f^5 - 21*a^3*sqrt(b)*c^2*d*e*f^5 + 5*a^3*sqrt(b)*c^3*f^6)*arctan
(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b
*e*f))/((b^2*d^4*e^9 - 4*b^2*c*d^3*e^8*f - 2*a*b*d^4*e^8*f + 6*b^2*c^2*d^2
*e^7*f^2 + 8*a*b*c*d^3*e^7*f^2 + a^2*d^4*e^7*f^2 - 4*b^2*c^3*d*e^6*f^3 - 1
2*a*b*c^2*d^2*e^6*f^3 - 4*a^2*c*d^3*e^6*f^3 + b^2*c^4*e^5*f^4 + 8*a*b*c^3*
d*e^5*f^4 + 6*a^2*c^2*d^2*e^5*f^4 - 2*a*b*c^4*e^4*f^5 - 4*a^2*c^3*d*e^4*f^
5 + a^2*c^4*e^3*f^6)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/24*(48*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*b^(7/2)*d^2*e^5*f^2 - 168*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a*b^(5/2)*d^2*e^4*f^3 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c*
d*e^3*f^4 + 174*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*d^2*e^3*f^4 -
 24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*e^2*f^5 - 114*(sqrt(b)*
x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*d*e^2*f^5 - 57*(sqrt(b)*x - sqrt(b*x
^2 + a))^10*a^3*sqrt(b)*d^2*e^2*f^5 + 36*(sqrt(b)*x - sqrt(b*x^2 + a))^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)(e+fx^2)^4}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)\,\left(f\,x^2+e\right)^4}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^4),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)*(e + f*x^2)^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)\left(e+fx^2\right)^4}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)\left(f\,x^2+e\right)^4}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^4,x)
```

3.284 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2 (e+fx^2)} dx$

Optimal result

Integrand size = 30, antiderivative size = 182

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2 (e+fx^2)} dx = \frac{dx\sqrt{a+bx^2}}{2c(de-cf)(c+dx^2)} + \frac{(2bc^2f+ad(de-3cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}(de-cf)^2} - \frac{f\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(de-cf)^2}$$

output
```
1/2*d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)+1/2*(2*b*c^2*f+a*d*(-3*c*f+
d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*
c)^(1/2)/(-c*f+d*e)^2-f*(-a*f+b*e)^(1/2)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2
)/(b*x^2+a)^(1/2))/e^(1/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx = \frac{1}{2}\left(-\frac{dx\sqrt{a+bx^2}}{c(-de+cf)(c+dx^2)} - \frac{(2bc^2f+ad(de-3cf))\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}\sqrt{-bc+ad}(de-cf)^2} - \frac{2f\sqrt{-be+af}\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(de-cf)^2}\right)$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)),x]
```

output
```
(-((d*x*Sqrt[a + b*x^2])/(c*(-(d*e) + c*f)*(c + d*x^2))) - ((2*b*c^2*f + a
*d*(d*e - 3*c*f))*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(S
qrt[c]*Sqrt[-(b*c) + a*d])])/(c^(3/2)*Sqrt[-(b*c) + a*d]*(d*e - c*f)^2) -
(2*f*Sqrt[-(b*e) + a*f]*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^
2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*(d*e - c*f)^2))/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {421, 25, 301, 224, 219, 291, 221, 401, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{(dx^2+c)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{f^2 \int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 301$$

$$\frac{f^2 \left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{f^2 \left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{f^2 \left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{f^2 \left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{d\int \frac{\sqrt{bx^2+a}\left(-dfx^2+de-2cf\right)}{\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2 \left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}-\frac{\int -\frac{d\left(a(de-3cf)-2bcfx^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2cd}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{\int \frac{d\left(a(de-3cf)-2bcfx^2\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{\int \frac{a(de-3cf)-2bcfx^2}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{d\left(\frac{\frac{\left(ad(de-3cf)+2bc^2f\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}-\frac{2bcf\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{d\left(\frac{\frac{\left(ad(de-3cf)+2bc^2f\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}-\frac{2bcf\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\frac{\left(ad(de-3cf)+2bc^2f\right)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}-\frac{2\sqrt{b}cf\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{\frac{\left(ad(de-3cf)+2bc^2f\right)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{2\sqrt{b}cf\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)\left(ad(de-3cf)+2bc^2f\right)}{\sqrt{c}d\sqrt{bc-ad}}-\frac{2\sqrt{b}cf\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{2c}+\frac{x\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)),x]
```

output
```
(d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + ((-2*Sqrt[b]*c*f*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d + ((2*b*c^2*f + a*d*(d*e - 3*c*f))*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d*Sqrt[b*
c - a*d]))/(2*c)))/(d*e - c*f)^2 + (f^2*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/f - (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sq
rt[a + b*x^2])])/(Sqrt[e]*f)))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{-3\left(x^2 d+c\right)\sqrt{\left(af-be\right)e}\,\left(acdf-\frac{1}{3}a\,d^2 e-\frac{2}{3}b\,c^2 f\right)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{\left(ad-bc\right)c}}\right)+\left(2cf\left(x^2 d+c\right)\left(af-be\right)\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{\left(af-be\right)e}}\right)+\cdots\right.}{2\sqrt{\left(ad-bc\right)c}\,\sqrt{\left(af-be\right)e}\,\left(x^2 d+c\right)\left(cf-de\right)^2 c}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*c)^(1/2)*(-3*(d*x^2+c)*((a*f-b*e)*e)^(1/2)*(a*c*d*f-1/3*a*
d^2*e-2/3*b*c^2*f)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+(2*c*f*
(d*x^2+c)*(a*f-b*e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+d*(c*f
-d*e)*(b*x^2+a)^(1/2)*x*((a*f-b*e)*e)^(1/2))*((a*d-b*c)*c)^(1/2))/((a*f-b*
e)*e)^(1/2)/(d*x^2+c)/(c*f-d*e)^2/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^2*(f*x^2 + e)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(156) = 312$.

Time = 0.87 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx$$
$$= -\frac{\left(a\sqrt{b}d^2e + 2\,b^{\frac{3}{2}}c^2f - 3\,a\sqrt{b}cdf\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,(cd^2e^2 - 2\,c^2def + c^3f^2)\sqrt{-b^2c^2+abcd}} + \frac{\left(b^{\frac{3}{2}}ef - a\sqrt{b}f^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{\sqrt{-b^2e^2+abef}\,(d^2e^2 - 2\,cdef + c^2f^2)} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d + a^2\sqrt{b}d}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad + a^2d\right)(cde-c^2f)}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
-1/2*(a*sqrt(b)*d^2*e + 2*b^(3/2)*c^2*f - 3*a*sqrt(b)*c*d*f)*arctan(1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/
((c*d^2*e^2 - 2*c^2*d*e*f + c^3*f^2)*sqrt(-b^2*c^2 + a*b*c*d)) + (b^(3/2)*
e*f - a*sqrt(b)*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e
 - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*(d^2*e^2 - 2*c
*d*e*f + c^2*f^2)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d + a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt
(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*d + a^2*d)*(c*d*e - c^2*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^2\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 1408, normalized size of antiderivative = 7.74

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e),x)
```

output
```
(3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2
) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*c**2*d*e*f - sqrt(c)*sqrt(a*d
- b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*
x)/(sqrt(c)*sqrt(b)))*a*c*d**2*e**2 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt
(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(
b)))*a*c*d**2*e*f*x**2 - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - s
qrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d**3*e**
2*x**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a
+ b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*c**3*e*f - 2*sqrt(c)*s
qrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*
sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*c**2*d*e*f*x**2 + 3*sqrt(c)*sqrt(a*d - b*c
)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(s
qrt(c)*sqrt(b)))*a*c**2*d*e*f - sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b
*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*c
*d**2*e**2 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqr
t(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*c*d**2*e*f*x**2 -
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) +
 sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d**3*e**2*x**2 - 2*sqrt(c)*sqrt(a
*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(
b)*x)/(sqrt(c)*sqrt(b)))*b*c**3*e*f - 2*sqrt(c)*sqrt(a*d - b*c)*atan((s...
```

3.285 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 249

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx = \frac{d^2x\sqrt{a+bx^2}}{2c(de-cf)^2\left(c+dx^2\right)}+\frac{f^2x\sqrt{a+bx^2}}{2e(de-cf)^2\left(e+fx^2\right)} + \frac{d(4bc^2f+ad(de-5cf))\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}(de-cf)^3} - \frac{f(4bde^2-af(5de-cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}\sqrt{be-af}(de-cf)^3}$$

output
```
1/2*d^2*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^2/(d*x^2+c)+1/2*f^2*x*(b*x^2+a)^(1/
2)/e/(-c*f+d*e)^2/(f*x^2+e)+1/2*d*(4*b*c^2*f+a*d*(-5*c*f+d*e))*arctanh((-a
*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(1/2)/(-c*f+d*
e)^3-1/2*f*(4*b*d*e^2-a*f*(-c*f+5*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)
/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 10.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx$$
$$= \frac{(de-cf)x\sqrt{a+bx^2}\left(\frac{d^2}{c^2+cdx^2}+\frac{f^2}{e^2+efx^2}\right)+\frac{d(4bc^2f+ad(de-5cf))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{-bc+ad}}-\frac{f(4bde^2+af(-5de+cf))\arctan\left(\cdots\right.}{e^{3/2}\sqrt{-be+af}}}{2(de-cf)^3}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2]*(d^2/(c^2 + c*d*x^2) + f^2/(e^2 + e*f*x^2))
 + (d*(4*b*c^2*f + a*d*(d*e - 5*c*f))*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[
c]*Sqrt[a + b*x^2])])/(c^(3/2)*Sqrt[-(b*c) + a*d]) - (f*(4*b*d*e^2 + a*f*(
-5*d*e + c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(
e^(3/2)*Sqrt[-(b*e) + a*f]))/(2*(d*e - c*f)^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 650 vs. 2(249) = 498.

Time = 0.96 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.61, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {425, 421, 291, 221, 402, 27, 291, 221, 426, 421, 25, 291, 221, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \ 421$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int-\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{\int\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f(\cdots}{\cdots}\right.}{\cdots}\right.}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}(\cdots}{\sqrt{c}\sqrt{bc-ad}\cdots}\right.}{\cdots}\right.}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctan}\ldots}{\sqrt{c}\sqrt{b}\ldots}\right.}{\ldots}\right.}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctan}\ldots}{\sqrt{c}\sqrt{bc}\ldots}\right.}{\ldots}\right.}{d}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(b*((d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*
Sqrt[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*x^2]
)/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c*f))
*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e -
 a*f)^(3/2))))/(d*e - c*f)^2))/d - ((b*c - a*d)*((d*((d*(-1/2*(d*(d*e - c*
f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) - ((a*d*(d*e - 3*c*f) -
2*b*c*(d*e - 2*c*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(2*c^(3/2)*(b*c - a*d)^(3/2))))/(d*e - c*f)^2 + (f^2*ArcTanh[(Sqrt[b*e
- a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f)
^2)))/(d*e - c*f) - (f*((d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a +
 b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*
f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f)
- a*f*(3*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(d*e - c*f)^2))/(d*e - c*f)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-5\left(acdf-\frac{1}{5}a\,d^{2}e-\frac{4}{5}b\,c^{2}f\right)\sqrt{\left(af-be\right)e}\,d\left(x^{2}d+c\right)\left(f\,x^{2}+e\right)e\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{\left(ad-bc\right)c}}\right)+\sqrt{\left(ad-bc\right)c}\left(-cf\left(f\,x^{2}+e\right)\left(x^{2}d+c\right)\left(a\right.\right.}{2\sqrt{\left(ad-bc\right)c}\,\sqrt{\left(af-be\right)e}\,\left(x^{2}d+c\right.}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(1/2)*(-5*(a*c*d*f-1/5*a*d^2*e-4/5*b
*c^2*f)*((a*f-b*e)*e)^(1/2)*d*(d*x^2+c)*(f*x^2+e)*e*arctan(c*(b*x^2+a)^(1/
2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(-c*f*(f*x^2+e)*(d*x^2+c)*(a
*c*f^2-5*a*d*e*f+4*b*d*e^2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)
)+(c^2*f^2+c*d*f^2*x^2+d^2*e*(f*x^2+e))*((a*f-b*e)*e)^(1/2)*(c*f-d*e)*(b*x
^2+a)^(1/2)*x))/(d*x^2+c)/(c*f-d*e)^3/c/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^2*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. 2(217) = 434.

Time = 1.99 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.34

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(a*sqrt(b)*d^3*e + 4*b^(3/2)*c^2*d*f - 5*a*sqrt(b)*c*d^2*f)*arctan(1/
2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
d))/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*sqrt(-b^2*c^2
 + a*b*c*d)) + 1/2*(4*b^(3/2)*d*e^2*f - 5*a*sqrt(b)*d*e*f^2 + a*sqrt(b)*c*
f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^
2*e^2 + a*b*e*f))/((d^3*e^4 - 3*c*d^2*e^3*f + 3*c^2*d*e^2*f^2 - c^3*e*f^3)
*sqrt(-b^2*e^2 + a*b*e*f)) + (4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*c*
d*e*f - (sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*d^2*e*f - (sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*sqrt(b)*c*d*f^2 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^
(5/2)*c*d*e^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d^2*e^2 + 8*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*b^(5/2)*c^2*e*f - 8*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a*b^(3/2)*c*d*e*f + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*
d^2*e*f - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*c^2*f^2 + 3*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*c*d*f^2 + 4*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^2*b^(3/2)*d^2*e^2 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*
c*d*e*f - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^2*e*f + 4*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*c^2*f^2 - 3*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^3*sqrt(b)*c*d*f^2 + a^4*sqrt(b)*d^2*e*f + a^4*sqrt(b)*c*d*f^2)/(
((sqrt(b)*x - sqrt(b*x^2 + a))^8*d*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b
*d*e + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b*c*f - 4*(sqrt(b)*x - sqrt(b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^2\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^2\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

3.286 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx = \frac{d^3x\sqrt{a+bx^2}}{2c(de-cf)^3\left(c+dx^2\right)} + \frac{f^2x\sqrt{a+bx^2}}{4e(de-cf)^2\left(e+fx^2\right)^2} - \frac{f^2(af(11de-3cf)-2be(5de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)^3\left(e+fx^2\right)} + \frac{d^2\left(6bc^2f+ad(de-7cf)\right)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}(de-cf)^4} - \frac{f\left(24b^2d^2e^4-4abef\left(15d^2e^2-4cdef+c^2f^2\right)+a^2f^2\left(35d^2e^2-14cdef+3c^2f^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{3/2}(de-cf)^4}$$

output

```
1/2*d^3*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^3/(d*x^2+c)+1/4*f^2*x*(b*x^2+a)^(1/
2)/e/(-c*f+d*e)^2/(f*x^2+e)^2-1/8*f^2*(a*f*(-3*c*f+11*d*e)-2*b*e*(-c*f+5*d
*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)^3/(f*x^2+e)+1/2*d^2*(6*b*
c^2*f+a*d*(-7*c*f+d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2)
)/c^(3/2)/(-a*d+b*c)^(1/2)/(-c*f+d*e)^4-1/8*f*(24*b^2*d^2*e^4-4*a*b*e*f*(c
^2*f^2-4*c*d*e*f+15*d^2*e^2)+a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2))*ar
ctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(3/2)
/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(x\sqrt{a+bx^2}\left(-\frac{4d^3}{c(-de+cf)^3(c+dx^2)} + \frac{2f^2}{e(de-cf)^2(e+fx^2)^2} + \frac{f^2(2be(5de-cf)+af(-11de+3cf))}{e^2(be-af)(de-cf)^3(e+fx^2)}\right) + \frac{4d^2(6bc^2f+ad(de-7cf))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{-bc+ad}(de-cf)^4} + \frac{f(24b^2d^2e^4-4abef(15d^2e^2-4cdef+c^2f^2)+a^2f^2(35d^2e^2-14cdef+3c^2f^2))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{5/2}(-be+af)^{3/2}(de-cf)^4}\right)$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((-4*d^3)/(c*(-(d*e) + c*f)^3*(c + d*x^2)) + (2*f^2)/(e
*(d*e - c*f)^2*(e + f*x^2)^2) + (f^2*(2*b*e*(5*d*e - c*f) + a*f*(-11*d*e +
 3*c*f)))/(e^2*(b*e - a*f)*(d*e - c*f)^3*(e + f*x^2))) + (4*d^2*(6*b*c^2*f
 + a*d*(d*e - 7*c*f))*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(c^(3/2)*Sqrt[-(b*c) + a*d]*(d*e - c*f)^4) + (f*(24*b^2*d^2*e^4 - 4*
a*b*e*f*(15*d^2*e^2 - 4*c*d*e*f + c^2*f^2) + a^2*f^2*(35*d^2*e^2 - 14*c*d*
e*f + 3*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
)/(e^(5/2)*(-(b*e) + a*f)^(3/2)*(d*e - c*f)^4))/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 421$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int \frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx}{4e(}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int \frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}-\frac{fx\sqrt{}}{4e(e+}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int \frac{}{\sqrt{bx^2+a}}\cdots}{2e(be-af)}}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 291

$$b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int \frac{1}{e-\frac{(be-\ldots}{bx^2\ldots}}}{2e(be-af)}}{4e(be-af)}\ldots\right)}{(de-cf)^2}\ldots\right)$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 221

$$b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8\ldots\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\ldots\right)}{(de-cf)^2}\ldots\right)$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 407

$$b\left(\frac{d^2\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3\ldots\right.}{2e^{3/2}(be\ldots}}{4e(be-af)}\ldots\right)}{(de-cf)^2}\ldots\right)$$

$$\overline{\qquad d \qquad}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2 \cdots\right.}{\cdots}}{4e(be-af)}\cdots}{(de-cf \cdots}\right.}{d}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2 \cdots\right.}{2e \cdots}}{4e(be-af)}\cdots}{(de-cf)^2 \cdots}\right.}{d}$$

$$\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 426

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2 \cdots\right.}{2e \cdots}}{4e(be-af)}\cdots}{(de-cf)^2 \cdots}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow 421$$

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2e}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}$$

$$\Big\downarrow 402$$

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2e}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx}{4e(\cdots}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\cdots\right.}{2e\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2\cdots}\right.$$

$$\overline{\qquad d\qquad}$$

$$(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\cdots}{4e\left(\cdots\right.}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.$$

$$\overline{\qquad d\qquad}$$

$$\Big\downarrow \text{402}$$

$$b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e(e+fx^2)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(a^2f^2}{2e}}{4e(be-af)}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad\qquad d \qquad\qquad}$$

$$(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int-\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf}{\sqrt{bx^2+a}(fx^2+e)}}{2e(be-af)}}{4e(be-a}\right.}{}\right.}{de-cf}\right.$$

$$\overline{\qquad\qquad d \qquad\qquad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2e}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int\frac{8b^2(2de}{}}{}}{4e(be-af}\right.}{}\right.}{de-cf}\right.}{d}$$

$\big\downarrow$ 27

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\cdots\right.}{2e\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2\cdots}\cdots\right.}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7d\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{de-cf\cdots}\cdots\right.}{d\cdots}$$

$\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\cdots\right.}{2e\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2\cdots}\cdots\right.$$

$$\overline{\qquad d\qquad}$$

$$(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7d\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{de-\cdots}\cdots\right.$$

$$\overline{\qquad d\qquad}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2 \cdots\right.}{2e \cdots}}{4e(be-af)}\cdots\right)}{(de-cf)^2 \cdots}\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctan}\cdots}{\cdots}}{\cdots}\cdots\right)}{\cdots}\right)}{de-\cdots}\cdots\right)}{d}$$

$\downarrow$ 407

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)(fx^2+e)}-\frac{(8b^2(2de}{}}{}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{f(2be(5}{}\right.}{}\right.}{}\right.}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\cdots}{\cdots}}{\cdots}\right)}{(de-cf)^2\cdots}\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\right.}{}\right.}{}\right.}{d}$$

$\downarrow$ 426

$$\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be}}{\sqrt{e}\sqrt{}}\right.}{\sqrt{e}\sqrt{be-af}(de-}\right.}{(de-cf)^2}\right.}{}\right.}{}$$

$\downarrow$ 421

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de \cdots\right.}{\cdots}\right.}{(de-cf)^2 \cdots}\right.}{d \cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}}{}\right.}{(de-cf)^2}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d}{-}\right.}{}\right.}{}\right.$$

$$\Big\downarrow\ 291$$

$$b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\right.$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de \cdots\right.}{\cdots}\right.}{(de-cf)^2 \cdots}\right.}{d \cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f \cdots}{\cdots}\right.}{\cdots}$$

↓ 402

$$b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}\right.}{(de-cf)^2}\right.$$

$$\frac{d}{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(\frac{\int-\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\right.}{}\right.}{}\right)}{de-cf}\right.}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{3\left(\left(8b^2d^2e^4-20ab\,d^2e^3f+\frac{35a\left(ad+\frac{16bc}{35}\right)d\,f^2e^2}{3}-\frac{14a\left(ad+\frac{2bc}{7}\right)c\,f^3e}{3}+a^2c^2f^4\right)\sqrt{(ad-bc)c}\,c\left(x^2d+c\right)\left(f\,x^2+e\right)^2f\arctan\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)*((8*b^2*d^2*e^4-20*a*b*d^2*e^3*f+35/3*a*(a*d+16/3
5*b*c)*d*f^2*e^2-14/3*a*(a*d+2/7*b*c)*c*f^3*e+a^2*c^2*f^4)*((a*d-b*c)*c)^(
1/2)*c*(d*x^2+c)*(f*x^2+e)^2*f*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1
/2))-28/3*((-1/7*a*d^2*e+c*f*(a*d-6/7*b*c))*d^2*(d*x^2+c)*(a*f-b*e)*(f*x^2
+e)^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+5/28*((a*d-b*c)*
c)^(1/2)*(4/5*b*d^3*e^5-4/5*d^3*f*(-2*b*x^2+a)*e^4-8/5*d*(-3/2*b*c^2-3/2*b
*c*d*x^2+d^2*x^2*(-1/2*b*x^2+a))*f^2*e^3-13/5*(4/13*c^3*b+d*(-6/13*b*x^2+a
)*c^2+d^2*x^2*(-10/13*b*x^2+a)*c+4/13*a*d^3*x^4)*f^3*e^2+(c*(-2/5*b*x^2+a)
-11/5*a*d*x^2)*c*(d*x^2+c)*f^4*e+3/5*a*c^2*f^5*x^2*(d*x^2+c))*(c*f-d*e)*(b
*x^2+a)^(1/2)*x)*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)/(d*x^2+c)/(c*f-d
*e)^4/c/(f*x^2+e)^2/(a*f-b*e)/e^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2 (e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^2*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(342) = 684$.

Time = 3.98 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.88

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/2*(a*sqrt(b)*d^4*e + 6*b^(3/2)*c^2*d^2*f - 7*a*sqrt(b)*c*d^3*f)*arctan(
1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*
c*d))/((c*d^4*e^4 - 4*c^2*d^3*e^3*f + 6*c^3*d^2*e^2*f^2 - 4*c^4*d*e*f^3 +
c^5*f^4)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/8*(24*b^(5/2)*d^2*e^4*f - 60*a*b^(3
/2)*d^2*e^3*f^2 + 16*a*b^(3/2)*c*d*e^2*f^3 + 35*a^2*sqrt(b)*d^2*e^2*f^3 -
4*a*b^(3/2)*c^2*e*f^4 - 14*a^2*sqrt(b)*c*d*e*f^4 + 3*a^2*sqrt(b)*c^2*f^5)*
arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2
 + a*b*e*f))/((b*d^4*e^7 - 4*b*c*d^3*e^6*f - a*d^4*e^6*f + 6*b*c^2*d^2*e^5
*f^2 + 4*a*c*d^3*e^5*f^2 - 4*b*c^3*d*e^4*f^3 - 6*a*c^2*d^2*e^4*f^3 + b*c^4
*e^3*f^4 + 4*a*c^3*d*e^3*f^4 - a*c^4*e^2*f^5)*sqrt(-b^2*e^2 + a*b*e*f)) +
(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((c*d^3*e^3 - 3*c^2*d^2*e^2*f +
3*c^3*d*e*f^2 - c^4*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)
) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d*e^3*f^2 - 28*(sqrt(b
)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*f^3 + 4*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a*b^(3/2)*c*e*f^4 + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*
d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*f^5 + 80*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*b^(7/2)*c*e^3*f^2 - 168*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^2\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^2*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^2\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

3.287 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 308

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx &= \frac{dx\sqrt{a+bx^2}}{4c(de-cf)(c+dx^2)^2} - \frac{d(ad(3de-7cf)-2bc(de-3cf))x\sqrt{a+bx^2}}{8c^2(bc-ad)(de-cf)^2(c+dx^2)} \\ &\quad - \frac{(8b^2c^4f^2-4abcd(d^2e^2-3cdef+6c^2f^2)+a^2d^2(3d^2e^2-10cdef+15c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}(de-cf)^3} \\ &\quad + \frac{f^2\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(de-cf)^3}\end{aligned}$$

output
```
1/4*d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^2-1/8*d*(a*d*(-7*c*f+3*d*e)
-2*b*c*(-3*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(-c*f+d*e)^2/(d*x^2+
c)-1/8*(8*b^2*c^4*f^2-4*a*b*c*d*(6*c^2*f^2-3*c*d*e*f+d^2*e^2)+a^2*d^2*(15*
c^2*f^2-10*c*d*e*f+3*d^2*e^2))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a
)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^3+f^2*(-a*f+b*e)^(1/2)*arctan
h((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 11.04 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx$$
$$= \frac{1}{8}\left(\frac{dx\sqrt{a+bx^2}\left(-2c(-de+cf)-\frac{(ad(3de-7cf)+2bc(-de+3cf))(c+dx^2)}{bc-ad}\right)}{c^2(de-cf)^2(c+dx^2)^2} - \frac{(8b^2c^4f^2-4abcd(d^2e^2-3cdef+6c^2f^2)+a^2d^2(3d^2e^2-10cdef+15c^2f^2))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(-bc+ad)^{3/2}(-de+cf)^3} - \frac{8f^2\sqrt{-be+af}\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(de-cf)^3}\right)$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)),x]
```

output
```
((d*x*Sqrt[a + b*x^2]*(-2*c*(-(d*e) + c*f) - ((a*d*(3*d*e - 7*c*f) + 2*b*c
*(-(d*e) + 3*c*f))*(c + d*x^2))/(b*c - a*d)))/(c^2*(d*e - c*f)^2*(c + d*x^
2)^2) - ((8*b^2*c^4*f^2 - 4*a*b*c*d*(d^2*e^2 - 3*c*d*e*f + 6*c^2*f^2) + a^
2*d^2*(3*d^2*e^2 - 10*c*d*e*f + 15*c^2*f^2))*ArcTan[(Sqrt[-(b*c) + a*d]*x)
/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5/2)*(-(b*c) + a*d)^(3/2)*(-(d*e) + c*f)^
3) - (8*f^2*Sqrt[-(b*e) + a*f]*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt
[a + b*x^2])])/(Sqrt[e]*(d*e - c*f)^3))/8
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {421, 25, 401, 25, 27, 402, 25, 27, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{(dx^2+c)^3}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\int \frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{(dx^2+c)^3}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}-\frac{\int -\frac{d\left(2b(de-3cf)x^2+a(3de-7cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4cd}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{d\left(2b(de-3cf)x^2+a(3de-7cf)\right)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int \frac{2b(de-3cf)x^2+a(3de-7cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+$$

$$\frac{d\left(\frac{\frac{\int -\frac{a(ad(3de-7cf)-4bc(de-2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{-\frac{\int \frac{a(ad(3de-7cf)-4bc(de-2cf))}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a(ad(3de-7cf)-4bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 291$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a(ad(3de-7cf)-4bc(de-2cf))\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 422$$

$$\frac{f^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{f^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{f^2\left(\frac{d\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{f^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 291

$$\frac{f^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}+\frac{d\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(3de-7cf)-4bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{x\sqrt{a+bx^2}(ad(3de-7cf)-2bc(de-3cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c}+\frac{x\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2}\right)}{(de-cf)^2}+\frac{f^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)),x]
```

output
```
(d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*c*(c + d*x^2)^2) + (-1/2*((a*d*(3*d
*e - 7*c*f) - 2*b*c*(d*e - 3*c*f))*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c +
d*x^2)) - (a*(a*d*(3*d*e - 7*c*f) - 4*b*c*(d*e - 2*c*f))*ArcTanh[(Sqrt[b*c
 - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(3/2)))/(4*c
)))/(d*e - c*f)^2 + (f^2*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/d - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(Sqrt[c]*d)))/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/f - (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(Sqrt[e]*f)))/(d*e - c*f)))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{-\frac{15\left(\frac{8b^2c^4f^2}{15}-\frac{8ab\,c^3d\,f^2}{5}+a\,d^2f\left(af+\frac{4be}{5}\right)c^2-\frac{2a\,d^3\left(af+\frac{2be}{5}\right)ec}{3}+\frac{a^2e^2d^4}{5}\right)\left(x^2d+c\right)^2\sqrt{(af-be)e}\,\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{8}+\left(\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-(-15/8*(8/15*b^2*c^4*f^2-8/5*a*b*c^3*d*f^2+a*d^2*f*(a*f+4/5*b*e)*c^2-2/3*
a*d^3*(a*f+2/5*b*e)*e*c+1/5*a^2*e^2*d^4)*(d*x^2+c)^2*((a*f-b*e)*e)^(1/2)*a
rctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((d*x^2+c)^2*(a*d-b*c)*(a*f
-b*e)*c^2*f^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+9/8*(-8/9*b*
c^3*f+(4/9*b*e+f*(-2/3*b*x^2+a))*d*c^2-5/9*d^2*((-2/5*b*x^2+a)*e-7/5*a*f*x
^2)*c-1/3*a*d^3*e*x^2)*d*(c*f-d*e)*((a*f-b*e)*e)^(1/2)*(b*x^2+a)^(1/2)*x)*
((a*d-b*c)*c)^(1/2))/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(1/2)/(d*x^2+c)^2/(
a*d-b*c)/(c*f-d*e)^3/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^3(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^3*(f*x^2 + e)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(279) = 558$.

Time = 2.06 (sec) , antiderivative size = 1174, normalized size of antiderivative = 3.81

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
-1/8*(4*a*b^(3/2)*c*d^3*e^2 - 3*a^2*sqrt(b)*d^4*e^2 - 12*a*b^(3/2)*c^2*d^2
*e*f + 10*a^2*sqrt(b)*c*d^3*e*f - 8*b^(5/2)*c^4*f^2 + 24*a*b^(3/2)*c^3*d*f
^2 - 15*a^2*sqrt(b)*c^2*d^2*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^3*d^3*e^3 - a*c^2*d^4*
e^3 - 3*b*c^4*d^2*e^2*f + 3*a*c^3*d^3*e^2*f + 3*b*c^5*d*e*f^2 - 3*a*c^4*d^
2*e*f^2 - b*c^6*f^3 + a*c^5*d*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (b^(3/2)*e*
f^2 - a*sqrt(b)*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e
 - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^
2 - c^3*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*a*b^(3/2)*c*d^3*e - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^
4*e + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^3*d*f - 16*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*a*b^(3/2)*c^2*d^2*f + 7*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^2*sqrt(b)*c*d^3*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3*d*e
+ 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2*e - 30*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3*e + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^3*sqrt(b)*d^4*e + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^4*f - 1
04*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^3*d*f + 74*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(3/2)*c^2*d^2*f - 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*sqrt(b)*c*d^3*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*
d^2*e + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3*e - 9*(sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^3\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 7591, normalized size of antiderivative = 24.65

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e),x)
```

output
```
(30*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**
2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*c**4*d**3*e*f**2 - 20*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqr
t(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*c**3*d**4*e**2*f + 60*sqrt(c)*sqrt
(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqr
t(b)*x)/(sqrt(c)*sqrt(b)))*a**3*c**3*d**4*e*f**2*x**2 + 6*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a**3*c**2*d**5*e**3 - 40*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(
c)*sqrt(b)))*a**3*c**2*d**5*e**2*f*x**2 + 30*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*
sqrt(b)))*a**3*c**2*d**5*e*f**2*x**4 + 12*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqr
t(b)))*a**3*c*d**6*e**3*x**2 - 20*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a
**3*c*d**6*e**2*f*x**4 + 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) -
 sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**
7*e**3*x**4 - 108*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*
sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*b*c**5*d**2*
e*f**2 + 64*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqr...
```

3.288 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 378

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^2}\,dx = \frac{d^2x\sqrt{a+bx^2}}{4c(de-cf)^2(c+dx^2)^2} - \frac{d^2(ad(3de-11cf)-2bc(de-5cf))x\sqrt{a+bx^2}}{8c^2(bc-ad)(de-cf)^3(c+dx^2)} - \frac{f^3x\sqrt{a+bx^2}}{2e(de-cf)^3(e+fx^2)} - \frac{d(24b^2c^4f^2-4abcd(d^2e^2-4cdef+15c^2f^2)+a^2d^2(3d^2e^2-14cdef+35c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}(de-cf)^4} + \frac{f^2(6bde^2-af(7de-cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}\sqrt{be-af}(de-cf)^4}$$

output

```
1/4*d^2*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^2/(d*x^2+c)^2-1/8*d^2*(a*d*(-11*c*f
+3*d*e)-2*b*c*(-5*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(-c*f+d*e)^3/
(d*x^2+c)-1/2*f^3*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)^3/(f*x^2+e)-1/8*d*(24*b^2
*c^4*f^2-4*a*b*c*d*(15*c^2*f^2-4*c*d*e*f+d^2*e^2)+a^2*d^2*(35*c^2*f^2-14*c
*d*e*f+3*d^2*e^2))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(
5/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^4+1/2*f^2*(6*b*d*e^2-a*f*(-c*f+7*d*e))*ar
ctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(1/2)
/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 11.69 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3 (e+fx^2)^2} \, dx = \frac{1}{8}\left(x\sqrt{a+bx^2}\left(\frac{2d^2}{c(de-cf)^2 (c+dx^2)^2} + \frac{d^2(ad(3de-11cf)+2bc(-de+5cf))}{c^2(bc-ad)(-de+cf)^3 (c+dx^2)} - \frac{4f^3}{e(de-cf)^3 (e+fx^2)}\right) + \frac{d(24b^2c^4f^2-4abcd(d^2e^2-4cdef+15c^2f^2)+a^2d^2(3d^2e^2-14cdef+35c^2f^2)) \arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(-bc+ad)^{3/2}(de-cf)^4} + \frac{4f^2(6bde^2+af(-7de+cf)) \arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{3/2}\sqrt{-be+af}(de-cf)^4}\right)$$

input
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Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)^2),x]
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output
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(x*Sqrt[a + b*x^2]*((2*d^2)/(c*(d*e - c*f)^2*(c + d*x^2)^2) + (d^2*(a*d*(3
*d*e - 11*c*f) + 2*b*c*(-(d*e) + 5*c*f)))/(c^2*(b*c - a*d)*(-(d*e) + c*f)^
3*(c + d*x^2)) - (4*f^3)/(e*(d*e - c*f)^3*(e + f*x^2))) + (d*(24*b^2*c^4*f
^2 - 4*a*b*c*d*(d^2*e^2 - 4*c*d*e*f + 15*c^2*f^2) + a^2*d^2*(3*d^2*e^2 - 1
4*c*d*e*f + 35*c^2*f^2))*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b
*x^2])])/(c^(5/2)*(-(b*c) + a*d)^(3/2)*(d*e - c*f)^4) + (4*f^2*(6*b*d*e^2
+ a*f*(-7*d*e + c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^
2])])/(e^(3/2)*Sqrt[-(b*e) + a*f]*(d*e - c*f)^4))/8
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Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1281 vs. 2(378) = 756.

Time = 1.97 (sec) , antiderivative size = 1281, normalized size of antiderivative = 3.39, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.967$, Rules used = {425, 426, 421, 25, 291, 221, 402, 25, 27, 291, 221, 402, 25, 27, 291, 221, 407, 291, 221, 426, 421, 25, 291, 221, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{b\left(\frac{d\left(\frac{f^2\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{d}$$

$\big\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\int -\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}}{2\cdots}\cdots\right)}{\cdots}\right)}{de-c\cdots}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}\cdots}{\cdots}\right)}{d}$$

$\big\downarrow$ 25

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\int \frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\int \frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}\cdots\right)}{2\cdots}\cdots\right)}{de-c\cdots}\cdots\right)}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int \frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right)}\cdots}{\cdots}\cdots\right)}{d}\cdots$$

$$\downarrow 27$$

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\cdots\right.}{\cdots}\right.}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int \frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\right.}{d\cdots}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-\cdots}}{\sqrt{c}\sqrt{a-\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf\cdots}\right.}{\cdots}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int \frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\right.}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-}}{\sqrt{c}\sqrt{a+}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{}{\sqrt{bx^2+a}\left(d\right.}}{}\right.}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c(c+dx^2)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-}}{\sqrt{c}\sqrt{a+}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{\frac{\int-\frac{8b^2(de-2cf)c^2-4abd(2de-5cf)c+a^2d^2(3de-7cf)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c(c+dx^2)^2(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\int}{}\right.}{de-cf}\right.}{d}$$

↓ 25

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-}}{\sqrt{c}\sqrt{a+}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{-\frac{\int\frac{8b^2(de-2cf)c^2-4abd(2de-5cf)c+a^2d^2(3de-7cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\int}{}\right.}{de-cf}\right.}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{d\left(\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-}}{\sqrt{c}\sqrt{a+}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(-\frac{-\frac{\left(a^2d^2(3de-7cf)-4abcd(2de-5cf)+8b^2c^2(de-2cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+b}}{4c\left(c+dx^2\right.}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2-4abd(2de-\right.}{}}{4c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}$$

↓ 221

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2-4abd(2de-\right.}{}}{4c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}$$

↓ 407

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)(dx^2+c)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)(dx^2+c)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+ax}}{2c(bc-ad)(dx^2+c)}-\frac{(8b}{}}{}\right.}{(de-}\right)}{de-cf}\right.}{d}$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a\cdots}}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right.}{\cdots}\cdots\right.}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+\cdots}}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\right.}{\cdots}\right)}{de-cf}\cdots\right.}{d\cdots}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.}{d}$$

$\downarrow$ 426

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right.}{\cdots}\cdots\right)}{d\cdots}$$

$$\frac{\cdots}{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\right)}{\cdots}\right)}{de-cf}\cdots\right)}$$

↓ 421

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.$$

↓ 25

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}\,(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2}\cdots}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)\cdots}\cdots\right.}{\cdots}\cdots}{d\cdots}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{de-cf}\cdots\right.$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$\frac{}{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+ax}}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.}$$

↓ 221

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.$$

↓ 402

$$b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)(dx^2+c)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.$$

$$\overline{\qquad d \qquad}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)(dx^2+c)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}}{2c(bc-ad)(dx^2+c)}}{}\right.}{}\right)}{de-cf}\right.$$

↓ 25

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a\cdots}}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right.}{\cdots}\cdots}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a\cdots}}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\right.}{\cdots}\right)}{de-cf\cdots}\cdots\right.}{\cdots}$$

↓ 27

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2}\cdots}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\right.}{\cdots}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\right.}{\cdots}\right)}{de-cf}\cdots\right.$$

↓ 221

$$\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right.}{d}$$

$$(bc-ad)\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}}{}\right.}{}\right)}{de-cf}\right.$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)^2),x]
```

output
```
(b*((d*((d*(-1/2*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x
^2)) - ((a*d*(d*e - 3*c*f) - 2*b*c*(d*e - 2*c*f))*ArcTanh[(Sqrt[b*c - a*d]
*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(3/2))))/(d*e - c*f
)^2 + (f^2*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e
]*Sqrt[b*e - a*f]*(d*e - c*f)^2)))/(d*e - c*f) - (f*((d^2*ArcTanh[(Sqrt[b*
c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*
f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x
^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]
*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(d*e - c*f
)^2))/(d*e - c*f)))/d - ((b*c - a*d)*((d*((d*(-1/4*(d*(d*e - c*f)*x*Sqrt[a
 + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^2) - (-1/2*(d*(a*d*(3*d*e - 7*c*f) -
 2*b*c*(3*d*e - 5*c*f))*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) - (
(a^2*d^2*(3*d*e - 7*c*f) - 4*a*b*c*d*(2*d*e - 5*c*f) + 8*b^2*c^2*(d*e - 2*
c*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(
b*c - a*d)^(3/2)))/(4*c*(b*c - a*d))))/(d*e - c*f)^2 + (f^2*((d*ArcTanh[(S
qrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*
e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sq
rt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(d*e - c*f)^2))/(d*e - c*f) - (f*((d*
((d*(-1/2*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) -
((a*d*(d*e - 3*c*f) - 2*b*c*(d*e - 2*c*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{\frac{35d\left(x^2d+c\right)^2\left(\frac{24b^2c^4f^2}{35}-\frac{12ab\,c^3d\,f^2}{7}+a\,d^2f\left(af+\frac{16be}{35}\right)c^2-\frac{2a\,d^3\left(af+\frac{2be}{7}\right)ec}{5}+\frac{3a^2e^2d^4}{35}\right)\sqrt{(af-be)e}\,\left(f\,x^2+e\right)e\arctan\left(\frac{c\sqrt{b}}{x\sqrt{(a}}\right.}{4}}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*c)^(1/2)*(35/4*d*(d*x^2+c)^2*(24/35*b^2*c^4*f^2-12/7*a*b*c
^3*d*f^2+a*d^2*f*(a*f+16/35*b*e)*c^2-2/5*a*d^3*(a*f+2/7*b*e)*e*c+3/35*a^2*
e^2*d^4)*((a*f-b*e)*e)^(1/2)*(f*x^2+e)*e*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-
b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*((a*d-b*c)*c^2*(a*c*f^2+(-7*a*e*f+6*b*e
^2)*d)*(d*x^2+c)^2*(f*x^2+e)*f^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^
(1/2))-(-b*c^5*f^3+d*f^3*(-2*b*x^2+a)*c^4+2*d^2*(-3/2*b*e^2-3/2*b*e*f*x^2+
f^2*x^2*(-1/2*b*x^2+a))*f*c^3+13/4*d^3*(4/13*b*e^3+f*(-6/13*b*x^2+a)*e^2+f
^2*x^2*(-10/13*b*x^2+a)*e+4/13*a*f^3*x^4)*c^2-5/4*d^4*((-2/5*b*x^2+a)*e-11
/5*a*f*x^2)*(f*x^2+e)*e*c-3/4*a*d^5*e^2*x^2*(f*x^2+e))*(c*f-d*e)*((a*f-b*e
)*e)^(1/2)*(b*x^2+a)^(1/2)*x))/((a*f-b*e)*e)^(1/2)/(d*x^2+c)^2/(a*d-b*c)/(
c*f-d*e)^4/c^2/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^3\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^3*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(343) = 686$.

Time = 3.85 (sec) , antiderivative size = 1462, normalized size of antiderivative = 3.87

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/8*(4*a*b^(3/2)*c*d^4*e^2 - 3*a^2*sqrt(b)*d^5*e^2 - 16*a*b^(3/2)*c^2*d^3
*e*f + 14*a^2*sqrt(b)*c*d^4*e*f - 24*b^(5/2)*c^4*d*f^2 + 60*a*b^(3/2)*c^3*
d^2*f^2 - 35*a^2*sqrt(b)*c^2*d^3*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^3*d^4*e^4 - a*c^2
*d^5*e^4 - 4*b*c^4*d^3*e^3*f + 4*a*c^3*d^4*e^3*f + 6*b*c^5*d^2*e^2*f^2 - 6
*a*c^4*d^3*e^2*f^2 - 4*b*c^6*d*e*f^3 + 4*a*c^5*d^2*e*f^3 + b*c^7*f^4 - a*c
^6*d*f^4)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/2*(6*b^(3/2)*d*e^2*f^2 - 7*a*sqrt(
b)*d*e*f^3 + a*sqrt(b)*c*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*
f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((d^4*e^5 - 4*c*d^3*e^4*f + 6*c
^2*d^2*e^3*f^2 - 4*c^3*d*e^2*f^3 + c^4*e*f^4)*sqrt(-b^2*e^2 + a*b*e*f)) -
(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*e*f^2 - (sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*f^3 + a^2*sqrt(b)*f^3)/((d^3*e^4 - 3*c*d^2*e^3*f + 3*c^
2*d*e^2*f^2 - c^3*e*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f)
) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4*e - 3*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^5*e + 16*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*b^(5/2)*c^3*d^2*f - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c^2*d
^3*f + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*d^4*f - 16*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3*d^2*e + 40*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a*b^(5/2)*c^2*d^3*e - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^3\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3 (e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^3 (f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^2,x)
```

3.289 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 507

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^3}\,dx \\
&= \frac{d^3x\sqrt{a+bx^2}}{4c(de-cf)^3(c+dx^2)^2} + \frac{d^3(2bc(de-7cf)-3ad(de-5cf))x\sqrt{a+bx^2}}{8c^2(bc-ad)(de-cf)^4(c+dx^2)} \\
&\quad - \frac{f^3x\sqrt{a+bx^2}}{4e(de-cf)^3(e+fx^2)^2} + \frac{f^3(3af(5de-cf)-2be(7de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)(de-cf)^4(e+fx^2)} \\
&\quad - \frac{d^2(48b^2c^4f^2+3a^2d^2(d^2e^2-6cdef+21c^2f^2)-4abcd(d^2e^2-5cdef+28c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}(de-cf)^5} \\
&\quad + \frac{f^2(48b^2d^2e^4+3a^2f^2(21d^2e^2-6cdef+c^2f^2)-4abef(28d^2e^2-5cdef+c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{3/2}(de-cf)^5}
\end{aligned}$$

output
```
1/4*d^3*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^3/(d*x^2+c)^2+1/8*d^3*(2*b*c*(-7*c*
f+d*e)-3*a*d*(-5*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(-c*f+d*e)^4/(
d*x^2+c)-1/4*f^3*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)^3/(f*x^2+e)^2+1/8*f^3*(3*a
*f*(-c*f+5*d*e)-2*b*e*(-c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)/(-c*f
+d*e)^4/(f*x^2+e)-1/8*d^2*(48*b^2*c^4*f^2+3*a^2*d^2*(21*c^2*f^2-6*c*d*e*f+
d^2*e^2)-4*a*b*c*d*(28*c^2*f^2-5*c*d*e*f+d^2*e^2))*arctanh((-a*d+b*c)^(1/2
)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^5+1/8*f^2
*(48*b^2*d^2*e^4+3*a^2*f^2*(c^2*f^2-6*c*d*e*f+21*d^2*e^2)-4*a*b*e*f*(c^2*f
^2-5*c*d*e*f+28*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/
2))/e^(5/2)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 13.28 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx$$
$$= \frac{1}{8}\Bigg(x\sqrt{a+bx^2}\Bigg(-\frac{2d^3}{c(-de+cf)^3\,(c+dx^2)^2}-\frac{d^3(3ad(de-5cf)+2bc(-de+7cf))}{c^2(bc-ad)(de-cf)^4\,(c+dx^2)} - \frac{2f^3}{e(de-cf)^3\,(e+fx^2)^2}-\frac{f^3(2be(7de-cf)+3af(-5de+cf))}{e^2(be-af)(de-cf)^4\,(e+fx^2)}\Bigg) - \frac{d^2(48b^2c^4f^2+3a^2d^2(d^2e^2-6cdef+21c^2f^2)-4abcd(d^2e^2-5cdef+28c^2f^2))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(-bc+ad)^{3/2}(-de+cf)^5} - \frac{f^2(48b^2d^2e^4+3a^2f^2(21d^2e^2-6cdef+c^2f^2)-4abef(28d^2e^2-5cdef+c^2f^2))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{5/2}(-be+af)^{3/2}(de-cf)^5}\Bigg)$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((-2*d^3)/(c*(-(d*e) + c*f)^3*(c + d*x^2)^2) - (d^3*(3*
a*d*(d*e - 5*c*f) + 2*b*c*(-(d*e) + 7*c*f)))/(c^2*(b*c - a*d)*(d*e - c*f)^
4*(c + d*x^2)) - (2*f^3)/(e*(d*e - c*f)^3*(e + f*x^2)^2) - (f^3*(2*b*e*(7*
d*e - c*f) + 3*a*f*(-5*d*e + c*f)))/(e^2*(b*e - a*f)*(d*e - c*f)^4*(e + f*
x^2))) - (d^2*(48*b^2*c^4*f^2 + 3*a^2*d^2*(d^2*e^2 - 6*c*d*e*f + 21*c^2*f^
2) - 4*a*b*c*d*(d^2*e^2 - 5*c*d*e*f + 28*c^2*f^2))*ArcTan[(Sqrt[-(b*c) + a
*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(5/2)*(-(b*c) + a*d)^(3/2)*(-(d*e) +
 c*f)^5) - (f^2*(48*b^2*d^2*e^4 + 3*a^2*f^2*(21*d^2*e^2 - 6*c*d*e*f + c^2*
f^2) - 4*a*b*e*f*(28*d^2*e^2 - 5*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[-(b*e) +
 a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*(-(b*e) + a*f)^(3/2)*(d*e -
c*f)^5))/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 426}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de\cdots}{4e\left(e+fx^2\right)^2(be\cdots}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}-\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int \frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de\cdots}{4e\left(e+fx^2\right)^2(be\cdots}\right.}{(de-cf)^2}\cdots}{de-cf}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a\cdots}}{\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots}{de-cf}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int \frac{8b^2(2de-cf)e^2-4ab \ldots}{\sqrt{b \ldots}}}{\ldots}}{4e(be-af)}\ldots\right)}{(de-cf)^2}\ldots\right)}{de-cf}\ldots\right.$$

$$\frac{\qquad}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4ab\cdots\right.}{\cdots}}{4e(be-af)\cdots}\cdots\right)}{\cdots}\cdots\right)}{de-cf\cdots}\cdots\right)}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4ab\right.}{}}{4e(be-a}\right.}{}\right.}{de-cf}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-a}}{\sqrt{e}\sqrt{a+b}}\right.}{}}{4e(be-af}\right.}{}\right.}{de-cf}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\downarrow 407$$

$$
b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{\cdots}\right.}{\cdots}\right.}{de-\cdots}\right.
$$

$$
\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}
$$

$$\downarrow 291$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7d}{2e\left(e+fx^2\right)(be-af)}\right.}{}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-}{2e\left(e+fx^2\right)(be-af)}\right.}{}\right)}{de}\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\cdots\right)}{\cdots}\right)}{(bc-ad)\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(\cdots\right.}}{\cdots}\right)}{de-cf}\right)}{d}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\text{arctanh}\ldots}{\sqrt{c}\sqrt{bc\ldots}}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{\ldots}{\sqrt{bx^2+a}\left(dx^2\ldots\right.}}{de-\ldots}\right.}{\ldots}\right.}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\cdots\right.}{\sqrt{c}\sqrt{bc-\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\cdots\right.}\cdots}{de-c\cdots}\cdots\right.}{\cdots}\cdots\right)}$$

$$d$$

$$\downarrow \; 291$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}f^2}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\text{ar}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\cdots\right)}}{de-c\cdots}\cdots\right)}{\cdots}\right)}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arc}\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\cdots\right.}}{de-c\cdots}\cdots\right.}{\cdots}\right.}{d\cdots}$$

↓ 402

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(\frac{\int -\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\int \frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}\cdots\right)}{2c\cdots}\right)}{de-c\cdots}\right)}{de-cf}\cdots\right)}{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{4c(bc-ad)\left(dx^2+c\right)^2}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right)}}{\cdots}\right)}{de-cf}\cdots\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\int\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\int\frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}\cdots\right)}{2c\cdots}\cdots\right)}{de-c\cdots}\right)}{de-cf}\cdots\right)}{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\cdots\right)}{de-cf}\cdots\right)}$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}\cdots}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\cdots\right.}{\cdots}\right)}{de-cf}\cdots\right.$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\right)}{de-cf}\cdots\right.$$

↓ 291

$$
b\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-\cdots}}{\sqrt{c}\sqrt{bx\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf\cdots}\right.}{}}{de-cf}\right.}{}\right.
$$

$$
(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\right.}{de-cf}\right.
$$

↓ 221

$$
\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right.}{\cdots}\right)}{de-cf}\cdots\right)}{(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{\cdots}\cdots\right)}{de-cf}\cdots\right)}
$$

↓ 402

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a\cdots}}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right)}{\cdots}\right)}{de-cf}\cdots\right)$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{\frac{\int-\frac{8b^2(de-2cf)c^2-4abd(2de-5cf)c+a^2d^2(3de-7cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(ad(3de-\cdots}{\cdots}}{4c(bc-ad)}\cdots\right)}{(de-cf)^2}\right)}{de-cf}\cdots\right)}{de-cf}\cdots\right.$$

$$\Big\downarrow \text{25}$$

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right)}{de-cf}\right)$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\int\frac{8b^2(de-2cf)c^2-4abd(}{\sqrt{bx^2}}}{2c}}{4c(bc-ad)}\right.}{(de-cf)^2}\right)}{de-cf}\right.}{de-cf}\right.$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right)}{de-cf}\right.$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2-4abd(2\right.}{}}{4c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}\right.$$

$\downarrow$ 291

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}}{de-cf}\right.$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2-4abd(2\right.}{}}{4c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}}{de-cf}\right.$$

↓ 221

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx^2}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right)}{de-cf}\right.$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2-4abd(2\right.}{}}{4c(bc-ad)}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}\right.$$

↓ 407

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2}\cdots}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\cdots\right.}{\cdots}}{de-cf}\cdots\right.$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\cdots}{\cdots}\right.}{(\cdots}\right)}{de-cf}\cdots}{de-cf}\cdots\right.$$

↓ 291

$$b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-a}\cdots}{\sqrt{c}\sqrt{bx^2\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\cdots\right.}{\cdots}\right)}{de-cf}\cdots\right.}{\cdots}$$

$$(bc-ad)\left(\frac{d\left(\frac{d\left(\frac{\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx\cdots}}{2c(bc-ad)\left(dx^2+c\right)}\cdots}{\cdots}\right.}{\cdots}\right)}{de-cf}\cdots\right)}{de-cf}\cdots\right.$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.78

method	result
pseudoelliptic	$-\frac{3\left(-21\sqrt{(af-be)e}\,d^2(x^2d+c)^2\left(\frac{16b^2c^4f^2}{21}-\frac{16ab\,c^3d\,f^2}{9}+a\,d^2f\left(af+\frac{20be}{63}\right)c^2-\frac{2a\left(af+\frac{2be}{9}\right)d^3ec}{7}+\frac{a^2e^2d^4}{21}\right)(af-be)(f\,x\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(1/2)*(-21*((a*f-b*e)*e)^(1/2)*d^2*
(d*x^2+c)^2*(16/21*b^2*c^4*f^2-16/9*a*b*c^3*d*f^2+a*d^2*f*(a*f+20/63*b*e)*
c^2-2/7*a*(a*f+2/9*b*e)*d^3*e*c+1/21*a^2*e^2*d^4)*(a*f-b*e)*(f*x^2+e)^2*e^
2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*((a*
d-b*c)*c^2*(d*x^2+c)^2*(f*x^2+e)^2*f^2*((21*a^2*e^2*f^2-112/3*a*b*e^3*f+16
*b^2*e^4)*d^2-6*a*d*f^2*e*(a*f-10/9*b*e)*c+a*f^3*(a*f-4/3*b*e)*c^2)*arctan
(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-5/3*((a*f-b*e)*e)^(1/2)*(-(-4/5*
b*e^2+f*(-2/5*b*x^2+a)*e+3/5*a*f^2*x^2)*b*f^4*c^6+d*(-16/5*e^3*b^2+13/5*(-
6/13*b*x^2+a)*b*f*e^2+f^2*(4/5*b^2*x^4+3/5*a*b*x^2+a^2)*e+3/5*a*f^3*x^2*(-
2*b*x^2+a))*f^3*c^5-17/5*(-16/17*b*(-2*b*x^2+a)*e^3+f*(24/17*b^2*x^4-40/17
*a*b*x^2+a^2)*e^2+5/17*(-2/5*b^2*x^4-21/5*a*b*x^2+a^2)*x^2*f^2*e-6/17*a*(-
1/2*b*x^2+a)*x^4*f^3)*d^2*f^3*c^4-34/5*d^3*(8/17*b^2*e^5-8/17*b*f*(-2*b*x^
2+a)*e^4-32/17*(-1/2*b*x^2+a)*b*x^2*f^2*e^3+f^3*x^2*(7/17*b^2*x^4-57/34*a*
b*x^2+a^2)*e^2+25/34*a*x^4*f^4*(-13/25*b*x^2+a)*e-3/34*a^2*f^5*x^6)*f*c^3-
17/5*d^4*(-4/17*b^2*e^5-13/17*(-6/13*b*x^2+a)*b*f*e^4+f^2*(24/17*b^2*x^4-4
0/17*a*b*x^2+a^2)*e^3+2*f^3*x^2*(7/17*b^2*x^4-57/34*a*b*x^2+a^2)*e^2+2*a*(
-14/17*b*x^2+a)*x^4*f^4*e+15/17*a^2*f^5*x^6)*e*c^2+((-2/5*b*x^2+a)*e-3*a*f
*x^2)*d^5*(a*f-b*e)*(f*x^2+e)^2*e^2*c+3/5*a*d^6*e^3*x^2*(f*x^2+e)^2*(a*f-b
*e))*(c*f-d*e)*(b*x^2+a)^(1/2)*x))/(c*f-d*e)^5/(a*d-b*c)/c^2/(a*f-b*e)/e^2
/(d*x^2+c)^2/(f*x^2+e)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^3(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^3*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8143 vs. $2(467) = 934$.

Time = 8.58 (sec) , antiderivative size = 8143, normalized size of antiderivative = 16.06

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*(4*a*b^(3/2)*c*d^5*e^2 - 3*a^2*sqrt(b)*d^6*e^2 - 20*a*b^(3/2)*c^2*d^4
*e*f + 18*a^2*sqrt(b)*c*d^5*e*f - 48*b^(5/2)*c^4*d^2*f^2 + 112*a*b^(3/2)*c
^3*d^3*f^2 - 63*a^2*sqrt(b)*c^2*d^4*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^3*d^5*e^5 - a*
c^2*d^6*e^5 - 5*b*c^4*d^4*e^4*f + 5*a*c^3*d^5*e^4*f + 10*b*c^5*d^3*e^3*f^2
 - 10*a*c^4*d^4*e^3*f^2 - 10*b*c^6*d^2*e^2*f^3 + 10*a*c^5*d^3*e^2*f^3 + 5*
b*c^7*d*e*f^4 - 5*a*c^6*d^2*e*f^4 - b*c^8*f^5 + a*c^7*d*f^5)*sqrt(-b^2*c^2
 + a*b*c*d)) - 1/8*(48*b^(5/2)*d^2*e^4*f^2 - 112*a*b^(3/2)*d^2*e^3*f^3 + 2
0*a*b^(3/2)*c*d*e^2*f^4 + 63*a^2*sqrt(b)*d^2*e^2*f^4 - 4*a*b^(3/2)*c^2*e*f
^5 - 18*a^2*sqrt(b)*c*d*e*f^5 + 3*a^2*sqrt(b)*c^2*f^6)*arctan(1/2*((sqrt(b
)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*d^
5*e^8 - 5*b*c*d^4*e^7*f - a*d^5*e^7*f + 10*b*c^2*d^3*e^6*f^2 + 5*a*c*d^4*e
^6*f^2 - 10*b*c^3*d^2*e^5*f^3 - 10*a*c^2*d^3*e^5*f^3 + 5*b*c^4*d*e^4*f^4 +
 10*a*c^3*d^2*e^4*f^4 - b*c^5*e^3*f^5 - 5*a*c^4*d*e^3*f^5 + a*c^5*e^2*f^6)
*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a*b^(
5/2)*c*d^5*e^4*f^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(3/2)*d^6*e^
4*f^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(7/2)*c^3*d^3*e^3*f^3 - 64*(
sqrt(b)*x - sqrt(b*x^2 + a))^14*a*b^(5/2)*c^2*d^4*e^3*f^3 + 11*(sqrt(b)*x
- sqrt(b*x^2 + a))^14*a^2*b^(3/2)*c*d^5*e^3*f^3 + 3*(sqrt(b)*x - sqrt(b*x^
2 + a))^14*a^3*sqrt(b)*d^6*e^3*f^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^3\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^3*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^3\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.290 $\int (a+bx^2)^{3/2} (c+dx^2) (e+fx^2)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 482

$$\int (a+bx^2)^{3/2} (c+dx^2) (e+fx^2)^3 \, dx = \frac{a(384b^4ce^3+7a^4df^3+72a^2b^2ef(de+cf)-12a^3bf^2(3de+cf)-64ab^3e^2(de+3cf))\,x\sqrt{a+}}{1024b^4}$$
$$+\frac{(384b^4ce^3+7a^4df^3+72a^2b^2ef(de+cf)-12a^3bf^2(3de+cf)-64ab^3e^2(de+3cf))\,x(a+bx^2)^{3/2}}{1536b^4}$$
$$-\frac{(7a^3df^3+72ab^2ef(de+cf)-12a^2bf^2(3de+cf)-64b^3e^2(de+3cf))\,x(a+bx^2)^{5/2}}{384b^4}$$
$$+\frac{f(7a^2df^2+72b^2e(de+cf)-12abf(3de+cf))\,x^3(a+bx^2)^{5/2}}{192b^3}$$
$$-\frac{f^2(7adf-12b(3de+cf))x^5(a+bx^2)^{5/2}}{120b^2}+\frac{df^3x^7(a+bx^2)^{5/2}}{12b}$$
$$+\frac{a^2(384b^4ce^3+7a^4df^3+72a^2b^2ef(de+cf)-12a^3bf^2(3de+cf)-64ab^3e^2(de+3cf))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{9/2}}$$

output

```
1/1024*a*(384*b^4*c*e^3+7*a^4*d*f^3+72*a^2*b^2*e*f*(c*f+d*e)-12*a^3*b*f^2*
(c*f+3*d*e)-64*a*b^3*e^2*(3*c*f+d*e))*x*(b*x^2+a)^(1/2)/b^4+1/1536*(384*b^
4*c*e^3+7*a^4*d*f^3+72*a^2*b^2*e*f*(c*f+d*e)-12*a^3*b*f^2*(c*f+3*d*e)-64*a
*b^3*e^2*(3*c*f+d*e))*x*(b*x^2+a)^(3/2)/b^4-1/384*(7*a^3*d*f^3+72*a*b^2*e*
f*(c*f+d*e)-12*a^2*b*f^2*(c*f+3*d*e)-64*b^3*e^2*(3*c*f+d*e))*x*(b*x^2+a)^(
5/2)/b^4+1/192*f*(7*a^2*d*f^2+72*b^2*e*(c*f+d*e)-12*a*b*f*(c*f+3*d*e))*x^3
*(b*x^2+a)^(5/2)/b^3-1/120*f^2*(7*a*d*f-12*b*(c*f+3*d*e))*x^5*(b*x^2+a)^(5
/2)/b^2+1/12*d*f^3*x^7*(b*x^2+a)^(5/2)/b+1/1024*a^2*(384*b^4*c*e^3+7*a^4*d
*f^3+72*a^2*b^2*e*f*(c*f+d*e)-12*a^3*b*f^2*(c*f+3*d*e)-64*a*b^3*e^2*(3*c*f
+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.89

$$\int (a+bx^2)^{3/2} (c+dx^2) (e+fx^2)^3 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^5df^3+10a^4bf^2(54de+18cf+7dfx^2)-8a^3b^2f(15cf(9e+fx^2)+d(13\ldots}{\ldots}$$

input

```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output

```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^5*d*f^3 + 10*a^4*b*f^2*(54*d*e + 18*c*f
 + 7*d*f*x^2) - 8*a^3*b^2*f*(15*c*f*(9*e + f*x^2) + d*(135*e^2 + 45*e*f*x^
2 + 7*f^2*x^4)) + 48*a^2*b^3*(c*f*(60*e^2 + 15*e*f*x^2 + 2*f^2*x^4) + d*(2
0*e^3 + 15*e^2*f*x^2 + 6*e*f^2*x^4 + f^3*x^6)) + 128*b^5*x^2*(3*c*(10*e^3
+ 20*e^2*f*x^2 + 15*e*f^2*x^4 + 4*f^3*x^6) + d*x^2*(20*e^3 + 45*e^2*f*x^2
+ 36*e*f^2*x^4 + 10*f^3*x^6)) + 64*a*b^4*(3*c*(50*e^3 + 70*e^2*f*x^2 + 45*
e*f^2*x^4 + 11*f^3*x^6) + d*x^2*(70*e^3 + 135*e^2*f*x^2 + 99*e*f^2*x^4 + 2
6*f^3*x^6))) - 15*a^2*(384*b^4*c*e^3 + 7*a^4*d*f^3 + 72*a^2*b^2*e*f*(d*e +
 c*f) - 12*a^3*b*f^2*(3*d*e + c*f) - 64*a*b^3*e^2*(d*e + 3*c*f))*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(9/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {403, 403, 403, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(e+fx^2\right)^3 \, dx$$

$$\Big\downarrow \ 403$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left(fx^2+e\right)^2 \left((12bc-ad)e-(7adf-6b(de+2cf))x^2\right)dx}{12b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\int \left(bx^2+a\right)^{3/2}\left(fx^2+e\right)\left(\left(24e(de+7cf)b^2-4af(17de+15cf)b+35a^2df^2\right)x^2+e\left(7dfa^2-16bdea-12bcfa+120b^2ce\right)\right)dx}{10b}+\frac{x\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2(}{10b}}{12b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{\frac{\int \left(bx^2+a\right)^{3/2}\left(e\left(-35df^2a^3+4bf(31de+15cf)a^2-8b^2e(19de+33cf)a+960b^3ce^2\right)-3\left(-16e^2(de+27cf)b^3+16aef(7de+15cf)b^2-10a^2f^2(11de+6cf)b+35a^3df^3\right)x}{8b}}{10b}}{12b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{\frac{5\left(7a^4df^3-12a^3bf^2(cf+3de)+72a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+384b^4ce^3\right)\int\left(bx^2+a\right)^{3/2}dx}{2b}}{8b}-\frac{x\left(a+bx^2\right)^{5/2}\left(35a^3df^3-10a^2bf^2(6cf+11de)+16ab^2ef(15c}{2b}}{10b}}{12b}+\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{\frac{5\left(7a^4df^3-12a^3bf^2(cf+3de)+72a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+384b^4ce^3\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}-\frac{x\left(a+bx^2\right)^{5/2}\left(35a^3df^3-10a^2bf^2(6cf+\right.}{}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{5\left(7a^4df^3-12a^3bf^2(cf+3de)+72a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+384b^4ce^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}-\frac{x\left(a+bx^2\right)^{5/2}\left(35a^3\right.}{}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{5\left(7a^4df^3-12a^3bf^2(cf+3de)+72a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+384b^4ce^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}-\frac{x\left(a+bx^2\right.}{}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)\left(35a^2df^2-4abf(15cf+17de)+24b^2e(7cf+de)\right)}{8b}+\frac{5\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(7a^4df^3-12a^3bf^2\right.}{2b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}{12b}$$

input

```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

output
```
(d*x*(a + b*x^2)^(5/2)*(e + f*x^2)^3)/(12*b) + (((6*b*d*e + 12*b*c*f - 7*a
*d*f)*x*(a + b*x^2)^(5/2)*(e + f*x^2)^2)/(10*b) + (((35*a^2*d*f^2 + 24*b^2
*e*(d*e + 7*c*f) - 4*a*b*f*(17*d*e + 15*c*f))*x*(a + b*x^2)^(5/2)*(e + f*x
^2))/(8*b) + (-1/2*((35*a^3*d*f^3 - 10*a^2*b*f^2*(11*d*e + 6*c*f) + 16*a*b
^2*e*f*(7*d*e + 15*c*f) - 16*b^3*e^2*(d*e + 27*c*f))*x*(a + b*x^2)^(5/2))/
b + (5*(384*b^4*c*e^3 + 7*a^4*d*f^3 + 72*a^2*b^2*e*f*(d*e + c*f) - 12*a^3*
b*f^2*(3*d*e + c*f) - 64*a*b^3*e^2*(d*e + 3*c*f))*((x*(a + b*x^2)^(3/2))/4
 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*Sqrt[b])))/4))/(2*b))/(8*b))/(10*b))/(12*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\frac{7a^{2}\left(a^{3}\left(-\frac{12bc}{7}+ad\right)f^{3}-\frac{36a^{2}be(ad-2bc)f^{2}}{7}+\frac{72a\left(ad-\frac{8bc}{3}\right)b^{2}e^{2}f}{7}-\frac{64e^{3}b^{3}(ad-6bc)}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{1024}-\frac{7\left(-\frac{256\left(\frac{2\left(\frac{5x^{2}d}{6}+c\right.}{5}\right.}{}\right.}{}}{}$
default	$c\,e^{3}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+f^{2}(cf+3de)\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^{3}\left(b\,x^{2}+\right.}{8b}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
7/1024*(a^2*(a^3*(-12/7*b*c+a*d)*f^3-36/7*a^2*b*e*(a*d-2*b*c)*f^2+72/7*a*(
a*d-8/3*b*c)*b^2*e^2*f-64/7*e^3*b^3*(a*d-6*b*c))*arctanh((b*x^2+a)^(1/2)/x
/b^(1/2))-(-256/7*(2/5*(5/6*x^2*d+c)*x^6*f^3+3/2*(4/5*x^2*d+c)*x^4*e*f^2+2
*(3/4*x^2*d+c)*x^2*e^2*f+e^3*(2/3*x^2*d+c))*x^2*b^(11/2)+a*(64*(-11/35*x^6
*(26/33*x^2*d+c)*f^3-9/7*(11/15*x^2*d+c)*x^4*e*f^2-2*x^2*(9/14*x^2*d+c)*e^
2*f-10/7*(7/15*x^2*d+c)*e^3)*b^(9/2)+a*(16/7*(-2/5*(1/2*x^2*d+c)*x^4*f^3-3
*(2/5*x^2*d+c)*x^2*e*f^2-12*(1/4*x^2*d+c)*e^2*f-4*d*e^3)*b^(7/2)+a*(8/7*((
7/15*x^2*d+c)*x^2*f^2+9*(1/3*x^2*d+c)*e*f+9*d*e^2)*b^(5/2)+a*(2*((-1/3*x^2
*d-6/7*c)*f-18/7*d*e)*b^(3/2)+a*d*f*b^(1/2))*f)*f)))*x*(b*x^2+a)^(1/2))/b^
(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.13

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(e+fx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/30720*(15*(64*(6*a^2*b^4*c - a^3*b^3*d)*e^3 - 24*(8*a^3*b^3*c - 3*a^4*b
^2*d)*e^2*f + 36*(2*a^4*b^2*c - a^5*b*d)*e*f^2 - (12*a^5*b*c - 7*a^6*d)*f^
3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(1280*b^6*d
*f^3*x^11 + 128*(36*b^6*d*e*f^2 + (12*b^6*c + 13*a*b^5*d)*f^3)*x^9 + 48*(1
20*b^6*d*e^2*f + 12*(10*b^6*c + 11*a*b^5*d)*e*f^2 + (44*a*b^5*c + a^2*b^4*
d)*f^3)*x^7 + 8*(320*b^6*d*e^3 + 120*(8*b^6*c + 9*a*b^5*d)*e^2*f + 36*(30*
a*b^5*c + a^2*b^4*d)*e*f^2 + (12*a^2*b^4*c - 7*a^3*b^3*d)*f^3)*x^5 + 10*(6
4*(6*b^6*c + 7*a*b^5*d)*e^3 + 24*(56*a*b^5*c + 3*a^2*b^4*d)*e^2*f + 36*(2*
a^2*b^4*c - a^3*b^3*d)*e*f^2 - (12*a^3*b^3*c - 7*a^4*b^2*d)*f^3)*x^3 + 15*
(64*(10*a*b^5*c + a^2*b^4*d)*e^3 + 24*(8*a^2*b^4*c - 3*a^3*b^3*d)*e^2*f -
36*(2*a^3*b^3*c - a^4*b^2*d)*e*f^2 + (12*a^4*b^2*c - 7*a^5*b*d)*f^3)*x)*sq
rt(b*x^2 + a))/b^5, -1/15360*(15*(64*(6*a^2*b^4*c - a^3*b^3*d)*e^3 - 24*(8
*a^3*b^3*c - 3*a^4*b^2*d)*e^2*f + 36*(2*a^4*b^2*c - a^5*b*d)*e*f^2 - (12*a
^5*b*c - 7*a^6*d)*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280
*b^6*d*f^3*x^11 + 128*(36*b^6*d*e*f^2 + (12*b^6*c + 13*a*b^5*d)*f^3)*x^9 +
 48*(120*b^6*d*e^2*f + 12*(10*b^6*c + 11*a*b^5*d)*e*f^2 + (44*a*b^5*c + a^
2*b^4*d)*f^3)*x^7 + 8*(320*b^6*d*e^3 + 120*(8*b^6*c + 9*a*b^5*d)*e^2*f + 3
6*(30*a*b^5*c + a^2*b^4*d)*e*f^2 + (12*a^2*b^4*c - 7*a^3*b^3*d)*f^3)*x^5 +
 10*(64*(6*b^6*c + 7*a*b^5*d)*e^3 + 24*(56*a*b^5*c + 3*a^2*b^4*d)*e^2*f +
36*(2*a^2*b^4*c - a^3*b^3*d)*e*f^2 - (12*a^3*b^3*c - 7*a^4*b^2*d)*f^3)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. $2(498) = 996$.

Time = 0.61 (sec) , antiderivative size = 1234, normalized size of antiderivative = 2.56

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(e+fx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)*(f*x**2+e)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d*f**3*x**11/12 + x**9*(13*a*b*d*f**3/12 +
b**2*c*f**3 + 3*b**2*d*e*f**2)/(10*b) + x**7*(a**2*d*f**3 + 2*a*b*c*f**3 +
 6*a*b*d*e*f**2 - 9*a*(13*a*b*d*f**3/12 + b**2*c*f**3 + 3*b**2*d*e*f**2)/(
10*b) + 3*b**2*c*e*f**2 + 3*b**2*d*e**2*f)/(8*b) + x**5*(a**2*c*f**3 + 3*a
**2*d*e*f**2 + 6*a*b*c*e*f**2 + 6*a*b*d*e**2*f - 7*a*(a**2*d*f**3 + 2*a*b*
c*f**3 + 6*a*b*d*e*f**2 - 9*a*(13*a*b*d*f**3/12 + b**2*c*f**3 + 3*b**2*d*e
*f**2)/(10*b) + 3*b**2*c*e*f**2 + 3*b**2*d*e**2*f)/(8*b) + 3*b**2*c*e**2*f
 + b**2*d*e**3)/(6*b) + x**3*(3*a**2*c*e*f**2 + 3*a**2*d*e**2*f + 6*a*b*c*
e**2*f + 2*a*b*d*e**3 - 5*a*(a**2*c*f**3 + 3*a**2*d*e*f**2 + 6*a*b*c*e*f**
2 + 6*a*b*d*e**2*f - 7*a*(a**2*d*f**3 + 2*a*b*c*f**3 + 6*a*b*d*e*f**2 - 9*
a*(13*a*b*d*f**3/12 + b**2*c*f**3 + 3*b**2*d*e*f**2)/(10*b) + 3*b**2*c*e*f
**2 + 3*b**2*d*e**2*f)/(8*b) + 3*b**2*c*e**2*f + b**2*d*e**3)/(6*b) + b**2
*c*e**3)/(4*b) + x*(3*a**2*c*e**2*f + a**2*d*e**3 + 2*a*b*c*e**3 - 3*a*(3*
a**2*c*e*f**2 + 3*a**2*d*e**2*f + 6*a*b*c*e**2*f + 2*a*b*d*e**3 - 5*a*(a**
2*c*f**3 + 3*a**2*d*e*f**2 + 6*a*b*c*e*f**2 + 6*a*b*d*e**2*f - 7*a*(a**2*d
*f**3 + 2*a*b*c*f**3 + 6*a*b*d*e*f**2 - 9*a*(13*a*b*d*f**3/12 + b**2*c*f**
3 + 3*b**2*d*e*f**2)/(10*b) + 3*b**2*c*e*f**2 + 3*b**2*d*e**2*f)/(8*b) + 3
*b**2*c*e**2*f + b**2*d*e**3)/(6*b) + b**2*c*e**3)/(4*b))/(2*b)) + (a**2*c
*e**3 - a*(3*a**2*c*e**2*f + a**2*d*e**3 + 2*a*b*c*e**3 - 3*a*(3*a**2*c*e*
f**2 + 3*a**2*d*e**2*f + 6*a*b*c*e**2*f + 2*a*b*d*e**3 - 5*a*(a**2*c*f*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.37

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(5/2)*d*f^3*x^7/b - 7/120*(b*x^2 + a)^(5/2)*a*d*f^3*x^5/b
^2 + 7/192*(b*x^2 + a)^(5/2)*a^2*d*f^3*x^3/b^3 + 1/10*(3*d*e*f^2 + c*f^3)*
(b*x^2 + a)^(5/2)*x^5/b + 1/4*(b*x^2 + a)^(3/2)*c*e^3*x + 3/8*sqrt(b*x^2 +
 a)*a*c*e^3*x - 7/384*(b*x^2 + a)^(5/2)*a^3*d*f^3*x/b^4 + 7/1536*(b*x^2 +
a)^(3/2)*a^4*d*f^3*x/b^4 + 7/1024*sqrt(b*x^2 + a)*a^5*d*f^3*x/b^4 + 3/8*a^
2*c*e^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 7/1024*a^6*d*f^3*arcsinh(b*x/sqrt
(a*b))/b^(9/2) - 1/16*(3*d*e*f^2 + c*f^3)*(b*x^2 + a)^(5/2)*a*x^3/b^2 + 3/
8*(d*e^2*f + c*e*f^2)*(b*x^2 + a)^(5/2)*x^3/b + 1/32*(3*d*e*f^2 + c*f^3)*(
b*x^2 + a)^(5/2)*a^2*x/b^3 - 1/128*(3*d*e*f^2 + c*f^3)*(b*x^2 + a)^(3/2)*a
^3*x/b^3 - 3/256*(3*d*e*f^2 + c*f^3)*sqrt(b*x^2 + a)*a^4*x/b^3 - 3/16*(d*e
^2*f + c*e*f^2)*(b*x^2 + a)^(5/2)*a*x/b^2 + 3/64*(d*e^2*f + c*e*f^2)*(b*x^
2 + a)^(3/2)*a^2*x/b^2 + 9/128*(d*e^2*f + c*e*f^2)*sqrt(b*x^2 + a)*a^3*x/b
^2 + 1/6*(d*e^3 + 3*c*e^2*f)*(b*x^2 + a)^(5/2)*x/b - 1/24*(d*e^3 + 3*c*e^2
*f)*(b*x^2 + a)^(3/2)*a*x/b - 1/16*(d*e^3 + 3*c*e^2*f)*sqrt(b*x^2 + a)*a^2
*x/b - 3/256*(3*d*e*f^2 + c*f^3)*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 9/12
8*(d*e^2*f + c*e*f^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*(d*e^3 + 3
*c*e^2*f)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.13

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,bdf^3x^2+\frac{36\,b^{11}def^2+12\,b^{11}cf^3+13\,ab^{10}df^3}{b^{10}}\right)x^2+\frac{3\,(120\,b^{11}de^2f+\right.\right.\right.\right.$$

$$-\frac{(384\,a^2b^4ce^3-64\,a^3b^3de^3-192\,a^3b^3ce^2f+72\,a^4b^2de^2f+72\,a^4b^2cef^2-36\,a^5bdef^2-12\,a^5bcf^3+7\,a^6df}{1024\,b^{\frac{9}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*b*d*f^3*x^2 + (36*b^11*d*e*f^2 + 12*b^11*c*f^3 + 1
3*a*b^10*d*f^3)/b^10)*x^2 + 3*(120*b^11*d*e^2*f + 120*b^11*c*e*f^2 + 132*a
*b^10*d*e*f^2 + 44*a*b^10*c*f^3 + a^2*b^9*d*f^3)/b^10)*x^2 + (320*b^11*d*e
^3 + 960*b^11*c*e^2*f + 1080*a*b^10*d*e^2*f + 1080*a*b^10*c*e*f^2 + 36*a^2
*b^9*d*e*f^2 + 12*a^2*b^9*c*f^3 - 7*a^3*b^8*d*f^3)/b^10)*x^2 + 5*(384*b^11
*c*e^3 + 448*a*b^10*d*e^3 + 1344*a*b^10*c*e^2*f + 72*a^2*b^9*d*e^2*f + 72*
a^2*b^9*c*e*f^2 - 36*a^3*b^8*d*e*f^2 - 12*a^3*b^8*c*f^3 + 7*a^4*b^7*d*f^3)
/b^10)*x^2 + 15*(640*a*b^10*c*e^3 + 64*a^2*b^9*d*e^3 + 192*a^2*b^9*c*e^2*f
 - 72*a^3*b^8*d*e^2*f - 72*a^3*b^8*c*e*f^2 + 36*a^4*b^7*d*e*f^2 + 12*a^4*b
^7*c*f^3 - 7*a^5*b^6*d*f^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(384*a^2*b^4*
c*e^3 - 64*a^3*b^3*d*e^3 - 192*a^3*b^3*c*e^2*f + 72*a^4*b^2*d*e^2*f + 72*a
^4*b^2*c*e*f^2 - 36*a^5*b*d*e*f^2 - 12*a^5*b*c*f^3 + 7*a^6*d*f^3)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx=\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^3\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx=\int \left(b\,x^2+a\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(f\,x^2+e\right)^3dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^3,x)
```

3.291 $\int (a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 337

$$\int (a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2\,dx = \frac{a(96b^3ce^2-3a^3df^2+6a^2bf(2de+cf)-16ab^2e(de+2cf))\,x\sqrt{a+bx^2}}{256b^3} + \frac{(96b^3ce^2-3a^3df^2+6a^2bf(2de+cf)-16ab^2e(de+2cf))\,x(a+bx^2)^{3/2}}{384b^3} + \frac{(3a^2df^2-6abf(2de+cf)+16b^2e(de+2cf))\,x(a+bx^2)^{5/2}}{96b^3} + \frac{f(4bde+2bcf-adf)x^3(a+bx^2)^{5/2}}{16b^2} + \frac{df^2x^5(a+bx^2)^{5/2}}{10b} + \frac{a^2(96b^3ce^2-3a^3df^2+6a^2bf(2de+cf)-16ab^2e(de+2cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output
```
1/256*a*(96*b^3*c*e^2-3*a^3*d*f^2+6*a^2*b*f*(c*f+2*d*e)-16*a*b^2*e*(2*c*f+
d*e))*x*(b*x^2+a)^(1/2)/b^3+1/384*(96*b^3*c*e^2-3*a^3*d*f^2+6*a^2*b*f*(c*f
+2*d*e)-16*a*b^2*e*(2*c*f+d*e))*x*(b*x^2+a)^(3/2)/b^3+1/96*(3*a^2*d*f^2-6*
a*b*f*(c*f+2*d*e)+16*b^2*e*(2*c*f+d*e))*x*(b*x^2+a)^(5/2)/b^3+1/16*f*(-a*d
*f+2*b*c*f+4*b*d*e)*x^3*(b*x^2+a)^(5/2)/b^2+1/10*d*f^2*x^5*(b*x^2+a)^(5/2)
/b+1/256*a^2*(96*b^3*c*e^2-3*a^3*d*f^2+6*a^2*b*f*(c*f+2*d*e)-16*a*b^2*e*(2
*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(e+fx^2\right)^2 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(45a^4df^2-30a^3bf(6de+3cf+dfx^2)+12a^2b^2(5cf(8e+fx^2)+2d(10e^2+5e$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(45*a^4*d*f^2 - 30*a^3*b*f*(6*d*e + 3*c*f + d*f
*x^2) + 12*a^2*b^2*(5*c*f*(8*e + f*x^2) + 2*d*(10*e^2 + 5*e*f*x^2 + f^2*x^
4)) + 32*b^4*x^2*(5*c*(6*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + 2*d*x^2*(10*e^2 +
15*e*f*x^2 + 6*f^2*x^4)) + 16*a*b^3*(5*c*(30*e^2 + 28*e*f*x^2 + 9*f^2*x^4)
 + d*x^2*(70*e^2 + 90*e*f*x^2 + 33*f^2*x^4))) + 15*a^2*(-96*b^3*c*e^2 + 3*
a^3*d*f^2 - 6*a^2*b*f*(2*d*e + c*f) + 16*a*b^2*e*(d*e + 2*c*f))*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(3840*b^(7/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {403, 403, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \left(e+fx^2\right)^2 \, dx$$

$$\Big\downarrow \; 403$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left(fx^2+e\right) \left((4bde+10bcf-5adf)x^2+(10bc-ad)e\right) dx}{10b} + \frac{dx\left(a+bx^2\right)^{5/2} \left(e+fx^2\right)^2}{10b}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int (bx^2+a)^{3/2}\left(\left(4e(2de+25cf)b^2-30af(de+cf)b+15a^2df^2\right)x^2+e\left(5dfa^2-12bdea-10bcfa+80b^2ce\right)\right)dx}{8b}+\frac{x(a+bx^2)^{5/2}(e+fx^2)(-5adf+10bcf}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{5\left(-3a^3df^2+6a^2bf(cf+2de)-16ab^2e(2cf+de)+96b^3ce^2\right)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{x\left(a+bx^2\right)^{5/2}\left(15a^2df^2-30abf(cf+de)+4b^2e(25cf+2de)\right)}{6b}}{8b}+\frac{x(a+bx^2)^{5/2}(e+}{}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{5\left(-3a^3df^2+6a^2bf(cf+2de)-16ab^2e(2cf+de)+96b^3ce^2\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}\left(15a^2df^2-30abf(cf+de)+4b^2e(25cf+2de)\right)}{6b}}{8b}+}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{5\left(-3a^3df^2+6a^2bf(cf+2de)-16ab^2e(2cf+de)+96b^3ce^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}\left(15a^2df^2-30abf(cf+de)+4\right.}{6b}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{5\left(-3a^3df^2+6a^2bf(cf+2de)-16ab^2e(2cf+de)+96b^3ce^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}\left(15a^2df^2-30abf\right.}{6b}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x\left(a+bx^2\right)^{5/2}\left(15a^2df^2-30abf(cf+de)+4b^2e(25cf+2de)\right)}{6b}+\frac{5\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(-3a^3df^2+6a^2bf(cf+2de)-1\right.}{6b}}{8b}}{10b}$$

$$\frac{dx\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}{10b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

output
```
(d*x*(a + b*x^2)^(5/2)*(e + f*x^2)^2)/(10*b) + (((4*b*d*e + 10*b*c*f - 5*a
*d*f)*x*(a + b*x^2)^(5/2)*(e + f*x^2))/(8*b) + (((15*a^2*d*f^2 - 30*a*b*f*
(d*e + c*f) + 4*b^2*e*(2*d*e + 25*c*f))*x*(a + b*x^2)^(5/2))/(6*b) + (5*(9
6*b^3*c*e^2 - 3*a^3*d*f^2 + 6*a^2*b*f*(2*d*e + c*f) - 16*a*b^2*e*(d*e + 2*
c*f))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b))/(8*b))/(10*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{3\left(a^2\left(a^3 d f^2-2a^2 b f(cf+2de)+\frac{32\left(cf+\frac{de}{2}\right)b^2 ea}{3}-32b^3 c e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)-\left(\frac{160\left(\frac{11f^2 d x^6}{50}+\frac{3f(cf+2de)x^4}{10}+\frac{7(2}{}\right.}{3}\right.\right.}{}$
risch	$\frac{x(384b^4 d f^2x^8+528a b^3 d f^2x^6+480b^4 c f^2x^6+960b^4 def x^6+24a^2b^2 d f^2x^4+720a b^3 c f^2x^4+1440a b^3 def x^4+1280b^4 cef x^4}{}$
default	$c e^2\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+f(cf+2de)\left(\frac{x^3(b x^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(b x^2+a}{6b}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/256*(a^2*(a^3*d*f^2-2*a^2*b*f*(c*f+2*d*e)+32/3*(c*f+1/2*d*e)*b^2*e*a-32
*b^3*c*e^2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(160/3*(11/50*f^2*d*x^6+3/1
0*f*(c*f+2*d*e)*x^4+7/15*(2*c*e*f+d*e^2)*x^2+c*e^2)*a*b^(7/2)+64/3*x^2*(2/
5*f^2*d*x^6+(1/2*c*f^2+d*e*f)*x^4+2/3*(2*c*e*f+d*e^2)*x^2+c*e^2)*b^(9/2)+a
^2*(4/3*(2/5*d*f^2*x^4+f*(c*f+2*d*e)*x^2+8*c*e*f+4*d*e^2)*b^(5/2)+a*(2*(-1
/3*d*f*x^2-c*f-2*d*e)*b^(3/2)+a*d*f*b^(1/2))*f))*(b*x^2+a)^(1/2)*x)/b^(7/2
)
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.07

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/7680*(15*(16*(6*a^2*b^3*c - a^3*b^2*d)*e^2 - 4*(8*a^3*b^2*c - 3*a^4*b*
d)*e*f + 3*(2*a^4*b*c - a^5*d)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(384*b^5*d*f^2*x^9 + 48*(20*b^5*d*e*f + (10*b^5*c +
11*a*b^4*d)*f^2)*x^7 + 8*(80*b^5*d*e^2 + 20*(8*b^5*c + 9*a*b^4*d)*e*f + 3*
(30*a*b^4*c + a^2*b^3*d)*f^2)*x^5 + 10*(16*(6*b^5*c + 7*a*b^4*d)*e^2 + 4*(
56*a*b^4*c + 3*a^2*b^3*d)*e*f + 3*(2*a^2*b^3*c - a^3*b^2*d)*f^2)*x^3 + 15*
(16*(10*a*b^4*c + a^2*b^3*d)*e^2 + 4*(8*a^2*b^3*c - 3*a^3*b^2*d)*e*f - 3*(
2*a^3*b^2*c - a^4*b*d)*f^2)*x)*sqrt(b*x^2 + a))/b^4, -1/3840*(15*(16*(6*a^
2*b^3*c - a^3*b^2*d)*e^2 - 4*(8*a^3*b^2*c - 3*a^4*b*d)*e*f + 3*(2*a^4*b*c
- a^5*d)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*d*f^2
*x^9 + 48*(20*b^5*d*e*f + (10*b^5*c + 11*a*b^4*d)*f^2)*x^7 + 8*(80*b^5*d*e
^2 + 20*(8*b^5*c + 9*a*b^4*d)*e*f + 3*(30*a*b^4*c + a^2*b^3*d)*f^2)*x^5 +
10*(16*(6*b^5*c + 7*a*b^4*d)*e^2 + 4*(56*a*b^4*c + 3*a^2*b^3*d)*e*f + 3*(2
*a^2*b^3*c - a^3*b^2*d)*f^2)*x^3 + 15*(16*(10*a*b^4*c + a^2*b^3*d)*e^2 + 4
*(8*a^2*b^3*c - 3*a^3*b^2*d)*e*f - 3*(2*a^3*b^2*c - a^4*b*d)*f^2)*x)*sqrt(
b*x^2 + a))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(342) = 684$.

Time = 0.56 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.26

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{bdf^2x^9}{10}+\frac{x^7\cdot\left(\frac{11abdf^2}{10}+b^2cf^2+2b^2def\right)}{8b}+\frac{x^5\left(a^2df^2+2abcf^2+4abdef-\frac{7a\left(\frac{11abdf^2}{10}+b^2cf^2+2b^2def\right)}{8b}+2b\right.}{6b}\right. \\ a^{\frac{3}{2}}\left(ce^2x+\frac{df^2x^7}{7}+\frac{x^5(cf^2+2def)}{5}+\frac{x^3\cdot(2cef+de^2)}{3}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)*(f*x**2+e)**2,x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(b*d*f**2*x**9/10 + x**7*(11*a*b*d*f**2/10 + b
**2*c*f**2 + 2*b**2*d*e*f)/(8*b) + x**5*(a**2*d*f**2 + 2*a*b*c*f**2 + 4*a*
b*d*e*f - 7*a*(11*a*b*d*f**2/10 + b**2*c*f**2 + 2*b**2*d*e*f)/(8*b) + 2*b*
*2*c*e*f + b**2*d*e**2)/(6*b) + x**3*(a**2*c*f**2 + 2*a**2*d*e*f + 4*a*b*c
*e*f + 2*a*b*d*e**2 - 5*a*(a**2*d*f**2 + 2*a*b*c*f**2 + 4*a*b*d*e*f - 7*a*
(11*a*b*d*f**2/10 + b**2*c*f**2 + 2*b**2*d*e*f)/(8*b) + 2*b**2*c*e*f + b**
2*d*e**2)/(6*b) + b**2*c*e**2)/(4*b) + x*(2*a**2*c*e*f + a**2*d*e**2 + 2*a
*b*c*e**2 - 3*a*(a**2*c*f**2 + 2*a**2*d*e*f + 4*a*b*c*e*f + 2*a*b*d*e**2 -
 5*a*(a**2*d*f**2 + 2*a*b*c*f**2 + 4*a*b*d*e*f - 7*a*(11*a*b*d*f**2/10 + b
**2*c*f**2 + 2*b**2*d*e*f)/(8*b) + 2*b**2*c*e*f + b**2*d*e**2)/(6*b) + b**
2*c*e**2)/(4*b))/(2*b)) + (a**2*c*e**2 - a*(2*a**2*c*e*f + a**2*d*e**2 + 2
*a*b*c*e**2 - 3*a*(a**2*c*f**2 + 2*a**2*d*e*f + 4*a*b*c*e*f + 2*a*b*d*e**2
 - 5*a*(a**2*d*f**2 + 2*a*b*c*f**2 + 4*a*b*d*e*f - 7*a*(11*a*b*d*f**2/10 +
 b**2*c*f**2 + 2*b**2*d*e*f)/(8*b) + 2*b**2*c*e*f + b**2*d*e**2)/(6*b) + b
**2*c*e**2)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*
x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2
)*(c*e**2*x + d*f**2*x**7/7 + x**5*(c*f**2 + 2*d*e*f)/5 + x**3*(2*c*e*f +
d*e**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.29

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\frac{(bx^2+a)^{\frac{5}{2}}df^2x^5}{10\,b}-\frac{(bx^2+a)^{\frac{5}{2}}adf^2x^3}{16\,b^2}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}ce^2x+\frac{3}{8}\,\sqrt{bx^2+a}ace^2x+\frac{(bx^2+a)^{\frac{5}{2}}a^2df^2x}{32\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}a^3df^2x}{128\,b^3}-\frac{3\,\sqrt{bx^2+a}a^4df^2x}{256\,b^3}+\frac{(2\,def+cf^2)(bx^2+a)^{\frac{5}{2}}x^3}{8\,b}+\frac{3\,a^2ce^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{3\,a^5df^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}-\frac{(2\,def+cf^2)(bx^2+a)^{\frac{5}{2}}ax}{16\,b^2}+\frac{(2\,def+cf^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,(2\,def+cf^2)\sqrt{bx^2+a}a^3x}{128\,b^2}+\frac{(de^2+2\,cef)(bx^2+a)^{\frac{5}{2}}x}{6\,b}-\frac{(de^2+2\,cef)(bx^2+a)^{\frac{3}{2}}ax}{24\,b}-\frac{(de^2+2\,cef)\sqrt{bx^2+a}a^2x}{16\,b}+\frac{3\,(2\,def+cf^2)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{(de^2+2\,cef)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*d*f^2*x^5/b - 1/16*(b*x^2 + a)^(5/2)*a*d*f^2*x^3/b^
2 + 1/4*(b*x^2 + a)^(3/2)*c*e^2*x + 3/8*sqrt(b*x^2 + a)*a*c*e^2*x + 1/32*(
b*x^2 + a)^(5/2)*a^2*d*f^2*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*a^3*d*f^2*x/b^3
 - 3/256*sqrt(b*x^2 + a)*a^4*d*f^2*x/b^3 + 1/8*(2*d*e*f + c*f^2)*(b*x^2 +
a)^(5/2)*x^3/b + 3/8*a^2*c*e^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/256*a^5*
d*f^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/16*(2*d*e*f + c*f^2)*(b*x^2 + a)^
(5/2)*a*x/b^2 + 1/64*(2*d*e*f + c*f^2)*(b*x^2 + a)^(3/2)*a^2*x/b^2 + 3/128
*(2*d*e*f + c*f^2)*sqrt(b*x^2 + a)*a^3*x/b^2 + 1/6*(d*e^2 + 2*c*e*f)*(b*x^
2 + a)^(5/2)*x/b - 1/24*(d*e^2 + 2*c*e*f)*(b*x^2 + a)^(3/2)*a*x/b - 1/16*(
d*e^2 + 2*c*e*f)*sqrt(b*x^2 + a)*a^2*x/b + 3/128*(2*d*e*f + c*f^2)*a^4*arc
sinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*(d*e^2 + 2*c*e*f)*a^3*arcsinh(b*x/sqrt(
a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\frac{1}{3840}\left(2\left(4\left(6\left(8\,bdf^2x^2+\frac{20\,b^9def+10\,b^9cf^2+11\,ab^8df^2}{b^8}\right)x^2+\frac{80\,b^9de^2+160\,b^9cef+18}{}\right.\right.\right.$$
$$-\frac{\left(96\,a^2b^3ce^2-16\,a^3b^2de^2-32\,a^3b^2cef+12\,a^4bdef+6\,a^4bcf^2-3\,a^5df^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*b*d*f^2*x^2 + (20*b^9*d*e*f + 10*b^9*c*f^2 + 11*a*b^8*d
*f^2)/b^8)*x^2 + (80*b^9*d*e^2 + 160*b^9*c*e*f + 180*a*b^8*d*e*f + 90*a*b^
8*c*f^2 + 3*a^2*b^7*d*f^2)/b^8)*x^2 + 5*(96*b^9*c*e^2 + 112*a*b^8*d*e^2 +
224*a*b^8*c*e*f + 12*a^2*b^7*d*e*f + 6*a^2*b^7*c*f^2 - 3*a^3*b^6*d*f^2)/b^
8)*x^2 + 15*(160*a*b^8*c*e^2 + 16*a^2*b^7*d*e^2 + 32*a^2*b^7*c*e*f - 12*a^
3*b^6*d*e*f - 6*a^3*b^6*c*f^2 + 3*a^4*b^5*d*f^2)/b^8)*sqrt(b*x^2 + a)*x -
1/256*(96*a^2*b^3*c*e^2 - 16*a^3*b^2*d*e^2 - 32*a^3*b^2*c*e*f + 12*a^4*b*d
*e*f + 6*a^4*b*c*f^2 - 3*a^5*d*f^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\int {\left(b\,x^2+a\right)}^{3/2}\,\left(d\,x^2+c\right)\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.86

$$\int (a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2\,dx = \frac{960\sqrt{b\,x^2+a}\,b^5c\,e^2x^3+480\sqrt{b\,x^2+a}\,b^5c\,f^2x^7+640\sqrt{b\,x^2+a}\,b^5d\,e^2x^5+384\sqrt{b\,x^2+a}\,b^5d\,f}{}$$

input

```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e)^2,x)
```

output

```
(45*sqrt(a + b*x**2)*a**4*b*d*f**2*x - 90*sqrt(a + b*x**2)*a**3*b**2*c*f**
2*x - 180*sqrt(a + b*x**2)*a**3*b**2*d*e*f*x - 30*sqrt(a + b*x**2)*a**3*b*
*2*d*f**2*x**3 + 480*sqrt(a + b*x**2)*a**2*b**3*c*e*f*x + 60*sqrt(a + b*x*
*2)*a**2*b**3*c*f**2*x**3 + 240*sqrt(a + b*x**2)*a**2*b**3*d*e**2*x + 120*
sqrt(a + b*x**2)*a**2*b**3*d*e*f*x**3 + 24*sqrt(a + b*x**2)*a**2*b**3*d*f*
*2*x**5 + 2400*sqrt(a + b*x**2)*a*b**4*c*e**2*x + 2240*sqrt(a + b*x**2)*a*
b**4*c*e*f*x**3 + 720*sqrt(a + b*x**2)*a*b**4*c*f**2*x**5 + 1120*sqrt(a +
b*x**2)*a*b**4*d*e**2*x**3 + 1440*sqrt(a + b*x**2)*a*b**4*d*e*f*x**5 + 528
*sqrt(a + b*x**2)*a*b**4*d*f**2*x**7 + 960*sqrt(a + b*x**2)*b**5*c*e**2*x*
*3 + 1280*sqrt(a + b*x**2)*b**5*c*e*f*x**5 + 480*sqrt(a + b*x**2)*b**5*c*f
**2*x**7 + 640*sqrt(a + b*x**2)*b**5*d*e**2*x**5 + 960*sqrt(a + b*x**2)*b*
*5*d*e*f*x**7 + 384*sqrt(a + b*x**2)*b**5*d*f**2*x**9 - 45*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d*f**2 + 90*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*f**2 + 180*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*e*f - 480*sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*e*f - 240*sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d*e**2 + 1440*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*c*e**2)/(3840*b**4)
```

3.292 $\int (a+bx^2)^{3/2} (c+dx^2) (e+fx^2) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 207

$$\int (a+bx^2)^{3/2} (c+dx^2) (e+fx^2) \, dx = \frac{a(48b^2ce+3a^2df-8ab(de+cf))\,x\sqrt{a+bx^2}}{128b^2} + \frac{(48b^2ce+3a^2df-8ab(de+cf))\,x(a+bx^2)^{3/2}}{192b^2} - \frac{(3adf-8b(de+cf))x(a+bx^2)^{5/2}}{48b^2} + \frac{dfx^3(a+bx^2)^{5/2}}{8b} + \frac{a^2(48b^2ce+3a^2df-8ab(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/128*a*(48*b^2*c*e+3*a^2*d*f-8*a*b*(c*f+d*e))*x*(b*x^2+a)^(1/2)/b^2+1/192
*(48*b^2*c*e+3*a^2*d*f-8*a*b*(c*f+d*e))*x*(b*x^2+a)^(3/2)/b^2-1/48*(3*a*d*
f-8*b*(c*f+d*e))*x*(b*x^2+a)^(5/2)/b^2+1/8*d*f*x^3*(b*x^2+a)^(5/2)/b+1/128
*a^2*(48*b^2*c*e+3*a^2*d*f-8*a*b*(c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-9a^3df+6a^2b(4de+4cf+dfx^2)+16b^3x^2(6ce+4dex^2+4cfx^2+3dfx^4)+8}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*d*f + 6*a^2*b*(4*d*e + 4*c*f + d*f*x^2)
 + 16*b^3*x^2*(6*c*e + 4*d*e*x^2 + 4*c*f*x^2 + 3*d*f*x^4) + 8*a*b^2*(30*c*
e + 14*d*e*x^2 + 14*c*f*x^2 + 9*d*f*x^4)) - 3*a^2*(48*b^2*c*e + 3*a^2*d*f
- 8*a*b*(d*e + c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {403, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \left(bx^2+a\right)^{3/2}\left((8bde+2bcf-3adf)x^2+c(8be-af)\right)dx}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(3a^2df-8ab(cf+de)+48b^2ce\right)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(-3adf+2bcf+8bde)}{6b}}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3a^2df-8ab(cf+de)+48b^2ce\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(-3adf+2bcf+8bde)}{6b}}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\left(3a^2df-8ab(cf+de)+48b^2ce\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(-3adf+2bcf+8bde)}{6b}}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\left(3a^2df-8ab(cf+de)+48b^2ce\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(-3adf+2bcf+8bde)}{6b}}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(3a^2df-8ab(cf+de)+48b^2ce\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(-3adf+2bcf+8bde)}{6b}}{8b}+\frac{fx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(8*b) + (((8*b*d*e + 2*b*c*f - 3*a*d*f
)*x*(a + b*x^2)^(5/2))/(6*b) + ((48*b^2*c*e + 3*a^2*d*f - 8*a*b*(d*e + c*f
))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{3a^2\left(a^2 df-\frac{8ab(cf+de)}{3}+16ce\,b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{128}-\frac{3\sqrt{b\,x^2+a}\,x\left(-\frac{80a\left(\frac{3df\,x^4}{10}+\frac{7(cf+de)x^2}{15}+ce\right)b^{\frac{5}{2}}}{3}-\frac{32x^2\left(\frac{df\,x^4}{2}+\frac{2(cf+de}{3}\cdots\right.}{3}\cdots\right.}{128}}{b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-48b^3df\,x^6-72adf\,b^2x^4-64b^3cf\,x^4-64de\,b^3x^4-6a^2bdf\,x^2-112acf\,b^2x^2-112ade\,b^2x^2-96b^3ce\,x^2+9a^3df-24a^2cfb\cdots\right.}{384b^2}$
default	$ce\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+(cf+de)\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\cdots}{\cdots}\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
3/128*(a^2*(a^2*d*f-8/3*a*b*(c*f+d*e)+16*c*e*b^2)*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))-(b*x^2+a)^(1/2)*x*(-80/3*a*(3/10*d*f*x^4+7/15*(c*f+d*e)*x^2+c*e
)*b^(5/2)-32/3*x^2*(1/2*d*f*x^4+2/3*(c*f+d*e)*x^2+c*e)*b^(7/2)+a^2*(2/3*(-
d*f*x^2-4*c*f-4*d*e)*b^(3/2)+a*d*f*b^(1/2))))/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx = \left[\frac{3\left(8\left(6a^2b^2c-a^3bd\right)e-\left(8a^3bc-3a^4d\right)f\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48b^4d\right.}{}\right.$$

$$-\frac{3\left(8\left(6a^2b^2c-a^3bd\right)e-\left(8a^3bc-3a^4d\right)f\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48b^4dfx^7+8\left(8b^4de+\left(8b^4c+9ab^3\right.\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/768*(3*(8*(6*a^2*b^2*c - a^3*b*d)*e - (8*a^3*b*c - 3*a^4*d)*f)*sqrt(b)*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d*f*x^7 + 8*(8
*b^4*d*e + (8*b^4*c + 9*a*b^3*d)*f)*x^5 + 2*(8*(6*b^4*c + 7*a*b^3*d)*e + (
56*a*b^3*c + 3*a^2*b^2*d)*f)*x^3 + 3*(8*(10*a*b^3*c + a^2*b^2*d)*e + (8*a^
2*b^2*c - 3*a^3*b*d)*f)*x)*sqrt(b*x^2 + a))/b^3, -1/384*(3*(8*(6*a^2*b^2*c
 - a^3*b*d)*e - (8*a^3*b*c - 3*a^4*d)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (48*b^4*d*f*x^7 + 8*(8*b^4*d*e + (8*b^4*c + 9*a*b^3*d)*f)*x^5
 + 2*(8*(6*b^4*c + 7*a*b^3*d)*e + (56*a*b^3*c + 3*a^2*b^2*d)*f)*x^3 + 3*(8
*(10*a*b^3*c + a^2*b^2*d)*e + (8*a^2*b^2*c - 3*a^3*b*d)*f)*x)*sqrt(b*x^2 +
 a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.87

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{bdfx^7}{8}+\frac{x^5\cdot\left(\frac{9abdf}{8}+b^2cf+b^2de\right)}{6b}+\frac{x^3\left(a^2df+2abcf+2abde-\frac{5a\left(\frac{9abdf}{8}+b^2cf+b^2de\right)}{6b}+b^2ce\right)}{4b}+\frac{x\left(a^2cf+\right.}{} \right. \\ a^{\frac{3}{2}}\left(cex+\frac{dfx^5}{5}+\frac{x^3(cf+de)}{3}\right)\end{cases}$$

input

```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)*(f*x**2+e),x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(b*d*f*x**7/8 + x**5*(9*a*b*d*f/8 + b**2*c*f +
 b**2*d*e)/(6*b) + x**3*(a**2*d*f + 2*a*b*c*f + 2*a*b*d*e - 5*a*(9*a*b*d*f
/8 + b**2*c*f + b**2*d*e)/(6*b) + b**2*c*e)/(4*b) + x*(a**2*c*f + a**2*d*e
 + 2*a*b*c*e - 3*a*(a**2*d*f + 2*a*b*c*f + 2*a*b*d*e - 5*a*(9*a*b*d*f/8 +
b**2*c*f + b**2*d*e)/(6*b) + b**2*c*e)/(4*b))/(2*b)) + (a**2*c*e - a*(a**2
*c*f + a**2*d*e + 2*a*b*c*e - 3*a*(a**2*d*f + 2*a*b*c*f + 2*a*b*d*e - 5*a*
(9*a*b*d*f/8 + b**2*c*f + b**2*d*e)/(6*b) + b**2*c*e)/(4*b))/(2*b))*Piecew
ise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)
/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(c*e*x + d*f*x**5/5 + x**3*(c*
f + d*e)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.15

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}dfx^3}{8\,b}+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}cex+\frac{3}{8}\sqrt{bx^2+a}\,acex-\frac{\left(bx^2+a\right)^{\frac{5}{2}}adfx}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^2dfx}{64\,b^2}+\frac{3\sqrt{bx^2+a}\,a^3dfx}{128\,b^2}+\frac{3\,a^2ce\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}+\frac{3\,a^4df\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}(de+cf)x}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}(de+cf)ax}{24\,b}-\frac{\sqrt{bx^2+a}\,(de+cf)a^2x}{16\,b}-\frac{(de+cf)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*d*f*x^3/b + 1/4*(b*x^2 + a)^(3/2)*c*e*x + 3/8*sqrt(b
*x^2 + a)*a*c*e*x - 1/16*(b*x^2 + a)^(5/2)*a*d*f*x/b^2 + 1/64*(b*x^2 + a)^
(3/2)*a^2*d*f*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*d*f*x/b^2 + 3/8*a^2*c*e*ar
csinh(b*x/sqrt(a*b))/sqrt(b) + 3/128*a^4*d*f*arcsinh(b*x/sqrt(a*b))/b^(5/2
) + 1/6*(b*x^2 + a)^(5/2)*(d*e + c*f)*x/b - 1/24*(b*x^2 + a)^(3/2)*(d*e +
c*f)*a*x/b - 1/16*sqrt(b*x^2 + a)*(d*e + c*f)*a^2*x/b - 1/16*(d*e + c*f)*a
^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\frac{1}{384}\left(2\left(4\left(6\,bdfx^2+\frac{8\,b^7de+8\,b^7cf+9\,ab^6df}{b^6}\right)x^2+\frac{48\,b^7ce+56\,ab^6de+56\,ab^6cf+3\,a^2b^5\cdots}{b^6}\right.\right.\cdots-\frac{\left(48\,a^2b^2ce-8\,a^3bde-8\,a^3bcf+3\,a^4df\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b*d*f*x^2 + (8*b^7*d*e + 8*b^7*c*f + 9*a*b^6*d*f)/b^6)*x^2
+ (48*b^7*c*e + 56*a*b^6*d*e + 56*a*b^6*c*f + 3*a^2*b^5*d*f)/b^6)*x^2 + 3*
(80*a*b^6*c*e + 8*a^2*b^5*d*e + 8*a^2*b^5*c*f - 3*a^3*b^4*d*f)/b^6)*sqrt(b
*x^2 + a)*x - 1/128*(48*a^2*b^2*c*e - 8*a^3*b*d*e - 8*a^3*b*c*f + 3*a^4*d*
f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2}(c+dx^2)(e+fx^2)\,dx = \int (b\,x^2+a)^{3/2}\,(d\,x^2+c)\,(f\,x^2+e)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.67

$$\int (a+bx^2)^{3/2}(c+dx^2)(e+fx^2)\,dx = \frac{-9\sqrt{b\,x^2+a}\,a^3bdfx+24\sqrt{b\,x^2+a}\,a^2b^2cfx+24\sqrt{b\,x^2+a}\,a^2b^2dex+6\sqrt{b\,x^2+a}\,a^2b^2df\,x^3+}{}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)*(f*x^2+e),x)
```

output

```
( - 9*sqrt(a + b*x**2)*a**3*b*d*f*x + 24*sqrt(a + b*x**2)*a**2*b**2*c*f*x
+ 24*sqrt(a + b*x**2)*a**2*b**2*d*e*x + 6*sqrt(a + b*x**2)*a**2*b**2*d*f*x
**3 + 240*sqrt(a + b*x**2)*a*b**3*c*e*x + 112*sqrt(a + b*x**2)*a*b**3*c*f*
x**3 + 112*sqrt(a + b*x**2)*a*b**3*d*e*x**3 + 72*sqrt(a + b*x**2)*a*b**3*d
*f*x**5 + 96*sqrt(a + b*x**2)*b**4*c*e*x**3 + 64*sqrt(a + b*x**2)*b**4*c*f
*x**5 + 64*sqrt(a + b*x**2)*b**4*d*e*x**5 + 48*sqrt(a + b*x**2)*b**4*d*f*x
**7 + 9*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d*f - 24*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*f - 24*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*e + 144*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*e)/(384*b**3)
```

3.293 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2} dx = -\frac{(4bde-4bcf-3adf)x\sqrt{a+bx^2}}{8f^2} + \frac{dx(a+bx^2)^{3/2}}{4f} + \frac{(3a^2df^2+8b^2e(de-cf)-12abf(de-cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}f^3} - \frac{(be-af)^{3/2}(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^3}$$

output
```
-1/8*(-3*a*d*f-4*b*c*f+4*b*d*e)*x*(b*x^2+a)^(1/2)/f^2+1/4*d*x*(b*x^2+a)^(3
/2)/f+1/8*(3*a^2*d*f^2+8*b^2*e*(-c*f+d*e)-12*a*b*f*(-c*f+d*e))*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^3-(-a*f+b*e)^(3/2)*(-c*f+d*e)*arctanh((-
a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx = \frac{fx\sqrt{a+bx^2}(5adf+b(-4de+4cf+2dfx^2))+\frac{8(-be+af)^{3/2}(de-cf)\arctan\left(\frac{-fx\sqrt{\cdots}}{\cdots}\right)}{\sqrt{e}}}{8f^3}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(5*a*d*f + b*(-4*d*e + 4*c*f + 2*d*f*x^2)) + (8*(-(b*
e) + a*f)^(3/2)*(d*e - c*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e +
f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/Sqrt[e] - ((3*a^2*d*f^2 + 8*b^2*e*(
d*e - c*f) + 12*a*b*f*(-(d*e) + c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/Sqrt[b])/(8*f^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {403, 25, 403, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}((4bde-4bcf-3adf)x^2+a(de-4cf))}{fx^2+e}dx}{4f}+\frac{dx(a+bx^2)^{3/2}}{4f}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\int \frac{\sqrt{bx^2+a}((4bde-4bcf-3adf)x^2+a(de-4cf))}{fx^2+e}dx}{4f}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\frac{\int \frac{a(af(5de-8cf)-4be(de-cf))-\left(8e(de-cf)b^2-12af(de-cf)b+3a^2df^2\right)x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}}{}$$

$\downarrow$ 398

$$\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\frac{\frac{8(be-af)^2(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{\left(3a^2df^2-12abf(de-cf)+8b^2e(de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{f}}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}$$

$\downarrow$ 224

$$\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\frac{\frac{8(be-af)^2(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{\left(3a^2df^2-12abf(de-cf)+8b^2e(de-cf)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}$$

$\downarrow$ 219

$$\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\frac{\frac{8(be-af)^2(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2df^2-12abf(de-cf)+8b^2e(de-cf)\right)}{\sqrt{b}f}}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}$$

$\downarrow$ 291

$$\frac{dx(a+bx^2)^{3/2}}{4f}-\frac{\frac{\frac{8(be-af)^2(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2df^2-12abf(de-cf)+8b^2e(de-cf)\right)}{\sqrt{b}f}}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}$$

$\downarrow$ 221

$$\frac{dx\left(a+bx^2\right)^{3/2}}{4f}-\frac{\frac{\frac{8(be-af)^{3/2}(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2df^2-12abf(de-cf)+8b^2e(de-cf)\right)}{\sqrt{b}f}}{2f}+\frac{x\sqrt{a+bx^2}(-3adf-4bcf+4bde)}{2f}}{4f}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2),x]
```

output
```
(d*x*(a + b*x^2)^(3/2))/(4*f) - (((4*b*d*e - 4*b*c*f - 3*a*d*f)*x*Sqrt[a +
 b*x^2])/(2*f) + (-(((3*a^2*d*f^2 + 8*b^2*e*(d*e - c*f) - 12*a*b*f*(d*e -
c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f)) + (8*(b*e - a*f)^
(3/2)*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/
(Sqrt[e]*f))/(2*f))/(4*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\frac{f\sqrt{b\,x^2+a}\left(2bdf\,x^2+5adf+4bcf-4bde\right)x}{4}-\frac{\left(3a^2d\,f^2+12abc\,f^2-12abdef-8b^2cef+8b^2d\,e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{4\sqrt{b}}+\frac{2(af-be)^2(cf\cdots}{\cdots}}{2f^3}$
risch	$\frac{x\left(2bdf\,x^2+5adf+4bcf-4bde\right)\sqrt{b\,x^2+a}}{8f^2}+\frac{\frac{\left(3a^2d\,f^2+12abc\,f^2-12abdef-8b^2cef+8b^2d\,e^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{f\sqrt{b}}+\frac{4\left(a^2c\,f^3-a^2d\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f^3*(-1/4*f*(b*x^2+a)^(1/2)*(2*b*d*f*x^2+5*a*d*f+4*b*c*f-4*b*d*e)*x-1
/4*(3*a^2*d*f^2+12*a*b*c*f^2-12*a*b*d*e*f-8*b^2*c*e*f+8*b^2*d*e^2)/b^(1/2)
*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+2*(a*f-b*e)^2*(c*f-d*e)/((a*f-b*e)*e)^
(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.98

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/16*((8*b^2*d*e^2 - 4*(2*b^2*c + 3*a*b*d)*e*f + 3*(4*a*b*c + a^2*d)*f^2)
*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*d*e^2 +
a*b*c*f^2 - (b^2*c + a*b*d)*e*f)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a
*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2
*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4
+ 2*e*f*x^2 + e^2)) + 2*(2*b^2*d*f^2*x^3 - (4*b^2*d*e*f - (4*b^2*c + 5*a*b
*d)*f^2)*x)*sqrt(b*x^2 + a))/(b*f^3), -1/8*((8*b^2*d*e^2 - 4*(2*b^2*c + 3*
a*b*d)*e*f + 3*(4*a*b*c + a^2*d)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - 2*(b^2*d*e^2 + a*b*c*f^2 - (b^2*c + a*b*d)*e*f)*sqrt((b*e - a*f)
/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 -
3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt(
(b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - (2*b^2*d*f^2*x^3 - (4*b^2*d
*e*f - (4*b^2*c + 5*a*b*d)*f^2)*x)*sqrt(b*x^2 + a))/(b*f^3), 1/16*(8*(b^2*
d*e^2 + a*b*c*f^2 - (b^2*c + a*b*d)*e*f)*sqrt(-(b*e - a*f)/e)*arctan(1/2*(
(2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(-(b*e - a*f)/e)/((b^2*e - a*
b*f)*x^3 + (a*b*e - a^2*f)*x)) + (8*b^2*d*e^2 - 4*(2*b^2*c + 3*a*b*d)*e*f
+ 3*(4*a*b*c + a^2*d)*f^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(2*b^2*d*f^2*x^3 - (4*b^2*d*e*f - (4*b^2*c + 5*a*b*d)*f^2)*x)
*sqrt(b*x^2 + a))/(b*f^3), -1/8*((8*b^2*d*e^2 - 4*(2*b^2*c + 3*a*b*d)*e*f
+ 3*(4*a*b*c + a^2*d)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) ...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.77

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{e+fx^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e),x)
```

output
```
( - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c*f**2 + 8*sqrt(e)*sqrt(a
*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(
b)*x)/(sqrt(e)*sqrt(b)))*a*b*d*e*f + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*b**2*c*e*f - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)
*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*d*e**2 - 8*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c*f**2 + 8*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a*b*d*e*f + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*
*2*c*e*f - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(
a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*d*e**2 + 5*sqrt(a
 + b*x**2)*a*b*d*e*f**2*x + 4*sqrt(a + b*x**2)*b**2*c*e*f**2*x - 4*sqrt(a
+ b*x**2)*b**2*d*e**2*f*x + 2*sqrt(a + b*x**2)*b**2*d*e*f**2*x**3 + 3*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*e*f**2 + 12*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*e*f**2 - 12*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*e**2*f - 8*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c*e**2*f + 8*sqrt(b)*log((sqrt...
```

3.294 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 199

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{b(2de-cf)x\sqrt{a+bx^2}}{2ef^2} - \frac{(de-cf)x(a+bx^2)^{3/2}}{2ef(e+fx^2)} - \frac{\sqrt{b}(4bde-2bcf-3adf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2f^3} + \frac{\sqrt{be-af}(2be(2de-cf)-af(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^3}$$

output

```
1/2*b*(-c*f+2*d*e)*x*(b*x^2+a)^(1/2)/e/f^2-1/2*(-c*f+d*e)*x*(b*x^2+a)^(3/2
)/e/f/(f*x^2+e)-1/2*b^(1/2)*(-3*a*d*f-2*b*c*f+4*b*d*e)*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/f^3+1/2*(-a*f+b*e)^(1/2)*(2*b*e*(-c*f+2*d*e)-a*f*(c*f+d*e)
)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^3
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{\frac{fx\sqrt{a+bx^2}(af(-de+cf)+be(2de-cf+dfx^2))}{e(e+fx^2)} + \frac{\sqrt{-be+af}(2be(2de-cf)-af(de+cf))\arctan\left(\frac{-fx\sqrt{\cdots}}{\cdots}\right.}{e^{3/2}}}{2f^3}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(a*f*(-(d*e) + c*f) + b*e*(2*d*e - c*f + d*f*x^2)))/
(e*(e + f*x^2)) + (Sqrt[-(b*e) + a*f]*(2*b*e*(2*d*e - c*f) - a*f*(d*e + c*
f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(
b*e) + a*f])])/e^(3/2) + Sqrt[b]*(4*b*d*e - 2*b*c*f - 3*a*d*f)*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*f^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {401, 25, 403, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(2b(2de-cf)x^2+a(de+cf))}{fx^2+e}dx}{2ef} - \frac{x(a+bx^2)^{3/2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(2b(2de-cf)x^2+a(de+cf))}{fx^2+e}dx}{2ef} - \frac{x(a+bx^2)^{3/2}(de-cf)}{2ef(e+fx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int -\frac{2\left(be(4bde-2bcf-3adf)x^2+a(be(2de-cf)-af(de+cf))\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\int \frac{be(4bde-2bcf-3adf)x^2+a(be(2de-cf)-af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\frac{be(-3adf-2bcf+4bde)\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)(2be(2de-cf)-af(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\frac{be(-3adf-2bcf+4bde)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)(2be(2de-cf)-af(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\frac{\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-3adf-2bcf+4bde)}{f}-\frac{(be-af)(2be(2de-cf)-af(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\frac{\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-3adf-2bcf+4bde)}{f}-\frac{(be-af)(2be(2de-cf)-af(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{bx\sqrt{a+bx^2}(2de-cf)}{f}-\frac{\frac{\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-3adf-2bcf+4bde)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(cf+de))}{\sqrt{e}f}}{f}}{2ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{2ef\left(e+fx^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

output
```
-1/2*((d*e - c*f)*x*(a + b*x^2)^(3/2))/(e*f*(e + f*x^2)) + ((b*(2*d*e - c*
f)*x*Sqrt[a + b*x^2])/f - ((Sqrt[b]*e*(4*b*d*e - 2*b*c*f - 3*a*d*f)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt[b*e - a*f]*(2*b*e*(2*d*e - c*f)
- a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])
/(Sqrt[e]*f))/f)/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\sqrt{b}\left(-2bd\,e^{2}+\frac{f(ad+2bc)e}{2}+\frac{ac\,f^{2}}{2}\right)\left(f\,x^{2}+e\right)(-af+be)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)-\frac{\left(-3\left(-\frac{4bde}{3}+f\left(ad+\frac{2bc}{3}\right)\right)b\left(f\,x^{2}+e\right)e\ \mathrm{arctanl}\right.}{}}{\sqrt{(af-be)e}\,\sqrt{b}\,f^{3}e(f\,x^{2}+e)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^(1/2)*(-2*b*d*e^2+1/2*f*(a*d+2*b*c)*e+1/2*a*c*f^2)*(f*x^2+e)*(-a*f+b*e)
*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-1/2*(-3*(-4/3*b*d*e+f*(a*
d+2/3*b*c))*b*(f*x^2+e)*e*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(-2*b*d*e^2+(
(-d*x^2+c)*b+a*d)*f*e-a*c*f^2)*(b*x^2+a)^(1/2)*b^(1/2)*x*f)*((a*f-b*e)*e)^
(1/2))/((a*f-b*e)*e)^(1/2)/b^(1/2)/f^3/e/(f*x^2+e)
```

Fricas [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 1324, normalized size of antiderivative = 6.65

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(4*b*d*e^3 - (2*b*c + 3*a*d)*e^2*f + (4*b*d*e^2*f - (2*b*c + 3*a*
d)*e*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (
4*b*d*e^3 - a*c*e*f^2 - (2*b*c + a*d)*e^2*f + (4*b*d*e^2*f - a*c*f^3 - (2*
b*c + a*d)*e*f^2)*x^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a
^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*
e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^
2 + e^2)) - 4*(b*d*e*f^2*x^3 + (2*b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)
*x)*sqrt(b*x^2 + a))/(e*f^4*x^2 + e^2*f^3), 1/8*(4*(4*b*d*e^3 - (2*b*c + 3
*a*d)*e^2*f + (4*b*d*e^2*f - (2*b*c + 3*a*d)*e*f^2)*x^2)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (4*b*d*e^3 - a*c*e*f^2 - (2*b*c + a*d)*e^2*f
+ (4*b*d*e^2*f - a*c*f^3 - (2*b*c + a*d)*e*f^2)*x^2)*sqrt((b*e - a*f)/e)*l
og(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2
*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e
- a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*(b*d*e*f^2*x^3 + (2*b*d*e^2*f
+ a*c*f^3 - (b*c + a*d)*e*f^2)*x)*sqrt(b*x^2 + a))/(e*f^4*x^2 + e^2*f^3),
-1/4*((4*b*d*e^3 - a*c*e*f^2 - (2*b*c + a*d)*e^2*f + (4*b*d*e^2*f - a*c*f^
3 - (2*b*c + a*d)*e*f^2)*x^2)*sqrt(-(b*e - a*f)/e)*arctan(1/2*((2*b*e - a*
f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(-(b*e - a*f)/e)/((b^2*e - a*b*f)*x^3 +
(a*b*e - a^2*f)*x)) + (4*b*d*e^3 - (2*b*c + 3*a*d)*e^2*f + (4*b*d*e^2*f -
(2*b*c + 3*a*d)*e*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{\sqrt{bx^2+a}bdx}{2f^2} + \frac{\left(4b^{\frac{3}{2}}de - 2b^{\frac{3}{2}}cf - 3a\sqrt{b}df\right)\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{4f^3} - \frac{\left(4b^{\frac{5}{2}}de^3 - 2b^{\frac{5}{2}}ce^2f - 5ab^{\frac{3}{2}}de^2f + ab^{\frac{3}{2}}cef^2 + a^2\sqrt{b}def^2 + a^2\sqrt{b}cf^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2be-af}{2\sqrt{-b^2e^2+abef}}\right)}{2\sqrt{-b^2e^2+abef}ef^3} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}de^3 - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}ce^2f - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}de^2f + 3\left(\sqrt{b}x\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f+4\left(\right.\right.}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b*d*x/f^2 + 1/4*(4*b^(3/2)*d*e - 2*b^(3/2)*c*f - 3*a*s
qrt(b)*d*f)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^3 - 1/2*(4*b^(5/2)*d*e^
3 - 2*b^(5/2)*c*e^2*f - 5*a*b^(3/2)*d*e^2*f + a*b^(3/2)*c*e*f^2 + a^2*sqrt
(b)*d*e*f^2 + a^2*sqrt(b)*c*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*e*
f^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*d*e^3 - 2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*b^(5/2)*c*e^2*f - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^
(3/2)*d*e^2*f + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2)*c*e*f^2 + (sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*d*e*f^2 - (sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^2*sqrt(b)*c*f^3 + a^2*b^(3/2)*d*e^2*f - a^2*b^(3/2)*c*e*f^2 - a^
3*sqrt(b)*d*e*f^2 + a^3*sqrt(b)*c*f^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*f
 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a*f + a^2*f)*e*f^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1227, normalized size of antiderivative = 6.17

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*e*f**2 - sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a*c*f**3*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a*d*e**2*f - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqr
t(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*d*e*f**2*x**2 - 2*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*e**2*f - 2*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*b*c*e*f**2*x**2 + 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*b*d*e**3 + 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*s
qrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*d*e**2*f*x**2 -
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*e*f**2 - sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a*c*f**3*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*
d*e**2*f - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt...
```

3.295 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 208

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = -\frac{(de-cf)x(a+bx^2)^{3/2}}{4ef(e+fx^2)^2} - \frac{(4bde^2-af(de+3cf))\,x\sqrt{a+bx^2}}{8e^2f^2(e+fx^2)} + \frac{b^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f^3} - \frac{(8b^2de^3-af(4bde^2+af(de+3cf)))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}f^3\sqrt{be-af}}$$

output

```
-1/4*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/e/f/(f*x^2+e)^2-1/8*(4*b*d*e^2-a*f*(3*c*
f+d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(f*x^2+e)+b^(3/2)*d*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/f^3-1/8*(8*b^2*d*e^3-a*f*(4*b*d*e^2+a*f*(3*c*f+d*e)))*arcta
nh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/f^3/(-a*f+b*e)^(1/2
)
```

Mathematica [A] (verified)

Time = 10.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{\frac{fx\sqrt{a+bx^2}(2be(cf^2x^2-de(2e+3fx^2))+af(de(-e+fx^2)+cf(5e+3fx^2)))}{e^2(e+fx^2)^2}+\frac{(-8b^2de^3+4abde^2f+a^2\ldots}{e\ldots}}{8f^3}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(2*b*e*(c*f^2*x^2 - d*e*(2*e + 3*f*x^2)) + a*f*(d*e*
(-e + f*x^2) + c*f*(5*e + 3*f*x^2))))/(e^2*(e + f*x^2)^2) + ((-8*b^2*d*e^3
 + 4*a*b*d*e^2*f + a^2*f^2*(d*e + 3*c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(S
qrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*Sqrt[-(b*e) + a*f]) + 8*b^(3/2)*d*Log[b
*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*f^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {401, 25, 401, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(4bdex^2+a(de+3cf))}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)^{3/2}(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(4bdex^2+a(de+3cf))}{(fx^2+e)^2}dx}{4ef}-\frac{x(a+bx^2)^{3/2}(de-cf)}{4ef(e+fx^2)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{-\frac{\int -\frac{8b^2de^2x^2+a\left(4bde^2+af(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8b^2de^2x^2+a\left(4bde^2+af(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\frac{8b^2de^2\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(8b^2de^3-af\left(af(3cf+de)+4bde^2\right)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{8b^2de^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(8b^2de^3-af\left(af(3cf+de)+4bde^2\right)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{8b^{3/2}de^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(8b^2de^3-af\left(af(3cf+de)+4bde^2\right)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{8b^{3/2}de^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(8b^2de^3-af\left(af(3cf+de)+4bde^2\right)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{8b^{3/2}de^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(8b^2de^3-af\left(af(3cf+de)+4bde^2\right)\right)}{\sqrt{e}f\sqrt{be-af}}}{2ef}-\frac{x\sqrt{a+bx^2}\left(\frac{4bde}{f}-\frac{a(3cf+de)}{e}\right)}{2(e+fx^2)}}{4ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{4ef\left(e+fx^2\right)^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

output
```
-1/4*((d*e - c*f)*x*(a + b*x^2)^(3/2))/(e*f*(e + f*x^2)^2) + (-1/2*(((4*b*
d*e)/f - (a*(d*e + 3*c*f))/e)*x*Sqrt[a + b*x^2])/(e + f*x^2) + ((8*b^(3/2)
*d*e^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((8*b^2*d*e^3 - a*f*(4*b*
d*e^2 + a*f*(d*e + 3*c*f)))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f]))/(2*e*f))/(4*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{3\left(\left(a^{2} c\, f^{3}+\frac{1}{3} a^{2} d e\, f^{2}+\frac{4}{3} a b d\, e^{2} f-\frac{8}{3} b^{2} d\, e^{3}\right)\left(f\, x^{2}+e\right)^{2} \arctan\left(\frac{e \sqrt{b\, x^{2}+a}}{x \sqrt{\left(a f-b e\right) e}}\right)-\frac{5 \sqrt{\left(a f-b e\right) e}\, \left(\frac{8 b^{\frac{3}{2}} d\, e^{2} \left(f\, x^{2}+e\right)^{2} \operatorname{arctanh}\left(\right.}{5}\right.}{}\right.}{8 \sqrt{\left(a f-b e\right) e}\, f^{3} \left(f\, x^{2}+e\right)^{2} e^{2}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*((a^2*c*f^3+1/3*a^2*d*e*f^2+4/3*a*b*d*e^2*f-8/3*b^2*d*e^3)*(f*x^2+e)^
2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-5/3*((a*f-b*e)*e)^(1/2)*
(8/5*b^(3/2)*d*e^2*(f*x^2+e)^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(-4/5*b*
d*e^3-1/5*d*f*(6*b*x^2+a)*e^2+(1/5*(a*d+2*b*c)*x^2+a*c)*f^2*e+3/5*a*c*f^3*
x^2)*(b*x^2+a)^(1/2)*x*f))/((a*f-b*e)*e)^(1/2)/f^3/(f*x^2+e)^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(182) = 364$.

Time = 1.95 (sec) , antiderivative size = 2206, normalized size of antiderivative = 10.61

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(b^2*d*e^6 - a*b*d*e^5*f + (b^2*d*e^4*f^2 - a*b*d*e^3*f^3)*x^4 +
 2*(b^2*d*e^5*f - a*b*d*e^4*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - (8*b^2*d*e^5 - 4*a*b*d*e^4*f - a^2*d*e^3*f^2 - 3*a^2
*c*e^2*f^3 + (8*b^2*d*e^3*f^2 - 4*a*b*d*e^2*f^3 - a^2*d*e*f^4 - 3*a^2*c*f^
5)*x^4 + 2*(8*b^2*d*e^4*f - 4*a*b*d*e^3*f^2 - a^2*d*e^2*f^3 - 3*a^2*c*e*f^
4)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a
^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqr
t(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((6*b^2
*d*e^4*f^2 + 3*a^2*c*e*f^5 - (2*b^2*c + 7*a*b*d)*e^3*f^3 - (a*b*c - a^2*d)
*e^2*f^4)*x^3 + (4*b^2*d*e^5*f - 3*a*b*d*e^4*f^2 + 5*a^2*c*e^2*f^4 - (5*a*
b*c + a^2*d)*e^3*f^3)*x)*sqrt(b*x^2 + a))/(b*e^6*f^3 - a*e^5*f^4 + (b*e^4*
f^5 - a*e^3*f^6)*x^4 + 2*(b*e^5*f^4 - a*e^4*f^5)*x^2), -1/32*(32*(b^2*d*e^
6 - a*b*d*e^5*f + (b^2*d*e^4*f^2 - a*b*d*e^3*f^3)*x^4 + 2*(b^2*d*e^5*f - a
*b*d*e^4*f^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^2*d*
e^5 - 4*a*b*d*e^4*f - a^2*d*e^3*f^2 - 3*a^2*c*e^2*f^3 + (8*b^2*d*e^3*f^2 -
 4*a*b*d*e^2*f^3 - a^2*d*e*f^4 - 3*a^2*c*f^5)*x^4 + 2*(8*b^2*d*e^4*f - 4*a
*b*d*e^3*f^2 - a^2*d*e^2*f^3 - 3*a^2*c*e*f^4)*x^2)*sqrt(b*e^2 - a*e*f)*log
(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e
*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a
))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((6*b^2*d*e^4*f^2 + 3*a^2*c*e*f^5 -...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(182) = 364$.

Time = 0.18 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.25

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/2*b^(3/2)*d*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^3 + 1/8*(8*b^(5/2)*d
*e^3 - 4*a*b^(3/2)*d*e^2*f - a^2*sqrt(b)*d*e*f^2 - 3*a^2*sqrt(b)*c*f^3)*ar
ctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 +
 a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*e^2*f^3) - 1/4*(16*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*b^(5/2)*d*e^3*f - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*
c*e^2*f^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*f^2 + (sqrt
(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2
 + a))^6*a^2*sqrt(b)*c*f^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*
e^4 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 56*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a*b^(5/2)*d*e^3*f - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
a*b^(5/2)*c*e^2*f^2 + 26*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*e^2
*f^2 + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 - 3*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d*e*f^3 - 9*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a^3*sqrt(b)*c*f^4 + 32*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*d
*e^3*f - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 - 20*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*d*e^2*f^2 - 16*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^3*b^(3/2)*c*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*s
qrt(b)*d*e*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*c*f^4 + 6*a
^4*b^(3/2)*d*e^2*f^2 - 2*a^4*b^(3/2)*c*e*f^3 - a^5*sqrt(b)*d*e*f^3 - 3*a^5
*sqrt(b)*c*f^4)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3328, normalized size of antiderivative = 16.00

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^3,x)
```

output

```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e**2*f**4 - 12*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e*f**5*x**2 - 6*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**3*c*f**6*x**4 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**3*d*e**3*f**3 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e
**2*f**4*x**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e*f**5*x**
4 + 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**3*f**3 + 24*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**2*f**4*x**2 + 12*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e*f**5*x**4 - 4*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a**2*b*d*e**4*f**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sq
rt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*...
```

3.296 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 227

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4} dx = \frac{(de-cf)x(a+bx^2)^{5/2}}{6e(be-af)(e+fx^2)^3} + \frac{(6bce-ade-5acf)x(a+bx^2)^{3/2}}{24e^2(be-af)(e+fx^2)^2} + \frac{a(6bce-ade-5acf)x\sqrt{a+bx^2}}{16e^3(be-af)(e+fx^2)} + \frac{a^2(6bce-ade-5acf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{16e^{7/2}(be-af)^{3/2}}$$

output
```
1/6*(-c*f+d*e)*x*(b*x^2+a)^(5/2)/e/(-a*f+b*e)/(f*x^2+e)^3+1/24*(-5*a*c*f-a
*d*e+6*b*c*e)*x*(b*x^2+a)^(3/2)/e^2/(-a*f+b*e)/(f*x^2+e)^2+1/16*a*(-5*a*c*
f-a*d*e+6*b*c*e)*x*(b*x^2+a)^(1/2)/e^3/(-a*f+b*e)/(f*x^2+e)+1/16*a^2*(-5*a
*c*f-a*d*e+6*b*c*e)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^
(7/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 13.24 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{x\left(\frac{6de(a+bx^2)\left(\sqrt{\frac{(-be+af)x^2}{ae}}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}(5ae+2bex^2+3afx^2)+3a(e+fx^2)\sqrt{1+\frac{fx^2}{e}}\arcsin\left(\frac{\sqrt{(-\cdots}}{\sqrt{1\cdots}}\right.\right.}{\sqrt{\frac{(-be+af)x^2}{ae}}\sqrt{1+\frac{bx^2}{a}}(e+fx^2)\sqrt{1+\frac{fx^2}{e}}}\right.}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
(x*((6*d*e*(a + b*x^2)*(Sqrt[((-(b*e) + a*f)*x^2)/(a*e)]*Sqrt[(e*(a + b*x^
2))/(a*(e + f*x^2))]*(5*a*e + 2*b*e*x^2 + 3*a*f*x^2) + 3*a*(e + f*x^2)*Sqr
t[1 + (f*x^2)/e]*ArcSin[Sqrt[(-(b/a) + f/e)*x^2]/Sqrt[1 + (f*x^2)/e]]))/(S
qrt[((-(b*e) + a*f)*x^2)/(a*e)]*Sqrt[1 + (b*x^2)/a]*(e + f*x^2)*Sqrt[1 + (
f*x^2)/e]) - ((d*e - c*f)*(e*(a + b*x^2)*(4*b^2*e^2*x^2*(3*e + f*x^2) + 2*
a*b*e*(15*e^2 + 11*e*f*x^2 + 4*f^2*x^4) - a^2*f*(33*e^2 + 40*e*f*x^2 + 15*
f^2*x^4)) + (3*a^2*(6*b*e - 5*a*f)*(e + f*x^2)^3*ArcTanh[Sqrt[((b*e - a*f)
*x^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/((b*e -
 a*f)*(e + f*x^2)^3)))/(48*e^4*f*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {401, 25, 401, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(2b(2de+cf)x^2+a(de+5cf)\right)}{(fx^2+e)^3}dx}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2b(2de+cf)x^2+a(de+5cf)\right)}{(fx^2+e)^3}dx}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 401$$

$$\frac{-\frac{\int -\frac{2b(2be(2de+cf)+af(de+5cf))x^2+a(2be(2de+cf)+3af(de+5cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2b(2be(2de+cf)+af(de+5cf))x^2+a(2be(2de+cf)+3af(de+5cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int \frac{3a^2f^2(6bce-ade-5acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)+2abef(de-4cf)-4b^2e^2(cf+2de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf+de))}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3a^2f^2(-5acf-ade+6bce)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)+2abef(de-4cf)-4b^2e^2(cf+2de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf+de}{4ef(e+fx^2)^2}}{6ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\frac{3a^2f^2(-5acf-ade+6bce)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)+2abef(de-4cf)-4b^2e^2(cf+2de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf}{4ef(e+fx^2)^2}}{6ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{3a^2f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-5acf-ade+6bce)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(5cf+de)+2abef(de-4cf)-4b^2e^2(cf+2de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4ef}-\frac{x\sqrt{a+bx^2}(2be(cf+2de)-af(5cf-}{4ef(e+fx^2)^2}}{6ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{6ef\left(e+fx^2\right)^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

output
```
-1/6*((d*e - c*f)*x*(a + b*x^2)^(3/2))/(e*f*(e + f*x^2)^3) + (-1/4*((2*b*e
*(2*d*e + c*f) - a*f*(d*e + 5*c*f))*x*Sqrt[a + b*x^2])/(e*f*(e + f*x^2)^2)
 + (-1/2*((2*a*b*e*f*(d*e - 4*c*f) - 4*b^2*e^2*(2*d*e + c*f) + 3*a^2*f^2*(
d*e + 5*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) + (3*a^2*f^2*
(6*b*c*e - a*d*e - 5*a*c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*f))/(6*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{5\left(a^{2}\left(\frac{(ad-6bc)e}{5}+acf\right)\left(f\,x^{2}+e\right)^{3}\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)-\frac{11\left(\frac{\left(-d\,a^{2}-10\left(\frac{7x^{2}d}{15}+c\right)ba-4\left(\frac{2x^{2}d}{3}+c\right)b^{2}x^{2}\right)e^{3}}{11}+\left(\left(\frac{8x^{2}d}{33}+c\right.\right.\right.}{}\right.}{16\sqrt{(af-be)e}\,(af-be)e}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16/((a*f-b*e)*e)^(1/2)*(a^2*(1/5*(a*d-6*b*c)*e+a*c*f)*(f*x^2+e)^3*arcta
n(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-11/5*(1/11*(-d*a^2-10*(7/15*x^2
*d+c)*b*a-4*(2/3*x^2*d+c)*b^2*x^2)*e^3+((8/33*x^2*d+c)*a^2-2/3*(-1/11*x^2*
d+c)*b*x^2*a-4/33*b^2*c*x^4)*f*e^2+40/33*((3/40*x^2*d+c)*a-1/5*x^2*b*c)*a*
x^2*f^2*e+5/11*a^2*c*f^3*x^4)*((a*f-b*e)*e)^(1/2)*(b*x^2+a)^(1/2)*x)/(a*f-
b*e)/e^3/(f*x^2+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(203) = 406$.

Time = 1.60 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.47

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*a^3*c*e^3*f + (5*a^3*c*f^4 - (6*a^2*b*c - a^3*d)*e*f^3)*x^6
- (6*a^2*b*c - a^3*d)*e^4 + 3*(5*a^3*c*e*f^3 - (6*a^2*b*c - a^3*d)*e^2*f^2
)*x^4 + 3*(5*a^3*c*e^2*f^2 - (6*a^2*b*c - a^3*d)*e^3*f)*x^2)*sqrt(b*e^2 -
a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2
 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt
(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((8*b^3*d*e^5 + 15*a^3*c*e*f
^4 + 2*(2*b^3*c - 5*a*b^2*d)*e^4*f + (4*a*b^2*c - a^2*b*d)*e^3*f^2 - (23*a
^2*b*c - 3*a^3*d)*e^2*f^3)*x^5 + 2*(20*a^3*c*e^2*f^3 + (6*b^3*c + 7*a*b^2*
d)*e^5 + (5*a*b^2*c - 11*a^2*b*d)*e^4*f - (31*a^2*b*c - 4*a^3*d)*e^3*f^2)*
x^3 + 3*(11*a^3*c*e^3*f^2 + (10*a*b^2*c + a^2*b*d)*e^5 - (21*a^2*b*c + a^3
*d)*e^4*f)*x)*sqrt(b*x^2 + a))/(b^2*e^9 - 2*a*b*e^8*f + a^2*e^7*f^2 + (b^2
*e^6*f^3 - 2*a*b*e^5*f^4 + a^2*e^4*f^5)*x^6 + 3*(b^2*e^7*f^2 - 2*a*b*e^6*f
^3 + a^2*e^5*f^4)*x^4 + 3*(b^2*e^8*f - 2*a*b*e^7*f^2 + a^2*e^6*f^3)*x^2),
1/96*(3*(5*a^3*c*e^3*f + (5*a^3*c*f^4 - (6*a^2*b*c - a^3*d)*e*f^3)*x^6 - (
6*a^2*b*c - a^3*d)*e^4 + 3*(5*a^3*c*e*f^3 - (6*a^2*b*c - a^3*d)*e^2*f^2)*x
^4 + 3*(5*a^3*c*e^2*f^2 - (6*a^2*b*c - a^3*d)*e^3*f)*x^2)*sqrt(-b*e^2 + a*
e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2
+ a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) + 2*((8*b^3*d*e^5
+ 15*a^3*c*e*f^4 + 2*(2*b^3*c - 5*a*b^2*d)*e^4*f + (4*a*b^2*c - a^2*b*d)*e
^3*f^2 - (23*a^2*b*c - 3*a^3*d)*e^2*f^3)*x^5 + 2*(20*a^3*c*e^2*f^3 + (6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1754 vs. $2(203) = 406$.

Time = 0.71 (sec) , antiderivative size = 1754, normalized size of antiderivative = 7.73

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-1/16*(6*a^2*b^(3/2)*c*e - a^3*sqrt(b)*d*e - 5*a^3*sqrt(b)*c*f)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f
))/((b*e^4 - a*e^3*f)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/24*(48*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*b^(7/2)*d*e^4*f^2 - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a*b^(5/2)*d*e^3*f^3 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*e*
f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d*e*f^5 + 15*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*f^6 + 192*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*b^(9/2)*d*e^5*f + 96*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*e^4*
f^2 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d*e^4*f^2 - 96*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c*e^3*f^3 + 48*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^2*b^(5/2)*d*e^3*f^3 - 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^
(5/2)*c*e^2*f^4 + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*d*e^2*f^4
 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*e*f^5 - 15*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d*e*f^5 - 75*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^4*sqrt(b)*c*f^6 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*d*e^
6 + 128*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c*e^5*f - 448*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*a*b^(9/2)*d*e^5*f + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*a*b^(9/2)*c*e^4*f^2 + 288*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*d*
e^4*f^2 - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c*e^3*f^3 - 8*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*d*e^3*f^3 + 968*(sqrt(b)*x - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}{(f\,x^2+e)^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 3988, normalized size of antiderivative = 17.57

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input

```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^4,x)
```

output

```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**3*f**4 - 45*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**2*f**5*x**2 - 45*sqrt(e)*sqrt(a
*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(
b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e*f**6*x**4 - 15*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a**4*c*f**7*x**6 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
- b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*
a**4*d*e**4*f**3 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**3*f*
*4*x**2 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**2*f**5*x**4 -
 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e*f**6*x**6 + 48*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**4*f**3 + 144*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**3*f**4*x**2 + 144*sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/...
```

3.297 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 379

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx &= -\frac{(de-cf)x(a+bx^2)^{3/2}}{8ef(e+fx^2)^4} \\
&- \frac{(4be(de+cf)-af(de+7cf))x\sqrt{a+bx^2}}{48e^2f^2(e+fx^2)^3} \\
&+ \frac{(24abcef^2+8b^2e^2(de+cf)-5a^2f^2(de+7cf))\,x\sqrt{a+bx^2}}{192e^3f^2(be-af)(e+fx^2)^2} \\
&- \frac{(8ab^2e^2f(de-5cf)-16b^3e^3(de+cf)-15a^3f^3(de+7cf)+2a^2bef^2(7de+85cf))\,x\sqrt{a+bx^2}}{384e^4f^2(be-af)^2(e+fx^2)} \\
&+ \frac{a^2(48b^2ce^2+5a^2f(de+7cf)-8abe(de+10cf))\,\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{128e^{9/2}(be-af)^{5/2}}
\end{aligned}$$

output
```
-1/8*(-c*f+d*e)*x*(b*x^2+a)^(3/2)/e/f/(f*x^2+e)^4-1/48*(4*b*e*(c*f+d*e)-a*
f*(7*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(f*x^2+e)^3+1/192*(24*a*b*c*e*f^2
+8*b^2*e^2*(c*f+d*e)-5*a^2*f^2*(7*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^3/f^2/(-a*
f+b*e)/(f*x^2+e)^2-1/384*(8*a*b^2*e^2*f*(-5*c*f+d*e)-16*b^3*e^3*(c*f+d*e)-
15*a^3*f^3*(7*c*f+d*e)+2*a^2*b*e*f^2*(85*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^4
/f^2/(-a*f+b*e)^2/(f*x^2+e)+1/128*a^2*(48*b^2*c*e^2+5*a^2*f*(7*c*f+d*e)-8*
a*b*e*(10*c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^
(9/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 12.75 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx = \frac{x\left(8de(be-af)(a+bx^2)(e+fx^2)\left(e(a+bx^2)(4b^2e^2x^2(3e+fx^2)+2abe\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^5,x]
```

output
```
(x*(8*d*e*(b*e - a*f)*(a + b*x^2)*(e + f*x^2)*(e*(a + b*x^2)*(4*b^2*e^2*x^
2*(3*e + f*x^2) + 2*a*b*e*(15*e^2 + 11*e*f*x^2 + 4*f^2*x^4) - a^2*f*(33*e^
2 + 40*e*f*x^2 + 15*f^2*x^4)) + (3*a^2*(6*b*e - 5*a*f)*(e + f*x^2)^3*ArcTa
nh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a
+ b*x^2))]) + a*(-(d*e) + c*f)*(1 + (b*x^2)/a)*(e*(a + b*x^2)*(16*b^3*e^3*
x^2*(6*e^2 + 4*e*f*x^2 + f^2*x^4) + 8*a*b^2*e^2*(30*e^3 + 26*e^2*f*x^2 + 1
9*e*f^2*x^4 + 5*f^3*x^6) - 2*a^2*b*e*f*(264*e^3 + 421*e^2*f*x^2 + 314*e*f^
2*x^4 + 85*f^3*x^6) + a^3*f^2*(279*e^3 + 511*e^2*f*x^2 + 385*e*f^2*x^4 + 1
05*f^3*x^6)) + (3*a^2*(48*b^2*e^2 - 80*a*b*e*f + 35*a^2*f^2)*(e + f*x^2)^4
*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/
(e*(a + b*x^2))])))/(384*e^5*f*(b*e - a*f)^2*(a + b*x^2)^(3/2)*(e + f*x^2)
^4)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {401, 25, 401, 25, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{\left(e+fx^2\right)^5}dx$$

$$\Big\downarrow \text{401}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(4b(de+cf)x^2+a(de+7cf)\right)}{\left(fx^2+e\right)^4}dx}{8ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(4b(de+cf)x^2+a(de+7cf)\right)}{\left(fx^2+e\right)^4}dx}{8ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$$\Big\downarrow \text{401}$$

$$-\frac{\frac{\int -\frac{4b(2be(de+cf)+af(de+7cf))x^2+a(4be(de+cf)+5af(de+7cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6ef}-\frac{x\sqrt{a+bx^2}(4be(cf+de)-af(7cf+de))}{6ef\left(e+fx^2\right)^3}}{8ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4b(2be(de+cf)+af(de+7cf))x^2+a(4be(de+cf)+5af(de+7cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6ef}-\frac{x\sqrt{a+bx^2}(4be(cf+de)-af(7cf+de))}{6ef\left(e+fx^2\right)^3}}{8ef}-\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{2b\left(8b^2(de+cf)e^2+24abcf^2e-5a^2f^2(de+7cf)\right)x^2+a\left(8b^2(de+cf)e^2+4abf(de+25cf)e-15a^2f^2(de+7cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-5a^2f^2(7cf+de)+24abcef^2+8b^2e^2\right.}{4e\left(e+fx^2\right)^2(be-af)}}{6ef}}{8ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{3a^2f^2\left(5f(de+7cf)a^2-8be(de+10cf)a+48b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(-15a^3f^3(7cf+de)+2a^2bef^2(85cf+7de)+8ab^2e^2f(de-5cf)-16b^3e^3(cf+de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}}{}}{6ef}}{8ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{3a^2f^2\left(5a^2f(7cf+de)-8abe(10cf+de)+48b^2ce^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(-15a^3f^3(7cf+de)+2a^2bef^2(85cf+7de)+8ab^2e^2f(de-5cf)-16b^3e^3(cf+\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6ef}}{8ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$\downarrow$ 291

$$\frac{\frac{\frac{\frac{3a^2f^2\left(5a^2f(7cf+de)-8abe(10cf+de)+48b^2ce^2\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(-15a^3f^3(7cf+de)+2a^2bef^2(85cf+7de)+8ab^2e^2f(de-5cf)-16b^3e^3\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6ef}}{8ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

$\downarrow$ 221

$$\frac{\frac{\frac{x\sqrt{a+bx^2}\left(-5a^2f^2(7cf+de)+24abcef^2+8b^2e^2(cf+de)\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{\frac{3a^2f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(5a^2f(7cf+de)-8abe(10cf+de)+48b^2ce^2\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(-15a^3f^3(\cdots\right.}{\cdots}}{4e(be-af)}}{6ef}}{8ef}$$

$$\frac{x\left(a+bx^2\right)^{3/2}(de-cf)}{8ef\left(e+fx^2\right)^4}$$

input

```
Int[((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^5,x]
```

output

```
-1/8*((d*e - c*f)*x*(a + b*x^2)^(3/2))/(e*f*(e + f*x^2)^4) + (-1/6*((4*b*e
*(d*e + c*f) - a*f*(d*e + 7*c*f))*x*Sqrt[a + b*x^2])/(e*f*(e + f*x^2)^3) +
 (((24*a*b*c*e*f^2 + 8*b^2*e^2*(d*e + c*f) - 5*a^2*f^2*(d*e + 7*c*f))*x*Sq
rt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (-1/2*((8*a*b^2*e^2*f*(d*
e - 5*c*f) - 16*b^3*e^3*(d*e + c*f) - 15*a^3*f^3*(d*e + 7*c*f) + 2*a^2*b*e
*f^2*(7*d*e + 85*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) + (3
*a^2*f^2*(48*b^2*c*e^2 + 5*a^2*f*(d*e + 7*c*f) - 8*a*b*e*(d*e + 10*c*f))*A
rcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a
*f)^(3/2)))/(4*e*(b*e - a*f)))/(6*e*f))/(8*e*f)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NegQ[b*c - a*d, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{35\left(a^2\left(-\frac{8b(ad-6bc)e^2}{35}+\frac{fa(ad-16bc)e}{7}+a^2cf^2\right)(fx^2+e)^4\arctan\left(\frac{e\sqrt{bx^2+a}}{x\sqrt{(af-be)e}}\right)-\frac{93\left(\frac{8\left(da^2+10\left(\frac{7x^2d}{15}+c\right)ba+4\left(\frac{2x^2d}{3}\right.\right.}{93}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-35/128/((a*f-b*e)*e)^(1/2)*(a^2*(-8/35*b*(a*d-6*b*c)*e^2+1/7*f*a*(a*d-16*
b*c)*e+a^2*c*f^2)*(f*x^2+e)^4*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/
2))-93/35*(8/93*(d*a^2+10*(7/15*x^2*d+c)*b*a+4*(2/3*x^2*d+c)*b^2*x^2)*b*e^
5-5/93*(a^3*d+176/5*b*(79/264*x^2*d+c)*a^2-208/15*(-5/26*x^2*d+c)*b^2*x^2*
a-64/15*(1/4*x^2*d+c)*b^3*x^4)*f*e^4+f^2*((73/279*x^2*d+c)*a^3-842/279*(26
/421*x^2*d+c)*b*x^2*a^2+152/279*b^2*x^4*(-1/19*x^2*d+c)*a+16/279*b^3*c*x^6
)*e^3+511/279*a*((55/511*x^2*d+c)*a^2-628/511*(7/314*x^2*d+c)*b*x^2*a+40/5
11*b^2*c*x^4)*x^2*f^3*e^2+385/279*a^2*x^4*((3/77*x^2*d+c)*a-34/77*x^2*b*c)
*f^4*e+35/93*a^3*c*f^5*x^6)*((a*f-b*e)*e)^(1/2)*(b*x^2+a)^(1/2)*x)/(f*x^2+
e)^4/(a*f-b*e)^2/e^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(351) = 702$.

Time = 11.00 (sec) , antiderivative size = 2204, normalized size of antiderivative = 5.82

$$\int \frac{(a + bx^2)^{3/2} (c + dx^2)}{(e + fx^2)^5} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x, algorithm="fricas")
```

output
```
[1/1536*(3*(35*a^4*c*e^4*f^2 + (35*a^4*c*f^6 + 8*(6*a^2*b^2*c - a^3*b*d)*e
^2*f^4 - 5*(16*a^3*b*c - a^4*d)*e*f^5)*x^8 + 8*(6*a^2*b^2*c - a^3*b*d)*e^6
 - 5*(16*a^3*b*c - a^4*d)*e^5*f + 4*(35*a^4*c*e*f^5 + 8*(6*a^2*b^2*c - a^3
*b*d)*e^3*f^3 - 5*(16*a^3*b*c - a^4*d)*e^2*f^4)*x^6 + 6*(35*a^4*c*e^2*f^4
+ 8*(6*a^2*b^2*c - a^3*b*d)*e^4*f^2 - 5*(16*a^3*b*c - a^4*d)*e^3*f^3)*x^4
+ 4*(35*a^4*c*e^3*f^3 + 8*(6*a^2*b^2*c - a^3*b*d)*e^5*f - 5*(16*a^3*b*c -
a^4*d)*e^4*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2
*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3
 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)
) + 4*((16*b^4*d*e^6*f - 105*a^4*c*e*f^6 + 8*(2*b^4*c - 3*a*b^3*d)*e^5*f^2
 + 6*(4*a*b^3*c - a^2*b^2*d)*e^4*f^3 - (210*a^2*b^2*c - 29*a^3*b*d)*e^3*f^
4 + 5*(55*a^3*b*c - 3*a^4*d)*e^2*f^5)*x^7 + (64*b^4*d*e^7 - 385*a^4*c*e^2*
f^5 + 8*(8*b^4*c - 13*a*b^3*d)*e^6*f + 4*(22*a*b^3*c - 3*a^2*b^2*d)*e^5*f^
2 - (780*a^2*b^2*c - 107*a^3*b*d)*e^4*f^3 + (1013*a^3*b*c - 55*a^4*d)*e^3*
f^4)*x^5 - (511*a^4*c*e^3*f^4 - 16*(6*b^4*c + 7*a*b^3*d)*e^7 - 2*(56*a*b^3
*c - 135*a^2*b^2*d)*e^6*f + 21*(50*a^2*b^2*c - 11*a^3*b*d)*e^5*f^2 - (1353
*a^3*b*c - 73*a^4*d)*e^4*f^3)*x^3 - 3*(93*a^4*c*e^4*f^3 - 8*(10*a*b^3*c +
a^2*b^2*d)*e^7 + (256*a^2*b^2*c + 13*a^3*b*d)*e^6*f - (269*a^3*b*c + 5*a^4
*d)*e^5*f^2)*x)*sqrt(b*x^2 + a))/(b^3*e^12 - 3*a*b^2*e^11*f + 3*a^2*b*e^10
*f^2 - a^3*e^9*f^3 + (b^3*e^8*f^4 - 3*a*b^2*e^7*f^5 + 3*a^2*b*e^6*f^6 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)/(f*x**2+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}{(fx^2+e)^5}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)/(f*x^2 + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3048 vs. $2(351) = 702$.

Time = 0.52 (sec) , antiderivative size = 3048, normalized size of antiderivative = 8.04

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x, algorithm="giac")
```

output
```
-1/128*(48*a^2*b^(5/2)*c*e^2 - 8*a^3*b^(3/2)*d*e^2 - 80*a^3*b^(3/2)*c*e*f
+ 5*a^4*sqrt(b)*d*e*f + 35*a^4*sqrt(b)*c*f^2)*arctan(1/2*((sqrt(b)*x - sqr
t(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^6 - 2*a
*b*e^5*f + a^2*e^4*f^2)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/192*(144*(sqrt(b)*x
- sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c*e^2*f^6 - 24*(sqrt(b)*x - sqrt(b*x^2 +
 a))^14*a^3*b^(3/2)*d*e^2*f^6 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b
^(3/2)*c*e*f^7 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d*e*f^7 +
 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*c*f^8 - 768*(sqrt(b)*x -
 sqrt(b*x^2 + a))^12*b^(11/2)*d*e^6*f^2 + 1536*(sqrt(b)*x - sqrt(b*x^2 + a
))^12*a*b^(9/2)*d*e^5*f^3 - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/
2)*d*e^4*f^4 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2)*c*e^3*f^5
 - 336*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*d*e^3*f^5 - 4368*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c*e^2*f^6 + 378*(sqrt(b)*x - sqrt(
b*x^2 + a))^12*a^4*b^(3/2)*d*e^2*f^6 + 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^
12*a^4*b^(3/2)*c*e*f^7 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*
d*e*f^7 - 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*c*f^8 - 2048*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*b^(13/2)*d*e^7*f - 2048*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*b^(13/2)*c*e^6*f^2 + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(11/2)*d*e^6*f^2 + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c*e
^5*f^3 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*d*e^5*f^3 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}{(f\,x^2+e)^5}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^5,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2))/(e + f*x^2)^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)}{(e+fx^2)^5}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)}{(f\,x^2+e)^5}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)/(f*x^2+e)^5,x)
```

3.298 $\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 821

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3 \, dx = \frac{a(768b^5c^2e^3 - 9a^5d^2f^3 + 14a^4bdf^2(3de+2cf) - 128ab^4ce^2(2de+3cf) - 24a^3b^2f(3d^2e^2+6c}{2048b^5}$$
$$+ \frac{(768b^5c^2e^3 - 9a^5d^2f^3 + 14a^4bdf^2(3de+2cf) - 128ab^4ce^2(2de+3cf) - 24a^3b^2f(3d^2e^2+6cdef+c^2f^2)}{3072b^5}$$
$$+ \frac{(9a^4d^2f^3 - 14a^3bdf^2(3de+2cf) + 128b^4ce^2(2de+3cf) + 24a^2b^2f(3d^2e^2+6cdef+c^2f^2) - 48ab^3e(d^2e^2}{768b^5}$$
$$- \frac{(9a^3d^2f^3 - 14a^2bdf^2(3de+2cf) + 24ab^2f(3d^2e^2+6cdef+c^2f^2) - 48b^3e(d^2e^2+6cdef+3c^2f^2))\,x^3(a}{384b^4}$$
$$+ \frac{f(9a^2d^2f^2 - 14abdf(3de+2cf) + 24b^2(3d^2e^2+6cdef+c^2f^2))\,x^5(a+bx^2)^{5/2}}{240b^3}$$
$$+ \frac{df^2(42bde+28bcf-9adf)x^7(a+bx^2)^{5/2}}{168b^2} + \frac{d^2f^3x^9(a+bx^2)^{5/2}}{14b}$$
$$+ \frac{a^2(768b^5c^2e^3 - 9a^5d^2f^3 + 14a^4bdf^2(3de+2cf) - 128ab^4ce^2(2de+3cf) - 24a^3b^2f(3d^2e^2+6cdef+c^2f}{2048b^{11/2}}$$

output
```
1/2048*a*(768*b^5*c^2*e^3-9*a^5*d^2*f^3+14*a^4*b*d*f^2*(2*c*f+3*d*e)-128*a
*b^4*c*e^2*(3*c*f+2*d*e)-24*a^3*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+48*a^2
*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/3072*(768*b^
5*c^2*e^3-9*a^5*d^2*f^3+14*a^4*b*d*f^2*(2*c*f+3*d*e)-128*a*b^4*c*e^2*(3*c*
f+2*d*e)-24*a^3*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+48*a^2*b^3*e*(3*c^2*f^
2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(3/2)/b^5+1/768*(9*a^4*d^2*f^3-14*a^3*b*
d*f^2*(2*c*f+3*d*e)+128*b^4*c*e^2*(3*c*f+2*d*e)+24*a^2*b^2*f*(c^2*f^2+6*c*
d*e*f+3*d^2*e^2)-48*a*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(5/
2)/b^5-1/384*(9*a^3*d^2*f^3-14*a^2*b*d*f^2*(2*c*f+3*d*e)+24*a*b^2*f*(c^2*f
^2+6*c*d*e*f+3*d^2*e^2)-48*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^3*(b*x^2
+a)^(5/2)/b^4+1/240*f*(9*a^2*d^2*f^2-14*a*b*d*f*(2*c*f+3*d*e)+24*b^2*(c^2*
f^2+6*c*d*e*f+3*d^2*e^2))*x^5*(b*x^2+a)^(5/2)/b^3+1/168*d*f^2*(-9*a*d*f+28
*b*c*f+42*b*d*e)*x^7*(b*x^2+a)^(5/2)/b^2+1/14*d^2*f^3*x^9*(b*x^2+a)^(5/2)/
b+1/2048*a^2*(768*b^5*c^2*e^3-9*a^5*d^2*f^3+14*a^4*b*d*f^2*(2*c*f+3*d*e)-1
28*a*b^4*c*e^2*(3*c*f+2*d*e)-24*a^3*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)+48
*a^2*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2
))/b^(11/2)
```

Mathematica [A] (verified)

Time = 3.07 (sec) , antiderivative size = 722, normalized size of antiderivative = 0.88

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(945a^6d^2f^3 - 210a^5bdf^2(14cf + 3d(7e + fx^2)) + 28a^4b^2f(90c^2f^2 + 10cdf(54e}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^6*d^2*f^3 - 210*a^5*b*d*f^2*(14*c*f + 3*
d*(7*e + f*x^2)) + 28*a^4*b^2*f*(90*c^2*f^2 + 10*c*d*f*(54*e + 7*f*x^2) +
3*d^2*(90*e^2 + 35*e*f*x^2 + 6*f^2*x^4)) + 96*a^2*b^4*(7*c^2*f*(60*e^2 + 1
5*e*f*x^2 + 2*f^2*x^4) + 14*c*d*(20*e^3 + 15*e^2*f*x^2 + 6*e*f^2*x^4 + f^3
*x^6) + d^2*x^2*(35*e^3 + 42*e^2*f*x^2 + 21*e*f^2*x^4 + 4*f^3*x^6)) - 16*a
^3*b^3*(105*c^2*f^2*(9*e + f*x^2) + 14*c*d*f*(135*e^2 + 45*e*f*x^2 + 7*f^2
*x^4) + 3*d^2*(105*e^3 + 105*e^2*f*x^2 + 49*e*f^2*x^4 + 9*f^3*x^6)) + 256*
b^6*x^2*(21*c^2*(10*e^3 + 20*e^2*f*x^2 + 15*e*f^2*x^4 + 4*f^3*x^6) + 14*c*
d*x^2*(20*e^3 + 45*e^2*f*x^2 + 36*e*f^2*x^4 + 10*f^3*x^6) + 3*d^2*x^4*(35*
e^3 + 84*e^2*f*x^2 + 70*e*f^2*x^4 + 20*f^3*x^6)) + 128*a*b^5*(21*c^2*(50*e
^3 + 70*e^2*f*x^2 + 45*e*f^2*x^4 + 11*f^3*x^6) + 14*c*d*x^2*(70*e^3 + 135*
e^2*f*x^2 + 99*e*f^2*x^4 + 26*f^3*x^6) + 3*d^2*x^4*(105*e^3 + 231*e^2*f*x^
2 + 182*e*f^2*x^4 + 50*f^3*x^6))) + 105*a^2*(-768*b^5*c^2*e^3 + 9*a^5*d^2*
f^3 - 14*a^4*b*d*f^2*(3*d*e + 2*c*f) + 128*a*b^4*c*e^2*(2*d*e + 3*c*f) + 2
4*a^3*b^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) - 48*a^2*b^3*e*(d^2*e^2 + 6*
c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(215040*b^(11/2
))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 1217, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \, dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(fx^6\left(a+bx^2\right)^{3/2}\left(c^2f^2+6cdef+3d^2e^2\right)+ex^4\left(a+bx^2\right)^{3/2}\left(3c^2f^2+6cdef+d^2e^2\right)+c^2e^3\left(a+bx^2\right)^{3/2}+ce$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{14}d^2f^3\left(bx^2+a\right)^{3/2}x^{11}+\frac{1}{56}ad^2f^3\sqrt{bx^2+a}x^{11}+\frac{1}{12}df^2(3de+2cf)\left(bx^2+a\right)^{3/2}x^9+\frac{a^2d^2f^3\sqrt{bx^2+a}x^9}{560b}+\frac{1}{40}adf^2(3de+2cf)\sqrt{bx^2+a}x^9+\frac{1}{10}f\left(3d^2e^2+6cdfe+c^2f^2\right)\left(bx^2+a\right)^{3/2}x^7-\frac{9a^3d^2f^3\sqrt{bx^2+a}x^7}{4480b^2}+\frac{a^2df^2(3de+2cf)\sqrt{bx^2+a}x^7}{320b}+\frac{3}{80}af\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^7+\frac{1}{8}e\left(d^2e^2+6cdfe+3c^2f^2\right)\left(bx^2+a\right)^{3/2}x^5+\frac{3a^4d^2f^3\sqrt{bx^2+a}x^5}{1280b^3}-\frac{7a^3df^2(3de+2cf)\sqrt{bx^2+a}x^5}{1920b^2}+\frac{a^2f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^5}{160b}+\frac{1}{16}ae\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^5+\frac{1}{6}ce^2(2de+3cf)\left(bx^2+a\right)^{3/2}x^3-\frac{3a^5d^2f^3\sqrt{bx^2+a}x^3}{1024b^4}+\frac{7a^4df^2(3de+2cf)\sqrt{bx^2+a}x^3}{1536b^3}+\frac{1}{8}ace^2(2de+3cf)\sqrt{bx^2+a}x^3-\frac{a^3f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{128b^2}+\frac{a^2e\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x^3}{64b}+\frac{1}{4}c^2e^3\left(bx^2+a\right)^{3/2}x+\frac{3}{8}ac^2e^3\sqrt{bx^2+a}x+\frac{9a^6d^2f^3\sqrt{bx^2+a}x}{2048b^5}-\frac{7a^5df^2(3de+2cf)\sqrt{bx^2+a}x}{1024b^4}+\frac{a^2ce^2(2de+3cf)\sqrt{bx^2+a}x}{16b}+\frac{3a^4f\left(3d^2e^2+6cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{256b^3}-\frac{3a^3e\left(d^2e^2+6cdfe+3c^2f^2\right)\sqrt{bx^2+a}x}{128b^2}+\frac{3a^2c^2e^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8\sqrt{b}}-\frac{9a^7d^2f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2048b^{11/2}}+\frac{7a^6df^2(3de+2cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{1024b^{9/2}}-\frac{a^3ce^2(2de+3cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{3/2}}-\frac{3a^5f\left(3d^2e^2+6cdfe+c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{256b^{7/2}}+\frac{3a^4e\left(d^2e^2+6cdfe+3c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{5/2}}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

output
```
(3*a*c^2*e^3*x*Sqrt[a + b*x^2])/8 + (9*a^6*d^2*f^3*x*Sqrt[a + b*x^2])/(204
8*b^5) - (7*a^5*d*f^2*(3*d*e + 2*c*f)*x*Sqrt[a + b*x^2])/(1024*b^4) + (a^2
*c*e^2*(2*d*e + 3*c*f)*x*Sqrt[a + b*x^2])/(16*b) + (3*a^4*f*(3*d^2*e^2 + 6
*c*d*e*f + c^2*f^2)*x*Sqrt[a + b*x^2])/(256*b^3) - (3*a^3*e*(d^2*e^2 + 6*c
*d*e*f + 3*c^2*f^2)*x*Sqrt[a + b*x^2])/(128*b^2) - (3*a^5*d^2*f^3*x^3*Sqrt
[a + b*x^2])/(1024*b^4) + (7*a^4*d*f^2*(3*d*e + 2*c*f)*x^3*Sqrt[a + b*x^2]
)/(1536*b^3) + (a*c*e^2*(2*d*e + 3*c*f)*x^3*Sqrt[a + b*x^2])/8 - (a^3*f*(3
*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(128*b^2) + (a^2*e*(d
^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^3*Sqrt[a + b*x^2])/(64*b) + (3*a^4*d^2*f
^3*x^5*Sqrt[a + b*x^2])/(1280*b^3) - (7*a^3*d*f^2*(3*d*e + 2*c*f)*x^5*Sqrt
[a + b*x^2])/(1920*b^2) + (a^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5*Sqr
t[a + b*x^2])/(160*b) + (a*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^5*Sqrt[a
+ b*x^2])/16 - (9*a^3*d^2*f^3*x^7*Sqrt[a + b*x^2])/(4480*b^2) + (a^2*d*f^2
*(3*d*e + 2*c*f)*x^7*Sqrt[a + b*x^2])/(320*b) + (3*a*f*(3*d^2*e^2 + 6*c*d*
e*f + c^2*f^2)*x^7*Sqrt[a + b*x^2])/80 + (a^2*d^2*f^3*x^9*Sqrt[a + b*x^2])
/(560*b) + (a*d*f^2*(3*d*e + 2*c*f)*x^9*Sqrt[a + b*x^2])/40 + (a*d^2*f^3*x
^11*Sqrt[a + b*x^2])/56 + (c^2*e^3*x*(a + b*x^2)^(3/2))/4 + (c*e^2*(2*d*e
+ 3*c*f)*x^3*(a + b*x^2)^(3/2))/6 + (e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x
^5*(a + b*x^2)^(3/2))/8 + (f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^7*(a + b*
x^2)^(3/2))/10 + (d*f^2*(3*d*e + 2*c*f)*x^9*(a + b*x^2)^(3/2))/12 + (d^...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 667, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{9\left(a^2\left(a^5d^2f^3-\frac{28\left(cf+\frac{3de}{2}\right)db\,f^2a^4}{9}+\frac{8a^3b^2f\left(c^2f^2+6cdef+3d^2e^2\right)}{3}-16b^3e\left(c^2f^2+2cdef+\frac{1}{3}d^2e^2\right)a^2+\frac{128c\,b^4e^2\left(cf+\frac{2de}{3}\right)}{3}\right.\right.}{}$ … $7a\ \frac{x^5}{}$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-9/2048/b^(11/2)*(a^2*(a^5*d^2*f^3-28/9*(c*f+3/2*d*e)*d*b*f^2*a^4+8/3*a^3*
b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-16*b^3*e*(c^2*f^2+2*c*d*e*f+1/3*d^2*e^
2)*a^2+128/3*c*b^4*e^2*(c*f+2/3*d*e)*a-256/3*b^5*c^2*e^3)*arctanh((b*x^2+a
)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*x*(1280/9*a*((1/7*d^2*x^10+26/75*c*d*x^
8+11/50*c^2*x^6)*f^3+9/10*(26/45*d^2*x^4+22/15*c*d*x^2+c^2)*x^4*e*f^2+7/5*
(33/70*d^2*x^4+9/7*c*d*x^2+c^2)*x^2*e^2*f+e^3*(3/10*d^2*x^4+14/15*c*d*x^2+
c^2))*b^(11/2)+512/9*(2*(1/7*d^2*x^10+1/3*c*d*x^8+1/5*c^2*x^6)*f^3+3/2*(2/
3*d^2*x^4+8/5*c*d*x^2+c^2)*x^4*e*f^2+2*(3/5*d^2*x^4+3/2*c*d*x^2+c^2)*x^2*e
^2*f+e^3*(1/2*d^2*x^4+4/3*c*d*x^2+c^2))*x^2*b^(13/2)+a^2*(32/3*(2/15*(2/7*
d^2*x^4+c*d*x^2+c^2)*x^4*f^3+x^2*e*(1/5*d^2*x^4+4/5*c*d*x^2+c^2)*f^2+4*e^2
*(1/10*d^2*x^4+1/2*c*d*x^2+c^2)*f+8/3*d*(1/8*x^2*d+c)*e^3)*b^(9/2)+a*(16*(
-1/9*(9/35*d^2*x^4+14/15*c*d*x^2+c^2)*x^2*f^3-(7/45*d^2*x^4+2/3*c*d*x^2+c^
2)*e*f^2-2*(1/6*x^2*d+c)*d*e^2*f-1/3*d^2*e^3)*b^(7/2)+a*(8*(1/3*(7/9*c*d*x
^2+c^2+1/5*d^2*x^4)*f^2+2*(7/36*x^2*d+c)*d*e*f+d^2*e^2)*b^(5/2)+a*d*(2/3*(
(-14/3*c-x^2*d)*f-7*d*e)*b^(3/2)+a*d*f*b^(1/2))*f)*f))))
```

Fricas [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 1740, normalized size of antiderivative = 2.12

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/430080*(105*(16*(48*a^2*b^5*c^2 - 16*a^3*b^4*c*d + 3*a^4*b^3*d^2)*e^3
- 24*(16*a^3*b^4*c^2 - 12*a^4*b^3*c*d + 3*a^5*b^2*d^2)*e^2*f + 6*(24*a^4*b
^3*c^2 - 24*a^5*b^2*c*d + 7*a^6*b*d^2)*e*f^2 - (24*a^5*b^2*c^2 - 28*a^6*b*
c*d + 9*a^7*d^2)*f^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 2*(15360*b^7*d^2*f^3*x^13 + 1280*(42*b^7*d^2*e*f^2 + (28*b^7*c*d + 1
5*a*b^6*d^2)*f^3)*x^11 + 128*(504*b^7*d^2*e^2*f + 42*(24*b^7*c*d + 13*a*b^
6*d^2)*e*f^2 + (168*b^7*c^2 + 364*a*b^6*c*d + 3*a^2*b^5*d^2)*f^3)*x^9 + 48
*(560*b^7*d^2*e^3 + 168*(20*b^7*c*d + 11*a*b^6*d^2)*e^2*f + 42*(40*b^7*c^2
 + 88*a*b^6*c*d + a^2*b^5*d^2)*e*f^2 + (616*a*b^6*c^2 + 28*a^2*b^5*c*d - 9
*a^3*b^4*d^2)*f^3)*x^7 + 56*(80*(16*b^7*c*d + 9*a*b^6*d^2)*e^3 + 24*(80*b^
7*c^2 + 180*a*b^6*c*d + 3*a^2*b^5*d^2)*e^2*f + 6*(360*a*b^6*c^2 + 24*a^2*b
^5*c*d - 7*a^3*b^4*d^2)*e*f^2 + (24*a^2*b^5*c^2 - 28*a^3*b^4*c*d + 9*a^4*b
^3*d^2)*f^3)*x^5 + 70*(16*(48*b^7*c^2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)*e^3
 + 24*(112*a*b^6*c^2 + 12*a^2*b^5*c*d - 3*a^3*b^4*d^2)*e^2*f + 6*(24*a^2*b
^5*c^2 - 24*a^3*b^4*c*d + 7*a^4*b^3*d^2)*e*f^2 - (24*a^3*b^4*c^2 - 28*a^4*
b^3*c*d + 9*a^5*b^2*d^2)*f^3)*x^3 + 105*(16*(80*a*b^6*c^2 + 16*a^2*b^5*c*d
 - 3*a^3*b^4*d^2)*e^3 + 24*(16*a^2*b^5*c^2 - 12*a^3*b^4*c*d + 3*a^4*b^3*d^
2)*e^2*f - 6*(24*a^3*b^4*c^2 - 24*a^4*b^3*c*d + 7*a^5*b^2*d^2)*e*f^2 + (24
*a^4*b^3*c^2 - 28*a^5*b^2*c*d + 9*a^6*b*d^2)*f^3)*x)*sqrt(b*x^2 + a))/b^6,
 -1/215040*(105*(16*(48*a^2*b^5*c^2 - 16*a^3*b^4*c*d + 3*a^4*b^3*d^2)*e...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2317 vs. $2(877) = 1754$.

Time = 0.71 (sec) , antiderivative size = 2317, normalized size of antiderivative = 2.82

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d**2*f**3*x**13/14 + x**11*(15*a*b*d**2*f**
3/14 + 2*b**2*c*d*f**3 + 3*b**2*d**2*e*f**2)/(12*b) + x**9*(a**2*d**2*f**3
 + 4*a*b*c*d*f**3 + 6*a*b*d**2*e*f**2 - 11*a*(15*a*b*d**2*f**3/14 + 2*b**2
*c*d*f**3 + 3*b**2*d**2*e*f**2)/(12*b) + b**2*c**2*f**3 + 6*b**2*c*d*e*f**
2 + 3*b**2*d**2*e**2*f)/(10*b) + x**7*(2*a**2*c*d*f**3 + 3*a**2*d**2*e*f**
2 + 2*a*b*c**2*f**3 + 12*a*b*c*d*e*f**2 + 6*a*b*d**2*e**2*f - 9*a*(a**2*d*
*2*f**3 + 4*a*b*c*d*f**3 + 6*a*b*d**2*e*f**2 - 11*a*(15*a*b*d**2*f**3/14 +
 2*b**2*c*d*f**3 + 3*b**2*d**2*e*f**2)/(12*b) + b**2*c**2*f**3 + 6*b**2*c*
d*e*f**2 + 3*b**2*d**2*e**2*f)/(10*b) + 3*b**2*c**2*e*f**2 + 6*b**2*c*d*e*
*2*f + b**2*d**2*e**3)/(8*b) + x**5*(a**2*c**2*f**3 + 6*a**2*c*d*e*f**2 +
3*a**2*d**2*e**2*f + 6*a*b*c**2*e*f**2 + 12*a*b*c*d*e**2*f + 2*a*b*d**2*e*
*3 - 7*a*(2*a**2*c*d*f**3 + 3*a**2*d**2*e*f**2 + 2*a*b*c**2*f**3 + 12*a*b*
c*d*e*f**2 + 6*a*b*d**2*e**2*f - 9*a*(a**2*d**2*f**3 + 4*a*b*c*d*f**3 + 6*
a*b*d**2*e*f**2 - 11*a*(15*a*b*d**2*f**3/14 + 2*b**2*c*d*f**3 + 3*b**2*d**
2*e*f**2)/(12*b) + b**2*c**2*f**3 + 6*b**2*c*d*e*f**2 + 3*b**2*d**2*e**2*f
)/(10*b) + 3*b**2*c**2*e*f**2 + 6*b**2*c*d*e**2*f + b**2*d**2*e**3)/(8*b)
+ 3*b**2*c**2*e**2*f + 2*b**2*c*d*e**3)/(6*b) + x**3*(3*a**2*c**2*e*f**2 +
 6*a**2*c*d*e**2*f + a**2*d**2*e**3 + 6*a*b*c**2*e**2*f + 4*a*b*c*d*e**3 -
 5*a*(a**2*c**2*f**3 + 6*a**2*c*d*e*f**2 + 3*a**2*d**2*e**2*f + 6*a*b*c**2
*e*f**2 + 12*a*b*c*d*e**2*f + 2*a*b*d**2*e**3 - 7*a*(2*a**2*c*d*f**3 + ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1101, normalized size of antiderivative = 1.34

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
1/14*(b*x^2 + a)^(5/2)*d^2*f^3*x^9/b - 3/56*(b*x^2 + a)^(5/2)*a*d^2*f^3*x^
7/b^2 + 3/80*(b*x^2 + a)^(5/2)*a^2*d^2*f^3*x^5/b^3 - 3/128*(b*x^2 + a)^(5/
2)*a^3*d^2*f^3*x^3/b^4 + 1/12*(3*d^2*e*f^2 + 2*c*d*f^3)*(b*x^2 + a)^(5/2)*
x^7/b + 1/4*(b*x^2 + a)^(3/2)*c^2*e^3*x + 3/8*sqrt(b*x^2 + a)*a*c^2*e^3*x
+ 3/256*(b*x^2 + a)^(5/2)*a^4*d^2*f^3*x/b^5 - 3/1024*(b*x^2 + a)^(3/2)*a^5
*d^2*f^3*x/b^5 - 9/2048*sqrt(b*x^2 + a)*a^6*d^2*f^3*x/b^5 - 7/120*(3*d^2*e
*f^2 + 2*c*d*f^3)*(b*x^2 + a)^(5/2)*a*x^5/b^2 + 1/10*(3*d^2*e^2*f + 6*c*d*
e*f^2 + c^2*f^3)*(b*x^2 + a)^(5/2)*x^5/b + 3/8*a^2*c^2*e^3*arcsinh(b*x/sqr
t(a*b))/sqrt(b) - 9/2048*a^7*d^2*f^3*arcsinh(b*x/sqrt(a*b))/b^(11/2) + 7/1
92*(3*d^2*e*f^2 + 2*c*d*f^3)*(b*x^2 + a)^(5/2)*a^2*x^3/b^3 - 1/16*(3*d^2*e
^2*f + 6*c*d*e*f^2 + c^2*f^3)*(b*x^2 + a)^(5/2)*a*x^3/b^2 + 1/8*(d^2*e^3 +
 6*c*d*e^2*f + 3*c^2*e*f^2)*(b*x^2 + a)^(5/2)*x^3/b - 7/384*(3*d^2*e*f^2 +
 2*c*d*f^3)*(b*x^2 + a)^(5/2)*a^3*x/b^4 + 7/1536*(3*d^2*e*f^2 + 2*c*d*f^3)
*(b*x^2 + a)^(3/2)*a^4*x/b^4 + 7/1024*(3*d^2*e*f^2 + 2*c*d*f^3)*sqrt(b*x^2
 + a)*a^5*x/b^4 + 1/32*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*(b*x^2 + a)^(
5/2)*a^2*x/b^3 - 1/128*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*(b*x^2 + a)^(
3/2)*a^3*x/b^3 - 3/256*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*sqrt(b*x^2 +
a)*a^4*x/b^3 - 1/16*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*(b*x^2 + a)^(5/2
)*a*x/b^2 + 1/64*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*(b*x^2 + a)^(3/2)*a
^2*x/b^2 + 3/128*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*sqrt(b*x^2 + a)*...
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 971, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/215040*(2*(4*(2*(8*(10*(12*b*d^2*f^3*x^2 + (42*b^13*d^2*e*f^2 + 28*b^13*
c*d*f^3 + 15*a*b^12*d^2*f^3)/b^12)*x^2 + (504*b^13*d^2*e^2*f + 1008*b^13*c
*d*e*f^2 + 546*a*b^12*d^2*e*f^2 + 168*b^13*c^2*f^3 + 364*a*b^12*c*d*f^3 +
3*a^2*b^11*d^2*f^3)/b^12)*x^2 + 3*(560*b^13*d^2*e^3 + 3360*b^13*c*d*e^2*f
+ 1848*a*b^12*d^2*e^2*f + 1680*b^13*c^2*e*f^2 + 3696*a*b^12*c*d*e*f^2 + 42
*a^2*b^11*d^2*e*f^2 + 616*a*b^12*c^2*f^3 + 28*a^2*b^11*c*d*f^3 - 9*a^3*b^1
0*d^2*f^3)/b^12)*x^2 + 7*(1280*b^13*c*d*e^3 + 720*a*b^12*d^2*e^3 + 1920*b^
13*c^2*e^2*f + 4320*a*b^12*c*d*e^2*f + 72*a^2*b^11*d^2*e^2*f + 2160*a*b^12
*c^2*e*f^2 + 144*a^2*b^11*c*d*e*f^2 - 42*a^3*b^10*d^2*e*f^2 + 24*a^2*b^11*
c^2*f^3 - 28*a^3*b^10*c*d*f^3 + 9*a^4*b^9*d^2*f^3)/b^12)*x^2 + 35*(768*b^1
3*c^2*e^3 + 1792*a*b^12*c*d*e^3 + 48*a^2*b^11*d^2*e^3 + 2688*a*b^12*c^2*e^
2*f + 288*a^2*b^11*c*d*e^2*f - 72*a^3*b^10*d^2*e^2*f + 144*a^2*b^11*c^2*e*
f^2 - 144*a^3*b^10*c*d*e*f^2 + 42*a^4*b^9*d^2*e*f^2 - 24*a^3*b^10*c^2*f^3
+ 28*a^4*b^9*c*d*f^3 - 9*a^5*b^8*d^2*f^3)/b^12)*x^2 + 105*(1280*a*b^12*c^2
*e^3 + 256*a^2*b^11*c*d*e^3 - 48*a^3*b^10*d^2*e^3 + 384*a^2*b^11*c^2*e^2*f
 - 288*a^3*b^10*c*d*e^2*f + 72*a^4*b^9*d^2*e^2*f - 144*a^3*b^10*c^2*e*f^2
+ 144*a^4*b^9*c*d*e*f^2 - 42*a^5*b^8*d^2*e*f^2 + 24*a^4*b^9*c^2*f^3 - 28*a
^5*b^8*c*d*f^3 + 9*a^6*b^7*d^2*f^3)/b^12)*sqrt(b*x^2 + a)*x - 1/2048*(768*
a^2*b^5*c^2*e^3 - 256*a^3*b^4*c*d*e^3 + 48*a^4*b^3*d^2*e^3 - 384*a^3*b^4*c
^2*e^2*f + 288*a^4*b^3*c*d*e^2*f - 72*a^5*b^2*d^2*e^2*f + 144*a^4*b^3*c...
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx=\int \left(b\,x^2+a\right)^{3/2}\left(d\,x^2+c\right)^2\left(f\,x^2+e\right)^3\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx=\int \left(b\,x^2+a\right)^{\frac{3}{2}}\left(d\,x^2+c\right)^2\left(f\,x^2+e\right)^3dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

3.299 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 542

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx = \frac{a(384b^4c^2e^2+7a^4d^2f^2-128ab^3ce(de+cf)-24a^3bdf(de+cf)+24a^2b^2(d^2e^2+4cdef+c^2}{1024b^4}$$
$$+\frac{(384b^4c^2e^2+7a^4d^2f^2-128ab^3ce(de+cf)-24a^3bdf(de+cf)+24a^2b^2(d^2e^2+4cdef+c^2f^2))\,x(a+bx}{1536b^4}$$
$$-\frac{(7a^3d^2f^2-128b^3ce(de+cf)-24a^2bdf(de+cf)+24ab^2(d^2e^2+4cdef+c^2f^2))\,x(a+bx^2)^{5/2}}{384b^4}$$
$$+\frac{(7a^2d^2f^2-24abdf(de+cf)+24b^2(d^2e^2+4cdef+c^2f^2))\,x^3(a+bx^2)^{5/2}}{192b^3}$$
$$-\frac{df(7adf-24b(de+cf))x^5(a+bx^2)^{5/2}}{120b^2}+\frac{d^2f^2x^7(a+bx^2)^{5/2}}{12b}$$
$$+\frac{a^2(384b^4c^2e^2+7a^4d^2f^2-128ab^3ce(de+cf)-24a^3bdf(de+cf)+24a^2b^2(d^2e^2+4cdef+c^2f^2))\arctan}{1024b^{9/2}}$$

output
```
1/1024*a*(384*b^4*c^2*e^2+7*a^4*d^2*f^2-128*a*b^3*c*e*(c*f+d*e)-24*a^3*b*d
*f*(c*f+d*e)+24*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^4
+1/1536*(384*b^4*c^2*e^2+7*a^4*d^2*f^2-128*a*b^3*c*e*(c*f+d*e)-24*a^3*b*d*
f*(c*f+d*e)+24*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(3/2)/b^4-
1/384*(7*a^3*d^2*f^2-128*b^3*c*e*(c*f+d*e)-24*a^2*b*d*f*(c*f+d*e)+24*a*b^2
*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(5/2)/b^4+1/192*(7*a^2*d^2*f^2-2
4*a*b*d*f*(c*f+d*e)+24*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(5/2
)/b^3-1/120*d*f*(7*a*d*f-24*b*(c*f+d*e))*x^5*(b*x^2+a)^(5/2)/b^2+1/12*d^2*
f^2*x^7*(b*x^2+a)^(5/2)/b+1/1024*a^2*(384*b^4*c^2*e^2+7*a^4*d^2*f^2-128*a*
b^3*c*e*(c*f+d*e)-24*a^3*b*d*f*(c*f+d*e)+24*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2
*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^5d^2f^2+10a^4bdf(36de+36cf+7dfx^2)+48a^2b^3(5c^2f(8e+fx^2)+d^2x}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^5*d^2*f^2 + 10*a^4*b*d*f*(36*d*e + 36*c
*f + 7*d*f*x^2) + 48*a^2*b^3*(5*c^2*f*(8*e + f*x^2) + d^2*x^2*(5*e^2 + 4*e
*f*x^2 + f^2*x^4) + 4*c*d*(10*e^2 + 5*e*f*x^2 + f^2*x^4)) - 8*a^3*b^2*(45*
c^2*f^2 + 30*c*d*f*(6*e + f*x^2) + d^2*(45*e^2 + 30*e*f*x^2 + 7*f^2*x^4))
+ 128*b^5*x^2*(5*c^2*(6*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + 4*c*d*x^2*(10*e^2 +
 15*e*f*x^2 + 6*f^2*x^4) + d^2*x^4*(15*e^2 + 24*e*f*x^2 + 10*f^2*x^4)) + 6
4*a*b^4*(5*c^2*(30*e^2 + 28*e*f*x^2 + 9*f^2*x^4) + d^2*x^4*(45*e^2 + 66*e*
f*x^2 + 26*f^2*x^4) + 2*c*d*x^2*(70*e^2 + 90*e*f*x^2 + 33*f^2*x^4))) - 15*
a^2*(384*b^4*c^2*e^2 + 7*a^4*d^2*f^2 - 128*a*b^3*c*e*(d*e + c*f) - 24*a^3*
b*d*f*(d*e + c*f) + 24*a^2*b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(9/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(x^4\left(a+bx^2\right)^{3/2}\left(c^2f^2+4cdef+d^2e^2\right)+c^2e^2\left(a+bx^2\right)^{3/2}+2cex^2\left(a+bx^2\right)^{3/2}(cf+de)+2dfx^6\left(a+bx^2\right)^{3/}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}d^2f^2\left(bx^2+a\right)^{3/2}x^9+\frac{1}{40}ad^2f^2\sqrt{bx^2+a}x^9+\frac{1}{5}df(de+cf)\left(bx^2+a\right)^{3/2}x^7+\frac{a^2d^2f^2\sqrt{bx^2+a}x^7}{320b}+\frac{3}{40}adf(de+cf)\sqrt{bx^2+a}x^7+\frac{1}{8}\left(d^2e^2+4cdfe+c^2f^2\right)\left(bx^2+a\right)^{3/2}x^5-\frac{7a^3d^2f^2\sqrt{bx^2+a}x^5}{1920b^2}+\frac{a^2df(de+cf)\sqrt{bx^2+a}x^5}{80b}+\frac{1}{16}a\left(d^2e^2+4cdfe+c^2f^2\right)\sqrt{bx^2+a}x^5+\frac{1}{3}ce(de+cf)\left(bx^2+a\right)^{3/2}x^3+\frac{7a^4d^2f^2\sqrt{bx^2+a}x^3}{1536b^3}+\frac{1}{4}ace(de+cf)\sqrt{bx^2+a}x^3-\frac{a^3df(de+cf)\sqrt{bx^2+a}x^3}{64b^2}+\frac{a^2\left(d^2e^2+4cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{64b}+\frac{1}{4}c^2e^2\left(bx^2+a\right)^{3/2}x+\frac{3}{8}ac^2e^2\sqrt{bx^2+a}x-\frac{7a^5d^2f^2\sqrt{bx^2+a}x}{1024b^4}+\frac{a^2ce(de+cf)\sqrt{bx^2+a}x}{8b}+\frac{3a^4df(de+cf)\sqrt{bx^2+a}x}{128b^3}-\frac{3a^3\left(d^2e^2+4cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{128b^2}+\frac{3a^2c^2e^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8\sqrt{b}}+\frac{7a^6d^2f^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{1024b^{9/2}}-\frac{a^3ce(de+cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{3/2}}-\frac{3a^5df(de+cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{7/2}}+\frac{3a^4\left(d^2e^2+4cdfe+c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{5/2}}$$

input
```
Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

output
```
(3*a*c^2*e^2*x*Sqrt[a + b*x^2])/8 - (7*a^5*d^2*f^2*x*Sqrt[a + b*x^2])/(102
4*b^4) + (a^2*c*e*(d*e + c*f)*x*Sqrt[a + b*x^2])/(8*b) + (3*a^4*d*f*(d*e +
 c*f)*x*Sqrt[a + b*x^2])/(128*b^3) - (3*a^3*(d^2*e^2 + 4*c*d*e*f + c^2*f^2
)*x*Sqrt[a + b*x^2])/(128*b^2) + (7*a^4*d^2*f^2*x^3*Sqrt[a + b*x^2])/(1536
*b^3) + (a*c*e*(d*e + c*f)*x^3*Sqrt[a + b*x^2])/4 - (a^3*d*f*(d*e + c*f)*x
^3*Sqrt[a + b*x^2])/(64*b^2) + (a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^3*Sq
rt[a + b*x^2])/(64*b) - (7*a^3*d^2*f^2*x^5*Sqrt[a + b*x^2])/(1920*b^2) + (
a^2*d*f*(d*e + c*f)*x^5*Sqrt[a + b*x^2])/(80*b) + (a*(d^2*e^2 + 4*c*d*e*f
+ c^2*f^2)*x^5*Sqrt[a + b*x^2])/16 + (a^2*d^2*f^2*x^7*Sqrt[a + b*x^2])/(32
0*b) + (3*a*d*f*(d*e + c*f)*x^7*Sqrt[a + b*x^2])/40 + (a*d^2*f^2*x^9*Sqrt[
a + b*x^2])/40 + (c^2*e^2*x*(a + b*x^2)^(3/2))/4 + (c*e*(d*e + c*f)*x^3*(a
 + b*x^2)^(3/2))/3 + ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^5*(a + b*x^2)^(3/2
))/8 + (d*f*(d*e + c*f)*x^7*(a + b*x^2)^(3/2))/5 + (d^2*f^2*x^9*(a + b*x^2
)^(3/2))/12 + (3*a^2*c^2*e^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt
[b]) + (7*a^6*d^2*f^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(9/2))
 - (a^3*c*e*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
- (3*a^5*d*f*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(7/2
)) + (3*a^4*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(128*b^(5/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{7a^2\left(a^4d^2f^2-\frac{24a^3bdf(cf+de)}{7}+\frac{24a^2b^2\left(c^2f^2+4cdef+d^2e^2\right)}{7}-\frac{128a\,b^3ce(cf+de)}{7}+\frac{384b^4c^2e^2}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{1024}-\frac{7\left(-\frac{256\left(d^2\right.}{}\right.}{}$
default	$c^2e^2\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+2df(cf+de)\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^3\left(b\,x^2\right.}{8b}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
7/1024*(a^2*(a^4*d^2*f^2-24/7*a^3*b*d*f*(c*f+d*e)+24/7*a^2*b^2*(c^2*f^2+4*
c*d*e*f+d^2*e^2)-128/7*a*b^3*c*e*(c*f+d*e)+384/7*b^4*c^2*e^2)*arctanh((b*x
^2+a)^(1/2)/x/b^(1/2))-(-256/7*(1/3*d^2*f^2*x^8+4/5*d*f*(c*f+d*e)*x^6+(1/2
*c^2*f^2+2*c*d*e*f+1/2*d^2*e^2)*x^4+4/3*c*e*(c*f+d*e)*x^2+c^2*e^2)*x^2*b^(
11/2)+a*(64*(-26/105*d^2*f^2*x^8-22/35*d*f*(c*f+d*e)*x^6+3/7*(-c^2*f^2-4*c
*d*e*f-d^2*e^2)*x^4-4/3*c*e*(c*f+d*e)*x^2-10/7*c^2*e^2)*b^(9/2)+a*(16/7*(-
1/5*d^2*f^2*x^6-4/5*d*f*(c*f+d*e)*x^4+(-c^2*f^2-4*c*d*e*f-d^2*e^2)*x^2-8*c
*e*(c*f+d*e))*b^(7/2)+a*(8*(1/15*d^2*f^2*x^4+2/7*d*f*(c*f+d*e)*x^2+12/7*c*
d*e*f+3/7*c^2*f^2+3/7*d^2*e^2)*b^(5/2)+(2*(-12/7*c*f-12/7*d*e-1/3*d*f*x^2)
*b^(3/2)+a*d*f*b^(1/2))*a*d*f))))*x*(b*x^2+a)^(1/2))/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 1182, normalized size of antiderivative = 2.18

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/30720*(15*(8*(48*a^2*b^4*c^2 - 16*a^3*b^3*c*d + 3*a^4*b^2*d^2)*e^2 - 8*
(16*a^3*b^3*c^2 - 12*a^4*b^2*c*d + 3*a^5*b*d^2)*e*f + (24*a^4*b^2*c^2 - 24
*a^5*b*c*d + 7*a^6*d^2)*f^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(1280*b^6*d^2*f^2*x^11 + 128*(24*b^6*d^2*e*f + (24*b^6*c*d
+ 13*a*b^5*d^2)*f^2)*x^9 + 48*(40*b^6*d^2*e^2 + 8*(20*b^6*c*d + 11*a*b^5*d
^2)*e*f + (40*b^6*c^2 + 88*a*b^5*c*d + a^2*b^4*d^2)*f^2)*x^7 + 8*(40*(16*b
^6*c*d + 9*a*b^5*d^2)*e^2 + 8*(80*b^6*c^2 + 180*a*b^5*c*d + 3*a^2*b^4*d^2)
*e*f + (360*a*b^5*c^2 + 24*a^2*b^4*c*d - 7*a^3*b^3*d^2)*f^2)*x^5 + 10*(8*(
48*b^6*c^2 + 112*a*b^5*c*d + 3*a^2*b^4*d^2)*e^2 + 8*(112*a*b^5*c^2 + 12*a^
2*b^4*c*d - 3*a^3*b^3*d^2)*e*f + (24*a^2*b^4*c^2 - 24*a^3*b^3*c*d + 7*a^4*
b^2*d^2)*f^2)*x^3 + 15*(8*(80*a*b^5*c^2 + 16*a^2*b^4*c*d - 3*a^3*b^3*d^2)*
e^2 + 8*(16*a^2*b^4*c^2 - 12*a^3*b^3*c*d + 3*a^4*b^2*d^2)*e*f - (24*a^3*b^
3*c^2 - 24*a^4*b^2*c*d + 7*a^5*b*d^2)*f^2)*x)*sqrt(b*x^2 + a))/b^5, -1/153
60*(15*(8*(48*a^2*b^4*c^2 - 16*a^3*b^3*c*d + 3*a^4*b^2*d^2)*e^2 - 8*(16*a^
3*b^3*c^2 - 12*a^4*b^2*c*d + 3*a^5*b*d^2)*e*f + (24*a^4*b^2*c^2 - 24*a^5*b
*c*d + 7*a^6*d^2)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280
*b^6*d^2*f^2*x^11 + 128*(24*b^6*d^2*e*f + (24*b^6*c*d + 13*a*b^5*d^2)*f^2)
*x^9 + 48*(40*b^6*d^2*e^2 + 8*(20*b^6*c*d + 11*a*b^5*d^2)*e*f + (40*b^6*c^
2 + 88*a*b^5*c*d + a^2*b^4*d^2)*f^2)*x^7 + 8*(40*(16*b^6*c*d + 9*a*b^5*d^2
)*e^2 + 8*(80*b^6*c^2 + 180*a*b^5*c*d + 3*a^2*b^4*d^2)*e*f + (360*a*b^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(554) = 1108$.

Time = 0.62 (sec) , antiderivative size = 1464, normalized size of antiderivative = 2.70

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(b*d**2*f**2*x**11/12 + x**9*(13*a*b*d**2*f**2
/12 + 2*b**2*c*d*f**2 + 2*b**2*d**2*e*f)/(10*b) + x**7*(a**2*d**2*f**2 + 4
*a*b*c*d*f**2 + 4*a*b*d**2*e*f - 9*a*(13*a*b*d**2*f**2/12 + 2*b**2*c*d*f**
2 + 2*b**2*d**2*e*f)/(10*b) + b**2*c**2*f**2 + 4*b**2*c*d*e*f + b**2*d**2*
e**2)/(8*b) + x**5*(2*a**2*c*d*f**2 + 2*a**2*d**2*e*f + 2*a*b*c**2*f**2 +
8*a*b*c*d*e*f + 2*a*b*d**2*e**2 - 7*a*(a**2*d**2*f**2 + 4*a*b*c*d*f**2 + 4
*a*b*d**2*e*f - 9*a*(13*a*b*d**2*f**2/12 + 2*b**2*c*d*f**2 + 2*b**2*d**2*e
*f)/(10*b) + b**2*c**2*f**2 + 4*b**2*c*d*e*f + b**2*d**2*e**2)/(8*b) + 2*b
**2*c**2*e*f + 2*b**2*c*d*e**2)/(6*b) + x**3*(a**2*c**2*f**2 + 4*a**2*c*d*
e*f + a**2*d**2*e**2 + 4*a*b*c**2*e*f + 4*a*b*c*d*e**2 - 5*a*(2*a**2*c*d*f
**2 + 2*a**2*d**2*e*f + 2*a*b*c**2*f**2 + 8*a*b*c*d*e*f + 2*a*b*d**2*e**2
- 7*a*(a**2*d**2*f**2 + 4*a*b*c*d*f**2 + 4*a*b*d**2*e*f - 9*a*(13*a*b*d**2
*f**2/12 + 2*b**2*c*d*f**2 + 2*b**2*d**2*e*f)/(10*b) + b**2*c**2*f**2 + 4*
b**2*c*d*e*f + b**2*d**2*e**2)/(8*b) + 2*b**2*c**2*e*f + 2*b**2*c*d*e**2)/
(6*b) + b**2*c**2*e**2)/(4*b) + x*(2*a**2*c**2*e*f + 2*a**2*c*d*e**2 + 2*a
*b*c**2*e**2 - 3*a*(a**2*c**2*f**2 + 4*a**2*c*d*e*f + a**2*d**2*e**2 + 4*a
*b*c**2*e*f + 4*a*b*c*d*e**2 - 5*a*(2*a**2*c*d*f**2 + 2*a**2*d**2*e*f + 2*
a*b*c**2*f**2 + 8*a*b*c*d*e*f + 2*a*b*d**2*e**2 - 7*a*(a**2*d**2*f**2 + 4*
a*b*c*d*f**2 + 4*a*b*d**2*e*f - 9*a*(13*a*b*d**2*f**2/12 + 2*b**2*c*d*f**2
 + 2*b**2*d**2*e*f)/(10*b) + b**2*c**2*f**2 + 4*b**2*c*d*e*f + b**2*d**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.33

$$\int (a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

output

```
1/12*(b*x^2 + a)^(5/2)*d^2*f^2*x^7/b - 7/120*(b*x^2 + a)^(5/2)*a*d^2*f^2*x
^5/b^2 + 7/192*(b*x^2 + a)^(5/2)*a^2*d^2*f^2*x^3/b^3 + 1/5*(d^2*e*f + c*d*
f^2)*(b*x^2 + a)^(5/2)*x^5/b + 1/4*(b*x^2 + a)^(3/2)*c^2*e^2*x + 3/8*sqrt(
b*x^2 + a)*a*c^2*e^2*x - 7/384*(b*x^2 + a)^(5/2)*a^3*d^2*f^2*x/b^4 + 7/153
6*(b*x^2 + a)^(3/2)*a^4*d^2*f^2*x/b^4 + 7/1024*sqrt(b*x^2 + a)*a^5*d^2*f^2
*x/b^4 + 3/8*a^2*c^2*e^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 7/1024*a^6*d^2*f
^2*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 1/8*(d^2*e*f + c*d*f^2)*(b*x^2 + a)^(5
/2)*a*x^3/b^2 + 1/8*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*(b*x^2 + a)^(5/2)*x^3/
b + 1/16*(d^2*e*f + c*d*f^2)*(b*x^2 + a)^(5/2)*a^2*x/b^3 - 1/64*(d^2*e*f +
 c*d*f^2)*(b*x^2 + a)^(3/2)*a^3*x/b^3 - 3/128*(d^2*e*f + c*d*f^2)*sqrt(b*x
^2 + a)*a^4*x/b^3 - 1/16*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*(b*x^2 + a)^(5/2)
*a*x/b^2 + 1/64*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*(b*x^2 + a)^(3/2)*a^2*x/b^
2 + 3/128*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*sqrt(b*x^2 + a)*a^3*x/b^2 + 1/3*
(c*d*e^2 + c^2*e*f)*(b*x^2 + a)^(5/2)*x/b - 1/12*(c*d*e^2 + c^2*e*f)*(b*x^
2 + a)^(3/2)*a*x/b - 1/8*(c*d*e^2 + c^2*e*f)*sqrt(b*x^2 + a)*a^2*x/b - 3/1
28*(d^2*e*f + c*d*f^2)*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/128*(d^2*e^2
 + 4*c*d*e*f + c^2*f^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(c*d*e^2
+ c^2*e*f)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \left(e+fx^2\right)^2 \, dx = \frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10\, bd^2f^2x^2 + \frac{24\, b^{11}d^2ef + 24\, b^{11}cdf^2 + 13\, ab^{10}d^2f^2}{b^{10}}\right)x^2 + \frac{3\,(40\, b^{11}d^2e^2}{} \right.\right.\right.\right.$$

$$- \frac{(384\, a^2b^4c^2e^2 - 128\, a^3b^3cde^2 + 24\, a^4b^2d^2e^2 - 128\, a^3b^3c^2ef + 96\, a^4b^2cdef - 24\, a^5bd^2ef + 24\, a^4b^2c^2f^2 - 2}{1024\, b^{\frac{9}{2}}}$$

input

```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*b*d^2*f^2*x^2 + (24*b^11*d^2*e*f + 24*b^11*c*d*f^2
 + 13*a*b^10*d^2*f^2)/b^10)*x^2 + 3*(40*b^11*d^2*e^2 + 160*b^11*c*d*e*f +
88*a*b^10*d^2*e*f + 40*b^11*c^2*f^2 + 88*a*b^10*c*d*f^2 + a^2*b^9*d^2*f^2)
/b^10)*x^2 + (640*b^11*c*d*e^2 + 360*a*b^10*d^2*e^2 + 640*b^11*c^2*e*f + 1
440*a*b^10*c*d*e*f + 24*a^2*b^9*d^2*e*f + 360*a*b^10*c^2*f^2 + 24*a^2*b^9*
c*d*f^2 - 7*a^3*b^8*d^2*f^2)/b^10)*x^2 + 5*(384*b^11*c^2*e^2 + 896*a*b^10*
c*d*e^2 + 24*a^2*b^9*d^2*e^2 + 896*a*b^10*c^2*e*f + 96*a^2*b^9*c*d*e*f - 2
4*a^3*b^8*d^2*e*f + 24*a^2*b^9*c^2*f^2 - 24*a^3*b^8*c*d*f^2 + 7*a^4*b^7*d^
2*f^2)/b^10)*x^2 + 15*(640*a*b^10*c^2*e^2 + 128*a^2*b^9*c*d*e^2 - 24*a^3*b
^8*d^2*e^2 + 128*a^2*b^9*c^2*e*f - 96*a^3*b^8*c*d*e*f + 24*a^4*b^7*d^2*e*f
 - 24*a^3*b^8*c^2*f^2 + 24*a^4*b^7*c*d*f^2 - 7*a^5*b^6*d^2*f^2)/b^10)*sqrt
(b*x^2 + a)*x - 1/1024*(384*a^2*b^4*c^2*e^2 - 128*a^3*b^3*c*d*e^2 + 24*a^4
*b^2*d^2*e^2 - 128*a^3*b^3*c^2*e*f + 96*a^4*b^2*c*d*e*f - 24*a^5*b*d^2*e*f
 + 24*a^4*b^2*c^2*f^2 - 24*a^5*b*c*d*f^2 + 7*a^6*d^2*f^2)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2\,dx=\int (b\,x^2+a)^{3/2}\,(d\,x^2+c)^2\,(f\,x^2+e)^2\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int (a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2\,dx=\int (b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^2\,(f\,x^2+e)^2\,dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

3.300 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 282

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2} dx = \frac{(a^2d^2f^2-10abdf(de-2cf)+8b^2(de-cf)^2)\,x\sqrt{a+bx^2}}{16bf^3} + \frac{d(7adf-6b(de-2cf))x^3\sqrt{a+bx^2}}{24f^2} + \frac{bd^2x^5\sqrt{a+bx^2}}{6f} - \frac{(a^3d^2f^3+6a^2bdf^2(de-2cf)+16b^3e(de-cf)^2-24ab^2f(de-cf)^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}f^4} + \frac{(be-af)^{3/2}(de-cf)^2\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^4}$$

output
```
1/16*(a^2*d^2*f^2-10*a*b*d*f*(-2*c*f+d*e)+8*b^2*(-c*f+d*e)^2)*x*(b*x^2+a)^
(1/2)/b/f^3+1/24*d*(7*a*d*f-6*b*(-2*c*f+d*e))*x^3*(b*x^2+a)^(1/2)/f^2+1/6*
b*d^2*x^5*(b*x^2+a)^(1/2)/f-1/16*(a^3*d^2*f^3+6*a^2*b*d*f^2*(-2*c*f+d*e)+1
6*b^3*e*(-c*f+d*e)^2-24*a*b^2*f*(-c*f+d*e)^2)*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(3/2)/f^4+(-a*f+b*e)^(3/2)*(-c*f+d*e)^2*arctanh((-a*f+b*e)^(1/2)*
x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^4
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \frac{\frac{fx\sqrt{a+bx^2}\left(3a^2d^2f^2+2abdf\left(-15de+30cf+7dfx^2\right)+4b^2\left(6c^2f^2+6cdf\left(-2e+fx^2\right)+d^2\left(6e^2-3efx^2+2f\right.\right.\right.}{b}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(3*a^2*d^2*f^2 + 2*a*b*d*f*(-15*d*e + 30*c*f + 7*d*f
*x^2) + 4*b^2*(6*c^2*f^2 + 6*c*d*f*(-2*e + f*x^2) + d^2*(6*e^2 - 3*e*f*x^2
 + 2*f^2*x^4))))/b - (48*(-(b*e) + a*f)^(3/2)*(d*e - c*f)^2*ArcTan[(-(f*x*
Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/Sqr
t[e] + (3*(a^3*d^2*f^3 + 6*a^2*b*d*f^2*(d*e - 2*c*f) + 16*b^3*e*(d*e - c*f
)^2 - 24*a*b^2*f*(d*e - c*f)^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/
2))/(48*f^4)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.40, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {420, 318, 299, 211, 224, 219, 420, 299, 211, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx$$

$$\Big\downarrow \ 420$$

$$\frac{b\int \sqrt{bx^2+a}(dx^2+c)^2\,dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \ 318$$

$$\frac{b\left(\frac{\int \sqrt{bx^2+a}(d(8bc-3ad)x^2+c(6bc-ad))dx}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{b\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}$$

$$\Big\downarrow \ 420$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(be-af)\left(\frac{d\int\sqrt{bx^2+a}\left(dx^2+c\right)dx}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 299$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{(4bc-ad)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 211$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{\int -\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{a+}}{2f}\right.}{f}\right.}{f}$$

$\big\downarrow$ 25

$$\frac{b\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int \frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2f}\right.}{f}\right.}{f}$$

$\big\downarrow$ 398

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{(-adf-2bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\cdots\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\cdots}{f}}{\cdots}\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc\cdots}{\sqrt{b}f}\cdots}{\cdots}\cdots\right)}{f}\cdots\right)}{f}\cdots$$

$\downarrow$ 291

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc\cdots}{\sqrt{b}f}\cdots}{\cdots}\cdots\right)}{f}\cdots\right)}{f}\cdots$$

$\downarrow$ 221

$$\frac{b\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bc}{\sqrt{b}f}\right.}{f}\right.}{f}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

output
```
(b*((d*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(8*b*c - 3*a*d)*x*(a +
 b*x^2)^(3/2))/(4*b) + (3*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b
*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(
6*b)))/f - ((b*e - a*f)*((d*((d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*d
)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b])))/(4*b)))/f - ((d*e - c*f)*((d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e
 - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2
*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
 b*x^2])])/(Sqrt[e]*f))/(2*f)))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-2b^{\frac{5}{2}}(-af+be)^2(cf-de)^2\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\left(-\frac{\left(16b^3d^2e^3-24\left(ad+\frac{4bc}{3}\right)d\,b^2f\,e^2+6b\,f^2\left(a^2d^2+8abcd+\frac{8}{3}b^2c^2\right)e+a\,f^3\left(a\right.\right.}{8}\right.}{}$
risch	$\frac{x\left(8f^2x^4b^2d^2+14ab\,d^2f^2x^2+24b^2cd\,f^2x^2-12b^2d^2ef\,x^2+3a^2d^2f^2+60abcd\,f^2-30ab\,d^2ef+24b^2c^2f^2-48b^2cdef+24b^2d^2e\right.}{48b\,f^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*b^(5/2)*(-a*f+b*e)^2*(c*f-d*e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-
b*e)*e)^(1/2))+(-1/8*(16*b^3*d^2*e^3-24*(a*d+4/3*b*c)*d*b^2*f*e^2+6*b*f^2*
(a^2*d^2+8*a*b*c*d+8/3*b^2*c^2)*e+a*f^3*(a^2*d^2-12*a*b*c*d-24*b^2*c^2))*b
*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+b^(3/2)*(b^2*d^2*e^2-5/4*d*b*f*(2/5*(d
*x^2+4*c)*b+a*d)*e+1/8*(8*(1/3*d^2*x^4+c*d*x^2+c^2)*b^2+20*a*d*(7/30*x^2*d
+c)*b+a^2*d^2)*f^2)*(b*x^2+a)^(1/2)*x*f)*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e
)^(1/2)/b^(5/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 97.36 (sec) , antiderivative size = 1801, normalized size of antiderivative = 6.39

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/96*(3*(16*b^3*d^2*e^3 - 8*(4*b^3*c*d + 3*a*b^2*d^2)*e^2*f + 2*(8*b^3*c^
2 + 24*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 - (24*a*b^2*c^2 + 12*a^2*b*c*d - a^3
*d^2)*f^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 24*(b
^3*d^2*e^3 - a*b^2*c^2*f^3 - (2*b^3*c*d + a*b^2*d^2)*e^2*f + (b^3*c^2 + 2*
a*b^2*c*d)*e*f^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^
2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 -
 a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e
^2)) + 2*(8*b^3*d^2*f^3*x^5 - 2*(6*b^3*d^2*e*f^2 - (12*b^3*c*d + 7*a*b^2*d
^2)*f^3)*x^3 + 3*(8*b^3*d^2*e^2*f - 2*(8*b^3*c*d + 5*a*b^2*d^2)*e*f^2 + (8
*b^3*c^2 + 20*a*b^2*c*d + a^2*b*d^2)*f^3)*x)*sqrt(b*x^2 + a))/(b^2*f^4), 1
/48*(3*(16*b^3*d^2*e^3 - 8*(4*b^3*c*d + 3*a*b^2*d^2)*e^2*f + 2*(8*b^3*c^2
+ 24*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 - (24*a*b^2*c^2 + 12*a^2*b*c*d - a^3*d
^2)*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*(b^3*d^2*e^3 - a
*b^2*c^2*f^3 - (2*b^3*c*d + a*b^2*d^2)*e^2*f + (b^3*c^2 + 2*a*b^2*c*d)*e*f
^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e
^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*b*e^2 - a*e*f)*x^3)*s
qrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + (8*b^3*
d^2*f^3*x^5 - 2*(6*b^3*d^2*e*f^2 - (12*b^3*c*d + 7*a*b^2*d^2)*f^3)*x^3 + 3
*(8*b^3*d^2*e^2*f - 2*(8*b^3*c*d + 5*a*b^2*d^2)*e*f^2 + (8*b^3*c^2 + 20*a*
b^2*c*d + a^2*b*d^2)*f^3)*x)*sqrt(b*x^2 + a))/(b^2*f^4), -1/96*(48*(b^3...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**2/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1271, normalized size of antiderivative = 4.51

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e),x)
```

output
```
( - 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**2*f**3 + 96*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c*d*e*f**2 - 48*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a*b**2*d**2*e**2*f + 48*sqrt(e)*sqrt(a*f - b*e)*atan(
(sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*
sqrt(b)))*b**3*c**2*e*f**2 - 96*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**
3*c*d*e**2*f + 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*d**2*e**3 -
48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2
) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**2*f**3 + 96*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c*d*e*f**2 - 48*sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a*b**2*d**2*e**2*f + 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*b**3*c**2*e*f**2 - 96*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e)
+ sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3...
```

3.301 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = -\frac{d(8bde-8bcf-5adf)x\sqrt{a+bx^2}}{8f^3} + \frac{bd^2x^3\sqrt{a+bx^2}}{4f^2} - \frac{(be-af)(de-cf)^2x\sqrt{a+bx^2}}{2ef^3(e+fx^2)} + \frac{(3a^2d^2f^2-24abdf(de-cf)+8b^2(3d^2e^2-4cdef+c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}f^4} - \frac{\sqrt{be-af}(de-cf)(2be(3de-cf)-af(3de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^4}$$

output
```
-1/8*d*(-5*a*d*f-8*b*c*f+8*b*d*e)*x*(b*x^2+a)^(1/2)/f^3+1/4*b*d^2*x^3*(b*x
^2+a)^(1/2)/f^2-1/2*(-a*f+b*e)*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f^3/(f*x^2
+e)+1/8*(3*a^2*d^2*f^2-24*a*b*d*f*(-c*f+d*e)+8*b^2*(c^2*f^2-4*c*d*e*f+3*d^
2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^4-1/2*(-a*f+b*e)^(1/2
)*(-c*f+d*e)*(2*b*e*(-c*f+3*d*e)-a*f*(c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)
*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^4
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{\frac{fx\sqrt{a+bx^2}\left(af\left(-8cdef+4c^2f^2+d^2e\left(9e+5fx^2\right)\right)-2be\left(2c^2f^2-4cdf\left(2e+fx^2\right)+d^2\left(6e^2+3efx^2-f^2x^4\right)\right.\right.}{e(e+fx^2)}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(a*f*(-8*c*d*e*f + 4*c^2*f^2 + d^2*e*(9*e + 5*f*x^2)
) - 2*b*e*(2*c^2*f^2 - 4*c*d*f*(2*e + f*x^2) + d^2*(6*e^2 + 3*e*f*x^2 - f^
2*x^4))))/(e*(e + f*x^2)) - (4*Sqrt[-(b*e) + a*f]*(d*e - c*f)*(2*b*e*(3*d*
e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e
+ f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/e^(3/2) - ((3*a^2*d^2*f^2 - 24*a*
b*d*f*(d*e - c*f) + 8*b^2*(3*d^2*e^2 - 4*c*d*e*f + c^2*f^2))*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(8*f^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{d\int \sqrt{bx^2+a}(dx^2+c)dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\left(\frac{d\left(\frac{(4bc-ad)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{b\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 403

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{\int-\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{a+bx^2}}{2f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 398

$$b\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{2(be-af)(d\cdots}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 224

$$b\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{2(\cdots}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$\downarrow$ 219

$$b\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(\cdots}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 291

$$b\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(}{}}{2f}}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 221

$$b\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right)}{f}\cdots\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 420$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right)}{f}\cdots\right)}{f}-\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\cdots}{\cdots}}{2f}\cdots\right)}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\Big\downarrow$ 224

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\cdots}{\cdots}}{2f}\cdots\right)}{f}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 398

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}\cdots}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{\cdots}{\sqrt{\cdots}}\cdots}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-}{}\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-}{}\right.}{f}$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\right.}{f}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\mathrm{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b\cdots}{\cdots}\cdots\right.}{f}$$

↓ 425

$$b\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b}{}\right.$$

$$\overline{\qquad f \qquad}$$

↓ 398

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b\cdots}{\cdots}\cdots\right.}{f}$$

$$\downarrow \; 224$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b}{}\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\mathrm{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b}{}\right.}{f}$$

↓ 291

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}\cdots}{}}{2f}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}\cdots$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{}}{}}{2f}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b}{}\right.}{f}$$

↓ 402

$$\frac{b\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{\cdots}}{\cdots}}{2f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(b\cdots}{\cdots}\cdots\right.}{f}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{-2\sqrt{b}\left(-3bd\,e^{2}+f\left(bc+\frac{3ad}{2}\right)e+\frac{ac\,f^{2}}{2}\right)(cf-de)(-af+be)(f\,x^{2}+e)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(-\frac{3\left(8b^{2}d^{2}e^{2}\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*f-b*e)*e)^(1/2)/b^(1/2)*(-2*b^(1/2)*(-3*b*d*e^2+f*(b*c+3/2*a*d)*e
+1/2*a*c*f^2)*(c*f-d*e)*(-a*f+b*e)*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((
a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(-3/4*(8*b^2*d^2*e^2-8*(a*d+4/3*b*c
)*d*b*f*e+f^2*(a^2*d^2+8*a*b*c*d+8/3*b^2*c^2))*(f*x^2+e)*e*arctanh((b*x^2+
a)^(1/2)/x/b^(1/2))+b^(1/2)*(b*x^2+a)^(1/2)*(3*e^3*b*d^2-9/4*d*((-2/3*x^2*
d+16/9*c)*b+a*d)*f*e^2+2*((-1/4*d^2*x^4-c*d*x^2+1/2*c^2)*b+d*a*(-5/8*x^2*d
+c))*f^2*e-a*c^2*f^3)*x*f))/f^4/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(252) = 504$.

Time = 11.62 (sec) , antiderivative size = 2263, normalized size of antiderivative = 7.97

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/16*((24*b^2*d^2*e^4 - 8*(4*b^2*c*d + 3*a*b*d^2)*e^3*f + (8*b^2*c^2 + 24
*a*b*c*d + 3*a^2*d^2)*e^2*f^2 + (24*b^2*d^2*e^3*f - 8*(4*b^2*c*d + 3*a*b*d
^2)*e^2*f^2 + (8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*e*f^3)*x^2)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*b^2*d^2*e^4 + a*b*c^2*
e*f^3 - (8*b^2*c*d + 3*a*b*d^2)*e^3*f + 2*(b^2*c^2 + a*b*c*d)*e^2*f^2 + (6
*b^2*d^2*e^3*f + a*b*c^2*f^4 - (8*b^2*c*d + 3*a*b*d^2)*e^2*f^2 + 2*(b^2*c^
2 + a*b*c*d)*e*f^3)*x^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*a*b*e*f +
 a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a*e^2*x + (2*
b*e^2 - a*e*f)*x^3)*sqrt(b*x^2 + a)*sqrt((b*e - a*f)/e))/(f^2*x^4 + 2*e*f*
x^2 + e^2)) + 2*(2*b^2*d^2*e*f^3*x^5 - (6*b^2*d^2*e^2*f^2 - (8*b^2*c*d + 5
*a*b*d^2)*e*f^3)*x^3 - (12*b^2*d^2*e^3*f - 4*a*b*c^2*f^4 - (16*b^2*c*d + 9
*a*b*d^2)*e^2*f^2 + 4*(b^2*c^2 + 2*a*b*c*d)*e*f^3)*x)*sqrt(b*x^2 + a))/(b*
e*f^5*x^2 + b*e^2*f^4), -1/8*((24*b^2*d^2*e^4 - 8*(4*b^2*c*d + 3*a*b*d^2)*
e^3*f + (8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*e^2*f^2 + (24*b^2*d^2*e^3*f -
 8*(4*b^2*c*d + 3*a*b*d^2)*e^2*f^2 + (8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*
e*f^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*b^2*d^2*e^4 +
 a*b*c^2*e*f^3 - (8*b^2*c*d + 3*a*b*d^2)*e^3*f + 2*(b^2*c^2 + a*b*c*d)*e^2
*f^2 + (6*b^2*d^2*e^3*f + a*b*c^2*f^4 - (8*b^2*c*d + 3*a*b*d^2)*e^2*f^2 +
2*(b^2*c^2 + a*b*c*d)*e*f^3)*x^2)*sqrt((b*e - a*f)/e)*log(((8*b^2*e^2 - 8*
a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 - 4*(a...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**2/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^2}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(252) = 504$.

Time = 0.17 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*b*d^2*x^2/f^2 - (8*b^3*d^2*e*f^6 - 8*b^3*c*d*f^7 -
5*a*b^2*d^2*f^7)/(b^2*f^9))*x - 1/16*(24*b^2*d^2*e^2 - 32*b^2*c*d*e*f - 24
*a*b*d^2*e*f + 8*b^2*c^2*f^2 + 24*a*b*c*d*f^2 + 3*a^2*d^2*f^2)*log((sqrt(b
)*x - sqrt(b*x^2 + a))^2)/(sqrt(b)*f^4) + 1/2*(6*b^(5/2)*d^2*e^4 - 8*b^(5/
2)*c*d*e^3*f - 9*a*b^(3/2)*d^2*e^3*f + 2*b^(5/2)*c^2*e^2*f^2 + 10*a*b^(3/2
)*c*d*e^2*f^2 + 3*a^2*sqrt(b)*d^2*e^2*f^2 - a*b^(3/2)*c^2*e*f^3 - 2*a^2*sq
rt(b)*c*d*e*f^3 - a^2*sqrt(b)*c^2*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e
*f)*e*f^4) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*d^2*e^4 - 4*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c*d*e^3*f - 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a*b^(3/2)*d^2*e^3*f + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2*
e^2*f^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2)*c*d*e^2*f^2 + (sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*d^2*e^2*f^2 - 3*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*b^(3/2)*c^2*e*f^3 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sq
rt(b)*c*d*e*f^3 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*c^2*f^4 + a^
2*b^(3/2)*d^2*e^3*f - 2*a^2*b^(3/2)*c*d*e^2*f^2 - a^3*sqrt(b)*d^2*e^2*f^2
+ a^2*b^(3/2)*c^2*e*f^3 + 2*a^3*sqrt(b)*c*d*e*f^3 - a^3*sqrt(b)*c^2*f^4)/(
((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e
 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f)*e*f^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 2161, normalized size of antiderivative = 7.61

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
( - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*e*f**3 - 4*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*f**4*x**2 - 8*sqrt(e)*sqrt(a*f - b*
e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(
sqrt(e)*sqrt(b)))*a*b*c*d*e**2*f**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt
(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(
b)))*a*b*c*d*e*f**3*x**2 + 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d
**2*e**3*f + 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sq
rt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d**2*e**2*f**2*
x**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*e**2*f**2 - 8*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - s
qrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*e*f**3*x**2 + 32*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e**3*f + 32*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*b**2*c*d*e**2*f**2*x**2 - 24*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e...
```

3.302 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 309

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3} dx = \frac{bd^2x\sqrt{a+bx^2}}{2f^3} - \frac{(be-af)(de-cf)^2x\sqrt{a+bx^2}}{4ef^3(e+fx^2)^2}$$
$$+ \frac{(de-cf)(2be(5de-cf)-af(5de+3cf))x\sqrt{a+bx^2}}{8e^2f^3(e+fx^2)}$$
$$- \frac{\sqrt{b}d(6bde-4bcf-3adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2f^4}$$
$$+ \frac{(8b^2de^3(3de-2cf)-8abde^2f(3de-cf)+a^2f^2(3d^2e^2+2cdef+3c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}f^4\sqrt{be-af}}$$

output
```
1/2*b*d^2*x*(b*x^2+a)^(1/2)/f^3-1/4*(-a*f+b*e)*(-c*f+d*e)^2*x*(b*x^2+a)^(1
/2)/e/f^3/(f*x^2+e)^2+1/8*(-c*f+d*e)*(2*b*e*(-c*f+5*d*e)-a*f*(3*c*f+5*d*e)
)*x*(b*x^2+a)^(1/2)/e^2/f^3/(f*x^2+e)-1/2*b^(1/2)*d*(-3*a*d*f-4*b*c*f+6*b*
d*e)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/f^4+1/8*(8*b^2*d*e^3*(-2*c*f+3*d*e
)-8*a*b*d*e^2*f*(-c*f+3*d*e)+a^2*f^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*arct
anh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/f^4/(-a*f+b*e)^(1/
2)
```

Mathematica [A] (verified)

Time = 10.65 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{fx\sqrt{a+bx^2}\left(4bd^2-\frac{2(be-af)(de-cf)^2}{e(e+fx^2)^2}+\frac{(de-cf)(2be(5de-cf)-af(5de+3cf))}{e^2(e+fx^2)}\right)+\frac{(8b^2\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(4*b*d^2 - (2*(b*e - a*f)*(d*e - c*f)^2)/(e*(e + f*x^
2)^2) + ((d*e - c*f)*(2*b*e*(5*d*e - c*f) - a*f*(5*d*e + 3*c*f)))/(e^2*(e
+ f*x^2))) + ((8*b^2*d*e^3*(3*d*e - 2*c*f) + 8*a*b*d*e^2*f*(-3*d*e + c*f)
+ a^2*f^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*
x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(5/2)*Sqrt[-(b*e) + a*f]) - 4*Sqrt[b]*d*
(6*b*d*e - 4*b*c*f - 3*a*d*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*f^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1024 vs. 2(309) = 618.

Time = 1.60 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.31, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {425, 425, 420, 299, 224, 219, 398, 224, 219, 291, 221, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{420}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{299}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 219

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\Big\downarrow$ 398

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{(\cdots}{\sqrt{bx^2\cdots}}}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 219

$$b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\frac{(\cdots}{\sqrt{bx^2\cdots}}}{f\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\frac{(be-af)\int\cdots}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(be-af)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}$$

$\downarrow$ 221

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\int \frac{}{\sqrt{}}}{}}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int}{}\right)}{}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}d}{f}\right)}{f}\right)}{f}$$

$\downarrow$ 398

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d(\cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\cdots\right.}{f}\right.}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d(\ldots}{}\right.}{}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+\ldots\right.}}{f}\right.}{}\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(d\left(\cdots\right.\right.}{\cdots}\right)}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}\cdots\right.}{\cdots}\right)}{f}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+\cdots}{\sqrt{bx^2+a}\left(f\cdots\right.}}{f}\right.}{\cdots}\right.}{f}$$

$\downarrow$ 221

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d}{\cdots}\right.}{\cdots}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+\cdots}{\sqrt{bx^2+a}\left(fx\cdots\right.}}{f}\right.}{\cdots}\right.}{f}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\frac{(be\cdots}{\cdots}\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+\cdots\right)}}{2e(be-af)}\cdots\right.}{f}\cdots\right.}{f}\cdots\right.$$

$\downarrow$ 291

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d(\cdots}{\cdots}\right.}{\cdots}}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)(fx^2+e)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\cdots}{2e(be-af)}\right.}{f}\right.}{f}\right.$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\right.}{}\right.}{}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be}}{\sqrt{e}\sqrt{}}\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.}{}$$

↓ 402

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(d\left(\cdots\right.\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\cdots\right.}{f}\cdots\right.}{\cdots}$$

↓ 27

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be\cdots}}{\sqrt{e}\sqrt{\cdots}}\right)}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be}\cdots}{\sqrt{e}\sqrt{\cdots}}\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.$$

↓ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\right.}{}\right.}{}\right.}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be}}{\sqrt{e}\sqrt{}}\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

output
```
(b*((b*((d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)
/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f - ((b*e -
 a*f)*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e -
c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sq
rt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b
*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*
f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f))/
f - ((b*e - a*f)*((b*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
)/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a +
b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh
[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3
/2))))/f))/f - ((b*e - a*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e
 - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]
*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e
- c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) +
(((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e -
 a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{2(f\,x^2+e)^2\left(-\frac{3a^2\left(c^2f^2+\frac{2}{3}cdef+d^2e^2\right)f^2\sqrt{b}}{16}+b^{\frac{3}{2}}d\left(-\frac{3bd\,e^2}{2}+f\left(bc+\frac{3ad}{2}\right)e-\frac{ac\,f^2}{2}\right)e^2\right)\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\frac{\sqrt{(af-be)}}{}}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
2*((f*x^2+e)^2*(-3/16*a^2*(c^2*f^2+2/3*c*d*e*f+d^2*e^2)*f^2*b^(1/2)+b^(3/2
)*d*(-3/2*b*d*e^2+f*(b*c+3/2*a*d)*e-1/2*a*c*f^2)*e^2)*arctan(e*(b*x^2+a)^(
1/2)/x/((a*f-b*e)*e)^(1/2))+1/8*((a*f-b*e)*e)^(1/2)*(6*(-2*b*d*e+f*(a*d+4/
3*b*c))*d*b*(f*x^2+e)^2*e^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(5/2*a*(3/5
*d*e^2+f*(d*x^2+c)*e+3/5*c*f^2*x^2)*(c*f-d*e)*f*b^(1/2)+b^(3/2)*(6*d^2*e^3
-4*d*(-9/4*x^2*d+c)*f*e^2-6*d*x^2*f^2*(-1/3*x^2*d+c)*e+f^3*x^2*c^2)*e)*(b*
x^2+a)^(1/2)*x*f))/((a*f-b*e)*e)^(1/2)/b^(1/2)/f^4/(f*x^2+e)^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. 2(277) = 554.

Time = 7.60 (sec) , antiderivative size = 3566, normalized size of antiderivative = 11.54

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**2/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^2}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(277) = 554$.

Time = 0.19 (sec) , antiderivative size = 1394, normalized size of antiderivative = 4.51

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b*d^2*x/f^3 + 1/4*(6*b^(3/2)*d^2*e - 4*b^(3/2)*c*d*f -
 3*a*sqrt(b)*d^2*f)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/f^4 - 1/8*(24*b^(
5/2)*d^2*e^4 - 16*b^(5/2)*c*d*e^3*f - 24*a*b^(3/2)*d^2*e^3*f + 8*a*b^(3/2)
*c*d*e^2*f^2 + 3*a^2*sqrt(b)*d^2*e^2*f^2 + 2*a^2*sqrt(b)*c*d*e*f^3 + 3*a^2
*sqrt(b)*c^2*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e -
a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*e^2*f^4) + 1/4*(2
4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d^2*e^4*f - 32*(sqrt(b)*x - sqrt
(b*x^2 + a))^6*b^(5/2)*c*d*e^3*f^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*
b^(3/2)*d^2*e^3*f^2 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*e^2*f^
3 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d*e^2*f^3 + 5*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^2*e^2*f^3 - 2*(sqrt(b)*x - sqrt(b*x^2
 + a))^6*a^2*sqrt(b)*c*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqr
t(b)*c^2*f^5 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^2*e^5 - 96*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*d*e^4*f - 120*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a*b^(5/2)*d^2*e^4*f + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2
)*c^2*e^3*f^2 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d*e^3*f^2
+ 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^2*e^3*f^2 + 8*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*e^2*f^3 - 52*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^2*b^(3/2)*c*d*e^2*f^3 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*s
qrt(b)*d^2*e^2*f^3 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 5944, normalized size of antiderivative = 19.24

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
( - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e**2*f**5 - 24*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e*f**6*x**2 - 12*sqrt(e)*sqr
t(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sq
rt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*f**7*x**4 - 8*sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a**3*c*d*e**3*f**4 - 16*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**3*c*d*e**2*f**5*x**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
 b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a
**3*c*d*e*f**6*x**4 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s
qrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*
e**4*f**3 - 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqr
t(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**3*f**4*
x**2 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
 b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**2*f**5*x**4
+ 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x*
*2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c**2*e**3*f**4 + 48*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -...
```

3.303 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 414

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)(de-cf)^2x\sqrt{a+bx^2}}{6ef^3(e+fx^2)^3}$$
$$+\frac{(de-cf)(2be(7de-cf)-af(7de+5cf))x\sqrt{a+bx^2}}{24e^2f^3(e+fx^2)^2}$$
$$-\frac{(4b^2e^2(11d^2e^2-4cdef-c^2f^2)-4abef(11d^2e^2-cdef+2c^2f^2)+3a^2f^2(d^2e^2+2cdef+5c^2f^2))\,x\sqrt{a+}}{48e^3f^3(be-af)(e+fx^2)}$$
$$+\frac{b^{3/2}d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f^4}$$
$$-\frac{(16b^3d^2e^5-24ab^2d^2e^4f+6a^2bef^2(d^2e^2-c^2f^2)+a^3f^3(d^2e^2+2cdef+5c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{16e^{7/2}f^4(be-af)^{3/2}}$$

output
```
-1/6*(-a*f+b*e)*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f^3/(f*x^2+e)^3+1/24*(-c*
f+d*e)*(2*b*e*(-c*f+7*d*e)-a*f*(5*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^3/(f
*x^2+e)^2-1/48*(4*b^2*e^2*(-c^2*f^2-4*c*d*e*f+11*d^2*e^2)-4*a*b*e*f*(2*c^2
*f^2-c*d*e*f+11*d^2*e^2)+3*a^2*f^2*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*x*(b*x^2
+a)^(1/2)/e^3/f^3/(-a*f+b*e)/(f*x^2+e)+b^(3/2)*d^2*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/f^4-1/16*(16*b^3*d^2*e^5-24*a*b^2*d^2*e^4*f+6*a^2*b*e*f^2*(-c^
2*f^2+d^2*e^2)+a^3*f^3*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*arctanh((-a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(7/2)/f^4/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 11.46 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \frac{\frac{fx\sqrt{a+bx^2}\left(-8e^2(be-af)(de-cf)^2+2e(de-cf)(2be(7de-cf)-af(7de+5cf))(e+fx^2)-\frac{(4abef(-11d\cdots}{\cdots}\right.}{e^3(e+fx^2)^3}\cdots}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(-8*e^2*(b*e - a*f)*(d*e - c*f)^2 + 2*e*(d*e - c*f)*
(2*b*e*(7*d*e - c*f) - a*f*(7*d*e + 5*c*f))*(e + f*x^2) - ((4*a*b*e*f*(-11
*d^2*e^2 + c*d*e*f - 2*c^2*f^2) + 4*b^2*e^2*(11*d^2*e^2 - 4*c*d*e*f - c^2*
f^2) + 3*a^2*f^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*(e + f*x^2)^2)/(b*e -
a*f)))/(e^3*(e + f*x^2)^3) + (3*(16*b^3*d^2*e^5 - 24*a*b^2*d^2*e^4*f + 6*a
^2*b*e*f^2*(d^2*e^2 - c^2*f^2) + a^3*f^3*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2)
)*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(7/2)*(-(b*
e) + a*f)^(3/2)) + 48*b^(3/2)*d^2*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*
f^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. 2(414) = 828.

Time = 2.05 (sec) , antiderivative size = 1559, normalized size of antiderivative = 3.77, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {425, 425, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{\left(fx^2+e\right)^4}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^4}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 398

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{}{\sqrt{}}}{}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^4}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 224

$$b\left(\frac{b\left(\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de\cdots}{\cdots}\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de\cdots}{\cdots}\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\right.}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\right.}{f}}{f}\right.}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\right.}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\right.}{f}}{f}\right.}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}\right)}{f}\right)}{f}-\frac{(be-a\cdots}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\cdots\right.}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}d\cdots}{4e(be-af)}\cdots\right.}{f}\cdots\right.}{f}\cdots\right.}$$

$\downarrow$ 291

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{f}}{f}}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}}{4e(be-af)}}{f}}{f}}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right)$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3a}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}}{4e(be-af)}\right.}{f}\right.}{f}\right.$$

↓ 402

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)(fx^2+e)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)(fx^2+e)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)(fx^2+e)}\right.}{f}\right.}{f}\right.}$$

$$\Big\downarrow 27$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{\cdots}}{2e(be-af)\left(fx^2+e\right)}\cdots\right.}{\cdots}}{f}\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}}{f}\right.$$

↓ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{\ }}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{f}\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}}{}\right.}{}\right)}{f}\right.$$

↓ 27

$$b\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{f}\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{f}\right.$$

↓ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}\right)}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))\sqrt{}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{f}\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

output
```
(b*((b*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e -
 c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*S
qrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(
b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a
*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f -
 ((b*e - a*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f
*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*S
qrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e
- c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e
- 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x
^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTan
h[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(
3/2)))/(4*e*(b*e - a*f))))/f))/f))/f - ((b*e - a*f)*((b*((d*(((d*e - c*f)*
x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*
f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*
e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(
b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*
Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(
d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*S
qrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/f)...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{5\left(\left(\frac{16b^3d^2e^5}{5}-\frac{24a\,b^2d^2e^4f}{5}+\frac{6a^2b\,d^2e^3f^2}{5}+\frac{a^3d^2e^2f^3}{5}+\frac{2a^2c\,f^4(ad-3bc)e}{5}+a^3c^2f^5\right)\left(f\,x^2+e\right)^3\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)--\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16*((16/5*b^3*d^2*e^5-24/5*a*b^2*d^2*e^4*f+6/5*a^2*b*d^2*e^3*f^2+1/5*a^
3*d^2*e^2*f^3+2/5*a^2*c*f^4*(a*d-3*b*c)*e+a^3*c^2*f^5)*(f*x^2+e)^3*arctan(
e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-11/5*(16/11*d^2*e^3*(f*x^2+e)^3*(
a*f*b^(3/2)-b^(5/2)*e)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(8/11*b^2*d^2*e^
6-6/11*(-10/3*b*x^2+a)*d^2*b*f*e^5-1/11*d^2*f^2*(-44/3*b^2*x^4+46/3*a*b*x^
2+a^2)*e^4-2/11*f^3*((22/3*a*b*d^2+8/3*b^2*c*d)*x^4+(4/3*a^2*d^2+14/3*a*b*
c*d+2*b^2*c^2)*x^2+a*c*(a*d+5*b*c))*e^3+f^4*((1/11*a^2*d^2-4/33*b^2*c^2+4/
33*a*b*c*d)*x^4+(-2/3*b*c^2*a+16/33*a^2*c*d)*x^2+a^2*c^2)*e^2+40/33*a*((3/
20*a*d-1/5*b*c)*x^2+a*c)*c*x^2*f^5*e+5/11*a^2*c^2*f^6*x^4)*(b*x^2+a)^(1/2)
*x*f)*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)/(f*x^2+e)^3/(a*f-b*e)/e^3/f
^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. 2(384) = 768.

Time = 29.43 (sec) , antiderivative size = 5081, normalized size of antiderivative = 12.27

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**2/(e + f*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^2}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2836 vs. $2(384) = 768$.

Time = 0.22 (sec) , antiderivative size = 2836, normalized size of antiderivative = 6.85

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/16*(16*b^(7/2)*d^2*e^5 - 24*a*b^(5/2)*d^2*e^4*f + 6*a^2*b^(3/2)*d^2*e^3*
f^2 + a^3*sqrt(b)*d^2*e^2*f^3 - 6*a^2*b^(3/2)*c^2*e*f^4 + 2*a^3*sqrt(b)*c*
d*e*f^4 + 5*a^3*sqrt(b)*c^2*f^5)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*e^4*f^4 - a*e^3*f^5)*sqr
t(-b^2*e^2 + a*b*e*f)) - 1/2*b^(3/2)*d^2*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2)/f^4 - 1/24*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d^2*e^5*f^2 -
 96*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c*d*e^4*f^3 - 216*(sqrt(b)*x
- sqrt(b*x^2 + a))^10*a*b^(5/2)*d^2*e^4*f^3 + 96*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*a*b^(5/2)*c*d*e^3*f^4 + 78*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^
(3/2)*d^2*e^3*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^2*e^2
*f^5 + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c^2*e*f^6 - 6*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*d*e*f^6 - 15*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^3*sqrt(b)*c^2*f^7 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^
(9/2)*d^2*e^6*f - 384*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*d*e^5*f^2
- 1728*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d^2*e^5*f^2 - 96*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^2*e^4*f^3 + 480*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a*b^(7/2)*c*d*e^4*f^3 + 1188*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b
^(5/2)*d^2*e^4*f^3 + 96*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^2*e^3*
f^4 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c*d*e^3*f^4 - 324*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*d^2*e^3*f^4 + 180*(sqrt(b)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}{(f\,x^2+e)^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^2)/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 8163, normalized size of antiderivative = 19.72

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e**3*f**6 - 45*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e**2*f**7*x**2 - 45*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*e*f**8*x**4 - 15*sqrt(e)*sqrt(a*f
 - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)
*x)/(sqrt(e)*sqrt(b)))*a**4*c**2*f**9*x**6 - 6*sqrt(e)*sqrt(a*f - b*e)*ata
n((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e
)*sqrt(b)))*a**4*c*d*e**4*f**5 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**4*c*d*e**3*f**6*x**2 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*
c*d*e**2*f**7*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqr
t(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d*e*f
**8*x**6 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(
a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d**2*e**5*f**4 -
9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2)
 - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d**2*e**4*f**5*x**2 - 9*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - ...
```

3.304 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 436

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2} dx =$$
$$-\frac{(3a^3d^3f^3+8a^2bd^2f^2(de-3cf)+64b^3(de-cf)^3-80ab^2df(d^2e^2-3cdef+3c^2f^2))\,x\sqrt{a+bx^2}}{128b^2f^4}$$
$$+\frac{d(3a^2d^2f^2-56abdf(de-3cf)+48b^2(d^2e^2-3cdef+3c^2f^2))\,x^3\sqrt{a+bx^2}}{192bf^3}$$
$$-\frac{d^2(8bde-24bcf-9adf)x^5\sqrt{a+bx^2}}{48f^2}+\frac{bd^3x^7\sqrt{a+bx^2}}{8f}$$
$$+\frac{(3a^4d^3f^4+8a^3bd^2f^3(de-3cf)+128b^4e(de-cf)^3-192ab^3f(de-cf)^3+48a^2b^2df^2(d^2e^2-3cdef+3c}{128b^{5/2}f^5}$$
$$-\frac{(be-af)^{3/2}(de-cf)^3\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^5}$$

output
```
-1/128*(3*a^3*d^3*f^3+8*a^2*b*d^2*f^2*(-3*c*f+d*e)+64*b^3*(-c*f+d*e)^3-80*
a*b^2*d*f*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^2/f^4+1/192*d
*(3*a^2*d^2*f^2-56*a*b*d*f*(-3*c*f+d*e)+48*b^2*(3*c^2*f^2-3*c*d*e*f+d^2*e^
2))*x^3*(b*x^2+a)^(1/2)/b/f^3-1/48*d^2*(-9*a*d*f-24*b*c*f+8*b*d*e)*x^5*(b*
x^2+a)^(1/2)/f^2+1/8*b*d^3*x^7*(b*x^2+a)^(1/2)/f+1/128*(3*a^4*d^3*f^4+8*a^
3*b*d^2*f^3*(-3*c*f+d*e)+128*b^4*e*(-c*f+d*e)^3-192*a*b^3*f*(-c*f+d*e)^3+4
8*a^2*b^2*d*f^2*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)
^(1/2))/b^(5/2)/f^5-(-a*f+b*e)^(3/2)*(-c*f+d*e)^3*arctanh((-a*f+b*e)^(1/2)
*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^5
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2}\,dx = \frac{fx\sqrt{a+bx^2}\left(-9a^3d^3f^3+6a^2bd^2f^2\left(-4de+12cf+dfx^2\right)+8ab^2df\left(90c^2f^2+6cdf\left(-15e+7fx^2\right)+d^2\left(30e^2\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(-9*a^3*d^3*f^3 + 6*a^2*b*d^2*f^2*(-4*d*e + 12*c*f +
 d*f*x^2) + 8*a*b^2*d*f*(90*c^2*f^2 + 6*c*d*f*(-15*e + 7*f*x^2) + d^2*(30*
e^2 - 14*e*f*x^2 + 9*f^2*x^4)) - 16*b^3*(-12*c^3*f^3 - 18*c^2*d*f^2*(-2*e
+ f*x^2) - 6*c*d^2*f*(6*e^2 - 3*e*f*x^2 + 2*f^2*x^4) + d^3*(12*e^3 - 6*e^2
*f*x^2 + 4*e*f^2*x^4 - 3*f^3*x^6))))/b^2 + (384*(-(b*e) + a*f)^(3/2)*(d*e
- c*f)^3*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sq
rt[-(b*e) + a*f])]/Sqrt[e] - (3*(3*a^4*d^3*f^4 + 8*a^3*b*d^2*f^3*(d*e - 3
*c*f) + 128*b^4*e*(d*e - c*f)^3 + 192*a*b^3*f*(-(d*e) + c*f)^3 + 48*a^2*b^
2*d*f^2*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/b^(5/2))/(384*f^5)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.44, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {420, 318, 403, 299, 211, 224, 219, 420, 318, 299, 211, 224, 219, 420, 299, 211, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{e+fx^2}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \sqrt{bx^2+a}\left(dx^2+c\right)^3dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{b\left(\frac{\int \sqrt{bx^2+a}\left(dx^2+c\right)\left(d(12bc-5ad)x^2+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{b\left(\frac{\frac{\int \sqrt{bx^2+a}\left(d\left(72b^2c^2-52abdc+15a^2d^2\right)x^2+c\left(48b^2c^2-18abdc+5a^2d^2\right)\right)dx}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-5ad)}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{fx^2+e}dx}{f}$$

$$\Big\downarrow \text{ 299}$$

$$b\left(\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-5ad)}{6b}}{8b}+\frac{dx(a+bx^2}{}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 211

$$b\left(\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-}{6b}}{8b}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 224

$$b\left(\frac{\frac{\frac{3\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+d\right.}{6b}}{8b}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 219

$$b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}}{8b}\right.}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{fx^2+e}dx}{f}$$

$\Big\downarrow$ 420

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\sqrt{bx^2+a}\left(dx^2+c\right)^2dx}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}$$

$\big\downarrow$ 318

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\int\sqrt{bx^2+a}\left(d(8bc-3ad)x^2+c(6bc-ad)\right)dx}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}$$

$\big\downarrow$ 299

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}$$

$\downarrow$ 211

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2\ldots}}{\ldots}}{f}\right.}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf\ldots}{}\right.}{f}$$

$\downarrow$ 219

$$b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.$$

$$\overline{\hspace{20em}f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$$\overline{\hspace{20em}f}$$

$\downarrow$ 420

$$b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.$$

$$\overline{\hspace{20em}f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$$\overline{\hspace{20em}f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.}{f}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}}{8b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\cdots}{}\right.}{f}$$

$\downarrow$ 403

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}}{8b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.}{f}$$

$\downarrow$ 398

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$\downarrow$ 224

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}}{8b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$\downarrow$ 219

$$\frac{b\left(\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$\downarrow$ 291

$$\frac{b\left(\frac{\frac{\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{4b}}{6b}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx\right.}{6b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

$$\downarrow 221$$

$$b\left(\frac{\frac{\frac{dx(a+bx^2)^{3/2}(15a^2d^2-52abcd+72b^2c^2)}{4b}+\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3)}{4b}}{6b}}{8b}+\frac{dx(a+bx^2)^{3/2}(c+dx}{6b}\right.}{f}$$

$$(be-af)\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(a^2d^2-4abcd+8b^2c^2)}{4b}+\frac{dx(a+bx^2)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b}\right)}{f}-\frac{(de-cf)}{}\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

output
```
(b*((d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + ((d*(12*b*c - 5*a*d)*x*(
a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^
2*d^2)*x*(a + b*x^2)^(3/2))/(4*b) + (3*(64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a
^2*b*c*d^2 - 5*a^3*d^3)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b))/(8*b)))/f - ((b*e - a*f)*((d*(
(d*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(6*b) + ((d*(8*b*c - 3*a*d)*x*(a + b*x
^2)^(3/2))/(4*b) + (3*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b*x^2
])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(6*b)
))/f - ((d*e - c*f)*((d*((d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*d)*((
x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)))/(4*b)))/f - ((d*e - c*f)*((d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e - 2
*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2*Sqr
t[b*e - a*f]*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x
^2])])/(Sqrt[e]*f))/(2*f)))/f))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-2(-af+be)^2(cf-de)^3 b^{\frac{9}{2}} \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(\frac{3\left(\frac{128b^4d^3e^4}{3}-64b^3d^2f(ad+2bc)e^3+16b^2d\,f^2\left(a^2d^2+12abcd-\right.\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(-a*f+b*e)^2*(c*f-d*e)^3*b^(9/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-
b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(3/64*(128/3*b^4*d^3*e^4-64*b^3*d^2*f*(
a*d+2*b*c)*e^3+16*b^2*d*f^2*(a^2*d^2+12*a*b*c*d+8*b^2*c^2)*e^2+8/3*b*f^3*(
a^3*d^3-18*a^2*b*c*d^2-72*a*b^2*c^2*d-16*b^3*c^3)*e+a*f^4*(a^3*d^3-8*a^2*b
*c*d^2+48*a*b^2*c^2*d+64*b^3*c^3))*b^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+
(b*x^2+a)^(1/2)*x*f*b^(5/2)*(-b^3*d^3*e^3+5/4*d^2*((2/5*x^2*d+12/5*c)*b+a*
d)*b^2*f*e^2-1/8*d*b*f^2*((8/3*d^2*x^4+12*c*d*x^2+24*c^2)*b^2+30*a*d*(7/45
*x^2*d+c)*b+a^2*d^2)*e-3/64*((-16/3*d^3*x^6-64/3*c*d^2*x^4-32*c^2*d*x^2-64
/3*c^3)*b^3-80*a*d*(1/10*d^2*x^4+7/15*c*d*x^2+c^2)*b^2-8*a^2*(1/12*x^2*d+c
)*d^2*b+a^3*d^3)*f^3)))/((a*f-b*e)*e)^(1/2)/b^(9/2)/f^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^3}{e+fx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(c+dx^2)^3}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**3/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{e+fx^2}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^3}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e),x)
```

3.305 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 375

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{d(a^2d^2f^2-10abdf(2de-3cf)+24b^2(de-cf)^2)x\sqrt{a+bx^2}}{16bf^4}$$
$$-\frac{d^2(12bde-18bcf-7adf)x^3\sqrt{a+bx^2}}{24f^3}$$
$$+\frac{bd^3x^5\sqrt{a+bx^2}}{6f^2}+\frac{(be-af)(de-cf)^3x\sqrt{a+bx^2}}{2ef^4(e+fx^2)}$$
$$-\frac{(a^3d^3f^3+6a^2bd^2f^2(2de-3cf)-72ab^2df(de-cf)^2+16b^3(de-cf)^2(4de-cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}f^5}$$
$$+\frac{\sqrt{be-af}(de-cf)^2(2be(4de-cf)-af(5de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^5}$$

output
```
1/16*d*(a^2*d^2*f^2-10*a*b*d*f*(-3*c*f+2*d*e)+24*b^2*(-c*f+d*e)^2)*x*(b*x^
2+a)^(1/2)/b/f^4-1/24*d^2*(-7*a*d*f-18*b*c*f+12*b*d*e)*x^3*(b*x^2+a)^(1/2)
/f^3+1/6*b*d^3*x^5*(b*x^2+a)^(1/2)/f^2+1/2*(-a*f+b*e)*(-c*f+d*e)^3*x*(b*x^
2+a)^(1/2)/e/f^4/(f*x^2+e)-1/16*(a^3*d^3*f^3+6*a^2*b*d^2*f^2*(-3*c*f+2*d*e
)-72*a*b^2*d*f*(-c*f+d*e)^2+16*b^3*(-c*f+d*e)^2*(-c*f+4*d*e))*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/f^5+1/2*(-a*f+b*e)^(1/2)*(-c*f+d*e)^2*(2*b*
e*(-c*f+4*d*e)-a*f*(c*f+5*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+
a)^(1/2))/e^(3/2)/f^5
```

Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{\frac{fx\sqrt{a+bx^2}\left(3a^2d^3ef^2(e+fx^2)+2abf\left(-36c^2def^2+12c^3f^3+9cd^2ef(9e+5fx^2)+d^3e\left(-42e^2-23efx^2+\right.\right.\right.}{}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
((f*x*Sqrt[a + b*x^2]*(3*a^2*d^3*e*f^2*(e + f*x^2) + 2*a*b*f*(-36*c^2*d*e*
f^2 + 12*c^3*f^3 + 9*c*d^2*e*f*(9*e + 5*f*x^2) + d^3*e*(-42*e^2 - 23*e*f*x
^2 + 7*f^2*x^4)) + 4*b^2*e*(-6*c^3*f^3 + 18*c^2*d*f^2*(2*e + f*x^2) + 9*c*
d^2*f*(-6*e^2 - 3*e*f*x^2 + f^2*x^4) + 2*d^3*(12*e^3 + 6*e^2*f*x^2 - 2*e*f
^2*x^4 + f^3*x^6))))/(b*e*(e + f*x^2)) + (24*Sqrt[-(b*e) + a*f]*(d*e - c*f
)^2*(2*b*e*(4*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2
]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/e^(3/2) + (3*(a^3
*d^3*f^3 + 6*a^2*b*d^2*f^2*(2*d*e - 3*c*f) - 72*a*b^2*d*f*(d*e - c*f)^2 +
16*b^3*(d*e - c*f)^2*(4*d*e - c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b
^(3/2))/(48*f^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3}{\left(e+fx^2\right)^2} \, dx$$

$$\Big\downarrow \ 425$$

$$\frac{b \int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{fx^2+e}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 420$$

$$\frac{b\left(\frac{d\int \sqrt{bx^2+a}\left(dx^2+c\right)^2dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 318$$

$$\frac{b\left(\frac{d\left(\frac{\int \sqrt{bx^2+a}\left(d(8bc-3ad)x^2+c(6bc-ad)\right)dx}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 299$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 211$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(a^2d^2-4abcd+8b^2c^2\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}}{fx}}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}(}{fx^2}}{f}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 420$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\sqrt{bx^2\cdots}}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 299

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(4bc-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 211

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\frac{(4bc-\cdots}{\cdots}\right.\right.\cdots}{\cdots}\right.\cdots}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 224

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\frac{(4bc-\cdots}{\cdots}\right.\right.\cdots}{\cdots}\right.\cdots}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\,\text{ar}\cdots}{\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow\ 403$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\,\text{ar}\cdots}{\cdots}\right.\right.\right.}{\cdots}\cdots\right)}{f}$$

$$\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{d\left(\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\frac{\left(\frac{a\,\text{ar}}{}\right.}{}\right.\right.}{}\right.}{f}$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 398$$

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\,\text{ar}\ldots\right.\right.\right.}{\ldots}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\big\downarrow$ 224

$$b\left(\frac{d\left(\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\,\text{ar}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{d\left(\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\,\text{ar}\cdots}{\cdots}\right.\right.\right.}{\cdots}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow\ 291$$

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\text{ar}}{}\right.\right.\right.}{}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 221

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\left(a\,\text{ar}\ldots\right.\right.\right.\right.}{\ldots}\right.$$

$$\frac{(be-af)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 425$$

$$b\left(\frac{d\left(\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\,\text{ar}\cdots}{\cdots}\right.\right.\right.}{\cdots}\right.$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\operatorname{ar}\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}}$$

$$\Big\downarrow \; 318$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{ar}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\int\frac{3d(2bc-ad)x^2+c(4bc-ad)}{\sqrt{bx^2+a}}dx}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\,\text{ar}\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-a\cdots}{\cdots}\right.}{f}$$

$\big\downarrow$ 224

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(\frac{a\,\text{ar}\cdots}{}\right.\right.\right.}{}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}$$

↓ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\operatorname{ar}\ldots}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\ldots\right.}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\,\text{ar}}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\cdots}{\cdots}\right.}{f}\right.}{f}\right.}{f}$$

$$\downarrow 299$$

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\operatorname{ar}\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx\cdots}{2b}\right.}{f}\cdots\right.}{\cdots}\right.}{f}\cdots\right.}{f}$$

↓ 224

$$b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(d\left(\left(a\operatorname{ar}\ldots\right.\right.\right.}{\ldots}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}\ldots}{2b}\right.}{\ldots}\right.}{\ldots}\right)}{f}\right.$$

$$f$$

↓ 219

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\frac{a\operatorname{ar}\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}\right.}{f}$$

↓ 398

$$\frac{b\left(\frac{d\left(\frac{\frac{3\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{4b}}{6b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\left(\left(\frac{a\operatorname{ar}\cdots}{\cdots}\right.\right.\right.}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)}{f}\cdots\right)}{f\cdots}$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 420

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$-\frac{-2b^{\frac{5}{2}}\left(-4bd\,e^{2}+\left(bcf+\frac{5}{2}adf\right)e+\frac{ac\,f^{2}}{2}\right)(-af+be)(cf-de)^{2}\left(f\,x^{2}+e\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(\frac{b\left(f\,x^{2}+e\right)e}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b^(5/2)*(-4*b*d*e^2+(b*c*f+5/2*a*d*f)*e+1/2*a*c*f^2)*(-a*f+b*e)*(
c*f-d*e)^2*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f
-b*e)*e)^(1/2)*(1/8*b*(f*x^2+e)*e*(64*b^3*d^3*e^3-72*b^2*d^2*f*(a*d+2*b*c)
*e^2+12*b*d*f^2*(a^2*d^2+12*a*b*c*d+8*b^2*c^2)*e+f^3*(a^3*d^3-18*a^2*b*c*d
^2-72*a*b^2*c^2*d-16*b^3*c^3))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+b^(3/2)*
(-4*b^2*d^3*e^4+7/2*((-4/7*x^2*d+18/7*c)*b+a*d)*d^2*b*f*e^3-1/8*d*f^2*((-1
6/3*d^2*x^4-36*c*d*x^2+48*c^2)*b^2+54*a*d*(-23/81*x^2*d+c)*b+a^2*d^2)*e^2-
1/8*((8/3*d^3*x^6+12*c*d^2*x^4+24*c^2*d*x^2-8*c^3)*b^2-24*a*d*(-7/36*d^2*x
^4-5/4*c*d*x^2+c^2)*b+a^2*d^3*x^2)*f^3*e-a*b*c^3*f^4)*(b*x^2+a)^(1/2)*x*f)
)/((a*f-b*e)*e)^(1/2)/b^(5/2)/f^5/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(339) = 678$.

Time = 72.70 (sec) , antiderivative size = 3557, normalized size of antiderivative = 9.49

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^3}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**3/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3/(f*x^2 + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. 2(339) = 678.

Time = 0.19 (sec) , antiderivative size = 1167, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/48*(2*(4*b*d^3*x^2/f^2 - (12*b^5*d^3*e*f^11 - 18*b^5*c*d^2*f^12 - 7*a*b^
4*d^3*f^12)/(b^4*f^14))*x^2 + 3*(24*b^5*d^3*e^2*f^10 - 48*b^5*c*d^2*e*f^11
 - 20*a*b^4*d^3*e*f^11 + 24*b^5*c^2*d*f^12 + 30*a*b^4*c*d^2*f^12 + a^2*b^3
*d^3*f^12)/(b^4*f^14))*sqrt(b*x^2 + a)*x + 1/2*(8*b^(5/2)*d^3*e^5 - 18*b^(
5/2)*c*d^2*e^4*f - 13*a*b^(3/2)*d^3*e^4*f + 12*b^(5/2)*c^2*d*e^3*f^2 + 27*
a*b^(3/2)*c*d^2*e^3*f^2 + 5*a^2*sqrt(b)*d^3*e^3*f^2 - 2*b^(5/2)*c^3*e^2*f^
3 - 15*a*b^(3/2)*c^2*d*e^2*f^3 - 9*a^2*sqrt(b)*c*d^2*e^2*f^3 + a*b^(3/2)*c
^3*e*f^4 + 3*a^2*sqrt(b)*c^2*d*e*f^4 + a^2*sqrt(b)*c^3*f^5)*arctan(-1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/
(sqrt(-b^2*e^2 + a*b*e*f)*e*f^5) + 1/32*(64*b^3*d^3*e^3 - 144*b^3*c*d^2*e^
2*f - 72*a*b^2*d^3*e^2*f + 96*b^3*c^2*d*e*f^2 + 144*a*b^2*c*d^2*e*f^2 + 12
*a^2*b*d^3*e*f^2 - 16*b^3*c^3*f^3 - 72*a*b^2*c^2*d*f^3 - 18*a^2*b*c*d^2*f^
3 + a^3*d^3*f^3)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/(b^(3/2)*f^5) + (2*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*b^3*d^3*e^5 - 6*(sqrt(b)*x - sqrt(b*x^2 + a
))^2*b^3*c*d^2*e^4*f - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^2*d^3*e^4*f +
 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^3*c^2*d*e^3*f^2 + 9*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^2*c*d^2*e^3*f^2 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b
*d^3*e^3*f^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^3*c^3*e^2*f^3 - 9*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*b^2*c^2*d*e^2*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b*c*d^2*e^2*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)^3}{(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

3.306 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 394

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx &= \frac{d^2(5adf-12b(de-cf))x\sqrt{a+bx^2}}{8f^4} \\
&+ \frac{bd^3x^3\sqrt{a+bx^2}}{4f^3} + \frac{(be-af)(de-cf)^3x\sqrt{a+bx^2}}{4ef^4(e+fx^2)^2} \\
&- \frac{(de-cf)^2(2be(7de-cf)-3af(3de+cf))x\sqrt{a+bx^2}}{8e^2f^4(e+fx^2)} \\
&+ \frac{3d(a^2d^2f^2-12abdf(de-cf)+8b^2(2d^2e^2-3cdef+c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}f^5} \\
&- \frac{3(de-cf)\left(8b^2de^3(2de-cf)-4abde^2f(5de-cf)+a^2f^2(5d^2e^2+2cdef+c^2f^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}f^5\sqrt{be-af}}
\end{aligned}
$$

output
```
1/8*d^2*(5*a*d*f-12*b*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/f^4+1/4*b*d^3*x^3*(b*x
^2+a)^(1/2)/f^3+1/4*(-a*f+b*e)*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^4/(f*x^2
+e)^2-1/8*(-c*f+d*e)^2*(2*b*e*(-c*f+7*d*e)-3*a*f*(c*f+3*d*e))*x*(b*x^2+a)^
(1/2)/e^2/f^4/(f*x^2+e)+3/8*d*(a^2*d^2*f^2-12*a*b*d*f*(-c*f+d*e)+8*b^2*(c^
2*f^2-3*c*d*e*f+2*d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^5
-3/8*(-c*f+d*e)*(8*b^2*d*e^3*(-c*f+2*d*e)-4*a*b*d*e^2*f*(-c*f+5*d*e)+a^2*f
^2*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^
2+a)^(1/2))/e^(5/2)/f^5/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 10.94 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{fx\sqrt{a+bx^2}\Big(d^2(-12bde+12bcf+5adf)+2bd^3fx^2+\frac{2(be-af)(de-cf)^3}{e(e+fx^2)^2}-\frac{(d}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(d^2*(-12*b*d*e + 12*b*c*f + 5*a*d*f) + 2*b*d^3*f*x^2
 + (2*(b*e - a*f)*(d*e - c*f)^3)/(e*(e + f*x^2)^2) - ((d*e - c*f)^2*(2*b*e
*(7*d*e - c*f) - 3*a*f*(3*d*e + c*f)))/(e^2*(e + f*x^2))) - (3*(d*e - c*f)
*(8*b^2*d*e^3*(2*d*e - c*f) + 4*a*b*d*e^2*f*(-5*d*e + c*f) + a^2*f^2*(5*d^
2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(e^(5/2)*Sqrt[-(b*e) + a*f]) + (3*d*(a^2*d^2*f^2 - 12*a*b*d*
f*(d*e - c*f) + 8*b^2*(2*d^2*e^2 - 3*c*d*e*f + c^2*f^2))*Log[b*x + Sqrt[b]
*Sqrt[a + b*x^2]])/Sqrt[b])/(8*f^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^2}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}(dx^2+c)^3}{(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\int \frac{3d(2bc-ad)x^2+c(4bc-ad)}{\sqrt{bx^2+a}}dx}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(d}{\sqrt{bx^2+}}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 224

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)}{}\cdots\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 219

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)}{}\cdots\right)}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx^2\ldots}{\sqrt{bx^2+a}\left(\ldots\right.}}{f}\right.}{f}\ldots}{f}\ldots\right.}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 299

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx\ldots}}{2b}\right.}{f}\ldots\right.}{f}\ldots}{f}\ldots\right.}{f}-\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 224$$

$$b\left(\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\cdots\right)}{f}\cdots\right.}{}}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\cdots\right)}{f}\cdots\right.}{\cdots}\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 398$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+}{f}\right.}{}\right.}{}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$
b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\cdots\right.}{f}\right.}{\cdots}\right.}{f}\right.
$$

$$
\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}
$$

$\downarrow$ 219

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}\right.}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}\right)}{f}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int\frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$\downarrow$ 425

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 420

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\cdots\right.}{f}\cdots\right.}{\cdots}\right.}{f}\cdots\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\cdots\right.}{f}\cdots\right.}{f}$$

$\downarrow$ 299

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\cdots\right)}{f}\cdots\right)}{\cdots}\right)}{f}\cdots\right)$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{f}$$

$\downarrow$ 224

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}\right.}{f}\right.$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-}{}\right.}{f}$$

$\downarrow$ 219

$$b\left(\frac{b\left(\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\right.}{f}\right.}{}\right.}{f}\right.}{}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-}{}\right.}{f}$$

$\downarrow$ 398

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+a}x}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\right.}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}\right.}{}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{}{\sqrt{b}}}{}\right.}{f}\right.}{f}$$

↓ 224

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+ax}}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\right.}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}\right.}{}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-}{}\right.}{f}\right.}{f}$$

$\Big\downarrow$ 219

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+ax}}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\right.}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-}{}\right.}{f}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+a}x}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\right.}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}\right)-}{f}\right.$$

$$f$$

↓ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+ax}}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\cdots\right.}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\right.}{f}\cdots\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}\cdots\right.}{f}$$

↓ 425

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+a}\,x}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\cdots\right.}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}\cdots\right.$$

↓ 398

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+ax}}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\cdots\right.}{2b^{3/2}}\cdots\right.}{f}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(\cdots}{\cdots}\cdots\right.}{f}\cdots\right.$$

↓ 224

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+a}x}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\right.}{2b^{3/2}}\right.}{f}\right.}{}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\right.}{f}\right.$$

$\downarrow$ 219

$$b\left(\frac{b\left(\frac{d\left(\frac{dx\sqrt{bx^2+a}\left(dx^2+c\right)}{4b}+\frac{\frac{3d(2bc-ad)\sqrt{bx^2+a}\,x}{2b}+\frac{\left(8b^2c^2-8abdc+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\cdots\right.}{2b^{3/2}}\right.}{f}\cdots\right.}{\cdots}\right.}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\cdots\right.}{f}\cdots\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{3(cf-de)\left(-\frac{f^{2}a^{2}\left(c^{2}f^{2}+2cdef+5d^{2}e^{2}\right)\sqrt{b}}{8}+b^{\frac{3}{2}}d\left(-2bde^{2}+f\left(bc+\frac{5ad}{2}\right)e-\frac{acf^{2}}{2}\right)e^{2}\right)\left(fx^{2}+e\right)^{2}\arctan\left(\frac{e\sqrt{bx^{2}+a}}{x\sqrt{(af-be)e}}\right)+-}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
3*((c*f-d*e)*(-1/8*f^2*a^2*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)*b^(1/2)+b^(3/2)*d
*(-2*b*d*e^2+f*(b*c+5/2*a*d)*e-1/2*a*c*f^2)*e^2)*(f*x^2+e)^2*arctan(e*(b*x
^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+1/12*((a*f-b*e)*e)^(1/2)*(3/2*(16*b^2*d
^2*e^2-12*b*d*f*(a*d+2*b*c)*e+f^2*(a^2*d^2+12*a*b*c*d+8*b^2*c^2))*d*(f*x^2
+e)^2*e^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(5/2*(12/5*e^4*d^3-9/5*d^2*(-
19/9*x^2*d+c)*f*e^3-3/5*d*(-5/3*d^2*x^4+5*c*d*x^2+c^2)*f^2*e^2+c^2*f^3*(3/
5*x^2*d+c)*e+3/5*c^3*f^4*x^2)*a*f*b^(1/2)+b^(3/2)*(-12*e^4*d^3+18*d^2*f*(-
d*x^2+c)*e^3-6*d*(2/3*d^2*x^4-9/2*c*d*x^2+c^2)*f^2*e^2-9*d*x^2*f^3*(-1/9*d
^2*x^4-2/3*c*d*x^2+c^2)*e+c^3*f^4*x^2)*e)*(b*x^2+a)^(1/2)*x*f))/((a*f-b*e)
*e)^(1/2)/b^(1/2)/(f*x^2+e)^2/f^5/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(358) = 716$.

Time = 38.26 (sec) , antiderivative size = 5597, normalized size of antiderivative = 14.21

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^3}{(e+fx^2)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**3/(e + f*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}{(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3/(f*x^2 + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1960 vs. $2(358) = 716$.

Time = 0.22 (sec) , antiderivative size = 1960, normalized size of antiderivative = 4.97

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*b*d^3*x^2/f^3 - (12*b^3*d^3*e*f^8 - 12*b^3*c*d^2*f^
9 - 5*a*b^2*d^3*f^9)/(b^2*f^12))*x - 3/16*(16*b^2*d^3*e^2 - 24*b^2*c*d^2*e
*f - 12*a*b*d^3*e*f + 8*b^2*c^2*d*f^2 + 12*a*b*c*d^2*f^2 + a^2*d^3*f^2)*lo
g((sqrt(b)*x - sqrt(b*x^2 + a))^2)/(sqrt(b)*f^5) + 3/8*(16*b^(5/2)*d^3*e^5
 - 24*b^(5/2)*c*d^2*e^4*f - 20*a*b^(3/2)*d^3*e^4*f + 8*b^(5/2)*c^2*d*e^3*f
^2 + 24*a*b^(3/2)*c*d^2*e^3*f^2 + 5*a^2*sqrt(b)*d^3*e^3*f^2 - 4*a*b^(3/2)*
c^2*d*e^2*f^3 - 3*a^2*sqrt(b)*c*d^2*e^2*f^3 - a^2*sqrt(b)*c^2*d*e*f^4 - a^
2*sqrt(b)*c^3*f^5)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e -
 a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*e^2*f^5) - 1/4*(
32*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d^3*e^5*f - 72*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*b^(5/2)*c*d^2*e^4*f^2 - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a*b^(3/2)*d^3*e^4*f^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d*
e^3*f^3 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^2*e^3*f^3 + 9*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3*e^3*f^3 - 8*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*b^(5/2)*c^3*e^2*f^4 - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a*b^(3/2)*c^2*d*e^2*f^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c
*d^2*e^2*f^4 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^2*d*e*f^5 +
 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^3*f^6 + 112*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*b^(7/2)*d^3*e^6 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b
^(7/2)*c*d^2*e^5*f - 184*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*d^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3}{(f\,x^2+e)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^3}{(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.307 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 510

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \frac{bd^3x\sqrt{a+bx^2}}{2f^4} + \frac{(be-af)(de-cf)^3x\sqrt{a+bx^2}}{6ef^4(e+fx^2)^3}$$
$$-\frac{(de-cf)^2(2be(10de-cf)-af(13de+5cf))x\sqrt{a+bx^2}}{24e^2f^4(e+fx^2)^2}$$
$$+\frac{(de-cf)\left(4b^2e^2(26d^2e^2-7cdef-c^2f^2)-2abef(67d^2e^2+cdef+4c^2f^2)+3a^2f^2(11d^2e^2+8cdef+5c\right.}{48e^3f^4(be-af)(e+fx^2)}$$
$$-\frac{\sqrt{b}d^2(8bde-6bcf-3adf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2f^5}$$
$$+\frac{(16b^3d^2e^5(4de-3cf)-24ab^2d^2e^4f(5de-3cf)+6a^2bef^2(10d^3e^3-3cd^2e^2f+c^3f^3)-a^3f^3(5d^3e^3+3cd}{16e^{7/2}f^5(be-af)^{3/2}}$$

output
```
1/2*b*d^3*x*(b*x^2+a)^(1/2)/f^4+1/6*(-a*f+b*e)*(-c*f+d*e)^3*x*(b*x^2+a)^(1
/2)/e/f^4/(f*x^2+e)^3-1/24*(-c*f+d*e)^2*(2*b*e*(-c*f+10*d*e)-a*f*(5*c*f+13
*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^4/(f*x^2+e)^2+1/48*(-c*f+d*e)*(4*b^2*e^2*(-
c^2*f^2-7*c*d*e*f+26*d^2*e^2)-2*a*b*e*f*(4*c^2*f^2+c*d*e*f+67*d^2*e^2)+3*a
^2*f^2*(5*c^2*f^2+8*c*d*e*f+11*d^2*e^2))*x*(b*x^2+a)^(1/2)/e^3/f^4/(-a*f+b
*e)/(f*x^2+e)-1/2*b^(1/2)*d^2*(-3*a*d*f-6*b*c*f+8*b*d*e)*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/f^5+1/16*(16*b^3*d^2*e^5*(-3*c*f+4*d*e)-24*a*b^2*d^2*e^4
*f*(-3*c*f+5*d*e)+6*a^2*b*e*f^2*(c^3*f^3-3*c*d^2*e^2*f+10*d^3*e^3)-a^3*f^3
*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arctanh((-a*f+b*e)^(1/
2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(7/2)/f^5/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \frac{fx\sqrt{a+bx^2}\Big(24bd^3+\frac{8(be-af)(de-cf)^3}{e(e+fx^2)^3}-\frac{2(de-cf)^2(2be(10de-cf)-af(13de+5cf))}{e^2(e+fx^2)^2}+\cdots$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(24*b*d^3 + (8*(b*e - a*f)*(d*e - c*f)^3)/(e*(e + f*x
^2)^3) - (2*(d*e - c*f)^2*(2*b*e*(10*d*e - c*f) - a*f*(13*d*e + 5*c*f)))/(
e^2*(e + f*x^2)^2) + ((d*e - c*f)*(4*b^2*e^2*(26*d^2*e^2 - 7*c*d*e*f - c^2
*f^2) - 2*a*b*e*f*(67*d^2*e^2 + c*d*e*f + 4*c^2*f^2) + 3*a^2*f^2*(11*d^2*e
^2 + 8*c*d*e*f + 5*c^2*f^2)))/(e^3*(b*e - a*f)*(e + f*x^2))) - (3*(16*b^3*
d^2*e^5*(4*d*e - 3*c*f) - 24*a*b^2*d^2*e^4*f*(5*d*e - 3*c*f) + 6*a^2*b*e*f
^2*(10*d^3*e^3 - 3*c*d^2*e^2*f + c^3*f^3) - a^3*f^3*(5*d^3*e^3 + 3*c*d^2*e
^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*
Sqrt[a + b*x^2]])])/(e^(7/2)*(-(b*e) + a*f)^(3/2)) - 24*Sqrt[b]*d^2*(8*b*d*
e - 6*b*c*f - 3*a*d*f)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*f^5)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2602 vs. $2(510) = 1020$.

Time = 2.45 (sec) , antiderivative size = 2602, normalized size of antiderivative = 5.10, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {425, 425, 425, 420, 299, 224, 219, 398, 224, 219, 291, 221, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}{\left(e+fx^2\right)^4}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^3}{\left(fx^2+e\right)^4}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{b\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^3}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{b\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}\right)}{f}-$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$\downarrow$ 420

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \cdots}{\cdots}\right.}{f}\cdots\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$\downarrow$ 299

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+\cdots\right.}{\sqrt{bx^2+a}\left(f\cdots\right.}}{f}\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4\cdots}}{f}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\downarrow 224$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(d\int \cdots\right.}{\cdots}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\right.}{f}\cdots\right)}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\right.}{f}\right.}{f}$$

$\downarrow$ 398

$$
\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)\cdots}\cdots}{f}\cdots}{f}\cdots\right)}{f}
$$

$$
\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}\cdots}{f}\cdots\right)}{f}\cdots\right)}{f}
$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{(\cdots}{\sqrt{bx^2\cdots}}}{f}\cdots}{f}\cdots}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^4}\cdots}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$\big\downarrow$ 219

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{\left(d\right.}{\sqrt{bx^2-}}}{f}}{f}\right)}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}}{f}\right.}{f}\right.}{f}$$

$$\downarrow 291$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{\quad}{\sqrt{\quad}}}{\quad}}{f}\right.$$

$$\frac{\quad}{f}$$

$$(be-af)\left(\frac{b\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}}{f}\right.}{f}\right.$$

$$\frac{\quad}{f}$$

$\downarrow$ 221

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\int \frac{}{\sqrt{b}}}{}}{f}}{f}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}-\frac{(be-af)\left(\frac{d\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^4}}{f}\right.}{f}\right.}{f}$$

$$\Big\downarrow \; 425$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int}{\cdots}\right.}{\cdots}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\cdots\right.}{f}\right.}{f}\right.$$

$\downarrow$ 398

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}}{f}\cdots\right)}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\cdots\right.}{f}\cdots}{f}\cdots\right)}$$

↓ 224

$$\frac{b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}}{f}\right.}{\cdots}$$

$$\frac{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\cdots\right.}}{f}\cdots\right.}{\cdots}}{f}\right.}{\cdots}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}}{f}\cdots\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\cdots\right.}}{f}\cdots\right.}{\cdots}}{f}\cdots\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}\cdots\right.}{(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+\cdots}{\sqrt{bx^2+a}\left(fx\cdots\right.}}{f}\cdots\right.}{\cdots}\cdots\right.}$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}}{f}\cdots\right.}{\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^{\cdots}\right.}}{f}\cdots\right.}{\cdots}}{f}\cdots\right.}{\cdots}$$

↓ 402

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}\right.}{\cdots}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{f}\right)}{f}-\frac{(d\cdots}{\cdots}\right.}{f}\right.$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx\cdots\right.}}{2e(be-af)}\right.}{f}\right.}{f}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{291}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\right.}{}\right.}{}}{f}\right.}{}$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\int\frac{1}{e-\frac{(be-af)x}{bx^2+a}}}{2e(be-af)}\right.}{f}\right.}{f}\right.}{}$$

$\downarrow$ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}}{f}\right.$$

$$(be-af)\left(\frac{b\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.$$

↓ 402

$$b\,\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}\right.}{f}$$

$$\left(b\,\frac{d\,\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right)}{2e^{3/2}(be-af)^{3/2}}\right.}{f}}{f}\right.$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d(\cdots}{\cdots}\right.}{\cdots}\right)}{f}\right.$$

$$\left(b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\frac{\cdots}{\cdots}\right)}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\right)}{f}\cdots\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}\cdots\right.}{\cdots}$$

$$\left(\left(\frac{b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}\,f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\,f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\cdots\right.}{f}\cdots\right.}{f}\cdots\right.\right.$$

$\downarrow$ 221

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}\right.}{f}\right.$$

$$\left(b\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.}{f}\right.\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+ax}}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}\right)}{f}\cdots\right.$$

$$\left(\left(\left(d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)\right.\right.\right.\cdots (de-cf)\left(\frac{(de-cf)\sqrt{bx^2+ax}}{2e(be-af)(ex^2+\cdots)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right)}{\cdots e^{3/2}(be-af)^{3/2}}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}\right.$$

$$\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{f}$$

↓ 291

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{f}\right.$$

$$\left(\left(d\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right)}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.\right.\right.$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{b\left(\frac{d\left(\frac{d\left(\frac{d\sqrt{bx^2+a}\,x}{2b}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(d\left(\cdots\right.\right.}{\cdots}}{f}\right.$$

$$\left(\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}+\frac{(2bce-a(de+cf))\operatorname{arctanh}\left(\cdots\right)}{2e^{3/2}(be-af)^{3/2}}\right.}{f}\right.\right.$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

output
```
(b*((b*((d*((d*((d*x*Sqrt[a + b*x^2]])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f
]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f - ((d
*e - c*f)*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*
e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*
f*Sqrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*
e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e
- a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/
f))/f - ((b*e - a*f)*((d*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sq
rt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^
2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[
a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*Arc
Tanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f
)^(3/2))))/f))/f - ((d*e - c*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*
(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e -
a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((
d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2
) + (((2*b*e*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b
*e - a*f)*(e + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$-\frac{3\left((f x^2+e)^3\left(\frac{5a^3\left(c^2f^2-\frac{2}{5}cdef+d^2e^2\right)f^3(cf+de)\sqrt{b}}{48}+b^{\frac{3}{2}}\left(-\frac{4b^2d^3e^5}{3}+\frac{5\left(ad+\frac{2bc}{5}\right)d^2bf\,e^4}{2}-\frac{5a\left(ad+\frac{6bc}{5}\right)d^2f^2e^3}{4}+\frac{3a^2c\,d^2e \ldots}{8} \ldots\right.\right.\right.}{\ldots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-3*((f*x^2+e)^3*(5/48*a^3*(c^2*f^2-2/5*c*d*e*f+d^2*e^2)*f^3*(c*f+d*e)*b^(1
/2)+b^(3/2)*(-4/3*b^2*d^3*e^5+5/2*(a*d+2/5*b*c)*d^2*b*f*e^4-5/4*a*(a*d+6/5
*b*c)*d^2*f^2*e^3+3/8*a^2*c*d^2*e^2*f^3-1/8*a^2*c^3*f^5)*e)*arctan(e*(b*x^
2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+1/12*((a*f-b*e)*e)^(1/2)*(-6*(-8/3*b*d*e
+f*(a*d+2*b*c))*d^2*(a*f-b*e)*b*(f*x^2+e)^3*e^3*arctanh((b*x^2+a)^(1/2)/x/
b^(1/2))+(-11/4*a^2*(c*f-d*e)*(5/11*e^4*d^2+8/11*d*(5/3*x^2*d+c)*f*e^3+f^2
*(64/33*c*d*x^2+d^2*x^4+c^2)*e^2+40/33*c*(3/5*x^2*d+c)*x^2*f^3*e+5/11*c^2*
f^4*x^4)*f^2*b^(1/2)+(8*b*d^3*e^6-9*(-20/9*b*d*x^2+2/3*b*c+a*d)*d^2*f*e^5+
9/2*(88/27*b*d*x^4+(-137/27*a*d-10/3*b*c)*x^2+a*c)*d^2*f^2*e^4+23/2*d^2*(4
/23*b*d*x^4+(-103/69*a*d-22/23*b*c)*x^2+a*c)*x^2*f^3*e^3+5/2*(-4/5*a*d^3*x
^6+(22/5*a*c*d^2+4/5*b*c^2*d)*x^4+(7/5*a*c^2*d+2/5*c^3*b)*x^2+c^3*a)*f^4*e
^2+11/6*c^2*((-3/11*a*d+2/11*b*c)*x^2+a*c)*x^2*f^5*e+2/3*a*c^3*f^6*x^4)*b^
(3/2)*e)*(b*x^2+a)^(1/2)*x*f))/((a*f-b*e)*e)^(1/2)/b^(1/2)/(a*f-b*e)/f^5/(
f*x^2+e)^3/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. $2(474) = 948$.

Time = 112.69 (sec) , antiderivative size = 7661, normalized size of antiderivative = 15.02

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^3}{(e+fx^2)^4}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**3/(e + f*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}{(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3/(f*x^2 + e)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3926 vs. $2(474) = 948$.

Time = 0.26 (sec) , antiderivative size = 3926, normalized size of antiderivative = 7.70

$$\int \frac{(a + bx^2)^{3/2} (c + dx^2)^3}{(e + fx^2)^4} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b*d^3*x/f^4 - 1/16*(64*b^(7/2)*d^3*e^6 - 48*b^(7/2)*c*
d^2*e^5*f - 120*a*b^(5/2)*d^3*e^5*f + 72*a*b^(5/2)*c*d^2*e^4*f^2 + 60*a^2*
b^(3/2)*d^3*e^4*f^2 - 18*a^2*b^(3/2)*c*d^2*e^3*f^3 - 5*a^3*sqrt(b)*d^3*e^3
*f^3 - 3*a^3*sqrt(b)*c*d^2*e^2*f^4 + 6*a^2*b^(3/2)*c^3*e*f^5 - 3*a^3*sqrt(
b)*c^2*d*e*f^5 - 5*a^3*sqrt(b)*c^3*f^6)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^
2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*e^4*f^5 - a*e^3*f
^6)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/24*(288*(sqrt(b)*x - sqrt(b*x^2 + a))^10
*b^(7/2)*d^3*e^6*f^2 - 432*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c*d^2*
e^5*f^3 - 504*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*d^3*e^5*f^3 + 144
*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^2*d*e^4*f^4 + 648*(sqrt(b)*x -
 sqrt(b*x^2 + a))^10*a*b^(5/2)*c*d^2*e^4*f^4 + 252*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*a^2*b^(3/2)*d^3*e^4*f^4 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(5/2)*c^2*d*e^3*f^5 - 234*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*
c*d^2*e^3*f^5 - 33*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^3*e^3*f^
5 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*d^2*e^2*f^6 - 18*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c^3*e*f^7 + 9*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^3*sqrt(b)*c^2*d*e*f^7 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a^3*sqrt(b)*c^3*f^8 + 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*d^3*e
^7*f - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*d^2*e^6*f^2 - 4368*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d^3*e^6*f^2 + 576*(sqrt(b)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3}{(f\,x^2+e)^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^3)/(e + f*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 12619, normalized size of antiderivative = 24.74

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e**3*f**7 - 45*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e**2*f**8*x**2 - 45*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*e*f**9*x**4 - 15*sqrt(e)*sqrt(a*f
 - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)
*x)/(sqrt(e)*sqrt(b)))*a**4*c**3*f**10*x**6 - 9*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a**4*c**2*d*e**4*f**6 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt
(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(
b)))*a**4*c**2*d*e**3*f**7*x**2 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**4*c**2*d*e**2*f**8*x**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*
*4*c**2*d*e*f**9*x**6 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d*
*2*e**5*f**5 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*d**2*e**4*
f**6*x**2 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*...
```

3.308 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)} dx$

Optimal result

Integrand size = 30, antiderivative size = 157

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)} dx = \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{df} + \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d(de-cf)} - \frac{(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f(de-cf)}$$

output
```
b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d/f+(-a*d+b*c)^(3/2)*arctanh((-
a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/d/(-c*f+d*e)-(-a*f+b*e)^
(3/2)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f/(-c*f+
d*e)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)} dx = \frac{(-bc+ad)^{3/2}\sqrt{e}f \arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right) + \sqrt{c}\left(-d(-be+af)^{3/2} \text{arcta}\right.}{\sqrt{c}d\sqrt{e}f(-de}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
((-(b*c) + a*d)^(3/2)*Sqrt[e]*f*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(
c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])] + Sqrt[c]*(-(d*(-(b*e) + a*f)^(3
/2)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(
b*e) + a*f])]) + b^(3/2)*Sqrt[e]*(d*e - c*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]]))/(Sqrt[c]*d*Sqrt[e]*f*(-(d*e) + c*f))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {420, 301, 224, 219, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{422}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{de-cf}\right)}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{de-cf}\right)}{d}$$

$\downarrow$ 219

$$\frac{b\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2}}}{f}\right)}{de-cf}\right)}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{d}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt[b*e - a*f]*Ar
cTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f)))/d - ((
b*c - a*d)*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]
*d)))/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f -
 (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/
(Sqrt[e]*f)))/(d*e - c*f)))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{(ad-bc)^2 \arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{d(cf-de)\sqrt{(ad-bc)c}}+\frac{b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{fd}-\frac{(af-be)^2 \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{f(cf-de)\sqrt{(af-be)e}}$	156
default	Expression too large to display	2624

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(a*d-b*c)^2/d/(c*f-d*e)/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a
*d-b*c)*c)^(1/2))+b^(3/2)/f/d*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(a*f-b*e)
^2/f/(c*f-d*e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e
)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral((a + b*x**2)**(3/2)/((c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.29

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)}\,dx = \frac{\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{b\,x^2+a}-\sqrt{d}\,\sqrt{b}\,x}{\sqrt{c}\,\sqrt{b}}\right)adef-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-\cdots}}{\cdots}\right)\cdots}{\cdots}$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x)
```

output

```
(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2)
- sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e*f - sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqr
t(c)*sqrt(b)))*b*c*e*f + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + s
qrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e*f -
sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) +
 sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*c*e*f - sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a*c*d*f + sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sq
rt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*d*e - s
qrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*d*f + sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*b*c*d*e + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*b*c**2*e*f - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*c*d*e*
*2)/(c*d*e*f*(c*f - d*e))
```

3.309 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 183

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2} dx = \frac{(be-af)x\sqrt{a+bx^2}}{2e(de-cf)(e+fx^2)} + \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^2} - \frac{\sqrt{be-af}(2bce-3ade+acf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(de-cf)^2}$$

output
```
1/2*(-a*f+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)+(-a*d+b*c)^(3/2)*a
rctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-c*f+d*e)^2-1/
2*(-a*f+b*e)^(1/2)*(a*c*f-3*a*d*e+2*b*c*e)*arctanh((-a*f+b*e)^(1/2)*x/e^(1
/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \frac{\frac{(be-af)(de-cf)x\sqrt{a+bx^2}}{e(e+fx^2)} - \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} - \frac{\sqrt{-be+af}(2bce-3a}{}}{2(de-cf)^2}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(((b*e - a*f)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(e + f*x^2)) - (2*(-(b*c)
+ a*d)^(3/2)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c
]*Sqrt[-(b*c) + a*d])])/Sqrt[c] - (Sqrt[-(b*e) + a*f]*(2*b*c*e - 3*a*d*e +
 a*c*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqr
t[-(b*e) + a*f])])/e^(3/2))/(2*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.46, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {419, 25, 301, 224, 219, 291, 221, 401, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx$$

$$\Big\downarrow \ 419$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2} - \frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2} - \frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}$$

$\downarrow$ 301

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 224

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 219

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 291

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^2}dx}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$\downarrow$ 401

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{f\left(a(af(3de-cf)-be(de+cf))-2b(bc-ad)efx^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\int \frac{a(af(3de-cf)-be(de+cf))-2b(bc-ad)efx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{(be-af)(acf-3ade+2bce)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx-2be(bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{(be-af)(acf-3ade+2bce)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx-2be(bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{(be-af)(acf-3ade+2bce)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx-2\sqrt{b}e\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(bc-ad)}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{(be-af)(acf-3ade+2bce)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(bc-ad)}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{2e(e+fx^2)}-\frac{\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(acf-3ade+2bce)}{\sqrt{e}}-2\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(bc-ad)}{2e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
-((d*(b*c - a*d)*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt
[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[
c]*d)))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e
*(e + f*x^2)) - (-2*Sqrt[b]*(b*c - a*d)*e*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]] + (Sqrt[b*e - a*f]*(2*b*c*e - 3*a*d*e + a*c*f)*ArcTanh[(Sqrt[b*e - a*
f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/Sqrt[e])/(2*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-\left((af+2be)c-3ade\right)\sqrt{(ad-bc)c}\,(af-be)(f\,x^2+e)\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\,\left(-2e(ad-bc)^2(f\,x^2+e)\arctan\left(\right.\right.}{2\sqrt{(ad-bc)c}\,\sqrt{(af-be)e}\,(cf-de)^2e(f\,x^2+e)}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-((a*f+2*b*e)*c-3*a*d*e)*((a*d-b*c)*c)^(1/2)*(a*f-b*e)*(f*x^2+e)*arct
an(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(-2*e*(a*d
-b*c)^2*(f*x^2+e)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*
c)*c)^(1/2)*(a*f-b*e)*(c*f-d*e)*(b*x^2+a)^(1/2)*x))/((a*d-b*c)*c)^(1/2)/((
a*f-b*e)*e)^(1/2)/(c*f-d*e)^2/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(157) = 314$.

Time = 0.87 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx =$$

$$-\frac{\left(b^{\frac{5}{2}}c^2-2\,ab^{\frac{3}{2}}cd+a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(d^2e^2-2\,cdef+c^2f^2)}$$
$$+\frac{\left(2\,b^{\frac{5}{2}}ce^2-3\,ab^{\frac{3}{2}}de^2-ab^{\frac{3}{2}}cef+3\,a^2\sqrt{b}def-a^2\sqrt{b}cf^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,(d^2e^3-2\,cde^2f+c^2ef^2)\sqrt{-b^2e^2+abef}}$$
$$+\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{5}{2}}e^2-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}ef+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2\sqrt{b}f^2+a^2b^{\frac{3}{2}}ef-a^{3}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2be-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2af+a^2f\right)(de^2f-cef^2}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-(b^(5/2)*c^2 - 2*a*b^(3/2)*c*d + a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2
*c^2 + a*b*c*d)*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)) + 1/2*(2*b^(5/2)*c*e^2 -
3*a*b^(3/2)*d*e^2 - a*b^(3/2)*c*e*f + 3*a^2*sqrt(b)*d*e*f - a^2*sqrt(b)*c*
f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^
2*e^2 + a*b*e*f))/((d^2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*sqrt(-b^2*e^2 + a*b
*e*f)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*e^2 - 3*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*a*b^(3/2)*e*f + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt
(b)*f^2 + a^2*b^(3/2)*e*f - a^3*sqrt(b)*f^2)/(((sqrt(b)*x - sqrt(b*x^2 + a
))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*f + a^2*f)*(d*e^2*f - c*e*f^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 1306, normalized size of antiderivative = 7.14

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**3 - 2*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a*d*e**2*f*x**2 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sq
rt(b)))*b*c*e**3 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(
d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*c*e**2*f*x**
2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x
**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e**3 - 2*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a*d*e**2*f*x**2 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sq
rt(b)))*b*c*e**3 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(
d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*c*e**2*f*x**
2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c**2*e*f - sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a*c**2*f**2*x**2 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sq
rt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqr
t(b)))*a*c*d*e**2 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s...
```

3.310 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 286

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \frac{(be-af)x\sqrt{a+bx^2}}{4e(de-cf)(e+fx^2)^2} - \frac{(af(7de-3cf)-2be(de+cf))x\sqrt{a+bx^2}}{8e^2(de-cf)^2(e+fx^2)} + \frac{d(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^3} - \frac{(8b^2cde^3-4abde^2(3de+cf)+a^2f(15d^2e^2-10cdef+3c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}\sqrt{be-af}(de-cf)^3}$$

output
```
1/4*(-a*f+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^2-1/8*(a*f*(-3*c*f
+7*d*e)-2*b*e*(c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/(-c*f+d*e)^2/(f*x^2+e)+d*(-
a*d+b*c)^(3/2)*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)
/(-c*f+d*e)^3-1/8*(8*b^2*c*d*e^3-4*a*b*d*e^2*(c*f+3*d*e)+a^2*f*(3*c^2*f^2-
10*c*d*e*f+15*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2)
)/e^(5/2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 10.58 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = -\frac{x\sqrt{a+bx^2}(-2be(cf^2x^2+de(2e+fx^2))+af(-cf(5e+3fx^2)+de(9e+7fx^2)))}{8e^2(de-cf)^2(e+fx^2)^2} - \frac{d(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(-de+cf)^3} - \frac{(8b^2cde^3-4abde^2(3de+cf)+a^2f(15d^2e^2-10cdef+3c^2f^2))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}\sqrt{-be+af}(de-cf)^3}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
-1/8*(x*Sqrt[a + b*x^2]*(-2*b*e*(c*f^2*x^2 + d*e*(2*e + f*x^2)) + a*f*(-(c
*f*(5*e + 3*f*x^2)) + d*e*(9*e + 7*f*x^2))))/(e^2*(d*e - c*f)^2*(e + f*x^2
)^2) - (d*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt
[a + b*x^2])])/(Sqrt[c]*(-(d*e) + c*f)^3) - ((8*b^2*c*d*e^3 - 4*a*b*d*e^2*
(3*d*e + c*f) + a^2*f*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[
-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(8*e^(5/2)*Sqrt[-(b*e) + a*f]
*(d*e - c*f)^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {419, 25, 401, 27, 402, 25, 27, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}dx$$

$$\Big\downarrow \text{ 419}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{(fx^2+e)^3}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 401

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{f\left(2b(af(3de-cf)-be(de+cf))x^2+a(af(7de-3cf)-be(3de+cf))\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\int \frac{2b(af(3de-cf)-be(de+cf))x^2+a(af(7de-3cf)-be(3de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 402

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{\int -\frac{a(be-af)\left(4bde^2-af(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{\int \frac{a(be-af)\left(4bde^2-af(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\left(4bde^2-af(7de-3cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\left(4bde^2-af(7de-3cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(4bde^2-af(7de-3cf)\right)}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{422}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(4bde^2-af(7de-3cf)\right)}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(4bde^2-af(7de-3cf)\right)}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 224

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(4bde^2-af(7de-3cf)\right)}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{d\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e\left(e+fx^2\right)}-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(4bde^2-af(7de-3cf)\right)}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-\frac{d(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 291

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e(e+fx^2)}-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(4bde^2-af(7de-3cf))}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-$$

$$\frac{d(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}$$

$$\downarrow \text{221}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{4e(e+fx^2)^2}-\frac{\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(cf+de))}{2e(e+fx^2)}-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(4bde^2-af(7de-3cf))}{2e^{3/2}\sqrt{be-af}}}{4e}}{(de-cf)^2}-$$

$$\frac{d(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}$$

input

```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^3),x]
```

output

```
-((d*(b*c - a*d)*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(S
qrt[c]*d)))/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/f - (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^
2])])/(Sqrt[e]*f)))/(d*e - c*f)))/(d*e - c*f)^2) + (((b*e - a*f)*(d*e - c*
f)*x*Sqrt[a + b*x^2])/(4*e*(e + f*x^2)^2) - (((a*f*(7*d*e - 3*c*f) - 2*b*e
*(d*e + c*f))*x*Sqrt[a + b*x^2])/(2*e*(e + f*x^2)) - (a*(4*b*d*e^2 - a*f*(
7*d*e - 3*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2
*e^(3/2)*Sqrt[b*e - a*f]))/(4*e))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{3\left(\left(\left(-4ab\,d^{2}+\frac{8}{3}b^{2}cd\right)e^{3}+5ad\left(ad-\frac{4bc}{15}\right)f\,e^{2}-\frac{10a^{2}cde\,f^{2}}{3}+a^{2}c^{2}f^{3}\right)\sqrt{(ad-bc)c}\,\left(f\,x^{2}+e\right)^{2}\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)-\cdots 8\sqrt{\cdots}\right)}{8\sqrt{(\cdots}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(1/2)*(((-4*a*b*d^2+8/3*b^2*c*d)*e^
3+5*a*d*(a*d-4/15*b*c)*f*e^2-10/3*a^2*c*d*e*f^2+a^2*c^2*f^3)*((a*d-b*c)*c)
^(1/2)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-8/3*((a
*f-b*e)*e)^(1/2)*(d*e^2*(f*x^2+e)^2*(a*d-b*c)^2*arctan(c*(b*x^2+a)^(1/2)/x
/((a*d-b*c)*c)^(1/2))+5/8*(c*f-d*e)*((a*d-b*c)*c)^(1/2)*(b*x^2+a)^(1/2)*x*
(4/5*b*d*e^3-9/5*(-2/9*b*x^2+a)*d*f*e^2+f^2*(-7/5*a*d*x^2+c*(2/5*b*x^2+a))
*e+3/5*a*c*f^3*x^2)))/(c*f-d*e)^3/(f*x^2+e)^2/e^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. 2(257) = 514.

Time = 1.92 (sec) , antiderivative size = 1054, normalized size of antiderivative = 3.69

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-(b^(5/2)*c^2*d - 2*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan(1/2*((sqrt(b
)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((d^3*
e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) +
 1/8*(8*b^(5/2)*c*d*e^3 - 12*a*b^(3/2)*d^2*e^3 - 4*a*b^(3/2)*c*d*e^2*f + 1
5*a^2*sqrt(b)*d^2*e^2*f - 10*a^2*sqrt(b)*c*d*e*f^2 + 3*a^2*sqrt(b)*c^2*f^3
)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e
^2 + a*b*e*f))/((d^3*e^5 - 3*c*d^2*e^4*f + 3*c^2*d*e^3*f^2 - c^3*e^2*f^3)*
sqrt(-b^2*e^2 + a*b*e*f)) + 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)
*c*e^2*f^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*f^2 + 7*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 - 3*(sqrt(b)*x - sqrt(b*
x^2 + a))^6*a^2*sqrt(b)*c*f^4 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)
*d*e^4 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 72*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a*b^(5/2)*d*e^3*f + 8*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a*b^(5/2)*c*e^2*f^2 + 62*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*
e^2*f^2 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 - 21*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d*e*f^3 + 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^3*sqrt(b)*c*f^4 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/
2)*d*e^3*f + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 - 52*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*d*e^2*f^2 + 16*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*a^3*b^(3/2)*c*e*f^3 + 21*(sqrt(b)*x - sqrt(b*x^2 + a))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 7621, normalized size of antiderivative = 26.65

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
(16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**
2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**5*f**3 + 32*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(
d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**4*f**4*x**2 + 16*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*d**2*e**3*f**5*x**4 - 16*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a**2*b*c*d*e**5*f**3 - 32*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt
(c)*sqrt(b)))*a**2*b*c*d*e**4*f**4*x**2 - 16*sqrt(c)*sqrt(a*d - b*c)*atan(
(sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*
sqrt(b)))*a**2*b*c*d*e**3*f**5*x**4 - 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqr
t(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt
(b)))*a**2*b*d**2*e**6*f**2 - 96*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*
*2*b*d**2*e**5*f**3*x**2 - 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c
) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*
b*d**2*e**4*f**4*x**4 + 48*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) -
 sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*b**2*c
*d*e**6*f**2 + 96*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(...
```

3.311 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 496

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \frac{(be-af)x\sqrt{a+bx^2}}{6e(de-cf)(e+fx^2)^3}$$
$$-\frac{(af(11de-5cf)-2be(2de+cf))x\sqrt{a+bx^2}}{24e^2(de-cf)^2(e+fx^2)^2}$$
$$+\frac{(4b^2e^2(2d^2e^2+5cdef-c^2f^2)-2abef(31d^2e^2-11cdef+4c^2f^2)+3a^2f^2(19d^2e^2-16cdef+5c^2f^2))\,x\sqrt{\quad}}{48e^3(be-af)(de-cf)^3(e+fx^2)}$$
$$+\frac{d^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(de-cf)^4}$$
$$-\frac{(16b^3cd^2e^5-24ab^2d^2e^4(de+cf)-a^3f^2(35d^3e^3-35cd^2e^2f+21c^2def^2-5c^3f^3)+6a^2bef(10d^3e^3-5c\cdots}{16e^{7/2}(be-af)^{3/2}(de-cf)^4}$$

output
```
1/6*(-a*f+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^3-1/24*(a*f*(-5*c*
f+11*d*e)-2*b*e*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-c*f+d*e)^2/(f*x^2+e)^
2+1/48*(4*b^2*e^2*(-c^2*f^2+5*c*d*e*f+2*d^2*e^2)-2*a*b*e*f*(4*c^2*f^2-11*c
*d*e*f+31*d^2*e^2)+3*a^2*f^2*(5*c^2*f^2-16*c*d*e*f+19*d^2*e^2))*x*(b*x^2+a
)^(1/2)/e^3/(-a*f+b*e)/(-c*f+d*e)^3/(f*x^2+e)+d^2*(-a*d+b*c)^(3/2)*arctanh
((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-c*f+d*e)^4-1/16*(16
*b^3*c*d^2*e^5-24*a*b^2*d^2*e^4*(c*f+d*e)-a^3*f^2*(-5*c^3*f^3+21*c^2*d*e*f
^2-35*c*d^2*e^2*f+35*d^3*e^3)+6*a^2*b*e*f*(-c^3*f^3+4*c^2*d*e*f^2-5*c*d^2*
e^2*f+10*d^3*e^3))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(
7/2)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \frac{\frac{(de-cf)x\sqrt{a+bx^2}\left(8e^2(be-af)(de-cf)^2+2e(de-cf)(2be(2de+cf)+af(-11de+5cf))(e+fx^2)+\frac{(4b^2e^2(}{}\right.}{e^3(e+fx^2)^3}}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^4),x]
```

output
```
(((d*e - c*f)*x*Sqrt[a + b*x^2]*(8*e^2*(b*e - a*f)*(d*e - c*f)^2 + 2*e*(d*
e - c*f)*(2*b*e*(2*d*e + c*f) + a*f*(-11*d*e + 5*c*f))*(e + f*x^2) + ((4*b
^2*e^2*(2*d^2*e^2 + 5*c*d*e*f - c^2*f^2) - 2*a*b*e*f*(31*d^2*e^2 - 11*c*d*
e*f + 4*c^2*f^2) + 3*a^2*f^2*(19*d^2*e^2 - 16*c*d*e*f + 5*c^2*f^2))*(e + f
*x^2)^2)/(b*e - a*f)))/(e^3*(e + f*x^2)^3) + (48*d^2*(-(b*c) + a*d)^(3/2)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + (3*(16
*b^3*c*d^2*e^5 - 24*a*b^2*d^2*e^4*(d*e + c*f) + 6*a^2*b*e*f*(10*d^3*e^3 -
5*c*d^2*e^2*f + 4*c^2*d*e*f^2 - c^3*f^3) + a^3*f^2*(-35*d^3*e^3 + 35*c*d^2
*e^2*f - 21*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(e^(7/2)*(-(b*e) + a*f)^(3/2)))/(48*(d*e - c*f)^4)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.19, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {419, 25, 401, 27, 402, 27, 402, 27, 291, 221, 421, 301, 224, 219, 291, 221, 401, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)\left(e+fx^2\right)^4}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^4}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(bde^2+(bc-ad)f^2x^2-af(2de-cf)\right)}{\left(fx^2+e\right)^4}dx}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e\left(e+fx^2\right)^3}-\frac{\int \frac{f\left(2b(af(5de-2cf)-be(2de+cf))x^2+a(af(11de-5cf)-be(5de+cf))\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6ef}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e\left(e+fx^2\right)^3}-\frac{\int \frac{2b(af(5de-2cf)-be(2de+cf))x^2+a(af(11de-5cf)-be(5de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{6e}}{(de-cf)^2}-\frac{d(bc-ad)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{\int\frac{(be-af)\left(2b(af(11de-5cf)-2be(2de+cf))x^2+a(3af(11de-5cf)-2be(8de+cf))\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(af(11de-5cf)-2be(cf+2d}{4e\left(e+fx^2\right)^2}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{\int\frac{2b(af(11de-5cf)-2be(2de+cf))x^2+a(3af(11de-5cf)-2be(8de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(af(11de-5cf)-2be(cf+2de))}{4e\left(e+fx^2\right)^2}}{6e}}{(de-cf)^2}-$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{\frac{\int-\frac{3a\left(8b^2de^3-2abf(10de-3cf)e+a^2f^2(11de-5cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-4cf)+4b^2e^2(cf+2de)\right)}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-4cf\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(de-cf)^2}$$

$$\Big\downarrow \text{421}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\downarrow$ 401

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int-\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}\right)}{(de-cf)^2}$$

$\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(de-cf)^2}$$

$$\Big\downarrow \textbf{398}$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2\right.}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(de-cf)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}\cdots}{4e}\cdots}{6e}\cdots}{(de-cf)^2}\cdots$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{\cdots}{\sqrt{bx^2\cdots}}}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(de-cf)^2}\cdots$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}\cdots}{4e}\cdots}{6e}\cdots}{(de-cf)^2}\cdots$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{\cdots}{\sqrt{b\cdots}}}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(de-cf)^2}\cdots$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{\cdots}{e-\cdots}}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{\frac{x\sqrt{a+bx^2}(be-af)(de-cf)}{6e(e+fx^2)^3}-\frac{\frac{-\frac{3a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(11de-5cf)-2abef(10de-3cf)+8b^2de^3\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(3a^2f^2(11de-5cf)-2abef(19de-\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e}}{6e}}{(de-cf)^2}$$

$$\frac{d(bc-ad)\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\cdots}{\sqrt{e}f\sqrt{be-\cdots}}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(de-cf)^2}$$

input

```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^4),x]
```

output
```
(((b*e - a*f)*(d*e - c*f)*x*Sqrt[a + b*x^2])/(6*e*(e + f*x^2)^3) - (((a*f*
(11*d*e - 5*c*f) - 2*b*e*(2*d*e + c*f))*x*Sqrt[a + b*x^2])/(4*e*(e + f*x^2
)^2) + (-1/2*((3*a^2*f^2*(11*d*e - 5*c*f) - 2*a*b*e*f*(19*d*e - 4*c*f) + 4
*b^2*e^2*(2*d*e + c*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) - (
3*a*(8*b^2*d*e^3 + a^2*f^2*(11*d*e - 5*c*f) - 2*a*b*e*f*(10*d*e - 3*c*f))*
ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e -
a*f)^(3/2)))/(4*e))/(6*e))/(d*e - c*f)^2 - (d*(b*c - a*d)*((d^2*((Sqrt[b]*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/(d*e - c*f)^2 - (f*
(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(e + f*x^2)) + ((2*Sqrt[b]*d*e*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((2*b*d*e^2 - a*f*(3*d*e - c*f))*ArcT
anh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e -
a*f]))/(2*e)))/(d*e - c*f)^2))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{5\left(\left(\frac{8\left(-3a\,b^2d^3+2b^3c\,d^2\right)e^5}{5}+12a\left(ad-\frac{2bc}{5}\right)d^2bf\,e^4+\left(-7a^3d^3-6a^2bc\,d^2\right)f^2e^3+7a^2c\left(ad+\frac{24bc}{35}\right)d\,f^3e^2-\frac{21a^2c^2\left(ad+\frac{2bc}{7}\right.}{5}\right.\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/16/((a*d-b*c)*c)^(1/2)*((8/5*(-3*a*b^2*d^3+2*b^3*c*d^2)*e^5+12*a*(a*d-2
/5*b*c)*d^2*b*f*e^4+(-7*a^3*d^3-6*a^2*b*c*d^2)*f^2*e^3+7*a^2*c*(a*d+24/35*
b*c)*d*f^3*e^2-21/5*a^2*c^2*(a*d+2/7*b*c)*f^4*e+a^3*c^3*f^5)*((a*d-b*c)*c)
^(1/2)*(f*x^2+e)^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+16/5*((
a*f-b*e)*e)^(1/2)*(d^2*e^3*(f*x^2+e)^3*(a*f-b*e)*(a*d-b*c)^2*arctan(c*(b*x
^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-11/16*(c*f-d*e)*((a*d-b*c)*c)^(1/2)*(b*
x^2+a)^(1/2)*x*(8/11*b^2*d^2*e^6-36/11*(-2/9*b*x^2+a)*d^2*b*f*e^5+29/11*d*
f^2*((8/87*b^2*x^4-154/87*a*b*x^2+a^2)*d+30/29*c*(2/5*b*x^2+a)*b)*e^4-32/1
1*(-17/12*a*x^2*(-31/68*b*x^2+a)*d^2+c*(-5/24*b^2*x^4-5/6*a*b*x^2+a^2)*d+5
/16*c^2*(2/5*b*x^2+a)*b)*f^3*e^3+(19/11*a^2*d^2*x^4-128/33*a*c*(-11/64*b*x
^2+a)*x^2*d+c^2*(-4/33*b^2*x^4-2/3*a*b*x^2+a^2))*f^4*e^2+40/33*a*c*x^2*f^5
*(-6/5*a*d*x^2+c*(-1/5*b*x^2+a))*e+5/11*a^2*c^2*f^6*x^4)))/((a*f-b*e)*e)^(
1/2)/(c*f-d*e)^4/(f*x^2+e)^3/(a*f-b*e)/e^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)(fx^2+e)^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)*(f*x^2 + e)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3177 vs. $2(463) = 926$.

Time = 6.35 (sec) , antiderivative size = 3177, normalized size of antiderivative = 6.41

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-(b^(5/2)*c^2*d^2 - 2*a*b^(3/2)*c*d^3 + a^2*sqrt(b)*d^4)*arctan(1/2*((sqrt
(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((d^
4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + c^4*f^4)*sqrt(
-b^2*c^2 + a*b*c*d)) + 1/16*(16*b^(7/2)*c*d^2*e^5 - 24*a*b^(5/2)*d^3*e^5 -
 24*a*b^(5/2)*c*d^2*e^4*f + 60*a^2*b^(3/2)*d^3*e^4*f - 30*a^2*b^(3/2)*c*d^
2*e^3*f^2 - 35*a^3*sqrt(b)*d^3*e^3*f^2 + 24*a^2*b^(3/2)*c^2*d*e^2*f^3 + 35
*a^3*sqrt(b)*c*d^2*e^2*f^3 - 6*a^2*b^(3/2)*c^3*e*f^4 - 21*a^3*sqrt(b)*c^2*
d*e*f^4 + 5*a^3*sqrt(b)*c^3*f^5)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*d^4*e^8 - 4*b*c*d^3*e^7*
f - a*d^4*e^7*f + 6*b*c^2*d^2*e^6*f^2 + 4*a*c*d^3*e^6*f^2 - 4*b*c^3*d*e^5*
f^3 - 6*a*c^2*d^2*e^5*f^3 + b*c^4*e^4*f^4 + 4*a*c^3*d*e^4*f^4 - a*c^4*e^3*
f^5)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/24*(48*(sqrt(b)*x - sqrt(b*x^2 + a))^10
*b^(7/2)*c*d*e^4*f^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*d^2*e
^4*f^3 - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c*d*e^3*f^4 + 132*(
sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*d^2*e^3*f^4 - 54*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^2*b^(3/2)*c*d*e^2*f^5 - 57*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*a^3*sqrt(b)*d^2*e^2*f^5 + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*
b^(3/2)*c^2*e*f^6 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*d*e*
f^6 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c^2*f^7 + 480*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*d*e^5*f^2 - 720*(sqrt(b)*x - sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)\,(f\,x^2+e)^4}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^4),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)*(e + f*x^2)^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)(e+fx^2)^4}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)\,(f\,x^2+e)^4}dx$$

input

```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x)
```

output

```
int((b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^4,x)
```

3.312 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 266

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx &= -\frac{d(bc-ad)x\sqrt{a+bx^2}}{2c(de-cf)^2(c+dx^2)} - \frac{f(be-af)x\sqrt{a+bx^2}}{2e(de-cf)^2(e+fx^2)} \\ &- \frac{\sqrt{bc-ad}(ad(de-5cf)+2bc(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(de-cf)^3} \\ &- \frac{\sqrt{be-af}(af(5de-cf)-2be(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(de-cf)^3}\end{aligned}$$

output
```
-1/2*d*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^2/(d*x^2+c)-1/2*f*(-a*f+b
*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)^2/(f*x^2+e)-1/2*(-a*d+b*c)^(1/2)*(a*d*(
-5*c*f+d*e)+2*b*c*(c*f+d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^
(1/2))/c^(3/2)/(-c*f+d*e)^3-1/2*(-a*f+b*e)^(1/2)*(a*f*(-c*f+5*d*e)-2*b*e*(
c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-c*
f+d*e)^3
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{1}{2}\left(\frac{x\sqrt{a+bx^2}(a(c^2f^2+cdf^2x^2+d^2e(e+fx^2))-bce(cf+d(e+2fx^2)))}{ce(de-cf)^2(c+dx^2)(e+fx^2)}\right.$$
$$+\frac{\sqrt{-bc+ad}(ad(de-5cf)+2bc(de+cf))\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}(-de+cf)^3}$$
$$\left.+\frac{\sqrt{-be+af}(af(-5de+cf)+2be(de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{e^{3/2}(de-cf)^3}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
((x*Sqrt[a + b*x^2]*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2)) - b*c*e
*(c*f + d*(e + 2*f*x^2))))/(c*e*(d*e - c*f)^2*(c + d*x^2)*(e + f*x^2)) + (
Sqrt[-(b*c) + a*d]*(a*d*(d*e - 5*c*f) + 2*b*c*(d*e + c*f))*ArcTan[(-(d*x*S
qrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(c^(
3/2)*(-(d*e) + c*f)^3) + (Sqrt[-(b*e) + a*f]*(a*f*(-5*d*e + c*f) + 2*b*e*(
d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]
*Sqrt[-(b*e) + a*f])])/(e^(3/2)*(d*e - c*f)^3))/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{ 301}$$

$$\frac{b\left(\frac{d^2\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\left(\frac{d^2\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$\Big\downarrow$ 401

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int -\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \ 398$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \ 224$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}\ldots}{f}}{2e}\ldots\right)}{(de-cf)^2}\ldots\right)}{d}\ldots$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow\ 219$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right.}\ldots}{f}}{2e}\ldots\right)}{(de-cf)^2}\ldots\right)}{d}\ldots$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow\ 291$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\ldots}{f}}{2e}\ldots\right)}{(de-cf)^2}\ldots\right)}{d}\ldots$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow\ 221$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}}{(de-cf)^2}}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^2}dx}{d}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}}{(de-cf)^2}}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}}{(de-cf)^2}}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}\right)}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}\right)}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{d}\right)}{d}$$

$\downarrow$ 402

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bde^2-af(3d}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$(bc-ad)\frac{\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int\frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}}{d}\right.}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bde^2-af(3d}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$(bc-ad)\frac{\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{}{\sqrt{bx^2+a}}}{}\right.}{d}$$

$\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bde^2-af(3d}{\sqrt{ef}\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.$$

$$\frac{}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{}{\sqrt{bx}}}{}\right.$$

$$\frac{}{d}$$

$\downarrow$ 221

$$b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bde^2-af(3d}{\sqrt{ef}\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.$$

$$\frac{}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{}{\sqrt{bx^2}}}{}\right.$$

$$\frac{}{d}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\cdots\right.}{\sqrt{e}f\sqrt{be-af}}\cdots}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int \cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{d}\cdots$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(f^2\right.\right.}{}\right.}{d}$$

$$\downarrow 25$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\cdots\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(f^2\cdots\right.\right.}{\cdots}\right.}{d}\cdots$$

$$\Big\downarrow \; 291$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\cdots\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(f^2\cdots\right.\right.}{\cdots}\cdots\right.}{d}\cdots$$

$$\Big\downarrow \; 221$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3d\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(\frac{d}{-}\right.\right.}{}\right.}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e\left(fx^2+e\right)}+\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\cdots\right.}{\sqrt{\cdots}}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(\mathrm{a}\cdots\right.\right.}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{b\left(\frac{d^2\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e\left(fx^2+e\right)}+\frac{\frac{2\sqrt{b}de\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right.}{\sqrt{}}}{2e}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\left(d\left(\mathrm{a}\ldots\right.\right.}{}\right.}{}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{-5(ad-bc)\sqrt{(af-be)e}\left(-\frac{2b\,c^2 f}{5}+d\left(af-\frac{2be}{5}\right)c-\frac{a\,d^2 e}{5}\right)(x^2 d+c)\,(f\,x^2+e)\,e\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\left(-c\left(\left(a\,f^2+2bef\right)c-\right.\right.}{2\sqrt{}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)*(-5*(a*d-b*c)*((a*f-b*e)*e)^(1/2)*(-2/5*b*c^2*f+d*
(a*f-2/5*b*e)*c-1/5*a*d^2*e)*(d*x^2+c)*(f*x^2+e)*e*arctan(c*(b*x^2+a)^(1/2
)/x/((a*d-b*c)*c)^(1/2))+(-c*((a*f^2+2*b*e*f)*c-5*a*d*e*f+2*b*d*e^2)*(d*x^
2+c)*(a*f-b*e)*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+(
(a*f-b*e)*e)^(1/2)*(c*f-d*e)*((a*f^2-b*e*f)*c^2+d*(a*f^2*x^2-2*b*e*f*x^2-b
*e^2)*c+a*d^2*e*(f*x^2+e))*(b*x^2+a)^(1/2)*x)*((a*d-b*c)*c)^(1/2))/((a*f-b
*e)*e)^(1/2)/(d*x^2+c)/(c*f-d*e)^3/c/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^2(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^2*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. $2(235) = 470$.

Time = 1.98 (sec) , antiderivative size = 1331, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^(5/2)*c^2*d*e - a*b^(3/2)*c*d^2*e - a^2*sqrt(b)*d^3*e + 2*b^(5/2)
*c^3*f - 7*a*b^(3/2)*c^2*d*f + 5*a^2*sqrt(b)*c*d^2*f)*arctan(1/2*((sqrt(b)
*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((c*d^3
*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*sqrt(-b^2*c^2 + a*b*c*d)
) - 1/2*(2*b^(5/2)*d*e^3 + 2*b^(5/2)*c*e^2*f - 7*a*b^(3/2)*d*e^2*f - a*b^(
3/2)*c*e*f^2 + 5*a^2*sqrt(b)*d*e*f^2 - a^2*sqrt(b)*c*f^3)*arctan(1/2*((sqr
t(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((d
^3*e^4 - 3*c*d^2*e^3*f + 3*c^2*d*e^2*f^2 - c^3*e*f^3)*sqrt(-b^2*e^2 + a*b*
e*f)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c*d*e^2 + 2*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*b^(5/2)*c^2*e*f - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
*b^(3/2)*c*d*e*f + (sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^2*e*f + (
sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*d*f^2 + 16*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*b^(7/2)*c^2*e^2 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/
2)*c*d*e^2 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^2*e^2 - 16*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*e*f + 14*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^2*b^(3/2)*c*d*e*f - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqr
t(b)*d^2*e*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c^2*f^2 - 3*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*c*d*f^2 + 6*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^2*b^(5/2)*c*d*e^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b
^(3/2)*d^2*e^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*e*f ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2\,(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^2\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^2\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

3.313 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 376

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = -\frac{d^2(bc-ad)x\sqrt{a+bx^2}}{2c(de-cf)^3(c+dx^2)} - \frac{f(be-af)x\sqrt{a+bx^2}}{4e(de-cf)^2(e+fx^2)^2} + \frac{f(af(11de-3cf)-2be(3de+cf))x\sqrt{a+bx^2}}{8e^2(de-cf)^3(e+fx^2)} - \frac{d\sqrt{bc-ad}(ad(de-7cf)+2bc(de+2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(de-cf)^4} - \frac{(8abde^2f(5de+cf)-8b^2de^3(de+2cf)-a^2f^2(35d^2e^2-14cdef+3c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}\sqrt{be-af}(de-cf)^4}$$

output
```
-1/2*d^2*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^3/(d*x^2+c)-1/4*f*(-a*f
+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)^2/(f*x^2+e)^2+1/8*f*(a*f*(-3*c*f+11*d
*e)-2*b*e*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-c*f+d*e)^3/(f*x^2+e)-1/2*d*
(-a*d+b*c)^(1/2)*(a*d*(-7*c*f+d*e)+2*b*c*(2*c*f+d*e))*arctanh((-a*d+b*c)^(
1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-c*f+d*e)^4-1/8*(8*a*b*d*e^2*f*(c
*f+5*d*e)-8*b^2*d*e^3*(2*c*f+d*e)-a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2
))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^
(1/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(x\sqrt{a+bx^2}\left(\frac{4d^2(bc-ad)}{c(-de+cf)^3(c+dx^2)}+\frac{2f(-be+af)}{e(de-cf)^2(e+fx^2)^2}-\frac{f(2be(3d\right.\right.$$
$$+\frac{4d\sqrt{-bc+ad}(ad(de-7cf)+2bc(de+2cf))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}(de-cf)^4}$$
$$\left.+\frac{(-8abde^2f(5de+cf)+8b^2de^3(de+2cf)+a^2f^2(35d^2e^2-14cdef+3c^2f^2))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{5/2}\sqrt{-be+af}(de-cf)^4}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((4*d^2*(b*c - a*d))/(c*(-(d*e) + c*f)^3*(c + d*x^2)) +
 (2*f*(-(b*e) + a*f))/(e*(d*e - c*f)^2*(e + f*x^2)^2) - (f*(2*b*e*(3*d*e +
 c*f) + a*f*(-11*d*e + 3*c*f)))/(e^2*(d*e - c*f)^3*(e + f*x^2))) + (4*d*Sq
rt[-(b*c) + a*d]*(a*d*(d*e - 7*c*f) + 2*b*c*(d*e + 2*c*f))*ArcTan[(Sqrt[-(
b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(3/2)*(d*e - c*f)^4) + ((-8*
a*b*d*e^2*f*(5*d*e + c*f) + 8*b^2*d*e^3*(d*e + 2*c*f) + a^2*f^2*(35*d^2*e^
2 - 14*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a
 + b*x^2])])/(e^(5/2)*Sqrt[-(b*e) + a*f]*(d*e - c*f)^4))/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^3}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 401

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}-\frac{\int-\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2b(3de-cf)x^2+a(7de-3cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\frac{\int -\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{\int \frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-}{4e\left(e+fx^2\right)}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-}{4e\left(e+fx^2\right)^2}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{422}$$

$$\frac{b\left(\frac{d^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x}{}\right.}{(de-cf)^2}\right.}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-a}}{\sqrt{e}\sqrt{a+bx}}\right)}{2e^{3/}}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 224

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{a\text{arc}}{}\right.}{}\right.}{d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\Big\downarrow$ 219

$$b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\ldots\right.}{}\right.$$

$$\overline{\qquad\qquad\qquad d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\ldots\right.$$

$$\overline{\qquad\qquad\qquad d}$$

$$\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right.}{d}$$

$$\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \ 425$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \ 421$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\ldots}{\ldots}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{\quad}{\sqrt{b}\cdots}}{\cdots}\right.}{d}$$

↓ 402

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2} - \cdots}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int-\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-\cdots}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}\cdots\right)}{(de-cf)^2}\cdots\right)}{d}\cdots\right)}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf\cdots}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2e(be-af)}}{4e(be-af)}\cdots\right)}{(de-cf)^2}\cdots\right)}{d}\cdots\right)}{d}$$

$\downarrow$ 27

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{-\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\cdots\right.}{2e(be-af)}}{4e(be-af)}\cdots\right)}{(de-cf)^2}\cdots\right)}{d}\cdots\right)}{d}$$

↓ 291

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d\cdots}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\cdots\right.}{2e(be-af)\cdots}}{4e(be-af)\cdots}\cdots\right)}{(de-cf)^2\cdots}\cdots\right)}{d\cdots}\cdots\right)}{d\cdots}$$

↓ 221

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2} -$$

$$d$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}\right.}{d}$$

↓ 407

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2\cdots\right.}{\cdots}}{\cdots}\right)}{(de-\cdots}\right)}{d}\cdots\right)}{d}$$

↓ 291

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+}}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right.}{d}\right.}{d}$$

$$\downarrow 221$$

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\cdots}{\cdots}\right.}{\cdots}\right.}{d}\cdots\right.}{d}$$

↓ 426

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\cdots}{\cdots}\right)}{\cdots}\right)}{d}\cdots\right)}{\cdots}$$

$\downarrow$ 421

$$\frac{b\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{d}$$

$$(bc-ad)\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\cdots}{\cdots}\right)}{\cdots}\right)}{d}\cdots\right)$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{3\left(c(x^2d+c)\left(\frac{8b^2d^2e^4}{3}-\frac{40\left(ad-\frac{2bc}{5}\right)dbf\,e^3}{3}+\frac{35ad\left(ad-\frac{8bc}{35}\right)f^2e^2}{3}-\frac{14a^2cde\,f^3}{3}+a^2c^2f^4\right)\sqrt{(ad-bc)c}\,(f\,x^2+e)^2\arctan\left(\ldots\right.\ldots\right)}{\ldots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)*(c*(d*x^2+c)*(8/3*b^2*d^2*e^4-40/3*(a*d-2/5*b*c)*
d*b*f*e^3+35/3*a*d*(a*d-8/35*b*c)*f^2*e^2-14/3*a^2*c*d*e*f^3+a^2*c^2*f^4)*
((a*d-b*c)*c)^(1/2)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(
1/2))-28/3*((a*f-b*e)*e)^(1/2)*((a*d-b*c)*d*(d*x^2+c)*(-1/7*d*(a*d+2*b*c)*
e+c*f*(a*d-4/7*b*c))*(f*x^2+e)^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)
*c)^(1/2))+5/28*(c*f-d*e)*((a*d-b*c)*c)^(1/2)*(b*x^2+a)^(1/2)*(-4/5*d^2*(a
*d-b*c)*e^4-8/5*d*f*(a*d^2*x^2-2*b*c*d*x^2-b*c^2)*e^3-13/5*d*f^2*((-6/13*b
*x^2+a)*c^2+d*x^2*(-10/13*b*x^2+a)*c+4/13*a*d^2*x^4)*e^2+c*(c*(2/5*b*x^2+a
)-11/5*a*d*x^2)*(d*x^2+c)*f^3*e+3/5*x^2*f^4*c^2*a*(d*x^2+c))*x))/((a*f-b*e
)*e)^(1/2)/(c*f-d*e)^4/(f*x^2+e)^2/e^2/c/(d*x^2+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^2(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^2*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(340) = 680$.

Time = 3.92 (sec) , antiderivative size = 1411, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/2*(2*b^(5/2)*c^2*d^2*e - a*b^(3/2)*c*d^3*e - a^2*sqrt(b)*d^4*e + 4*b^(5/
2)*c^3*d*f - 11*a*b^(3/2)*c^2*d^2*f + 7*a^2*sqrt(b)*c*d^3*f)*arctan(1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/
((c*d^4*e^4 - 4*c^2*d^3*e^3*f + 6*c^3*d^2*e^2*f^2 - 4*c^4*d*e*f^3 + c^5*f^
4)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/8*(8*b^(5/2)*d^2*e^4 + 16*b^(5/2)*c*d*e^3
*f - 40*a*b^(3/2)*d^2*e^3*f - 8*a*b^(3/2)*c*d*e^2*f^2 + 35*a^2*sqrt(b)*d^2
*e^2*f^2 - 14*a^2*sqrt(b)*c*d*e*f^3 + 3*a^2*sqrt(b)*c^2*f^4)*arctan(1/2*((
sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/
((d^4*e^6 - 4*c*d^3*e^5*f + 6*c^2*d^2*e^4*f^2 - 4*c^3*d*e^3*f^3 + c^4*e^2*
f^4)*sqrt(-b^2*e^2 + a*b*e*f)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2
)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2)*c*d^2 + (sqrt(b)*x -
 sqrt(b*x^2 + a))^2*a^2*sqrt(b)*d^3 + a^2*b^(3/2)*c*d^2 - a^3*sqrt(b)*d^3)
/((c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*((sqrt(b)*x - sq
rt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*a*d + a^2*d)) - 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*b^(5/2)*d*e^3*f + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c*e^2*f^2 -
24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*f^2 + 11*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^2*sqrt(b)*c*f^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4 + 16*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 136*(sqrt(b)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^2\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^2*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^2\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

3.314 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 484

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = -\frac{d^2(bc-ad)x\sqrt{a+bx^2}}{4c(de-cf)^3(c+dx^2)^2} + \frac{d^2(3ad(de-5cf)+2bc(de+5cf))x\sqrt{a+bx^2}}{8c^2(de-cf)^4(c+dx^2)} + \frac{f^2(be-af)x\sqrt{a+bx^2}}{4e(de-cf)^3(e+fx^2)^2} - \frac{f^2(3af(5de-cf)-2be(5de+cf))x\sqrt{a+bx^2}}{8e^2(de-cf)^4(e+fx^2)} + \frac{3d(8b^2c^3f(de+cf)-4abc^2df(de+7cf)+a^2d^2(d^2e^2-6cdef+21c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}(de-cf)^5} - \frac{3f(8b^2de^3(de+cf)-4abde^2f(7de+cf)+a^2f^2(21d^2e^2-6cdef+c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}\sqrt{be-af}(de-cf)^5}$$

output
```
-1/4*d^2*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)^3/(d*x^2+c)^2+1/8*d^2*(
3*a*d*(-5*c*f+d*e)+2*b*c*(5*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-c*f+d*e)^4/(
d*x^2+c)+1/4*f^2*(-a*f+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)^3/(f*x^2+e)^2-1
/8*f^2*(3*a*f*(-c*f+5*d*e)-2*b*e*(c*f+5*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-c*f+
d*e)^4/(f*x^2+e)+3/8*d*(8*b^2*c^3*f*(c*f+d*e)-4*a*b*c^2*d*f*(7*c*f+d*e)+a^
2*d^2*(21*c^2*f^2-6*c*d*e*f+d^2*e^2))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(
b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(1/2)/(-c*f+d*e)^5-3/8*f*(8*b^2*d*e^3*(
c*f+d*e)-4*a*b*d*e^2*f*(c*f+7*d*e)+a^2*f^2*(c^2*f^2-6*c*d*e*f+21*d^2*e^2))
*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(1
/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 12.14 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \frac{1}{8}\left(x\sqrt{a+bx^2}\left(\frac{2d^2(bc-ad)}{c(-de+cf)^3(c+dx^2)^2}+\frac{d^2(3ad(de-5cf)+2bc(de+5cf\right.\right.$$
$$-\frac{3d(8b^2c^3f(de+cf)-4abc^2df(de+7cf)+a^2d^2(d^2e^2-6cdef+21c^2f^2))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}\sqrt{-bc+ad}(-de+cf)^5}$$
$$\left.-\frac{3f(8b^2de^3(de+cf)-4abde^2f(7de+cf)+a^2f^2(21d^2e^2-6cdef+c^2f^2))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{5/2}\sqrt{-be+af}(de-cf)^5}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((2*d^2*(b*c - a*d))/(c*(-(d*e) + c*f)^3*(c + d*x^2)^2)
 + (d^2*(3*a*d*(d*e - 5*c*f) + 2*b*c*(d*e + 5*c*f)))/(c^2*(d*e - c*f)^4*(c
 + d*x^2)) + (2*f^2*(b*e - a*f))/(e*(d*e - c*f)^3*(e + f*x^2)^2) + (f^2*(3
*a*f*(-5*d*e + c*f) + 2*b*e*(5*d*e + c*f)))/(e^2*(d*e - c*f)^4*(e + f*x^2)
)) - (3*d*(8*b^2*c^3*f*(d*e + c*f) - 4*a*b*c^2*d*f*(d*e + 7*c*f) + a^2*d^2
*(d^2*e^2 - 6*c*d*e*f + 21*c^2*f^2))*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c
]*Sqrt[a + b*x^2])])/(c^(5/2)*Sqrt[-(b*c) + a*d]*(-(d*e) + c*f)^5) - (3*f*
(8*b^2*d*e^3*(d*e + c*f) - 4*a*b*d*e^2*f*(7*d*e + c*f) + a^2*f^2*(21*d^2*e
^2 - 6*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a +
 b*x^2])])/(e^(5/2)*Sqrt[-(b*e) + a*f]*(d*e - c*f)^5))/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^3\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2(fx^2+e)^3}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^3(fx^2+e)^3}dx}{d}$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{ 421}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+\cdots\right.}}{d}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+\cdots\right.}}{d}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 402

$$b\left(\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx\sqrt{\ldots}}{4e\left(e\ldots\right.}\right.}{(de-cf)^2}\right.}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow 25$$

$$b\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}-\frac{fx\sqrt{\cdots}}{4e\left(e+\cdots\right.}\cdots\right.}{(de-cf)^2}\cdots\right.}{d}\cdots\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int\frac{}{\sqrt{bx^2+a}}}{2e(be-af)}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int\frac{1}{e-\frac{(be-a}{bx^2}}}{2e(be-af)}}{4e(be-af)}}{(de-cf)^2}}{d}}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right.}{(de-cf)^2}\right)}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$\downarrow$ 407

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3\cdots\right.}{2e^{3/2}(be-\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right.}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\downarrow \text{291}$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f \cdots\right.}{2\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)\cdots}\cdots\right)}{d}\cdots\right.}{d}\cdots$$

$$\frac{(bc-ad)\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2e}}{4e(be-af)}\right.}{(de-cf)^2}}{d}\right.}{d}$$

$$\frac{(bc-ad)\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2e}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{d}\right.}{(bc-ad)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{b}}\right.}{d}\right.}{d}}$$

$\downarrow$ 421

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}\right.}{(de-cf)^2 \cdots}\right.}{d \cdots}\right.$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int\frac{\cdots}{\sqrt{bx^2+a}\cdots}}{\cdots}\right.}{\cdots}\right.}{d}$$

↓ 402

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{d}\cdots\right.}{\cdots}$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{f(\cdots}{4e(\cdots}\right.}{(de-cf)^2}\right.}{de-cf}\cdots\right.}{d}\cdots\right.}{d}$$

↓ 25

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}}{}\right.}{(de-cf)^2}}{d}\right.$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right.}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{d}\right.}{d}$$

↓ 402

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}}{}\right.}{(de-cf)^2}\right)}{d}\right.$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf)}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right)}{de-cf}\right.\right.$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)(fx^2+e)}-\frac{(8b^2(2de-}{}}{}\right.}{(de-cf)^2}\right.}{d}\right.$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf)}{2e(be-af)(fx^2+e)}\right.}{}\right.}{de-cf}\right.\right.$$

$$\Big\downarrow \, 27$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right)$$

$$\overline{}$$

$$(bc-ad)\left(\frac{b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf)\cdots}{2e(be-af)\left(fx^2+e\right)\cdots}\cdots\right.}{\cdots}\right.}{de-cf\cdots}\cdots\right)}{d}\cdots\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}\right.}{(de-cf)^2}\right.}{d}\right.$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf)}{2e(be-af)\left(fx^2+e\right)}\right.}{}\right)}{de-}\right.\right.$$

↓ 221

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right.$$

$$\overline{\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf)\cdots}{2e(be-af)\left(fx^2+e\right)}\cdots}{\cdots}\right.}{\cdots}\right.}{de-\cdots}\right.\right.}$$

↓ 407

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right)$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5\cdots}{\cdots}}{\cdots}\right.}{\cdots}\right)}{\cdots}\cdots\right.\right.$$

↓ 291

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right)$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}\cdots\right.}{\cdots}\right)}{\cdots}\cdots\right.\right.$$

$$\downarrow 221$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}}{}\right.}{(de-cf)^2}\right)}{d}\right.$$

$$\left(b\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\right.}{}\right.}{}\right.\right.$$

↓ 426

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right)$$

$$\left(\left(\frac{b\,d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be}\cdots}{\sqrt{e}\sqrt{\cdots}}\right)}{\sqrt{e}\sqrt{be-af}(de-\cdots)}\right)}{(de-cf)^2}\cdots\right.}{\cdots}\right.\right.$$

↓ 421

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2 \cdots}\right.}{d \cdots}\right.$$

$$\left(b\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)\Big/(de-cf)-\frac{f\left(\frac{d^2\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \ 25$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\right.}{d}\right.$$

$$\overline{\left(b\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(d^2\left(\frac{d\cdots}{\cdots}-\cdots\right.\right.}{\cdots}\right.\right.}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right.$$

$$\left(\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}\,b-\cdots\right.\right.$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{d}\cdots\right)$$

$$\overline{\left(\left(\frac{b\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}\right.\right.\right.\,f\cdots-\cdots}$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{3\left(-21d\left(x^2d+c\right)^2\left(\frac{8b^2c^4f^2}{21}-\frac{4d\left(af-\frac{2be}{7}\right)bf\,c^3}{3}+a\,d^2f\left(af-\frac{4be}{21}\right)c^2-\frac{2a^2cef\,d^3}{7}+\frac{a^2e^2d^4}{21}\right)\sqrt{(af-be)e}\,\left(f\,x^2+e\right)^2e^2\,\mathrm{arc}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)*(-21*d*(d*x^2+c)^2*(8/21*b^2*c^4*f^2-4/3*d*(a*f-2
/7*b*e)*b*f*c^3+a*d^2*f*(a*f-4/21*b*e)*c^2-2/7*a^2*c*e*f*d^3+1/21*a^2*e^2*
d^4)*((a*f-b*e)*e)^(1/2)*(f*x^2+e)^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-
b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(c^2*(a^2*c^2*f^4-6*(a^2*f^2+2/3*a*b*f*
e-4/3*b^2*e^2)*d*f*e*c+21*d^2*(a^2*f^2-4/3*a*b*f*e+8/21*b^2*e^2)*e^2)*(d*x
^2+c)^2*(f*x^2+e)^2*f*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-5/3*
((a*f-b*e)*e)^(1/2)*(c*f-d*e)*(((2/5*b*x^2+a)*e+3/5*a*f*x^2)*f^4*c^5-17/5*
d*(-12/17*b*e^3+f*(-10/17*b*x^2+a)*e^2+5/17*(-4/5*b*x^2+a)*x^2*f^2*e-6/17*
a*f^3*x^4)*f^2*c^4-34/5*d^2*f*(-6/17*e^4*b-24/17*b*e^3*f*x^2+f^2*x^2*(-16/
17*b*x^2+a)*e^2+25/34*x^4*f^3*(-2/25*b*x^2+a)*e-3/34*a*f^4*x^6)*c^3-17/5*d
^3*((-10/17*b*x^2+a)*e^3+2*x^2*f*(-16/17*b*x^2+a)*e^2+2*(-10/17*b*x^2+a)*x
^4*f^2*e+15/17*a*f^3*x^6)*f*e*c^2+((2/5*b*x^2+a)*e-3*a*f*x^2)*d^4*(f*x^2+e
)^2*e^2*c+3/5*a*d^5*e^3*x^2*(f*x^2+e)^2)*(b*x^2+a)^(1/2)*x))/((a*f-b*e)*e)
^(1/2)/(f*x^2+e)^2/e^2/(c*f-d*e)^5/(d*x^2+c)^2/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^3(fx^2+e)^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^3*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5515 vs. $2(444) = 888$.

Time = 8.50 (sec) , antiderivative size = 5515, normalized size of antiderivative = 11.39

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-3/8*(a^2*sqrt(b)*d^5*e^2 + 8*b^(5/2)*c^3*d^2*e*f - 4*a*b^(3/2)*c^2*d^3*e*
f - 6*a^2*sqrt(b)*c*d^4*e*f + 8*b^(5/2)*c^4*d*f^2 - 28*a*b^(3/2)*c^3*d^2*f
^2 + 21*a^2*sqrt(b)*c^2*d^3*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((c^2*d^5*e^5 - 5*c^3*d^4*e^
4*f + 10*c^4*d^3*e^3*f^2 - 10*c^5*d^2*e^2*f^3 + 5*c^6*d*e*f^4 - c^7*f^5)*s
qrt(-b^2*c^2 + a*b*c*d)) + 3/8*(8*b^(5/2)*d^2*e^4*f + 8*b^(5/2)*c*d*e^3*f^
2 - 28*a*b^(3/2)*d^2*e^3*f^2 - 4*a*b^(3/2)*c*d*e^2*f^3 + 21*a^2*sqrt(b)*d^
2*e^2*f^3 - 6*a^2*sqrt(b)*c*d*e*f^4 + a^2*sqrt(b)*c^2*f^5)*arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((
d^5*e^7 - 5*c*d^4*e^6*f + 10*c^2*d^3*e^5*f^2 - 10*c^3*d^2*e^4*f^3 + 5*c^4*
d*e^3*f^4 - c^5*e^2*f^5)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/4*(24*(sqrt(b)*x -
sqrt(b*x^2 + a))^14*b^(5/2)*c^2*d^3*e^3*f^2 - 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^14*a^2*sqrt(b)*d^5*e^3*f^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(5/
2)*c^3*d^2*e^2*f^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a*b^(3/2)*c^2*d^3
*e^2*f^3 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*sqrt(b)*c*d^4*e^2*f^3 +
 15*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*sqrt(b)*c^2*d^3*e*f^4 - 3*(sqrt(b
)*x - sqrt(b*x^2 + a))^14*a^2*sqrt(b)*c^3*d^2*f^5 + 144*(sqrt(b)*x - sqrt(
b*x^2 + a))^12*b^(7/2)*c^2*d^3*e^4*f - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*a^2*b^(3/2)*d^5*e^4*f + 288*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2)*c^3*
d^2*e^3*f^2 - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a*b^(5/2)*c^2*d^3*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^3\,(f\,x^2+e)^3}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^3*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3\,(e+fx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^3\,(f\,x^2+e)^3}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.315 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 201

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x\sqrt{a+bx^2}}{2df} + \frac{b^{3/2}(5adf-2b(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2f^2} - \frac{(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2(de-cf)} + \frac{(be-af)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^2(de-cf)}$$

output
```
1/2*b^2*x*(b*x^2+a)^(1/2)/d/f+1/2*b^(3/2)*(5*a*d*f-2*b*(c*f+d*e))*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/d^2/f^2-(-a*d+b*c)^(5/2)*arctanh((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/d^2/(-c*f+d*e)+(-a*f+b*e)^(5/2)*arct
anh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^2/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x\sqrt{a+bx^2}}{2df} + \frac{(-bc+ad)^{5/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}d^2(-de+cf)} + \frac{(-be+af)^{5/2}\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}f^2(de-cf)} + \frac{b^{3/2}(-5adf+2b(de+cf))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2d^2f^2}$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b^2*x*Sqrt[a + b*x^2])/(2*d*f) + ((-(b*c) + a*d)^(5/2)*ArcTan[(-(d*x*Sqrt
[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c
]*d^2*(-(d*e) + c*f)) + ((-(b*e) + a*f)^(5/2)*ArcTan[(-(f*x*Sqrt[a + b*x^2
]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*f^2*(d*e
 - c*f)) + (b^(3/2)*(-5*a*d*f + 2*b*(d*e + c*f))*Log[-(Sqrt[b]*x) + Sqrt[a
 + b*x^2]])/(2*d^2*f^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 429 vs. $2(201) = 402$.

Time = 0.76 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.13, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {420, 318, 25, 398, 224, 219, 291, 221, 420, 301, 224, 219, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\int \frac{(bx^2+a)^{3/2}}{fx^2+e}dx}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{b\left(\frac{\int -\frac{b(2be-3af)x^2+a(be-2af)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2f}+\frac{bx\sqrt{a+bx^2}}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\int \frac{b(2be-3af)x^2+a(be-2af)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{b(2be-3af)\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{2(be-af)^2\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{b(2be-3af)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{2(be-af)^2\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^2\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}}{2f}\right)}{d}-\frac{(bc-ad)\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^2\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2f}\right)}{d}-\frac{(bc-ad)\int\frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\int\frac{(bx^2+a)^{3/2}}{(dx^2+c)(fx^2+e)}dx}{d}$$

$$\Big\downarrow \text{420}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$\Big\downarrow$ 219

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{d}\right)}{d}$$

$$\Big\downarrow \text{ 422}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{d}\right)}{d}$$

$$\Big\downarrow \text{ 301}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\cdots\right.}{d}\cdots}{d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2-\right.}}{d}\right.}{de-cf}\right.}{}\right.}{d}$$

$\downarrow$ 219

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-$$

$$\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx\right.}}{d}\right.}{de-cf}\right.}{}\right.}{d}$$

$\downarrow$ 291

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x}{bx^2+a}}}{d}\right.}{de-cf}\right.}{}\right.}{d}$$

$\downarrow$ 221

$$\frac{b\left(\frac{bx\sqrt{a+bx^2}}{2f}-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-3af)}{f}-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{d}-\frac{(bc-ad)\left(\frac{b\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{d}-\frac{(bc-ad)\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\cdots\right)}{\sqrt{c}d}\right.}{de-cf}\right.}{}\right.}{d}$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*((b*x*Sqrt[a + b*x^2])/(2*f) - ((Sqrt[b]*(2*b*e - 3*a*f)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/f - (2*(b*e - a*f)^(3/2)*ArcTanh[(Sqrt[b*e - a*f]*
x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))/(2*f)))/d - ((b*c - a*d)*((b*(
(Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt[b*e - a*f]*ArcTan
h[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f)))/d - ((b*c
- a*d)*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d))
)/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sq
rt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqr
t[e]*f)))/(d*e - c*f)))/d))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{(ad-bc)^3 \arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{d^2(cf-de)\sqrt{(ad-bc)c}}-\frac{b^2\left(-\sqrt{b\,x^2+a}\,dfx-\frac{(5adf-2bcf-2bde)\,\text{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{2d^2f^2}-\frac{(af-be)^3 \arctan\left(\frac{e\ldots}{x\sqrt{\ldots}}\right)}{f^2(cf-de)\sqrt{(af\ldots}}$ …
risch	$\frac{b^2x\sqrt{b\,x^2+a}}{2df}+\frac{\frac{b^{\frac{3}{2}}(5adf-2bcf-2bde)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{df}-\frac{f^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-c\ldots}}{d}\right)}{d}\ldots}{\ldots}\right)}{\sqrt{-cd}\left(\sqrt{-cd}\,f+\sqrt{-ef}\,d\right)(\sqrt{\ldots}}}{\ldots}$ …
default	Expression too large to display

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(a*d-b*c)^3/d^2/(c*f-d*e)/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/(
(a*d-b*c)*c)^(1/2))-1/2*b^2*(-(b*x^2+a)^(1/2)*d*f*x-(5*a*d*f-2*b*c*f-2*b*d
*e)/b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)))/d^2/f^2-(a*f-b*e)^3/f^2/(c
*f-d*e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)\,(f\,x^2+e)}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 924, normalized size of antiderivative = 4.60

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
(2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2
) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*d**2*e*f**2 - 4*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a*b*c*d*e*f**2 + 2*sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c
)*sqrt(b)))*b**2*c**2*e*f**2 + 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*
*2*d**2*e*f**2 - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)
*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*b*c*d*e*f**2 +
 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2
) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b**2*c**2*e*f**2 - 2*sqrt(e)*sqr
t(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sq
rt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*c*d**2*f**2 + 4*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a*b*c*d**2*e*f - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*
*2*c*d**2*e**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)
*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*c*d**2*f**2
 + 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x*
*2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c*d**2*e*f - 2*sqrt(e)*...
```

3.316 $\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 300

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx = -\frac{(35a^3df^3+144ab^2ef(de+cf)-40a^2bf^2(3de+cf)-64b^3e^2(de+3cf))\,x\sqrt{a+bx^2}}{128b^4} + \frac{f(35a^2df^2+144b^2e(de+cf)-40abf(3de+cf))\,x^3\sqrt{a+bx^2}}{192b^3} - \frac{f^2(7adf-8b(3de+cf))x^5\sqrt{a+bx^2}}{48b^2} + \frac{df^3x^7\sqrt{a+bx^2}}{8b} + \frac{(128b^4ce^3+35a^4df^3+144a^2b^2ef(de+cf)-40a^3bf^2(3de+cf)-64ab^3e^2(de+3cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}$$

output
```
-1/128*(35*a^3*d*f^3+144*a*b^2*e*f*(c*f+d*e)-40*a^2*b*f^2*(c*f+3*d*e)-64*b
^3*e^2*(3*c*f+d*e))*x*(b*x^2+a)^(1/2)/b^4+1/192*f*(35*a^2*d*f^2+144*b^2*e*
(c*f+d*e)-40*a*b*f*(c*f+3*d*e))*x^3*(b*x^2+a)^(1/2)/b^3-1/48*f^2*(7*a*d*f-
8*b*(c*f+3*d*e))*x^5*(b*x^2+a)^(1/2)/b^2+1/8*d*f^3*x^7*(b*x^2+a)^(1/2)/b+1
/128*(128*b^4*c*e^3+35*a^4*d*f^3+144*a^2*b^2*e*f*(c*f+d*e)-40*a^3*b*f^2*(c
*f+3*d*e)-64*a*b^3*e^2*(3*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(
9/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^3df^3+10a^2bf^2(36de+12cf+7dfx^2)-8ab^2f(2cf(27e+5fx^2)+d(54e^2+30efx}{}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*d*f^3 + 10*a^2*b*f^2*(36*d*e + 12*c*f
 + 7*d*f*x^2) - 8*a*b^2*f*(2*c*f*(27*e + 5*f*x^2) + d*(54*e^2 + 30*e*f*x^2
 + 7*f^2*x^4)) + 16*b^3*(2*c*f*(18*e^2 + 9*e*f*x^2 + 2*f^2*x^4) + 3*d*(4*e
^3 + 6*e^2*f*x^2 + 4*e*f^2*x^4 + f^3*x^6))) - 3*(128*b^4*c*e^3 + 35*a^4*d*
f^3 + 144*a^2*b^2*e*f*(d*e + c*f) - 40*a^3*b*f^2*(3*d*e + c*f) - 64*a*b^3*
e^2*(d*e + 3*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {403, 403, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{403}$$
$$\frac{\int \frac{(fx^2+e)^2((6bde+8bcf-7adf)x^2+(8bc-ad)e)}{\sqrt{bx^2+a}}dx}{8b}+\frac{dx\sqrt{a+bx^2}(e+fx^2)^3}{8b}$$
$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{\left(fx^2+e\right)\left(\left(8e(3de+10cf)b^2-8af(8de+5cf)b+35a^2df^2\right)x^2+e\left(7dfa^2-12bdea-8bcfa+48b^2ce\right)\right)}{\sqrt{bx^2+a}}dx}{6b}+\frac{x\sqrt{a+bx^2}\left(e+fx^2\right)^2(-7adf+8bcf+6bde)}{6b}}{8b}+$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^3}{8b}$$

$\downarrow$ 403

$$\frac{\frac{\frac{\int \frac{e\left(-35df^2a^3+4bf(23de+10cf)a^2-8b^2e(9de+14cf)a+192b^3ce^2\right)-\left(-16e^2(3de+22cf)b^3+8aef(31de+44cf)b^2-10a^2f^2(29de+12cf)b+105a^3df^3\right)x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x\sqrt{a-}}{}}{6b}}{8b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^3}{8b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{\frac{3\left(35a^4df^3-40a^3bf^2(cf+3de)+144a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+128b^4ce^3\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{x\sqrt{a+bx^2}\left(105a^3df^3-10a^2bf^2(12cf+29de)+8ab^2ef(44cf+3\right.}{2b}}{4b}}{6b}}{8b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^3}{8b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\frac{3\left(35a^4df^3-40a^3bf^2(cf+3de)+144a^2b^2ef(cf+de)-64ab^3e^2(3cf+de)+128b^4ce^3\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{x\sqrt{a+bx^2}\left(105a^3df^3-10a^2bf^2(12cf+29de)+8ab^2e\right.}{2b}}{4b}}{6b}}{8b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^3}{8b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df^2-8abf(5cf+8de)+8b^2e(10cf+3de)\right)}{4b}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(35a^4df^3-40a^3bf^2(cf+3de)+144a^2b^2ef(cf+de)-64ab^3e^2(3cf+d\right.}{2b^{3/2}}}{6b}}{8b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^3}{8b}$$

input

```
Int[((c + d*x^2)*(e + f*x^2)^3)/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(e + f*x^2)^3)/(8*b) + (((6*b*d*e + 8*b*c*f - 7*a*d*f
)*x*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(6*b) + (((35*a^2*d*f^2 - 8*a*b*f*(8*d*
e + 5*c*f) + 8*b^2*e*(3*d*e + 10*c*f))*x*Sqrt[a + b*x^2]*(e + f*x^2))/(4*b
) + (-1/2*((105*a^3*d*f^3 - 10*a^2*b*f^2*(29*d*e + 12*c*f) - 16*b^3*e^2*(3
*d*e + 22*c*f) + 8*a*b^2*e*f*(31*d*e + 44*c*f))*x*Sqrt[a + b*x^2])/b + (3*
(128*b^4*c*e^3 + 35*a^4*d*f^3 + 144*a^2*b^2*e*f*(d*e + c*f) - 40*a^3*b*f^2
*(3*d*e + c*f) - 64*a*b^3*e^2*(d*e + 3*c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2)))/(4*b))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{35\left(a^3\left(ad-\frac{8bc}{7}\right)f^3-\frac{24a^2\left(ad-\frac{6bc}{5}\right)bef^2}{7}+\frac{144a\left(ad-\frac{4bc}{3}\right)b^2e^2f}{35}-\frac{64b^3e^3(ad-2bc)}{35}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{128}-\frac{35\sqrt{bx^2+a}\,x\left(\left(-\frac{64(}{}\right.\right.}{}$
risch	$-\frac{x\left(-48d f^3b^3x^6+56a b^2d f^3x^4-64b^3c f^3x^4-192b^3de f^2x^4-70a^2f^3db x^2+80a b^2c f^3x^2+240a b^2de f^2x^2-288b^3ce f^2x^2\right.}{384}$
default	$\frac{ce^3\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{\sqrt{b}}+f^2(cf+3de)\left(\frac{x^5\sqrt{bx^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right.}{6b}\right.$

input
```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/128/b^(9/2)*((a^3*(a*d-8/7*b*c)*f^3-24/7*a^2*(a*d-6/5*b*c)*b*e*f^2+144/
35*a*(a*d-4/3*b*c)*b^2*e^2*f-64/35*b^3*e^3*(a*d-2*b*c))*arctanh((b*x^2+a)^
(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*x*((-64/105*(3/4*x^2*d+c)*x^4*f^3-96/35*(
2/3*x^2*d+c)*x^2*e*f^2-192/35*e^2*(1/2*x^2*d+c)*f-64/35*d*e^3)*b^(7/2)+a*(
(16/21*(7/10*x^2*d+c)*x^2*f^2+144/35*(5/9*x^2*d+c)*e*f+144/35*d*e^2)*b^(5/
2)+a*(((-2/3*x^2*d-8/7*c)*f-24/7*d*e)*b^(3/2)+a*d*f*b^(1/2))*f)*f))
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\,(64\,(2\,b^4c-ab^3d)e^3-48\,(4\,ab^3c-3\,a^2b^2d)e^2f+24\,(6\,a^2b^2c-5\,a^3bd)ef^2-5\,(8\,a^3bc-7\,a^4d)f^3)\sqrt{b}\,1\right.$$
$$-\frac{3\,(64\,(2\,b^4c-ab^3d)e^3-48\,(4\,ab^3c-3\,a^2b^2d)e^2f+24\,(6\,a^2b^2c-5\,a^3bd)ef^2-5\,(8\,a^3bc-7\,a^4d)f^3)\sqrt{-}}{}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(64*(2*b^4*c - a*b^3*d)*e^3 - 48*(4*a*b^3*c - 3*a^2*b^2*d)*e^2*f
 + 24*(6*a^2*b^2*c - 5*a^3*b*d)*e*f^2 - 5*(8*a^3*b*c - 7*a^4*d)*f^3)*sqrt(
b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d*f^3*x^7 +
 8*(24*b^4*d*e*f^2 + (8*b^4*c - 7*a*b^3*d)*f^3)*x^5 + 2*(144*b^4*d*e^2*f +
 24*(6*b^4*c - 5*a*b^3*d)*e*f^2 - 5*(8*a*b^3*c - 7*a^2*b^2*d)*f^3)*x^3 + 3
*(64*b^4*d*e^3 + 48*(4*b^4*c - 3*a*b^3*d)*e^2*f - 24*(6*a*b^3*c - 5*a^2*b^
2*d)*e*f^2 + 5*(8*a^2*b^2*c - 7*a^3*b*d)*f^3)*x)*sqrt(b*x^2 + a))/b^5, -1/
384*(3*(64*(2*b^4*c - a*b^3*d)*e^3 - 48*(4*a*b^3*c - 3*a^2*b^2*d)*e^2*f +
24*(6*a^2*b^2*c - 5*a^3*b*d)*e*f^2 - 5*(8*a^3*b*c - 7*a^4*d)*f^3)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*b^4*d*f^3*x^7 + 8*(24*b^4*d*e*f^
2 + (8*b^4*c - 7*a*b^3*d)*f^3)*x^5 + 2*(144*b^4*d*e^2*f + 24*(6*b^4*c - 5*
a*b^3*d)*e*f^2 - 5*(8*a*b^3*c - 7*a^2*b^2*d)*f^3)*x^3 + 3*(64*b^4*d*e^3 +
48*(4*b^4*c - 3*a*b^3*d)*e^2*f - 24*(6*a*b^3*c - 5*a^2*b^2*d)*e*f^2 + 5*(8
*a^2*b^2*c - 7*a^3*b*d)*f^3)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{df^3x^7}{8b} + \frac{x^5\left(-\frac{7adf^3}{8b}+cf^3+3def^2\right)}{6b} + \frac{x^3\left(-\frac{5a\left(-\frac{7adf^3}{8b}+cf^3+3def^2\right)}{6b}+3cef^2+3de^2f\right)}{4b} + \frac{x\left(-\frac{3a\left(-\frac{5a\left(-\frac{7adf^3}{8b}+cf^3+\right.}{6b}\right.}{4}\right.}{}\right. \\ \frac{ce^3x+\frac{df^3x^9}{9}+\frac{x^7\left(cf^3+3def^2\right)}{7}+\frac{x^5\cdot\left(3cef^2+3de^2f\right)}{5}+\frac{x^3\cdot\left(3ce^2f+de^3\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f**3*x**7/(8*b) + x**5*(-7*a*d*f**3/(8*b) +
 c*f**3 + 3*d*e*f**2)/(6*b) + x**3*(-5*a*(-7*a*d*f**3/(8*b) + c*f**3 + 3*d
*e*f**2)/(6*b) + 3*c*e*f**2 + 3*d*e**2*f)/(4*b) + x*(-3*a*(-5*a*(-7*a*d*f*
*3/(8*b) + c*f**3 + 3*d*e*f**2)/(6*b) + 3*c*e*f**2 + 3*d*e**2*f)/(4*b) + 3
*c*e**2*f + d*e**3)/(2*b)) + (-a*(-3*a*(-5*a*(-7*a*d*f**3/(8*b) + c*f**3 +
 3*d*e*f**2)/(6*b) + 3*c*e*f**2 + 3*d*e**2*f)/(4*b) + 3*c*e**2*f + d*e**3)
/(2*b) + c*e**3)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b
), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c*e**3*x + d*f*
*3*x**9/9 + x**7*(c*f**3 + 3*d*e*f**2)/7 + x**5*(3*c*e*f**2 + 3*d*e**2*f)/
5 + x**3*(3*c*e**2*f + d*e**3)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}df^3x^7}{8\,b} - \frac{7\,\sqrt{bx^2+a}adf^3x^5}{48\,b^2} + \frac{35\,\sqrt{bx^2+a}a^2df^3x^3}{192\,b^3} + \frac{(3\,def^2+cf^3)\sqrt{bx^2+a}x^5}{6\,b} - \frac{35\,\sqrt{bx^2+a}a^3df^3x}{128\,b^4} + \frac{ce^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{35\,a^4df^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{5\,(3\,def^2+cf^3)\sqrt{bx^2+a}ax^3}{24\,b^2} + \frac{3\,(de^2f+cef^2)\sqrt{bx^2+a}x^3}{4\,b} + \frac{5\,(3\,def^2+cf^3)\sqrt{bx^2+a}a^2x}{16\,b^3} - \frac{9\,(de^2f+cef^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(de^3+3\,ce^2f)\sqrt{bx^2+a}x}{2\,b} - \frac{5\,(3\,def^2+cf^3)a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{9\,(de^2f+cef^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(de^3+3\,ce^2f)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(b*x^2 + a)*d*f^3*x^7/b - 7/48*sqrt(b*x^2 + a)*a*d*f^3*x^5/b^2 + 3
5/192*sqrt(b*x^2 + a)*a^2*d*f^3*x^3/b^3 + 1/6*(3*d*e*f^2 + c*f^3)*sqrt(b*x
^2 + a)*x^5/b - 35/128*sqrt(b*x^2 + a)*a^3*d*f^3*x/b^4 + c*e^3*arcsinh(b*x
/sqrt(a*b))/sqrt(b) + 35/128*a^4*d*f^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/
24*(3*d*e*f^2 + c*f^3)*sqrt(b*x^2 + a)*a*x^3/b^2 + 3/4*(d*e^2*f + c*e*f^2)
*sqrt(b*x^2 + a)*x^3/b + 5/16*(3*d*e*f^2 + c*f^3)*sqrt(b*x^2 + a)*a^2*x/b^
3 - 9/8*(d*e^2*f + c*e*f^2)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2*(d*e^3 + 3*c*e^2
*f)*sqrt(b*x^2 + a)*x/b - 5/16*(3*d*e*f^2 + c*f^3)*a^3*arcsinh(b*x/sqrt(a*
b))/b^(7/2) + 9/8*(d*e^2*f + c*e*f^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) -
 1/2*(d*e^3 + 3*c*e^2*f)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^2)(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{384}\left(2\left(4\left(\frac{6\,df^3x^2}{b}+\frac{24\,b^6def^2+8\,b^6cf^3-7\,ab^5df^3}{b^7}\right)x^2+\frac{144\,b^6de^2f+144\,b^6cef^2-120\,ab^5def^2-4\right.\right.\right.}{b^7}$$
$$-\frac{(128\,b^4ce^3-64\,ab^3de^3-192\,ab^3ce^2f+144\,a^2b^2de^2f+144\,a^2b^2cef^2-120\,a^3bdef^2-40\,a^3bcf^3+35\,a}{128\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*d*f^3*x^2/b + (24*b^6*d*e*f^2 + 8*b^6*c*f^3 - 7*a*b^5*d*f^3
)/b^7)*x^2 + (144*b^6*d*e^2*f + 144*b^6*c*e*f^2 - 120*a*b^5*d*e*f^2 - 40*a
*b^5*c*f^3 + 35*a^2*b^4*d*f^3)/b^7)*x^2 + 3*(64*b^6*d*e^3 + 192*b^6*c*e^2*
f - 144*a*b^5*d*e^2*f - 144*a*b^5*c*e*f^2 + 120*a^2*b^4*d*e*f^2 + 40*a^2*b
^4*c*f^3 - 35*a^3*b^3*d*f^3)/b^7)*sqrt(b*x^2 + a)*x - 1/128*(128*b^4*c*e^3
 - 64*a*b^3*d*e^3 - 192*a*b^3*c*e^2*f + 144*a^2*b^2*d*e^2*f + 144*a^2*b^2*
c*e*f^2 - 120*a^3*b*d*e*f^2 - 40*a^3*b*c*f^3 + 35*a^4*d*f^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)\,(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)\,\left(f\,x^2+e\right)^3}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.96

$$\int \frac{(c+dx^2)\,(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-105\sqrt{b\,x^2+a}\,a^3bd\,f^3x+120\sqrt{b\,x^2+a}\,a^2b^2c\,f^3x+360\sqrt{b\,x^2+a}\,a^2b^2de\,f^2x+70\sqrt{b\,x^2+a}\,a^2b^2d\,f^3x^3}{}$$

input
```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(1/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**3*b*d*f**3*x + 120*sqrt(a + b*x**2)*a**2*b**2*
c*f**3*x + 360*sqrt(a + b*x**2)*a**2*b**2*d*e*f**2*x + 70*sqrt(a + b*x**2)
*a**2*b**2*d*f**3*x**3 - 432*sqrt(a + b*x**2)*a*b**3*c*e*f**2*x - 80*sqrt(
a + b*x**2)*a*b**3*c*f**3*x**3 - 432*sqrt(a + b*x**2)*a*b**3*d*e**2*f*x -
240*sqrt(a + b*x**2)*a*b**3*d*e*f**2*x**3 - 56*sqrt(a + b*x**2)*a*b**3*d*f
**3*x**5 + 576*sqrt(a + b*x**2)*b**4*c*e**2*f*x + 288*sqrt(a + b*x**2)*b**
4*c*e*f**2*x**3 + 64*sqrt(a + b*x**2)*b**4*c*f**3*x**5 + 192*sqrt(a + b*x*
*2)*b**4*d*e**3*x + 288*sqrt(a + b*x**2)*b**4*d*e**2*f*x**3 + 192*sqrt(a +
 b*x**2)*b**4*d*e*f**2*x**5 + 48*sqrt(a + b*x**2)*b**4*d*f**3*x**7 + 105*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d*f**3 - 120*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*f**3 - 360*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*e*f**2 + 432*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*e*f**2 + 432*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*e**2*f - 576*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c*e**2*f - 192*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*d*e**3 + 384*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*c*e**3)/(384*b**5)
```

3.317 $\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{(5a^2df^2-6abf(2de+cf)+8b^2e(de+2cf))\,x\sqrt{a+bx^2}}{16b^3} - \frac{f(5adf-6b(2de+cf))x^3\sqrt{a+bx^2}}{24b^2} + \frac{df^2x^5\sqrt{a+bx^2}}{6b} + \frac{(16b^3ce^2-5a^3df^2+6a^2bf(2de+cf)-8ab^2e(de+2cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
1/16*(5*a^2*d*f^2-6*a*b*f*(c*f+2*d*e)+8*b^2*e*(2*c*f+d*e))*x*(b*x^2+a)^(1/
2)/b^3-1/24*f*(5*a*d*f-6*b*(c*f+2*d*e))*x^3*(b*x^2+a)^(1/2)/b^2+1/6*d*f^2*
x^5*(b*x^2+a)^(1/2)/b+1/16*(16*b^3*c*e^2-5*a^3*d*f^2+6*a^2*b*f*(c*f+2*d*e)
-8*a*b^2*e*(2*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(15a^2df^2 - 2abf(18de+9cf+5dfx^2) + 4b^2(3cf(4e+fx^2) + 2d(3e^2+3efx^2+f^2x^4))) +}{48b^{7/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*d*f^2 - 2*a*b*f*(18*d*e + 9*c*f + 5*d*f
*x^2) + 4*b^2*(3*c*f*(4*e + f*x^2) + 2*d*(3*e^2 + 3*e*f*x^2 + f^2*x^4))) +
 3*(-16*b^3*c*e^2 + 5*a^3*d*f^2 - 6*a^2*b*f*(2*d*e + c*f) + 8*a*b^2*e*(d*e
 + 2*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {403, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \ 403$$
$$\frac{\int \frac{(fx^2+e)((4bde+6bcf-5adf)x^2+(6bc-ad)e)}{\sqrt{bx^2+a}}dx}{6b} + \frac{dx\sqrt{a+bx^2}(e+fx^2)^2}{6b}$$
$$\Big\downarrow \ 403$$

$$\frac{\frac{\int \frac{\left(4e(2de+9cf)b^2-2af(13de+9cf)b+15a^2df^2\right)x^2+e\left(5dfa^2-8bdea-6bcfa+24b^2ce\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{x\sqrt{a+bx^2}(e+fx^2)(-5adf+6bcf+4bde)}{4b}}{6b}+\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^2}{6b}$$

$\Big\downarrow$ 299

$$\frac{\frac{\frac{3\left(-5a^3df^2+6a^2bf(cf+2de)-8ab^2e(2cf+de)+16b^3ce^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}\left(15a^2df^2-2abf(9cf+13de)+4b^2e(9cf+2de)\right)}{2b}}{4b}+\frac{x\sqrt{a+bx^2}(e+fx^2)(-5a}{4b}}{6b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^2}{6b}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{3\left(-5a^3df^2+6a^2bf(cf+2de)-8ab^2e(2cf+de)+16b^3ce^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}\left(15a^2df^2-2abf(9cf+13de)+4b^2e(9cf+2de)\right)}{2b}}{4b}+\frac{x\sqrt{a+bx^2}(e+f}{}}{6b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^2}{6b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{x\sqrt{a+bx^2}\left(15a^2df^2-2abf(9cf+13de)+4b^2e(9cf+2de)\right)}{2b}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3df^2+6a^2bf(cf+2de)-8ab^2e(2cf+de)+16b^3ce^2\right)}{2b^{3/2}}}{4b}+\frac{x\sqrt{a+bx^2}(e+f}{}}{6b}$$

$$\frac{dx\sqrt{a+bx^2}\left(e+fx^2\right)^2}{6b}$$

input
```
Int[((c + d*x^2)*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(6*b) + (((4*b*d*e + 6*b*c*f - 5*a*d*f
)*x*Sqrt[a + b*x^2]*(e + f*x^2))/(4*b) + (((15*a^2*d*f^2 + 4*b^2*e*(2*d*e
+ 9*c*f) - 2*a*b*f*(13*d*e + 9*c*f))*x*Sqrt[a + b*x^2])/(2*b) + (3*(16*b^3
*c*e^2 - 5*a^3*d*f^2 + 6*a^2*b*f*(2*d*e + c*f) - 8*a*b^2*e*(d*e + 2*c*f))*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{5\left(\left(\left(a^2\left(ad-\frac{6bc}{5}\right)f^2-\frac{12a\left(ad-\frac{4bc}{3}\right)bef}{5}+\frac{8b^2e^2(ad-2bc)}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\sqrt{b\,x^2+a}\left(\frac{4\left(\left(\frac{2x^2d}{3}+c\right)x^2f^2+4e\left(\frac{x^2d}{2}+\right.\right.}{5}\right.\right.}{16b^{\frac{7}{2}}}$
risch	$\frac{x\left(8f^2d\,b^2x^4-10abd\,f^2x^2+12b^2c\,f^2x^2+24b^2def\,x^2+15a^2d\,f^2-18abc\,f^2-36abdef+48b^2cef+24b^2d\,e^2\right)\sqrt{b\,x^2+a}}{48b^3}-\frac{(5a^3}{}$
default	$\frac{c\,e^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+f(cf+2de)\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+e(2cf+$

input

```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-5/16*((a^2*(a*d-6/5*b*c)*f^2-12/5*a*(a*d-4/3*b*c)*b*e*f+8/5*b^2*e^2*(a*d-
2*b*c))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*(4/5*((2/3*x^2*
d+c)*x^2*f^2+4*e*(1/2*x^2*d+c)*f+2*d*e^2)*b^(5/2)+a*f*(2*((-1/3*x^2*d-3/5*
c)*f-6/5*d*e)*b^(3/2)+a*d*f*b^(1/2)))*x)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$

$$= \left[-\frac{3\left(8\left(2b^3c-ab^2d\right)e^2-4\left(4ab^2c-3a^2bd\right)ef+\left(6a^2bc-5a^3d\right)f^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-\right.}{}\right.$$

$$-\frac{3\left(8\left(2b^3c-ab^2d\right)e^2-4\left(4ab^2c-3a^2bd\right)ef+\left(6a^2bc-5a^3d\right)f^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8b^3df^2x^5\right.}{}$$

input

```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(8*(2*b^3*c - a*b^2*d)*e^2 - 4*(4*a*b^2*c - 3*a^2*b*d)*e*f + (6*
a^2*b*c - 5*a^3*d)*f^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(8*b^3*d*f^2*x^5 + 2*(12*b^3*d*e*f + (6*b^3*c - 5*a*b^2*d)*f^2)*
x^3 + 3*(8*b^3*d*e^2 + 4*(4*b^3*c - 3*a*b^2*d)*e*f - (6*a*b^2*c - 5*a^2*b*
d)*f^2)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(8*(2*b^3*c - a*b^2*d)*e^2 - 4*(
4*a*b^2*c - 3*a^2*b*d)*e*f + (6*a^2*b*c - 5*a^3*d)*f^2)*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*f^2*x^5 + 2*(12*b^3*d*e*f + (6*b^3*c
- 5*a*b^2*d)*f^2)*x^3 + 3*(8*b^3*d*e^2 + 4*(4*b^3*c - 3*a*b^2*d)*e*f - (6*
a*b^2*c - 5*a^2*b*d)*f^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{df^2x^5}{6b} + \frac{x^3\left(-\frac{5adf^2}{6b}+cf^2+2def\right)}{4b} + \frac{x\left(-\frac{3a\left(-\frac{5adf^2}{6b}+cf^2+2def\right)}{4b}+2cef+de^2\right)}{2b}\right) + \left(-\frac{a\left(-\frac{3a\left(-\frac{5adf^2}{6b}+cf^2+2def\right.}{4b}\right.}{2b}\right. \\ \frac{ce^2x+\frac{df^2x^7}{7}+\frac{x^5\left(cf^2+2def\right)}{5}+\frac{x^3\cdot\left(2cef+de^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**2/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f**2*x**5/(6*b) + x**3*(-5*a*d*f**2/(6*b) +
 c*f**2 + 2*d*e*f)/(4*b) + x*(-3*a*(-5*a*d*f**2/(6*b) + c*f**2 + 2*d*e*f)/
(4*b) + 2*c*e*f + d*e**2)/(2*b)) + (-a*(-3*a*(-5*a*d*f**2/(6*b) + c*f**2 +
 2*d*e*f)/(4*b) + 2*c*e*f + d*e**2)/(2*b) + c*e**2)*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), T
rue)), Ne(b, 0)), ((c*e**2*x + d*f**2*x**7/7 + x**5*(c*f**2 + 2*d*e*f)/5 +
 x**3*(2*c*e*f + d*e**2)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}df^2x^5}{6\,b} - \frac{5\sqrt{bx^2+a}adf^2x^3}{24\,b^2} + \frac{5\sqrt{bx^2+a}a^2df^2x}{16\,b^3} + \frac{(2\,def+cf^2)\sqrt{bx^2+a}x^3}{4\,b} + \frac{ce^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{5\,a^3df^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} - \frac{3\,(2\,def+cf^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(de^2+2\,cef)\sqrt{bx^2+a}x}{2\,b} + \frac{3\,(2\,def+cf^2)a^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(de^2+2\,cef)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*d*f^2*x^5/b - 5/24*sqrt(b*x^2 + a)*a*d*f^2*x^3/b^2 + 5
/16*sqrt(b*x^2 + a)*a^2*d*f^2*x/b^3 + 1/4*(2*d*e*f + c*f^2)*sqrt(b*x^2 + a
)*x^3/b + c*e^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/16*a^3*d*f^2*arcsinh(b*
x/sqrt(a*b))/b^(7/2) - 3/8*(2*d*e*f + c*f^2)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2
*(d*e^2 + 2*c*e*f)*sqrt(b*x^2 + a)*x/b + 3/8*(2*d*e*f + c*f^2)*a^2*arcsinh
(b*x/sqrt(a*b))/b^(5/2) - 1/2*(d*e^2 + 2*c*e*f)*a*arcsinh(b*x/sqrt(a*b))/b
^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,df^2x^2}{b} + \frac{12\,b^4def+6\,b^4cf^2-5\,ab^3df^2}{b^5}\right)x^2 + \frac{3\,(8\,b^4de^2+16\,b^4cef-12\,ab^3def-6\,ab^3cf^2+5\right. - \frac{(16\,b^3ce^2-8\,ab^2de^2-16\,ab^2cef+12\,a^2bdef+6\,a^2bcf^2-5\,a^3df^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*d*f^2*x^2/b + (12*b^4*d*e*f + 6*b^4*c*f^2 - 5*a*b^3*d*f^2)/b^5)
*x^2 + 3*(8*b^4*d*e^2 + 16*b^4*c*e*f - 12*a*b^3*d*e*f - 6*a*b^3*c*f^2 + 5*
a^2*b^2*d*f^2)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c*e^2 - 8*a*b^2*d*e^2
 - 16*a*b^2*c*e*f + 12*a^2*b*d*e*f + 6*a^2*b*c*f^2 - 5*a^3*d*f^2)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)\,\left(f\,x^2+e\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.81

$$\int \frac{(c+dx^2)(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2bd\,f^2x - 18\sqrt{b\,x^2+a}\,a\,b^2c\,f^2x - 36\sqrt{b\,x^2+a}\,a\,b^2def\,x - 10\sqrt{b\,x^2+a}\,a\,b^2d\,f^2x^3 + 48\sqrt{b}}{}$$

input
```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(1/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*d*f**2*x - 18*sqrt(a + b*x**2)*a*b**2*c*f**2*x
 - 36*sqrt(a + b*x**2)*a*b**2*d*e*f*x - 10*sqrt(a + b*x**2)*a*b**2*d*f**2*
x**3 + 48*sqrt(a + b*x**2)*b**3*c*e*f*x + 12*sqrt(a + b*x**2)*b**3*c*f**2*
x**3 + 24*sqrt(a + b*x**2)*b**3*d*e**2*x + 24*sqrt(a + b*x**2)*b**3*d*e*f*
x**3 + 8*sqrt(a + b*x**2)*b**3*d*f**2*x**5 - 15*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a**3*d*f**2 + 18*sqrt(b)*log((sqrt(a + b*x**2) +
 sqrt(b)*x)/sqrt(a))*a**2*b*c*f**2 + 36*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a**2*b*d*e*f - 48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a*b**2*c*e*f - 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)
/sqrt(a))*a*b**2*d*e**2 + 48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*b**3*c*e**2)/(48*b**4)
```

3.318 $\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx = -\frac{(3adf-4b(de+cf))x\sqrt{a+bx^2}}{8b^2} + \frac{dfx^3\sqrt{a+bx^2}}{4b} + \frac{(8b^2ce+3a^2df-4ab(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-1/8*(3*a*d*f-4*b*(c*f+d*e))*x*(b*x^2+a)^(1/2)/b^2+1/4*d*f*x^3*(b*x^2+a)^(
1/2)/b+1/8*(8*b^2*c*e+3*a^2*d*f-4*a*b*(c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(4bde+4bcf-3adf+2bdfx^2)}{8b^2} + \frac{(-8b^2ce+4abde+4abcf-3a^2df)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*b*d*e + 4*b*c*f - 3*a*d*f + 2*b*d*f*x^2))/(8*b^2) +
((-8*b^2*c*e + 4*a*b*d*e + 4*a*b*c*f - 3*a^2*d*f)*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\int \frac{(4bde+2bcf-3adf)x^2+c(4be-af)}{\sqrt{bx^2+a}}dx}{4b}+\frac{fx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{(3a^2df-4ab(cf+de)+8b^2ce)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}(-3adf+2bcf+4bde)}{2b}}{4b}+\frac{fx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{(3a^2df-4ab(cf+de)+8b^2ce)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}(-3adf+2bcf+4bde)}{2b}}{4b}+\frac{fx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2df-4ab(cf+de)+8b^2ce)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}(-3adf+2bcf+4bde)}{2b}}{4b}+\frac{fx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

input
```
Int[((c + d*x^2)*(e + f*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*b) + (((4*b*d*e + 2*b*c*f - 3*a*d*f)*
x*Sqrt[a + b*x^2])/(2*b) + ((8*b^2*c*e + 3*a^2*d*f - 4*a*b*(d*e + c*f))*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{x\left(-2bdf\,x^2+3adf-4bcf-4bde\right)\sqrt{b\,x^2+a}}{8b^2}+\frac{\left(3a^2df-4abcf-4abde+8ce\,b^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{3\left(a\left(af-\frac{4be}{3}\right)d-\frac{4bc(af-2be)}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}-\frac{3\sqrt{b\,x^2+a}\,x\left(\frac{2\left(\left(-f\,x^2-2e\right)d-2cf\right)b^{\frac{3}{2}}}{3}+adf\sqrt{b}\right)}{8}}{b^{\frac{5}{2}}}$
default	$\frac{ce\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+(cf+de)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+df\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2 \cdots}}{2b}\right.}{\cdots}\right.$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-2*b*d*f*x^2+3*a*d*f-4*b*c*f-4*b*d*e)/b^2*(b*x^2+a)^(1/2)+1/8*(3*a
^2*d*f-4*a*b*c*f-4*a*b*d*e+8*b^2*c*e)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$

$$=\left[\frac{(4\,(2\,b^2c-abd)e-(4\,abc-3\,a^2d)f)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2dfx^3+(4\,b^2de+(\cdots}{16\,b^3}\right.$$

$$-\frac{(4\,(2\,b^2c-abd)e-(4\,abc-3\,a^2d)f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2dfx^3+(4\,b^2de+(4\,b^2c-3\,abd)f)x\cdots}{8\,b^3}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((4*(2*b^2*c - a*b*d)*e - (4*a*b*c - 3*a^2*d)*f)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d*f*x^3 + (4*b^2*d*e + (4*
b^2*c - 3*a*b*d)*f)*x)*sqrt(b*x^2 + a))/b^3, -1/8*((4*(2*b^2*c - a*b*d)*e
- (4*a*b*c - 3*a^2*d)*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*
b^2*d*f*x^3 + (4*b^2*d*e + (4*b^2*c - 3*a*b*d)*f)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{dfx^3}{4b}+\frac{x\left(-\frac{3adf}{4b}+cf+de\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3adf}{4b}+cf+de\right)}{2b}+ce\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{f} \\ \frac{cex+\frac{dfx^5}{5}+\frac{x^3(cf+de)}{3}}{\sqrt{a}} & \text{o} \end{cases}$$

input
```
integrate((d*x**2+c)*(f*x**2+e)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d*f*x**3/(4*b) + x*(-3*a*d*f/(4*b) + c*f + d*
e)/(2*b)) + (-a*(-3*a*d*f/(4*b) + c*f + d*e)/(2*b) + c*e)*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True)), Ne(b, 0)), ((c*e*x + d*f*x**5/5 + x**3*(c*f + d*e)/3)/sqrt(a)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}\,adfx^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}\,adfx}{8\,b^2} + \frac{ce\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{3\,a^2df\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}(de+cf)x}{2\,b} - \frac{(de+cf)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*d*f*x^3/b - 3/8*sqrt(b*x^2 + a)*a*d*f*x/b^2 + c*e*arcs
inh(b*x/sqrt(a*b))/sqrt(b) + 3/8*a^2*d*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) +
1/2*sqrt(b*x^2 + a)*(d*e + c*f)*x/b - 1/2*(d*e + c*f)*a*arcsinh(b*x/sqrt(a
*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,dfx^2}{b}+\frac{4\,b^2de+4\,b^2cf-3\,abdf}{b^3}\right)x \\ &\quad - \frac{(8\,b^2ce-4\,abde-4\,abcf+3\,a^2df)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*d*f*x^2/b + (4*b^2*d*e + 4*b^2*c*f - 3*a*b*d*f)/b^3
)*x - 1/8*(8*b^2*c*e - 4*a*b*d*e - 4*a*b*c*f + 3*a^2*d*f)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx^2)(e+fx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,abdf\,x+4\sqrt{b\,x^2+a}\,b^2cf\,x+4\sqrt{b\,x^2+a}\,b^2dex+2\sqrt{b\,x^2+a}\,b^2df\,x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right.}{8b^3}$$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*d*f*x + 4*sqrt(a + b*x**2)*b**2*c*f*x + 4*sqrt(
a + b*x**2)*b**2*d*e*x + 2*sqrt(a + b*x**2)*b**2*d*f*x**3 + 3*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*f - 4*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*f - 4*sqrt(b)*log((sqrt(a + b*x**2) +
 sqrt(b)*x)/sqrt(a))*a*b*d*e + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*b**2*c*e)/(8*b**3)
```

3.319 $\int \frac{c+dx^2}{\sqrt{a+bx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx=\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}$$

output
```
d*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f-(-c*f+d*e)*arctanh((-a*f+b*
e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx=\frac{\frac{(de-cf)\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}\sqrt{-be+af}}-\frac{d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{f}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output
```
(((d*e - c*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[
e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*Sqrt[-(b*e) + a*f]) - (d*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/Sqrt[b])/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)}dx$$

$$\Big\downarrow \text{398}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}$$

input
```
Int[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTan
h[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*
f])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}-\frac{(cf-de)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{\sqrt{(af-be)e}}}{f}$
default	$\frac{d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{f\sqrt{b}}+\frac{(cf-de)\ln\left(\frac{\frac{2af-2be}{f}-\frac{2b\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{af-be}{f}}\sqrt{\left(x+\frac{\sqrt{-ef}}{f}\right)^{2}b-\frac{2b\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+a\cdots}}{x+\frac{\sqrt{-ef}}{f}}\cdots\right.}{2\sqrt{-ef}\,f\sqrt{\frac{af-be}{f}}}\cdots$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(d/b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(c*f-d*e)/((a*f-b*e)*e)^
(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(75) = 150.

Time = 0.61 (sec) , antiderivative size = 741, normalized size of antiderivative = 8.14

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$
$$=\left[\frac{2\,(bde^2-adef)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(bde-bcf)\sqrt{be^2-aef}\log\left(\frac{(8\,b^2e^2-8\,abef+a^2f^2\cdots}{\cdots}\right.}{4\,(b^2e^2f-abef^2)}\cdots\right.$$
$$-\frac{4\,(bde^2-adef)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(bde-bcf)\sqrt{be^2-aef}\log\left(\frac{(8\,b^2e^2-8\,abef+a^2f^2)x^4+a^2e^2+2\,(4\,abe^2-\cdots}{f^2x\cdots}\right.}{4\,(b^2e^2f-abef^2)}\cdots$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(2*(b*d*e^2 - a*d*e*f)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) - (b*d*e - b*c*f)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*
f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a
*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2
 + e^2)))/(b^2*e^2*f - a*b*e*f^2), -1/4*(4*(b*d*e^2 - a*d*e*f)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b*d*e - b*c*f)*sqrt(b*e^2 - a*e*f)*log
(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e
*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a
))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(b^2*e^2*f - a*b*e*f^2), 1/2*((b*d*e - b*
c*f)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x
^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x
)) + (b*d*e^2 - a*d*e*f)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a))/(b^2*e^2*f - a*b*e*f^2), 1/2*((b*d*e - b*c*f)*sqrt(-b*e^2 + a*e*f)
*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)
/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) - 2*(b*d*e^2 - a*d*e*f
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^2*e^2*f - a*b*e*f^2)]
```

Sympy [F]

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx=\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \int \frac{d\,x^2+c}{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.18

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)bcf+\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)bde}{}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*f + sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sq
rt(e)*sqrt(b)))*b*d*e - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sq
rt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*f + sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*d*e + sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a*d*e*f - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)
/sqrt(a))*b*d*e**2)/(b*e*f*(a*f - b*e))
```

3.320 $\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 115

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^2}\,dx=\frac{(de-cf)x\sqrt{a+bx^2}}{2e(be-af)\left(e+fx^2\right)}+\frac{(2bce-a(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{3/2}}$$

output
```
1/2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(f*x^2+e)+1/2*(2*b*c*e-a*(c*
f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+
b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^2}\,dx=\frac{\frac{\sqrt{e}(de-cf)x\sqrt{a+bx^2}}{(be-af)(e+fx^2)}+\frac{(2bce-a(de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}\left(e+fx^2\right)}{\sqrt{e}\sqrt{-be+af}}\right)}{(-be+af)^{3/2}}}{2e^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
((Sqrt[e]*(d*e - c*f)*x*Sqrt[a + b*x^2])/((b*e - a*f)*(e + f*x^2)) + ((2*b
*c*e - a*(d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2)
)/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(-(b*e) + a*f)^(3/2))/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{\int \frac{2bce-a(de+cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2bce-a(cf+de))\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(2bce-a(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}$$

input
```
Int[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*c*e
- a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(
2*e^(3/2)*(b*e - a*f)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{\frac{(cf-de)\sqrt{b\,x^{2}+a}\,x}{f\,x^{2}+e}-\frac{(acf+ade-2bce)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{\sqrt{(af-be)e}}}{2(af-be)e}$
default	$-\frac{(cf-de)\left(-\frac{f\sqrt{\left(x-\frac{\sqrt{-ef}}{f}\right)^{2}b+\frac{2b\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{af-be}{f}}}{(af-be)\left(x-\frac{\sqrt{-ef}}{f}\right)}+\frac{b\sqrt{-ef}\,\ln\left(\frac{\frac{2af-2be}{f}+\frac{2b\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{af-be}{f}}\sqrt{\left(x \cdots\right.}}{x-\frac{\sqrt{-e \cdots}}{f}}\right.}{(af-be)\sqrt{\frac{af-be \cdots}{f}}}\right.}{4e\,f^{2}}$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*f-b*e)/e*((c*f-d*e)*(b*x^2+a)^(1/2)*x/(f*x^2+e)-(a*c*f+a*d*e-2*b*c*
e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(99) = 198$.

Time = 1.13 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.53

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,(bde^3+acef^2-(bc+ad)e^2f)\sqrt{bx^2+a}x-(acef-(2\,bc-ad)e^2+(acf^2-(2\,bc-ad)ef)x^2)\sqrt{be^2 \cdots}}{8\,(b^2e^5-2\,abe^4f+a^2e^3f^2+(b^2e^4f-2 \cdots}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(b*x^2 + a)*x - (a*c
*e*f - (2*b*c - a*d)*e^2 + (a*c*f^2 - (2*b*c - a*d)*e*f)*x^2)*sqrt(b*e^2 -
 a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^
2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqr
t(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(b^2*e^5 - 2*a*b*e^4*f + a^2*e
^3*f^2 + (b^2*e^4*f - 2*a*b*e^3*f^2 + a^2*e^2*f^3)*x^2), 1/4*(2*(b*d*e^3 +
 a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(b*x^2 + a)*x + (a*c*e*f - (2*b*c - a*
d)*e^2 + (a*c*f^2 - (2*b*c - a*d)*e*f)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/
2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2
 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)))/(b^2*e^5 - 2*a*b*e^4*f + a^2*e^
3*f^2 + (b^2*e^4*f - 2*a*b*e^3*f^2 + a^2*e^2*f^3)*x^2)]
```

Sympy [F]

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(99) = 198$.

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.88

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$
$$= -\frac{\left(2\,b^{\frac{3}{2}}ce - a\sqrt{b}de - a\sqrt{b}cf\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,\sqrt{-b^2e^2+abef}\,(be^2-aef)}$$
$$+\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}de^2 - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}cef - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a\sqrt{b}def + \left(\sqrt{b}x - \right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 f + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 be - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 af\right.}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b^(3/2)*c*e - a*sqrt(b)*d*e - a*sqrt(b)*c*f)*arctan(1/2*((sqrt(b)*
x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b
^2*e^2 + a*b*e*f)*(b*e^2 - a*e*f)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^
(3/2)*d*e^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*e*f - (sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a*sqrt(b)*d*e*f + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*
sqrt(b)*c*f^2 + a^2*sqrt(b)*d*e*f - a^2*sqrt(b)*c*f^2)/(((sqrt(b)*x - sqrt
(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*f + a^2*f)*(b*e^2*f - a*e*f^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{d\,x^2+c}{\sqrt{b\,x^2+a}\,{(f\,x^2+e)}^2}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 813, normalized size of antiderivative = 7.07

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*e*f - sqrt(e)*sqrt(a*f - b*
e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(
sqrt(e)*sqrt(b)))*a*c*f**2*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
 b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a
*d*e**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
 b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*d*e*f*x**2 + 2*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*e**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*s
qrt(b)))*b*c*e*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sq
rt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*e*f - s
qrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*f**2*x**2 - sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a*d*e**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*d*e
*f*x**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a
 + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c*e**2 + 2*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f...
```

3.321 $\int \frac{c+dx^2}{\sqrt{a+bx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^3} dx = \frac{(de-cf)x\sqrt{a+bx^2}}{4e(be-af)\left(e+fx^2\right)^2}+\frac{(2be(de-3cf)+af(de+3cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2\left(e+fx^2\right)} + \frac{(8b^2ce^2-4abe(de+2cf)+a^2f(de+3cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{5/2}}$$

output
```
1/4*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(f*x^2+e)^2+1/8*(2*b*e*(-3*c
*f+d*e)+a*f*(3*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)^2/(f*x^2+e)+1/8*
(8*b^2*c*e^2-4*a*b*e*(2*c*f+d*e)+a^2*f*(3*c*f+d*e))*arctanh((-a*f+b*e)^(1/
2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.48

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^3}\,dx = \frac{x\left(-4de^2(be-af)\left(e+fx^2\right)\left(f\left(a+bx^2\right)-\frac{(2be-af)(e+fx^2)\text{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e\left(a+bx^2\right)}}\right)}{e\sqrt{\frac{(be-af)x^2}{e\left(a+bx^2\right)}}}\right)+(de-cf)\left(ef(a+\right.\right.}{8e^3f(be-af)^2\sqrt{a+bx}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
(x*(-4*d*e^2*(b*e - a*f)*(e + f*x^2)*(f*(a + b*x^2) - ((2*b*e - a*f)*(e +
f*x^2)*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a
*f)*x^2)/(e*(a + b*x^2))])) + (d*e - c*f)*(e*f*(a + b*x^2)*(2*b*e*(4*e + 3
*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*b^2*e^2 - 8*a*b*e*f + 3*a^2*f^2)*(e +
 f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a
*f)*x^2)/(e*(a + b*x^2))])))/(8*e^3*f*(b*e - a*f)^2*Sqrt[a + b*x^2]*(e + f
*x^2)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^3}\,dx$$

$$\Big\downarrow \text{402}$$

$$\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e(e+fx^2)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e(e+fx^2)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e(e+fx^2)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e(e+fx^2)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}$$

input
```
Int[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + ((((2*b*e
*(d*e - 3*c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e
 + f*x^2)) + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*
ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e -
a*f)^(3/2)))/(4*e*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-\frac{3\left(-\frac{4b(ad-2bc)e^2}{3}+\frac{fa(ad-8bc)e}{3}+a^2 c f^2\right)\left(f x^2+e\right)^2 \arctan\left(\frac{e\sqrt{b x^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\frac{5\sqrt{(af-be)e}\left(\frac{4bd e^3}{5}-\frac{\left(ad+8b\left(-\frac{x^2 d}{4}+c\right)\right)f e^2}{5}+\right.}{8}}{\sqrt{(af-be)e}\,(af-be)^2 e^2 (f x^2+e)^2}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)*(-3/5*(-4/3*b*(a*d-2*b*c)*e^2+1/3*f*a*(a*d-8*b*c)*
e+a^2*c*f^2)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+(
(a*f-b*e)*e)^(1/2)*(4/5*b*d*e^3-1/5*(a*d+8*b*(-1/4*x^2*d+c))*f*e^2+((1/5*x
^2*d+c)*a-6/5*x^2*b*c)*f^2*e+3/5*a*c*f^3*x^2)*(b*x^2+a)^(1/2)*x)/(a*f-b*e)
^2/e^2/(f*x^2+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(177) = 354$.

Time = 2.75 (sec) , antiderivative size = 1094, normalized size of antiderivative = 5.55

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/32*((3*a^2*c*e^2*f^2 + 4*(2*b^2*c - a*b*d)*e^4 - (8*a*b*c - a^2*d)*e^3*
f + (3*a^2*c*f^4 + 4*(2*b^2*c - a*b*d)*e^2*f^2 - (8*a*b*c - a^2*d)*e*f^3)*
x^4 + 2*(3*a^2*c*e*f^3 + 4*(2*b^2*c - a*b*d)*e^3*f - (8*a*b*c - a^2*d)*e^2
*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4
+ a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*
sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((2*
b^2*d*e^4*f - 3*a^2*c*e*f^4 - (6*b^2*c + a*b*d)*e^3*f^2 + (9*a*b*c - a^2*d
)*e^2*f^3)*x^3 + (4*b^2*d*e^5 - 5*a^2*c*e^2*f^3 - (8*b^2*c + 5*a*b*d)*e^4*
f + (13*a*b*c + a^2*d)*e^3*f^2)*x)*sqrt(b*x^2 + a))/(b^3*e^8 - 3*a*b^2*e^7
*f + 3*a^2*b*e^6*f^2 - a^3*e^5*f^3 + (b^3*e^6*f^2 - 3*a*b^2*e^5*f^3 + 3*a^
2*b*e^4*f^4 - a^3*e^3*f^5)*x^4 + 2*(b^3*e^7*f - 3*a*b^2*e^6*f^2 + 3*a^2*b*
e^5*f^3 - a^3*e^4*f^4)*x^2), -1/16*((3*a^2*c*e^2*f^2 + 4*(2*b^2*c - a*b*d)
*e^4 - (8*a*b*c - a^2*d)*e^3*f + (3*a^2*c*f^4 + 4*(2*b^2*c - a*b*d)*e^2*f^
2 - (8*a*b*c - a^2*d)*e*f^3)*x^4 + 2*(3*a^2*c*e*f^3 + 4*(2*b^2*c - a*b*d)*
e^3*f - (8*a*b*c - a^2*d)*e^2*f^2)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sq
rt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a
*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) - 2*((2*b^2*d*e^4*f - 3*a^2*c*e*f^4
- (6*b^2*c + a*b*d)*e^3*f^2 + (9*a*b*c - a^2*d)*e^2*f^3)*x^3 + (4*b^2*d*e^
5 - 5*a^2*c*e^2*f^3 - (8*b^2*c + 5*a*b*d)*e^4*f + (13*a*b*c + a^2*d)*e^3*f
^2)*x)*sqrt(b*x^2 + a))/(b^3*e^8 - 3*a*b^2*e^7*f + 3*a^2*b*e^6*f^2 - a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(177) = 354$.

Time = 0.37 (sec) , antiderivative size = 907, normalized size of antiderivative = 4.60

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*(8*b^(5/2)*c*e^2 - 4*a*b^(3/2)*d*e^2 - 8*a*b^(3/2)*c*e*f + a^2*sqrt(b
)*d*e*f + 3*a^2*sqrt(b)*c*f^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2
*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^4 - 2*a*b*e^3*f + a^2*
e^2*f^2)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*b^(5/2)*c*e^2*f^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^2*f^
2 - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^3 + (sqrt(b)*x - sqr
t(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^
2*sqrt(b)*c*f^4 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4 + 48*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f + 8*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a*b^(5/2)*d*e^3*f - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c
*e^2*f^2 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*e^2*f^2 + 42*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 - 3*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^3*sqrt(b)*d*e*f^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sq
rt(b)*c*f^4 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*d*e^3*f + 40*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^3*b^(3/2)*d*e^2*f^2 - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*a^3*b^(3/2)*c*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d*e*f
^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*c*f^4 - 2*a^4*b^(3/2)*d
*e^2*f^2 + 6*a^4*b^(3/2)*c*e*f^3 - a^5*sqrt(b)*d*e*f^3 - 3*a^5*sqrt(b)*c*f
^4)/((b^2*e^4*f - 2*a*b*e^3*f^2 + a^2*e^2*f^3)*((sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx=\int \frac{d\,x^2+c}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3591, normalized size of antiderivative = 18.23

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e**2*f**4 - 12*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c*e*f**5*x**2 - 6*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**3*c*f**6*x**4 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**3*d*e**3*f**3 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e
**2*f**4*x**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d*e*f**5*x**
4 + 28*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**3*f**3 + 56*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e**2*f**4*x**2 + 28*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*e*f**5*x**4 + 12*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**2*b*d*e**4*f**2 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e...
```

3.322 $\int \frac{c+dx^2}{\sqrt{a+bx^2}(e+fx^2)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 309

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \frac{(de-cf)x\sqrt{a+bx^2}}{6e(be-af)\,(e+fx^2)^3} + \frac{(2be(2de-5cf)+af(de+5cf))x\sqrt{a+bx^2}}{24e^2(be-af)^2\,(e+fx^2)^2} + \frac{(4b^2e^2(2de-11cf)-3a^2f^2(de+5cf)+2abef(5de+22cf))\,x\sqrt{a+bx^2}}{48e^3(be-af)^3\,(e+fx^2)} + \frac{(16b^3ce^3-8ab^2e^2(de+3cf)-a^3f^2(de+5cf)+2a^2bef(2de+9cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{16e^{7/2}(be-af)^{7/2}}$$

output

```
1/6*(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(f*x^2+e)^3+1/24*(2*b*e*(-5*
c*f+2*d*e)+a*f*(5*c*f+d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)^2/(f*x^2+e)^2
+1/48*(4*b^2*e^2*(-11*c*f+2*d*e)-3*a^2*f^2*(5*c*f+d*e)+2*a*b*e*f*(22*c*f+5
*d*e))*x*(b*x^2+a)^(1/2)/e^3/(-a*f+b*e)^3/(f*x^2+e)+1/16*(16*b^3*c*e^3-8*a
*b^2*e^2*(3*c*f+d*e)-a^3*f^2*(5*c*f+d*e)+2*a^2*b*e*f*(9*c*f+2*d*e))*arctan
h((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(7/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx$$

$$= \frac{x\left(-6de(be-af)\,(e+fx^2)\left(ef(a+bx^2)\,(2be(4e+3fx^2)-af(5e+3fx^2))-\frac{(8b^2e^2-8abef+3a^2f^2)\,(e+fx^2 \cdots}{\sqrt{\frac{(be- \cdots}{e(a+ \cdots}}}\right.\right.}{}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^4),x]
```

output
```
(x*(-6*d*e*(b*e - a*f)*(e + f*x^2)*(e*f*(a + b*x^2)*(2*b*e*(4*e + 3*f*x^2)
 - a*f*(5*e + 3*f*x^2)) - ((8*b^2*e^2 - 8*a*b*e*f + 3*a^2*f^2)*(e + f*x^2)
^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2
)/(e*(a + b*x^2))]) + (d*e - c*f)*(e*f*(a + b*x^2)*(4*b^2*e^2*(18*e^2 + 27
*e*f*x^2 + 11*f^2*x^4) + a^2*f^2*(33*e^2 + 40*e*f*x^2 + 15*f^2*x^4) - 2*a*
b*e*f*(45*e^2 + 59*e*f*x^2 + 22*f^2*x^4)) - (3*(2*b*e - a*f)*(8*b^2*e^2 -
8*a*b*e*f + 5*a^2*f^2)*(e + f*x^2)^3*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a
+ b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])))/(48*e^4*f*(b*e - a
*f)^3*Sqrt[a + b*x^2]*(e + f*x^2)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {402, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{\int \frac{4b(de-cf)x^2+6bce-ade-5acf}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\int \frac{3f(de+5cf)a^2-2be(4de+17cf)a+24b^2ce^2+2b(2be(2de-5cf)+af(de+5cf))x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(af(5cf+de)+2be(2de-5cf))}{4e(e+fx^2)^2(be-af)}}{6e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{3\left(-f^2(de+5cf)a^3+2bef(2de+9cf)a^2-8b^2e^2(de+3cf)a+16b^3ce^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(5cf+de)+2abef(22cf+5de)+4b^2e^2(2de-11cf)\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(}{4}}{6e(be-af)}$$
$$\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\left(a^3\left(-f^2\right)(5cf+de)+2a^2bef(9cf+2de)-8ab^2e^2(3cf+de)+16b^3ce^3\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(5cf+de)+2abef(22cf+5de)+4b^2e^2(2de-11c\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e(be-af)}$$
$$\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{3\left(a^3\left(-f^2\right)(5cf+de)+2a^2bef(9cf+2de)-8ab^2e^2(3cf+de)+16b^3ce^3\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(5cf+de)+2abef(22cf+5de)+4b^2e^2(2de\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{6e(be-af)}$$
$$\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{x\sqrt{a+bx^2}\left(-3a^2f^2(5cf+de)+2abef(22cf+5de)+4b^2e^2(2de-11cf)\right)}{2e\left(e+fx^2\right)(be-af)}+\frac{3\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^3\left(-f^2\right)(5cf+de)+2a^2bef(9cf+2de)-8ab^2e^2(3cf+de)+16}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}}{6e(be-af)}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{6e\left(e+fx^2\right)^3(be-af)}$$

input
```
Int[(c + d*x^2)/(Sqrt[a + b*x^2]*(e + f*x^2)^4),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(6*e*(b*e - a*f)*(e + f*x^2)^3) + (((2*b*e
*(2*d*e - 5*c*f) + a*f*(d*e + 5*c*f))*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*
(e + f*x^2)^2) + (((4*b^2*e^2*(2*d*e - 11*c*f) - 3*a^2*f^2*(d*e + 5*c*f) +
 2*a*b*e*f*(5*d*e + 22*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^
2)) + (3*(16*b^3*c*e^3 - 8*a*b^2*e^2*(d*e + 3*c*f) - a^3*f^2*(d*e + 5*c*f)
 + 2*a^2*b*e*f*(2*d*e + 9*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f)))/(6*e*(b*e
- a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{11\sqrt{(af-be)e}\,x\left(\left(-\frac{d\,e^{3}}{11}+f\left(\frac{8x^{2}d}{33}+c\right)e^{2}+\frac{40\left(\frac{3x^{2}d}{40}+c\right)x^{2}f^{2}e}{33}+\frac{5c\,f^{3}x^{4}}{11}\right)f^{2}a^{2}-\frac{30\left(-\frac{2d\,e^{3}}{15}+f\left(\frac{7x^{2}d}{45}+c\right)e^{2}+\frac{59\left(\frac{5x^{2}d}{59}+c\right)x^{2}f^{2}\cdots}{45}\cdots\right.}{11}\cdots\right.}{16\cdots}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

output
```
11/16/((a*f-b*e)*e)^(1/2)*(((a*f-b*e)*e)^(1/2)*x*((-1/11*d*e^3+f*(8/33*x^2
*d+c)*e^2+40/33*(3/40*x^2*d+c)*x^2*f^2*e+5/11*c*f^3*x^4)*f^2*a^2-30/11*(-2
/15*d*e^3+f*(7/45*x^2*d+c)*e^2+59/45*(5/59*x^2*d+c)*x^2*f^2*e+22/45*c*f^3*
x^4)*b*f*e*a+24/11*(-1/3*d*e^3+f*(-1/3*x^2*d+c)*e^2+3/2*(-2/27*x^2*d+c)*x^
2*f^2*e+11/18*c*f^3*x^4)*b^2*e^2)*(b*x^2+a)^(1/2)-5/11*arctan(e*(b*x^2+a)^
(1/2)/x/((a*f-b*e)*e)^(1/2))*(f*x^2+e)^3*(f^2*(c*f+1/5*d*e)*a^3-18/5*(c*f+
2/9*d*e)*b*f*e*a^2+24/5*(c*f+1/3*d*e)*b^2*e^2*a-16/5*b^3*c*e^3))/(f*x^2+e)
^3/(a*f-b*e)^3/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(285) = 570$.

Time = 17.68 (sec) , antiderivative size = 1936, normalized size of antiderivative = 6.27

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^4,x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*a^3*c*e^3*f^3 - 8*(2*b^3*c - a*b^2*d)*e^6 + 4*(6*a*b^2*c - a
^2*b*d)*e^5*f - (18*a^2*b*c - a^3*d)*e^4*f^2 + (5*a^3*c*f^6 - 8*(2*b^3*c -
 a*b^2*d)*e^3*f^3 + 4*(6*a*b^2*c - a^2*b*d)*e^2*f^4 - (18*a^2*b*c - a^3*d)
*e*f^5)*x^6 + 3*(5*a^3*c*e*f^5 - 8*(2*b^3*c - a*b^2*d)*e^4*f^2 + 4*(6*a*b^
2*c - a^2*b*d)*e^3*f^3 - (18*a^2*b*c - a^3*d)*e^2*f^4)*x^4 + 3*(5*a^3*c*e^
2*f^4 - 8*(2*b^3*c - a*b^2*d)*e^5*f + 4*(6*a*b^2*c - a^2*b*d)*e^4*f^2 - (1
8*a^2*b*c - a^3*d)*e^3*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a
*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*
e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*
f*x^2 + e^2)) - 4*((8*b^3*d*e^5*f^2 + 15*a^3*c*e*f^6 - 2*(22*b^3*c - a*b^2
*d)*e^4*f^3 + (88*a*b^2*c - 13*a^2*b*d)*e^3*f^4 - (59*a^2*b*c - 3*a^3*d)*e
^2*f^5)*x^5 + 2*(12*b^3*d*e^6*f + 20*a^3*c*e^2*f^5 - (54*b^3*c + 5*a*b^2*d
)*e^5*f^2 + (113*a*b^2*c - 11*a^2*b*d)*e^4*f^3 - (79*a^2*b*c - 4*a^3*d)*e^
3*f^4)*x^3 + 3*(8*b^3*d*e^7 + 11*a^3*c*e^3*f^4 - 12*(2*b^3*c + a*b^2*d)*e^
6*f + (54*a*b^2*c + 5*a^2*b*d)*e^5*f^2 - (41*a^2*b*c + a^3*d)*e^4*f^3)*x)*
sqrt(b*x^2 + a))/(b^4*e^11 - 4*a*b^3*e^10*f + 6*a^2*b^2*e^9*f^2 - 4*a^3*b*
e^8*f^3 + a^4*e^7*f^4 + (b^4*e^8*f^3 - 4*a*b^3*e^7*f^4 + 6*a^2*b^2*e^6*f^5
 - 4*a^3*b*e^5*f^6 + a^4*e^4*f^7)*x^6 + 3*(b^4*e^9*f^2 - 4*a*b^3*e^8*f^3 +
 6*a^2*b^2*e^7*f^4 - 4*a^3*b*e^6*f^5 + a^4*e^5*f^6)*x^4 + 3*(b^4*e^10*f -
4*a*b^3*e^9*f^2 + 6*a^2*b^2*e^8*f^3 - 4*a^3*b*e^7*f^4 + a^4*e^6*f^5)*x^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(1/2)/(f*x**2+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^4}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1896 vs. $2(285) = 570$.

Time $= 0.71$ (sec) , antiderivative size $= 1896$, normalized size of antiderivative $= 6.14$

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^4,x, algorithm="giac")
```

output
```
-1/16*(16*b^(7/2)*c*e^3 - 8*a*b^(5/2)*d*e^3 - 24*a*b^(5/2)*c*e^2*f + 4*a^2
*b^(3/2)*d*e^2*f + 18*a^2*b^(3/2)*c*e*f^2 - a^3*sqrt(b)*d*e*f^2 - 5*a^3*sq
rt(b)*c*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/
sqrt(-b^2*e^2 + a*b*e*f))/((b^3*e^6 - 3*a*b^2*e^5*f + 3*a^2*b*e^4*f^2 - a^
3*e^3*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/24*(48*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*b^(7/2)*c*e^3*f^3 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*d
*e^3*f^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c*e^2*f^4 + 12*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(3/2)*d*e^2*f^4 + 54*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*c*e*f^5 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a^3*sqrt(b)*d*e*f^5 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*c*f^
6 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c*e^4*f^2 - 240*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a*b^(7/2)*d*e^4*f^2 - 960*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*a*b^(7/2)*c*e^3*f^3 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2
)*d*e^3*f^3 + 900*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c*e^2*f^4 -
90*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*d*e^2*f^4 - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*e*f^5 + 15*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*a^4*sqrt(b)*d*e*f^5 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*c
*f^6 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*d*e^6 + 1408*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*b^(11/2)*c*e^5*f + 64*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*a*b^(9/2)*d*e^5*f - 3520*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx=\int \frac{d\,x^2+c}{\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^4}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^4),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(1/2)*(e + f*x^2)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 6329, normalized size of antiderivative = 20.48

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}\,(e+fx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(1/2)/(f*x^2+e)^4,x)
```

output
```
( - 15*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**3*f**5 - 45*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e**2*f**6*x**2 - 45*sqrt(e)*sqrt(a
*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(
b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e*f**7*x**4 - 15*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a**4*c*f**8*x**6 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
- b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*
a**4*d*e**4*f**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**3*f*
*5*x**2 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e**2*f**6*x**4 -
 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e*f**7*x**6 + 84*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**4*f**4 + 252*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**3*f**5*x**2 + 252*sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/...
```

3.323 $\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 533

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{(63a^4d^2f^3 - 70a^3bdf^2(3de+2cf) + 128b^4ce^2(2de+3cf) + 80a^2b^2f(3d^2e^2+6cdef+c^2f^2) - 96ab^3e(d^2}{256b^5}$$

$$- \frac{(63a^3d^2f^3 - 70a^2bdf^2(3de+2cf) + 80ab^2f(3d^2e^2+6cdef+c^2f^2) - 96b^3e(d^2e^2+6cdef+3c^2f^2))\,x}{384b^4}$$

$$+ \frac{f(63a^2d^2f^2 - 70abdf(3de+2cf) + 80b^2(3d^2e^2+6cdef+c^2f^2))\,x^5\sqrt{a+bx^2}}{480b^3}$$

$$+ \frac{df^2(30bde+20bcf-9adf)x^7\sqrt{a+bx^2}}{80b^2} + \frac{d^2f^3x^9\sqrt{a+bx^2}}{10b}$$

$$+ \frac{(256b^5c^2e^3 - 63a^5d^2f^3 + 70a^4bdf^2(3de+2cf) - 128ab^4ce^2(2de+3cf) - 80a^3b^2f(3d^2e^2+6cdef+c^2}{256b^{11/2}}$$

output
```
1/256*(63*a^4*d^2*f^3-70*a^3*b*d*f^2*(2*c*f+3*d*e)+128*b^4*c*e^2*(3*c*f+2*
d*e)+80*a^2*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-96*a*b^3*e*(3*c^2*f^2+6*c*
d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5-1/384*(63*a^3*d^2*f^3-70*a^2*b*d*f^2
*(2*c*f+3*d*e)+80*a*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-96*b^3*e*(3*c^2*f^
2+6*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(1/2)/b^4+1/480*f*(63*a^2*d^2*f^2-70*a
*b*d*f*(2*c*f+3*d*e)+80*b^2*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^5*(b*x^2+a)^(
1/2)/b^3+1/80*d*f^2*(-9*a*d*f+20*b*c*f+30*b*d*e)*x^7*(b*x^2+a)^(1/2)/b^2+1
/10*d^2*f^3*x^9*(b*x^2+a)^(1/2)/b+1/256*(256*b^5*c^2*e^3-63*a^5*d^2*f^3+70
*a^4*b*d*f^2*(2*c*f+3*d*e)-128*a*b^4*c*e^2*(3*c*f+2*d*e)-80*a^3*b^2*f*(c^2
*f^2+6*c*d*e*f+3*d^2*e^2)+96*a^2*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(945a^4d^2f^3-210a^3bdf^2(10cf+3d(5e+fx^2))+4a^2b^2f(300c^2f^2+50cdf(36e+7fx^2)+}{}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^4*d^2*f^3 - 210*a^3*b*d*f^2*(10*c*f + 3*
d*(5*e + f*x^2)) + 4*a^2*b^2*f*(300*c^2*f^2 + 50*c*d*f*(36*e + 7*f*x^2) +
3*d^2*(300*e^2 + 175*e*f*x^2 + 42*f^2*x^4)) + 32*b^4*(10*c^2*f*(18*e^2 + 9
*e*f*x^2 + 2*f^2*x^4) + 30*c*d*(4*e^3 + 6*e^2*f*x^2 + 4*e*f^2*x^4 + f^3*x^
6) + 3*d^2*x^2*(10*e^3 + 20*e^2*f*x^2 + 15*e*f^2*x^4 + 4*f^3*x^6)) - 16*a*
b^3*(10*c^2*f^2*(27*e + 5*f*x^2) + 10*c*d*f*(54*e^2 + 30*e*f*x^2 + 7*f^2*x
^4) + 3*d^2*(30*e^3 + 50*e^2*f*x^2 + 35*e*f^2*x^4 + 9*f^3*x^6))) - 15*(256
*b^5*c^2*e^3 - 63*a^5*d^2*f^3 + 70*a^4*b*d*f^2*(3*d*e + 2*c*f) - 128*a*b^4
*c*e^2*(2*d*e + 3*c*f) - 80*a^3*b^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) +
96*a^2*b^3*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(3840*b^(11/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{fx^6(c^2f^2+6cdef+3d^2e^2)}{\sqrt{a+bx^2}}+\frac{ex^4(3c^2f^2+6cdef+d^2e^2)}{\sqrt{a+bx^2}}+\frac{c^2e^3}{\sqrt{a+bx^2}}+\frac{ce^2x^2(3cf+2de)}{\sqrt{a+bx^2}}+\frac{df^2x^8(2cf+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2f^3\sqrt{bx^2+a}x^9}{10b}-\frac{9ad^2f^3\sqrt{bx^2+a}x^7}{80b^2}+\frac{df^2(3de+2cf)\sqrt{bx^2+a}x^7}{8b}+\frac{21a^2d^2f^3\sqrt{bx^2+a}x^5}{160b^3}-\frac{7adf^2(3de+2cf)\sqrt{bx^2+a}x^5}{48b^2}+\frac{f(3d^2e^2+6cdfe+c^2f^2)\sqrt{bx^2+a}x^5}{6b}-\frac{21a^3d^2f^3\sqrt{bx^2+a}x^3}{128b^4}+\frac{35a^2df^2(3de+2cf)\sqrt{bx^2+a}x^3}{192b^3}-\frac{5af(3d^2e^2+6cdfe+c^2f^2)\sqrt{bx^2+a}x^3}{24b^2}+\frac{e(d^2e^2+6cdfe+3c^2f^2)\sqrt{bx^2+a}x^3}{4b}+\frac{63a^4d^2f^3\sqrt{bx^2+a}x}{256b^5}-\frac{35a^3df^2(3de+2cf)\sqrt{bx^2+a}x}{128b^4}+\frac{ce^2(2de+3cf)\sqrt{bx^2+a}x}{2b}+\frac{5a^2f(3d^2e^2+6cdfe+c^2f^2)\sqrt{bx^2+a}x}{16b^3}-\frac{3ae(d^2e^2+6cdfe+3c^2f^2)\sqrt{bx^2+a}x}{8b^2}+\frac{c^2e^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}}-\frac{63a^5d^2f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{256b^{11/2}}+\frac{35a^4df^2(3de+2cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{9/2}}-\frac{ace^2(2de+3cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{3/2}}-\frac{5a^3f(3d^2e^2+6cdfe+c^2f^2)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{7/2}}+\frac{3a^2e(d^2e^2+6cdfe+3c^2f^2)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{5/2}}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^3)/Sqrt[a + b*x^2],x]
```

output
```
(63*a^4*d^2*f^3*x*Sqrt[a + b*x^2])/(256*b^5) - (35*a^3*d*f^2*(3*d*e + 2*c*
f)*x*Sqrt[a + b*x^2])/(128*b^4) + (c*e^2*(2*d*e + 3*c*f)*x*Sqrt[a + b*x^2]
)/(2*b) + (5*a^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x*Sqrt[a + b*x^2])/(1
6*b^3) - (3*a*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x*Sqrt[a + b*x^2])/(8*b^
2) - (21*a^3*d^2*f^3*x^3*Sqrt[a + b*x^2])/(128*b^4) + (35*a^2*d*f^2*(3*d*e
 + 2*c*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) - (5*a*f*(3*d^2*e^2 + 6*c*d*e*f +
 c^2*f^2)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*
f^2)*x^3*Sqrt[a + b*x^2])/(4*b) + (21*a^2*d^2*f^3*x^5*Sqrt[a + b*x^2])/(16
0*b^3) - (7*a*d*f^2*(3*d*e + 2*c*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*(3*
d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5*Sqrt[a + b*x^2])/(6*b) - (9*a*d^2*f^3*x
^7*Sqrt[a + b*x^2])/(80*b^2) + (d*f^2*(3*d*e + 2*c*f)*x^7*Sqrt[a + b*x^2])
/(8*b) + (d^2*f^3*x^9*Sqrt[a + b*x^2])/(10*b) + (c^2*e^3*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/Sqrt[b] - (63*a^5*d^2*f^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(256*b^(11/2)) + (35*a^4*d*f^2*(3*d*e + 2*c*f)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(9/2)) - (a*c*e^2*(2*d*e + 3*c*f)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)) - (5*a^3*f*(3*d^2*e^2 + 6*c*d*e*f +
 c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2)) + (3*a^2*e*(d
^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b
^(5/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{63\left(\left(a^3\left(a^2d^2+\frac{80}{63}b^2c^2-\frac{20}{9}abcd\right)f^3-\frac{10a^2b\left(a^2d^2-\frac{16}{7}abcd+\frac{48}{35}b^2c^2\right)ef^2}{3}+\frac{80ab^2e^2\left(a^2d^2-\frac{12}{5}abcd+\frac{8}{5}b^2c^2\right)f}{21}-\frac{32\left(a^2d^2-\frac{8}{3}c\right.}{}\right.\right.}{}$
default	$\frac{c^2e^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+f^2d(2cf+3de)\left(\frac{x^7\sqrt{b\,x^2+a}}{8b}-\frac{7a\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\right.}{}\right)}{6b}\right)}{8b}\right.$
risch	$\frac{x\left(384d^2f^3b^4x^8-432a\,b^3d^2f^3x^6+960b^4cd\,f^3x^6+1440b^4d^2e\,f^2x^6+504a^2b^2d^2f^3x^4-1120a\,b^3cd\,f^3x^4-1680a\,b^3d^2e\,f^2x^4+\right.}{}$

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-63/256/b^(11/2)*((a^3*(a^2*d^2+80/63*b^2*c^2-20/9*a*b*c*d)*f^3-10/3*a^2*b
*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*e*f^2+80/21*a*b^2*e^2*(a^2*d^2-12/5*
a*b*c*d+8/5*b^2*c^2)*f-32/21*(a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2)*b^3*e^3)*ar
ctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*(64/21*(2/9*x^4*(3/5*d^2*
x^4+3/2*c*d*x^2+c^2)*f^3+(1/2*d^2*x^4+4/3*c*d*x^2+c^2)*x^2*e*f^2+2*e^2*(1/
3*d^2*x^4+c*d*x^2+c^2)*f+4/3*d*(1/4*x^2*d+c)*e^3)*b^(9/2)+a*(32/7*(-5/27*(
27/50*d^2*x^4+7/5*c*d*x^2+c^2)*x^2*f^3-(7/18*d^2*x^4+10/9*c*d*x^2+c^2)*e*f
^2-2*d*(5/18*x^2*d+c)*e^2*f-1/3*d^2*e^3)*b^(7/2)+a*f*(8/3*((5/9*c*d*x^2+10
/21*c^2+1/5*d^2*x^4)*f^2+20/7*(7/24*x^2*d+c)*d*e*f+10/7*d^2*e^2)*b^(5/2)+(
2/3*((-10/3*c-x^2*d)*f-5*d*e)*b^(3/2)+a*d*f*b^(1/2))*a*d*f)))*x)
```

Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1110, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{\sqrt{a+bx^2}} dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output

```
[-1/7680*(15*(32*(8*b^5*c^2 - 8*a*b^4*c*d + 3*a^2*b^3*d^2)*e^3 - 48*(8*a*b
^4*c^2 - 12*a^2*b^3*c*d + 5*a^3*b^2*d^2)*e^2*f + 6*(48*a^2*b^3*c^2 - 80*a^
3*b^2*c*d + 35*a^4*b*d^2)*e*f^2 - (80*a^3*b^2*c^2 - 140*a^4*b*c*d + 63*a^5
*d^2)*f^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(38
4*b^5*d^2*f^3*x^9 + 48*(30*b^5*d^2*e*f^2 + (20*b^5*c*d - 9*a*b^4*d^2)*f^3)
*x^7 + 8*(240*b^5*d^2*e^2*f + 30*(16*b^5*c*d - 7*a*b^4*d^2)*e*f^2 + (80*b^
5*c^2 - 140*a*b^4*c*d + 63*a^2*b^3*d^2)*f^3)*x^5 + 10*(96*b^5*d^2*e^3 + 48
*(12*b^5*c*d - 5*a*b^4*d^2)*e^2*f + 6*(48*b^5*c^2 - 80*a*b^4*c*d + 35*a^2*
b^3*d^2)*e*f^2 - (80*a*b^4*c^2 - 140*a^2*b^3*c*d + 63*a^3*b^2*d^2)*f^3)*x^
3 + 15*(32*(8*b^5*c*d - 3*a*b^4*d^2)*e^3 + 48*(8*b^5*c^2 - 12*a*b^4*c*d +
5*a^2*b^3*d^2)*e^2*f - 6*(48*a*b^4*c^2 - 80*a^2*b^3*c*d + 35*a^3*b^2*d^2)*
e*f^2 + (80*a^2*b^3*c^2 - 140*a^3*b^2*c*d + 63*a^4*b*d^2)*f^3)*x)*sqrt(b*x
^2 + a))/b^6, -1/3840*(15*(32*(8*b^5*c^2 - 8*a*b^4*c*d + 3*a^2*b^3*d^2)*e^
3 - 48*(8*a*b^4*c^2 - 12*a^2*b^3*c*d + 5*a^3*b^2*d^2)*e^2*f + 6*(48*a^2*b^
3*c^2 - 80*a^3*b^2*c*d + 35*a^4*b*d^2)*e*f^2 - (80*a^3*b^2*c^2 - 140*a^4*b
*c*d + 63*a^5*d^2)*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384
*b^5*d^2*f^3*x^9 + 48*(30*b^5*d^2*e*f^2 + (20*b^5*c*d - 9*a*b^4*d^2)*f^3)*
x^7 + 8*(240*b^5*d^2*e^2*f + 30*(16*b^5*c*d - 7*a*b^4*d^2)*e*f^2 + (80*b^5
*c^2 - 140*a*b^4*c*d + 63*a^2*b^3*d^2)*f^3)*x^5 + 10*(96*b^5*d^2*e^3 + 48*
(12*b^5*c*d - 5*a*b^4*d^2)*e^2*f + 6*(48*b^5*c^2 - 80*a*b^4*c*d + 35*a^...
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2f^3x^9}{10b} + \frac{x^7\left(-\frac{9ad^2f^3}{10b}+2cdf^3+3d^2ef^2\right)}{8b} + \frac{x^5\left(-\frac{7a\left(-\frac{9ad^2f^3}{10b}+2cdf^3+3d^2ef^2\right)}{8b}+c^2f^3+6cdef^2+3d^2e^2f\right)}{6b} + \cdots \right. \\ \frac{c^2e^3x+\frac{d^2f^3x^{11}}{11}+\frac{x^9\cdot\left(2cdf^3+3d^2ef^2\right)}{9}+\frac{x^7\left(c^2f^3+6cdef^2+3d^2e^2f\right)}{7}+\frac{x^5\cdot\left(3c^2ef^2+6cde^2f+d^2e^3\right)}{5}+\frac{x^3\cdot\left(3c^2e^2f+2cde^3\right)}{3}}{\sqrt{a}} \end{cases}$$

input

```
integrate((d*x**2+c)**2*(f*x**2+e)**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*f**3*x**9/(10*b) + x**7*(-9*a*d**2*f**3/
(10*b) + 2*c*d*f**3 + 3*d**2*e*f**2)/(8*b) + x**5*(-7*a*(-9*a*d**2*f**3/(1
0*b) + 2*c*d*f**3 + 3*d**2*e*f**2)/(8*b) + c**2*f**3 + 6*c*d*e*f**2 + 3*d*
*2*e**2*f)/(6*b) + x**3*(-5*a*(-7*a*(-9*a*d**2*f**3/(10*b) + 2*c*d*f**3 +
3*d**2*e*f**2)/(8*b) + c**2*f**3 + 6*c*d*e*f**2 + 3*d**2*e**2*f)/(6*b) + 3
*c**2*e*f**2 + 6*c*d*e**2*f + d**2*e**3)/(4*b) + x*(-3*a*(-5*a*(-7*a*(-9*a
*d**2*f**3/(10*b) + 2*c*d*f**3 + 3*d**2*e*f**2)/(8*b) + c**2*f**3 + 6*c*d*
e*f**2 + 3*d**2*e**2*f)/(6*b) + 3*c**2*e*f**2 + 6*c*d*e**2*f + d**2*e**3)/
(4*b) + 3*c**2*e**2*f + 2*c*d*e**3)/(2*b)) + (-a*(-3*a*(-5*a*(-7*a*(-9*a*d
**2*f**3/(10*b) + 2*c*d*f**3 + 3*d**2*e*f**2)/(8*b) + c**2*f**3 + 6*c*d*e*
f**2 + 3*d**2*e**2*f)/(6*b) + 3*c**2*e*f**2 + 6*c*d*e**2*f + d**2*e**3)/(4
*b) + 3*c**2*e**2*f + 2*c*d*e**3)/(2*b) + c**2*e**3)*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
True)), Ne(b, 0)), ((c**2*e**3*x + d**2*f**3*x**11/11 + x**9*(2*c*d*f**3 +
 3*d**2*e*f**2)/9 + x**7*(c**2*f**3 + 6*c*d*e*f**2 + 3*d**2*e**2*f)/7 + x*
*5*(3*c**2*e*f**2 + 6*c*d*e**2*f + d**2*e**3)/5 + x**3*(3*c**2*e**2*f + 2*
c*d*e**3)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{\sqrt{a+bx^2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*sqrt(b*x^2 + a)*d^2*f^3*x^9/b - 9/80*sqrt(b*x^2 + a)*a*d^2*f^3*x^7/b^
2 + 21/160*sqrt(b*x^2 + a)*a^2*d^2*f^3*x^5/b^3 - 21/128*sqrt(b*x^2 + a)*a^
3*d^2*f^3*x^3/b^4 + 1/8*(3*d^2*e*f^2 + 2*c*d*f^3)*sqrt(b*x^2 + a)*x^7/b +
63/256*sqrt(b*x^2 + a)*a^4*d^2*f^3*x/b^5 - 7/48*(3*d^2*e*f^2 + 2*c*d*f^3)*
sqrt(b*x^2 + a)*a*x^5/b^2 + 1/6*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*sqrt
(b*x^2 + a)*x^5/b + c^2*e^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 63/256*a^5*d^
2*f^3*arcsinh(b*x/sqrt(a*b))/b^(11/2) + 35/192*(3*d^2*e*f^2 + 2*c*d*f^3)*s
qrt(b*x^2 + a)*a^2*x^3/b^3 - 5/24*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*sq
rt(b*x^2 + a)*a*x^3/b^2 + 1/4*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*sqrt(b
*x^2 + a)*x^3/b - 35/128*(3*d^2*e*f^2 + 2*c*d*f^3)*sqrt(b*x^2 + a)*a^3*x/b
^4 + 5/16*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*sqrt(b*x^2 + a)*a^2*x/b^3
- 3/8*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2*
(2*c*d*e^3 + 3*c^2*e^2*f)*sqrt(b*x^2 + a)*x/b + 35/128*(3*d^2*e*f^2 + 2*c*
d*f^3)*a^4*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/16*(3*d^2*e^2*f + 6*c*d*e*f^
2 + c^2*f^3)*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*(d^2*e^3 + 6*c*d*e^2
*f + 3*c^2*e*f^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*(2*c*d*e^3 + 3*
c^2*e^2*f)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2\,(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,d^2f^3x^2}{b}+\frac{30\,b^8d^2ef^2+20\,b^8cdf^3-9\,ab^7d^2f^3}{b^9}\right)x^2+\frac{240\,b^8d^2e^2f+480\,b^8cdef^2-210\right.\right.\right.$$
$$-\frac{(256\,b^5c^2e^3-256\,ab^4cde^3+96\,a^2b^3d^2e^3-384\,ab^4c^2e^2f+576\,a^2b^3cde^2f-240\,a^3b^2d^2e^2f+288\,a^2b^3c^2}{25}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*d^2*f^3*x^2/b + (30*b^8*d^2*e*f^2 + 20*b^8*c*d*f^3 - 9*
a*b^7*d^2*f^3)/b^9)*x^2 + (240*b^8*d^2*e^2*f + 480*b^8*c*d*e*f^2 - 210*a*b
^7*d^2*e*f^2 + 80*b^8*c^2*f^3 - 140*a*b^7*c*d*f^3 + 63*a^2*b^6*d^2*f^3)/b^
9)*x^2 + 5*(96*b^8*d^2*e^3 + 576*b^8*c*d*e^2*f - 240*a*b^7*d^2*e^2*f + 288
*b^8*c^2*e*f^2 - 480*a*b^7*c*d*e*f^2 + 210*a^2*b^6*d^2*e*f^2 - 80*a*b^7*c^
2*f^3 + 140*a^2*b^6*c*d*f^3 - 63*a^3*b^5*d^2*f^3)/b^9)*x^2 + 15*(256*b^8*c
*d*e^3 - 96*a*b^7*d^2*e^3 + 384*b^8*c^2*e^2*f - 576*a*b^7*c*d*e^2*f + 240*
a^2*b^6*d^2*e^2*f - 288*a*b^7*c^2*e*f^2 + 480*a^2*b^6*c*d*e*f^2 - 210*a^3*
b^5*d^2*e*f^2 + 80*a^2*b^6*c^2*f^3 - 140*a^3*b^5*c*d*f^3 + 63*a^4*b^4*d^2*
f^3)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(256*b^5*c^2*e^3 - 256*a*b^4*c*d*e^3 +
 96*a^2*b^3*d^2*e^3 - 384*a*b^4*c^2*e^2*f + 576*a^2*b^3*c*d*e^2*f - 240*a^
3*b^2*d^2*e^2*f + 288*a^2*b^3*c^2*e*f^2 - 480*a^3*b^2*c*d*e*f^2 + 210*a^4*
b*d^2*e*f^2 - 80*a^3*b^2*c^2*f^3 + 140*a^4*b*c*d*f^3 - 63*a^5*d^2*f^3)*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx=\int \frac{{\left(d\,x^2+c\right)}^2\,{\left(f\,x^2+e\right)}^3}{\sqrt{b\,x^2+a}} \,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{\sqrt{a+bx^2}}\,dx=\int \frac{\left(d\,x^{2}+c\right)^{2} \left(f\,x^{2}+e\right)^{3}}{\sqrt{b\,x^{2}+a}}d x$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(1/2),x)
```

3.324 $\int \frac{(c+dx^2)^2(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 336

$$\int \frac{(c+dx^2)^2(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = -\frac{(35a^3d^2f^2-128b^3ce(de+cf)-80a^2bdf(de+cf)+48ab^2(d^2e^2+4cdef+c^2f^2))\,x\sqrt{a+bx^2}}{128b^4} + \frac{(35a^2d^2f^2-80abdf(de+cf)+48b^2(d^2e^2+4cdef+c^2f^2))\,x^3\sqrt{a+bx^2}}{192b^3} - \frac{df(7adf-16b(de+cf))x^5\sqrt{a+bx^2}}{48b^2} + \frac{d^2f^2x^7\sqrt{a+bx^2}}{8b} + \frac{(128b^4c^2e^2+35a^4d^2f^2-128ab^3ce(de+cf)-80a^3bdf(de+cf)+48a^2b^2(d^2e^2+4cdef+c^2f^2))\,\text{arcta}}{128b^{9/2}}$$

output
```
-1/128*(35*a^3*d^2*f^2-128*b^3*c*e*(c*f+d*e)-80*a^2*b*d*f*(c*f+d*e)+48*a*b
^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^4+1/192*(35*a^2*d^2*f^
2-80*a*b*d*f*(c*f+d*e)+48*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(
1/2)/b^3-1/48*d*f*(7*a*d*f-16*b*(c*f+d*e))*x^5*(b*x^2+a)^(1/2)/b^2+1/8*d^2
*f^2*x^7*(b*x^2+a)^(1/2)/b+1/128*(128*b^4*c^2*e^2+35*a^4*d^2*f^2-128*a*b^3
*c*e*(c*f+d*e)-80*a^3*b*d*f*(c*f+d*e)+48*a^2*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^
2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}} dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^3d^2f^2+10a^2bdf(24de+24cf+7dfx^2)+16b^3(6c^2f(4e+fx^2)+8cd(3e^2+3efx^2}{}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*d^2*f^2 + 10*a^2*b*d*f*(24*d*e + 24*c
*f + 7*d*f*x^2) + 16*b^3*(6*c^2*f*(4*e + f*x^2) + 8*c*d*(3*e^2 + 3*e*f*x^2
 + f^2*x^4) + d^2*x^2*(6*e^2 + 8*e*f*x^2 + 3*f^2*x^4)) - 8*a*b^2*(18*c^2*f
^2 + 4*c*d*f*(18*e + 5*f*x^2) + d^2*(18*e^2 + 20*e*f*x^2 + 7*f^2*x^4))) -
3*(128*b^4*c^2*e^2 + 35*a^4*d^2*f^2 - 128*a*b^3*c*e*(d*e + c*f) - 80*a^3*b
*d*f*(d*e + c*f) + 48*a^2*b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(384*b^(9/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}} dx$$
$$\Big\downarrow \text{ 433}$$
$$\int \left(\frac{x^4(c^2f^2+4cdef+d^2e^2)}{\sqrt{a+bx^2}}+\frac{c^2e^2}{\sqrt{a+bx^2}}+\frac{2cex^2(cf+de)}{\sqrt{a+bx^2}}+\frac{2dfx^6(cf+de)}{\sqrt{a+bx^2}}+\frac{d^2f^2x^8}{\sqrt{a+bx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{35a^4d^2f^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}} - \frac{5a^3df\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{8b^{7/2}} - \frac{35a^3d^2f^2x\sqrt{a+bx^2}}{128b^4} + \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(c^2f^2+4cdef+d^2e^2)}{8b^{5/2}} + \frac{5a^2dfx\sqrt{a+bx^2}(cf+de)}{8b^3} + \frac{35a^2d^2f^2x^3\sqrt{a+bx^2}}{192b^3} - \frac{ace\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{b^{3/2}} + \frac{c^2e^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{3ax\sqrt{a+bx^2}(c^2f^2+4cdef+d^2e^2)}{8b^2} - \frac{5adfx^3\sqrt{a+bx^2}(cf+de)}{12b^2} - \frac{7ad^2f^2x^5\sqrt{a+bx^2}}{48b^2} + \frac{x^3\sqrt{a+bx^2}(c^2f^2+4cdef+d^2e^2)}{4b} + \frac{cex\sqrt{a+bx^2}(cf+de)}{b} + \frac{dfx^5\sqrt{a+bx^2}(cf+de)}{3b} + \frac{d^2f^2x^7\sqrt{a+bx^2}}{8b}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^2)/Sqrt[a + b*x^2],x]
```

output
```
(-35*a^3*d^2*f^2*x*Sqrt[a + b*x^2])/(128*b^4) + (c*e*(d*e + c*f)*x*Sqrt[a
+ b*x^2])/b + (5*a^2*d*f*(d*e + c*f)*x*Sqrt[a + b*x^2])/(8*b^3) - (3*a*(d^
2*e^2 + 4*c*d*e*f + c^2*f^2)*x*Sqrt[a + b*x^2])/(8*b^2) + (35*a^2*d^2*f^2*
x^3*Sqrt[a + b*x^2])/(192*b^3) - (5*a*d*f*(d*e + c*f)*x^3*Sqrt[a + b*x^2])
/(12*b^2) + ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(4*b) -
(7*a*d^2*f^2*x^5*Sqrt[a + b*x^2])/(48*b^2) + (d*f*(d*e + c*f)*x^5*Sqrt[a +
 b*x^2])/(3*b) + (d^2*f^2*x^7*Sqrt[a + b*x^2])/(8*b) + (c^2*e^2*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] + (35*a^4*d^2*f^2*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(128*b^(9/2)) - (a*c*e*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(3/2) - (5*a^3*d*f*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(8*b^(7/2)) + (3*a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{35\left(a^{2}\left(a^{2}f^{2}+\frac{48}{35}b^{2}e^{2}-\frac{16}{7}abfe\right)d^{2}-\frac{16\left(a^{2}f^{2}-\frac{12}{5}abfe+\frac{8}{5}b^{2}e^{2}\right)acbd}{7}+\frac{48c^{2}\left(a^{2}f^{2}-\frac{8}{3}abfe+\frac{8}{3}b^{2}e^{2}\right)b^{2}}{35}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\frac{35\sqrt{b}}{}}{128}$
risch	$-\frac{x\left(-48b^{3}d^{2}f^{2}x^{6}+56a\,b^{2}d^{2}f^{2}x^{4}-128b^{3}cd\,f^{2}x^{4}-128b^{3}d^{2}ef\,x^{4}-70a^{2}b\,x^{2}d^{2}f^{2}+160a\,b^{2}cd\,f^{2}x^{2}+160a\,b^{2}d^{2}ef\,x^{2}-96b^{3}c\right.}{}$
default	$\frac{c^{2}e^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+2df(cf+de)\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right.$

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/128*((a^2*(a^2*f^2+48/35*b^2*e^2-16/7*a*b*f*e)*d^2-16/7*(a^2*f^2-12/5*a
*b*f*e+8/5*b^2*e^2)*a*c*b*d+48/35*c^2*(a^2*f^2-8/3*a*b*f*e+8/3*b^2*e^2)*b^
2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^(1/2)*((-32/35*(1/2*f^2*x^
4+4/3*e*f*x^2+e^2)*x^2*d^2-128/35*(1/3*f^2*x^4+e*f*x^2+e^2)*c*d-128/35*(1/
4*f*x^2+e)*c^2*f)*b^(7/2)+a*(((8/15*f^2*x^4+32/21*e*f*x^2+48/35*e^2)*d^2+1
92/35*c*(5/18*f*x^2+e)*f*d+48/35*c^2*f^2)*b^(5/2)+a*d*(((-2/3*f*x^2-16/7*e
)*d-16/7*c*f)*b^(3/2)+a*d*f*b^(1/2))*f))*x)/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(8*b^4*c^2 - 8*a*b^3*c*d + 3*a^2*b^2*d^2)*e^2 - 16*(8*a*b^3*
c^2 - 12*a^2*b^2*c*d + 5*a^3*b*d^2)*e*f + (48*a^2*b^2*c^2 - 80*a^3*b*c*d +
 35*a^4*d^2)*f^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(48*b^4*d^2*f^2*x^7 + 8*(16*b^4*d^2*e*f + (16*b^4*c*d - 7*a*b^3*d^2)*f
^2)*x^5 + 2*(48*b^4*d^2*e^2 + 16*(12*b^4*c*d - 5*a*b^3*d^2)*e*f + (48*b^4*
c^2 - 80*a*b^3*c*d + 35*a^2*b^2*d^2)*f^2)*x^3 + 3*(16*(8*b^4*c*d - 3*a*b^3
*d^2)*e^2 + 16*(8*b^4*c^2 - 12*a*b^3*c*d + 5*a^2*b^2*d^2)*e*f - (48*a*b^3*
c^2 - 80*a^2*b^2*c*d + 35*a^3*b*d^2)*f^2)*x)*sqrt(b*x^2 + a))/b^5, -1/384*
(3*(16*(8*b^4*c^2 - 8*a*b^3*c*d + 3*a^2*b^2*d^2)*e^2 - 16*(8*a*b^3*c^2 - 1
2*a^2*b^2*c*d + 5*a^3*b*d^2)*e*f + (48*a^2*b^2*c^2 - 80*a^3*b*c*d + 35*a^4
*d^2)*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*b^4*d^2*f^2*x
^7 + 8*(16*b^4*d^2*e*f + (16*b^4*c*d - 7*a*b^3*d^2)*f^2)*x^5 + 2*(48*b^4*d
^2*e^2 + 16*(12*b^4*c*d - 5*a*b^3*d^2)*e*f + (48*b^4*c^2 - 80*a*b^3*c*d +
35*a^2*b^2*d^2)*f^2)*x^3 + 3*(16*(8*b^4*c*d - 3*a*b^3*d^2)*e^2 + 16*(8*b^4
*c^2 - 12*a*b^3*c*d + 5*a^2*b^2*d^2)*e*f - (48*a*b^3*c^2 - 80*a^2*b^2*c*d
+ 35*a^3*b*d^2)*f^2)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2f^2x^7}{8b} + \frac{x^5\left(-\frac{7ad^2f^2}{8b}+2cdf^2+2d^2ef\right)}{6b} + \frac{x^3\left(-\frac{5a\left(-\frac{7ad^2f^2}{8b}+2cdf^2+2d^2ef\right)}{6b}+c^2f^2+4cdef+d^2e^2\right)}{4b} + \frac{x\left(-\frac{3a\left(-\frac{5a\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots} \cdots\right. \\ \frac{c^2e^2x+\frac{d^2f^2x^9}{9}+\frac{x^7\cdot\left(2cdf^2+2d^2ef\right)}{7}+\frac{x^5\left(c^2f^2+4cdef+d^2e^2\right)}{5}+\frac{x^3\cdot\left(2c^2ef+2cde^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)**2/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(d**2*f**2*x**7/(8*b) + x**5*(-7*a*d**2*f**2/(
8*b) + 2*c*d*f**2 + 2*d**2*e*f)/(6*b) + x**3*(-5*a*(-7*a*d**2*f**2/(8*b) +
 2*c*d*f**2 + 2*d**2*e*f)/(6*b) + c**2*f**2 + 4*c*d*e*f + d**2*e**2)/(4*b)
 + x*(-3*a*(-5*a*(-7*a*d**2*f**2/(8*b) + 2*c*d*f**2 + 2*d**2*e*f)/(6*b) +
c**2*f**2 + 4*c*d*e*f + d**2*e**2)/(4*b) + 2*c**2*e*f + 2*c*d*e**2)/(2*b))
 + (-a*(-3*a*(-5*a*(-7*a*d**2*f**2/(8*b) + 2*c*d*f**2 + 2*d**2*e*f)/(6*b)
+ c**2*f**2 + 4*c*d*e*f + d**2*e**2)/(4*b) + 2*c**2*e*f + 2*c*d*e**2)/(2*b
) + c**2*e**2)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c**2*e**2*x + d**
2*f**2*x**9/9 + x**7*(2*c*d*f**2 + 2*d**2*e*f)/7 + x**5*(c**2*f**2 + 4*c*d
*e*f + d**2*e**2)/5 + x**3*(2*c**2*e*f + 2*c*d*e**2)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^2)^2(e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^2f^2x^7}{8\,b} - \frac{7\sqrt{bx^2+a}ad^2f^2x^5}{48\,b^2} + \frac{35\sqrt{bx^2+a}a^2d^2f^2x^3}{192\,b^3} + \frac{(d^2ef+cdf^2)\sqrt{bx^2+a}x^5}{3\,b} - \frac{35\sqrt{bx^2+a}a^3d^2f^2x}{128\,b^4} + \frac{c^2e^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{35\,a^4d^2f^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{5\,(d^2ef+cdf^2)\sqrt{bx^2+a}ax^3}{12\,b^2} + \frac{(d^2e^2+4\,cdef+c^2f^2)\sqrt{bx^2+a}x^3}{4\,b} + \frac{5\,(d^2ef+cdf^2)\sqrt{bx^2+a}a^2x}{8\,b^3} - \frac{3\,(d^2e^2+4\,cdef+c^2f^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(cde^2+c^2ef)\sqrt{bx^2+a}x}{b} - \frac{5\,(d^2ef+cdf^2)a^3\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} + \frac{3\,(d^2e^2+4\,cdef+c^2f^2)a^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(cde^2+c^2ef)a\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input

```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(b*x^2 + a)*d^2*f^2*x^7/b - 7/48*sqrt(b*x^2 + a)*a*d^2*f^2*x^5/b^2
 + 35/192*sqrt(b*x^2 + a)*a^2*d^2*f^2*x^3/b^3 + 1/3*(d^2*e*f + c*d*f^2)*sq
rt(b*x^2 + a)*x^5/b - 35/128*sqrt(b*x^2 + a)*a^3*d^2*f^2*x/b^4 + c^2*e^2*a
rcsinh(b*x/sqrt(a*b))/sqrt(b) + 35/128*a^4*d^2*f^2*arcsinh(b*x/sqrt(a*b))/
b^(9/2) - 5/12*(d^2*e*f + c*d*f^2)*sqrt(b*x^2 + a)*a*x^3/b^2 + 1/4*(d^2*e^
2 + 4*c*d*e*f + c^2*f^2)*sqrt(b*x^2 + a)*x^3/b + 5/8*(d^2*e*f + c*d*f^2)*s
qrt(b*x^2 + a)*a^2*x/b^3 - 3/8*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*sqrt(b*x^2
+ a)*a*x/b^2 + (c*d*e^2 + c^2*e*f)*sqrt(b*x^2 + a)*x/b - 5/8*(d^2*e*f + c*
d*f^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - (c*d*e^2 + c^2*e*f)*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{384}\left(2\left(4\left(\frac{6\,d^2f^2x^2}{b} + \frac{16\,b^6d^2ef + 16\,b^6cdf^2 - 7\,ab^5d^2f^2}{b^7}\right)x^2 + \frac{48\,b^6d^2e^2 + 192\,b^6cdef - 80\,ab^5d^2ef + \cdots}{b \cdots}\right.\right.$$
$$- \frac{(128\,b^4c^2e^2 - 128\,ab^3cde^2 + 48\,a^2b^2d^2e^2 - 128\,ab^3c^2ef + 192\,a^2b^2cdef - 80\,a^3bd^2ef + 48\,a^2b^2c^2f^2 - 8\cdots}{128\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*d^2*f^2*x^2/b + (16*b^6*d^2*e*f + 16*b^6*c*d*f^2 - 7*a*b^5*
d^2*f^2)/b^7)*x^2 + (48*b^6*d^2*e^2 + 192*b^6*c*d*e*f - 80*a*b^5*d^2*e*f +
 48*b^6*c^2*f^2 - 80*a*b^5*c*d*f^2 + 35*a^2*b^4*d^2*f^2)/b^7)*x^2 + 3*(128
*b^6*c*d*e^2 - 48*a*b^5*d^2*e^2 + 128*b^6*c^2*e*f - 192*a*b^5*c*d*e*f + 80
*a^2*b^4*d^2*e*f - 48*a*b^5*c^2*f^2 + 80*a^2*b^4*c*d*f^2 - 35*a^3*b^3*d^2*
f^2)/b^7)*sqrt(b*x^2 + a)*x - 1/128*(128*b^4*c^2*e^2 - 128*a*b^3*c*d*e^2 +
 48*a^2*b^2*d^2*e^2 - 128*a*b^3*c^2*e*f + 192*a^2*b^2*c*d*e*f - 80*a^3*b*d
^2*e*f + 48*a^2*b^2*c^2*f^2 - 80*a^3*b*c*d*f^2 + 35*a^4*d^2*f^2)*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^2}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(1/2),x)
```

3.325 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 135

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \frac{d^2x\sqrt{a+bx^2}}{2bf} - \frac{d(2bde-4bcf+adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}f^2} + \frac{(de-cf)^2\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^2\sqrt{be-af}}$$

output
```
1/2*d^2*x*(b*x^2+a)^(1/2)/b/f-1/2*d*(a*d*f-4*b*c*f+2*b*d*e)*arctanh(b^(1/2
)*x/(b*x^2+a)^(1/2))/b^(3/2)/f^2+(-c*f+d*e)^2*arctanh((-a*f+b*e)^(1/2)*x/e
^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f^2/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \frac{\frac{d^2fx\sqrt{a+bx^2}}{b} - \frac{2(de-cf)^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}\sqrt{-be+af}} + \frac{d(2bde-4bcf+adf)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}}{2f^2}$$

input
```
Integrate[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output
```
((d^2*f*x*Sqrt[a + b*x^2])/b - (2*(d*e - c*f)^2*ArcTan[(-(f*x*Sqrt[a + b*x
^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*Sqrt[-
(b*e) + a*f]) + (d*(2*b*d*e - 4*b*c*f + a*d*f)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/b^(3/2))/(2*f^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {420, 299, 224, 219, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \text{420}$$

$$\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}$$

input
```
Int[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output
```
(d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e
]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NegQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{b^{\frac{5}{2}} (cf-de)^2 \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)-\frac{d\left(\left((4cf-2de)b^2-dfab\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+df\sqrt{b\,x^2+a}\,x\,b^{\frac{3}{2}}\right)\sqrt{(af-be)e}}{2}}{\sqrt{(af-be)e}\,b^{\frac{5}{2}} f^2}$
risch	$\frac{d^2 x\sqrt{b\,x^2+a}}{2bf}-\frac{\frac{d(adf-4bcf+2bde)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{f\sqrt{b}}-\frac{b\left(c^2f^2-2cdef+d^2e^2\right)\ln\left(\frac{\frac{2af-2be}{f}-\frac{2b\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{af-be}{f}}}{x+}\right)}{\sqrt{-ef}\,f\sqrt{\frac{af-be}{f}}}}{}$
default	$\frac{d\left(df\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{2cf\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{de\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}\right)}{f^2}+\frac{(c^2f^2-2cdef+d^2e^2)\ln\left(\frac{2c}{}\right)}{}$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-(b^(5/2)*(c*f-d*e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-1/2*
d*(((4*c*f-2*d*e)*b^2-d*f*a*b)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+d*f*(b*x
^2+a)^(1/2)*x*b^(3/2))*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)/b^(5/2)/f^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(113) = 226$.

Time = 1.04 (sec) , antiderivative size = 1131, normalized size of antiderivative = 8.38

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*d^2*e^2*f - a*b*d^2*e*f^2)*sqrt(b*x^2 + a)*x + (2*b^2*d^2*e^3
 - (4*b^2*c*d + a*b*d^2)*e^2*f + (4*a*b*c*d - a^2*d^2)*e*f^2)*sqrt(b)*log(
-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f
 + b^2*c^2*f^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)
*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*
e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(b
^3*e^2*f^2 - a*b^2*e*f^3), 1/4*(2*(b^2*d^2*e^2*f - a*b*d^2*e*f^2)*sqrt(b*x
^2 + a)*x + 2*(2*b^2*d^2*e^3 - (4*b^2*c*d + a*b*d^2)*e^2*f + (4*a*b*c*d -
a^2*d^2)*e*f^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*d^2*e^2
 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*
b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e
 - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f
*x^2 + e^2)))/(b^3*e^2*f^2 - a*b^2*e*f^3), 1/4*(2*(b^2*d^2*e^2*f - a*b*d^2
*e*f^2)*sqrt(b*x^2 + a)*x - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*
sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 +
a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) +
(2*b^2*d^2*e^3 - (4*b^2*c*d + a*b*d^2)*e^2*f + (4*a*b*c*d - a^2*d^2)*e*f^2
)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^3*e^2*f^2 -
a*b^2*e*f^3), 1/2*((b^2*d^2*e^2*f - a*b*d^2*e*f^2)*sqrt(b*x^2 + a)*x - (b^
2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**2/(sqrt(a + b*x**2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^2}{\sqrt{b\,x^2+a}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.28

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \frac{-2\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)b^2c^2f^2+4\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)}{}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e),x)
```

output
```
( - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*f**2 + 4*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e*f - 2*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*b**2*d**2*e**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*
c**2*f**2 + 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt
(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c*d*e*f - 2*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*d**2*e**2 + sqrt(a + b*x**2)*a*b*d
**2*e*f**2*x - sqrt(a + b*x**2)*b**2*d**2*e**2*f*x - sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d**2*e*f**2 + 4*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d*e*f**2 - sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*a*b*d**2*e**2*f - 4*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*b**2*c*d*e**2*f + 2*sqrt(b)*log((sqrt(a + b*x**2) +
 sqrt(b)*x)/sqrt(a))*b**2*d**2*e**3)/(2*b**2*e*f**2*(a*f - b*e))
```

3.326 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 170

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = -\frac{(de-cf)^2x\sqrt{a+bx^2}}{2ef(be-af)\,(e+fx^2)}+\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f^2} - \frac{(de-cf)(2be(de+cf)-af(3de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^2(be-af)^{3/2}}$$

output
```
-1/2*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)/e/f/(-a*f+b*e)/(f*x^2+e)+d^2*arctanh(b
^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^2-1/2*(-c*f+d*e)*(2*b*e*(c*f+d*e)-a*f*
(c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f
^2/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx =$$

$$-\frac{\frac{f(de-cf)^2x\sqrt{a+bx^2}}{e(be-af)(e+fx^2)}+\frac{(de-cf)(2be(de+cf)-af(3de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}\left(e+fx^2\right)}{\sqrt{e}\sqrt{-be+af}}\right)}{e^{3/2}(-be+af)^{3/2}}+\frac{2d^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{2f^2}$$

input
```
Integrate[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
-1/2*((f*(d*e - c*f)^2*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) + ((
d*e - c*f)*(2*b*e*(d*e + c*f) - a*f*(3*d*e + c*f))*ArcTan[(-(f*x*Sqrt[a +
b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])]])/(e^(3/2)*(-(
b*e) + a*f)^(3/2)) + (2*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/
f^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 398, 224, 219, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 221$$

$$\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\mathrm{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{d\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\mathrm{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}$$

input
```
Int[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
(d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*Ar
cTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e
- a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*
f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(
Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{2\left(bd\,e^{2}+f\left(bc-\frac{3ad}{2}\right)e-\frac{ac\,f^{2}}{2}\right)(cf-de)\sqrt{b}\,(f\,x^{2}+e)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\,\left(2d^{2}e(f\,x^{2}+e)(af-be)\,\mathrm{arctanh} \ldots\right.}{2\sqrt{(af-be)e}\,\sqrt{b}\,f^{2}(af-be)e(f\,x^{2}+e)}$
default	$\frac{d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{f^{2}\sqrt{b}}-\frac{(c^{2}f^{2}-2cdef+d^{2}e^{2})\left(-\frac{f\sqrt{\left(x-\frac{\sqrt{-ef}}{f}\right)^{2}b+\frac{2b\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{af-be}{f}}}{(af-be)\left(x-\frac{\sqrt{-ef}}{f}\right)}+\frac{b\sqrt{-ef}\,\ln\left(\frac{\frac{2af-2be}{f}\ldots}{\ldots}\right)}{\ldots}\ldots\right)}{4f^{3}e\ldots}$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*(b*d*e^2+f*(b*c-3/2*a*d)*e-1/2*a*c*f^2)*(c*f-d*e)*b^(1/2)*(f*x^2+e)
*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(2*d^
2*e*(f*x^2+e)*(a*f-b*e)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)
*(c*f-d*e)^2*b^(1/2)*x*f))/((a*f-b*e)*e)^(1/2)/b^(1/2)/f^2/(a*f-b*e)/e/(f*
x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(148) = 296$.

Time = 4.40 (sec) , antiderivative size = 1947, normalized size of antiderivative = 11.45

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b^2*d^2*e^4*f - a*b*c^2*e*f^4 - (2*b^2*c*d + a*b*d^2)*e^3*f^2 +
(b^2*c^2 + 2*a*b*c*d)*e^2*f^3)*sqrt(b*x^2 + a)*x - 4*(b^2*d^2*e^5 - 2*a*b*
d^2*e^4*f + a^2*d^2*e^3*f^2 + (b^2*d^2*e^4*f - 2*a*b*d^2*e^3*f^2 + a^2*d^2
*e^2*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (
2*b^2*d^2*e^4 - 3*a*b*d^2*e^3*f + a*b*c^2*e*f^3 - 2*(b^2*c^2 - a*b*c*d)*e^
2*f^2 + (2*b^2*d^2*e^3*f - 3*a*b*d^2*e^2*f^2 + a*b*c^2*f^4 - 2*(b^2*c^2 -
a*b*c*d)*e*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2
*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3
 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)
))/(b^3*e^5*f^2 - 2*a*b^2*e^4*f^3 + a^2*b*e^3*f^4 + (b^3*e^4*f^3 - 2*a*b^2
*e^3*f^4 + a^2*b*e^2*f^5)*x^2), -1/8*(4*(b^2*d^2*e^4*f - a*b*c^2*e*f^4 - (
2*b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 2*a*b*c*d)*e^2*f^3)*sqrt(b*x^2 +
 a)*x + 8*(b^2*d^2*e^5 - 2*a*b*d^2*e^4*f + a^2*d^2*e^3*f^2 + (b^2*d^2*e^4*
f - 2*a*b*d^2*e^3*f^2 + a^2*d^2*e^2*f^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (2*b^2*d^2*e^4 - 3*a*b*d^2*e^3*f + a*b*c^2*e*f^3 - 2*(b^
2*c^2 - a*b*c*d)*e^2*f^2 + (2*b^2*d^2*e^3*f - 3*a*b*d^2*e^2*f^2 + a*b*c^2*
f^4 - 2*(b^2*c^2 - a*b*c*d)*e*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^
2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4
*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4
 + 2*e*f*x^2 + e^2)))/(b^3*e^5*f^2 - 2*a*b^2*e^4*f^3 + a^2*b*e^3*f^4 + ...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((c + d*x**2)**2/(sqrt(a + b*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/(sqrt(b*x^2 + a)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(148) = 296$.

Time = 0.16 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.94

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\left(2\,b^{\frac{3}{2}}d^2e^3 - 3\,a\sqrt{b}d^2e^2f - 2\,b^{\frac{3}{2}}c^2ef^2 + 2\,a\sqrt{b}cdef^2 + a\sqrt{b}c^2f^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,(be^2f^2-aef^3)\sqrt{-b^2e^2+abef}}$$
$$- \frac{d^2\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,\sqrt{b}f^2}$$
$$- \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}d^2e^3 - 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cde^2f - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d^2e^2f + 2\,(}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f + 4\left(\sqrt{b}x\right.\right.}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^(3/2)*d^2*e^3 - 3*a*sqrt(b)*d^2*e^2*f - 2*b^(3/2)*c^2*e*f^2 + 2*a
*sqrt(b)*c*d*e*f^2 + a*sqrt(b)*c^2*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^
2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*e^2*f^2 - a*e*f^3
)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/2*d^2*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)
/(sqrt(b)*f^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*d^2*e^3 - 4*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d*e^2*f - (sqrt(b)*x - sqrt(b*x^2
+ a))^2*a*sqrt(b)*d^2*e^2*f + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c^
2*e*f^2 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*d*e*f^2 - (sqrt(b)
*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c^2*f^3 + a^2*sqrt(b)*d^2*e^2*f - 2*a^2*
sqrt(b)*c*d*e*f^2 + a^2*sqrt(b)*c^2*f^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4
*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a
))^2*a*f + a^2*f)*(b*e^2*f^2 - a*e*f^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^2}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1649, normalized size of antiderivative = 9.70

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*e*f**3 - sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a*b*c**2*f**4*x**2 - 2*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a*b*c*d*e**2*f**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a*b*c*d*e*f**3*x**2 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s
qrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d**2*e
**3*f + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
 b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b*d**2*e**2*f**2*x**2 +
 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*e**2*f**2 + 2*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**2*c**2*e*f**3*x**2 - 2*sqrt(e)*sqrt(a*f
- b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*
x)/(sqrt(e)*sqrt(b)))*b**2*d**2*e**4 - 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*b**2*d**2*e**3*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*...
```

3.327 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 261

$$\begin{aligned}\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx &= \frac{d^2x\sqrt{a+bx^2}}{4ef(be-af)} - \frac{fx\sqrt{a+bx^2}(c+dx^2)^2}{4e(be-af)\,(e+fx^2)^2} \\ &\quad - \frac{(de-cf)(2be(de-3cf)+af(de+3cf))x\sqrt{a+bx^2}}{8e^2f(be-af)^2\,(e+fx^2)} \\ &\quad + \frac{(8b^2c^2e^2-8abce(de+cf)+a^2(3d^2e^2+2cdef+3c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{5/2}}\end{aligned}$$

output
```
1/4*d^2*x*(b*x^2+a)^(1/2)/e/f/(-a*f+b*e)-1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)
^2/e/(-a*f+b*e)/(f*x^2+e)^2-1/8*(-c*f+d*e)*(2*b*e*(-3*c*f+d*e)+a*f*(3*c*f+
d*e))*x*(b*x^2+a)^(1/2)/e^2/f/(-a*f+b*e)^2/(f*x^2+e)+1/8*(8*b^2*c^2*e^2-8*
a*b*c*e*(c*f+d*e)+a^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*arctanh((-a*f+b*e)^
(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 11.30 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{d(de-cf)x\left(f(a+bx^2)-\frac{(2be-af)(e+fx^2)\text{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{e\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{ef^2(be-af)\sqrt{a+bx^2}\,(e+fx^2)}$$
$$- \frac{(de-cf)^2x\left(ef(a+bx^2)\left(2be(4e+3fx^2)-af(5e+3fx^2)\right)-\frac{(8b^2e^2-8abef+3a^2f^2)(e+fx^2)^2\text{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{8e^3f^2(be-af)^2\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$+ \frac{d^2\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^2\sqrt{be-af}}$$

input
```
Integrate[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
(d*(d*e - c*f)*x*(f*(a + b*x^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTanh[Sqrt[
((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x
^2))])))/(e*f^2*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) - ((d*e - c*f)^2*
x*(e*f*(a + b*x^2)*(2*b*e*(4*e + 3*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*b^2
*e^2 - 8*a*b*e*f + 3*a^2*f^2)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)
/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(8*e^3*f^2*(
b*e - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (d^2*ArcTanh[(Sqrt[b*e - a*f
]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f^2*Sqrt[b*e - a*f])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {425, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{\sqrt{a+bx^2}\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{\int \frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(af(3cf+de)+2be(de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{f}$$

input
```
Int[(c + d*x^2)^2/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
(d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((2*b*
c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a
 + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*c*f) + a*f*
(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + ((8*b^2*
c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a
*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*
e - a*f))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{5\sqrt{(af-be)e}\left(\frac{\left(3ad-8\left(\frac{x^2d}{4}+c\right)b\right)e^2}{5}+f\left(\left(x^2d+c\right)a-\frac{6x^2bc}{5}\right)e+\frac{3acf^2x^2}{5}\right)(cf-de)\sqrt{bx^2+a}\,x-3\left(\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)e\right.}{8\sqrt{(af-be)e}\,(af-be)^2e^2(fx^2+e)^2}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(5*((a*f-b*e)*e)^(1/2)*(1/5*(3*a*d-8*(1/4*x^2*d+c)*b)*e^2+f*((d*x^2+c)
*a-6/5*x^2*b*c)*e+3/5*a*c*f^2*x^2)*(c*f-d*e)*(b*x^2+a)^(1/2)*x-3*((a^2*d^2
-8/3*a*b*c*d+8/3*b^2*c^2)*e^2+2/3*a*c*f*(a*d-4*b*c)*e+a^2*c^2*f^2)*(f*x^2+
e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))/((a*f-b*e)*e)^(1/2)/
(a*f-b*e)^2/e^2/(f*x^2+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(237) = 474$.

Time = 2.88 (sec) , antiderivative size = 1318, normalized size of antiderivative = 5.05

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/32*((3*a^2*c^2*e^2*f^2 + (8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*e^4 - 2*(4
*a*b*c^2 - a^2*c*d)*e^3*f + (3*a^2*c^2*f^4 + (8*b^2*c^2 - 8*a*b*c*d + 3*a^
2*d^2)*e^2*f^2 - 2*(4*a*b*c^2 - a^2*c*d)*e*f^3)*x^4 + 2*(3*a^2*c^2*e*f^3 +
 (8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*e^3*f - 2*(4*a*b*c^2 - a^2*c*d)*e^2*f
^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 +
a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sq
rt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((2*b^
2*d^2*e^5 - 3*a^2*c^2*e*f^4 + (4*b^2*c*d - 7*a*b*d^2)*e^4*f - (6*b^2*c^2 +
 2*a*b*c*d - 5*a^2*d^2)*e^3*f^2 + (9*a*b*c^2 - 2*a^2*c*d)*e^2*f^3)*x^3 - (
5*a^2*c^2*e^2*f^3 - (8*b^2*c*d - 3*a*b*d^2)*e^5 + (8*b^2*c^2 + 10*a*b*c*d
- 3*a^2*d^2)*e^4*f - (13*a*b*c^2 + 2*a^2*c*d)*e^3*f^2)*x)*sqrt(b*x^2 + a))
/(b^3*e^8 - 3*a*b^2*e^7*f + 3*a^2*b*e^6*f^2 - a^3*e^5*f^3 + (b^3*e^6*f^2 -
 3*a*b^2*e^5*f^3 + 3*a^2*b*e^4*f^4 - a^3*e^3*f^5)*x^4 + 2*(b^3*e^7*f - 3*a
*b^2*e^6*f^2 + 3*a^2*b*e^5*f^3 - a^3*e^4*f^4)*x^2), -1/16*((3*a^2*c^2*e^2*
f^2 + (8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*e^4 - 2*(4*a*b*c^2 - a^2*c*d)*e^
3*f + (3*a^2*c^2*f^4 + (8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*e^2*f^2 - 2*(4*
a*b*c^2 - a^2*c*d)*e*f^3)*x^4 + 2*(3*a^2*c^2*e*f^3 + (8*b^2*c^2 - 8*a*b*c*
d + 3*a^2*d^2)*e^3*f - 2*(4*a*b*c^2 - a^2*c*d)*e^2*f^2)*x^2)*sqrt(-b*e^2 +
 a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x
^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) - 2*((2*b^2*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Integral((c + d*x**2)**2/(sqrt(a + b*x**2)*(e + f*x**2)**3), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/(sqrt(b*x^2 + a)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(237) = 474$.

Time = 0.36 (sec) , antiderivative size = 1350, normalized size of antiderivative = 5.17

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*(8*b^(5/2)*c^2*e^2 - 8*a*b^(3/2)*c*d*e^2 + 3*a^2*sqrt(b)*d^2*e^2 - 8*
a*b^(3/2)*c^2*e*f + 2*a^2*sqrt(b)*c*d*e*f + 3*a^2*sqrt(b)*c^2*f^2)*arctan(
1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*
e*f))/((b^2*e^4 - 2*a*b*e^3*f + a^2*e^2*f^2)*sqrt(-b^2*e^2 + a*b*e*f)) + 1
/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d^2*e^4*f - 16*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*a*b^(3/2)*d^2*e^3*f^2 - 8*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*b^(5/2)*c^2*e^2*f^3 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d*e
^2*f^3 + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^2*e^2*f^3 + 8*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c^2*e*f^4 - 2*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*a^2*sqrt(b)*c*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*s
qrt(b)*c^2*f^5 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^2*e^5 + 32*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c*d*e^4*f - 56*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a*b^(5/2)*d^2*e^4*f - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/
2)*c^2*e^3*f^2 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d*e^3*f^2
+ 46*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^2*e^3*f^2 + 72*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*e^2*f^3 - 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^2*b^(3/2)*c*d*e^2*f^3 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*s
qrt(b)*d^2*e^2*f^3 - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c^2*e*
f^4 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*c*d*e*f^4 + 9*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*c^2*f^5 + 8*(sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^2}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 4829, normalized size of antiderivative = 18.50

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e**2*f**4 - 12*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*e*f**5*x**2 - 6*sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a**3*c**2*f**6*x**4 - 4*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**3*c*d*e**3*f**3 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a**3*c*d*e**2*f**4*x**2 - 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*
e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3
*c*d*e*f**5*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**4
*f**2 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
+ b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**3*f**3*x**2
 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x*
*2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*d**2*e**2*f**4*x**4 + 28*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c**2*e**3*f**3 + 56*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt...
```

3.328 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}(e+fx^2)} dx$

Optimal result

Integrand size = 30, antiderivative size = 217

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = -\frac{d^2(4bde-12bcf+3adf)x\sqrt{a+bx^2}}{8b^2f^2}+\frac{d^3x^3\sqrt{a+bx^2}}{4bf} + \frac{d(3a^2d^2f^2+4abdf(de-3cf)+8b^2(d^2e^2-3cdef+3c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}f^3} - \frac{(de-cf)^3\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^3\sqrt{be-af}}$$

output
```
-1/8*d^2*(3*a*d*f-12*b*c*f+4*b*d*e)*x*(b*x^2+a)^(1/2)/b^2/f^2+1/4*d^3*x^3*
(b*x^2+a)^(1/2)/b/f+1/8*d*(3*a^2*d^2*f^2+4*a*b*d*f*(-3*c*f+d*e)+8*b^2*(3*c
^2*f^2-3*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)/f^3-
(-c*f+d*e)^3*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f
^3/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\frac{d^2fx\sqrt{a+bx^2}(-3adf+2b(-2de+6cf+dfx^2))}{b^2} + \frac{8(de-cf)^3\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}\sqrt{-be+af}} - \frac{d(3a^2d^2f^2+4abdf(de-3cf)+8b^2(d^2e^2-3}{b^{5/2}}}{8f^3}$$

input
```
Integrate[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output
```
((d^2*f*x*Sqrt[a + b*x^2]*(-3*a*d*f + 2*b*(-2*d*e + 6*c*f + d*f*x^2)))/b^2
 + (8*(d*e - c*f)^3*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/
(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*Sqrt[-(b*e) + a*f]) - (d*(3*a^2*d^
2*f^2 + 4*a*b*d*f*(d*e - 3*c*f) + 8*b^2*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.40, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {420, 318, 299, 224, 219, 420, 299, 224, 219, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 420$$
$$\frac{d\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}}dx}{f} - \frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$
$$\Big\downarrow \ 318$$

$$\frac{d\left(\frac{\int \frac{3d(2bc-ad)x^2+c(4bc-ad)}{\sqrt{bx^2+a}}dx}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{d\left(\frac{\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}$$

$\Big\downarrow$ 398

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 291

$$\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-8abcd+8b^2c^2\right)}{2b^{3/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{2b}}{4b}+\frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}$$

input

```
Int[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)),x]
```

output

```
(d*((d*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*b) + ((3*d*(2*b*c - a*d)*x*Sqrt[a
 + b*x^2])/(2*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/f - ((d*e - c*f)*((d*((d*x*Sqrt[a
 + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sq
rt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^
2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{-b^{\frac{9}{2}}(cf-de)^3 \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\frac{3\left(\frac{\left(\frac{8b^2d^2e^2}{3}+\frac{4bdf(ad-6bc)e}{3}+f^2\left(a^2d^2-4abcd+8b^2c^2\right)\right)b^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{4}+d\,b^{\frac{5}{2}}\left(\cdots\right.\right.}{2}}{\sqrt{(af-be)e}\,b^{\frac{9}{2}}f^3}$
risch	$-\frac{d^2x\left(-2bdf\,x^2+3adf-12bcf+4bde\right)\sqrt{b\,x^2+a}}{8b^2f^2}+\frac{\frac{d\left(3a^2d^2f^2-12abcd\,f^2+4ab\,d^2ef+24b^2c^2f^2-24b^2cdef+8b^2d^2e^2\right)\ln\left(\sqrt{b}\,x+\cdots\right.}{f\sqrt{b}}}{\cdots}$
default	$\frac{d\left(\frac{d^2e^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+df(3cf-de)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+d^2f^2\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x\cdots\right.}{\cdots}\right)}{4b}\cdots\right.\right.}{f^3}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
3/2*(-2/3*b^(9/2)*(c*f-d*e)^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/
2))+(1/4*(8/3*b^2*d^2*e^2+4/3*b*d*f*(a*d-6*b*c)*e+f^2*(a^2*d^2-4*a*b*c*d+8
*b^2*c^2))*b^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+d*b^(5/2)*(-1/3*b*d*e-1/
4*((-2/3*x^2*d-4*c)*b+a*d)*f)*(b*x^2+a)^(1/2)*x*f)*d*((a*f-b*e)*e)^(1/2))/
((a*f-b*e)*e)^(1/2)/b^(9/2)/f^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(191) = 382$.

Time = 6.02 (sec) , antiderivative size = 1729, normalized size of antiderivative = 7.97

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/16*((8*b^3*d^3*e^4 - 4*(6*b^3*c*d^2 + a*b^2*d^3)*e^3*f + (24*b^3*c^2*d
+ 12*a*b^2*c*d^2 - a^2*b*d^3)*e^2*f^2 - 3*(8*a*b^2*c^2*d - 4*a^2*b*c*d^2 +
 a^3*d^3)*e*f^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
 4*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqr
t(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*
(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a
*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 2*(2*(b^3*d^3*e^2*f^
2 - a*b^2*d^3*e*f^3)*x^3 - (4*b^3*d^3*e^3*f - (12*b^3*c*d^2 + a*b^2*d^3)*e
^2*f^2 + 3*(4*a*b^2*c*d^2 - a^2*b*d^3)*e*f^3)*x)*sqrt(b*x^2 + a))/(b^4*e^2
*f^3 - a*b^3*e*f^4), -1/8*((8*b^3*d^3*e^4 - 4*(6*b^3*c*d^2 + a*b^2*d^3)*e^
3*f + (24*b^3*c^2*d + 12*a*b^2*c*d^2 - a^2*b*d^3)*e^2*f^2 - 3*(8*a*b^2*c^2
*d - 4*a^2*b*c*d^2 + a^3*d^3)*e*f^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*
f^3)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*
e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b
*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - (2*(b^3*d^3*
e^2*f^2 - a*b^2*d^3*e*f^3)*x^3 - (4*b^3*d^3*e^3*f - (12*b^3*c*d^2 + a*b^2*
d^3)*e^2*f^2 + 3*(4*a*b^2*c*d^2 - a^2*b*d^3)*e*f^3)*x)*sqrt(b*x^2 + a))/(b
^4*e^2*f^3 - a*b^3*e*f^4), 1/16*(8*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^
3*c^2*d*e*f^2 - b^3*c^3*f^3)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^...
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(1/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**3/(sqrt(a + b*x**2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^3}{\sqrt{b\,x^2+a}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.53

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e),x)
```

output
```
( - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**3*f**3 + 24*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**2*d*e*f**2 - 24*sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*b**3*c*d**2*e**2*f + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sq
rt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqr
t(b)))*b**3*d**3*e**3 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) +
sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**3
*f**3 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a
+ b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**2*d*e*f**2 - 24*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c*d**2*e**2*f + 8*sqrt(e)*sqrt
(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqr
t(b)*x)/(sqrt(e)*sqrt(b)))*b**3*d**3*e**3 - 3*sqrt(a + b*x**2)*a**2*b*d**3
*e*f**3*x + 12*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f**3*x - sqrt(a + b*x**2)*
a*b**2*d**3*e**2*f**2*x + 2*sqrt(a + b*x**2)*a*b**2*d**3*e*f**3*x**3 - 12*
sqrt(a + b*x**2)*b**3*c*d**2*e**2*f**2*x + 4*sqrt(a + b*x**2)*b**3*d**3*e*
*3*f*x - 2*sqrt(a + b*x**2)*b**3*d**3*e**2*f**2*x**3 + 3*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d**3*e*f**3 - 12*sqrt(b)*log((s...
```

3.329 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 216

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \frac{d^3x\sqrt{a+bx^2}}{2bf^2}+\frac{(de-cf)^3x\sqrt{a+bx^2}}{2ef^2(be-af)\,(e+fx^2)}-\frac{d^2(4bde-6bcf+adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}f^3}+\frac{(de-cf)^2(2be(2de+cf)-af(5de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^3(be-af)^{3/2}}$$

output
```
1/2*d^3*x*(b*x^2+a)^(1/2)/b/f^2+1/2*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^2/(
-a*f+b*e)/(f*x^2+e)-1/2*d^2*(a*d*f-6*b*c*f+4*b*d*e)*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(3/2)/f^3+1/2*(-c*f+d*e)^2*(2*b*e*(c*f+2*d*e)-a*f*(c*f+5*d*
e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/f^3/(-a*f+
b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

$$= \frac{-\frac{fx\sqrt{a+bx^2}(ad^3ef(e+fx^2)+b(3cd^2e^2f-3c^2def^2+c^3f^3-d^3e^2(2e+fx^2)))}{be(be-af)(e+fx^2)} + \frac{(de-cf)^2(2be(2de+cf)-af(5de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+}{\sqrt{e}\sqrt{-b}}\right.}{e^{3/2}(-be+af)^{3/2}}}{2f^3}$$

input
```
Integrate[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
(-((f*x*Sqrt[a + b*x^2]*(a*d^3*e*f*(e + f*x^2) + b*(3*c*d^2*e^2*f - 3*c^2*
d*e*f^2 + c^3*f^3 - d^3*e^2*(2*e + f*x^2))))/(b*e*(b*e - a*f)*(e + f*x^2))
) + ((d*e - c*f)^2*(2*b*e*(2*d*e + c*f) - a*f*(5*d*e + c*f))*ArcTan[(-(f*x
*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(e
^(3/2)*(-(b*e) + a*f)^(3/2)) + (d^2*(4*b*d*e - 6*b*c*f + a*d*f)*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(2*f^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.91, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {425, 420, 299, 224, 219, 398, 224, 219, 291, 221, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{f} - \frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{420}$$

$$\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 299

$$\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{d\left(\frac{(2bc-ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 398

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \ 221$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 398

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}\right)}{f}$$

$\downarrow$ 224

$$\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 291

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}}{2e\left(e+fx\right.}\right.}{f}\right.}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}}{2e\left(e+fx^2\right.}\right.}{f}\right.}{f}$$

input

```
Int[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)^2),x]
```

output
```
(d*((d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sq
rt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/f - ((d*e - c*f
)*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)
*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b
*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e -
 a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x
)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\frac{\left(2bd\,e^2+f\left(-\frac{5ad}{2}+bc\right)e-\frac{ac\,f^2}{2}\right)(cf-de)^2\left(f\,x^2+e\right)b^{\frac{5}{2}}\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\frac{\left(-d^2(af-be)b\left(f\,x^2+e\right)(4bde+f(ad-6bc))e\ a\cdots\right.}{\cdots}}{\sqrt{(af-be)e}\,b^{\frac{5}{2}}f^3(af-be\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((2*b*d*e^2+f*(-5/2*a*d+b*c)*e-1/2*a*c*f^2)*(c*f-d*e)^2*(f*x^2+e)*b^(5/2)*
arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+1/2*(-d^2*(a*f-b*e)*b*(f*x
^2+e)*(4*b*d*e+f*(a*d-6*b*c))*e*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+
a)^(1/2)*x*f*(-2*b*d^3*e^3+((-d*x^2+3*c)*b+a*d)*d^2*f*e^2+d*f^2*(a*d^2*x^2
-3*b*c^2)*e+b*c^3*f^3)*b^(3/2))*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)/b
^(5/2)/f^3/(a*f-b*e)/e/(f*x^2+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(188) = 376$.

Time = 22.76 (sec) , antiderivative size = 3081, normalized size of antiderivative = 14.26

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral((c + d*x**2)**3/(sqrt(a + b*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/(sqrt(b*x^2 + a)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(188) = 376.

Time = 0.17 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.02

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{bx^2+a}d^3x}{2\,bf^2}$$
$$-\frac{\left(4\,b^{\frac{3}{2}}d^3e^4-6\,b^{\frac{3}{2}}cd^2e^3f-5\,a\sqrt{b}d^3e^3f+9\,a\sqrt{b}cd^2e^2f^2+2\,b^{\frac{3}{2}}c^3ef^3-3\,a\sqrt{b}c^2def^3-a\sqrt{b}c^3f^4\right)\arctan}{2\,(be^2f^3-aef^4)\sqrt{-b^2e^2+abef}}$$
$$+\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^2d^3e^4-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^2cd^2e^3f-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2abd^3e^3f+6\left(\sqrt{\right.}\right.}{}$$
$$+\frac{(4\,bd^3e-6\,bcd^2f+ad^3f)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^{\frac{3}{2}}f^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*d^3*x/(b*f^2) - 1/2*(4*b^(3/2)*d^3*e^4 - 6*b^(3/2)*c*d
^2*e^3*f - 5*a*sqrt(b)*d^3*e^3*f + 9*a*sqrt(b)*c*d^2*e^2*f^2 + 2*b^(3/2)*c
^3*e*f^3 - 3*a*sqrt(b)*c^2*d*e*f^3 - a*sqrt(b)*c^3*f^4)*arctan(1/2*((sqrt(
b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b*e
^2*f^3 - a*e*f^4)*sqrt(-b^2*e^2 + a*b*e*f)) + (2*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*b^2*d^3*e^4 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*c*d^2*e^3*f - (
sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*d^3*e^3*f + 6*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*b^2*c^2*d*e^2*f^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*c*d^2*e^2
*f^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*c^3*e*f^3 - 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*a*b*c^2*d*e*f^3 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*c^3
*f^4 + a^2*b*d^3*e^3*f - 3*a^2*b*c*d^2*e^2*f^2 + 3*a^2*b*c^2*d*e*f^3 - a^2
*b*c^3*f^4)/((b^(3/2)*e^2*f^3 - a*sqrt(b)*e*f^4)*((sqrt(b)*x - sqrt(b*x^2
+ a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*f + a^2*f)) + 1/4*(4*b*d^3*e - 6*b*c*d^2*f + a*d^3*f)*log((s
qrt(b)*x - sqrt(b*x^2 + a))^2)/(b^(3/2)*f^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^3}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 2669, normalized size of antiderivative = 12.36

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**3*e*f**4 - sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*c**3*f**5*x**2 - 3*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a*b**2*c**2*d*e**2*f**3 - 3*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a*b**2*c**2*d*e*f**4*x**2 + 9*sqrt(e)*sqrt(a*f - b*e)*atan((s
qrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sq
rt(b)))*a*b**2*c*d**2*e**3*f**2 + 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a*b**2*c*d**2*e**2*f**3*x**2 - 5*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
 b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a
*b**2*d**3*e**4*f - 5*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt
(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*d**3*e
**3*f**2*x**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**3*e**2*f*
*3 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b**3*c**3*e*f**4*x**2 - 6*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) ...
```

3.330 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 309

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \frac{(de-cf)^3x\sqrt{a+bx^2}}{4ef^2(be-af)\,(e+fx^2)^2}$$
$$-\frac{3(de-cf)^2(2be(de+cf)-af(3de+cf))x\sqrt{a+bx^2}}{8e^2f^2(be-af)^2\,(e+fx^2)}+\frac{d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f^3}$$
$$-\frac{(de-cf)\,(8b^2e^2(d^2e^2+cdef+c^2f^2)+3a^2f^2(5d^2e^2+2cdef+c^2f^2)-4abef(5d^2e^2+5cdef+2c^2f^2}{8e^{5/2}f^3(be-af)^{5/2}}$$

output
```
1/4*(-c*f+d*e)^3*x*(b*x^2+a)^(1/2)/e/f^2/(-a*f+b*e)/(f*x^2+e)^2-3/8*(-c*f+
d*e)^2*(2*b*e*(c*f+d*e)-a*f*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)/e^2/f^2/(-a*f+b
*e)^2/(f*x^2+e)+d^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/f^3-1/8*(-c
*f+d*e)*(8*b^2*e^2*(c^2*f^2+c*d*e*f+d^2*e^2)+3*a^2*f^2*(c^2*f^2+2*c*d*e*f+
5*d^2*e^2)-4*a*b*e*f*(2*c^2*f^2+5*c*d*e*f+5*d^2*e^2))*arctanh((-a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/f^3/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 11.49 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \frac{-\frac{12d(de-cf)^2x\left(f(a+bx^2)-\frac{(2be-af)(e+fx^2)\operatorname{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{e\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{e(be-af)\sqrt{a+bx^2}(e+fx^2)} + \frac{(de-cf)^3x\left(ef(a+bx^2)(2be(4e+3fx^2)-af(5e+3fx^2))-\cdots\right)}{e^3(be-af)^2\sqrt{a+\cdots}}}{8f^3}$$

input
```
Integrate[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
((-12*d*(d*e - c*f)^2*x*(f*(a + b*x^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTan
h[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(
a + b*x^2))])))/(e*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + ((d*e - c*f)
^3*x*(e*f*(a + b*x^2)*(2*b*e*(4*e + 3*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*
b^2*e^2 - 8*a*b*e*f + 3*a^2*f^2)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x
^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(e^3*(b*e
 - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (8*d^3*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/Sqrt[b] - (24*d^2*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(S
qrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {425, 425, 398, 224, 219, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 425$$

$$\frac{d\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{d\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{d\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{d\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{d\left(\frac{d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\int\frac{2bce-a(de+cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{(2bce-a(cf+de))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de-c}{4e\left(e+fx^2\right)^2(be-}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{(2bce-a(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(2bce-a(cf+de))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(d\cdots}{4e\left(e+fx^2\right)^2(\cdots}\right)}{f}\cdots\right)}{f}$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2b(de-cf)x^2+4bce-ade-3acf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}(de\cdots}{4e\left(e+fx^2\right)^2(b\cdots}\right)}{f}\cdots\right)}{f}$$

$\Big\downarrow$ 402

$$d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.$$

$$\overline{f}$$

$$(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{f(de+3cf)a^2-4be(de+2cf)a+8b^2ce^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+b}}{}}{4e(be-af)}\right.}{f}\right.$$

$$\overline{f}$$

$\downarrow$ 27

$$d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.$$

$$\overline{f}$$

$$(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(\right.}}{2e(be-af)}}{4e(be-}\right.}{}\right.$$

$$\overline{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\left(a^2f(3cf+de)-4abe(2cf+de)+8b^2ce^2\right)\int\frac{1}{e-\frac{(be-af)}{bx^2+}}}{2e(be-af)}}{4e(b}\right.}{}\right.}{f}$$

$\big\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}-\frac{(de-cf)\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2bce-a(cf+de))}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f(3cf+de)-4abe(2cf+\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(b}\right.}{}\right.}{f}$$

input
```
Int[(c + d*x^2)^3/(Sqrt[a + b*x^2]*(e + f*x^2)^3),x]
```

output
```
(d*((d*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f
)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[
b*e - a*f])))/f - ((d*e - c*f)*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e
- a*f)*(e + f*x^2)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*
x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f))/f - ((d
*e - c*f)*((d*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2
)) + ((2*b*c*e - a*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((d*e - c*
f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((2*b*e*(d*e - 3*
c*f) + a*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2))
 + ((8*b^2*c*e^2 - 4*a*b*e*(d*e + 2*c*f) + a^2*f*(d*e + 3*c*f))*ArcTanh[(S
qrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)
))/(4*e*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{b}\,(af-be)\left(3a^2c^2f^4+6a^2cde\,f^3+15a^2d^2e^2f^2-8ab\,c^2e\,f^3-20abcd\,e^2f^2-20ab\,d^2e^3f+8b^2c^2e^2f^2+8b^2cd\,e^3f+8b^2d^2e^4\right)(cf-de)\left(f\right.}{8}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*b^(1/2)*(a*f-b*e)*(3*a^2*c^2*f^4+6*a^2*c*d*e*f^3+15*a^2*d^2*e^2*f^2-
8*a*b*c^2*e*f^3-20*a*b*c*d*e^2*f^2-20*a*b*d^2*e^3*f+8*b^2*c^2*e^2*f^2+8*b^
2*c*d*e^3*f+8*b^2*d^2*e^4)*(c*f-d*e)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/
x/((a*f-b*e)*e)^(1/2))+(d^3*e^2*(f*x^2+e)^2*(a*f-b*e)^3*arctanh((b*x^2+a)^
(1/2)/x/b^(1/2))+1/8*b^(1/2)*(b*x^2+a)^(1/2)*(a*f-b*e)*(3*a*c*f^3*x^2+9*a*
d*e*f^2*x^2-6*b*c*e*f^2*x^2-6*b*d*e^2*f*x^2+5*a*c*e*f^2+7*a*d*e^2*f-8*b*c*
e^2*f-4*b*d*e^3)*(c*f-d*e)^2*x*f)*((a*f-b*e)*e)^(1/2))/((a*f-b*e)*e)^(1/2)
/b^(1/2)/f^3/(f*x^2+e)^2/e^2/(a*f-b*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1138 vs. $2(283) = 566$.

Time = 66.38 (sec) , antiderivative size = 4646, normalized size of antiderivative = 15.04

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Integral((c + d*x**2)**3/(sqrt(a + b*x**2)*(e + f*x**2)**3), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^3}{\sqrt{bx^2+a}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/(sqrt(b*x^2 + a)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1863 vs. 2(283) = 566.

Time = 0.18 (sec) , antiderivative size = 1863, normalized size of antiderivative = 6.03

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/8*(8*b^(5/2)*d^3*e^5 - 20*a*b^(3/2)*d^3*e^4*f + 15*a^2*sqrt(b)*d^3*e^3*f
^2 - 8*b^(5/2)*c^3*e^2*f^3 + 12*a*b^(3/2)*c^2*d*e^2*f^3 - 9*a^2*sqrt(b)*c*
d^2*e^2*f^3 + 8*a*b^(3/2)*c^3*e*f^4 - 3*a^2*sqrt(b)*c^2*d*e*f^4 - 3*a^2*sq
rt(b)*c^3*f^5)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f
)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^4*f^3 - 2*a*b*e^3*f^4 + a^2*e^2*f^5)*s
qrt(-b^2*e^2 + a*b*e*f)) - 1/2*d^3*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/(s
qrt(b)*f^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d^3*e^5*f -
24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c*d^2*e^4*f^2 - 28*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*a*b^(3/2)*d^3*e^4*f^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a*b^(3/2)*c*d^2*e^3*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(
b)*d^3*e^3*f^3 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^3*e^2*f^4 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c^2*d*e^2*f^4 - 15*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*d^2*e^2*f^4 - 8*(sqrt(b)*x - sqrt(b*x^2
 + a))^6*a*b^(3/2)*c^3*e*f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(
b)*c^2*d*e*f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^3*f^6 + 4
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^3*e^6 - 48*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(7/2)*c*d^2*e^5*f - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b
^(5/2)*d^3*e^5*f - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^2*d*e^4*f^
2 + 168*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d^2*e^4*f^2 + 90*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^3*e^4*f^2 + 48*(sqrt(b)*x - sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^3}{\sqrt{b\,x^2+a}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 7632, normalized size of antiderivative = 24.70

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*e**2*f**6 - 12*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - s
qrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*e*f**7*x**2 - 6*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c**3*f**8*x**4 - 6*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**3*b*c**2*d*e**3*f**5 - 12*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**3*b*c**2*d*e**2*f**6*x**2 - 6*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a**3*b*c**2*d*e*f**7*x**4 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*s
qrt(b)))*a**3*b*c*d**2*e**4*f**4 - 36*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a**3*b*c*d**2*e**3*f**5*x**2 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**3*b*c*d**2*e**2*f**6*x**4 + 30*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**3*b*d**3*e**5*f**3 + 60*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*...
```

3.331 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)} - \frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}$$

output
```
d*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(
1/2)/(-c*f+d*e)-f*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1
/2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \frac{\frac{d\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}} - \frac{f\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}\sqrt{-be+af}}}{-de+cf}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
((d*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(
b*c) + a*d])])/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (f*ArcTan[(-(f*x*Sqrt[a + b*
x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*Sqrt[
-(b*e) + a*f]))/(-(d*e) + c*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {407, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)}dx$$

$$\Big\downarrow \text{407}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{de-cf}$$

$$\Big\downarrow \text{291}$$

$$\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}$$

$$\Big\downarrow \text{221}$$

$$\frac{d\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b
*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{d \arctan\left(\frac{c \sqrt{b\,x^{2}+a}}{x \sqrt{(a d-b c) c}}\right)}{\sqrt{(a d-b c) c}}-\frac{f \arctan\left(\frac{e \sqrt{b\,x^{2}+a}}{x \sqrt{(a f-b e) e}}\right)}{\sqrt{(a f-b e) e}}}{c f-d e}$
default	$-\frac{d\,f^{2} \ln\left(\frac{\frac{2 a f-2 b e}{f}+\frac{2 b \sqrt{-e f}\,\left(x-\frac{\sqrt{-e f}}{f}\right)}{f}+2 \sqrt{\frac{a f-b e}{f}}\,\sqrt{\left(x-\frac{\sqrt{-e f}}{f}\right)^{2} b+\frac{2 b \sqrt{-e f}\,\left(x-\frac{\sqrt{-e f}}{f}\right)}{f}+\frac{a f-b e}{f}}}{x-\frac{\sqrt{-e f}}{f}}\right)}{2\left(\sqrt{-c d}\,f+\sqrt{-e f}\,d\right)\left(\sqrt{-e f}\,d-\sqrt{-c d}\,f\right)\sqrt{-e f}\,\sqrt{\frac{a f-b e}{f}}}+\frac{d\,f^{2} \ln\left(\frac{\frac{2 a f-2 b}{f}}{}\right.}{}$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/(c*f-d*e)*(d/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c
)^(1/2))-f/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(102) = 204$.

Time = 52.72 (sec) , antiderivative size = 1289, normalized size of antiderivative = 10.57

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/4*((b*c^2 - a*c*d)*sqrt(b*e^2 - a*e*f)*f*log(((8*b^2*e^2 - 8*a*b*e*f +
 a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)
*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 +
e^2)) + (b*d*e^2 - a*d*e*f)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*((2*b*c - a
*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2
 + c^2)))/((b^2*c^2*d - a*b*c*d^2)*e^3 - (b^2*c^3 - a^2*c*d^2)*e^2*f + (a*
b*c^3 - a^2*c^2*d)*e*f^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(-b*e^2 + a*e*f)*f*a
rctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/(
(b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) - (b*d*e^2 - a*d*e*f)*sq
rt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 -
a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((b^2*c^2*d - a*b*c*
d^2)*e^3 - (b^2*c^3 - a^2*c*d^2)*e^2*f + (a*b*c^3 - a^2*c^2*d)*e*f^2), -1/
4*((b*c^2 - a*c*d)*sqrt(b*e^2 - a*e*f)*f*log(((8*b^2*e^2 - 8*a*b*e*f + a^2
*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3
 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)
) + 2*(b*d*e^2 - a*d*e*f)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*
c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 +
(a*b*c^2 - a^2*c*d)*x)))/((b^2*c^2*d - a*b*c*d^2)*e^3 - (b^2*c^3 - a^2*...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = -b^{\frac{3}{2}}\left(\frac{d\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bde-bcf)}-\frac{f\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{\sqrt{-b^2e^2+abef}(bde-bcf)}\right)$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
-b^(3/2)*(d*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*d*e - b*c*f)) - f*ar
ctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 +
 a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*(b*d*e - b*c*f)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 508, normalized size of antiderivative = 4.16

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)}\,dx = \frac{\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{b\,x^2+a}-\sqrt{d}\,\sqrt{b}\,x}{\sqrt{c}\,\sqrt{b}}\right)adef-\sqrt{c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{ad-bc}-\sqrt{d}\,\sqrt{b\,x^2+a}-\sqrt{d}\,\sqrt{b}\,x}{\sqrt{c}\,\sqrt{b}}\right)bd\,e^2$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e),x)
```

output

```
(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2)
- sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e*f - sqrt(c)*sqrt(a*d - b*c)*
atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqr
t(c)*sqrt(b)))*b*d*e**2 + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) +
sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*d*e*f -
 sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2)
+ sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b*d*e**2 - sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sq
rt(e)*sqrt(b)))*a*c*d*f + sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*b*c**2*f
- sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2)
 + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*c*d*f + sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sq
rt(e)*sqrt(b)))*b*c**2*f)/(c*e*(a**2*c*d*f**2 - a**2*d**2*e*f - a*b*c**2*f
**2 + a*b*d**2*e**2 + b**2*c**2*e*f - b**2*c*d*e**2))
```

3.332 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 204

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx &= \frac{f^2x\sqrt{a+bx^2}}{2e(be-af)(de-cf)\,(e+fx^2)}+\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\\&\quad-\frac{f(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{3/2}(de-cf)^2}\end{aligned}$$

output
```
1/2*f^2*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+d^2*arctanh((-
a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(1/2)/(-c*f+d
*e)^2-1/2*f*(2*b*e*(-c*f+2*d*e)-a*f*(-c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)
*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 498 vs. $2(204) = 408$.

Time = 12.79 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx$$

$$= \frac{-\frac{2\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}} - \frac{2c^{3/2}f^2\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}(de-cf)^2} + \frac{2c\sqrt{e}f^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{-be+af}(de-cf)^2} + \frac{f\left(\cdots\right.}{}}{2e\cdots}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
((-2*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-
(b*c) + a*d])])/(Sqrt[c]*Sqrt[-(b*c) + a*d]) - (2*c^(3/2)*f^2*ArcTan[(-(d*
x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(
Sqrt[-(b*c) + a*d]*(d*e - c*f)^2) + (2*c*Sqrt[e]*f^2*ArcTan[(-(f*x*Sqrt[a
 + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[-(b*
e) + a*f]*(d*e - c*f)^2) + (f*((e*(a*f*x*Sqrt[a + b*x^2] + a*Sqrt[b]*(e -
f*x^2) + 2*b*e*x*(Sqrt[b]*x - Sqrt[a + b*x^2])))/((-(b*e) + a*f)*(e + f*x^
2)*(a + 2*b*x^2 - 2*Sqrt[b]*x*Sqrt[a + b*x^2])) + (4*Sqrt[c]*ArcTan[(-(d*x
*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sq
rt[-(b*c) + a*d] + (Sqrt[e]*(4*b*e - 3*a*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2])
 + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(-(b*e) + a*f)^(3/2
)))/(-(d*e) + c*f))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {421, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{ 421}$$
$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{(de-cf)^2}$$
$$\Big\downarrow \text{ 291}$$
$$\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{(de-cf)^2}$$
$$\Big\downarrow \text{ 221}$$
$$\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{(de-cf)^2}$$
$$\Big\downarrow \text{ 402}$$
$$\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}$$
$$\Big\downarrow \text{ 291}$$
$$\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}$$
$$\Big\downarrow \text{ 221}$$
$$\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e(e+fx^2)(be-af)}\right)}{(de-cf)^2}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt
[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e
*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c*f))*Arc
Tanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f
)^(3/2))))/(d*e - c*f)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-(4bd\,e^2+f(-3ad-2bc)e+ac\,f^2)(f\,x^2+e)f\sqrt{(ad-bc)c}\,\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\left(-2d^2e(f\,x^2+e)(af-be)\arctan\left(\frac{c\sqrt{b\,x^2}}{x\sqrt{(ac}}\right.\right.}{2\sqrt{(ad-bc)c}\,\sqrt{(af-be)e}\,(cf-de)^2(af-be)e(f\,x^2+e)}$
default	Expression too large to display

input

```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/((a*d-b*c)*c)^(1/2)*(-(4*b*d*e^2+f*(-3*a*d-2*b*c)*e+a*c*f^2)*(f*x^2+e)
*f*((a*d-b*c)*c)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+(-2
*d^2*e*(f*x^2+e)*(a*f-b*e)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))
+(c*f-d*e)*(b*x^2+a)^(1/2)*x*f^2*((a*d-b*c)*c)^(1/2))*((a*f-b*e)*e)^(1/2))
/((a*f-b*e)*e)^(1/2)/(c*f-d*e)^2/(a*f-b*e)/e/(f*x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(178) = 356.

Time = 0.84 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = -\frac{1}{2}\left(\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{(b^2d^2e^2-2\,b^2cdef+b^2c^2f^2)\sqrt{-b^2c^2+abcd}} - \frac{(4\,bde^2f-2\,bcef^2-3\,adef^2+acf^3)\,\mathrm{arcta}}{(b^3d^2e^4-2\,b^3cde^3f-ab^2d^2e^3f+b^3c^2e^2f^2+2\,ab^2}\right.$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*(2*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt
(-b^2*c^2 + a*b*c*d)) - (4*b*d*e^2*f - 2*b*c*e*f^2 - 3*a*d*e*f^2 + a*c*f^3
)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e
^2 + a*b*e*f))/((b^3*d^2*e^4 - 2*b^3*c*d*e^3*f - a*b^2*d^2*e^3*f + b^3*c^2
*e^2*f^2 + 2*a*b^2*c*d*e^2*f^2 - a*b^2*c^2*e*f^3)*sqrt(-b^2*e^2 + a*b*e*f)
) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e*f - (sqrt(b)*x - sqrt(b*x^2 +
 a))^2*a*f^2 + a^2*f^2)/((b^3*d*e^3 - b^3*c*e^2*f - a*b^2*d*e^2*f + a*b^2*
c*e*f^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f)))*b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 3033, normalized size of antiderivative = 14.87

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*d**2*e**3*f**2 - 2*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt
(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*d**2*e**2*f**3*x**2 + 4*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a*b*d**2*e**4*f + 4*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(
c)*sqrt(b)))*a*b*d**2*e**3*f**2*x**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqr
t(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt
(b)))*b**2*d**2*e**5 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - s
qrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*b**2*d**2*
e**4*f*x**2 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sq
rt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*d**2*e**3*f**2
 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x*
*2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*d**2*e**2*f**3*x**2 + 4*s
qrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) +
sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*b*d**2*e**4*f + 4*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a*b*d**2*e**3*f**2*x**2 - 2*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/...
```

3.333 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx = \frac{f^2x\sqrt{a+bx^2}}{4e(be-af)(de-cf)\,(e+fx^2)^2} + \frac{f^2(2be(5de-3cf)-af(7de-3cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)^2\,(e+fx^2)} + \frac{d^3\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^3} - \frac{f(8b^2e^2(3d^2e^2-3cdef+c^2f^2)-4abef(9d^2e^2-7cdef+2c^2f^2)+a^2f^2(15d^2e^2-10cdef+3c^2f^2))\text{ a}}{8e^{5/2}(be-af)^{5/2}(de-cf)^3}$$

output

```
1/4*f^2*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2+1/8*f^2*(2*b
*e*(-3*c*f+5*d*e)-a*f*(-3*c*f+7*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)^2/(
-c*f+d*e)^2/(f*x^2+e)+d^3*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/
2))/c^(1/2)/(-a*d+b*c)^(1/2)/(-c*f+d*e)^3-1/8*f*(8*b^2*e^2*(c^2*f^2-3*c*d*
e*f+3*d^2*e^2)-4*a*b*e*f*(2*c^2*f^2-7*c*d*e*f+9*d^2*e^2)+a^2*f^2*(3*c^2*f^
2-10*c*d*e*f+15*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/
2))/e^(5/2)/(-a*f+b*e)^(5/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 11.49 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx$$
$$= \frac{\frac{4df(de-cf)x\left(f(a+bx^2)-\frac{(2be-af)(e+fx^2)\operatorname{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{e\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{e(be-af)\sqrt{a+bx^2}(e+fx^2)}+\frac{f(de-cf)^2x\left(ef(a+bx^2)(2be(4e+3fx^2)-af(5e+3fx^2))-\frac{(8b^2\cdots}{\cdots}\right)}{e^3(be-af)^2\sqrt{a+bx^2}\cdots}}{8(de-cf)^3}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
((4*d*f*(d*e - c*f)*x*(f*(a + b*x^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTanh[
Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(a
+ b*x^2))])))/(e*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + (f*(d*e - c*f)
^2*x*(e*f*(a + b*x^2)*(2*b*e*(4*e + 3*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*
b^2*e^2 - 8*a*b*e*f + 3*a^2*f^2)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x
^2)/(e*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(e^3*(b*e
 - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (8*d^3*ArcTanh[(Sqrt[b*c - a*d]
*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]) - (8*d^2*f*ArcTa
nh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f
]))/(8*(d*e - c*f)^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {421, 402, 25, 402, 25, 27, 291, 221, 407, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{(de-cf)^2}$$

$\Big\downarrow$ 402

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int \frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 402

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 25

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int \frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$\Big\downarrow$ 27

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 291$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 407$$

$$\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$$\Big\downarrow \ 291$$

$$\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

$\downarrow$ 221

$$\frac{d^2\left(\frac{d\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2(be-af)}\right)}{(de-cf)^2}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(d^2*((d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*
Sqrt[b*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqr
t[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(d*e - c*f)^2 - (f
*(-1/4*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)^2) - (
(f*(2*b*e*(5*d*e - 3*c*f) - a*f*(7*d*e - 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(
b*e - a*f)*(e + f*x^2)) - ((a^2*f^2*(7*d*e - 3*c*f) - 4*a*b*e*f*(5*d*e - 2
*c*f) + 8*b^2*e^2*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt
[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/(d*e - c
*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{5\left(\left(\left(\frac{12bd\,e^{3}}{5}-\frac{9\left(ad+\frac{8b\left(-\frac{5x^{2}d}{4}+c\right)}{9}\right)f\,e^{2}}{5}+\left(\left(-\frac{7x^{2}d}{5}+c\right)a-\frac{6x^{2}bc}{5}\right)f^{2}e+\frac{3ac\,f^{3}x^{2}}{5}\right)\sqrt{(af-be)e}\,(cf-de)xf\sqrt{b\,x^{2}+a}-\frac{3\arctan\left(\frac{\ }{x}\right.}{\ }\right.\right.}{\ }$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/8*((12/5*b*d*e^3-9/5*(a*d+8/9*b*(-5/4*x^2*d+c))*f*e^2+((-7/5*x^2*d+c)*a
-6/5*x^2*b*c)*f^2*e+3/5*a*c*f^3*x^2)*((a*f-b*e)*e)^(1/2)*(c*f-d*e)*x*f*(b*
x^2+a)^(1/2)-3/5*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))*(f*x^2+e)
^2*(8*b^2*d^2*e^4+(-12*a*b*d^2-8*b^2*c*d)*f*e^3+5*(a^2*d^2+28/15*a*b*c*d+8
/15*b^2*c^2)*f^2*e^2-10/3*(a*d+4/5*b*c)*a*c*f^3*e+a^2*c^2*f^4))*f*((a*d-b*
c)*c)^(1/2)+((a*f-b*e)*e)^(1/2)*d^3*e^2*(f*x^2+e)^2*(a*f-b*e)^2*arctan(c*(
b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/((a*d-b*c)*c)^(1/2)/((a*f-b*e)*e)^(
1/2)/(a*f-b*e)^2/(c*f-d*e)^3/(f*x^2+e)^2/e^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(315) = 630$.

Time = 1.92 (sec) , antiderivative size = 1344, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)\left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*(8*d^3*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e
*f^2 - b^3*c^3*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (24*b^2*d^2*e^4*f - 24*b^2
*c*d*e^3*f^2 - 36*a*b*d^2*e^3*f^2 + 8*b^2*c^2*e^2*f^3 + 28*a*b*c*d*e^2*f^3
 + 15*a^2*d^2*e^2*f^3 - 8*a*b*c^2*e*f^4 - 10*a^2*c*d*e*f^4 + 3*a^2*c^2*f^5
)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e
^2 + a*b*e*f))/((b^5*d^3*e^7 - 3*b^5*c*d^2*e^6*f - 2*a*b^4*d^3*e^6*f + 3*b
^5*c^2*d*e^5*f^2 + 6*a*b^4*c*d^2*e^5*f^2 + a^2*b^3*d^3*e^5*f^2 - b^5*c^3*e
^4*f^3 - 6*a*b^4*c^2*d*e^4*f^3 - 3*a^2*b^3*c*d^2*e^4*f^3 + 2*a*b^4*c^3*e^3
*f^4 + 3*a^2*b^3*c^2*d*e^3*f^4 - a^2*b^3*c^3*e^2*f^5)*sqrt(-b^2*e^2 + a*b*
e*f)) - 2*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^2*d*e^3*f^2 - 8*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*b^2*c*e^2*f^3 - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
*b*d*e^2*f^3 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*e*f^4 + 7*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*a^2*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2
*c*f^5 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^3*d*e^4*f - 48*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*b^3*c*e^3*f^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b
^2*d*e^3*f^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c*e^2*f^3 + 86*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*a^2*b*d*e^2*f^3 - 42*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^2*b*c*e*f^4 - 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*d*e*f^4 + 9
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*c*f^5 + 64*(sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 12786, normalized size of antiderivative = 37.06

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output

```
(16*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**
2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*d**3*e**5*f**4 + 32*sqrt(c
)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(
d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*d**3*e**4*f**5*x**2 + 16*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*d**3*e**3*f**6*x**4 - 80*sqrt(c)*sqrt(a*d
 - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)
*x)/(sqrt(c)*sqrt(b)))*a**3*b*d**3*e**6*f**3 - 160*sqrt(c)*sqrt(a*d - b*c)
*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sq
rt(c)*sqrt(b)))*a**3*b*d**3*e**5*f**4*x**2 - 80*sqrt(c)*sqrt(a*d - b*c)*at
an((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(
c)*sqrt(b)))*a**3*b*d**3*e**4*f**5*x**4 + 144*sqrt(c)*sqrt(a*d - b*c)*atan
((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)
*sqrt(b)))*a**2*b**2*d**3*e**7*f**2 + 288*sqrt(c)*sqrt(a*d - b*c)*atan((sq
rt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqr
t(b)))*a**2*b**2*d**3*e**6*f**3*x**2 + 144*sqrt(c)*sqrt(a*d - b*c)*atan((s
qrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sq
rt(b)))*a**2*b**2*d**3*e**5*f**4*x**4 - 112*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*s
qrt(b)))*a*b**3*d**3*e**8*f - 224*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*...
```

3.334 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx &= -\frac{d^3x\sqrt{a+bx^2}}{2c(bc-ad)(de-cf)^2\,(c+dx^2)}-\frac{f^3x\sqrt{a+bx^2}}{2e(be-af)(de-cf)^2\,(e+fx^2)}\\ &\quad-\frac{d^2(ad(de-5cf)-2bc(de-3cf))\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}(de-cf)^3}\\ &\quad+\frac{f^2(2be(3de-cf)-af(5de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{3/2}(de-cf)^3}\end{aligned}$$

output
```
-1/2*d^3*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)^2/(d*x^2+c)-1/2*f^3*x*(
b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2+e)-1/2*d^2*(a*d*(-5*c*f+d*
e)-2*b*c*(-3*c*f+d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))
/c^(3/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^3+1/2*f^2*(2*b*e*(-c*f+3*d*e)-a*f*(-c
*f+5*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a
*f+b*e)^(3/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 13.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx=$$

$$-\frac{\frac{d^2(de-cf)x\left(d(a+bx^2)-\frac{(2bc-ad)\left(c+dx^2\right)\operatorname{arctanh}\left(\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}\right)}{c\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}}\right)}{c(bc-ad)\sqrt{a+bx^2}(c+dx^2)}+\frac{f^2(de-cf)x\left(f(a+bx^2)-\frac{(2be-af)\left(e+fx^2\right)\operatorname{arctanh}\left(\sqrt{\frac{(be-\ldots}{e(a+\ldots}}\right.}{e\sqrt{\frac{(be-af)x^2}{e\left(a+bx^2\right)}}}\right.}{e(be-af)\sqrt{a+bx^2}(e+fx^2)}\ldots}{2(de-cf)^3}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
-1/2*((d^2*(d*e - c*f)*x*(d*(a + b*x^2) - ((2*b*c - a*d)*(c + d*x^2)*ArcTa
nh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c*Sqrt[((b*c - a*d)*x^2)/(c*
(a + b*x^2))])))/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) + (f^2*(d*e -
 c*f)*x*(f*(a + b*x^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTanh[Sqrt[((b*e - a
*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])))/
(e*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + (4*d^2*f*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]) - (4*d*f^2*
ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e
- a*f]))/(d*e - c*f)^3
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {426, 421, 25, 291, 221, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{de-cf}$$

$$\Big\downarrow \ 421$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}$$

$$\Big\downarrow \ 25$$

$$\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}$$

$$\Big\downarrow \ 291$$

$$\frac{d\left(\frac{f^2\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}$$

$$\Big\downarrow \ 221$$

$$\frac{d\left(\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}$$

$\Big\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{\int -\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{d\left(-\frac{\int \frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$\Big\downarrow$ 291

$$\frac{d\left(\frac{d\left(-\frac{(ad(de-3cf)-2bc(de-2cf))\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{d\left(-\frac{\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)(ad(de-3cf)-2bc(de-2cf))}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(de-cf)}{2c\left(c+dx^2\right)(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(d*((d*(-1/2*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2))
 - ((a*d*(d*e - 3*c*f) - 2*b*c*(d*e - 2*c*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(3/2))))/(d*e - c*f)^2
+ (f^2*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sq
rt[b*e - a*f]*(d*e - c*f)^2)))/(d*e - c*f) - (f*((d^2*ArcTanh[(Sqrt[b*c -
a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*f)^2
) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2))
 + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/
(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(d*e - c*f)^2)
)/(d*e - c*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{-5d^2\left(-\frac{6b\,c^2 f}{5}+d\left(af+\frac{2be}{5}\right)c-\frac{a\,d^2 e}{5}\right)(x^2 d+c)\sqrt{(af-be)e}\,(af-be)(f\,x^2+e)e\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\sqrt{(ad-bc)c}\left(-(ac\right.}{2\sqrt{(ac}}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*c)^(1/2)*(-5*d^2*(-6/5*b*c^2*f+d*(a*f+2/5*b*e)*c-1/5*a*d^2*
e)*(d*x^2+c)*((a*f-b*e)*e)^(1/2)*(a*f-b*e)*(f*x^2+e)*e*arctan(c*(b*x^2+a)^
(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(-(a*d-b*c)*c*((a*f^2-2*b
*e*f)*c-5*a*d*e*f+6*b*d*e^2)*(d*x^2+c)*(f*x^2+e)*f^2*arctan(e*(b*x^2+a)^(1
/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(c*f-d*e)*(b*x^2+a)^(1/2)*x
*(-b*c^3*f^3+d*f^3*(-b*x^2+a)*c^2+a*c*d^2*f^3*x^2+d^3*e*(f*x^2+e)*(a*f-b*e
))))/((a*f-b*e)*e)^(1/2)/(a*d-b*c)/(d*x^2+c)/(c*f-d*e)^3/c/(a*f-b*e)/e/(f*
x^2+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**2*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. $2(253) = 506$.

Time = 1.76 (sec) , antiderivative size = 1772, normalized size of antiderivative = 6.24

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-1/2*b^(7/2)*((2*b*c*d^3*e - a*d^4*e - 6*b*c^2*d^2*f + 5*a*c*d^3*f)*arctan
(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^4*c^2*d^3*e^3 - a*b^3*c*d^4*e^3 - 3*b^4*c^3*d^2*e^2*f + 3*a*b^3
*c^2*d^3*e^2*f + 3*b^4*c^4*d*e*f^2 - 3*a*b^3*c^3*d^2*e*f^2 - b^4*c^5*f^3 +
 a*b^3*c^4*d*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) + (6*b*d*e^2*f^2 - 2*b*c*e*f^3
 - 5*a*d*e*f^3 + a*c*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f +
2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^4*d^3*e^5 - 3*b^4*c*d^2*e^4*f -
 a*b^3*d^3*e^4*f + 3*b^4*c^2*d*e^3*f^2 + 3*a*b^3*c*d^2*e^3*f^2 - b^4*c^3*e
^2*f^3 - 3*a*b^3*c^2*d*e^2*f^3 + a*b^3*c^3*e*f^4)*sqrt(-b^2*e^2 + a*b*e*f)
) + 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^2*c*d^2*e^2*f - (sqrt(b)*x - sq
rt(b*x^2 + a))^6*a*b*d^3*e^2*f + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^2*c^2
*d*e*f^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d^2*e*f^2 + (sqrt(b)*x
- sqrt(b*x^2 + a))^6*a^2*d^3*e*f^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c
^2*d*f^3 + (sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*c*d^2*f^3 + 8*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*b^3*c*d^2*e^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2
*d^3*e^3 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c*d^2*e^2*f + 7*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^2*b*d^3*e^2*f + 8*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*b^3*c^3*e*f^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d*e*f^2 +
 8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2*e*f^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*a^3*d^3*e*f^2 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

3.335 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^2(e+fx^2)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 425

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3} dx \\
&= -\frac{d^4x\sqrt{a+bx^2}}{2c(bc-ad)(de-cf)^3\left(c+dx^2\right)} - \frac{f^3x\sqrt{a+bx^2}}{4e(be-af)(de-cf)^2\left(e+fx^2\right)^2} \\
&\quad - \frac{f^3(2be(7de-3cf)-af(11de-3cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)^3\left(e+fx^2\right)} \\
&\quad - \frac{d^3(ad(de-7cf)-2bc(de-4cf))\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}(de-cf)^4} \\
&\quad - \frac{f^2(8abef(10d^2e^2-5cdef+c^2f^2)-8b^2e^2(6d^2e^2-4cdef+c^2f^2)-a^2f^2(35d^2e^2-14cdef+3c^2f^2))}{8e^{5/2}(be-af)^{5/2}(de-cf)^4}
\end{aligned}$$

output

```
-1/2*d^4*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)^3/(d*x^2+c)-1/4*f^3*x*(
b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2+e)^2-1/8*f^3*(2*b*e*(-3*c*
f+7*d*e)-a*f*(-3*c*f+11*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*f+b*e)^2/(-c*f+d*e
)^3/(f*x^2+e)-1/2*d^3*(a*d*(-7*c*f+d*e)-2*b*c*(-4*c*f+d*e))*arctanh((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^
4-1/8*f^2*(8*a*b*e*f*(c^2*f^2-5*c*d*e*f+10*d^2*e^2)-8*b^2*e^2*(c^2*f^2-4*c
*d*e*f+6*d^2*e^2)-a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2))*arctanh((-a*f
+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(5/2)/(-c*f+d*e)
^4
```

Mathematica [A] (verified)

Time = 14.91 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx = -\frac{\frac{4d^3(de-cf)x\left(d(a+bx^2)-\frac{(2bc-ad)\left(c+dx^2\right)\operatorname{arctanh}\left(\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}\right)}{c\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}}\right)}{c(bc-ad)\sqrt{a+bx^2}(c+dx^2)}+\frac{8df^2(de-cf)x\left(f(a+bx^2)-\frac{(2be-af)\left(e+fx^2\right)\operatorname{arctanh}\left(\sqrt{\cdots}\right)}{e\sqrt{\frac{(be-af)x^2}{e\left(a+bx^2\right)}}}\right)}{e(be-af)\sqrt{a+bx^2}(e+fx^2)}\cdots}{\cdots}$$

input

```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output

```
-1/8*((4*d^3*(d*e - c*f)*x*(d*(a + b*x^2) - ((2*b*c - a*d)*(c + d*x^2)*Arc
Tanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c*Sqrt[((b*c - a*d)*x^2)/(
c*(a + b*x^2))])))/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) + (8*d*f^2*
(d*e - c*f)*x*(f*(a + b*x^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTanh[Sqrt[((b
*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2)
)])))/(e*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + (f^2*(d*e - c*f)^2*x*(
e*f*(a + b*x^2)*(2*b*e*(4*e + 3*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*b^2*e^
2 - 8*a*b*e*f + 3*a^2*f^2)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e
*(a + b*x^2))]])/Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(e^3*(b*e - a*f
)^2*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (24*d^3*f*ArcTanh[(Sqrt[b*c - a*d]*x)
/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]) - (24*d^2*f^2*ArcTa
nh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f
]))/(d*e - c*f)^4
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.97, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {426, 421, 402, 25, 402, 25, 27, 291, 221, 407, 291, 221, 426, 421, 25, 291, 221, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{421}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2} - \frac{f\left(-\frac{\int \frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)} - \frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{de-cf}$$

$\downarrow$ 402

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2} - \frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)} + \frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)} - \frac{fx \cdots}{4e \cdots} \cdots\right)}{(de-cf)^2} \cdots\right)}{de-cf}$$

$\downarrow$ 25

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2} - \frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)} - \frac{\int \frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)} - \frac{fx\sqrt{\cdots}}{4e(e \cdots} \cdots\right)}{(de-cf)^2} \cdots\right)}{de-cf}$$

$\downarrow$ 27

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-$$

$$\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int \frac{}{\sqrt{bx^2+c}}}{2e(be-af)}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{de-cf}$$

$\downarrow$ 291

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-$$

$$\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int \frac{1}{e-\frac{(be-}{bx^2}}}{2e(be-af)}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{de-cf}$$

$\downarrow$ 221

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf}-$$

$$\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+\right.}{2e^{3/2}(be-af)^{3/2}}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{de-cf}$$

$\downarrow$ 407

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2} - \frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2(7de-3\right.}{2e^{3/2}(be}}{4e(be-af)}\right.}{(de-cf)^2}}{de-cf}$$

$\downarrow$ 291

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2} - \frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2\right.}{}}{4e(be-af)}\right.}{(de-cf}}{de-cf}$$

$\downarrow$ 221

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{de-cf} - \frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2} - \frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2}}{4e(be-af)}\right.}{(de-cf)^2}}{de-cf}$$

$\downarrow$ 426

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-$$

$$\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2\cdots}\right.}{de-cf\cdots}$$

$\downarrow$ 421

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-$$

$$\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\right.}{2\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2\cdots}\right.}{de-cf\cdots}$$

$\downarrow$ 25

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-$$

$$\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^{\cdots}\right.}{2\cdots}}{4e(be-af)}\cdots\right)}{(de-cf)^{\cdots}}\cdots\right)}{de-cf}\cdots$$

$\Big\downarrow$ 291

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-$$

$$\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^{\cdots}\right.}{2\cdots}}{4e(be-af)}\cdots\right)}{(de-cf)^{\cdots}}\cdots\right)}{de-cf}\cdots$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-$$

$$\frac{f\left(\frac{d^2\left(\frac{d\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(a^2f^2\cdots\right.}{2\cdots}}{4e(be-af)}\cdots\right)}{(de-cf)^2\cdots}\right)}{de-cf}$$

$\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(\frac{\int -\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}}{2\cdots}\cdots\right)}{\cdots}\right)}{de-c\cdots}\right)}{de-cf}$$

$$\frac{f\left(\frac{d^2\left(\frac{d\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\cdots\right.}{\cdots}}{\cdots}\right)}{(de-cf)^2\cdots}\right)}{de-cf}$$

$\downarrow$ 25

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)(dx^2+c)}-\frac{\int\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\int\frac{2be(2de-}{\sqrt{bx^2}}\cdots\right)}{2\cdots}\right)}{de-c\cdots}\right)}{de-cf}$$

$$\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)(fx^2+e)}-\frac{(8b^2(2de\cdots}{\cdots}}{\cdots}\right)}{(de-cf)^{\cdots}}\right)}{de-cf}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\cdots\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\cdots\right.}{\cdots}\right.}{de-cf\cdots}$$

$$\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{de-cf\cdots}$$

$$\downarrow 291$$

$$d\left(\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-\cdots}}{\sqrt{c}\sqrt{bx\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf\cdots}\cdots\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad de-cf \qquad\cdots}$$

$$f\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{(de-cf)^{\cdots}}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad de-cf \qquad\cdots}$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-a}}{\sqrt{c}\sqrt{bx}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)}\right.}{}\right)}{de-cf}$$

$$\frac{f\left(\frac{d^2\left(\frac{d\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\right.}{}}{}\right.}{(de-cf)^2}\right)}{de-cf}$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
-((f*((d^2*((d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sq
rt[c]*Sqrt[b*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(d*e - c*f)^
2 - (f*(-1/4*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)^
2) - ((f*(2*b*e*(5*d*e - 3*c*f) - a*f*(7*d*e - 3*c*f))*x*Sqrt[a + b*x^2])/
(2*e*(b*e - a*f)*(e + f*x^2)) - ((a^2*f^2*(7*d*e - 3*c*f) - 4*a*b*e*f*(5*d
*e - 2*c*f) + 8*b^2*e^2*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e
]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/(d
*e - c*f)^2))/(d*e - c*f)) + (d*((d*((d*(-1/2*(d*(d*e - c*f)*x*Sqrt[a + b*
x^2])/(c*(b*c - a*d)*(c + d*x^2)) - ((a*d*(d*e - 3*c*f) - 2*b*c*(d*e - 2*c
*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b
*c - a*d)^(3/2))))/(d*e - c*f)^2 + (f^2*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f)^2)))/(d*e - c*f
) - (f*((d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt
[c]*Sqrt[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*
x^2])/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c
*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b
*e - a*f)^(3/2))))/(d*e - c*f)^2))/(d*e - c*f)))/(d*e - c*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.50

method	result
pseudoelliptic	$-\frac{3\left((ad-bc)\left(a^2c^2f^4-\frac{14ac\left(ad+\frac{4bc}{7}\right)e\,f^3}{3}+\frac{35\left(a^2d^2+\frac{8}{7}abcd+\frac{8}{35}b^2c^2\right)e^2f^2}{3}-\frac{80\left(ad+\frac{2bc}{5}\right)db\,e^3f}{3}+16b^2d^2e^4\right)c\left(x^2d+c\right)\sqrt{(a\right.}{}$
default	Expression too large to display

input

```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output

```
-3/8*((a*d-b*c)*(a^2*c^2*f^4-14/3*a*c*(a*d+4/7*b*c)*e*f^3+35/3*(a^2*d^2+8/
7*a*b*c*d+8/35*b^2*c^2)*e^2*f^2-80/3*(a*d+2/5*b*c)*d*b*e^3*f+16*b^2*d^2*e^
4)*c*(d*x^2+c)*((a*d-b*c)*c)^(1/2)*(f*x^2+e)^2*f^2*arctan(e*(b*x^2+a)^(1/2
)/x/((a*f-b*e)*e)^(1/2))-28/3*((a*f-b*e)*e)^(1/2)*(d^3*(d*x^2+c)*(a*f-b*e)
^2*(f*x^2+e)^2*(c*(a*d-8/7*b*c)*f-1/7*d*e*(a*d-2*b*c))*e^2*arctan(c*(b*x^2
+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+5/28*(-4/5*b^2*d^4*e^6+8/5*b*d^4*f*(-b*x^
2+a)*e^5-4/5*d^4*f^2*(b^2*x^4-4*a*b*x^2+a^2)*e^4-8/5*d*(2*b^2*c^3-2*b*d*(-
b*x^2+a)*c^2-2*a*b*c*d^2*x^2+a*d^3*x^2*(-b*x^2+a))*f^3*e^3-13/5*(-8/13*b^2
*c^4-5/13*(-6/5*b*x^2+a)*d*b*c^3+d^2*(14/13*b^2*x^4-19/13*a*b*x^2+a^2)*c^2
+a*d^3*x^2*(-14/13*b*x^2+a)*c+4/13*a^2*d^4*x^4)*f^4*e^2+(a*d-b*c)*((-6/5*b
*x^2+a)*c-11/5*a*d*x^2)*c*(d*x^2+c)*f^5*e+3/5*a*c^2*f^6*x^2*(d*x^2+c)*(a*d
-b*c))*(c*f-d*e)*((a*d-b*c)*c)^(1/2)*(b*x^2+a)^(1/2)*x))/((a*d-b*c)*c)^(1/
2)/((a*f-b*e)*e)^(1/2)/(a*f-b*e)^2/(f*x^2+e)^2/e^2/(c*f-d*e)^4/(a*d-b*c)/c
/(d*x^2+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(389) = 778$.

Time = 3.67 (sec) , antiderivative size = 1833, normalized size of antiderivative = 4.31

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*b^(9/2)*(4*(2*b*c*d^4*e - a*d^5*e - 8*b*c^2*d^3*f + 7*a*c*d^4*f)*arct
an(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a
*b*c*d))/((b^5*c^2*d^4*e^4 - a*b^4*c*d^5*e^4 - 4*b^5*c^3*d^3*e^3*f + 4*a*b
^4*c^2*d^4*e^3*f + 6*b^5*c^4*d^2*e^2*f^2 - 6*a*b^4*c^3*d^3*e^2*f^2 - 4*b^5
*c^5*d*e*f^3 + 4*a*b^4*c^4*d^2*e*f^3 + b^5*c^6*f^4 - a*b^4*c^5*d*f^4)*sqrt
(-b^2*c^2 + a*b*c*d)) + (48*b^2*d^2*e^4*f^2 - 32*b^2*c*d*e^3*f^3 - 80*a*b*
d^2*e^3*f^3 + 8*b^2*c^2*e^2*f^4 + 40*a*b*c*d*e^2*f^4 + 35*a^2*d^2*e^2*f^4
- 8*a*b*c^2*e*f^5 - 14*a^2*c*d*e*f^5 + 3*a^2*c^2*f^6)*arctan(1/2*((sqrt(b)
*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^6*d
^4*e^8 - 4*b^6*c*d^3*e^7*f - 2*a*b^5*d^4*e^7*f + 6*b^6*c^2*d^2*e^6*f^2 + 8
*a*b^5*c*d^3*e^6*f^2 + a^2*b^4*d^4*e^6*f^2 - 4*b^6*c^3*d*e^5*f^3 - 12*a*b^
5*c^2*d^2*e^5*f^3 - 4*a^2*b^4*c*d^3*e^5*f^3 + b^6*c^4*e^4*f^4 + 8*a*b^5*c^
3*d*e^4*f^4 + 6*a^2*b^4*c^2*d^2*e^4*f^4 - 2*a*b^5*c^4*e^3*f^5 - 4*a^2*b^4*
c^3*d*e^3*f^5 + a^2*b^4*c^4*e^2*f^6)*sqrt(-b^2*e^2 + a*b*e*f)) + 8*(2*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*b*c*d^3 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d^
4 + a^2*d^4)/((b^5*c^2*d^3*e^3 - a*b^4*c*d^4*e^3 - 3*b^5*c^3*d^2*e^2*f + 3
*a*b^4*c^2*d^3*e^2*f + 3*b^5*c^4*d*e*f^2 - 3*a*b^4*c^3*d^2*e*f^2 - b^5*c^5
*f^3 + a*b^4*c^4*d*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
 + 2*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^2*d*e^3*f^3 - 8*(sqrt(b)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\;(d\,x^2+c)^2\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

3.336 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 566

$$
\begin{aligned}
&\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx \\
&= -\frac{d^4x\sqrt{a+bx^2}}{4c(bc-ad)(de-cf)^3\,(c+dx^2)^2} + \frac{3d^4(ad(de-5cf)-2bc(de-3cf))x\sqrt{a+bx^2}}{8c^2(bc-ad)^2(de-cf)^4\,(c+dx^2)} \\
&\quad + \frac{f^4x\sqrt{a+bx^2}}{4e(be-af)(de-cf)^3\,(e+fx^2)^2} + \frac{3f^4(2be(3de-cf)-af(5de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)^4\,(e+fx^2)} \\
&\quad + \frac{d^3(8b^2c^2(d^2e^2-5cdef+10c^2f^2)+3a^2d^2(d^2e^2-6cdef+21c^2f^2)-4abcd(2d^2e^2-13cdef+35c^2f^2)}{8c^{5/2}(bc-ad)^{5/2}(de-cf)^5} \\
&\quad - \frac{f^3(3a^2f^2(21d^2e^2-6cdef+c^2f^2)+8b^2e^2(10d^2e^2-5cdef+c^2f^2)-4abef(35d^2e^2-13cdef+2c^2f^2}{8e^{5/2}(be-af)^{5/2}(de-cf)^5}
\end{aligned}
$$

output
```
-1/4*d^4*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)^3/(d*x^2+c)^2+3/8*d^4*(
a*d*(-5*c*f+d*e)-2*b*c*(-3*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)^2/(-
c*f+d*e)^4/(d*x^2+c)+1/4*f^4*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)^3/(
f*x^2+e)^2+3/8*f^4*(2*b*e*(-c*f+3*d*e)-a*f*(-c*f+5*d*e))*x*(b*x^2+a)^(1/2)
/e^2/(-a*f+b*e)^2/(-c*f+d*e)^4/(f*x^2+e)+1/8*d^3*(8*b^2*c^2*(10*c^2*f^2-5*
c*d*e*f+d^2*e^2)+3*a^2*d^2*(21*c^2*f^2-6*c*d*e*f+d^2*e^2)-4*a*b*c*d*(35*c^
2*f^2-13*c*d*e*f+2*d^2*e^2))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^
(1/2))/c^(5/2)/(-a*d+b*c)^(5/2)/(-c*f+d*e)^5-1/8*f^3*(3*a^2*f^2*(c^2*f^2-6
*c*d*e*f+21*d^2*e^2)+8*b^2*e^2*(c^2*f^2-5*c*d*e*f+10*d^2*e^2)-4*a*b*e*f*(2
*c^2*f^2-13*c*d*e*f+35*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2
+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(5/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 17.27 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx$$

$$= \frac{3d^3fx\left(d(a+bx^2)-\frac{(2bc-ad)(c+dx^2)\operatorname{arctanh}\left(\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}\right)}{c\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}}\right)}{2c(bc-ad)(de-cf)^4\sqrt{a+bx^2}\,(c+dx^2)}$$
$$-\frac{d^3x\left(cd(a+bx^2)\,(2bc(4c+3dx^2)-ad(5c+3dx^2))-\frac{(8b^2c^2-8abcd+3a^2d^2)(c+dx^2)^2\operatorname{arctanh}\left(\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}\right)}{\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}}}\right)}{8c^3(bc-ad)^2(de-cf)^3\sqrt{a+bx^2}\,(c+dx^2)^2}$$
$$+\frac{3df^3x\left(f(a+bx^2)-\frac{(2be-af)(e+fx^2)\operatorname{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{e\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{2e(be-af)(de-cf)^4\sqrt{a+bx^2}\,(e+fx^2)}$$
$$+\frac{f^3x\left(ef(a+bx^2)\,(2be(4e+3fx^2)-af(5e+3fx^2))-\frac{(8b^2e^2-8abef+3a^2f^2)(e+fx^2)^2\operatorname{arctanh}\left(\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}\right)}{\sqrt{\frac{(be-af)x^2}{e(a+bx^2)}}}\right)}{8e^3(be-af)^2(de-cf)^3\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$+\frac{6d^3f^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^5}-\frac{6d^2f^3\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^5}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(3*d^3*f*x*(d*(a + b*x^2) - ((2*b*c - a*d)*(c + d*x^2)*ArcTanh[Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))]])/(c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))])
))/(2*c*(b*c - a*d)*(d*e - c*f)^4*Sqrt[a + b*x^2]*(c + d*x^2)) - (d^3*x*(c
*d*(a + b*x^2)*(2*b*c*(4*c + 3*d*x^2) - a*d*(5*c + 3*d*x^2)) - ((8*b^2*c^2
 - 8*a*b*c*d + 3*a^2*d^2)*(c + d*x^2)^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*
(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(8*c^3*(b*c - a*
d)^2*(d*e - c*f)^3*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (3*d*f^3*x*(f*(a + b*x
^2) - ((2*b*e - a*f)*(e + f*x^2)*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*
x^2))]])/(e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])))/(2*e*(b*e - a*f)*(d
*e - c*f)^4*Sqrt[a + b*x^2]*(e + f*x^2)) + (f^3*x*(e*f*(a + b*x^2)*(2*b*e*
(4*e + 3*f*x^2) - a*f*(5*e + 3*f*x^2)) - ((8*b^2*e^2 - 8*a*b*e*f + 3*a^2*f
^2)*(e + f*x^2)^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/Sqrt[(
(b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(8*e^3*(b*e - a*f)^2*(d*e - c*f)^3*Sqr
t[a + b*x^2]*(e + f*x^2)^2) + (6*d^3*f^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt
[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*f)^5) - (6*d^2*f^
3*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*
e - a*f]*(d*e - c*f)^5)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1697 vs. $2(566) = 1132$.

Time = 2.32 (sec) , antiderivative size = 1697, normalized size of antiderivative = 3.00, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {426, 426, 421, 25, 402, 25, 402, 25, 27, 291, 221, 407, 291, 221, 426, 421, 25, 291, 221, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}$$

$$\Big\downarrow \text{426}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{de-cf}\right)}{de-cf}$$

$$\Big\downarrow \text{421}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\int -\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx\right.}}{de-cf}\right.}{de-cf}$$

$$\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de}{4e\left(e+fx^2\right)^2(b}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{de-cf}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{\int\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx\cdots\right.}}{de-cf}\cdots\right.}{de-cf}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de\cdots}{4e\left(e+fx^2\right)^2(b\cdots}\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{de-cf}$$

$\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{d\left(-\frac{\frac{\int -\frac{8b^2(de-2cf)c^2-4abd(2de-5cf)c+a^2d^2(3de-7cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\int \frac{\cdots}{\sqrt{bx^2+a}\left(d\cdots\right.}}{(de-\cdots}\cdots\right)}{de-cf}\cdots\right)}{de-cf}\cdots$$

$$\frac{f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int -\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a\cdots}}{\cdots}}{4e(be-af)}\cdots\right)}{(de-cf)^2\cdots}\right)}{de-cf}\cdots\right)}{de-cf}$$

$\downarrow$ 25

$$d\left(\frac{d\left(\frac{d\left(-\frac{-\frac{\int \frac{8b^2(de-2cf)c^2-4abd(2de-5cf)c+a^2d^2(3de-7cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right)}{(de-cf)^2}+\frac{f^2\int \frac{}{\sqrt{bx^2+a}\left(d\right.}}{(de-}}{de-cf}\right)}{de-cf}$$

$$f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int \frac{8b^2(2de-cf)e^2-4a}{\sqrt{b}}}{}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\overline{\qquad de-cf \qquad}$$

↓ 27

$$d\left(\frac{d\left(\frac{d\left(-\frac{-\frac{\left(a^2d^2(3de-7cf)-4abcd(2de-5cf)+8b^2c^2(de-2cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-cf)}{4c\left(c+dx^2\right)^2(bc-ad)}\right.}{(de-cf)^2}}{de-cf}\right.}{de-cf}$$

$$f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4ab\right.}{}}{4e(be-af)}\right.}{}\right.}{de-cf}\right.}{de-cf}$$

↓ 291

$$\frac{d\left(\frac{d\left(\frac{d\left(-\frac{-\frac{\left(a^2d^2(3de-7cf)-4abcd(2de-5cf)+8b^2c^2(de-2cf)\right)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf)-2bc(3de-5cf))}{2c\left(c+dx^2\right)(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^2}(de-\cdots}{4c\left(c+dx^2\right)^2(bc-\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{de-cf}$$

$$\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4ab\cdots\right.}{\cdots}}{4e(be-a\cdots}\cdots\right)}{\cdots}\right)}{de-cf}\cdots\right)}{de-cf}$$

↓ 221

$$
\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(-\frac{-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)\left(a^2d^2(3de-7cf)-4abcd(2de-5cf)+8b^2c^2(de-2cf)\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}(ad(3de-7cf\cdots}{2c\left(c+dx^2\right)(b\cdots}}{4c(bc-ad)}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{de-cf}\cdots
$$

$$
\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-\cdots}}{\sqrt{e}\sqrt{a+\cdots}}\right)\cdots}{\cdots}}{4e(be-a\cdots}\cdots\right)}{\cdots}\right)}{de-cf}\cdots\right)}{de-cf}\cdots
$$

↓ 407

$$d\left(\frac{d\left(\frac{\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+ax}}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-2cf)c^2 \cdots\right.}{\cdots}}{4c(b \cdots}\right.}{(de-cf)^2}\right)}{de-cf}\right.}{de-cf}$$

$$f\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-\cdots}{2e(be-\cdots}}{\cdots}\right.}{\cdots}\right.}{de-\cdots}\right.}{de-cf}$$

↓ 291

$$d\left(\frac{d\left(\frac{\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(d\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf\cdots}\right.}{de-cf}\cdots\right.}{de-cf}$$

$$f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de\cdots}{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{de-cf}$$

$\downarrow$ 221

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}}{de-cf}\cdots\right.}{de-cf\cdots}$$

$$f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-\cdots}{\cdots}\right.}{\cdots}\right.}{d\cdots}\right.}{de-cf\cdots}$$

↓ 426

$$\frac{d\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\right.}{}}{}\right.}{(de-cf)^2}\right)}{de-cf}}{de}$$

$$\frac{f\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}\right.}{}\right.}{de}$$

$$\downarrow 421$$

$$\frac{d\left(\frac{d\left(\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+ax}}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right)}{de-cf\cdots}\cdots}{f\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\operatorname{arctan}\cdots}{\sqrt{c}\sqrt{b\cdots}}\right.}{\cdots}\right.}{\cdots}\right)}$$

↓ 25

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{\cdots}$$

$$\frac{f\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\text{arctanh}\cdots}{\sqrt{c}\sqrt{bc-\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

↓ 291

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{de-cf}\cdots\right.}{\cdots}$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\mathrm{ar}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

↓ 221

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{de-cf\cdots}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}\,(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d^2\left(\frac{d\,\text{arc}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 402

$$
d\left(\frac{d\left(\frac{\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.
$$

$$
f\left(\frac{d\left(\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(\frac{\int -\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}}{2\cdots}\right.}{\cdots}\right.}{de-c\cdots}\right)}{de-cf}\cdots\right.
$$

$$\Big\downarrow \text{25}$$

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}\,(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}\,(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\int\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)^2}-\frac{f\left(\int\frac{2be(2de\cdots}{\sqrt{bx^2\cdots}}\cdots\right.}{2\cdots}\right.}{de-c\cdots}\right)}{de-cf}\cdots\right.$$

$\Big\downarrow$ 27

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{4c(bc-ad)(dx^2+c)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}x}{2c(bc-ad)(dx^2+c)}-\frac{\left(8b^2(de-\right.}{}}{}\right.}{(de-cf)^2}}{de-cf}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)(dx^2+c)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}\right)-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\right.}{}}{de-cf}\right.$$

$\Big\downarrow$ 291

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+ax}}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots}{\cdots}}{\cdots}\right)}{(de-cf)^2}\right)}{de-cf}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+ax}}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-\cdots}}{\sqrt{c}\sqrt{bx\cdots}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf\cdots}\right.}{\cdots}}{de-cf}\right.$$

↓ 221

$$d\left(\frac{d\left(\frac{\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)f^2}{(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{4c(bc-ad)\left(dx^2+c\right)^2}-\frac{-\frac{d(ad(3de-7cf)-2bc(3de-5cf))\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\left(8b^2(de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2\cdots}\right.}{de-cf\cdots}\right.$$

$$f\left(\frac{d\left(\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}\,x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{2c^{3/2}(bc-ad)^{3/2}}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\mathrm{arctanh}\left(\frac{\sqrt{bc-\cdots}}{\sqrt{c}\sqrt{bx\cdots}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)\cdots}\right.}{\cdots}\right.}{de-cf\cdots}\right.$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
-((f*(-((f*((d^2*((d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/
(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(d*e -
 c*f)^2 - (f*(-1/4*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f
*x^2)^2) - ((f*(2*b*e*(5*d*e - 3*c*f) - a*f*(7*d*e - 3*c*f))*x*Sqrt[a + b*
x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - ((a^2*f^2*(7*d*e - 3*c*f) - 4*a*b*e*
f*(5*d*e - 2*c*f) + 8*b^2*e^2*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(
Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f)
)))/(d*e - c*f)^2))/(d*e - c*f)) + (d*((d*((d*(-1/2*(d*(d*e - c*f)*x*Sqrt[
a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) - ((a*d*(d*e - 3*c*f) - 2*b*c*(d*e
 - 2*c*f))*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3
/2)*(b*c - a*d)^(3/2))))/(d*e - c*f)^2 + (f^2*ArcTanh[(Sqrt[b*e - a*f]*x)/
(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f)^2)))/(d*e
 - c*f) - (f*((d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])
/(Sqrt[c]*Sqrt[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[
a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d
*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3
/2)*(b*e - a*f)^(3/2))))/(d*e - c*f)^2))/(d*e - c*f)))/(d*e - c*f)))/(d*e
- c*f)) + (d*((d*((d*(-1/4*(d*(d*e - c*f)*x*Sqrt[a + b*x^2])/(c*(b*c - a*d
)*(c + d*x^2)^2) - (-1/2*(d*(a*d*(3*d*e - 7*c*f) - 2*b*c*(3*d*e - 5*c*f...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. 2(526) = 1052.

Time = 17.01 (sec) , antiderivative size = 1168, normalized size of antiderivative = 2.06

method	result	size
pseudoelliptic	Expression too large to display	1168
default	Expression too large to display	4322

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8/((a*d-b*c)*c)^(1/2)*(-21*((a*f-b*e)*e)^(1/2)*d^3*(80/63*b^2*c^4*f^2-2
0/9*d*(a*f+2/7*b*e)*b*f*c^3+d^2*(a^2*f^2+52/63*a*b*f*e+8/63*b^2*e^2)*c^2-2
/7*a*(a*f+4/9*b*e)*d^3*e*c+1/21*a^2*e^2*d^4)*(d*x^2+c)^2*(a*f-b*e)^2*(f*x^
2+e)^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)^2*c^
2*(d*x^2+c)^2*(f*x^2+e)^2*f^3*((21*a^2*e^2*f^2-140/3*a*b*e^3*f+80/3*b^2*e^
4)*d^2-6*d*(a^2*f^2-26/9*a*b*f*e+20/9*b^2*e^2)*f*e*c+f^2*(a^2*f^2-8/3*a*b*
f*e+8/3*b^2*e^2)*c^2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-5/3*
((a*f-b*e)*e)^(1/2)*(c*f-d*e)*(b*x^2+a)^(1/2)*(3/5*a*d^7*e^3*x^2*(f*x^2+e)
^2*(a*f-b*e)^2+d^6*(a*f-b*e)^2*(f*x^2+e)^2*((-6/5*b*x^2+a)*e-3*a*f*x^2)*e^
2*c-17/5*d^5*(8/17*b^3*e^6+1/17*b^2*f*(-2*b*x^2+a)*e^5-26/17*b*f^2*(14/13*
b^2*x^4-19/13*a*b*x^2+a^2)*e^4+f^3*(-18/17*b^3*x^6+73/17*a*b^2*x^4-70/17*a
^2*b*x^2+a^3)*e^3+2*a*f^4*x^2*(18/17*b^2*x^4-41/17*a*b*x^2+a^2)*e^2+2*a^2*
(-18/17*b*x^2+a)*x^4*f^5*e+15/17*a^3*f^6*x^6)*e*c^2-34/5*d^4*(-10/17*b^3*e
^6+20/17*b^2*f*(-b*x^2+a)*e^5-10/17*b*f^2*(b^2*x^4-4*a*b*x^2+a^2)*e^4-40/1
7*a*b*f^3*x^2*(-b*x^2+a)*e^3+a*f^4*x^2*(18/17*b^2*x^4-41/17*a*b*x^2+a^2)*e
^2+25/34*a^2*(-24/25*b*x^2+a)*x^4*f^5*e-3/34*a^3*f^6*x^6)*f*c^3-17/5*d^3*f
^4*(-20/17*b*(b^2*x^4-4*a*b*x^2+a^2)*e^3+f*(-18/17*b^3*x^6+73/17*a*b^2*x^4
-70/17*a^2*b*x^2+a^3)*e^2+5/17*a*(3/5*b^2*x^4-38/5*a*b*x^2+a^2)*x^2*f^2*e-
6/17*a^2*x^4*f^3*(-b*x^2+a))*c^4+c^5*f^4*(3/5*a*f^3*x^2*(b^2*x^4-4*a*b*x^2
+a^2)+f^2*(4/5*a^2*b*x^2-6/5*b^3*x^6-1/5*a*b^2*x^4+a^3)*e+26/5*b*f*(14/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11237 vs. 2(527) = 1054.

Time = 7.70 (sec) , antiderivative size = 11237, normalized size of antiderivative = 19.85

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-1/8*b^(11/2)*((8*b^2*c^2*d^5*e^2 - 8*a*b*c*d^6*e^2 + 3*a^2*d^7*e^2 - 40*b
^2*c^3*d^4*e*f + 52*a*b*c^2*d^5*e*f - 18*a^2*c*d^6*e*f + 80*b^2*c^4*d^3*f^
2 - 140*a*b*c^3*d^4*f^2 + 63*a^2*c^2*d^5*f^2)*arctan(1/2*((sqrt(b)*x - sqr
t(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^7*c^4*d^5*e
^5 - 2*a*b^6*c^3*d^6*e^5 + a^2*b^5*c^2*d^7*e^5 - 5*b^7*c^5*d^4*e^4*f + 10*
a*b^6*c^4*d^5*e^4*f - 5*a^2*b^5*c^3*d^6*e^4*f + 10*b^7*c^6*d^3*e^3*f^2 - 2
0*a*b^6*c^5*d^4*e^3*f^2 + 10*a^2*b^5*c^4*d^5*e^3*f^2 - 10*b^7*c^7*d^2*e^2*
f^3 + 20*a*b^6*c^6*d^3*e^2*f^3 - 10*a^2*b^5*c^5*d^4*e^2*f^3 + 5*b^7*c^8*d*
e*f^4 - 10*a*b^6*c^7*d^2*e*f^4 + 5*a^2*b^5*c^6*d^3*e*f^4 - b^7*c^9*f^5 + 2
*a*b^6*c^8*d*f^5 - a^2*b^5*c^7*d^2*f^5)*sqrt(-b^2*c^2 + a*b*c*d)) - (80*b^
2*d^2*e^4*f^3 - 40*b^2*c*d*e^3*f^4 - 140*a*b*d^2*e^3*f^4 + 8*b^2*c^2*e^2*f
^5 + 52*a*b*c*d*e^2*f^5 + 63*a^2*d^2*e^2*f^5 - 8*a*b*c^2*e*f^6 - 18*a^2*c*
d*e*f^6 + 3*a^2*c^2*f^7)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2
*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^7*d^5*e^9 - 5*b^7*c*d^4*e^8*f -
2*a*b^6*d^5*e^8*f + 10*b^7*c^2*d^3*e^7*f^2 + 10*a*b^6*c*d^4*e^7*f^2 + a^2*
b^5*d^5*e^7*f^2 - 10*b^7*c^3*d^2*e^6*f^3 - 20*a*b^6*c^2*d^3*e^6*f^3 - 5*a^
2*b^5*c*d^4*e^6*f^3 + 5*b^7*c^4*d*e^5*f^4 + 20*a*b^6*c^3*d^2*e^5*f^4 + 10*
a^2*b^5*c^2*d^3*e^5*f^4 - b^7*c^5*e^4*f^5 - 10*a*b^6*c^4*d*e^4*f^5 - 10*a^
2*b^5*c^3*d^2*e^4*f^5 + 2*a*b^6*c^5*e^3*f^6 + 5*a^2*b^5*c^4*d*e^3*f^6 - a^
2*b^5*c^5*e^2*f^7)*sqrt(-b^2*e^2 + a*b*e*f)) + 2*(8*(sqrt(b)*x - sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^3\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\left(f\,x^2+e\right)^3}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.337 $\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)(be-af)^3x}{ab^4\sqrt{a+bx^2}} + \frac{f(19a^2df^2+24b^2e(de+cf)-14abf(3de+cf))\,x\sqrt{a+bx^2}}{16b^4} + \frac{f^2(18bde+6bcf-11adf)x^3\sqrt{a+bx^2}}{24b^3} + \frac{df^3x^5\sqrt{a+bx^2}}{6b^2} - \frac{(35a^3df^3+72ab^2ef(de+cf)-30a^2bf^2(3de+cf)-16b^3e^2(de+3cf))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}$$

output
```
(-a*d+b*c)*(-a*f+b*e)^3*x/a/b^4/(b*x^2+a)^(1/2)+1/16*f*(19*a^2*d*f^2+24*b^
2*e*(c*f+d*e)-14*a*b*f*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)/b^4+1/24*f^2*(-11*a*
d*f+6*b*c*f+18*b*d*e)*x^3*(b*x^2+a)^(1/2)/b^3+1/6*d*f^3*x^5*(b*x^2+a)^(1/2
)/b^2-1/16*(35*a^3*d*f^3+72*a*b^2*e*f*(c*f+d*e)-30*a^2*b*f^2*(c*f+3*d*e)-1
6*b^3*e^2*(3*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{b}x\left(48b^4ce^3+105a^4df^3+5a^3bf^2\left(-54de-18cf+7dfx^2\right)+2a^2b^2f\left(3cf\left(36e-5fx^2\right)+d\left(108e^2-45efx^2-7f\right.\right.\right.}{a\sqrt{a+bx^2}}}{}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(48*b^4*c*e^3 + 105*a^4*d*f^3 + 5*a^3*b*f^2*(-54*d*e - 18*c*f
+ 7*d*f*x^2) + 2*a^2*b^2*f*(3*c*f*(36*e - 5*f*x^2) + d*(108*e^2 - 45*e*f*x
^2 - 7*f^2*x^4)) + 4*a*b^3*(3*c*f*(-12*e^2 + 6*e*f*x^2 + f^2*x^4) + d*(-12
*e^3 + 18*e^2*f*x^2 + 9*e*f^2*x^4 + 2*f^3*x^6))))/(a*Sqrt[a + b*x^2]) + 3*
(35*a^3*d*f^3 + 72*a*b^2*e*f*(d*e + c*f) - 30*a^2*b*f^2*(3*d*e + c*f) - 16
*b^3*e^2*(d*e + 3*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {401, 25, 403, 25, 403, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 401$$

$$\frac{x(e+fx^2)^3(bc-ad)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{(fx^2+e)^2(ade-(6bc-7ad)fx^2)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(fx^2+e)^2(ade-(6bc-7ad)fx^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(e+fx^2)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int -\frac{\left(fx^2+e\right)\left(f\left(35dfa^2-2b(17de+15cf)a+24b^2ce\right)x^2+ae(7adf-6b(de+cf))\right)}{\sqrt{bx^2+a}}dx}{6b}-\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)^2(6bc-7ad)}{6b}}{ab}+\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\left(fx^2+e\right)\left(f\left(35dfa^2-2b(17de+15cf)a+24b^2ce\right)x^2+ae(7adf-6b(de+cf))\right)}{\sqrt{bx^2+a}}dx}{6b}-\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)^2(6bc-7ad)}{6b}}{ab}+\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 403$$

$$-\frac{\frac{\int -\frac{ae\left(24e(de+2cf)b^2-2af(31de+15cf)b+35a^2df^2\right)-f\left(-105df^2a^3+10bf(20de+9cf)a^2-4b^2e(23de+39cf)a+48b^3ce^2\right)x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df \cdots\right.}{4b}}{6b} \cdots}{ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df-30abcf-34abde+24b^2ce\right)}{4b}-\frac{\int \frac{ae\left(24e(de+2cf)b^2-2af(31de+15cf)b+35a^2df^2\right)-f\left(-105df^2a^3+10bf(20de+9cf)a^2-4b^2e(23de \cdots\right.}{\sqrt{bx^2+a}}}{4b}}{6b} \cdots}{ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 299$$

$$-\frac{\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df-30abcf-34abde+24b^2ce\right)}{4b}-\frac{-\frac{3a\left(35a^3df^3-30a^2bf^2(cf+3de)+72ab^2ef(cf+de)-16b^3e^2(3cf+de)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{fx\sqrt{a+bx^2} \cdots}{4b}}{6b} \cdots}{ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{-\frac{\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df-30abcf-34abde+24b^2ce\right)}{4b}-\frac{-\frac{3a\left(35a^3df^3-30a^2bf^2(cf+3de)+72ab^2ef(cf+de)-16b^3e^2(3cf+de)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{f}{\ldots}}{4b}}{6b}}{ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

$\downarrow$ 219

$$\frac{-\frac{\frac{fx\sqrt{a+bx^2}\left(e+fx^2\right)\left(35a^2df-30abcf-34abde+24b^2ce\right)}{4b}-\frac{-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(35a^3df^3-30a^2bf^2(cf+3de)+72ab^2ef(cf+de)-16b^3e^2(3cf+de)\right)}{2b^{3/2}}-\frac{f}{\ldots}}{4b}}{6b}}{ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*(e + f*x^2)^3)/(a*b*Sqrt[a + b*x^2]) + (-1/6*((6*b*c - 7*a*
d)*f*x*Sqrt[a + b*x^2]*(e + f*x^2)^2)/b - ((f*(24*b^2*c*e - 34*a*b*d*e - 3
0*a*b*c*f + 35*a^2*d*f)*x*Sqrt[a + b*x^2]*(e + f*x^2))/(4*b) - (-1/2*(f*(4
8*b^3*c*e^2 - 105*a^3*d*f^2 + 10*a^2*b*f*(20*d*e + 9*c*f) - 4*a*b^2*e*(23*
d*e + 39*c*f))*x*Sqrt[a + b*x^2])/b - (3*a*(35*a^3*d*f^3 + 72*a*b^2*e*f*(d
*e + c*f) - 30*a^2*b*f^2*(3*d*e + c*f) - 16*b^3*e^2*(d*e + 3*c*f))*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b))/(6*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-\frac{35a\left(a^3 d f^3-\frac{6a^2 b f^2(cf+3de)}{7}+\frac{72a b^2 e f(cf+de)}{35}-\frac{48\left(cf+\frac{de}{3}\right)b^3e^2}{35}\right)\sqrt{b x^2+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)}{16}+\left(\frac{35a^4 d f^3}{16}-\frac{15\left(-\frac{7}{18}df x^2\right.}{}\right.}{}$
risch	$\frac{fx\left(8x^4 d f^2 b^2-22abd f^2x^2+12b^2c f^2x^2+36b^2def x^2+57a^2d f^2-42abc f^2-126abdef+72b^2cef+72b^2d e^2\right)\sqrt{b x^2+a}}{48b^4}-\frac{b(}{}$
default	$\frac{c e^3 x}{a\sqrt{b x^2+a}}+f^2(cf+3de)\left(\frac{x^5}{4b\sqrt{b x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+3ef$

input

```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/(b*x^2+a)^(1/2)*(-35/16*a*(a^3*d*f^3-6/7*a^2*b*f^2*(c*f+3*d*e)+72/35*a*b
^2*e*f*(c*f+d*e)-48/35*(c*f+1/3*d*e)*b^3*e^2)*(b*x^2+a)^(1/2)*arctanh((b*x
^2+a)^(1/2)/x/b^(1/2))+(35/16*a^4*d*f^3-15/8*(-7/18*d*f*x^2+c*f+3*d*e)*b*f
^2*a^3+9/2*b^2*f*(-7/108*d*f^2*x^4-5/36*f*(c*f+3*d*e)*x^2+c*e*f+d*e^2)*a^2
-3*(-1/18*d*f^3*x^6-1/12*f^2*(c*f+3*d*e)*x^4-1/2*e*f*(c*f+d*e)*x^2+e^2*(c*
f+1/3*d*e))*b^3*a+b^4*c*e^3)*x*b^(1/2))/b^(9/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 864, normalized size of antiderivative = 3.53

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*a^2*b^3*d*e^3 + 24*(2*a^2*b^3*c - 3*a^3*b^2*d)*e^2*f - 18*(4
*a^3*b^2*c - 5*a^4*b*d)*e*f^2 + 5*(6*a^4*b*c - 7*a^5*d)*f^3 + (16*a*b^4*d*
e^3 + 24*(2*a*b^4*c - 3*a^2*b^3*d)*e^2*f - 18*(4*a^2*b^3*c - 5*a^3*b^2*d)*
e*f^2 + 5*(6*a^3*b^2*c - 7*a^4*b*d)*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*a*b^4*d*f^3*x^7 + 2*(18*a*b^4*d*e*f^2 +
 (6*a*b^4*c - 7*a^2*b^3*d)*f^3)*x^5 + (72*a*b^4*d*e^2*f + 18*(4*a*b^4*c -
5*a^2*b^3*d)*e*f^2 - 5*(6*a^2*b^3*c - 7*a^3*b^2*d)*f^3)*x^3 + 3*(16*(b^5*c
 - a*b^4*d)*e^3 - 24*(2*a*b^4*c - 3*a^2*b^3*d)*e^2*f + 18*(4*a^2*b^3*c - 5
*a^3*b^2*d)*e*f^2 - 5*(6*a^3*b^2*c - 7*a^4*b*d)*f^3)*x)*sqrt(b*x^2 + a))/(
a*b^6*x^2 + a^2*b^5), -1/48*(3*(16*a^2*b^3*d*e^3 + 24*(2*a^2*b^3*c - 3*a^3
*b^2*d)*e^2*f - 18*(4*a^3*b^2*c - 5*a^4*b*d)*e*f^2 + 5*(6*a^4*b*c - 7*a^5*
d)*f^3 + (16*a*b^4*d*e^3 + 24*(2*a*b^4*c - 3*a^2*b^3*d)*e^2*f - 18*(4*a^2*
b^3*c - 5*a^3*b^2*d)*e*f^2 + 5*(6*a^3*b^2*c - 7*a^4*b*d)*f^3)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*a*b^4*d*f^3*x^7 + 2*(18*a*b^4*d*
e*f^2 + (6*a*b^4*c - 7*a^2*b^3*d)*f^3)*x^5 + (72*a*b^4*d*e^2*f + 18*(4*a*b
^4*c - 5*a^2*b^3*d)*e*f^2 - 5*(6*a^2*b^3*c - 7*a^3*b^2*d)*f^3)*x^3 + 3*(16
*(b^5*c - a*b^4*d)*e^3 - 24*(2*a*b^4*c - 3*a^2*b^3*d)*e^2*f + 18*(4*a^2*b^
3*c - 5*a^3*b^2*d)*e*f^2 - 5*(6*a^3*b^2*c - 7*a^4*b*d)*f^3)*x)*sqrt(b*x^2
+ a))/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [F]

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)*(e + f*x**2)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{df^3x^7}{6\sqrt{bx^2+a}b} - \frac{7\,adf^3x^5}{24\sqrt{bx^2+a}b^2} + \frac{35\,a^2df^3x^3}{48\sqrt{bx^2+a}b^3} + \frac{(3\,def^2+cf^3)x^5}{4\sqrt{bx^2+a}b} + \frac{ce^3x}{\sqrt{bx^2+a}a} + \frac{35\,a^3df^3x}{16\sqrt{bx^2+a}b^4} - \frac{35\,a^3df^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}} - \frac{5\,(3\,def^2+cf^3)ax^3}{8\sqrt{bx^2+a}b^2} + \frac{3\,(de^2f+cef^2)x^3}{2\sqrt{bx^2+a}b} - \frac{15\,(3\,def^2+cf^3)a^2x}{8\sqrt{bx^2+a}b^3} + \frac{9\,(de^2f+cef^2)ax}{2\sqrt{bx^2+a}b^2} - \frac{(de^3+3\,ce^2f)x}{\sqrt{bx^2+a}b} + \frac{15\,(3\,def^2+cf^3)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{9\,(de^2f+cef^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{(de^3+3\,ce^2f)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*d*f^3*x^7/(sqrt(b*x^2 + a)*b) - 7/24*a*d*f^3*x^5/(sqrt(b*x^2 + a)*b^2)
 + 35/48*a^2*d*f^3*x^3/(sqrt(b*x^2 + a)*b^3) + 1/4*(3*d*e*f^2 + c*f^3)*x^5
/(sqrt(b*x^2 + a)*b) + c*e^3*x/(sqrt(b*x^2 + a)*a) + 35/16*a^3*d*f^3*x/(sq
rt(b*x^2 + a)*b^4) - 35/16*a^3*d*f^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/8*
(3*d*e*f^2 + c*f^3)*a*x^3/(sqrt(b*x^2 + a)*b^2) + 3/2*(d*e^2*f + c*e*f^2)*
x^3/(sqrt(b*x^2 + a)*b) - 15/8*(3*d*e*f^2 + c*f^3)*a^2*x/(sqrt(b*x^2 + a)*
b^3) + 9/2*(d*e^2*f + c*e*f^2)*a*x/(sqrt(b*x^2 + a)*b^2) - (d*e^3 + 3*c*e^
2*f)*x/(sqrt(b*x^2 + a)*b) + 15/8*(3*d*e*f^2 + c*f^3)*a^2*arcsinh(b*x/sqrt
(a*b))/b^(7/2) - 9/2*(d*e^2*f + c*e*f^2)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2)
+ (d*e^3 + 3*c*e^2*f)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,df^3x^2}{b}+\frac{18\,ab^6def^2+6\,ab^6cf^3-7\,a^2b^5df^3}{ab^7}\right)x^2+\frac{72\,ab^6de^2f+72\,ab^6cef^2-90\,a^2b^5def^2-30\,a\cdots}{ab^7}\cdots\right.\right.}{\cdots}$$

$$-\frac{(16\,b^3de^3+48\,b^3ce^2f-72\,ab^2de^2f-72\,ab^2cef^2+90\,a^2bdef^2+30\,a^2bcf^3-35\,a^3df^3)\log\left(\left|-\sqrt{b}x+\sqrt{b\cdots}\right.\right.}{16\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*d*f^3*x^2/b + (18*a*b^6*d*e*f^2 + 6*a*b^6*c*f^3 - 7*a^2*b^5*d*
f^3)/(a*b^7))*x^2 + (72*a*b^6*d*e^2*f + 72*a*b^6*c*e*f^2 - 90*a^2*b^5*d*e*
f^2 - 30*a^2*b^5*c*f^3 + 35*a^3*b^4*d*f^3)/(a*b^7))*x^2 + 3*(16*b^7*c*e^3
- 16*a*b^6*d*e^3 - 48*a*b^6*c*e^2*f + 72*a^2*b^5*d*e^2*f + 72*a^2*b^5*c*e*
f^2 - 90*a^3*b^4*d*e*f^2 - 30*a^3*b^4*c*f^3 + 35*a^4*b^3*d*f^3)/(a*b^7))*x
/sqrt(b*x^2 + a) - 1/16*(16*b^3*d*e^3 + 48*b^3*c*e^2*f - 72*a*b^2*d*e^2*f
- 72*a*b^2*c*e*f^2 + 90*a^2*b*d*e*f^2 + 30*a^2*b*c*f^3 - 35*a^3*d*f^3)*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 31.96 (sec) , antiderivative size = 1098, normalized size of antiderivative = 4.48

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(3/2),x)
```

output
```
(840*sqrt(a + b*x**2)*a**4*b*d*f**3*x - 720*sqrt(a + b*x**2)*a**3*b**2*c*f
**3*x - 2160*sqrt(a + b*x**2)*a**3*b**2*d*e*f**2*x + 280*sqrt(a + b*x**2)*
a**3*b**2*d*f**3*x**3 + 1728*sqrt(a + b*x**2)*a**2*b**3*c*e*f**2*x - 240*s
qrt(a + b*x**2)*a**2*b**3*c*f**3*x**3 + 1728*sqrt(a + b*x**2)*a**2*b**3*d*
e**2*f*x - 720*sqrt(a + b*x**2)*a**2*b**3*d*e*f**2*x**3 - 112*sqrt(a + b*x
**2)*a**2*b**3*d*f**3*x**5 - 1152*sqrt(a + b*x**2)*a*b**4*c*e**2*f*x + 576
*sqrt(a + b*x**2)*a*b**4*c*e*f**2*x**3 + 96*sqrt(a + b*x**2)*a*b**4*c*f**3
*x**5 - 384*sqrt(a + b*x**2)*a*b**4*d*e**3*x + 576*sqrt(a + b*x**2)*a*b**4
*d*e**2*f*x**3 + 288*sqrt(a + b*x**2)*a*b**4*d*e*f**2*x**5 + 64*sqrt(a + b
*x**2)*a*b**4*d*f**3*x**7 + 384*sqrt(a + b*x**2)*b**5*c*e**3*x - 840*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d*f**3 + 720*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*f**3 + 2160*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*e*f**2 - 840*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*f**3*x**2 - 1728*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*e*f**2 + 720*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*f**3*x**2 - 1
728*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d*e**2*f
 + 2160*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d*e*
f**2*x**2 + 1152*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b**3*c*e**2*f - 1728*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)...
```

3.338 $\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 164

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)(be-af)^2x}{ab^3\sqrt{a+bx^2}} + \frac{f(8bde+4bcf-7adf)x\sqrt{a+bx^2}}{8b^3} + \frac{df^2x^3\sqrt{a+bx^2}}{4b^2} + \frac{(15a^2df^2-12abf(2de+cf)+8b^2e(de+2cf))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output

```
(-a*d+b*c)*(-a*f+b*e)^2*x/a/b^3/(b*x^2+a)^(1/2)+1/8*f*(-7*a*d*f+4*b*c*f+8*
b*d*e)*x*(b*x^2+a)^(1/2)/b^3+1/4*d*f^2*x^3*(b*x^2+a)^(1/2)/b^2+1/8*(15*a^2
*d*f^2-12*a*b*f*(c*f+2*d*e)+8*b^2*e*(2*c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{b}x(8b^3ce^2-15a^3df^2+a^2bf(24de+12cf-5dfx^2)+2ab^2(2cf(-4e+fx^2)+d(-4e^2+4efx^2+f^2x^4)))}{a\sqrt{a+bx^2}}+(}{8b^{7/}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(8*b^3*c*e^2 - 15*a^3*d*f^2 + a^2*b*f*(24*d*e + 12*c*f - 5*d*f
*x^2) + 2*a*b^2*(2*c*f*(-4*e + f*x^2) + d*(-4*e^2 + 4*e*f*x^2 + f^2*x^4)))
)/(a*Sqrt[a + b*x^2]) + (-15*a^2*d*f^2 + 12*a*b*f*(2*d*e + c*f) - 8*b^2*e*
(d*e + 2*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {401, 25, 403, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(e+fx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{(fx^2+e)(ade-(4bc-5ad)fx^2)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(fx^2+e)(ade-(4bc-5ad)fx^2)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(e+fx^2)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int -\frac{f\left(15dfa^2-2b(7de+6cf)a+8b^2ce\right)x^2+ae(5adf-4b(de+cf))}{\sqrt{bx^2+a}}dx}{4b}-\frac{fx\sqrt{a+bx^2}(e+fx^2)(4bc-5ad)}{4b}}{ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{f\left(15dfa^2-2b(7de+6cf)a+8b^2ce\right)x^2+ae(5adf-4b(de+cf))}{\sqrt{bx^2+a}}dx}{4b}-\frac{fx\sqrt{a+bx^2}(e+fx^2)(4bc-5ad)}{4b}}{ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 299

$$\frac{-\frac{\frac{fx\sqrt{a+bx^2}\left(15a^2df-12abcf-14abde+8b^2ce\right)}{2b}-\frac{a\left(15a^2df^2-12abf(cf+2de)+8b^2e(2cf+de)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}}{4b}-\frac{fx\sqrt{a+bx^2}(e+fx^2)(4bc-5ad)}{4b}}{ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 224

$$\frac{-\frac{\frac{fx\sqrt{a+bx^2}\left(15a^2df-12abcf-14abde+8b^2ce\right)}{2b}-\frac{a\left(15a^2df^2-12abf(cf+2de)+8b^2e(2cf+de)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}}{4b}-\frac{fx\sqrt{a+bx^2}(e+fx^2)(4bc-5ad)}{4b}}{ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

$\Big\downarrow$ 219

$$\frac{-\frac{\frac{fx\sqrt{a+bx^2}\left(15a^2df-12abcf-14abde+8b^2ce\right)}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2df^2-12abf(cf+2de)+8b^2e(2cf+de)\right)}{2b^{3/2}}}{4b}-\frac{fx\sqrt{a+bx^2}(e+fx^2)(4bc-5ad)}{4b}}{ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
((b*c - a*d)*x*(e + f*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (-1/4*((4*b*c - 5*a*
d)*f*x*Sqrt[a + b*x^2]*(e + f*x^2))/b - ((f*(8*b^2*c*e - 14*a*b*d*e - 12*a
*b*c*f + 15*a^2*d*f)*x*Sqrt[a + b*x^2])/(2*b) - (a*(15*a^2*d*f^2 - 12*a*b*
f*(2*d*e + c*f) + 8*b^2*e*(d*e + 2*c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(3/2)))/(4*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{15a\left(a^{2} d f^{2}-\frac{4abf(cf+2de)}{5}+\frac{16\left(cf+\frac{de}{2}\right)b^{2}e}{15}\right)\sqrt{b\,x^{2}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{8}+\sqrt{b}\,x\left(-\frac{15a^{3}d\,f^{2}}{8}+\frac{3bf\left(-\frac{5}{12}df\,x^{2}+cf+2de\right)a^{2}}{2}-2(\right.}{\sqrt{b\,x^{2}+a}\,a\,b^{\frac{7}{2}}}$
risch	$-\frac{fx\left(-2bdf\,x^{2}+7adf-4bcf-8bde\right)\sqrt{b\,x^{2}+a}}{8b^{3}}+\frac{b\left(15a^{2}d\,f^{2}-12abc\,f^{2}-24abdef+16b^{2}cef+8b^{2}d\,e^{2}\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln(\sqrt{}}{}\right.}{8b^{3}}$
default	$\frac{c\,e^{2}x}{a\sqrt{b\,x^{2}+a}}+f(cf+2de)\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+e(2cf+de)\left(-\frac{}{b\sqrt{}}\right.$

input
```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(15/8*a*(a^2*d*f^2-4/5*a*b*f*(c*f+2*d*e)+16/15*(c*f+1/2*d*e)*b^2*e)*(b*x^2
+a)^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+b^(1/2)*x*(-15/8*a^3*d*f^2+3/
2*b*f*(-5/12*d*f*x^2+c*f+2*d*e)*a^2-2*(-1/8*d*f^2*x^4-1/4*f*(c*f+2*d*e)*x^
2+e*(c*f+1/2*d*e))*b^2*a+b^3*c*e^2))/(b*x^2+a)^(1/2)/a/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.45

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \Bigg[\frac{(8a^2b^2de^2 + 8(2a^2b^2c - 3a^3bd)ef - 3(4a^3bc - 5a^4d)f^2 + (8ab^3de^2 + 8(2\ldots}{}$$

$$-\frac{(8a^2b^2de^2 + 8(2a^2b^2c - 3a^3bd)ef - 3(4a^3bc - 5a^4d)f^2 + (8ab^3de^2 + 8(2ab^3c - 3a^2b^2d)ef - 3(4a^2b\ldots}{}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((8*a^2*b^2*d*e^2 + 8*(2*a^2*b^2*c - 3*a^3*b*d)*e*f - 3*(4*a^3*b*c -
 5*a^4*d)*f^2 + (8*a*b^3*d*e^2 + 8*(2*a*b^3*c - 3*a^2*b^2*d)*e*f - 3*(4*a^
2*b^2*c - 5*a^3*b*d)*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(2*a*b^3*d*f^2*x^5 + (8*a*b^3*d*e*f + (4*a*b^3*c - 5*a^2*
b^2*d)*f^2)*x^3 + (8*(b^4*c - a*b^3*d)*e^2 - 8*(2*a*b^3*c - 3*a^2*b^2*d)*e
*f + 3*(4*a^2*b^2*c - 5*a^3*b*d)*f^2)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2
*b^4), -1/8*((8*a^2*b^2*d*e^2 + 8*(2*a^2*b^2*c - 3*a^3*b*d)*e*f - 3*(4*a^3
*b*c - 5*a^4*d)*f^2 + (8*a*b^3*d*e^2 + 8*(2*a*b^3*c - 3*a^2*b^2*d)*e*f - 3
*(4*a^2*b^2*c - 5*a^3*b*d)*f^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (2*a*b^3*d*f^2*x^5 + (8*a*b^3*d*e*f + (4*a*b^3*c - 5*a^2*b^2*d)*f
^2)*x^3 + (8*(b^4*c - a*b^3*d)*e^2 - 8*(2*a*b^3*c - 3*a^2*b^2*d)*e*f + 3*(
4*a^2*b^2*c - 5*a^3*b*d)*f^2)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [F]

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)*(e + f*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{df^2x^5}{4\sqrt{bx^2+a}b} - \frac{5\,adf^2x^3}{8\sqrt{bx^2+a}b^2} + \frac{ce^2x}{\sqrt{bx^2+a}a} - \frac{15\,a^2df^2x}{8\sqrt{bx^2+a}b^3} + \frac{(2\,def+cf^2)x^3}{2\sqrt{bx^2+a}b} + \frac{15\,a^2df^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} + \frac{3\,(2\,def+cf^2)ax}{2\sqrt{bx^2+a}b^2} - \frac{(de^2+2\,cef)x}{\sqrt{bx^2+a}b} - \frac{3\,(2\,def+cf^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{(de^2+2\,cef)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*d*f^2*x^5/(sqrt(b*x^2 + a)*b) - 5/8*a*d*f^2*x^3/(sqrt(b*x^2 + a)*b^2)
+ c*e^2*x/(sqrt(b*x^2 + a)*a) - 15/8*a^2*d*f^2*x/(sqrt(b*x^2 + a)*b^3) + 1
/2*(2*d*e*f + c*f^2)*x^3/(sqrt(b*x^2 + a)*b) + 15/8*a^2*d*f^2*arcsinh(b*x/
sqrt(a*b))/b^(7/2) + 3/2*(2*d*e*f + c*f^2)*a*x/(sqrt(b*x^2 + a)*b^2) - (d*
e^2 + 2*c*e*f)*x/(sqrt(b*x^2 + a)*b) - 3/2*(2*d*e*f + c*f^2)*a*arcsinh(b*x
/sqrt(a*b))/b^(5/2) + (d*e^2 + 2*c*e*f)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,df^2x^2}{b} + \frac{8\,ab^4def+4\,ab^4cf^2-5\,a^2b^3df^2}{ab^5}\right)x^2 + \frac{8\,b^5ce^2-8\,ab^4de^2-16\,ab^4cef+24\,a^2b^3def+12\,a^2b}{ab^5}\right.}{8\sqrt{bx^2+a}} - \frac{(8\,b^2de^2+16\,b^2cef-24\,abdef-12\,abcf^2+15\,a^2df^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*d*f^2*x^2/b + (8*a*b^4*d*e*f + 4*a*b^4*c*f^2 - 5*a^2*b^3*d*f^2)/(a
*b^5))*x^2 + (8*b^5*c*e^2 - 8*a*b^4*d*e^2 - 16*a*b^4*c*e*f + 24*a^2*b^3*d*
e*f + 12*a^2*b^3*c*f^2 - 15*a^3*b^2*d*f^2)/(a*b^5))*x/sqrt(b*x^2 + a) - 1/
8*(8*b^2*d*e^2 + 16*b^2*c*e*f - 24*a*b*d*e*f - 12*a*b*c*f^2 + 15*a^2*d*f^2
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx=\int \frac{(d\,x^2+c)\,\left(f\,x^2+e\right)^2}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 710, normalized size of antiderivative = 4.33

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**3*b*d*f**2*x + 12*sqrt(a + b*x**2)*a**2*b**2*c*
f**2*x + 24*sqrt(a + b*x**2)*a**2*b**2*d*e*f*x - 5*sqrt(a + b*x**2)*a**2*b
**2*d*f**2*x**3 - 16*sqrt(a + b*x**2)*a*b**3*c*e*f*x + 4*sqrt(a + b*x**2)*
a*b**3*c*f**2*x**3 - 8*sqrt(a + b*x**2)*a*b**3*d*e**2*x + 8*sqrt(a + b*x**
2)*a*b**3*d*e*f*x**3 + 2*sqrt(a + b*x**2)*a*b**3*d*f**2*x**5 + 8*sqrt(a +
b*x**2)*b**4*c*e**2*x + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**4*d*f**2 - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**3*b*c*f**2 - 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
3*b*d*e*f + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*
d*f**2*x**2 + 16*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b**2*c*e*f - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
**2*c*f**2*x**2 + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*2*b**2*d*e**2 - 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*2*b**2*d*e*f*x**2 + 16*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a*b**3*c*e*f*x**2 + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a*b**3*d*e**2*x**2 - 10*sqrt(b)*a**4*d*f**2 + 9*sqrt(b)*a**3*b*c*f**2 +
18*sqrt(b)*a**3*b*d*e*f - 10*sqrt(b)*a**3*b*d*f**2*x**2 - 16*sqrt(b)*a**2*
b**2*c*e*f + 9*sqrt(b)*a**2*b**2*c*f**2*x**2 - 8*sqrt(b)*a**2*b**2*d*e**2
+ 18*sqrt(b)*a**2*b**2*d*e*f*x**2 + 8*sqrt(b)*a*b**3*c*e**2 - 16*sqrt(b)*a
*b**3*c*e*f*x**2 - 8*sqrt(b)*a*b**3*d*e**2*x**2 + 8*sqrt(b)*b**4*c*e**2...
```

3.339 $\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)(be-af)x}{ab^2\sqrt{a+bx^2}} + \frac{dfx\sqrt{a+bx^2}}{2b^2} - \frac{(3adf-2b(de+cf))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
(-a*d+b*c)*(-a*f+b*e)*x/a/b^2/(b*x^2+a)^(1/2)+1/2*d*f*x*(b*x^2+a)^(1/2)/b^
2-1/2*(3*a*d*f-2*b*(c*f+d*e))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{x(2b^2ce-2abde-2abcf+3a^2df+abdfx^2)}{2ab^2\sqrt{a+bx^2}} + \frac{(-2bde-2bcf+3adf)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(x*(2*b^2*c*e - 2*a*b*d*e - 2*a*b*c*f + 3*a^2*d*f + a*b*d*f*x^2))/(2*a*b^2
*Sqrt[a + b*x^2]) + ((-2*b*d*e - 2*b*c*f + 3*a*d*f)*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {401, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(c+dx^2)(be-af)}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{acf-d(2be-3af)x^2}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acf-d(2be-3af)x^2}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{-\frac{a(3adf-2b(cf+de))\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}(2be-3af)}{2b}}{ab}+\frac{x(c+dx^2)(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{-\frac{a(3adf-2b(cf+de))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}(2be-3af)}{2b}}{ab}+\frac{x(c+dx^2)(be-af)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3adf-2b(cf+de))}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}(2be-3af)}{2b}}{ab}+\frac{x(c+dx^2)(be-af)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2))/(a*b*Sqrt[a + b*x^2]) + (-1/2*(d*(2*b*e - 3*a*
f)*x*Sqrt[a + b*x^2])/b - (a*(3*a*d*f - 2*b*(d*e + c*f))*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{-\frac{3a\left(adf-\frac{2b(cf+de)}{3}\right)\sqrt{b\,x^{2}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{2}+\sqrt{b}\,x\left(\frac{3a^{2}df}{2}-\left(-\frac{1}{2}df\,x^{2}+cf+de\right)ba+ce\,b^{2}\right)}{\sqrt{b\,x^{2}+a}\,b^{\frac{5}{2}}\,a}$
risch	$\frac{dfx\sqrt{b\,x^{2}+a}}{2b^{2}}-\frac{b(3adf-2bcf-2bde)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{adfx}{\sqrt{b\,x^{2}+a}}-\frac{2ce\,b^{2}x}{a\sqrt{b\,x^{2}+a}}}{2b^{2}}$
default	$\frac{cex}{a\sqrt{b\,x^{2}+a}}+(cf+de)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+df\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\right.}{}\right.}{2b}\right.$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(-3/2*a*(a*d*f-2/3*b*(c*f+d*e))*(b*x^2+a)^(1/2)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))+b^(1/2)*x*(3/2*a^2*d*f-(-1/2*d*f*x^2+c*f+d*e)*b
*a+c*e*b^2))/b^(5/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.21

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{(2\,a^2bde+(2\,ab^2de+(2\,ab^2c-3\,a^2bd)f)x^2+(2\,a^2bc-3\,a^3d)f)\sqrt{b}\log\left(-\right.}{}\right.$$

$$-\frac{(2\,a^2bde+(2\,ab^2de+(2\,ab^2c-3\,a^2bd)f)x^2+(2\,a^2bc-3\,a^3d)f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(ab^2dfx^3+(2\,(}{2\,(ab^4x^2+a^2b^3)}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*a^2*b*d*e + (2*a*b^2*d*e + (2*a*b^2*c - 3*a^2*b*d)*f)*x^2 + (2*a
^2*b*c - 3*a^3*d)*f)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(a*b^2*d*f*x^3 + (2*(b^3*c - a*b^2*d)*e - (2*a*b^2*c - 3*a^2*b*d)*f
)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/2*((2*a^2*b*d*e + (2*a*b^2
*d*e + (2*a*b^2*c - 3*a^2*b*d)*f)*x^2 + (2*a^2*b*c - 3*a^3*d)*f)*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d*f*x^3 + (2*(b^3*c - a*b^2*d)
*e - (2*a*b^2*c - 3*a^2*b*d)*f)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(90) = 180$.

Time = 5.66 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = cf\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + de\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + df\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{cex}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x**2+c)*(f*x**2+e)/(b*x**2+a)**(3/2),x)
```

output
```
c*f*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
 + d*e*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a
))) + d*f*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/s
qrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + c*e*x/(a**
(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{dfx^3}{2\sqrt{bx^2+a}b} + \frac{cex}{\sqrt{bx^2+a}a} + \frac{3\,adfx}{2\sqrt{bx^2+a}b^2} - \frac{3\,adf\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{(de+cf)x}{\sqrt{bx^2+a}b} + \frac{(de+cf)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d*f*x^3/(sqrt(b*x^2 + a)*b) + c*e*x/(sqrt(b*x^2 + a)*a) + 3/2*a*d*f*x/
(sqrt(b*x^2 + a)*b^2) - 3/2*a*d*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - (d*e +
c*f)*x/(sqrt(b*x^2 + a)*b) + (d*e + c*f)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{dfx^2}{b} + \frac{2\,b^3ce-2\,ab^2de-2\,ab^2cf+3\,a^2bdf}{ab^3}\right)x}{2\sqrt{bx^2+a}} - \frac{(2\,bde+2\,bcf-3\,adf)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(d*f*x^2/b + (2*b^3*c*e - 2*a*b^2*d*e - 2*a*b^2*c*f + 3*a^2*b*d*f)/(a*
b^3))*x/sqrt(b*x^2 + a) - 1/2*(2*b*d*e + 2*b*c*f - 3*a*d*f)*log(abs(-sqrt(
b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.72

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{12\sqrt{b\,x^2+a}\,a^2bdfx - 8\sqrt{b\,x^2+a}\,a\,b^2cfx - 8\sqrt{b\,x^2+a}\,a\,b^2dex + 4\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(3/2),x)
```

output
```
(12*sqrt(a + b*x**2)*a**2*b*d*f*x - 8*sqrt(a + b*x**2)*a*b**2*c*f*x - 8*sq
rt(a + b*x**2)*a*b**2*d*e*x + 4*sqrt(a + b*x**2)*a*b**2*d*f*x**3 + 8*sqrt(
a + b*x**2)*b**3*c*e*x - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqr
t(a))*a**3*d*f + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
2*b*c*f + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*e
 - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*f*x**2
+ 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*f*x**2 +
8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*d*e*x**2 + 9*
sqrt(b)*a**3*d*f - 8*sqrt(b)*a**2*b*c*f - 8*sqrt(b)*a**2*b*d*e + 9*sqrt(b)
*a**2*b*d*f*x**2 + 8*sqrt(b)*a*b**2*c*e - 8*sqrt(b)*a*b**2*c*f*x**2 - 8*sq
rt(b)*a*b**2*d*e*x**2 + 8*sqrt(b)*b**3*c*e*x**2)/(8*a*b**3*(a + b*x**2))
```

3.340 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \frac{(bc-ad)x}{a(be-af)\sqrt{a+bx^2}} + \frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}$$

output
```
(-a*d+b*c)*x/a/(-a*f+b*e)/(b*x^2+a)^(1/2)+(-c*f+d*e)*arctanh((-a*f+b*e)^(1
/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \frac{(-bc+ad)x}{a(-be+af)\sqrt{a+bx^2}} + \frac{(de-cf)\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(-be+af)^{3/2}}$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((-(b*c) + a*d)*x)/(a*(-(b*e) + a*f)*Sqrt[a + b*x^2]) + ((d*e - c*f)*ArcTa
n[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*
f])])/(Sqrt[e]*(-(b*e) + a*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}dx$$
$$\Big\downarrow \text{402}$$
$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int -\frac{a(de-cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{a(be-af)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{a(de-cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}$$
$$\Big\downarrow \text{27}$$
$$\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}$$
$$\Big\downarrow \text{291}$$
$$\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}$$
$$\Big\downarrow \text{221}$$
$$\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) + ((d*e - c*f)*ArcTanh[(Sq
rt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*(b*e - a*f)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{\frac{(ad-bc)x}{\sqrt{b\,x^2+a}}-\frac{(cf-de)a\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{\sqrt{(af-be)e}}}{(af-be)a}$
default	$\frac{dx}{fa\sqrt{b\,x^2+a}}-\frac{(cf-de)\left(\frac{f}{(af-be)\sqrt{\left(x+\frac{\sqrt{-ef}}{f}\right)^2 b-\frac{2b\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{af-be}{f}}}+\frac{2b\sqrt{-ef}\left(2b\left(x+\frac{\sqrt{-e}}{f}\right.\right.}{(af-be)\left(\frac{4b(af-be)}{f}+\frac{4b^2e}{f}\right)\sqrt{\left(x+\frac{\sqrt{-ef}}{f}\right.}}\right.}{}$

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/(a*f-b*e)*((a*d-b*c)*x/(b*x^2+a)^(1/2)-(c*f-d*e)*a/((a*f-b*e)*e)^(1/2)*a
rctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2)))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(80) = 160.

Time = 0.99 (sec) , antiderivative size = 509, normalized size of antiderivative = 5.53

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}\,(e+fx^2)}\,dx = \left[\frac{4\left((b^2c-abd)e^2-(abc-a^2d)ef\right)\sqrt{bx^2+a}\,x+\left(a^2de-a^2cf+(abde-ab\right.}{4\left(a^2b^2e^3-2\,a^3be^2f\right.}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(4*((b^2*c - a*b*d)*e^2 - (a*b*c - a^2*d)*e*f)*sqrt(b*x^2 + a)*x + (a
^2*d*e - a^2*c*f + (a*b*d*e - a*b*c*f)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^
2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2
 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2
*x^4 + 2*e*f*x^2 + e^2)))/(a^2*b^2*e^3 - 2*a^3*b*e^2*f + a^4*e*f^2 + (a*b^
3*e^3 - 2*a^2*b^2*e^2*f + a^3*b*e*f^2)*x^2), 1/2*(2*((b^2*c - a*b*d)*e^2 -
 (a*b*c - a^2*d)*e*f)*sqrt(b*x^2 + a)*x - (a^2*d*e - a^2*c*f + (a*b*d*e -
a*b*c*f)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e
 - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a
^2*e*f)*x)))/(a^2*b^2*e^3 - 2*a^3*b*e^2*f + a^4*e*f^2 + (a*b^3*e^3 - 2*a^2
*b^2*e^2*f + a^3*b*e*f^2)*x^2)]
```

Sympy [F]

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{c+dx^2}{(a+bx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)/((a + b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.43

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \frac{(bc-ad)x}{(abe-a^2f)\sqrt{bx^2+a}} + \frac{(bd^2e^2-2\,bcdef+bc^2f^2)\arctan\left(-\frac{2\,b^{\frac{3}{2}}de^2-2\,b^{\frac{3}{2}}cef-a\sqrt{b}def+a\sqrt{b}cf^2+\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\sqrt{b}de-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\sqrt{b}cf\right)f}{2\left(\sqrt{-be^2+aef}bde-\sqrt{-be^2+aef}bcf\right)}\right.}{(bde-bcf)\sqrt{-be^2+aef}(be-af)}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
(b*c - a*d)*x/((a*b*e - a^2*f)*sqrt(b*x^2 + a)) + (b*d^2*e^2 - 2*b*c*d*e*f
 + b*c^2*f^2)*arctan(-1/2*(2*b^(3/2)*d*e^2 - 2*b^(3/2)*c*e*f - a*sqrt(b)*d
*e*f + a*sqrt(b)*c*f^2 + ((sqrt(b)*x - sqrt(b*x^2 + a))^2*sqrt(b)*d*e - (s
qrt(b)*x - sqrt(b*x^2 + a))^2*sqrt(b)*c*f)*f)/(sqrt(-b*e^2 + a*e*f)*b*d*e
- sqrt(-b*e^2 + a*e*f)*b*c*f))/((b*d*e - b*c*f)*sqrt(-b*e^2 + a*e*f)*(b*e
- a*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{d\,x^2+c}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 702, normalized size of antiderivative = 7.63

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \frac{-\sqrt{e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{af-be}-\sqrt{f}\,\sqrt{b\,x^2+a}-\sqrt{f}\,\sqrt{b}\,x}{\sqrt{e}\,\sqrt{b}}\right)a^2bcf+\sqrt{e}\,\sqrt{af-be}\,atan}{}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*f + sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**2*b*d*e - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
 b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a
*b**2*c*f*x**2 + sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*s
qrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*d*e*x**2 -
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) +
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b*c*f + sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a**2*b*d*e - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**
2*c*f*x**2 + sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(
a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**2*d*e*x**2 + sqrt
(a + b*x**2)*a**2*b*d*e*f*x - sqrt(a + b*x**2)*a*b**2*c*e*f*x - sqrt(a + b
*x**2)*a*b**2*d*e**2*x + sqrt(a + b*x**2)*b**3*c*e**2*x + sqrt(b)*a**3*d*e
*f - sqrt(b)*a**2*b*c*e*f - sqrt(b)*a**2*b*d*e**2 + sqrt(b)*a**2*b*d*e*f*x
**2 + sqrt(b)*a*b**2*c*e**2 - sqrt(b)*a*b**2*c*e*f*x**2 - sqrt(b)*a*b**2*d
*e**2*x**2 + sqrt(b)*b**3*c*e**2*x**2)/(a*b*e*(a**3*f**2 - 2*a**2*b*e*f +
a**2*b*f**2*x**2 + a*b**2*e**2 - 2*a*b**2*e*f*x**2 + b**3*e**2*x**2))
```

3.341 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 170

$$\begin{aligned}\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx &= \frac{b(2bce-3ade+acf)x}{2ae(be-af)^2\sqrt{a+bx^2}} \\ &+ \frac{(de-cf)x}{2e(be-af)\sqrt{a+bx^2}\,(e+fx^2)} \\ &+ \frac{(2be(de-2cf)+af(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{5/2}}\end{aligned}$$

output
```
1/2*b*(a*c*f-3*a*d*e+2*b*c*e)*x/a/e/(-a*f+b*e)^2/(b*x^2+a)^(1/2)+1/2*(-c*f
+d*e)*x/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)+1/2*(2*b*e*(-2*c*f+d*e)+a*f
*(c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-
a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{x(a^2f(-de+cf)+2b^2ce(e+fx^2)+ab(cf^2x^2-de(2e+3fx^2)))}{2ae(be-af)^2\sqrt{a+bx^2}\,(e+fx^2)} - \frac{(2be(de-2cf)+af(de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{2e^{3/2}(-be+af)^{5/2}}$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(x*(a^2*f*(-(d*e) + c*f) + 2*b^2*c*e*(e + f*x^2) + a*b*(c*f^2*x^2 - d*e*(2
*e + 3*f*x^2))))/(2*a*e*(b*e - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)) - ((2*b
*e*(d*e - 2*c*f) + a*f*(d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[
b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(2*e^(3/2)*(-(b*e) + a*f)^(
5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {402, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)(be-af)} - \frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{a(be-af)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{a(be-af)} + \frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)(be-af)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e(e+fx^2)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)\,(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a(af(cf+de)+2be(de-2cf))\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e(e+fx^2)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)\,(be-af)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{a(af(cf+de)+2be(de-2cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e(e+fx^2)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)\,(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e(e+fx^2)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)\,(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + ((f*(2*b*c*e
 - 3*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*
(2*b*e*(d*e - 2*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(a*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-a\sqrt{b\,x^{2}+a}\,\left(2bd\,e^{2}+f\left(ad-4bc\right)e+ac\,f^{2}\right)\left(f\,x^{2}+e\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{\left(af-be\right)e}}\right)+\left(\left(-2abd+2b^{2}c\right)e^{2}-f\left(3abd\,x^{2}-2b^{2}c\,x^{2}+d\,a^{2}\right)\right.}{2\sqrt{\left(af-be\right)e}\,\sqrt{b\,x^{2}+a}\,e\left(f\,x^{2}+e\right)\left(af-be\right)^{2}a}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*(b*x^2+a)^(1/2)*(2*b*d*e^2+f*(a*d-4*b*c)*e+a*c*f^2)*(f*x^2+e)*arct
an(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((-2*a*b*d+2*b^2*c)*e^2-f*(3*a
*b*d*x^2-2*b^2*c*x^2+a^2*d)*e+a*c*f^2*(b*x^2+a))*((a*f-b*e)*e)^(1/2)*x)/((
a*f-b*e)*e)^(1/2)/(b*x^2+a)^(1/2)/e/(f*x^2+e)/(a*f-b*e)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(150) = 300$.

Time = 3.27 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.52

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/8*((2*a^2*b*d*e^3 + a^3*c*e*f^2 + (2*a*b^2*d*e^2*f + a^2*b*c*f^3 - (4*a
*b^2*c - a^2*b*d)*e*f^2)*x^4 - (4*a^2*b*c - a^3*d)*e^2*f + (2*a*b^2*d*e^3
+ a^3*c*f^3 - (4*a*b^2*c - 3*a^2*b*d)*e^2*f - (3*a^2*b*c - a^3*d)*e*f^2)*x
^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e
^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*
e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((a^2*b*c*e
*f^3 - (2*b^3*c - 3*a*b^2*d)*e^3*f + (a*b^2*c - 3*a^2*b*d)*e^2*f^2)*x^3 +
(a^3*c*e*f^3 - 2*(b^3*c - a*b^2*d)*e^4 + (2*a*b^2*c - a^2*b*d)*e^3*f - (a^
2*b*c + a^3*d)*e^2*f^2)*x)*sqrt(b*x^2 + a))/(a^2*b^3*e^6 - 3*a^3*b^2*e^5*f
 + 3*a^4*b*e^4*f^2 - a^5*e^3*f^3 + (a*b^4*e^5*f - 3*a^2*b^3*e^4*f^2 + 3*a^
3*b^2*e^3*f^3 - a^4*b*e^2*f^4)*x^4 + (a*b^4*e^6 - 2*a^2*b^3*e^5*f + 2*a^4*
b*e^3*f^3 - a^5*e^2*f^4)*x^2), -1/4*((2*a^2*b*d*e^3 + a^3*c*e*f^2 + (2*a*b
^2*d*e^2*f + a^2*b*c*f^3 - (4*a*b^2*c - a^2*b*d)*e*f^2)*x^4 - (4*a^2*b*c -
 a^3*d)*e^2*f + (2*a*b^2*d*e^3 + a^3*c*f^3 - (4*a*b^2*c - 3*a^2*b*d)*e^2*f
 - (3*a^2*b*c - a^3*d)*e*f^2)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b
*e^2 + a*e*f)*((2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*
f)*x^3 + (a*b*e^2 - a^2*e*f)*x)) + 2*((a^2*b*c*e*f^3 - (2*b^3*c - 3*a*b^2*
d)*e^3*f + (a*b^2*c - 3*a^2*b*d)*e^2*f^2)*x^3 + (a^3*c*e*f^3 - 2*(b^3*c -
a*b^2*d)*e^4 + (2*a*b^2*c - a^2*b*d)*e^3*f - (a^2*b*c + a^3*d)*e^2*f^2)*x)
*sqrt(b*x^2 + a))/(a^2*b^3*e^6 - 3*a^3*b^2*e^5*f + 3*a^4*b*e^4*f^2 - a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(150) = 300$.

Time = 0.44 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.44

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{(b^2c-abd)x}{(ab^2e^2-2\,a^2bef+a^3f^2)\sqrt{bx^2+a}}$$
$$-\frac{\left(2\,b^{\frac{3}{2}}de^2-4\,b^{\frac{3}{2}}cef+a\sqrt{b}def+a\sqrt{b}cf^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{2\,(b^2e^3-2\,abe^2f+a^2ef^2)\sqrt{-b^2e^2+abef}}$$
$$-\frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}de^2-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cef-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}def+\left(\sqrt{b}x-\sqrt{\right.}}{(b^2e^3-2\,abe^2f+a^2ef^2)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2be-2\left(\sqrt{b}x-\right.\right.}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
(b^2*c - a*b*d)*x/((a*b^2*e^2 - 2*a^2*b*e*f + a^3*f^2)*sqrt(b*x^2 + a)) -
1/2*(2*b^(3/2)*d*e^2 - 4*b^(3/2)*c*e*f + a*sqrt(b)*d*e*f + a*sqrt(b)*c*f^2
)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e
^2 + a*b*e*f))/((b^2*e^3 - 2*a*b*e^2*f + a^2*e*f^2)*sqrt(-b^2*e^2 + a*b*e*
f)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*d*e^2 - 2*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*b^(3/2)*c*e*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)
*d*e*f + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*f^2 + a^2*sqrt(b)*d*e
*f - a^2*sqrt(b)*c*f^2)/((b^2*e^3 - 2*a*b*e^2*f + a^2*e*f^2)*((sqrt(b)*x -
 sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*a*f + a^2*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{d\,x^2+c}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3450, normalized size of antiderivative = 20.29

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*c*e*f**3 - sqrt(e)*sqrt(a*
f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b
)*x)/(sqrt(e)*sqrt(b)))*a**4*c*f**4*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((s
qrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sq
rt(b)))*a**4*d*e**2*f**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*d*e
*f**3*x**2 + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqr
t(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*e**2*f**2 +
 7*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*e*f**3*x**2 - sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*c*f**4*x**4 + 2*sqrt(e)*sqrt(a*f - b*
e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(
sqrt(e)*sqrt(b)))*a**3*b*d*e**3*f + sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**3*b*d*e**2*f**2*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b*d
*e*f**3*x**4 - 16*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**2*c*e*...
```

3.342 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 280

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{b(8b^2ce^2-2abe(7de-5cf)-a^2f(de+3cf))\,x}{8ae^2(be-af)^3\sqrt{a+bx^2}} + \frac{(de-cf)x}{4e(be-af)\sqrt{a+bx^2}\,(e+fx^2)^2} + \frac{(4be(de-2cf)+af(de+3cf))x}{8e^2(be-af)^2\sqrt{a+bx^2}\,(e+fx^2)} + \frac{(8b^2e^2(de-3cf)-a^2f^2(de+3cf)+4abef(2de+3cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{7/2}}$$

output
```
1/8*b*(8*b^2*c*e^2-2*a*b*e*(-5*c*f+7*d*e)-a^2*f*(3*c*f+d*e))*x/a/e^2/(-a*f
+b*e)^3/(b*x^2+a)^(1/2)+1/4*(-c*f+d*e)*x/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x
^2+e)^2+1/8*(4*b*e*(-2*c*f+d*e)+a*f*(3*c*f+d*e))*x/e^2/(-a*f+b*e)^2/(b*x^2
+a)^(1/2)/(f*x^2+e)+1/8*(8*b^2*e^2*(-3*c*f+d*e)-a^2*f^2*(3*c*f+d*e)+4*a*b*
e*f*(3*c*f+2*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(
5/2)/(-a*f+b*e)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 18.85 (sec) , antiderivative size = 2140, normalized size of antiderivative = 7.64

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(x*((-12*d*(-(b*e) + a*f)*x^2*(1 + (f*x^2)/e)*(-2625*Sqrt[((b*e - a*f)*x^2
)/(e*(a + b*x^2))] - (5250*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/
e - (2310*f^2*x^4*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e^2 + 70*(((b*e
 - a*f)*x^2)/(e*(a + b*x^2)))^(3/2) + (560*f*x^2*(((b*e - a*f)*x^2)/(e*(a
+ b*x^2)))^(3/2))/e + (280*f^2*x^4*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/
2))/e^2 + 2625*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + (5250*f*
x^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/e + (2310*f^2*x^4*Ar
cTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/e^2 - (945*(b*e - a*f)*x^2
*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*(a + b*x^2)) + (2310
*f*(-(b*e) + a*f)*x^4*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e
^2*(a + b*x^2)) + (1050*f^2*(-(b*e) + a*f)*x^6*ArcTanh[Sqrt[((b*e - a*f)*x
^2)/(e*(a + b*x^2))]])/(e^3*(a + b*x^2)) + 24*(((b*e - a*f)*x^2)/(e*(a + b
*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*e - a*f)*x^2)/(
e*(a + b*x^2))] + (48*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(7/2)*Hype
rgeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e
 + (24*f^2*x^4*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(7/2)*HypergeometricPFQ
[{2, 2, 5/2}, {1, 9/2}, ((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e^2))/(a + b*x
^2) + (-(d*e) + c*f)*(-108045*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - (3
24135*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e - (324135*f^2*x^4*S
qrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e^2 - (103320*f^3*x^6*Sqrt[((b*...
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {402, 25, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \text{ 402}$$

$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{2bf(4bce-5ade+acf)x^2+a(4be(de-2cf)+af(de+3cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e(e+fx^2)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{\int \frac{a\left(8b^2(de-3cf)e^2+4abf(2de+3cf)e-a^2f^2(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e(e+fx^2)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{a\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf}{4e(e+fx^2)^2}}{a(be-af)}$$
$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)^2\,(be-af)}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{a\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(}{4e(e+f}}{a(be-af)}$$
$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)^2\,(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(a}{4e(e+fx}}{a(be-af)}$$
$$\frac{x(bc-ad)}{a\sqrt{a+bx^2}\,(e+fx^2)^2\,(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2) + ((f*(4*b*c
*e - 5*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) +
 ((f*(8*b^2*c*e^2 - 2*a*b*e*(7*d*e - 5*c*f) - a^2*f*(d*e + 3*c*f))*x*Sqrt[
a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*(8*b^2*e^2*(d*e - 3*c*f) -
a^2*f^2*(d*e + 3*c*f) + 4*a*b*e*f*(2*d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*f
]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e
- a*f)))/(a*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{-\frac{3a\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^2\left(-\frac{8b^2d\,e^3}{3}-\frac{8bf(ad-3bc)e^2}{3}+\frac{a\,f^2(ad-12bc)e}{3}+a^2c\,f^3\right)\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\frac{5\left(\frac{8b^2(ad-bc)e^4}{5}+\frac{8bf\left(3a\right.}{}\right.}{}}{(f\,x^2+}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(-3/5*a*(b*x^2+a)^(1/2)*(f*x^2+e)^2*(-8/3*b^2*d*e^3-8/3*b*f*(a*d-3*b*c
)*e^2+1/3*a*f^2*(a*d-12*b*c)*e+a^2*c*f^3)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f
-b*e)*e)^(1/2))+(8/5*b^2*(a*d-b*c)*e^4+8/5*b*f*(3*a*b*d*x^2-2*b^2*c*x^2+a^
2*d)*e^3-1/5*(a^3*d+12*(-5/12*x^2*d+c)*b*a^2+12*b^2*x^2*(-7/6*x^2*d+c)*a+8
*b^3*c*x^4)*f^2*e^2+a*((1/5*x^2*d+c)*a-2*x^2*b*c)*(b*x^2+a)*f^3*e+3/5*a^2*
c*f^4*x^2*(b*x^2+a))*((a*f-b*e)*e)^(1/2)*x)/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^
(1/2)/(f*x^2+e)^2/e^2/(a*f-b*e)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(256) = 512$.

Time = 17.04 (sec) , antiderivative size = 2032, normalized size of antiderivative = 7.26

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[1/32*((8*a^2*b^2*d*e^5 - 3*a^4*c*e^2*f^3 + (8*a*b^3*d*e^3*f^2 - 3*a^3*b*c
*f^5 - 8*(3*a*b^3*c - a^2*b^2*d)*e^2*f^3 + (12*a^2*b^2*c - a^3*b*d)*e*f^4)
*x^6 - 8*(3*a^2*b^2*c - a^3*b*d)*e^4*f + (12*a^3*b*c - a^4*d)*e^3*f^2 + (1
6*a*b^3*d*e^4*f + 6*a^3*b*d*e^2*f^3 - 3*a^4*c*f^5 - 24*(2*a*b^3*c - a^2*b^
2*d)*e^3*f^2 + (6*a^3*b*c - a^4*d)*e*f^4)*x^4 + (8*a*b^3*d*e^5 - 6*a^4*c*e
*f^4 - 24*(a*b^3*c - a^2*b^2*d)*e^4*f - 3*(12*a^2*b^2*c - 5*a^3*b*d)*e^3*f
^2 + (21*a^3*b*c - 2*a^4*d)*e^2*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*
e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 +
 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x
^4 + 2*e*f*x^2 + e^2)) + 4*((3*a^3*b*c*e*f^5 + 2*(4*b^4*c - 7*a*b^3*d)*e^4
*f^2 + (2*a*b^3*c + 13*a^2*b^2*d)*e^3*f^3 - (13*a^2*b^2*c - a^3*b*d)*e^2*f
^4)*x^5 + (3*a^4*c*e*f^5 + 8*(2*b^4*c - 3*a*b^3*d)*e^5*f - (4*a*b^3*c - 19
*a^2*b^2*d)*e^4*f^2 - (7*a^2*b^2*c - 4*a^3*b*d)*e^3*f^3 - (8*a^3*b*c - a^4
*d)*e^2*f^4)*x^3 - (8*a*b^3*c*e^5*f - 5*a^4*c*e^2*f^4 - 8*(b^4*c - a*b^3*d
)*e^6 - 3*(4*a^2*b^2*c + 3*a^3*b*d)*e^4*f^2 + (17*a^3*b*c + a^4*d)*e^3*f^3
)*x)*sqrt(b*x^2 + a))/(a^2*b^4*e^9 - 4*a^3*b^3*e^8*f + 6*a^4*b^2*e^7*f^2 -
 4*a^5*b*e^6*f^3 + a^6*e^5*f^4 + (a*b^5*e^7*f^2 - 4*a^2*b^4*e^6*f^3 + 6*a^
3*b^3*e^5*f^4 - 4*a^4*b^2*e^4*f^5 + a^5*b*e^3*f^6)*x^6 + (2*a*b^5*e^8*f -
7*a^2*b^4*e^7*f^2 + 8*a^3*b^3*e^6*f^3 - 2*a^4*b^2*e^5*f^4 - 2*a^5*b*e^4*f^
5 + a^6*e^3*f^6)*x^4 + (a*b^5*e^9 - 2*a^2*b^4*e^8*f - 2*a^3*b^3*e^7*f^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(256) = 512$.

Time = 0.69 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.61

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
(b^3*c - a*b^2*d)*x/((a*b^3*e^3 - 3*a^2*b^2*e^2*f + 3*a^3*b*e*f^2 - a^4*f^
3)*sqrt(b*x^2 + a)) - 1/8*(8*b^(5/2)*d*e^3 - 24*b^(5/2)*c*e^2*f + 8*a*b^(3
/2)*d*e^2*f + 12*a*b^(3/2)*c*e*f^2 - a^2*sqrt(b)*d*e*f^2 - 3*a^2*sqrt(b)*c
*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b
^2*e^2 + a*b*e*f))/((b^3*e^5 - 3*a*b^2*e^4*f + 3*a^2*b*e^3*f^2 - a^3*e^2*f
^3)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(
5/2)*d*e^3*f - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c*e^2*f^2 + 12*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^3 - (sqrt(b)*x - sqrt(b*x^2
 + a))^6*a^2*sqrt(b)*d*e*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(
b)*c*f^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d*e^4 - 80*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*b^(7/2)*c*e^3*f - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a*b^(5/2)*d*e^3*f + 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*e^2*
f^2 + 10*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d*e^2*f^2 - 54*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*e*f^3 + 3*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^3*sqrt(b)*d*e*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b
)*c*f^4 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*d*e^3*f - 64*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c*e^2*f^2 - 16*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^3*b^(3/2)*d*e^2*f^2 + 52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^
3*b^(3/2)*c*e*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d*e*f^3
- 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*c*f^4 + 6*a^4*b^(3/2)*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}\,(e+fx^2)^3}\,dx = \int \frac{d\,x^2+c}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 6407, normalized size of antiderivative = 22.88

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x)
```

output
```
( - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*c*e**2*f**4 - 18*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*c*e*f**5*x**2 - 9*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**5*c*f**6*x**4 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**5*d*e**3*f**3 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*d*e
**2*f**4*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*d*e*f**5*x**
4 + 60*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*
x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c*e**3*f**3 + 111*sqr
t(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq
rt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c*e**2*f**4*x**2 + 42*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c*e*f**5*x**4 - 9*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**4*b*c*f**6*x**6 + 32*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e...
```

3.343 $\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 428

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}} \, dx = \frac{(bc-ad)^2(be-af)^3 x}{ab^5\sqrt{a+bx^2}}$$
$$- \frac{(187a^3d^2f^3 - 152a^2bdf^2(3de+2cf) + 112ab^2f(3d^2e^2+6cdef+c^2f^2) - 64b^3e(d^2e^2+6cdef+3c^2f^2))}{128b^5}$$
$$+ \frac{f(123a^2d^2f^2 - 88abdf(3de+2cf) + 48b^2(3d^2e^2+6cdef+c^2f^2))\, x^3\sqrt{a+bx^2}}{192b^4}$$
$$+ \frac{df^2(24bde+16bcf-15adf)x^5\sqrt{a+bx^2}}{48b^3} + \frac{d^2f^3x^7\sqrt{a+bx^2}}{8b^2}$$
$$+ \frac{(315a^4d^2f^3 - 280a^3bdf^2(3de+2cf) + 128b^4ce^2(2de+3cf) + 240a^2b^2f(3d^2e^2+6cdef+c^2f^2) - 192ab^3}{128b^{11/2}}$$

output
```
(-a*d+b*c)^2*(-a*f+b*e)^3*x/a/b^5/(b*x^2+a)^(1/2)-1/128*(187*a^3*d^2*f^3-1
52*a^2*b*d*f^2*(2*c*f+3*d*e)+112*a*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-64*
b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/192*f*(123*a^
2*d^2*f^2-88*a*b*d*f*(2*c*f+3*d*e)+48*b^2*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x
^3*(b*x^2+a)^(1/2)/b^4+1/48*d*f^2*(-15*a*d*f+16*b*c*f+24*b*d*e)*x^5*(b*x^2
+a)^(1/2)/b^3+1/8*d^2*f^3*x^7*(b*x^2+a)^(1/2)/b^2+1/128*(315*a^4*d^2*f^3-2
80*a^3*b*d*f^2*(2*c*f+3*d*e)+128*b^4*c*e^2*(3*c*f+2*d*e)+240*a^2*b^2*f*(c^
2*f^2+6*c*d*e*f+3*d^2*e^2)-192*a*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}x(384b^5c^2e^3-945a^5d^2f^3+105a^4bdf^2(24de+16cf-3dfx^2)+2a^3b^2f(-360c^2f^2+40cdf(-54e+7fx^2)+ \cdots}{\cdots}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(384*b^5*c^2*e^3 - 945*a^5*d^2*f^3 + 105*a^4*b*d*f^2*(24*d*e +
 16*c*f - 3*d*f*x^2) + 2*a^3*b^2*f*(-360*c^2*f^2 + 40*c*d*f*(-54*e + 7*f*x
^2) + d^2*(-1080*e^2 + 420*e*f*x^2 + 63*f^2*x^4)) + 8*a^2*b^3*(6*c^2*f^2*(
36*e - 5*f*x^2) - 4*c*d*f*(-108*e^2 + 45*e*f*x^2 + 7*f^2*x^4) + d^2*(72*e^
3 - 90*e^2*f*x^2 - 42*e*f^2*x^4 - 9*f^3*x^6)) - 16*a*b^4*(-6*c^2*f*(-12*e^
2 + 6*e*f*x^2 + f^2*x^4) + 4*c*d*(12*e^3 - 18*e^2*f*x^2 - 9*e*f^2*x^4 - 2*
f^3*x^6) - 3*d^2*x^2*(4*e^3 + 6*e^2*f*x^2 + 4*e*f^2*x^4 + f^3*x^6))))/(a*S
qrt[a + b*x^2]) - 3*(315*a^4*d^2*f^3 - 280*a^3*b*d*f^2*(3*d*e + 2*c*f) + 1
28*b^4*c*e^2*(2*d*e + 3*c*f) + 240*a^2*b^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*
f^2) - 192*a*b^3*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + S
qrt[a + b*x^2]])/(384*b^(11/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{fx^6(c^2f^2+6cdef+3d^2e^2)}{(a+bx^2)^{3/2}}+\frac{ex^4(3c^2f^2+6cdef+d^2e^2)}{(a+bx^2)^{3/2}}+\frac{c^2e^3}{(a+bx^2)^{3/2}}+\frac{ce^2x^2(3cf+2de)}{(a+bx^2)^{3/2}}+\frac{df^2x^8(2c\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{315a^4d^2f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{11/2}}-\frac{35a^3df^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cf+3de)}{16b^{9/2}}-\frac{315a^3d^2f^3x\sqrt{a+bx^2}}{128b^5}+\frac{15a^2f\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(c^2f^2+6cdef+3d^2e^2)}{8b^{7/2}}+\frac{35a^2df^2x\sqrt{a+bx^2}(2cf+3de)}{16b^4}+\frac{105a^2d^2f^3x^3\sqrt{a+bx^2}}{64b^4}-\frac{3ae\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3c^2f^2+6cdef+d^2e^2)}{2b^{5/2}}+\frac{ce^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3cf+2de)}{b^{3/2}}-\frac{15afx\sqrt{a+bx^2}(c^2f^2+6cdef+3d^2e^2)}{8b^3}-\frac{35adf^2x^3\sqrt{a+bx^2}(2cf+3de)}{24b^3}-\frac{21ad^2f^3x^5\sqrt{a+bx^2}}{16b^3}+\frac{3ex\sqrt{a+bx^2}(3c^2f^2+6cdef+d^2e^2)}{2b^2}+\frac{5fx^3\sqrt{a+bx^2}(c^2f^2+6cdef+3d^2e^2)}{4b^2}+\frac{7df^2x^5\sqrt{a+bx^2}(2cf+3de)}{6b^2}+\frac{9d^2f^3x^7\sqrt{a+bx^2}}{8b^2}-\frac{fx^5(c^2f^2+6cdef+3d^2e^2)}{b\sqrt{a+bx^2}}-\frac{ex^3(3c^2f^2+6cdef+d^2e^2)}{b\sqrt{a+bx^2}}+\frac{c^2e^3x}{a\sqrt{a+bx^2}}-\frac{ce^2x(3cf+2de)}{b\sqrt{a+bx^2}}-\frac{df^2x^7(2cf+3de)}{b\sqrt{a+bx^2}}-\frac{d^2f^3x^9}{b\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
(c^2*e^3*x)/(a*Sqrt[a + b*x^2]) - (c*e^2*(2*d*e + 3*c*f)*x)/(b*Sqrt[a + b*
x^2]) - (e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^3)/(b*Sqrt[a + b*x^2]) - (f
*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5)/(b*Sqrt[a + b*x^2]) - (d*f^2*(3*d*
e + 2*c*f)*x^7)/(b*Sqrt[a + b*x^2]) - (d^2*f^3*x^9)/(b*Sqrt[a + b*x^2]) -
(315*a^3*d^2*f^3*x*Sqrt[a + b*x^2])/(128*b^5) + (35*a^2*d*f^2*(3*d*e + 2*c
*f)*x*Sqrt[a + b*x^2])/(16*b^4) - (15*a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2
)*x*Sqrt[a + b*x^2])/(8*b^3) + (3*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x*Sq
rt[a + b*x^2])/(2*b^2) + (105*a^2*d^2*f^3*x^3*Sqrt[a + b*x^2])/(64*b^4) -
(35*a*d*f^2*(3*d*e + 2*c*f)*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*f*(3*d^2*e^
2 + 6*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(4*b^2) - (21*a*d^2*f^3*x^5*
Sqrt[a + b*x^2])/(16*b^3) + (7*d*f^2*(3*d*e + 2*c*f)*x^5*Sqrt[a + b*x^2])/
(6*b^2) + (9*d^2*f^3*x^7*Sqrt[a + b*x^2])/(8*b^2) + (315*a^4*d^2*f^3*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(11/2)) - (35*a^3*d*f^2*(3*d*e + 2
*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2)) + (c*e^2*(2*d*e +
 3*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2) + (15*a^2*f*(3*d^2*e
^2 + 6*c*d*e*f + c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2)
) - (3*a*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(5/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{315a\left(a^4d^2f^3-\frac{16dbf^2\left(cf+\frac{3de}{2}\right)a^3}{9}+\frac{16a^2b^2f\left(c^2f^2+6cdef+3d^2e^2\right)}{21}-\frac{64\left(c^2f^2+2cdef+\frac{1}{3}d^2e^2\right)b^3ea}{35}+\frac{128cb^4e^2\left(cf+\frac{2de}{3}\right)}{105}\right)\sqrt{bx^2}}{128}$
default	$\frac{c^2e^3x}{a\sqrt{bx^2+a}}+f^2d(2cf+3de)\left(\frac{x^7}{6b\sqrt{bx^2+a}}-\frac{7a\left(\frac{x^5}{4b\sqrt{bx^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{bx^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(\sqrt{b}x+\sqrt{bx^2+}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)}{6b}\right)$
risch	$-\frac{x\left(-48d^2f^3x^6b^3+120ab^2d^2f^3x^4-128b^3cdf^3x^4-192b^3d^2ef^2x^4-246a^2bd^2f^3x^2+352ab^2cdf^3x^2+528ab^2d^2ef^2x^2-9\right.}{}$

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
315/128/b^(11/2)*(a*(a^4*d^2*f^3-16/9*d*b*f^2*(c*f+3/2*d*e)*a^3+16/21*a^2*
b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-64/35*(c^2*f^2+2*c*d*e*f+1/3*d^2*e^2)*
b^3*e*a+128/105*c*b^4*e^2*(c*f+2/3*d*e))*(b*x^2+a)^(1/2)*arctanh((b*x^2+a)
^(1/2)/x/b^(1/2))-x*(128/105*a*((-1/24*d^2*x^8-1/9*c*d*x^6-1/12*c^2*x^4)*f
^3-1/2*x^2*e*(1/3*d^2*x^4+c*d*x^2+c^2)*f^2+e^2*(-1/4*d^2*x^4-c*d*x^2+c^2)*
f+2/3*d*(-1/4*x^2*d+c)*e^3)*b^(9/2)-64/35*a^2*((-1/24*d^2*x^6-7/54*c*d*x^4
-5/36*c^2*x^2)*f^3+e*(-7/36*d^2*x^4-5/6*c*d*x^2+c^2)*f^2+2*d*(-5/24*x^2*d+
c)*e^2*f+1/3*d^2*e^3)*b^(7/2)+16/21*a^3*((-7/40*d^2*x^4-7/9*c*d*x^2+c^2)*f
^2+6*(-7/36*x^2*d+c)*d*e*f+3*d^2*e^2)*f*b^(5/2)-16/9*a^4*d*((-3/16*x^2*d+c
)*f+3/2*d*e)*f^2*b^(3/2)-128/315*b^(11/2)*c^2*e^3+b^(1/2)*a^5*d^2*f^3))/(b
*x^2+a)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 1490, normalized size of antiderivative = 3.48

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(64*(4*a^2*b^4*c*d - 3*a^3*b^3*d^2)*e^3 + 48*(8*a^2*b^4*c^2 - 24
*a^3*b^3*c*d + 15*a^4*b^2*d^2)*e^2*f - 24*(24*a^3*b^3*c^2 - 60*a^4*b^2*c*d
 + 35*a^5*b*d^2)*e*f^2 + 5*(48*a^4*b^2*c^2 - 112*a^5*b*c*d + 63*a^6*d^2)*f
^3 + (64*(4*a*b^5*c*d - 3*a^2*b^4*d^2)*e^3 + 48*(8*a*b^5*c^2 - 24*a^2*b^4*
c*d + 15*a^3*b^3*d^2)*e^2*f - 24*(24*a^2*b^4*c^2 - 60*a^3*b^3*c*d + 35*a^4
*b^2*d^2)*e*f^2 + 5*(48*a^3*b^3*c^2 - 112*a^4*b^2*c*d + 63*a^5*b*d^2)*f^3)
*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*a*b^
5*d^2*f^3*x^9 + 8*(24*a*b^5*d^2*e*f^2 + (16*a*b^5*c*d - 9*a^2*b^4*d^2)*f^3
)*x^7 + 2*(144*a*b^5*d^2*e^2*f + 24*(12*a*b^5*c*d - 7*a^2*b^4*d^2)*e*f^2 +
 (48*a*b^5*c^2 - 112*a^2*b^4*c*d + 63*a^3*b^3*d^2)*f^3)*x^5 + (192*a*b^5*d
^2*e^3 + 144*(8*a*b^5*c*d - 5*a^2*b^4*d^2)*e^2*f + 24*(24*a*b^5*c^2 - 60*a
^2*b^4*c*d + 35*a^3*b^3*d^2)*e*f^2 - 5*(48*a^2*b^4*c^2 - 112*a^3*b^3*c*d +
 63*a^4*b^2*d^2)*f^3)*x^3 + 3*(64*(2*b^6*c^2 - 4*a*b^5*c*d + 3*a^2*b^4*d^2
)*e^3 - 48*(8*a*b^5*c^2 - 24*a^2*b^4*c*d + 15*a^3*b^3*d^2)*e^2*f + 24*(24*
a^2*b^4*c^2 - 60*a^3*b^3*c*d + 35*a^4*b^2*d^2)*e*f^2 - 5*(48*a^3*b^3*c^2 -
 112*a^4*b^2*c*d + 63*a^5*b*d^2)*f^3)*x)*sqrt(b*x^2 + a))/(a*b^7*x^2 + a^2
*b^6), -1/384*(3*(64*(4*a^2*b^4*c*d - 3*a^3*b^3*d^2)*e^3 + 48*(8*a^2*b^4*c
^2 - 24*a^3*b^3*c*d + 15*a^4*b^2*d^2)*e^2*f - 24*(24*a^3*b^3*c^2 - 60*a^4*
b^2*c*d + 35*a^5*b*d^2)*e*f^2 + 5*(48*a^4*b^2*c^2 - 112*a^5*b*c*d + 63*a^6
*d^2)*f^3 + (64*(4*a*b^5*c*d - 3*a^2*b^4*d^2)*e^3 + 48*(8*a*b^5*c^2 - 2...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/8*d^2*f^3*x^9/(sqrt(b*x^2 + a)*b) - 3/16*a*d^2*f^3*x^7/(sqrt(b*x^2 + a)*
b^2) + 21/64*a^2*d^2*f^3*x^5/(sqrt(b*x^2 + a)*b^3) - 105/128*a^3*d^2*f^3*x
^3/(sqrt(b*x^2 + a)*b^4) + 1/6*(3*d^2*e*f^2 + 2*c*d*f^3)*x^7/(sqrt(b*x^2 +
 a)*b) + c^2*e^3*x/(sqrt(b*x^2 + a)*a) - 315/128*a^4*d^2*f^3*x/(sqrt(b*x^2
 + a)*b^5) - 7/24*(3*d^2*e*f^2 + 2*c*d*f^3)*a*x^5/(sqrt(b*x^2 + a)*b^2) +
1/4*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*x^5/(sqrt(b*x^2 + a)*b) + 315/12
8*a^4*d^2*f^3*arcsinh(b*x/sqrt(a*b))/b^(11/2) + 35/48*(3*d^2*e*f^2 + 2*c*d
*f^3)*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2
*f^3)*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f
^2)*x^3/(sqrt(b*x^2 + a)*b) + 35/16*(3*d^2*e*f^2 + 2*c*d*f^3)*a^3*x/(sqrt(
b*x^2 + a)*b^4) - 15/8*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*a^2*x/(sqrt(b
*x^2 + a)*b^3) + 3/2*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*a*x/(sqrt(b*x^2
 + a)*b^2) - (2*c*d*e^3 + 3*c^2*e^2*f)*x/(sqrt(b*x^2 + a)*b) - 35/16*(3*d^
2*e*f^2 + 2*c*d*f^3)*a^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 15/8*(3*d^2*e^2*
f + 6*c*d*e*f^2 + c^2*f^3)*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*(d^2*e
^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + (2*c*d*
e^3 + 3*c^2*e^2*f)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{((2(4(\frac{6d^2f^3x^2}{b} + \frac{24ab^8d^2ef^2+16ab^8cdf^3-9a^2b^7d^2f^3}{ab^9})x^2 + \frac{144ab^8d^2e^2f+288ab^8cdef^2-1\cdots}{\cdots}}{\cdots} - \frac{(256b^4cde^3 - 192ab^3d^2e^3 + 384b^4c^2e^2f - 1152ab^3cde^2f + 720a^2b^2d^2e^2f - 576ab^3c^2ef^2 + 1440a^2b^2cde\cdots}{128b^{\frac{11}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/384*((2*(4*(6*d^2*f^3*x^2/b + (24*a*b^8*d^2*e*f^2 + 16*a*b^8*c*d*f^3 - 9
*a^2*b^7*d^2*f^3)/(a*b^9))*x^2 + (144*a*b^8*d^2*e^2*f + 288*a*b^8*c*d*e*f^
2 - 168*a^2*b^7*d^2*e*f^2 + 48*a*b^8*c^2*f^3 - 112*a^2*b^7*c*d*f^3 + 63*a^
3*b^6*d^2*f^3)/(a*b^9))*x^2 + (192*a*b^8*d^2*e^3 + 1152*a*b^8*c*d*e^2*f -
720*a^2*b^7*d^2*e^2*f + 576*a*b^8*c^2*e*f^2 - 1440*a^2*b^7*c*d*e*f^2 + 840
*a^3*b^6*d^2*e*f^2 - 240*a^2*b^7*c^2*f^3 + 560*a^3*b^6*c*d*f^3 - 315*a^4*b
^5*d^2*f^3)/(a*b^9))*x^2 + 3*(128*b^9*c^2*e^3 - 256*a*b^8*c*d*e^3 + 192*a^
2*b^7*d^2*e^3 - 384*a*b^8*c^2*e^2*f + 1152*a^2*b^7*c*d*e^2*f - 720*a^3*b^6
*d^2*e^2*f + 576*a^2*b^7*c^2*e*f^2 - 1440*a^3*b^6*c*d*e*f^2 + 840*a^4*b^5*
d^2*e*f^2 - 240*a^3*b^6*c^2*f^3 + 560*a^4*b^5*c*d*f^3 - 315*a^5*b^4*d^2*f^
3)/(a*b^9))*x/sqrt(b*x^2 + a) - 1/128*(256*b^4*c*d*e^3 - 192*a*b^3*d^2*e^3
 + 384*b^4*c^2*e^2*f - 1152*a*b^3*c*d*e^2*f + 720*a^2*b^2*d^2*e^2*f - 576*
a*b^3*c^2*e*f^2 + 1440*a^2*b^2*c*d*e*f^2 - 840*a^3*b*d^2*e*f^2 + 240*a^2*b
^2*c^2*f^3 - 560*a^3*b*c*d*f^3 + 315*a^4*d^2*f^3)*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^3}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(3/2),x)
```

3.344 $\int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 272

$$\int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{(bc-ad)^2(be-af)^2x}{ab^4\sqrt{a+bx^2}} + \frac{(19a^2d^2f^2-28abdf(de+cf)+8b^2(d^2e^2+4cdef+c^2f^2))\,x\sqrt{a+bx^2}}{16b^4} - \frac{df(11adf-12b(de+cf))x^3\sqrt{a+bx^2}}{24b^3} + \frac{d^2f^2x^5\sqrt{a+bx^2}}{6b^2} - \frac{(35a^3d^2f^2-32b^3ce(de+cf)-60a^2bdf(de+cf)+24ab^2(d^2e^2+4cdef+c^2f^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}$$

output
```
(-a*d+b*c)^2*(-a*f+b*e)^2*x/a/b^4/(b*x^2+a)^(1/2)+1/16*(19*a^2*d^2*f^2-28*
a*b*d*f*(c*f+d*e)+8*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^4
-1/24*d*f*(11*a*d*f-12*b*(c*f+d*e))*x^3*(b*x^2+a)^(1/2)/b^3+1/6*d^2*f^2*x^
5*(b*x^2+a)^(1/2)/b^2-1/16*(35*a^3*d^2*f^2-32*b^3*c*e*(c*f+d*e)-60*a^2*b*d
*f*(c*f+d*e)+24*a*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}} \, dx = \frac{\frac{\sqrt{b}x(48b^4c^2e^2+105a^4d^2f^2+5a^3bdf(-36de-36cf+7dfx^2)+2a^2b^2(36c^2f^2+6cdf(24e-5fx^2)+d^2(36e^2-}{a\sqrt{a+b}}}{}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(48*b^4*c^2*e^2 + 105*a^4*d^2*f^2 + 5*a^3*b*d*f*(-36*d*e - 36*
c*f + 7*d*f*x^2) + 2*a^2*b^2*(36*c^2*f^2 + 6*c*d*f*(24*e - 5*f*x^2) + d^2*
(36*e^2 - 30*e*f*x^2 - 7*f^2*x^4)) + 8*a*b^3*(3*c^2*f*(-4*e + f*x^2) + d^2
*x^2*(3*e^2 + 3*e*f*x^2 + f^2*x^4) + 3*c*d*(-4*e^2 + 4*e*f*x^2 + f^2*x^4))
))/(a*Sqrt[a + b*x^2]) + 3*(35*a^3*d^2*f^2 - 32*b^3*c*e*(d*e + c*f) - 60*a
^2*b*d*f*(d*e + c*f) + 24*a*b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(48*b^(9/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{x^4 (c^2f^2+4cdef+d^2e^2)}{(a+bx^2)^{3/2}}+\frac{c^2e^2}{(a+bx^2)^{3/2}}+\frac{2cex^2(cf+de)}{(a+bx^2)^{3/2}}+\frac{2dfx^6(cf+de)}{(a+bx^2)^{3/2}}+\frac{d^2f^2x^8}{(a+bx^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{35a^3d^2f^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}+\frac{15a^2df\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{4b^{7/2}}+\frac{35a^2d^2f^2x\sqrt{a+bx^2}}{16b^4}-\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(c^2f^2+4cdef+d^2e^2)}{2b^{5/2}}+\frac{2ce\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{b^{3/2}}-\frac{15adfx\sqrt{a+bx^2}(cf+de)}{4b^3}-\frac{35ad^2f^2x^3\sqrt{a+bx^2}}{24b^3}+\frac{3x\sqrt{a+bx^2}(c^2f^2+4cdef+d^2e^2)}{2b^2}+\frac{5dfx^3\sqrt{a+bx^2}(cf+de)}{2b^2}+\frac{7d^2f^2x^5\sqrt{a+bx^2}}{6b^2}-\frac{x^3(c^2f^2+4cdef+d^2e^2)}{b\sqrt{a+bx^2}}+\frac{c^2e^2x}{a\sqrt{a+bx^2}}-\frac{2cex(cf+de)}{b\sqrt{a+bx^2}}-\frac{2dfx^5(cf+de)}{b\sqrt{a+bx^2}}-\frac{d^2f^2x^7}{b\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x]
```

output
```
(c^2*e^2*x)/(a*Sqrt[a + b*x^2]) - (2*c*e*(d*e + c*f)*x)/(b*Sqrt[a + b*x^2]
) - ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^3)/(b*Sqrt[a + b*x^2]) - (2*d*f*(d*
e + c*f)*x^5)/(b*Sqrt[a + b*x^2]) - (d^2*f^2*x^7)/(b*Sqrt[a + b*x^2]) + (3
5*a^2*d^2*f^2*x*Sqrt[a + b*x^2])/(16*b^4) - (15*a*d*f*(d*e + c*f)*x*Sqrt[a
 + b*x^2])/(4*b^3) + (3*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x*Sqrt[a + b*x^2])
/(2*b^2) - (35*a*d^2*f^2*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*d*f*(d*e + c*f
)*x^3*Sqrt[a + b*x^2])/(2*b^2) + (7*d^2*f^2*x^5*Sqrt[a + b*x^2])/(6*b^2) -
 (35*a^3*d^2*f^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2)) + (2*c
*e*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2) + (15*a^2*d*f
*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*b^(7/2)) - (3*a*(d^2
*e^2 + 4*c*d*e*f + c^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/
2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{35a\left(a^3d^2f^2-\frac{12a^2bdf(cf+de)}{7}+\frac{24a\,b^2\left(c^2f^2+4cdef+d^2e^2\right)}{35}-\frac{32b^3ce(cf+de)}{35}\right)\sqrt{b\,x^2+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{16}+\left(\frac{35a^4d^2f^2}{16}-\frac{15(}{}\right.}{}$
default	$\frac{c^2e^2x}{a\sqrt{b\,x^2+a}}+2df(cf+de)\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+2ce($
risch	$\frac{x\left(8f^2x^4b^2d^2-22ab\,d^2f^2x^2+24b^2cd\,f^2x^2+24b^2d^2ef\,x^2+57a^2d^2f^2-84abcd\,f^2-84ab\,d^2ef+24b^2c^2f^2+96b^2cdef+24b^2d^2\right.}{48b^4}$

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(-35/16*a*(a^3*d^2*f^2-12/7*a^2*b*d*f*(c*f+d*e)+24/35*a*
b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2)-32/35*b^3*c*e*(c*f+d*e))*(b*x^2+a)^(1/2)*a
rctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(35/16*a^4*d^2*f^2-15/4*(-7/36*d*f*x^2+c
*f+d*e)*d*b*f*a^3+3/2*b^2*(-7/36*d^2*f^2*x^4-5/6*d*f*(c*f+d*e)*x^2+c^2*f^2
+4*c*d*e*f+d^2*e^2)*a^2-2*(-1/12*d^2*f^2*x^6-1/4*d*f*(c*f+d*e)*x^4+(-1/4*c
^2*f^2-c*d*e*f-1/4*d^2*e^2)*x^2+c*e*(c*f+d*e))*b^3*a+b^4*c^2*e^2)*x*b^(1/2
))/b^(9/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.65

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(8*(4*a^2*b^3*c*d - 3*a^3*b^2*d^2)*e^2 + 4*(8*a^2*b^3*c^2 - 24*a
^3*b^2*c*d + 15*a^4*b*d^2)*e*f - (24*a^3*b^2*c^2 - 60*a^4*b*c*d + 35*a^5*d
^2)*f^2 + (8*(4*a*b^4*c*d - 3*a^2*b^3*d^2)*e^2 + 4*(8*a*b^4*c^2 - 24*a^2*b
^3*c*d + 15*a^3*b^2*d^2)*e*f - (24*a^2*b^3*c^2 - 60*a^3*b^2*c*d + 35*a^4*b
*d^2)*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(8*a*b^4*d^2*f^2*x^7 + 2*(12*a*b^4*d^2*e*f + (12*a*b^4*c*d - 7*a^2*b^3*d
^2)*f^2)*x^5 + (24*a*b^4*d^2*e^2 + 12*(8*a*b^4*c*d - 5*a^2*b^3*d^2)*e*f +
(24*a*b^4*c^2 - 60*a^2*b^3*c*d + 35*a^3*b^2*d^2)*f^2)*x^3 + 3*(8*(2*b^5*c^
2 - 4*a*b^4*c*d + 3*a^2*b^3*d^2)*e^2 - 4*(8*a*b^4*c^2 - 24*a^2*b^3*c*d + 1
5*a^3*b^2*d^2)*e*f + (24*a^2*b^3*c^2 - 60*a^3*b^2*c*d + 35*a^4*b*d^2)*f^2)
*x)*sqrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(8*(4*a^2*b^3*c*d - 3
*a^3*b^2*d^2)*e^2 + 4*(8*a^2*b^3*c^2 - 24*a^3*b^2*c*d + 15*a^4*b*d^2)*e*f
- (24*a^3*b^2*c^2 - 60*a^4*b*c*d + 35*a^5*d^2)*f^2 + (8*(4*a*b^4*c*d - 3*a
^2*b^3*d^2)*e^2 + 4*(8*a*b^4*c^2 - 24*a^2*b^3*c*d + 15*a^3*b^2*d^2)*e*f -
(24*a^2*b^3*c^2 - 60*a^3*b^2*c*d + 35*a^4*b*d^2)*f^2)*x^2)*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*a*b^4*d^2*f^2*x^7 + 2*(12*a*b^4*d^2*e*f
+ (12*a*b^4*c*d - 7*a^2*b^3*d^2)*f^2)*x^5 + (24*a*b^4*d^2*e^2 + 12*(8*a*b^
4*c*d - 5*a^2*b^3*d^2)*e*f + (24*a*b^4*c^2 - 60*a^2*b^3*c*d + 35*a^3*b^2*d
^2)*f^2)*x^3 + 3*(8*(2*b^5*c^2 - 4*a*b^4*c*d + 3*a^2*b^3*d^2)*e^2 - 4*(8*a
*b^4*c^2 - 24*a^2*b^3*c*d + 15*a^3*b^2*d^2)*e*f + (24*a^2*b^3*c^2 - 60*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}} \, dx = \int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{d^2f^2x^7}{6\sqrt{bx^2+a}b} - \frac{7\,ad^2f^2x^5}{24\sqrt{bx^2+a}b^2} + \frac{35\,a^2d^2f^2x^3}{48\sqrt{bx^2+a}b^3} + \frac{(d^2ef+cdf^2)x^5}{2\sqrt{bx^2+a}b} + \frac{c^2e^2x}{\sqrt{bx^2+a}a} + \frac{35\,a^3d^2f^2x}{16\sqrt{bx^2+a}b^4} - \frac{35\,a^3d^2f^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}} - \frac{5\,(d^2ef+cdf^2)ax^3}{4\sqrt{bx^2+a}b^2} + \frac{(d^2e^2+4\,cdef+c^2f^2)x^3}{2\sqrt{bx^2+a}b} - \frac{15\,(d^2ef+cdf^2)a^2x}{4\sqrt{bx^2+a}b^3} + \frac{3\,(d^2e^2+4\,cdef+c^2f^2)ax}{2\sqrt{bx^2+a}b^2} - \frac{2\,(cde^2+c^2ef)x}{\sqrt{bx^2+a}b} + \frac{15\,(d^2ef+cdf^2)a^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{7}{2}}} - \frac{3\,(d^2e^2+4\,cdef+c^2f^2)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{2\,(cde^2+c^2ef)\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*d^2*f^2*x^7/(sqrt(b*x^2 + a)*b) - 7/24*a*d^2*f^2*x^5/(sqrt(b*x^2 + a)*
b^2) + 35/48*a^2*d^2*f^2*x^3/(sqrt(b*x^2 + a)*b^3) + 1/2*(d^2*e*f + c*d*f^
2)*x^5/(sqrt(b*x^2 + a)*b) + c^2*e^2*x/(sqrt(b*x^2 + a)*a) + 35/16*a^3*d^2
*f^2*x/(sqrt(b*x^2 + a)*b^4) - 35/16*a^3*d^2*f^2*arcsinh(b*x/sqrt(a*b))/b^
(9/2) - 5/4*(d^2*e*f + c*d*f^2)*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*(d^2*e^2
 + 4*c*d*e*f + c^2*f^2)*x^3/(sqrt(b*x^2 + a)*b) - 15/4*(d^2*e*f + c*d*f^2)
*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*a*x/(sq
rt(b*x^2 + a)*b^2) - 2*(c*d*e^2 + c^2*e*f)*x/(sqrt(b*x^2 + a)*b) + 15/4*(d
^2*e*f + c*d*f^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*(d^2*e^2 + 4*c*
d*e*f + c^2*f^2)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*(c*d*e^2 + c^2*e*f)*
arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,d^2f^2x^2}{b} + \frac{12\,ab^6d^2ef+12\,ab^6cdf^2-7\,a^2b^5d^2f^2}{ab^7}\right)x^2 + \frac{24\,ab^6d^2e^2+96\,ab^6cdef-60\,a^2b^5d^2e}{}\right.\right.}{}$$

$$-\frac{(32\,b^3cde^2 - 24\,ab^2d^2e^2 + 32\,b^3c^2ef - 96\,ab^2cdef + 60\,a^2bd^2ef - 24\,ab^2c^2f^2 + 60\,a^2bcdf^2 - 35\,a^3d^2f^2)\,\mathrm{lo}}{16\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*d^2*f^2*x^2/b + (12*a*b^6*d^2*e*f + 12*a*b^6*c*d*f^2 - 7*a^2*b
^5*d^2*f^2)/(a*b^7))*x^2 + (24*a*b^6*d^2*e^2 + 96*a*b^6*c*d*e*f - 60*a^2*b
^5*d^2*e*f + 24*a*b^6*c^2*f^2 - 60*a^2*b^5*c*d*f^2 + 35*a^3*b^4*d^2*f^2)/(
a*b^7))*x^2 + 3*(16*b^7*c^2*e^2 - 32*a*b^6*c*d*e^2 + 24*a^2*b^5*d^2*e^2 -
32*a*b^6*c^2*e*f + 96*a^2*b^5*c*d*e*f - 60*a^3*b^4*d^2*e*f + 24*a^2*b^5*c^
2*f^2 - 60*a^3*b^4*c*d*f^2 + 35*a^4*b^3*d^2*f^2)/(a*b^7))*x/sqrt(b*x^2 + a
) - 1/16*(32*b^3*c*d*e^2 - 24*a*b^2*d^2*e^2 + 32*b^3*c^2*e*f - 96*a*b^2*c*
d*e*f + 60*a^2*b*d^2*e*f - 24*a*b^2*c^2*f^2 + 60*a^2*b*c*d*f^2 - 35*a^3*d^
2*f^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{3/2}} \, dx = \int \frac{(d\,x^2+c)^2 \,(f\,x^2+e)^2}{(b\,x^2+a)^{\frac{3}{2}}} dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(3/2),x)
```

3.345 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \frac{(bc-ad)^2x}{ab(be-af)\sqrt{a+bx^2}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}f} - \frac{(de-cf)^2\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f(be-af)^{3/2}}$$

output
```
(-a*d+b*c)^2*x/a/b/(-a*f+b*e)/(b*x^2+a)^(1/2)+d^2*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(3/2)/f-(-c*f+d*e)^2*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^
2+a)^(1/2))/e^(1/2)/f/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = -\frac{(bc-ad)^2x}{ab(-be+af)\sqrt{a+bx^2}} - \frac{(de-cf)^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}f(-be+af)^{3/2}} - \frac{d^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}f}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
-(((b*c - a*d)^2*x)/(a*b*(-(b*e) + a*f)*Sqrt[a + b*x^2])) - ((d*e - c*f)^2
*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e
) + a*f])])/(Sqrt[e]*f*(-(b*e) + a*f)^(3/2)) - (d^2*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(b^(3/2)*f)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(134) = 268.

Time = 0.65 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.43, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {419, 25, 401, 25, 27, 299, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{b\left(abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2}{\sqrt{bx^2+a}}dx}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{a\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{a\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{\int -\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{a+bx^2}}{2f}\right)}{(be-af)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int \frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}\right)}{(be-af)^2}$$

$\Big\downarrow$ 398

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2f}\right)}{(be-af)^2}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{(-adf-2bcf+2bde)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{2(be-af)(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2f}\right)}{(be-af)^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2f}\right)}{(be-af)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2(be-af)(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}}{2f}\right)}{(be-af)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2bcf+2bde)}{\sqrt{b}f}-\frac{2\sqrt{be-af}(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}}{2f}\right)}{(be-af)^2}$$

input

```
Int[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2))/(a*Sqrt[a + b*x^2]) + (-1/2*(d*(2
*b^2*c*e + 2*a^2*d*f - a*b*(3*d*e + c*f))*x*Sqrt[a + b*x^2])/b + (a*(2*a^2
*d^2*f + 2*b^2*c*(2*d*e - c*f) - a*b*d*(3*d*e + c*f))*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*b^(3/2)))/a)/(b*e - a*f)^2 - (f*(d*e - c*f)*((d*x*Sqr
t[a + b*x^2])/(2*f) - (((2*b*d*e - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(Sqrt[b]*f) - (2*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTanh[(Sqrt
[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))/(2*f)))/(b*e - a*f
)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$-\frac{a\,b^{\frac{5}{2}}\,(cf-de)^{2}\sqrt{b\,x^{2}+a}\,\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(-a\,d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)(af-be)b\sqrt{b\,x^{2}+a}+(-ad+bc)^{2}\right.}{\sqrt{(af-be)e}\,\sqrt{b\,x^{2}+a}\,(af-be)b^{\frac{5}{2}}\,fa}$
default	$\frac{d\left(df\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{2cfx}{a\sqrt{b\,x^{2}+a}}-\frac{dex}{a\sqrt{b\,x^{2}+a}}\right)}{f^{2}}-\frac{(c^{2}f^{2}-2cdef+d^{2}e^{2})\left(\frac{f}{(af-be)\sqrt{\left(x+\frac{\sqrt{-ef}}{f}\right)^{2}b-\frac{2b}{}}}\right.}{}$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-(a*b^(5/2)*(c*f-d*e)^2*(b*x^2+a)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b
*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(-a*d^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2
))*(a*f-b*e)*b*(b*x^2+a)^(1/2)+(-a*d+b*c)^2*b^(3/2)*x*f))/((a*f-b*e)*e)^(1
/2)/(b*x^2+a)^(1/2)/(a*f-b*e)/b^(5/2)/f/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(116) = 232.

Time = 2.14 (sec) , antiderivative size = 1822, normalized size of antiderivative = 13.60

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/4*(4*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e^2*f - (a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2)*e*f^2)*sqrt(b*x^2 + a)*x + 2*(a^2*b^2*d^2*e^3 - 2*a^3
*b*d^2*e^2*f + a^4*d^2*e*f^2 + (a*b^3*d^2*e^3 - 2*a^2*b^2*d^2*e^2*f + a^3*
b*d^2*e*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- (a^2*b^2*d^2*e^2 - 2*a^2*b^2*c*d*e*f + a^2*b^2*c^2*f^2 + (a*b^3*d^2*e^2
- 2*a*b^3*c*d*e*f + a*b^3*c^2*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^
2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4
*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4
 + 2*e*f*x^2 + e^2)))/(a^2*b^4*e^3*f - 2*a^3*b^3*e^2*f^2 + a^4*b^2*e*f^3 +
 (a*b^5*e^3*f - 2*a^2*b^4*e^2*f^2 + a^3*b^3*e*f^3)*x^2), 1/4*(4*((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*e^2*f - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^
2)*e*f^2)*sqrt(b*x^2 + a)*x - 4*(a^2*b^2*d^2*e^3 - 2*a^3*b*d^2*e^2*f + a^4
*d^2*e*f^2 + (a*b^3*d^2*e^3 - 2*a^2*b^2*d^2*e^2*f + a^3*b*d^2*e*f^2)*x^2)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a^2*b^2*d^2*e^2 - 2*a^2*b^2
*c*d*e*f + a^2*b^2*c^2*f^2 + (a*b^3*d^2*e^2 - 2*a*b^3*c*d*e*f + a*b^3*c^2*
f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 +
 a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*s
qrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(a^2*b^4
*e^3*f - 2*a^3*b^3*e^2*f^2 + a^4*b^2*e*f^3 + (a*b^5*e^3*f - 2*a^2*b^4*e^2*
f^2 + a^3*b^3*e*f^3)*x^2), 1/2*(2*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**2/((a + b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1363, normalized size of antiderivative = 10.17

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**2*c**2*f**2 + 2*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**2*c*d*e*f - sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a**2*b**2*d**2*e**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*a*b**3*c**2*f**2*x**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**
3*c*d*e*f*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*s
qrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**3*d**2*e**2*x
**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x
**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**2*c**2*f**2 + 2*sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt
(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**2*c*d*e*f - sqrt(e)*sqrt(a*f - b
*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/
(sqrt(e)*sqrt(b)))*a**2*b**2*d**2*e**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a*b**3*c**2*f**2*x**2 + 2*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*...
```

3.346 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 218

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{d^2x}{af^2\sqrt{a+bx^2}} + \frac{b(de-cf)(af(5de-cf)-2be(de+cf))x}{2aef^2(be-af)^2\sqrt{a+bx^2}} - \frac{(de-cf)^2x}{2ef(be-af)\sqrt{a+bx^2}\,(e+fx^2)} + \frac{(de-cf)(4bce-3ade-acf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{5/2}}$$

output
```
d^2*x/a/f^2/(b*x^2+a)^(1/2)+1/2*b*(-c*f+d*e)*(a*f*(-c*f+5*d*e)-2*b*e*(c*f+
d*e))*x/a/e/f^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)-1/2*(-c*f+d*e)^2*x/e/f/(-a*f+
b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)+1/2*(-c*f+d*e)*(-a*c*f-3*a*d*e+4*b*c*e)*arc
tanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{\frac{\sqrt{e}x(2b^2c^2e(e+fx^2)+a^2(-2cdef+c^2f^2+d^2e(3e+2fx^2))+ab(d^2e^2x^2+c^2f^2x^2-2cde(2e+3fx^2)))}{a(be-af)^2\sqrt{a+bx^2}(e+fx^2)}+}{2e^{3/2}}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
((Sqrt[e]*x*(2*b^2*c^2*e*(e + f*x^2) + a^2*(-2*c*d*e*f + c^2*f^2 + d^2*e*(
3*e + 2*f*x^2)) + a*b*(d^2*e^2*x^2 + c^2*f^2*x^2 - 2*c*d*e*(2*e + 3*f*x^2)
)))/(a*(b*e - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)) + ((d*e - c*f)*(-4*b*c*e
 + 3*a*d*e + a*c*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/
(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(-(b*e) + a*f)^(5/2))/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 402, 25, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{(bx^2+a)^{3/2}(fx^2+e)}dx}{f} - \frac{(de-cf)\int \frac{dx^2+c}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int -\frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{\int \frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d\left(\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e(e+fx^2)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}$$

input
```
Int[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(d*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) + ((d*e - c*f)*ArcTanh
[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*(b*e - a*f)^(3/2
))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e +
 f*x^2)) + ((f*(2*b*c*e - 3*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e -
a*f)*(e + f*x^2)) + (a*(2*b*e*(d*e - 2*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sq
rt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))
)/(a*(b*e - a*f))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-a\sqrt{b\,x^{2}+a}\,((3ad-4bc)e+acf)(cf-de)(f\,x^{2}+e)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\left(\left(3a^{2}d^{2}-4\left(-\frac{x^{2}d}{4}+c\right)dba+2b^{2}c^{2}\right)\right.}{2\sqrt{(af-be)e}\,\sqrt{b\,x^{2}+a}\,e(f\,x^{2}+e)(af-be)^{2}a}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(1/2)*(-a*(b*x^2+a)^(1/2)*((3*a*d-4*b*c)
*e+a*c*f)*(c*f-d*e)*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/
2))+((a*f-b*e)*e)^(1/2)*((3*a^2*d^2-4*(-1/4*x^2*d+c)*d*b*a+2*b^2*c^2)*e^2-
2*(d*(-d*x^2+c)*a^2+3*a*b*c*d*x^2-b^2*c^2*x^2)*f*e+a*c^2*f^2*(b*x^2+a))*x)
/e/(f*x^2+e)/(a*f-b*e)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(197) = 394$.

Time = 3.42 (sec) , antiderivative size = 1364, normalized size of antiderivative = 6.26

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/8*((a^3*c^2*e*f^2 + (a^2*b*c^2*f^3 + (4*a*b^2*c*d - 3*a^2*b*d^2)*e^2*f
- 2*(2*a*b^2*c^2 - a^2*b*c*d)*e*f^2)*x^4 + (4*a^2*b*c*d - 3*a^3*d^2)*e^3 -
 2*(2*a^2*b*c^2 - a^3*c*d)*e^2*f + (a^3*c^2*f^3 + (4*a*b^2*c*d - 3*a^2*b*d
^2)*e^3 - (4*a*b^2*c^2 - 6*a^2*b*c*d + 3*a^3*d^2)*e^2*f - (3*a^2*b*c^2 - 2
*a^3*c*d)*e*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^
2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^
3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2
)) + 4*((a*b^2*d^2*e^4 - a^2*b*c^2*e*f^3 + (2*b^3*c^2 - 6*a*b^2*c*d + a^2*
b*d^2)*e^3*f - (a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2)*e^2*f^2)*x^3 - (a^3*c
^2*e*f^3 - (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*e^4 + (2*a*b^2*c^2 - 2*
a^2*b*c*d + 3*a^3*d^2)*e^3*f - (a^2*b*c^2 + 2*a^3*c*d)*e^2*f^2)*x)*sqrt(b*
x^2 + a))/(a^2*b^3*e^6 - 3*a^3*b^2*e^5*f + 3*a^4*b*e^4*f^2 - a^5*e^3*f^3 +
 (a*b^4*e^5*f - 3*a^2*b^3*e^4*f^2 + 3*a^3*b^2*e^3*f^3 - a^4*b*e^2*f^4)*x^4
 + (a*b^4*e^6 - 2*a^2*b^3*e^5*f + 2*a^4*b*e^3*f^3 - a^5*e^2*f^4)*x^2), -1/
4*((a^3*c^2*e*f^2 + (a^2*b*c^2*f^3 + (4*a*b^2*c*d - 3*a^2*b*d^2)*e^2*f - 2
*(2*a*b^2*c^2 - a^2*b*c*d)*e*f^2)*x^4 + (4*a^2*b*c*d - 3*a^3*d^2)*e^3 - 2*
(2*a^2*b*c^2 - a^3*c*d)*e^2*f + (a^3*c^2*f^3 + (4*a*b^2*c*d - 3*a^2*b*d^2)
*e^3 - (4*a*b^2*c^2 - 6*a^2*b*c*d + 3*a^3*d^2)*e^2*f - (3*a^2*b*c^2 - 2*a^
3*c*d)*e*f^2)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((
2*b*e - a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^2}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/((b*x^2 + a)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(197) = 394$.

Time = 0.43 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.47

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{(ab^2e^2 - 2\,a^2bef + a^3f^2)\sqrt{bx^2+a}}$$
$$- \frac{\left(4\,b^{\frac{3}{2}}cde^2 - 3\,a\sqrt{b}d^2e^2 - 4\,b^{\frac{3}{2}}c^2ef + 2\,a\sqrt{b}cdef + a\sqrt{b}c^2f^2\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 f + 2\,be - af}{2\sqrt{-b^2e^2+abef}}\right)}{2\,(b^2e^3 - 2\,abe^2f + a^2ef^2)\sqrt{-b^2e^2+abef}}$$
$$+ \frac{2\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}d^2e^3 - 4\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}cde^2f - \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 a\sqrt{b}d^2e^2f + 2\left(\sqrt{b}\right.}{(b^2e^3f - 2\,abe^2f^2 + a^2ef^3)\left(\left(\sqrt{b}x - \sqrt{bx^2}\right.\right.}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/((a*b^2*e^2 - 2*a^2*b*e*f + a^3*f^2)*sqr
t(b*x^2 + a)) - 1/2*(4*b^(3/2)*c*d*e^2 - 3*a*sqrt(b)*d^2*e^2 - 4*b^(3/2)*c
^2*e*f + 2*a*sqrt(b)*c*d*e*f + a*sqrt(b)*c^2*f^2)*arctan(1/2*((sqrt(b)*x -
 sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*e^3 -
 2*a*b*e^2*f + a^2*e*f^2)*sqrt(-b^2*e^2 + a*b*e*f)) + (2*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*b^(3/2)*d^2*e^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)
*c*d*e^2*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2*e^2*f + 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c^2*e*f^2 + 2*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*a*sqrt(b)*c*d*e*f^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c^2
*f^3 + a^2*sqrt(b)*d^2*e^2*f - 2*a^2*sqrt(b)*c*d*e*f^2 + a^2*sqrt(b)*c^2*f
^3)/((b^2*e^3*f - 2*a*b*e^2*f^2 + a^2*e*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a)
)^4*f + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a*f + a^2*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4917, normalized size of antiderivative = 22.56

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c**2*e*f**3 - sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c**2*f**4*x**2 - 2*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**4*b*c*d*e**2*f**2 - 2*sqrt(e)*sqrt(a*f - b*e)*atan(
(sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*
sqrt(b)))*a**4*b*c*d*e*f**3*x**2 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**4*b*d**2*e**3*f + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s
qrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*d**
2*e**2*f**2*x**2 + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(
f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b**2*c**2
*e**2*f**2 + 7*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqr
t(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b**2*c**2*e*f**
3*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a +
b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b**2*c**2*f**4*x**4 +
 4*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2
) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**3*b**2*c*d*e**3*f + 2*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr...
```

3.347 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 427

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{d(4bde-4bcf+adf)x}{4aef^2(be-af)\sqrt{a+bx^2}} + \frac{b(a^2f^2(5d^2e^2+6cdef-3c^2f^2)-8b^2e^2(d^2e^2-cdef-c^2f^2)+2abef(9d^2e^2-22cdef+5c^2f^2))x}{8ae^2f^2(be-af)^3\sqrt{a+bx^2}} - \frac{fx(c+dx^2)^2}{4e(be-af)\sqrt{a+bx^2}(e+fx^2)^2} - \frac{(de-cf)(4be(de-2cf)+af(de+3cf))x}{8e^2f(be-af)^2\sqrt{a+bx^2}(e+fx^2)} + \frac{(8b^2ce^2(2de-3cf)-4abe(3d^2e^2-4cdef-3c^2f^2)-a^2f(3d^2e^2+2cdef+3c^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{7/2}}$$

output

```
1/4*d*(a*d*f-4*b*c*f+4*b*d*e)*x/a/e/f^2/(-a*f+b*e)/(b*x^2+a)^(1/2)+1/8*b*(
a^2*f^2*(-3*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-8*b^2*e^2*(-c^2*f^2-c*d*e*f+d^2*e
^2)+2*a*b*e*f*(5*c^2*f^2-22*c*d*e*f+9*d^2*e^2))*x/a/e^2/f^2/(-a*f+b*e)^3/(
b*x^2+a)^(1/2)-1/4*f*x*(d*x^2+c)^2/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^
2-1/8*(-c*f+d*e)*(4*b*e*(-2*c*f+d*e)+a*f*(3*c*f+d*e))*x/e^2/f/(-a*f+b*e)^2
/(b*x^2+a)^(1/2)/(f*x^2+e)+1/8*(8*b^2*c*e^2*(-3*c*f+2*d*e)-4*a*b*e*(-3*c^2
*f^2-4*c*d*e*f+3*d^2*e^2)-a^2*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*arctanh((
-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 20.66 (sec) , antiderivative size = 2542, normalized size of antiderivative = 5.95

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(d^2*x*(-15*e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - 10*f*x^2*Sqrt[((b*
e - a*f)*x^2)/(e*(a + b*x^2))] + 15*e*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a
 + b*x^2))]] + 10*f*x^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] +
 2*e*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7
/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2))] + 2*f*x^2*(((b*e - a*f)*x^2)/(e*(a
+ b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a +
b*x^2))]))/(5*a*e^2*f^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*Sqrt[a +
 b*x^2]*(1 + (b*x^2)/a)) - (d*(d*e - c*f)*x*(-2625*Sqrt[((b*e - a*f)*x^2)/
(e*(a + b*x^2))] - (5250*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e
- (2310*f^2*x^4*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e^2 + 70*(((b*e -
 a*f)*x^2)/(e*(a + b*x^2)))^(3/2) + (560*f*x^2*(((b*e - a*f)*x^2)/(e*(a +
b*x^2)))^(3/2))/e + (280*f^2*x^4*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)
)/e^2 + 2625*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + (5250*f*x^
2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/e + (2310*f^2*x^4*ArcT
anh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/e^2 - (945*(b*e - a*f)*x^2*A
rcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e*(a + b*x^2)) + (2310*f
*(-(b*e) + a*f)*x^4*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e^2
*(a + b*x^2)) + (1050*f^2*(-(b*e) + a*f)*x^6*ArcTanh[Sqrt[((b*e - a*f)*x^2
)/(e*(a + b*x^2))]])/(e^3*(a + b*x^2)) + 24*(((b*e - a*f)*x^2)/(e*(a + b*x
^2)))^(7/2)*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, ((b*e - a*f)*x^2)/...
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {425, 402, 25, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{\frac{\int \frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{2bf(4bce-5ade+acf)x^2+a(4be(de-2cf)+af(de+3cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e\left(e+fx^2\right)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}\right)}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{2bf(4bce-5ade+acf)x^2+a(4be(de-2cf)+af(de+3cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e\left(e+fx^2\right)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{2bf(4bce-5ade+acf)x^2+a(4be(de-2cf)+af(de+3cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e\left(e+fx^2\right)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{2bf(4bce-5ade+acf)x^2+a(4be(de-2cf)+af(de+3cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade+4bce)}{4e\left(e+fx^2\right)^2(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{\frac{\frac{\int\frac{a\left(8b^2(de-3cf)e^2+4abf(2de+3cf)e-a^2f^2(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-5ade-}{4e\left(e+fx^2\right)^2(be-a}}{a(be-af)}\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{\frac{\frac{a\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+fx}{a(be-af)}\right.}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\frac{a\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}}{a(be-af)}\right.}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(-a^2f^2(3cf+de)+4abef(3cf+2de)+8b^2e^2(de-3cf)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}\left(a^2(-f)(3cf+de)-2abe(7de-5cf)+8b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+}{a(be-af)}\right.}{f}$$

input

```
Int[(c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(d*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + ((f*(2*b
*c*e - 3*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) +
 (a*(2*b*e*(d*e - 2*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(S
qrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(a*(b*e - a*f))))
/f - ((d*e - c*f)*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x
^2)^2) + ((f*(4*b*c*e - 5*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*
f)*(e + f*x^2)^2) + ((f*(8*b^2*c*e^2 - 2*a*b*e*(7*d*e - 5*c*f) - a^2*f*(d*
e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*(8*b^2*e
^2*(d*e - 3*c*f) - a^2*f^2*(d*e + 3*c*f) + 4*a*b*e*f*(2*d*e + 3*c*f))*ArcT
anh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)
^(3/2)))/(4*e*(b*e - a*f)))/(a*(b*e - a*f))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{3a\sqrt{b\,x^2+a}\left(4\left(ab\,d^2-\frac{4}{3}b^2cd\right)e^3+f\left(a^2d^2-\frac{16}{3}abcd+8b^2c^2\right)e^2+\frac{2ac\,f^2(ad-6bc)e}{3}+a^2c^2f^3\right)\left(f\,x^2+e\right)^2\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\frac{5\left(\ldots\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(1/2)*(-3/5*a*(b*x^2+a)^(1/2)*(4*(a*b*d^
2-4/3*b^2*c*d)*e^3+f*(a^2*d^2-16/3*a*b*c*d+8*b^2*c^2)*e^2+2/3*a*c*f^2*(a*d
-6*b*c)*e+a^2*c^2*f^3)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e
)^(1/2))+(-12/5*(a^2*d^2-4/3*(-1/4*x^2*d+c)*d*b*a+2/3*b^2*c^2)*b*e^4-3/5*f
*(a^3*d^2-16/3*d*(-21/16*x^2*d+c)*b*a^2-16*d*(-1/24*x^2*d+c)*b^2*x^2*a+16/
3*x^2*b^3*c^2)*e^3-2/5*(d*(5/2*x^2*d+c)*a^3+6*(13/12*d^2*x^4-5/6*c*d*x^2+c
^2)*b*a^2+6*(-7/3*x^2*d+c)*c*b^2*x^2*a+4*c^2*b^3*x^4)*f^2*e^2+a*c*(b*x^2+a
)*(a*(2/5*x^2*d+c)-2*x^2*b*c)*f^3*e+3/5*a^2*c^2*f^4*x^2*(b*x^2+a))*((a*f-b
*e)*e)^(1/2)*x)/(f*x^2+e)^2/e^2/(a*f-b*e)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(400) = 800$.

Time = 9.58 (sec) , antiderivative size = 2616, normalized size of antiderivative = 6.13

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
[-1/32*((3*a^4*c^2*e^2*f^3 + (3*a^3*b*c^2*f^5 - 4*(4*a*b^3*c*d - 3*a^2*b^2
*d^2)*e^3*f^2 + (24*a*b^3*c^2 - 16*a^2*b^2*c*d + 3*a^3*b*d^2)*e^2*f^3 - 2*
(6*a^2*b^2*c^2 - a^3*b*c*d)*e*f^4)*x^6 - 4*(4*a^2*b^2*c*d - 3*a^3*b*d^2)*e
^5 + (24*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*e^4*f - 2*(6*a^3*b*c^2 -
a^4*c*d)*e^3*f^2 + (3*a^4*c^2*f^5 - 8*(4*a*b^3*c*d - 3*a^2*b^2*d^2)*e^4*f
+ 6*(8*a*b^3*c^2 - 8*a^2*b^2*c*d + 3*a^3*b*d^2)*e^3*f^2 - 3*(4*a^3*b*c*d -
 a^4*d^2)*e^2*f^3 - 2*(3*a^3*b*c^2 - a^4*c*d)*e*f^4)*x^4 + (6*a^4*c^2*e*f^
4 - 4*(4*a*b^3*c*d - 3*a^2*b^2*d^2)*e^5 + 3*(8*a*b^3*c^2 - 16*a^2*b^2*c*d
+ 9*a^3*b*d^2)*e^4*f + 6*(6*a^2*b^2*c^2 - 5*a^3*b*c*d + a^4*d^2)*e^3*f^2 -
 (21*a^3*b*c^2 - 4*a^4*c*d)*e^2*f^3)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*
e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 +
 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x
^4 + 2*e*f*x^2 + e^2)) - 4*((2*a*b^3*d^2*e^5*f + 3*a^3*b*c^2*e*f^5 + (8*b^
4*c^2 - 28*a*b^3*c*d + 11*a^2*b^2*d^2)*e^4*f^2 + (2*a*b^3*c^2 + 26*a^2*b^2
*c*d - 13*a^3*b*d^2)*e^3*f^3 - (13*a^2*b^2*c^2 - 2*a^3*b*c*d)*e^2*f^4)*x^5
 + (4*a*b^3*d^2*e^6 + 3*a^4*c^2*e*f^5 + (16*b^4*c^2 - 48*a*b^3*c*d + 17*a^
2*b^2*d^2)*e^5*f - 2*(2*a*b^3*c^2 - 19*a^2*b^2*c*d + 8*a^3*b*d^2)*e^4*f^2
- (7*a^2*b^2*c^2 - 8*a^3*b*c*d + 5*a^4*d^2)*e^3*f^3 - 2*(4*a^3*b*c^2 - a^4
*c*d)*e^2*f^4)*x^3 + (5*a^4*c^2*e^2*f^4 + 4*(2*b^4*c^2 - 4*a*b^3*c*d + 3*a
^2*b^2*d^2)*e^6 - (8*a*b^3*c^2 + 9*a^3*b*d^2)*e^5*f + 3*(4*a^2*b^2*c^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^2}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/((b*x^2 + a)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1451 vs. $2(400) = 800$.

Time = 0.71 (sec) , antiderivative size = 1451, normalized size of antiderivative = 3.40

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x/((a*b^3*e^3 - 3*a^2*b^2*e^2*f + 3*a^
3*b*e*f^2 - a^4*f^3)*sqrt(b*x^2 + a)) - 1/8*(16*b^(5/2)*c*d*e^3 - 12*a*b^(
3/2)*d^2*e^3 - 24*b^(5/2)*c^2*e^2*f + 16*a*b^(3/2)*c*d*e^2*f - 3*a^2*sqrt(
b)*d^2*e^2*f + 12*a*b^(3/2)*c^2*e*f^2 - 2*a^2*sqrt(b)*c*d*e*f^2 - 3*a^2*sq
rt(b)*c^2*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f
)/sqrt(-b^2*e^2 + a*b*e*f))/((b^3*e^5 - 3*a*b^2*e^4*f + 3*a^2*b*e^3*f^2 -
a^3*e^2*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*b^(5/2)*c*d*e^3*f^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*
d^2*e^3*f^2 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*e^2*f^3 + 5*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^2*e^2*f^3 + 12*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(3/2)*c^2*e*f^4 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^2*sqrt(b)*c*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^2*
f^5 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^2*e^5 + 96*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*b^(7/2)*c*d*e^4*f - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a*b^(5/2)*d^2*e^4*f - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^2*e^3*
f^2 - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d*e^3*f^2 + 34*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^2*e^3*f^2 + 104*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a*b^(5/2)*c^2*e^2*f^3 + 20*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a
^2*b^(3/2)*c*d*e^2*f^3 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^
2*e^2*f^3 - 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c^2*e*f^4 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 9235, normalized size of antiderivative = 21.63

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x)
```

output
```
( - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*c**2*e**2*f**5 - 18*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*c**2*e*f**6*x**2 - 9*sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a**5*c**2*f**7*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**5*c*d*e**3*f**4 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*a**5*c*d*e**2*f**5*x**2 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**
5*c*d*e*f**6*x**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt
(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*d**2*e**
4*f**3 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*d**2*e**3*f**4*x**
2 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*d**2*e**2*f**5*x**4 + 60
*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2)
- sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c**2*e**3*f**4 + 111*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sq...
```

3.348 $\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 635

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}} \, dx = \frac{(bc-ad)^3(be-af)^3x}{ab^6\sqrt{a+bx^2}}$$
$$+ \frac{(437a^4d^3f^3 - 1122a^3bd^2f^2(de+cf) + 384b^4ce(d^2e^2+3cdef+c^2f^2) + 912a^2b^2df(d^2e^2+3cdef+c^2f^2)}{256b^6}$$
$$- \frac{(103a^3d^3f^3 - 246a^2bd^2f^2(de+cf) + 176ab^2df(d^2e^2+3cdef+c^2f^2) - 32b^3(d^3e^3+9cd^2e^2f+9c^2def^2}{128b^5}$$
$$+ \frac{df(71a^2d^2f^2 - 150abdf(de+cf) + 80b^2(d^2e^2+3cdef+c^2f^2))\,x^5\sqrt{a+bx^2}}{160b^4}$$
$$- \frac{d^2f^2(19adf - 30b(de+cf))x^7\sqrt{a+bx^2}}{80b^3} + \frac{d^3f^3x^9\sqrt{a+bx^2}}{10b^2}$$
$$- \frac{3(231a^5d^3f^3 - 256b^5c^2e^2(de+cf) - 630a^4bd^2f^2(de+cf) + 384ab^4ce(d^2e^2+3cdef+c^2f^2) + 560a^3b^2d}{256b^{13/2}}$$

output
```
(-a*d+b*c)^3*(-a*f+b*e)^3*x/a/b^6/(b*x^2+a)^(1/2)+1/256*(437*a^4*d^3*f^3-1
122*a^3*b*d^2*f^2*(c*f+d*e)+384*b^4*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)+912*a^
2*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-224*a*b^3*(c^3*f^3+9*c^2*d*e*f^2+9*c
*d^2*e^2*f+d^3*e^3))*x*(b*x^2+a)^(1/2)/b^6-1/128*(103*a^3*d^3*f^3-246*a^2*
b*d^2*f^2*(c*f+d*e)+176*a*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-32*b^3*(c^3*
f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^3*(b*x^2+a)^(1/2)/b^5+1/160*d*
f*(71*a^2*d^2*f^2-150*a*b*d*f*(c*f+d*e)+80*b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2)
)*x^5*(b*x^2+a)^(1/2)/b^4-1/80*d^2*f^2*(19*a*d*f-30*b*(c*f+d*e))*x^7*(b*x^
2+a)^(1/2)/b^3+1/10*d^3*f^3*x^9*(b*x^2+a)^(1/2)/b^2-3/256*(231*a^5*d^3*f^3
-256*b^5*c^2*e^2*(c*f+d*e)-630*a^4*b*d^2*f^2*(c*f+d*e)+384*a*b^4*c*e*(c^2*
f^2+3*c*d*e*f+d^2*e^2)+560*a^3*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-160*a^2
*b^3*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}x\left(1280b^6c^3e^3+3465a^6d^3f^3+105a^5bd^2f^2\left(-90de-90cf+11dfx^2\right)+42a^4b^2df\left(200c^2f^2-75cdf\left(-8e+f\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(1280*b^6*c^3*e^3 + 3465*a^6*d^3*f^3 + 105*a^5*b*d^2*f^2*(-90*
d*e - 90*c*f + 11*d*f*x^2) + 42*a^4*b^2*d*f*(200*c^2*f^2 - 75*c*d*f*(-8*e
+ f*x^2) + d^2*(200*e^2 - 75*e*f*x^2 - 11*f^2*x^4)) - 4*a^3*b^3*(600*c^3*f
^3 + 100*c^2*d*f^2*(54*e - 7*f*x^2) - 15*c*d^2*f*(-360*e^2 + 140*e*f*x^2 +
 21*f^2*x^4) + d^3*(600*e^3 - 700*e^2*f*x^2 - 315*e*f^2*x^4 - 66*f^3*x^6))
 - 32*a*b^5*(-10*c^3*f*(-12*e^2 + 6*e*f*x^2 + f^2*x^4) + 10*c^2*d*(12*e^3
- 18*e^2*f*x^2 - 9*e*f^2*x^4 - 2*f^3*x^6) - 15*c*d^2*x^2*(4*e^3 + 6*e^2*f*
x^2 + 4*e*f^2*x^4 + f^3*x^6) - d^3*x^4*(10*e^3 + 20*e^2*f*x^2 + 15*e*f^2*x
^4 + 4*f^3*x^6)) + 16*a^2*b^4*(10*c^3*f^2*(36*e - 5*f*x^2) - 10*c^2*d*f*(-
108*e^2 + 45*e*f*x^2 + 7*f^2*x^4) + 15*c*d^2*(24*e^3 - 30*e^2*f*x^2 - 14*e
*f^2*x^4 - 3*f^3*x^6) - d^3*x^2*(50*e^3 + 70*e^2*f*x^2 + 45*e*f^2*x^4 + 11
*f^3*x^6))))/(a*Sqrt[a + b*x^2]) + 15*(231*a^5*d^3*f^3 - 256*b^5*c^2*e^2*(
d*e + c*f) - 630*a^4*b*d^2*f^2*(d*e + c*f) + 384*a*b^4*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + 560*a^3*b^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) - 160*a
^2*b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(1280*b^(13/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 1083, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{c^3e^3}{(a+bx^2)^{3/2}}+\frac{3dfx^8(c^2f^2+3cdef+d^2e^2)}{(a+bx^2)^{3/2}}+\frac{x^6(cf+de)(c^2f^2+8cdef+d^2e^2)}{(a+bx^2)^{3/2}}+\frac{3cex^4(c^2f^2+3cdef+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d^3f^3x^{11}}{b\sqrt{bx^2+a}}+\frac{11d^3f^3\sqrt{bx^2+a}x^9}{10b^2}-\frac{3d^2f^2(de+cf)x^9}{b\sqrt{bx^2+a}}-\frac{99ad^3f^3\sqrt{bx^2+a}x^7}{80b^3}+\frac{27d^2f^2(de+cf)\sqrt{bx^2+a}x^7}{8b^2}-\frac{3df\left(d^2e^2+3cdfe+c^2f^2\right)x^7}{b\sqrt{bx^2+a}}+\frac{231a^2d^3f^3\sqrt{bx^2+a}x^5}{160b^4}-\frac{63ad^2f^2(de+cf)\sqrt{bx^2+a}x^5}{16b^3}+\frac{7df\left(d^2e^2+3cdfe+c^2f^2\right)\sqrt{bx^2+a}x^5}{2b^2}-\frac{(de+cf)\left(d^2e^2+8cdfe+c^2f^2\right)x^5}{b\sqrt{bx^2+a}}-\frac{231a^3d^3f^3\sqrt{bx^2+a}x^3}{128b^5}+\frac{315a^2d^2f^2(de+cf)\sqrt{bx^2+a}x^3}{64b^4}-\frac{35adf\left(d^2e^2+3cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{8b^3}+\frac{5(de+cf)\left(d^2e^2+8cdfe+c^2f^2\right)\sqrt{bx^2+a}x^3}{4b^2}-\frac{3ce\left(d^2e^2+3cdfe+c^2f^2\right)x^3}{b\sqrt{bx^2+a}}+\frac{693a^4d^3f^3\sqrt{bx^2+a}x}{256b^6}-\frac{945a^3d^2f^2(de+cf)\sqrt{bx^2+a}x}{128b^5}+\frac{9ce\left(d^2e^2+3cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{2b^2}+\frac{105a^2df\left(d^2e^2+3cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{16b^4}-\frac{15a(de+cf)\left(d^2e^2+8cdfe+c^2f^2\right)\sqrt{bx^2+a}x}{8b^3}+\frac{c^3e^3x}{a\sqrt{bx^2+a}}-\frac{3c^2e^2(de+cf)x}{b\sqrt{bx^2+a}}-\frac{693a^5d^3f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{256b^{13/2}}+\frac{3c^2e^2(de+cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{b^{3/2}}+\frac{945a^4d^2f^2(de+cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{11/2}}-\frac{9ace\left(d^2e^2+3cdfe+c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{5/2}}-\frac{105a^3df\left(d^2e^2+3cdfe+c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{9/2}}+\frac{15a^2(de+cf)\left(d^2e^2+8cdfe+c^2f^2\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{7/2}}$$

input
```
Int[((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x]
```

output
```
(c^3*e^3*x)/(a*Sqrt[a + b*x^2]) - (3*c^2*e^2*(d*e + c*f)*x)/(b*Sqrt[a + b*
x^2]) - (3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x^3)/(b*Sqrt[a + b*x^2]) -
((d*e + c*f)*(d^2*e^2 + 8*c*d*e*f + c^2*f^2)*x^5)/(b*Sqrt[a + b*x^2]) - (3
*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x^7)/(b*Sqrt[a + b*x^2]) - (3*d^2*f^2
*(d*e + c*f)*x^9)/(b*Sqrt[a + b*x^2]) - (d^3*f^3*x^11)/(b*Sqrt[a + b*x^2])
 + (693*a^4*d^3*f^3*x*Sqrt[a + b*x^2])/(256*b^6) - (945*a^3*d^2*f^2*(d*e +
 c*f)*x*Sqrt[a + b*x^2])/(128*b^5) + (9*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2
)*x*Sqrt[a + b*x^2])/(2*b^2) + (105*a^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2
)*x*Sqrt[a + b*x^2])/(16*b^4) - (15*a*(d*e + c*f)*(d^2*e^2 + 8*c*d*e*f + c
^2*f^2)*x*Sqrt[a + b*x^2])/(8*b^3) - (231*a^3*d^3*f^3*x^3*Sqrt[a + b*x^2])
/(128*b^5) + (315*a^2*d^2*f^2*(d*e + c*f)*x^3*Sqrt[a + b*x^2])/(64*b^4) -
(35*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(8*b^3) + (
5*(d*e + c*f)*(d^2*e^2 + 8*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(4*b^2)
 + (231*a^2*d^3*f^3*x^5*Sqrt[a + b*x^2])/(160*b^4) - (63*a*d^2*f^2*(d*e +
c*f)*x^5*Sqrt[a + b*x^2])/(16*b^3) + (7*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2
)*x^5*Sqrt[a + b*x^2])/(2*b^2) - (99*a*d^3*f^3*x^7*Sqrt[a + b*x^2])/(80*b^
3) + (27*d^2*f^2*(d*e + c*f)*x^7*Sqrt[a + b*x^2])/(8*b^2) + (11*d^3*f^3*x^
9*Sqrt[a + b*x^2])/(10*b^2) - (693*a^5*d^3*f^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(256*b^(13/2)) + (3*c^2*e^2*(d*e + c*f)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/b^(3/2) + (945*a^4*d^2*f^2*(d*e + c*f)*ArcTanh[(Sqrt[b]*...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{693\left(a\sqrt{b\,x^2+a}\left(a^5 d^3 f^3-\frac{30a^4 b\,d^2 f^2(cf+de)}{11}+\frac{80a^3 b^2 df\left(c^2f^2+3cdef+d^2e^2\right)}{33}-\frac{160b^3(cf+de)\left(c^2f^2+8cdef+d^2e^2\right)a^2}{231}+\frac{128a\cdots}{\cdots}\right.\right.}{\cdots}$
default	$\frac{c^3e^3x}{a\sqrt{b\,x^2+a}}+3d^2f^2(cf+de)\left(\frac{x^9}{8b\sqrt{b\,x^2+a}}-\frac{9a\left(\frac{x^7}{6b\sqrt{b\,x^2+a}}-\frac{7a\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}\cdots\right)}{\cdots}\right)}{4b}\right)}{6b}\right)}{8b}\cdots\right.$

input
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-693/256/b^(13/2)*(a*(b*x^2+a)^(1/2)*(a^5*d^3*f^3-30/11*a^4*b*d^2*f^2*(c*f
+d*e)+80/33*a^3*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-160/231*b^3*(c*f+d*e)*
(c^2*f^2+8*c*d*e*f+d^2*e^2)*a^2+128/77*a*b^4*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^
2)-256/231*b^5*c^2*e^2*(c*f+d*e))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(-2
56/231*a*(-1/12*(2/5*f^3*x^6+3/2*e*f^2*x^4+2*e^2*f*x^2+e^3)*x^4*d^3-1/2*c*
(1/2*f*x^2+e)*(1/2*f^2*x^4+e*f*x^2+e^2)*x^2*d^2+c^2*(-1/6*f^3*x^6-3/4*e*f^
2*x^4-3/2*e^2*f*x^2+e^3)*d+c^3*f*(-1/12*f^2*x^4-1/2*e*f*x^2+e^2))*b^(11/2)
+128/77*a^2*(-5/36*(11/50*f^3*x^6+9/10*e*f^2*x^4+7/5*e^2*f*x^2+e^3)*x^2*d^
3+c*(-1/8*f^3*x^6-7/12*e*f^2*x^4-5/4*e^2*f*x^2+e^3)*d^2+3*c^2*f*(-7/108*f^
2*x^4-5/12*e*f*x^2+e^2)*d+c^3*f^2*(-5/36*f*x^2+e))*b^(9/2)-160/231*a^3*((-
11/100*f^3*x^6-21/40*e*f^2*x^4-7/6*e^2*f*x^2+e^3)*d^3+9*c*(-7/120*f^2*x^4-
7/18*e*f*x^2+e^2)*f*d^2+9*c^2*f^2*(-7/54*f*x^2+e)*d+c^3*f^3)*b^(7/2)+80/33
*a^4*((-11/200*f^2*x^4-3/8*e*f*x^2+e^2)*d^2+3*c*(-1/8*f*x^2+e)*f*d+c^2*f^2
)*d*f*b^(5/2)-30/11*a^5*d^2*((-11/90*f*x^2+e)*d+c*f)*f^2*b^(3/2)+b^(1/2)*a
^6*d^3*f^3+256/693*b^(13/2)*c^3*e^3))/(b*x^2+a)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 2222, normalized size of antiderivative = 3.50

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2560*(15*(32*(8*a^2*b^5*c^2*d - 12*a^3*b^4*c*d^2 + 5*a^4*b^3*d^3)*e^3
+ 16*(16*a^2*b^5*c^3 - 72*a^3*b^4*c^2*d + 90*a^4*b^3*c*d^2 - 35*a^5*b^2*d^
3)*e^2*f - 6*(64*a^3*b^4*c^3 - 240*a^4*b^3*c^2*d + 280*a^5*b^2*c*d^2 - 105
*a^6*b*d^3)*e*f^2 + (160*a^4*b^3*c^3 - 560*a^5*b^2*c^2*d + 630*a^6*b*c*d^2
 - 231*a^7*d^3)*f^3 + (32*(8*a*b^6*c^2*d - 12*a^2*b^5*c*d^2 + 5*a^3*b^4*d^
3)*e^3 + 16*(16*a*b^6*c^3 - 72*a^2*b^5*c^2*d + 90*a^3*b^4*c*d^2 - 35*a^4*b
^3*d^3)*e^2*f - 6*(64*a^2*b^5*c^3 - 240*a^3*b^4*c^2*d + 280*a^4*b^3*c*d^2
- 105*a^5*b^2*d^3)*e*f^2 + (160*a^3*b^4*c^3 - 560*a^4*b^3*c^2*d + 630*a^5*
b^2*c*d^2 - 231*a^6*b*d^3)*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(128*a*b^6*d^3*f^3*x^11 + 16*(30*a*b^6*d^3*e*f^2 +
(30*a*b^6*c*d^2 - 11*a^2*b^5*d^3)*f^3)*x^9 + 8*(80*a*b^6*d^3*e^2*f + 30*(8
*a*b^6*c*d^2 - 3*a^2*b^5*d^3)*e*f^2 + (80*a*b^6*c^2*d - 90*a^2*b^5*c*d^2 +
 33*a^3*b^4*d^3)*f^3)*x^7 + 2*(160*a*b^6*d^3*e^3 + 80*(18*a*b^6*c*d^2 - 7*
a^2*b^5*d^3)*e^2*f + 30*(48*a*b^6*c^2*d - 56*a^2*b^5*c*d^2 + 21*a^3*b^4*d^
3)*e*f^2 + (160*a*b^6*c^3 - 560*a^2*b^5*c^2*d + 630*a^3*b^4*c*d^2 - 231*a^
4*b^3*d^3)*f^3)*x^5 + 5*(32*(12*a*b^6*c*d^2 - 5*a^2*b^5*d^3)*e^3 + 16*(72*
a*b^6*c^2*d - 90*a^2*b^5*c*d^2 + 35*a^3*b^4*d^3)*e^2*f + 6*(64*a*b^6*c^3 -
 240*a^2*b^5*c^2*d + 280*a^3*b^4*c*d^2 - 105*a^4*b^3*d^3)*e*f^2 - (160*a^2
*b^5*c^3 - 560*a^3*b^4*c^2*d + 630*a^4*b^3*c*d^2 - 231*a^5*b^2*d^3)*f^3)*x
^3 + 5*(32*(8*b^7*c^3 - 24*a*b^6*c^2*d + 36*a^2*b^5*c*d^2 - 15*a^3*b^4*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx=\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3*(f*x**2+e)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**3*(e + f*x**2)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/10*d^3*f^3*x^11/(sqrt(b*x^2 + a)*b) - 11/80*a*d^3*f^3*x^9/(sqrt(b*x^2 +
a)*b^2) + 33/160*a^2*d^3*f^3*x^7/(sqrt(b*x^2 + a)*b^3) - 231/640*a^3*d^3*f
^3*x^5/(sqrt(b*x^2 + a)*b^4) + 3/8*(d^3*e*f^2 + c*d^2*f^3)*x^9/(sqrt(b*x^2
 + a)*b) + 231/256*a^4*d^3*f^3*x^3/(sqrt(b*x^2 + a)*b^5) - 9/16*(d^3*e*f^2
 + c*d^2*f^3)*a*x^7/(sqrt(b*x^2 + a)*b^2) + 1/2*(d^3*e^2*f + 3*c*d^2*e*f^2
 + c^2*d*f^3)*x^7/(sqrt(b*x^2 + a)*b) + c^3*e^3*x/(sqrt(b*x^2 + a)*a) + 69
3/256*a^5*d^3*f^3*x/(sqrt(b*x^2 + a)*b^6) - 693/256*a^5*d^3*f^3*arcsinh(b*
x/sqrt(a*b))/b^(13/2) + 63/64*(d^3*e*f^2 + c*d^2*f^3)*a^2*x^5/(sqrt(b*x^2
+ a)*b^3) - 7/8*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3)*a*x^5/(sqrt(b*x^2
+ a)*b^2) + 1/4*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*x^5/(s
qrt(b*x^2 + a)*b) - 315/128*(d^3*e*f^2 + c*d^2*f^3)*a^3*x^3/(sqrt(b*x^2 +
a)*b^4) + 35/16*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3)*a^2*x^3/(sqrt(b*x^
2 + a)*b^3) - 5/8*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*a*x^
3/(sqrt(b*x^2 + a)*b^2) + 3/2*(c*d^2*e^3 + 3*c^2*d*e^2*f + c^3*e*f^2)*x^3/
(sqrt(b*x^2 + a)*b) - 945/128*(d^3*e*f^2 + c*d^2*f^3)*a^4*x/(sqrt(b*x^2 +
a)*b^5) + 105/16*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3)*a^3*x/(sqrt(b*x^2
 + a)*b^4) - 15/8*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*a^2*
x/(sqrt(b*x^2 + a)*b^3) + 9/2*(c*d^2*e^3 + 3*c^2*d*e^2*f + c^3*e*f^2)*a*x/
(sqrt(b*x^2 + a)*b^2) - 3*(c^2*d*e^3 + c^3*e^2*f)*x/(sqrt(b*x^2 + a)*b) +
945/128*(d^3*e*f^2 + c*d^2*f^3)*a^4*arcsinh(b*x/sqrt(a*b))/b^(11/2) - 1...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/1280*((2*(4*(2*(8*d^3*f^3*x^2/b + (30*a*b^10*d^3*e*f^2 + 30*a*b^10*c*d^2
*f^3 - 11*a^2*b^9*d^3*f^3)/(a*b^11))*x^2 + (80*a*b^10*d^3*e^2*f + 240*a*b^
10*c*d^2*e*f^2 - 90*a^2*b^9*d^3*e*f^2 + 80*a*b^10*c^2*d*f^3 - 90*a^2*b^9*c
*d^2*f^3 + 33*a^3*b^8*d^3*f^3)/(a*b^11))*x^2 + (160*a*b^10*d^3*e^3 + 1440*
a*b^10*c*d^2*e^2*f - 560*a^2*b^9*d^3*e^2*f + 1440*a*b^10*c^2*d*e*f^2 - 168
0*a^2*b^9*c*d^2*e*f^2 + 630*a^3*b^8*d^3*e*f^2 + 160*a*b^10*c^3*f^3 - 560*a
^2*b^9*c^2*d*f^3 + 630*a^3*b^8*c*d^2*f^3 - 231*a^4*b^7*d^3*f^3)/(a*b^11))*
x^2 + 5*(384*a*b^10*c*d^2*e^3 - 160*a^2*b^9*d^3*e^3 + 1152*a*b^10*c^2*d*e^
2*f - 1440*a^2*b^9*c*d^2*e^2*f + 560*a^3*b^8*d^3*e^2*f + 384*a*b^10*c^3*e*
f^2 - 1440*a^2*b^9*c^2*d*e*f^2 + 1680*a^3*b^8*c*d^2*e*f^2 - 630*a^4*b^7*d^
3*e*f^2 - 160*a^2*b^9*c^3*f^3 + 560*a^3*b^8*c^2*d*f^3 - 630*a^4*b^7*c*d^2*
f^3 + 231*a^5*b^6*d^3*f^3)/(a*b^11))*x^2 + 5*(256*b^11*c^3*e^3 - 768*a*b^1
0*c^2*d*e^3 + 1152*a^2*b^9*c*d^2*e^3 - 480*a^3*b^8*d^3*e^3 - 768*a*b^10*c^
3*e^2*f + 3456*a^2*b^9*c^2*d*e^2*f - 4320*a^3*b^8*c*d^2*e^2*f + 1680*a^4*b
^7*d^3*e^2*f + 1152*a^2*b^9*c^3*e*f^2 - 4320*a^3*b^8*c^2*d*e*f^2 + 5040*a^
4*b^7*c*d^2*e*f^2 - 1890*a^5*b^6*d^3*e*f^2 - 480*a^3*b^8*c^3*f^3 + 1680*a^
4*b^7*c^2*d*f^3 - 1890*a^5*b^6*c*d^2*f^3 + 693*a^6*b^5*d^3*f^3)/(a*b^11))*
x/sqrt(b*x^2 + a) - 3/256*(256*b^5*c^2*d*e^3 - 384*a*b^4*c*d^2*e^3 + 160*a
^2*b^3*d^3*e^3 + 256*b^5*c^3*e^2*f - 1152*a*b^4*c^2*d*e^2*f + 1440*a^2*b^3
*c*d^2*e^2*f - 560*a^3*b^2*d^3*e^2*f - 384*a*b^4*c^3*e*f^2 + 1440*a^2*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^3\,(f\,x^2+e)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^3 (f\,x^2+e)^3}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(3/2),x)
```

3.349 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 177

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)} \, dx = \frac{(bc-ad)^3x}{ab^2(be-af)\sqrt{a+bx^2}} + \frac{d^3x\sqrt{a+bx^2}}{2b^2f} - \frac{d^2(2bde-6bcf+3adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}f^2} + \frac{(de-cf)^3\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f^2(be-af)^{3/2}}$$

output

```
(-a*d+b*c)^3*x/a/b^2/(-a*f+b*e)/(b*x^2+a)^(1/2)+1/2*d^3*x*(b*x^2+a)^(1/2)/
b^2/f-1/2*d^2*(3*a*d*f-6*b*c*f+2*b*d*e)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(5/2)/f^2+(-c*f+d*e)^3*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/
2))/e^(1/2)/f^2/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)} \, dx = \frac{\frac{fx(-2b^3c^3f+3a^3d^3f+ab^2d(6c^2f-d^2ex^2)+a^2bd^2(-de-6cf+dfx^2))}{ab^2(-be+af)\sqrt{a+bx^2}} + \frac{2(de-cf)^3\arctan\left(\frac{-fx\sqrt{a+b}}{\sqrt{e}}\right.}{\sqrt{e}(-be+af)^{3}}}{2f^2}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((f*x*(-2*b^3*c^3*f + 3*a^3*d^3*f + a*b^2*d*(6*c^2*f - d^2*e*x^2) + a^2*b*
d^2*(-(d*e) - 6*c*f + d*f*x^2)))/(a*b^2*(-(b*e) + a*f)*Sqrt[a + b*x^2]) +
(2*(d*e - c*f)^3*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sq
rt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*(-(b*e) + a*f)^(3/2)) + (d^2*(2*b*d*e
 - 6*b*c*f + 3*a*d*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2))/(2*f^2
)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 538 vs. $2(177) = 354$.

Time = 1.01 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.04, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {419, 25, 401, 25, 27, 403, 299, 224, 219, 420, 299, 211, 224, 219, 403, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{419}$$

$$-\frac{\int -\frac{(dx^2+c)^2(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)^2(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x(c+dx^2)^2(bc-ad)(be-af)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{b(dx^2+c)\left(abc(de-cf)-d\left(4dfa^2-5bdea-3bcfa+4b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{b(dx^2+c)\left(abc(de-cf)-d\left(4dfa^2-b(5de+3cf)a+4b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)^2(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(dx^2+c)\left(abc(de-cf)-d\left(4dfa^2-b(5de+3cf)a+4b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{a}+\frac{x(c+dx^2)^2(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$\Big\downarrow$ 403

$$\frac{\frac{\frac{\int \frac{ac\left(4c(2de-cf)b^2-ad(5de+3cf)b+4a^2d^2f\right)-d\left(-12d^2fa^3+bd(15de+17cf)a^2-2b^2c(13de+cf)a+8b^3c^2e\right)x^2}{\sqrt{bx^2+a}}dx}{4b}-\frac{dx\sqrt{a+bx^2}(c+dx^2)\left(4a^2df-3abcf-5abde+\cdots\right)}{4b}\cdots}{a}\cdots}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$\Big\downarrow$ 299

$$\frac{\frac{\frac{-\frac{a\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b^3c^2e\right)}{2b}}{4b}-\cdots}{a}\cdots}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)^2}{fx^2+e}dx}{(be-af)^2}$$

$\Big\downarrow$ 224

$$\frac{\frac{-\frac{-\frac{a\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{-\frac{-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^2}{fx^2+e}dx}{(be-af)^2}$$

$$\Big\downarrow \text{420}$$

$$\frac{\frac{-\frac{-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\int \sqrt{bx^2+a}\left(dx^2+c\right)dx}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{-\frac{-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\int \sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{-\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{-\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{(4bc-ad)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{-\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 403$$

$$\frac{\frac{-\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\cdots\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{\int -\frac{(2bde-2bcf-adf)x^2+a(de-2cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2f}+\frac{dx\sqrt{\cdots}}{2\cdots}\right.}{f}\right.}{(be-af)^2}$$

$\downarrow$ 25

$$\frac{\frac{-\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\cdots\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\int \frac{(2bde-2bcf-adf)x^2+a(de-2cf\cdots}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2f}\right.}{f}\right.}{(be-af)^2}$$

$\downarrow$ 398

$$\frac{\frac{-\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{(-adf-2bcf+2bde)\int\frac{1}{\sqrt{bx^2+a}}dx}{f}\right.}{f}\right.}{(be-af)^2}$$

$\downarrow$ 224

$$\frac{\frac{-\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{(-adf-2bcf+2bde)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d}{f}\right.}{f}\right.}{(be-af)^2}$$

$\downarrow$ 219

$$\frac{\frac{\frac{-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\cdots\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2\cdots}{\sqrt{b}f}\cdots\right.}{f}\cdots\right.}{(be-af)^2}$$

$\downarrow$ 291

$$\frac{\frac{\frac{-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\cdots\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2\cdots}{\sqrt{b}f}\cdots\right.}{\cdots}\cdots\right.}{(be-af)^2}$$

$\downarrow$ 221

$$\frac{\frac{\frac{-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(12a^3d^3f-5a^2bd^2(5cf+3de)+4ab^2cd(2cf+9de)-8b^3c^2(3de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(-12a^3d^2f+a^2bd(17cf+15de)-2ab^2c(cf+13de)+8b\right.}{2b}}{4b}}{a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4bc-ad)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}\right)}{f}-\frac{(de-cf)\left(\frac{dx\sqrt{a+bx^2}}{2f}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-adf-2\ldots}{\sqrt{b}f}}{}\right.}{}\right.}{(be-af)^2}$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2)^2)/(a*Sqrt[a + b*x^2]) + (-1/4*(d*
(4*b^2*c*e - 5*a*b*d*e - 3*a*b*c*f + 4*a^2*d*f)*x*Sqrt[a + b*x^2]*(c + d*x
^2))/b + (-1/2*(d*(8*b^3*c^2*e - 12*a^3*d^2*f - 2*a*b^2*c*(13*d*e + c*f) +
 a^2*b*d*(15*d*e + 17*c*f))*x*Sqrt[a + b*x^2])/b - (a*(12*a^3*d^3*f - 8*b^
3*c^2*(3*d*e - c*f) + 4*a*b^2*c*d*(9*d*e + 2*c*f) - 5*a^2*b*d^2*(3*d*e + 5
*c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b))/a)/(b*e -
 a*f)^2 - (f*(d*e - c*f)*((d*((d*x*(a + b*x^2)^(3/2))/(4*b) + ((4*b*c - a*
d)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sq
rt[b])))/(4*b)))/f - ((d*e - c*f)*((d*x*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*
e - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (
2*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a
+ b*x^2])])/(Sqrt[e]*f))/(2*f)))/f))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$\frac{\frac{a\,d^{2}\left(\sqrt{b\,x^{2}+a}\,d f x-\frac{(3adf-6bcf+2bde)\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{2f^{2}b^{2}}-\frac{a\left(c^{3}f^{3}-3c^{2}e\,f^{2}d+3c\,d^{2}e^{2}f-e^{3}d^{3}\right)\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{(af-be)f^{2}\sqrt{(af-be)e}}+\frac{\left(a^{3}d^{3}\right.}{}}{a}$
risch	$\frac{d^{3}x\sqrt{b\,x^{2}+a}}{2b^{2}f}-\frac{\frac{d^{2}(3adf-6bcf+2bde)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{f\sqrt{b}}-\frac{b^{3}\left(c^{3}f^{3}-3c^{2}e\,f^{2}d+3c\,d^{2}e^{2}f-e^{3}d^{3}\right)\ln\left(\frac{\frac{2af-2be}{f}+\frac{2b\sqrt{-ef}\left(x-\frac{\sqrt{-}}{f}\right.}{f}}{}\right.}{\sqrt{-ef}\,\left(\sqrt{-ab}\,f+b\sqrt{-ef}\right)(\sqrt{}}}{}$
default	$\frac{d\left(\frac{d^{2}e^{2}x}{a\sqrt{b\,x^{2}+a}}+df(3cf-de)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+d^{2}f^{2}\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right.\right.}{f^{3}}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(1/2*a*d^2*((b*x^2+a)^(1/2)*d*f*x-(3*a*d*f-6*b*c*f+2*b*d*e)/b^(1/2)*arctan
h((b*x^2+a)^(1/2)/x/b^(1/2)))/f^2/b^2-1/(a*f-b*e)*a*(c^3*f^3-3*c^2*d*e*f^2
+3*c*d^2*e^2*f-d^3*e^3)/f^2/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x
/((a*f-b*e)*e)^(1/2))+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^2/(a
*f-b*e)*x/(b*x^2+a)^(1/2))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(153) = 306$.

Time = 9.83 (sec) , antiderivative size = 2877, normalized size of antiderivative = 16.25

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input

```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output

```
[1/4*((2*a^2*b^3*d^3*e^4 - (6*a^2*b^3*c*d^2 + a^3*b^2*d^3)*e^3*f + 4*(3*a^
3*b^2*c*d^2 - a^4*b*d^3)*e^2*f^2 - 3*(2*a^4*b*c*d^2 - a^5*d^3)*e*f^3 + (2*
a*b^4*d^3*e^4 - (6*a*b^4*c*d^2 + a^2*b^3*d^3)*e^3*f + 4*(3*a^2*b^3*c*d^2 -
 a^3*b^2*d^3)*e^2*f^2 - 3*(2*a^3*b^2*c*d^2 - a^4*b*d^3)*e*f^3)*x^2)*sqrt(b
)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (a^2*b^3*d^3*e^3 - 3*a
^2*b^3*c*d^2*e^2*f + 3*a^2*b^3*c^2*d*e*f^2 - a^2*b^3*c^3*f^3 + (a*b^4*d^3*
e^3 - 3*a*b^4*c*d^2*e^2*f + 3*a*b^4*c^2*d*e*f^2 - a*b^4*c^3*f^3)*x^2)*sqrt
(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(
4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*
e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 2*((a*b^4*d^3*e^3*f -
 2*a^2*b^3*d^3*e^2*f^2 + a^3*b^2*d^3*e*f^3)*x^3 + (a^2*b^3*d^3*e^3*f + 2*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 2*a^3*b^2*d^3)*e^2*f^2 - (2*a*
b^4*c^3 - 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2 - 3*a^4*b*d^3)*e*f^3)*x)*sqrt(
b*x^2 + a))/(a^2*b^5*e^3*f^2 - 2*a^3*b^4*e^2*f^3 + a^4*b^3*e*f^4 + (a*b^6*
e^3*f^2 - 2*a^2*b^5*e^2*f^3 + a^3*b^4*e*f^4)*x^2), 1/4*(2*(2*a^2*b^3*d^3*e
^4 - (6*a^2*b^3*c*d^2 + a^3*b^2*d^3)*e^3*f + 4*(3*a^3*b^2*c*d^2 - a^4*b*d^
3)*e^2*f^2 - 3*(2*a^4*b*c*d^2 - a^5*d^3)*e*f^3 + (2*a*b^4*d^3*e^4 - (6*a*b
^4*c*d^2 + a^2*b^3*d^3)*e^3*f + 4*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*e^2*f^2
- 3*(2*a^3*b^2*c*d^2 - a^4*b*d^3)*e*f^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (a^2*b^3*d^3*e^3 - 3*a^2*b^3*c*d^2*e^2*f + 3*a^2*b^3*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(c+dx^2)^3}{(a+bx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(3/2)/(f*x**2+e),x)
```

output
```
Integral((c + d*x**2)**3/((a + b*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2263, normalized size of antiderivative = 12.79

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e),x)
```

output

```
( - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*c**3*f**3 + 24*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*c**2*d*e*f**2 - 24*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*c*d**2*e**2*f + 8*sqrt(e)*sqrt(a*f
- b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*
x)/(sqrt(e)*sqrt(b)))*a**2*b**3*d**3*e**3 - 8*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*a*b**4*c**3*f**3*x**2 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a*b**4*c**2*d*e*f**2*x**2 - 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f -
 b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a
*b**4*c*d**2*e**2*f*x**2 + 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e)
 - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a*b**4
*d**3*e**3*x**2 - 8*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f
)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*c**3*
f**3 + 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a +
 b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*c**2*d*e*f**2 -
 24*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*...
```

3.350 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = &-\frac{d^2(2bde-3bcf+adf)x}{abf^3\sqrt{a+bx^2}} \\ &-\frac{b(de-cf)^2(af(7de-cf)-2be(2de+cf))x}{2aef^3(be-af)^2\sqrt{a+bx^2}} \\ &+\frac{(de-cf)^3x}{2ef^2(be-af)\sqrt{a+bx^2}\,(e+fx^2)}+\frac{d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}f^2} \\ &+\frac{(de-cf)^2(af(5de+cf)-2be(de+2cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}f^2(be-af)^{5/2}}\end{aligned}$$

output
```
-d^2*(a*d*f-3*b*c*f+2*b*d*e)*x/a/b/f^3/(b*x^2+a)^(1/2)-1/2*b*(-c*f+d*e)^2*
(a*f*(-c*f+7*d*e)-2*b*e*(c*f+2*d*e))*x/a/e/f^3/(-a*f+b*e)^2/(b*x^2+a)^(1/2
)+1/2*(-c*f+d*e)^3*x/e/f^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)+d^3*arctan
h(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/f^2+1/2*(-c*f+d*e)^2*(a*f*(c*f+5*d*e)
-2*b*e*(2*c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^
(3/2)/f^2/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{\frac{fx(2b^3c^3ef(e+fx^2)-2a^3d^3ef(e+fx^2)+a^2b(-d^3e^3-3c^2def^2+c^3f^3+3cd^2ef(3e+2fx^2))+ab^2(-d^3e\cdots}{abe(be-af)^2\sqrt{a+bx^2}(e+fx^2)}\cdots}{\cdots}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
((f*x*(2*b^3*c^3*e*f*(e + f*x^2) - 2*a^3*d^3*e*f*(e + f*x^2) + a^2*b*(-(d^
3*e^3) - 3*c^2*d*e*f^2 + c^3*f^3 + 3*c*d^2*e*f*(3*e + 2*f*x^2)) + a*b^2*(-
(d^3*e^3*x^2) + 3*c*d^2*e^2*f*x^2 + c^3*f^3*x^2 - 3*c^2*d*e*f*(2*e + 3*f*x
^2))))/(a*b*e*(b*e - a*f)^2*Sqrt[a + b*x^2]*(e + f*x^2)) + ((d*e - c*f)^2*
(-(a*f*(5*d*e + c*f)) + 2*b*e*(d*e + 2*c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]
) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(e^(3/2)*(-(b*e) +
 a*f)^(5/2)) - (2*d^3*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(2*f^2
)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 645 vs. 2(284) = 568.

Time = 1.15 (sec) , antiderivative size = 645, normalized size of antiderivative = 2.27, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {425, 419, 25, 401, 25, 27, 299, 224, 219, 403, 25, 398, 224, 219, 291, 221, 425, 402, 25, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \; 425$$

$$\frac{d\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow 419$$

$$\frac{d\left(-\frac{\int -\frac{(dx^2+c)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow 25$$

$$\frac{d\left(\frac{\int \frac{(dx^2+c)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{(bx^2+a)^{3/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow 401$$

$$\frac{d\left(\frac{\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{b\left(abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow 25$$

$$\frac{d\left(\frac{\frac{\int \frac{b\left(abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2\right)}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(c+dx^2)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}(dx^2+c)}{fx^2+e}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow 27$$

$$\frac{d\left(\frac{\frac{\int \frac{abc(de-cf)-d\left(2dfa^2-b(3de+cf)a+2b^2ce\right)x^2}{\sqrt{bx^2+a}}dx}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)}{fx^2+e}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 299

$$\frac{d\left(\frac{\frac{\frac{a\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{\sqrt{bx^2+}}{fx}}{(be-af)}\right.}{f}$$

$$\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\frac{\frac{a\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{}\right.}{f}$$

$$\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{}\right.}{f}$$

$$\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 403

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 398

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\cdots\right.}{\cdots}\right.}{f}$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 224

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{\cdots}\right.$$

$$f$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 219

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{\cdots}\right.$$

$$f$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 291

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{\cdots}\right.$$

$$f$$

$$\frac{(de-cf)\int\frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}$$

$\Big\downarrow$ 221

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\ldots}{\ldots}\right.$$

$$\frac{\ldots}{f}$$

$$\frac{(de-cf)\int\frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\ldots}{\ldots}\right.$$

$$\frac{\ldots}{f}$$

$$\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\cdots\right.}{\cdots}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int -\frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\cdots\right.}{\cdots}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\int \frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\right.}{}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 291

$$\frac{d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\left(\right.}{}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\cdots}{\cdots}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\cdots}{\cdots}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bc\cdots}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\cdots\right)}{f}\cdots\right)}{f}$$

$\downarrow$ 27

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\ldots}{f}\right.$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx\ldots}}{2e(e\ldots}}{a(be-af)}\ldots\right.}{f}\ldots\right.}{f}$$

$\downarrow$ 291

$$d\left(\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)\ldots}{f}\right.$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{\ldots}}{\ldots}}{a(be-af)}\ldots\right.}{f}\ldots\right.}{f}$$

$\downarrow$ 221

$$d\left(\frac{\frac{\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f-abd(cf+3de)+2b^2c(2de-cf)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(2a^2df-ab(cf+3de)+2b^2ce\right)}{2b}}{a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{a\sqrt{a+bx^2}}}{(be-af)^2}-\frac{f(de-cf)}{\cdots}\right)$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

$$(de-cf)\left(\frac{d\left(\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a}\cdots}{2\cdots}}{a(be-af)}\cdots\right)}{f}\cdots\right)$$

$$\overline{\qquad\qquad f \qquad\qquad}$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(d*((((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2))/(a*Sqrt[a + b*x^2]) + (-1/2*(
d*(2*b^2*c*e + 2*a^2*d*f - a*b*(3*d*e + c*f))*x*Sqrt[a + b*x^2])/b + (a*(2
*a^2*d^2*f + 2*b^2*c*(2*d*e - c*f) - a*b*d*(3*d*e + c*f))*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/a)/(b*e - a*f)^2 - (f*(d*e - c*f)*((d*x
*Sqrt[a + b*x^2])/(2*f) - (((2*b*d*e - 2*b*c*f - a*d*f)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) - (2*Sqrt[b*e - a*f]*(d*e - c*f)*ArcTanh[(
Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f))/(2*f)))/(b*e -
 a*f)^2))/f - ((d*e - c*f)*((d*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*
x^2]) + ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
])/(Sqrt[e]*(b*e - a*f)^(3/2))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(a*(b*e
 - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + ((f*(2*b*c*e - 3*a*d*e + a*c*f)*x*S
qrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*(2*b*e*(d*e - 2*c*f) +
a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(
2*e^(3/2)*(b*e - a*f)^(3/2)))/(a*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\frac{2 a\, b^{\frac{5}{2}} \left(\frac{b d\, e^{2}}{2}+f \left(b c-\frac{5 a d}{4}\right) e-\frac{a c\, f^{2}}{4}\right) \left(c f-d e\right)^{2} \sqrt{b\, x^{2}+a}\, \left(f\, x^{2}+e\right) \arctan\left(\frac{e \sqrt{b\, x^{2}+a}}{x \sqrt{\left(a f-b e\right) e}}\right)+\sqrt{\left(a f-b e\right) e}\, \left(a\, d^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{x \sqrt{}}\right.\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*f-b*e)*e)^(1/2)*(2*a*b^(5/2)*(1/2*b*d*e^2+f*(b*c-5/4*a*d)*e-1/4*a*c*
f^2)*(c*f-d*e)^2*(b*x^2+a)^(1/2)*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*
f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(a*d^3*arctanh((b*x^2+a)^(1/2)/x/b^(1
/2))*e*(f*x^2+e)*(a*f-b*e)^2*b*(b*x^2+a)^(1/2)+(-1/2*b*a*d^3*(b*x^2+a)*e^3
-(-b^3*c^3+3*a*c*d*(-1/2*x^2*d+c)*b^2-9/2*a^2*b*c*d^2+a^3*d^3)*f*e^2-(-b^3
*c^3*x^2+9/2*a*b^2*c^2*d*x^2+3/2*a^2*c*d*(-2*d*x^2+c)*b+a^3*d^3*x^2)*f^2*e
+1/2*b*a*c^3*f^3*(b*x^2+a))*b^(3/2)*x*f))/(b*x^2+a)^(1/2)/b^(5/2)/f^2/e/(f
*x^2+e)/(a*f-b*e)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(258) = 516.

Time = 40.21 (sec) , antiderivative size = 4834, normalized size of antiderivative = 17.02

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^3}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/((b*x^2 + a)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(258) = 516$.

Time = 0.17 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.58

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{(b^3c^3f^4 - 3\,ab^2c^2df^4 + 3\,a^2bcd^2f^4 - a^3d^3f^4)x}{(ab^3e^2f^4 - 2\,a^2b^2ef^5 + a^3bf^6)\sqrt{bx^2+a}}$$

$$+ \frac{\left(2\,b^{\frac{3}{2}}d^3e^4 - 5\,a\sqrt{b}d^3e^3f - 6\,b^{\frac{3}{2}}c^2de^2f^2 + 9\,a\sqrt{b}cd^2e^2f^2 + 4\,b^{\frac{3}{2}}c^3ef^3 - 3\,a\sqrt{b}c^2def^3 - a\sqrt{b}c^3f^4\right)\arctan\Bigg(}{2\,(b^2e^3f^2 - 2\,abe^2f^3 + a^2ef^4)\sqrt{-b^2e^2 + abef}}$$

$$- \frac{d^3\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{2\,b^{\frac{3}{2}}f^2}$$

$$- \frac{2\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^2d^3e^4 - 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^2cd^2e^3f - \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 abd^3e^3f + 6\left(\sqrt{b}x\right.}{\left(b^{\frac{5}{2}}e^3f^2\right.}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
(b^3*c^3*f^4 - 3*a*b^2*c^2*d*f^4 + 3*a^2*b*c*d^2*f^4 - a^3*d^3*f^4)*x/((a*
b^3*e^2*f^4 - 2*a^2*b^2*e*f^5 + a^3*b*f^6)*sqrt(b*x^2 + a)) + 1/2*(2*b^(3/
2)*d^3*e^4 - 5*a*sqrt(b)*d^3*e^3*f - 6*b^(3/2)*c^2*d*e^2*f^2 + 9*a*sqrt(b)
*c*d^2*e^2*f^2 + 4*b^(3/2)*c^3*e*f^3 - 3*a*sqrt(b)*c^2*d*e*f^3 - a*sqrt(b)
*c^3*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqr
t(-b^2*e^2 + a*b*e*f))/((b^2*e^3*f^2 - 2*a*b*e^2*f^3 + a^2*e*f^4)*sqrt(-b^
2*e^2 + a*b*e*f)) - 1/2*d^3*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/(b^(3/2)*
f^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*d^3*e^4 - 6*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b^2*c*d^2*e^3*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*d^3*
e^3*f + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^2*c^2*d*e^2*f^2 + 3*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a*b*c*d^2*e^2*f^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*b^2*c^3*e*f^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b*c^2*d*e*f^3 + (sqr
t(b)*x - sqrt(b*x^2 + a))^2*a*b*c^3*f^4 + a^2*b*d^3*e^3*f - 3*a^2*b*c*d^2*
e^2*f^2 + 3*a^2*b*c^2*d*e*f^3 - a^2*b*c^3*f^4)/((b^(5/2)*e^3*f^2 - 2*a*b^(
3/2)*e^2*f^3 + a^2*sqrt(b)*e*f^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f
 + a^2*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{3/2}(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 7621, normalized size of antiderivative = 26.83

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**
2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**3*e*f**5 - sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**3*f**6*x**2 - 3*sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**2*d*e**2*f**4 - 3*sqrt(e)*sqrt(a*f
- b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*
x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**2*d*e*f**5*x**2 + 9*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**4*b**2*c*d**2*e**3*f**3 + 9*sqrt(e)*sqrt(a*f - b*e)
*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sq
rt(e)*sqrt(b)))*a**4*b**2*c*d**2*e**2*f**4*x**2 - 5*sqrt(e)*sqrt(a*f - b*e
)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(s
qrt(e)*sqrt(b)))*a**4*b**2*d**3*e**4*f**2 - 5*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*a**4*b**2*d**3*e**3*f**3*x**2 + 8*sqrt(e)*sqrt(a*f - b*e)*atan(
(sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*
sqrt(b)))*a**3*b**3*c**3*e**2*f**4 + 7*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*a**3*b**3*c**3*e*f**5*x**2 - sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f...
```

3.351 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 395

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \frac{d^3x}{af^3\sqrt{a+bx^2}}$$
$$+\frac{b(de-cf)\left(2abef(13d^2e^2+16cdef-5c^2f^2)-3a^2f^2(11d^2e^2-2cdef-c^2f^2)-8b^2e^2(d^2e^2+cdef+c^2\right.}{8ae^2f^3(be-af)^3\sqrt{a+bx^2}}$$
$$+\frac{(de-cf)^3x}{4ef^2(be-af)\sqrt{a+bx^2}\,(e+fx^2)^2}+\frac{(de-cf)^2(3af(3de+cf)-4be(de+2cf))x}{8e^2f^2(be-af)^2\sqrt{a+bx^2}\,(e+fx^2)}$$
$$+\frac{3(de-cf)\left(8b^2c^2e^2-4abce(3de+cf)+a^2(5d^2e^2+2cdef+c^2f^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{7/2}}$$

output
```
d^3*x/a/f^3/(b*x^2+a)^(1/2)+1/8*b*(-c*f+d*e)*(2*a*b*e*f*(-5*c^2*f^2+16*c*d
*e*f+13*d^2*e^2)-3*a^2*f^2*(-c^2*f^2-2*c*d*e*f+11*d^2*e^2)-8*b^2*e^2*(c^2*
f^2+c*d*e*f+d^2*e^2))*x/a/e^2/f^3/(-a*f+b*e)^3/(b*x^2+a)^(1/2)+1/4*(-c*f+d
*e)^3*x/e/f^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^2+1/8*(-c*f+d*e)^2*(3*a
*f*(c*f+3*d*e)-4*b*e*(2*c*f+d*e))*x/e^2/f^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(
f*x^2+e)+3/8*(-c*f+d*e)*(8*b^2*c^2*e^2-4*a*b*c*e*(c*f+3*d*e)+a^2*(c^2*f^2+
2*c*d*e*f+5*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/
e^(5/2)/(-a*f+b*e)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 20.03 (sec) , antiderivative size = 2523, normalized size of antiderivative = 6.39

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(d^3*x)/(a*f^3*Sqrt[a + b*x^2]) - (3*d^2*(d*e - c*f)*x*(-15*e*Sqrt[((b*e -
 a*f)*x^2)/(e*(a + b*x^2))] - 10*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^
2))] + 15*e*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 10*f*x^2*Ar
cTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 2*e*(((b*e - a*f)*x^2)/(e
*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(
a + b*x^2))] + 2*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Hypergeom
etric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(5*e^2*f^3*(((b
*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*(a + b*x^2)^(3/2)) + (d*(d*e - c*f)^
2*x*(-2625*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - (5250*f*x^2*Sqrt[((b*
e - a*f)*x^2)/(e*(a + b*x^2))])/e - (2310*f^2*x^4*Sqrt[((b*e - a*f)*x^2)/(
e*(a + b*x^2))])/e^2 + 70*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2) + (560
*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e + (280*f^2*x^4*(((b*e
- a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e^2 + 2625*ArcTanh[Sqrt[((b*e - a*f)*x
^2)/(e*(a + b*x^2))]] + (5250*f*x^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a +
 b*x^2))]])/e + (2310*f^2*x^4*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2
))]])/e^2 - (945*(b*e - a*f)*x^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*
x^2))]])/(e*(a + b*x^2)) + (2310*f*(-(b*e) + a*f)*x^4*ArcTanh[Sqrt[((b*e -
 a*f)*x^2)/(e*(a + b*x^2))]])/(e^2*(a + b*x^2)) + (1050*f^2*(-(b*e) + a*f)
*x^6*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]])/(e^3*(a + b*x^2)) +
 24*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, ...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 836 vs. 2(395) = 790.

Time = 1.13 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {425, 425, 402, 25, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{d\left(\frac{d\int \frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{d\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int -\frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{f}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+a(de-c}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}}{a(be-af)}\right.}{f}\right.}{f}$$

$\downarrow$ 25

$$\frac{d\left(\frac{d\left(\frac{\int \frac{a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af}\right.}{f}\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af}\right.}{f}\right.}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}\right)}{f}-$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\int\frac{2(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{4(bc-ad)fx^2+a(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af}\right.}{f}\right.}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int\frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{}{a\sqrt{a+b}}\right.}{f}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{\int\frac{a(2be(de-2cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2bf(4bce-5ade+acf)}{\sqrt{b}}}{}\right.}{}\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade}{2e\left(e+fx^2\right)(be-a}}{a(be-af)}}{f}}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{2bf}{}}{}\right.}{}}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-\cdots}{2e\left(e+fx^2\right)(\cdots}}{a(be-af)}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{a(af(cf+de)+2be(de-2cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(\frac{d\left(\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3\cdots}{2e\left(e+fx^2\right)(b\cdots}}{a(be-af)}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(de-2cf))}{2e^{3/2}(be-af)^{3/2}}+\frac{fx\sqrt{a+bx^2}(acf-3ade+2bce)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}+\frac{x(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\int\cdots\right.}{\cdots}\cdots\right.}{f}\cdots$$

$\downarrow$ 402

$$\frac{d\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}}+\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}(fx^2+e)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)(fx^2+e)}+\frac{a(2be(de-2cf}{}}{a(be-a}\right.}{f}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}(fx^2+e)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)(fx^2+e)}+\frac{a(2be(de-2cf)+af(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{}{a(}\right.}{}\right.}{}$$

$\downarrow$ 27

$$d\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}}+\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf}{}}{a(be-a}\right.}{f}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf)+af(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{}{a(}\right.}{}\right.$$

↓ 291

$$\frac{d\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}}+\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf}{\cdots}}{a(be-\cdots}\right.}{f}\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf)+af(de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\cdots}{a(\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 221

$$\frac{d\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}}+\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf}{}}{a(be-a}\right.}{f}\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\frac{f(2bce-3ade+acf)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{a(2be(de-2cf)+af(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}}{a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{}{a(}\right.}{}\right.$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x]
```

output
```
(d*((d*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) + ((d*e - c*f)*Arc
Tanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*(b*e - a*f)^
(3/2))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*
(e + f*x^2)) + ((f*(2*b*c*e - 3*a*d*e + a*c*f)*x*Sqrt[a + b*x^2])/(2*e*(b*
e - a*f)*(e + f*x^2)) + (a*(2*b*e*(d*e - 2*c*f) + a*f*(d*e + c*f))*ArcTanh
[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3
/2)))/(a*(b*e - a*f))))/f))/f - ((d*e - c*f)*((d*(((b*c - a*d)*x)/(a*(b*e
- a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + ((f*(2*b*c*e - 3*a*d*e + a*c*f)*x*Sq
rt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*(2*b*e*(d*e - 2*c*f) + a
*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2
*e^(3/2)*(b*e - a*f)^(3/2)))/(a*(b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a
*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2) + ((f*(4*b*c*e - 5*a*
d*e + a*c*f)*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + ((f*(8*b
^2*c*e^2 - 2*a*b*e*(7*d*e - 5*c*f) - a^2*f*(d*e + 3*c*f))*x*Sqrt[a + b*x^2
])/(2*e*(b*e - a*f)*(e + f*x^2)) + (a*(8*b^2*e^2*(d*e - 3*c*f) - a^2*f^2*(
d*e + 3*c*f) + 4*a*b*e*f*(2*d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqr
t[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f)))/
(a*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{3\left(\left(5a^2d^2-12abcd+8b^2c^2\right)e^2+2acf(ad-2bc)e+a^2c^2f^2\right)a\sqrt{b\,x^2+a}\,(cf-de)\left(f\,x^2+e\right)^2\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\frac{5\sqrt{(af-be)e}\left(\left(3a^3\right.\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(-3/5*((5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*e^2+2*a*c*f*(a*d-2*b*c)*e+a^2*
c^2*f^2)*a*(b*x^2+a)^(1/2)*(c*f-d*e)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/
x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*((3*a^3*d^3-36/5*(-5/36*x^2*d+c
)*d^2*b*a^2+24/5*d*b^2*(-1/12*d^2*x^4-1/2*c*d*x^2+c^2)*a-8/5*b^3*c^3)*e^4-
9/5*(d^2*(-25/9*x^2*d+c)*a^3-8/3*d*(3/8*d^2*x^4-21/8*c*d*x^2+c^2)*b*a^2-8*
(-1/12*x^2*d+c)*c*d*b^2*x^2*a+16/9*b^3*c^3*x^2)*f*e^3-3/5*f^2*(d*(-8/3*d^2
*x^4+5*c*d*x^2+c^2)*a^3+4*c*(13/4*d^2*x^4-5/4*c*d*x^2+c^2)*b*a^2+4*(-7/2*x
^2*d+c)*c^2*b^2*x^2*a+8/3*c^3*b^3*x^4)*e^2+a*c^2*(b*x^2+a)*f^3*(a*(3/5*x^2
*d+c)-2*x^2*b*c)*e+3/5*a^2*c^3*f^4*x^2*(b*x^2+a))*x)/((a*f-b*e)*e)^(1/2)/(
b*x^2+a)^(1/2)/(f*x^2+e)^2/e^2/(a*f-b*e)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1491 vs. 2(369) = 738.

Time = 20.43 (sec) , antiderivative size = 3022, normalized size of antiderivative = 7.65

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(3/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^3}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/((b*x^2 + a)^(3/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1902 vs. $2(369) = 738$.

Time = 0.71 (sec) , antiderivative size = 1902, normalized size of antiderivative = 4.82

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/((a*b^3*e^3 - 3*a^2*
b^2*e^2*f + 3*a^3*b*e*f^2 - a^4*f^3)*sqrt(b*x^2 + a)) - 3/8*(8*b^(5/2)*c^2
*d*e^3 - 12*a*b^(3/2)*c*d^2*e^3 + 5*a^2*sqrt(b)*d^3*e^3 - 8*b^(5/2)*c^3*e^
2*f + 8*a*b^(3/2)*c^2*d*e^2*f - 3*a^2*sqrt(b)*c*d^2*e^2*f + 4*a*b^(3/2)*c^
3*e*f^2 - a^2*sqrt(b)*c^2*d*e*f^2 - a^2*sqrt(b)*c^3*f^3)*arctan(1/2*((sqrt
(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^
3*e^5 - 3*a*b^2*e^4*f + 3*a^2*b*e^3*f^2 - a^3*e^2*f^3)*sqrt(-b^2*e^2 + a*b
*e*f)) + 1/4*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d^3*e^5*f - 24*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d^3*e^4*f^2 - 24*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*b^(5/2)*c^2*d*e^3*f^3 + 36*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
*b^(3/2)*c*d^2*e^3*f^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3
*e^3*f^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^3*e^2*f^4 - 15*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c*d^2*e^2*f^4 - 12*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(3/2)*c^3*e*f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^2*sqrt(b)*c^2*d*e*f^5 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*c^
3*f^6 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*d^3*e^6 + 48*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*b^(7/2)*c*d^2*e^5*f - 88*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a*b^(5/2)*d^3*e^5*f - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^2
*d*e^4*f^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c*d^2*e^4*f^2 +
78*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*d^3*e^4*f^2 + 80*(sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(3/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^3}dx$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x)
```

output
```
int((d*x^2+c)^3/(b*x^2+a)^(3/2)/(f*x^2+e)^3,x)
```

3.352 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x}{a(bc-ad)(be-af)\sqrt{a+bx^2}} - \frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}(de-cf)} + \frac{f^2\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}(de-cf)}$$

output
```
b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)-d^2*arctanh((-a*d+b*c)^(1/2)
*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)+f^2*arctan
h((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/(-a*f+b*e)^(3/2)/(-c
*f+d*e)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x}{a(-bc+ad)(-be+af)\sqrt{a+bx^2}} - \frac{d^2\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{3/2}(de-cf)} + \frac{f^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(-be+af)^{3/2}(de-cf)}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b^2*x)/(a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[a + b*x^2]) - (d^2*ArcTan[(-
(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])]
)/(Sqrt[c]*(-(b*c) + a*d)^(3/2)*(d*e - c*f)) + (f^2*ArcTan[(-(f*x*Sqrt[a +
 b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*(-
(b*e) + a*f)^(3/2)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {421, 25, 402, 27, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{(-2adf+bcf+bde)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{422}$$

$$\frac{d^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{d^2\left(\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{ef}}\right)}{de-cf}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(be-af)}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}\right)}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b*((b*(b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) - ((b*d*e + b*c*f -
2*a*d*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*
(b*e - a*f)^(3/2))))/(b*c - a*d)^2 + (d^2*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt
[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/f - (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*
x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f)))/(d*e - c*f)))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{b^{2} x}{a(af-be)(ad-bc)\sqrt{b\,x^{2}+a}}+\frac{d^{2} \arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{(ad-bc)(cf-de)\sqrt{(ad-bc)c}}-\frac{f^{2} \arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)}{(af-be)(cf-de)\sqrt{(af-be)e}}$	166
default	Expression too large to display	1616

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/a*b^2/(a*f-b*e)/(a*d-b*c)*x/(b*x^2+a)^(1/2)+1/(a*d-b*c)*d^2/(c*f-d*e)/((
a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-1/(a*f-b
*e)*f^2/(c*f-d*e)/((a*f-b*e)*e)^(1/2)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e
)*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx=\int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \frac{\sqrt{b}d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bcde-ad^2e-bc^2f+acdf)} - \frac{\sqrt{b}f^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\,\sqrt{-b^2e^2+abef}}\right)}{\sqrt{-b^2e^2+abef}(bde^2-bcef-adef+acf^2)} + \frac{b^2x}{(ab^2ce-a^2bde-a^2bcf+a^3df)\sqrt{bx^2+a}}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c*d*e - a*d^2*e - b*
c^2*f + a*c*d*f)) - sqrt(b)*f^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^
2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/(sqrt(-b^2*e^2 + a*b*e*f)*(b*
d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)) + b^2*x/((a*b^2*c*e - a^2*b*d*e - a^
2*b*c*f + a^3*d*f)*sqrt(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 2231, normalized size of antiderivative = 13.52

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
(sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2)
- sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*d**2*e*f**2 - 2*sqrt(c)*sqrt(
a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt
(b)*x)/(sqrt(c)*sqrt(b)))*a**3*b*d**2*e**2*f + sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c
)*sqrt(b)))*a**3*b*d**2*e*f**2*x**2 + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a
*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)
))*a**2*b**2*d**2*e**3 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) -
 sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**2*b**
2*d**2*e**2*f*x**2 + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(
d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a*b**3*d**2*e*
*3*x**2 + sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a +
 b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*d**2*e*f**2 - 2*sqrt
(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqr
t(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*b*d**2*e**2*f + sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x
)/(sqrt(c)*sqrt(b)))*a**3*b*d**2*e*f**2*x**2 + sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c
)*sqrt(b)))*a**2*b**2*d**2*e**3 - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
 - b*c) + sqrt(d)*sqrt(a + b*x**2) + sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b...
```

3.353 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 291

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2} \, dx = -\frac{b(abcf^2-a^2df^2-2b^2e(de-cf))x}{2a(bc-ad)e(be-af)^2(de-cf)\sqrt{a+bx^2}} + \frac{f^2x}{2e(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)} - \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}(de-cf)^2} + \frac{f^2(2be(3de-2cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{5/2}(de-cf)^2}$$

output
```
-1/2*b*(a*b*c*f^2-a^2*d*f^2-2*b^2*e*(-c*f+d*e))*x/a/(-a*d+b*c)/e/(-a*f+b*e
)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/2*f^2*x/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)
^(1/2)/(f*x^2+e)-d^3*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c
^(1/2)/(-a*d+b*c)^(3/2)/(-c*f+d*e)^2+1/2*f^2*(2*b*e*(-2*c*f+3*d*e)-a*f*(-c
*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a
*f+b*e)^(5/2)/(-c*f+d*e)^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 15.53 (sec) , antiderivative size = 1386, normalized size of antiderivative = 4.76

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(x*((42*d^2*(-15*c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 10*d*x^2*Sqrt
[((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 15*c*ArcTanh[Sqrt[((b*c - a*d)*x^2)/
(c*(a + b*x^2))]] + 10*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]] + 2*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5
/2, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 2*d*x^2*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*c - a*d)*x^2)/(c*
(a + b*x^2))]))/(c^2*(d*e - c*f)^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/
2)) - (42*d*f*(-15*e*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - 10*f*x^2*Sq
rt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] + 15*e*ArcTanh[Sqrt[((b*e - a*f)*x^2
)/(e*(a + b*x^2))]] + 10*f*x^2*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^
2))]] + 2*e*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2,
 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2))] + 2*f*x^2*(((b*e - a*f)*x^2)
/(e*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(
e*(a + b*x^2))]))/(e^2*(d*e - c*f)^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(
3/2)) + (f*(-2625*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - (5250*f*x^2*Sq
rt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e - (2310*f^2*x^4*Sqrt[((b*e - a*f)
*x^2)/(e*(a + b*x^2))])/e^2 + 70*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)
 + (560*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e + (280*f^2*x^4*
(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e^2 + 2625*ArcTanh[Sqrt[((b*e -
 a*f)*x^2)/(e*(a + b*x^2))]] + (5250*f*x^2*ArcTanh[Sqrt[((b*e - a*f)*x^...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.73, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {421, 25, 402, 402, 27, 291, 221, 421, 301, 224, 219, 291, 221, 401, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)-2b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{\int \frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{301}$$

$$\frac{d^2\left(\frac{d^2\left(\frac{b\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 224

$$\frac{d^2\left(\frac{d^2\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 219

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\downarrow$ 401

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int-\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\downarrow$ 25

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \; 398$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{}}{}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 224

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right.}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$\Big\downarrow$ 219

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}\cdots}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+$$

$$\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2bde^2-af(3\right.}{\sqrt{e}f\sqrt{be-af}}}{2e}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(b*((b*(b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) - (-1/2*
(f*(2*b^2*c*e - 2*a^2*d*f - a*b*(d*e - c*f))*x*Sqrt[a + b*x^2])/(e*(b*e -
a*f)*(e + f*x^2)) + (a*(2*a^2*d*f^2 - a*b*f*(7*d*e + c*f) + 2*b^2*e*(d*e +
 2*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2
)*(b*e - a*f)^(3/2)))/(a*(b*e - a*f))))/(b*c - a*d)^2 + (d^2*((d^2*((Sqrt[
b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)))/(d*e - c*f)^2 -
(f*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(2*e*(e + f*x^2)) + ((2*Sqrt[b]*d*e*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f - ((2*b*d*e^2 - a*f*(3*d*e - c*f))*A
rcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e
 - a*f]))/(2*e)))/(d*e - c*f)^2))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{-(ad-bc)\left(\left(c\,f^{2}-3def\right)a+\left(-4cef+6d\,e^{2}\right)b\right)a\sqrt{(ad-bc)c}\,\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)f^{2}\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\,\left(-2a\right.}{2\sqrt{(af-be)e}\,\sqrt{(ad-bc)c}}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*f-b*e)*e)^(1/2)/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(1/2)*(-(a*d-b*c)*((
c*f^2-3*d*e*f)*a+(-4*c*e*f+6*d*e^2)*b)*a*((a*d-b*c)*c)^(1/2)*(b*x^2+a)^(1/
2)*(f*x^2+e)*f^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e
)*e)^(1/2)*(-2*a*d^3*e*(b*x^2+a)^(1/2)*(f*x^2+e)*(a*f-b*e)^2*arctan(c*(b*x
^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(c*f-d*e)*(a^3*d*f^
3-b*f^3*(-d*x^2+c)*a^2-a*b^2*c*f^3*x^2-2*b^3*e*(f*x^2+e)*(c*f-d*e))*x))/(c
*f-d*e)^2/(a*d-b*c)/e/(f*x^2+e)/(a*f-b*e)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(261) = 522.

Time = 2.19 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.17

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx = \frac{\sqrt{b}d^3\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{(bcd^2e^2-ad^3e^2-2\,bc^2def+2\,acd^2ef+bc^3f^2-ac^2df^2)\sqrt{-b^2c^2}} + \frac{b^3x}{(ab^3ce^2-a^2b^2de^2-2\,a^2b^2cef+2\,a^3bdef+a^3bcf^2-a^4df^2)\sqrt{bx^2+a}} - \frac{\left(6\,b^{\frac{3}{2}}de^2f^2-4\,b^{\frac{3}{2}}cef^3-3\,a\sqrt{b}def^3+a\sqrt{b}cf^4\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2f+2\,be-af}{2\sqrt{-b^2e^2+abef}}\right)}{2\,(b^2d^2e^5-2\,b^2cde^4f-2\,abd^2e^4f+b^2c^2e^3f^2+4\,abcde^3f^2+a^2d^2e^3f^2-2\,abc^2e^2f^3-2\,a^2cde^2f^3+a^2c^2e} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}ef^2-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}f^3+a^2\sqrt{}}{(b^2de^4-b^2ce^3f-2\,abde^3f+2\,abce^2f^2+a^2de^2f^2-a^2cef^3)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4f+4\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}$$

input

```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output

```
sqrt(b)*d^3*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b*c*d^2*e^2 - a*d^3*e^2 - 2*b*c^2*d*e*f + 2*a*c
*d^2*e*f + b*c^3*f^2 - a*c^2*d*f^2)*sqrt(-b^2*c^2 + a*b*c*d)) + b^3*x/((a*
b^3*c*e^2 - a^2*b^2*d*e^2 - 2*a^2*b^2*c*e*f + 2*a^3*b*d*e*f + a^3*b*c*f^2
- a^4*d*f^2)*sqrt(b*x^2 + a)) - 1/2*(6*b^(3/2)*d*e^2*f^2 - 4*b^(3/2)*c*e*f
^3 - 3*a*sqrt(b)*d*e*f^3 + a*sqrt(b)*c*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*d^2*e^5 - 2
*b^2*c*d*e^4*f - 2*a*b*d^2*e^4*f + b^2*c^2*e^3*f^2 + 4*a*b*c*d*e^3*f^2 + a
^2*d^2*e^3*f^2 - 2*a*b*c^2*e^2*f^3 - 2*a^2*c*d*e^2*f^3 + a^2*c^2*e*f^4)*sq
rt(-b^2*e^2 + a*b*e*f)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*e*f^2
 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*f^3 + a^2*sqrt(b)*f^3)/((b^2*
d*e^4 - b^2*c*e^3*f - 2*a*b*d*e^3*f + 2*a*b*c*e^2*f^2 + a^2*d*e^2*f^2 - a^
2*c*e*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 12518, normalized size of antiderivative = 43.02

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
( - 2*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x
**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**6*d**3*e**3*f**4 - 2*sqrt(
c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt
(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**6*d**3*e**2*f**5*x**2 + 14*sqrt(c)*sq
rt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*s
qrt(b)*x)/(sqrt(c)*sqrt(b)))*a**5*b*d**3*e**4*f**3 + 12*sqrt(c)*sqrt(a*d -
 b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x
)/(sqrt(c)*sqrt(b)))*a**5*b*d**3*e**3*f**4*x**2 - 2*sqrt(c)*sqrt(a*d - b*c
)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(s
qrt(c)*sqrt(b)))*a**5*b*d**3*e**2*f**5*x**4 - 30*sqrt(c)*sqrt(a*d - b*c)*a
tan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt
(c)*sqrt(b)))*a**4*b**2*d**3*e**5*f**2 - 16*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*s
qrt(b)))*a**4*b**2*d**3*e**4*f**3*x**2 + 14*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*s
qrt(b)))*a**4*b**2*d**3*e**3*f**4*x**4 + 26*sqrt(c)*sqrt(a*d - b*c)*atan((
sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*s
qrt(b)))*a**3*b**3*d**3*e**6*f - 4*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d
- b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*
a**3*b**3*d**3*e**5*f**2*x**2 - 30*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a...
```

3.354 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 490

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx = -\frac{b(2ab^2cef^2(7de-5cf)+a^3df^3(7de-3cf)-8b^3e^2(de-cf)^2-a^2bf^2(14d^2e^2-3cdef-3c^2f^2))\,x}{8a(bc-ad)e^2(be-af)^3(de-cf)^2\sqrt{a+bx^2}}$$
$$+\frac{f^2x}{4e(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$-\frac{f^2(af(7de-3cf)-4be(3de-2cf))x}{8e^2(be-af)^2(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)}-\frac{d^4\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}(de-cf)^3}$$
$$-\frac{f^2(4abef(12d^2e^2-11cdef+3c^2f^2)-a^2f^2(15d^2e^2-10cdef+3c^2f^2)-8b^2e^2(6d^2e^2-8cdef+3c^2f^2))}{8e^{5/2}(be-af)^{7/2}(de-cf)^3}$$

output
```
-1/8*b*(2*a*b^2*c*e*f^2*(-5*c*f+7*d*e)+a^3*d*f^3*(-3*c*f+7*d*e)-8*b^3*e^2*
(-c*f+d*e)^2-a^2*b*f^2*(-3*c^2*f^2-3*c*d*e*f+14*d^2*e^2))*x/a/(-a*d+b*c)/e
^2/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)+1/4*f^2*x/e/(-a*f+b*e)/(-c*f+
d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^2-1/8*f^2*(a*f*(-3*c*f+7*d*e)-4*b*e*(-2*c*f
+3*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)-d^4*arc
tanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(3/2)/
(-c*f+d*e)^3-1/8*f^2*(4*a*b*e*f*(3*c^2*f^2-11*c*d*e*f+12*d^2*e^2)-a^2*f^2*
(3*c^2*f^2-10*c*d*e*f+15*d^2*e^2)-8*b^2*e^2*(3*c^2*f^2-8*c*d*e*f+6*d^2*e^2
))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^
(7/2)/(-c*f+d*e)^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 20.50 (sec) , antiderivative size = 2885, normalized size of antiderivative = 5.89

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(d^3*x*(-15*c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 10*d*x^2*Sqrt[((b*
c - a*d)*x^2)/(c*(a + b*x^2))] + 15*c*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a
 + b*x^2))]] + 10*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] +
 2*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7
/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 2*d*x^2*(((b*c - a*d)*x^2)/(c*(a
+ b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*c - a*d)*x^2)/(c*(a +
b*x^2))]))/(5*a*c^2*(d*e - c*f)^3*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2
)*Sqrt[a + b*x^2]*(1 + (b*x^2)/a)) + (d^2*f*x*(-15*e*Sqrt[((b*e - a*f)*x^2
)/(e*(a + b*x^2))] - 10*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] + 15
*e*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 10*f*x^2*ArcTanh[Sqr
t[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 2*e*(((b*e - a*f)*x^2)/(e*(a + b*x
^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2
))] + 2*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Hypergeometric2F1[
2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(5*a*e^2*(-(d*e) + c*f)^
3*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*Sqrt[a + b*x^2]*(1 + (b*x^2)/a
)) - (d*f*x*(-2625*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))] - (5250*f*x^2*S
qrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))])/e - (2310*f^2*x^4*Sqrt[((b*e - a*f
)*x^2)/(e*(a + b*x^2))])/e^2 + 70*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2
) + (560*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e + (280*f^2*x^4
*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2))/e^2 + 2625*ArcTanh[Sqrt[((b...
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.60, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {421, 25, 402, 402, 402, 25, 27, 291, 221, 421, 401, 25, 27, 402, 25, 27, 291, 221, 422, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)-4b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{\int \frac{a\left(4e(de+2cf)b^2-3af(5de+cf)b+6a^2df^2\right)-2bf\left(-2dfa^2-b(3de-cf)a+4b^2ce\right)x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(3de-\cdots\right)}{4e\left(e+fx^2\right)^2(be-af\cdots)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{\frac{\int -\frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)-2\cdots\right)}{2e\left(e+fx^2\right)(be-a\cdots)}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{\int \frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)-2\ldots\right.}{2e\left(e+fx^2\right)(be-a\ldots}}{4e(be-af)}}{a(be-af)}\ldots\right.}{(bc-ad)^2}\ldots$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 27

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2\ldots\right.}{2\ldots}}{4e(be-af)}}{a(be-af)}\ldots\right.}{(bc-ad)^2}\ldots$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 291

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\ldots\right.}{\ldots}}{4e(be-af)}}{a(be-af)}\ldots\right.}{(bc-ad)^2}\ldots$$

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 421$$

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 401$$

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}-\frac{\int -\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 27

$$\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2b(3de-cf)x^2+a(7de-3cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 402

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\frac{\int -\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(6a^3df^2\cdots}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 25

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{\int \frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(6a^3df^2\cdots}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 27

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf\cdots}{4e\left(e+fx^2\right)^2}\cdots\right)}{(de-cf)^2}\cdots\right)}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(d\cdots}{4e\left(e+fx^2\cdots\right.}\cdots\right)}{(de-cf)^2}\cdots\right)}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2} - \frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}} - \frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e} + \frac{x\sqrt{a+bx^2}(de\cdots}{4e\left(e+fx^2\right)\cdots}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)} - \frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}} - \frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$\downarrow$ 422

$$\frac{d^2\left(\frac{d^2\left(\frac{d\int \frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf} - \frac{f\int \frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2} - \frac{f\left(\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}} - \frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e} + \cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)} - \frac{\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}} - \frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$\downarrow$ 301

$$\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-}}{\sqrt{e}\sqrt{a+}}\right.}{2e^3}\right.}{}\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\right.}{}}{4e(be-af)}}{a(be-af)}\right.}{(bc-ad)^2}$$

$\downarrow$ 224

$$\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{a\text{a}}{}\right.}{}\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\right.}{}}{4e(be-af)}}{a(be-af)}\right.}{(bc-ad)^2}$$

$\downarrow$ 219

$$\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\cdots\right.}{\cdots}\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{(be-af)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}$$

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{-\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}$$

$$\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(d^2*((d^2*((d*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]
*d)))/(d*e - c*f) - (f*((Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/f -
 (Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/
(Sqrt[e]*f)))/(d*e - c*f)))/(d*e - c*f)^2 - (f*((((d*e - c*f)*x*Sqrt[a + b*
x^2])/(4*e*(e + f*x^2)^2) + (-1/2*((a*f*(7*d*e - 3*c*f) - 2*b*e*(3*d*e - c
*f))*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)) - (a*(a*f*(7*d*e - 3*c
*f) - 4*b*e*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b
*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e)))/(d*e - c*f)^2))/(b*c - a*d
)^2 + (b*((b*(b*c - a*d)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2)
 - (-1/4*(f*(4*b^2*c*e - 2*a^2*d*f - a*b*(3*d*e - c*f))*x*Sqrt[a + b*x^2])/
(e*(b*e - a*f)*(e + f*x^2)^2) + (-1/2*(f*(8*b^3*c*e^2 + 6*a^3*d*f^2 - 2*a*
b^2*e*(d*e - 5*c*f) - a^2*b*f*(19*d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(e*(b*e
 - a*f)*(e + f*x^2)) - (a*(6*a^3*d*f^3 - 8*b^3*e^2*(d*e + 3*c*f) + 4*a*b^2
*e*f*(10*d*e + 3*c*f) - a^2*b*f^2*(23*d*e + 3*c*f))*ArcTanh[(Sqrt[b*e - a*
f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e
 - a*f)))/(a*(b*e - a*f))))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$-\frac{\frac{3(ad-bc)a\sqrt{b\,x^2+a}\,\sqrt{(ad-bc)c}\left(f^2\left(c^2f^2-\frac{10}{3}cdef+5d^2e^2\right)a^2-4\left(c^2f^2-\frac{11}{3}cdef+4d^2e^2\right)bfea+8b^2\left(c^2f^2-\frac{8}{3}cdef+2d^2e^2\right)e^2\right)\left(f\,x^2\right.}{8}}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(-3/5*(a*d-b*c)*a*(b*x^2+a)^(1/2)*((a*d-b*c)*c)^(1/2)*(f^2*(c^2*f^2-10
/3*c*d*e*f+5*d^2*e^2)*a^2-4*(c^2*f^2-11/3*c*d*e*f+4*d^2*e^2)*b*f*e*a+8*b^2
*(c^2*f^2-8/3*c*d*e*f+2*d^2*e^2)*e^2)*(f*x^2+e)^2*f^2*arctan(e*(b*x^2+a)^(
1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(8/5*(b*x^2+a)^(1/2)*a*d^4
*e^2*(f*x^2+e)^2*(a*f-b*e)^3*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2
))+((a*d-b*c)*c)^(1/2)*(c*f-d*e)*((f*(3/5*f*x^2+e)*c-9/5*(7/9*f*x^2+e)*d*e
)*d*f^4*a^4-(f^2*(3/5*f*x^2+e)*c^2+3/5*d*f*(-f^2*x^4-2/3*e*f*x^2+e^2)*c-16
/5*(-7/16*f^2*x^4+5/16*e*f*x^2+e^2)*d^2*e)*b*f^3*a^3+12/5*((3/4*f*x^2+e)*f
*(-1/3*f*x^2+e)*c^2-4/3*d*e*(3/16*f^2*x^4+17/16*e*f*x^2+e^2)*c+4/3*(7/8*f*
x^2+e)*d^2*x^2*e^2)*b^2*f^3*a^2+12/5*c*(f*(5/6*f*x^2+e)*c-4/3*(7/8*f*x^2+e
)*d*e)*b^3*x^2*f^3*e*a+8/5*b^4*e^2*(f*x^2+e)^2*(c*f-d*e)^2)*x))/((a*f-b*e)
*e)^(1/2)/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^2/(c*f-d*e)^3/(a*f
-b*e)^3/e^2/(a*d-b*c)/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2} (c+dx^2) (e+fx^2)^3} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)(fx^2+e)^3} \, dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1630 vs. 2(456) = 912.

Time = 9.21 (sec) , antiderivative size = 1630, normalized size of antiderivative = 3.33

$$\int \frac{1}{(a+bx^2)^{3/2} (c+dx^2) (e+fx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
sqrt(b)*d^4*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b*c*d^3*e^3 - a*d^4*e^3 - 3*b*c^2*d^2*e^2*f + 3
*a*c*d^3*e^2*f + 3*b*c^3*d*e*f^2 - 3*a*c^2*d^2*e*f^2 - b*c^4*f^3 + a*c^3*d
*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) + b^4*x/((a*b^4*c*e^3 - a^2*b^3*d*e^3 - 3*
a^2*b^3*c*e^2*f + 3*a^3*b^2*d*e^2*f + 3*a^3*b^2*c*e*f^2 - 3*a^4*b*d*e*f^2
- a^4*b*c*f^3 + a^5*d*f^3)*sqrt(b*x^2 + a)) - 1/8*(48*b^(5/2)*d^2*e^4*f^2
- 64*b^(5/2)*c*d*e^3*f^3 - 48*a*b^(3/2)*d^2*e^3*f^3 + 24*b^(5/2)*c^2*e^2*f
^4 + 44*a*b^(3/2)*c*d*e^2*f^4 + 15*a^2*sqrt(b)*d^2*e^2*f^4 - 12*a*b^(3/2)*
c^2*e*f^5 - 10*a^2*sqrt(b)*c*d*e*f^5 + 3*a^2*sqrt(b)*c^2*f^6)*arctan(1/2*(
(sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))
/((b^3*d^3*e^8 - 3*b^3*c*d^2*e^7*f - 3*a*b^2*d^3*e^7*f + 3*b^3*c^2*d*e^6*f
^2 + 9*a*b^2*c*d^2*e^6*f^2 + 3*a^2*b*d^3*e^6*f^2 - b^3*c^3*e^5*f^3 - 9*a*b
^2*c^2*d*e^5*f^3 - 9*a^2*b*c*d^2*e^5*f^3 - a^3*d^3*e^5*f^3 + 3*a*b^2*c^3*e
^4*f^4 + 9*a^2*b*c^2*d*e^4*f^4 + 3*a^3*c*d^2*e^4*f^4 - 3*a^2*b*c^3*e^3*f^5
 - 3*a^3*c^2*d*e^3*f^5 + a^3*c^3*e^2*f^6)*sqrt(-b^2*e^2 + a*b*e*f)) - 1/4*
(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d*e^3*f^3 - 16*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*b^(5/2)*c*e^2*f^4 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*
b^(3/2)*d*e^2*f^4 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^5 +
 7*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^5 - 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*c*f^6 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

3.355 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 418

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{b(2b^3ce(de-cf)^2+a^3d^2f^2(de+cf)-2a^2bdf(d^2e^2+c^2f^2)+a}{2ac(bc-ad)^2e(be-af)^2(de-cf)^2\sqrt{a+bx^2}} - \frac{d^3x}{2c(bc-ad)(de-cf)^2\sqrt{a+bx^2}\,(c+dx^2)} - \frac{f^3x}{2e(be-af)(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)} + \frac{d^3(ad(de-5cf)-4bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}(de-cf)^3} - \frac{f^3(4be(2de-cf)-af(5de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{5/2}(de-cf)^3}$$

output
```
1/2*b*(2*b^3*c*e*(-c*f+d*e)^2+a^3*d^2*f^2*(c*f+d*e)-2*a^2*b*d*f*(c^2*f^2+d
^2*e^2)+a*b^2*(c^3*f^3+d^3*e^3))*x/a/c/(-a*d+b*c)^2/e/(-a*f+b*e)^2/(-c*f+d
*e)^2/(b*x^2+a)^(1/2)-1/2*d^3*x/c/(-a*d+b*c)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/
(d*x^2+c)-1/2*f^3*x/e/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)+1/
2*d^3*(a*d*(-5*c*f+d*e)-4*b*c*(-2*c*f+d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(
1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(5/2)/(-c*f+d*e)^3-1/2*f^3*(4*b*e
*(-c*f+2*d*e)-a*f*(-c*f+5*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+
a)^(1/2))/e^(3/2)/(-a*f+b*e)^(5/2)/(-c*f+d*e)^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 19.17 (sec) , antiderivative size = 2255, normalized size of antiderivative = 5.39

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(-2*d^2*f*x*(-15*c*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - 10*d*x^2*Sqrt
[((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 15*c*ArcTanh[Sqrt[((b*c - a*d)*x^2)/
(c*(a + b*x^2))]] + 10*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]] + 2*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5
/2, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 2*d*x^2*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*c - a*d)*x^2)/(c*
(a + b*x^2))]))/(5*a*c^2*(d*e - c*f)^3*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))
^(3/2)*Sqrt[a + b*x^2]*(1 + (b*x^2)/a)) - (2*d*f^2*x*(-15*e*Sqrt[((b*e - a
*f)*x^2)/(e*(a + b*x^2))] - 10*f*x^2*Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2)
)] + 15*e*ArcTanh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 10*f*x^2*ArcT
anh[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]] + 2*e*(((b*e - a*f)*x^2)/(e*(
a + b*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a
+ b*x^2))] + 2*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Hypergeomet
ric2F1[2, 5/2, 7/2, ((b*e - a*f)*x^2)/(e*(a + b*x^2))]))/(5*a*e^2*(-(d*e)
+ c*f)^3*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*Sqrt[a + b*x^2]*(1 + (b
*x^2)/a)) + (d^2*x*(-2625*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (5250*
d*x^2*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (2310*d^2*x^4*Sqrt[((b*
c - a*d)*x^2)/(c*(a + b*x^2))])/c^2 + 70*(((b*c - a*d)*x^2)/(c*(a + b*x^2)
))^(3/2) + (560*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (280*
d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + 2625*ArcTanh[S...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)(fx^2+e)^2}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)-2b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{\int \frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}}{(bc-ad)^2}}{bc-ad}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int}{}}{bc-ad}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 291

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\frac{a}{}}{bc-ad}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(-2a^2}{2e(e+}}{a(be-af)}\right.}{(bc-ad)^2}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 421

$$\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\frac{a\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2a^2df^2-abf(cf+7de)+2b^2e}{2e^{3/2}(be-af)^{3/2}}}{a(be-}\right.}{(bc-ad)^2}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 301

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{a\text{arctanh}\left(\frac{x\sqrt{\cdots}}{\sqrt{e\cdots}}\right.}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 224

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{a\text{arcta}\cdots}{\cdots}\right.}{\cdots}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 219

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{a\,\text{arct}\cdots}{\cdots}\right)}{\cdots}\right)}{bc-ad\cdots}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\cdots\right)}{\cdots}\right)}{bc-ad\cdots}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 221

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{a\text{a}\ldots}{\ldots}\right.}{\ldots}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 401

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}-\frac{\int-\frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\ldots}{a\sqrt{a+bx\ldots}}\right.}{\ldots}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f\left(2bdex^2+a(3de-cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\cdots}{a\sqrt{a+bx^2}(\cdots}\cdots\right.}{\cdots}\cdots\right.}{bc-ad\cdots}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{2bdex^2+a(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e}+\frac{x\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-\cdots}{a\sqrt{a+bx^2}\left(e+\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{bc-ad\cdots}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \; 398$$

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}}{2e}+\frac{x\sqrt{}}{}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 224

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2bde\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{f}}{2e}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 219

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+\cdots\right.}}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\cdots}{f}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 221

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+ \ldots d^2\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 426

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+ \ldots d^2\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 421

$$
b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+ - \quad d^2
$$

$$
\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{bc-ad}
$$

$$
\Big\downarrow \; 25
$$

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+\cdots\right.$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 291

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+-\ d^2\cdots\right)}{}$$

$$\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \ 221$$

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+7de)+2b^2e(2cf+de)\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(de-cf)+2b^2ce\right)}{2e\left(e+fx^2\right)(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}+-\ d^2\cdots\right)}{d\left(\frac{d\left(\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{(de-cf)^2}+\frac{f^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{de-cf}\right)}$$

$$bc-ad$$

$\downarrow$ 402

$$\frac{b\left(\frac{\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e\left(fx^2+e\right)}+\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)}{\sqrt{e}f}}{2e}\right.}{(de-cf)^2}}{(bc-ad)^2}}{\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(\frac{\int-\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{2c(bc-ad)\left(dx^2+c\right)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int\frac{2be(2de-}{\sqrt{bx^2}}}{2}\right.}{}\right.}{de-c}}{bc-ad}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\left(\frac{d^2\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e\left(fx^2+e\right)}+\frac{\frac{2\sqrt{b}de\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)\cdots}{\sqrt{e}f\cdots}}{2e}\cdots\right.}{(de-cf)^2}\cdots\right)}{(bc-ad)^2}\cdots}{\cdots}$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{\int\frac{ad(de-3cf)-2bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\int\frac{2be(2de-\cdots}{\sqrt{bx^2\cdots}}\cdots\right)}{2\cdots}\cdots\right)}{de-c\cdots}\cdots\right)}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{\left(\frac{d^2\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}x}{2e\left(fx^2+e\right)}+\frac{\frac{2\sqrt{b}de\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\left(2bde^2-af(3de-cf)\right)}{\sqrt{e}f}}{2e}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)f^2}{\sqrt{e}\sqrt{be-af}(de-cf)^2}+\frac{d\left(-\frac{d(de-cf)\sqrt{bx^2+a}x}{2c(bc-ad)\left(dx^2+c\right)}-\frac{(ad(de-3cf)-2bc(de-2cf))\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right.}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}\right.}{}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-5a\sqrt{b\,x^2+a}\,\sqrt{(af-be)e}\,d^3\,(x^2d+c)\,(af-be)^2\left(d\left(cf-\frac{de}{5}\right)a-\frac{8\left(cf-\frac{de}{2}\right)cb}{5}\right)(f\,x^2+e)\,e\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\sqrt{(ad-bc)}}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-5*a*(b*x^2+a)^(1/2)*((a*f-b*e)*e)^(1/2)*d^3*(d*x^2+c)*(a*f-b*e)^2*(d
*(c*f-1/5*d*e)*a-8/5*(c*f-1/2*d*e)*c*b)*(f*x^2+e)*e*arctan(c*(b*x^2+a)^(1/
2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(-(a*d-b*c)^2*a*(b*x^2+a)^(1
/2)*c*(d*x^2+c)*((c*f^2-5*d*e*f)*a-4*b*e*(c*f-2*d*e))*(f*x^2+e)*f^3*arctan
(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(d^2*(c^2*f^
2+c*d*f^2*x^2+d^2*e*(f*x^2+e))*f^2*a^4-2*d*(c^3*f^3+1/2*c^2*d*f^3*x^2-1/2*
c*d^2*f^3*x^4+d^3*(f*x^2+e)*(-1/2*f*x^2+e)*e)*b*f*a^3+(c^4*f^4-c^3*d*f^4*x
^2-2*c^2*d^2*f^4*x^4+d^4*e^2*(f*x^2+e)*(-2*f*x^2+e))*b^2*a^2+(c^4*f^4+c^3*
d*f^4*x^2+d^4*e^3*(f*x^2+e))*b^3*x^2*a+2*b^4*c*e*(f*x^2+e)*(d*x^2+c)*(c*f-
d*e)^2)*(c*f-d*e)*x))/((a*f-b*e)*e)^(1/2)/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(1
/2)/(d*x^2+c)/(c*f-d*e)^3/(a*d-b*c)^2/c/e/(f*x^2+e)/(a*f-b*e)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^2(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. 2(383) = 766.

Time = 6.91 (sec) , antiderivative size = 2677, normalized size of antiderivative = 6.40

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
b^4*x/((a*b^4*c^2*e^2 - 2*a^2*b^3*c*d*e^2 + a^3*b^2*d^2*e^2 - 2*a^2*b^3*c^
2*e*f + 4*a^3*b^2*c*d*e*f - 2*a^4*b*d^2*e*f + a^3*b^2*c^2*f^2 - 2*a^4*b*c*
d*f^2 + a^5*d^2*f^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2)*c*d^4*e - a*sqrt(b)
*d^5*e - 8*b^(3/2)*c^2*d^3*f + 5*a*sqrt(b)*c*d^4*f)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3
*d^3*e^3 - 2*a*b*c^2*d^4*e^3 + a^2*c*d^5*e^3 - 3*b^2*c^4*d^2*e^2*f + 6*a*b
*c^3*d^3*e^2*f - 3*a^2*c^2*d^4*e^2*f + 3*b^2*c^5*d*e*f^2 - 6*a*b*c^4*d^2*e
*f^2 + 3*a^2*c^3*d^3*e*f^2 - b^2*c^6*f^3 + 2*a*b*c^5*d*f^3 - a^2*c^4*d^2*f
^3)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/2*(8*b^(3/2)*d*e^2*f^3 - 4*b^(3/2)*c*e*f
^4 - 5*a*sqrt(b)*d*e*f^4 + a*sqrt(b)*c*f^5)*arctan(1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^2*d^3*e^6 - 3
*b^2*c*d^2*e^5*f - 2*a*b*d^3*e^5*f + 3*b^2*c^2*d*e^4*f^2 + 6*a*b*c*d^2*e^4
*f^2 + a^2*d^3*e^4*f^2 - b^2*c^3*e^3*f^3 - 6*a*b*c^2*d*e^3*f^3 - 3*a^2*c*d
^2*e^3*f^3 + 2*a*b*c^3*e^2*f^4 + 3*a^2*c^2*d*e^2*f^4 - a^2*c^3*e*f^5)*sqrt
(-b^2*e^2 + a*b*e*f)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c*d^3*e
^3*f - (sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*d^4*e^3*f - 4*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*a*b^(5/2)*c*d^3*e^2*f^2 + 2*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(3/2)*d^4*e^2*f^2 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)
*c^3*d*e*f^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2*e*f^3 +
 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3*e*f^3 - (sqrt(b)*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)^2(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

3.356 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 668

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{b(8b^4ce^2(de-cf)^3 - a^4d^2f^3(4d^2e^2+11cdef-3c^2f^2)+6a^3bdf}{}$$

$$-\frac{d^4x}{2c(bc-ad)(de-cf)^3\sqrt{a+bx^2}\,(c+dx^2)}$$

$$-\frac{f^3x}{4e(be-af)(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)^2}$$

$$+\frac{f^3(af(11de-3cf)-8be(2de-cf))x}{8e^2(be-af)^2(de-cf)^3\sqrt{a+bx^2}\,(e+fx^2)}$$

$$+\frac{d^4(ad(de-7cf)-2bc(2de-5cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}(de-cf)^4}$$

$$+\frac{f^3(4abef(25d^2e^2-16cdef+3c^2f^2)-a^2f^2(35d^2e^2-14cdef+3c^2f^2)-8b^2e^2(10d^2e^2-10cdef+3c^2f^2}{8e^{5/2}(be-af)^{7/2}(de-cf)^4}$$

output
```
1/8*b*(8*b^4*c*e^2*(-c*f+d*e)^3-a^4*d^2*f^3*(-3*c^2*f^2+11*c*d*e*f+4*d^2*e
^2)+6*a^3*b*d*f^2*(-c^3*f^3+2*c^2*d*e*f^2+3*c*d^2*e^2*f+2*d^3*e^3)+2*a*b^3
*e*(-5*c^4*f^4+9*c^3*d*e*f^3+2*d^4*e^4)-3*a^2*b^2*f*(-c^4*f^4-3*c^3*d*e*f^
3+12*c^2*d^2*e^2*f^2+4*d^4*e^4))*x/a/c/(-a*d+b*c)^2/e^2/(-a*f+b*e)^3/(-c*f
+d*e)^3/(b*x^2+a)^(1/2)-1/2*d^4*x/c/(-a*d+b*c)/(-c*f+d*e)^3/(b*x^2+a)^(1/2
)/(d*x^2+c)-1/4*f^3*x/e/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^
2+1/8*f^3*(a*f*(-3*c*f+11*d*e)-8*b*e*(-c*f+2*d*e))*x/e^2/(-a*f+b*e)^2/(-c*
f+d*e)^3/(b*x^2+a)^(1/2)/(f*x^2+e)+1/2*d^4*(a*d*(-7*c*f+d*e)-2*b*c*(-5*c*f
+2*d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d
+b*c)^(5/2)/(-c*f+d*e)^4+1/8*f^3*(4*a*b*e*f*(3*c^2*f^2-16*c*d*e*f+25*d^2*e
^2)-a^2*f^2*(3*c^2*f^2-14*c*d*e*f+35*d^2*e^2)-8*b^2*e^2*(3*c^2*f^2-10*c*d*
e*f+10*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/
2)/(-a*f+b*e)^(7/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 19.89 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{1}{8}\left(x\sqrt{a+bx^2}\left(-\frac{8b^5}{a(bc-ad)^2(-be+af)^3(a+bx^2)} - \frac{}{c(bc-ad}\right.\right.$$

$$+\frac{4d^4(ad(de-7cf)+2bc(-2de+5cf))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}(-bc+ad)^{5/2}(de-cf)^4}$$

$$+\frac{f^3(-4abef(25d^2e^2-16cdef+3c^2f^2)+a^2f^2(35d^2e^2-14cdef+3c^2f^2)+8b^2e^2(10d^2e^2-10cdef+3c^2}{e^{5/2}(-be+af)^{7/2}(de-cf)^4}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((-8*b^5)/(a*(b*c - a*d)^2*(-(b*e) + a*f)^3*(a + b*x^2)
) - (4*d^5)/(c*(b*c - a*d)^2*(-(d*e) + c*f)^3*(c + d*x^2)) + (2*f^4)/(e*(b
*e - a*f)^2*(d*e - c*f)^2*(e + f*x^2)^2) + (f^4*(2*b*e*(9*d*e - 5*c*f) + a
*f*(-11*d*e + 3*c*f)))/(e^2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x^2))) + (4
*d^4*(a*d*(d*e - 7*c*f) + 2*b*c*(-2*d*e + 5*c*f))*ArcTan[(Sqrt[-(b*c) + a*
d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(3/2)*(-(b*c) + a*d)^(5/2)*(d*e - c*f
)^4) + (f^3*(-4*a*b*e*f*(25*d^2*e^2 - 16*c*d*e*f + 3*c^2*f^2) + a^2*f^2*(3
5*d^2*e^2 - 14*c*d*e*f + 3*c^2*f^2) + 8*b^2*e^2*(10*d^2*e^2 - 10*c*d*e*f +
 3*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^
(5/2)*(-(b*e) + a*f)^(7/2)*(d*e - c*f)^4))/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^3}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)-4b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{\int \frac{a\left(4e(de+2cf)b^2-3af(5de+cf)b+6a^2df^2\right)-2bf\left(-2dfa^2-b(3de-cf)a+4b^2ce\right)x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(3de-c\right.}{4e\left(e+fx^2\right)^2(be-af)}}{a(be-af)}\right.}{(bc-ad)^2}}{bc-ad}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 402

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{\frac{\int -\frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)-2\cdots\right.}{2e\left(e+fx^2\right)(be-af\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 25

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{\int \frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)-2\cdots\right.}{2e\left(e+fx^2\right)(be-af\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2b\cdots\right.}{2e\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 291$$

$$b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2\cdots\right.}{\cdots}}{4e(be-af)}\cdots}{a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\cdots}{4e(be-af)\cdots}\right)}{(bc\cdots}\right)}{bc-ad} \cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 421$$

$$b\left(\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2\cdots\right.}{2e^{3/2}\cdots}\cdots}{\cdots}\right)}{\cdots}\right)}{b\cdots} \cdots$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 401$$

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}-\frac{\int-\frac{f(2b(3de-cf)x^2+a(7de-3cf))}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{a\text{arctanh}\ldots}{\ldots}}{\ldots}\right.}{\ldots}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 25

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{f(2b(3de-cf)x^2+a(7de-3cf))}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{a\text{arctanh}(\ldots}{\ldots}}{\ldots}\right.}{\ldots}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2b(3de-cf)x^2+a(7de-3cf)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{a\operatorname{arctanh}\left(\frac{x\cdots}{\sqrt{e\cdots}}\right)}{\cdots}}{\cdots}\right)}{\cdots}\right)$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\frac{\int-\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\cdots}{a\sqrt{a\cdots}}\cdots\right)}{\cdots}\right)$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{\int \frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\cdots}{a\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf\cdots}{4e\left(e+fx^2\right)^2}\cdots\right)}{(de-cf)^2}\cdots\right)}{(bc-ad)^2}\cdots\right)}{\cdots}$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de}{4e(e+fx^2}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 221

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-}{4e(e+fx^2)}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 422

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(7de-3cf)-4be(2de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{301}$$

$$b\left(\frac{d^2\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-\cdots}}{\sqrt{e}\sqrt{a+\cdots}}\right)\cdots}{2e^3\cdots}\cdots\right)}{\cdots}\cdots\right)}{(bc-ad)^2}\cdots\right)}{\cdots}$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\cdots}{4e\left(fx\cdots\right.}\cdots\right.}{\cdots}\cdots}{(bc-ad)^2\cdots}\cdots\right.$$

$$\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{ 219}$$

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-}{4e}\right.}{}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \ 291$$

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{(be-af)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right)}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 426

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}- \cdots}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 421

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \; 402$$

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{f(de-cf)x\sqrt{bx\cdots}}{4e(be-af)\left(fx^2\cdots\right.}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{bc-ad}$$

$\downarrow$ 25

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right.}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-\cdots}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}\cdots}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.}{bc-ad}$$

↓ 402

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}-\cdots\right.}{(bc-ad)^2}\cdots\right.$$

$$d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}\cdots}{\cdots}\right.}{(de-cf)^2}\cdots\right.}{de-cf}\cdots\right.$$

$$\overline{\qquad bc-ad \qquad}$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 17.48 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.68

method	result	size
pseudoelliptic	Expression too large to display	1119
default	Expression too large to display	6453

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)*(-3/5*(a*d-b*c)^2*a*(b*x^2+a)^(1/2)*c*(d*x^2+c)*(a
^2*c^2*f^4-14/3*a*c*(a*d+6/7*b*c)*e*f^3+35/3*(a^2*d^2+64/35*a*b*c*d+24/35*
b^2*c^2)*e^2*f^2-100/3*d*(a*d+4/5*b*c)*b*e^3*f+80/3*b^2*d^2*e^4)*((a*d-b*c
)*c)^(1/2)*(f*x^2+e)^2*f^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))
+((a*f-b*e)*e)^(1/2)*(28/5*a*(b*x^2+a)^(1/2)*d^4*(d*x^2+c)*(a*f-b*e)^3*(f*
x^2+e)^2*e^2*(c*(a*d-10/7*b*c)*f-1/7*d*e*(a*d-4*b*c))*arctan(c*(b*x^2+a)^(
1/2)/x/((a*d-b*c)*c)^(1/2))+(3/5*a^2*c^2*x^2*(d*x^2+c)*(b*x^2+a)*(a*d-b*c)
^2*f^7+(a*d-b*c)^2*a*c*(b*x^2+a)*(d*x^2+c)*(-11/5*a*d*x^2+(-2*b*x^2+a)*c)*
e*f^6-13/5*(4/13*a^4*x^4*(b*x^2+a)*d^5+a^3*c*(b*x^2+a)*x^2*(-18/13*b*x^2+a
)*d^4+a^2*(36/13*b^2*x^4-32/13*a*b*x^2+a^2)*c^2*(b*x^2+a)*d^3-14/13*a*(9/7
*b^2*x^4-25/14*a*b*x^2+a^2)*c^3*(b*x^2+a)*b*d^2-11/13*c^4*(-8/11*b^3*x^6+6
/11*a*b^2*x^4+17/11*a^2*b*x^2+a^3)*b^2*d+12/13*c^5*(2/3*b^2*x^4+a*b*x^2+a^
2)*b^3)*e^2*f^5-8/5*(a^3*(b*x^2+a)*x^2*(-3/2*b*x^2+a)*d^5-5/2*a^3*b*c*x^2*
(b*x^2+a)*d^4-5/2*a^2*b*c^2*(b*x^2+a)*(-2*b*x^2+a)*d^3+5*(-3/5*b^3*x^6-1/2
*a*b^2*x^4+1/2*a^2*b*x^2+a^3)*c^3*b^2*d^2-5/2*c^4*(2/5*b^2*x^4+a*b*x^2+a^2
)*b^3*d+2*b^5*c^5*x^2)*e^3*f^4-4/5*((3*a^2*b^3*x^6-3*a^3*b^2*x^4-5*a^4*b*x
^2+a^5)*d^5+6*b^5*c^2*d^3*x^6-6*b^5*c^3*d^2*x^4-10*b^5*c^4*d*x^2+2*c^5*b^5
)*e^4*f^3+12/5*d*((1/3*b^2*x^4-2*a*b*x^2+a^2)*a*(b*x^2+a)*d^4+2/3*b^4*c*d^
3*x^6-10/3*b^4*c^2*d^2*x^4-2*b^4*c^3*d*x^2+2*c^4*b^4)*b*e^5*f^2-12/5*(a*(-
2/3*b*x^2+a)*(b*x^2+a)*d^3-4/3*b^3*c*d^2*x^4+2/3*x^2*b^3*c^2*d+2*b^3*c^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2307 vs. $2(629) = 1258$.

Time = 24.10 (sec) , antiderivative size = 2307, normalized size of antiderivative = 3.45

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b^5*x/((a*b^5*c^2*e^3 - 2*a^2*b^4*c*d*e^3 + a^3*b^3*d^2*e^3 - 3*a^2*b^4*c^
2*e^2*f + 6*a^3*b^3*c*d*e^2*f - 3*a^4*b^2*d^2*e^2*f + 3*a^3*b^3*c^2*e*f^2
- 6*a^4*b^2*c*d*e*f^2 + 3*a^5*b*d^2*e*f^2 - a^4*b^2*c^2*f^3 + 2*a^5*b*c*d*
f^3 - a^6*d^2*f^3)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2)*c*d^5*e - a*sqrt(b)*d
^6*e - 10*b^(3/2)*c^2*d^4*f + 7*a*sqrt(b)*c*d^5*f)*arctan(1/2*((sqrt(b)*x
- sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3*
d^4*e^4 - 2*a*b*c^2*d^5*e^4 + a^2*c*d^6*e^4 - 4*b^2*c^4*d^3*e^3*f + 8*a*b*
c^3*d^4*e^3*f - 4*a^2*c^2*d^5*e^3*f + 6*b^2*c^5*d^2*e^2*f^2 - 12*a*b*c^4*d
^3*e^2*f^2 + 6*a^2*c^3*d^4*e^2*f^2 - 4*b^2*c^6*d*e*f^3 + 8*a*b*c^5*d^2*e*f
^3 - 4*a^2*c^4*d^3*e*f^3 + b^2*c^7*f^4 - 2*a*b*c^6*d*f^4 + a^2*c^5*d^2*f^4
)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/8*(80*b^(5/2)*d^2*e^4*f^3 - 80*b^(5/2)*c*d
*e^3*f^4 - 100*a*b^(3/2)*d^2*e^3*f^4 + 24*b^(5/2)*c^2*e^2*f^5 + 64*a*b^(3/
2)*c*d*e^2*f^5 + 35*a^2*sqrt(b)*d^2*e^2*f^5 - 12*a*b^(3/2)*c^2*e*f^6 - 14*
a^2*sqrt(b)*c*d*e*f^6 + 3*a^2*sqrt(b)*c^2*f^7)*arctan(1/2*((sqrt(b)*x - sq
rt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^3*d^4*e^9
- 4*b^3*c*d^3*e^8*f - 3*a*b^2*d^4*e^8*f + 6*b^3*c^2*d^2*e^7*f^2 + 12*a*b^2
*c*d^3*e^7*f^2 + 3*a^2*b*d^4*e^7*f^2 - 4*b^3*c^3*d*e^6*f^3 - 18*a*b^2*c^2*
d^2*e^6*f^3 - 12*a^2*b*c*d^3*e^6*f^3 - a^3*d^4*e^6*f^3 + b^3*c^4*e^5*f^4 +
 12*a*b^2*c^3*d*e^5*f^4 + 18*a^2*b*c^2*d^2*e^5*f^4 + 4*a^3*c*d^3*e^5*f^4 -
 3*a*b^2*c^4*e^4*f^5 - 12*a^2*b*c^3*d*e^4*f^5 - 6*a^3*c^2*d^2*e^4*f^5 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3} \, dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2\,(f\,x^2+e)^3} \, dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2} (c+dx^2)^2 (e+fx^2)^3} \, dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^2\,(f\,x^2+e)^3} dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

3.357 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 943

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \frac{b(8b^5c^2e^2(de-cf)^4+3a^5d^3f^3(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f}{}$$

$$-\frac{d^4x}{4c(bc-ad)(de-cf)^3\sqrt{a+bx^2}\,(c+dx^2)^2}$$

$$+\frac{d^4(3ad(de-5cf)-4bc(2de-5cf))x}{8c^2(bc-ad)^2(de-cf)^4\sqrt{a+bx^2}\,(c+dx^2)}$$

$$+\frac{f^4x}{4e(be-af)(de-cf)^3\sqrt{a+bx^2}\,(e+fx^2)^2}$$

$$+\frac{f^4(4be(5de-2cf)-3af(5de-cf))x}{8e^2(be-af)^2(de-cf)^4\sqrt{a+bx^2}\,(e+fx^2)}$$

$$-\frac{3d^4(8b^2c^2(d^2e^2-4cdef+5c^2f^2)-4abcd(d^2e^2-7cdef+14c^2f^2)+a^2d^2(d^2e^2-6cdef+21c^2f^2))\,\mathrm{arcta}}{8c^{5/2}(bc-ad)^{7/2}(de-cf)^5}$$

$$-\frac{3f^4(4abef(14d^2e^2-7cdef+c^2f^2)-a^2f^2(21d^2e^2-6cdef+c^2f^2)-8b^2e^2(5d^2e^2-4cdef+c^2f^2))\,\mathrm{arct}}{8e^{5/2}(be-af)^{7/2}(de-cf)^5}$$

output
```
1/8*b*(8*b^5*c^2*e^2*(-c*f+d*e)^4+3*a^5*d^3*f^3*(c^3*f^3-5*c^2*d*e*f^2-5*c
*d^2*e^2*f+d^3*e^3)-a^4*b*d^2*f^2*(9*c^4*f^4-35*c^3*d*e*f^3-44*c^2*d^2*e^2
*f^2-35*c*d^3*e^3*f+9*d^4*e^4)+3*a^3*b^2*d*f*(3*c^5*f^5-5*c^4*d*e*f^4-22*c
^3*d^2*e^2*f^3-22*c^2*d^3*e^3*f^2-5*c*d^4*e^4*f+3*d^5*e^5)+2*a*b^4*c*e*(5*
c^5*f^5-11*c^4*d*e*f^4-11*c*d^4*e^4*f+5*d^5*e^5)-3*a^2*b^3*(c^6*f^6+5*c^5*
d*e*f^5-22*c^4*d^2*e^2*f^4-22*c^2*d^4*e^4*f^2+5*c*d^5*e^5*f+d^6*e^6))*x/a/
c^2/(-a*d+b*c)^3/e^2/(-a*f+b*e)^3/(-c*f+d*e)^4/(b*x^2+a)^(1/2)-1/4*d^4*x/c
/(-a*d+b*c)/(-c*f+d*e)^3/(b*x^2+a)^(1/2)/(d*x^2+c)^2+1/8*d^4*(3*a*d*(-5*c*
f+d*e)-4*b*c*(-5*c*f+2*d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*e)^4/(b*x^2+a)^(1/
2)/(d*x^2+c)+1/4*f^4*x/e/(-a*f+b*e)/(-c*f+d*e)^3/(b*x^2+a)^(1/2)/(f*x^2+e)
^2+1/8*f^4*(4*b*e*(-2*c*f+5*d*e)-3*a*f*(-c*f+5*d*e))*x/e^2/(-a*f+b*e)^2/(-
c*f+d*e)^4/(b*x^2+a)^(1/2)/(f*x^2+e)-3/8*d^4*(8*b^2*c^2*(5*c^2*f^2-4*c*d*e
*f+d^2*e^2)-4*a*b*c*d*(14*c^2*f^2-7*c*d*e*f+d^2*e^2)+a^2*d^2*(21*c^2*f^2-6
*c*d*e*f+d^2*e^2))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(
5/2)/(-a*d+b*c)^(7/2)/(-c*f+d*e)^5-3/8*f^4*(4*a*b*e*f*(c^2*f^2-7*c*d*e*f+1
4*d^2*e^2)-a^2*f^2*(c^2*f^2-6*c*d*e*f+21*d^2*e^2)-8*b^2*e^2*(c^2*f^2-4*c*d
*e*f+5*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/
2)/(-a*f+b*e)^(7/2)/(-c*f+d*e)^5
```

Mathematica [A] (verified)

Time = 24.14 (sec) , antiderivative size = 635, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx=\sqrt{a+bx^2}\left(\frac{b^6x}{a(-bc+ad)^3(-be+af)^3(a+bx^2)}-\frac{d^5x}{4c(bc-ad)^2(-de+cf)^3(c+dx^2)^2}-\frac{d^5(-10bcde+3ad^2e+22bc^2f-15acdf)\,x}{8c^2(bc-ad)^3(-de+cf)^4(c+dx^2)}-\frac{f^5x}{4e(be-af)^2(de-cf)^3(e+fx^2)^2}-\frac{f^5(22bde^2-10bcef-15adef+3acf^2)\,x}{8e^2(be-af)^3(de-cf)^4(e+fx^2)}\right)+\frac{3(8b^2c^2d^6e^2-4abcd^7e^2+a^2d^8e^2-32b^2c^3d^5ef+28abc^2d^6ef-6a^2cd^7ef+40b^2c^4d^4f^2-56abc^3d^5f^2+2}{8c^{5/2}(bc-ad)^3\sqrt{-bc+ad}(-de+cf)^5}+\frac{3(40b^2d^2e^4f^4-32b^2cde^3f^5-56abd^2e^3f^5+8b^2c^2e^2f^6+28abcde^2f^6+21a^2d^2e^2f^6-4abc^2ef^7-6a^2cdef}{8e^{5/2}(be-af)^3\sqrt{-be+af}(de-cf)^5}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
Sqrt[a + b*x^2]*((b^6*x)/(a*(-(b*c) + a*d)^3*(-(b*e) + a*f)^3*(a + b*x^2))
 - (d^5*x)/(4*c*(b*c - a*d)^2*(-(d*e) + c*f)^3*(c + d*x^2)^2) - (d^5*(-10*
b*c*d*e + 3*a*d^2*e + 22*b*c^2*f - 15*a*c*d*f)*x)/(8*c^2*(b*c - a*d)^3*(-(
d*e) + c*f)^4*(c + d*x^2)) - (f^5*x)/(4*e*(b*e - a*f)^2*(d*e - c*f)^3*(e +
 f*x^2)^2) - (f^5*(22*b*d*e^2 - 10*b*c*e*f - 15*a*d*e*f + 3*a*c*f^2)*x)/(8
*e^2*(b*e - a*f)^3*(d*e - c*f)^4*(e + f*x^2))) + (3*(8*b^2*c^2*d^6*e^2 - 4
*a*b*c*d^7*e^2 + a^2*d^8*e^2 - 32*b^2*c^3*d^5*e*f + 28*a*b*c^2*d^6*e*f - 6
*a^2*c*d^7*e*f + 40*b^2*c^4*d^4*f^2 - 56*a*b*c^3*d^5*f^2 + 21*a^2*c^2*d^6*
f^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*
(b*c - a*d)^3*Sqrt[-(b*c) + a*d]*(-(d*e) + c*f)^5) + (3*(40*b^2*d^2*e^4*f^
4 - 32*b^2*c*d*e^3*f^5 - 56*a*b*d^2*e^3*f^5 + 8*b^2*c^2*e^2*f^6 + 28*a*b*c
*d*e^2*f^6 + 21*a^2*d^2*e^2*f^6 - 4*a*b*c^2*e*f^7 - 6*a^2*c*d*e*f^7 + a^2*
c^2*f^8)*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(8*e^(5
/2)*(b*e - a*f)^3*Sqrt[-(b*e) + a*f]*(d*e - c*f)^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \textbf{402}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\int \frac{a(bde+bcf-2adf)-4b(bc-ad)fx^2}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{\int\frac{a\left(4e(de+2cf)b^2-3af(5de+cf)b+6a^2df^2\right)-2bf\left(-2dfa^2-b(3de-cf)a+4b^2ce\right)x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(-2a^2df-ab(3de\right.}{4e\left(e+fx^2\right)^2(be-\right.}}{a(be-af)}\right.}{(bc-ad)^2}}{bc-ad}\right.}{bc-ad}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{\frac{\int -\frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)\right.}{2e\left(e+fx^2\right)(be-}}{4e(be-af)}}{a(be-af)}\right.}{(bc-ad)^2}}{bc-ad}}{bc-ad}\right.$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow 25$$

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{-\frac{\int \frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-a^2bf(3cf+19de)\right.}{2e\left(e+fx^2\right)(be-}}{4e(be-af)}}{a(be-af)}}{(bc-ad)^2}}{bc-ad}}{bc-ad}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3df^2-\cdots\right.}{\cdots}}{4e(be-af)}}{a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right)}{bc-ad}\cdots\right)}{b\cdots}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \; 291$$

$$b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\frac{-\frac{a\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(3cf+de)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}\left(6a^3\right.}{}}{4e(be-af)}}{a(be-af)}\right)}{(bc-ad)^2}}{bc-ad}\right)}{}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\downarrow \text{221}$$

$$b\left(\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(6a^3df^3-a^2bf^2(3cf+23de)+4ab^2ef(3cf+10de)-8b^3e^2(\cdots\right.}{2e^{3/2}(be-af)^{3/2}}\cdots}{4e(be-af)\cdots}\cdots\right)}{(bc\cdots}\cdots\right)}{bc-ad}\cdots\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\big\downarrow$ 421

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}(dfx^2+2de-cf)}{(fx^2+e)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(6a^3df^3-a^2bf^2}{2e^{3/2}}}{}\right.}{}\right.}{}\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 401

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}-\frac{\int -\frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{4ef}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{-\frac{a\,\text{arctanh}\ldots}{\ldots}}{\ldots}\ldots\right.}{\ldots}}{\ldots}\right.$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\downarrow 25$$

$$b\left(\frac{b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int \frac{f\left(2b(3de-cf)x^2+a(7de-3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4ef}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{a\text{arctanh}\Big(}{}}{}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int\frac{2b(3de-cf)x^2+a(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx(bc-ad)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{-\frac{a\,\text{arctanh}\left(\frac{x\cdots}{\sqrt{e}\cdots}\right.\cdots}{\cdots}}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}}$$

$$\Big\downarrow\ 402$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{\frac{\int-\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\cdots}{a\sqrt{a\cdots}}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 25

$$b\left(b\left(\frac{b\left(d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{\int\frac{a(af(7de-3cf)-4be(2de-cf))}{\sqrt{bx^2+a}(fx^2+e)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e(e+fx^2)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-cf)}{4e(e+fx^2)^2}\right)}{(de-cf)^2}\right)\right)}{(bc-ad)^2}+\frac{b\left(\frac{\cdots}{a\sqrt{a}\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{-\frac{a(af(7de-3cf)-4be(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{x\sqrt{a+bx^2}(af(7de-3cf)-2be(3de-cf))}{2e\left(e+fx^2\right)(be-af)}}{4e}+\frac{x\sqrt{a+bx^2}(de-\cdots}{4e\left(e+fx^2\right)^{\cdots}}\cdots\right)}{(de-cf)^2}\cdots\right)}{(bc-ad)^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{\frac{d^2\int \frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e(fx^2+e)^2}+\frac{-\frac{(af(7de-3cf)-2be(3de-cf))\sqrt{bx^2+a}\,x}{2e(be-af)(fx^2+e)}-\frac{a(af(7de-3cf)-4be(2de-cf))\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}}{4e}\right.}{(de-cf)^2}}{(bc-ad)^2}}{}\right.}{}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\downarrow 221$$

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e\left(fx^2+e\right)^2}+\frac{-\frac{(af(7de-3cf)-2be(3de-cf))\sqrt{bx^2+a}\,x}{2e(be-af)\left(fx^2+e\right)}-\frac{a(af(7de-3cf)-4be(2de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}}{\sqrt{e}\sqrt{bx^2}\cdots}\right.}{2e^{3/2}(be-af)^{3/2}}}{4e}\cdots\right.}{(de-cf)^2}\cdots\right.}{(bc-ad)^2}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\downarrow$ 422

$$b\left(\frac{b\left(\frac{\frac{d^2\left(\frac{d\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{de-cf}-\frac{f\int\frac{\sqrt{bx^2+a}}{fx^2+e}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+ax}}{4e\left(fx^2+e\right)^2}+\frac{-\frac{(af(7de-3cf)-2be(3de-cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}-\frac{a(af(7de-3cf)-4be(2de-cf))\arctan}{2e^{3/2}(be-af)^{3/2}}}{4e}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.}{}\right.}{\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}}$$

$\downarrow$ 301

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-cf)\sqrt{bx^2+a}\,x}{4e\left(fx^2+e\right)^2}+\frac{-(a\ldots}{\ldots}\right.}{\ldots}\right.}{(bc-ad)^2}\ldots}{\ldots}\right.}{\ldots}$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$$\Big\downarrow\ 224$$

$$b\left(b\left(\left(\frac{d^2\left(\frac{d\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(de-\cdots}{4e(\cdots}\cdots\right.}{\cdots}\right.\right.\right.$$

$$\cdots\frac{\cdots}{(bc-ad)^2}\cdots$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

$\Big\downarrow$ 219

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(\frac{(c\ldots}{\ldots}\right.}{\ldots}\right.}{(bc-ad)^2}\right.}{\ldots}\right.}{\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^2(fx^2+e)^3}dx}{de-cf}\right)}{bc-ad}}$$

$\downarrow$ 291

$$b\left(b\left(\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{(be-af)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{de-cf}\right)}{(de-cf)^2}\right.\right.\right.$$

$$\frac{\cdots}{(bc-ad)^2}$$

$$\frac{d\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

↓ 221

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}\right.}{(bc-ad)^2}\right.}{}\right.$$

$$\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{de-cf}\right)}{bc-ad}$$

↓ 426

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}}{(bc-ad)^2}}{}\right.}{}\right.$$

$$\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2\right.}}{de-cf}\right.}{de-cf}\right.}{bc-ad}$$

↓ 421

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}\right.}{(bc-ad)^2}\right.}{}\right.$$

$$d\left(\frac{d\left(\frac{d\left(\frac{f^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+\right.}}{de-cf}\right.}{}\right.\right.$$

↓ 25

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}\right.}{(bc-ad)^2}\right.}{}\right.$$

$$d\left(\frac{d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}\,dx\,f^2}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^3}\,dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}\,dx}{de-cf}\right)}{de-cf}-\frac{f\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)}}{de-cf}\right.}{}\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{d\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}d}\right)}{de-cf}-\frac{f\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{f}-\frac{\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}f}\right)}{de-cf}\right)}{(de-cf)^2}}{(bc-ad)^2}}{}\right)}{}\right)$$

$$\left(d\left(\frac{d\left(\frac{\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dxf^2}{(de-cf)^2}+\frac{d\left(\frac{\int-\frac{2bd(de-cf)x^2+ad(3de-7cf)-4bc(de-2cf)}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{d(de-cf)x\sqrt{bx^2+a}}{4c(bc-ad)\left(dx^2+c\right)^2}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2\cdots\right.}}{de-cf}\right.\right.$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 422
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[-d/(b*c - a*d)   Int[(c + d*x^2)^q*(e + f*x^2)^r,
 x], x] + Simp[b/(b*c - a*d)   Int[(c + d*x^2)^(q + 1)*((e + f*x^2)^r/(a +
b*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && LeQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. 2(899) = 1798.

Time = 208.01 (sec) , antiderivative size = 2018, normalized size of antiderivative = 2.14

method	result	size
pseudoelliptic	Expression too large to display	2018
default	Expression too large to display	8706

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)*(63/5*a*(b*x^2+a)^(1/2)*d^4*((a*f-b*e)*e)^(1/2)*(d
*x^2+c)^2*(a*f-b*e)^3*(1/21*a^2*e^2*d^4-2/7*a*(a*f+2/3*b*e)*c*e*d^3+c^2*(a
^2*f^2+4/3*a*b*f*e+8/21*b^2*e^2)*d^2-8/3*c^3*(a*f+4/7*b*e)*b*f*d+40/21*b^2
*c^4*f^2)*(f*x^2+e)^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+
(-3/5*(a*d-b*c)^3*a*(b*x^2+a)^(1/2)*c^2*(d*x^2+c)^2*((21*a^2*e^2*f^2-56*a*
b*e^3*f+40*b^2*e^4)*d^2-6*(a*f-8/3*b*e)*c*f*(a*f-2*b*e)*e*d+c^2*f^2*(a^2*f
^2-4*a*b*e*f+8*b^2*e^2))*(f*x^2+e)^2*f^4*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-
b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(3/5*a^2*e^3*x^2*(f*x^2+e)^2*(b*x^2+a)*
(a*f-b*e)^3*d^8+a*c*(b*x^2+a)*(a*f-b*e)^3*(-3*a*f*x^2+e*(-2*b*x^2+a))*(f*x
^2+e)^2*e^2*d^7-17/5*c^2*(15/17*a^5*x^6*(b*x^2+a)*f^7+2*a^4*(b*x^2+a)*(-22
/17*b*x^2+a)*x^4*e*f^6+2*a^3*(b*x^2+a)*(33/17*b^2*x^4-107/34*a*b*x^2+a^2)*
x^2*e^2*f^5+a^2*(b*x^2+a)*(-66/17*b^3*x^6+147/17*a*b^2*x^4-100/17*a^2*b*x^
2+a^3)*e^3*f^4-39/17*a*(b*x^2+a)*b*e^4*(-22/39*b^3*x^6+113/39*a*b^2*x^4-32
/13*a^2*b*x^2+a^3)*f^3+15/17*b^2*(-8/15*b^4*x^8+32/15*a*b^3*x^6+4/15*a^2*b
^2*x^4-13/15*a^3*b*x^2+a^4)*e^5*f^2+19/17*(-16/19*b^3*x^6-2/19*a*b^2*x^4+1
7/19*a^2*b*x^2+a^3)*b^3*e^6*f-12/17*(2/3*b^2*x^4+a*b*x^2+a^2)*b^4*e^7)*e*d
^6-34/5*c^3*(-3/34*a^5*x^6*(b*x^2+a)*f^8+25/34*a^4*(-7/5*b*x^2+a)*(b*x^2+a
)*x^4*e*f^7+a^3*(b*x^2+a)*(33/17*b^2*x^4-107/34*a*b*x^2+a^2)*x^2*e^2*f^6-4
8/17*a^3*(b*x^2+a)*b*x^2*(-3/2*b*x^2+a)*e^3*f^5-12/17*a^2*b*e^4*(b*x^2+a)*
(3*b^2*x^4-6*a*b*x^2+a^2)*f^4+36/17*(-1/3*b^2*x^4+a^2)*b^2*(-4/3*b^2*x^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14609 vs. 2(898) = 1796.

Time = 82.01 (sec) , antiderivative size = 14609, normalized size of antiderivative = 15.49

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
b^6*x/((a*b^6*c^3*e^3 - 3*a^2*b^5*c^2*d*e^3 + 3*a^3*b^4*c*d^2*e^3 - a^4*b^
3*d^3*e^3 - 3*a^2*b^5*c^3*e^2*f + 9*a^3*b^4*c^2*d*e^2*f - 9*a^4*b^3*c*d^2*
e^2*f + 3*a^5*b^2*d^3*e^2*f + 3*a^3*b^4*c^3*e*f^2 - 9*a^4*b^3*c^2*d*e*f^2
+ 9*a^5*b^2*c*d^2*e*f^2 - 3*a^6*b*d^3*e*f^2 - a^4*b^3*c^3*f^3 + 3*a^5*b^2*
c^2*d*f^3 - 3*a^6*b*c*d^2*f^3 + a^7*d^3*f^3)*sqrt(b*x^2 + a)) + 3/8*(8*b^(
5/2)*c^2*d^6*e^2 - 4*a*b^(3/2)*c*d^7*e^2 + a^2*sqrt(b)*d^8*e^2 - 32*b^(5/2
)*c^3*d^5*e*f + 28*a*b^(3/2)*c^2*d^6*e*f - 6*a^2*sqrt(b)*c*d^7*e*f + 40*b^
(5/2)*c^4*d^4*f^2 - 56*a*b^(3/2)*c^3*d^5*f^2 + 21*a^2*sqrt(b)*c^2*d^6*f^2)
*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^
2 + a*b*c*d))/((b^3*c^5*d^5*e^5 - 3*a*b^2*c^4*d^6*e^5 + 3*a^2*b*c^3*d^7*e^
5 - a^3*c^2*d^8*e^5 - 5*b^3*c^6*d^4*e^4*f + 15*a*b^2*c^5*d^5*e^4*f - 15*a^
2*b*c^4*d^6*e^4*f + 5*a^3*c^3*d^7*e^4*f + 10*b^3*c^7*d^3*e^3*f^2 - 30*a*b^
2*c^6*d^4*e^3*f^2 + 30*a^2*b*c^5*d^5*e^3*f^2 - 10*a^3*c^4*d^6*e^3*f^2 - 10
*b^3*c^8*d^2*e^2*f^3 + 30*a*b^2*c^7*d^3*e^2*f^3 - 30*a^2*b*c^6*d^4*e^2*f^3
 + 10*a^3*c^5*d^5*e^2*f^3 + 5*b^3*c^9*d*e*f^4 - 15*a*b^2*c^8*d^2*e*f^4 + 1
5*a^2*b*c^7*d^3*e*f^4 - 5*a^3*c^6*d^4*e*f^4 - b^3*c^10*f^5 + 3*a*b^2*c^9*d
*f^5 - 3*a^2*b*c^8*d^2*f^5 + a^3*c^7*d^3*f^5)*sqrt(-b^2*c^2 + a*b*c*d)) -
3/8*(40*b^(5/2)*d^2*e^4*f^4 - 32*b^(5/2)*c*d*e^3*f^5 - 56*a*b^(3/2)*d^2*e^
3*f^5 + 8*b^(5/2)*c^2*e^2*f^6 + 28*a*b^(3/2)*c*d*e^2*f^6 + 21*a^2*sqrt(b)*
d^2*e^2*f^6 - 4*a*b^(3/2)*c^2*e*f^7 - 6*a^2*sqrt(b)*c*d*e*f^7 + a^2*sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^3\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx=\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^3\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.358 $\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 227

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)(be-af)^3x}{3ab^4(a+bx^2)^{3/2}} + \frac{(be-af)^2(2b^2ce-10a^2df+ab(de+7cf))x}{3a^2b^4\sqrt{a+bx^2}} + \frac{f^2(12bde+4bcf-11adf)x\sqrt{a+bx^2}}{8b^4} + \frac{df^3x^3\sqrt{a+bx^2}}{4b^3} + \frac{f(35a^2df^2+24b^2e(de+cf)-20abf(3de+cf))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

output
```
1/3*(-a*d+b*c)*(-a*f+b*e)^3*x/a/b^4/(b*x^2+a)^(3/2)+1/3*(-a*f+b*e)^2*(2*b^
2*c*e-10*a^2*d*f+a*b*(7*c*f+d*e))*x/a^2/b^4/(b*x^2+a)^(1/2)+1/8*f^2*(-11*a
*d*f+4*b*c*f+12*b*d*e)*x*(b*x^2+a)^(1/2)/b^4+1/4*d*f^3*x^3*(b*x^2+a)^(1/2)
/b^3+1/8*f*(35*a^2*d*f^2+24*b^2*e*(c*f+d*e)-20*a*b*f*(c*f+3*d*e))*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{x(-105a^5df^3+16b^5ce^3x^2+20a^4bf^2(9de+3cf-7dfx^2)+8ab^4e^2(dex^2+3c}{}$$
$$+\frac{f(-35a^2df^2-24b^2e(de+cf)+20abf(3de+cf))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(-105*a^5*d*f^3 + 16*b^5*c*e^3*x^2 + 20*a^4*b*f^2*(9*d*e + 3*c*f - 7*d*
f*x^2) + 8*a*b^4*e^2*(d*e*x^2 + 3*c*(e + f*x^2)) + 6*a^2*b^3*f*x^2*(2*c*f*
(-8*e + f*x^2) + d*(-16*e^2 + 6*e*f*x^2 + f^2*x^4)) + a^3*b^2*f*(8*c*f*(-9
*e + 10*f*x^2) - 3*d*(24*e^2 - 80*e*f*x^2 + 7*f^2*x^4))))/(24*a^2*b^4*(a +
 b*x^2)^(3/2)) + (f*(-35*a^2*d*f^2 - 24*b^2*e*(d*e + c*f) + 20*a*b*f*(3*d*
e + c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {401, 25, 401, 27, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(e+fx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{(fx^2+e)^2((2bc+ad)e-(4bc-7ad)fx^2)}{(bx^2+a)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(fx^2+e\right)^2\left((2bc+ad)e-(4bc-7ad)fx^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(e+fx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 401

$$\frac{\frac{x\left(e+fx^2\right)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{\int \frac{f\left(fx^2+e\right)\left(\left(-35dfa^2+4b(de+5cf)a+8b^2ce\right)x^2+a(4bc-7ad)e\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{x\left(e+fx^2\right)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{f\int \frac{\left(fx^2+e\right)\left(\left(-35dfa^2+4b(de+5cf)a+8b^2ce\right)x^2+a(4bc-7ad)e\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 403

$$\frac{\frac{x\left(e+fx^2\right)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{\int \frac{\left(105df^2a^3-10bf(11de+6cf)a^2+8b^2e(de+4cf)a+16b^3ce^2\right)x^2+ae\left(35dfa^2-32bdea-20bcfa+8b^2ce\right)}{\sqrt{bx^2+a}}dx}{4b}+\right.}{ab}}{3ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 299

$$\frac{\frac{x\left(e+fx^2\right)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3df^2-10a^2bf(6cf+11de)+8ab^2e(4cf+de)+16b^3ce^2\right)}{2b}-\frac{3a^2\left(35a^2df^2-20abf(cf+3de)+24b\right.}{2b}}{4b}\right.}{ab}}{3ab}$$

$$\frac{x\left(e+fx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{x(e+fx^2)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3df^2-10a^2bf(6cf+11de)+8ab^2e(4cf+de)+16b^3ce^2\right)}{2b}-\frac{3a^2\left(35a^2df^2-20abf(cf+3de)+24b\right.}{2}}{4b}}{ab}}{3ab}$$

$$\frac{x(e+fx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{x(e+fx^2)^2(be(ad+2bc)+af(4bc-7ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{x\sqrt{a+bx^2}(e+fx^2)\left(-35a^2df+4ab(5cf+de)+8b^2ce\right)}{4b}+\frac{\frac{x\sqrt{a+bx^2}\left(105a^3df^2-10a^2bf(6cf+11de)+8ab^2e(4c\right.}{2b}}{}\right)}{ab}}{3ab}$$

$$\frac{x(e+fx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*(e + f*x^2)^3)/(3*a*b*(a + b*x^2)^(3/2)) + (((b*(2*b*c + a*
d)*e + a*(4*b*c - 7*a*d)*f)*x*(e + f*x^2)^2)/(a*b*Sqrt[a + b*x^2]) - (f*((
(8*b^2*c*e - 35*a^2*d*f + 4*a*b*(d*e + 5*c*f))*x*Sqrt[a + b*x^2]*(e + f*x^
2))/(4*b) + (((16*b^3*c*e^2 + 105*a^3*d*f^2 + 8*a*b^2*e*(d*e + 4*c*f) - 10
*a^2*b*f*(11*d*e + 6*c*f))*x*Sqrt[a + b*x^2])/(2*b) - (3*a^2*(35*a^2*d*f^2
 + 24*b^2*e*(d*e + c*f) - 20*a*b*f*(3*d*e + c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{35\left(a^2 d f^2-\frac{4abf(cf+3de)}{7}+\frac{24b^2 e(cf+de)}{35}\right)a^2\left(b x^2+a\right)^{\frac{3}{2}} f \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)}{8}+\frac{2x\left(-\frac{9\left(\frac{\left(\frac{7}{8} d x^4-\frac{10}{3} c x^2\right) f^2}{3}+e\left(-\frac{10x^2 d}{3}+c\right) f+\right.}{2}\right.}{}$
default	$c e^3\left(\frac{x}{3a(b x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b x^2+a}}\right)+f^2(cf+3de)\left(\frac{x^5}{2b(b x^2+a)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln(}{}}{b}\right.}{2b}\right.$
risch	$-\frac{f^2 x(-2bdf x^2+11adf-4bcf-12bde)\sqrt{b x^2+a}}{8b^4}+\frac{\frac{f\left(35a^2 d f^2-20abc f^2-60abdef+24b^2 cef+24b^2 d e^2\right)\ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{\sqrt{b}}-}{}$

input
```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)*(105/16*(a^2*d*f^2-4/7*a*b*f*(c*f+3*d*e)+24/35*b^2*e*(
c*f+d*e))*a^2*(b*x^2+a)^(3/2)*f*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x*(-9/2
*(1/3*(7/8*d*x^4-10/3*c*x^2)*f^2+e*(-10/3*x^2*d+c)*f+d*e^2)*f*a^3*b^(5/2)-
6*x^2*f*(-1/8*(1/2*x^2*d+c)*x^2*f^2+e*(-3/8*x^2*d+c)*f+d*e^2)*a^2*b^(7/2)+
15/4*((-7/3*x^2*d+c)*f+3*d*e)*f^2*a^4*b^(3/2)-105/16*a^5*d*f^3*b^(1/2)+b^(
9/2)*(3/2*(c*f*x^2+e*(1/3*x^2*d+c))*a+b*c*e*x^2)*e^2))/b^(9/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(203) = 406.

Time = 0.40 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.31

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(24*a^4*b^2*d*e^2*f + (24*a^2*b^4*d*e^2*f + 12*(2*a^2*b^4*c - 5*a
^3*b^3*d)*e*f^2 - 5*(4*a^3*b^3*c - 7*a^4*b^2*d)*f^3)*x^4 + 12*(2*a^4*b^2*c
 - 5*a^5*b*d)*e*f^2 - 5*(4*a^5*b*c - 7*a^6*d)*f^3 + 2*(24*a^3*b^3*d*e^2*f
+ 12*(2*a^3*b^3*c - 5*a^4*b^2*d)*e*f^2 - 5*(4*a^4*b^2*c - 7*a^5*b*d)*f^3)*
x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*a^2*b^
4*d*f^3*x^7 + 3*(12*a^2*b^4*d*e*f^2 + (4*a^2*b^4*c - 7*a^3*b^3*d)*f^3)*x^5
 + 4*(2*(2*b^6*c + a*b^5*d)*e^3 + 6*(a*b^5*c - 4*a^2*b^4*d)*e^2*f - 12*(2*
a^2*b^4*c - 5*a^3*b^3*d)*e*f^2 + 5*(4*a^3*b^3*c - 7*a^4*b^2*d)*f^3)*x^3 +
3*(8*a*b^5*c*e^3 - 24*a^3*b^3*d*e^2*f - 12*(2*a^3*b^3*c - 5*a^4*b^2*d)*e*f
^2 + 5*(4*a^4*b^2*c - 7*a^5*b*d)*f^3)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2
*a^3*b^6*x^2 + a^4*b^5), -1/24*(3*(24*a^4*b^2*d*e^2*f + (24*a^2*b^4*d*e^2*
f + 12*(2*a^2*b^4*c - 5*a^3*b^3*d)*e*f^2 - 5*(4*a^3*b^3*c - 7*a^4*b^2*d)*f
^3)*x^4 + 12*(2*a^4*b^2*c - 5*a^5*b*d)*e*f^2 - 5*(4*a^5*b*c - 7*a^6*d)*f^3
 + 2*(24*a^3*b^3*d*e^2*f + 12*(2*a^3*b^3*c - 5*a^4*b^2*d)*e*f^2 - 5*(4*a^4
*b^2*c - 7*a^5*b*d)*f^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
- (6*a^2*b^4*d*f^3*x^7 + 3*(12*a^2*b^4*d*e*f^2 + (4*a^2*b^4*c - 7*a^3*b^3*
d)*f^3)*x^5 + 4*(2*(2*b^6*c + a*b^5*d)*e^3 + 6*(a*b^5*c - 4*a^2*b^4*d)*e^2
*f - 12*(2*a^2*b^4*c - 5*a^3*b^3*d)*e*f^2 + 5*(4*a^3*b^3*c - 7*a^4*b^2*d)*
f^3)*x^3 + 3*(8*a*b^5*c*e^3 - 24*a^3*b^3*d*e^2*f - 12*(2*a^3*b^3*c - 5*a^4
*b^2*d)*e*f^2 + 5*(4*a^4*b^2*c - 7*a^5*b*d)*f^3)*x)*sqrt(b*x^2 + a))/(a...
```

Sympy [F]

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)*(e + f*x**2)**3/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(203) = 406$.

Time = 0.04 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx &= \frac{df^3x^7}{4\,(bx^2+a)^{\frac{3}{2}}b} - \frac{7\,adf^3x^5}{8\,(bx^2+a)^{\frac{3}{2}}b^2} \\
&- \frac{35\,a^2df^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24\,b^2} + \frac{(3\,def^2+cf^3)x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} \\
&- (de^2f+cef^2)x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) \\
&+ \frac{5\,(3\,def^2+cf^3)ax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{2\,ce^3x}{3\,\sqrt{bx^2+a}a^2} \\
&+ \frac{ce^3x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{35\,a^2df^3x}{24\,\sqrt{bx^2+a}b^4} + \frac{35\,a^2df^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}} \\
&+ \frac{5\,(3\,def^2+cf^3)ax}{6\,\sqrt{bx^2+a}b^3} - \frac{(de^2f+cef^2)x}{\sqrt{bx^2+a}b^2} - \frac{(de^3+3\,ce^2f)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{(de^3+3\,ce^2f)x}{3\,\sqrt{bx^2+a}ab} \\
&- \frac{5\,(3\,def^2+cf^3)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} + \frac{3\,(de^2f+cef^2)\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/4*d*f^3*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*a*d*f^3*x^5/((b*x^2 + a)^(3/2)*b
^2) - 35/24*a^2*d*f^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3
/2)*b^2))/b^2 + 1/2*(3*d*e*f^2 + c*f^3)*x^5/((b*x^2 + a)^(3/2)*b) - (d*e^2
*f + c*e*f^2)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)
) + 5/6*(3*d*e*f^2 + c*f^3)*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2
 + a)^(3/2)*b^2))/b + 2/3*c*e^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*e^3*x/((b*
x^2 + a)^(3/2)*a) - 35/24*a^2*d*f^3*x/(sqrt(b*x^2 + a)*b^4) + 35/8*a^2*d*f
^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 5/6*(3*d*e*f^2 + c*f^3)*a*x/(sqrt(b*x^
2 + a)*b^3) - (d*e^2*f + c*e*f^2)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(d*e^3 + 3
*c*e^2*f)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(d*e^3 + 3*c*e^2*f)*x/(sqrt(b*x^2
+ a)*a*b) - 5/2*(3*d*e*f^2 + c*f^3)*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3*(
d*e^2*f + c*e*f^2)*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx^2)(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,df^3x^2}{b}+\frac{12\,a^2b^6def^2+4\,a^2b^6cf^3-7\,a^3b^5df^3}{a^2b^7}\right)x^2+\frac{4\,(4\,b^8ce^3+2\,ab^7de^3+6\,ab^7ce^2f-24\,a^2b^6}{}\right.\right.}{} - \frac{(24\,b^2de^2f+24\,b^2cef^2-60\,abdef^2-20\,abcf^3+35\,a^2df^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*d*f^3*x^2/b + (12*a^2*b^6*d*e*f^2 + 4*a^2*b^6*c*f^3 - 7*a^3*b^
5*d*f^3)/(a^2*b^7))*x^2 + 4*(4*b^8*c*e^3 + 2*a*b^7*d*e^3 + 6*a*b^7*c*e^2*f
 - 24*a^2*b^6*d*e^2*f - 24*a^2*b^6*c*e*f^2 + 60*a^3*b^5*d*e*f^2 + 20*a^3*b
^5*c*f^3 - 35*a^4*b^4*d*f^3)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c*e^3 - 24*a^3*b^
5*d*e^2*f - 24*a^3*b^5*c*e*f^2 + 60*a^4*b^4*d*e*f^2 + 20*a^4*b^4*c*f^3 - 3
5*a^5*b^3*d*f^3)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(24*b^2*d*e^2*f + 24
*b^2*c*e*f^2 - 60*a*b*d*e*f^2 - 20*a*b*c*f^3 + 35*a^2*d*f^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)\left(e+fx^2\right)^3}{(a+bx^2)^{5/2}}\,dx=\int \frac{\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^3}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^3)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.41

$$\int \frac{(c+dx^2)\left(e+fx^2\right)^3}{(a+bx^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x^2+c)*(f*x^2+e)^3/(b*x^2+a)^(5/2),x)
```

output
```
( - 840*sqrt(a + b*x**2)*a**5*b*d*f**3*x + 480*sqrt(a + b*x**2)*a**4*b**2*
c*f**3*x + 1440*sqrt(a + b*x**2)*a**4*b**2*d*e*f**2*x - 1120*sqrt(a + b*x*
*2)*a**4*b**2*d*f**3*x**3 - 576*sqrt(a + b*x**2)*a**3*b**3*c*e*f**2*x + 64
0*sqrt(a + b*x**2)*a**3*b**3*c*f**3*x**3 - 576*sqrt(a + b*x**2)*a**3*b**3*
d*e**2*f*x + 1920*sqrt(a + b*x**2)*a**3*b**3*d*e*f**2*x**3 - 168*sqrt(a +
b*x**2)*a**3*b**3*d*f**3*x**5 - 768*sqrt(a + b*x**2)*a**2*b**4*c*e*f**2*x*
*3 + 96*sqrt(a + b*x**2)*a**2*b**4*c*f**3*x**5 - 768*sqrt(a + b*x**2)*a**2
*b**4*d*e**2*f*x**3 + 288*sqrt(a + b*x**2)*a**2*b**4*d*e*f**2*x**5 + 48*sq
rt(a + b*x**2)*a**2*b**4*d*f**3*x**7 + 192*sqrt(a + b*x**2)*a*b**5*c*e**3*
x + 192*sqrt(a + b*x**2)*a*b**5*c*e**2*f*x**3 + 64*sqrt(a + b*x**2)*a*b**5
*d*e**3*x**3 + 128*sqrt(a + b*x**2)*b**6*c*e**3*x**3 + 840*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**6*d*f**3 - 480*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*c*f**3 - 1440*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*d*e*f**2 + 1680*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*d*f**3*x**2 + 576*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c*e*f**2 - 960*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c*f**3*x**2 + 576*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d*e**2*f - 2880*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d*e*f**2*x**2 +
 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d*f*...
```

3.359 $\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)(be-af)^2x}{3ab^3(a+bx^2)^{3/2}} + \frac{(be-af)(2b^2ce-7a^2df+ab(de+4cf))x}{3a^2b^3\sqrt{a+bx^2}} + \frac{df^2x\sqrt{a+bx^2}}{2b^3} + \frac{f(4bde+2bcf-5adf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*(-a*d+b*c)*(-a*f+b*e)^2*x/a/b^3/(b*x^2+a)^(3/2)+1/3*(-a*f+b*e)*(2*b^2*
c*e-7*a^2*d*f+a*b*(4*c*f+d*e))*x/a^2/b^3/(b*x^2+a)^(1/2)+1/2*d*f^2*x*(b*x^
2+a)^(1/2)/b^3+1/2*f*(-5*a*d*f+2*b*c*f+4*b*d*e)*arctanh(b^(1/2)*x/(b*x^2+a
)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{x(15a^4df^2+4b^4ce^2x^2+2ab^3e(3ce+dex^2+2cfx^2)+a^2b^2fx^2(-16de-8cf}{6a^2b^3(a+bx^2)^{3/2}}$$
$$-\frac{f(4bde+2bcf-5adf)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(15*a^4*d*f^2 + 4*b^4*c*e^2*x^2 + 2*a*b^3*e*(3*c*e + d*e*x^2 + 2*c*f*x^
2) + a^2*b^2*f*x^2*(-16*d*e - 8*c*f + 3*d*f*x^2) + 2*a^3*b*f*(-6*d*e - 3*c
*f + 10*d*f*x^2)))/(6*a^2*b^3*(a + b*x^2)^(3/2)) - (f*(4*b*d*e + 2*b*c*f -
 5*a*d*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {401, 25, 401, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{x(e+fx^2)^2(bc-ad)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{(fx^2+e)((2bc+ad)e-(2bc-5ad)fx^2)}{(bx^2+a)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(fx^2+e)((2bc+ad)e-(2bc-5ad)fx^2)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(e+fx^2)^2(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{x(e+fx^2)(be(ad+2bc)+af(2bc-5ad))}{ab\sqrt{a+bx^2}}-\frac{\int \frac{f\left(\left(-15dfa^2+2b(de+3cf)a+4b^2ce\right)x^2+a(2bc-5ad)e\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x(e+fx^2)(be(ad+2bc)+af(2bc-5ad))}{ab\sqrt{a+bx^2}}-\frac{f\int \frac{\left(-15dfa^2+2b(de+3cf)a+4b^2ce\right)x^2+a(2bc-5ad)e}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{x(e+fx^2)(be(ad+2bc)+af(2bc-5ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(3cf+de)+4b^2ce\right)}{2b}-\frac{3a^2(-5adf+2bcf+4bde)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x(e+fx^2)(be(ad+2bc)+af(2bc-5ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(3cf+de)+4b^2ce\right)}{2b}-\frac{3a^2(-5adf+2bcf+4bde)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x(e+fx^2)(be(ad+2bc)+af(2bc-5ad))}{ab\sqrt{a+bx^2}}-\frac{f\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(3cf+de)+4b^2ce\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-5adf+2bcf+4bde)}{2b^{3/2}}\right)}{ab}}{3ab}+\frac{x\left(e+fx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
((b*c - a*d)*x*(e + f*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (((b*(2*b*c + a*
d)*e + a*(2*b*c - 5*a*d)*f)*x*(e + f*x^2))/(a*b*Sqrt[a + b*x^2]) - (f*(((4
*b^2*c*e - 15*a^2*d*f + 2*a*b*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*b) - (3
*a^2*(4*b*d*e + 2*b*c*f - 5*a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
2*b^(3/2))))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-\frac{5a^2(b x^2+a)^{\frac{3}{2}}\left(adf-\frac{2b(cf+2de)}{5}\right)f \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)}{2}+\frac{2\left(-\frac{3a^3\left(\left(-\frac{10x^2 d}{3}+c\right)f+2de\right)f b^{\frac{3}{2}}}{2}-2\left(\left(-\frac{3x^2 d}{8}+c\right)f+2de\right)x^2 f a^2 b}{3}}{b^{\frac{7}{2}}(b x^2+a)^{\frac{3}{2}}a^2}$
default	$c e^2\left(\frac{x}{3a(b x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b x^2+a}}\right)+f(cf+2de)\left(-\frac{x^3}{3b(b x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)$
risch	$\frac{d f^2 x\sqrt{b x^2+a}}{2b^3}-\frac{\frac{f(5adf-2bcf-4bde)\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{\sqrt{b}}-\frac{\left(a^3 d f^2-a^2 c f^2 b-2a^2 bdef+2acef b^2+a b^2 d e^2-b^3 c e^2\right)\left(-\frac{\sqrt{\left(x-\frac{\sqrt{-}}{}\right.}}{3}\right.}{}}{}$

input
```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)/b^(7/2)*(-15/4*a^2*(b*x^2+a)^(3/2)*(a*d*f-2/5*b*(c*f+2
*d*e))*f*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(-3/2*a^3*((-10/3*x^2*d+c)*f+2
*d*e)*f*b^(3/2)-2*((-3/8*x^2*d+c)*f+2*d*e)*x^2*f*a^2*b^(5/2)+3/2*a*(2/3*c*
f*x^2+e*(1/3*x^2*d+c))*e*b^(7/2)+b^(9/2)*e^2*c*x^2+15/4*a^4*d*f^2*b^(1/2))
*x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(145) = 290$.

Time = 0.18 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.73

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{3\,(4\,a^4bdef + (4\,a^2b^3def + (2\,a^2b^3c - 5\,a^3b^2d)f^2)x^4 + (2\,a^4bc - 5\,a^5d)f^2 \cdots}{\cdots}\right.$$

$$-\frac{3\,(4\,a^4bdef + (4\,a^2b^3def + (2\,a^2b^3c - 5\,a^3b^2d)f^2)x^4 + (2\,a^4bc - 5\,a^5d)f^2 + 2\,(4\,a^3b^2def + (2\,a^3b^2c - 5\,a^4 \cdots}{\cdots}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(4*a^4*b*d*e*f + (4*a^2*b^3*d*e*f + (2*a^2*b^3*c - 5*a^3*b^2*d)*
f^2)*x^4 + (2*a^4*b*c - 5*a^5*d)*f^2 + 2*(4*a^3*b^2*d*e*f + (2*a^3*b^2*c -
 5*a^4*b*d)*f^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 2*(3*a^2*b^3*d*f^2*x^5 + 2*((2*b^5*c + a*b^4*d)*e^2 + 2*(a*b^4*c - 4
*a^2*b^3*d)*e*f - 2*(2*a^2*b^3*c - 5*a^3*b^2*d)*f^2)*x^3 + 3*(2*a*b^4*c*e^
2 - 4*a^3*b^2*d*e*f - (2*a^3*b^2*c - 5*a^4*b*d)*f^2)*x)*sqrt(b*x^2 + a))/(
a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4), -1/6*(3*(4*a^4*b*d*e*f + (4*a^2*b^
3*d*e*f + (2*a^2*b^3*c - 5*a^3*b^2*d)*f^2)*x^4 + (2*a^4*b*c - 5*a^5*d)*f^2
 + 2*(4*a^3*b^2*d*e*f + (2*a^3*b^2*c - 5*a^4*b*d)*f^2)*x^2)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b^3*d*f^2*x^5 + 2*((2*b^5*c + a*b^4
*d)*e^2 + 2*(a*b^4*c - 4*a^2*b^3*d)*e*f - 2*(2*a^2*b^3*c - 5*a^3*b^2*d)*f^
2)*x^3 + 3*(2*a*b^4*c*e^2 - 4*a^3*b^2*d*e*f - (2*a^3*b^2*c - 5*a^4*b*d)*f^
2)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [F]

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)*(e + f*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(145) = 290.

Time = 0.04 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{df^2x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,adf^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} - \frac{1}{3}\left(2\,def+cf^2\right)x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{2\,ce^2x}{3\,\sqrt{bx^2+a}a^2} + \frac{ce^2x}{3\,(bx^2+a)^{\frac{3}{2}}a} + \frac{5\,adf^2x}{6\,\sqrt{bx^2+a}b^3} - \frac{5\,adf^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} - \frac{(2\,def+cf^2)x}{3\,\sqrt{bx^2+a}b^2} - \frac{(de^2+2\,cef)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{(de^2+2\,cef)x}{3\,\sqrt{bx^2+a}ab} + \frac{(2\,def+cf^2)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*d*f^2*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*d*f^2*x*(3*x^2/((b*x^2 + a)^(3
/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 1/3*(2*d*e*f + c*f^2)*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/3*c*e^2*x/(sqrt(b*
x^2 + a)*a^2) + 1/3*c*e^2*x/((b*x^2 + a)^(3/2)*a) + 5/6*a*d*f^2*x/(sqrt(b*
x^2 + a)*b^3) - 5/2*a*d*f^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/3*(2*d*e*f
+ c*f^2)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(d*e^2 + 2*c*e*f)*x/((b*x^2 + a)^(3
/2)*b) + 1/3*(d*e^2 + 2*c*e*f)*x/(sqrt(b*x^2 + a)*a*b) + (2*d*e*f + c*f^2)
*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,df^2x^2}{b}+\frac{2\,(2\,b^6ce^2+ab^5de^2+2\,ab^5cef-8\,a^2b^4def-4\,a^2b^4cf^2+10\,a^3b^3df^2)}{a^2b^5}\right)x^2+\frac{3\,(2\,ab^5ce^2-4}{}\right.}{6\,(bx^2+a)^{\frac{3}{2}}}$$
$$-\frac{(4\,bdef+2\,bcf^2-5\,adf^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*d*f^2*x^2/b + 2*(2*b^6*c*e^2 + a*b^5*d*e^2 + 2*a*b^5*c*e*f - 8*a^2
*b^4*d*e*f - 4*a^2*b^4*c*f^2 + 10*a^3*b^3*d*f^2)/(a^2*b^5))*x^2 + 3*(2*a*b
^5*c*e^2 - 4*a^3*b^3*d*e*f - 2*a^3*b^3*c*f^2 + 5*a^4*b^2*d*f^2)/(a^2*b^5))
*x/(b*x^2 + a)^(3/2) - 1/2*(4*b*d*e*f + 2*b*c*f^2 - 5*a*d*f^2)*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 738, normalized size of antiderivative = 4.47

$$\int \frac{(c+dx^2)(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output
```
(30*sqrt(a + b*x**2)*a**4*b*d*f**2*x - 12*sqrt(a + b*x**2)*a**3*b**2*c*f**
2*x - 24*sqrt(a + b*x**2)*a**3*b**2*d*e*f*x + 40*sqrt(a + b*x**2)*a**3*b**
2*d*f**2*x**3 - 16*sqrt(a + b*x**2)*a**2*b**3*c*f**2*x**3 - 32*sqrt(a + b*
x**2)*a**2*b**3*d*e*f*x**3 + 6*sqrt(a + b*x**2)*a**2*b**3*d*f**2*x**5 + 12
*sqrt(a + b*x**2)*a*b**4*c*e**2*x + 8*sqrt(a + b*x**2)*a*b**4*c*e*f*x**3 +
 4*sqrt(a + b*x**2)*a*b**4*d*e**2*x**3 + 8*sqrt(a + b*x**2)*b**5*c*e**2*x*
*3 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d*f**2 +
12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*f**2 + 24*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*e*f - 60*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*f**2*x**2 + 24*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*f**2*x**2 +
48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d*e*f*x**
2 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d*f**
2*x**4 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*
c*f**2*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b**3*d*e*f*x**4 - 5*sqrt(b)*a**5*d*f**2 - 10*sqrt(b)*a**4*b*d*f**2*x**2 +
8*sqrt(b)*a**3*b**2*c*e*f + 4*sqrt(b)*a**3*b**2*d*e**2 - 5*sqrt(b)*a**3*b*
*2*d*f**2*x**4 - 8*sqrt(b)*a**2*b**3*c*e**2 + 16*sqrt(b)*a**2*b**3*c*e*f*x
**2 + 8*sqrt(b)*a**2*b**3*d*e**2*x**2 - 16*sqrt(b)*a*b**4*c*e**2*x**2 + 8*
sqrt(b)*a*b**4*c*e*f*x**4 + 4*sqrt(b)*a*b**4*d*e**2*x**4 - 8*sqrt(b)*b*...
```

3.360 $\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx=\frac{(bc-ad)(be-af)x}{3ab^2(a+bx^2)^{3/2}}+\frac{(2b^2ce-4a^2df+ab(de+cf))\,x}{3a^2b^2\sqrt{a+bx^2}}+\frac{df\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
1/3*(-a*d+b*c)*(-a*f+b*e)*x/a/b^2/(b*x^2+a)^(3/2)+1/3*(2*b^2*c*e-4*a^2*d*f
+a*b*(c*f+d*e))*x/a^2/b^2/(b*x^2+a)^(1/2)+d*f*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx=\frac{-3a^3dfx+2b^3cex^3-4a^2bdfx^3+ab^2x(3ce+dex^2+cfx^2)}{3a^2b^2(a+bx^2)^{3/2}}-\frac{df\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-3*a^3*d*f*x + 2*b^3*c*e*x^3 - 4*a^2*b*d*f*x^3 + a*b^2*x*(3*c*e + d*e*x^2
 + c*f*x^2))/(3*a^2*b^2*(a + b*x^2)^(3/2)) - (d*f*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {401, 25, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(c+dx^2)(be-af)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{3adfx^2+c(2be+af)}{(bx^2+a)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3adfx^2+c(2be+af)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(c+dx^2)(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{3adf\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x\left(\frac{3adf}{b}-\frac{c(af+2be)}{a}\right)}{\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3adf\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x\left(\frac{3adf}{b}-\frac{c(af+2be)}{a}\right)}{\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3adf\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x\left(\frac{3adf}{b}-\frac{c(af+2be)}{a}\right)}{\sqrt{a+bx^2}}}{3ab}+\frac{x(c+dx^2)\,(be-af)}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (-((((3*a*d*f)/b -
 (c*(2*b*e + a*f))/a)*x)/Sqrt[a + b*x^2]) + (3*a*d*f*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(3/2))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{\frac{a^{2} d f \,\operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{x \sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{\left(-a^{2} d f +c e \,b^{2}\right) x}{b^{2} \sqrt{b \,x^{2}+a}}-\frac{\left(a^{2} d f -a b c f -a b d e +c e \,b^{2}\right) x^{3}}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}}{a^{2}}$
default	$c e \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)+\left(c f +d e \right) \left(-\frac{x}{2 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{a \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{2 b}\right)+d f$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a^2*d*f/b^(5/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+1/b^2*(-a^2*d*f+b^2*c*
e)*x/(b*x^2+a)^(1/2)-1/3*(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*x^3/b/(b*x^2+a)
^(3/2))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.85

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(a^2b^2dfx^4+2\,a^3bdfx^2+a^4df)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(}{6\,(a^2b^5x^4+2\,a^3b}\right.$$

$$-\frac{3\,(a^2b^2dfx^4+2\,a^3bdfx^2+a^4df)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(((2\,b^4c+ab^3d)e+(ab^3c-4\,a^2b^2d)f)x^3+3\,(ab}{3\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*b^2*d*f*x^4 + 2*a^3*b*d*f*x^2 + a^4*d*f)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(((2*b^4*c + a*b^3*d)*e + (a*b^3*c
 - 4*a^2*b^2*d)*f)*x^3 + 3*(a*b^3*c*e - a^3*b*d*f)*x)*sqrt(b*x^2 + a))/(a^
2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d*f*x^4 + 2*a^3*b*d
*f*x^2 + a^4*d*f)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (((2*b^4*c
 + a*b^3*d)*e + (a*b^3*c - 4*a^2*b^2*d)*f)*x^3 + 3*(a*b^3*c*e - a^3*b*d*f)
*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(110) = 220$.

Time = 7.93 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.47

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = ce\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{cfx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{dex^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + df\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((d*x**2+c)*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
c*e*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x
**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt
(1 + b*x**2/a))) + c*f*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*
x**2*sqrt(1 + b*x**2/a)) + d*e*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**
(3/2)*b*x**2*sqrt(1 + b*x**2/a)) + d*f*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*
a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt
(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*
x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/
2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**
2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt
(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}dfx\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)+\frac{2cex}{3\sqrt{bx^2+a}a^2}+\frac{cex}{3(bx^2+a)^{\frac{3}{2}}a}-\frac{dfx}{3\sqrt{bx^2+a}b^2}+\frac{df\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{(de+cf)x}{3(bx^2+a)^{\frac{3}{2}}b}+\frac{(de+cf)x}{3\sqrt{bx^2+a}ab}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*d*f*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2
/3*c*e*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*e*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*f
*x/(sqrt(b*x^2 + a)*b^2) + d*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/3*(d*e +
 c*f)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(d*e + c*f)*x/(sqrt(b*x^2 + a)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{(2\,b^4ce+ab^3de+ab^3cf-4\,a^2b^2df)x^2}{a^2b^3}+\frac{3\,(ab^3ce-a^3bdf)}{a^2b^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{df\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*((2*b^4*c*e + a*b^3*d*e + a*b^3*c*f - 4*a^2*b^2*d*f)*x^2/(a^2*b^3) +
 3*(a*b^3*c*e - a^3*b*d*f)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d*f*log(abs(-sqr
t(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.02

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^3bdfx-4\sqrt{b\,x^2+a}\,a^2b^2df\,x^3+3\sqrt{b\,x^2+a}\,a\,b^3cex+\sqrt{b\,x^2+a}}{}$$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**3*b*d*f*x - 4*sqrt(a + b*x**2)*a**2*b**2*d*f*x**
3 + 3*sqrt(a + b*x**2)*a*b**3*c*e*x + sqrt(a + b*x**2)*a*b**3*c*f*x**3 + s
qrt(a + b*x**2)*a*b**3*d*e*x**3 + 2*sqrt(a + b*x**2)*b**4*c*e*x**3 + 3*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d*f + 6*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*f*x**2 + 3*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*f*x**4 + sqrt(b)*a**3*b*
c*f + sqrt(b)*a**3*b*d*e - 2*sqrt(b)*a**2*b**2*c*e + 2*sqrt(b)*a**2*b**2*c
*f*x**2 + 2*sqrt(b)*a**2*b**2*d*e*x**2 - 4*sqrt(b)*a*b**3*c*e*x**2 + sqrt(
b)*a*b**3*c*f*x**4 + sqrt(b)*a*b**3*d*e*x**4 - 2*sqrt(b)*b**4*c*e*x**4)/(3
*a**2*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.361 $\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)} dx = \frac{(bc-ad)x}{3a(be-af)(a+bx^2)^{3/2}} + \frac{(2b^2ce+2a^2df+ab(de-5cf))x}{3a^2(be-af)^2\sqrt{a+bx^2}} - \frac{f(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{5/2}}$$

output
```
1/3*(-a*d+b*c)*x/a/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/3*(2*b^2*c*e+2*a^2*d*f+a*b
*(-5*c*f+d*e))*x/a^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)-f*(-c*f+d*e)*arctanh((-a
*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)} dx = \frac{x(3a^3df+2b^3cex^2+2a^2bf(-3c+dx^2)+ab^2(3ce+dex^2-5cfx^2))}{3a^2(be-af)^2(a+bx^2)^{3/2}} + \frac{f(de-cf)\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(-be+af)^{5/2}}$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(x*(3*a^3*d*f + 2*b^3*c*e*x^2 + 2*a^2*b*f*(-3*c + d*x^2) + a*b^2*(3*c*e +
d*e*x^2 - 5*c*f*x^2)))/(3*a^2*(b*e - a*f)^2*(a + b*x^2)^(3/2)) + (f*(d*e -
 c*f)*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[
-(b*e) + a*f])])/(Sqrt[e]*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {402, 25, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)}dx}{3a(be-af)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{x(2a^2df+ab(de-5cf)+2b^2ce)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int \frac{3a^2f(de-cf)}{\sqrt{bx^2+a}(fx^2+e)}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x(2a^2df+ab(de-5cf)+2b^2ce)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x(2a^2df+ab(de-5cf)+2b^2ce)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x(2a^2df+ab(de-5cf)+2b^2ce)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)) + (((2*b^2*c*e + 2*a^2
*d*f + a*b*(d*e - 5*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) - (3*a*f*(d*e
 - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*(
b*e - a*f)^(3/2)))/(3*a*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-a^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}} \left(c f -d e \right) f \arctan \left(\frac{e \sqrt{b \,x^{2}+a}}{x \sqrt{\left(a f -b e \right) e}}\right)+\left(a^{3} d f -2 \left(-\frac{x^{2} d}{3}+c \right) b f \,a^{2}+\left(e \left(\frac{x^{2} d}{3}+c \right)-\frac{5 c f \,x^{2}}{3}\right) b^{2} a +\frac{2 b^{3} c e \,x^{2}}{3}\right) \sqrt{\left(a f\right.}}{\sqrt{\left(a f -b e \right) e}\, \left(b \,x^{2}+a \right)^{\frac{3}{2}} \left(a f -b e \right)^{2} a^{2}}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-a^2*(b*x^2+a)^(3/2)*(c*f-d*e)*f*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)
^(1/2))+(a^3*d*f-2*(-1/3*x^2*d+c)*b*f*a^2+(e*(1/3*x^2*d+c)-5/3*c*f*x^2)*b^
2*a+2/3*b^3*c*e*x^2)*((a*f-b*e)*e)^(1/2)*x)/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^
(3/2)/(a*f-b*e)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(134) = 268.

Time = 2.80 (sec) , antiderivative size = 940, normalized size of antiderivative = 6.18

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^4*d*e*f - a^4*c*f^2 + (a^2*b^2*d*e*f - a^2*b^2*c*f^2)*x^4 + 2
*(a^3*b*d*e*f - a^3*b*c*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*
a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b
*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e
*f*x^2 + e^2)) - 4*(((2*b^4*c + a*b^3*d)*e^3 - (7*a*b^3*c - a^2*b^2*d)*e^2
*f + (5*a^2*b^2*c - 2*a^3*b*d)*e*f^2)*x^3 + 3*(a*b^3*c*e^3 - (3*a^2*b^2*c
- a^3*b*d)*e^2*f + (2*a^3*b*c - a^4*d)*e*f^2)*x)*sqrt(b*x^2 + a))/(a^4*b^3
*e^4 - 3*a^5*b^2*e^3*f + 3*a^6*b*e^2*f^2 - a^7*e*f^3 + (a^2*b^5*e^4 - 3*a^
3*b^4*e^3*f + 3*a^4*b^3*e^2*f^2 - a^5*b^2*e*f^3)*x^4 + 2*(a^3*b^4*e^4 - 3*
a^4*b^3*e^3*f + 3*a^5*b^2*e^2*f^2 - a^6*b*e*f^3)*x^2), 1/6*(3*(a^4*d*e*f -
 a^4*c*f^2 + (a^2*b^2*d*e*f - a^2*b^2*c*f^2)*x^4 + 2*(a^3*b*d*e*f - a^3*b*
c*f^2)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e -
 a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2
*e*f)*x)) + 2*(((2*b^4*c + a*b^3*d)*e^3 - (7*a*b^3*c - a^2*b^2*d)*e^2*f +
(5*a^2*b^2*c - 2*a^3*b*d)*e*f^2)*x^3 + 3*(a*b^3*c*e^3 - (3*a^2*b^2*c - a^3
*b*d)*e^2*f + (2*a^3*b*c - a^4*d)*e*f^2)*x)*sqrt(b*x^2 + a))/(a^4*b^3*e^4
- 3*a^5*b^2*e^3*f + 3*a^6*b*e^2*f^2 - a^7*e*f^3 + (a^2*b^5*e^4 - 3*a^3*b^4
*e^3*f + 3*a^4*b^3*e^2*f^2 - a^5*b^2*e*f^3)*x^4 + 2*(a^3*b^4*e^4 - 3*a^4*b
^3*e^3*f + 3*a^5*b^2*e^2*f^2 - a^6*b*e*f^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^2}{(a + bx^2)^{5/2} (e + fx^2)}\, dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(134) = 268$.

Time = 0.14 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.34

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{\left(\frac{(2\,b^6ce^3+ab^5de^3-9\,ab^5ce^2f+12\,a^2b^4cef^2-3\,a^3b^3def^2-5\,a^3b^3cf^3+2\,a^4b^2df^3)x^2}{a^2b^5e^4-4\,a^3b^4e^3f+6\,a^4b^3e^2f^2-4\,a^5b^2ef^3+a^6bf^4}+\frac{3\,(ab^5ce^3-4\,a^2}{a}\right.}{3\,(bx^2+a)^{\frac{3}{2}}}$$
$$+\frac{(bd^2e^2f-2\,bcdef^2+bc^2f^3)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\sqrt{b}def-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\sqrt{b}cf^2+2\,b^{\frac{3}{2}}de^2-2\,b^{\frac{3}{2}}cef-a\sqrt{b}def+a\sqrt{b}cf^2}{2\left(\sqrt{-be^2+aef}bde-\sqrt{-be^2+aef}bcf\right)}\right)}{(b^2e^2-2\,abef+a^2f^2)(bde-bcf)\sqrt{-be^2+aef}}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
1/3*((2*b^6*c*e^3 + a*b^5*d*e^3 - 9*a*b^5*c*e^2*f + 12*a^2*b^4*c*e*f^2 - 3
*a^3*b^3*d*e*f^2 - 5*a^3*b^3*c*f^3 + 2*a^4*b^2*d*f^3)*x^2/(a^2*b^5*e^4 - 4
*a^3*b^4*e^3*f + 6*a^4*b^3*e^2*f^2 - 4*a^5*b^2*e*f^3 + a^6*b*f^4) + 3*(a*b
^5*c*e^3 - 4*a^2*b^4*c*e^2*f + a^3*b^3*d*e^2*f + 5*a^3*b^3*c*e*f^2 - 2*a^4
*b^2*d*e*f^2 - 2*a^4*b^2*c*f^3 + a^5*b*d*f^3)/(a^2*b^5*e^4 - 4*a^3*b^4*e^3
*f + 6*a^4*b^3*e^2*f^2 - 4*a^5*b^2*e*f^3 + a^6*b*f^4))*x/(b*x^2 + a)^(3/2)
 + (b*d^2*e^2*f - 2*b*c*d*e*f^2 + b*c^2*f^3)*arctan(1/2*((sqrt(b)*x - sqrt
(b*x^2 + a))^2*sqrt(b)*d*e*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*sqrt(b)*c*f
^2 + 2*b^(3/2)*d*e^2 - 2*b^(3/2)*c*e*f - a*sqrt(b)*d*e*f + a*sqrt(b)*c*f^2
)/(sqrt(-b*e^2 + a*e*f)*b*d*e - sqrt(-b*e^2 + a*e*f)*b*c*f))/((b^2*e^2 - 2
*a*b*e*f + a^2*f^2)*(b*d*e - b*c*f)*sqrt(-b*e^2 + a*e*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{d\,x^2+c}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1425, normalized size of antiderivative = 9.38

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e),x)
```

output

```
( - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c*f**2 + 3*sqrt(e)*sqr
t(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sq
rt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*d*e*f - 6*sqrt(e)*sqrt(a*f - b*e)*atan(
(sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*
sqrt(b)))*a**3*b**2*c*f**2*x**2 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**3*b**2*d*e*f*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3
*c*f**2*x**4 + 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*s
qrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**3*d*e*f*x*
*4 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*
x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*c*f**2 + 3*sqrt(e)*sq
rt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*s
qrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b*d*e*f - 6*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)
*sqrt(b)))*a**3*b**2*c*f**2*x**2 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) + sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**3*b**2*d*e*f*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) +
 sqrt(f)*sqrt(a + b*x**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*...
```

3.362 $\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 243

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^2} dx = \frac{b(2bce-5ade+3acf)x}{6ae(be-af)^2(a+bx^2)^{3/2}} + \frac{b(4b^2ce^2+2abe(de-8cf)+a^2f(13de-3cf))x}{6a^2e(be-af)^3\sqrt{a+bx^2}} + \frac{(de-cf)x}{2e(be-af)(a+bx^2)^{3/2}(e+fx^2)} - \frac{f(2be(2de-3cf)+af(de+cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{7/2}}$$

output
```
1/6*b*(3*a*c*f-5*a*d*e+2*b*c*e)*x/a/e/(-a*f+b*e)^2/(b*x^2+a)^(3/2)+1/6*b*(
4*b^2*c*e^2+2*a*b*e*(-8*c*f+d*e)+a^2*f*(-3*c*f+13*d*e))*x/a^2/e/(-a*f+b*e)
^3/(b*x^2+a)^(1/2)+1/2*(-c*f+d*e)*x/e/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e)
-1/2*f*(2*b*e*(-3*c*f+2*d*e)+a*f*(c*f+d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(
1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 15.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{1}{6}\left(x\sqrt{a+bx^2}\left(\frac{2b(bc-ad)}{a(be-af)^2(a+bx^2)^2}-\frac{2b(2b^2ce+5a^2df+ab(de-8cf)}{a^2(-be+af)^3(a+bx^2)}\right.\right.$$
$$\left.+\frac{3f(2be(2de-3cf)+af(de+cf))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{3/2}(-be+af)^{7/2}}\right)$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*((2*b*(b*c - a*d))/(a*(b*e - a*f)^2*(a + b*x^2)^2) - (2
*b*(2*b^2*c*e + 5*a^2*d*f + a*b*(d*e - 8*c*f)))/(a^2*(-(b*e) + a*f)^3*(a +
 b*x^2)) + (3*f^2*(d*e - c*f))/(e*(b*e - a*f)^3*(e + f*x^2))) + (3*f*(2*b*
e*(2*d*e - 3*c*f) + a*f*(d*e + c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e
]*Sqrt[a + b*x^2])])/(e^(3/2)*(-(b*e) + a*f)^(7/2)))/6
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {402, 25, 402, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{3a(be-af)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{x(4a^2df+ab(de-7cf)+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}-\frac{\int -\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(4a^2df+ab(de-7cf)+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{f\int \frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x(4a^2df+ab(de-7cf)+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{f\left(\frac{\int -\frac{3a^2(2be(2de-3cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{x(4a^2df+ab(de-7cf)+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$\Big\downarrow$ 291

$$\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + (((2*b^2
*c*e + 4*a^2*d*f + a*b*(d*e - 7*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e
 + f*x^2)) + (f*(((4*b^2*c*e^2 + 2*a*b*e*(d*e - 8*c*f) + a^2*f*(13*d*e - 3
*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(2*b*e*(2
*d*e - 3*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt
[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(a*(b*e - a*f)))/(3*a*(b*e
- a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-a^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(\left(c\,f^{2}+def\right)a-6bcef+4bd\,e^{2}\right)\left(f\,x^{2}+e\right)f\arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{\left(af-be\right)e}}\right)+\left(\left(c\,f^{3}-e\,f^{2}d\right)a^{4}+2bf\left(c\,f^{2}x^{2}-3def\,x^{2}\right.\right.}{2\sqrt{\left(af-be\right)e}}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a^2*(b*x^2+a)^(3/2)*((c*f^2+d*e*f)*a-6*b*c*e*f+4*b*d*e^2)*(f*x^2+e)*
f*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((c*f^3-d*e*f^2)*a^4+2*b
*f*(c*f^2*x^2-3*d*e*f*x^2-2*d*e^2)*a^3+6*((-5/9*x^2*d+c)*e^2+f*x^2*(-13/18
*x^2*d+c)*e+1/6*c*f^2*x^4)*b^2*f*a^2-2*(e*(1/3*x^2*d+c)-8/3*c*f*x^2)*b^3*(
f*x^2+e)*e*a-4/3*b^4*c*e^2*x^2*(f*x^2+e))*((a*f-b*e)*e)^(1/2)*x)/((a*f-b*e
)*e)^(1/2)/(b*x^2+a)^(3/2)/e/(f*x^2+e)/(a*f-b*e)^3/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(219) = 438$.

Time = 7.25 (sec) , antiderivative size = 1914, normalized size of antiderivative = 7.88

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*(4*a^4*b*d*e^3*f + a^5*c*e*f^3 + (4*a^2*b^3*d*e^2*f^2 + a^3*b^2*
c*f^4 - (6*a^2*b^3*c - a^3*b^2*d)*e*f^3)*x^6 - (6*a^4*b*c - a^5*d)*e^2*f^2
 + (4*a^2*b^3*d*e^3*f + 2*a^4*b*c*f^4 - 3*(2*a^2*b^3*c - 3*a^3*b^2*d)*e^2*
f^2 - (11*a^3*b^2*c - 2*a^4*b*d)*e*f^3)*x^4 + (8*a^3*b^2*d*e^3*f + a^5*c*f
^4 - 6*(2*a^3*b^2*c - a^4*b*d)*e^2*f^2 - (4*a^4*b*c - a^5*d)*e*f^3)*x^2)*s
qrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 +
2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 -
 a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((3*a^3*b^2*c*e*
f^4 + 2*(2*b^5*c + a*b^4*d)*e^4*f - (20*a*b^4*c - 11*a^2*b^3*d)*e^3*f^2 +
13*(a^2*b^3*c - a^3*b^2*d)*e^2*f^3)*x^5 + 2*(3*a^4*b*c*e*f^4 + (2*b^5*c +
a*b^4*d)*e^5 - (7*a*b^4*c - 4*a^2*b^3*d)*e^4*f - 4*(a^2*b^3*c - a^3*b^2*d)
*e^3*f^2 + 3*(2*a^3*b^2*c - 3*a^4*b*d)*e^2*f^3)*x^3 + 3*(2*a*b^4*c*e^5 + a
^5*c*e*f^4 - 4*(2*a^2*b^3*c - a^3*b^2*d)*e^4*f + 3*(2*a^3*b^2*c - a^4*b*d)
*e^3*f^2 - (a^4*b*c + a^5*d)*e^2*f^3)*x)*sqrt(b*x^2 + a))/(a^4*b^4*e^7 - 4
*a^5*b^3*e^6*f + 6*a^6*b^2*e^5*f^2 - 4*a^7*b*e^4*f^3 + a^8*e^3*f^4 + (a^2*
b^6*e^6*f - 4*a^3*b^5*e^5*f^2 + 6*a^4*b^4*e^4*f^3 - 4*a^5*b^3*e^3*f^4 + a^
6*b^2*e^2*f^5)*x^6 + (a^2*b^6*e^7 - 2*a^3*b^5*e^6*f - 2*a^4*b^4*e^5*f^2 +
8*a^5*b^3*e^4*f^3 - 7*a^6*b^2*e^3*f^4 + 2*a^7*b*e^2*f^5)*x^4 + (2*a^3*b^5*
e^7 - 7*a^4*b^4*e^6*f + 8*a^5*b^3*e^5*f^2 - 2*a^6*b^2*e^4*f^3 - 2*a^7*b*e^
3*f^4 + a^8*e^2*f^5)*x^2), 1/12*(3*(4*a^4*b*d*e^3*f + a^5*c*e*f^3 + (4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{2}}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(219) = 438$.

Time = 0.42 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.45

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3*((2*b^8*c*e^4 + a*b^7*d*e^4 - 14*a*b^7*c*e^3*f + 2*a^2*b^6*d*e^3*f + 3
0*a^2*b^6*c*e^2*f^2 - 12*a^3*b^5*d*e^2*f^2 - 26*a^3*b^5*c*e*f^3 + 14*a^4*b
^4*d*e*f^3 + 8*a^4*b^4*c*f^4 - 5*a^5*b^3*d*f^4)*x^2/(a^2*b^7*e^6 - 6*a^3*b
^6*e^5*f + 15*a^4*b^5*e^4*f^2 - 20*a^5*b^4*e^3*f^3 + 15*a^6*b^3*e^2*f^4 -
6*a^7*b^2*e*f^5 + a^8*b*f^6) + 3*(a*b^7*c*e^4 - 6*a^2*b^6*c*e^3*f + 2*a^3*
b^5*d*e^3*f + 12*a^3*b^5*c*e^2*f^2 - 6*a^4*b^4*d*e^2*f^2 - 10*a^4*b^4*c*e*
f^3 + 6*a^5*b^3*d*e*f^3 + 3*a^5*b^3*c*f^4 - 2*a^6*b^2*d*f^4)/(a^2*b^7*e^6
- 6*a^3*b^6*e^5*f + 15*a^4*b^5*e^4*f^2 - 20*a^5*b^4*e^3*f^3 + 15*a^6*b^3*e
^2*f^4 - 6*a^7*b^2*e*f^5 + a^8*b*f^6))*x/(b*x^2 + a)^(3/2) + 1/2*(4*b^(3/2
)*d*e^2*f - 6*b^(3/2)*c*e*f^2 + a*sqrt(b)*d*e*f^2 + a*sqrt(b)*c*f^3)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*
b*e*f))/((b^3*e^4 - 3*a*b^2*e^3*f + 3*a^2*b*e^2*f^2 - a^3*e*f^3)*sqrt(-b^2
*e^2 + a*b*e*f)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*d*e^2*f - 2*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*e*f^2 - (sqrt(b)*x - sqrt(b*x^2
+ a))^2*a*sqrt(b)*d*e*f^2 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*f^
3 + a^2*sqrt(b)*d*e*f^2 - a^2*sqrt(b)*c*f^3)/((b^3*e^4 - 3*a*b^2*e^3*f + 3
*a^2*b*e^2*f^2 - a^3*e*f^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*f + 4*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*b*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*f + a^2
*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}\,(e+fx^2)^2}\,dx=\int \frac{d\,x^2+c}{{\left(b\,x^2+a\right)}^{5/2}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 5210, normalized size of antiderivative = 21.44

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x)
```

output

```
( - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*c*e*f**4 - 3*sqrt(e)*sqr
t(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sq
rt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*c*f**5*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a**6*d*e**2*f**3 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
- b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*
a**6*d*e*f**4*x**2 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqr
t(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b*c*e**
2*f**3 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
+ b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b*c*f**5*x**4 - 24*
sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) -
 sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b*d*e**3*f**2 - 30*sqrt(e)*sqr
t(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sq
rt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b*d*e**2*f**3*x**2 - 6*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**5*b*d*e*f**4*x**4 + 72*sqrt(e)*sqrt(a*f - b*e)*ata
n((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e
)*sqrt(b)))*a**4*b**2*c*e**3*f**2 + 84*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqr...
```

3.363 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 375

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^3}\,dx = \frac{b(8b^2ce^2-4abe(8de-9cf)-3a^2f(de+3cf))\,x}{24ae^2(be-af)^3\left(a+bx^2\right)^{3/2}} + \frac{b(16b^3ce^3+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+3a^3f^2(de+3cf))\,x}{24a^2e^2(be-af)^4\sqrt{a+bx^2}} + \frac{(de-cf)x}{4e(be-af)\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2} + \frac{(2be(3de-5cf)+af(de+3cf))x}{8e^2(be-af)^2\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)} - \frac{f(24b^2e^2(de-2cf)-a^2f^2(de+3cf)+4abef(3de+4cf))\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{8e^{5/2}(be-af)^{9/2}}$$

output
```
1/24*b*(8*b^2*c*e^2-4*a*b*e*(-9*c*f+8*d*e)-3*a^2*f*(3*c*f+d*e))*x/a/e^2/(-
a*f+b*e)^3/(b*x^2+a)^(3/2)+1/24*b*(16*b^3*c*e^3+2*a^2*b*e*f*(-21*c*f+47*d*
e)+8*a*b^2*e^2*(-11*c*f+d*e)+3*a^3*f^2*(3*c*f+d*e))*x/a^2/e^2/(-a*f+b*e)^4
/(b*x^2+a)^(1/2)+1/4*(-c*f+d*e)*x/e/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e)^2
+1/8*(2*b*e*(-5*c*f+3*d*e)+a*f*(3*c*f+d*e))*x/e^2/(-a*f+b*e)^2/(b*x^2+a)^(
3/2)/(f*x^2+e)-1/8*f*(24*b^2*e^2*(-2*c*f+d*e)-a^2*f^2*(3*c*f+d*e)+4*a*b*e*
f*(4*c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(5/
2)/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 15.98 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{1}{24}\left(x\sqrt{a+bx^2}\left(\frac{8b^2(-bc+ad)}{a(-be+af)^3(a+bx^2)^2}+\frac{8b^2(2b^2ce+8a^2df+ab(de-1}{a^2(be-af)^4(a+bx^2)}\right.\right.$$
$$\left.+\frac{3f(-24b^2e^2(de-2cf)+a^2f^2(de+3cf)-4abef(3de+4cf))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{5/2}(-be+af)^{9/2}}\right)$$

input
```
Integrate[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((8*b^2*(-(b*c) + a*d))/(a*(-(b*e) + a*f)^3*(a + b*x^2)
^2) + (8*b^2*(2*b^2*c*e + 8*a^2*d*f + a*b*(d*e - 11*c*f)))/(a^2*(b*e - a*f
)^4*(a + b*x^2)) + (6*f^2*(d*e - c*f))/(e*(b*e - a*f)^3*(e + f*x^2)^2) + (
3*f^2*(2*b*e*(5*d*e - 7*c*f) + a*f*(d*e + 3*c*f)))/(e^2*(b*e - a*f)^4*(e +
 f*x^2))) + (3*f*(-24*b^2*e^2*(d*e - 2*c*f) + a^2*f^2*(d*e + 3*c*f) - 4*a*
b*e*f*(3*d*e + 4*c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x
^2])])/(e^(5/2)*(-(b*e) + a*f)^(9/2)))/24
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {402, 25, 402, 25, 27, 402, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}-\frac{\int -\frac{6(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{3a(be-af)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6(bc-ad)fx^2+2bce+ade-3acf}{(bx^2+a)^{3/2}(fx^2+e)^3}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}-\frac{\int -\frac{f\left(4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{f\left(4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{f\int \frac{4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{f\left(\frac{\int \frac{2b\left(f(31de-3cf)a^2+4be(de-10cf)a+8b^2ce^2\right)x^2+a\left(-3f(de+3cf)a^2-4be(8de-9cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-10cf)+8b^2ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}\right.}{a(be-af)}}{3a(be-af)}$$

$$\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$$\Big\downarrow \; 402$$

$$\frac{f\left(\frac{\frac{\int -\frac{3a^2\left(24b^2(de-2cf)e^2+4abf(3de+4cf)e-a^2f^2(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^3ce^3\right)}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}+\frac{x\sqrt{a+b}}{}\right)}{a(be-af)}}{3a(be-af)}$$

$$\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$\downarrow$ 27

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^3ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(-a^2f^2(3cf+de)+4abef(4cf+3de)+24b^2e^2(de-2cf)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2\right.}}{2e(be-af)}}{4e(be-af)}\right)}{a(be-af)}}{3a(be-af)}$$

$$\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$\downarrow$ 291

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^3ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(-a^2f^2(3cf+de)+4abef(4cf+3de)+24b^2e^2(de-2cf)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}}{2e(be-af)}}{4e(be-af)}\right)}{a(be-af)}}{3a(be-af)}$$

$$\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

$\downarrow$ 221

$$\frac{\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-10cf)+8b^2ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-\right.}{2e\left(e+fx^2\right)(be-af)}}{}\right)}{a(be-af)}}{3a(be-af)}$$

$$\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}$$

input
```
Int[(c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)^2) + (((2*b
^2*c*e + 6*a^2*d*f + a*b*(d*e - 9*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*
(e + f*x^2)^2) + (f*(((8*b^2*c*e^2 + 4*a*b*e*(d*e - 10*c*f) + a^2*f*(31*d*
e - 3*c*f))*x*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((16*b^3
*c*e^3 + 2*a^2*b*e*f*(47*d*e - 21*c*f) + 8*a*b^2*e^2*(d*e - 11*c*f) + 3*a^
3*f^2*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - (3
*a^2*(24*b^2*e^2*(d*e - 2*c*f) - a^2*f^2*(d*e + 3*c*f) + 4*a*b*e*f*(3*d*e
+ 4*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/
2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/(a*(b*e - a*f)))/(3*a*(b*e - a*
f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{\frac{3a^2(bx^2+a)^{\frac{3}{2}}\left(f^2\left(cf+\frac{de}{3}\right)a^2-\frac{16bfe\left(cf+\frac{3de}{4}\right)a}{3}+16b^2ce^2f-8b^2de^3\right)(fx^2+e)^2 f\arctan\left(\frac{e\sqrt{bx^2+a}}{x\sqrt{(af-be)e}}\right)}{8}+\frac{5\left(\left(\frac{3cf^2x^2}{5}+e\left(\frac{x^2}{5}\right.\right.\right.}{}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(3/2)*(-3/5*a^2*(b*x^2+a)^(3/2)*(f^2*(c*
f+1/3*d*e)*a^2-16/3*b*f*e*(c*f+3/4*d*e)*a+16*b^2*c*e^2*f-8*b^2*d*e^3)*(f*x
^2+e)^2*f*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((3/5*c*f^2*x^2+
e*(1/5*x^2*d+c)*f-1/5*d*e^2)*f^3*a^5-16/5*(-3/8*c*f^3*x^4+1/4*(-1/2*x^2*d+
c)*x^2*e*f^2+e^2*(-1/2*x^2*d+c)*f-3/4*d*e^3)*b*f^2*a^4-32/5*(-3/32*c*f^4*x
^6+23/32*x^4*e*(-1/23*x^2*d+c)*f^3+e^2*x^2*(-43/32*x^2*d+c)*f^2-9/4*d*e^3*
f*x^2-3/4*d*e^4)*b^2*f*a^3-32/5*b^3*(7/16*c*f^3*x^6+3/2*(-47/72*x^2*d+c)*x
^4*e*f^2+2*(-41/48*x^2*d+c)*x^2*e^2*f+e^3*(-2/3*x^2*d+c))*f*e*a^2+8/5*(-11
/3*c*f*x^2+e*(1/3*x^2*d+c))*b^4*(f*x^2+e)^2*e^2*a+16/15*b^5*c*e^3*x^2*(f*x
^2+e)^2)*((a*f-b*e)*e)^(1/2)*x)/(f*x^2+e)^2/e^2/(a*f-b*e)^4/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(347) = 694.

Time = 31.98 (sec) , antiderivative size = 3168, normalized size of antiderivative = 8.45

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to display
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(347) = 694$.

Time = 0.44 (sec) , antiderivative size = 1567, normalized size of antiderivative = 4.18

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^10*c*e^5 + a*b^9*d*e^5 - 19*a*b^9*c*e^4*f + 4*a^2*b^8*d*e^4*f +
56*a^2*b^8*c*e^3*f^2 - 26*a^3*b^7*d*e^3*f^2 - 74*a^3*b^7*c*e^2*f^3 + 44*a^
4*b^6*d*e^2*f^3 + 46*a^4*b^6*c*e*f^4 - 31*a^5*b^5*d*e*f^4 - 11*a^5*b^5*c*f
^5 + 8*a^6*b^4*d*f^5)*x^2/(a^2*b^9*e^8 - 8*a^3*b^8*e^7*f + 28*a^4*b^7*e^6*
f^2 - 56*a^5*b^6*e^5*f^3 + 70*a^6*b^5*e^4*f^4 - 56*a^7*b^4*e^3*f^5 + 28*a^
8*b^3*e^2*f^6 - 8*a^9*b^2*e*f^7 + a^10*b*f^8) + 3*(a*b^9*c*e^5 - 8*a^2*b^8
*c*e^4*f + 3*a^3*b^7*d*e^4*f + 22*a^3*b^7*c*e^3*f^2 - 12*a^4*b^6*d*e^3*f^2
 - 28*a^4*b^6*c*e^2*f^3 + 18*a^5*b^5*d*e^2*f^3 + 17*a^5*b^5*c*e*f^4 - 12*a
^6*b^4*d*e*f^4 - 4*a^6*b^4*c*f^5 + 3*a^7*b^3*d*f^5)/(a^2*b^9*e^8 - 8*a^3*b
^8*e^7*f + 28*a^4*b^7*e^6*f^2 - 56*a^5*b^6*e^5*f^3 + 70*a^6*b^5*e^4*f^4 -
56*a^7*b^4*e^3*f^5 + 28*a^8*b^3*e^2*f^6 - 8*a^9*b^2*e*f^7 + a^10*b*f^8))*x
/(b*x^2 + a)^(3/2) + 1/8*(24*b^(5/2)*d*e^3*f - 48*b^(5/2)*c*e^2*f^2 + 12*a
*b^(3/2)*d*e^2*f^2 + 16*a*b^(3/2)*c*e*f^3 - a^2*sqrt(b)*d*e*f^3 - 3*a^2*sq
rt(b)*c*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/
sqrt(-b^2*e^2 + a*b*e*f))/((b^4*e^6 - 4*a*b^3*e^5*f + 6*a^2*b^2*e^4*f^2 -
4*a^3*b*e^3*f^3 + a^4*e^2*f^4)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/4*(16*(sqrt(b
)*x - sqrt(b*x^2 + a))^6*b^(5/2)*d*e^3*f^2 - 24*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*b^(5/2)*c*e^2*f^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*d*e^
2*f^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*e*f^4 - (sqrt(b)*x
- sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d*e*f^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx=\int \frac{d\,x^2+c}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)/((a + b*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 9199, normalized size of antiderivative = 24.53

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x^2+c)/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x)
```

output

```
( - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*c*e**2*f**5 - 18*sqrt(e)
*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f
)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*c*e*f**6*x**2 - 9*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**7*c*f**7*x**4 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**7*d*e**3*f**4 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
 sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*d*e
**2*f**5*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*
sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*d*e*f**6*x**
4 + 120*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b
*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c*e**3*f**4 + 222*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - s
qrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c*e**2*f**5*x**2 + 84*sqrt(e)*
sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)
*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c*e*f**6*x**4 - 18*sqrt(e)*sqrt(a*f
- b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*
x)/(sqrt(e)*sqrt(b)))*a**6*b*c*f**7*x**6 + 60*sqrt(e)*sqrt(a*f - b*e)*atan
((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt...
```

3.364 $\int \frac{(c+dx^2)^2(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 368

$$\int \frac{(c+dx^2)^2(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(be-af)^3x}{3ab^5(a+bx^2)^{3/2}}$$
$$+\frac{(bc-ad)(be-af)^2(2b^2ce-13a^2df+ab(4de+7cf))x}{3a^2b^5\sqrt{a+bx^2}}$$
$$+\frac{f(41a^2d^2f^2-22abdf(3de+2cf)+8b^2(3d^2e^2+6cdef+c^2f^2))x\sqrt{a+bx^2}}{16b^5}$$
$$+\frac{df^2(18bde+12bcf-17adf)x^3\sqrt{a+bx^2}}{24b^4}+\frac{d^2f^3x^5\sqrt{a+bx^2}}{6b^3}$$
$$-\frac{(105a^3d^2f^3-70a^2bdf^2(3de+2cf)+40ab^2f(3d^2e^2+6cdef+c^2f^2)-16b^3e(d^2e^2+6cdef+3c^2f^2))\,\mathrm{arc}}{16b^{11/2}}$$

output
```
1/3*(-a*d+b*c)^2*(-a*f+b*e)^3*x/a/b^5/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)*(-a*f
+b*e)^2*(2*b^2*c*e-13*a^2*d*f+a*b*(7*c*f+4*d*e))*x/a^2/b^5/(b*x^2+a)^(1/2)
+1/16*f*(41*a^2*d^2*f^2-22*a*b*d*f*(2*c*f+3*d*e)+8*b^2*(c^2*f^2+6*c*d*e*f+
3*d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/24*d*f^2*(-17*a*d*f+12*b*c*f+18*b*d*e)
*x^3*(b*x^2+a)^(1/2)/b^4+1/6*d^2*f^3*x^5*(b*x^2+a)^(1/2)/b^3-1/16*(105*a^3
*d^2*f^3-70*a^2*b*d*f^2*(2*c*f+3*d*e)+40*a*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*
e^2)-16*b^3*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(
1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\frac{\sqrt{b}x(315a^6d^2f^3+32b^6c^2e^3x^2+210a^5bdf^2(-3de-2cf+2dfx^2)+16ab^5ce^2(2dex^2+3c(e+fx^2))+a^4b^2f(1}{}}{}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
((Sqrt[b]*x*(315*a^6*d^2*f^3 + 32*b^6*c^2*e^3*x^2 + 210*a^5*b*d*f^2*(-3*d*
e - 2*c*f + 2*d*f*x^2) + 16*a*b^5*c*e^2*(2*d*e*x^2 + 3*c*(e + f*x^2)) + a^
4*b^2*f*(120*c^2*f^2 + 80*c*d*f*(9*e - 7*f*x^2) + d^2*(360*e^2 - 840*e*f*x
^2 + 63*f^2*x^4)) + 4*a^2*b^4*x^2*(6*c^2*f^2*(-8*e + f*x^2) + 6*c*d*f*(-16
*e^2 + 6*e*f*x^2 + f^2*x^4) + d^2*(-16*e^3 + 18*e^2*f*x^2 + 9*e*f^2*x^4 +
2*f^3*x^6)) - 2*a^3*b^3*(8*c^2*f^2*(9*e - 10*f*x^2) + 6*c*d*f*(24*e^2 - 80
*e*f*x^2 + 7*f^2*x^4) + 3*d^2*(8*e^3 - 80*e^2*f*x^2 + 21*e*f^2*x^4 + 3*f^3
*x^6))))/(a^2*(a + b*x^2)^(3/2)) - 3*(-105*a^3*d^2*f^3 + 70*a^2*b*d*f^2*(3
*d*e + 2*c*f) - 40*a*b^2*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + 16*b^3*e*(d
^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*
b^(11/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 750 vs. 2(368) = 736.

Time = 0.83 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{fx^6(c^2f^2+6cdef+3d^2e^2)}{(a+bx^2)^{5/2}}+\frac{ex^4(3c^2f^2+6cdef+d^2e^2)}{(a+bx^2)^{5/2}}+\frac{c^2e^3}{(a+bx^2)^{5/2}}+\frac{ce^2x^2(3cf+2de)}{(a+bx^2)^{5/2}}+\frac{df^2x^8(2c}{(a+bx}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{105a^3d^2f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{11/2}}+\frac{35a^2df^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cf+3de)}{8b^{9/2}}+\frac{105a^2d^2f^3x\sqrt{a+bx^2}}{16b^5}+\frac{2c^2e^3x}{3a^2\sqrt{a+bx^2}}-\frac{5af\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(c^2f^2+6cdef+3d^2e^2)}{2b^{7/2}}+\frac{e\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3c^2f^2+6cdef+d^2e^2)}{b^{5/2}}-\frac{35adf^2x\sqrt{a+bx^2}(2cf+3de)}{8b^4}-\frac{35ad^2f^3x^3\sqrt{a+bx^2}}{8b^4}+\frac{5fx\sqrt{a+bx^2}(c^2f^2+6cdef+3d^2e^2)}{2b^3}+\frac{35df^2x^3\sqrt{a+bx^2}(2cf+3de)}{12b^3}+\frac{7d^2f^3x^5\sqrt{a+bx^2}}{2b^3}-\frac{ex(3c^2f^2+6cdef+d^2e^2)}{b^2\sqrt{a+bx^2}}-\frac{5fx^3(c^2f^2+6cdef+3d^2e^2)}{3b^2\sqrt{a+bx^2}}-\frac{7df^2x^5(2cf+3de)}{3b^2\sqrt{a+bx^2}}-\frac{3d^2f^3x^7}{b^2\sqrt{a+bx^2}}-\frac{fx^5(c^2f^2+6cdef+3d^2e^2)}{3b(a+bx^2)^{3/2}}-\frac{ex^3(3c^2f^2+6cdef+d^2e^2)}{3b(a+bx^2)^{3/2}}+\frac{c^2e^3x}{3a(a+bx^2)^{3/2}}+\frac{ce^2x^3(3cf+2de)}{3a(a+bx^2)^{3/2}}-\frac{df^2x^7(2cf+3de)}{3b(a+bx^2)^{3/2}}-\frac{d^2f^3x^9}{3b(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
(c^2*e^3*x)/(3*a*(a + b*x^2)^(3/2)) + (c*e^2*(2*d*e + 3*c*f)*x^3)/(3*a*(a
+ b*x^2)^(3/2)) - (e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^3)/(3*b*(a + b*x^
2)^(3/2)) - (f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^5)/(3*b*(a + b*x^2)^(3/
2)) - (d*f^2*(3*d*e + 2*c*f)*x^7)/(3*b*(a + b*x^2)^(3/2)) - (d^2*f^3*x^9)/
(3*b*(a + b*x^2)^(3/2)) + (2*c^2*e^3*x)/(3*a^2*Sqrt[a + b*x^2]) - (e*(d^2*
e^2 + 6*c*d*e*f + 3*c^2*f^2)*x)/(b^2*Sqrt[a + b*x^2]) - (5*f*(3*d^2*e^2 +
6*c*d*e*f + c^2*f^2)*x^3)/(3*b^2*Sqrt[a + b*x^2]) - (7*d*f^2*(3*d*e + 2*c*
f)*x^5)/(3*b^2*Sqrt[a + b*x^2]) - (3*d^2*f^3*x^7)/(b^2*Sqrt[a + b*x^2]) +
(105*a^2*d^2*f^3*x*Sqrt[a + b*x^2])/(16*b^5) - (35*a*d*f^2*(3*d*e + 2*c*f)
*x*Sqrt[a + b*x^2])/(8*b^4) + (5*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x*Sqr
t[a + b*x^2])/(2*b^3) - (35*a*d^2*f^3*x^3*Sqrt[a + b*x^2])/(8*b^4) + (35*d
*f^2*(3*d*e + 2*c*f)*x^3*Sqrt[a + b*x^2])/(12*b^3) + (7*d^2*f^3*x^5*Sqrt[a
 + b*x^2])/(2*b^3) - (105*a^3*d^2*f^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(16*b^(11/2)) + (35*a^2*d*f^2*(3*d*e + 2*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(9/2)) - (5*a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2)) + (e*(d^2*e^2 + 6*c*d*e*f + 3*c
^2*f^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$-\frac{105\left(a^{3} d^{2} f^{3}-\frac{4\left(c f+\frac{3 d e}{2}\right) d b f^{2} a^{2}}{3}+\frac{8 a b^{2} f\left(c^{2} f^{2}+6 c d e f+3 d^{2} e^{2}\right)}{21}-\frac{16\left(c^{2} f^{2}+2 c d e f+\frac{1}{3} d^{2} e^{2}\right) b^{3} e}{35}\right) a^{2}\left(b x^{2}+a\right)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x^{2}+c}}{x \sqrt{b}}\right.}{16}$
default	$c^{2} e^{3}\left(\frac{x}{3 a(b x^{2}+a)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b x^{2}+a}}\right)+f^{2} d(2 c f+3 d e)\left(\frac{x^{7}}{4 b(b x^{2}+a)^{\frac{3}{2}}}-\frac{7 a\left(\frac{x^{5}}{2 b\left(b x^{2}+a\right)^{\frac{3}{2}}}-\frac{5 a\left(-\frac{x^{3}}{3 b\left(b x^{2}+\right.}\right.}{}\right.}{}\right.$
risch	Expression too large to display

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)/b^(11/2)*(-315/32*(a^3*d^2*f^3-4/3*(c*f+3/2*d*e)*d*b*f
^2*a^2+8/21*a*b^2*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^2)-16/35*(c^2*f^2+2*c*d*e*f
+1/3*d^2*e^2)*b^3*e)*a^2*(b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)
)+(-9/2*a^3*((1/8*d^2*x^6+7/12*c*d*x^4-10/9*c^2*x^2)*f^3+e*(7/8*d^2*x^4-20
/3*c*d*x^2+c^2)*f^2+2*d*(-5/3*x^2*d+c)*e^2*f+1/3*d^2*e^3)*b^(7/2)+15/4*((2
1/40*d^2*x^4-14/3*c*d*x^2+c^2)*f^2+6*(-7/6*x^2*d+c)*d*e*f+3*d^2*e^2)*f*a^4
*b^(5/2)-105/8*d*((-d*x^2+c)*f+3/2*d*e)*f^2*a^5*b^(3/2)+315/32*a^6*d^2*f^3
*b^(1/2)+(-6*(-1/8*(1/3*d^2*x^4+c*d*x^2+c^2)*x^2*f^3+e*(-3/16*d^2*x^4-3/4*
c*d*x^2+c^2)*f^2+2*(-3/16*x^2*d+c)*d*e^2*f+1/3*d^2*e^3)*x^2*a^2+3/2*c*b*(c
*f*x^2+e*(2/3*x^2*d+c))*e^2*a+b^2*c^2*e^3*x^2)*b^(9/2))*x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(340) = 680$.

Time = 0.94 (sec) , antiderivative size = 1732, normalized size of antiderivative = 4.71

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*a^4*b^3*d^2*e^3 + (16*a^2*b^5*d^2*e^3 + 24*(4*a^2*b^5*c*d -
5*a^3*b^4*d^2)*e^2*f + 6*(8*a^2*b^5*c^2 - 40*a^3*b^4*c*d + 35*a^4*b^3*d^2)
*e*f^2 - 5*(8*a^3*b^4*c^2 - 28*a^4*b^3*c*d + 21*a^5*b^2*d^2)*f^3)*x^4 + 24
*(4*a^4*b^3*c*d - 5*a^5*b^2*d^2)*e^2*f + 6*(8*a^4*b^3*c^2 - 40*a^5*b^2*c*d
 + 35*a^6*b*d^2)*e*f^2 - 5*(8*a^5*b^2*c^2 - 28*a^6*b*c*d + 21*a^7*d^2)*f^3
 + 2*(16*a^3*b^4*d^2*e^3 + 24*(4*a^3*b^4*c*d - 5*a^4*b^3*d^2)*e^2*f + 6*(8
*a^3*b^4*c^2 - 40*a^4*b^3*c*d + 35*a^5*b^2*d^2)*e*f^2 - 5*(8*a^4*b^3*c^2 -
 28*a^5*b^2*c*d + 21*a^6*b*d^2)*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(8*a^2*b^5*d^2*f^3*x^9 + 6*(6*a^2*b^5*d^2*e*f^
2 + (4*a^2*b^5*c*d - 3*a^3*b^4*d^2)*f^3)*x^7 + 3*(24*a^2*b^5*d^2*e^2*f + 6
*(8*a^2*b^5*c*d - 7*a^3*b^4*d^2)*e*f^2 + (8*a^2*b^5*c^2 - 28*a^3*b^4*c*d +
 21*a^4*b^3*d^2)*f^3)*x^5 + 4*(8*(b^7*c^2 + a*b^6*c*d - 2*a^2*b^5*d^2)*e^3
 + 12*(a*b^6*c^2 - 8*a^2*b^5*c*d + 10*a^3*b^4*d^2)*e^2*f - 6*(8*a^2*b^5*c^
2 - 40*a^3*b^4*c*d + 35*a^4*b^3*d^2)*e*f^2 + 5*(8*a^3*b^4*c^2 - 28*a^4*b^3
*c*d + 21*a^5*b^2*d^2)*f^3)*x^3 + 3*(16*(a*b^6*c^2 - a^3*b^4*d^2)*e^3 - 24
*(4*a^3*b^4*c*d - 5*a^4*b^3*d^2)*e^2*f - 6*(8*a^3*b^4*c^2 - 40*a^4*b^3*c*d
 + 35*a^5*b^2*d^2)*e*f^2 + 5*(8*a^4*b^3*c^2 - 28*a^5*b^2*c*d + 21*a^6*b*d^
2)*f^3)*x)*sqrt(b*x^2 + a))/(a^2*b^8*x^4 + 2*a^3*b^7*x^2 + a^4*b^6), -1/48
*(3*(16*a^4*b^3*d^2*e^3 + (16*a^2*b^5*d^2*e^3 + 24*(4*a^2*b^5*c*d - 5*a^3*
b^4*d^2)*e^2*f + 6*(8*a^2*b^5*c^2 - 40*a^3*b^4*c*d + 35*a^4*b^3*d^2)*e*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)**3/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(340) = 680$.

Time = 0.05 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*d^2*f^3*x^9/((b*x^2 + a)^(3/2)*b) - 3/8*a*d^2*f^3*x^7/((b*x^2 + a)^(3/
2)*b^2) + 21/16*a^2*d^2*f^3*x^5/((b*x^2 + a)^(3/2)*b^3) + 35/16*a^3*d^2*f^
3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 1/4*
(3*d^2*e*f^2 + 2*c*d*f^3)*x^7/((b*x^2 + a)^(3/2)*b) + 2/3*c^2*e^3*x/(sqrt(
b*x^2 + a)*a^2) + 1/3*c^2*e^3*x/((b*x^2 + a)^(3/2)*a) + 35/16*a^3*d^2*f^3*
x/(sqrt(b*x^2 + a)*b^5) - 7/8*(3*d^2*e*f^2 + 2*c*d*f^3)*a*x^5/((b*x^2 + a)
^(3/2)*b^2) + 1/2*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*x^5/((b*x^2 + a)^(
3/2)*b) - 105/16*a^3*d^2*f^3*arcsinh(b*x/sqrt(a*b))/b^(11/2) - 1/3*(d^2*e^
3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^
2 + a)^(3/2)*b^2)) - 35/24*(3*d^2*e*f^2 + 2*c*d*f^3)*a^2*x*(3*x^2/((b*x^2
+ a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 5/6*(3*d^2*e^2*f + 6*c*
d*e*f^2 + c^2*f^3)*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/
2)*b^2))/b - 35/24*(3*d^2*e*f^2 + 2*c*d*f^3)*a^2*x/(sqrt(b*x^2 + a)*b^4) +
 5/6*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3
*(d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(2*c*
d*e^3 + 3*c^2*e^2*f)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(2*c*d*e^3 + 3*c^2*e^2*
f)*x/(sqrt(b*x^2 + a)*a*b) + 35/8*(3*d^2*e*f^2 + 2*c*d*f^3)*a^2*arcsinh(b*
x/sqrt(a*b))/b^(9/2) - 5/2*(3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*a*arcsinh
(b*x/sqrt(a*b))/b^(7/2) + (d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*arcsinh(b*
x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{4d^2f^3x^2}{b}+\frac{3(6a^2b^8d^2ef^2+4a^2b^8cdf^3-3a^3b^7d^2f^3)}{a^2b^9}\right)x^2+\frac{3(24a^2b^8d^2e^2f+48a^2b^8cdef\cdots}{\cdots}\right.\right.\right.}{\cdots}$$
$$-\frac{(16b^3d^2e^3+96b^3cde^2f-120ab^2d^2e^2f+48b^3c^2ef^2-240ab^2cdef^2+210a^2bd^2ef^2-40ab^2c^2f^3+140\cdots}{16b^{\frac{11}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/48*(((2*(4*d^2*f^3*x^2/b + 3*(6*a^2*b^8*d^2*e*f^2 + 4*a^2*b^8*c*d*f^3 -
3*a^3*b^7*d^2*f^3)/(a^2*b^9))*x^2 + 3*(24*a^2*b^8*d^2*e^2*f + 48*a^2*b^8*c
*d*e*f^2 - 42*a^3*b^7*d^2*e*f^2 + 8*a^2*b^8*c^2*f^3 - 28*a^3*b^7*c*d*f^3 +
 21*a^4*b^6*d^2*f^3)/(a^2*b^9))*x^2 + 4*(8*b^10*c^2*e^3 + 8*a*b^9*c*d*e^3
- 16*a^2*b^8*d^2*e^3 + 12*a*b^9*c^2*e^2*f - 96*a^2*b^8*c*d*e^2*f + 120*a^3
*b^7*d^2*e^2*f - 48*a^2*b^8*c^2*e*f^2 + 240*a^3*b^7*c*d*e*f^2 - 210*a^4*b^
6*d^2*e*f^2 + 40*a^3*b^7*c^2*f^3 - 140*a^4*b^6*c*d*f^3 + 105*a^5*b^5*d^2*f
^3)/(a^2*b^9))*x^2 + 3*(16*a*b^9*c^2*e^3 - 16*a^3*b^7*d^2*e^3 - 96*a^3*b^7
*c*d*e^2*f + 120*a^4*b^6*d^2*e^2*f - 48*a^3*b^7*c^2*e*f^2 + 240*a^4*b^6*c*
d*e*f^2 - 210*a^5*b^5*d^2*e*f^2 + 40*a^4*b^6*c^2*f^3 - 140*a^5*b^5*c*d*f^3
 + 105*a^6*b^4*d^2*f^3)/(a^2*b^9))*x/(b*x^2 + a)^(3/2) - 1/16*(16*b^3*d^2*
e^3 + 96*b^3*c*d*e^2*f - 120*a*b^2*d^2*e^2*f + 48*b^3*c^2*e*f^2 - 240*a*b^
2*c*d*e*f^2 + 210*a^2*b*d^2*e*f^2 - 40*a*b^2*c^2*f^3 + 140*a^2*b*c*d*f^3 -
 105*a^3*d^2*f^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^3}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^3)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2 (f\,x^2+e)^3}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^3/(b*x^2+a)^(5/2),x)
```

3.365 $\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 249

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(be-af)^2x}{3ab^4(a+bx^2)^{3/2}} + \frac{2(bc-ad)(be-af)\left(b^2ce-5a^2df+2ab(de+cf)\right)x}{3a^2b^4\sqrt{a+bx^2}} - \frac{df(11adf-8b(de+cf))x\sqrt{a+bx^2}}{8b^4} + \frac{d^2f^2x^3\sqrt{a+bx^2}}{4b^3} + \frac{\left(35a^2d^2f^2-40abdf(de+cf)+8b^2(d^2e^2+4cdef+c^2f^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

output
```
1/3*(-a*d+b*c)^2*(-a*f+b*e)^2*x/a/b^4/(b*x^2+a)^(3/2)+2/3*(-a*d+b*c)*(-a*f
+b*e)*(b^2*c*e-5*a^2*d*f+2*a*b*(c*f+d*e))*x/a^2/b^4/(b*x^2+a)^(1/2)-1/8*d*
f*(11*a*d*f-8*b*(c*f+d*e))*x*(b*x^2+a)^(1/2)/b^4+1/4*d^2*f^2*x^3*(b*x^2+a)
^(1/2)/b^3+1/8*(35*a^2*d^2*f^2-40*a*b*d*f*(c*f+d*e)+8*b^2*(c^2*f^2+4*c*d*e
*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{x(-105a^5d^2f^2+16b^5c^2e^2x^2+8ab^4ce(3ce+2dex^2+2cfx^2)+20a^4bdf(6de}{} \\ - \frac{(35a^2d^2f^2-40abdf(de+cf)+8b^2(d^2e^2+4cdef+c^2f^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(-105*a^5*d^2*f^2 + 16*b^5*c^2*e^2*x^2 + 8*a*b^4*c*e*(3*c*e + 2*d*e*x^2
 + 2*c*f*x^2) + 20*a^4*b*d*f*(6*d*e + 6*c*f - 7*d*f*x^2) + 2*a^2*b^3*x^2*(
-16*c^2*f^2 + 4*c*d*f*(-16*e + 3*f*x^2) + d^2*(-16*e^2 + 12*e*f*x^2 + 3*f^
2*x^4)) - a^3*b^2*(24*c^2*f^2 + 32*c*d*f*(3*e - 5*f*x^2) + d^2*(24*e^2 - 1
60*e*f*x^2 + 21*f^2*x^4))))/(24*a^2*b^4*(a + b*x^2)^(3/2)) - ((35*a^2*d^2*
f^2 - 40*a*b*d*f*(d*e + c*f) + 8*b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 433}$$

$$\int \left(\frac{x^4(c^2f^2+4cdef+d^2e^2)}{(a+bx^2)^{5/2}}+\frac{c^2e^2}{(a+bx^2)^{5/2}}+\frac{2cex^2(cf+de)}{(a+bx^2)^{5/2}}+\frac{2dfx^6(cf+de)}{(a+bx^2)^{5/2}}+\frac{d^2f^2x^8}{(a+bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
\frac{35a^2d^2f^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{2c^2e^2x}{3a^2\sqrt{a+bx^2}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(c^2f^2+4cdef+d^2e^2\right)}{b^{5/2}}-\frac{5adf\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cf+de)}{b^{7/2}}-\frac{35ad^2f^2x\sqrt{a+bx^2}}{8b^4}+\frac{5dfx\sqrt{a+bx^2}(cf+de)}{b^3}+\frac{35d^2f^2x^3\sqrt{a+bx^2}}{12b^3}-\frac{x\left(c^2f^2+4cdef+d^2e^2\right)}{b^2\sqrt{a+bx^2}}-\frac{10dfx^3(cf+de)}{3b^2\sqrt{a+bx^2}}-\frac{7d^2f^2x^5}{3b^2\sqrt{a+bx^2}}-\frac{x^3\left(c^2f^2+4cdef+d^2e^2\right)}{3b\left(a+bx^2\right)^{3/2}}+\frac{c^2e^2x}{3a\left(a+bx^2\right)^{3/2}}-\frac{2dfx^5(cf+de)}{3b\left(a+bx^2\right)^{3/2}}+\frac{2cex^3(cf+de)}{3a\left(a+bx^2\right)^{3/2}}-\frac{d^2f^2x^7}{3b\left(a+bx^2\right)^{3/2}}
$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(c^2*e^2*x)/(3*a*(a + b*x^2)^(3/2)) + (2*c*e*(d*e + c*f)*x^3)/(3*a*(a + b*
x^2)^(3/2)) - ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^3)/(3*b*(a + b*x^2)^(3/2)
) - (2*d*f*(d*e + c*f)*x^5)/(3*b*(a + b*x^2)^(3/2)) - (d^2*f^2*x^7)/(3*b*(
a + b*x^2)^(3/2)) + (2*c^2*e^2*x)/(3*a^2*Sqrt[a + b*x^2]) - ((d^2*e^2 + 4*
c*d*e*f + c^2*f^2)*x)/(b^2*Sqrt[a + b*x^2]) - (10*d*f*(d*e + c*f)*x^3)/(3*
b^2*Sqrt[a + b*x^2]) - (7*d^2*f^2*x^5)/(3*b^2*Sqrt[a + b*x^2]) - (35*a*d^2
*f^2*x*Sqrt[a + b*x^2])/(8*b^4) + (5*d*f*(d*e + c*f)*x*Sqrt[a + b*x^2])/b^
3 + (35*d^2*f^2*x^3*Sqrt[a + b*x^2])/(12*b^3) + (35*a^2*d^2*f^2*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2)) - (5*a*d*f*(d*e + c*f)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/b^(7/2) + ((d^2*e^2 + 4*c*d*e*f + c^2*f^2)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{\frac{35a^2\left(b\,x^2+a\right)^{\frac{3}{2}}\left(a^2d^2f^2-\frac{8abdf(cf+de)}{7}+\frac{8b^2\left(c^2f^2+4cdef+d^2e^2\right)}{35}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}+2\left(-\frac{3a^3\left(\left(\frac{7}{8}f^2x^4-\frac{20}{3}ef\,x^2+e^2\right)d^2+4\cdots\right.}{2}\cdots\right.}{\cdots}$
default	$c^2e^2\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+2df(cf+de)\left(\frac{x^5}{2b(b\,x^2+a)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln(\cdots}{\cdots}}{b}\cdots\right.}{2b}\cdots\right.$
risch	Expression too large to display

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)*(105/16*a^2*(b*x^2+a)^(3/2)*(a^2*d^2*f^2-8/7*a*b*d*f*(
c*f+d*e)+8/35*b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*arctanh((b*x^2+a)^(1/2)/x/b
^(1/2))+(-3/2*a^3*((7/8*f^2*x^4-20/3*e*f*x^2+e^2)*d^2+4*c*(-5/3*f*x^2+e)*f
*d+c^2*f^2)*b^(5/2)-2*a^2*x^2*((-3/16*f^2*x^4-3/4*e*f*x^2+e^2)*d^2+4*c*(-3
/16*f*x^2+e)*f*d+c^2*f^2)*b^(7/2)+15/2*a^4*d*((-7/6*f*x^2+e)*d+c*f)*f*b^(3
/2)-105/16*a^5*d^2*f^2*b^(1/2)+((d*e*x^2+3/2*c*(2/3*f*x^2+e))*a+b*c*e*x^2)
*c*b^(9/2)*e)*x)/b^(9/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(225) = 450.

Time = 0.39 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.58

$$\int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^4*b^2*d^2*e^2 + (8*a^2*b^4*d^2*e^2 + 8*(4*a^2*b^4*c*d - 5*a^
3*b^3*d^2)*e*f + (8*a^2*b^4*c^2 - 40*a^3*b^3*c*d + 35*a^4*b^2*d^2)*f^2)*x^
4 + 8*(4*a^4*b^2*c*d - 5*a^5*b*d^2)*e*f + (8*a^4*b^2*c^2 - 40*a^5*b*c*d +
35*a^6*d^2)*f^2 + 2*(8*a^3*b^3*d^2*e^2 + 8*(4*a^3*b^3*c*d - 5*a^4*b^2*d^2)
*e*f + (8*a^3*b^3*c^2 - 40*a^4*b^2*c*d + 35*a^5*b*d^2)*f^2)*x^2)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^2*f^2*x^7
+ 3*(8*a^2*b^4*d^2*e*f + (8*a^2*b^4*c*d - 7*a^3*b^3*d^2)*f^2)*x^5 + 4*(4*(
b^6*c^2 + a*b^5*c*d - 2*a^2*b^4*d^2)*e^2 + 4*(a*b^5*c^2 - 8*a^2*b^4*c*d +
10*a^3*b^3*d^2)*e*f - (8*a^2*b^4*c^2 - 40*a^3*b^3*c*d + 35*a^4*b^2*d^2)*f^
2)*x^3 + 3*(8*(a*b^5*c^2 - a^3*b^3*d^2)*e^2 - 8*(4*a^3*b^3*c*d - 5*a^4*b^2
*d^2)*e*f - (8*a^3*b^3*c^2 - 40*a^4*b^2*c*d + 35*a^5*b*d^2)*f^2)*x)*sqrt(b
*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5), -1/24*(3*(8*a^4*b^2*d^
2*e^2 + (8*a^2*b^4*d^2*e^2 + 8*(4*a^2*b^4*c*d - 5*a^3*b^3*d^2)*e*f + (8*a^
2*b^4*c^2 - 40*a^3*b^3*c*d + 35*a^4*b^2*d^2)*f^2)*x^4 + 8*(4*a^4*b^2*c*d -
 5*a^5*b*d^2)*e*f + (8*a^4*b^2*c^2 - 40*a^5*b*c*d + 35*a^6*d^2)*f^2 + 2*(8
*a^3*b^3*d^2*e^2 + 8*(4*a^3*b^3*c*d - 5*a^4*b^2*d^2)*e*f + (8*a^3*b^3*c^2
- 40*a^4*b^2*c*d + 35*a^5*b*d^2)*f^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (6*a^2*b^4*d^2*f^2*x^7 + 3*(8*a^2*b^4*d^2*e*f + (8*a^2*b^4*
c*d - 7*a^3*b^3*d^2)*f^2)*x^5 + 4*(4*(b^6*c^2 + a*b^5*c*d - 2*a^2*b^4*d^2)
*e^2 + 4*(a*b^5*c^2 - 8*a^2*b^4*c*d + 10*a^3*b^3*d^2)*e*f - (8*a^2*b^4*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(225) = 450$.

Time = 0.05 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \frac{(c+dx^2)^2(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx &= \frac{d^2f^2x^7}{4(bx^2+a)^{\frac{3}{2}}b} - \frac{7ad^2f^2x^5}{8(bx^2+a)^{\frac{3}{2}}b^2} \\ &- \frac{35a^2d^2f^2x\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24b^2} + \frac{(d^2ef+cdf^2)x^5}{(bx^2+a)^{\frac{3}{2}}b} \\ &- \frac{1}{3}(d^2e^2+4cdef+c^2f^2)x\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) \\ &+ \frac{5(d^2ef+cdf^2)ax\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3b} + \frac{2c^2e^2x}{3\sqrt{bx^2+a}a^2} + \frac{c^2e^2x}{3(bx^2+a)^{\frac{3}{2}}a} \\ &- \frac{35a^2d^2f^2x}{24\sqrt{bx^2+a}b^4} + \frac{35a^2d^2f^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{\frac{9}{2}}} + \frac{5(d^2ef+cdf^2)ax}{3\sqrt{bx^2+a}b^3} \\ &- \frac{(d^2e^2+4cdef+c^2f^2)x}{3\sqrt{bx^2+a}b^2} - \frac{2(cde^2+c^2ef)x}{3(bx^2+a)^{\frac{3}{2}}b} + \frac{2(cde^2+c^2ef)x}{3\sqrt{bx^2+a}ab} \\ &- \frac{5(d^2ef+cdf^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}} + \frac{(d^2e^2+4cdef+c^2f^2)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/4*d^2*f^2*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*a*d^2*f^2*x^5/((b*x^2 + a)^(3/
2)*b^2) - 35/24*a^2*d^2*f^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 +
 a)^(3/2)*b^2))/b^2 + (d^2*e*f + c*d*f^2)*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*
(d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x
^2 + a)^(3/2)*b^2)) + 5/3*(d^2*e*f + c*d*f^2)*a*x*(3*x^2/((b*x^2 + a)^(3/2
)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*c^2*e^2*x/(sqrt(b*x^2 + a)*a^2
) + 1/3*c^2*e^2*x/((b*x^2 + a)^(3/2)*a) - 35/24*a^2*d^2*f^2*x/(sqrt(b*x^2
+ a)*b^4) + 35/8*a^2*d^2*f^2*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 5/3*(d^2*e*f
 + c*d*f^2)*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3*(d^2*e^2 + 4*c*d*e*f + c^2*f^2
)*x/(sqrt(b*x^2 + a)*b^2) - 2/3*(c*d*e^2 + c^2*e*f)*x/((b*x^2 + a)^(3/2)*b
) + 2/3*(c*d*e^2 + c^2*e*f)*x/(sqrt(b*x^2 + a)*a*b) - 5*(d^2*e*f + c*d*f^2
)*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + (d^2*e^2 + 4*c*d*e*f + c^2*f^2)*arcsi
nh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,d^2f^2x^2}{b}+\frac{8\,a^2b^6d^2ef+8\,a^2b^6cdf^2-7\,a^3b^5d^2f^2}{a^2b^7}\right)x^2+\frac{4\,(4\,b^8c^2e^2+4\,ab^7cde^2-8\,a^2b^6d^2e^2+\cdots}{\cdots}\right.\right.}{\cdots} - \frac{(8\,b^2d^2e^2+32\,b^2cdef-40\,abd^2ef+8\,b^2c^2f^2-40\,abcdf^2+35\,a^2d^2f^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*d^2*f^2*x^2/b + (8*a^2*b^6*d^2*e*f + 8*a^2*b^6*c*d*f^2 - 7*a^3
*b^5*d^2*f^2)/(a^2*b^7))*x^2 + 4*(4*b^8*c^2*e^2 + 4*a*b^7*c*d*e^2 - 8*a^2*
b^6*d^2*e^2 + 4*a*b^7*c^2*e*f - 32*a^2*b^6*c*d*e*f + 40*a^3*b^5*d^2*e*f -
8*a^2*b^6*c^2*f^2 + 40*a^3*b^5*c*d*f^2 - 35*a^4*b^4*d^2*f^2)/(a^2*b^7))*x^
2 + 3*(8*a*b^7*c^2*e^2 - 8*a^3*b^5*d^2*e^2 - 32*a^3*b^5*c*d*e*f + 40*a^4*b
^4*d^2*e*f - 8*a^3*b^5*c^2*f^2 + 40*a^4*b^4*c*d*f^2 - 35*a^5*b^3*d^2*f^2)/
(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(8*b^2*d^2*e^2 + 32*b^2*c*d*e*f - 40*
a*b*d^2*e*f + 8*b^2*c^2*f^2 - 40*a*b*c*d*f^2 + 35*a^2*d^2*f^2)*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)^2}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

3.366 $\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(be-af)x}{3ab^3(a+bx^2)^{3/2}} + \frac{(bc-ad)(2b^2ce-7a^2df+ab(4de+cf))x}{3a^2b^3\sqrt{a+bx^2}} + \frac{d^2fx\sqrt{a+bx^2}}{2b^3} + \frac{d(2bde+4bcf-5adf)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*(-a*d+b*c)^2*(-a*f+b*e)*x/a/b^3/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)*(2*b^2*
c*e-7*a^2*d*f+a*b*(c*f+4*d*e))*x/a^2/b^3/(b*x^2+a)^(1/2)+1/2*d^2*f*x*(b*x^
2+a)^(1/2)/b^3+1/2*d*(-5*a*d*f+4*b*c*f+2*b*d*e)*arctanh(b^(1/2)*x/(b*x^2+a
)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{x(15a^4d^2f+4b^4c^2ex^2+2ab^3c(3ce+2dex^2+cfx^2)-2a^3bd(3de+6cf-10d}{6a^2b^3(a+bx^2)^{3/2}}$$
$$-\frac{d(2bde+4bcf-5adf)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(x*(15*a^4*d^2*f + 4*b^4*c^2*e*x^2 + 2*a*b^3*c*(3*c*e + 2*d*e*x^2 + c*f*x^
2) - 2*a^3*b*d*(3*d*e + 6*c*f - 10*d*f*x^2) + a^2*b^2*d*x^2*(-8*d*e - 16*c
*f + 3*d*f*x^2)))/(6*a^2*b^3*(a + b*x^2)^(3/2)) - (d*(2*b*d*e + 4*b*c*f -
5*a*d*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {401, 25, 401, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{401}$$
$$\frac{x(c+dx^2)^2(be-af)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{(dx^2+c)(c(2be+af)-d(2be-5af)x^2)}{(bx^2+a)^{3/2}}dx}{3ab}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{(dx^2+c)(c(2be+af)-d(2be-5af)x^2)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(c+dx^2)^2(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \textbf{401}$$

$$\frac{\frac{x(c+dx^2)(bc(af+2be)+ad(2be-5af))}{ab\sqrt{a+bx^2}}-\frac{\int \frac{d\left(\left(-15dfa^2+2b(3de+cf)a+4b^2ce\right)x^2+ac(2be-5af)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{x(c+dx^2)(bc(af+2be)+ad(2be-5af))}{ab\sqrt{a+bx^2}}-\frac{d\int \frac{\left(-15dfa^2+2b(3de+cf)a+4b^2ce\right)x^2+ac(2be-5af)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{299}$$

$$\frac{\frac{x(c+dx^2)(bc(af+2be)+ad(2be-5af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(cf+3de)+4b^2ce\right)}{2b}-\frac{3a^2(-5adf+4bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{x(c+dx^2)(bc(af+2be)+ad(2be-5af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(cf+3de)+4b^2ce\right)}{2b}-\frac{3a^2(-5adf+4bcf+2bde)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{x(c+dx^2)(bc(af+2be)+ad(2be-5af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(-15a^2df+2ab(cf+3de)+4b^2ce\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-5adf+4bcf+2bde)}{2b^{3/2}}\right)}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^2(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (((a*d*(2*b*e -
5*a*f) + b*c*(2*b*e + a*f))*x*(c + d*x^2))/(a*b*Sqrt[a + b*x^2]) - (d*(((4
*b^2*c*e - 15*a^2*d*f + 2*a*b*(3*d*e + c*f))*x*Sqrt[a + b*x^2])/(2*b) - (3
*a^2*(2*b*d*e + 4*b*c*f - 5*a*d*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
2*b^(3/2))))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-\frac{5a^2\left(adf-\frac{4\left(cf+\frac{de}{2}\right)b}{5}\right)d\left(b\,x^2+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{2}+\frac{2\left(-3a^3d\left(\left(-\frac{5f\,x^2}{3}+\frac{e}{2}\right)d+cf\right)b^{\frac{3}{2}}-4\left(\frac{\left(-\frac{3f\,x^2}{8}+e\right)d}{2}+cf\right)d\,x^2a^2b^{\frac{5}{2}}\cdots\right.}{3}}{b^{\frac{7}{2}}\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}$
default	$c^2e\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d(2cf+de)\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+\cdots$
risch	$\frac{d^2fx\sqrt{b\,x^2+a}}{2b^3}-\frac{\frac{d(5adf-4bcf-2bde)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{\left(a^3d^2f-2a^2bcdf-a^2b\,d^2e+b^2c^2fa+2a\,b^2cde-b^3c^2e\right)\left(-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)\cdots}}{3\sqrt{\cdots}}\cdots\right.}{2b\cdots}}{\cdots}$

input
```
int((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)*(-15/4*a^2*(a*d*f-4/5*(c*f+1/2*d*e)*b)*d*(b*x^2+a)^(3/
2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(-3*a^3*d*((-5/3*f*x^2+1/2*e)*d+c*f)
*b^(3/2)-4*(1/2*(-3/8*f*x^2+e)*d+c*f)*d*x^2*a^2*b^(5/2)+3/2*a*(2/3*d*e*x^2
+c*(1/3*f*x^2+e))*c*b^(7/2)+b^(9/2)*c^2*e*x^2+15/4*a^4*d^2*f*b^(1/2))*x)/b
^(7/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(145) = 290$.

Time = 0.16 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.84

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \Bigg[-\frac{3\,(2\,a^4bd^2e + (2\,a^2b^3d^2e + (4\,a^2b^3cd - 5\,a^3b^2d^2)f)x^4 + 2\,(2\,a^3b^2d^2e + (4\,a^3}{}$$

$$-\frac{3\,(2\,a^4bd^2e + (2\,a^2b^3d^2e + (4\,a^2b^3cd - 5\,a^3b^2d^2)f)x^4 + 2\,(2\,a^3b^2d^2e + (4\,a^3b^2cd - 5\,a^4bd^2)f)x^2 + (4\,a^4bcd}{}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*a^4*b*d^2*e + (2*a^2*b^3*d^2*e + (4*a^2*b^3*c*d - 5*a^3*b^2*d
^2)*f)*x^4 + 2*(2*a^3*b^2*d^2*e + (4*a^3*b^2*c*d - 5*a^4*b*d^2)*f)*x^2 + (
4*a^4*b*c*d - 5*a^5*d^2)*f)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) - 2*(3*a^2*b^3*d^2*f*x^5 + 2*(2*(b^5*c^2 + a*b^4*c*d - 2*a^2*b^3
*d^2)*e + (a*b^4*c^2 - 8*a^2*b^3*c*d + 10*a^3*b^2*d^2)*f)*x^3 + 3*(2*(a*b^
4*c^2 - a^3*b^2*d^2)*e - (4*a^3*b^2*c*d - 5*a^4*b*d^2)*f)*x)*sqrt(b*x^2 +
a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4), -1/6*(3*(2*a^4*b*d^2*e + (2*a
^2*b^3*d^2*e + (4*a^2*b^3*c*d - 5*a^3*b^2*d^2)*f)*x^4 + 2*(2*a^3*b^2*d^2*e
 + (4*a^3*b^2*c*d - 5*a^4*b*d^2)*f)*x^2 + (4*a^4*b*c*d - 5*a^5*d^2)*f)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b^3*d^2*f*x^5 + 2*(2*(b^
5*c^2 + a*b^4*c*d - 2*a^2*b^3*d^2)*e + (a*b^4*c^2 - 8*a^2*b^3*c*d + 10*a^3
*b^2*d^2)*f)*x^3 + 3*(2*(a*b^4*c^2 - a^3*b^2*d^2)*e - (4*a^3*b^2*c*d - 5*a
^4*b*d^2)*f)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{d^2fx^5}{2\,(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,ad^2fx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} - \frac{1}{3}\,(d^2e+2\,cdf)x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{2\,c^2ex}{3\,\sqrt{bx^2+a}a^2} + \frac{c^2ex}{3\,(bx^2+a)^{\frac{3}{2}}a} + \frac{5\,ad^2fx}{6\,\sqrt{bx^2+a}b^3} - \frac{5\,ad^2f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} - \frac{(d^2e+2\,cdf)x}{3\,\sqrt{bx^2+a}b^2} - \frac{(2\,cde+c^2f)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{(2\,cde+c^2f)x}{3\,\sqrt{bx^2+a}ab} + \frac{(d^2e+2\,cdf)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*d^2*f*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*d^2*f*x*(3*x^2/((b*x^2 + a)^(3
/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 1/3*(d^2*e + 2*c*d*f)*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/3*c^2*e*x/(sqrt(b*
x^2 + a)*a^2) + 1/3*c^2*e*x/((b*x^2 + a)^(3/2)*a) + 5/6*a*d^2*f*x/(sqrt(b*
x^2 + a)*b^3) - 5/2*a*d^2*f*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/3*(d^2*e +
2*c*d*f)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(2*c*d*e + c^2*f)*x/((b*x^2 + a)^(3
/2)*b) + 1/3*(2*c*d*e + c^2*f)*x/(sqrt(b*x^2 + a)*a*b) + (d^2*e + 2*c*d*f)
*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^2)^2\,(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,d^2fx^2}{b}+\frac{2\,(2\,b^6c^2e+2\,ab^5cde-4\,a^2b^4d^2e+ab^5c^2f-8\,a^2b^4cdf+10\,a^3b^3d^2f)}{a^2b^5}\right)x^2+\frac{3\,(2\,ab^5c^2e-2}{}\right.}{6\,(bx^2+a)^{\frac{3}{2}}}$$
$$-\frac{(2\,bd^2e+4\,bcdf-5\,ad^2f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*d^2*f*x^2/b + 2*(2*b^6*c^2*e + 2*a*b^5*c*d*e - 4*a^2*b^4*d^2*e + a
*b^5*c^2*f - 8*a^2*b^4*c*d*f + 10*a^3*b^3*d^2*f)/(a^2*b^5))*x^2 + 3*(2*a*b
^5*c^2*e - 2*a^3*b^3*d^2*e - 4*a^3*b^3*c*d*f + 5*a^4*b^2*d^2*f)/(a^2*b^5))
*x/(b*x^2 + a)^(3/2) - 1/2*(2*b*d^2*e + 4*b*c*d*f - 5*a*d^2*f)*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2\,(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 738, normalized size of antiderivative = 4.47

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output
```
(30*sqrt(a + b*x**2)*a**4*b*d**2*f*x - 24*sqrt(a + b*x**2)*a**3*b**2*c*d*f
*x - 12*sqrt(a + b*x**2)*a**3*b**2*d**2*e*x + 40*sqrt(a + b*x**2)*a**3*b**
2*d**2*f*x**3 - 32*sqrt(a + b*x**2)*a**2*b**3*c*d*f*x**3 - 16*sqrt(a + b*x
**2)*a**2*b**3*d**2*e*x**3 + 6*sqrt(a + b*x**2)*a**2*b**3*d**2*f*x**5 + 12
*sqrt(a + b*x**2)*a*b**4*c**2*e*x + 4*sqrt(a + b*x**2)*a*b**4*c**2*f*x**3
+ 8*sqrt(a + b*x**2)*a*b**4*c*d*e*x**3 + 8*sqrt(a + b*x**2)*b**5*c**2*e*x*
*3 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d**2*f +
24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*d*f + 12*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d**2*e - 60*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d**2*f*x**2 + 48*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*d*f*x**2 + 2
4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d**2*e*x**
2 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*d**2*
f*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*
c*d*f*x**4 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
**3*d**2*e*x**4 - 5*sqrt(b)*a**5*d**2*f - 10*sqrt(b)*a**4*b*d**2*f*x**2 +
4*sqrt(b)*a**3*b**2*c**2*f + 8*sqrt(b)*a**3*b**2*c*d*e - 5*sqrt(b)*a**3*b*
*2*d**2*f*x**4 - 8*sqrt(b)*a**2*b**3*c**2*e + 8*sqrt(b)*a**2*b**3*c**2*f*x
**2 + 16*sqrt(b)*a**2*b**3*c*d*e*x**2 - 16*sqrt(b)*a*b**4*c**2*e*x**2 + 4*
sqrt(b)*a*b**4*c**2*f*x**4 + 8*sqrt(b)*a*b**4*c*d*e*x**4 - 8*sqrt(b)*b*...
```

3.367 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 169

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{(bc-ad)^2x}{3ab(be-af)(a+bx^2)^{3/2}} + \frac{(bc-ad)(2b^2ce-a^2df+ab(4de-5cf))x}{3a^2b(be-af)^2\sqrt{a+bx^2}} + \frac{(de-cf)^2\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{5/2}}$$

output
```
1/3*(-a*d+b*c)^2*x/a/b/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)*(2*b^2*c*
e-a^2*d*f+a*b*(-5*c*f+4*d*e))*x/a^2/b/(-a*f+b*e)^2/(b*x^2+a)^(1/2)+(-c*f+d
*e)^2*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/(-a*f+b*
e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{(bc-ad)x(2b^2cex^2+ab(3ce+4dex^2-5cfx^2)+a^2(3de-6cf-dfx^2))}{3a^2(be-af)^2(a+bx^2)^{3/2}} - \frac{(de-cf)^2\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(-be+af)^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
((b*c - a*d)*x*(2*b^2*c*e*x^2 + a*b*(3*c*e + 4*d*e*x^2 - 5*c*f*x^2) + a^2*
(3*d*e - 6*c*f - d*f*x^2)))/(3*a^2*(b*e - a*f)^2*(a + b*x^2)^(3/2)) - ((d*
e - c*f)^2*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*
Sqrt[-(b*e) + a*f])])/(Sqrt[e]*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {419, 25, 398, 224, 219, 291, 221, 401, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}dx$$

$$\Big\downarrow \; 419$$

$$-\frac{\int -\frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \; 398$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x(c+dx^2)(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}-\frac{\int -\frac{b\left(3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{b\left(3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{3ad(de-cf)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3ad(de-cf)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}$$

input
```
Int[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2)) + (((b*c
- a*d)*(2*b*c*e + 3*a*d*e - 5*a*c*f)*x)/(a*Sqrt[a + b*x^2]) + (3*a*d*(d*e
- c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a))/(b*e - a*f)^2
 - (f*(d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f) -
((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt
[e]*f*Sqrt[b*e - a*f])))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-a^2(b\,x^2+a)^{\frac{3}{2}}(cf-de)^2\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+2(ad-bc)\left(\left(-\frac{1}{2}de+\frac{1}{6}df\,x^2+cf\right)a^2-\frac{\left(\frac{(-5cf+4de)x^2}{3}+ce\right)ba}{2}-\frac{e\,b^2c\,x^2}{3}\right)\sqrt{}}{(af-be)^2\sqrt{(af-be)e}\,(b\,x^2+a)^{\frac{3}{2}}a^2}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*a^2*(b*x^2+a)^(3/2)*(c*f-d*e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b
*e)*e)^(1/2))+(a*d-b*c)*((-1/2*d*e+1/6*d*f*x^2+c*f)*a^2-1/2*(1/3*(-5*c*f+4
*d*e)*x^2+c*e)*b*a-1/3*e*b^2*c*x^2)*((a*f-b*e)*e)^(1/2)*x)/((a*f-b*e)*e)^(
1/2)/(b*x^2+a)^(3/2)/(a*f-b*e)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(151) = 302.

Time = 2.75 (sec) , antiderivative size = 1154, normalized size of antiderivative = 6.83

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^4*d^2*e^2 - 2*a^4*c*d*e*f + a^4*c^2*f^2 + (a^2*b^2*d^2*e^2 - 2
*a^2*b^2*c*d*e*f + a^2*b^2*c^2*f^2)*x^4 + 2*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e
*f + a^3*b*c^2*f^2)*x^2)*sqrt(b*e^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f +
 a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)
*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 +
e^2)) + 4*((2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*e^3 - (7*a*b^3*c^2 - 2
*a^2*b^2*c*d - 5*a^3*b*d^2)*e^2*f + (5*a^2*b^2*c^2 - 4*a^3*b*c*d - a^4*d^2
)*e*f^2)*x^3 + 3*((a*b^3*c^2 - a^3*b*d^2)*e^3 - (3*a^2*b^2*c^2 - 2*a^3*b*c
*d - a^4*d^2)*e^2*f + 2*(a^3*b*c^2 - a^4*c*d)*e*f^2)*x)*sqrt(b*x^2 + a))/(
a^4*b^3*e^4 - 3*a^5*b^2*e^3*f + 3*a^6*b*e^2*f^2 - a^7*e*f^3 + (a^2*b^5*e^4
 - 3*a^3*b^4*e^3*f + 3*a^4*b^3*e^2*f^2 - a^5*b^2*e*f^3)*x^4 + 2*(a^3*b^4*e
^4 - 3*a^4*b^3*e^3*f + 3*a^5*b^2*e^2*f^2 - a^6*b*e*f^3)*x^2), -1/6*(3*(a^4
*d^2*e^2 - 2*a^4*c*d*e*f + a^4*c^2*f^2 + (a^2*b^2*d^2*e^2 - 2*a^2*b^2*c*d*
e*f + a^2*b^2*c^2*f^2)*x^4 + 2*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^
2*f^2)*x^2)*sqrt(-b*e^2 + a*e*f)*arctan(1/2*sqrt(-b*e^2 + a*e*f)*((2*b*e -
 a*f)*x^2 + a*e)*sqrt(b*x^2 + a)/((b^2*e^2 - a*b*e*f)*x^3 + (a*b*e^2 - a^2
*e*f)*x)) - 2*((2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*e^3 - (7*a*b^3*c^2
 - 2*a^2*b^2*c*d - 5*a^3*b*d^2)*e^2*f + (5*a^2*b^2*c^2 - 4*a^3*b*c*d - a^4
*d^2)*e*f^2)*x^3 + 3*((a*b^3*c^2 - a^3*b*d^2)*e^3 - (3*a^2*b^2*c^2 - 2*a^3
*b*c*d - a^4*d^2)*e^2*f + 2*(a^3*b*c^2 - a^4*c*d)*e*f^2)*x)*sqrt(b*x^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(151) = 302.

Time = 0.15 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.54

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{\left(\frac{(2\,b^6c^2e^3+2\,ab^5cde^3-4\,a^2b^4d^2e^3-9\,ab^5c^2e^2f+9\,a^3b^3d^2e^2f+12\,a^2b^4c^2ef^2-6\,a^3b^3cdef^2-6\,a^4b^2d^2ef\cdots}{a^2b^5e^4-4\,a^3b^4e^3f+6\,a^4b^3e^2f^2-4\,a^5b^2ef^3+a^6bf^4}\cdots\right.}{\cdots} + \frac{(bd^4e^4-4\,bcd^3e^3f+6\,bc^2d^2e^2f^2-4\,bc^3def^3+bc^4f^4)\arctan\left(-\frac{2\,b^{\frac{3}{2}}d^2e^3-4\,b^{\frac{3}{2}}cde^2f-a\sqrt{b}d^2e^2f+2\,b^{\frac{3}{2}}c^2ef^2+2\,a\sqrt{b}c\cdots}{2\left(\sqrt{-\cdots}\right.\cdots}\right.}{(bd^2e^2-2\,bcdef+bc^2f^2)(b^2e^2-2\,abef+a\cdots}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
1/3*((2*b^6*c^2*e^3 + 2*a*b^5*c*d*e^3 - 4*a^2*b^4*d^2*e^3 - 9*a*b^5*c^2*e^
2*f + 9*a^3*b^3*d^2*e^2*f + 12*a^2*b^4*c^2*e*f^2 - 6*a^3*b^3*c*d*e*f^2 - 6
*a^4*b^2*d^2*e*f^2 - 5*a^3*b^3*c^2*f^3 + 4*a^4*b^2*c*d*f^3 + a^5*b*d^2*f^3
)*x^2/(a^2*b^5*e^4 - 4*a^3*b^4*e^3*f + 6*a^4*b^3*e^2*f^2 - 4*a^5*b^2*e*f^3
 + a^6*b*f^4) + 3*(a*b^5*c^2*e^3 - a^3*b^3*d^2*e^3 - 4*a^2*b^4*c^2*e^2*f +
 2*a^3*b^3*c*d*e^2*f + 2*a^4*b^2*d^2*e^2*f + 5*a^3*b^3*c^2*e*f^2 - 4*a^4*b
^2*c*d*e*f^2 - a^5*b*d^2*e*f^2 - 2*a^4*b^2*c^2*f^3 + 2*a^5*b*c*d*f^3)/(a^2
*b^5*e^4 - 4*a^3*b^4*e^3*f + 6*a^4*b^3*e^2*f^2 - 4*a^5*b^2*e*f^3 + a^6*b*f
^4))*x/(b*x^2 + a)^(3/2) + (b*d^4*e^4 - 4*b*c*d^3*e^3*f + 6*b*c^2*d^2*e^2*
f^2 - 4*b*c^3*d*e*f^3 + b*c^4*f^4)*arctan(-1/2*(2*b^(3/2)*d^2*e^3 - 4*b^(3
/2)*c*d*e^2*f - a*sqrt(b)*d^2*e^2*f + 2*b^(3/2)*c^2*e*f^2 + 2*a*sqrt(b)*c*
d*e*f^2 - a*sqrt(b)*c^2*f^3 + ((sqrt(b)*x - sqrt(b*x^2 + a))^2*sqrt(b)*d^2
*e^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*sqrt(b)*c*d*e*f + (sqrt(b)*x - sq
rt(b*x^2 + a))^2*sqrt(b)*c^2*f^2)*f)/(sqrt(-b*e^2 + a*e*f)*b*d^2*e^2 - 2*s
qrt(-b*e^2 + a*e*f)*b*c*d*e*f + sqrt(-b*e^2 + a*e*f)*b*c^2*f^2))/((b*d^2*e
^2 - 2*b*c*d*e*f + b*c^2*f^2)*(b^2*e^2 - 2*a*b*e*f + a^2*f^2)*sqrt(-b*e^2
+ a*e*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 2122, normalized size of antiderivative = 12.56

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**2*f**2 + 6*sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt
(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c*d*e*f - 3*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**4*b**2*d**2*e**2 - 6*sqrt(e)*sqrt(a*f - b*e)*atan(
(sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*
sqrt(b)))*a**3*b**3*c**2*f**2*x**2 + 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt
(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(
b)))*a**3*b**3*c*d*e*f*x**2 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**
3*b**3*d**2*e**2*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) -
sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**4
*c**2*f**2*x**4 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f
)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**4*c*d*e
*f*x**4 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**4*d**2*e**2*x**
4 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x
**2) + sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**2*c**2*f**2 + 6*sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) + sqrt(f)*sqrt(a + b*x**2) + s...
```

3.368 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx &= \frac{d^2x}{3af^2(a+bx^2)^{3/2}}\\
&+\frac{b(de-cf)(af(7de-3cf)-2be(de+cf))x}{6aef^2(be-af)^2(a+bx^2)^{3/2}}+\frac{2d^2x}{3a^2f^2\sqrt{a+bx^2}}\\
&-\frac{b(de-cf)\left(a^2f^2(23de-3cf)+4b^2e^2(de+cf)-4abef(3de+4cf)\right)x}{6a^2ef^2(be-af)^3\sqrt{a+bx^2}}\\
&-\frac{(de-cf)^2x}{2ef(be-af)(a+bx^2)^{3/2}(e+fx^2)}\\
&+\frac{(de-cf)(2be(de-3cf)+af(3de+cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{7/2}}\end{aligned}$$

output
```
1/3*d^2*x/a/f^2/(b*x^2+a)^(3/2)+1/6*b*(-c*f+d*e)*(a*f*(-3*c*f+7*d*e)-2*b*e
*(c*f+d*e))*x/a/e/f^2/(-a*f+b*e)^2/(b*x^2+a)^(3/2)+2/3*d^2*x/a^2/f^2/(b*x^
2+a)^(1/2)-1/6*b*(-c*f+d*e)*(a^2*f^2*(-3*c*f+23*d*e)+4*b^2*e^2*(c*f+d*e)-4
*a*b*e*f*(4*c*f+3*d*e))*x/a^2/e/f^2/(-a*f+b*e)^3/(b*x^2+a)^(1/2)-1/2*(-c*f
+d*e)^2*x/e/f/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e)+1/2*(-c*f+d*e)*(2*b*e*(
-3*c*f+d*e)+a*f*(c*f+3*d*e))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^
(1/2))/e^(3/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{x(-4b^4c^2e^2x^2(e+fx^2)-2ab^3ce(e+fx^2)(3ce+2dex^2-8cfx^2)+3a^4f(\cdots}{\cdots}$$
$$+\frac{(de-cf)(2be(de-3cf)+af(3de+cf))\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{2e^{3/2}(-be+af)^{7/2}}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(x*(-4*b^4*c^2*e^2*x^2*(e + f*x^2) - 2*a*b^3*c*e*(e + f*x^2)*(3*c*e + 2*d*
e*x^2 - 8*c*f*x^2) + 3*a^4*f*(-2*c*d*e*f + c^2*f^2 + d^2*e*(3*e + 2*f*x^2)
) + a^2*b^2*(d^2*e^2*x^2*(8*e + 11*f*x^2) - 2*c*d*e*f*x^2*(10*e + 13*f*x^2
) + 3*c^2*f*(6*e^2 + 6*e*f*x^2 + f^2*x^4)) + 2*a^3*b*(3*c^2*f^3*x^2 - 6*c*
d*e*f*(2*e + 3*f*x^2) + d^2*e*(3*e^2 + 8*e*f*x^2 + 2*f^2*x^4))))/(6*a^2*e*
(-(b*e) + a*f)^3*(a + b*x^2)^(3/2)*(e + f*x^2)) + ((d*e - c*f)*(2*b*e*(d*e
 - 3*c*f) + a*f*(3*d*e + c*f))*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e
 + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(2*e^(3/2)*(-(b*e) + a*f)^(7/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.40, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {425, 402, 25, 402, 25, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{(bx^2+a)^{5/2}(fx^2+e)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \textbf{402}$$

$$\frac{d\left(\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}\right)}{f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \textbf{402}$$

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int \frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int \frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\int \frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\int \frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\int\frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\int-\frac{3a^2(2be(2de-3cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{\cdots}{3a(a\cdots}\right)}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2\cdots\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-\cdots}}{3a(be-af)}\cdots\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7c\ldots\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)\ldots}}{3a(be-af)}\ldots\right)}{f}$$

$\Big\downarrow$ 221

$$\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf\ldots\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)\ldots}}{3a(be-af)}\ldots\right)}{f}$$

input
```
Int[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(d*(((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)) + (((2*b^2*c*e + 2
*a^2*d*f + a*b*(d*e - 5*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) - (3*a*f*
(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[
e]*(b*e - a*f)^(3/2)))/(3*a*(b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a*d)*
x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + (((2*b^2*c*e + 4*a^2*
d*f + a*b*(d*e - 7*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) +
(f*(((4*b^2*c*e^2 + 2*a*b*e*(d*e - 8*c*f) + a^2*f*(13*d*e - 3*c*f))*x*Sqrt
[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(2*b*e*(2*d*e - 3*c*f)
 + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
)/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(a*(b*e - a*f)))/(3*a*(b*e - a*f))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-a^2\left(\left(c f^2+3def\right)a-6bcef+2bd e^2\right)\left(b x^2+a\right)^{\frac{3}{2}}(cf-de)\left(f x^2+e\right)\arctan\left(\frac{e\sqrt{b x^2+a}}{x\sqrt{(af-be)e}}\right)+\sqrt{(af-be)e}\,x\left(\left(3d^2e^2-2df\left(-x^2\right.\right.\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a^2*((c*f^2+3*d*e*f)*a-6*b*c*e*f+2*b*d*e^2)*(b*x^2+a)^(3/2)*(c*f-d*e
)*(f*x^2+e)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^
(1/2)*x*((3*d^2*e^2-2*d*f*(-d*x^2+c)*e+c^2*f^2)*f*a^4+2*(d^2*e^3-4*d*(-2/3
*x^2*d+c)*f*e^2-6*d*x^2*f^2*(-1/9*x^2*d+c)*e+f^3*x^2*c^2)*b*a^3+6*(4/9*d^2
*e^3*x^2+f*(11/18*d^2*x^4-10/9*c*d*x^2+c^2)*e^2+c*f^2*x^2*(-13/9*x^2*d+c)*
e+1/6*c^2*f^3*x^4)*b^2*a^2-2*c*(e*(2/3*x^2*d+c)-8/3*c*f*x^2)*b^3*(f*x^2+e)
*e*a-4/3*b^4*c^2*e^2*x^2*(f*x^2+e)))/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(3/2)/e
/(f*x^2+e)/(a*f-b*e)^3/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(314) = 628.

Time = 7.71 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.22

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[1/24*(3*(2*a^4*b*d^2*e^4 - a^5*c^2*e*f^3 + (2*a^2*b^3*d^2*e^3*f - a^3*b^2
*c^2*f^4 - (8*a^2*b^3*c*d - 3*a^3*b^2*d^2)*e^2*f^2 + 2*(3*a^2*b^3*c^2 - a^
3*b^2*c*d)*e*f^3)*x^6 - (8*a^4*b*c*d - 3*a^5*d^2)*e^3*f + 2*(3*a^4*b*c^2 -
 a^5*c*d)*e^2*f^2 + (2*a^2*b^3*d^2*e^4 - 2*a^4*b*c^2*f^4 - (8*a^2*b^3*c*d
- 7*a^3*b^2*d^2)*e^3*f + 6*(a^2*b^3*c^2 - 3*a^3*b^2*c*d + a^4*b*d^2)*e^2*f
^2 + (11*a^3*b^2*c^2 - 4*a^4*b*c*d)*e*f^3)*x^4 + (4*a^3*b^2*d^2*e^4 - a^5*
c^2*f^4 - 8*(2*a^3*b^2*c*d - a^4*b*d^2)*e^3*f + 3*(4*a^3*b^2*c^2 - 4*a^4*b
*c*d + a^5*d^2)*e^2*f^2 + 2*(2*a^4*b*c^2 - a^5*c*d)*e*f^3)*x^2)*sqrt(b*e^2
 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*b*
e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)*s
qrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) + 4*((3*a^3*b^2*c^2*e*f^4 + (
4*b^5*c^2 + 4*a*b^4*c*d - 11*a^2*b^3*d^2)*e^4*f - (20*a*b^4*c^2 - 22*a^2*b
^3*c*d - 7*a^3*b^2*d^2)*e^3*f^2 + (13*a^2*b^3*c^2 - 26*a^3*b^2*c*d + 4*a^4
*b*d^2)*e^2*f^3)*x^5 + 2*(3*a^4*b*c^2*e*f^4 + 2*(b^5*c^2 + a*b^4*c*d - 2*a
^2*b^3*d^2)*e^5 - (7*a*b^4*c^2 - 8*a^2*b^3*c*d + 4*a^3*b^2*d^2)*e^4*f - (4
*a^2*b^3*c^2 - 8*a^3*b^2*c*d - 5*a^4*b*d^2)*e^3*f^2 + 3*(2*a^3*b^2*c^2 - 6
*a^4*b*c*d + a^5*d^2)*e^2*f^3)*x^3 + 3*(a^5*c^2*e*f^4 + 2*(a*b^4*c^2 - a^3
*b^2*d^2)*e^5 - (8*a^2*b^3*c^2 - 8*a^3*b^2*c*d + a^4*b*d^2)*e^4*f + 3*(2*a
^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*e^3*f^2 - (a^4*b*c^2 + 2*a^5*c*d)*e^2*
f^3)*x)*sqrt(b*x^2 + a))/(a^4*b^4*e^7 - 4*a^5*b^3*e^6*f + 6*a^6*b^2*e^5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^2}{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/((b*x^2 + a)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(314) = 628$.

Time = 0.44 (sec) , antiderivative size = 1116, normalized size of antiderivative = 3.23

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3*(2*(b^8*c^2*e^4 + a*b^7*c*d*e^4 - 2*a^2*b^6*d^2*e^4 - 7*a*b^7*c^2*e^3*
f + 2*a^2*b^6*c*d*e^3*f + 5*a^3*b^5*d^2*e^3*f + 15*a^2*b^6*c^2*e^2*f^2 - 1
2*a^3*b^5*c*d*e^2*f^2 - 3*a^4*b^4*d^2*e^2*f^2 - 13*a^3*b^5*c^2*e*f^3 + 14*
a^4*b^4*c*d*e*f^3 - a^5*b^3*d^2*e*f^3 + 4*a^4*b^4*c^2*f^4 - 5*a^5*b^3*c*d*
f^4 + a^6*b^2*d^2*f^4)*x^2/(a^2*b^7*e^6 - 6*a^3*b^6*e^5*f + 15*a^4*b^5*e^4
*f^2 - 20*a^5*b^4*e^3*f^3 + 15*a^6*b^3*e^2*f^4 - 6*a^7*b^2*e*f^5 + a^8*b*f
^6) + 3*(a*b^7*c^2*e^4 - a^3*b^5*d^2*e^4 - 6*a^2*b^6*c^2*e^3*f + 4*a^3*b^5
*c*d*e^3*f + 2*a^4*b^4*d^2*e^3*f + 12*a^3*b^5*c^2*e^2*f^2 - 12*a^4*b^4*c*d
*e^2*f^2 - 10*a^4*b^4*c^2*e*f^3 + 12*a^5*b^3*c*d*e*f^3 - 2*a^6*b^2*d^2*e*f
^3 + 3*a^5*b^3*c^2*f^4 - 4*a^6*b^2*c*d*f^4 + a^7*b*d^2*f^4)/(a^2*b^7*e^6 -
 6*a^3*b^6*e^5*f + 15*a^4*b^5*e^4*f^2 - 20*a^5*b^4*e^3*f^3 + 15*a^6*b^3*e^
2*f^4 - 6*a^7*b^2*e*f^5 + a^8*b*f^6))*x/(b*x^2 + a)^(3/2) - 1/2*(2*b^(3/2)
*d^2*e^3 - 8*b^(3/2)*c*d*e^2*f + 3*a*sqrt(b)*d^2*e^2*f + 6*b^(3/2)*c^2*e*f
^2 - 2*a*sqrt(b)*c*d*e*f^2 - a*sqrt(b)*c^2*f^3)*arctan(1/2*((sqrt(b)*x - s
qrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^3*e^4 - 3
*a*b^2*e^3*f + 3*a^2*b*e^2*f^2 - a^3*e*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) - (2
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*d^2*e^3 - 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b^(3/2)*c*d*e^2*f - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d
^2*e^2*f + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c^2*e*f^2 + 2*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*c*d*e*f^2 - (sqrt(b)*x - sqrt(b*x^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx=\int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 8023, normalized size of antiderivative = 23.26

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c**2*e*f**4 - 3*sqrt(e
)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(
f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c**2*f**5*x**2 - 6*sqrt(e)*sqrt(a*
f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b
)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c*d*e**2*f**3 - 6*sqrt(e)*sqrt(a*f - b*e)*a
tan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt
(e)*sqrt(b)))*a**6*b*c*d*e*f**4*x**2 + 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqr
t(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt
(b)))*a**6*b*d**2*e**3*f**2 + 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**
6*b*d**2*e**2*f**3*x**2 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e)
- sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b*
*2*c**2*e**2*f**3 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt
(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b**2*c**
2*f**5*x**4 - 48*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*s
qrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b**2*c*d*e**3
*f**2 - 60*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
+ b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**5*b**2*c*d*e**2*f**3*
x**2 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(...
```

3.369 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 654

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{d(8bde-8bcf+3adf)x}{12aef^2(be-af)(a+bx^2)^{3/2}}$$
$$+\frac{b(a^2f^2(7d^2e^2+10cdef-9c^2f^2)-8b^2e^2(2d^2e^2-2cdef-c^2f^2)+4abef(11d^2e^2-24cdef+9c^2f^2))x}{24ae^2f^2(be-af)^3(a+bx^2)^{3/2}}$$
$$+\frac{4bd(de-cf)x}{3a^2ef^2(be-af)\sqrt{a+bx^2}}$$
$$+\frac{b(8ab^2e^2f(12d^2e^2-10cdef-11c^2f^2)-a^3f^3(23d^2e^2+26cdef-9c^2f^2)-16b^3e^3(2d^2e^2-2cdef-c^2f^2}{24a^2e^2f^2(be-af)^4\sqrt{a+bx^2}}$$
$$-\frac{fx(c+dx^2)^2}{4e(be-af)(a+bx^2)^{3/2}(e+fx^2)^2}-\frac{(de-cf)(2be(3de-5cf)+af(de+3cf))x}{8e^2f(be-af)^2(a+bx^2)^{3/2}(e+fx^2)}$$
$$+\frac{(8abef(3d^2e^2-3cdef-2c^2f^2)+a^2f^2(3d^2e^2+2cdef+3c^2f^2)+8b^2e^2(d^2e^2-6cdef+6c^2f^2))\,\mathrm{arctanh}}{8e^{5/2}(be-af)^{9/2}}$$

output
```
1/12*d*(3*a*d*f-8*b*c*f+8*b*d*e)*x/a/e/f^2/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/24
*b*(a^2*f^2*(-9*c^2*f^2+10*c*d*e*f+7*d^2*e^2)-8*b^2*e^2*(-c^2*f^2-2*c*d*e*
f+2*d^2*e^2)+4*a*b*e*f*(9*c^2*f^2-24*c*d*e*f+11*d^2*e^2))*x/a/e^2/f^2/(-a*
f+b*e)^3/(b*x^2+a)^(3/2)+4/3*b*d*(-c*f+d*e)*x/a^2/e/f^2/(-a*f+b*e)/(b*x^2+
a)^(1/2)+1/24*b*(8*a*b^2*e^2*f*(-11*c^2*f^2-10*c*d*e*f+12*d^2*e^2)-a^3*f^3
*(-9*c^2*f^2+26*c*d*e*f+23*d^2*e^2)-16*b^3*e^3*(-c^2*f^2-2*c*d*e*f+2*d^2*e
^2)-2*a^2*b*e*f^2*(21*c^2*f^2-142*c*d*e*f+73*d^2*e^2))*x/a^2/e^2/f^2/(-a*f
+b*e)^4/(b*x^2+a)^(1/2)-1/4*f*x*(d*x^2+c)^2/e/(-a*f+b*e)/(b*x^2+a)^(3/2)/(
f*x^2+e)^2-1/8*(-c*f+d*e)*(2*b*e*(-5*c*f+3*d*e)+a*f*(3*c*f+d*e))*x/e^2/f/(
-a*f+b*e)^2/(b*x^2+a)^(3/2)/(f*x^2+e)+1/8*(8*a*b*e*f*(-2*c^2*f^2-3*c*d*e*f
+3*d^2*e^2)+a^2*f^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+8*b^2*e^2*(6*c^2*f^2-6
*c*d*e*f+d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(
5/2)/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 16.16 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.53

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{1}{24}\left(x\sqrt{a+bx^2}\left(-\frac{8b(bc-ad)^2}{a(-be+af)^3(a+bx^2)^2}+\frac{8b(bc-ad)(2b^2ce+5a^2df-\right.\right.}{a^2(be-af)^4(a\cdot}$$

$$+\frac{3(8abef(3d^2e^2-3cdef-2c^2f^2)+a^2f^2(3d^2e^2+2cdef+3c^2f^2)+8b^2e^2(d^2e^2-6cdef+6c^2f^2))\arctan}{e^{5/2}(-be+af)^{9/2}}$$

input
```
Integrate[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((-8*b*(b*c - a*d)^2)/(a*(-(b*e) + a*f)^3*(a + b*x^2)^2
) + (8*b*(b*c - a*d)*(2*b^2*c*e + 5*a^2*d*f + a*b*(4*d*e - 11*c*f)))/(a^2*
(b*e - a*f)^4*(a + b*x^2)) - (6*f*(d*e - c*f)^2)/(e*(b*e - a*f)^3*(e + f*x
^2)^2) + (3*f*(-(d*e) + c*f)*(2*b*e*(3*d*e - 7*c*f) + a*f*(5*d*e + 3*c*f))
)/(e^2*(b*e - a*f)^4*(e + f*x^2))) + (3*(8*a*b*e*f*(3*d^2*e^2 - 3*c*d*e*f
- 2*c^2*f^2) + a^2*f^2*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 8*b^2*e^2*(d^
2*e^2 - 6*c*d*e*f + 6*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqr
t[a + b*x^2])])/(e^(5/2)*(-(b*e) + a*f)^(9/2)))/24
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {425, 402, 25, 402, 25, 27, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{5/2} \left(e+fx^2\right)^3} \, dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{dx^2+c}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{(bx^2+a)^{5/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}-\frac{\int -\frac{6(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{\int \frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{6(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{f\left(4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{\frac{\int \frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{\int \frac{f\left(4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{\frac{f\int \frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{f\int \frac{4\left(6dfa^2+b(de-9cf)a+2b^2ce\right)x^2+a(4bce-7ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{f\left(\frac{\int -\frac{3a^2(2be(2de-3cf)+af(de+cf))}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{x(bc-\cdots}{3a(a+bx^2)^{3/2}(e\cdots}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\int \frac{2b\left(f(31de-3cf)a^2+4be(de-10cf)a+8b^2ce^2\right)x^2+a\left(-3f(de+3cf)a^2-4be(8de-9cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-\cdots\right.}{4e\left(e+fx^2\right)^2(be-af\cdots}\right.}{a(be-af)}\cdots}{3a(be-af)}\cdots\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{}{3}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\int\frac{2b\left(f(31de-3cf)a^2+4be(de-10cf)a+8b^2ce^2\right)x^2+a\left(-3f(de+3cf)a^2-4be(8de-9cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-\right.}{4e\left(e+fx^2\right)^2(be-af)}\right)}{a(be-af)}}{3a(be-af)}\right)}{f}$$

$\downarrow$ 291

$$\frac{d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2(af(cf+de)+2be(2de-3cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}\right)}{f}$$

$$\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\int\frac{2b\left(f(31de-3cf)a^2+4be(de-10cf)a+8b^2ce^2\right)x^2+a\left(-3f(de+3cf)a^2-4be(8de-9cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-\right.}{4e\left(e+fx^2\right)^2(be-af)}\right)}{a(be-af)}}{3a(be-af)}\right)}{f}$$

$\downarrow$ 221

$$\frac{d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\int\frac{2b\left(f(31de-3cf)a^2+4be(de-10cf)a+8b^2ce^2\right)x^2+a\left(-3f(de+3cf)a^2-4be(8de-9cf)a+8b^2ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-\cdots\right.}{4e\left(e+fx^2\right)^2(be-af)}\cdots\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right)}{f}$$

$\Big\downarrow$ 402

$$\frac{d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\cdots\right)}{f}$$

$$\frac{(de-cf)\left(\frac{\frac{f\left(\frac{\frac{\int-\frac{3a^2\left(24b^2(de-2cf)e^2+4abf(3de+4cf)e-a^2f^2(de+3cf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^{\cdots}\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right)}{f}$$

$\Big\downarrow$ 27

$$d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\cdots\right.$$

$$f$$

$$(de-cf)\left(\frac{\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^3ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(-a^2f^2(3cf+de)+4abef(4cf+3de)+24b^2e^2(de-2cf)\right)\cdots}{2e(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right.$$

$$f\cdots$$

$\Big\downarrow$ 291

$$d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\cdots\right.$$

$$f$$

$$(de-cf)\left(\frac{\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+8ab^2e^2(de-11cf)+16b^3ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(-a^2f^2(3cf+de)+4abef(4cf+3de)+24b^2e^2(de-2cf)\right)\cdots}{2e(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right.$$

$$\Big\downarrow 221$$

$$\frac{d\left(\frac{\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(13de-3cf)+2abe(de-8cf)+4b^2ce^2\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(af(cf+de)+2be(2de-3cf))}{2e^{3/2}(be-af)^{3/2}}\right)}{a(be-af)}+\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\right.}{f}$$

$$(de-cf)\left(\frac{\frac{x\left(6a^2df+ab(de-9cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(31de-3cf)+4abe(de-10cf)+8b^2ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(3a^3f^2(3cf+de)+2a^2bef(47de-21cf)+\right.}{2e\left(e+fx^2\right)(be-af)}\right.}{}}{3a(be-af)}\right.$$

input
```
Int[(c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(d*(((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + (((2
*b^2*c*e + 4*a^2*d*f + a*b*(d*e - 7*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2
]*(e + f*x^2)) + (f*(((4*b^2*c*e^2 + 2*a*b*e*(d*e - 8*c*f) + a^2*f*(13*d*e
 - 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(2*b*
e*(2*d*e - 3*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*
Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(a*(b*e - a*f)))/(3*a*(
b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^
2)^(3/2)*(e + f*x^2)^2) + (((2*b^2*c*e + 6*a^2*d*f + a*b*(d*e - 9*c*f))*x)
/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (f*(((8*b^2*c*e^2 + 4*a*b
*e*(d*e - 10*c*f) + a^2*f*(31*d*e - 3*c*f))*x*Sqrt[a + b*x^2])/(4*e*(b*e -
 a*f)*(e + f*x^2)^2) + (((16*b^3*c*e^3 + 2*a^2*b*e*f*(47*d*e - 21*c*f) + 8
*a*b^2*e^2*(d*e - 11*c*f) + 3*a^3*f^2*(d*e + 3*c*f))*x*Sqrt[a + b*x^2])/(2
*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(24*b^2*e^2*(d*e - 2*c*f) - a^2*f^2*(
d*e + 3*c*f) + 4*a*b*e*f*(3*d*e + 4*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqr
t[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f)))
/(a*(b*e - a*f)))/(3*a*(b*e - a*f))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 603, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{-\frac{3a^2\left(b x^2+a\right)^{\frac{3}{2}}\left(f^2\left(c^2f^2+\frac{2}{3}cdef+d^2e^2\right)a^2-\frac{16\left(c^2f^2+\frac{3}{2}cdef-\frac{3}{2}d^2e^2\right)bfea}{3}+16\left(c^2f^2-cdef+\frac{1}{6}d^2e^2\right)b^2e^2\right)\left(f x^2+e\right)^2 \arctan\left(\frac{e}{x \cdots}\right.}{\cdots}}{8}$
default	Expression too large to display

input
```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(-3/5*a^2*(b*x^2+a)^(3/2)*(f^2*(c^2*f^2+2/3*c*d*e*f+d^2*e^2)*a^2-16/3*
(c^2*f^2+3/2*c*d*e*f-3/2*d^2*e^2)*b*f*e*a+16*(c^2*f^2-c*d*e*f+1/6*d^2*e^2)
*b^2*e^2)*(f*x^2+e)^2*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((c*
f-d*e)*f^2*(3/5*d*e^2+f*(d*x^2+c)*e+3/5*c*f^2*x^2)*a^5-16/5*(3/2*e^4*d^2-3
/2*(-11/6*x^2*d+c)*d*f*e^3+f^2*(13/8*d^2*x^4-c*d*x^2+c^2)*e^2+1/4*c*f^3*x^
2*(-d*x^2+c)*e-3/8*c^2*f^4*x^4)*b*f*a^4-32/5*b^2*(1/4*d^2*e^5-3/2*(-17/18*
x^2*d+c)*d*f*e^4-9/2*d*x^2*f^2*(-149/432*x^2*d+c)*e^3+f^3*x^2*(55/96*d^2*x
^4-43/16*c*d*x^2+c^2)*e^2+23/32*c*(-2/23*x^2*d+c)*x^4*f^4*e-3/32*c^2*f^5*x
^6)*a^3-32/5*(1/3*d^2*e^4*x^2+f*(11/12*d^2*x^4-4/3*c*d*x^2+c^2)*e^3+2*x^2*
f^2*(25/96*d^2*x^4-41/24*c*d*x^2+c^2)*e^2+3/2*c*(-47/36*x^2*d+c)*x^4*f^3*e
+7/16*c^2*f^4*x^6)*b^3*e*a^2+8/5*c*b^4*(f*x^2+e)^2*e^2*(e*(2/3*x^2*d+c)-11
/3*c*f*x^2)*a+16/15*b^5*c^2*e^3*x^2*(f*x^2+e)^2)*((a*f-b*e)*e)^(1/2)*x)/((
a*f-b*e)*e)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^2/e^2/(a*f-b*e)^4/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2084 vs. $2(618) = 1236$.

Time = 34.31 (sec) , antiderivative size = 4208, normalized size of antiderivative = 6.43

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^2}{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2/((b*x^2 + a)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2249 vs. 2(618) = 1236.

Time = 0.46 (sec) , antiderivative size = 2249, normalized size of antiderivative = 3.44

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^10*c^2*e^5 + 2*a*b^9*c*d*e^5 - 4*a^2*b^8*d^2*e^5 - 19*a*b^9*c^2*
e^4*f + 8*a^2*b^8*c*d*e^4*f + 11*a^3*b^7*d^2*e^4*f + 56*a^2*b^8*c^2*e^3*f^
2 - 52*a^3*b^7*c*d*e^3*f^2 - 4*a^4*b^6*d^2*e^3*f^2 - 74*a^3*b^7*c^2*e^2*f^
3 + 88*a^4*b^6*c*d*e^2*f^3 - 14*a^5*b^5*d^2*e^2*f^3 + 46*a^4*b^6*c^2*e*f^4
 - 62*a^5*b^5*c*d*e*f^4 + 16*a^6*b^4*d^2*e*f^4 - 11*a^5*b^5*c^2*f^5 + 16*a
^6*b^4*c*d*f^5 - 5*a^7*b^3*d^2*f^5)*x^2/(a^2*b^9*e^8 - 8*a^3*b^8*e^7*f + 2
8*a^4*b^7*e^6*f^2 - 56*a^5*b^6*e^5*f^3 + 70*a^6*b^5*e^4*f^4 - 56*a^7*b^4*e
^3*f^5 + 28*a^8*b^3*e^2*f^6 - 8*a^9*b^2*e*f^7 + a^10*b*f^8) + 3*(a*b^9*c^2
*e^5 - a^3*b^7*d^2*e^5 - 8*a^2*b^8*c^2*e^4*f + 6*a^3*b^7*c*d*e^4*f + 2*a^4
*b^6*d^2*e^4*f + 22*a^3*b^7*c^2*e^3*f^2 - 24*a^4*b^6*c*d*e^3*f^2 + 2*a^5*b
^5*d^2*e^3*f^2 - 28*a^4*b^6*c^2*e^2*f^3 + 36*a^5*b^5*c*d*e^2*f^3 - 8*a^6*b
^4*d^2*e^2*f^3 + 17*a^5*b^5*c^2*e*f^4 - 24*a^6*b^4*c*d*e*f^4 + 7*a^7*b^3*d
^2*e*f^4 - 4*a^6*b^4*c^2*f^5 + 6*a^7*b^3*c*d*f^5 - 2*a^8*b^2*d^2*f^5)/(a^2
*b^9*e^8 - 8*a^3*b^8*e^7*f + 28*a^4*b^7*e^6*f^2 - 56*a^5*b^6*e^5*f^3 + 70*
a^6*b^5*e^4*f^4 - 56*a^7*b^4*e^3*f^5 + 28*a^8*b^3*e^2*f^6 - 8*a^9*b^2*e*f^
7 + a^10*b*f^8))*x/(b*x^2 + a)^(3/2) - 1/8*(8*b^(5/2)*d^2*e^4 - 48*b^(5/2)
*c*d*e^3*f + 24*a*b^(3/2)*d^2*e^3*f + 48*b^(5/2)*c^2*e^2*f^2 - 24*a*b^(3/2
)*c*d*e^2*f^2 + 3*a^2*sqrt(b)*d^2*e^2*f^2 - 16*a*b^(3/2)*c^2*e*f^3 + 2*a^2
*sqrt(b)*c*d*e*f^3 + 3*a^2*sqrt(b)*c^2*f^4)*arctan(1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^4*e^6 - 4*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^2/((a + b*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 13481, normalized size of antiderivative = 20.61

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}(e+fx^2)^3} dx = \text{Too large to display}$$

input

```
int((d*x^2+c)^2/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x)
```

output

```
( - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*c**2*e**2*f**5 - 18*sqrt
(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqr
t(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*c**2*e*f**6*x**2 - 9*sqrt(e)*sqrt(
a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt
(b)*x)/(sqrt(e)*sqrt(b)))*a**7*c**2*f**7*x**4 - 6*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**7*c*d*e**3*f**4 - 12*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*a**7*c*d*e**2*f**5*x**2 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b
*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**
7*c*d*e*f**6*x**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt
(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*d**2*e**
4*f**3 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a
 + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*d**2*e**3*f**4*x**
2 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*d**2*e**2*f**5*x**4 + 12
0*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2)
 - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**6*b*c**2*e**3*f**4 + 222*sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - s...
```

3.370 $\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 533

$$\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^3(be-af)^3x}{3ab^6(a+bx^2)^{3/2}}$$
$$+\frac{(bc-ad)^2(be-af)^2(2b^2ce-16a^2df+7ab(de+cf))\,x}{3a^2b^6\sqrt{a+bx^2}}$$
$$-\frac{(515a^3d^3f^3-984a^2bd^2f^2(de+cf)+528ab^2df(d^2e^2+3cdef+c^2f^2)-64b^3(d^3e^3+9cd^2e^2f+9c^2def^2}{128b^6}$$
$$+\frac{df(259a^2d^2f^2-408abdf(de+cf)+144b^2(d^2e^2+3cdef+c^2f^2))\,x^3\sqrt{a+bx^2}}{192b^5}$$
$$-\frac{d^2f^2(23adf-24b(de+cf))x^5\sqrt{a+bx^2}}{48b^4}+\frac{d^3f^3x^7\sqrt{a+bx^2}}{8b^3}$$
$$+\frac{(1155a^4d^3f^3-2520a^3bd^2f^2(de+cf)+384b^4ce(d^2e^2+3cdef+c^2f^2)+1680a^2b^2df(d^2e^2+3cdef+c^2f}{128b^{13/2}}$$

output
```
1/3*(-a*d+b*c)^3*(-a*f+b*e)^3*x/a/b^6/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)^2*(-a
*f+b*e)^2*(2*b^2*c*e-16*a^2*d*f+7*a*b*(c*f+d*e))*x/a^2/b^6/(b*x^2+a)^(1/2)
-1/128*(515*a^3*d^3*f^3-984*a^2*b*d^2*f^2*(c*f+d*e)+528*a*b^2*d*f*(c^2*f^2
+3*c*d*e*f+d^2*e^2)-64*b^3*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*
x*(b*x^2+a)^(1/2)/b^6+1/192*d*f*(259*a^2*d^2*f^2-408*a*b*d*f*(c*f+d*e)+144
*b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(1/2)/b^5-1/48*d^2*f^2*(23
*a*d*f-24*b*(c*f+d*e))*x^5*(b*x^2+a)^(1/2)/b^4+1/8*d^3*f^3*x^7*(b*x^2+a)^(
1/2)/b^3+1/128*(1155*a^4*d^3*f^3-2520*a^3*b*d^2*f^2*(c*f+d*e)+384*b^4*c*e*
(c^2*f^2+3*c*d*e*f+d^2*e^2)+1680*a^2*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-3
20*a*b^3*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^2)^3(e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b}x(-3465a^7d^3f^3+256b^7c^3e^3x^2+420a^6bd^2f^2(18de+18cf-11dfx^2)+384ab^6c^2e^2(dex^2+c(e+fx^2))-6}{}$$

input
```
Integrate[((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
((Sqrt[b]*x*(-3465*a^7*d^3*f^3 + 256*b^7*c^3*e^3*x^2 + 420*a^6*b*d^2*f^2*(
18*d*e + 18*c*f - 11*d*f*x^2) + 384*a*b^6*c^2*e^2*(d*e*x^2 + c*(e + f*x^2)
) - 63*a^5*b^2*d*f*(80*c^2*f^2 + 80*c*d*f*(3*e - 2*f*x^2) + d^2*(80*e^2 -
160*e*f*x^2 + 11*f^2*x^4)) + 48*a^2*b^5*x^2*(4*c^3*f^2*(-8*e + f*x^2) + 6*
c^2*d*f*(-16*e^2 + 6*e*f*x^2 + f^2*x^4) + d^3*x^2*(4*e^3 + 6*e^2*f*x^2 + 4
*e*f^2*x^4 + f^3*x^6) + 2*c*d^2*(-16*e^3 + 18*e^2*f*x^2 + 9*e*f^2*x^4 + 2*
f^3*x^6)) - 8*a^3*b^4*(16*c^3*f^2*(9*e - 10*f*x^2) + 18*c^2*d*f*(24*e^2 -
80*e*f*x^2 + 7*f^2*x^4) + 18*c*d^2*(8*e^3 - 80*e^2*f*x^2 + 21*e*f^2*x^4 +
3*f^3*x^6) + d^3*x^2*(-160*e^3 + 126*e^2*f*x^2 + 54*e*f^2*x^4 + 11*f^3*x^6
)) + 6*a^4*b^3*(160*c^3*f^3 + 160*c^2*d*f^2*(9*e - 7*f*x^2) + 12*c*d^2*f*(
120*e^2 - 280*e*f*x^2 + 21*f^2*x^4) + d^3*(160*e^3 - 1120*e^2*f*x^2 + 252*
e*f^2*x^4 + 33*f^3*x^6))))/(a^2*(a + b*x^2)^(3/2)) - 3*(1155*a^4*d^3*f^3 -
 2520*a^3*b*d^2*f^2*(d*e + c*f) + 384*b^4*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f
^2) + 1680*a^2*b^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) - 320*a*b^3*(d^3*e^
3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(384*b^(13/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 1056, normalized size of antiderivative = 1.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{c^3e^3}{(a+bx^2)^{5/2}} + \frac{3dfx^8(c^2f^2+3cdef+d^2e^2)}{(a+bx^2)^{5/2}} + \frac{x^6(cf+de)(c^2f^2+8cdef+d^2e^2)}{(a+bx^2)^{5/2}} + \frac{3cex^4(c^2f^2+3cdef+}{(a+bx^2)^{5/2}}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^3f^3x^{11}}{3b(bx^2+a)^{3/2}} - \frac{11d^3f^3x^9}{3b^2\sqrt{bx^2+a}} - \frac{d^2f^2(de+cf)x^9}{b(bx^2+a)^{3/2}} + \frac{33d^3f^3\sqrt{bx^2+a}\,x^7}{8b^3} - \frac{9d^2f^2(de+cf)x^7}{b^2\sqrt{bx^2+a}} - \frac{df(d^2e^2+3cdfe+c^2f^2)x^7}{b(bx^2+a)^{3/2}} - \frac{77ad^3f^3\sqrt{bx^2+a}\,x^5}{16b^4} + \frac{21d^2f^2(de+cf)\sqrt{bx^2+a}\,x^5}{2b^3} - \frac{7df(d^2e^2+3cdfe+c^2f^2)x^5}{b^2\sqrt{bx^2+a}} - \frac{(de+cf)(d^2e^2+8cdfe+c^2f^2)x^5}{3b(bx^2+a)^{3/2}} + \frac{385a^2d^3f^3\sqrt{bx^2+a}\,x^3}{64b^5} - \frac{105ad^2f^2(de+cf)\sqrt{bx^2+a}\,x^3}{8b^4} + \frac{35df(d^2e^2+3cdfe+c^2f^2)\sqrt{bx^2+a}\,x^3}{4b^3} - \frac{5(de+cf)(d^2e^2+8cdfe+c^2f^2)x^3}{3b^2\sqrt{bx^2+a}} + \frac{c^2e^2(de+cf)x^3}{a(bx^2+a)^{3/2}} - \frac{ce(d^2e^2+3cdfe+c^2f^2)x^3}{b(bx^2+a)^{3/2}} - \frac{1155a^3d^3f^3\sqrt{bx^2+a}\,x}{128b^6} + \frac{315a^2d^2f^2(de+cf)\sqrt{bx^2+a}\,x}{16b^5} - \frac{105adf(d^2e^2+3cdfe+c^2f^2)\sqrt{bx^2+a}\,x}{8b^4} + \frac{5(de+cf)(d^2e^2+8cdfe+c^2f^2)\sqrt{bx^2+a}\,x}{2b^3} + \frac{2c^3e^3x}{3a^2\sqrt{bx^2+a}} - \frac{3ce(d^2e^2+3cdfe+c^2f^2)x}{b^2\sqrt{bx^2+a}} + \frac{c^3e^3x}{3a(bx^2+a)^{3/2}} + \frac{1155a^4d^3f^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{128b^{13/2}} - \frac{315a^3d^2f^2(de+cf)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{16b^{11/2}} + \frac{3ce(d^2e^2+3cdfe+c^2f^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{b^{5/2}} + \frac{105a^2df(d^2e^2+3cdfe+c^2f^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{8b^{9/2}} - \frac{5a(de+cf)(d^2e^2+8cdfe+c^2f^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2b^{7/2}}$$

input
```
Int[((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x]
```

output
```
(c^3*e^3*x)/(3*a*(a + b*x^2)^(3/2)) + (c^2*e^2*(d*e + c*f)*x^3)/(a*(a + b*
x^2)^(3/2)) - (c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x^3)/(b*(a + b*x^2)^(3/
2)) - ((d*e + c*f)*(d^2*e^2 + 8*c*d*e*f + c^2*f^2)*x^5)/(3*b*(a + b*x^2)^(
3/2)) - (d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x^7)/(b*(a + b*x^2)^(3/2)) -
(d^2*f^2*(d*e + c*f)*x^9)/(b*(a + b*x^2)^(3/2)) - (d^3*f^3*x^11)/(3*b*(a +
 b*x^2)^(3/2)) + (2*c^3*e^3*x)/(3*a^2*Sqrt[a + b*x^2]) - (3*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2)*x)/(b^2*Sqrt[a + b*x^2]) - (5*(d*e + c*f)*(d^2*e^2 +
 8*c*d*e*f + c^2*f^2)*x^3)/(3*b^2*Sqrt[a + b*x^2]) - (7*d*f*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2)*x^5)/(b^2*Sqrt[a + b*x^2]) - (9*d^2*f^2*(d*e + c*f)*x^7)
/(b^2*Sqrt[a + b*x^2]) - (11*d^3*f^3*x^9)/(3*b^2*Sqrt[a + b*x^2]) - (1155*
a^3*d^3*f^3*x*Sqrt[a + b*x^2])/(128*b^6) + (315*a^2*d^2*f^2*(d*e + c*f)*x*
Sqrt[a + b*x^2])/(16*b^5) - (105*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)*x*S
qrt[a + b*x^2])/(8*b^4) + (5*(d*e + c*f)*(d^2*e^2 + 8*c*d*e*f + c^2*f^2)*x
*Sqrt[a + b*x^2])/(2*b^3) + (385*a^2*d^3*f^3*x^3*Sqrt[a + b*x^2])/(64*b^5)
 - (105*a*d^2*f^2*(d*e + c*f)*x^3*Sqrt[a + b*x^2])/(8*b^4) + (35*d*f*(d^2*
e^2 + 3*c*d*e*f + c^2*f^2)*x^3*Sqrt[a + b*x^2])/(4*b^3) - (77*a*d^3*f^3*x^
5*Sqrt[a + b*x^2])/(16*b^4) + (21*d^2*f^2*(d*e + c*f)*x^5*Sqrt[a + b*x^2])
/(2*b^3) + (33*d^3*f^3*x^7*Sqrt[a + b*x^2])/(8*b^3) + (1155*a^4*d^3*f^3*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(13/2)) - (315*a^3*d^2*f^2*(d*e
 + c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(11/2)) + (3*c*e*(d...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{1155a^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a^{4}d^{3}f^{3}-\frac{24a^{3}b\,d^{2}f^{2}(cf+de)}{11}+\frac{16a^{2}b^{2}df\left(c^{2}f^{2}+3cdef+d^{2}e^{2}\right)}{11}-\frac{64b^{3}(cf+de)\left(c^{2}f^{2}+8cdef+d^{2}e^{2}\right)a}{231}+\frac{128b^{4}ce\left(c^{2}\right.}{}\right.}{128}$
default	$c^{3}e^{3}\left(\frac{x}{3a(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)+3d^{2}f^{2}(cf+de)\left(\frac{x^{9}}{6b(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{3a\left(\frac{x^{7}}{4b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^{5}}{2b\left(b\,x^{2}+a\right)}\right.}{}\right)}{}\right.$

input
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1155/128/b^(13/2)/(b*x^2+a)^(3/2)*(a^2*(b*x^2+a)^(3/2)*(a^4*d^3*f^3-24/11*
a^3*b*d^2*f^2*(c*f+d*e)+16/11*a^2*b^2*d*f*(c^2*f^2+3*c*d*e*f+d^2*e^2)-64/2
31*b^3*(c*f+d*e)*(c^2*f^2+8*c*d*e*f+d^2*e^2)*a+128/385*b^4*c*e*(c^2*f^2+3*
c*d*e*f+d^2*e^2))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(512/1155*a^2*x^2*(-1
/8*(1/2*f^2*x^4+e*f*x^2+e^2)*x^2*(1/2*f*x^2+e)*d^3+c*(-1/8*f^3*x^6-9/16*e*
f^2*x^4-9/8*e^2*f*x^2+e^3)*d^2+3*(-1/2*f*x^2+e)*(1/8*f*x^2+e)*c^2*f*d+c^3*
f^2*(-1/8*f*x^2+e))*b^(11/2)+128/385*a^3*((11/144*f^3*x^8+3/8*e*f^2*x^6+7/
8*e^2*f*x^4-10/9*e^3*x^2)*d^3+c*(3/8*f^3*x^6+21/8*e*f^2*x^4-10*e^2*f*x^2+e
^3)*d^2+3*(7/24*f^2*x^4-10/3*e*f*x^2+e^2)*c^2*f*d+c^3*f^2*(-10/9*f*x^2+e))
*b^(9/2)-64/231*a^4*((33/160*f^3*x^6+63/40*e*f^2*x^4-7*e^2*f*x^2+e^3)*d^3+
9*(7/40*f^2*x^4-7/3*e*f*x^2+e^2)*c*f*d^2+9*c^2*(-7/9*f*x^2+e)*f^2*d+c^3*f^
3)*b^(7/2)+16/11*a^5*d*((11/80*f^2*x^4-2*e*f*x^2+e^2)*d^2+3*c*(-2/3*f*x^2+
e)*f*d+c^2*f^2)*f*b^(5/2)-128/1155*a*c^2*e^2*(d*e*x^2+c*(f*x^2+e))*b^(13/2
)-24/11*a^6*d^2*f^2*((-11/18*f*x^2+e)*d+c*f)*b^(3/2)+b^(1/2)*a^7*d^3*f^3-2
56/3465*b^(15/2)*c^3*e^3*x^2)*x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(501) = 1002$.

Time = 3.09 (sec) , antiderivative size = 2602, normalized size of antiderivative = 4.88

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(3*((64*(6*a^2*b^6*c*d^2 - 5*a^3*b^5*d^3)*e^3 + 48*(24*a^2*b^6*c^2*
d - 60*a^3*b^5*c*d^2 + 35*a^4*b^4*d^3)*e^2*f + 24*(16*a^2*b^6*c^3 - 120*a^
3*b^5*c^2*d + 210*a^4*b^4*c*d^2 - 105*a^5*b^3*d^3)*e*f^2 - 5*(64*a^3*b^5*c
^3 - 336*a^4*b^4*c^2*d + 504*a^5*b^3*c*d^2 - 231*a^6*b^2*d^3)*f^3)*x^4 + 6
4*(6*a^4*b^4*c*d^2 - 5*a^5*b^3*d^3)*e^3 + 48*(24*a^4*b^4*c^2*d - 60*a^5*b^
3*c*d^2 + 35*a^6*b^2*d^3)*e^2*f + 24*(16*a^4*b^4*c^3 - 120*a^5*b^3*c^2*d +
 210*a^6*b^2*c*d^2 - 105*a^7*b*d^3)*e*f^2 - 5*(64*a^5*b^3*c^3 - 336*a^6*b^
2*c^2*d + 504*a^7*b*c*d^2 - 231*a^8*d^3)*f^3 + 2*(64*(6*a^3*b^5*c*d^2 - 5*
a^4*b^4*d^3)*e^3 + 48*(24*a^3*b^5*c^2*d - 60*a^4*b^4*c*d^2 + 35*a^5*b^3*d^
3)*e^2*f + 24*(16*a^3*b^5*c^3 - 120*a^4*b^4*c^2*d + 210*a^5*b^3*c*d^2 - 10
5*a^6*b^2*d^3)*e*f^2 - 5*(64*a^4*b^4*c^3 - 336*a^5*b^3*c^2*d + 504*a^6*b^2
*c*d^2 - 231*a^7*b*d^3)*f^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(48*a^2*b^6*d^3*f^3*x^11 + 8*(24*a^2*b^6*d^3*e*f^2 + (
24*a^2*b^6*c*d^2 - 11*a^3*b^5*d^3)*f^3)*x^9 + 18*(16*a^2*b^6*d^3*e^2*f + 2
4*(2*a^2*b^6*c*d^2 - a^3*b^5*d^3)*e*f^2 + (16*a^2*b^6*c^2*d - 24*a^3*b^5*c
*d^2 + 11*a^4*b^4*d^3)*f^3)*x^7 + 3*(64*a^2*b^6*d^3*e^3 + 48*(12*a^2*b^6*c
*d^2 - 7*a^3*b^5*d^3)*e^2*f + 72*(8*a^2*b^6*c^2*d - 14*a^3*b^5*c*d^2 + 7*a
^4*b^4*d^3)*e*f^2 + (64*a^2*b^6*c^3 - 336*a^3*b^5*c^2*d + 504*a^4*b^4*c*d^
2 - 231*a^5*b^3*d^3)*f^3)*x^5 + 4*(32*(2*b^8*c^3 + 3*a*b^7*c^2*d - 12*a^2*
b^6*c*d^2 + 10*a^3*b^5*d^3)*e^3 + 48*(2*a*b^7*c^3 - 24*a^2*b^6*c^2*d + ...
```

Sympy [F]

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3*(f*x**2+e)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**3*(e + f*x**2)**3/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(501) = 1002$.

Time = 0.05 (sec) , antiderivative size = 1158, normalized size of antiderivative = 2.17

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/8*d^3*f^3*x^11/((b*x^2 + a)^(3/2)*b) - 11/48*a*d^3*f^3*x^9/((b*x^2 + a)^
(3/2)*b^2) + 33/64*a^2*d^3*f^3*x^7/((b*x^2 + a)^(3/2)*b^3) - 231/128*a^3*d
^3*f^3*x^5/((b*x^2 + a)^(3/2)*b^4) - 385/128*a^4*d^3*f^3*x*(3*x^2/((b*x^2
+ a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 1/2*(d^3*e*f^2 + c*d^2*
f^3)*x^9/((b*x^2 + a)^(3/2)*b) - 9/8*(d^3*e*f^2 + c*d^2*f^3)*a*x^7/((b*x^2
 + a)^(3/2)*b^2) + 3/4*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3)*x^7/((b*x^2
 + a)^(3/2)*b) + 2/3*c^3*e^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3*e^3*x/((b*x
^2 + a)^(3/2)*a) - 385/128*a^4*d^3*f^3*x/(sqrt(b*x^2 + a)*b^6) + 1155/128*
a^4*d^3*f^3*arcsinh(b*x/sqrt(a*b))/b^(13/2) + 63/16*(d^3*e*f^2 + c*d^2*f^3
)*a^2*x^5/((b*x^2 + a)^(3/2)*b^3) - 21/8*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*
d*f^3)*a*x^5/((b*x^2 + a)^(3/2)*b^2) + 1/2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + c^3*f^3)*x^5/((b*x^2 + a)^(3/2)*b) - (c*d^2*e^3 + 3*c^2*d*e^2*
f + c^3*e*f^2)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2
)) + 105/16*(d^3*e*f^2 + c*d^2*f^3)*a^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 35/8*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*
f^3)*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2
 + 5/6*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*a*x*(3*x^2/((b*
x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b + 105/16*(d^3*e*f^2 + c
*d^2*f^3)*a^3*x/(sqrt(b*x^2 + a)*b^5) - 35/8*(d^3*e^2*f + 3*c*d^2*e*f^2 +
c^2*d*f^3)*a^2*x/(sqrt(b*x^2 + a)*b^4) + 5/6*(d^3*e^3 + 9*c*d^2*e^2*f +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(501) = 1002$.

Time = 0.18 (sec) , antiderivative size = 1015, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/384*(((2*(4*(6*d^3*f^3*x^2/b + (24*a^2*b^10*d^3*e*f^2 + 24*a^2*b^10*c*d^
2*f^3 - 11*a^3*b^9*d^3*f^3)/(a^2*b^11))*x^2 + 9*(16*a^2*b^10*d^3*e^2*f + 4
8*a^2*b^10*c*d^2*e*f^2 - 24*a^3*b^9*d^3*e*f^2 + 16*a^2*b^10*c^2*d*f^3 - 24
*a^3*b^9*c*d^2*f^3 + 11*a^4*b^8*d^3*f^3)/(a^2*b^11))*x^2 + 3*(64*a^2*b^10*
d^3*e^3 + 576*a^2*b^10*c*d^2*e^2*f - 336*a^3*b^9*d^3*e^2*f + 576*a^2*b^10*
c^2*d*e*f^2 - 1008*a^3*b^9*c*d^2*e*f^2 + 504*a^4*b^8*d^3*e*f^2 + 64*a^2*b^
10*c^3*f^3 - 336*a^3*b^9*c^2*d*f^3 + 504*a^4*b^8*c*d^2*f^3 - 231*a^5*b^7*d
^3*f^3)/(a^2*b^11))*x^2 + 4*(64*b^12*c^3*e^3 + 96*a*b^11*c^2*d*e^3 - 384*a
^2*b^10*c*d^2*e^3 + 320*a^3*b^9*d^3*e^3 + 96*a*b^11*c^3*e^2*f - 1152*a^2*b
^10*c^2*d*e^2*f + 2880*a^3*b^9*c*d^2*e^2*f - 1680*a^4*b^8*d^3*e^2*f - 384*
a^2*b^10*c^3*e*f^2 + 2880*a^3*b^9*c^2*d*e*f^2 - 5040*a^4*b^8*c*d^2*e*f^2 +
 2520*a^5*b^7*d^3*e*f^2 + 320*a^3*b^9*c^3*f^3 - 1680*a^4*b^8*c^2*d*f^3 + 2
520*a^5*b^7*c*d^2*f^3 - 1155*a^6*b^6*d^3*f^3)/(a^2*b^11))*x^2 + 3*(128*a*b
^11*c^3*e^3 - 384*a^3*b^9*c*d^2*e^3 + 320*a^4*b^8*d^3*e^3 - 1152*a^3*b^9*c
^2*d*e^2*f + 2880*a^4*b^8*c*d^2*e^2*f - 1680*a^5*b^7*d^3*e^2*f - 384*a^3*b
^9*c^3*e*f^2 + 2880*a^4*b^8*c^2*d*e*f^2 - 5040*a^5*b^7*c*d^2*e*f^2 + 2520*
a^6*b^6*d^3*e*f^2 + 320*a^4*b^8*c^3*f^3 - 1680*a^5*b^7*c^2*d*f^3 + 2520*a^
6*b^6*c*d^2*f^3 - 1155*a^7*b^5*d^3*f^3)/(a^2*b^11))*x/(b*x^2 + a)^(3/2) -
1/128*(384*b^4*c*d^2*e^3 - 320*a*b^3*d^3*e^3 + 1152*b^4*c^2*d*e^2*f - 2880
*a*b^3*c*d^2*e^2*f + 1680*a^2*b^2*d^3*e^2*f + 384*b^4*c^3*e*f^2 - 2880*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3\,(f\,x^2+e)^3}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^3*(e + f*x^2)^3)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^3 (e+fx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3\,(f\,x^2+e)^3}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^3*(f*x^2+e)^3/(b*x^2+a)^(5/2),x)
```

3.371 $\int \frac{(c+dx^2)^3(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 368

$$\int \frac{(c+dx^2)^3(e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^3(be-af)^2x}{3ab^5(a+bx^2)^{3/2}}$$
$$+\frac{(bc-ad)^2(be-af)\left(2b^2ce-13a^2df+ab(7de+4cf)\right)x}{3a^2b^5\sqrt{a+bx^2}}$$
$$+\frac{d(41a^2d^2f^2-22abdf(2de+3cf)+8b^2(d^2e^2+6cdef+3c^2f^2))\,x\sqrt{a+bx^2}}{16b^5}$$
$$+\frac{d^2f(12bde+18bcf-17adf)x^3\sqrt{a+bx^2}}{24b^4}+\frac{d^3f^2x^5\sqrt{a+bx^2}}{6b^3}$$
$$-\frac{(105a^3d^3f^2-70a^2bd^2f(2de+3cf)-16b^3c(3d^2e^2+6cdef+c^2f^2)+40ab^2d(d^2e^2+6cdef+3c^2f^2))\,\mathrm{arc}}{16b^{11/2}}$$

output

```
1/3*(-a*d+b*c)^3*(-a*f+b*e)^2*x/a/b^5/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)^2*(-a
*f+b*e)*(2*b^2*c*e-13*a^2*d*f+a*b*(4*c*f+7*d*e))*x/a^2/b^5/(b*x^2+a)^(1/2)
+1/16*d*(41*a^2*d^2*f^2-22*a*b*d*f*(3*c*f+2*d*e)+8*b^2*(3*c^2*f^2+6*c*d*e*
f+d^2*e^2))*x*(b*x^2+a)^(1/2)/b^5+1/24*d^2*f*(-17*a*d*f+18*b*c*f+12*b*d*e)
*x^3*(b*x^2+a)^(1/2)/b^4+1/6*d^3*f^2*x^5*(b*x^2+a)^(1/2)/b^3-1/16*(105*a^3
*d^3*f^2-70*a^2*b*d^2*f*(3*c*f+2*d*e)-16*b^3*c*(c^2*f^2+6*c*d*e*f+3*d^2*e^
2)+40*a*b^2*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(
1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b}x(315a^6d^3f^2+32b^6c^3e^2x^2+16ab^5c^2e(3ce+3dex^2+2cfx^2)+210a^5bd^2f(-2de-3cf+2dfx^2)+4a^2b^4x^2}{}$$

input
```
Integrate[((c + d*x^2)^3*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
((Sqrt[b]*x*(315*a^6*d^3*f^2 + 32*b^6*c^3*e^2*x^2 + 16*a*b^5*c^2*e*(3*c*e
+ 3*d*e*x^2 + 2*c*f*x^2) + 210*a^5*b*d^2*f*(-2*d*e - 3*c*f + 2*d*f*x^2) +
4*a^2*b^4*x^2*(-16*c^3*f^2 + 6*c^2*d*f*(-16*e + 3*f*x^2) + 2*d^3*x^2*(3*e^
2 + 3*e*f*x^2 + f^2*x^4) + 3*c*d^2*(-16*e^2 + 12*e*f*x^2 + 3*f^2*x^4)) - 2
*a^3*b^3*(24*c^3*f^2 + 48*c^2*d*f*(3*e - 5*f*x^2) + d^3*x^2*(-80*e^2 + 42*
e*f*x^2 + 9*f^2*x^4) + 3*c*d^2*(24*e^2 - 160*e*f*x^2 + 21*f^2*x^4)) + a^4*
b^2*d*(360*c^2*f^2 + 120*c*d*f*(6*e - 7*f*x^2) + d^2*(120*e^2 - 560*e*f*x^
2 + 63*f^2*x^4))))/(a^2*(a + b*x^2)^(3/2)) - 3*(-105*a^3*d^3*f^2 + 70*a^2*
b*d^2*f*(2*d*e + 3*c*f) + 16*b^3*c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) - 40*
a*b^2*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(48*b^(11/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 750 vs. 2(368) = 736.

Time = 0.83 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{c^3e^2}{(a+bx^2)^{5/2}}+\frac{dx^6(3c^2f^2+6cdef+d^2e^2)}{(a+bx^2)^{5/2}}+\frac{cx^4(c^2f^2+6cdef+3d^2e^2)}{(a+bx^2)^{5/2}}+\frac{c^2ex^2(2cf+3de)}{(a+bx^2)^{5/2}}+\frac{d^2fx^8(3c}{(a+bx}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{105a^3d^3f^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{11/2}}+\frac{35a^2d^2f\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3cf+2de)}{8b^{9/2}}+\frac{105a^2d^3f^2x\sqrt{a+bx^2}}{16b^5}+\frac{2c^3e^2x}{3a^2\sqrt{a+bx^2}}+\frac{c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(c^2f^2+6cdef+3d^2e^2)}{b^{5/2}}-\frac{5ad\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3c^2f^2+6cdef+d^2e^2)}{2b^{7/2}}-\frac{35ad^2fx\sqrt{a+bx^2}(3cf+2de)}{8b^4}-\frac{35ad^3f^2x^3\sqrt{a+bx^2}}{8b^4}+\frac{5dx\sqrt{a+bx^2}(3c^2f^2+6cdef+d^2e^2)}{2b^3}+\frac{35d^2fx^3\sqrt{a+bx^2}(3cf+2de)}{12b^3}+\frac{7d^3f^2x^5\sqrt{a+bx^2}}{2b^3}-\frac{cx(c^2f^2+6cdef+3d^2e^2)}{b^2\sqrt{a+bx^2}}-\frac{5dx^3(3c^2f^2+6cdef+d^2e^2)}{3b^2\sqrt{a+bx^2}}-\frac{7d^2fx^5(3cf+2de)}{3b^2\sqrt{a+bx^2}}-\frac{3d^3f^2x^7}{b^2\sqrt{a+bx^2}}+\frac{c^3e^2x}{3a(a+bx^2)^{3/2}}-\frac{dx^5(3c^2f^2+6cdef+d^2e^2)}{3b(a+bx^2)^{3/2}}-\frac{cx^3(c^2f^2+6cdef+3d^2e^2)}{3b(a+bx^2)^{3/2}}+\frac{c^2ex^3(2cf+3de)}{3a(a+bx^2)^{3/2}}-\frac{d^2fx^7(3cf+2de)}{3b(a+bx^2)^{3/2}}-\frac{d^3f^2x^9}{3b(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)^3*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(c^3*e^2*x)/(3*a*(a + b*x^2)^(3/2)) + (c^2*e*(3*d*e + 2*c*f)*x^3)/(3*a*(a
+ b*x^2)^(3/2)) - (c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*x^3)/(3*b*(a + b*x^
2)^(3/2)) - (d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x^5)/(3*b*(a + b*x^2)^(3/
2)) - (d^2*f*(2*d*e + 3*c*f)*x^7)/(3*b*(a + b*x^2)^(3/2)) - (d^3*f^2*x^9)/
(3*b*(a + b*x^2)^(3/2)) + (2*c^3*e^2*x)/(3*a^2*Sqrt[a + b*x^2]) - (c*(3*d^
2*e^2 + 6*c*d*e*f + c^2*f^2)*x)/(b^2*Sqrt[a + b*x^2]) - (5*d*(d^2*e^2 + 6*
c*d*e*f + 3*c^2*f^2)*x^3)/(3*b^2*Sqrt[a + b*x^2]) - (7*d^2*f*(2*d*e + 3*c*
f)*x^5)/(3*b^2*Sqrt[a + b*x^2]) - (3*d^3*f^2*x^7)/(b^2*Sqrt[a + b*x^2]) +
(105*a^2*d^3*f^2*x*Sqrt[a + b*x^2])/(16*b^5) - (35*a*d^2*f*(2*d*e + 3*c*f)
*x*Sqrt[a + b*x^2])/(8*b^4) + (5*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2)*x*Sqr
t[a + b*x^2])/(2*b^3) - (35*a*d^3*f^2*x^3*Sqrt[a + b*x^2])/(8*b^4) + (35*d
^2*f*(2*d*e + 3*c*f)*x^3*Sqrt[a + b*x^2])/(12*b^3) + (7*d^3*f^2*x^5*Sqrt[a
 + b*x^2])/(2*b^3) - (105*a^3*d^3*f^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(16*b^(11/2)) + (35*a^2*d^2*f*(2*d*e + 3*c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(9/2)) + (c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2) - (5*a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f
^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{105\left(a^3d^3f^2-2d^2\left(cf+\frac{2de}{3}\right)bf\,a^2+\frac{8d\left(c^2f^2+2cdef+\frac{1}{3}d^2e^2\right)b^2a}{7}-\frac{16b^3c\left(c^2f^2+6cdef+3d^2e^2\right)}{105}\right)a^2\left(b\,x^2+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right.}{16}$
default	$e^2c^3\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^2f(3cf+2de)\left(\frac{x^7}{4b(b\,x^2+a)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^5}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+\right.}\right.}{}\right.}{}\right.$
risch	Expression too large to display

input
```
int((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)/b^(11/2)*(-315/32*(a^3*d^3*f^2-2*d^2*(c*f+2/3*d*e)*b*f
*a^2+8/7*d*(c^2*f^2+2*c*d*e*f+1/3*d^2*e^2)*b^2*a-16/105*b^3*c*(c^2*f^2+6*c
*d*e*f+3*d^2*e^2))*a^2*(b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+
(-3/2*a^3*((3/8*f^2*x^6+7/4*e*f*x^4-10/3*e^2*x^2)*d^3+3*c*(7/8*f^2*x^4-20/
3*e*f*x^2+e^2)*d^2+6*c^2*(-5/3*f*x^2+e)*f*d+f^2*c^3)*b^(7/2)+45/4*((7/40*f
^2*x^4-14/9*e*f*x^2+1/3*e^2)*d^2+2*c*(-7/6*f*x^2+e)*f*d+c^2*f^2)*d*a^4*b^(
5/2)-315/16*d^2*(2/3*(-f*x^2+e)*d+c*f)*f*a^5*b^(3/2)+315/32*a^6*d^3*f^2*b^
(1/2)+(-2*(-3/8*(1/3*f^2*x^4+e*f*x^2+e^2)*x^2*d^3+3*c*(-3/16*f^2*x^4-3/4*e
*f*x^2+e^2)*d^2+6*c^2*(-3/16*f*x^2+e)*f*d+f^2*c^3)*x^2*a^2+3/2*c^2*b*(d*e*
x^2+c*(2/3*f*x^2+e))*e*a+b^2*c^3*e^2*x^2)*b^(9/2))*x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(340) = 680$.

Time = 1.19 (sec) , antiderivative size = 1754, normalized size of antiderivative = 4.77

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*((8*(6*a^2*b^5*c*d^2 - 5*a^3*b^4*d^3)*e^2 + 4*(24*a^2*b^5*c^2*d
- 60*a^3*b^4*c*d^2 + 35*a^4*b^3*d^3)*e*f + (16*a^2*b^5*c^3 - 120*a^3*b^4*c
^2*d + 210*a^4*b^3*c*d^2 - 105*a^5*b^2*d^3)*f^2)*x^4 + 8*(6*a^4*b^3*c*d^2
- 5*a^5*b^2*d^3)*e^2 + 4*(24*a^4*b^3*c^2*d - 60*a^5*b^2*c*d^2 + 35*a^6*b*d
^3)*e*f + (16*a^4*b^3*c^3 - 120*a^5*b^2*c^2*d + 210*a^6*b*c*d^2 - 105*a^7*
d^3)*f^2 + 2*(8*(6*a^3*b^4*c*d^2 - 5*a^4*b^3*d^3)*e^2 + 4*(24*a^3*b^4*c^2*
d - 60*a^4*b^3*c*d^2 + 35*a^5*b^2*d^3)*e*f + (16*a^3*b^4*c^3 - 120*a^4*b^3
*c^2*d + 210*a^5*b^2*c*d^2 - 105*a^6*b*d^3)*f^2)*x^2)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*a^2*b^5*d^3*f^2*x^9 + 6*(4*a^2*
b^5*d^3*e*f + 3*(2*a^2*b^5*c*d^2 - a^3*b^4*d^3)*f^2)*x^7 + 3*(8*a^2*b^5*d^
3*e^2 + 4*(12*a^2*b^5*c*d^2 - 7*a^3*b^4*d^3)*e*f + 3*(8*a^2*b^5*c^2*d - 14
*a^3*b^4*c*d^2 + 7*a^4*b^3*d^3)*f^2)*x^5 + 4*(4*(2*b^7*c^3 + 3*a*b^6*c^2*d
 - 12*a^2*b^5*c*d^2 + 10*a^3*b^4*d^3)*e^2 + 4*(2*a*b^6*c^3 - 24*a^2*b^5*c^
2*d + 60*a^3*b^4*c*d^2 - 35*a^4*b^3*d^3)*e*f - (16*a^2*b^5*c^3 - 120*a^3*b
^4*c^2*d + 210*a^4*b^3*c*d^2 - 105*a^5*b^2*d^3)*f^2)*x^3 + 3*(8*(2*a*b^6*c
^3 - 6*a^3*b^4*c*d^2 + 5*a^4*b^3*d^3)*e^2 - 4*(24*a^3*b^4*c^2*d - 60*a^4*b
^3*c*d^2 + 35*a^5*b^2*d^3)*e*f - (16*a^3*b^4*c^3 - 120*a^4*b^3*c^2*d + 210
*a^5*b^2*c*d^2 - 105*a^6*b*d^3)*f^2)*x)*sqrt(b*x^2 + a))/(a^2*b^8*x^4 + 2*
a^3*b^7*x^2 + a^4*b^6), -1/48*(3*((8*(6*a^2*b^5*c*d^2 - 5*a^3*b^4*d^3)*e^2
 + 4*(24*a^2*b^5*c^2*d - 60*a^3*b^4*c*d^2 + 35*a^4*b^3*d^3)*e*f + (16*a...
```

Sympy [F]

$$\int \frac{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{\left(c+dx^2\right)^3\left(e+fx^2\right)^2}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3*(f*x**2+e)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**3*(e + f*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(340) = 680$.

Time = 0.07 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*d^3*f^2*x^9/((b*x^2 + a)^(3/2)*b) - 3/8*a*d^3*f^2*x^7/((b*x^2 + a)^(3/
2)*b^2) + 21/16*a^2*d^3*f^2*x^5/((b*x^2 + a)^(3/2)*b^3) + 35/16*a^3*d^3*f^
2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 1/4*
(2*d^3*e*f + 3*c*d^2*f^2)*x^7/((b*x^2 + a)^(3/2)*b) + 2/3*c^3*e^2*x/(sqrt(
b*x^2 + a)*a^2) + 1/3*c^3*e^2*x/((b*x^2 + a)^(3/2)*a) + 35/16*a^3*d^3*f^2*
x/(sqrt(b*x^2 + a)*b^5) - 7/8*(2*d^3*e*f + 3*c*d^2*f^2)*a*x^5/((b*x^2 + a)
^(3/2)*b^2) + 1/2*(d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*x^5/((b*x^2 + a)^(
3/2)*b) - 105/16*a^3*d^3*f^2*arcsinh(b*x/sqrt(a*b))/b^(11/2) - 1/3*(3*c*d^
2*e^2 + 6*c^2*d*e*f + c^3*f^2)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^
2 + a)^(3/2)*b^2)) - 35/24*(2*d^3*e*f + 3*c*d^2*f^2)*a^2*x*(3*x^2/((b*x^2
+ a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 5/6*(d^3*e^2 + 6*c*d^2*
e*f + 3*c^2*d*f^2)*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/
2)*b^2))/b - 35/24*(2*d^3*e*f + 3*c*d^2*f^2)*a^2*x/(sqrt(b*x^2 + a)*b^4) +
 5/6*(d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3
*(3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(3*c^
2*d*e^2 + 2*c^3*e*f)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(3*c^2*d*e^2 + 2*c^3*e*
f)*x/(sqrt(b*x^2 + a)*a*b) + 35/8*(2*d^3*e*f + 3*c*d^2*f^2)*a^2*arcsinh(b*
x/sqrt(a*b))/b^(9/2) - 5/2*(d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*a*arcsinh
(b*x/sqrt(a*b))/b^(7/2) + (3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*arcsinh(b*
x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{4d^3f^2x^2}{b}+\frac{3(4a^2b^8d^3ef+6a^2b^8cd^2f^2-3a^3b^7d^3f^2)}{a^2b^9}\right)x^2+\frac{3(8a^2b^8d^3e^2+48a^2b^8cd^2ef-}{}\right.\right.\right.}{}$$

$$-\frac{(48b^3cd^2e^2-40ab^2d^3e^2+96b^3c^2def-240ab^2cd^2ef+140a^2bd^3ef+16b^3c^3f^2-120ab^2c^2df^2+210a^2}{16\,b^{\frac{11}{2}}}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/48*(((2*(4*d^3*f^2*x^2/b + 3*(4*a^2*b^8*d^3*e*f + 6*a^2*b^8*c*d^2*f^2 -
3*a^3*b^7*d^3*f^2)/(a^2*b^9))*x^2 + 3*(8*a^2*b^8*d^3*e^2 + 48*a^2*b^8*c*d^
2*e*f - 28*a^3*b^7*d^3*e*f + 24*a^2*b^8*c^2*d*f^2 - 42*a^3*b^7*c*d^2*f^2 +
 21*a^4*b^6*d^3*f^2)/(a^2*b^9))*x^2 + 4*(8*b^10*c^3*e^2 + 12*a*b^9*c^2*d*e
^2 - 48*a^2*b^8*c*d^2*e^2 + 40*a^3*b^7*d^3*e^2 + 8*a*b^9*c^3*e*f - 96*a^2*
b^8*c^2*d*e*f + 240*a^3*b^7*c*d^2*e*f - 140*a^4*b^6*d^3*e*f - 16*a^2*b^8*c
^3*f^2 + 120*a^3*b^7*c^2*d*f^2 - 210*a^4*b^6*c*d^2*f^2 + 105*a^5*b^5*d^3*f
^2)/(a^2*b^9))*x^2 + 3*(16*a*b^9*c^3*e^2 - 48*a^3*b^7*c*d^2*e^2 + 40*a^4*b
^6*d^3*e^2 - 96*a^3*b^7*c^2*d*e*f + 240*a^4*b^6*c*d^2*e*f - 140*a^5*b^5*d^
3*e*f - 16*a^3*b^7*c^3*f^2 + 120*a^4*b^6*c^2*d*f^2 - 210*a^5*b^5*c*d^2*f^2
 + 105*a^6*b^4*d^3*f^2)/(a^2*b^9))*x/(b*x^2 + a)^(3/2) - 1/16*(48*b^3*c*d^
2*e^2 - 40*a*b^2*d^3*e^2 + 96*b^3*c^2*d*e*f - 240*a*b^2*c*d^2*e*f + 140*a^
2*b*d^3*e*f + 16*b^3*c^3*f^2 - 120*a*b^2*c^2*d*f^2 + 210*a^2*b*c*d^2*f^2 -
 105*a^3*d^3*f^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3\,(f\,x^2+e)^2}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^3*(e + f*x^2)^2)/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^3*(e + f*x^2)^2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^3 (e+fx^2)^2}{(a+bx^2)^{5/2}} \, dx = \int \frac{(d\,x^2+c)^3 (f\,x^2+e)^2}{(b\,x^2+a)^{\frac{5}{2}}} dx$$

input
```
int((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^3*(f*x^2+e)^2/(b*x^2+a)^(5/2),x)
```

3.372 $\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 227

$$\begin{aligned}\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{(bc-ad)^3(be-af)x}{3ab^4(a+bx^2)^{3/2}} \\ &+ \frac{(bc-ad)^2(2b^2ce-10a^2df+ab(7de+cf))x}{3a^2b^4\sqrt{a+bx^2}} \\ &+ \frac{d^2(4bde+12bcf-11adf)x\sqrt{a+bx^2}}{8b^4} + \frac{d^3fx^3\sqrt{a+bx^2}}{4b^3} \\ &+ \frac{d(35a^2d^2f+24b^2c(de+cf)-20abd(de+3cf))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}\end{aligned}$$

output
```
1/3*(-a*d+b*c)^3*(-a*f+b*e)*x/a/b^4/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)^2*(2*b^
2*c*e-10*a^2*d*f+a*b*(c*f+7*d*e))*x/a^2/b^4/(b*x^2+a)^(1/2)+1/8*d^2*(-11*a
*d*f+12*b*c*f+4*b*d*e)*x*(b*x^2+a)^(1/2)/b^4+1/4*d^3*f*x^3*(b*x^2+a)^(1/2)
/b^3+1/8*d*(35*a^2*d^2*f+24*b^2*c*(c*f+d*e)-20*a*b*d*(3*c*f+d*e))*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{x(-105a^5d^3f+16b^5c^3ex^2+8ab^4c^2(3ce+3dex^2+cfx^2)+20a^4bd^2(3de+9c}{}$$
$$+\frac{d(-35a^2d^2f-24b^2c(de+cf)+20abd(de+3cf))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

input
```
Integrate[((c + d*x^2)^3*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(x*(-105*a^5*d^3*f + 16*b^5*c^3*e*x^2 + 8*a*b^4*c^2*(3*c*e + 3*d*e*x^2 + c
*f*x^2) + 20*a^4*b*d^2*(3*d*e + 9*c*f - 7*d*f*x^2) + 6*a^2*b^3*d*x^2*(-16*
c^2*f + d^2*x^2*(2*e + f*x^2) + c*d*(-16*e + 6*f*x^2)) + a^3*b^2*d*(-72*c^
2*f + d^2*x^2*(80*e - 21*f*x^2) + c*d*(-72*e + 240*f*x^2))))/(24*a^2*b^4*(
a + b*x^2)^(3/2)) + (d*(-35*a^2*d^2*f - 24*b^2*c*(d*e + c*f) + 20*a*b*d*(d
*e + 3*c*f))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {401, 25, 401, 27, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x(c+dx^2)^3(be-af)}{3ab(a+bx^2)^{3/2}}-\frac{\int -\frac{(dx^2+c)^2(c(2be+af)-d(4be-7af)x^2)}{(bx^2+a)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)^2\left(c(2be+af)-d(4be-7af)x^2\right)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)^3(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{x\left(c+dx^2\right)^2\left(bc(af+2be)+ad(4be-7af)\right)}{ab\sqrt{a+bx^2}}-\frac{\int \frac{d\left(dx^2+c\right)\left(\left(-35dfa^2+4b(5de+cf)a+8b^2ce\right)x^2+ac(4be-7af)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^3(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x\left(c+dx^2\right)^2\left(bc(af+2be)+ad(4be-7af)\right)}{ab\sqrt{a+bx^2}}-\frac{d\int \frac{\left(dx^2+c\right)\left(\left(-35dfa^2+4b(5de+cf)a+8b^2ce\right)x^2+ac(4be-7af)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}+\frac{x\left(c+dx^2\right)^3(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{x\left(c+dx^2\right)^2\left(bc(af+2be)+ad(4be-7af)\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{\int \frac{\left(105d^2fa^3-10bd(6de+11cf)a^2+8b^2c(4de+cf)a+16b^3c^2e\right)x^2+ac\left(35dfa^2-20bdea-32bcfa+8b^2ce\right)}{\sqrt{bx^2+a}}dx}{4b}+\cdots\right)}{ab}}{3ab}\;\cdots\;\frac{x\left(c+dx^2\right)^3(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{x\left(c+dx^2\right)^2\left(bc(af+2be)+ad(4be-7af)\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^2f-10a^2bd(11cf+6de)+8ab^2c(cf+4de)+16b^3c^2e\right)}{2b}-\frac{3a^2\left(35a^2d^2f-20abd(3cf+de)+24b\cdots\right.}{2b}}{4b}\cdots\right)}{ab}}{3ab}\;\cdots\;\frac{x\left(c+dx^2\right)^3(be-af)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{x(c+dx^2)^2(bc(af+2be)+ad(4be-7af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^2f-10a^2bd(11cf+6de)+8ab^2c(cf+4de)+16b^3c^2e\right)}{2b}-\frac{3a^2\left(35a^2d^2f-20abd(3cf+de)+24b\cdots\right.}{2\cdots}}{4b}\cdots\right.}{ab}}{3ab}$$

$$\frac{x(c+dx^2)^3(be-af)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x(c+dx^2)^2(bc(af+2be)+ad(4be-7af))}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(c+dx^2)\left(-35a^2df+4ab(cf+5de)+8b^2ce\right)}{4b}+\frac{\frac{x\sqrt{a+bx^2}\left(105a^3d^2f-10a^2bd(11cf+6de)+8ab^2c(cf\cdots\right.}{2b}\cdots}{\cdots}\right.}{ab}}{3ab}$$

$$\frac{x(c+dx^2)^3(be-af)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[((c + d*x^2)^3*(e + f*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((b*e - a*f)*x*(c + d*x^2)^3)/(3*a*b*(a + b*x^2)^(3/2)) + (((a*d*(4*b*e -
7*a*f) + b*c*(2*b*e + a*f))*x*(c + d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) - (d*((
(8*b^2*c*e - 35*a^2*d*f + 4*a*b*(5*d*e + c*f))*x*Sqrt[a + b*x^2]*(c + d*x^
2))/(4*b) + (((16*b^3*c^2*e + 105*a^3*d^2*f + 8*a*b^2*c*(4*d*e + c*f) - 10
*a^2*b*d*(6*d*e + 11*c*f))*x*Sqrt[a + b*x^2])/(2*b) - (3*a^2*(35*a^2*d^2*f
 + 24*b^2*c*(d*e + c*f) - 20*a*b*d*(d*e + 3*c*f))*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{\frac{35a^2d(b\,x^2+a)^{\frac{3}{2}}\left(f\,d^2a^2-\frac{12\left(cf+\frac{de}{3}\right)dba}{7}+\frac{24b^2c(cf+de)}{35}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}+\frac{2\left(-\frac{9d\left(\frac{\left(\frac{7}{8}f\,x^4-\frac{10}{3}e\,x^2\right)d^2}{3}+c\left(-\frac{10f\,x^2}{3}+e\right)d}{2}\right.}{}}{}$
default	$e\,c^3\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^2(3cf+de)\left(\frac{x^5}{2b(b\,x^2+a)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln(}{}}{b}\right.}{2b}\right.$
risch	$-\frac{d^2x\left(-2bdf\,x^2+11adf-12bcf-4bde\right)\sqrt{b\,x^2+a}}{8b^4}+\frac{\frac{d\left(35f\,d^2a^2-60fdcba-20ab\,d^2e+24f\,c^2b^2+24d\,b^2ce\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-}{}$

input
```
int((d*x^2+c)^3*(f*x^2+e)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)*(105/16*a^2*d*(b*x^2+a)^(3/2)*(f*d^2*a^2-12/7*(c*f+1/3
*d*e)*d*b*a+24/35*b^2*c*(c*f+d*e))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(-9/
2*d*(1/3*(7/8*f*x^4-10/3*e*x^2)*d^2+c*(-10/3*f*x^2+e)*d+c^2*f)*a^3*b^(5/2)
-6*d*(-1/8*x^2*(1/2*f*x^2+e)*d^2+c*(-3/8*f*x^2+e)*d+c^2*f)*x^2*a^2*b^(7/2)
+45/4*d^2*(1/3*(-7/3*f*x^2+e)*d+c*f)*a^4*b^(3/2)-105/16*a^5*d^3*f*b^(1/2)+
b^(9/2)*c^2*(3/2*(d*e*x^2+c*(1/3*f*x^2+e))*a+b*c*e*x^2))*x)/b^(9/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(203) = 406$.

Time = 0.43 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.46

$$\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((4*(6*a^2*b^4*c*d^2 - 5*a^3*b^3*d^3)*e + (24*a^2*b^4*c^2*d - 60*
a^3*b^3*c*d^2 + 35*a^4*b^2*d^3)*f)*x^4 + 2*(4*(6*a^3*b^3*c*d^2 - 5*a^4*b^2
*d^3)*e + (24*a^3*b^3*c^2*d - 60*a^4*b^2*c*d^2 + 35*a^5*b*d^3)*f)*x^2 + 4*
(6*a^4*b^2*c*d^2 - 5*a^5*b*d^3)*e + (24*a^4*b^2*c^2*d - 60*a^5*b*c*d^2 + 3
5*a^6*d^3)*f)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*a^2*b^4*d^3*f*x^7 + 3*(4*a^2*b^4*d^3*e + (12*a^2*b^4*c*d^2 - 7*a^3*b^3*
d^3)*f)*x^5 + 4*(2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a^3*
b^3*d^3)*e + (2*a*b^5*c^3 - 24*a^2*b^4*c^2*d + 60*a^3*b^3*c*d^2 - 35*a^4*b
^2*d^3)*f)*x^3 + 3*(4*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*d^3)*e -
(24*a^3*b^3*c^2*d - 60*a^4*b^2*c*d^2 + 35*a^5*b*d^3)*f)*x)*sqrt(b*x^2 + a)
)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5), -1/24*(3*((4*(6*a^2*b^4*c*d^2 -
 5*a^3*b^3*d^3)*e + (24*a^2*b^4*c^2*d - 60*a^3*b^3*c*d^2 + 35*a^4*b^2*d^3)
*f)*x^4 + 2*(4*(6*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*e + (24*a^3*b^3*c^2*d - 6
0*a^4*b^2*c*d^2 + 35*a^5*b*d^3)*f)*x^2 + 4*(6*a^4*b^2*c*d^2 - 5*a^5*b*d^3)
*e + (24*a^4*b^2*c^2*d - 60*a^5*b*c*d^2 + 35*a^6*d^3)*f)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (6*a^2*b^4*d^3*f*x^7 + 3*(4*a^2*b^4*d^3*e + (
12*a^2*b^4*c*d^2 - 7*a^3*b^3*d^3)*f)*x^5 + 4*(2*(2*b^6*c^3 + 3*a*b^5*c^2*d
 - 12*a^2*b^4*c*d^2 + 10*a^3*b^3*d^3)*e + (2*a*b^5*c^3 - 24*a^2*b^4*c^2*d
+ 60*a^3*b^3*c*d^2 - 35*a^4*b^2*d^3)*f)*x^3 + 3*(4*(2*a*b^5*c^3 - 6*a^3*b^
3*c*d^2 + 5*a^4*b^2*d^3)*e - (24*a^3*b^3*c^2*d - 60*a^4*b^2*c*d^2 + 35*...
```

Sympy [F]

$$\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**3*(f*x**2+e)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**3*(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(203) = 406.

Time = 0.08 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{d^3fx^7}{4(bx^2+a)^{\frac{3}{2}}b} - \frac{7ad^3fx^5}{8(bx^2+a)^{\frac{3}{2}}b^2} \\ &\quad - \frac{35a^2d^3fx\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24b^2} + \frac{(d^3e+3cd^2f)x^5}{2(bx^2+a)^{\frac{3}{2}}b} \\ &\quad - (cd^2e+c^2df)x\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) \\ &\quad + \frac{5(d^3e+3cd^2f)ax\left(\frac{3x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6b} + \frac{2c^3ex}{3\sqrt{bx^2+a}a^2} \\ &\quad + \frac{c^3ex}{3(bx^2+a)^{\frac{3}{2}}a} - \frac{35a^2d^3fx}{24\sqrt{bx^2+a}b^4} + \frac{35a^2d^3f\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{\frac{9}{2}}} \\ &\quad + \frac{5(d^3e+3cd^2f)ax}{6\sqrt{bx^2+a}b^3} - \frac{(cd^2e+c^2df)x}{\sqrt{bx^2+a}b^2} - \frac{(3c^2de+c^3f)x}{3(bx^2+a)^{\frac{3}{2}}b} + \frac{(3c^2de+c^3f)x}{3\sqrt{bx^2+a}ab} \\ &\quad - \frac{5(d^3e+3cd^2f)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b^{\frac{7}{2}}} + \frac{3(cd^2e+c^2df)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/4*d^3*f*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*a*d^3*f*x^5/((b*x^2 + a)^(3/2)*b
^2) - 35/24*a^2*d^3*f*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3
/2)*b^2))/b^2 + 1/2*(d^3*e + 3*c*d^2*f)*x^5/((b*x^2 + a)^(3/2)*b) - (c*d^2
*e + c^2*d*f)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)
) + 5/6*(d^3*e + 3*c*d^2*f)*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2
 + a)^(3/2)*b^2))/b + 2/3*c^3*e*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3*e*x/((b*
x^2 + a)^(3/2)*a) - 35/24*a^2*d^3*f*x/(sqrt(b*x^2 + a)*b^4) + 35/8*a^2*d^3
*f*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 5/6*(d^3*e + 3*c*d^2*f)*a*x/(sqrt(b*x^
2 + a)*b^3) - (c*d^2*e + c^2*d*f)*x/(sqrt(b*x^2 + a)*b^2) - 1/3*(3*c^2*d*e
 + c^3*f)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(3*c^2*d*e + c^3*f)*x/(sqrt(b*x^2
+ a)*a*b) - 5/2*(d^3*e + 3*c*d^2*f)*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3*(
c*d^2*e + c^2*d*f)*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx^2)^3(e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,d^3fx^2}{b} + \frac{4\,a^2b^6d^3e+12\,a^2b^6cd^2f-7\,a^3b^5d^3f}{a^2b^7}\right)x^2 + \frac{4\,(4\,b^8c^3e+6\,ab^7c^2de-24\,a^2b^6cd^2e+20\,a}{}\right.\right.}{}$$
$$- \frac{(24\,b^2cd^2e - 20\,abd^3e + 24\,b^2c^2df - 60\,abcd^2f + 35\,a^2d^3f)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^3*(f*x^2+e)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*d^3*f*x^2/b + (4*a^2*b^6*d^3*e + 12*a^2*b^6*c*d^2*f - 7*a^3*b^
5*d^3*f)/(a^2*b^7))*x^2 + 4*(4*b^8*c^3*e + 6*a*b^7*c^2*d*e - 24*a^2*b^6*c*
d^2*e + 20*a^3*b^5*d^3*e + 2*a*b^7*c^3*f - 24*a^2*b^6*c^2*d*f + 60*a^3*b^5
*c*d^2*f - 35*a^4*b^4*d^3*f)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^3*e - 24*a^3*b^
5*c*d^2*e + 20*a^4*b^4*d^3*e - 24*a^3*b^5*c^2*d*f + 60*a^4*b^4*c*d^2*f - 3
5*a^5*b^3*d^3*f)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(24*b^2*c*d^2*e - 20
*a*b*d^3*e + 24*b^2*c^2*d*f - 60*a*b*c*d^2*f + 35*a^2*d^3*f)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3 (e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3\,(f\,x^2+e)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^3*(e + f*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^3*(e + f*x^2))/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.41

$$\int \frac{(c+dx^2)^3 (e+fx^2)}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^3*(f*x^2+e)/(b*x^2+a)^(5/2),x)
```

output

```
( - 840*sqrt(a + b*x**2)*a**5*b*d**3*f*x + 1440*sqrt(a + b*x**2)*a**4*b**2
*c*d**2*f*x + 480*sqrt(a + b*x**2)*a**4*b**2*d**3*e*x - 1120*sqrt(a + b*x*
*2)*a**4*b**2*d**3*f*x**3 - 576*sqrt(a + b*x**2)*a**3*b**3*c**2*d*f*x - 57
6*sqrt(a + b*x**2)*a**3*b**3*c*d**2*e*x + 1920*sqrt(a + b*x**2)*a**3*b**3*
c*d**2*f*x**3 + 640*sqrt(a + b*x**2)*a**3*b**3*d**3*e*x**3 - 168*sqrt(a +
b*x**2)*a**3*b**3*d**3*f*x**5 - 768*sqrt(a + b*x**2)*a**2*b**4*c**2*d*f*x*
*3 - 768*sqrt(a + b*x**2)*a**2*b**4*c*d**2*e*x**3 + 288*sqrt(a + b*x**2)*a
**2*b**4*c*d**2*f*x**5 + 96*sqrt(a + b*x**2)*a**2*b**4*d**3*e*x**5 + 48*sq
rt(a + b*x**2)*a**2*b**4*d**3*f*x**7 + 192*sqrt(a + b*x**2)*a*b**5*c**3*e*
x + 64*sqrt(a + b*x**2)*a*b**5*c**3*f*x**3 + 192*sqrt(a + b*x**2)*a*b**5*c
**2*d*e*x**3 + 128*sqrt(a + b*x**2)*b**6*c**3*e*x**3 + 840*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**6*d**3*f - 1440*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*c*d**2*f - 480*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*d**3*e + 1680*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*d**3*f*x**2 + 576*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c**2*d*f + 576*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c*d**2*e - 2880*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c*d**2*f*x**2 - 960*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d**3*e*x**2 +
 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d**3...
```

3.373 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{(bc-ad)^3x}{3ab^2(be-af)\,(a+bx^2)^{3/2}} + \frac{(bc-ad)^2\,(2b^2ce-4a^2df+ab(7de-5cf))\,x}{3a^2b^2(be-af)^2\sqrt{a+bx^2}} + \frac{d^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}f} - \frac{(de-cf)^3\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f(be-af)^{5/2}}$$

output
```
1/3*(-a*d+b*c)^3*x/a/b^2/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/3*(-a*d+b*c)^2*(2*b^
2*c*e-4*a^2*d*f+a*b*(-5*c*f+7*d*e))*x/a^2/b^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)
+d^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)/f-(-c*f+d*e)^3*arctanh((-a
*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1/2)/f/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \frac{(bc-ad)^2x(-3a^3df+2b^3cex^2+ab^2(3ce+7dex^2-5cfx^2)+2a^2b(3de-3}{3a^2b^2(be-af)^2(a+bx^2)^{3/2}}$$
$$+\frac{(de-cf)^3\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}f(-be+af)^{5/2}}-\frac{d^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}f}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
((b*c - a*d)^2*x*(-3*a^3*d*f + 2*b^3*c*e*x^2 + a*b^2*(3*c*e + 7*d*e*x^2 -
5*c*f*x^2) + 2*a^2*b*(3*d*e - 3*c*f - 2*d*f*x^2)))/(3*a^2*b^2*(b*e - a*f)^
2*(a + b*x^2)^(3/2)) + ((d*e - c*f)^3*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqr
t[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*f*(-(b*e) + a*f)
^(5/2)) - (d^3*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(5/2)*f)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 466 vs. $2(206) = 412$.

Time = 0.89 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.26, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.633$, Rules used = {419, 25, 401, 25, 27, 401, 27, 299, 224, 219, 420, 299, 224, 219, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \ 419$$

$$-\frac{\int -\frac{(dx^2+c)^2(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(dx^2+c)^2(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 401

$$\frac{\frac{x(c+dx^2)^2(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}-\frac{\int -\frac{b\left(dx^2+c\right)\left(c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)-d\left(2dfa^2-b(5de-cf)a+2b^2ce\right)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{b\left(dx^2+c\right)\left(c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)-d\left(2dfa^2-b(5de-cf)a+2b^2ce\right)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x(c+dx^2)^2(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(dx^2+c\right)\left(c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)-d\left(2dfa^2-b(5de-cf)a+2b^2ce\right)x^2\right)}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x(c+dx^2)^2(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 401

$$\frac{\frac{\frac{x\left(c+dx^2\right)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{\int \frac{d\left(\left(6d^2fa^3-bd(15de-7cf)a^2+2b^2c(4de-5cf)a+4b^3c^2e\right)x^2+ac\left(2dfa^2-b(5de-cf)a+2b^2ce\right)\right)}{\sqrt{bx^2+a}}dx}{ab}}{3a}+x(\cdots}{(be-af)^2}-\frac{f(de-cf)\int \frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\int\frac{(6d^2fa^3-bd(15de-7cf)a^2+2b^2c(4de-5cf)a+4b^3c^2e)x^2+ac(2dfa^2-b(5de-cf)a+2b^2ce)}{\sqrt{bx^2+a}}dx}{ab}}{3a}+\frac{x(c-}{}}{(be-af)^2}$$

$$\frac{f(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 299

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2(2a^2d^2f-abd(5de-cf)+b^2(6cde}{2b}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 224

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2(2a^2d^2f-abd(5de-cf)+b^2(6cde}{2b}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2a^2d^2f}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\int\frac{(dx^2+c)^2}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}$$

$\downarrow$ 420

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2a^2d^2f}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\int\frac{dx^2+c}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$\downarrow$ 299

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2a^2d^2f}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$\downarrow$ 224

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)(-2a^2df+ab(5de-5cf)+2b^2ce)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2a^2d^2f}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{(2bc-ad)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}$$

$\downarrow$ 219

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\cdots\right.}{2b^{3/\cdots}}\right.}{ab}}{3a}\cdots}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 398$$

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\cdots\right.}{2b^{3/\cdots}}\right.}{ab}}{3a}\cdots}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{x(c+dx^2)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\cdots\right.}{2b^{3/\cdots}}\right.}{ab}}{3a}\cdots}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{(be-af)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{x\left(c+dx^2\right)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\right.}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}\right)}{(be-af)^2}$$

$\downarrow$ 291

$$\frac{\frac{\frac{x\left(c+dx^2\right)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\right.}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{f}\right)}{(be-af)^2}$$

$\downarrow$ 221

$$\frac{\frac{\frac{x\left(c+dx^2\right)(bc-ad)\left(-2a^2df+ab(5de-5cf)+2b^2ce\right)}{ab\sqrt{a+bx^2}}-\frac{d\left(\frac{x\sqrt{a+bx^2}\left(6a^3d^2f-a^2bd(15de-7cf)+2ab^2c(4de-5cf)+4b^3c^2e\right)}{2b}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2a^2d^2f\right.}{2b^{3/}}\right.}{ab}}{3a}}{(be-af)^2}$$

$$\frac{f(de-cf)\left(\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}\right)}{f}-\frac{(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{f}\right)}{(be-af)^2}$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)),x]
```

output
```
(((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2)^2)/(3*a*(a + b*x^2)^(3/2)) + (((b*
c - a*d)*(2*b^2*c*e - 2*a^2*d*f + a*b*(5*d*e - 5*c*f))*x*(c + d*x^2))/(a*b
*Sqrt[a + b*x^2]) - (d*(((4*b^3*c^2*e + 6*a^3*d^2*f - a^2*b*d*(15*d*e - 7*
c*f) + 2*a*b^2*c*(4*d*e - 5*c*f))*x*Sqrt[a + b*x^2])/(2*b) - (3*a^2*(2*a^2
*d^2*f - a*b*d*(5*d*e - c*f) + b^2*(6*c*d*e - 4*c^2*f))*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(a*b))/(3*a))/(b*e - a*f)^2 - (f*(d*e -
c*f)*((d*((d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(2*b^(3/2))))/f - ((d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(Sqrt[b]*f) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(
Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*f*Sqrt[b*e - a*f])))/f))/(b*e - a*f)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{-a^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{\frac{9}{2}} \left(cf-de\right)^{3} \arctan\left(\frac{e\sqrt{b\,x^{2}+a}}{x\sqrt{\left(af-be\right)e}}\right)+\sqrt{\left(af-be\right)e}\,\left(a^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}} d^{3}\left(af-be\right)^{2} b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-(ad-}{\sqrt{\left(af-be\right)e}\,\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{\frac{9}{2}} f\left(af-be\right)^{2}}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(3/2)*(-a^2*(b*x^2+a)^(3/2)*b^(9/2)*(c*f-d
*e)^3*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*
(a^2*(b*x^2+a)^(3/2)*d^3*(a*f-b*e)^2*b^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)
)-(a*d-b*c)^2*b^(5/2)*(a^3*d*f+2*((2/3*f*x^2-e)*d+c*f)*b*a^2-(7/3*d*e*x^2+
(-5/3*f*x^2+e)*c)*b^2*a-2/3*b^3*c*e*x^2)*x*f))/b^(9/2)/f/(a*f-b*e)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(182) = 364$.

Time = 17.38 (sec) , antiderivative size = 3941, normalized size of antiderivative = 19.13

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(5/2)/(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx=\int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3361, normalized size of antiderivative = 16.32

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)}\,dx=\text{Too large to display}$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**3*c**3*f**3 + 9*sqrt(
e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt
(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**3*c**2*d*e*f**2 - 9*sqrt(e)*sqrt
(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqr
t(b)*x)/(sqrt(e)*sqrt(b)))*a**4*b**3*c*d**2*e**2*f + 3*sqrt(e)*sqrt(a*f -
b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)
/(sqrt(e)*sqrt(b)))*a**4*b**3*d**3*e**3 - 6*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*s
qrt(b)))*a**3*b**4*c**3*f**3*x**2 + 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(
a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b
)))*a**3*b**4*c**2*d*e*f**2*x**2 - 18*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a**3*b**4*c*d**2*e**2*f*x**2 + 6*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**3*b**4*d**3*e**3*x**2 - 3*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e
) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*
b**5*c**3*f**3*x**4 + 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sq
rt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**2*b**5*c
**2*d*e*f**2*x**4 - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - s...
```

3.374 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 384

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx &= -\frac{d^2(2bde-3bcf+adf)x}{3abf^3(a+bx^2)^{3/2}} \\ &\quad -\frac{b(de-cf)^2(3af(3de-cf)-2be(2de+cf))x}{6aef^3(be-af)^2(a+bx^2)^{3/2}}-\frac{d^2(4bde-6bcf-adf)x}{3a^2bf^3\sqrt{a+bx^2}} \\ &\quad +\frac{b(de-cf)^2\left(3a^2f^2(11de-cf)+4b^2e^2(2de+cf)-2abef(13de+8cf)\right)x}{6a^2ef^3(be-af)^3\sqrt{a+bx^2}} \\ &\quad +\frac{(de-cf)^3x}{2ef^2(be-af)(a+bx^2)^{3/2}(e+fx^2)} \\ &\quad +\frac{(de-cf)^2(6bce-5ade-acf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{7/2}}\end{aligned}$$

output

```
-1/3*d^2*(a*d*f-3*b*c*f+2*b*d*e)*x/a/b/f^3/(b*x^2+a)^(3/2)-1/6*b*(-c*f+d*e
)^2*(3*a*f*(-c*f+3*d*e)-2*b*e*(c*f+2*d*e))*x/a/e/f^3/(-a*f+b*e)^2/(b*x^2+a
)^(3/2)-1/3*d^2*(-a*d*f-6*b*c*f+4*b*d*e)*x/a^2/b/f^3/(b*x^2+a)^(1/2)+1/6*b
*(-c*f+d*e)^2*(3*a^2*f^2*(-c*f+11*d*e)+4*b^2*e^2*(c*f+2*d*e)-2*a*b*e*f*(8*
c*f+13*d*e))*x/a^2/e/f^3/(-a*f+b*e)^3/(b*x^2+a)^(1/2)+1/2*(-c*f+d*e)^3*x/e
/f^2/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e)+1/2*(-c*f+d*e)^2*(-a*c*f-5*a*d*e
+6*b*c*e)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*
f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \frac{x(-4b^4c^3e^2x^2(e+fx^2)-2ab^3c^2e(e+fx^2)(3ce+3dex^2-8cfx^2)+3a^2b^2}{}$$
$$-\frac{(de-cf)^2(-6bce+5ade+acf)\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{2e^{3/2}(-be+af)^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(x*(-4*b^4*c^3*e^2*x^2*(e + f*x^2) - 2*a*b^3*c^2*e*(e + f*x^2)*(3*c*e + 3*
d*e*x^2 - 8*c*f*x^2) + 3*a^2*b^2*(-(d^3*e^3*x^4) + c*d^2*e^2*x^2*(8*e + 11
*f*x^2) - c^2*d*e*f*x^2*(10*e + 13*f*x^2) + c^3*f*(6*e^2 + 6*e*f*x^2 + f^2
*x^4)) + a^4*(-9*c^2*d*e*f^2 + 3*c^3*f^3 + 9*c*d^2*e*f*(3*e + 2*f*x^2) + d
^3*e*(-15*e^2 - 10*e*f*x^2 + 2*f^2*x^4)) + 2*a^3*b*(3*c^3*f^3*x^2 - 9*c^2*
d*e*f*(2*e + 3*f*x^2) - d^3*e^2*x^2*(10*e + 7*f*x^2) + 3*c*d^2*e*(3*e^2 +
8*e*f*x^2 + 2*f^2*x^4))))/(6*a^2*e*(-(b*e) + a*f)^3*(a + b*x^2)^(3/2)*(e +
 f*x^2)) - ((d*e - c*f)^2*(-6*b*c*e + 5*a*d*e + a*c*f)*ArcTan[(-(f*x*Sqrt[
a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(b*e) + a*f])])/(2*e^(3/
2)*(-(b*e) + a*f)^(7/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.96, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {425, 419, 25, 398, 224, 219, 291, 221, 401, 25, 27, 298, 224, 219, 425, 402, 25, 402, 25, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx$$

$$\Big\downarrow \ 425$$

$$\frac{d\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 419

$$\frac{d\left(-\frac{\int -\frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\int \frac{dx^2+c}{\sqrt{bx^2+a}(fx^2+e)}dx}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 398

$$\frac{d\left(\frac{\int \frac{(dx^2+c)(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce)}{(bx^2+a)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\int \frac{\left(dx^2+c\right)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(bx^2+a\right)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{\int \frac{\left(dx^2+c\right)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(bx^2+a\right)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{d\left(\frac{\int \frac{\left(dx^2+c\right)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(bx^2+a\right)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{f}\right)}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{\left(dx^2+c\right)^2}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(\frac{\int \frac{\left(dx^2+c\right)\left(dfa^2-2bcfa+b^2(de-cf)x^2+b^2ce\right)}{\left(bx^2+a\right)^{5/2}}dx}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}f\sqrt{be-af}}\right)}{(be-af)^2}\right)}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 401

$$\frac{d\left(\frac{\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{b\left(3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\cdots\right)}{\sqrt{e}f\sqrt{be-\cdots}}\cdots\right)}{(be-af)^2}\cdots\right)}{f}$$

$$\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 25

$$\frac{d\left(\frac{\frac{\int \frac{b\left(3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\cdots\right)}{\sqrt{e}f\sqrt{be-af}}\cdots\right)}{(be-af)^2}\cdots\right)}{f}$$

$$\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{\int \frac{3abd(de-cf)x^2+c\left(2dfa^2+b(de-5cf)a+2b^2ce\right)}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{e}\sqrt{}}\right.}{\sqrt{e}f\sqrt{be-af}}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 298

$$\frac{d\left(\frac{\frac{3ad(de-cf)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{arctanh}\left(\right.}{\sqrt{e}f\sqrt{be-}}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{\frac{3ad(de-cf)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}\cdots}{\cdots}\right.}{(be-af)^2}\cdots}{f}$$

$$\frac{(de-cf)\int\frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}\cdots}{\cdots}\right.}{(be-af)^2}\cdots}{f}$$

$$\frac{(de-cf)\left(\frac{d\int\frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}\cdots}{\cdots}\right.}{(be-af)^2}\cdots}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}-\frac{\int-\frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}-\frac{\int-\frac{4(bc-ad)fx^2+2\cdots}{\left(bx^2+a\right)^{3/2}\cdots}}{3a(be-\cdots}\right.}{f}\cdots\right.}{f}$$

$\downarrow$ 25

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\cdots\right.}{(be-af)^2}\cdots\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\int\frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\cdots\right.}{f}\cdots\right.}{f}$$

$\Big\downarrow$ 402

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\cdots\right.}{(be-af)^2}\cdots\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int\frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\frac{x\left(4a^2df+ab(de-7cf)+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int\frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2\right.}{\sqrt{bx^2+a}\left(fx^2+\right.}}{a(be-af)}\right.}{}\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\int\frac{2\left(4dfa^2+b(de-7cf)\right.}{\sqrt{b}}}{}\right.}{}\right.}{f}$$

$\downarrow$ 291

$$d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{a}}{}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad f \qquad}$$

$$(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\int\frac{2\left(4dfa^2+b(de-\right.}{}}{}\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 221

$$d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{a}}{}\right.}{(be-af)^2}\right.$$

$$\overline{\qquad f \qquad}$$

$$(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\int\frac{2\left(4dfa^2+b(de-7\right.}{}}{}\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 402

$$\frac{d\left(\frac{\frac{\frac{3ad\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$\frac{(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\left(\frac{\int -\frac{3a^2(2be(2de-}{\sqrt{bx^2-}}}{2e(}\right.}{}\right.}{}\right.}{f}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\frac{\frac{3ad\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\mathrm{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\mathrm{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(1\right.}{2}\right.}{}\right.}{}\right.$$

↓ 291

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x\left(c+dx^2\right)(bc-ad)(be-af)}{3a\left(a+bx^2\right)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}\ldots}{\ldots}\right.}{(be-af)^2}\ldots\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{\frac{x\left(2a^2df+ab(de-5cf)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\left(\frac{x\sqrt{a+bx^2}\left(a^2f(1\ldots\right.}{2\ldots}\right.}{\ldots}\right.}{\ldots}\ldots\right.$$

↓ 221

$$\frac{d\left(\frac{\frac{\frac{3ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(de-cf)}{\sqrt{b}}+\frac{x(bc-ad)(-5acf+3ade+2bce)}{a\sqrt{a+bx^2}}}{3a}+\frac{x(c+dx^2)(bc-ad)(be-af)}{3a(a+bx^2)^{3/2}}}{(be-af)^2}-\frac{f(de-cf)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}f}-\frac{(de-cf)\text{ar}}{}\right.}{(be-af)^2}\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{\frac{x(2a^2df+ab(de-5cf)+2b^2ce)}{a\sqrt{a+bx^2}(be-af)}-\frac{3af(de-cf)\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{x(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{f\left(\frac{x\sqrt{a+bx^2}(a^2f(1}{2}\right.}{}\right.}{}\right.$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x]
```

output
```
(d*((((b*c - a*d)*(b*e - a*f)*x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2)) + (((
b*c - a*d)*(2*b*c*e + 3*a*d*e - 5*a*c*f)*x)/(a*Sqrt[a + b*x^2]) + (3*a*d*(
d*e - c*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a))/(b*e - a*
f)^2 - (f*(d*e - c*f)*((d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*f
) - ((d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(
Sqrt[e]*f*Sqrt[b*e - a*f])))/(b*e - a*f)^2))/f - ((d*e - c*f)*((d*(((b*c -
 a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)) + (((2*b^2*c*e + 2*a^2*d*f +
a*b*(d*e - 5*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) - (3*a*f*(d*e - c*f)
*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[e]*(b*e - a
*f)^(3/2)))/(3*a*(b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(3*a*(b*
e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + (((2*b^2*c*e + 4*a^2*d*f + a*b*(
d*e - 7*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + (f*(((4*b^2
*c*e^2 + 2*a*b*e*(d*e - 8*c*f) + a^2*f*(13*d*e - 3*c*f))*x*Sqrt[a + b*x^2]
)/(2*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(2*b*e*(2*d*e - 3*c*f) + a*f*(d*e
 + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2
)*(b*e - a*f)^(3/2))))/(a*(b*e - a*f)))/(3*a*(b*e - a*f))))/f))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 419
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b*((b*e - a*f)/(b*c - a*d)^2)   Int[(c + d*x^2)^(
q + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Simp[1/(b*c - a*d)^2   I
nt[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(
b*e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && Gt
Q[r, 1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-a^2(cf-de)^2(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)((cf+5de)a-6bce)\arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\left(\left(-5e^3d^3+9\left(-\frac{10x^2d}{27}+c\right)d^2f\,e^2-3d\left(-\frac{2}{9}d\right.\right.\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a^2*(c*f-d*e)^2*(b*x^2+a)^(3/2)*(f*x^2+e)*((c*f+5*d*e)*a-6*b*c*e)*ar
ctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((-5*e^3*d^3+9*(-10/27*x^2*d
+c)*d^2*f*e^2-3*d*(-2/9*d^2*x^4-2*c*d*x^2+c^2)*f^2*e+c^3*f^3)*a^4+2*((-10/
3*x^2*d^3+3*c*d^2)*e^3-6*d*(7/18*d^2*x^4-4/3*c*d*x^2+c^2)*f*e^2-9*(-2/9*x^
2*d+c)*c*d*x^2*f^2*e+x^2*f^3*c^3)*b*a^3+6*(4/3*d^2*(-1/8*x^2*d+c)*x^2*e^3+
c*f*(11/6*d^2*x^4-5/3*c*d*x^2+c^2)*e^2+c^2*f^2*x^2*(-13/6*x^2*d+c)*e+1/6*c
^3*f^3*x^4)*b^2*a^2-2*(e*(d*x^2+c)-8/3*c*f*x^2)*c^2*b^3*(f*x^2+e)*e*a-4/3*
b^4*c^3*e^2*x^2*(f*x^2+e))*((a*f-b*e)*e)^(1/2)*x)/((a*f-b*e)*e)^(1/2)/(b*x
^2+a)^(3/2)/e/(f*x^2+e)/(a*f-b*e)^3/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. 2(352) = 704.

Time = 19.50 (sec) , antiderivative size = 2928, normalized size of antiderivative = 7.62

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^5*c^3*e*f^3 + (a^3*b^2*c^3*f^4 - (6*a^2*b^3*c*d^2 - 5*a^3*b^2
*d^3)*e^3*f + 3*(4*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2)*e^2*f^2 - 3*(2*a^2*b^3
*c^3 - a^3*b^2*c^2*d)*e*f^3)*x^6 - (6*a^4*b*c*d^2 - 5*a^5*d^3)*e^4 + 3*(4*
a^4*b*c^2*d - 3*a^5*c*d^2)*e^3*f - 3*(2*a^4*b*c^3 - a^5*c^2*d)*e^2*f^2 + (
2*a^4*b*c^3*f^4 - (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*e^4 + (12*a^2*b^3*c^2*
d - 21*a^3*b^2*c*d^2 + 10*a^4*b*d^3)*e^3*f - 3*(2*a^2*b^3*c^3 - 9*a^3*b^2*
c^2*d + 6*a^4*b*c*d^2)*e^2*f^2 - (11*a^3*b^2*c^3 - 6*a^4*b*c^2*d)*e*f^3)*x
^4 + (a^5*c^3*f^4 - 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*e^4 + (24*a^3*b^2*c^
2*d - 24*a^4*b*c*d^2 + 5*a^5*d^3)*e^3*f - 3*(4*a^3*b^2*c^3 - 6*a^4*b*c^2*d
 + 3*a^5*c*d^2)*e^2*f^2 - (4*a^4*b*c^3 - 3*a^5*c^2*d)*e*f^3)*x^2)*sqrt(b*e
^2 - a*e*f)*log(((8*b^2*e^2 - 8*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*(4*a*
b*e^2 - 3*a^2*e*f)*x^2 + 4*((2*b*e - a*f)*x^3 + a*e*x)*sqrt(b*e^2 - a*e*f)
*sqrt(b*x^2 + a))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 4*((3*a^2*b^3*d^3*e^5 + 3
*a^3*b^2*c^3*e*f^4 + (4*b^5*c^3 + 6*a*b^4*c^2*d - 33*a^2*b^3*c*d^2 + 11*a^
3*b^2*d^3)*e^4*f - (20*a*b^4*c^3 - 33*a^2*b^3*c^2*d - 21*a^3*b^2*c*d^2 + 1
6*a^4*b*d^3)*e^3*f^2 + (13*a^2*b^3*c^3 - 39*a^3*b^2*c^2*d + 12*a^4*b*c*d^2
 + 2*a^5*d^3)*e^2*f^3)*x^5 + 2*(3*a^4*b*c^3*e*f^4 + (2*b^5*c^3 + 3*a*b^4*c
^2*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*e^5 - (7*a*b^4*c^3 - 12*a^2*b^3*
c^2*d + 12*a^3*b^2*c*d^2 + 5*a^4*b*d^3)*e^4*f - (4*a^2*b^3*c^3 - 12*a^3*b^
2*c^2*d - 15*a^4*b*c*d^2 + 5*a^5*d^3)*e^3*f^2 + 3*(2*a^3*b^2*c^3 - 9*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(5/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(dx^2+c)^3}{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/((b*x^2 + a)^(5/2)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(352) = 704$.

Time = 0.39 (sec) , antiderivative size = 1390, normalized size of antiderivative = 3.62

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3*((2*b^8*c^3*e^4 + 3*a*b^7*c^2*d*e^4 - 12*a^2*b^6*c*d^2*e^4 + 7*a^3*b^5
*d^3*e^4 - 14*a*b^7*c^3*e^3*f + 6*a^2*b^6*c^2*d*e^3*f + 30*a^3*b^5*c*d^2*e
^3*f - 22*a^4*b^4*d^3*e^3*f + 30*a^2*b^6*c^3*e^2*f^2 - 36*a^3*b^5*c^2*d*e^
2*f^2 - 18*a^4*b^4*c*d^2*e^2*f^2 + 24*a^5*b^3*d^3*e^2*f^2 - 26*a^3*b^5*c^3
*e*f^3 + 42*a^4*b^4*c^2*d*e*f^3 - 6*a^5*b^3*c*d^2*e*f^3 - 10*a^6*b^2*d^3*e
*f^3 + 8*a^4*b^4*c^3*f^4 - 15*a^5*b^3*c^2*d*f^4 + 6*a^6*b^2*c*d^2*f^4 + a^
7*b*d^3*f^4)*x^2/(a^2*b^7*e^6 - 6*a^3*b^6*e^5*f + 15*a^4*b^5*e^4*f^2 - 20*
a^5*b^4*e^3*f^3 + 15*a^6*b^3*e^2*f^4 - 6*a^7*b^2*e*f^5 + a^8*b*f^6) + 3*(a
*b^7*c^3*e^4 - 3*a^3*b^5*c*d^2*e^4 + 2*a^4*b^4*d^3*e^4 - 6*a^2*b^6*c^3*e^3
*f + 6*a^3*b^5*c^2*d*e^3*f + 6*a^4*b^4*c*d^2*e^3*f - 6*a^5*b^3*d^3*e^3*f +
 12*a^3*b^5*c^3*e^2*f^2 - 18*a^4*b^4*c^2*d*e^2*f^2 + 6*a^6*b^2*d^3*e^2*f^2
 - 10*a^4*b^4*c^3*e*f^3 + 18*a^5*b^3*c^2*d*e*f^3 - 6*a^6*b^2*c*d^2*e*f^3 -
 2*a^7*b*d^3*e*f^3 + 3*a^5*b^3*c^3*f^4 - 6*a^6*b^2*c^2*d*f^4 + 3*a^7*b*c*d
^2*f^4)/(a^2*b^7*e^6 - 6*a^3*b^6*e^5*f + 15*a^4*b^5*e^4*f^2 - 20*a^5*b^4*e
^3*f^3 + 15*a^6*b^3*e^2*f^4 - 6*a^7*b^2*e*f^5 + a^8*b*f^6))*x/(b*x^2 + a)^
(3/2) - 1/2*(6*b^(3/2)*c*d^2*e^3 - 5*a*sqrt(b)*d^3*e^3 - 12*b^(3/2)*c^2*d*
e^2*f + 9*a*sqrt(b)*c*d^2*e^2*f + 6*b^(3/2)*c^3*e*f^2 - 3*a*sqrt(b)*c^2*d*
e*f^2 - a*sqrt(b)*c^3*f^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f +
 2*b*e - a*f)/sqrt(-b^2*e^2 + a*b*e*f))/((b^3*e^4 - 3*a*b^2*e^3*f + 3*a^2*
b*e^2*f^2 - a^3*e*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) + (2*(sqrt(b)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^2),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}\,(e+fx^2)^2}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{\frac{5}{2}}\,(f\,x^2+e)^2}dx$$

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x)
```

output
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^2,x)
```

3.375 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 615

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{d^3x}{3af^3(a+bx^2)^{3/2}}$$
$$-\frac{b(de-cf)\left(3a^2f^2(17d^2e^2-6cdef-3c^2f^2)-12abef(2d^2e^2+5cdef-3c^2f^2)+8b^2e^2(d^2e^2+cdef+c^2\right.}{24ae^2f^3(be-af)^3(a+bx^2)^{3/2}}$$
$$+\frac{2d^3x}{3a^2f^3\sqrt{a+bx^2}}$$
$$-\frac{b(de-cf)\left(6a^2bef^2(15d^2e^2+40cdef-7c^2f^2)-3a^3f^3(49d^2e^2-6cdef-3c^2f^2)+16b^3e^3(d^2e^2+cdef\right.}{24a^2e^2f^3(be-af)^4\sqrt{a+bx^2}}$$
$$+\frac{(de-cf)^3x}{4ef^2(be-af)(a+bx^2)^{3/2}(e+fx^2)^2}+\frac{(de-cf)^2(3af(3de+cf)-2be(de+5cf))x}{8e^2f^2(be-af)^2(a+bx^2)^{3/2}(e+fx^2)}$$
$$+\frac{(de-cf)\left(24b^2ce^2(de-2cf)-4abe(5d^2e^2-13cdef-4c^2f^2)-3a^2f(5d^2e^2+2cdef+c^2f^2)\right)\operatorname{arctanh}\left(\right.}{8e^{5/2}(be-af)^{9/2}}$$

output
```
1/3*d^3*x/a/f^3/(b*x^2+a)^(3/2)-1/24*b*(-c*f+d*e)*(3*a^2*f^2*(-3*c^2*f^2-6
*c*d*e*f+17*d^2*e^2)-12*a*b*e*f*(-3*c^2*f^2+5*c*d*e*f+2*d^2*e^2)+8*b^2*e^2
*(c^2*f^2+c*d*e*f+d^2*e^2))*x/a/e^2/f^3/(-a*f+b*e)^3/(b*x^2+a)^(3/2)+2/3*d
^3*x/a^2/f^3/(b*x^2+a)^(1/2)-1/24*b*(-c*f+d*e)*(6*a^2*b*e*f^2*(-7*c^2*f^2+
40*c*d*e*f+15*d^2*e^2)-3*a^3*f^3*(-3*c^2*f^2-6*c*d*e*f+49*d^2*e^2)+16*b^3*
e^3*(c^2*f^2+c*d*e*f+d^2*e^2)-8*a*b^2*e^2*f*(11*c^2*f^2+8*c*d*e*f+8*d^2*e^
2))*x/a^2/e^2/f^3/(-a*f+b*e)^4/(b*x^2+a)^(1/2)+1/4*(-c*f+d*e)^3*x/e/f^2/(-
a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e)^2+1/8*(-c*f+d*e)^2*(3*a*f*(c*f+3*d*e)-2
*b*e*(5*c*f+d*e))*x/e^2/f^2/(-a*f+b*e)^2/(b*x^2+a)^(3/2)/(f*x^2+e)+1/8*(-c
*f+d*e)*(24*b^2*c*e^2*(-2*c*f+d*e)-4*a*b*e*(-4*c^2*f^2-13*c*d*e*f+5*d^2*e^
2)-3*a^2*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/
2)/(b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 16.34 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.55

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \frac{1}{24}\left(x\sqrt{a+bx^2}\left(\frac{8(-bc+ad)^3}{a(-be+af)^3(a+bx^2)^2}+\frac{8(bc-ad)^2(2b^2ce+2a^2df+\cdots}{a^2(be-af)^4(a+\cdots}\right.\right.$$
$$-\frac{3(de-cf)(24b^2ce^2(-de+2cf)+4abe(5d^2e^2-13cdef-4c^2f^2)+3a^2f(5d^2e^2+2cdef+c^2f^2))\arctan\cdots}{e^{5/2}(-be+af)^{9/2}}$$

input
```
Integrate[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((8*(-(b*c) + a*d)^3)/(a*(-(b*e) + a*f)^3*(a + b*x^2)^2
) + (8*(b*c - a*d)^2*(2*b^2*c*e + 2*a^2*d*f + a*b*(7*d*e - 11*c*f)))/(a^2*
(b*e - a*f)^4*(a + b*x^2)) + (6*(d*e - c*f)^3)/(e*(b*e - a*f)^3*(e + f*x^2
)^2) + (3*(d*e - c*f)^2*(2*b*e*(d*e - 7*c*f) + 3*a*f*(3*d*e + c*f)))/(e^2*
(b*e - a*f)^4*(e + f*x^2))) - (3*(d*e - c*f)*(24*b^2*c*e^2*(-(d*e) + 2*c*f
) + 4*a*b*e*(5*d^2*e^2 - 13*c*d*e*f - 4*c^2*f^2) + 3*a^2*f*(5*d^2*e^2 + 2*
c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2]
)])/(e^(5/2)*(-(b*e) + a*f)^(9/2)))/24
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. 2(615) = 1230.

Time = 1.61 (sec) , antiderivative size = 1253, normalized size of antiderivative = 2.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.567, Rules used = {425, 425, 402, 25, 402, 25, 27, 291, 221, 402, 27, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{d\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{f}-\frac{(de-cf)\int \frac{(dx^2+c)^2}{(bx^2+a)^{5/2}(fx^2+e)^3}dx}{f}$$

$$\Big\downarrow \text{425}$$

$$\frac{d\left(\frac{d\int \frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}\right)}{f}-\frac{(de-cf)\left(\frac{d\int \frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{f}-\frac{(de-cf)\int \frac{dx^2+c}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^3}dx}{f}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{d\left(\frac{d\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}-\frac{\int -\frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+2bce+ade-3a\cdots}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}\cdots}{3a(be-af)}\cdots}{f}\cdots\right.}{f}\cdots$$

$$(de-cf)\left(\frac{d\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{6(bc-\cdots}{\left(b\cdots\right.}\cdots}{\cdots}\cdots\right.}{f}\cdots\right.$$

$$\cdots \Big/ f$$

$\big\downarrow$ 25

$$\frac{d\left(\frac{d\left(\frac{\int \frac{2(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-a\cdots}\cdots\right.}{f}\cdots\right.}{f}\cdots$$

$$(de-cf)\left(\frac{d\left(\frac{\int \frac{4(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\frac{x(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{\int \frac{6(bc-ad)fx^2+2bce+ade-3acf}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}+\frac{\cdots}{3a\left(a+bx^{2}\cdots\right.}\cdots\right.}{f}\cdots\right.$$

$$\cdots \Big/ f$$

$\big\downarrow$ 402

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{\int\frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-\cdots}{a(be-af)\sqrt{\cdots}}\cdots\right.}{\cdots}\right)}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\int-\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\cdots\right)$$
$$\overline{\qquad\qquad f \qquad\qquad}$$

$\downarrow$ 25

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{\int\frac{3a^2f(de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-\cdots\right.}{a(be-af)\sqrt{\cdots}}}{\cdots}\right.}{f}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{\int\frac{f\left(2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\cdots\right.$$

$$\frac{}{f}$$

$\downarrow$ 27

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\cdots\right.}{}\right.$$

$$\frac{}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\int\frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\cdots\right.$$

$$\frac{}{f}$$

$\downarrow$ 291

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\cdots\right.}{f}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\int\frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\cdots\right.$$

$$\downarrow 221$$

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$\overline{\qquad f \qquad}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\int\frac{2\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x^2+a(2bce-5ade+3acf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}\right)}{f}-\right.$$

$$\overline{\qquad f \qquad}$$

$$\downarrow 402$$

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\cdots\right.}{\cdots}\right.$$

$$\frac{\cdots}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int-\frac{3a^2(2be(\cdots}{\sqrt{\cdots}}}{\cdots}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\cdots\right.$$

$\downarrow$ 27

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$\overline{\qquad\qquad f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de\cdots}{\cdots}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\right.$$

↓ 291

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de\cdots}{}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\right.$$

↓ 221

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de\cdots}{}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\right.$$

↓ 402

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de}{}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\right.$$

↓ 27

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.$$

$$f$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de}{}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\right.$$

↓ 291

$$d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\cdots\right.}{\cdots}\right.$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de\cdots}{\cdots}\right.}{a(be-af)}}{3a(be-af)}\right.}{f}\cdots\right.$$

↓ 221

$$\frac{d\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}}+\frac{\frac{\left(2dfa^2+b(de-5cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}}-\frac{3af(de-cf)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}\right)}{f}-\frac{(de-cf)\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}\right.}{}\right.}{f}$$

$$(de-cf)\left(\frac{d\left(\frac{(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)}+\frac{\frac{\left(4dfa^2+b(de-7cf)a+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}+\frac{f\left(\frac{\left(f(13de-3cf)a^2+2be(de-8cf)a+4b^2ce^2\right)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{3a^2(2be(2de}{}\right.}{a(be-af)}}{3a(be-af)}\right)}{f}\right.$$

input
```
Int[(c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x]
```

output
```
(d*((d*(((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)) + (((2*b^2*c*e
 + 2*a^2*d*f + a*b*(d*e - 5*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) - (3*
a*f*(d*e - c*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(S
qrt[e]*(b*e - a*f)^(3/2)))/(3*a*(b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a
*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + (((2*b^2*c*e + 4*
a^2*d*f + a*b*(d*e - 7*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)
) + (f*(((4*b^2*c*e^2 + 2*a*b*e*(d*e - 8*c*f) + a^2*f*(13*d*e - 3*c*f))*x*
Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) - (3*a^2*(2*b*e*(2*d*e - 3*
c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^
2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/(a*(b*e - a*f)))/(3*a*(b*e - a*f)))
/f))/f - ((d*e - c*f)*((d*(((b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3
/2)*(e + f*x^2)) + (((2*b^2*c*e + 4*a^2*d*f + a*b*(d*e - 7*c*f))*x)/(a*(b*
e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) + (f*(((4*b^2*c*e^2 + 2*a*b*e*(d*e -
 8*c*f) + a^2*f*(13*d*e - 3*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e +
 f*x^2)) - (3*a^2*(2*b*e*(2*d*e - 3*c*f) + a*f*(d*e + c*f))*ArcTanh[(Sqrt[
b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2))))/
(a*(b*e - a*f)))/(3*a*(b*e - a*f))))/f - ((d*e - c*f)*(((b*c - a*d)*x)/(3*
a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)^2) + (((2*b^2*c*e + 6*a^2*d*f
+ a*b*(d*e - 9*c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (f
*(((8*b^2*c*e^2 + 4*a*b*e*(d*e - 10*c*f) + a^2*f*(31*d*e - 3*c*f))*x*Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{3a^2\left(bx^2+a\right)^{\frac{3}{2}}(cf-de)\left(fx^2+e\right)^2\left(a^2f\left(c^2f^2+2cdef+5d^2e^2\right)-\frac{16\left(c^2f^2+\frac{13}{4}cdef-\frac{5}{4}d^2e^2\right)bea}{3}+16b^2c^2e^2f-8b^2de^3c\right)\arctan\left(\frac{e\sqrt{}}{x\sqrt{}}\right.}{8}$
default	Expression too large to display

input
```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(-3/5*a^2*(b*x^2+a)^(3/2)*(c*f-d*e)*(f*x^2+e)^2*(a^2*f*(c^2*f^2+2*c*d*
e*f+5*d^2*e^2)-16/3*(c^2*f^2+13/4*c*d*e*f-5/4*d^2*e^2)*b*e*a+16*b^2*c^2*e^
2*f-8*b^2*d*e^3*c)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((3*e^4
*d^3-9/5*d^2*(-25/9*x^2*d+c)*f*e^3-3/5*d*(-8/3*d^2*x^4+5*c*d*x^2+c^2)*f^2*
e^2+c^2*f^3*(3/5*x^2*d+c)*e+3/5*c^3*f^4*x^2)*f*a^5-16/5*(-5/4*d^3*e^5+9/2*
d^2*(-20/27*x^2*d+c)*f*e^4-9/4*d*(67/54*d^2*x^4-11/3*c*d*x^2+c^2)*f^2*e^3+
f^3*(-1/3*d^3*x^6+39/8*c*d^2*x^4-3/2*c^2*d*x^2+c^3)*e^2+1/4*c^2*(-3/2*x^2*
d+c)*x^2*f^4*e-3/8*c^3*f^5*x^4)*b*a^4-32/5*b^2*((-5/6*x^2*d^3+3/4*c*d^2)*e
^5-9/4*d*(145/216*d^2*x^4-17/9*c*d*x^2+c^2)*f*e^4-27/4*d*(83/648*d^2*x^4-1
49/216*c*d*x^2+c^2)*x^2*f^2*e^3+f^3*x^2*c*(55/32*d^2*x^4-129/32*c*d*x^2+c^
2)*e^2+23/32*c^2*(-3/23*x^2*d+c)*x^4*f^4*e-3/32*c^3*f^5*x^6)*a^3-32/5*b^3*
e*(d^2*x^2*(-1/8*x^2*d+c)*e^4+f*(-1/16*d^3*x^6+11/4*c*d^2*x^4-2*c^2*d*x^2+
c^3)*e^3+2*c*(25/32*d^2*x^4-41/16*c*d*x^2+c^2)*x^2*f^2*e^2+3/2*c^2*(-47/24
*x^2*d+c)*x^4*f^3*e+7/16*c^3*f^4*x^6)*a^2+8/5*c^2*(e*(d*x^2+c)-11/3*c*f*x^
2)*b^4*(f*x^2+e)^2*e^2*a+16/15*b^5*c^3*e^3*x^2*(f*x^2+e)^2)*((a*f-b*e)*e)^
(1/2)*x)/((a*f-b*e)*e)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^2/e^2/(a*f-b*e)^4/a
^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2513 vs. $2(579) = 1158$.

Time = 88.18 (sec) , antiderivative size = 5066, normalized size of antiderivative = 8.24

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**(5/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \int \frac{(dx^2+c)^3}{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^3}\,dx$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3/((b*x^2 + a)^(5/2)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2892 vs. 2(579) = 1158.

Time = 0.49 (sec) , antiderivative size = 2892, normalized size of antiderivative = 4.70

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^10*c^3*e^5 + 3*a*b^9*c^2*d*e^5 - 12*a^2*b^8*c*d^2*e^5 + 7*a^3*b^
7*d^3*e^5 - 19*a*b^9*c^3*e^4*f + 12*a^2*b^8*c^2*d*e^4*f + 33*a^3*b^7*c*d^2
*e^4*f - 26*a^4*b^6*d^3*e^4*f + 56*a^2*b^8*c^3*e^3*f^2 - 78*a^3*b^7*c^2*d*
e^3*f^2 - 12*a^4*b^6*c*d^2*e^3*f^2 + 34*a^5*b^5*d^3*e^3*f^2 - 74*a^3*b^7*c
^3*e^2*f^3 + 132*a^4*b^6*c^2*d*e^2*f^3 - 42*a^5*b^5*c*d^2*e^2*f^3 - 16*a^6
*b^4*d^3*e^2*f^3 + 46*a^4*b^6*c^3*e*f^4 - 93*a^5*b^5*c^2*d*e*f^4 + 48*a^6*
b^4*c*d^2*e*f^4 - a^7*b^3*d^3*e*f^4 - 11*a^5*b^5*c^3*f^5 + 24*a^6*b^4*c^2*
d*f^5 - 15*a^7*b^3*c*d^2*f^5 + 2*a^8*b^2*d^3*f^5)*x^2/(a^2*b^9*e^8 - 8*a^3
*b^8*e^7*f + 28*a^4*b^7*e^6*f^2 - 56*a^5*b^6*e^5*f^3 + 70*a^6*b^5*e^4*f^4
- 56*a^7*b^4*e^3*f^5 + 28*a^8*b^3*e^2*f^6 - 8*a^9*b^2*e*f^7 + a^10*b*f^8)
+ 3*(a*b^9*c^3*e^5 - 3*a^3*b^7*c*d^2*e^5 + 2*a^4*b^6*d^3*e^5 - 8*a^2*b^8*c
^3*e^4*f + 9*a^3*b^7*c^2*d*e^4*f + 6*a^4*b^6*c*d^2*e^4*f - 7*a^5*b^5*d^3*e
^4*f + 22*a^3*b^7*c^3*e^3*f^2 - 36*a^4*b^6*c^2*d*e^3*f^2 + 6*a^5*b^5*c*d^2
*e^3*f^2 + 8*a^6*b^4*d^3*e^3*f^2 - 28*a^4*b^6*c^3*e^2*f^3 + 54*a^5*b^5*c^2
*d*e^2*f^3 - 24*a^6*b^4*c*d^2*e^2*f^3 - 2*a^7*b^3*d^3*e^2*f^3 + 17*a^5*b^5
*c^3*e*f^4 - 36*a^6*b^4*c^2*d*e*f^4 + 21*a^7*b^3*c*d^2*e*f^4 - 2*a^8*b^2*d
^3*e*f^4 - 4*a^6*b^4*c^3*f^5 + 9*a^7*b^3*c^2*d*f^5 - 6*a^8*b^2*c*d^2*f^5 +
 a^9*b*d^3*f^5)/(a^2*b^9*e^8 - 8*a^3*b^8*e^7*f + 28*a^4*b^7*e^6*f^2 - 56*a
^5*b^6*e^5*f^3 + 70*a^6*b^5*e^4*f^4 - 56*a^7*b^4*e^3*f^5 + 28*a^8*b^3*e^2*
f^6 - 8*a^9*b^2*e*f^7 + a^10*b*f^8))*x/(b*x^2 + a)^(3/2) - 1/8*(24*b^(5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx=\int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^3}\,dx$$

input
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^3),x)
```

output
```
int((c + d*x^2)^3/((a + b*x^2)^(5/2)*(e + f*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 41.22 (sec) , antiderivative size = 17693, normalized size of antiderivative = 28.77

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}(e+fx^2)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x^2+c)^3/(b*x^2+a)^(5/2)/(f*x^2+e)^3,x)
```

output

```
( - 9*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x
**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*b*c**3*e**2*f**6 - 18*sq
rt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - s
qrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*b*c**3*e*f**7*x**2 - 9*sqrt(e)*s
qrt(a*f - b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*
sqrt(b)*x)/(sqrt(e)*sqrt(b)))*a**7*b*c**3*f**8*x**4 - 9*sqrt(e)*sqrt(a*f -
 b*e)*atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x
)/(sqrt(e)*sqrt(b)))*a**7*b*c**2*d*e**3*f**5 - 18*sqrt(e)*sqrt(a*f - b*e)*
atan((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqr
t(e)*sqrt(b)))*a**7*b*c**2*d*e**2*f**6*x**2 - 9*sqrt(e)*sqrt(a*f - b*e)*at
an((sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(
e)*sqrt(b)))*a**7*b*c**2*d*e*f**7*x**4 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((
sqrt(a*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*s
qrt(b)))*a**7*b*c*d**2*e**4*f**4 - 54*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a
*f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)
))*a**7*b*c*d**2*e**3*f**5*x**2 - 27*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*
f - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b))
)*a**7*b*c*d**2*e**2*f**6*x**4 + 45*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f
 - b*e) - sqrt(f)*sqrt(a + b*x**2) - sqrt(f)*sqrt(b)*x)/(sqrt(e)*sqrt(b)))
*a**7*b*d**3*e**5*f**3 + 90*sqrt(e)*sqrt(a*f - b*e)*atan((sqrt(a*f - b*...
```

3.376 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 236

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}} + \frac{b^2(2b^2ce+8a^2df-5ab(de+cf))x}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}} + \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}(de-cf)} - \frac{f^3\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}(be-af)^{5/2}(de-cf)}$$

output
```
1/3*b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)+1/3*b^2*(2*b^2*c*e+8*a^2
*d*f-5*a*b*(c*f+d*e))*x/a^2/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)+d^3*
arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(5/
2)/(-c*f+d*e)-f^3*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(1
/2)/(-a*f+b*e)^(5/2)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \frac{b^2x(9a^3df+2b^3cex^2+ab^2(3ce-5dex^2-5cfx^2)-2a^2b(3de+3c}{3a^2(bc-ad)^2(be-af)^2(a+bx^2)^{3/2}}$$
$$+\frac{d^3\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{5/2}(-de+cf)}+\frac{f^3\arctan\left(\frac{-fx\sqrt{a+bx^2}+\sqrt{b}(e+fx^2)}{\sqrt{e}\sqrt{-be+af}}\right)}{\sqrt{e}(-be+af)^{5/2}(de-cf)}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(b^2*x*(9*a^3*d*f + 2*b^3*c*e*x^2 + a*b^2*(3*c*e - 5*d*e*x^2 - 5*c*f*x^2)
- 2*a^2*b*(3*d*e + 3*c*f - 4*d*f*x^2)))/(3*a^2*(b*c - a*d)^2*(b*e - a*f)^2
*(a + b*x^2)^(3/2)) + (d^3*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d
*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2)*(-(d*e
) + c*f)) + (f^3*ArcTan[(-(f*x*Sqrt[a + b*x^2]) + Sqrt[b]*(e + f*x^2))/(Sq
rt[e]*Sqrt[-(b*e) + a*f])])/(Sqrt[e]*(-(b*e) + a*f)^(5/2)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {421, 25, 402, 25, 402, 27, 291, 221, 407, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+2b(bc-ad)fx^2+2b^2ce}{(bx^2+a)^{3/2}(fx^2+e)}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+2b(bc-ad)fx^2+2b^2ce}{(bx^2+a)^{3/2}(fx^2+e)}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{b\left(\frac{\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}-\frac{\int -\frac{3a^2f(bde+bcf-2adf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{\frac{3af(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{be-af}+\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{b\left(\frac{\frac{3af(-2adf+bcf+bde)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{be-af}+\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}+\frac{3af\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{407}$$

$$\frac{d^2\left(\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}+\frac{3af\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}+\frac{3af\,\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{\frac{bx\left(8a^2df-5ab(cf+de)+2b^2ce\right)}{a\sqrt{a+bx^2}(be-af)}+\frac{3af\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(-2adf+bcf+bde)}{\sqrt{e}(be-af)^{3/2}}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(bc-ad)^2}$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)),x]
```

output
```
(d^2*((d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*
Sqrt[b*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqr
t[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(b*c - a*d)^2 + (b
*((b*(b*c - a*d)*x)/(3*a*(b*e - a*f)*(a + b*x^2)^(3/2)) + ((b*(2*b^2*c*e +
 8*a^2*d*f - 5*a*b*(d*e + c*f))*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]) + (3*a*
f*(b*d*e + b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a +
b*x^2])])/(Sqrt[e]*(b*e - a*f)^(3/2)))/(3*a*(b*e - a*f))))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

method	result	s
pseudoelliptic	$\frac{\frac{a^{2} d^{3} \arctan\left(\frac{c \sqrt{b\,x^{2}+a}}{x \sqrt{\left(a d -b c \right) c}}\right)}{\left(c f -d e \right) \left(a d -b c \right)^{2} \sqrt{\left(a d -b c \right) c}}-\frac{f^{3} a^{2} \arctan\left(\frac{e \sqrt{b\,x^{2}+a}}{x \sqrt{\left(a f -b e \right) e}}\right)}{\left(c f -d e \right) \left(a f -b e \right)^{2} \sqrt{\left(a f -b e \right) e}}+\frac{b^{2} \left(3 a^{2} d f -2 a b c f -2 a b d e +c e\,b^{2}\right) x}{\left(a f -b e \right)^{2} \left(a d -b c \right)^{2} \sqrt{b\,x^{2}+a}}-\frac{b^{3} x^{3}}{3 \left(a f -b e \right) \left(a d -b c \right) \left(b\,x^{2}+a \right)^{\frac{3}{2}}}}{a^{2}}$	2
default	Expression too large to display	2

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(a^2*d^3/(c*f-d*e)/(a*d-b*c)^2/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2
)/x/((a*d-b*c)*c)^(1/2))-f^3*a^2/(c*f-d*e)/(a*f-b*e)^2/((a*f-b*e)*e)^(1/2)
*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+b^2*(3*a^2*d*f-2*a*b*c*f-
2*a*b*d*e+b^2*c*e)/(a*f-b*e)^2/(a*d-b*c)^2*x/(b*x^2+a)^(1/2)-1/3*b^3/(a*f-
b*e)/(a*d-b*c)*x^3/(b*x^2+a)^(3/2))/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(208) = 416$.

Time = 1.19 (sec) , antiderivative size = 1545, normalized size of antiderivative = 6.55

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

output
```
-sqrt(b)*d^3*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/
sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2*d*e - 2*a*b*c*d^2*e + a^2*d^3*e - b^2*
c^3*f + 2*a*b*c^2*d*f - a^2*c*d^2*f)*sqrt(-b^2*c^2 + a*b*c*d)) + sqrt(b)*f
^3*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*f + 2*b*e - a*f)/sqrt(-b^2*
e^2 + a*b*e*f))/((b^2*d*e^3 - b^2*c*e^2*f - 2*a*b*d*e^2*f + 2*a*b*c*e*f^2
+ a^2*d*e*f^2 - a^2*c*f^3)*sqrt(-b^2*e^2 + a*b*e*f)) + 1/3*((2*b^10*c^3*e^
3 - 9*a*b^9*c^2*d*e^3 + 12*a^2*b^8*c*d^2*e^3 - 5*a^3*b^7*d^3*e^3 - 9*a*b^9
*c^3*e^2*f + 36*a^2*b^8*c^2*d*e^2*f - 45*a^3*b^7*c*d^2*e^2*f + 18*a^4*b^6*
d^3*e^2*f + 12*a^2*b^8*c^3*e*f^2 - 45*a^3*b^7*c^2*d*e*f^2 + 54*a^4*b^6*c*d
^2*e*f^2 - 21*a^5*b^5*d^3*e*f^2 - 5*a^3*b^7*c^3*f^3 + 18*a^4*b^6*c^2*d*f^3
 - 21*a^5*b^5*c*d^2*f^3 + 8*a^6*b^4*d^3*f^3)*x^2/(a^2*b^9*c^4*e^4 - 4*a^3*
b^8*c^3*d*e^4 + 6*a^4*b^7*c^2*d^2*e^4 - 4*a^5*b^6*c*d^3*e^4 + a^6*b^5*d^4*
e^4 - 4*a^3*b^8*c^4*e^3*f + 16*a^4*b^7*c^3*d*e^3*f - 24*a^5*b^6*c^2*d^2*e^
3*f + 16*a^6*b^5*c*d^3*e^3*f - 4*a^7*b^4*d^4*e^3*f + 6*a^4*b^7*c^4*e^2*f^2
 - 24*a^5*b^6*c^3*d*e^2*f^2 + 36*a^6*b^5*c^2*d^2*e^2*f^2 - 24*a^7*b^4*c*d^
3*e^2*f^2 + 6*a^8*b^3*d^4*e^2*f^2 - 4*a^5*b^6*c^4*e*f^3 + 16*a^6*b^5*c^3*d
*e*f^3 - 24*a^7*b^4*c^2*d^2*e*f^3 + 16*a^8*b^3*c*d^3*e*f^3 - 4*a^9*b^2*d^4
*e*f^3 + a^6*b^5*c^4*f^4 - 4*a^7*b^4*c^3*d*f^4 + 6*a^8*b^3*c^2*d^2*f^4 - 4
*a^9*b^2*c*d^3*f^4 + a^10*b*d^4*f^4) + 3*(a*b^9*c^3*e^3 - 4*a^2*b^8*c^2*d*
e^3 + 5*a^3*b^7*c*d^2*e^3 - 2*a^4*b^6*d^3*e^3 - 4*a^2*b^8*c^3*e^2*f + 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 5555, normalized size of antiderivative = 23.54

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e),x)
```

output
```
(3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2
) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**7*d**3*e*f**3 - 9*sqrt(c)*sqr
t(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sq
rt(b)*x)/(sqrt(c)*sqrt(b)))*a**6*b*d**3*e**2*f**2 + 6*sqrt(c)*sqrt(a*d - b
*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/
(sqrt(c)*sqrt(b)))*a**6*b*d**3*e*f**3*x**2 + 9*sqrt(c)*sqrt(a*d - b*c)*ata
n((sqrt(a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c
)*sqrt(b)))*a**5*b**2*d**3*e**3*f - 18*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(
a*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b
)))*a**5*b**2*d**3*e**2*f**2*x**2 + 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a
*d - b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)
))*a**5*b**2*d**3*e*f**3*x**4 - 3*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d -
 b*c) - sqrt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a
**4*b**3*d**3*e**4 + 18*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sq
rt(d)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*b**3*d
**3*e**3*f*x**2 - 9*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d
)*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**4*b**3*d**3*
e**2*f**2*x**4 - 6*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)
*sqrt(a + b*x**2) - sqrt(d)*sqrt(b)*x)/(sqrt(c)*sqrt(b)))*a**3*b**4*d**3*e
**4*x**2 + 9*sqrt(c)*sqrt(a*d - b*c)*atan((sqrt(a*d - b*c) - sqrt(d)*sq...
```

3.377 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 446

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2} dx = -\frac{b(3abcf^2-3a^2df^2-2b^2e(de-cf))x}{6a(bc-ad)e(be-af)^2(de-cf)(a+bx^2)^{3/2}} - \frac{b(6a^3bcdf^3-3a^4d^2f^3-4b^4ce^2(de-cf)+2ab^3e(5d^2e^2+3cdef-8c^2f^2)-a^2b^2f(22d^2e^2-22cdef+3c}{6a^2(bc-ad)^2e(be-af)^3(de-cf)\sqrt{a+bx^2}} + \frac{f^2x}{2e(be-af)(de-cf)(a+bx^2)^{3/2}(e+fx^2)} + \frac{d^4\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}(de-cf)^2} - \frac{f^3(2be(4de-3cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{7/2}(de-cf)^2}$$

output
```
-1/6*b*(3*a*b*c*f^2-3*a^2*d*f^2-2*b^2*e*(-c*f+d*e))*x/a/(-a*d+b*c)/e/(-a*f
+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(3/2)-1/6*b*(6*a^3*b*c*d*f^3-3*a^4*d^2*f^3-4*
b^4*c*e^2*(-c*f+d*e)+2*a*b^3*e*(-8*c^2*f^2+3*c*d*e*f+5*d^2*e^2)-a^2*b^2*f*
(3*c^2*f^2-22*c*d*e*f+22*d^2*e^2))*x/a^2/(-a*d+b*c)^2/e/(-a*f+b*e)^3/(-c*f
+d*e)/(b*x^2+a)^(1/2)+1/2*f^2*x/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(3/2)/(f
*x^2+e)+d^4*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-
a*d+b*c)^(5/2)/(-c*f+d*e)^2-1/2*f^3*(2*b*e*(-3*c*f+4*d*e)-a*f*(-c*f+3*d*e)
)*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(
7/2)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 18.52 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2}\,dx = \frac{1}{6}x\sqrt{a+bx^2}\left(-\frac{2b^3}{a(-bc+ad)(be-af)^2(a+bx^2)^2} - \frac{2b^3(2b^2ce+11a^2df-ab(5de+8cf))}{a^2(bc-ad)^2(-be+af)^3(a+bx^2)} + \frac{3f^4}{e(be-af)^3(de-cf)(e+fx^2)}\right) + \frac{d^4\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(-bc+ad)^{5/2}(de-cf)^2} + \frac{f^3(2be(4de-3cf)+af(-3de+cf))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(-be+af)^{7/2}(de-cf)^2}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*((-2*b^3)/(a*(-(b*c) + a*d)*(b*e - a*f)^2*(a + b*x^2)^2
) - (2*b^3*(2*b^2*c*e + 11*a^2*d*f - a*b*(5*d*e + 8*c*f)))/(a^2*(b*c - a*d
)^2*(-(b*e) + a*f)^3*(a + b*x^2)) + (3*f^4)/(e*(b*e - a*f)^3*(d*e - c*f)*(
e + f*x^2))))/6 + (d^4*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x
^2])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2)*(d*e - c*f)^2) + (f^3*(2*b*e*(4*d*e -
 3*c*f) + a*f*(-3*d*e + c*f))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(2*e^(3/2)*(-(b*e) + a*f)^(7/2)*(d*e - c*f)^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.25, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {421, 25, 402, 25, 402, 25, 27, 402, 27, 291, 221, 421, 291, 221, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+4b(bc-ad)fx^2+2b^2ce}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+4b(bc-ad)fx^2+2b^2ce}{(bx^2+a)^{3/2}(fx^2+e)^2}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{f\left(2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 25

$$\frac{b\left(\frac{\frac{\int \frac{f\left(2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{f\int \frac{2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)(be-af)}\right)}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 402

$$b\left(\frac{\frac{f\left(\frac{\int \frac{3a^2\left(2e(2de+3cf)b^2-af(11de+cf)b+2a^2df^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde+2b\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-a}}{3a(be-af)}\right.}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 27

$$b\left(\frac{\frac{f\left(\frac{3a^2\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)\int \frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx\left(10a^2df\right.}{a\sqrt{a+b}}}{3a(be-af)}\right.}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 291

$$b\left(\frac{\frac{f\left(\frac{3a^2\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx\left(10\right.}{a\sqrt{}}}{3a(be-af)}\right.}{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^2}dx}{(bc-ad)^2}+\frac{b\Bigg(\frac{\frac{bx(10a^2df-7abcf-5abde+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}+\frac{f\Big(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\ldots\right.}{2e(e+fx^2)(be-a\ldots}}{a(be-af)}}{3a(be-af)}}{(bc-ad)^2}$$

$$\Big\downarrow \; 421$$

$$\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\Bigg(\frac{\frac{bx(10a^2df-7abcf-5abde+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}+\frac{f\Big(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\ldots\right.}{2e(e+fx^2)(be-a\ldots}}{a(be-af)}}{3a(be-af)}}{(bc-ad)^2}$$

$$\Big\downarrow \; 291$$

$$\frac{d^2\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\Bigg(\frac{\frac{bx(10a^2df-7abcf-5abde+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)(be-af)}+\frac{f\Big(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\ldots\right.}{2e(e+fx^2)(be-a\ldots}}{a(be-af)}}{3a(be-af)}}{(bc-ad)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\cdots\right.}{2e\left(e+fx^2\right)(be-a\cdots}\right.}{a(be-af)}}{3a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int \frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\cdots\right.}{2e\left(e+fx^2\right)(be-a\cdots}\right.}{a(be-af)}}{3a(be-af)}\cdots\right.}{(bc-ad)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\right.}{2e\left(e+fx^2\right)(be-a}\right.}{a(be-af)}}{3a(be-af)}\right.}{(bc-ad)^2}$$

$\downarrow$ 291

$$\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2\right.}{2e\left(e+fx^2\right)(be-a}\right.}{a(be-af)}}{3a(be-af)}\right.}{(bc-ad)^2}$$

$\downarrow$ 221

$$b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2a\ldots\right.}{2e\left(e+fx^2\right)(be-a\ldots}\right.}{a(be-af)}}{3a(be-af)}\right.$$

$$\overline{\qquad\qquad\qquad(bc-ad)^2}$$

$$\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)(2be(2de-cf)-af(3de-cf))}{2e^{3/2}(be-af)^{3/2}}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}$$

input

```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^2),x]
```

output

```
(d^2*((d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c
]*Sqrt[b*c - a*d]*(d*e - c*f)^2) - (f*(-1/2*(f*(d*e - c*f)*x*Sqrt[a + b*x^
2])/(e*(b*e - a*f)*(e + f*x^2)) + ((2*b*e*(2*d*e - c*f) - a*f*(3*d*e - c*f
))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e
 - a*f)^(3/2))))/(d*e - c*f)^2))/(b*c - a*d)^2 + (b*((b*(b*c - a*d)*x)/(3*
a*(b*e - a*f)*(a + b*x^2)^(3/2)*(e + f*x^2)) + ((b*(2*b^2*c*e - 5*a*b*d*e
- 7*a*b*c*f + 10*a^2*d*f)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)) +
 (f*(((4*b^3*c*e^2 + 6*a^3*d*f^2 + a^2*b*f*(19*d*e - 3*c*f) - 2*a*b^2*e*(5
*d*e + 8*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^2)) + (3*a^2*(
2*a^2*d*f^2 - a*b*f*(11*d*e + c*f) + 2*b^2*e*(2*d*e + 3*c*f))*ArcTanh[(Sqr
t[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))
)/(a*(b*e - a*f)))/(3*a*(b*e - a*f))))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{-(ad-bc)^2 a^2 \sqrt{(ad-bc)c}\,(ac\,f^2-3f(ad+2bc)e+8bd\,e^2)\,(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)\,f^3 \arctan\left(\frac{e\sqrt{b\,x^2+a}}{x\sqrt{(af-be)e}}\right)+\left(-2(b\,x^2+a)^{\frac{3}{2}} a^2 \right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*f-b*e)*e)^(1/2)*(-(a*d-b*c)^2*a^2*((a*d-b*c)*c)^(1/2)*(a*c*f^2-3*f
*(a*d+2*b*c)*e+8*b*d*e^2)*(b*x^2+a)^(3/2)*(f*x^2+e)*f^3*arctan(e*(b*x^2+a)
^(1/2)/x/((a*f-b*e)*e)^(1/2))+(-2*(b*x^2+a)^(3/2)*a^2*d^4*(a*f-b*e)^3*(f*x
^2+e)*e*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2
)*(a^2*f^4*(b*x^2+a)^2*(a*d-b*c)^2-8*a*(d*a^2-3/4*(-11/9*x^2*d+c)*b*a-2/3*
b^2*c*x^2)*c*b^3*x^2*e*f^3-8*(d*(-d*x^2+c)*a^3-3/4*b*(11/9*d^2*x^4-14/9*c*
d*x^2+c^2)*a^2-5/12*c*b^2*x^2*(-3/5*x^2*d+c)*a+1/6*c^2*b^3*x^4)*b^3*e^2*f^
2+8*b^3*e^3*(a^3*d^2-1/4*d*b*(-5/3*x^2*d+c)*a^2-1/4*(5/3*d^2*x^4+c^2)*b^2*
a-1/6*b^3*c*x^2*(-d*x^2+c))*f-4*d*(d*a^2-1/2*(-5/3*x^2*d+c)*b*a-1/3*b^2*c*
x^2)*b^4*e^4)*(c*f-d*e)*x)*((a*f-b*e)*e)^(1/2))/((a*d-b*c)*c)^(1/2)/(b*x^2
+a)^(3/2)/(c*f-d*e)^2/(a*d-b*c)^2/e/(f*x^2+e)/(a*f-b*e)^3/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2471 vs. $2(412) = 824$.

Time = 4.29 (sec) , antiderivative size = 2471, normalized size of antiderivative = 5.54

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
-sqrt(b)*d^4*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/
sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2*d^2*e^2 - 2*a*b*c*d^3*e^2 + a^2*d^4*e^
2 - 2*b^2*c^3*d*e*f + 4*a*b*c^2*d^2*e*f - 2*a^2*c*d^3*e*f + b^2*c^4*f^2 -
2*a*b*c^3*d*f^2 + a^2*c^2*d^2*f^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/3*((2*b^1
2*c^3*e^4 - 9*a*b^11*c^2*d*e^4 + 12*a^2*b^10*c*d^2*e^4 - 5*a^3*b^9*d^3*e^4
 - 14*a*b^11*c^3*e^3*f + 54*a^2*b^10*c^2*d*e^3*f - 66*a^3*b^9*c*d^2*e^3*f
+ 26*a^4*b^8*d^3*e^3*f + 30*a^2*b^10*c^3*e^2*f^2 - 108*a^3*b^9*c^2*d*e^2*f
^2 + 126*a^4*b^8*c*d^2*e^2*f^2 - 48*a^5*b^7*d^3*e^2*f^2 - 26*a^3*b^9*c^3*e
*f^3 + 90*a^4*b^8*c^2*d*e*f^3 - 102*a^5*b^7*c*d^2*e*f^3 + 38*a^6*b^6*d^3*e
*f^3 + 8*a^4*b^8*c^3*f^4 - 27*a^5*b^7*c^2*d*f^4 + 30*a^6*b^6*c*d^2*f^4 - 1
1*a^7*b^5*d^3*f^4)*x^2/(a^2*b^11*c^4*e^6 - 4*a^3*b^10*c^3*d*e^6 + 6*a^4*b^
9*c^2*d^2*e^6 - 4*a^5*b^8*c*d^3*e^6 + a^6*b^7*d^4*e^6 - 6*a^3*b^10*c^4*e^5
*f + 24*a^4*b^9*c^3*d*e^5*f - 36*a^5*b^8*c^2*d^2*e^5*f + 24*a^6*b^7*c*d^3*
e^5*f - 6*a^7*b^6*d^4*e^5*f + 15*a^4*b^9*c^4*e^4*f^2 - 60*a^5*b^8*c^3*d*e^
4*f^2 + 90*a^6*b^7*c^2*d^2*e^4*f^2 - 60*a^7*b^6*c*d^3*e^4*f^2 + 15*a^8*b^5
*d^4*e^4*f^2 - 20*a^5*b^8*c^4*e^3*f^3 + 80*a^6*b^7*c^3*d*e^3*f^3 - 120*a^7
*b^6*c^2*d^2*e^3*f^3 + 80*a^8*b^5*c*d^3*e^3*f^3 - 20*a^9*b^4*d^4*e^3*f^3 +
 15*a^6*b^7*c^4*e^2*f^4 - 60*a^7*b^6*c^3*d*e^2*f^4 + 90*a^8*b^5*c^2*d^2*e^
2*f^4 - 60*a^9*b^4*c*d^3*e^2*f^4 + 15*a^10*b^3*d^4*e^2*f^4 - 6*a^7*b^6*c^4
*e*f^5 + 24*a^8*b^5*c^3*d*e*f^5 - 36*a^9*b^4*c^2*d^2*e*f^5 + 24*a^10*b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)\,(e+fx^2)^2}\,dx=\int \frac{1}{(b\,x^2+a)^{5/2}\,\left(d\,x^2+c\right)\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^2,x)
```

3.378 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 720

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)\,(e+fx^2)^3}\,dx =$$
$$-\frac{b(12ab^2cef^2(4de-3cf)+3a^3df^3(7de-3cf)-8b^3e^2(de-cf)^2-3a^2bf^2(16d^2e^2-5cdef-3c^2f^2))\,x}{24a(bc-ad)e^2(be-af)^3(de-cf)^2\,(a+bx^2)^{3/2}}$$
$$-\frac{b(3a^5d^2f^4(7de-3cf)-16b^5ce^3(de-cf)^2+8ab^4e^2(de-cf)^2(5de+11cf)-18a^4bdf^3(3d^2e^2-c^2f^2)+}{24a^2(bc-ad)^2e^2(be-af)^4(de}$$
$$+\frac{f^2x}{4e(be-af)(de-cf)\,(a+bx^2)^{3/2}\,(e+fx^2)^2}$$
$$+\frac{f^2(2be(7de-5cf)-af(7de-3cf))x}{8e^2(be-af)^2(de-cf)^2\,(a+bx^2)^{3/2}\,(e+fx^2)}+\frac{d^5\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}(de-cf)^3}$$
$$-\frac{f^3(a^2f^2(15d^2e^2-10cdef+3c^2f^2)-4abef(15d^2e^2-15cdef+4c^2f^2)+8b^2e^2(10d^2e^2-15cdef+6c^2f}{8e^{5/2}(be-af)^{9/2}(de-cf)^3}$$

output
```
-1/24*b*(12*a*b^2*c*e*f^2*(-3*c*f+4*d*e)+3*a^3*d*f^3*(-3*c*f+7*d*e)-8*b^3*
e^2*(-c*f+d*e)^2-3*a^2*b*f^2*(-3*c^2*f^2-5*c*d*e*f+16*d^2*e^2))*x/a/(-a*d+
b*c)/e^2/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(3/2)-1/24*b*(3*a^5*d^2*f^4*(
-3*c*f+7*d*e)-16*b^5*c*e^3*(-c*f+d*e)^2+8*a*b^4*e^2*(-c*f+d*e)^2*(11*c*f+5
*d*e)-18*a^4*b*d*f^3*(-c^2*f^2+3*d^2*e^2)+9*a^3*b^2*c*f^3*(-c^2*f^2-7*c*d*
e*f+12*d^2*e^2)-2*a^2*b^3*e*f*(-21*c^3*f^3+83*c^2*d*e*f^2-112*c*d^2*e^2*f+
56*d^3*e^3))*x/a^2/(-a*d+b*c)^2/e^2/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1
/2)+1/4*f^2*x/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(3/2)/(f*x^2+e)^2+1/8*f^2*
(2*b*e*(-5*c*f+7*d*e)-a*f*(-3*c*f+7*d*e))*x/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/
(b*x^2+a)^(3/2)/(f*x^2+e)+d^5*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)
^(1/2))/c^(1/2)/(-a*d+b*c)^(5/2)/(-c*f+d*e)^3-1/8*f^3*(a^2*f^2*(3*c^2*f^2-
10*c*d*e*f+15*d^2*e^2)-4*a*b*e*f*(4*c^2*f^2-15*c*d*e*f+15*d^2*e^2)+8*b^2*e
^2*(6*c^2*f^2-15*c*d*e*f+10*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(
b*x^2+a)^(1/2))/e^(5/2)/(-a*f+b*e)^(9/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 17.76 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \frac{1}{24}x\sqrt{a+bx^2}\left(\frac{8b^4}{a(-bc+ad)(-be+af)^3(a+bx^2)^2}\right.$$
$$+\frac{8b^4(2b^2ce+14a^2df-ab(5de+11cf))}{a^2(bc-ad)^2(be-af)^4(a+bx^2)}+\frac{6f^4}{e(be-af)^3(de-cf)(e+fx^2)^2}$$
$$\left.+\frac{3f^4(2be(9de-7cf)+af(-7de+3cf))}{e^2(be-af)^4(de-cf)^2(e+fx^2)}\right)-\frac{d^5\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(-bc+ad)^{5/2}(-de+cf)^3}$$
$$-\frac{f^3(a^2f^2(15d^2e^2-10cdef+3c^2f^2)-4abef(15d^2e^2-15cdef+4c^2f^2)+8b^2e^2(10d^2e^2-15cdef+6c^2f}{8e^{5/2}(-be+af)^{9/2}(de-cf)^3}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((8*b^4)/(a*(-(b*c) + a*d)*(-(b*e) + a*f)^3*(a + b*x^2)
^2) + (8*b^4*(2*b^2*c*e + 14*a^2*d*f - a*b*(5*d*e + 11*c*f)))/(a^2*(b*c -
a*d)^2*(b*e - a*f)^4*(a + b*x^2)) + (6*f^4)/(e*(b*e - a*f)^3*(d*e - c*f)*(
e + f*x^2)^2) + (3*f^4*(2*b*e*(9*d*e - 7*c*f) + a*f*(-7*d*e + 3*c*f)))/(e^
2*(b*e - a*f)^4*(d*e - c*f)^2*(e + f*x^2))))/24 - (d^5*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2)*(-(d*
e) + c*f)^3) - (f^3*(a^2*f^2*(15*d^2*e^2 - 10*c*d*e*f + 3*c^2*f^2) - 4*a*b
*e*f*(15*d^2*e^2 - 15*c*d*e*f + 4*c^2*f^2) + 8*b^2*e^2*(10*d^2*e^2 - 15*c*
d*e*f + 6*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2]
)])/(8*e^(5/2)*(-(b*e) + a*f)^(9/2)*(d*e - c*f)^3)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 887, normalized size of antiderivative = 1.23, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.767$, Rules used = {421, 25, 402, 25, 402, 25, 27, 402, 402, 27, 291, 221, 421, 402, 25, 402, 25, 27, 291, 221, 407, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}dx$$

$$\Big\downarrow \text{421}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{(bx^2+a)^{5/2}(fx^2+e)^3}dx}{(bc-ad)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\frac{\int \frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\Big\downarrow$ 27

$$b\left(\frac{\frac{f\int \frac{4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a(a+bx^2)^{3/2}(e+fx^2)^2(be-af)}\right.$$
$$\overline{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 402

$$b\left(\frac{\frac{f\left(\frac{\int \frac{2b\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)x^2+a\left(18df^2a^3-3bf(23de+3cf)a^2+4b^2e(4de+9cf)a+8b^3ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(\right.}{4e\left(\right.}\right.}{a(be-af)}}{3a(be-af)}\right.$$
$$\overline{(bc-ad)^2}$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 402

$$b\left(\frac{\frac{f\left(\frac{\frac{\int -\frac{3a^2\left(-24e^2(de+2cf)b^3+4aef(21de+4cf)b^2-a^2f^2(31de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}\right.}{a(be-af)}}{3a(be-af)}\right.$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 27

$$b\left(\frac{\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21de)-2\cdots\right.}{2e(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}}{3a(be-af\cdots}\cdots\right)$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 291

$$b\left(\frac{\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21de)-2\cdots\right.}{2e(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}}{3a(be-\cdots}\cdots\right)$$

$$\frac{d^2\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}$$

$\downarrow$ 221

$$\frac{d^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)^3}dx}{(bc-ad)^2}+$$

$$b\left(\frac{\frac{bx(12a^2df-9abcf-5abde+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2)}{4e(e+fx^2)^2(be-af)}+\frac{x\sqrt{a+bx^2}(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e}{2e(e+fx^2)}\right.}{}}{3a(be-a}\right.$$

↓ 421

$$\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)(fx^2+e)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}(fx^2+e)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$b\left(\frac{\frac{bx(12a^2df-9abcf-5abde+2b^2ce)}{a\sqrt{a+bx^2}(e+fx^2)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2)}{4e(e+fx^2)^2(be-af)}+\frac{x\sqrt{a+bx^2}(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e}{2e(e+fx^2)}\right.}{}}{3a(be-a}\right.$$

↓ 402

$$\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e\right.}{2e\left(e+fx^2\right)}\right.}{}}{3a(be-a}\right.$$

$\downarrow$ 25

$$\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+$$

$$b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e\right.}{2e\left(e+fx^2\right)}\right.}{}}{3a(be-a}\right.$$

$\downarrow$ 402

$$d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{\int-\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}-\frac{f}{4e}\cdots\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad\qquad(bc-ad)^2\qquad\qquad}$$

$$b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e\cdots\right.}{2e\left(e+fx^2\right)\cdots}\right.}{\cdots}}{3a(be-a\cdots}\right.$$

$\downarrow$ 25

$$\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\int \frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(7de-3cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}}{4e(be-af)}-\frac{fx}{4e(}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

$$b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e\right.}{2e\left(e+fx^2\right)}\right.}{}}{3a(be-a}\right.$$

$\downarrow$ 27

$$d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\frac{fx\sqrt{a+bx^2}(2be(5de-3cf)-af(7de-3cf))}{2e\left(e+fx^2\right)(be-af)}-\frac{\left(a^2f^2(7de-3cf)-4abef(5de-2cf)+8b^2e^2(2de-cf)\right)\int\frac{}{\sqrt{bx^2+}}}{2e(be-af)}}{4e(be-af)}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad(bc-ad)^2\qquad}$$

$$b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2e\right.}{2e\left(e+fx^2\right)}\right.}{}}{3a(be-a}\right.$$

$\downarrow$ 291

$$\frac{\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(\cdots}{2e(be-\cdots}}{4e(be-af)}\cdots}{(de-cf)^2}\cdots}{(bc-ad)^2}$$

$$b\left(\frac{b(bc-ad)x}{3a(be-af)(bx^2+a)^{3/2}(fx^2+e)^2}+\frac{\frac{b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}+\frac{f\left(\frac{\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\left(-18d\cdots\right.}{\cdots}\cdots}{\cdots}}{\cdots}\cdots\right.$$

$$\downarrow 221$$

$$\frac{\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(\right.}{2e^{3/2}(be-}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

$$b\left(\frac{b(bc-ad)x}{3a(be-af)(bx^2+a)^{3/2}(fx^2+e)^2}+\frac{\frac{b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}+\frac{f\left(\frac{\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\left(-18df\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 407

$$\left(\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-\right.}{}}{4e(be}\right.}{(de-cf)^2}\right.$$

$$\overline{(bc-ad)^2}$$

$$b\left(\frac{b(bc-ad)x}{3a(be-af)(bx^2+a)^{3/2}(fx^2+e)^2}+\frac{\frac{b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}+\frac{f\left(\frac{\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\left(-18df\right.}{}\right.}{}}{}\right.$$

↓ 291

$$\frac{\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de \cdots\right.}{\cdots}\right.}{(de-cf)^2 \cdots}}{(bc-ad)^2}$$

$$\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)(bx^2+a)^{3/2}(fx^2+e)^2}+\frac{\frac{b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}+\frac{f\left(\frac{\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\left(-18df \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

$$\downarrow 221$$

$$\frac{\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}$$

$$b\left(\frac{b(bc-ad)x}{3a(be-af)(bx^2+a)^{3/2}(fx^2+e)^2}+\frac{\frac{b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}+\frac{f\left(\frac{\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}+\frac{\left(-18df\right.}{}\right.}{}}{}\right.$$

input

```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^3),x]
```

output
```
(d^2*((d^2*((d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sq
rt[c]*Sqrt[b*c - a*d]*(d*e - c*f)) - (f*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(Sqrt[e]*Sqrt[b*e - a*f]*(d*e - c*f))))/(d*e - c*f)^
2 - (f*(-1/4*(f*(d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*(e + f*x^2)^
2) - ((f*(2*b*e*(5*d*e - 3*c*f) - a*f*(7*d*e - 3*c*f))*x*Sqrt[a + b*x^2])/
(2*e*(b*e - a*f)*(e + f*x^2)) - ((a^2*f^2*(7*d*e - 3*c*f) - 4*a*b*e*f*(5*d
*e - 2*c*f) + 8*b^2*e^2*(2*d*e - c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[e
]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/(d
*e - c*f)^2))/(b*c - a*d)^2 + (b*((b*(b*c - a*d)*x)/(3*a*(b*e - a*f)*(a +
b*x^2)^(3/2)*(e + f*x^2)^2) + ((b*(2*b^2*c*e - 5*a*b*d*e - 9*a*b*c*f + 12*
a^2*d*f)*x)/(a*(b*e - a*f)*Sqrt[a + b*x^2]*(e + f*x^2)^2) + (f*(((8*b^3*c*
e^2 + 6*a^3*d*f^2 + a^2*b*f*(49*d*e - 3*c*f) - 20*a*b^2*e*(d*e + 2*c*f))*x
*Sqrt[a + b*x^2])/(4*e*(b*e - a*f)*(e + f*x^2)^2) + (((16*b^4*c*e^3 - 18*a
^4*d*f^3 + 2*a^2*b^2*e*f*(41*d*e - 21*c*f) + 9*a^3*b*f^2*(9*d*e + c*f) - 8
*a*b^3*e^2*(5*d*e + 11*c*f))*x*Sqrt[a + b*x^2])/(2*e*(b*e - a*f)*(e + f*x^
2)) - (3*a^2*(6*a^3*d*f^3 - 24*b^3*e^2*(d*e + 2*c*f) - a^2*b*f^2*(31*d*e +
 3*c*f) + 4*a*b^2*e*f*(21*d*e + 4*c*f))*ArcTanh[(Sqrt[b*e - a*f]*x)/(Sqrt[
e]*Sqrt[a + b*x^2])])/(2*e^(3/2)*(b*e - a*f)^(3/2)))/(4*e*(b*e - a*f))))/(
a*(b*e - a*f)))/(3*a*(b*e - a*f))))/(b*c - a*d)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.48

method	result	size
pseudoelliptic	Expression too large to display	1063
default	Expression too large to display	8901

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)*(-3/5*(a*d-b*c)^2*a^2*(b*x^2+a)^(3/2)*((a*d-b*c)*c
)^(1/2)*(f*x^2+e)^2*f^3*(a^2*c^2*f^4-10/3*a*c*(a*d+8/5*b*c)*e*f^3+5*(a^2*d
^2+4*a*b*c*d+16/5*b^2*c^2)*e^2*f^2-20*b*d*e^3*(a*d+2*b*c)*f+80/3*b^2*d^2*e
^4)*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(8
/5*a^2*(b*x^2+a)^(3/2)*d^5*(a*f-b*e)^4*(f*x^2+e)^2*e^2*arctan(c*(b*x^2+a)^
(1/2)/x/((a*d-b*c)*c)^(1/2))+(3/5*a^3*c*x^2*(b*x^2+a)^2*(a*d-b*c)^2*f^7+((
-7/5*x^2*d+c)*a-14/5*x^2*b*c)*(a*d-b*c)^2*a^2*(b*x^2+a)^2*e*f^6-9/5*a*(a^6
*d^3-2/9*a^5*b*c*d^2-23/9*d*(27/23*d^2*x^4-32/23*c*d*x^2+c^2)*b^2*a^4+16/9
*(-9/8*d^3*x^6+35/8*c*d^2*x^4-4*c^2*d*x^2+c^3)*b^3*a^3+32/9*(9/8*d^2*x^4-9
9/32*c*d*x^2+c^2)*c*b^4*x^2*a^2+16/3*c^2*b^5*x^4*(-83/72*x^2*d+c)*a+88/27*
b^6*x^6*c^3)*e^2*f^5+4*(a^6*d^3-2*b*d^2*(-d*x^2+c)*a^5+b^2*d*(d^2*x^4-4*c*
d*x^2+c^2)*a^4+6*b^3*c*d*x^2*(-d*x^2+c)*a^3-16/5*c*(7/6*d^2*x^4-107/48*c*d
*x^2+c^2)*b^4*x^2*a^2-38/15*c^2*(-17/19*x^2*d+c)*b^5*x^4*a+4/15*b^6*x^6*c^
3)*b*e^3*f^4+8*(d*(d^2*x^4-4*c*d*x^2+c^2)*a^3-4/5*b*(-7/6*d^3*x^6+14/3*c*d
^2*x^4-25/6*c^2*d*x^2+c^3)*a^2-1/3*c*(1/5*d^2*x^4-28/5*c*d*x^2+c^2)*b^2*x^
2*a+4/15*c^2*x^4*b^3*(-d*x^2+c))*b^4*e^4*f^3-16*(d^2*(-d*x^2+c)*a^3-3/5*d*
b*(11/9*d^2*x^4-14/9*c*d*x^2+c^2)*a^2-1/10*(-5/3*d^3*x^6+1/3*c*d^2*x^4+5/3
*c^2*d*x^2+c^3)*b^2*a-1/15*b^3*c*x^2*(d^2*x^4-4*c*d*x^2+c^2))*b^4*e^5*f^2+
8*d*(a^3*d^2+2/15*x^2*a^2*b*d^2-2/5*b^2*(5/3*d^2*x^4-5/6*c*d*x^2+c^2)*a-4/
15*b^3*c*x^2*(-d*x^2+c))*b^4*e^6*f-16/5*d^2*(d*a^2-1/2*(-5/3*x^2*d+c)*b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3998 vs. $2(682) = 1364$.

Time $= 17.59$ (sec) , antiderivative size $= 3998$, normalized size of antiderivative $= 5.55$

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
-sqrt(b)*d^5*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/
sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2*d^3*e^3 - 2*a*b*c*d^4*e^3 + a^2*d^5*e^
3 - 3*b^2*c^3*d^2*e^2*f + 6*a*b*c^2*d^3*e^2*f - 3*a^2*c*d^4*e^2*f + 3*b^2*
c^4*d*e*f^2 - 6*a*b*c^3*d^2*e*f^2 + 3*a^2*c^2*d^3*e*f^2 - b^2*c^5*f^3 + 2*
a*b*c^4*d*f^3 - a^2*c^3*d^2*f^3)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/3*((2*b^14*
c^3*e^5 - 9*a*b^13*c^2*d*e^5 + 12*a^2*b^12*c*d^2*e^5 - 5*a^3*b^11*d^3*e^5
- 19*a*b^13*c^3*e^4*f + 72*a^2*b^12*c^2*d*e^4*f - 87*a^3*b^11*c*d^2*e^4*f
+ 34*a^4*b^10*d^3*e^4*f + 56*a^2*b^12*c^3*e^3*f^2 - 198*a^3*b^11*c^2*d*e^3
*f^2 + 228*a^4*b^10*c*d^2*e^3*f^2 - 86*a^5*b^9*d^3*e^3*f^2 - 74*a^3*b^11*c
^3*e^2*f^3 + 252*a^4*b^10*c^2*d*e^2*f^3 - 282*a^5*b^9*c*d^2*e^2*f^3 + 104*
a^6*b^8*d^3*e^2*f^3 + 46*a^4*b^10*c^3*e*f^4 - 153*a^5*b^9*c^2*d*e*f^4 + 16
8*a^6*b^8*c*d^2*e*f^4 - 61*a^7*b^7*d^3*e*f^4 - 11*a^5*b^9*c^3*f^5 + 36*a^6
*b^8*c^2*d*f^5 - 39*a^7*b^7*c*d^2*f^5 + 14*a^8*b^6*d^3*f^5)*x^2/(a^2*b^13*
c^4*e^8 - 4*a^3*b^12*c^3*d*e^8 + 6*a^4*b^11*c^2*d^2*e^8 - 4*a^5*b^10*c*d^3
*e^8 + a^6*b^9*d^4*e^8 - 8*a^3*b^12*c^4*e^7*f + 32*a^4*b^11*c^3*d*e^7*f -
48*a^5*b^10*c^2*d^2*e^7*f + 32*a^6*b^9*c*d^3*e^7*f - 8*a^7*b^8*d^4*e^7*f +
 28*a^4*b^11*c^4*e^6*f^2 - 112*a^5*b^10*c^3*d*e^6*f^2 + 168*a^6*b^9*c^2*d^
2*e^6*f^2 - 112*a^7*b^8*c*d^3*e^6*f^2 + 28*a^8*b^7*d^4*e^6*f^2 - 56*a^5*b^
10*c^4*e^5*f^3 + 224*a^6*b^9*c^3*d*e^5*f^3 - 336*a^7*b^8*c^2*d^2*e^5*f^3 +
 224*a^8*b^7*c*d^3*e^5*f^3 - 56*a^9*b^6*d^4*e^5*f^3 + 70*a^6*b^9*c^4*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}(d\,x^2+c)(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)/(f*x^2+e)^3,x)
```

3.379 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 653

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{b(2b^3ce(de-cf)^2+3a^3d^2f^2(de+cf)-6a^2bdf(d^2e^2+c^2f^2)+}{6ac(bc-ad)^2e(be-af)^2(de-cf)^2(a+bx^2)}$$

$$+\frac{b(4b^5c^2e^2(de-cf)^2+3a^5d^3f^3(de+cf)-16ab^4ce(de-cf)^2(de+cf)-9a^4bd^2f^2(d^2e^2+c^2f^2)+9a^3b^2}{6a^2c(bc-ad)^3e(be-af)^3(de-cf)}$$

$$-\frac{d^3x}{2c(bc-ad)(de-cf)^2(a+bx^2)^{3/2}(c+dx^2)}$$

$$-\frac{f^3x}{2e(be-af)(de-cf)^2(a+bx^2)^{3/2}(e+fx^2)}$$

$$-\frac{d^4(ad(de-5cf)-2bc(3de-5cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}(de-cf)^3}$$

$$+\frac{f^4(2be(5de-3cf)-af(5de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{2e^{3/2}(be-af)^{7/2}(de-cf)^3}$$

output
```
1/6*b*(2*b^3*c*e*(-c*f+d*e)^2+3*a^3*d^2*f^2*(c*f+d*e)-6*a^2*b*d*f*(c^2*f^2
+d^2*e^2)+3*a*b^2*(c^3*f^3+d^3*e^3))*x/a/c/(-a*d+b*c)^2/e/(-a*f+b*e)^2/(-c
*f+d*e)^2/(b*x^2+a)^(3/2)+1/6*b*(4*b^5*c^2*e^2*(-c*f+d*e)^2+3*a^5*d^3*f^3*
(c*f+d*e)-16*a*b^4*c*e*(-c*f+d*e)^2*(c*f+d*e)-9*a^4*b*d^2*f^2*(c^2*f^2+d^2
*e^2)+9*a^3*b^2*d*f*(c^3*f^3+d^3*e^3)-a^2*b^3*(3*c^4*f^4-28*c^3*d*e*f^3+56
*c^2*d^2*e^2*f^2-28*c*d^3*e^3*f+3*d^4*e^4))*x/a^2/c/(-a*d+b*c)^3/e/(-a*f+b
*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)-1/2*d^3*x/c/(-a*d+b*c)/(-c*f+d*e)^2/(b*
x^2+a)^(3/2)/(d*x^2+c)-1/2*f^3*x/e/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(3/2)
/(f*x^2+e)-1/2*d^4*(a*d*(-5*c*f+d*e)-2*b*c*(-5*c*f+3*d*e))*arctanh((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(7/2)/(-c*f+d*e)^3
+1/2*f^4*(2*b*e*(-3*c*f+5*d*e)-a*f*(-c*f+5*d*e))*arctanh((-a*f+b*e)^(1/2)*
x/e^(1/2)/(b*x^2+a)^(1/2))/e^(3/2)/(-a*f+b*e)^(7/2)/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 19.56 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \frac{1}{6}\left(x\sqrt{a+bx^2}\left(\frac{2b^4}{a(bc-ad)^2(be-af)^2(a+bx^2)^2}+\frac{4b^4(b^2ce+7}{a^2(bc-ad)}\right.\right.$$
$$-\frac{3d^4(ad(de-5cf)+2bc(-3de+5cf))\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}(-bc+ad)^{7/2}(-de+cf)^3}$$
$$\left.-\frac{3f^4(2be(5de-3cf)+af(-5de+cf))\arctan\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)}{e^{3/2}(-be+af)^{7/2}(de-cf)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*((2*b^4)/(a*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x^2)^2)
+ (4*b^4*(b^2*c*e + 7*a^2*d*f - 4*a*b*(d*e + c*f)))/(a^2*(b*c - a*d)^3*(b*
e - a*f)^3*(a + b*x^2)) - (3*d^5)/(c*(b*c - a*d)^3*(d*e - c*f)^2*(c + d*x^
2)) - (3*f^5)/(e*(b*e - a*f)^3*(d*e - c*f)^2*(e + f*x^2))) - (3*d^4*(a*d*(
d*e - 5*c*f) + 2*b*c*(-3*d*e + 5*c*f))*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt
[c]*Sqrt[a + b*x^2])])/(c^(3/2)*(-(b*c) + a*d)^(7/2)*(-(d*e) + c*f)^3) - (
3*f^4*(2*b*e*(5*d*e - 3*c*f) + a*f*(-5*d*e + c*f))*ArcTan[(Sqrt[-(b*e) + a
*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(e^(3/2)*(-(b*e) + a*f)^(7/2)*(d*e - c*
f)^3))/6
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)(fx^2+e)^2}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+4b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+4b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}-\frac{\int -\frac{f\left(2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-a}\right)}{(bc-ad)^2}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{f\left(2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{b\left(\frac{\frac{f\int \frac{2b\left(10dfa^2-5bdea-7bcfa+2b^2ce\right)x^2+a\left(-6dfa^2+b(de+3cf)a+2b^2ce\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)(be-af)}\right)}{(bc-ad)^2}\right.$$

$$\overline{\qquad bc-ad \qquad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow 402$$

$$b\left(\frac{b\left(\frac{\frac{f\left(\frac{\int \frac{3a^2\left(2e(2de+3cf)b^2-af(11de+cf)b+2a^2df^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx\left(10a^2df-7abcf-5abde\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be}}{3a(be-af)}\right.}{(bc-ad)^2}\right.$$

$$\overline{\qquad bc-ad \qquad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$$\Big\downarrow 27$$

$$b\left(\frac{b\left(\frac{\frac{f\left(\frac{3a^2\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx\left(10a\right.}{a\sqrt{a}}}{3a(be-af)}}{(bc-ad)^2}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}$$

$\downarrow$ 291

$$b\left(\frac{b\left(\frac{\frac{f\left(\frac{3a^2\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)-2ab^2e(8cf+5de)+4b^3ce^2\right)}{2e\left(e+fx^2\right)(be-af)}\right)}{a(be-af)}+\frac{bx(}{}}{3a(be-af)}}{(bc-ad)^2}\right.}{bc-ad}$$

$$\frac{d\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2d\right.}{2e^{3/2}(be-af)^{3/2}}\right.}{a(be-}}{3a(be-af)}\right)}{(bc-a}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 421

$$\frac{b\left(\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf\right.}{2e^{3/2}(be-af)^{3/}}\right.}{}}{}\right.}{}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 291

$$\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-\right.}{2e^{3/2}(be-a}\right.}{}\right.}{}\right)}{bc-ad}$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 221

$$\frac{b\left(\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-\right.}{2e^{3/2}(be-af}\right.}{}\right.}{}\right)}{bc-ad}$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{\int\frac{2be(2de-cf)-af(3de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arc}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 27

$$b\left(\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+\ldots\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-\ldots}\right.}{\ldots}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\Big\downarrow$ 291

$$b\left(\frac{d^2\left(\frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{2e\left(e+fx^2\right)(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(10a^2df-7abcf-5a\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)}\right.}{}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 221

$$b\left(\frac{b\left(\frac{\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)\right.}{2e\left(e+fx^2\right)(be\right.}\right)}{a(be-af)}}{3a(be-af)}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^2}dx}{bc-ad}$$

$\downarrow$ 426

$$\frac{b\left(\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)\right.}{2e\left(e+fx^2\right)(be}\right)}{a(be-af)}}{3a(be-af)}}{(bc-ad)^2}}{}$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\operatorname{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)\right.}{2e\left(e+fx^2\right)(be}\right)}{a(be-af)}}{3a(be-af)}}{(bc-ad)^2}}{}$$

$$\frac{d\left(\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}-\frac{b\int-\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{bx\left(10a^2df-7abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)(be-af)}+\frac{f\left(\frac{3a^2\text{arctanh}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right)\left(2a^2df^2-abf(cf+11de)+2b^2e(3cf+2de)\right)}{2e^{3/2}(be-af)^{3/2}}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(19de-3cf)\cdots\right.}{2e\left(e+fx^2\right)(be\cdots}\right)}{a(be-af)}}{3a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots}{\cdots}$$

$$\frac{d\left(\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx}{(bc-ad)^2}+\frac{b\int\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^2}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \; 402$$

$$\frac{b\left(\frac{\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(}\right.}{}\right.}{}$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dxd^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\int\frac{a(bde+bcf-2adf)-2b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^2}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 402

$$\frac{b\left(\frac{\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}\left(\cdots\right.}\right.}{\cdots}\right)}{\cdots}$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{\int\frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{f\left(-2dfa^2-b(de-cf)a+2b^2ce\right)x\sqrt{bx^2+\cdots}}{2e(be-af)\left(fx^2+e\right)}}{a(be-af)}\cdots\right.}{(bc-ad)^2}\cdots\right)}{bc-ad}\cdots\right)}{bc-ad}$$

$\downarrow$ 27

$$b\left(\frac{\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\cdots}\cdots\right.}{\cdots}\right.$$

$$\overline{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{f\left(-2dfa^2-b(de-cf)\cdots\right.}{2e(be-af)\cdots}}{a(be-af)}\cdots\right.}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots\right.}$$

$$bc-ad$$

↓ 291

$$b\left(\frac{\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\cdots}\right.}{\cdots}\right.$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)\int\frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2e(be-af)}-\frac{f\left(-2dfa^2-b(de\cdots\right.}{2e(be-\cdots}}{a(be-af)}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.}{bc-ad}$$

↓ 221

$$\frac{b\left(\frac{\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\ldots}\right.}{}\right.}{}$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^2}dxd^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f\left(-2dfa^2-b(de-\ldots\right.}{2e(be-\ldots}}{a(be-af)}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.}{bc-ad}$$

↓ 421

$$b\left(\frac{\left(\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}\,(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$\overline{}$$

$$d\left(\frac{b\left(\frac{\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{dx^2+c}dx}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{a\left(2e(de+2cf)b^2-af(7de+cf)b+2a^2df^2\right)\operatorname{arctanh}\left(\frac{\sqrt{\cdots}}{\sqrt{e}\cdots}\right.}{2e^{3/2}(be-af)^{3/2}}\cdots}{a(be-\cdots}\cdots\right.}{(bc-ad)^2}\cdots\right.}{bc-ad}\cdots\right.$$

$$bc-ad$$

↓ 301

$$b\left(\frac{\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\cdots}\right.}{}\right.$$

$$d\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{b\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{a\left(2e(de+2cf)b^2-\cdots\right.}{}\right.}{}\right)}{bc-ad}\right.$$

$$bc-ad$$

↓ 224

$$b\left(\frac{\left(\frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)^2}-\frac{f\left(\frac{(2be(2de-cf)-af(3de-cf))\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{2e^{3/2}(be-af)^{3/2}}-\frac{f(de-cf)x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/2}(\cdots}\right.}{\cdots}\right.$$

$$d\left(\frac{b\left(\frac{\left(\frac{d^2\left(\frac{b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{d}\right)}{(de-cf)^2}-\frac{f\int\frac{\sqrt{bx^2+a}\left(dfx^2+2de-cf\right)}{\left(fx^2+e\right)^2}dx}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)}-\frac{\frac{a\left(2e(de-\cdots\right.}{\cdots}}{\cdots}\right.}{bc-ad}\right.$$

$$bc-a\cdots$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 1048, normalized size of antiderivative = 1.60

method	result	size
pseudoelliptic	Expression too large to display	1048
default	Expression too large to display	7278

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2/((a*f-b*e)*e)^(1/2)*(((a*f-b*e)*e)^(1/2)*a^2*(b*x^2+a)^(3/2)*d^4*(a*f
-b*e)^3*(d*x^2+c)*((-2*c^2*f+6/5*c*e*d)*b+d*(c*f-1/5*d*e)*a)*(f*x^2+e)*e*a
rctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+1/5*((a*d-b*c)*c)^(1/2)*((a
*d-b*c)^3*a^2*(b*x^2+a)^(3/2)*c*((-6*c*e*f+10*d*e^2)*b+f*a*(c*f-5*d*e))*(d
*x^2+c)*(f*x^2+e)*f^4*arctan(e*(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))-((a*
f-b*e)*e)^(1/2)*(d^3*(c^2*f^2+c*d*f^2*x^2+d^2*e*(f*x^2+e))*f^3*a^7-3*d^2*(
c^3*f^3+1/3*c^2*d*f^3*x^2-2/3*c*d^2*f^3*x^4+d^3*(-2/3*f*x^2+e)*(f*x^2+e)*e
)*b*f^2*a^6+3*(c^4*f^4-c^3*d*f^4*x^2-5/3*c^2*d^2*f^4*x^4+1/3*c*d^3*f^4*x^6
+d^4*(1/3*f^2*x^4-2*e*f*x^2+e^2)*(f*x^2+e)*e)*d*b^2*f*a^5-(c^5*f^5-5*c^4*d
*f^5*x^2-3*c^3*d^2*f^5*x^4+3*c^2*d^3*f^5*x^6+d^5*e^2*(f*x^2+e)*(3*f^2*x^4-
6*e*f*x^2+e^2))*b^3*a^4-2*b^4*(c^5*f^5*x^2-5*(1/10*f^2*x^4+e*f*x^2+e^2)*d*
f^3*c^4+10*(-3/20*f^3*x^6-1/2*e*f^2*x^4+1/2*e^2*f*x^2+e^3)*d^2*f^2*c^3-5*f
*e^2*d^3*(f*x^2+e)*(-2*f*x^2+e)*c^2-5*d^4*e^3*f*x^2*(f*x^2+e)*c+d^5*(-3/2*
f*x^2+e)*x^2*(f*x^2+e)*e^3)*a^3-6*(f^3*(1/6*f^2*x^4+e*f*x^2+e^2)*c^5-d*(-1
/6*f^3*x^6+5/9*e*f^2*x^4+14/9*e^2*f*x^2+e^3)*f^2*c^4-(14/9*f^2*x^4-19/9*e*
f*x^2+e^2)*d^2*(f*x^2+e)*f*e*c^3+d^3*(28/9*f^2*x^4-23/9*e*f*x^2+e^2)*(f*x^
2+e)*e^2*c^2+d^4*(-14/9*f*x^2+e)*x^2*(f*x^2+e)*e^3*c+1/6*d^5*e^4*x^4*(f*x^
2+e))*b^5*a^2+2*c*((-8/3*f*x^2+e)*c-8/3*d*e*x^2)*(c*f-d*e)^2*(d*x^2+c)*b^6
*(f*x^2+e)*e*a+4/3*b^7*c^2*e^2*x^2*(f*x^2+e)*(d*x^2+c)*(c*f-d*e)^2)*(c*f-d
*e)*x))/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)/(c*f-d*e)^3/(a*d-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**2*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^2(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2*(f*x^2 + e)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5859 vs. $2(614) = 1228$.

Time = 13.16 (sec) , antiderivative size = 5859, normalized size of antiderivative = 8.97

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^2\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
1/3*(2*(b^14*c^4*e^4 - 7*a*b^13*c^3*d*e^4 + 15*a^2*b^12*c^2*d^2*e^4 - 13*a
^3*b^11*c*d^3*e^4 + 4*a^4*b^10*d^4*e^4 - 7*a*b^13*c^4*e^3*f + 40*a^2*b^12*
c^3*d*e^3*f - 78*a^3*b^11*c^2*d^2*e^3*f + 64*a^4*b^10*c*d^3*e^3*f - 19*a^5
*b^9*d^4*e^3*f + 15*a^2*b^12*c^4*e^2*f^2 - 78*a^3*b^11*c^3*d*e^2*f^2 + 144
*a^4*b^10*c^2*d^2*e^2*f^2 - 114*a^5*b^9*c*d^3*e^2*f^2 + 33*a^6*b^8*d^4*e^2
*f^2 - 13*a^3*b^11*c^4*e*f^3 + 64*a^4*b^10*c^3*d*e*f^3 - 114*a^5*b^9*c^2*d
^2*e*f^3 + 88*a^6*b^8*c*d^3*e*f^3 - 25*a^7*b^7*d^4*e*f^3 + 4*a^4*b^10*c^4*
f^4 - 19*a^5*b^9*c^3*d*f^4 + 33*a^6*b^8*c^2*d^2*f^4 - 25*a^7*b^7*c*d^3*f^4
 + 7*a^8*b^6*d^4*f^4)*x^2/(a^2*b^13*c^6*e^6 - 6*a^3*b^12*c^5*d*e^6 + 15*a^
4*b^11*c^4*d^2*e^6 - 20*a^5*b^10*c^3*d^3*e^6 + 15*a^6*b^9*c^2*d^4*e^6 - 6*
a^7*b^8*c*d^5*e^6 + a^8*b^7*d^6*e^6 - 6*a^3*b^12*c^6*e^5*f + 36*a^4*b^11*c
^5*d*e^5*f - 90*a^5*b^10*c^4*d^2*e^5*f + 120*a^6*b^9*c^3*d^3*e^5*f - 90*a^
7*b^8*c^2*d^4*e^5*f + 36*a^8*b^7*c*d^5*e^5*f - 6*a^9*b^6*d^6*e^5*f + 15*a^
4*b^11*c^6*e^4*f^2 - 90*a^5*b^10*c^5*d*e^4*f^2 + 225*a^6*b^9*c^4*d^2*e^4*f
^2 - 300*a^7*b^8*c^3*d^3*e^4*f^2 + 225*a^8*b^7*c^2*d^4*e^4*f^2 - 90*a^9*b^
6*c*d^5*e^4*f^2 + 15*a^10*b^5*d^6*e^4*f^2 - 20*a^5*b^10*c^6*e^3*f^3 + 120*
a^6*b^9*c^5*d*e^3*f^3 - 300*a^7*b^8*c^4*d^2*e^3*f^3 + 400*a^8*b^7*c^3*d^3*
e^3*f^3 - 300*a^9*b^6*c^2*d^4*e^3*f^3 + 120*a^10*b^5*c*d^5*e^3*f^3 - 20*a^
11*b^4*d^6*e^3*f^3 + 15*a^6*b^9*c^6*e^2*f^4 - 90*a^7*b^8*c^5*d*e^2*f^4 + 2
25*a^8*b^7*c^4*d^2*e^2*f^4 - 300*a^9*b^6*c^3*d^3*e^2*f^4 + 225*a^10*b^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^2\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}(d\,x^2+c)^2(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^2,x)
```

3.380 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 1034

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \frac{b(8b^4ce^2(de-cf)^3-3a^4d^2f^3(4d^2e^2+11cdef-3c^2f^2)+6a^3ba\cdots}{\cdots}$$

$$+\frac{b(16b^6c^2e^3(de-cf)^3-8ab^5ce^2(de-cf)^3(8de+11cf)-3a^6d^3f^4(4d^2e^2+11cdef-3c^2f^2)+3a^5bd^2f^3(\cdots}{\cdots}$$

$$-\frac{d^4x}{2c(bc-ad)(de-cf)^3(a+bx^2)^{3/2}(c+dx^2)}$$

$$-\frac{f^3x}{4e(be-af)(de-cf)^2(a+bx^2)^{3/2}(e+fx^2)^2}$$

$$-\frac{f^3(2be(9de-5cf)-af(11de-3cf))x}{8e^2(be-af)^2(de-cf)^3(a+bx^2)^{3/2}(e+fx^2)}$$

$$-\frac{d^5(ad(de-7cf)-6bc(de-2cf))\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}(de-cf)^4}$$

$$-\frac{f^4(8abef(15d^2e^2-11cdef+2c^2f^2)-24b^2e^2(5d^2e^2-6cdef+2c^2f^2)-a^2f^2(35d^2e^2-14cdef+3c^2f^2\cdots}{8e^{5/2}(be-af)^{9/2}(de-cf)^4}$$

output
```
1/24*b*(8*b^4*c*e^2*(-c*f+d*e)^3-3*a^4*d^2*f^3*(-3*c^2*f^2+11*c*d*e*f+4*d^
2*e^2)+6*a^3*b*d*f^2*(-3*c^3*f^3+5*c^2*d*e*f^2+10*c*d^2*e^2*f+6*d^3*e^3)-3
*a^2*b^2*f*(-3*c^4*f^4-13*c^3*d*e*f^3+40*c^2*d^2*e^2*f^2+12*d^4*e^4)+12*a*
b^3*e*(-3*c^4*f^4+5*c^3*d*e*f^3+d^4*e^4))*x/a/c/(-a*d+b*c)^2/e^2/(-a*f+b*e
)^3/(-c*f+d*e)^3/(b*x^2+a)^(3/2)+1/24*b*(16*b^6*c^2*e^3*(-c*f+d*e)^3-8*a*b
^5*c*e^2*(-c*f+d*e)^3*(11*c*f+8*d*e)-3*a^6*d^3*f^4*(-3*c^2*f^2+11*c*d*e*f+
4*d^2*e^2)+3*a^5*b*d^2*f^3*(-9*c^3*f^3+19*c^2*d*e*f^2+22*c*d^2*e^2*f+16*d^
3*e^3)-9*a^4*b^2*d*f^2*(-3*c^4*f^4-3*c^3*d*e*f^3+22*c^2*d^2*e^2*f^2+8*d^4*
e^4)-2*a^2*b^4*e*(-21*c^5*f^5+101*c^4*d*e*f^4-204*c^3*d^2*e^2*f^3+204*c^2*
d^3*e^3*f^2-68*c*d^4*e^4*f+6*d^5*e^5)+3*a^3*b^3*f*(-3*c^5*f^5-31*c^4*d*e*f
^4+66*c^3*d^2*e^2*f^3+16*d^5*e^5))*x/a^2/c/(-a*d+b*c)^3/e^2/(-a*f+b*e)^4/(
-c*f+d*e)^3/(b*x^2+a)^(1/2)-1/2*d^4*x/c/(-a*d+b*c)/(-c*f+d*e)^3/(b*x^2+a)^
(3/2)/(d*x^2+c)-1/4*f^3*x/e/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(3/2)/(f*x^2
+e)^2-1/8*f^3*(2*b*e*(-5*c*f+9*d*e)-a*f*(-3*c*f+11*d*e))*x/e^2/(-a*f+b*e)^
2/(-c*f+d*e)^3/(b*x^2+a)^(3/2)/(f*x^2+e)-1/2*d^5*(a*d*(-7*c*f+d*e)-6*b*c*(
-2*c*f+d*e))*arctanh((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(
-a*d+b*c)^(7/2)/(-c*f+d*e)^4-1/8*f^4*(8*a*b*e*f*(2*c^2*f^2-11*c*d*e*f+15*d
^2*e^2)-24*b^2*e^2*(2*c^2*f^2-6*c*d*e*f+5*d^2*e^2)-a^2*f^2*(3*c^2*f^2-14*c
*d*e*f+35*d^2*e^2))*arctanh((-a*f+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2))/e^
(5/2)/(-a*f+b*e)^(9/2)/(-c*f+d*e)^4
```

Mathematica [A] (verified)

Time = 24.72 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \sqrt{a+bx^2}\left(-\frac{b^5x}{3a(-bc+ad)^2(-be+af)^3(a+bx^2)^2} - \frac{b^5(2b^2ce-8abde-11abcf+17a^2df)\,x}{3a^2(-bc+ad)^3(-be+af)^4(a+bx^2)} + \frac{d^6x}{2c(bc-ad)^3(-de+cf)^3(c+dx^2)} - \frac{f^5x}{4e(be-af)^3(de-cf)^2(e+fx^2)^2} - \frac{f^5(22bde^2-14bcef-11adef+3acf^2)\,x}{8e^2(be-af)^4(de-cf)^3(e+fx^2)}\right) - \frac{(-6bcd^6e+ad^7e+12bc^2d^5f-7acd^6f)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^3\sqrt{-bc+ad}(-de+cf)^4} + \frac{(120b^2d^2e^4f^4-144b^2cde^3f^5-120abd^2e^3f^5+48b^2c^2e^2f^6+88abcde^2f^6+35a^2d^2e^2f^6-16abc^2ef^7-14a}{8e^{5/2}(be-af)^4\sqrt{-be+af}(de-cf)^4}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
Sqrt[a + b*x^2]*(-1/3*(b^5*x)/(a*(-(b*c) + a*d)^2*(-(b*e) + a*f)^3*(a + b*
x^2)^2) - (b^5*(2*b^2*c*e - 8*a*b*d*e - 11*a*b*c*f + 17*a^2*d*f)*x)/(3*a^2
*(-(b*c) + a*d)^3*(-(b*e) + a*f)^4*(a + b*x^2)) + (d^6*x)/(2*c*(b*c - a*d)
^3*(-(d*e) + c*f)^3*(c + d*x^2)) - (f^5*x)/(4*e*(b*e - a*f)^3*(d*e - c*f)^
2*(e + f*x^2)^2) - (f^5*(22*b*d*e^2 - 14*b*c*e*f - 11*a*d*e*f + 3*a*c*f^2)
*x)/(8*e^2*(b*e - a*f)^4*(d*e - c*f)^3*(e + f*x^2))) - ((-6*b*c*d^6*e + a*
d^7*e + 12*b*c^2*d^5*f - 7*a*c*d^6*f)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[
c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^3*Sqrt[-(b*c) + a*d]*(-(d*e)
+ c*f)^4) + ((120*b^2*d^2*e^4*f^4 - 144*b^2*c*d*e^3*f^5 - 120*a*b*d^2*e^3*
f^5 + 48*b^2*c^2*e^2*f^6 + 88*a*b*c*d*e^2*f^6 + 35*a^2*d^2*e^2*f^6 - 16*a*
b*c^2*e*f^7 - 14*a^2*c*d*e*f^7 + 3*a^2*c^2*f^8)*ArcTan[(Sqrt[-(b*e) + a*f]
*x)/(Sqrt[e]*Sqrt[a + b*x^2])])/(8*e^(5/2)*(b*e - a*f)^4*Sqrt[-(b*e) + a*f
]*(d*e - c*f)^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\left(e+fx^2\right)^3}dx$$

$$\Big\downarrow \ 426$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \ 421$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \ 402$$

$$\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-\cdots}\right)}{(bc-ad)^2}\cdots}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af}\right)}{(bc-ad)^2}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{b\left(\frac{\frac{f\int \frac{4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$b\left(\frac{b\left(\frac{b\left(\frac{f\left(\frac{\int \frac{2b\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)x^2+a\left(18df^2a^3-3bf(23de+3cf)a^2+4b^2e(4de+9cf)a+8b^3ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2\cdots\right.}{\cdots}\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right)}{(bc-ad)^2}\cdots\right)}{bc\cdots}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 402$$

$$b\left(\frac{b\left(\frac{b\left(\frac{f\left(\frac{\frac{\int -\frac{3a^2\left(-24e^2(de+2cf)b^3+4aef(21de+4cf)b^2-a^2f^2(31de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8\cdots\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}\cdots\right)}{a(be-af)}\cdots}{3a(be-af)}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 27

$$b\left(\frac{b\left(\frac{f\left(\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21de)\right.}{2e(be-af}\right.}{4e(be-af)}}{a(be-af)}}{3a(be-}}{}\right.$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\downarrow 291$$

$$b\left(\frac{b\left(\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21de)\right.}{2e(be-}}{4e(be-af)}\right.}{a(be-af)}}{3a(b}\right.}{}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\downarrow 221$$

$$b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{\cdots}}{\cdots}\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{\cdots}{\cdots}$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\big\downarrow$ 421

$$b\left(\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int \frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3c\right.}{4e\left(e+fx^2\right)^2(\right.}\right.}{}\right.}{}\right.$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{ 402}$$

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int-\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(12a^2df-9abcf-5c\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)}\right.}{}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$b\left(\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{\int\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{fx\sqrt{a+bx^2}(de-cf)}{4e\left(e+fx^2\right)^2(be-af)}\right)}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{\frac{bx\left(12a^2df-9abcf-5c\cdots\right.}{a\sqrt{a+bx^2}\left(e+fx^2\right)\cdots}}{\cdots}\cdots\right)}{\cdots}\right)$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))\sqrt{bx^2+a}x}{2e(be-af)\left(fx^2+e\right)}+\frac{\int-\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2e(be-af)}}{4e(be-af)}}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow 25$$

$$b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\int\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2e(be-af)}}{4e(be-af)}}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{\dfrac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\dfrac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(\cdots}{2e(be-af\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \ 291$$

$$b\left(\frac{\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(\right.}{2e(be-}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2f^2(\right.}{2e^{3/2}(be-}}{4e(be-af)}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\Big\downarrow$ 407

$$b\left(\frac{\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-\cdots\right.}{\cdots}}{4e(be-\cdots}\cdots\right.}{(de-cf)^2}\cdots}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$\downarrow$ 291

$$b\left(\frac{\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 221$$

$$b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}}{(bc-ad)^2}\cdots\right.$$

$$\frac{d\int\frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \text{426}$$

$$b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\cdots\right)}{(de-cf)^2}\cdots}{(bc-ad)^2}\cdots\right)$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 421}$$

$$b\left(\frac{\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}}{}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}-\frac{b\int-\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.}{\cdots}$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx\,d^2}{(bc-ad)^2}+\frac{b\int\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}\right.}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}-\frac{\int\frac{a(bde+bcf-2adf)-4b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 402

$$b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$\frac{d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}-\frac{\frac{\int\frac{a\left(4e(de+2cf)b^2-3af(5de+cf)b+6a^2df^2\right)-2bf\left(-2dfa^2-b(3de-cf)a+4b^2ce\right)x^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}\cdots}{a(be-af)}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.}{bc-ad}$$

↓ 402

$$b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-\right.}{}}{}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.$$

$$d\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}-\frac{\frac{\int-\frac{a\left(-8e^2(de+3cf)b^3+4aef(10de+3cf)b^2-a^2f^2(23de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}-\frac{f\left(6df^2\right.}{}}{4e(be-af)}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.$$

$bc-ad$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [A] (verified)

Time = 99.27 (sec) , antiderivative size = 1980, normalized size of antiderivative = 1.91

method	result	size
pseudoelliptic	Expression too large to display	1980
default	Expression too large to display	11082

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*e)^(1/2)/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(3/2)*(-3/5*(a*d-b*c
)^3*a^2*(b*x^2+a)^(3/2)*c*(a^2*c^2*f^4-14/3*(a*d+8/7*b*c)*a*c*e*f^3+35/3*(
a*d+12/7*b*c)*e^2*(a*d+4/5*b*c)*f^2-40*(a*d+6/5*b*c)*d*b*e^3*f+40*b^2*d^2*
e^4)*(d*x^2+c)*((a*d-b*c)*c)^(1/2)*(f*x^2+e)^2*f^4*arctan(e*(b*x^2+a)^(1/2
)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(28/5*a^2*(c*(-12/7*b*c+a*d)*
f-1/7*d*e*(a*d-6*b*c))*d^5*(b*x^2+a)^(3/2)*(a*f-b*e)^4*(d*x^2+c)*(f*x^2+e)
^2*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+(3/5*a^3*c^2*x^2*(d
*x^2+c)*(b*x^2+a)^2*(a*d-b*c)^3*f^8+(a*d-b*c)^3*a^2*c*((-11/5*x^2*d+c)*a-1
4/5*x^2*b*c)*(b*x^2+a)^2*(d*x^2+c)*e*f^7-13/5*a*(d^4*(4/13*d^2*x^4+c*d*x^2
+c^2)*a^7-23/13*d^3*(-8/23*d^3*x^6-4/23*c*d^2*x^4+19/23*c^2*d*x^2+c^3)*b*a
^6-9/13*d^2*(-4/9*d^4*x^8+31/9*c*d^3*x^6+11/9*c^2*d^2*x^4-11/9*c^3*d*x^2+c
^4)*b^2*a^5+35/13*(-22/35*d^3*x^6+109/35*c*d^2*x^4-12/5*c^2*d*x^2+c^3)*c*d
*(d*x^2+c)*b^3*a^4-16/13*c^2*(d*x^2+c)*b^4*(-33/8*d^3*x^6+141/16*c*d^2*x^4
-23/4*c^2*d*x^2+c^3)*a^3-32/13*(33/16*d^2*x^4-127/32*c*d*x^2+c^2)*c^3*(d*x
^2+c)*b^5*x^2*a^2-48/13*c^4*(-101/72*x^2*d+c)*(d*x^2+c)*b^6*x^4*a-88/39*b^
7*c^5*x^6*(d*x^2+c))*e^2*f^6-8/5*(a^8*d^6*x^2-3*b*c*d^4*(d*x^2+c)*a^7+9*(-
1/3*d^3*x^6-2/3*c*d^2*x^4+1/3*c^2*d*x^2+c^3)*d^3*b^2*a^6-9*d^2*b^3*(2/9*d^
4*x^8+1/3*c*d^3*x^6-5/3*c^2*d^2*x^4-c^3*d*x^2+c^4)*a^5+3*b^4*c^2*d*(d*x^2+
c)*(3*d^2*x^4-6*c*d*x^2+c^2)*a^4+18*(-3/2*x^2*d+c)*c^3*d*(d*x^2+c)*b^5*x^2
*a^3-8*(-3/2*x^2*d+c)*c^3*(d*x^2+c)*b^6*x^2*(-17/12*x^2*d+c)*a^2-19/3*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^2(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5637 vs. $2(990) = 1980$.

Time = 47.97 (sec) , antiderivative size = 5637, normalized size of antiderivative = 5.45

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^16*c^4*e^5 - 14*a*b^15*c^3*d*e^5 + 30*a^2*b^14*c^2*d^2*e^5 - 26*
a^3*b^13*c*d^3*e^5 + 8*a^4*b^12*d^4*e^5 - 19*a*b^15*c^4*e^4*f + 106*a^2*b^
14*c^3*d*e^4*f - 204*a^3*b^13*c^2*d^2*e^4*f + 166*a^4*b^12*c*d^3*e^4*f - 4
9*a^5*b^11*d^4*e^4*f + 56*a^2*b^14*c^4*e^3*f^2 - 284*a^3*b^13*c^3*d*e^3*f^
2 + 516*a^4*b^12*c^2*d^2*e^3*f^2 - 404*a^5*b^11*c*d^3*e^3*f^2 + 116*a^6*b^
10*d^4*e^3*f^2 - 74*a^3*b^13*c^4*e^2*f^3 + 356*a^4*b^12*c^3*d*e^2*f^3 - 62
4*a^5*b^11*c^2*d^2*e^2*f^3 + 476*a^6*b^10*c*d^3*e^2*f^3 - 134*a^7*b^9*d^4*
e^2*f^3 + 46*a^4*b^12*c^4*e*f^4 - 214*a^5*b^11*c^3*d*e*f^4 + 366*a^6*b^10*
c^2*d^2*e*f^4 - 274*a^7*b^9*c*d^3*e*f^4 + 76*a^8*b^8*d^4*e*f^4 - 11*a^5*b^
11*c^4*f^5 + 50*a^6*b^10*c^3*d*f^5 - 84*a^7*b^9*c^2*d^2*f^5 + 62*a^8*b^8*c
*d^3*f^5 - 17*a^9*b^7*d^4*f^5)*x^2/(a^2*b^15*c^6*e^8 - 6*a^3*b^14*c^5*d*e^
8 + 15*a^4*b^13*c^4*d^2*e^8 - 20*a^5*b^12*c^3*d^3*e^8 + 15*a^6*b^11*c^2*d^
4*e^8 - 6*a^7*b^10*c*d^5*e^8 + a^8*b^9*d^6*e^8 - 8*a^3*b^14*c^6*e^7*f + 48
*a^4*b^13*c^5*d*e^7*f - 120*a^5*b^12*c^4*d^2*e^7*f + 160*a^6*b^11*c^3*d^3*
e^7*f - 120*a^7*b^10*c^2*d^4*e^7*f + 48*a^8*b^9*c*d^5*e^7*f - 8*a^9*b^8*d^
6*e^7*f + 28*a^4*b^13*c^6*e^6*f^2 - 168*a^5*b^12*c^5*d*e^6*f^2 + 420*a^6*b
^11*c^4*d^2*e^6*f^2 - 560*a^7*b^10*c^3*d^3*e^6*f^2 + 420*a^8*b^9*c^2*d^4*e
^6*f^2 - 168*a^9*b^8*c*d^5*e^6*f^2 + 28*a^10*b^7*d^6*e^6*f^2 - 56*a^5*b^12
*c^6*e^5*f^3 + 336*a^6*b^11*c^5*d*e^5*f^3 - 840*a^7*b^10*c^4*d^2*e^5*f^3 +
 1120*a^8*b^9*c^3*d^3*e^5*f^3 - 840*a^9*b^8*c^2*d^4*e^5*f^3 + 336*a^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^2\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}(d\,x^2+c)^2(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2/(f*x^2+e)^3,x)
```

3.381 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 1441

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \text{Too large to display}$$

output
```
1/24*b*(8*b^5*c^2*e^2*(-c*f+d*e)^4+27*a^3*b^2*d*f*(c*f+d*e)^3*(c^2*f^2-4*c
*d*e*f+d^2*e^2)+9*a^5*d^3*f^3*(c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3
)-9*a^4*b*d^2*f^2*(3*c^4*f^4-11*c^3*d*e*f^3-16*c^2*d^2*e^2*f^2-11*c*d^3*e^
3*f+3*d^4*e^4)+36*a*b^4*c*e*(c^5*f^5-2*c^4*d*e*f^4-2*c*d^4*e^4*f+d^5*e^5)-
9*a^2*b^3*(c^6*f^6+7*c^5*d*e*f^5-24*c^4*d^2*e^2*f^4-24*c^2*d^4*e^4*f^2+7*c
*d^5*e^5*f+d^6*e^6))*x/a/c^2/(-a*d+b*c)^3/e^2/(-a*f+b*e)^3/(-c*f+d*e)^4/(b
*x^2+a)^(3/2)+1/24*b*(16*b^7*c^3*e^3*(-c*f+d*e)^4-88*a*b^6*c^2*e^2*(-c*f+d
*e)^4*(c*f+d*e)+9*a^7*d^4*f^4*(c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3
)-6*a^6*b*d^3*f^3*(6*c^4*f^4-23*c^3*d*e*f^3-26*c^2*d^2*e^2*f^2-23*c*d^3*e^
3*f+6*d^4*e^4)+6*a^5*b^2*d^2*f^2*(9*c^5*f^5-17*c^4*d*e*f^4-52*c^3*d^2*e^2*
f^3-52*c^2*d^3*e^3*f^2-17*c*d^4*e^4*f+9*d^5*e^5)-36*a^4*b^3*d*f*(c^6*f^6+2
*c^5*d*e*f^5-13*c^4*d^2*e^2*f^4-13*c^2*d^4*e^4*f^2+2*c*d^5*e^5*f+d^6*e^6)-
2*a^2*b^5*c*e*(21*c^6*f^6-119*c^5*d*e*f^5+320*c^4*d^2*e^2*f^4-480*c^3*d^3*
e^3*f^3+320*c^2*d^4*e^4*f^2-119*c*d^5*e^5*f+21*d^6*e^6)+3*a^3*b^4*(3*c^7*f
^7+41*c^6*d*e*f^6-104*c^5*d^2*e^2*f^5-104*c^2*d^5*e^5*f^2+41*c*d^6*e^6*f+3
*d^7*e^7))*x/a^2/c^2/(-a*d+b*c)^4/e^2/(-a*f+b*e)^4/(-c*f+d*e)^4/(b*x^2+a)^
(1/2)-1/4*d^4*x/c/(-a*d+b*c)/(-c*f+d*e)^3/(b*x^2+a)^(3/2)/(d*x^2+c)^2-1/8*
d^4*(2*b*c*(-11*c*f+5*d*e)-3*a*d*(-5*c*f+d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*
e)^4/(b*x^2+a)^(3/2)/(d*x^2+c)+1/4*f^4*x/e/(-a*f+b*e)/(-c*f+d*e)^3/(b*x^2+
a)^(3/2)/(f*x^2+e)^2+1/8*f^4*(2*b*e*(-5*c*f+11*d*e)-3*a*f*(-c*f+5*d*e))...
```

Mathematica [A] (verified)

Time = 27.26 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.44

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3(e+fx^2)^3}\,dx = \frac{1}{24}x\sqrt{a+bx^2}\left(\frac{8b^6}{a(bc-ad)^3(be-af)^3(a+bx^2)^2}\right.$$
$$+\frac{8b^6(2b^2ce+20a^2df-11ab(de+cf))}{a^2(bc-ad)^4(be-af)^4(a+bx^2)}+\frac{6d^6}{c(bc-ad)^3(-de+cf)^3(c+dx^2)^2}$$
$$+\frac{3d^6(3ad(de-5cf)+2bc(-7de+13cf))}{c^2(bc-ad)^4(de-cf)^4(c+dx^2)}+\frac{6f^6}{e(be-af)^3(de-cf)^3(e+fx^2)^2}$$
$$\left.+\frac{3f^6(2be(13de-7cf)+3af(-5de+cf))}{e^2(be-af)^4(de-cf)^4(e+fx^2)}\right)$$
$$-\frac{d^5(24b^2c^2(2d^2e^2-7cdef+7c^2f^2)+3a^2d^2(d^2e^2-6cdef+21c^2f^2)-4abcd(4d^2e^2-29cdef+49c^2f^2))}{8c^{5/2}(-bc+ad)^{9/2}(-de+cf)^5}$$
$$-\frac{f^5(3a^2f^2(21d^2e^2-6cdef+c^2f^2)+24b^2e^2(7d^2e^2-7cdef+2c^2f^2)-4abef(49d^2e^2-29cdef+4c^2f^2))}{8e^{5/2}(-be+af)^{9/2}(de-cf)^5}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
(x*Sqrt[a + b*x^2]*((8*b^6)/(a*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x^2)^2)
+ (8*b^6*(2*b^2*c*e + 20*a^2*d*f - 11*a*b*(d*e + c*f)))/(a^2*(b*c - a*d)^4
*(b*e - a*f)^4*(a + b*x^2)) + (6*d^6)/(c*(b*c - a*d)^3*(-(d*e) + c*f)^3*(c
 + d*x^2)^2) + (3*d^6*(3*a*d*(d*e - 5*c*f) + 2*b*c*(-7*d*e + 13*c*f)))/(c^
2*(b*c - a*d)^4*(d*e - c*f)^4*(c + d*x^2)) + (6*f^6)/(e*(b*e - a*f)^3*(d*e
 - c*f)^3*(e + f*x^2)^2) + (3*f^6*(2*b*e*(13*d*e - 7*c*f) + 3*a*f*(-5*d*e
+ c*f)))/(e^2*(b*e - a*f)^4*(d*e - c*f)^4*(e + f*x^2))))/24 - (d^5*(24*b^2
*c^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f^2) + 3*a^2*d^2*(d^2*e^2 - 6*c*d*e*f
+ 21*c^2*f^2) - 4*a*b*c*d*(4*d^2*e^2 - 29*c*d*e*f + 49*c^2*f^2))*ArcTan[(S
qrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(-(b*c) + a*d)
^(9/2)*(-(d*e) + c*f)^5) - (f^5*(3*a^2*f^2*(21*d^2*e^2 - 6*c*d*e*f + c^2*f
^2) + 24*b^2*e^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2) - 4*a*b*e*f*(49*d^2*e
^2 - 29*c*d*e*f + 4*c^2*f^2))*ArcTan[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[e]*Sqrt[
a + b*x^2])])/(8*e^(5/2)*(-(b*e) + a*f)^(9/2)*(d*e - c*f)^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 426$$

$$\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^3(fx^2+e)^3}dx}{bc-ad}$$

$$\Big\downarrow \; 426$$

$$\frac{b\left(\frac{b\int \frac{1}{(bx^2+a)^{5/2}(dx^2+c)(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{b\int \frac{1}{(bx^2+a)^{3/2}(dx^2+c)^2(fx^2+e)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3(fx^2+e)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}-\frac{b\int -\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{b\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\frac{b\int \frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{5/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{6dfa^2-b(5de+3cf)a+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{}{\left(bx^2+a\right)}}{}\right)}{bc-ad}$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\int \frac{6dfa^2-5bdea-3bcfa+6b(bc-ad)fx^2+2b^2ce}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-af)}\right)}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int \frac{}{\left(bx^2+a\right)^3}}{}\right)}{bc-ad}$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 402

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}-\frac{\int -\frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(b\cdots}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.}{bc-ad}$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\frac{\int \frac{f\left(4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-}\right.}{(bc-ad)^2}}{bc-ad}}{bc-ad}$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{b\left(\frac{b\left(\frac{\frac{f\int \frac{4b\left(12dfa^2-5bdea-9bcfa+2b^2ce\right)x^2+a\left(-6dfa^2-b(de-3cf)a+4b^2ce\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}+\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}}{3a(be-af)}+\frac{bx(bc-ad)}{3a\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^2(be-a}\right.}{(bc-ad)^2}\right.}{bc-ad}\right.}{bc-ad}$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 402}$$

$$b\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{f\left(\frac{\int \frac{2b\left(6df^2a^3+bf(49de-3cf)a^2-20b^2e(de+2cf)a+8b^3ce^2\right)x^2+a\left(18df^2a^3-3bf(23de+3cf)a^2+4b^2e(4de+9cf)a+8b^3ce^2\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}+\frac{x\sqrt{a+bx^2}\left(6a^3df^2 \cdots\right.}{\cdots}\right)}{a(be-af)}}{3a(be-af)}\cdots}{(bc-ad)^2}\cdots}{\cdots}\cdots}{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}\right.$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 402

$$b\left(\frac{b\left(\frac{b\left(\frac{f\left(\frac{\int -\frac{3a^2\left(-24e^2(de+2cf)b^3+4aef(21de+4cf)b^2-a^2f^2(31de+3cf)b+6a^3df^3\right)}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{2e(be-af)}+\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)\right.}{2e\left(e+fx^2\right)(be-af)}}{4e(be-af)}\right.}{a(be-af)}}{3a(be-af)}}{}}{}\right.}{}$$

$$d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{b\left(\frac{b\left(\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)}-\frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21\cdots\right.}{2e(be-\cdots}}{4e(be-af)}\right.}{a(be-af)}\right.}{3a(\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\downarrow 291$$

$$\frac{x\sqrt{a+bx^2}\left(-18a^4df^3+9a^3bf^2(cf+9de)+2a^2b^2ef(41de-21cf)-8ab^3e^2(11cf+5de)+16b^4ce^3\right)}{2e\left(e+fx^2\right)(be-af)} - \frac{3a^2\left(6a^3df^3-a^2bf^2(3cf+31de)+4ab^2ef(4cf+21\right.}{2e(b}$$

$$4e(be-af)$$

$$a(be-af)$$

$$3$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\downarrow 221$$

$$f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf)-20ab^2e(2cf+de)+8b^3ce^2\right)}{4e\left(e+fx^2\right)^2(be-af)}+\frac{x\sqrt{\ }}{}\right.$$

$$b\left(\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\right.$$

$$b\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{(bc-ad)^2}+\right.$$

$$b\left(\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\downarrow 421$$

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\int\frac{dfx^2+2de-cf}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{(de-cf)^2}\right)}{(bc-ad)^2}+\frac{b\left(\frac{bx\left(12a^2df-9abcf-5abde+2b^2ce\right)}{a\sqrt{a+bx^2}\left(e+fx^2\right)^2(be-af)}+\frac{f\left(\frac{x\sqrt{a+bx^2}\left(6a^3df^2+a^2bf(49de-3cf\right.}{4e\left(e+fx^2\right)^2(\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 402

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(\frac{\int -\frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}-\frac{f(de-cf)x\sqrt{bx^2+a}}{4e(be-af)\left(fx^2+e\right)^2}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/\cdots}}\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 25

$$b\left(\frac{b\left(\frac{\left(\frac{d^2\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\int \frac{2bf(de-cf)x^2+af(7de-3cf)-4be(2de-cf)}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{4e(be-af)}\right)}{(de-cf)^2}\right)d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{3a(be-af)\left(bx^2+a\right)^{3/}}\right.}{}\right.}{}\right.$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow 402$$

$$b\left(\frac{b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+ax}}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))\sqrt{bx^2+ax}}{2e(be-af)\left(fx^2+e\right)}+\frac{\int-\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+}{\sqrt{bx^2+a}\left(fx^2+e\right)}}{2e(be-af)}}{4e(be-af)}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.}{}\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 25

$$b\left(\frac{b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\int\frac{8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2\cdots}{\sqrt{bx^2+a}\left(fx^2+e\right)\cdots}\cdots}{2e(be-af)\cdots}\cdots}{4e(be-af)\cdots}\cdots\right)}{(de-cf)^2\cdots}\cdots}{(bc-ad)^2\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2\cdots\right.}{2e(be-\cdots}}{4e(be-af)}\cdots\right.}{(de-cf)^2}\cdots}{(bc-ad)^2}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 291

$$b\left(b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2\ldots\right.}{2e(\ldots}}{4e(be-af)}\ldots\right.}{(de-cf)^2}\ldots}{(bc-ad)^2}\ldots\right.\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\downarrow$ 221

$$b\left(\frac{b\left(\frac{\frac{d^2\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e^2-4abf(5de-2cf)e+a^2\right.}{2e^{3/2}(b}}{4e(be-af)}\right.}{(de-cf)^2}}{(bc-ad)^2}\right.}{}\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 407

$$b\left(\frac{b\left(\frac{\frac{d^2\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2de-cf)e\cdots\right.}{\cdots}}{4e\cdots}\cdots\right)}{(de-cf)^2}\cdots}{(bc-ad)^2}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

↓ 291

$$b\left(b\left(\frac{\frac{d^2\left(\frac{d\int \frac{1}{c-\frac{(bc-ad)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}-\frac{f\int \frac{1}{e-\frac{(be-af)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{de-cf}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2\cdots\right.}{\cdots}}{\cdots}\right.}{(de-c\cdots}}{(bc-ad)^2}\cdots\right.\right.$$

$$\frac{d\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$$\Big\downarrow \text{221}$$

$$b\left(b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2d\right.}{}}{}\right.}{(de-cf)}}{(bc-ad)^2}\right.\right.$$

$$\frac{d\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}$$

$\Big\downarrow$ 426

$$b\left(b\left(\frac{\frac{d^2\left(\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2d\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)\cdots}}{(bc-ad)^2}\cdots\right.\right.$$

$$\frac{\cdots}{d\left(\frac{b\left(\frac{b\int\frac{1}{\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)\left(fx^2+e\right)^3}dx}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\cdots}}{de-cf}\cdots\right.}{bc-ad}\cdots\right.}$$

$$bc-ad$$

↓ 421

$$b\left(\frac{b\left(\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2d\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)\cdots}}{(bc-ad)^2}\cdots\right.}{\cdots}\cdots\right.$$

$$d\left(\frac{b\left(\frac{b\left(\frac{d^2\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}\,dx}{(bc-ad)^2}-\frac{b\int-\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}\,dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}\,dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}\cdots}{de-cf}\cdots\right.}{b\cdots}\cdots\right.$$

↓ 25

$$b\Biggl(b\Biggl(\frac{d^2\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2d}{}}{}\right.}{(de-cf)}}{(bc-ad)^2}$$

$$d\left(\frac{b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dxd^2}{(bc-ad)^2}+\frac{b\int\frac{-bdx^2+bc-2ad}{\left(bx^2+a\right)^{3/2}\left(fx^2+e\right)^3}dx}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)}{bc-ad}-\frac{d\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^3\left(fx^2+e\right)^2}dx}{de-cf}\right.}{bc-\cdots}\right.$$

↓ 402

$$b\left(\frac{b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}\,x}{4e(be-af)\left(fx^2+e\right)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)\left(fx^2+e\right)}-\frac{\left(8b^2(2d\cdots\right.}{\cdots}}{\cdots}\right.}{(de-cf)\cdots}}{(bc-ad)^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\left(b\left(\frac{b\left(\frac{\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)\left(fx^2+e\right)^3}dx\,d^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}\left(fx^2+e\right)^2}-\frac{\int\frac{a(bde+bcf-2adf)-4b(bc-ad)fx^2}{\sqrt{bx^2+a}\left(fx^2+e\right)^3}dx}{a(be-af)}\right)}{(bc-ad)^2}\right)}{bc-ad}-\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2\left(fx^2+e\right)^3}dx}{bc-ad}\right)\right.\cdots$$

↓ 402

$$b\left(b\left(\frac{\frac{d^2\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)}{\sqrt{c}\sqrt{bc-ad}(de-cf)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)}{\sqrt{e}\sqrt{be-af}(de-cf)}\right)}{(de-cf)^2}-\frac{f\left(-\frac{f(de-cf)\sqrt{bx^2+a}x}{4e(be-af)(fx^2+e)^2}-\frac{\frac{f(2be(5de-3cf)-af(7de-3cf))x\sqrt{bx^2+a}}{2e(be-af)(fx^2+e)}-\frac{(8b^2(2d}{}}{}\right.}{(de-cf)}}{(bc-ad)^2}\right.\right.$$

$$\left(\left(\left(b\frac{\int\frac{\sqrt{bx^2+a}}{(dx^2+c)(fx^2+e)^3}dxd^2}{(bc-ad)^2}+\frac{b\left(\frac{b(bc-ad)x}{a(be-af)\sqrt{bx^2+a}(fx^2+e)^2}-\frac{\frac{\int\frac{a(4e(de+2cf)b^2-3af(5de+cf)b+6a^2df^2)-2bf(-2dfa^2-b(3de-cf)a+4b^2ce)x^2}{\sqrt{bx^2+a}(fx^2+e)^2}}{4e(be-af)}}{a(be-af)}\right.}{(bc-ad)^2}\right.\right.\right.$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 407
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/((a + b*x^2)*Sqrt[e + f*x^2]),
x], x] - Simp[d/(b*c - a*d)   Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /
; FreeQ[{a, b, c, d, e, f}, x]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. 2(1393) = 2786.

Time = 400.98 (sec) , antiderivative size = 3369, normalized size of antiderivative = 2.34

method	result	size
pseudoelliptic	Expression too large to display	3369
default	Expression too large to display	14878

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(63/5*((a*f-b*e)*e)^(1/2)*a^2*d^5*(a*f-b*e)^4*(d*x^2+c)^2*(b*x^2+a)^(3
/2)*(f*x^2+e)^2*(1/21*a^2*e^2*d^4-2/7*a*e*(a*f+8/9*b*e)*c*d^3+c^2*(a^2*f^2
+116/63*a*b*f*e+16/21*b^2*e^2)*d^2-28/9*b*f*c^3*(a*f+6/7*b*e)*d+8/3*b^2*c^
4*f^2)*e^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(
1/2)*(-3/5*(a*d-b*c)^4*a^2*c^2*((21*a^2*e^2*f^2-196/3*a*b*e^3*f+56*b^2*e^4
)*d^2-6*c*f*e*(a^2*f^2-58/9*a*b*f*e+28/3*b^2*e^2)*d+c^2*f^2*(a^2*f^2-16/3*
a*b*f*e+16*b^2*e^2))*(d*x^2+c)^2*(b*x^2+a)^(3/2)*(f*x^2+e)^2*f^5*arctan(e*
(b*x^2+a)^(1/2)/x/((a*f-b*e)*e)^(1/2))+((a*f-b*e)*e)^(1/2)*(c*f-d*e)*x*(3/
5*a^3*e^3*x^2*(f*x^2+e)^2*(b*x^2+a)^2*(a*f-b*e)^4*d^9+a^2*c*(b*x^2+a)^2*(a
*f-b*e)^4*(-3*a*f*x^2+e*(-14/5*b*x^2+a))*(f*x^2+e)^2*e^2*d^8-17/5*a*c^2*(1
5/17*a^6*x^6*(b*x^2+a)^2*f^8+2*a^5*(-26/17*b*x^2+a)*(b*x^2+a)^2*x^4*e*f^7+
2*a^4*(b*x^2+a)^2*(52/17*b^2*x^4-66/17*a*b*x^2+a^2)*x^2*e^2*f^6+a^3*(b*x^2
+a)^2*(-156/17*b^3*x^6+246/17*a*b^2*x^4-130/17*a^2*b*x^2+a^3)*e^3*f^5-52/1
7*a^2*(b*x^2+a)^2*(-2*b^3*x^6+71/13*a*b^2*x^4-45/13*a^2*b*x^2+a^3)*b*e^4*f
^4+38/17*a*(-119/57*b^5*x^10+53/38*a*b^4*x^8+98/19*a^2*b^3*x^6-1/38*a^3*b^
2*x^4-12/19*a^4*b*x^2+a^5)*b^2*e^5*f^3+28/17*(22/21*b^5*x^10-83/21*a*b^4*x
^8-87/14*a^2*b^3*x^6+3/7*a^3*b^2*x^4+33/14*a^4*b*x^2+a^5)*b^3*e^6*f^2-47/1
7*b^4*e^7*(-176/141*b^4*x^8-50/141*a*b^3*x^6+91/47*a^2*b^2*x^4+88/47*a^3*b
*x^2+a^4)*f+16/17*(11/6*b^3*x^6+3*a*b^2*x^4+2*a^2*b*x^2+a^3)*b^5*e^8)*e*d^
7-34/5*c^3*(-3/34*a^7*x^6*(b*x^2+a)^2*f^9+25/34*a^6*(b*x^2+a)^2*(-46/25...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \left(e+fx^2\right)^3}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\left(dx^2+c\right)^3\left(fx^2+e\right)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3*(f*x^2 + e)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21752 vs. 2(1393) = 2786.

Time = 150.54 (sec) , antiderivative size = 21752, normalized size of antiderivative = 15.10

$$\int \frac{1}{\left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \left(e+fx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

output
```
1/3*((2*b^18*c^5*e^5 - 19*a*b^17*c^4*d*e^5 + 56*a^2*b^16*c^3*d^2*e^5 - 74*
a^3*b^15*c^2*d^3*e^5 + 46*a^4*b^14*c*d^4*e^5 - 11*a^5*b^13*d^5*e^5 - 19*a*
b^17*c^5*e^4*f + 140*a^2*b^16*c^4*d*e^4*f - 370*a^3*b^15*c^3*d^2*e^4*f + 4
60*a^4*b^14*c^2*d^3*e^4*f - 275*a^5*b^13*c*d^4*e^4*f + 64*a^6*b^12*d^5*e^4
*f + 56*a^2*b^16*c^5*e^3*f^2 - 370*a^3*b^15*c^4*d*e^3*f^2 + 920*a^4*b^14*c
^3*d^2*e^3*f^2 - 1100*a^5*b^13*c^2*d^3*e^3*f^2 + 640*a^6*b^12*c*d^4*e^3*f^
2 - 146*a^7*b^11*d^5*e^3*f^2 - 74*a^3*b^15*c^5*e^2*f^3 + 460*a^4*b^14*c^4*
d*e^2*f^3 - 1100*a^5*b^13*c^3*d^2*e^2*f^3 + 1280*a^6*b^12*c^2*d^3*e^2*f^3
- 730*a^7*b^11*c*d^4*e^2*f^3 + 164*a^8*b^10*d^5*e^2*f^3 + 46*a^4*b^14*c^5*
e*f^4 - 275*a^5*b^13*c^4*d*e*f^4 + 640*a^6*b^12*c^3*d^2*e*f^4 - 730*a^7*b^
11*c^2*d^3*e*f^4 + 410*a^8*b^10*c*d^4*e*f^4 - 91*a^9*b^9*d^5*e*f^4 - 11*a^
5*b^13*c^5*f^5 + 64*a^6*b^12*c^4*d*f^5 - 146*a^7*b^11*c^3*d^2*f^5 + 164*a^
8*b^10*c^2*d^3*f^5 - 91*a^9*b^9*c*d^4*f^5 + 20*a^10*b^8*d^5*f^5)*x^2/(a^2*
b^17*c^8*e^8 - 8*a^3*b^16*c^7*d*e^8 + 28*a^4*b^15*c^6*d^2*e^8 - 56*a^5*b^1
4*c^5*d^3*e^8 + 70*a^6*b^13*c^4*d^4*e^8 - 56*a^7*b^12*c^3*d^5*e^8 + 28*a^8
*b^11*c^2*d^6*e^8 - 8*a^9*b^10*c*d^7*e^8 + a^10*b^9*d^8*e^8 - 8*a^3*b^16*c
^8*e^7*f + 64*a^4*b^15*c^7*d*e^7*f - 224*a^5*b^14*c^6*d^2*e^7*f + 448*a^6*
b^13*c^5*d^3*e^7*f - 560*a^7*b^12*c^4*d^4*e^7*f + 448*a^8*b^11*c^3*d^5*e^7
*f - 224*a^9*b^10*c^2*d^6*e^7*f + 64*a^10*b^9*c*d^7*e^7*f - 8*a^11*b^8*d^8
*e^7*f + 28*a^4*b^15*c^8*e^6*f^2 - 224*a^5*b^14*c^7*d*e^6*f^2 + 784*a^6...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3(e+fx^2)^3}\,dx=\int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^3\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^3\,(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^3\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3/(f*x^2+e)^3,x)
```

3.382 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 854

$$\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx = \frac{\left(8ac+\frac{3bc^2}{d}-\frac{3a^2d}{b}-\frac{3bde^2}{f^2}+\frac{8bce}{f}+\frac{2ade}{f}\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48\sqrt{a+bx^2}}$$
$$+\frac{(bde+7bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24bf}+\frac{1}{6}dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$
$$+\frac{\sqrt{bc-ad}\,e(3a^2d^2f^2-2abdf(de+4cf)+b^2(3d^2e^2-8cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-}}{\sqrt{c}\sqrt{}}\right.\right.}{48b^2\sqrt{c}df^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a\sqrt{bc-ad}(3a^2d^2f^2-6abdf(de+cf)-b^2(d^2e^2-22cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{EllipticF}\left(\mathrm{arcsi}\right.}{48b^3\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(a^3d^3f^3+b^3(de-cf)^3-a^2bd^2f^2(de+3cf)-ab^2df(d^2e^2-6cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{Ellipt}}{16b^3\sqrt{c}d\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(8*a*c+3*b*c^2/d-3*a^2*d/b-3*b*d*e^2/f^2+8*b*c*e/f+2*a*d*e/f)*x*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)+1/24*(a*d*f+7*b*c*f+b*d*e)*x*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f+1/6*d*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/48*(-a*d+b*c)^(1/2)*e*(3*a^2*d^2*f^2-2
*a*b*d*f*(4*c*f+d*e)+b^2*(-3*c^2*f^2-8*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/f^2/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/48*a*(-a*d+b*c)^(1/2)*(3*a^2*d^2*
f^2-6*a*b*d*f*(c*f+d*e)-b^2*(-3*c^2*f^2-22*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/
2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b
*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d/f/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/16*a*(a^3*d^3*f^3+b^3*(-c*f+d*e)^
3-a^2*b*d^2*f^2*(3*c*f+d*e)-a*b^2*d*f*(-3*c^2*f^2-6*c*d*e*f+d^2*e^2))*(d*x
^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/
c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/
b^3/c^(1/2)/d/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+
e)^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

output
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}dx$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}}\sqrt{f\,x^{2}+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

3.383 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 646

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$= \frac{(bde+bcf+adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8df\sqrt{a+bx^2}} + \frac{1}{4}x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$

$$- \frac{\sqrt{bc-ad}e(bde+bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+ \frac{a\sqrt{bc-ad}(3bde+bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$- \frac{a\left(a^2d^2f^2+b^2(de-cf)^2-2abdf(de+cf)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\right.}{8b^2\sqrt{c}d\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(a*d*f+b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f/(b*x^2+a)^(1
/2)+1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/8*(-a*d+b*c)^(
1/2)*e*(a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+
b*c)/e)^(1/2))/b/c^(1/2)/d/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/
2)+1/8*a*(-a*d+b*c)^(1/2)*(-a*d*f+b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2
+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2+b^2*(-c*f+d*e)^2-2*a*b*d*f*(c*
f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c
)/e)^(1/2))/b^2/c^(1/2)/d/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}dx$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx \\
&= \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bcf}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bde}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)acf}{2} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)ade}{2} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bce}{2} \\
&+ \frac{3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)ace}{4}
\end{aligned}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(e + f*x*
*2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
d*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
 + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*
x**4 + b*d*f*x**6),x)*b*c*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x*
*2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e + 2*int((sqrt(e + f*x*
*2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
c*f + 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*a*d*e + 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*
e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e + 3*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c
*e)/4
```

3.384 $\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 533

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}} - \frac{\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{a\sqrt{bc-ad}f\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{a(bde-bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}d\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/(b*x^2+a)^(1/2)-1/2*(-a*d+b*c)^(
1/2)*e*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c
)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/
2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(-a*d+b*c)^(1/2
)*f*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(
1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2
)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(a*d*f-b*c*f+b*d
*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^
(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)
^(1/2))/b/c^(1/2)/d/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^
2+e)^(1/2)
```

Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

$$= \frac{\frac{x\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|\frac{(-bc+ad)e}{a(de-cf)}\right)}{d\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{(bc-2ad)\sqrt{e}(-de+cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{E}}{cd^2\sqrt{be-af}\sqrt{}}}{2\sqrt{e+}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
((x*Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2] - (Sqrt[e]*Sqrt[d*e - c*f
]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE[ArcSin[(
Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], ((-(b*c) + a*d)*e)/(a*(d*e
- c*f))]])/(d*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + ((b*c - 2*a*d)*Sqrt[
e]*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*El
lipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*
d*e)/(b*c*e - a*c*f)])/(c*d^2*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a +
 b*x^2))]) + (c*Sqrt[e]*(b*d*e - b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e
 + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e
 - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], ((-(b*c) + a*d)*e)/(a*(d*e - c*f))]
)/(a*d^2*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]))/(2*Sqrt[e
 + f*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {430, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 430}$$

$$\frac{af(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2bd}+\frac{(adf-bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2bd}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2d}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 427}$$

$$\frac{(adf-bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2bd}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2d}+\frac{af\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2bcd\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{(adf-bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2bd}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2d}+\frac{a\sqrt{e}f\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2bcd\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 428}$$

$$-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2d}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-bcf+bde)\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)\sqrt{1-\frac{(bc-ad)x^2}{c(bx^2+a)}}\sqrt{1-\frac{(be-af)x^2}{e(bx^2+a)}}}d\frac{x}{\sqrt{bx^2+a}}}{2bcd\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{e}f\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2bcd\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$\big\downarrow$ 412

$$-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2d}+\frac{a\sqrt{e}f\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2bcd\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-bcf+bde)\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}d\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$\big\downarrow$ 429

$$-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\int \frac{\sqrt{1-\frac{(be-af)x^2}{e(bx^2+a)}}}{\sqrt{1-\frac{(bc-ad)x^2}{c(bx^2+a)}}}d\frac{x}{\sqrt{bx^2+a}}}{2d\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{a\sqrt{e}f\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2bcd\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-bcf+bde)\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}d\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

$\big\downarrow$ 327

$$\frac{a\sqrt{e}f\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2bcd\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2d\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-bcf+bde)\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}d\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2d\sqrt{a+bx^2}}$$

input

```
Int[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output

```
(b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*d*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqr
t[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*Ellipti
cE[ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/
((b*c - a*d)*e)])/(2*d*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2)
)]) + (a*(b*c - a*d)*Sqrt[e]*f*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
+ b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
, ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b*c*d*Sqrt[b*e - a*f]*Sqrt[(a*(c +
d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) + (a*(b*d*e - b*c*f + a*d*f)*Sqr
t[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c -
 a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*
f))/((b*c - a*d)*e)])/(2*b*Sqrt[c]*d*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}{\sqrt{x^{2} d+c}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,x^2+c}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c + d*x**2),x)
```

3.385 $\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 478

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx &= \frac{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{cd\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &- \frac{(bc-ad)f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{d^2\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &+ \frac{bcf\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{d^2\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

output
```
a*(-c*f+d*e)^(1/2)*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*Ellipti
cE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))
^(1/2))/c/d/e^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-(-a*d+
b*c)*f*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*EllipticF((-c*f+d*e
)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/d^2/
e^(1/2)/(-c*f+d*e)^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b
*c*f*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*EllipticPi((-c*f+d*e)
^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e
))^(1/2))/d^2/e^(1/2)/(-c*f+d*e)^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {432, 428, 412, 429, 326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \; 432$$

$$\frac{b\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{d}$$

$\Big\downarrow$ 428

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\int \frac{1}{\left(1-\frac{fx^2}{fx^2+e}\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{ad\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{d}$$

$\Big\downarrow$ 412

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}d\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{3/2}}dx}{d}$$

$\Big\downarrow$ 429

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}d\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\int \frac{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}d\frac{x}{\sqrt{dx^2+c}}}{cd\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$\Big\downarrow$ 326

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}d\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\int \frac{1}{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}-\frac{a(de-cf)\int \frac{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}\right)}{cd\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \; 321$$

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-bex}}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}d\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{e}(bc-ad)\sqrt{de-cf}}-\frac{a(de-cf)\int\frac{\sqrt{\frac{(bc-ad)x^2}{a(dx^2+c)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e(dx^2+c)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}\right)}{cd\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \; 327$$

$$\frac{be\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-bex}}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}d\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{e}(bc-ad)\sqrt{de-cf}}-\frac{a\sqrt{de-cf}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)}{a(de-cf}\right.}{\sqrt{e}(bc-ad)}\right.}{cd\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2]*(-((a
*Sqrt[d*e - c*f]*EllipticE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*
x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b*c - a*d)*Sqrt[e])) + (c*(
b*e - a*f)*EllipticF[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])]
, -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b*c - a*d)*Sqrt[e]*Sqrt[d*e - c*f
])))/(c*d*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) + (b*e*S
qrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticPi[-((a*f)/(b
*e - a*f)), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*(
d*e - c*f))/(c*(b*e - a*f))])/(Sqrt[a]*d*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x^2
]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 432
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Simp[b/f   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sq
rt[e + f*x^2]), x], x] - Simp[(b*e - a*f)/f   Int[Sqrt[c + d*x^2]/(Sqrt[a +
 b*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}{\left(x^{2} d+c\right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*x^4 + 2*c*d*
x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

3.386 $\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 388

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(c+dx^2)^{3/2}} + \frac{\sqrt{e}(bce-2ade+acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(bc-ad)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{e}(be-af)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c(bc-ad)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(d*x^2+c)^(3/2)+1/3*e^(1/2)*(a*c*f
-2*a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE(
(-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1
/2))/c^2/(-a*d+b*c)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^
2+e)^(1/2)+1/3*e^(1/2)*(-a*f+b*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c)
)^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*
e/a/(-c*f+d*e))^(1/2))/c/(-a*d+b*c)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}{\left(x^{2} d+c\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*x^6 + 3*c*d^
2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\left(d\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

3.387 $\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 561

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx &= \frac{x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5c\,(c+dx^2)^{5/2}} \\
&- \frac{(ad(4de-3cf)-bc(3de-2cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2(bc-ad)(de-cf)\,(c+dx^2)^{3/2}} \\
&+ \frac{\sqrt{e}(b^2c^2e(3de-5cf)+a^2d(8d^2e^2-13cdef+3c^2f^2)-abc(13d^2e^2-22cdef+5c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+f}{e(c+d}}}{15c^3(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{\sqrt{e}(be-af)(bc(6de-5cf)-ad(4de-3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)}{a(de-cf}\right.}{15c^2(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/5*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(d*x^2+c)^(5/2)-1/15*(a*d*(-3*c*f+
4*d*e)-b*c*(-2*c*f+3*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/(-a*d+b*c
)/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(b^2*c^2*e*(-5*c*f+3*d*e)+a^2*d*
(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2)-a*b*c*(5*c^2*f^2-22*c*d*e*f+13*d^2*e^2))*
(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)
*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b
*c)^2/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/1
5*e^(1/2)*(-a*f+b*e)*(b*c*(-5*c*f+6*d*e)-a*d*(-3*c*f+4*d*e))*(b*x^2+a)^(1/
2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d
*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)^2/(-c*f+d
*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}{\left(x^{2} d+c\right)^{\frac{7}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*x^8 + 4*c*d^
3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{{\left(d\,x^2+c\right)}^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\left(d\,x^2+c\right)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

3.388 $\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 870

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7c\,(c+dx^2)^{7/2}}$$
$$-\frac{(ad(6de-5cf)-bc(5de-4cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2(bc-ad)(de-cf)\,(c+dx^2)^{5/2}}$$
$$+\frac{(b^2c^2(15d^2e^2-27cdef+8c^2f^2)+a^2d^2(24d^2e^2-43cdef+15c^2f^2)-abcd(43d^2e^2-78cdef+27c^2f^2))\ldots}{105c^3(bc-ad)^2(de-cf)^2\,(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(b^3c^3e(15d^2e^2-42cdef+35c^2f^2)-ab^2c^2(103d^3e^3-282cd^2e^2f+238c^2def^2-35c^3f^3)+2a^2bcd(64c\ldots}{105\ldots}$$
$$+\frac{\sqrt{e}(be-af)\,(a^2d^2(24d^2e^2-43cdef+15c^2f^2)+b^2c^2(45d^2e^2-84cdef+35c^2f^2)-abcd(61d^2e^2-111c\ldots}{105c^3(bc-ad)^3(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\ldots}$$

output
```
1/7*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(d*x^2+c)^(7/2)-1/35*(a*d*(-5*c*f+
6*d*e)-b*c*(-4*c*f+5*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/(-a*d+b*c
)/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/105*(b^2*c^2*(8*c^2*f^2-27*c*d*e*f+15*d^2*e
^2)+a^2*d^2*(15*c^2*f^2-43*c*d*e*f+24*d^2*e^2)-a*b*c*d*(27*c^2*f^2-78*c*d*
e*f+43*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+
d*e)^2/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(b^3*c^3*e*(35*c^2*f^2-42*c*d*e*f+15*
d^2*e^2)-a*b^2*c^2*(-35*c^3*f^3+238*c^2*d*e*f^2-282*c*d^2*e^2*f+103*d^3*e^
3)+2*a^2*b*c*d*(-21*c^3*f^3+141*c^2*d*e*f^2-172*c*d^2*e^2*f+64*d^3*e^3)-a^
3*d^2*(-15*c^3*f^3+103*c^2*d*e*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)
/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)^3/(-c*
f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/105*e^(1/2)
*(-a*f+b*e)*(a^2*d^2*(15*c^2*f^2-43*c*d*e*f+24*d^2*e^2)+b^2*c^2*(35*c^2*f^
2-84*c*d*e*f+45*d^2*e^2)-a*b*c*d*(42*c^2*f^2-111*c*d*e*f+61*d^2*e^2))*(b*x
^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e
^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b*c)^
3/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^5*x^10 + 5*c*d
^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4 + 5*c^4*d*x^2 + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\left(d\,x^2+c\right)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}{\left(d\,x^2+c\right)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

3.389 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 859

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \frac{(3a^2d^2f^2+8abdf(de+cf)-b^2(3d^2e^2-2cdef+3c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e}}{48d^2f^2\sqrt{a+bx^2}}$$

$$+\frac{(bde+bcf+7adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24df}+\frac{1}{6}bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$

$$-\frac{\sqrt{bc-ad}\,e(3a^2d^2f^2+8abdf(de+cf)-b^2(3d^2e^2-2cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-}}{\sqrt{c}\sqrt{a}}\right.\right.}{48b\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a\sqrt{bc-ad}(3a^2d^2f^2-6abdf(2de+cf)+b^2(d^2e^2-4cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\text{arcsi}\right.}{48b^2\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a(a^3d^3f^3+3ab^2df(de-cf)^2-3a^2bd^2f^2(de+cf)-b^3(de-cf)^2(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{Elliptic}}{16b^2\sqrt{c}d^2\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(3*a^2*d^2*f^2+8*a*b*d*f*(c*f+d*e)-b^2*(3*c^2*f^2-2*c*d*e*f+3*d^2*e^2
))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/f^2/(b*x^2+a)^(1/2)+1/24*(7*a*d*f
+b*c*f+b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f+1/6*b*
x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/48*(-a*d+b*c)^(1/2)*
e*(3*a^2*d^2*f^2+8*a*b*d*f*(c*f+d*e)-b^2*(3*c^2*f^2-2*c*d*e*f+3*d^2*e^2))*
(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)
*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^
2/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/48*a*(-a*d+b*c)^(1
/2)*(3*a^2*d^2*f^2-6*a*b*d*f*(c*f+2*d*e)+b^2*(3*c^2*f^2-4*c*d*e*f+d^2*e^2)
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2
)/d^2/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/16*a*(a^3*d^3*f^
3+3*a*b^2*d*f*(-c*f+d*e)^2-3*a^2*b*d^2*f^2*(c*f+d*e)-b^3*(-c*f+d*e)^2*(c*f
+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*
c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)
/e)^(1/2))/b^2/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}dx$$

input
```
Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(a+bx^2\right)^{\frac{3}{2}} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(b\,x^2+a\right)^{3/2}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

3.390 $\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 666

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b(bde-3bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8d^2f\sqrt{a+bx^2}}$$
$$+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4d}$$
$$-\frac{\sqrt{bc-ad}e(bde-3bcf+5adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{3a\sqrt{bc-ad}(bde-bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(3a^2d^2f^2+6abdf(de-cf)-b^2(d^2e^2+2cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b\sqrt{c}d^2\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*b*(5*a*d*f-3*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/f/(b*x
^2+a)^(1/2)+1/4*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d-1/8*
(-a*d+b*c)^(1/2)*e*(5*a*d*f-3*b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/
(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/d^2/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)/(f*x^2+e)^(1/2)+3/8*a*(-a*d+b*c)^(1/2)*(a*d*f-b*c*f+b*d*e)*(d*x^2+c)^(1
/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(
b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^2/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^2*f^2+6*a*b*d*f*(-c*f
+d*e)-b^2*(-3*c^2*f^2+2*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(
b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/
(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^2/(-a*d+b*c)^(1/
2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$
$$\big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}\sqrt{f\,x^{2}+e}}{\sqrt{x^{2}d+c}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{f\,x^2+e}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bx+5\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2}\right.}{}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*x + 5*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*b*d*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4
+ b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*f + int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d*e + 4*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a**2*d*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*f + 6*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d
*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*
e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e
*x**4 + b*d*f*x**6),x)*b**2*c*e + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d*e - int((sqrt(e +...
```

3.391 $\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 584

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{bx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2d\sqrt{c+dx^2}}$$

$$-\frac{(3bc-2ad)\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2cd^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}(2a^2d^2f-b^2c(de-3cf)+2abd(de-3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-a}{a(de-}\right.}{2ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{bc\sqrt{e}(bde-3bcf+3adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*b*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/(d*x^2+c)^(1/2)-1/2*(-2*a*d+3*b*
c)*e^(1/2)*(-c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2
)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-
c*f+d*e))^(1/2))/c/d^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/2
*e^(1/2)*(2*a^2*d^2*f-b^2*c*(-3*c*f+d*e)+2*a*b*d*(-3*c*f+d*e))*(b*x^2+a)^(
1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/
(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^3/(-c*f+d*e)^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/2*b*c*e^(1/2)*(3*a*d*f-
3*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi(
(-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a
/(-c*f+d*e))^(1/2))/a/d^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*x^4 + 2*c*
d*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

3.392 $\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 590

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{\left(\frac{a}{c}-\frac{b}{d}\right)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3\,(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(bc(2de-3cf)+ad(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2d^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(3b^2c^2f-a^2d^2f+abd(de-3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3acd^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{b^2c\sqrt{e}f\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(a/c-b/d)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2)+1/3*e^(1/2
)*(b*c*(-3*c*f+2*d*e)+a*d*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*
x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d
+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(3*b^2*c^2*f-a^2*d^2*f+a*b*d*(-3*c
*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d
*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/
c/d^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+b^2
*c*e^(1/2)*f*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-
c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(
-c*f+d*e))^(1/2))/a/d^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\,\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*x^6 + 3*c*
d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

3.393 $\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 531

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \frac{\left(\frac{a}{c}-\frac{b}{d}\right)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5\left(c+dx^2\right)^{5/2}}$$
$$+\frac{(bc(2de-3cf)+ad(4de-3cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$+\frac{\sqrt{e}(2b^2c^2e^2+abce(3de-7cf)-a^2(8d^2e^2-13cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)\right|}{15c^3(bc-ad)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(be-af)(bce-4ade+3acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{15c^2(bc-ad)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*(a/c-b/d)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2)+1/15*(b*c*
(-3*c*f+2*d*e)+a*d*(-3*c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/d
/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(2*b^2*c^2*e^2+a*b*c*e*(-7*c*f+3*
d*e)-a^2*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/
(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b*c)/(-c*f+d*e)^(3/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/15*e^(1/2)*(-a*f+b*e)*(3*a*c*f-4*a
*d*e+b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*
f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))
/c^2/(-a*d+b*c)/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)
^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}\sqrt{f\,x^{2}+e}}{(x^{2}d+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*x^8 + 4*c*
d^3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}\,dx = \int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{fx^2+e}}{\left(dx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}\,dx = \int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{fx^2+e}}{\left(dx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

3.394 $\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 830

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \frac{\left(\frac{a}{c}-\frac{b}{d}\right)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7(c+dx^2)^{7/2}}$$

$$+\frac{(ad(6de-5cf)+bc(2de-3cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2d(de-cf)(c+dx^2)^{5/2}}$$

$$+\frac{(2b^2c^2(3d^2e^2-4cdef+3c^2f^2)+abcd(15d^2e^2-29cdef+6c^2f^2)-a^2d^2(24d^2e^2-43cdef+15c^2f^2))\,x\sqrt{}}{105c^3d(bc-ad)(de-cf)^2(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(2b^3c^3e^2(3de-7cf)+ab^2c^2e(12d^2e^2-37cdef+49c^2f^2)-a^2bc(72d^3e^3-197cd^2e^2f+170c^2def^2-}{105c^4(bc-ad)^2(de}$$

$$-\frac{\sqrt{e}(be-af)\,(b^2c^2e(3de-7cf)+a^2d(24d^2e^2-43cdef+15c^2f^2)-abc(33d^2e^2-62cdef+21c^2f^2))\,\sqrt{a}}{105c^3(bc-ad)^2(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx}}$$

output
```
1/7*(a/c-b/d)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2)+1/35*(a*d*
(-5*c*f+6*d*e)+b*c*(-3*c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/d
/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/105*(2*b^2*c^2*(3*c^2*f^2-4*c*d*e*f+3*d^2*e^
2)+a*b*c*d*(6*c^2*f^2-29*c*d*e*f+15*d^2*e^2)-a^2*d^2*(15*c^2*f^2-43*c*d*e*
f+24*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/d/(-a*d+b*c)/(-c*f+d*
e)^2/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(2*b^3*c^3*e^2*(-7*c*f+3*d*e)+a*b^2*c^2
*e*(49*c^2*f^2-37*c*d*e*f+12*d^2*e^2)-a^2*b*c*(-21*c^3*f^3+170*c^2*d*e*f^2
-197*c*d^2*e^2*f+72*d^3*e^3)+a^3*d*(-15*c^3*f^3+103*c^2*d*e*f^2-128*c*d^2*
e^2*f+48*d^3*e^3))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*Ellipti
cE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))
^(1/2))/c^4/(-a*d+b*c)^2/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(f*x^2+e)^(1/2)-1/105*e^(1/2)*(-a*f+b*e)*(b^2*c^2*e*(-7*c*f+3*d*e)+a^2*d*(
15*c^2*f^2-43*c*d*e*f+24*d^2*e^2)-a*b*c*(21*c^2*f^2-62*c*d*e*f+33*d^2*e^2)
)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/
2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d
+b*c)^2/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow 434$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^5*x^10 + 5*c
*d^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4 + 5*c^4*d*x^2 + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

3.395 $\int (a+bx^2)^{5/2} \sqrt{c+dx^2} \sqrt{e+fx^2} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 1161

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \sqrt{e+fx^2} \, dx = \text{Too large to display}$$

output
```
1/384*(15*a^3*d^3*f^3+73*a^2*b*d^2*f^2*(c*f+d*e)-a*b^2*d*f*(55*c^2*f^2-38*
c*d*e*f+55*d^2*e^2)+b^3*(15*c^3*f^3-7*c^2*d*e*f^2-7*c*d^2*e^2*f+15*d^3*e^3
))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^3/f^3/(b*x^2+a)^(1/2)+1/192*(59*a^2
*d^2*f^2+18*a*b*d*f*(c*f+d*e)-b^2*(5*c^2*f^2-2*c*d*e*f+5*d^2*e^2))*x*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/f^2+1/48*b*(17*a*d*f+b*c*f+
b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f+1/8*b^2*x^5
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/384*(-a*d+b*c)^(1/2)*e*
(15*a^3*d^3*f^3+73*a^2*b*d^2*f^2*(c*f+d*e)-a*b^2*d*f*(55*c^2*f^2-38*c*d*e*
f+55*d^2*e^2)+b^3*(15*c^3*f^3-7*c^2*d*e*f^2-7*c*d^2*e^2*f+15*d^3*e^3))*(d*
x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/
c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^3/f
^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/384*a*(-a*d+b*c)^(1/2
)*(15*a^3*d^3*f^3-15*a^2*b*d^2*f^2*(3*c*f+5*d*e)+a*b^2*d*f*(45*c^2*f^2-62*
c*d*e*f+17*d^2*e^2)-b^3*(15*c^3*f^3-17*c^2*d*e*f^2-3*c*d^2*e^2*f+5*d^3*e^3
))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/
2)/d^3/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/128*a*(5*a^4*
d^4*f^4+30*a^2*b^2*d^2*f^2*(-c*f+d*e)^2-20*a^3*b*d^3*f^3*(c*f+d*e)-20*a*b^
3*d*f*(-c*f+d*e)^2*(c*f+d*e)+b^4*(-c*f+d*e)^2*(5*c^2*f^2+6*c*d*e*f+5*d^2*e
^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b...
```

Mathematica [F]

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \sqrt{e+fx^2}\, dx = \int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2} \sqrt{e+fx^2}\, dx$$

input
```
Integrate[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\sqrt{e+fx^2}dx$$

input
```
Int[(a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{2}} \sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(bx^2+a\right)^{\frac{5}{2}} \sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(b\,x^2+a\right)^{5/2} \sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/2} \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{5}{2}} \sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

3.396 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 900

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b(33a^2d^2f^2+2abdf(7de-20cf)-b^2(3d^2e^2+4cdef-15c^2f^2))\,x\sqrt{c+dx^2}}{48d^3f^2\sqrt{a+bx^2}}$$

$$+\frac{b(bde-5bcf+13adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24d^2f}$$

$$+\frac{b^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{6d}$$

$$-\frac{\sqrt{bc-ad}\,e(33a^2d^2f^2+2abdf(7de-20cf)-b^2(3d^2e^2+4cdef-15c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\right.}{48\sqrt{c}d^3f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{a\sqrt{bc-ad}(15a^2d^2f^2+30abdf(de-cf)-b^2(d^2e^2+14cdef-15c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\mathrm{ar}\right.}{48b\sqrt{c}d^3f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{a(5a^3d^3f^3+15a^2bd^2f^2(de-cf)-5ab^2df(d^2e^2+2cdef-3c^2f^2)+b^3(d^3e^3+cd^2e^2f+3c^2def^2-5c^3f^3}{16b\sqrt{c}d^3\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+f}}$$

output
```
1/48*b*(33*a^2*d^2*f^2+2*a*b*d*f*(-20*c*f+7*d*e)-b^2*(-15*c^2*f^2+4*c*d*e*
f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^3/f^2/(b*x^2+a)^(1/2)+1/
24*b*(13*a*d*f-5*b*c*f+b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/d^2/f+1/6*b^2*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d-
1/48*(-a*d+b*c)^(1/2)*e*(33*a^2*d^2*f^2+2*a*b*d*f*(-20*c*f+7*d*e)-b^2*(-15
*c^2*f^2+4*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(
1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-
a*d+b*c)/e)^(1/2))/c^(1/2)/d^3/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+
e)^(1/2)+1/48*a*(-a*d+b*c)^(1/2)*(15*a^2*d^2*f^2+30*a*b*d*f*(-c*f+d*e)-b^2
*(-15*c^2*f^2+14*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a
))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e
)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^3/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*
x^2+e)^(1/2)+1/16*a*(5*a^3*d^3*f^3+15*a^2*b*d^2*f^2*(-c*f+d*e)-5*a*b^2*d*f
*(-3*c^2*f^2+2*c*d*e*f+d^2*e^2)+b^3*(-5*c^3*f^3+3*c^2*d*e*f^2+c*d^2*e^2*f+
d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d
+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b
*c)/e)^(1/2))/b/c^(1/2)/d^3/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{\sqrt{x^2d+c}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
(13*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x - 5*sqrt(
e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*f*x + sqrt(e + f*x**2
)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**2*d*f*x**3 + 33*int((sqrt(e + f*x**2)*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*b*d**2
*f**2 - 40*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b
*d*e*x**4 + b*d*f*x**6),x)*a*b**2*c*d*f**2 + 14*int((sqrt(e + f*x**2)*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d
*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*d**
2*e*f + 15*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b
*d*e*x**4 + b*d*f*x**6),x)*b**3*c**2*f**2 - 4*int((sqrt(e + f*x**2)*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f
*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*d*e*f
 - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
+ a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x
**4 + b*d*f*x**6),x)*b**3*d**2*e**2 + 24*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x...
```

3.397 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 755

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{b(bde-5bcf+9adf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{8d^2f\sqrt{c+dx^2}}$$

$$+\frac{b^2x^3\sqrt{a+bx^2}\sqrt{e+fx^2}}{4d\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{e}\sqrt{de-cf}(25abcdf-8a^2d^2f+b^2c(de-15cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{8cd^3f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}(8a^3d^3f^2-3ab^2cdf(7de-15cf)+8a^2bd^2f(2de-5cf)+b^3c(d^2e^2+6cdef-15c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e}{e(c}}}{8ad^4f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{bc\sqrt{e}(15a^2d^2f^2+10abdf(de-3cf)-b^2(d^2e^2+6cdef-15c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-cf},\right.}{8ad^4f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*b*(9*a*d*f-5*b*c*f+b*d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/f/(d*x
^2+c)^(1/2)+1/4*b^2*x^3*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/(d*x^2+c)^(1/2)-
1/8*e^(1/2)*(-c*f+d*e)^(1/2)*(25*a*b*c*d*f-8*a^2*d^2*f+b^2*c*(-15*c*f+d*e)
)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/
2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/d^3/f/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/8*e^(1/2)*(8*a^3*d^3*f^2-
3*a*b^2*c*d*f*(-15*c*f+7*d*e)+8*a^2*b*d^2*f*(-5*c*f+2*d*e)+b^3*c*(-15*c^2*
f^2+6*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*El
lipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+
d*e))^(1/2))/a/d^4/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x
^2+e)^(1/2)+1/8*b*c*e^(1/2)*(15*a^2*d^2*f^2+10*a*b*d*f*(-3*c*f+d*e)-b^2*(-
15*c^2*f^2+6*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(
1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),
(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\left(a+bx^2\right)^{5/2}\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\sqrt{f\,x^{2}+e}}{\left(x^{2}d+c\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{5/2}\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

3.398 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 708

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3cd^2(c+dx^2)^{3/2}} + \frac{b^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{2d^2\sqrt{c+dx^2}}$$
$$- \frac{\sqrt{e}(b^2c^2(13de-15cf)-2abcd(3de-5cf)-2a^2d^2(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)\right.}{6c^2d^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{\sqrt{e}(2a^3d^3f+2ab^2cd(4de-15cf)-3b^3c^2(de-5cf)-2a^2bd^2(de-5cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\right.}{6acd^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{b^2c\sqrt{e}(bde-5bcf+5adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+c)^(3/2)+1
/2*b^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/(d*x^2+c)^(1/2)-1/6*e^(1/2)*(
b^2*c^2*(-15*c*f+13*d*e)-2*a*b*c*d*(-5*c*f+3*d*e)-2*a^2*d^2*(-c*f+2*d*e))*
(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)
*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^3/(-c
*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/6*e^(1/2)*
(2*a^3*d^3*f+2*a*b^2*c*d*(-15*c*f+4*d*e)-3*b^3*c^2*(-5*c*f+d*e)-2*a^2*b*d^
2*(-5*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF(
(-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1
/2))/a/c/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)+1/2*b^2*c*e^(1/2)*(5*a*d*f-5*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)
/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),
d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(-c*f+d*e)^(1/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

3.399 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 812

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5cd^2(c+dx^2)^{5/2}}$$

$$-\frac{(bc-ad)(bc(7de-8cf)+ad(4de-3cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d^2(de-cf)(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(a^2d^2(8d^2e^2-13cdef+3c^2f^2)+abcd(7d^2e^2-8cdef+5c^2f^2)+b^2c^2(8d^2e^2-25cdef+15c^2f^2))\sqrt{a}}{15c^3d^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}(a^3d^3f(4de-3cf)-15b^3c^3f(de-cf)-a^2bd^2(4d^2e^2-7cdef+5c^2f^2)-ab^2cd(4d^2e^2-20cdef+15}{15ac^2d^4(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+f}}$$

$$+\frac{b^3c\sqrt{e}f\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+c)^(5/2)-1
/15*(-a*d+b*c)*(b*c*(-8*c*f+7*d*e)+a*d*(-3*c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(
f*x^2+e)^(1/2)/c^2/d^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(a^2*d^2*(3
*c^2*f^2-13*c*d*e*f+8*d^2*e^2)+a*b*c*d*(5*c^2*f^2-8*c*d*e*f+7*d^2*e^2)+b^2
*c^2*(15*c^2*f^2-25*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*
x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d
+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/d^3/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(f*x^2+e)^(1/2)+1/15*e^(1/2)*(a^3*d^3*f*(-3*c*f+4*d*e)-15*b^3*
c^3*f*(-c*f+d*e)-a^2*b*d^2*(5*c^2*f^2-7*c*d*e*f+4*d^2*e^2)-a*b^2*c*d*(15*c
^2*f^2-20*c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1
/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/
(-c*f+d*e))^(1/2))/a/c^2/d^4/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(f*x^2+e)^(1/2)+b^3*c*e^(1/2)*f*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+
c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+
d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

3.400 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 777

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7cd^2(c+dx^2)^{7/2}}$$

$$-\frac{(bc-ad)(bc(9de-10cf)+ad(6de-5cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2d^2(de-cf)(c+dx^2)^{5/2}}$$

$$+\frac{(a^2d^2(24d^2e^2-43cdef+15c^2f^2)+b^2c^2(8d^2e^2-27cdef+15c^2f^2)+abcd(13d^2e^2-20cdef+15c^2f^2))}{105c^3d^2(de-cf)^2(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(8b^3c^3e^3+3ab^2c^2e^2(3de-11cf)+2a^2bce(8d^2e^2-25cdef+29c^2f^2)-a^3(48d^3e^3-128cd^2e^2f+103}{105c^4(bc-ad)(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e}}$$

$$-\frac{\sqrt{e}(be-af)(4b^2c^2e^2+abce(5de-13cf)-a^2(24d^2e^2-43cdef+15c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{Elliptic}}{105c^3(bc-ad)(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/7*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+c)^(7/2)-1
/35*(-a*d+b*c)*(b*c*(-10*c*f+9*d*e)+a*d*(-5*c*f+6*d*e))*x*(b*x^2+a)^(1/2)*
(f*x^2+e)^(1/2)/c^2/d^2/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/105*(a^2*d^2*(15*c^2*
f^2-43*c*d*e*f+24*d^2*e^2)+b^2*c^2*(15*c^2*f^2-27*c*d*e*f+8*d^2*e^2)+a*b*c
*d*(15*c^2*f^2-20*c*d*e*f+13*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c
^3/d^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(8*b^3*c^3*e^3+3*a*b^2*c
^2*e^2*(-11*c*f+3*d*e)+2*a^2*b*c*e*(29*c^2*f^2-25*c*d*e*f+8*d^2*e^2)-a^3*(
-15*c^3*f^3+103*c^2*d*e*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*(b*x^2+a)^(1/2)*(
c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2
+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)/(-c*f+d*e)^(5
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/105*e^(1/2)*(-a*f+b*
e)*(4*b^2*c^2*e^2+a*b*c*e*(-13*c*f+5*d*e)-a^2*(15*c^2*f^2-43*c*d*e*f+24*d^
2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*
e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3
/(-a*d+b*c)/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/
2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^5*x^10 + 5*c*d^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4 + 5*c
^4*d*x^2 + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(9/2),x)
```

3.401 $\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1221

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx = \text{Too large to display}$$

output
```
1/9*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+c)^(9/2)-1
/63*(-a*d+b*c)*(b*c*(-12*c*f+11*d*e)+a*d*(-7*c*f+8*d*e))*x*(b*x^2+a)^(1/2)
*(f*x^2+e)^(1/2)/c^2/d^2/(-c*f+d*e)/(d*x^2+c)^(7/2)+1/315*(b^2*c^2*(15*c^2
*f^2-29*c*d*e*f+8*d^2*e^2)+a*b*c*d*(25*c^2*f^2-32*c*d*e*f+19*d^2*e^2)+a^2*
d^2*(35*c^2*f^2-89*c*d*e*f+48*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/
c^3/d^2/(-c*f+d*e)^2/(d*x^2+c)^(5/2)-1/315*(a^3*d^3*(-35*c^3*f^3+159*c^2*d
*e*f^2-180*c*d^2*e^2*f+64*d^3*e^3)-a*b^2*c^2*d*(-10*c^3*f^3+14*c^2*d*e*f^2
-43*c*d^2*e^2*f+15*d^3*e^3)-2*b^3*c^3*(-5*c^3*f^3+13*c^2*d*e*f^2-8*c*d^2*e
^2*f+4*d^3*e^3)-2*a^2*b*c*d^2*(-5*c^3*f^3+52*c^2*d*e*f^2-53*c*d^2*e^2*f+18
*d^3*e^3))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^4/d^2/(-a*d+b*c)/(-c*f+d*e)
^3/(d*x^2+c)^(3/2)+1/315*e^(1/2)*(8*b^4*c^4*e^3*(-3*c*f+d*e)+a*b^3*c^3*e^2
*(99*c^2*f^2-46*c*d*e*f+11*d^2*e^2)+3*a^2*b^2*c^2*e*(-58*c^3*f^3+53*c^2*d*
e*f^2-36*c*d^2*e^2*f+9*d^3*e^3)+a^4*d*(35*c^4*f^4-334*c^3*d*e*f^3+627*c^2*
d^2*e^2*f^2-472*c*d^3*e^3*f+128*d^4*e^4)-a^3*b*c*(45*c^4*f^4-548*c^3*d*e*f
^3+945*c^2*d^2*e^2*f^2-690*c*d^3*e^3*f+184*d^4*e^4))*(b*x^2+a)^(1/2)*(c*(f
*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^
(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^5/(-a*d+b*c)^2/(-c*f+d*e)^(7/2
)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/315*e^(1/2)*(-a*f+b*e)
*(4*b^3*c^3*e^2*(-3*c*f+d*e)+3*a*b^2*c^2*e*(13*c^2*f^2-7*c*d*e*f+2*d^2*e^2
)+a^3*d*(-35*c^3*f^3+159*c^2*d*e*f^2-180*c*d^2*e^2*f+64*d^3*e^3)-3*a^2*...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(11/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[e + f*x^2])/(c + d*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{(x^2d+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x, algorithm="f
ricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^6*x^12 + 6*c*d^5*x^10 + 15*c^2*d^4*x^8 + 20*c^3*d^3*x^6 + 15
*c^4*d^2*x^4 + 6*c^5*d*x^2 + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(1/2)/(d*x**2+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x, algorithm="m
axima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(11/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{fx^2+e}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x, algorithm="g
iac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(f*x^2 + e)/(d*x^2 + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{11/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(11/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{e+fx^2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(1/2)/(d*x^2+c)^(11/2),x)
```

3.402 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 671

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \frac{(bde+5bcf-3adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bf\sqrt{a+bx^2}}$$
$$+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4b}$$
$$-\frac{\sqrt{bc-ad}e(bde+5bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{bc-ad}(8b^2ce-5abde-3abcf+3a^2df)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^3\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(3a^2d^2f^2-2abdf(de+3cf)-b^2(d^2e^2-6cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b^3\sqrt{c}\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(-3*a*d*f+5*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f/(b*x^2+
a)^(1/2)+1/4*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/8*(-a
*d+b*c)^(1/2)*e*(-3*a*d*f+5*b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b
*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a
*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(f*x^2+e)^(1/2)+1/8*(-a*d+b*c)^(1/2)*(3*a^2*d*f-3*a*b*c*f-5*a*b*d*e+8*b^2
*c*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)
^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^2*f^2
-2*a*b*d*f*(3*c*f+d*e)-b^2*(-3*c^2*f^2-6*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*
x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)
/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[a + b*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(x^2 d + c)^{\frac{3}{2}} \sqrt{f\,x^2 + e}}{\sqrt{b\,x^2 + a}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c + dx^2)^{3/2} \sqrt{e + fx^2}}{\sqrt{a + bx^2}}\, dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx=\int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx=\int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx=\int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*d*x - 3*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*d**2*f + 5*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4
 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*f + int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d**2*e - 2*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*c*d*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d**2*e + 4*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c**
2*f + 6*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*b*c*d*e - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**
2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*d*e + 4*int((sqrt(e + ...
```

3.403 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 537

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx &= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2b\sqrt{c+dx^2}} \\ &\quad - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\ &\quad + \frac{c\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\ &\quad + \frac{c\sqrt{e}(bde+bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2abd\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
1/2*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/b/(d*x^2+c)^(1/2)-1/2*e^(1/2)*(-c*
f+d*e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c
*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2)
)/b/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/2*c*e^(1/2)*(-c*f+d*
e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d
*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/
d/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/2*c*e^(1/2)*(-a*d*f+b*
c*f+b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*
f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c
*f+d*e))^(1/2))/a/b/d/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*
x^2+e)^(1/2)
```

Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(b^2c\sqrt{bc-ad}x\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}(e+fx^2)-bc\sqrt{bc-ad}\sqrt{e}\sqrt{be-af}\sqrt{a+bx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(b^2*c*Sqrt[b*c - a
*d]*x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(e + f*x^2) - b*c*Sqrt[b*c - a
*d]*Sqrt[e]*Sqrt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b
*x^2))]*EllipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (
b*c*e - a*d*e)/(b*c*e - a*c*f)] + Sqrt[b*c - a*d]*(2*b*c - a*d)*Sqrt[e]*Sq
rt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Ellipt
icF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)
/(b*c*e - a*c*f)] - a*Sqrt[c]*(a*d*f - b*(d*e + c*f))*Sqrt[a + b*x^2]*Sqrt
[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*d), ArcSin[(Sq
rt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (b*c*e - a*c*f)/(b*c*e - a*d*
e)]))/(2*a*b^2*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {431, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{431}$$

$$\frac{(bc-ad)(2be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}+\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{427}$$

$$\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}+\frac{\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{(-adf+bcf+bde)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2b^2}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{428}$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{a(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{2b}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 429}$$

$$-\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\int \frac{\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(-adf+bcf+bde)\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*
c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*
c - a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))])
 + ((b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/
(e*(a + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*
x^2])], ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a
*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) + (a*(b*d*e + b*c*f - a*d*
f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 431
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Simp[e*((b*e - a*f)/(2*f))   Int[Sqrt[c + d*x^2]/(Sqrt[a +
b*x^2]*(e + f*x^2)^(3/2)), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*f^2)
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[(b*e -
 a*f)*((d*e - 2*c*f)/(2*f^2))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt
[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c
]
```

Maple [F]

$$\int \frac{\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a + b*x**2),x)
```

3.404 $\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 160

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(
1/2))/a^(1/2)/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(\frac{af}{-be+af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(-de+cf)}{c(-be+af)}\right)}{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticPi[(a*f)/
(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])],
(a*(-(d*e) + c*f))/(c*(-(b*e) + a*f))])/(Sqrt[a]*Sqrt[-(b*e) + a*f]*Sqrt[c
 + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {428, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \textbf{428}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\int \frac{1}{\left(1-\frac{fx^2}{fx^2+e}\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{a\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \textbf{412}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticPi[-((a*f
)/(b*e - a*f)), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])],
(a*(d*e - c*f))/(c*(b*e - a*f))])/(Sqrt[a]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x
^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)
```

3.405 $\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 149

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)|-\frac{a(de-cf)}{(bc-ad)e}\right)}{c\sqrt{-bc+ad}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
a^(1/2)*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*EllipticE((a*d-b*c
)^(1/2)*x/a^(1/2)/(d*x^2+c)^(1/2),(-a*(-c*f+d*e)/(-a*d+b*c)/e)^(1/2))/c/(a
*d-b*c)^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)|\frac{a(de-cf)}{(-bc+ad)e}\right)}{c\sqrt{-bc+ad}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])], (a*(d*e - c*f))/(
(-(b*c) + a*d)*e)])/(c*Sqrt[-(b*c) + a*d]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x
^2))/(e*(c + d*x^2))])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.72, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {429, 326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{429}$$

$$\frac{\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\int \frac{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}d\frac{x}{\sqrt{dx^2+c}}}{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\int \frac{1}{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}-\frac{a(de-cf)\int \frac{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}\right)}{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{e}(bc-ad)\sqrt{de-cf}}-\frac{a(de-cf)\int\frac{\sqrt{\frac{(bc-ad)x^2}{a(dx^2+c)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e(dx^2+c)}}}d\frac{x}{\sqrt{dx^2+c}}}{e(bc-ad)}\right)}{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(\frac{c(be-af)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{e}(bc-ad)\sqrt{de-cf}}-\frac{a\sqrt{de-cf}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{e}(bc-ad)}\right)}{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2]*(-((a*Sqrt[d*e - c*
f]*EllipticE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*
c - a*d)*e)/(a*(d*e - c*f)))])/((b*c - a*d)*Sqrt[e])) + (c*(b*e - a*f)*Ell
ipticF[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*
d)*e)/(a*(d*e - c*f)))])/((b*c - a*d)*Sqrt[e]*Sqrt[d*e - c*f])))/(c*Sqrt[a
 + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}}}d x$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^2*x^6 + (2*b
*c*d + a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*a*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d
*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)
```

3.406 $\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 423

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}\,dx = -\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{\sqrt{e}(bc(4de-3cf)-ad(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{(3bc-ad)\sqrt{e}(be-af)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3ac(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*
e^(1/2)*(b*c*(-3*c*f+4*d*e)-a*d*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)
/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(
-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-a*d+3*b*c)*e^(1/2)*(-a*f+
b*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^
(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-
a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2
)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{5}{2}}}d x$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^3*x^8 + (3*b
*c*d^2 + a*d^3)*x^6 + 3*(b*c^2*d + a*c*d^2)*x^4 + a*c^3 + (b*c^3 + 3*a*c^2
*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

3.407 $\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 614

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = -\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{5c(bc-ad)\,(c+dx^2)^{5/2}}$$
$$-\frac{d(bc(8de-7cf)-ad(4de-3cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\,(c+dx^2)^{3/2}}$$
$$-\frac{\sqrt{e}(a^2d^2(8d^2e^2-13cdef+3c^2f^2)-abcd(23d^2e^2-37cdef+10c^2f^2)+b^2c^2(23d^2e^2-40cdef+15c^2f^2}{15c^3(bc-ad)^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(be-af)\,(abcd(11de-10cf)-a^2d^2(4de-3cf)-15b^2c^2(de-cf))\,\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(a\right.}{15ac^2(bc-ad)^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(5/2)-1/15
*d*(b*c*(-7*c*f+8*d*e)-a*d*(-3*c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/
2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/15*e^(1/2)*(a^2*d^2*(3*c^
2*f^2-13*c*d*e*f+8*d^2*e^2)-a*b*c*d*(10*c^2*f^2-37*c*d*e*f+23*d^2*e^2)+b^2
*c^2*(15*c^2*f^2-40*c*d*e*f+23*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d
*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*
d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/15*e^(1/2)*(-a*f+b*e)*(a*b*c*d*(-1
0*c*f+11*d*e)-a^2*d^2*(-3*c*f+4*d*e)-15*b^2*c^2*(-c*f+d*e))*(b*x^2+a)^(1/2
)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*
x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c^2/(-a*d+b*c)^3/(-c*f+
d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{7}{2}}}d x$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^4*x^10 + (4*
b*c*d^3 + a*d^4)*x^8 + 2*(3*b*c^2*d^2 + 2*a*c*d^3)*x^6 + a*c^4 + 2*(2*b*c^
3*d + 3*a*c^2*d^2)*x^4 + (b*c^4 + 4*a*c^3*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

3.408 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 549

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2b\sqrt{a+bx^2}} + \frac{(2bc-3ad)\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2ab^2\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{bc-ad}(4be-3af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^3\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{ad(bde+3bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*d*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/(b*x^2+a)^(1/2)+1/2*(-3*a*d+2*b*
c)*(-a*d+b*c)^(1/2)*e*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Elli
pticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/
e)^(1/2))/a/b^2/c^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/
2*d*(-a*d+b*c)^(1/2)*(-3*a*f+4*b*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+
a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*
e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2
+e)^(1/2)+1/2*a*d*(-3*a*d*f+3*b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/
(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c
/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/(-a*d+b*c)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2),x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{3}{2}}\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b^2*x^4 + 2*a*
b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx=\int \frac{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx=\int \frac{(d\,x^2+c)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

3.409 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 479

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$
$$+\frac{\sqrt{c}d(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^2\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$
$$+\frac{a\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^2\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

output
```
c^(1/2)*(-a*d+b*c)^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*E
llipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*
c)/e)^(1/2))/a/b/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)+c^(1/2)*d
*(-a*f+b*e)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*EllipticF((-a*
d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/
b^2/(-a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)+a*c
^(1/2)*d*f*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*EllipticPi((-a*
d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+
b*c)/e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^
2+a))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {432, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 432$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{b}+\frac{d\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow 428$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{b}+\frac{de\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\int \frac{1}{\left(1-\frac{fx^2}{fx^2+e}\right)\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{ab\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow 412$$

$$\frac{(bc-ad)\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{b}+\frac{de\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}b\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow 429$$

$$\frac{\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\int \frac{\sqrt{1-\frac{(be-af)x^2}{e(bx^2+a)}}}{\sqrt{1-\frac{(bc-ad)x^2}{c(bx^2+a)}}}d\frac{x}{\sqrt{bx^2+a}}}{ab\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{de\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}b\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}+\frac{de\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}b\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*
x^2]*EllipticE[ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(
b*e - a*f))/((b*c - a*d)*e)])/(a*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e
*(a + b*x^2))]) + (d*e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*EllipticPi[-((a*f)/(b*e - a*f)), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]
*Sqrt[e + f*x^2])], (a*(d*e - c*f))/(c*(b*e - a*f))])/(Sqrt[a]*b*Sqrt[-(b*
e) + a*f]*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 432
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Simp[b/f   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sq
rt[e + f*x^2]), x], x] - Simp[(b*e - a*f)/f   Int[Sqrt[c + d*x^2]/(Sqrt[a +
 b*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(3/2),x)
```

3.410 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

output
```
c^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*EllipticE((-a*d+b*
c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/(-
a*d+b*c)^(1/2)/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
```

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*
c - a*d)*e)])/(a*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(
a + b*x^2))])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 429$$

$$\frac{\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\int \frac{\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{a\sqrt{c+dx^2}\sqrt{bc-ad}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*
c - a*d)*e)])/(a*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(
a + b*x^2))])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*x^6 + (b^2
*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*
x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
```

3.411 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 395

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{bx\sqrt{e+fx^2}}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{(bc+ad)\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ac(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(2bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
b*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+(a*d+b*c)
*e^(1/2)*(-c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*
EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*
f+d*e))^(1/2))/a/c/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^
(1/2)-e^(1/2)*(-a*d*f-b*c*f+2*b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/a/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(x^{2} d+c\right)^{\frac{3}{2}}}d x$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^{2}+e}}{{\left(bx^{2}+a\right)}^{\frac{3}{2}}{\left(dx^{2}+c\right)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^2*x^8 + 2*
(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 +
2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{3}{2}}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

3.412 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 531

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}+\frac{b(3bc+ad)x\sqrt{e+fx^2}}{3ac(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{e}(abcd(7de-6cf)+3b^2c^2(de-cf)-a^2d^2(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-\cdots}{a(de\cdots}\right.}{3ac^2(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(a^2d^2f+3b^2c(3de-cf)-abd(de+6cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad\cdots}{a(de-c\cdots}\right.}{3ac(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*d*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*
b*(a*d+3*b*c)*x*(f*x^2+e)^(1/2)/a/c/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c)
^(1/2)+1/3*e^(1/2)*(a*b*c*d*(-6*c*f+7*d*e)+3*b^2*c^2*(-c*f+d*e)-a^2*d^2*(-
c*f+2*d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*
f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))
/a/c^2/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^
2+e)^(1/2)-1/3*e^(1/2)*(a^2*d^2*f+3*b^2*c*(-c*f+3*d*e)-a*b*d*(6*c*f+d*e))*
(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)
*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-a*d+b
*c)^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(x^{2}d+c\right)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^3*x^10 + (
3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 +
 a^2*c^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + (2*a*b*c^3 + 3*a^2*
c^2*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(3/2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

3.413 $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 716

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ab^2(a+bx^2)^{3/2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2b^2\sqrt{a+bx^2}}$$
$$+ \frac{\sqrt{bc-ad}\,e(4b^3c^2e+15a^3d^2f+2ab^2c(3de-cf)-a^2bd(13de+10cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{c}}\right.\right.}{6a^2b^3\sqrt{c}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\sqrt{bc-ad}(33a^2bd^2ef-15a^3d^2f^2-2b^3ce(de-cf)-2ab^2de(8de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(a\right.}{6ab^4\sqrt{c}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{ad^2(bde+5bcf-5adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^4\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/b^2/(b*x^2+a)^(3/2)+1
/2*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/(b*x^2+a)^(1/2)+1/6*(-a*d+b*c
)^(1/2)*e*(4*b^3*c^2*e+15*a^3*d^2*f+2*a*b^2*c*(-c*f+3*d*e)-a^2*b*d*(10*c*f
+13*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^
2/b^3/c^(1/2)/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1
/6*(-a*d+b*c)^(1/2)*(33*a^2*b*d^2*e*f-15*a^3*d^2*f^2-2*b^3*c*e*(-c*f+d*e)-
2*a*b^2*d*e*(c*f+8*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*E
llipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*
c)/e)^(1/2))/a/b^4/c^(1/2)/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x
^2+e)^(1/2)+1/2*a*d^2*(-5*a*d*f+5*b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e
)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2)
,b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/(-a*d+b*c)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}dx$$

input
```
Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{5}{2}}\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

3.414 $\int \frac{\left(c+dx^2\right)^{3/2} \sqrt{e+fx^2}}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 629

$$\int \frac{\left(c+dx^2\right)^{3/2} \sqrt{e+fx^2}}{\left(a+bx^2\right)^{5/2}} \, dx = \frac{(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ab\left(a+bx^2\right)^{3/2}}$$

$$+\frac{\sqrt{bc-ad}e(2b^2ce+2abde-abcf-3a^2df)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2b^2\sqrt{c}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{(6a^2bd^2ef-3a^3d^2f^2-b^3ce(de-cf)-ab^2de(2de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}\cdots}{\sqrt{c}\sqrt{a+b\cdots}}\right.\right.}{3ab^3\sqrt{c}\sqrt{bc-ad}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{ad^2f\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/b/(b*x^2+a)^(3/2)+1/3*(
-a*d+b*c)^(1/2)*e*(-3*a^2*d*f-a*b*c*f+2*a*b*d*e+2*b^2*c*e)*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/b^2/c^(1/2)/(-a*f+b*e)/
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/3*(6*a^2*b*d^2*e*f-3*a^3
*d^2*f^2-b^3*c*e*(-c*f+d*e)-a*b^2*d*e*(c*f+2*d*e))*(d*x^2+c)^(1/2)*(a*(f*x
^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1
/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/b^3/c^(1/2)/(-a*d+b*c)^(1/2)/(-a*
f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+a*d^2*f*(d*x^2+c)^(
1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)
/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(
1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(x^2 d+c)^{\frac{3}{2}} \sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

3.415 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 390

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3a(a+bx^2)^{3/2}} + \frac{e(2bce-a(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2\sqrt{c}\sqrt{bc-ad}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{e(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3a\sqrt{c}\sqrt{bc-ad}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)^(3/2)+1/3*e*(2*b*c*e-a*(
c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^
2/c^(1/2)/(-a*d+b*c)^(1/2)/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x
^2+e)^(1/2)+1/3*e*(-c*f+d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/
2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*
d+b*c)/e)^(1/2))/a/c^(1/2)/(-a*d+b*c)^(1/2)/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*x^6 + 3*a*b^
2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(5/2),x)
```

3.416 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 413

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{e(2b^2ce+3a^2df-ab(4de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-adx}}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2\sqrt{c}(bc-ad)^{3/2}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{be(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-adx}}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3a\sqrt{c}(bc-ad)^{3/2}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+1/3*e
*(2*b^2*c*e+3*a^2*d*f-a*b*(c*f+4*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x
^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f
+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^(3/2)/(-a*f+b*e)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*e*(-c*f+d*e)*(d*x^2+c)^(1/2
)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*
x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(1/2)/(-a*d+b*c)^(3/2)
/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d*x^8 + (b^3
*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a^3
*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{{\left(b\,x^2+a\right)}^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)^{\frac{5}{2}}\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

3.417 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 565

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{bx\sqrt{e+fx^2}}{3a(bc-ad)(a+bx^2)^{3/2}\sqrt{c+dx^2}}$$
$$+\frac{b(2b^2ce+5a^2df-ab(6de+cf))x\sqrt{e+fx^2}}{3a^2(bc-ad)^2(be-af)\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{e}\sqrt{de-cf}(2b^3c^2e+3a^3d^2f-3a^2bd(de-2cf)-ab^2c(7de+cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-c}}{\sqrt{e}\sqrt{c+}}\right.\right.}{3a^2c(bc-ad)^3(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(3a^2d^2f-3abd(3de-2cf)+b^2c(de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-}{a(de-}\right.}{3a^2(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*b*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+1/3*b
*(2*b^2*c*e+5*a^2*d*f-a*b*(c*f+6*d*e))*x*(f*x^2+e)^(1/2)/a^2/(-a*d+b*c)^2/
(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*e^(1/2)*(-c*f+d*e)^(1/2)*(2
*b^3*c^2*e+3*a^3*d^2*f-3*a^2*b*d*(-2*c*f+d*e)-a*b^2*c*(c*f+7*d*e))*(b*x^2+
a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1
/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^3
/(-a*f+b*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(3
*a^2*d^2*f-3*a*b*d*(-2*c*f+3*d*e)+b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(
1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(x^{2} d+c\right)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^2*x^10 + (
2*b^3*c*d + 3*a*b^2*d^2)*x^8 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^6 +
 a^3*c^2 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^4 + (3*a^2*b*c^2 + 2*a^
3*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

3.418 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 786

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{bx\sqrt{e+fx^2}}{3a(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^{3/2}}$$
$$+\frac{b(2b^2ce+7a^2df-ab(8de+cf))x\sqrt{e+fx^2}}{3a^2(bc-ad)^2(be-af)\sqrt{a+bx^2}(c+dx^2)^{3/2}}$$
$$+\frac{d(2b^3c^2e+a^3d^2f-a^2bd(de-8cf)-ab^2c(9de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3a^2c(bc-ad)^3(be-af)(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(2b^4c^3e(de-cf)-a^4d^3f(2de-cf)+a^3bd^2(2d^2e^2+9cdef-9c^2f^2)-ab^3c^2(10d^2e^2-9cdef-c^2f^2)}{3a^2c^2(bc-ad)^4(be-af)\sqrt{de-c}}$$
$$+\frac{\sqrt{e}(a^3d^3f-b^3c^2(de-cf)+9ab^2cd(2de-cf)-a^2bd^2(de+9cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\right.}{3a^2c(bc-ad)^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*b*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)+1/3*b
*(2*b^2*c*e+7*a^2*d*f-a*b*(c*f+8*d*e))*x*(f*x^2+e)^(1/2)/a^2/(-a*d+b*c)^2/
(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*d*(2*b^3*c^2*e+a^3*d^2*f-a^
2*b*d*(-8*c*f+d*e)-a*b^2*c*(c*f+9*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/
a^2/c/(-a*d+b*c)^3/(-a*f+b*e)/(d*x^2+c)^(3/2)+1/3*e^(1/2)*(2*b^4*c^3*e*(-c
*f+d*e)-a^4*d^3*f*(-c*f+2*d*e)+a^3*b*d^2*(-9*c^2*f^2+9*c*d*e*f+2*d^2*e^2)-
a*b^3*c^2*(-c^2*f^2-9*c*d*e*f+10*d^2*e^2)-a^2*b^2*c*d*(9*c^2*f^2-18*c*d*e*
f+10*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((
-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/
2))/a^2/c^2/(-a*d+b*c)^4/(-a*f+b*e)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*e^(1/2)*(a^3*d^3*f-b^3*c^2*(-c*f+d*e)+9*a*b
^2*c*d*(-c*f+2*d*e)-a^2*b*d^2*(9*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/
(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^4/(-c*f+d*e)^(1/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}\,(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^3*x^12 + 3
*(b^3*c*d^2 + a*b^2*d^3)*x^10 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*
x^8 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^6 + a^3*c^3 +
3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^4 + 3*(a^2*b*c^3 + a^3*c^2*d)*
x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{\frac{5}{2}}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/((a + b*x**2)**(5/2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

3.419 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1146

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx = \text{Too large to display}$$

output
```
1/384*(15*a^3*d^2*f^2/b-31*a^2*d*f*(c*f+d*e)+a*b*(9*c^2*f^2+86*c*d*e*f+9*d
^2*e^2)+b^2*(33*c*d*e^2-9*d^2*e^3/f+33*c^2*e*f-9*c^3*f^2/d))*x*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/b/d/f/(b*x^2+a)^(1/2)+1/192*(66*c*e+3*d*e^2/f+3*c^2*f
/d-5*a^2*d*f/b^2+10*a*(c*f+d*e)/b)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)+1/48*(a*d*f+9*b*(c*f+d*e))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*
(f*x^2+e)^(1/2)/b+1/8*d*f*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)-1/384*(-a*d+b*c)^(1/2)*e*(15*a^3*d^3*f^3-31*a^2*b*d^2*f^2*(c*f+d*e)+a*
b^2*d*f*(9*c^2*f^2+86*c*d*e*f+9*d^2*e^2)-b^3*(9*c^3*f^3-33*c^2*d*e*f^2-33*
c*d^2*e^2*f+9*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*El
lipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c
)/e)^(1/2))/b^3/c^(1/2)/d^2/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^
(1/2)-1/384*a*(-a*d+b*c)^(1/2)*(15*a^3*d^3*f^3-3*a^2*b*d^2*f^2*(7*c*f+17*d
*e)+a*b^2*d*f*(-3*c^2*f^2+98*c*d*e*f+49*d^2*e^2)+3*b^3*(3*c^3*f^3-13*c^2*d
*e*f^2-55*c*d^2*e^2*f+d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d^2/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x
^2+e)^(1/2)-1/128*a*(5*a^4*d^4*f^4-3*b^4*(-c*f+d*e)^4-12*a^3*b*d^3*f^3*(c*
f+d*e)+6*a^2*b^2*d^2*f^2*(c^2*f^2+6*c*d*e*f+d^2*e^2)+4*a*b^3*d*f*(c^3*f^3-
9*c^2*d*e*f^2-9*c*d^2*e^2*f+d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^
2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(...
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}dx$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{\frac{3}{2}}\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2),x)
```

3.420 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 850

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{\left(8ae+\frac{8bce}{d}+\frac{3be^2}{f}-\frac{3a^2f}{b}-\frac{3bc^2f}{d^2}+\frac{2acf}{d}\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48\sqrt{a+bx^2}}$$
$$+\frac{(7bde+bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24bd}+\frac{1}{6}fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$
$$+\frac{\sqrt{bc-ad}e(3a^2d^2f^2-2abdf(4de+cf)-b^2(3d^2e^2+8cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-}}{\sqrt{c}\sqrt{}}\right.\right.}{48b^2\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a\sqrt{bc-ad}(12abd^2ef-3a^2d^2f^2-b^2(17d^2e^2+10cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{48b^3\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(a^3d^3f^3-b^3(de-cf)^3-a^2bd^2f^2(3de+cf)+ab^2df(3d^2e^2+6cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{Ellipt}}{16b^3\sqrt{c}d^2\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(8*a*e+8*b*c*e/d+3*b*e^2/f-3*a^2*f/b-3*b*c^2*f/d^2+2*a*c*f/d)*x*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)+1/24*(a*d*f+b*c*f+7*b*d*e)*x*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d+1/6*f*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/48*(-a*d+b*c)^(1/2)*e*(3*a^2*d^2*f^2-2
*a*b*d*f*(c*f+4*d*e)-b^2*(-3*c^2*f^2+8*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/48*a*(-a*d+b*c)^(1/2)*(12*a*b*d^2
*e*f-3*a^2*d^2*f^2-b^2*(-3*c^2*f^2+10*c*d*e*f+17*d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/16*a*(a^3*d^3*f^3-b^3*(-c*f+d*e)^3-
a^2*b*d^2*f^2*(c*f+3*d*e)+a*b^2*d*f*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^
3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)
^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

3.421 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 650

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{(5bde-3bcf+adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8d^2\sqrt{a+bx^2}}$$
$$+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4d}$$
$$-\frac{\sqrt{bc-ad}\,e(5bde-3bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)\middle|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a\sqrt{bc-ad}\,f(7bde-3bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a(a^2d^2f^2-3b^2(de-cf)^2-2abdf(3de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\right.}{8b^2\sqrt{c}d^2\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(a*d*f-3*b*c*f+5*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/(b*x^2+a
)^(1/2)+1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d-1/8*(-a*
d+b*c)^(1/2)*e*(a*d*f-3*b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x
^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f
+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(
f*x^2+e)^(1/2)+1/8*a*(-a*d+b*c)^(1/2)*f*(-a*d*f-3*b*c*f+7*b*d*e)*(d*x^2+c)
^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2
)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2-3*b^2*(-c*f+
d*e)^2-2*a*b*d*f*(-c*f+3*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(
1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),
(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\sqrt{x^{2} d+c}}d x$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx+\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+}\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x + int((sqrt(e + f*
x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*d*f**2 - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 +
 b*d*e*x**4 + b*d*f*x**6),x)*b*c*f**2 + 5*int((sqrt(e + f*x**2)*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**
4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e*f - 2*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*c*f**2 + 6*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*e*f - 2*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e
*f + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*
e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e
*x**4 + b*d*f*x**6),x)*b*d*e**2 - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**
2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*e*f + 4*int((sqrt(e + ...
```

3.422 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 557

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{fx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2d\sqrt{c+dx^2}} + \frac{\sqrt{e}(2de-3cf)\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2cd^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{(3bc-4ad)\sqrt{e}f\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{c\sqrt{e}f(3bde-3bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*f*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/(d*x^2+c)^(1/2)+1/2*e^(1/2)*(-3*
c*f+2*d*e)*(-c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2
)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-
c*f+d*e))^(1/2))/c/d^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/2
*(-4*a*d+3*b*c)*e^(1/2)*f*(-c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/
(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x
^2+e)^(1/2)+1/2*c*e^(1/2)*f*(a*d*f-3*b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^
(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^3/(-c*f+d*e)^
(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2} d+c\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

3.423 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 635

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}}$$

$$-\frac{\sqrt{e}\sqrt{de-cf}(2ad(de+cf)-bc(de+3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2d^2(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(3b^2c^3f^2+a^2d^2f(de+2cf)-abd(d^2e^2-cdef+6c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-}}{\sqrt{e}\sqrt{c}}\right.\right.}{3acd^3(bc-ad)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{bc\sqrt{e}f^2\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(3/2)-1/3*e
^(1/2)*(-c*f+d*e)^(1/2)*(2*a*d*(c*f+d*e)-b*c*(3*c*f+d*e))*(b*x^2+a)^(1/2)*
(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^
2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^2/(-a*d+b*c)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(3*b^2*c^3*f^2+a^2*d^2
*f*(2*c*f+d*e)-a*b*d*(6*c^2*f^2-c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^
2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/
2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/d^3/(-a*d+b*c)/(-c*f+d*e)^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+b*c*e^(1/2)*f^2*(b*x^2+a)
^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/
2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^
3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2} d+c\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

3.424 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 533

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx=\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}}$$

$$+\frac{(3bc(de+cf)-2ad(2de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d(bc-ad)\left(c+dx^2\right)^{3/2}}$$

$$+\frac{\sqrt{e}(3b^2c^2e^2-abce(13de-7cf)+a^2(8d^2e^2-3cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)\middle|\right.}{15c^3(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{2\sqrt{e}(be-af)(3bce-2ade-acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{15c^2(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(5/2)+1/15*
(3*b*c*(c*f+d*e)-2*a*d*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/
d/(-a*d+b*c)/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(3*b^2*c^2*e^2-a*b*c*e*(-7*c*f+1
3*d*e)+a^2*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/
e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-
(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+2/15*e^(1/2)*(-a*f+b*e)*(-a*c*f-
2*a*d*e+3*b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF
((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(
1/2))/c^2/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f
*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2} d+c\right)^{\frac{7}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(d^4*x^8 + 4*c*
d^3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

3.425 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 835

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7cd\,(c+dx^2)^{7/2}}$$

$$-\frac{(2ad(3de+cf)-bc(5de+3cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2d(bc-ad)\,(c+dx^2)^{5/2}}$$

$$-\frac{(abcd(43d^2e^2-29cdef-8c^2f^2)-3a^2d^2(8d^2e^2-5cdef-2c^2f^2)-3b^2c^2(5d^2e^2-2cdef-2c^2f^2))\,x\sqrt{a}}{105c^3d(bc-ad)^2(de-cf)\,(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(3b^3c^3e^2(5de-7cf)-ab^2c^2e(103d^2e^2-170cdef+49c^2f^2)-6a^3d(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3}{105c^4(bc-ad)^3(de-}$$

$$+\frac{\sqrt{e}(be-af)\,(3b^2c^2e(15de-14cf)-abc(61d^2e^2-41cdef-14c^2f^2)+3a^2d(8d^2e^2-5cdef-2c^2f^2))\,\sqrt{}}{105c^3(bc-ad)^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx}}$$

output
```
1/7*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(7/2)-1/35*
(2*a*d*(c*f+3*d*e)-b*c*(3*c*f+5*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^
2/d/(-a*d+b*c)/(d*x^2+c)^(5/2)-1/105*(a*b*c*d*(-8*c^2*f^2-29*c*d*e*f+43*d^
2*e^2)-3*a^2*d^2*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2)-3*b^2*c^2*(-2*c^2*f^2-2*
c*d*e*f+5*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/d/(-a*d+b*c)^2/(
-c*f+d*e)/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(3*b^3*c^3*e^2*(-7*c*f+5*d*e)-a*b^
2*c^2*e*(49*c^2*f^2-170*c*d*e*f+103*d^2*e^2)-6*a^3*d*(c^3*f^3+2*c^2*d*e*f^
2-12*c*d^2*e^2*f+8*d^3*e^3)+a^2*b*c*(14*c^3*f^3+37*c^2*d*e*f^2-197*c*d^2*e
^2*f+128*d^3*e^3))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*Ellipti
cE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))
^(1/2))/c^4/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(f*x^2+e)^(1/2)+1/105*e^(1/2)*(-a*f+b*e)*(3*b^2*c^2*e*(-14*c*f+15*d*e)-a*b
*c*(-14*c^2*f^2-41*c*d*e*f+61*d^2*e^2)+3*a^2*d*(-2*c^2*f^2-5*c*d*e*f+8*d^2
*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e
)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/
(-a*d+b*c)^3/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}{\left(x^2d+c\right)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx=\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(d^5*x^10 + 5*c
*d^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4 + 5*c^4*d*x^2 + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

3.426 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1296

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \text{Too large to display}$$

output
```
1/9*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(9/2)-1/63*
(2*a*d*(c*f+4*d*e)-b*c*(3*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^
2/d/(-a*d+b*c)/(d*x^2+c)^(7/2)-1/315*(a*b*c*d*(-16*c^2*f^2-67*c*d*e*f+89*d
^2*e^2)-b^2*c^2*(-12*c^2*f^2-20*c*d*e*f+35*d^2*e^2)-a^2*d^2*(-10*c^2*f^2-3
5*c*d*e*f+48*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/d/(-a*d+b*c)^
2/(-c*f+d*e)/(d*x^2+c)^(5/2)-1/315*(2*a^3*d^3*(5*c^3*f^3+15*c^2*d*e*f^2-54
*c*d^2*e^2*f+32*d^3*e^3)-b^3*c^3*(8*c^3*f^3+8*c^2*d*e*f^2-55*c*d^2*e^2*f+3
5*d^3*e^3)+a*b^2*c^2*d*(16*c^3*f^3+91*c^2*d*e*f^2-278*c*d^2*e^2*f+159*d^3*
e^3)-a^2*b*c*d^2*(26*c^3*f^3+89*c^2*d*e*f^2-307*c*d^2*e^2*f+180*d^3*e^3))*
x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^4/d/(-a*d+b*c)^3/(-c*f+d*e)^2/(d*x^2+c
)^(3/2)+1/315*e^(1/2)*(b^4*c^4*e^2*(63*c^2*f^2-90*c*d*e*f+35*d^2*e^2)-a*b^
3*c^3*e*(-147*c^3*f^3+747*c^2*d*e*f^2-902*c*d^2*e^2*f+334*d^3*e^3)-a^3*b*c
*d*(-36*c^4*f^4-101*c^3*d*e*f^3+915*c^2*d^2*e^2*f^2-1218*c*d^3*e^3*f+472*d
^4*e^4)+3*a^2*b^2*c^2*(-14*c^4*f^4-53*c^3*d*e*f^3+420*c^2*d^2*e^2*f^2-546*
c*d^3*e^3*f+209*d^4*e^4)+a^4*d^2*(-10*c^4*f^4-25*c^3*d*e*f^3+243*c^2*d^2*e
^2*f^2-328*c*d^3*e^3*f+128*d^4*e^4))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/c^5/(-a*d+b*c)^4/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+2/315*e^(1/2)*(-a*f+b*e)*(b^3*c^3*e*(63*
c^2*f^2-135*c*d*e*f+70*d^2*e^2)-a^3*d^2*(5*c^3*f^3+15*c^2*d*e*f^2-54*c*...
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{11/2}} \, dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{11/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2} d+c\right)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(d^6*x^12 + 6*c
*d^5*x^10 + 15*c^2*d^4*x^8 + 20*c^3*d^3*x^6 + 15*c^4*d^2*x^4 + 6*c^5*d*x^2
 + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{11/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

3.427 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1147

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \text{Too large to display}$$

output
```
-1/384*(9*a^3*d^3*f^3-3*a^2*b*d^2*f^2*(3*c*f+11*d*e)-a*b^2*d*f*(-31*c^2*f^
2+86*c*d*e*f+33*d^2*e^2)+b^3*(-15*c^3*f^3+31*c^2*d*e*f^2-9*c*d^2*e^2*f+9*d
^3*e^3))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d^3/f^2/(b*x^2+a)^(1/2)+1/192
*(66*a*e+10*b*c*e/d+3*b*e^2/f+3*a^2*f/b-5*b*c^2*f/d^2+10*a*c*f/d)*x*(b*x^2
+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/48*(9*a*d*f+b*c*f+9*b*d*e)*x^3
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d+1/8*b*f*x^5*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/384*(-a*d+b*c)^(1/2)*e*(9*a^3*d^3*f
^3-3*a^2*b*d^2*f^2*(3*c*f+11*d*e)-a*b^2*d*f*(-31*c^2*f^2+86*c*d*e*f+33*d^2
*e^2)+b^3*(-15*c^3*f^3+31*c^2*d*e*f^2-9*c*d^2*e^2*f+9*d^3*e^3))*(d*x^2+c)^
(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)
/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^3/f^2/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/384*a*(-a*d+b*c)^(1/2)*(9*
a^3*d^3*f^3-3*a^2*b*d^2*f^2*(c*f+15*d*e)+3*a*b^2*d*f*(-7*c^2*f^2+26*c*d*e*
f+29*d^2*e^2)-b^3*(-15*c^3*f^3+41*c^2*d*e*f^2-29*c*d^2*e^2*f+3*d^3*e^3))*(
d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*
x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d
^3/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/128*a*(3*a^4*d^4*f^
4-12*a*b^3*d*f*(-c*f+d*e)^3-4*a^3*b*d^3*f^3*(c*f+3*d*e)+b^4*(-c*f+d*e)^3*(
5*c*f+3*d*e)+6*a^2*b^2*d^2*f^2*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(
1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1...
```

Mathematica [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \; 434$$

$$\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)^{3/2} \, dx$$

input
```
Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{2}} \sqrt{x^{2} d+c}\, \left(f\,x^{2}+e\right)^{\frac{3}{2}}\, dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(b\,x^2+a\right)^{3/2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \left(b\,x^2+a\right)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

3.428 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 866

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{(3a^2d^2f^2+2abdf(19de-11cf)+b^2(3d^2e^2-22cdef+15c^2f^2))\,x\sqrt{c+}}{48d^3f\sqrt{a+bx^2}}$$

$$+\frac{(7bde-5bcf+7adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24d^2}+\frac{bfx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{6d}$$

$$-\frac{\sqrt{bc-ad}e(3a^2d^2f^2+2abdf(19de-11cf)+b^2(3d^2e^2-22cdef+15c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\right.}{48b\sqrt{c}d^3f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a\sqrt{bc-ad}(3a^2d^2f^2-6abdf(5de-2cf)-b^2(17d^2e^2-32cdef+15c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}}{48b^2\sqrt{c}d^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a(a^3d^3f^3-9ab^2df(de-cf)^2-3a^2bd^2f^2(3de-cf)+b^3(de-cf)^2(de+5cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{Ellip}}{16b^2\sqrt{c}d^3\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(3*a^2*d^2*f^2+2*a*b*d*f*(-11*c*f+19*d*e)+b^2*(15*c^2*f^2-22*c*d*e*f+
3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^3/f/(b*x^2+a)^(1/2)+1/24*(
7*a*d*f-5*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)
/d^2+1/6*b*f*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d-1/48*(-
a*d+b*c)^(1/2)*e*(3*a^2*d^2*f^2+2*a*b*d*f*(-11*c*f+19*d*e)+b^2*(15*c^2*f^2
-22*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*El
lipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c
)/e)^(1/2))/b/c^(1/2)/d^3/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2
)-1/48*a*(-a*d+b*c)^(1/2)*(3*a^2*d^2*f^2-6*a*b*d*f*(-2*c*f+5*d*e)-b^2*(15*
c^2*f^2-32*c*d*e*f+17*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2
+e)^(1/2)-1/16*a*(a^3*d^3*f^3-9*a*b^2*d*f*(-c*f+d*e)^2-3*a^2*b*d^2*f^2*(-c
*f+3*d*e)+b^3*(-c*f+d*e)^2*(5*c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*
x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-
a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^3/(-a*d+b*c)^(1/
2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{x^2d+c}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(7*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 5*sqrt(e +
 f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x + 7*sqrt(e + f*x**2)*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b*d*f*x**3 + 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 - 22*in
t((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f
*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b
*d*f*x**6),x)*a*b*c*d*f**2 + 38*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d**2*e*f + 15*int((sqr
t(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2
+ a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x
**6),x)*b**2*c**2*f**2 - 22*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2
 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 3*int((sqrt(e +
 f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d
*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),
x)*b**2*d**2*e**2 - 14*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +...
```

3.429 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 694

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{5(bde-bcf+adf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{8d^2\sqrt{c+dx^2}}$$

$$+\frac{bfx^3\sqrt{a+bx^2}\sqrt{e+fx^2}}{4d\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{e}\sqrt{de-cf}(bc(13de-15cf)-ad(8de-13cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{8cd^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}\sqrt{de-cf}(16a^2d^2f+abd(8de-33cf)-3b^2c(de-5cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-}}{\sqrt{e}\sqrt{c+}}\right.\right.}{8ad^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{3c\sqrt{e}(a^2d^2f^2+6abdf(de-cf)+b^2(d^2e^2-6cdef+5c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\right.}{8ad^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
5/8*(a*d*f-b*c*f+b*d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/(d*x^2+c)^(1
/2)+1/4*b*f*x^3*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/(d*x^2+c)^(1/2)-1/8*e^(1
/2)*(-c*f+d*e)^(1/2)*(b*c*(-15*c*f+13*d*e)-a*d*(-13*c*f+8*d*e))*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)
/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/d^3/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/8*e^(1/2)*(-c*f+d*e)^(1/2)*(16*a^2*d^2*
f+a*b*d*(-33*c*f+8*d*e)-3*b^2*c*(-5*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)
/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(
-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
f*x^2+e)^(1/2)+3/8*c*e^(1/2)*(a^2*d^2*f^2+6*a*b*d*f*(-c*f+d*e)+b^2*(5*c^2*
f^2-6*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*El
lipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d
+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

3.430 $\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 691

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}}\,dx =$$

$$-\frac{(bc-ad)(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{bfx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2d^2\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{e}\sqrt{de-cf}(bc(4de-15cf)+4ad(de+cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{6c^2d^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(3b^2c^2f(3de-5cf)-2a^2d^2f(de+2cf)+2abd(d^2e^2-7cdef+12c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{Elliptic}}{6acd^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{bc\sqrt{e}f(3bde-5bcf+3adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*(-a*d+b*c)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+
c)^(3/2)+1/2*b*f*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/(d*x^2+c)^(1/2)+1/6
*e^(1/2)*(-c*f+d*e)^(1/2)*(b*c*(-15*c*f+4*d*e)+4*a*d*(c*f+d*e))*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)
/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^3/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/6*e^(1/2)*(3*b^2*c^2*f*(-5*c*f+3*d*e)
-2*a^2*d^2*f*(2*c*f+d*e)+2*a*b*d*(12*c^2*f^2-7*c*d*e*f+d^2*e^2))*(b*x^2+a)
^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2
)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/d^4/(-c*f+d*e)^(
1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/2*b*c*e^(1/2)*f*(3*
a*d*f-5*b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*Ell
ipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+
b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

3.431 $\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 813

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx = -\frac{(bc-ad)(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5cd^2\left(c+dx^2\right)^{5/2}}$$

$$+\frac{2(bc(de-4cf)+ad(2de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d^2\left(c+dx^2\right)^{3/2}}$$

$$+\frac{\sqrt{e}(b^2c^2(2d^2e^2+10cdef-15c^2f^2)-a^2d^2(8d^2e^2-3cdef-2c^2f^2)+abcd(3d^2e^2-7cdef+10c^2f^2))\sqrt{a}}{15c^3d^3(bc-ad)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(15b^3c^4f^2+2a^3d^3f(2de+cf)+ab^2cd(d^2e^2+5cdef-30c^2f^2)-a^2bd^2(4d^2e^2+3cdef-10c^2f^2))\sqrt{}}{15ac^2d^4(bc-ad)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx}}$$

$$+\frac{b^2c\sqrt{e}f^2\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*(-a*d+b*c)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+
c)^(5/2)+2/15*(b*c*(-4*c*f+d*e)+a*d*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+
e)^(1/2)/c^2/d^2/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(b^2*c^2*(-15*c^2*f^2+10*c*d
*e*f+2*d^2*e^2)-a^2*d^2*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)+a*b*c*d*(10*c^2*f
^2-7*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*E
llipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f
+d*e))^(1/2))/c^3/d^3/(-a*d+b*c)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(f*x^2+e)^(1/2)-1/15*e^(1/2)*(15*b^3*c^4*f^2+2*a^3*d^3*f*(c*f+2*d*
e)+a*b^2*c*d*(-30*c^2*f^2+5*c*d*e*f+d^2*e^2)-a^2*b*d^2*(-10*c^2*f^2+3*c*d*
e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF(
(-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1
/2))/a/c^2/d^4/(-a*d+b*c)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(f*x^2+e)^(1/2)+b^2*c*e^(1/2)*f^2*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c
))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d
*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(d^4*x^8 + 4*c*d^3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^
4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

3.432 $\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 735

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = -\frac{(bc-ad)(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7cd^2(c+dx^2)^{7/2}}$$
$$+\frac{2(bc(de-5cf)+ad(3de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2d^2(c+dx^2)^{5/2}}$$
$$+\frac{(b^2c^2(2d^2e^2+2cdef-5c^2f^2)-a^2d^2(8d^2e^2-5cdef-2c^2f^2)+abcd(5d^2e^2-5cdef+2c^2f^2))\,x\sqrt{a+bx}}{35c^3d^2(bc-ad)(de-cf)(c+dx^2)^{3/2}}$$
$$+\frac{2\sqrt{e}(bce-2ade+acf)(b^2c^2e^2+2abce(2de-3cf)-a^2(4d^2e^2-4cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\Big(a}{35c^4(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(be-af)(b^2c^2e^2-abce(11de-9cf)+a^2(8d^2e^2-5cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\Big(\text{ar}}{35c^3(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/7*(-a*d+b*c)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+
c)^(7/2)+2/35*(b*c*(-5*c*f+d*e)+a*d*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+
e)^(1/2)/c^2/d^2/(d*x^2+c)^(5/2)+1/35*(b^2*c^2*(-5*c^2*f^2+2*c*d*e*f+2*d^2
*e^2)-a^2*d^2*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2)+a*b*c*d*(2*c^2*f^2-5*c*d*e*
f+5*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/d^2/(-a*d+b*c)/(-c*f+d
*e)/(d*x^2+c)^(3/2)+2/35*e^(1/2)*(a*c*f-2*a*d*e+b*c*e)*(b^2*c^2*e^2+2*a*b*
c*e*(-3*c*f+2*d*e)-a^2*(-c^2*f^2-4*c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*(c*
(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c
)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)^2/(-c*f+d*e)^(3
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/35*e^(1/2)*(-a*f+b*e
)*(b^2*c^2*e^2-a*b*c*e*(-9*c*f+11*d*e)+a^2*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2
))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1
/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*
d+b*c)^2/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx=\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(d^5*x^10 + 5*c*d^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4
 + 5*c^4*d*x^2 + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

3.433 $\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1218

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{11/2}}\,dx=\text{Too large to display}$$

output
```
-1/9*(-a*d+b*c)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^2/(d*x^2+
c)^(9/2)+2/63*(b*c*(-6*c*f+d*e)+a*d*(c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+
e)^(1/2)/c^2/d^2/(d*x^2+c)^(7/2)+1/315*(b^2*c^2*(-15*c^2*f^2+2*c*d*e*f+10*
d^2*e^2)-a^2*d^2*(-10*c^2*f^2-35*c*d*e*f+48*d^2*e^2)+a*b*c*d*(2*c^2*f^2-31
*c*d*e*f+35*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/d^2/(-a*d+b*c)
/(-c*f+d*e)/(d*x^2+c)^(5/2)+2/315*(a*b^2*c^2*d*(-4*c^3*f^3+14*c^2*d*e*f^2-
31*c*d^2*e^2*f+15*d^3*e^3)-a^2*b*c*d^2*(4*c^3*f^3+31*c^2*d*e*f^2-95*c*d^2*
e^2*f+54*d^3*e^3)+b^3*c^3*(5*c^3*f^3-4*c^2*d*e*f^2-4*c*d^2*e^2*f+5*d^3*e^3
)+a^3*d^3*(5*c^3*f^3+15*c^2*d*e*f^2-54*c*d^2*e^2*f+32*d^3*e^3))*x*(b*x^2+a
)^(1/2)*(f*x^2+e)^(1/2)/c^4/d^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+
1/315*e^(1/2)*(2*b^4*c^4*e^3*(-9*c*f+5*d*e)+a*b^3*c^3*e^2*(90*c^2*f^2-83*c
*d*e*f+25*d^2*e^2)-3*a^2*b^2*c^2*e*(-30*c^3*f^3+186*c^2*d*e*f^2-221*c*d^2*
e^2*f+81*d^3*e^3)+a^3*b*c*(-18*c^4*f^4-83*c^3*d*e*f^3+663*c^2*d^2*e^2*f^2-
858*c*d^3*e^3*f+328*d^4*e^4)-a^4*d*(-10*c^4*f^4-25*c^3*d*e*f^3+243*c^2*d^2
*e^2*f^2-328*c*d^3*e^3*f+128*d^4*e^4))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+
b*c)*e/a/(-c*f+d*e))^(1/2))/c^5/(-a*d+b*c)^3/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/315*e^(1/2)*(-a*f+b*e)*(b^3*c^3*e^2*
(-9*c*f+5*d*e)-3*a*b^2*c^2*e*(27*c^2*f^2-66*c*d*e*f+35*d^2*e^2)-2*a^3*d*(5
*c^3*f^3+15*c^2*d*e*f^2-54*c*d^2*e^2*f+32*d^3*e^3)+3*a^2*b*c*(6*c^3*f^3...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="f
ricas")
```

output
```
integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(d^6*x^12 + 6*c*d^5*x^10 + 15*c^2*d^4*x^8 + 20*c^3*d^3*x^
6 + 15*c^4*d^2*x^4 + 6*c^5*d*x^2 + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="m
axima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="g
iac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{11/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

3.434 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}} dx$

Optimal result

Integrand size = 34, antiderivative size = 1171

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}} dx = \text{Too large to display}$$

output
```
1/384*(15*a^3*d^3*f^3+a^2*b*d^2*f^2*(-191*c*f+337*d*e)+a*b^2*d*f*(265*c^2*
f^2-394*c*d*e*f+57*d^2*e^2)-b^3*(105*c^3*f^3-145*c^2*d*e*f^2+15*c*d^2*e^2*
f+9*d^3*e^3))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^4/f^2/(b*x^2+a)^(1/2)+1/
192*(59*a^2*d^2*f^2+2*a*b*d*f*(-43*c*f+61*d*e)+b^2*(35*c^2*f^2-46*c*d*e*f+
3*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^3/f+1/48*b
*(17*a*d*f-7*b*c*f+9*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/d^2+1/8*b^2*f*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d-
1/384*(-a*d+b*c)^(1/2)*e*(15*a^3*d^3*f^3+a^2*b*d^2*f^2*(-191*c*f+337*d*e)+
a*b^2*d*f*(265*c^2*f^2-394*c*d*e*f+57*d^2*e^2)-b^3*(105*c^3*f^3-145*c^2*d*
e*f^2+15*c*d^2*e^2*f+9*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))
^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/
(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^4/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*
x^2+e)^(1/2)-1/384*a*(-a*d+b*c)^(1/2)*(15*a^3*d^3*f^3-15*a^2*b*d^2*f^2*(-5
*c*f+13*d*e)-a*b^2*d*f*(195*c^2*f^2-418*c*d*e*f+223*d^2*e^2)+b^3*(105*c^3*
f^3-215*c^2*d*e*f^2+107*c*d^2*e^2*f+3*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+
e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2)
,(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^4/f/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)/(f*x^2+e)^(1/2)-1/128*a*(5*a^4*d^4*f^4-90*a^2*b^2*d^2*f^2*(-c*
f+d*e)^2-20*a^3*b*d^3*f^3*(-c*f+3*d*e)+20*a*b^3*d*f*(-c*f+d*e)^2*(5*c*f+d*
e)-b^4*(-c*f+d*e)^2*(35*c^2*f^2+10*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{x^2d+c}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{d\,x^2+c}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

3.435 $\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 934

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{\left(33a^2d^2f^2+68abdf(de-cf)+b^2\left(3d^2e^2-38cdef+35c^2f^2\right)\right)x\sqrt{a+bx}}{48d^3f\sqrt{c+dx^2}}$$

$$+\frac{b(7bde-7bcf+13adf)x^3\sqrt{a+bx^2}\sqrt{e+fx^2}}{24d^2\sqrt{c+dx^2}}+\frac{b^2fx^5\sqrt{a+bx^2}\sqrt{e+fx^2}}{6d\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{e}\sqrt{de-cf}\left(2abcdf(82de-95cf)-3a^2d^2f(16de-27cf)+b^2c\left(3d^2e^2-100cdef+105c^2f^2\right)\right)\sqrt{a+bx}}{48cd^4f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}\sqrt{de-cf}\left(96a^3d^3f^2+a^2bd^2f(96de-325cf)-110ab^2cdf(de-3cf)+3b^3c\left(d^2e^2+10cdef-35c^2f^2\right)\right.}{48ad^5f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{c\sqrt{e}\left(5a^3d^3f^3+45a^2bd^2f^2(de-cf)+15ab^2df\left(d^2e^2-6cdef+5c^2f^2\right)-b^3\left(d^3e^3+9cd^2e^2f-45c^2def^2+\right.\right.}{16ad^5f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+}}$$

output
```
1/48*(33*a^2*d^2*f^2+68*a*b*d*f*(-c*f+d*e)+b^2*(35*c^2*f^2-38*c*d*e*f+3*d^
2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^3/f/(d*x^2+c)^(1/2)+1/24*b*(13
*a*d*f-7*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/(d*x^2+c)^
(1/2)+1/6*b^2*f*x^5*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/(d*x^2+c)^(1/2)-1/48
*e^(1/2)*(-c*f+d*e)^(1/2)*(2*a*b*c*d*f*(-95*c*f+82*d*e)-3*a^2*d^2*f*(-27*c
*f+16*d*e)+b^2*c*(105*c^2*f^2-100*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(
f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)
^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/d^4/f/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(f*x^2+e)^(1/2)+1/48*e^(1/2)*(-c*f+d*e)^(1/2)*(96*a^3*d^3*f^2+a^2
*b*d^2*f*(-325*c*f+96*d*e)-110*a*b^2*c*d*f*(-3*c*f+d*e)+3*b^3*c*(-35*c^2*f
^2+10*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*El
lipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+
d*e))^(1/2))/a/d^5/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/16*
c*e^(1/2)*(5*a^3*d^3*f^3+45*a^2*b*d^2*f^2*(-c*f+d*e)+15*a*b^2*d*f*(5*c^2*f
^2-6*c*d*e*f+d^2*e^2)-b^3*(35*c^3*f^3-45*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3
))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(
1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))
^(1/2))/a/d^5/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)
^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2}d+c\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*f*x^6 + (b^2*e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*x^4 + 2*c*d*x^2 +
c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

3.436 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 862

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3cd^3(c+dx^2)^{3/2}}$$
$$+\frac{b(5bde-9bcf+9adf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{8d^3\sqrt{c+dx^2}}+\frac{b^2fx^3\sqrt{a+bx^2}\sqrt{e+fx^2}}{4d^2\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{e}\sqrt{de-cf}(abcd(24de-115cf)-5b^2c^2(11de-21cf)+16a^2d^2(de+cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\right.}{24c^2d^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{e}(8a^3d^3f(de+2cf)-8a^2bd^2(d^2e^2-12cdef+20c^2f^2)-3b^3c^2(3d^2e^2-30cdef+35c^2f^2)+ab^2cd(32d}{24acd^5\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{bc\sqrt{e}(15a^2d^2f^2+10abdf(3de-5cf)+b^2(3d^2e^2-30cdef+35c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticPi}\left(\frac{de}{de-}\right.}{8ad^5\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^3/(d*x^2
+c)^(3/2)+1/8*b*(9*a*d*f-9*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2
)/d^3/(d*x^2+c)^(1/2)+1/4*b^2*f*x^3*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d^2/(d
*x^2+c)^(1/2)+1/24*e^(1/2)*(-c*f+d*e)^(1/2)*(a*b*c*d*(-115*c*f+24*d*e)-5*b
^2*c^2*(-21*c*f+11*d*e)+16*a^2*d^2*(c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)
/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(
-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^4/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(f*x^2+e)^(1/2)+1/24*e^(1/2)*(8*a^3*d^3*f*(2*c*f+d*e)-8*a^2*b*d^2*(20*c^2
*f^2-12*c*d*e*f+d^2*e^2)-3*b^3*c^2*(35*c^2*f^2-30*c*d*e*f+3*d^2*e^2)+a*b^2
*c*d*(255*c^2*f^2-215*c*d*e*f+32*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/
(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/d^5/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d
*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/8*b*c*e^(1/2)*(15*a^2*d^2*f^2+10*a*b*d*f*
(-5*c*f+3*d*e)+b^2*(35*c^2*f^2-30*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(
f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c
)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^5/(-c*f+d*e
)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$$

$$\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2} (e+fx^2)^{3/2}}{(c+dx^2)^{5/2}} dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}} (f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{5}{2}}} dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2} (e+fx^2)^{3/2}}{(c+dx^2)^{5/2}} dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*f*x^6 + (b^2*e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*x^6 + 3*c*d^2*x^4
+ 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

3.437 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 877

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \frac{(bc-ad)^2(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5cd^3(c+dx^2)^{5/2}}$$

$$-\frac{(bc-ad)(bc(7de-13cf)+2ad(2de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d^3(c+dx^2)^{3/2}}$$

$$+\frac{b^2fx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2d^3\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{e}(2abcd(7d^2e^2+7cdef-20c^2f^2)+2a^2d^2(8d^2e^2-3cdef-2c^2f^2)+b^2c^2(16d^2e^2-115cdef+105c^2f}{30c^3d^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(15b^3c^3f(3de-7cf)-4a^3d^3f(2de+cf)+4a^2bd^2(2d^2e^2-cdef-10c^2f^2)+4ab^2cd(2d^2e^2-20cdef}{30ac^2d^5\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+}}$$

$$+\frac{b^2c\sqrt{e}f(3bde-7bcf+5adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^5\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*(-a*d+b*c)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^3/(d*x^2
+c)^(5/2)-1/15*(-a*d+b*c)*(b*c*(-13*c*f+7*d*e)+2*a*d*(c*f+2*d*e))*x*(b*x^2
+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/d^3/(d*x^2+c)^(3/2)+1/2*b^2*f*x*(b*x^2+a)^(1
/2)*(f*x^2+e)^(1/2)/d^3/(d*x^2+c)^(1/2)+1/30*e^(1/2)*(2*a*b*c*d*(-20*c^2*f
^2+7*c*d*e*f+7*d^2*e^2)+2*a^2*d^2*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)+b^2*c^2
*(105*c^2*f^2-115*c*d*e*f+16*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+
b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/d^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(f*x^2+e)^(1/2)-1/30*e^(1/2)*(15*b^3*c^3*f*(-7*c*f+3*d*e)-4*a^3
*d^3*f*(c*f+2*d*e)+4*a^2*b*d^2*(-10*c^2*f^2-c*d*e*f+2*d^2*e^2)+4*a*b^2*c*d
*(45*c^2*f^2-20*c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+
c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c
)*e/a/(-c*f+d*e))^(1/2))/a/c^2/d^5/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(f*x^2+e)^(1/2)+1/2*b^2*c*e^(1/2)*f*(5*a*d*f-7*b*c*f+3*b*d*e)*(b
*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*
x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2
))/a/d^5/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$$

$$\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2} (e+fx^2)^{3/2}}{(c+dx^2)^{7/2}} \, dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{7/2}} \, dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2} (e+fx^2)^{3/2}}{(c+dx^2)^{7/2}} \, dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{7}{2}}} dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

3.438 $\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1161

$$\int \frac{(a+bx^2)^{5/2}\,(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

output
```
1/7*(-a*d+b*c)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^3/(d*x^2
+c)^(7/2)-1/35*(-a*d+b*c)*(b*c*(-17*c*f+9*d*e)+2*a*d*(c*f+3*d*e))*x*(b*x^2
+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/d^3/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-20*c^2*
f^2+c*d*e*f+13*d^2*e^2)+3*a^2*d^2*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2)+b^2*c^2
*(71*c^2*f^2-76*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/
d^3/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(a*b^2*c^2*d*(-70*c^3*f^3+119
*c^2*d*e*f^2-40*c*d^2*e^2*f+9*d^3*e^3)+a^2*b*c*d^2*(-14*c^3*f^3+23*c^2*d*e
*f^2-43*c*d^2*e^2*f+16*d^3*e^3)-6*a^3*d^3*(c^3*f^3+2*c^2*d*e*f^2-12*c*d^2*
e^2*f+8*d^3*e^3)+b^3*c^3*(105*c^3*f^3-175*c^2*d*e*f^2+56*c*d^2*e^2*f+8*d^3
*e^3))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e
)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/
d^4/(-a*d+b*c)/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^
(1/2)-1/105*e^(1/2)*(105*b^4*c^5*f^2*(-c*f+d*e)+3*a^4*d^4*f*(-2*c^2*f^2-5*
c*d*e*f+8*d^2*e^2)+a^2*b^2*c*d^2*(-70*c^3*f^3+91*c^2*d*e*f^2-17*c*d^2*e^2*
f+5*d^3*e^3)-a^3*b*d^3*(14*c^3*f^3-19*c^2*d*e*f^2-10*c*d^2*e^2*f+24*d^3*e^
3)+a*b^3*c^2*d*(210*c^3*f^3-245*c^2*d*e*f^2+28*c*d^2*e^2*f+4*d^3*e^3))*(b*
x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/
e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c^3/d^5/(-a*
d+b*c)/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+b^
3*c*e^(1/2)*f^2*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*Ellipti...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{\left(a+bx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(x^{2} d+c\right)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

3.439 $\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}} dx$

Optimal result

Integrand size = 34, antiderivative size = 1134

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}} dx = \text{Too large to display}$$

output
```
1/9*(-a*d+b*c)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d^3/(d*x^2
+c)^(9/2)-1/63*(-a*d+b*c)*(b*c*(-21*c*f+11*d*e)+2*a*d*(c*f+4*d*e))*x*(b*x^
2+a)^(1/2)*(f*x^2+e)^(1/2)/c^2/d^3/(d*x^2+c)^(7/2)+1/315*(a*b*c*d*(-20*c^2
*f^2-5*c*d*e*f+19*d^2*e^2)+a^2*d^2*(-10*c^2*f^2-35*c*d*e*f+48*d^2*e^2)+b^2
*c^2*(105*c^2*f^2-110*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2
)/c^3/d^3/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/315*(a^2*b*c*d^2*(-10*c^3*f^3+35*c^
2*d*e*f^2-73*c*d^2*e^2*f+36*d^3*e^3)+a*b^2*c^2*d*(-10*c^3*f^3+35*c^2*d*e*f
^2-28*c*d^2*e^2*f+15*d^3*e^3)-2*a^3*d^3*(5*c^3*f^3+15*c^2*d*e*f^2-54*c*d^2
*e^2*f+32*d^3*e^3)+b^3*c^3*(35*c^3*f^3-55*c^2*d*e*f^2+8*c*d^2*e^2*f+8*d^3*
e^3))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^4/d^3/(-a*d+b*c)/(-c*f+d*e)^2/(d
*x^2+c)^(3/2)+1/315*e^(1/2)*(8*b^4*c^4*e^4+a*b^3*c^3*e^3*(-43*c*f+11*d*e)+
3*a^2*b^2*c^2*e^2*(36*c^2*f^2-29*c*d*e*f+9*d^2*e^2)-a^3*b*c*e*(-65*c^3*f^3
+411*c^2*d*e*f^2-498*c*d^2*e^2*f+184*d^3*e^3)+a^4*(-10*c^4*f^4-25*c^3*d*e*
f^3+243*c^2*d^2*e^2*f^2-328*c*d^3*e^3*f+128*d^4*e^4))*(b*x^2+a)^(1/2)*(c*(
f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)
^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^5/(-a*d+b*c)^2/(-c*f+d*e)^(5/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-2/315*e^(1/2)*(-a*f+b*e
)*(2*b^3*c^3*e^3+3*a*b^2*c^2*e^2*(-3*c*f+d*e)-6*a^2*b*c*e*(5*c^2*f^2-13*c*
d*e*f+7*d^2*e^2)+a^3*(5*c^3*f^3+15*c^2*d*e*f^2-54*c*d^2*e^2*f+32*d^3*e^3))
*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}} \, dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}{(x^2d+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="f
ricas")
```

output
```
integral((b^2*f*x^6 + (b^2*e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^6*x^12 + 6*c*d^5*x^1
0 + 15*c^2*d^4*x^8 + 20*c^3*d^3*x^6 + 15*c^4*d^2*x^4 + 6*c^5*d*x^2 + c^6),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(f*x**2+e)**(3/2)/(d*x**2+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="m
axima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x, algorithm="g
iac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{11/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(e+fx^2)^{3/2}}{(c+dx^2)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(f*x^2+e)^(3/2)/(d*x^2+c)^(11/2),x)
```

3.440 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 858

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{\left(38ce-\frac{22ade}{b}+\frac{3de^2}{f}-\frac{22acf}{b}+\frac{3c^2f}{d}+\frac{15a^2df}{b^2}\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48\sqrt{a+bx^2}}$$

$$-\frac{(5adf-7b(de+cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24b^2}+\frac{dfx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{6b}$$

$$-\frac{\sqrt{bc-ad}e(15a^2d^2f^2-22abdf(de+cf)+b^2(3d^2e^2+38cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\cdots}{\sqrt{\cdots}}\right.\right.}{48b^3\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{bc-ad}(48b^3cde^2-15a^3d^2f^2+6a^2bdf(7de+2cf)-ab^2(31d^2e^2+44cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\cdots}{48b^4\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{a(bde+bcf-adf)(5a^2d^2f^2-4abdf(de+cf)-b^2(d^2e^2-10cdef+c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticP}\cdots}{16b^4\sqrt{c}d\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(38*c*e-22*a*d*e/b+3*d*e^2/f-22*a*c*f/b+3*c^2*f/d+15*a^2*d*f/b^2)*x*(
d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)-1/24*(5*a*d*f-7*b*(c*f+d*e)
)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2+1/6*d*f*x^3*(b*x^2
+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/48*(-a*d+b*c)^(1/2)*e*(15*a^
2*d^2*f^2-22*a*b*d*f*(c*f+d*e)+b^2*(3*c^2*f^2+38*c*d*e*f+3*d^2*e^2))*(d*x^
2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d/f/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/48*(-a*d+b*c)^(1/2)*(48*b
^3*c*d*e^2-15*a^3*d^2*f^2+6*a^2*b*d*f*(2*c*f+7*d*e)-a*b^2*(-3*c^2*f^2+44*c
*d*e*f+31*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Ellipt
icF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)
^(1/2))/b^4/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/16
*a*(-a*d*f+b*c*f+b*d*e)*(5*a^2*d^2*f^2-4*a*b*d*f*(c*f+d*e)-b^2*(c^2*f^2-10
*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Ellipti
cPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e
)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2} (e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{3/2} (e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2],x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2} (e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \text{too large to display}$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

output
```
( - 5*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + 7*sqrt(
e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x + 7*sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 15*int((sqrt(e + f*x**2)*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 - 2
2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4
 + b*d*f*x**6),x)*a*b*c*d*f**2 - 22*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d**2*e*f + 3*int((
sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x*
*2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*
f*x**6),x)*b**2*c**2*f**2 + 38*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 3*int((sqrt(
e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 +
a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**
6),x)*b**2*d**2*e**2 + 10*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**...
```

3.441 $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 680

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{(5bde+bcf-3adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bd\sqrt{a+bx^2}}$$
$$+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4b}$$
$$-\frac{\sqrt{bc-ad}\,e(5bde+bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{bc-ad}(8b^2de^2+3a^2df^2-abf(9de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^3\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(3a^2d^2f^2-2abdf(3de+cf)+b^2(3d^2e^2+6cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b^3\sqrt{c}d\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(-3*a*d*f+b*c*f+5*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/(b*x^2+
a)^(1/2)+1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/8*(-a
*d+b*c)^(1/2)*e*(-3*a*d*f+b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b
*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a
*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(f*x^2+e)^(1/2)+1/8*(-a*d+b*c)^(1/2)*(8*b^2*d*e^2+3*a^2*d*f^2-a*b*f*(-c*f
+9*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3
/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^
2*f^2-2*a*b*d*f*(c*f+3*d*e)+b^2*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^
(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2
)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^
(1/2)/d/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2],x]
```

output
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}dx$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/Sqrt[a + b*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx=\int \frac{\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx-3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2}\right.}{}$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x - 3*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*d*f**2 + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 +
 b*d*e*x**4 + b*d*f*x**6),x)*b*c*f**2 + 5*int((sqrt(e + f*x**2)*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**
4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e*f - 2*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*c*f**2 - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*e*f + 6*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e
*f + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*
e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e
*x**4 + b*d*f*x**6),x)*b*d*e**2 - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**
2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*e*f + 4*int((sqrt(e + ...
```

3.442 $\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 541

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd\sqrt{e+fx^2}} - \frac{cf\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bd\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}b\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{e(3bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bd\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/2*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-1/2*c*f*(a*f
-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(1/2)/b/d/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(a*f-b*e)
^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)
^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/b/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(-a*d*f-b*c*f+3*
b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(1/2)/b/d/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right) f + \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right) e$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x
**2 + b*c*x**2 + b*d*x**4),x)*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*e
```

3.443 $\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 504

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{a(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{cd(bc-ad)\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{\sqrt{de-cf}\,(bde+bcf-2adf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{d^2(bc-ad)\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{cf^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{d^2\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
-a*(-c*f+d*e)^(3/2)*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*Ellipt
icE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e)
)^(1/2))/c/d/(-a*d+b*c)/e^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^
(1/2)+(-c*f+d*e)^(1/2)*(-2*a*d*f+b*c*f+b*d*e)*(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)*(f*x^2+e)^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(
-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/d^2/(-a*d+b*c)/e^(1/2)/(b*x^2+a)^(1/2)/
(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+c*f^2*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x
^2+e)^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*
f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/d^2/e^(1/2)/(-c*f+d*e)^(1/2)/(b
*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\;(x^2d+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)f + \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)e$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 + 2*
a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*f
+ int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c
*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*e
```

3.444 $\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 427

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(bc-ad)\,(c+dx^2)^{3/2}} - \frac{2\sqrt{e}\sqrt{de-cf}(2bce-ade-acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{e}(be-af)(3bce-ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3ac(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(
3/2)-2/3*e^(1/2)*(-c*f+d*e)^(1/2)*(-a*c*f-a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(
c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2
+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)^2/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*e^(1/2)*(-a*f+b*e)*(-2*a*c*f-a*d*
e+3*b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f
+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/
a/c/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e
)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\sqrt{b\,x^{2}+a}\;\left(x^{2} d+c\right)^{\frac{5}{2}}}d x$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx=\int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d^3*x^8 + (3
*b*c*d^2 + a*d^3)*x^6 + 3*(b*c^2*d + a*c*d^2)*x^4 + a*c^3 + (b*c^3 + 3*a*c
^2*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx=\int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}\,dx=\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\;(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2),x)
```

3.445 $\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 583

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5c(bc-ad)\,(c+dx^2)^{5/2}}$$
$$-\frac{2(bc(4de-cf)-ad(2de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2(bc-ad)^2\,(c+dx^2)^{3/2}}$$
$$-\frac{\sqrt{e}(b^2c^2e(23de-20cf)-abc(23d^2e^2-7cdef-10c^2f^2)+a^2d(8d^2e^2-3cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+\cdots}{e(c+\cdots}}}{15c^3(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{e}(be-af)\,(15b^2c^2e+2a^2d(2de+cf)-abc(11de+10cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-\cdots}}{\sqrt{e}\sqrt{c\cdots}}\right.\right.}{15ac^2(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(
5/2)-2/15*(b*c*(-c*f+4*d*e)-a*d*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(
1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/15*e^(1/2)*(b^2*c^2*e*(-20*c*f+23*
d*e)-a*b*c*(-10*c^2*f^2-7*c*d*e*f+23*d^2*e^2)+a^2*d*(-2*c^2*f^2-3*c*d*e*f+
8*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*
f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))
/c^3/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+
e)^(1/2)+1/15*e^(1/2)*(-a*f+b*e)*(15*b^2*c^2*e+2*a^2*d*(c*f+2*d*e)-a*b*c*(
10*c*f+11*d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF(
(-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1
/2))/a/c^2/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(f \,x^{2}+e \right)^{\frac{3}{2}}}{\sqrt{b \,x^{2}+a}\, \left(x^{2} d +c \right)^{\frac{7}{2}}}d x$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d^4*x^10 + (
4*b*c*d^3 + a*d^4)*x^8 + 2*(3*b*c^2*d^2 + 2*a*c*d^3)*x^6 + a*c^4 + 2*(2*b*
c^3*d + 3*a*c^2*d^2)*x^4 + (b*c^4 + 4*a*c^3*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(7/2),x)
```

3.446 $\int \frac{\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 901

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}\left(c+dx^2\right)^{9/2}}\,dx=-\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7c(bc-ad)\left(c+dx^2\right)^{7/2}}$$

$$-\frac{2(2bc(3de-cf)-ad(3de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2(bc-ad)^2\left(c+dx^2\right)^{5/2}}$$

$$+\frac{(abcd(71d^2e^2-43cdef-22c^2f^2)-3a^2d^2(8d^2e^2-5cdef-2c^2f^2)-b^2c^2(71d^2e^2-76cdef+8c^2f^2))\,x\sqrt{}}{105c^3(bc-ad)^3(de-cf)\left(c+dx^2\right)^{3/2}}$$

$$-\frac{2\sqrt{e}(b^3c^3e(88d^2e^2-161cdef+70c^2f^2)-3a^3d^2(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+a^2bcd(92d^3e^3-}{105c^4(}$$

$$+\frac{\sqrt{e}(be-af)\,(105b^3c^3e(de-cf)-ab^2c^2(122d^2e^2-49cdef-70c^2f^2)+a^2bcd(89d^2e^2-55cdef-28c^2f}{105ac^3(bc-ad)^4(de-cf)^{3/}}$$

output
```
-1/7*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(
7/2)-2/35*(2*b*c*(-c*f+3*d*e)-a*d*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)
^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-22*c^2*f^2-43*c*d
*e*f+71*d^2*e^2)-3*a^2*d^2*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2)-b^2*c^2*(8*c^2
*f^2-76*c*d*e*f+71*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/(-a*d+b
*c)^3/(-c*f+d*e)/(d*x^2+c)^(3/2)-2/105*e^(1/2)*(b^3*c^3*e*(70*c^2*f^2-161*
c*d*e*f+88*d^2*e^2)-3*a^3*d^2*(c^3*f^3+2*c^2*d*e*f^2-12*c*d^2*e^2*f+8*d^3*
e^3)+a^2*b*c*d*(14*c^3*f^3+22*c^2*d*e*f^2-137*c*d^2*e^2*f+92*d^3*e^3)-a*b^
2*c^2*(35*c^3*f^3+14*c^2*d*e*f^2-190*c*d^2*e^2*f+132*d^3*e^3))*(b*x^2+a)^(
1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/
(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)^4/(-c*f
+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/105*e^(1/2)*
(-a*f+b*e)*(105*b^3*c^3*e*(-c*f+d*e)-a*b^2*c^2*(-70*c^2*f^2-49*c*d*e*f+122
*d^2*e^2)+a^2*b*c*d*(-28*c^2*f^2-55*c*d*e*f+89*d^2*e^2)-3*a^3*d^2*(-2*c^2*
f^2-5*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*
EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*
f+d*e))^(1/2))/a/c^3/(-a*d+b*c)^4/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx=\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(9/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(9/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(9/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\,\left(x^2d+c\right)^{\frac{9}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d^5*x^12 + (
5*b*c*d^4 + a*d^5)*x^10 + 5*(2*b*c^2*d^3 + a*c*d^4)*x^8 + 10*(b*c^3*d^2 +
a*c^2*d^3)*x^6 + a*c^5 + 5*(b*c^4*d + 2*a*c^3*d^2)*x^4 + (b*c^5 + 5*a*c^4*
d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(9/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{9/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(9/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(9/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\,(c+dx^2)^{9/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(9/2),x)
```

3.447 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 686

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx =$$

$$-\frac{5(adf-b(de+cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8b^2\sqrt{a+bx^2}}+\frac{dfx^3\sqrt{c+dx^2}\sqrt{e+fx^2}}{4b\sqrt{a+bx^2}}$$

$$+\frac{\sqrt{bc-ad}\,e(8b^2ce+15a^2df-13ab(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8ab^3\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{bc-ad}(15a^2df^2+8b^2e(2de+cf)-3abf(11de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right.\right.}{8b^4\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{3a(5a^2d^2f^2-6abdf(de+cf)+b^2(d^2e^2+6cdef+c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b^4\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-5/8*(a*d*f-b*(c*f+d*e))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/(b*x^2+a)^(
1/2)+1/4*d*f*x^3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/(b*x^2+a)^(1/2)+1/8*(-a
*d+b*c)^(1/2)*e*(8*b^2*c*e+15*a^2*d*f-13*a*b*(c*f+d*e))*(d*x^2+c)^(1/2)*(a
*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+
a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/b^3/c^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*(-a*d+b*c)^(1/2)*(15*a^2*d*f^2+8*b^2*e
*(c*f+2*d*e)-3*a*b*f*(c*f+11*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a
))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e
)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+
e)^(1/2)+3/8*a*(5*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+6*c*d*e*f+d
^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*
c)/e)^(1/2))/b^4/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx=\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{{\left(dx^{2} + c\right)}^{\frac{3}{2}} {\left(fx^{2} + e\right)}^{\frac{3}{2}}}{{\left(bx^{2} + a\right)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^{3/2}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

3.448 $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 575

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2b\sqrt{a+bx^2}}$$
$$+ \frac{\sqrt{bc-ad}\,e(2be-3af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2ab^2\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{\left(3a^2df^2+2b^2e(de+cf)-abf(6de+cf)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{af(3bde+bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/(b*x^2+a)^(1/2)+1/2*(-a*d+b*c)^(
1/2)*e*(-3*a*f+2*b*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Elli
pticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/
e)^(1/2))/a/b^2/c^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/
2*(3*a^2*d*f^2+2*b^2*e*(c*f+d*e)-a*b*f*(c*f+6*d*e))*(d*x^2+c)^(1/2)*(a*(f*
x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(
1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*f*(-3*a*d*f+b*c*f+3*b*d*e
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(
1/2))/b^3/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+
e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}dx$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{x^{2} d +c}\, \left(f \,x^{2}+e \right)^{\frac{3}{2}}}{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx=\int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^2*x^4 + 2*a*
b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(3/2),x)
```

3.449 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 488

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{ab\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$
$$+\frac{\sqrt{c}f(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^2\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$
$$+\frac{a\sqrt{c}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^2\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

output
```
c^(1/2)*(-a*f+b*e)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*Ellipti
cE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^
(1/2))/a/b/(-a*d+b*c)^(1/2)/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2
)+c^(1/2)*f*(-a*f+b*e)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*Ell
ipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)
/e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a)
)^(1/2)+a*c^(1/2)*f^2*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*Elli
pticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+
b*e)/(-a*d+b*c)/e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(a*(f*x^2
+e)/e/(b*x^2+a))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)f + \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)e$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c + a*
*2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*f
+ int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*
d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*e
```

3.450 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 401

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{(be-af)x\sqrt{e+fx^2}}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{e}\sqrt{de-cf}(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ac(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{2\sqrt{e}(be-af)\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
(-a*f+b*e)*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+
e^(1/2)*(-c*f+d*e)^(1/2)*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+
e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2)
,(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(f*x^2+e)^(1/2)-2*e^(1/2)*(-a*f+b*e)*(-c*f+d*e)^(1/2)*(b*x^2+a
)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/
2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/(-a*d+b*c)^2/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^{2}+e)^{\frac{3}{2}}}{(b\,x^{2}+a)^{\frac{3}{2}}\,(x^{2}d+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^2*d^2*x^8 +
2*(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

3.451 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 530

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}+\frac{b(3bce+ade-4acf)x\sqrt{e+fx^2}}{3ac(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{e}\sqrt{de-cf}(3b^2c^2e+abc(7de-6cf)-2a^2d(de+cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-}{a(de-}\right.}{3ac^2(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{e}(be-af)(3bc(3de-2cf)-ad(de+2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)}{a(de-c}\right.}{3ac(bc-ad)^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*(-c*f+d*e)*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(
3/2)+1/3*b*(-4*a*c*f+a*d*e+3*b*c*e)*x*(f*x^2+e)^(1/2)/a/c/(-a*d+b*c)^2/(b*
x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*e^(1/2)*(-c*f+d*e)^(1/2)*(3*b^2*c^2*e+a*b
*c*(-6*c*f+7*d*e)-2*a^2*d*(c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/a/c^2/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(-a*f+b*e)*(3*b*c*(-2*c*f+3*d*e)-a*d*(2*c*f
+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e
)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/
(-a*d+b*c)^3/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}\,(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^2*d^3*x^10 +
 (3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6
 + a^2*c^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + (2*a*b*c^3 + 3*a^
2*c^2*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2),x)
```

3.452 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 718

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^{5/2}}$$

$$+\frac{b(5bce+ade-6acf)x\sqrt{e+fx^2}}{5ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}$$

$$+\frac{d(15b^2c^2e+abc(13de-22cf)-2a^2d(2de+cf))\,x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15ac^2(bc-ad)^3\,(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(15b^3c^3e(de-cf)-a^2bcd(33d^2e^2-7cdef-20c^2f^2)+a^3d^2(8d^2e^2-3cdef-2c^2f^2)+ab^2c^2(58d^2e^2}{15ac^3(bc-ad)^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{}}$$

$$-\frac{2\sqrt{e}(be-af)\,(15b^2c^2(2de-cf)+a^2d^2(2de+cf)-2abcd(4de+5cf))\,\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticF}\left(\mathrm{a}\right.}{15ac^2(bc-ad)^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*(-c*f+d*e)*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(
5/2)+1/5*b*(-6*a*c*f+a*d*e+5*b*c*e)*x*(f*x^2+e)^(1/2)/a/c/(-a*d+b*c)^2/(b*
x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/15*d*(15*b^2*c^2*e+a*b*c*(-22*c*f+13*d*e)-2
*a^2*d*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/a/c^2/(-a*d+b*c)^3/(
d*x^2+c)^(3/2)+1/15*e^(1/2)*(15*b^3*c^3*e*(-c*f+d*e)-a^2*b*c*d*(-20*c^2*f^
2-7*c*d*e*f+33*d^2*e^2)+a^3*d^2*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)+a*b^2*c^2
*(30*c^2*f^2-85*c*d*e*f+58*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/a/c^3/(-a*d+b*c)^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-2/15*e^(1/2)*(-a*f+b*e)*(15*b^2*c^2*(-
c*f+2*d*e)+a^2*d^2*(c*f+2*d*e)-2*a*b*c*d*(5*c*f+4*d*e))*(b*x^2+a)^(1/2)*(c
*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+
c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c^2/(-a*d+b*c)^4/(-c*f+d*e)
^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(7/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(7/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}(x^2d+c)^{\frac{7}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^2*d^4*x^12 +
 2*(2*b^2*c*d^3 + a*b*d^4)*x^10 + (6*b^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*
x^8 + 4*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^3)*x^6 + a^2*c^4 + (b^2*c^4 +
 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^4 + 2*(a*b*c^4 + 2*a^2*c^3*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(3/2)/(d*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}\,(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{7/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(7/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{3/2}(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(3/2)/(d*x^2+c)^(7/2),x)
```

3.453 $\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 683

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{(bc-ad)(be-af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ab^2\left(a+bx^2\right)^{3/2}}$$

$$+\frac{dfx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2b^2\sqrt{a+bx^2}}$$

$$+\frac{\sqrt{bc-ad}e(4b^2ce-15a^2df+4ab(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{6a^2b^3\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{(15a^3d^2f^2+2b^3ce(de-cf)-3a^2bdf(8de+3cf)+2ab^2de(2de+7cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\text{ar}\right.}{6ab^4\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{adf(5adf-3b(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^4\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)*(-a*f+b*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/b^2/(b*x^2+a
)^(3/2)+1/2*d*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/(b*x^2+a)^(1/2)+1/6*
(-a*d+b*c)^(1/2)*e*(4*b^2*c*e-15*a^2*d*f+4*a*b*(c*f+d*e))*(d*x^2+c)^(1/2)*
(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^
2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/b^3/c^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/6*(15*a^3*d^2*f^2+2*b^3*c*e*(-c*f+d*
e)-3*a^2*b*d*f*(3*c*f+8*d*e)+2*a*b^2*d*e*(7*c*f+2*d*e))*(d*x^2+c)^(1/2)*(a
*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+
a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/b^4/c^(1/2)/(-a*d+b*c)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*a*d*f*(5*a*d*f-3*b*(c
*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*
c)/e)^(1/2))/b^4/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

input
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x]
```

output
```
Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}dx$$

input
```
Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(x^2 d+c\right)^{\frac{3}{2}}\left(f\,x^2+e\right)^{\frac{3}{2}}}{\left(b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

3.454 $\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 585

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{(be-af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ab\,(a+bx^2)^{3/2}}$$
$$+\frac{e(2b^2ce-3a^2df-ab(de-2cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2b^2\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{(3abdef-3a^2df^2+b^2e(de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3ab^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{adf^2\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b^3\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*f+b*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/b/(b*x^2+a)^(3/2)+1/3*e
*(2*b^2*c*e-3*a^2*d*f-a*b*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*
x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*
f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/b^2/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/3*(3*a*b*d*e*f-3*a^2*d*f^2+b^2*e*(-c*
f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*
c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/b^
3/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
+a*d*f^2*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*
c)/e)^(1/2))/b^3/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e \right)^{\frac{3}{2}}}{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a)^(5/2),x)
```

3.455 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 386

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{(be-af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3a(bc-ad)(a+bx^2)^{3/2}} + \frac{2e(bce-2ade+acf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2\sqrt{c}(bc-ad)^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{e(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3a\sqrt{c}(bc-ad)^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*f+b*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3
/2)+2/3*e*(a*c*f-2*a*d*e+b*c*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^(3/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)/(f*x^2+e)^(1/2)+1/3*e*(-c*f+d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*
x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*
f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(1/2)/(-a*d+b*c)^(3/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^3*d*x^8 + (b
^3*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a
^3*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

3.456 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 528

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(be-af)x\sqrt{e+fx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$
$$+\frac{2(b^2ce+a^2df-ab(3de-cf))\,x\sqrt{e+fx^2}}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{e}\sqrt{de-cf}(2b^2c^2e-3a^2d(de-2cf)-abc(7de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-}{a(de}\right.}{3a^2c(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{de-cf}(b^2ce-9abde+2abcf+6a^2df)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)}{a(de-cf}\right.}{3a^2(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*f+b*e)*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1
/2)+2/3*(b^2*c*e+a^2*d*f-a*b*(-c*f+3*d*e))*x*(f*x^2+e)^(1/2)/a^2/(-a*d+b*c
)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*e^(1/2)*(-c*f+d*e)^(1/2)*(2*b^2*c^
2*e-3*a^2*d*(-2*c*f+d*e)-a*b*c*(-2*c*f+7*d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e
)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),
(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(-c*f+d*e)^(1/2)*(6*a^2*d*f+2*a*b
*c*f-9*a*b*d*e+b^2*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*El
lipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+
d*e))^(1/2))/a^2/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^{2}+e)^{\frac{3}{2}}}{(b\,x^{2}+a)^{\frac{5}{2}}\,(x^{2}d+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^3*d^2*x^10 +
 (2*b^3*c*d + 3*a*b^2*d^2)*x^8 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^6
 + a^3*c^2 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^4 + (3*a^2*b*c^2 + 2*
a^3*c*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

3.457 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 711

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{(be-af)x\sqrt{e+fx^2}}{3a(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^{3/2}}$$
$$+ \frac{2(b^2ce+2a^2df-ab(4de-cf))x\sqrt{e+fx^2}}{3a^2(bc-ad)^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}$$
$$+ \frac{d(2b^2c^2e-a^2d(de-6cf)-abc(9de-2cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3a^2c(bc-ad)^3(c+dx^2)^{3/2}}$$
$$+ \frac{2\sqrt{e}\sqrt{de-cf}(b^3c^3e-a^2bcd(5de-6cf)-ab^2c^2(5de-cf)+a^3d^2(de+cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\Big(\text{arcsi}}{3a^2c^2(bc-ad)^4\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\sqrt{e}(b^3c^2e(de-cf)-a^3d^2f(de+2cf)+a^2bd(d^2e^2+17cdef-12c^2f^2)-ab^2c(18d^2e^2-17cdef+2c^2f^2}{3a^2c(bc-ad)^4\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+f}}$$

output
```
1/3*(-a*f+b*e)*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(3
/2)+2/3*(b^2*c*e+2*a^2*d*f-a*b*(-c*f+4*d*e))*x*(f*x^2+e)^(1/2)/a^2/(-a*d+b
*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*d*(2*b^2*c^2*e-a^2*d*(-6*c*f+d*e
)-a*b*c*(-2*c*f+9*d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/a^2/c/(-a*d+b*c)
^3/(d*x^2+c)^(3/2)+2/3*e^(1/2)*(-c*f+d*e)^(1/2)*(b^3*c^3*e-a^2*b*c*d*(-6*c
*f+5*d*e)-a*b^2*c^2*(-c*f+5*d*e)+a^3*d^2*(c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(
1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/c^2/(-a*d+b*c)^4/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(b^3*c^2*e*(-c*f+d*e)-a^3*
d^2*f*(2*c*f+d*e)+a^2*b*d*(-12*c^2*f^2+17*c*d*e*f+d^2*e^2)-a*b^2*c*(2*c^2*
f^2-17*c*d*e*f+18*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2
)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-
c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^4/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}(x^2d+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b^3*d^3*x^12 +
 3*(b^3*c*d^2 + a*b^2*d^3)*x^10 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*x^8 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^6 + a^3*c^3
+ 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^4 + 3*(a^2*b*c^3 + a^3*c^2*d
)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2),x)
```

3.458 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 648

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}\,dx = -\frac{(3bde-5bcf-adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8f^2\sqrt{a+bx^2}}$$
$$+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4f}$$
$$+\frac{\sqrt{bc-ad}e(3bde-5bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a\sqrt{bc-ad}(bde-5bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a(a^2d^2f^2+2abdf(de-3cf)-3b^2(de-cf)^2)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),}{8b^2\sqrt{c}\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/8*(-a*d*f-5*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f^2/(b*x^2
+a)^(1/2)+1/4*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f+1/8*(-
a*d+b*c)^(1/2)*e*(-a*d*f-5*b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(
b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-
a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)/(f*x^2+e)^(1/2)-1/8*a*(-a*d+b*c)^(1/2)*(a*d*f-5*b*c*f+b*d*e)*(d*x^2+c)^(
1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/
(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/f/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2+2*a*b*d*f*(-3*c*
f+d*e)-3*b^2*(-c*f+d*e)^2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}}}{\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output

```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*d*x + int((sqrt(e + f*
x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*d**2*f + 5*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 +
 b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d**2*e + 6*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*c*d*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d**2*e + 4*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c**
2*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*b*c*d*e + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c**2*f - int((sqrt(e +...
```

3.459 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 530

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2f\sqrt{a+bx^2}} - \frac{\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{a(bde-bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(b*x^2+a)^(1/2)-1/2*(-a*d+b*c)^(
1/2)*e*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c
)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/
2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(-a*d+b*c)^(1/2
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*a*(-a*d*f-b*c*f+b*d*e)
*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1
/2))/b/c^(1/2)/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e
)^(1/2)
```

Mathematica [A] (verified)

Time = 4.22 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{\frac{x\sqrt{a+bx^2}(c+dx^2)}{\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{-de+cf}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{bce-acf}{ade-acf}\right)}{f\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(be-2af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\mathrm{Ellip}}{\sqrt{e}f^2\sqrt{be-af}\sqrt{}}}{2\sqrt{c+d}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) +
c*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSi
n[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*
e - a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d
*e - c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[
ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*
c*e - a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b
*x^2))]) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e)
+ a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(S
qrt[a]*f^2*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*S
qrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {430, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{ 430}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{bc(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2df}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 427}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2df\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 428}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{2adf\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 429}$$

$$-\frac{\sqrt{a+bx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqr
t[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*Ellipti
cE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e
)/(a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f
*x^2]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
], ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d
*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)
/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b
*c - a*d)*e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e + f*x**2),x)
```

3.460 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 159

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
a*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(
1/2))/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(
1/2)
```

Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {428, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{ 428}$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/
(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*
e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 +
 d*e*x**2 + d*f*x**4),x)
```

3.461 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{c\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*
e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/(
-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [A] (verified)

Time = 5.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|\frac{(-bc+ad)e}{a(de-cf)}\right)}{c\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE[A
rcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], ((-(b*c) + a*d)*e)/(
a*(d*e - c*f))])/(c*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{429}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{c\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{c\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE[A
rcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a
*(d*e - c*f)))])/(c*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{3}{2}} \sqrt{f\,x^{2}+e}}d x$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^{2}+a}}{{\left(dx^{2}+c\right)}^{\frac{3}{2}}\sqrt{fx^{2}+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f*x^6 + (d^2
*e + 2*c*d*f)*x^4 + c^2*e + (2*c*d*e + c^2*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e + c**2*f*
x**2 + 2*c*d*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.462 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 410

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(de-cf)\,(c+dx^2)^{3/2}} + \frac{\sqrt{e}(bc(de-3cf)-2ad(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(bc-ad)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}(be-af)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c(bc-ad)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/3*e
^(1/2)*(b*c*(-3*c*f+d*e)-2*a*d*(-2*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/
e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-
(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)/(-c*f+d*e)^(3/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*e^(1/2)*(-a*f+b*e)*(b*x^2+a)
^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2
)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/(-a*d+b*c)/(-c*f+d
*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{5}{2}} \sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f*x^8 + (d^3
*e + 3*c*d^2*f)*x^6 + 3*(c*d^2*e + c^2*d*f)*x^4 + c^3*e + (3*c^2*d*e + c^3
*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.463 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 588

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{5c(de-cf)\,(c+dx^2)^{5/2}}$$

$$+\frac{d(bc(3de-7cf)-4ad(de-2cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2(bc-ad)(de-cf)^2\,(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(b^2c^2(3d^2e^2-10cdef+15c^2f^2)+a^2d^2(8d^2e^2-23cdef+23c^2f^2)-abcd(13d^2e^2-37cdef+40c^2f^2)}{15c^3(bc-ad)^2(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{2d\sqrt{e}(be-af)(bc(3de-5cf)-2ad(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-}{a(de-}\right.}{15c^2(bc-ad)^2(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(5/2)+1/15*
d*(b*c*(-7*c*f+3*d*e)-4*a*d*(-2*c*f+d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2
)/c^2/(-a*d+b*c)/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(b^2*c^2*(15*c^
2*f^2-10*c*d*e*f+3*d^2*e^2)+a^2*d^2*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-a*b*
c*d*(40*c^2*f^2-37*c*d*e*f+13*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*
x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d
+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b*c)^2/(-c*f+d*e)^(5/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+2/15*d*e^(1/2)*(-a*f+b*e)*(b*c*(-5*c*
f+3*d*e)-2*a*d*(-2*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/
2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(
-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)^2/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{7}{2}} \sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*f*x^10 + (d^
4*e + 4*c*d^3*f)*x^8 + 2*(2*c*d^3*e + 3*c^2*d^2*f)*x^6 + c^4*e + 2*(3*c^2*
d^2*e + 2*c^3*d*f)*x^4 + (4*c^3*d*e + c^4*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{7}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(7/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{\frac{7}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

3.464 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 672

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = -\frac{b(3bde-bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8df^2\sqrt{a+bx^2}}$$
$$+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4f}$$
$$+\frac{\sqrt{bc-ad}\,e(3bde-bcf-5adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8\sqrt{c}\,df^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a\sqrt{bc-ad}(bde-bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}\,df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(3a^2d^2f^2-6abdf(de-cf)+b^2(3d^2e^2-2cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b\sqrt{c}\,d\sqrt{bc-ad}\,f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/8*b*(-5*a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f^2/(b
*x^2+a)^(1/2)+1/4*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f+1/
8*(-a*d+b*c)^(1/2)*e*(-5*a*d*f-b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)
/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(
c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/d/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)/(f*x^2+e)^(1/2)-1/8*a*(-a*d+b*c)^(1/2)*(-3*a*d*f-b*c*f+b*d*e)*(d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(
1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d/f/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^2*f^2-6*a*b*d*f
*(-c*f+d*e)+b^2*(-c^2*f^2-2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+
e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d/(-a*d+b*c)
^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{x^2d+c}}{\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bx+5\Big(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2}$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*x + 5*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*b*d*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 +
b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d*e + 4*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a**2*d*f + 6*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*f - 2*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d
*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*
e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e
*x**4 + b*d*f*x**6),x)*b**2*c*e + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*c*f - int((sqrt(e +...
```

3.465 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 538

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx &= \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2df\sqrt{a+bx^2}} \\ &- \frac{b\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\ &- \frac{a(bde+bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}d\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
1/2*b^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f/(b*x^2+a)^(1/2)-1/2*b*(-a*d+
b*c)^(1/2)*e*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a
*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))
/c^(1/2)/d/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(-a*d+b
*c)^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^
(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*a*(-3*a*d*f+b*
c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*
d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+
b*c)/e)^(1/2))/c^(1/2)/d/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f*x^4 + (d*e
 + c*f)*x^2 + c*e), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)b+\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e + c*f*x
**2 + d*e*x**2 + d*f*x**4),x)*b + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(c*e + c*f*x**2 + d*e*x**2 + d*f*x**4),x)*a
```

3.466 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 493

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{cd\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{b(bc-ad)\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{b^2c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-(-a*d+b*c)*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*Ellipt
icE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e)
)^(1/2))/c/d/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)-b*(-a*d+b*c)*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*E
llipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f
+d*e))^(1/2))/a/d^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^
2+e)^(1/2)+b^2*c*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*E
llipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-a*
d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2
+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx\right)b + \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e + c*
*2*f*x**2 + 2*c*d*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)*b
+ int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e + c**2*
f*x**2 + 2*c*d*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)*a
```

3.467 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 386

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx &= -\frac{(bc-ad)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} \\ &+ \frac{2\sqrt{e}(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\ &- \frac{\sqrt{e}(be-af)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(
3/2)+2/3*e^(1/2)*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*
x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d
+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(-a*f+b*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e
)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),
(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f*x^8 + (d
^3*e + 3*c*d^2*f)*x^6 + 3*(c*d^2*e + c^2*d*f)*x^4 + c^3*e + (3*c^2*d*e + c
^3*f)*x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.468 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 564

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5c(de-cf)\,(c+dx^2)^{5/2}}$$
$$+\frac{2(2ad(de-2cf)+bc(de+cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2(de-cf)^2\,(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(2b^2c^2e(de-5cf)+abc(3d^2e^2-7cdef+20c^2f^2)-a^2d(8d^2e^2-23cdef+23c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+f}{e(c+d}}}{15c^3(bc-ad)(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(be-af)(bc(de-5cf)-4ad(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right.}{15c^2(bc-ad)(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/5*(-a*d+b*c)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(
5/2)+2/15*(2*a*d*(-2*c*f+d*e)+b*c*(c*f+d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(
1/2)/c^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(2*b^2*c^2*e*(-5*c*f+d*
e)+a*b*c*(20*c^2*f^2-7*c*d*e*f+3*d^2*e^2)-a^2*d*(23*c^2*f^2-23*c*d*e*f+8*d
^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d
*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^
3/(-a*d+b*c)/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1
/2)-1/15*e^(1/2)*(-a*f+b*e)*(b*c*(-5*c*f+d*e)-4*a*d*(-2*c*f+d*e))*(b*x^2+a
)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/
2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)/(-c*
f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{7}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*f*x^10 + (
d^4*e + 4*c*d^3*f)*x^8 + 2*(2*c*d^3*e + 3*c^2*d^2*f)*x^6 + c^4*e + 2*(3*c^
2*d^2*e + 2*c^3*d*f)*x^4 + (4*c^3*d*e + c^4*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{7}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

3.469 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 877

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7c(de-cf)\left(c+dx^2\right)^{7/2}}$$
$$+\frac{2(3ad(de-2cf)+bc(de+2cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2(de-cf)^2\left(c+dx^2\right)^{5/2}}$$
$$+\frac{(2b^2c^2(3d^2e^2-11cdef-4c^2f^2)-a^2d^2(24d^2e^2-71cdef+71c^2f^2)+abcd(15d^2e^2-43cdef+76c^2f^2))\ldots}{105c^3(bc-ad)(de-cf)^3\left(c+dx^2\right)^{3/2}}$$
$$+\frac{2\sqrt{e}(b^3c^3e(3d^2e^2-14cdef+35c^2f^2)-a^2bcd(36d^3e^3-137cd^2e^2f+190c^2def^2-161c^3f^3)+2ab^2c^2(3d^3\ldots}{105c^4(\ldots}$$
$$-\frac{\sqrt{e}(be-af)\left(b^2c^2(3d^2e^2-14cdef+35c^2f^2)+a^2d^2(24d^2e^2-71cdef+71c^2f^2)-abcd(33d^2e^2-97cde\ldots\right.}{105c^3(bc-ad)^2(de-cf)^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\ldots}$$

output
```
-1/7*(-a*d+b*c)*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(
7/2)+2/35*(3*a*d*(-2*c*f+d*e)+b*c*(2*c*f+d*e))*x*(b*x^2+a)^(1/2)*(f*x^2+e)
^(1/2)/c^2/(-c*f+d*e)^2/(d*x^2+c)^(5/2)+1/105*(2*b^2*c^2*(-4*c^2*f^2-11*c*
d*e*f+3*d^2*e^2)-a^2*d^2*(71*c^2*f^2-71*c*d*e*f+24*d^2*e^2)+a*b*c*d*(76*c^
2*f^2-43*c*d*e*f+15*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c^3/(-a*d+
b*c)/(-c*f+d*e)^3/(d*x^2+c)^(3/2)+2/105*e^(1/2)*(b^3*c^3*e*(35*c^2*f^2-14*
c*d*e*f+3*d^2*e^2)-a^2*b*c*d*(-161*c^3*f^3+190*c^2*d*e*f^2-137*c*d^2*e^2*f
+36*d^3*e^3)+2*a*b^2*c^2*(-35*c^3*f^3+7*c^2*d*e*f^2-11*c*d^2*e^2*f+3*d^3*e
^3)+4*a^3*d^2*(-22*c^3*f^3+33*c^2*d*e*f^2-23*c*d^2*e^2*f+6*d^3*e^3))*(b*x^
2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^
(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)^2
/(-c*f+d*e)^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/105*e^
(1/2)*(-a*f+b*e)*(b^2*c^2*(35*c^2*f^2-14*c*d*e*f+3*d^2*e^2)+a^2*d^2*(71*c^
2*f^2-71*c*d*e*f+24*d^2*e^2)-a*b*c*d*(112*c^2*f^2-97*c*d*e*f+33*d^2*e^2))*
(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)
*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-a*d+b
*c)^2/(-c*f+d*e)^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{9}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^5*f*x^12 + (
d^5*e + 5*c*d^4*f)*x^10 + 5*(c*d^4*e + 2*c^2*d^3*f)*x^8 + 10*(c^2*d^3*e +
c^3*d^2*f)*x^6 + c^5*e + 5*(2*c^3*d^2*e + c^4*d*f)*x^4 + (5*c^4*d*e + c^5*
f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(9/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(9/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(9/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{9/2}\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{9}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

3.470 $\int \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 903

$$\int \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{b(33a^2d^2f^2-2abdf(20de-7cf)+b^2(15d^2e^2-4cdef-3c^2f^2))\,x\sqrt{c+dx^2}}{48d^2f^3\sqrt{a+bx^2}}$$
$$-\frac{b(5bde-bcf-13adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24df^2}$$
$$+\frac{b^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{6f}$$
$$-\frac{\sqrt{bc-ad}e(33a^2d^2f^2-2abdf(20de-7cf)+b^2(15d^2e^2-4cdef-3c^2f^2))\,\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\Big(\arcsin}{48\sqrt{c}d^2f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a\sqrt{bc-ad}(15a^2d^2f^2-12abdf(de-cf)+b^2(5d^2e^2-2cdef-3c^2f^2))\,\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\Big(\text{arc}}{48b\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(5a^3d^3f^3-15a^2bd^2f^2(de-cf)+5ab^2df(3d^2e^2-2cdef-c^2f^2)-b^3(5d^3e^3-3cd^2e^2f-c^2def^2-c^3f^3}{16b\sqrt{c}d^2\sqrt{bc-ad}f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+f}}$$

output
```
1/48*b*(33*a^2*d^2*f^2-2*a*b*d*f*(-7*c*f+20*d*e)+b^2*(-3*c^2*f^2-4*c*d*e*f
+15*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/f^3/(b*x^2+a)^(1/2)-1/
24*b*(-13*a*d*f-b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)
^(1/2)/d/f^2+1/6*b^2*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f
-1/48*(-a*d+b*c)^(1/2)*e*(33*a^2*d^2*f^2-2*a*b*d*f*(-7*c*f+20*d*e)+b^2*(-3
*c^2*f^2-4*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/c^(1/2)/d^2/f^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2
+e)^(1/2)+1/48*a*(-a*d+b*c)^(1/2)*(15*a^2*d^2*f^2-12*a*b*d*f*(-c*f+d*e)+b^
2*(-3*c^2*f^2-2*c*d*e*f+5*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+
a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*
e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d^2/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)+1/16*a*(5*a^3*d^3*f^3-15*a^2*b*d^2*f^2*(-c*f+d*e)+5*a*b^2*
d*f*(-c^2*f^2-2*c*d*e*f+3*d^2*e^2)-b^3*(-c^3*f^3-c^2*d*e*f^2-3*c*d^2*e^2*f
+5*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-
a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*
d+b*c)/e)^(1/2))/b/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^3/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2d+c}}{\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(
f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
(13*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x + sqrt(e
+ f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*f*x - 5*sqrt(e + f*x**2
)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**2*d*f*x**3 + 33*int((sqrt(e + f*x**2)*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*b*d**2
*f**2 + 14*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b
*d*e*x**4 + b*d*f*x**6),x)*a*b**2*c*d*f**2 - 40*int((sqrt(e + f*x**2)*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d
*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*d**
2*e*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*
d*e*x**4 + b*d*f*x**6),x)*b**3*c**2*f**2 - 4*int((sqrt(e + f*x**2)*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*
x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*d*e*f
+ 15*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
+ a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x
**4 + b*d*f*x**6),x)*b**3*d**2*e**2 + 24*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x...
```

3.471 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 669

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{3b^2(bde+bcf-3adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8d^2f^2\sqrt{a+bx^2}}$$
$$+\frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4df}$$
$$+\frac{3b\sqrt{bc-ad}e(bde+bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a\sqrt{bc-ad}(bde+3bcf-7adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(15a^2d^2f^2-10abdf(de+cf)+b^2(3d^2e^2+2cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticPi}\left(\frac{bc}{bc-ad},\text{arcsin}\right.}{8\sqrt{c}d^2\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-3/8*b^2*(-3*a*d*f+b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^2/f^2/
(b*x^2+a)^(1/2)+1/4*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/
d/f+3/8*b*(-a*d+b*c)^(1/2)*e*(-3*a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*
x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(
1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/d^2/f^2/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(-a*d+b*c)^(1/2)*(-7*a*d*f+3*b*c*f+b*d
*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(
1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/
d^2/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(15*a^2*d^2*f^
2-10*a*b*d*f*(c*f+d*e)+b^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2
)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b
*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/d^
2/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d*f*x^4 + (d*e + c*f)*x^2 + c*e), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*x + 9*int((sqrt(e
 + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a
*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6
),x)*a*b**2*d*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f
*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*f - 3*int((sqrt(e + f*x**2)*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*
d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*d*e
 + 12*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e
 + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x
**4 + b*d*f*x**6),x)*a**2*b*d*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b**2*c*f - 2*int((sq
rt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*a*b**2*d*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**3*c*e + 4*int((sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d
*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**3*d*...
```

3.472 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 616

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{2df\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{e}(4abcdf-2a^2d^2f+b^2c(de-3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2cd^2f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{b\sqrt{e}(8abcdf-4a^2d^2f-b^2c(de+3cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^3f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{b^2c\sqrt{e}(bde+3bcf-5adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2ad^3f\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/2*b^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/d/f/(d*x^2+c)^(1/2)-1/2*e^(1/2)*
(4*a*b*c*d*f-2*a^2*d^2*f+b^2*c*(-3*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/
e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-
(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/d^2/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/2*b*e^(1/2)*(8*a*b*c*d*f-4*a^2*d^2*f-b^
2*c*(3*c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF
((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(
1/2))/a/d^3/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(
1/2)-1/2*b^2*c*e^(1/2)*(-5*a*d*f+3*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+
e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2
),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^3/f/(-c*f+d*e)^(1
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{3}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^2*f*x^6 + (d^2*e + 2*c*d*f)*x^4 + c^2*e + (2*c*d*e + c^2*f)*
x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
(3*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 5*int((sq
rt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c**2*d*e*f +
a**2*c**2*d*f**2*x**2 + 2*a**2*c*d**2*e*f*x**2 + 2*a**2*c*d**2*f**2*x**4 +
 a**2*d**3*e*f*x**4 + a**2*d**3*f**2*x**6 + 3*a*b*c**3*e*f + 3*a*b*c**3*f*
*2*x**2 + a*b*c**2*d*e**2 + 8*a*b*c**2*d*e*f*x**2 + 7*a*b*c**2*d*f**2*x**4
 + 2*a*b*c*d**2*e**2*x**2 + 7*a*b*c*d**2*e*f*x**4 + 5*a*b*c*d**2*f**2*x**6
 + a*b*d**3*e**2*x**4 + 2*a*b*d**3*e*f*x**6 + a*b*d**3*f**2*x**8 + 3*b**2*
c**3*e*f*x**2 + 3*b**2*c**3*f**2*x**4 + b**2*c**2*d*e**2*x**2 + 7*b**2*c**
2*d*e*f*x**4 + 6*b**2*c**2*d*f**2*x**6 + 2*b**2*c*d**2*e**2*x**4 + 5*b**2*
c*d**2*e*f*x**6 + 3*b**2*c*d**2*f**2*x**8 + b**2*d**3*e**2*x**6 + b**2*d**
3*e*f*x**8),x)*a**2*b**3*c*d**2*f**2 - 5*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c**2*d*e*f + a**2*c**2*d*f**2*x**2 + 2*
a**2*c*d**2*e*f*x**2 + 2*a**2*c*d**2*f**2*x**4 + a**2*d**3*e*f*x**4 + a**2
*d**3*f**2*x**6 + 3*a*b*c**3*e*f + 3*a*b*c**3*f**2*x**2 + a*b*c**2*d*e**2
+ 8*a*b*c**2*d*e*f*x**2 + 7*a*b*c**2*d*f**2*x**4 + 2*a*b*c*d**2*e**2*x**2
+ 7*a*b*c*d**2*e*f*x**4 + 5*a*b*c*d**2*f**2*x**6 + a*b*d**3*e**2*x**4 + 2*
a*b*d**3*e*f*x**6 + a*b*d**3*f**2*x**8 + 3*b**2*c**3*e*f*x**2 + 3*b**2*c**
3*f**2*x**4 + b**2*c**2*d*e**2*x**2 + 7*b**2*c**2*d*e*f*x**4 + 6*b**2*c**2
*d*f**2*x**6 + 2*b**2*c*d**2*e**2*x**4 + 5*b**2*c*d**2*e*f*x**6 + 3*b**2*c
*d**2*f**2*x**8 + b**2*d**3*e**2*x**6 + b**2*d**3*e*f*x**8),x)*a**2*b**...
```

3.473 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 619

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3cd(de-cf)\,(c+dx^2)^{3/2}}$$

$$-\frac{(bc-ad)\sqrt{e}(bc(5de-3cf)+2ad(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2d^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{(bc-ad)\sqrt{e}(abd^2e-a^2d^2f-3b^2c(de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3acd^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{b^3c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ad^3\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c
)^(3/2)-1/3*(-a*d+b*c)*e^(1/2)*(b*c*(-3*c*f+5*d*e)+2*a*d*(-2*c*f+d*e))*(b*
x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/
e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/d^2/(-c*f+
d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*
e^(1/2)*(a*b*d^2*e-a^2*d^2*f-3*b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2
+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2
),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/d^3/(-c*f+d*e)^(3/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+b^3*c*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x
^2+e)/e/(d*x^2+c))^(1/2)*EllipticPi((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(
1/2),d*e/(-c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/d^3/(-c*f+d*e)^(
1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{5}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^3*f*x^8 + (d^3*e + 3*c*d^2*f)*x^6 + 3*(c*d^2*e + c^2*d*f)*x^
4 + c^3*e + (3*c^2*d*e + c^3*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.474 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 531

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{5cd(de-cf)\,(c+dx^2)^{5/2}}$$
$$-\frac{(bc-ad)(bc(7de-3cf)+4ad(de-2cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{15c^2d(de-cf)^2\,(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(8b^2c^2e^2+abce(7de-23cf)+a^2(8d^2e^2-23cdef+23c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\Bigg(\arcsin\bigg(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}}{15c^3(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{4\sqrt{e}(be-af)(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{15c^2(de-cf)^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/5*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c
)^(5/2)-1/15*(-a*d+b*c)*(b*c*(-3*c*f+7*d*e)+4*a*d*(-2*c*f+d*e))*x*(b*x^2+a
)^(1/2)*(f*x^2+e)^(1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*e^(1/2)*(8
*b^2*c^2*e^2+a*b*c*e*(-23*c*f+7*d*e)+a^2*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)
)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/
2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^3/(-c*f
+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-4/15*e^(1/2)*(
-a*f+b*e)*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))
^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e
/a/(-c*f+d*e))^(1/2))/c^2/(-c*f+d*e)^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{7}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^4*f*x^10 + (d^4*e + 4*c*d^3*f)*x^8 + 2*(2*c*d^3*e + 3*c^2*d^
2*f)*x^6 + c^4*e + 2*(3*c^2*d^2*e + 2*c^3*d*f)*x^4 + (4*c^3*d*e + c^4*f)*x
^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{7}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

3.475 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 840

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{7cd(de-cf)\,(c+dx^2)^{7/2}}$$

$$-\frac{3(bc-ad)(2ad(de-2cf)+bc(3de-cf))x\sqrt{a+bx^2}\sqrt{e+fx^2}}{35c^2d(de-cf)^2\,(c+dx^2)^{5/2}}$$

$$+\frac{(abcd(13d^2e^2-41cdef-20c^2f^2)+2b^2c^2(4d^2e^2+11cdef-3c^2f^2)+a^2d^2(24d^2e^2-71cdef+71c^2f^2))\,\cdots}{105c^3d(de-cf)^3\,(c+dx^2)^{3/2}}$$

$$+\frac{\sqrt{e}(8b^3c^3e^2(de-7cf)+ab^2c^2e(9d^2e^2-26cdef+161c^2f^2)+a^2bc(16d^3e^3-57cd^2e^2f+58c^2def^2-161\cdots}{105c^4(bc-ad)(de-c\cdots}$$

$$-\frac{\sqrt{e}(be-af)\,(4b^2c^2e(de-7cf)+abc(5d^2e^2-13cdef+56c^2f^2)-a^2d(24d^2e^2-71cdef+71c^2f^2))\,\sqrt{a\cdots}}{105c^3(bc-ad)(de-cf)^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2\cdots}}$$

output
```
1/7*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c
)^(7/2)-3/35*(-a*d+b*c)*(2*a*d*(-2*c*f+d*e)+b*c*(-c*f+3*d*e))*x*(b*x^2+a)^
(1/2)*(f*x^2+e)^(1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(5/2)+1/105*(a*b*c*d*(-
20*c^2*f^2-41*c*d*e*f+13*d^2*e^2)+2*b^2*c^2*(-3*c^2*f^2+11*c*d*e*f+4*d^2*e
^2)+a^2*d^2*(71*c^2*f^2-71*c*d*e*f+24*d^2*e^2))*x*(b*x^2+a)^(1/2)*(f*x^2+e
)^(1/2)/c^3/d/(-c*f+d*e)^3/(d*x^2+c)^(3/2)+1/105*e^(1/2)*(8*b^3*c^3*e^2*(-
7*c*f+d*e)+a*b^2*c^2*e*(161*c^2*f^2-26*c*d*e*f+9*d^2*e^2)+a^2*b*c*(-161*c^
3*f^3+58*c^2*d*e*f^2-57*c*d^2*e^2*f+16*d^3*e^3)-8*a^3*d*(-22*c^3*f^3+33*c^
2*d*e*f^2-23*c*d^2*e^2*f+6*d^3*e^3))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/c^4/(-a*d+b*c)/(-c*f+d*e)^(7/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/105*e^(1/2)*(-a*f+b*e)*(4*b^2*c^2*e*(-7*
c*f+d*e)+a*b*c*(56*c^2*f^2-13*c*d*e*f+5*d^2*e^2)-a^2*d*(71*c^2*f^2-71*c*d*
e*f+24*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF
((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(
1/2))/c^3/(-a*d+b*c)/(-c*f+d*e)^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x
^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{9}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^5*f*x^12 + (d^5*e + 5*c*d^4*f)*x^10 + 5*(c*d^4*e + 2*c^2*d^3
*f)*x^8 + 10*(c^2*d^3*e + c^3*d^2*f)*x^6 + c^5*e + 5*(2*c^3*d^2*e + c^4*d*
f)*x^4 + (5*c^4*d*e + c^5*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(9/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{9/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\sqrt{e+fx^2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{9}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(1/2),x)
```

3.476 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 145

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)
*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/(-a*
d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {427, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output

```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*d
*e + (b*c + a*d)*f)*x^4 + a*c*e + (a*c*f + (b*c + a*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x*
*2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*
f*x**6),x)
```

3.477 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 320

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{ad\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{c(bc-ad)\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{(bc-ad)\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
-a*d*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*EllipticE((-c*f+d*e)^
(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c/(-a*
d+b*c)/e^(1/2)/(-c*f+d*e)^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^
(1/2)+b*(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)*(f*x^2+e)^(1/2)*EllipticF((-c*f+d*
e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/(-a
*d+b*c)/e^(1/2)/(-c*f+d*e)^(1/2)/(b*x^2+a)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))
^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}} \sqrt{f\,x^{2}+e}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^2*f*x^8 + (b
*d^2*e + (2*b*c*d + a*d^2)*f)*x^6 + ((2*b*c*d + a*d^2)*e + (b*c^2 + 2*a*c*
d)*f)*x^4 + a*c^2*e + (a*c^2*f + (b*c^2 + 2*a*c*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}\,\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2}}{b\,d^2f\,x^8+a\,d^2f\,x^6+2bcdf\,x^6+b\,d^2e\,x^6+2acdf\,x^4+a\,d^2e\,x^4}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2*e + a*c**
2*f*x**2 + 2*a*c*d*e*x**2 + 2*a*c*d*f*x**4 + a*d**2*e*x**4 + a*d**2*f*x**6
 + b*c**2*e*x**2 + b*c**2*f*x**4 + 2*b*c*d*e*x**4 + 2*b*c*d*f*x**6 + b*d**
2*e*x**6 + b*d**2*f*x**8),x)
```

3.478 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 449

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx = -\frac{d^2x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{2d\sqrt{e}(bc(2de-3cf)-ad(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{3c^2(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}\left(abd^2e-a^2d^2f-3b^2c(de-cf)\right)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3ac(bc-ad)^2(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/3*d^2*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+
c)^(3/2)-2/3*d*e^(1/2)*(b*c*(-3*c*f+2*d*e)-a*d*(-2*c*f+d*e))*(b*x^2+a)^(1/
2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d
*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/c^2/(-a*d+b*c)^2/(-c*f+d
*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-1/3*e^(1/2)*(a*b
*d^2*e-a^2*d^2*f-3*b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*
c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-a*d+b*c)^2/(-c*f+d*e)^(3/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{5}{2}} \sqrt{f\,x^{2}+e}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^3*f*x^10 + (
b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^8 + ((3*b*c*d^2 + a*d^3)*e + 3*(b*c^2*d
 + a*c*d^2)*f)*x^6 + a*c^3*e + (3*(b*c^2*d + a*c*d^2)*e + (b*c^3 + 3*a*c^2
*d)*f)*x^4 + (a*c^3*f + (b*c^3 + 3*a*c^2*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}\,\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.479 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 320

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{b\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{a\sqrt{bc-ad}(be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} - \frac{\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{bc-ad}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

output
```
b*c^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*EllipticE((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/
(-a*d+b*c)^(1/2)/(-a*f+b*e)/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2
)-c^(1/2)*f*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)*EllipticF((-a*
d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/
(-a*d+b*c)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1
/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f*x^8 + (b
^2*d*e + (b^2*c + 2*a*b*d)*f)*x^6 + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*
d)*f)*x^4 + a^2*c*e + (a^2*c*f + (2*a*b*c + a^2*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*e + a**2*
c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4
 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*
d*e*x**6 + b**2*d*f*x**8),x)
```

3.480 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 426

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{a(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{e}(abd^2e-a^2d^2f+b^2c(de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ac(bc-ad)^2(be-af)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{2bd\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a(bc-ad)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1
/2)+e^(1/2)*(a*b*d^2*e-a^2*d^2*f+b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x
^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1
/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+
d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-2*b*d*e^(1/2)*(
b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*
x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/(-a*d+b*c)
^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(x^{2} d+c\right)^{\frac{3}{2}} \sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \sqrt{e+fx^2}}\,dx=\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{2}}\left(dx^{2}+c\right)^{\frac{3}{2}}\sqrt{fx^{2}+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^2*f*x^10 +
 (b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)*x^8 + (2*(b^2*c*d + a*b*d^2)*e + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*x^6 + a^2*c^2*e + ((b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*e + 2*(a*b*c^2 + a^2*c*d)*f)*x^4 + (a^2*c^2*f + 2*(a*b*c^2 + a^2
*c*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \sqrt{e+fx^2}}\,dx=\int \frac{1}{\left(a+bx^{2}\right)^{\frac{3}{2}} \left(c+dx^{2}\right)^{\frac{3}{2}} \sqrt{e+fx^{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)^{\frac{3}{2}}\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

3.481 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 627

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{a(bc-ad)(be-af)\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}$$
$$+\frac{d(abd^2e-a^2d^2f+3b^2c(de-cf))\,x\sqrt{a+bx^2}\sqrt{e+fx^2}}{3ac(bc-ad)^2(be-af)(de-cf)\,(c+dx^2)^{3/2}}$$
$$+\frac{\sqrt{e}(ab^2cd^2e(7de-9cf)+2a^3d^3f(de-2cf)+3b^3c^2(de-cf)^2-a^2bd^2(2d^2e^2+3cdef-9c^2f^2))\,\sqrt{a+b}}{3ac^2(bc-ad)^3(be-af)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{d\sqrt{e}(abd^2e-a^2d^2f-9b^2c(de-cf))\,\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3ac(bc-ad)^3(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3
/2)+1/3*d*(a*b*d^2*e-a^2*d^2*f+3*b^2*c*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(f*x^
2+e)^(1/2)/a/c/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/3*e^(1
/2)*(a*b^2*c*d^2*e*(-9*c*f+7*d*e)+2*a^3*d^3*f*(-2*c*f+d*e)+3*b^3*c^2*(-c*f
+d*e)^2-a^2*b*d^2*(-9*c^2*f^2+3*c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(
1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c^2/(-a*d+b*c)^3/(-a*f+b*e)/(-c
*f+d*e)^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*e^(1/2
)*(a*b*d^2*e-a^2*d^2*f-9*b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/
(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a/c/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(x^2d+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^3*f*x^12 +
 (b^2*d^3*e + (3*b^2*c*d^2 + 2*a*b*d^3)*f)*x^10 + ((3*b^2*c*d^2 + 2*a*b*d^
3)*e + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*f)*x^8 + ((3*b^2*c^2*d + 6*a*
b*c*d^2 + a^2*d^3)*e + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*f)*x^6 + a^2*
c^3*e + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e + (2*a*b*c^3 + 3*a^2*c^2*
d)*f)*x^4 + (a^2*c^3*f + (2*a*b*c^3 + 3*a^2*c^2*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.482 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 445

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}} + \frac{2be(b^2ce+3a^2df-2ab(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3a^2\sqrt{c}(bc-ad)^{3/2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{(3abcf^2-3a^2df^2+b^2e(de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3a\sqrt{c}(bc-ad)^{3/2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*b^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a
)^(3/2)+2/3*b*e*(b^2*c*e+3*a^2*d*f-2*a*b*(c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*
x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(
1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/c^(1/2)/(-a*d+b*c)^(3/2)/(-a*f
+b*e)^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/3*(3*a*b*c*f^2-3
*a^2*d*f^2+b^2*e*(-c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/
2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*
d+b*c)/e)^(1/2))/a/c^(1/2)/(-a*d+b*c)^(3/2)/(-a*f+b*e)^2/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d*f*x^10 + (
b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*x^8 + ((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c
 + a^2*b*d)*f)*x^6 + a^3*c*e + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3
*d)*f)*x^4 + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.483 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 636

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{3a(bc-ad)(be-af)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$
$$+\frac{2b^2(b^2ce-3abde-2abcf+4a^2df)\,x\sqrt{e+fx^2}}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{e}(6a^3bd^3ef-3a^4d^3f^2+2b^4c^2e(de-cf)-ab^3c(7d^2e^2-3cdef-4c^2f^2)-3a^2b^2d(d^2e^2-3cdef+3c^2}{3a^2c(bc-ad)^3(be-af)^2\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+}}$$
$$+\frac{b\sqrt{e}(9abd^2e-9a^2d^2f-b^2c(de-cf))\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{3a^2(bc-ad)^3(be-af)\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

output
```
1/3*b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c
)^(1/2)+2/3*b^2*(4*a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)*x*(f*x^2+e)^(1/2)/
a^2/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*e^(1/2)*
(6*a^3*b*d^3*e*f-3*a^4*d^3*f^2+2*b^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-4*c^2*f^2-
3*c*d*e*f+7*d^2*e^2)-3*a^2*b^2*d*(3*c^2*f^2-3*c*d*e*f+d^2*e^2))*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)
/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^3/(-
a*f+b*e)^2/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2
)+1/3*b*e^(1/2)*(9*a*b*d^2*e-9*a^2*d^2*f-b^2*c*(-c*f+d*e))*(b*x^2+a)^(1/2)
*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x
^2+c)^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))/a^2/(-a*d+b*c)^3/(-a*f+b*e
)/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^{2}+a)^{\frac{5}{2}}\,(x^{2}d+c)^{\frac{3}{2}}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^2*f*x^12 +
 (b^3*d^2*e + (2*b^3*c*d + 3*a*b^2*d^2)*f)*x^10 + ((2*b^3*c*d + 3*a*b^2*d^
2)*e + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f)*x^8 + ((b^3*c^2 + 6*a*b^2*
c*d + 3*a^2*b*d^2)*e + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f)*x^6 + a^3*
c^2*e + ((3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e + (3*a^2*b*c^2 + 2*a^3*c*
d)*f)*x^4 + (a^3*c^2*f + (3*a^2*b*c^2 + 2*a^3*c*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

3.484 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 951

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}(c+dx^2)^{3/2}}$$

$$+\frac{2b^2(b^2ce+5a^2df-2ab(2de+cf))x\sqrt{e+fx^2}}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}$$

$$+\frac{d(2a^3bd^3ef-a^4d^3f^2+2b^4c^2e(de-cf)-ab^3c(9d^2e^2-5cdef-4c^2f^2)-a^2b^2d(d^2e^2-11cdef+11c^2f^2}{3a^2c(bc-ad)^3(be-af)^2(de-cf)(c+dx^2)^{3/2}}$$

$$+\frac{2\sqrt{e}(a^5d^4f^2(de-2cf)+b^5c^3e(de-cf)^2-ab^4c^2(de-cf)^2(5de+2cf)+a^3b^2d^3e(d^2e^2+8cdef-12c^2f}{3a^2c^2(bc-a}$$

$$+\frac{\sqrt{e}(2a^3bd^4ef-a^4d^4f^2+18ab^3cd^2e(de-cf)-b^4c^2(de-cf)^2-a^2b^2d^2(d^2e^2+18cdef-18c^2f^2))\sqrt{a+}}{3a^2c(bc-ad)^4(be-af)(de-cf)^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+}}$$

output
```
1/3*b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c
)^(3/2)+2/3*b^2*(b^2*c*e+5*a^2*d*f-2*a*b*(c*f+2*d*e))*x*(f*x^2+e)^(1/2)/a^
2/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)+1/3*d*(2*a^3*b
*d^3*e*f-a^4*d^3*f^2+2*b^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-4*c^2*f^2-5*c*d*e*f+
9*d^2*e^2)-a^2*b^2*d*(11*c^2*f^2-11*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(f
*x^2+e)^(1/2)/a^2/c/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+2
/3*e^(1/2)*(a^5*d^4*f^2*(-2*c*f+d*e)+b^5*c^3*e*(-c*f+d*e)^2-a*b^4*c^2*(-c*
f+d*e)^2*(2*c*f+5*d*e)+a^3*b^2*d^3*e*(-12*c^2*f^2+8*c*d*e*f+d^2*e^2)-a^4*b
*d^3*f*(-6*c^2*f^2+c*d*e*f+2*d^2*e^2)-a^2*b^3*c*d*(-6*c^3*f^3+12*c^2*d*e*f
^2-12*c*d^2*e^2*f+5*d^3*e^3))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1
/2)*EllipticE((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/a/
(-c*f+d*e))^(1/2))/a^2/c^2/(-a*d+b*c)^4/(-a*f+b*e)^2/(-c*f+d*e)^(3/2)/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)+1/3*e^(1/2)*(2*a^3*b*d^4*e*f-a
^4*d^4*f^2+18*a*b^3*c*d^2*e*(-c*f+d*e)-b^4*c^2*(-c*f+d*e)^2-a^2*b^2*d^2*(-
18*c^2*f^2+18*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^
(1/2)*EllipticF((-c*f+d*e)^(1/2)*x/e^(1/2)/(d*x^2+c)^(1/2),(-(-a*d+b*c)*e/
a/(-c*f+d*e))^(1/2))/a^2/c/(-a*d+b*c)^4/(-a*f+b*e)/(-c*f+d*e)^(3/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(x^2d+c)^{\frac{5}{2}}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^3*f*x^14 +
 (b^3*d^3*e + 3*(b^3*c*d^2 + a*b^2*d^3)*f)*x^12 + 3*((b^3*c*d^2 + a*b^2*d^
3)*e + (b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f)*x^10 + (3*(b^3*c^2*d + 3
*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a
^3*d^3)*f)*x^8 + a^3*c^3*e + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a
^3*d^3)*e + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f)*x^6 + 3*((a*b^2*c
^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e + (a^2*b*c^3 + a^3*c^2*d)*f)*x^4 + (a^3*
c^3*f + 3*(a^2*b*c^3 + a^3*c^2*d)*e)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2} (c+dx^2)^{5/2} \sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}} (d\,x^2+c)^{\frac{5}{2}} \sqrt{f\,x^2+e}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

3.485 $\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/7*(5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticF(1/3*x*3^(1/2)/(
2*x^2+1)^(1/2),3/7*7^(1/2))*7^(1/2)/((5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^2+7)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3+6x^2}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[1/(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticF[ArcSin[x/Sqrt[3
+ 6*x^2]], 9/7])/(Sqrt[7]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*Sqrt[7 + 11*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {427, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2+1}\sqrt{5x^2+3}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{3x^2}{2x^2+1}}\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{21}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{3x^2}{2x^2+1}}\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

input
```
Int[1/(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticF[ArcSin[x/(Sqrt[3
]*Sqrt[1 + 2*x^2])], 9/7])/(Sqrt[7]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*Sqrt[7 +
 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}\,dx$$

input
```
integrate(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/(110*x^6 + 191*x
^4 + 110*x^2 + 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{2x^2+1}\sqrt{5x^2+3}\sqrt{11x^2+7}}\,dx$$

input
```
integrate(1/(2*x**2+1)**(1/2)/(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}\,dx$$

input
```
integrate(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}\,dx$$

input
```
integrate(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{2\,x^2+1}\,\sqrt{5\,x^2+3}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int(1/((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{11x^2+7}}{110x^6+191x^4+110x^2+21}dx$$

input
```
int(1/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7))/(110*x**6 + 191*
x**4 + 110*x**2 + 21),x)
```

3.486 $\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3+5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right),\frac{2}{77}\right)}{\sqrt{77}\sqrt{\frac{1-2x^2}{3+5x^2}}\sqrt{7+11x^2}}$$

output
```
1/77*(-2*x^2+1)^(1/2)*((11*x^2+7)/(5*x^2+3))^(1/2)*EllipticF(11^(1/2)*x/(5
*x^2+3)^(1/2),1/77*154^(1/2))*77^(1/2)/((-2*x^2+1)/(5*x^2+3))^(1/2)/(11*x^
2+7)^(1/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = -\frac{x\sqrt{\frac{-1+2x^2}{3+5x^2}}\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{3+5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{5}\sqrt{\frac{77+121x^2}{3+5x^2}}\right),\frac{75}{77}\right)}{\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{x^2}{3+5x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[1/(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
-((x*Sqrt[(-1 + 2*x^2)/(3 + 5*x^2)]*Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(3 +
 5*x^2)]*EllipticF[ArcSin[Sqrt[(77 + 121*x^2)/(3 + 5*x^2)]/5], 75/77])/(Sq
rt[77]*Sqrt[1 - 2*x^2]*Sqrt[x^2/(3 + 5*x^2)]*Sqrt[7 + 11*x^2]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {427, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{5x^2+3}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{\sqrt{21}}{\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{21}\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{1}{\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{5\sqrt{3}\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}$$

input
```
Int[1/(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*Sqrt[3 + (11*x^2)/(1 - 2*x
^2)]*EllipticF[ArcTan[(5*x)/(Sqrt[7]*Sqrt[1 - 2*x^2])], -2/75])/(5*Sqrt[3]
*Sqrt[(3 + 5*x^2)/(1 - 2*x^2)]*Sqrt[7 + 11*x^2]*Sqrt[(3 + (11*x^2)/(1 - 2*
x^2))/(7 + (25*x^2)/(1 - 2*x^2))]*Sqrt[7 + (25*x^2)/(1 - 2*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/(110*x^6 + 81*
x^4 - 26*x^2 - 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{1-2x^2}\sqrt{5x^2+3}\sqrt{11x^2+7}}\,dx$$

input
```
integrate(1/(-2*x**2+1)**(1/2)/(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(1/(sqrt(1 - 2*x**2)*sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}\,dx$$

input
```
integrate(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{1-2\,x^2}\,\sqrt{5\,x^2+3}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int(1/((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-2x^2}\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = -\left(\int \frac{\sqrt{5x^2+3}\,\sqrt{11x^2+7}\,\sqrt{-2x^2+1}}{110x^6+81x^4-26x^2-21}dx\right)$$

input
```
int(1/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
 - int((sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1))/(110*x**6
+ 81*x**4 - 26*x**2 - 21),x)
```

3.487 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 88

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{7+11x^2}}$$

output
```
1/7*(-2*x^2+1)^(1/2)*((11*x^2+7)/(-5*x^2+3))^(1/2)*EllipticF(x/(-5*x^2+3)^
(1/2),2/7*I*119^(1/2))*7^(1/2)/((-2*x^2+1)/(-5*x^2+3))^(1/2)/(11*x^2+7)^(1
/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{\sqrt{\frac{7-14x^2}{3-5x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[(7 + 11*x^2)/(3 - 5*x^2)]*EllipticF[ArcSin[x/Sqrt[3
- 5*x^2]], -68/7])/(Sqrt[(7 - 14*x^2)/(3 - 5*x^2)]*Sqrt[7 + 11*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {427, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{\sqrt{7}}{\sqrt{1-\frac{x^2}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{1}{\sqrt{1-\frac{x^2}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[(7 + 11*x^2)/(3 - 5*x^2)]*EllipticF[ArcSin[x/Sqrt[3
- 5*x^2]], -68/7])/(Sqrt[7]*Sqrt[(1 - 2*x^2)/(3 - 5*x^2)]*Sqrt[7 + 11*x^2]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-5x^2+3}\,\sqrt{-2x^2+1}\,\sqrt{11x^2+7}}dx$$

input
```
int(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(11*x^2 + 7)*sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/(110*x^6 - 51*
x^4 - 44*x^2 + 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{1-2x^2}\sqrt{3-5x^2}\sqrt{11x^2+7}}\,dx$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(-2*x**2+1)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(1/(sqrt(1 - 2*x**2)*sqrt(3 - 5*x**2)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorith
m="giac")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{1-2\,x^2}\,\sqrt{3-5\,x^2}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int(1/((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{11x^2+7}\,\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}}{110x^6-51x^4-44x^2+21}\,dx$$

input
```
int(1/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3))/(110*x**6
- 51*x**4 - 44*x**2 + 21),x)
```

3.488 $\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 95

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx = \frac{\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/77*(-5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticF(1/3*33^(1/2)*
x/(2*x^2+1)^(1/2),3/77*77^(1/2))*77^(1/2)/((-5*x^2+3)/(2*x^2+1))^(1/2)/(11
*x^2+7)^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx = -\frac{x\sqrt{1+2x^2}\sqrt{\frac{-3+5x^2}{1+2x^2}}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{77+121x^2}{17+34x^2}}\right),\frac{68}{77}\right)}{\sqrt{77}\sqrt{3-5x^2}\sqrt{\frac{x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[1/(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
-((x*Sqrt[1 + 2*x^2]*Sqrt[(-3 + 5*x^2)/(1 + 2*x^2)]*Sqrt[(7 + 11*x^2)/(1 +
 2*x^2)]*EllipticF[ArcSin[Sqrt[(77 + 121*x^2)/(17 + 34*x^2)]/2], 68/77])/(
Sqrt[77]*Sqrt[3 - 5*x^2]*Sqrt[x^2/(1 + 2*x^2)]*Sqrt[7 + 11*x^2]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {427, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+1}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{\sqrt{7}}{\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{\sqrt{7}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{1}{\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}$$

input
```
Int[1/(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[1 + 2*x^2]*Sqrt[(7 + 11*x^2)/(3 - 5*x^2)]*Sqrt[7 + (68*x^2)/(3 - 5*x
^2)]*EllipticF[ArcTan[(Sqrt[11]*x)/Sqrt[3 - 5*x^2]], 9/77])/(Sqrt[77]*Sqrt
[(1 + 2*x^2)/(3 - 5*x^2)]*Sqrt[7 + 11*x^2]*Sqrt[1 + (11*x^2)/(3 - 5*x^2)]*
Sqrt[(7 + (68*x^2)/(3 - 5*x^2))/(1 + (11*x^2)/(3 - 5*x^2))])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-5x^2+3}\,\sqrt{2x^2+1}\,\sqrt{11x^2+7}}dx$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-sqrt(11*x^2 + 7)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/(110*x^6 + 59*
x^4 - 40*x^2 - 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+1}\sqrt{11x^2+7}}\,dx$$

input
```
integrate(1/(-5*x**2+3)**(1/2)/(2*x**2+1)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(1/(sqrt(3 - 5*x**2)*sqrt(2*x**2 + 1)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx = \int \frac{1}{\sqrt{11\,x^2+7}\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{11\,x^2+7}\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/(sqrt(11*x^2 + 7)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx=\int \frac{1}{\sqrt{2\,x^2+1}\,\sqrt{3-5\,x^2}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int(1/((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-5x^2}\sqrt{1+2x^2}\sqrt{7+11x^2}}\,dx=-\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{11x^2+7}\,\sqrt{-5x^2+3}}{110x^6+59x^4-40x^2-21}dx\right)$$

input
```
int(1/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt(2*x**2 + 1)*sqrt(11*x**2 + 7)*sqrt( - 5*x**2 + 3))/(110*x**6
+ 59*x**4 - 40*x**2 - 21),x)
```

3.489 $\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 95

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx = \frac{\sqrt{3-5x^2}\sqrt{\frac{1+2x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1+2x^2}}$$

output
```
1/2*(-5*x^2+3)^(1/2)*((2*x^2+1)/(-11*x^2+7))^(1/2)*EllipticF(1/3*6^(1/2)*x
/(-11*x^2+7)^(1/2),5/2*I*6^(1/2))*2^(1/2)/((-5*x^2+3)/(-11*x^2+7))^(1/2)/(
2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx = \frac{\sqrt{3-5x^2}\sqrt{\frac{1+2x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2+4x^2}}$$

input
```
Integrate[1/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[2/3
]*x)/Sqrt[7 - 11*x^2]], -75/2])/(Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*Sqrt[2 + 4
*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {427, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{2x^2+1}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{3-\frac{2x^2}{7-11x^2}}\sqrt{\frac{25x^2}{7-11x^2}+1}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{3}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\int \frac{1}{\sqrt{3-\frac{2x^2}{7-11x^2}}\sqrt{\frac{25x^2}{7-11x^2}+1}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}$$

input
```
Int[1/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[2/3
]*x)/Sqrt[7 - 11*x^2]], -75/2])/(Sqrt[2]*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*Sq
rt[1 + 2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-11x^2+7}\,\sqrt{-5x^2+3}\,\sqrt{2x^2+1}}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x)
```

output
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx=\int \frac{1}{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(110*x^6 - 81*
x^4 - 26*x^2 + 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx=\int \frac{1}{\sqrt{3-5x^2}\sqrt{7-11x^2}\sqrt{2x^2+1}}\,dx$$

input
```
integrate(1/(-11*x**2+7)**(1/2)/(-5*x**2+3)**(1/2)/(2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(3 - 5*x**2)*sqrt(7 - 11*x**2)*sqrt(2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx=\int \frac{1}{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x, algorith
m="giac")
```

output
```
integrate(1/(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+1}\,\sqrt{3-5\,x^2}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int(1/((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1+2x^2}}\,dx = \int \frac{\sqrt{2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{110x^6-81x^4-26x^2+21}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(110*x**6
- 81*x**4 - 26*x**2 + 21),x)
```

3.490 $\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \frac{\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{3\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}$$

output
```
1/3*(-5*x^2+3)^(1/2)*((-2*x^2+1)/(-11*x^2+7))^(1/2)*EllipticF(3^(1/2)*x/(-
11*x^2+7)^(1/2),1/3*2^(1/2))/((-5*x^2+3)/(-11*x^2+7))^(1/2)/(-2*x^2+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \frac{\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{3\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}$$

input
```
Integrate[1/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[3]*
x)/Sqrt[7 - 11*x^2]], 2/9])/(3*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*Sqrt[1 - 2*x
^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {427, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{3}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{1}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{3\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}$$

input
```
Int[1/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[3]*
x)/Sqrt[7 - 11*x^2]], 2/9])/(3*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*Sqrt[1 - 2*x
^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-11x^2+7}\,\sqrt{-5x^2+3}\,\sqrt{-2x^2+1}}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x)
```

output
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \int \frac{1}{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(110*x^6 - 1
91*x^4 + 110*x^2 - 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \int \frac{1}{\sqrt{1-2x^2}\sqrt{3-5x^2}\sqrt{7-11x^2}}\,dx$$

input
```
integrate(1/(-11*x**2+7)**(1/2)/(-5*x**2+3)**(1/2)/(-2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(1 - 2*x**2)*sqrt(3 - 5*x**2)*sqrt(7 - 11*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \int \frac{1}{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \int \frac{1}{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = \int \frac{1}{\sqrt{1-2\,x^2}\,\sqrt{3-5\,x^2}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int(1/((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}\,dx = -\left(\int \frac{\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{110x^6-191x^4+110x^2-21}dx\right)$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)/(-2*x^2+1)^(1/2),x)
```

output
```
- int((sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(110
*x**6 - 191*x**4 + 110*x**2 - 21),x)
```

3.491 $\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{\frac{3+5x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{3\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{3+5x^2}}$$

output
```
1/3*(-2*x^2+1)^(1/2)*((5*x^2+3)/(-11*x^2+7))^(1/2)*EllipticF(3^(1/2)*x/(-1
1*x^2+7)^(1/2),2/3*I*17^(1/2))/((-2*x^2+1)/(-11*x^2+7))^(1/2)/(5*x^2+3)^(1
/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx = \frac{\sqrt{1-2x^2}\sqrt{\frac{3+5x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{3\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{3+5x^2}}$$

input
```
Integrate[1/(Sqrt[7 - 11*x^2]*Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[3]*
x)/Sqrt[7 - 11*x^2]], -68/9])/(3*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*Sqrt[3 + 5
*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = $\{427, 27, 321\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{5x^2+3}}dx$$

$$\Big\downarrow \ 427$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{3}\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{1}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{3\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}$$

input
```
Int[1/(Sqrt[7 - 11*x^2]*Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[3]*
x)/Sqrt[7 - 11*x^2]], -68/9])/(3*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*Sqrt[3 + 5
*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-11x^2+7}\,\sqrt{-2x^2+1}\,\sqrt{5x^2+3}}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
int(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx=\int \frac{1}{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)*sqrt(-11*x^2 + 7)/(110*x^6 - 59*
x^4 - 40*x^2 + 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx=\int \frac{1}{\sqrt{1-2x^2}\sqrt{7-11x^2}\sqrt{5x^2+3}}\,dx$$

input
```
integrate(1/(-11*x**2+7)**(1/2)/(-2*x**2+1)**(1/2)/(5*x**2+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(1 - 2*x**2)*sqrt(7 - 11*x**2)*sqrt(5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx=\int \frac{1}{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorith
m="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{1-2\,x^2}\,\sqrt{5\,x^2+3}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int(1/((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int(1/((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{5x^2+3}\,\sqrt{-2x^2+1}\,\sqrt{-11x^2+7}}{110x^6-59x^4-40x^2+21}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
int((sqrt(5*x**2 + 3)*sqrt( - 2*x**2 + 1)*sqrt( - 11*x**2 + 7))/(110*x**6
- 59*x**4 - 40*x**2 + 21),x)
```

3.492 $\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \frac{\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{75}{7}\right)}{\sqrt{7}\sqrt{\frac{7-11x^2}{1+2x^2}}\sqrt{3+5x^2}}$$

output
```
1/7*(-11*x^2+7)^(1/2)*((5*x^2+3)/(2*x^2+1))^(1/2)*EllipticF(1/3*x*3^(1/2)/
(2*x^2+1)^(1/2),5/7*21^(1/2))*7^(1/2)/((-11*x^2+7)/(2*x^2+1))^(1/2)/(5*x^2
+3)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = -\frac{x\sqrt{3+5x^2}\sqrt{\frac{-7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{5}{2}\sqrt{\frac{3+5x^2}{17+34x^2}}\right),\frac{68}{75}\right)}{5\sqrt{21-33x^2}\sqrt{1+2x^2}\sqrt{\frac{x^2(3+5x^2)}{(1+2x^2)^2}}}$$

input
```
Integrate[1/(Sqrt[7 - 11*x^2]*Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
-1/5*(x*Sqrt[3 + 5*x^2]*Sqrt[(-7 + 11*x^2)/(1 + 2*x^2)]*EllipticF[ArcSin[(
5*Sqrt[(3 + 5*x^2)/(17 + 34*x^2)])/2], 68/75])/(Sqrt[21 - 33*x^2]*Sqrt[1 +
 2*x^2]*Sqrt[(x^2*(3 + 5*x^2))/(1 + 2*x^2)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {427, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{2x^2+1}\sqrt{5x^2+3}}dx$$
$$\Big\downarrow \text{427}$$
$$\frac{\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{3}\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{1}{\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}}$$
$$\Big\downarrow \text{320}$$
$$\frac{\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{5\sqrt{3}\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}$$

input
```
Int[1/(Sqrt[7 - 11*x^2]*Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[1 + 2*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*Sqrt[3 + (68*x^2)/(7 - 11*
x^2)]*EllipticF[ArcTan[(5*x)/Sqrt[7 - 11*x^2]], 7/75])/(5*Sqrt[3]*Sqrt[(1
+ 2*x^2)/(7 - 11*x^2)]*Sqrt[3 + 5*x^2]*Sqrt[1 + (25*x^2)/(7 - 11*x^2)]*Sqr
t[(3 + (68*x^2)/(7 - 11*x^2))/(1 + (25*x^2)/(7 - 11*x^2))])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{-11x^2+7}\,\sqrt{2x^2+1}\,\sqrt{5x^2+3}}dx$$

input
```
int(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
int(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)*sqrt(-11*x^2 + 7)/(110*x^6 + 51*
x^4 - 44*x^2 - 21), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{7-11x^2}\sqrt{2x^2+1}\sqrt{5x^2+3}}\,dx$$

input
```
integrate(1/(-11*x**2+7)**(1/2)/(2*x**2+1)**(1/2)/(5*x**2+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(7 - 11*x**2)*sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+1}\,\sqrt{5\,x^2+3}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int(1/((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{7-11x^2}\sqrt{1+2x^2}\sqrt{3+5x^2}}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{-11x^2+7}}{110x^6+51x^4-44x^2-21}dx\right)$$

input
```
int(1/(-11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
 - int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(110*x**6
+ 51*x**4 - 44*x**2 - 21),x)
```

3.493 $\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\frac{\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/7*(5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticPi(1/3*x*3^(1/2)/
(2*x^2+1)^(1/2),6,3/7*7^(1/2))*7^(1/2)/((5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^2
+7)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx=\frac{\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3+6x^2}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticPi[6, ArcSin[x/Sqr
t[3 + 6*x^2]], 9/7])/(Sqrt[7]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*Sqrt[7 + 11*x^
2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {428, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{5x^2+3}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{ 428}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{21}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}$$

input
```
Int[Sqrt[1 + 2*x^2]/(Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticPi[6, ArcSin[x/(Sq
rt[3]*Sqrt[1 + 2*x^2])], 9/7])/(Sqrt[7]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*Sqrt
[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/(55*x^4 + 68*x^2
 + 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}}{\sqrt{5x^2+3}\sqrt{11x^2+7}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{5\,x^2+3}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int((2*x^2 + 1)^(1/2)/((5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int((2*x^2 + 1)^(1/2)/((5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{11x^2+7}}{55x^4+68x^2+21}dx$$

input
```
int((2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7))/(55*x**4 + 68*x*
*2 + 21),x)
```

3.494 $\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 98

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx=\frac{\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/77*(-5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticPi(1/3*33^(1/2)
*x/(2*x^2+1)^(1/2),6/11,3/77*77^(1/2))*77^(1/2)/((-5*x^2+3)/(2*x^2+1))^(1/
2)/(11*x^2+7)^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx=\frac{\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(Sqrt[3 - 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticPi[6/11, ArcSin[(S
qrt[11/3]*x)/Sqrt[1 + 2*x^2]], 9/77])/(Sqrt[77]*Sqrt[(3 - 5*x^2)/(1 + 2*x^
2)]*Sqrt[7 + 11*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {428, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{428}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{3-\frac{11x^2}{2x^2+1}}\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{21}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{1}{\sqrt{3-\frac{11x^2}{2x^2+1}}\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{2x^2+1}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}$$

input
```
Int[Sqrt[1 + 2*x^2]/(Sqrt[3 - 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticPi[6/11, ArcSin[(S
qrt[11/3]*x)/Sqrt[1 + 2*x^2]], 9/77])/(Sqrt[77]*Sqrt[(3 - 5*x^2)/(1 + 2*x^
2)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{-5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(11*x^2 + 7)*sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/(55*x^4 + 2*x^
2 - 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(-5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(3 - 5*x**2)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{3-5\,x^2}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int((2*x^2 + 1)^(1/2)/((3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int((2*x^2 + 1)^(1/2)/((3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{11x^2+7}\,\sqrt{-5x^2+3}}{55x^4+2x^2-21}dx\right)$$

input
```
int((2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt(2*x**2 + 1)*sqrt(11*x**2 + 7)*sqrt( - 5*x**2 + 3))/(55*x**4 +
2*x**2 - 21),x)
```

3.495 $\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx=\frac{\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{75}{7}\right)}{\sqrt{7}\sqrt{\frac{7-11x^2}{1+2x^2}}\sqrt{3+5x^2}}$$

output
```
1/7*(-11*x^2+7)^(1/2)*((5*x^2+3)/(2*x^2+1))^(1/2)*EllipticPi(1/3*x*3^(1/2)
/(2*x^2+1)^(1/2),6,5/7*21^(1/2))*7^(1/2)/((-11*x^2+7)/(2*x^2+1))^(1/2)/(5*
x^2+3)^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx=\frac{\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3+6x^2}}\right),\frac{75}{7}\right)}{\sqrt{\frac{49-77x^2}{1+2x^2}}\sqrt{3+5x^2}}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*EllipticPi[6, ArcSin[x/Sqr
t[3 + 6*x^2]], 75/7])/(Sqrt[(49 - 77*x^2)/(1 + 2*x^2)]*Sqrt[3 + 5*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {428, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{5x^2+3}}dx$$

$$\Big\downarrow \text{428}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{25x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{21}\sqrt{\frac{7-11x^2}{2x^2+1}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{25x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{\frac{7-11x^2}{2x^2+1}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{2x^2+1}}\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{75}{7}\right)}{\sqrt{7}\sqrt{\frac{7-11x^2}{2x^2+1}}\sqrt{5x^2+3}}$$

input
```
Int[Sqrt[1 + 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2)]*EllipticPi[6, ArcSin[x/(Sq
rt[3]*Sqrt[1 + 2*x^2])], 75/7])/(Sqrt[7]*Sqrt[(7 - 11*x^2)/(1 + 2*x^2)]*Sq
rt[3 + 5*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{-11x^2+7}\,\sqrt{5x^2+3}}dx$$

input
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)*sqrt(-11*x^2 + 7)/(55*x^4 - 2*x^
2 - 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{5x^2+3}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2)/(5*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(7 - 11*x**2)*sqrt(5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{5\,x^2+3}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int((2*x^2 + 1)^(1/2)/((5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int((2*x^2 + 1)^(1/2)/((5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{-11x^2+7}}{55x^4-2x^2-21}dx\right)$$

input
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
- int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(55*x**4 -
2*x**2 - 21),x)
```

3.496 $\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 98

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{75}{77}\right)}{\sqrt{77}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1+2x^2}}}$$

output
```
1/77*(-11*x^2+7)^(1/2)*((-5*x^2+3)/(2*x^2+1))^(1/2)*EllipticPi(1/3*33^(1/2
)*x/(2*x^2+1)^(1/2),6/11,5/77*231^(1/2))*77^(1/2)/(-5*x^2+3)^(1/2)/((-11*x
^2+7)/(2*x^2+1))^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{75}{77}\right)}{\sqrt{77}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1+2x^2}}}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(1 + 2*x^2)]*EllipticPi[6/11, ArcSin[(S
qrt[11/3]*x)/Sqrt[1 + 2*x^2]], 75/77])/(Sqrt[77]*Sqrt[3 - 5*x^2]*Sqrt[(7 -
 11*x^2)/(1 + 2*x^2)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {428, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}dx$$

$$\Big\downarrow \text{ 428}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{25x^2}{2x^2+1}}\sqrt{3-\frac{11x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{21}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{25x^2}{2x^2+1}}\sqrt{3-\frac{11x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\text{EllipticPi}\left(\frac{14}{25},\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right),\frac{77}{75}\right)}{5\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}$$

input
```
Int[Sqrt[1 + 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(1 + 2*x^2)]*EllipticPi[14/25, ArcSin[(
5*x)/(Sqrt[7]*Sqrt[1 + 2*x^2])], 77/75])/(5*Sqrt[3]*Sqrt[3 - 5*x^2]*Sqrt[(
7 - 11*x^2)/(1 + 2*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{-11x^2+7}\,\sqrt{-5x^2+3}}dx$$

input
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(55*x^4 - 68*x
^2 + 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{7-11x^2}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2)/(-5*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(3 - 5*x**2)*sqrt(7 - 11*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx=\int \frac{\sqrt{2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(2*x^2 + 1)/(sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx=\int \frac{\sqrt{2\,x^2+1}}{\sqrt{3-5\,x^2}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int((2*x^2 + 1)^(1/2)/((3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int((2*x^2 + 1)^(1/2)/((3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx=\int \frac{\sqrt{2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{55x^4-68x^2+21}dx$$

input
```
int((2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(55*x**4 -
 68*x**2 + 21),x)
```

3.497 $\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 198

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \frac{5\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{11\sqrt{3}\sqrt{\frac{3+5x^2}{1-2x^2}}\sqrt{7+11x^2}} - \frac{14\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticPi}\left(\frac{11}{25},\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{55\sqrt{3}\sqrt{\frac{3+5x^2}{1-2x^2}}\sqrt{7+11x^2}}$$

output
```
5/33*(5*x^2+3)^(1/2)*((11*x^2+7)/(-2*x^2+1))^(1/2)*InverseJacobiAM(arctan(
5/7*x*7^(1/2)/(-2*x^2+1)^(1/2)),1/15*I*6^(1/2))*3^(1/2)/((5*x^2+3)/(-2*x^2
+1))^(1/2)/(11*x^2+7)^(1/2)-14/165*(5*x^2+3)^(1/2)*((11*x^2+7)/(-2*x^2+1))
^(1/2)*EllipticPi(5*x*7^(1/2)/(-2*x^2+1)^(1/2)/(49+175*x^2/(-2*x^2+1))^(1/
2),11/25,1/15*I*6^(1/2))*3^(1/2)/((5*x^2+3)/(-2*x^2+1))^(1/2)/(11*x^2+7)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = -\frac{i\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticPi}\left(\frac{14}{25}, i\text{arcsinh}\left(\frac{5x}{\sqrt{7-14x^2}}\right), \frac{77}{75}\right)}{5\sqrt{7+11x^2}\sqrt{\frac{9+15x^2}{1-2x^2}}}$$

input
```
Integrate[Sqrt[1 - 2*x^2]/(Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
((-1/5*I)*Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*EllipticPi[14/25,
 I*ArcSinh[(5*x)/Sqrt[7 - 14*x^2]], 77/75])/(Sqrt[7 + 11*x^2]*Sqrt[(9 + 15
*x^2)/(1 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {428, 27, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{5x^2+3}\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{428}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{\sqrt{21}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{21}\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{1}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{411}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{25}{11}\int \frac{1}{\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}-\frac{2}{11}\int \frac{\sqrt{\frac{25x^2}{1-2x^2}+7}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{11x^2}{1-2x^2}+3}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{5\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{11\sqrt{3}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}-\frac{2}{11}\int \frac{\sqrt{\frac{25x^2}{1-2x^2}+7}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{11x^2}{1-2x^2}+3}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{5\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{11\sqrt{3}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}-\frac{14\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticPi}\left(\frac{11}{25},\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{55\sqrt{3}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}\right)}{\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}$$

input
```
Int[Sqrt[1 - 2*x^2]/(Sqrt[3 + 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*((5*Sqrt[3 + (11*x^2)/(1 -
 2*x^2)]*EllipticF[ArcTan[(5*x)/(Sqrt[7]*Sqrt[1 - 2*x^2])], -2/75])/(11*Sq
rt[3]*Sqrt[(3 + (11*x^2)/(1 - 2*x^2))/(7 + (25*x^2)/(1 - 2*x^2))]*Sqrt[7 +
 (25*x^2)/(1 - 2*x^2)]) - (14*Sqrt[3 + (11*x^2)/(1 - 2*x^2)]*EllipticPi[11
/25, ArcTan[(5*x)/(Sqrt[7]*Sqrt[1 - 2*x^2])], -2/75])/(55*Sqrt[3]*Sqrt[(3
+ (11*x^2)/(1 - 2*x^2))/(7 + (25*x^2)/(1 - 2*x^2))]*Sqrt[7 + (25*x^2)/(1 -
 2*x^2)])))/(Sqrt[(3 + 5*x^2)/(1 - 2*x^2)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}}{\sqrt{5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/(55*x^4 + 68*x^
2 + 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}}{\sqrt{5x^2+3}\sqrt{11x^2+7}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)/(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)/(sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}}{\sqrt{5\,x^2+3}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int((1 - 2*x^2)^(1/2)/((5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int((1 - 2*x^2)^(1/2)/((5*x^2 + 3)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3+5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5x^2+3}\,\sqrt{11x^2+7}\,\sqrt{-2x^2+1}}{55x^4+68x^2+21}dx$$

input
```
int((-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1))/(55*x**4 + 68
*x**2 + 21),x)
```

3.498 $\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 194

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = -\frac{\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{68}{7}\right)}{5\sqrt{7}\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{7+11x^2}} + \frac{6\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticPi}\left(-5,\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{68}{7}\right)}{5\sqrt{7}\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{7+11x^2}}$$

output
```
-1/35*(-5*x^2+3)^(1/2)*((11*x^2+7)/(-2*x^2+1))^(1/2)*InverseJacobiAM(arcta
n(1/3*x*3^(1/2)/(-2*x^2+1)^(1/2)),2/7*I*119^(1/2))*7^(1/2)/((-5*x^2+3)/(-2
*x^2+1))^(1/2)/(11*x^2+7)^(1/2)+6/35*(-5*x^2+3)^(1/2)*((11*x^2+7)/(-2*x^2+
1))^(1/2)*EllipticPi(x*3^(1/2)/(-2*x^2+1)^(1/2)/(9+3*x^2/(-2*x^2+1))^(1/2)
,-5,2/7*I*119^(1/2))*7^(1/2)/((-5*x^2+3)/(-2*x^2+1))^(1/2)/(11*x^2+7)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = -\frac{i\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1-2x^2}}\,\text{EllipticPi}\left(\frac{14}{25},i\text{arcsinh}\left(\frac{5x}{\sqrt{7-14x^2}}\right),\frac{7}{75}\right)}{5\sqrt{\frac{9-15x^2}{1-2x^2}}\sqrt{7+11x^2}}$$

input
```
Integrate[Sqrt[1 - 2*x^2]/(Sqrt[3 - 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
((-1/5*I)*Sqrt[3 - 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*EllipticPi[14/25,
 I*ArcSinh[(5*x)/Sqrt[7 - 14*x^2]], 7/75])/(Sqrt[(9 - 15*x^2)/(1 - 2*x^2)]
*Sqrt[7 + 11*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {428, 27, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}\,dx$$

$$\Big\downarrow \text{428}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{\sqrt{21}}{\sqrt{\frac{x^2}{1-2x^2}+3}\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{21}\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{1}{\sqrt{\frac{x^2}{1-2x^2}+3}\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \textbf{411}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{2}{5}\int \frac{\sqrt{\frac{x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}-\frac{1}{5}\int \frac{1}{\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{2}{5}\int \frac{\sqrt{\frac{x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}-\frac{\sqrt{\frac{25x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{68}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{25x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}\right)}{\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{11x^2+7}}$$

$$\Big\downarrow \textbf{414}$$

$$\frac{\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{1-2x^2}}\left(\frac{6\sqrt{\frac{25x^2}{1-2x^2}+7}\,\text{EllipticPi}\left(-5,\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{68}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{25x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}-\frac{\sqrt{\frac{25x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{68}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{25x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}\right)}{\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{11x^2+7}}$$

input
```
Int[Sqrt[1 - 2*x^2]/(Sqrt[3 - 5*x^2]*Sqrt[7 + 11*x^2]),x]
```

output
```
(Sqrt[3 - 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*(-1/5*(Sqrt[7 + (25*x^2)/(
1 - 2*x^2)]*EllipticF[ArcTan[x/(Sqrt[3]*Sqrt[1 - 2*x^2])], -68/7])/(Sqrt[7
]*Sqrt[3 + x^2/(1 - 2*x^2)]*Sqrt[(7 + (25*x^2)/(1 - 2*x^2))/(3 + x^2/(1 -
2*x^2))]) + (6*Sqrt[7 + (25*x^2)/(1 - 2*x^2)]*EllipticPi[-5, ArcTan[x/(Sqr
t[3]*Sqrt[1 - 2*x^2])], -68/7])/(5*Sqrt[7]*Sqrt[3 + x^2/(1 - 2*x^2)]*Sqrt[
(7 + (25*x^2)/(1 - 2*x^2))/(3 + x^2/(1 - 2*x^2))])))/(Sqrt[(3 - 5*x^2)/(1
- 2*x^2)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}}{\sqrt{-5x^2+3}\,\sqrt{11x^2+7}}dx$$

input
```
int((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(11*x^2 + 7)*sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/(55*x^4 + 2*x
^2 - 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)/(-5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)/(sqrt(3 - 5*x**2)*sqrt(11*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(-5*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}\sqrt{-5\,x^2+3}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(11*x^2 + 7)*sqrt(-5*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}}{\sqrt{3-5\,x^2}\,\sqrt{11\,x^2+7}}\,dx$$

input
```
int((1 - 2*x^2)^(1/2)/((3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)),x)
```

output
```
int((1 - 2*x^2)^(1/2)/((3 - 5*x^2)^(1/2)*(11*x^2 + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7+11x^2}}\,dx = -\left(\int \frac{\sqrt{11x^2+7}\,\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}}{55x^4+2x^2-21}dx\right)$$

input
```
int((-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
 - int((sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3))/(55*x**
4 + 2*x**2 - 21),x)
```

3.499 $\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 190

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = -\frac{\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1-2x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{3}{7}}x}{\sqrt{1-2x^2}}\right),-\frac{68}{9}\right)}{11\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{3+5x^2}} + \frac{14\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1-2x^2}}\,\text{EllipticPi}\left(-\frac{11}{3},\arctan\left(\frac{\sqrt{\frac{3}{7}}x}{\sqrt{1-2x^2}}\right),-\frac{68}{9}\right)}{33\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{3+5x^2}}$$

output
```
-1/11*(-11*x^2+7)^(1/2)*((5*x^2+3)/(-2*x^2+1))^(1/2)*InverseJacobiAM(arcta
n(1/7*21^(1/2)*x/(-2*x^2+1)^(1/2)),2/3*I*17^(1/2))/((-11*x^2+7)/(-2*x^2+1)
)^(1/2)/(5*x^2+3)^(1/2)+14/33*(-11*x^2+7)^(1/2)*((5*x^2+3)/(-2*x^2+1))^(1/
2)*EllipticPi(21^(1/2)*x/(-2*x^2+1)^(1/2)/(49+21*x^2/(-2*x^2+1))^(1/2),-11
/3,2/3*I*17^(1/2))/((-11*x^2+7)/(-2*x^2+1))^(1/2)/(5*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = -\frac{i\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1-2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1-2x^2}}\right),\frac{9}{77}\right)}{\sqrt{77}\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{3+5x^2}}$$

input
```
Integrate[Sqrt[1 - 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
((-I)*Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(1 - 2*x^2)]*EllipticPi[6/11, I*Ar
cSinh[(Sqrt[11/3]*x)/Sqrt[1 - 2*x^2]], 9/77])/(Sqrt[77]*Sqrt[(7 - 11*x^2)/
(1 - 2*x^2)]*Sqrt[3 + 5*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {428, 27, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{5x^2+3}}\,dx$$

$$\Big\downarrow \text{428}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{1-2x^2}}\int \frac{\sqrt{21}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}\sqrt{\frac{11x^2}{1-2x^2}+3}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{21}\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{1-2x^2}}\int \frac{1}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}\sqrt{\frac{11x^2}{1-2x^2}+3}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{411}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{1-2x^2}}\left(\frac{11}{5}\int \frac{1}{\sqrt{\frac{3x^2}{1-2x^2}+7}\sqrt{\frac{11x^2}{1-2x^2}+3}}d\frac{x}{\sqrt{1-2x^2}}-\frac{2}{5}\int \frac{\sqrt{\frac{11x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{1-2x^2}}\left(\frac{\sqrt{\frac{11}{7}}\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1-2x^2}}\right),\frac{68}{77}\right)}{5\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{11x^2}{1-2x^2}+3}}\sqrt{\frac{11x^2}{1-2x^2}+3}}-\frac{2}{5}\int \frac{\sqrt{\frac{11x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{5x^2+3}{1-2x^2}}\left(\frac{\sqrt{\frac{11}{7}}\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1-2x^2}}\right),\frac{68}{77}\right)}{5\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{11x^2}{1-2x^2}+3}}\sqrt{\frac{11x^2}{1-2x^2}+3}}-\frac{6\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticPi}\left(\frac{5}{11},\arctan\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1-2x^2}}\right),\frac{68}{77}\right)}{5\sqrt{77}\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{11x^2}{1-2x^2}+3}}\sqrt{\frac{11x^2}{1-2x^2}+3}}\right)}{\sqrt{\frac{7-11x^2}{1-2x^2}}\sqrt{5x^2+3}}$$

input
```
Int[Sqrt[1 - 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 + 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(1 - 2*x^2)]*((Sqrt[11/7]*Sqrt[7 + (3*x
^2)/(1 - 2*x^2)]*EllipticF[ArcTan[(Sqrt[11/3]*x)/Sqrt[1 - 2*x^2]], 68/77])
/(5*Sqrt[(7 + (3*x^2)/(1 - 2*x^2))/(3 + (11*x^2)/(1 - 2*x^2))]*Sqrt[3 + (1
1*x^2)/(1 - 2*x^2)]) - (6*Sqrt[7 + (3*x^2)/(1 - 2*x^2)]*EllipticPi[5/11, A
rcTan[(Sqrt[11/3]*x)/Sqrt[1 - 2*x^2]], 68/77])/(5*Sqrt[77]*Sqrt[(7 + (3*x^
2)/(1 - 2*x^2))/(3 + (11*x^2)/(1 - 2*x^2))]*Sqrt[3 + (11*x^2)/(1 - 2*x^2)]
)))/(Sqrt[(7 - 11*x^2)/(1 - 2*x^2)]*Sqrt[3 + 5*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}}{\sqrt{-11x^2+7}\,\sqrt{5x^2+3}}dx$$

input
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)*sqrt(-11*x^2 + 7)/(55*x^4 - 2*x
^2 - 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{5x^2+3}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2)/(5*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)/(sqrt(7 - 11*x**2)*sqrt(5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}}{\sqrt{5\,x^2+3}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int((1 - 2*x^2)^(1/2)/((5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int((1 - 2*x^2)^(1/2)/((5*x^2 + 3)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3+5x^2}}\,dx = -\left(\int \frac{\sqrt{5x^2+3}\,\sqrt{-2x^2+1}\,\sqrt{-11x^2+7}}{55x^4-2x^2-21}dx\right)$$

input
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(5*x^2+3)^(1/2),x)
```

output
```
- int((sqrt(5*x**2 + 3)*sqrt( - 2*x**2 + 1)*sqrt( - 11*x**2 + 7))/(55*x**
4 - 2*x**2 - 21),x)
```

3.500 $\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 194

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = -\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{2}{7}\right)}{5\sqrt{7}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}} + \frac{6\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\,\text{EllipticPi}\left(-5,\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{2}{7}\right)}{5\sqrt{7}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

output

```
-1/35*(-11*x^2+7)^(1/2)*((-5*x^2+3)/(-2*x^2+1))^(1/2)*InverseJacobiAM(arct
an(1/3*x*3^(1/2)/(-2*x^2+1)^(1/2)),1/7*I*14^(1/2))*7^(1/2)/(-5*x^2+3)^(1/2
)/((-11*x^2+7)/(-2*x^2+1))^(1/2)+6/35*(-11*x^2+7)^(1/2)*((-5*x^2+3)/(-2*x^
2+1))^(1/2)*EllipticPi(x*3^(1/2)/(-2*x^2+1)^(1/2)/(9+3*x^2/(-2*x^2+1))^(1/
2),-5,1/7*I*14^(1/2))*7^(1/2)/(-5*x^2+3)^(1/2)/((-11*x^2+7)/(-2*x^2+1))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = -\frac{i\sqrt{\frac{3-5x^2}{1-2x^2}}\sqrt{1-\frac{11x^2}{7}}\,\text{EllipticPi}\left(6,i\text{arcsinh}\left(\frac{x}{\sqrt{3-6x^2}}\right),\frac{9}{7}\right)}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

input
```
Integrate[Sqrt[1 - 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]),x]
```

output
```
((-I)*Sqrt[(3 - 5*x^2)/(1 - 2*x^2)]*Sqrt[1 - (11*x^2)/7]*EllipticPi[6, I*A
rcSinh[x/Sqrt[3 - 6*x^2]], 9/7])/(Sqrt[3 - 5*x^2]*Sqrt[(7 - 11*x^2)/(1 - 2
*x^2)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {428, 27, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx$$

$$\Big\downarrow \ 428$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\int \frac{\sqrt{21}}{\sqrt{\frac{x^2}{1-2x^2}+3}\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{21}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\int \frac{1}{\sqrt{\frac{x^2}{1-2x^2}+3}\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

$$\Big\downarrow \text{411}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\left(\frac{2}{5}\int \frac{\sqrt{\frac{x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}-\frac{1}{5}\int \frac{1}{\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}\right)}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\left(\frac{2}{5}\int \frac{\sqrt{\frac{x^2}{1-2x^2}+3}}{\left(\frac{2x^2}{1-2x^2}+1\right)\sqrt{\frac{3x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}-\frac{\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{2}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}\right)}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1-2x^2}}\left(\frac{6\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticPi}\left(-5,\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{2}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}-\frac{\sqrt{\frac{3x^2}{1-2x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{3}\sqrt{1-2x^2}}\right),-\frac{2}{7}\right)}{5\sqrt{7}\sqrt{\frac{x^2}{1-2x^2}+3}\sqrt{\frac{\frac{3x^2}{1-2x^2}+7}{\frac{x^2}{1-2x^2}+3}}}\right)}{\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1-2x^2}}}$$

input
```
Int[Sqrt[1 - 2*x^2]/(Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2]),x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(1 - 2*x^2)]*(-1/5*(Sqrt[7 + (3*x^2)/(1
 - 2*x^2)]*EllipticF[ArcTan[x/(Sqrt[3]*Sqrt[1 - 2*x^2])], -2/7])/(Sqrt[7]*
Sqrt[3 + x^2/(1 - 2*x^2)]*Sqrt[(7 + (3*x^2)/(1 - 2*x^2))/(3 + x^2/(1 - 2*x
^2))]) + (6*Sqrt[7 + (3*x^2)/(1 - 2*x^2)]*EllipticPi[-5, ArcTan[x/(Sqrt[3]
*Sqrt[1 - 2*x^2])], -2/7])/(5*Sqrt[7]*Sqrt[3 + x^2/(1 - 2*x^2)]*Sqrt[(7 +
(3*x^2)/(1 - 2*x^2))/(3 + x^2/(1 - 2*x^2))])))/(Sqrt[3 - 5*x^2]*Sqrt[(7 -
11*x^2)/(1 - 2*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}}{\sqrt{-11x^2+7}\,\sqrt{-5x^2+3}}dx$$

input
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(55*x^4 - 68*
x^2 + 21), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\sqrt{7-11x^2}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2)/(-5*x**2+3)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)/(sqrt(3 - 5*x**2)*sqrt(7 - 11*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}}{\sqrt{-5\,x^2+3}\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)/(sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}}{\sqrt{3-5\,x^2}\,\sqrt{7-11\,x^2}}\,dx$$

input
```
int((1 - 2*x^2)^(1/2)/((3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)),x)
```

output
```
int((1 - 2*x^2)^(1/2)/((3 - 5*x^2)^(1/2)*(7 - 11*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}\,dx = \int \frac{\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{55x^4-68x^2+21}dx$$

input
```
int((-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2),x)
```

output
```
int((sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(55*x**
4 - 68*x**2 + 21),x)
```

3.501 $\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 328

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \frac{x\sqrt{3+5x^2}\sqrt{7+11x^2}}{11\sqrt{1+2x^2}} - \frac{\sqrt{7}\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}E\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right)|\frac{9}{7}\right)}{22\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}} + \frac{\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{9}{7}\right)}{4\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}} + \frac{51\sqrt{3+5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{9}{7}\right)}{44\sqrt{7}\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/11*x*(5*x^2+3)^(1/2)*(11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)-1/22*7^(1/2)*(5*x^
2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticE(1/3*x*3^(1/2)/(2*x^2+1)^
(1/2),3/7*7^(1/2))/((5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^2+7)^(1/2)+1/28*(5*x^
2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticF(1/3*x*3^(1/2)/(2*x^2+1)^
(1/2),3/7*7^(1/2))*7^(1/2)/((5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^2+7)^(1/2)+51
/308*(5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticPi(1/3*x*3^(1/2)
/(2*x^2+1)^(1/2),6,3/7*7^(1/2))*7^(1/2)/((5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^
2+7)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx$$
$$= \frac{\frac{847x\sqrt{1+2x^2}(3+5x^2)}{\sqrt{7+11x^2}} - \frac{77i\sqrt{2+4x^2}\sqrt{\frac{3+5x^2}{7+11x^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7+11x^2}}\right)|\frac{9}{2}\right)}{\sqrt{\frac{1+2x^2}{7+11x^2}}} - \frac{16(1+2x^2)\sqrt{\frac{3+5x^2}{1+2x^2}}\sqrt{\frac{49+77x^2}{1+2x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3+6x}}\right.\right.}{\sqrt{7+11x^2}}}{1694\sqrt{3+5x^2}}$$

input
```
Integrate[(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
((847*x*Sqrt[1 + 2*x^2]*(3 + 5*x^2))/Sqrt[7 + 11*x^2] - ((77*I)*Sqrt[2 + 4
*x^2]*Sqrt[(3 + 5*x^2)/(7 + 11*x^2)]*EllipticE[I*ArcSinh[(Sqrt[2/3]*x)/Sqr
t[7 + 11*x^2]], 9/2])/Sqrt[(1 + 2*x^2)/(7 + 11*x^2)] - (16*(1 + 2*x^2)*Sqr
t[(3 + 5*x^2)/(1 + 2*x^2)]*Sqrt[(49 + 77*x^2)/(1 + 2*x^2)]*EllipticF[ArcSi
n[x/Sqrt[3 + 6*x^2]], 9/7])/Sqrt[7 + 11*x^2] - ((833*I)*Sqrt[1 + 2*x^2]*Sq
rt[(3 + 5*x^2)/(7 + 11*x^2)]*EllipticPi[-11/3, I*ArcSinh[(Sqrt[3]*x)/Sqrt[
7 + 11*x^2]], 2/9])/Sqrt[(1 + 2*x^2)/(7 + 11*x^2)])/(1694*Sqrt[3 + 5*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 321, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}\sqrt{5x^2+3}}{\sqrt{11x^2+7}}\,dx$$
$$\Big\downarrow \; 430$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{\left(5x^2+3\right)^{3/2}\sqrt{11x^2+7}}dx+\frac{6}{55}\int \frac{1}{\sqrt{2x^2+1}\sqrt{5x^2+3}\sqrt{11x^2+7}}dx+\frac{51}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{2x^2+1}\sqrt{11x^2+7}}dx+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 427}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{\left(5x^2+3\right)^{3/2}\sqrt{11x^2+7}}dx+\frac{51}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{2x^2+1}\sqrt{11x^2+7}}dx+\frac{2\sqrt{\frac{3}{7}}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{3x^2}{2x^2+1}}\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{55\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{\left(5x^2+3\right)^{3/2}\sqrt{11x^2+7}}dx+\frac{51}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{2x^2+1}\sqrt{11x^2+7}}dx+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{3x^2}{2x^2+1}}\sqrt{3-\frac{x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{55\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{\left(5x^2+3\right)^{3/2}\sqrt{11x^2+7}}dx+\frac{51}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{2x^2+1}\sqrt{11x^2+7}}dx+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 428}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{\left(5x^2+3\right)^{3/2}\sqrt{11x^2+7}}dx+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{7}}{\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{7-\frac{2x^2}{5x^2+3}}\sqrt{\frac{x^2}{5x^2+3}+1}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{7}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{1}{\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{7-\frac{2x^2}{5x^2+3}}\sqrt{\frac{x^2}{5x^2+3}+1}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{3}{11}\int \frac{\sqrt{2x^2+1}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\text{EllipticPi}\left(\frac{35}{2},\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{2}\right)}{110\sqrt{2}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{429}$$

$$-\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{7}\sqrt{\frac{x^2}{5x^2+3}+1}}{\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\text{EllipticPi}\left(\frac{35}{2},\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{2}\right)}{110\sqrt{2}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{\frac{x^2}{5x^2+3}+1}}{\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{35}{2},\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{2}\right)}{110\sqrt{2}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 327$$

$$\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{2x^2+1}}\right),\frac{9}{7}\right)}{55\sqrt{7}\sqrt{\frac{5x^2+3}{2x^2+1}}\sqrt{11x^2+7}}-\frac{\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}E\left(\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right)|-\frac{7}{2}\right)}{11\sqrt{2}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{153\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{35}{2},\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{2}\right)}{110\sqrt{2}\sqrt{\frac{2x^2+1}{5x^2+3}}\sqrt{11x^2+7}}+\frac{5\sqrt{2x^2+1}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

input
```
Int[(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
(5*x*Sqrt[1 + 2*x^2]*Sqrt[7 + 11*x^2])/(22*Sqrt[3 + 5*x^2]) - (Sqrt[1 + 2*
x^2]*Sqrt[(7 + 11*x^2)/(3 + 5*x^2)]*EllipticE[ArcSin[(Sqrt[2/7]*x)/Sqrt[3
+ 5*x^2]], -7/2])/(11*Sqrt[2]*Sqrt[(1 + 2*x^2)/(3 + 5*x^2)]*Sqrt[7 + 11*x^
2]) + (6*Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticF[ArcSin[x
/(Sqrt[3]*Sqrt[1 + 2*x^2])], 9/7])/(55*Sqrt[7]*Sqrt[(3 + 5*x^2)/(1 + 2*x^2
)]*Sqrt[7 + 11*x^2]) + (153*Sqrt[1 + 2*x^2]*Sqrt[(7 + 11*x^2)/(3 + 5*x^2)]
*EllipticPi[35/2, ArcSin[(Sqrt[2/7]*x)/Sqrt[3 + 5*x^2]], -7/2])/(110*Sqrt[
2]*Sqrt[(1 + 2*x^2)/(3 + 5*x^2)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}}{\sqrt{11x^2+7}}dx$$

input
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}\sqrt{5x^2+3}}{\sqrt{11x^2+7}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)*(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)/sqrt(11*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\,\sqrt{5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
int(((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2))/(11*x^2 + 7)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2))/(11*x^2 + 7)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{11x^2+7}}{11x^2+7}dx$$

input
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7))/(11*x**2 + 7),x)
```

3.502 $\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 334

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \frac{5x\sqrt{1-2x^2}\sqrt{7+11x^2}}{22\sqrt{3+5x^2}} - \frac{\sqrt{\frac{7}{11}}\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3+5x^2}}\,E\left(\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right)|\frac{2}{77}\right)}{2\sqrt{\frac{1-2x^2}{3+5x^2}}\sqrt{7+11x^2}} + \frac{3\sqrt{\frac{11}{7}}\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3+5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right),\frac{2}{77}\right)}{10\sqrt{\frac{1-2x^2}{3+5x^2}}\sqrt{7+11x^2}} + \frac{177\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3+5x^2}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right),\frac{2}{77}\right)}{110\sqrt{77}\sqrt{\frac{1-2x^2}{3+5x^2}}\sqrt{7+11x^2}}$$

output
```
5/22*x*(-2*x^2+1)^(1/2)*(11*x^2+7)^(1/2)/(5*x^2+3)^(1/2)-1/22*77^(1/2)*(-2
*x^2+1)^(1/2)*((11*x^2+7)/(5*x^2+3))^(1/2)*EllipticE(11^(1/2)*x/(5*x^2+3)^
(1/2),1/77*154^(1/2))/((-2*x^2+1)/(5*x^2+3))^(1/2)/(11*x^2+7)^(1/2)+3/70*(
-2*x^2+1)^(1/2)*((11*x^2+7)/(5*x^2+3))^(1/2)*EllipticF(11^(1/2)*x/(5*x^2+3
)^(1/2),1/77*154^(1/2))*77^(1/2)/((-2*x^2+1)/(5*x^2+3))^(1/2)/(11*x^2+7)^(
1/2)+177/8470*(-2*x^2+1)^(1/2)*((11*x^2+7)/(5*x^2+3))^(1/2)*EllipticPi(11^
(1/2)*x/(5*x^2+3)^(1/2),5/11,1/77*154^(1/2))*77^(1/2)/((-2*x^2+1)/(5*x^2+3
))^(1/2)/(11*x^2+7)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx$$

input
```
Integrate[(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
Integrate[(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 + 11*x^2], x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 320, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}\sqrt{5x^2+3}}{\sqrt{11x^2+7}}dx$$

$$\Big\downarrow \text{430}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx-\frac{6}{55}\int \frac{1}{\sqrt{1-2x^2}\sqrt{5x^2+3}\sqrt{11x^2+7}}dx+\frac{59}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{1-2x^2}\sqrt{11x^2+7}}dx+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{427}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{59}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{1-2x^2}\sqrt{11x^2+7}}dx-\frac{2\sqrt{\frac{3}{7}}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{\sqrt{21}}{\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{55\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{59}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{1-2x^2}\sqrt{11x^2+7}}dx-\frac{6\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\int \frac{1}{\sqrt{\frac{11x^2}{1-2x^2}+3}\sqrt{\frac{25x^2}{1-2x^2}+7}}d\frac{x}{\sqrt{1-2x^2}}}{55\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 320}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{59}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{1-2x^2}\sqrt{11x^2+7}}dx-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 428}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{7}}{\sqrt{1-\frac{11x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{7}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{1}{\sqrt{1-\frac{11x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 412$$

$$-\frac{3}{11}\int \frac{\sqrt{1-2x^2}}{(5x^2+3)^{3/2}\sqrt{11x^2+7}}dx+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{2}{77}\right)}{110\sqrt{77}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 429$$

$$-\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{7}\sqrt{1-\frac{11x^2}{5x^2+3}}}{\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{2}{77}\right)}{110\sqrt{77}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\int \frac{\sqrt{1-\frac{11x^2}{5x^2+3}}}{\sqrt{7-\frac{2x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{2}{77}\right)}{110\sqrt{77}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}E\left(\arcsin\left(\frac{\sqrt{\frac{2}{7}}x}{\sqrt{5x^2+3}}\right)|\frac{77}{2}\right)}{11\sqrt{2}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}+\frac{177\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{2}{77}\right)}{110\sqrt{77}\sqrt{\frac{1-2x^2}{5x^2+3}}\sqrt{11x^2+7}}-\frac{2\sqrt{3}\sqrt{5x^2+3}\sqrt{\frac{11x^2+7}{1-2x^2}}\sqrt{\frac{11x^2}{1-2x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7}\sqrt{1-2x^2}}\right),-\frac{2}{75}\right)}{275\sqrt{\frac{5x^2+3}{1-2x^2}}\sqrt{11x^2+7}\sqrt{\frac{\frac{11x^2}{1-2x^2}+3}{\frac{25x^2}{1-2x^2}+7}}\sqrt{\frac{25x^2}{1-2x^2}+7}}+\frac{5\sqrt{1-2x^2}\sqrt{11x^2+7}x}{22\sqrt{5x^2+3}}$$

input
```
Int[(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
(5*x*Sqrt[1 - 2*x^2]*Sqrt[7 + 11*x^2])/(22*Sqrt[3 + 5*x^2]) - (Sqrt[1 - 2*
x^2]*Sqrt[(7 + 11*x^2)/(3 + 5*x^2)]*EllipticE[ArcSin[(Sqrt[2/7]*x)/Sqrt[3
+ 5*x^2]], 77/2])/(11*Sqrt[2]*Sqrt[(1 - 2*x^2)/(3 + 5*x^2)]*Sqrt[7 + 11*x^
2]) - (2*Sqrt[3]*Sqrt[3 + 5*x^2]*Sqrt[(7 + 11*x^2)/(1 - 2*x^2)]*Sqrt[3 + (
11*x^2)/(1 - 2*x^2)]*EllipticF[ArcTan[(5*x)/(Sqrt[7]*Sqrt[1 - 2*x^2])], -2
/75])/(275*Sqrt[(3 + 5*x^2)/(1 - 2*x^2)]*Sqrt[7 + 11*x^2]*Sqrt[(3 + (11*x^
2)/(1 - 2*x^2))/(7 + (25*x^2)/(1 - 2*x^2))]*Sqrt[7 + (25*x^2)/(1 - 2*x^2)]
) + (177*Sqrt[1 - 2*x^2]*Sqrt[(7 + 11*x^2)/(3 + 5*x^2)]*EllipticPi[5/11, A
rcSin[(Sqrt[11]*x)/Sqrt[3 + 5*x^2]], 2/77])/(110*Sqrt[77]*Sqrt[(1 - 2*x^2)
/(3 + 5*x^2)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}\,\sqrt{5x^2+3}}{\sqrt{11x^2+7}}dx$$

input
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}\sqrt{5x^2+3}}{\sqrt{11x^2+7}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)*(5*x**2+3)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)*sqrt(5*x**2 + 3)/sqrt(11*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx=\int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/sqrt(11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx=\int \frac{\sqrt{1-2\,x^2}\,\sqrt{5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
int(((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2))/(11*x^2 + 7)^(1/2),x)
```

output
```
int(((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2))/(11*x^2 + 7)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7+11x^2}}\,dx=\int \frac{\sqrt{5x^2+3}\,\sqrt{11x^2+7}\,\sqrt{-2x^2+1}}{11x^2+7}dx$$

input
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(5*x**2 + 3)*sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1))/(11*x**2 + 7)
,x)
```

3.503 $\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 340

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \frac{x\sqrt{3-5x^2}\sqrt{7+11x^2}}{11\sqrt{1+2x^2}} - \frac{\sqrt{\frac{7}{11}}\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right)|\frac{9}{77}\right)}{2\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}} + \frac{\sqrt{\frac{11}{7}}\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{9}{77}\right)}{4\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}} + \frac{81\sqrt{3-5x^2}\sqrt{\frac{7+11x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{3-5x^2}{1+2x^2}}\sqrt{7+11x^2}}$$

output
```
1/11*x*(-5*x^2+3)^(1/2)*(11*x^2+7)^(1/2)/(2*x^2+1)^(1/2)-1/22*77^(1/2)*(-5
*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticE(1/3*33^(1/2)*x/(2*x^2
+1)^(1/2),3/77*77^(1/2))/((-5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^2+7)^(1/2)+1/2
8*(-5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticF(1/3*33^(1/2)*x/(
2*x^2+1)^(1/2),3/77*77^(1/2))*77^(1/2)/((-5*x^2+3)/(2*x^2+1))^(1/2)/(11*x^
2+7)^(1/2)+81/3388*(-5*x^2+3)^(1/2)*((11*x^2+7)/(2*x^2+1))^(1/2)*EllipticP
i(1/3*33^(1/2)*x/(2*x^2+1)^(1/2),6/11,3/77*77^(1/2))*77^(1/2)/((-5*x^2+3)/
(2*x^2+1))^(1/2)/(11*x^2+7)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx$$

input
```
Integrate[(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
Integrate[(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])/Sqrt[7 + 11*x^2], x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 320, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-5x^2}\sqrt{2x^2+1}}{\sqrt{11x^2+7}}\,dx$$

$$\Big\downarrow \text{ 430}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{\left(2x^2+1\right)^{3/2}\sqrt{11x^2+7}}dx-\frac{15}{44}\int \frac{1}{\sqrt{3-5x^2}\sqrt{2x^2+1}\sqrt{11x^2+7}}dx+\frac{81}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}dx+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{427}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{81}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}dx-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{\sqrt{7}}{\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{44\sqrt{7}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{81}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}dx-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{1}{\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{44\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{320}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{81}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{3-5x^2}\sqrt{11x^2+7}}dx-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{428}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{27\sqrt{\frac{3}{7}}\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{3-\frac{11x^2}{2x^2+1}}\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{44\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{81\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{1}{\sqrt{3-\frac{11x^2}{2x^2+1}}\sqrt{7-\frac{3x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{44\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{3}{22}\int \frac{\sqrt{3-5x^2}}{(2x^2+1)^{3/2}\sqrt{11x^2+7}}dx+\frac{81\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{2x^2+1}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{429}$$

$$-\frac{3\sqrt{\frac{3}{7}}\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{\frac{7}{3}}\sqrt{3-\frac{11x^2}{2x^2+1}}}{\sqrt{7-\frac{3x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{22\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}+\frac{81\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{2x^2+1}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\int \frac{\sqrt{3-\frac{11x^2}{2x^2+1}}}{\sqrt{7-\frac{3x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{22\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}+\frac{81\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{2x^2+1}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \ 327$$

$$-\frac{3\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}E\left(\arcsin\left(\frac{\sqrt{\frac{3}{7}}x}{\sqrt{2x^2+1}}\right)|\frac{77}{9}\right)}{22\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}+\frac{81\sqrt{3-5x^2}\sqrt{\frac{11x^2+7}{2x^2+1}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{2x^2+1}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{3-5x^2}{2x^2+1}}\sqrt{11x^2+7}}-\frac{15\sqrt{2x^2+1}\sqrt{\frac{11x^2+7}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{11}x}{\sqrt{3-5x^2}}\right),\frac{9}{77}\right)}{44\sqrt{77}\sqrt{\frac{2x^2+1}{3-5x^2}}\sqrt{11x^2+7}\sqrt{\frac{11x^2}{3-5x^2}+1}\sqrt{\frac{\frac{68x^2}{3-5x^2}+7}{\frac{11x^2}{3-5x^2}+1}}}+\frac{\sqrt{3-5x^2}\sqrt{11x^2+7}x}{11\sqrt{2x^2+1}}$$

input

```
Int[(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])/Sqrt[7 + 11*x^2],x]
```

output

```
(x*Sqrt[3 - 5*x^2]*Sqrt[7 + 11*x^2])/(11*Sqrt[1 + 2*x^2]) - (3*Sqrt[3 - 5*
x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticE[ArcSin[(Sqrt[3/7]*x)/Sqrt[1
+ 2*x^2]], 77/9])/(22*Sqrt[(3 - 5*x^2)/(1 + 2*x^2)]*Sqrt[7 + 11*x^2]) - (1
5*Sqrt[1 + 2*x^2]*Sqrt[(7 + 11*x^2)/(3 - 5*x^2)]*Sqrt[7 + (68*x^2)/(3 - 5*
x^2)]*EllipticF[ArcTan[(Sqrt[11]*x)/Sqrt[3 - 5*x^2]], 9/77])/(44*Sqrt[77]*
Sqrt[(1 + 2*x^2)/(3 - 5*x^2)]*Sqrt[7 + 11*x^2]*Sqrt[1 + (11*x^2)/(3 - 5*x^
2)]*Sqrt[(7 + (68*x^2)/(3 - 5*x^2))/(1 + (11*x^2)/(3 - 5*x^2))]) + (81*Sqr
t[3 - 5*x^2]*Sqrt[(7 + 11*x^2)/(1 + 2*x^2)]*EllipticPi[6/11, ArcSin[(Sqrt[
11/3]*x)/Sqrt[1 + 2*x^2]], 9/77])/(44*Sqrt[77]*Sqrt[(3 - 5*x^2)/(1 + 2*x^2
)]*Sqrt[7 + 11*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{-5x^2+3}\,\sqrt{2x^2+1}}{\sqrt{11x^2+7}}dx$$

input
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{3-5x^2}\sqrt{2x^2+1}}{\sqrt{11x^2+7}}\,dx$$

input
```
integrate((-5*x**2+3)**(1/2)*(2*x**2+1)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(3 - 5*x**2)*sqrt(2*x**2 + 1)/sqrt(11*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\,\sqrt{3-5\,x^2}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
int(((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2))/(11*x^2 + 7)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2))/(11*x^2 + 7)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{2x^2+1}\,\sqrt{11x^2+7}\,\sqrt{-5x^2+3}}{11x^2+7}dx$$

input
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(2*x**2 + 1)*sqrt(11*x**2 + 7)*sqrt( - 5*x**2 + 3))/(11*x**2 + 7)
,x)
```

3.504 $\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 313

$$\begin{aligned}\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx &= -\frac{5x\sqrt{1-2x^2}\sqrt{7+11x^2}}{22\sqrt{3-5x^2}}-\frac{\sqrt{7}\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3-5x^2}}E\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right)|-\frac{68}{7}\right)}{22\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{7+11x^2}}\\ &\quad-\frac{3\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{10\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{7+11x^2}}\\ &\quad+\frac{573\sqrt{1-2x^2}\sqrt{\frac{7+11x^2}{3-5x^2}}\,\text{EllipticPi}\left(-5,\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{110\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{7+11x^2}}\end{aligned}$$

output
```
-5/22*x*(-2*x^2+1)^(1/2)*(11*x^2+7)^(1/2)/(-5*x^2+3)^(1/2)-1/22*7^(1/2)*(-
2*x^2+1)^(1/2)*((11*x^2+7)/(-5*x^2+3))^(1/2)*EllipticE(x/(-5*x^2+3)^(1/2),
2/7*I*119^(1/2))/((-2*x^2+1)/(-5*x^2+3))^(1/2)/(11*x^2+7)^(1/2)-3/70*(-2*x
^2+1)^(1/2)*((11*x^2+7)/(-5*x^2+3))^(1/2)*EllipticF(x/(-5*x^2+3)^(1/2),2/7
*I*119^(1/2))*7^(1/2)/((-2*x^2+1)/(-5*x^2+3))^(1/2)/(11*x^2+7)^(1/2)+573/7
70*(-2*x^2+1)^(1/2)*((11*x^2+7)/(-5*x^2+3))^(1/2)*EllipticPi(x/(-5*x^2+3)^
(1/2),-5,2/7*I*119^(1/2))*7^(1/2)/((-2*x^2+1)/(-5*x^2+3))^(1/2)/(11*x^2+7)
^(1/2)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx$$
$$= \frac{\frac{12705x\sqrt{3-5x^2}(1-2x^2)}{\sqrt{7+11x^2}} - \frac{5775\sqrt{9-15x^2}\sqrt{\frac{1-2x^2}{7+11x^2}}E\left(\arcsin\left(\frac{5x}{\sqrt{7+11x^2}}\right)|\frac{68}{75}\right)}{\sqrt{\frac{3-5x^2}{7+11x^2}}} - \frac{37875(-1+2x^2)\sqrt{\frac{49+77x^2}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right)\right.}{\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{7+11x^2}}}{25410\sqrt{1-2x^2}}$$

input
```
Integrate[(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
((12705*x*Sqrt[3 - 5*x^2]*(1 - 2*x^2))/Sqrt[7 + 11*x^2] - (5775*Sqrt[9 - 1
5*x^2]*Sqrt[(1 - 2*x^2)/(7 + 11*x^2)]*EllipticE[ArcSin[(5*x)/Sqrt[7 + 11*x
^2]], 68/75])/Sqrt[(3 - 5*x^2)/(7 + 11*x^2)] - (37875*(-1 + 2*x^2)*Sqrt[(4
9 + 77*x^2)/(3 - 5*x^2)]*EllipticF[ArcSin[x/Sqrt[3 - 5*x^2]], -68/7])/(Sqr
t[(1 - 2*x^2)/(3 - 5*x^2)]*Sqrt[7 + 11*x^2]) - (9359*Sqrt[9 - 15*x^2]*Sqrt
[(1 - 2*x^2)/(7 + 11*x^2)]*EllipticPi[11/25, ArcSin[(5*x)/Sqrt[7 + 11*x^2]
], 68/75])/Sqrt[(3 - 5*x^2)/(7 + 11*x^2)])/(25410*Sqrt[1 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {431, 427, 27, 321, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{11x^2+7}}dx$$
$$\Big\downarrow \text{431}$$
$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\,(11x^2+7)^{3/2}}dx+\frac{1224}{121}\int \frac{1}{\sqrt{3-5x^2}\sqrt{1-2x^2}\sqrt{11x^2+7}}dx-\frac{191}{242}\int \frac{\sqrt{11x^2+7}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{427}$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\left(11x^2+7\right)^{3/2}}dx-\frac{191}{242}\int \frac{\sqrt{11x^2+7}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{\sqrt{7}}{\sqrt{1-\frac{x^2}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\left(11x^2+7\right)^{3/2}}dx-\frac{191}{242}\int \frac{\sqrt{11x^2+7}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\int \frac{1}{\sqrt{1-\frac{x^2}{3-5x^2}}\sqrt{\frac{68x^2}{3-5x^2}+7}}d\frac{x}{\sqrt{3-5x^2}}}{121\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\left(11x^2+7\right)^{3/2}}dx-\frac{191}{242}\int \frac{\sqrt{11x^2+7}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{428}$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\left(11x^2+7\right)^{3/2}}dx-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\int \frac{\sqrt{3}}{\sqrt{3-\frac{68x^2}{11x^2+7}}\sqrt{1-\frac{25x^2}{11x^2+7}}\left(1-\frac{11x^2}{11x^2+7}\right)}d\frac{x}{\sqrt{11x^2+7}}}{242\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\,(11x^2+7)^{3/2}}dx-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\int \frac{1}{\sqrt{3-\frac{68x^2}{11x^2+7}}\sqrt{1-\frac{25x^2}{11x^2+7}}\left(1-\frac{11x^2}{11x^2+7}\right)}d\frac{x}{\sqrt{11x^2+7}}}{242\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \ 412$$

$$-\frac{238}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{3-5x^2}\,(11x^2+7)^{3/2}}dx+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\,\text{EllipticPi}\left(\frac{11}{25},\arcsin\left(\frac{5x}{\sqrt{11x^2+7}}\right),\frac{68}{75}\right)}{1210\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \ 429$$

$$-\frac{34\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}\int \frac{\sqrt{3}\sqrt{1-\frac{25x^2}{11x^2+7}}}{\sqrt{3-\frac{68x^2}{11x^2+7}}}d\frac{x}{\sqrt{11x^2+7}}}{11\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}}+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\,\text{EllipticPi}\left(\frac{11}{25},\arcsin\left(\frac{5x}{\sqrt{11x^2+7}}\right),\frac{68}{75}\right)}{1210\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \ 27$$

$$-\frac{34\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}\int \frac{\sqrt{1-\frac{25x^2}{11x^2+7}}}{\sqrt{3-\frac{68x^2}{11x^2+7}}}d\frac{x}{\sqrt{11x^2+7}}}{11\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}}+\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\,\text{EllipticPi}\left(\frac{11}{25},\arcsin\left(\frac{5x}{\sqrt{11x^2+7}}\right),\frac{68}{75}\right)}{1210\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

$$\Big\downarrow \; 327$$

$$\frac{1224\sqrt{1-2x^2}\sqrt{\frac{11x^2+7}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{68}{7}\right)}{121\sqrt{7}\sqrt{\frac{1-2x^2}{3-5x^2}}\sqrt{11x^2+7}}-\frac{\sqrt{17}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}E\left(\arcsin\left(\frac{2\sqrt{\frac{17}{3}}x}{\sqrt{11x^2+7}}\right)|\frac{75}{68}\right)}{11\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}}-\frac{1337\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{11x^2+7}}\,\text{EllipticPi}\left(\frac{11}{25},\arcsin\left(\frac{5x}{\sqrt{11x^2+7}}\right),\frac{68}{75}\right)}{1210\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{3-5x^2}{11x^2+7}}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{11x^2+7}}$$

input
```
Int[(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/Sqrt[7 + 11*x^2],x]
```

output
```
(x*Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/(2*Sqrt[7 + 11*x^2]) - (Sqrt[17]*Sqrt[
1 - 2*x^2]*Sqrt[(3 - 5*x^2)/(7 + 11*x^2)]*EllipticE[ArcSin[(2*Sqrt[17/3]*x
)/Sqrt[7 + 11*x^2]], 75/68])/(11*Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 + 11*
x^2)]) + (1224*Sqrt[1 - 2*x^2]*Sqrt[(7 + 11*x^2)/(3 - 5*x^2)]*EllipticF[Ar
cSin[x/Sqrt[3 - 5*x^2]], -68/7])/(121*Sqrt[7]*Sqrt[(1 - 2*x^2)/(3 - 5*x^2)
]*Sqrt[7 + 11*x^2]) - (1337*Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 + 11*x^2)]
*EllipticPi[11/25, ArcSin[(5*x)/Sqrt[7 + 11*x^2]], 68/75])/(1210*Sqrt[3]*S
qrt[1 - 2*x^2]*Sqrt[(3 - 5*x^2)/(7 + 11*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 431
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Simp[e*((b*e - a*f)/(2*f))   Int[Sqrt[c + d*x^2]/(Sqrt[a +
b*x^2]*(e + f*x^2)^(3/2)), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*f^2)
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[(b*e -
 a*f)*((d*e - 2*c*f)/(2*f^2))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt
[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c
]
```

Maple [F]

$$\int \frac{\sqrt{-5x^2+3}\,\sqrt{-2x^2+1}}{\sqrt{11x^2+7}}dx$$

input
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}\sqrt{3-5x^2}}{\sqrt{11x^2+7}}\,dx$$

input
```
integrate((-5*x**2+3)**(1/2)*(-2*x**2+1)**(1/2)/(11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)*sqrt(3 - 5*x**2)/sqrt(11*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}\,\sqrt{3-5\,x^2}}{\sqrt{11\,x^2+7}}\,dx$$

input
```
int(((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2))/(11*x^2 + 7)^(1/2),x)
```

output
```
int(((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2))/(11*x^2 + 7)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7+11x^2}}\,dx = \int \frac{\sqrt{11x^2+7}\,\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}}{11x^2+7}dx$$

input
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(11*x^2+7)^(1/2),x)
```

output
```
int((sqrt(11*x**2 + 7)*sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3))/(11*x**2 +
 7),x)
```

3.505 $\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 328

$$\begin{aligned}
\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx &= -\frac{x\sqrt{7-11x^2}\sqrt{3+5x^2}}{11\sqrt{1+2x^2}} + \frac{\sqrt{7}\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}E\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right)|\frac{75}{7}\right)}{22\sqrt{\frac{7-11x^2}{1+2x^2}}\sqrt{3+5x^2}} \\
&\quad + \frac{\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{75}{7}\right)}{4\sqrt{7}\sqrt{\frac{7-11x^2}{1+2x^2}}\sqrt{3+5x^2}} \\
&\quad + \frac{191\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{1+2x^2}}\,\text{EllipticPi}\left(6,\arcsin\left(\frac{x}{\sqrt{3}\sqrt{1+2x^2}}\right),\frac{75}{7}\right)}{44\sqrt{7}\sqrt{\frac{7-11x^2}{1+2x^2}}\sqrt{3+5x^2}}
\end{aligned}$$

output

```
-1/11*x*(-11*x^2+7)^(1/2)*(5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)+1/22*7^(1/2)*(-1
1*x^2+7)^(1/2)*((5*x^2+3)/(2*x^2+1))^(1/2)*EllipticE(1/3*x*3^(1/2)/(2*x^2+
1)^(1/2),5/7*21^(1/2))/((-11*x^2+7)/(2*x^2+1))^(1/2)/(5*x^2+3)^(1/2)+1/28*
(-11*x^2+7)^(1/2)*((5*x^2+3)/(2*x^2+1))^(1/2)*EllipticF(1/3*x*3^(1/2)/(2*x
^2+1)^(1/2),5/7*21^(1/2))*7^(1/2)/((-11*x^2+7)/(2*x^2+1))^(1/2)/(5*x^2+3)^
(1/2)+191/308*(-11*x^2+7)^(1/2)*((5*x^2+3)/(2*x^2+1))^(1/2)*EllipticPi(1/3
*x*3^(1/2)/(2*x^2+1)^(1/2),6,5/7*21^(1/2))*7^(1/2)/((-11*x^2+7)/(2*x^2+1))
^(1/2)/(5*x^2+3)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx$$

input
```
Integrate[(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
Integrate[(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 - 11*x^2], x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 320, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}\sqrt{5x^2+3}}{\sqrt{7-11x^2}}dx$$

$$\Big\downarrow \text{ 430}$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx-\frac{204}{55}\int \frac{1}{\sqrt{7-11x^2}\sqrt{2x^2+1}\sqrt{5x^2+3}}dx+\frac{191}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{2x^2+1}}dx-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 427}$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx+\frac{191}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{2x^2+1}}dx-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{55\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\left(5x^2+3\right)^{3/2}}dx+\frac{191}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{2x^2+1}}dx-\frac{204\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{1}{\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{55\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \ 320$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\left(5x^2+3\right)^{3/2}}dx+\frac{191}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{2x^2+1}}dx-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \ 428$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\left(5x^2+3\right)^{3/2}}dx+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\int \frac{\sqrt{7}}{\sqrt{7-\frac{68x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{\frac{x^2}{5x^2+3}+1}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{7}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\int \frac{1}{\sqrt{7-\frac{68x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)\sqrt{\frac{x^2}{5x^2+3}+1}}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 412$$

$$\frac{102}{11}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\,\text{EllipticPi}\left(\frac{35}{68},\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{68}\right)}{220\sqrt{17}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \; 429$$

$$\frac{34\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}\int \frac{\sqrt{7}\sqrt{\frac{x^2}{5x^2+3}+1}}{\sqrt{7-\frac{68x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7}\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}}+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\,\text{EllipticPi}\left(\frac{35}{68},\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{68}\right)}{220\sqrt{17}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\,\text{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{34\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}\int\frac{\sqrt{\frac{x^2}{5x^2+3}+1}}{\sqrt{7-\frac{68x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}}+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\operatorname{EllipticPi}\left(\frac{35}{68},\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{68}\right)}{220\sqrt{17}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\operatorname{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{17}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}E\left(\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right)|-\frac{7}{68}\right)}{11\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}}+\frac{573\sqrt{7-11x^2}\sqrt{\frac{2x^2+1}{5x^2+3}}\operatorname{EllipticPi}\left(\frac{35}{68},\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right),-\frac{7}{68}\right)}{220\sqrt{17}\sqrt{2x^2+1}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{68\sqrt{3}\sqrt{2x^2+1}\sqrt{\frac{5x^2+3}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}\operatorname{EllipticF}\left(\arctan\left(\frac{5x}{\sqrt{7-11x^2}}\right),\frac{7}{75}\right)}{275\sqrt{\frac{2x^2+1}{7-11x^2}}\sqrt{5x^2+3}\sqrt{\frac{25x^2}{7-11x^2}+1}\sqrt{\frac{\frac{68x^2}{7-11x^2}+3}{\frac{25x^2}{7-11x^2}+1}}}-\frac{5\sqrt{7-11x^2}\sqrt{2x^2+1}x}{22\sqrt{5x^2+3}}$$

input
```
Int[(Sqrt[1 + 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(-5*x*Sqrt[7 - 11*x^2]*Sqrt[1 + 2*x^2])/(22*Sqrt[3 + 5*x^2]) + (Sqrt[17]*S
qrt[1 + 2*x^2]*Sqrt[(7 - 11*x^2)/(3 + 5*x^2)]*EllipticE[ArcSin[(2*Sqrt[17/
7]*x)/Sqrt[3 + 5*x^2]], -7/68])/(11*Sqrt[7 - 11*x^2]*Sqrt[(1 + 2*x^2)/(3 +
 5*x^2)]) - (68*Sqrt[3]*Sqrt[1 + 2*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*Sqr
t[3 + (68*x^2)/(7 - 11*x^2)]*EllipticF[ArcTan[(5*x)/Sqrt[7 - 11*x^2]], 7/7
5])/(275*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*Sqrt[3 + 5*x^2]*Sqrt[1 + (25*x^2)/
(7 - 11*x^2)]*Sqrt[(3 + (68*x^2)/(7 - 11*x^2))/(1 + (25*x^2)/(7 - 11*x^2))
]) + (573*Sqrt[7 - 11*x^2]*Sqrt[(1 + 2*x^2)/(3 + 5*x^2)]*EllipticPi[35/68,
 ArcSin[(2*Sqrt[17/7]*x)/Sqrt[3 + 5*x^2]], -7/68])/(220*Sqrt[17]*Sqrt[1 +
2*x^2]*Sqrt[(7 - 11*x^2)/(3 + 5*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}}{\sqrt{-11x^2+7}}dx$$

input
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx=\int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)*sqrt(-11*x^2 + 7)/(11*x^2 - 7),
x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx=\int \frac{\sqrt{2x^2+1}\sqrt{5x^2+3}}{\sqrt{7-11x^2}}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)*(5*x**2+3)**(1/2)/(-11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)/sqrt(7 - 11*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx=\int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/sqrt(-11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(2*x^2 + 1)/sqrt(-11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\,\sqrt{5\,x^2+3}}{\sqrt{7-11\,x^2}}\,dx$$

input
```
int(((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2))/(7 - 11*x^2)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^(1/2)*(5*x^2 + 3)^(1/2))/(7 - 11*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{5x^2+3}\,\sqrt{-11x^2+7}}{11x^2-7}dx\right)$$

input
```
int((2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt(2*x**2 + 1)*sqrt(5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(11*x**2 -
7),x)
```

3.506 $\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 334

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = -\frac{5x\sqrt{7-11x^2}\sqrt{1-2x^2}}{22\sqrt{3+5x^2}} + \frac{\sqrt{\frac{7}{11}}\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3+5x^2}}\,E\left(\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right)|\frac{68}{77}\right)}{2\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{3+5x^2}}} + \frac{3\sqrt{\frac{11}{7}}\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3+5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right),\frac{68}{77}\right)}{10\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{3+5x^2}}} - \frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3+5x^2}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{3+5x^2}}\right),\frac{68}{77}\right)}{110\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{3+5x^2}}}$$

output
```
-5/22*x*(-11*x^2+7)^(1/2)*(-2*x^2+1)^(1/2)/(5*x^2+3)^(1/2)+1/22*77^(1/2)*(
-11*x^2+7)^(1/2)*((-2*x^2+1)/(5*x^2+3))^(1/2)*EllipticE(11^(1/2)*x/(5*x^2+
3)^(1/2),2/77*1309^(1/2))/(-2*x^2+1)^(1/2)/((-11*x^2+7)/(5*x^2+3))^(1/2)+3
/70*77^(1/2)*(-11*x^2+7)^(1/2)*((-2*x^2+1)/(5*x^2+3))^(1/2)*EllipticF(11^(
1/2)*x/(5*x^2+3)^(1/2),2/77*1309^(1/2))/(-2*x^2+1)^(1/2)/((-11*x^2+7)/(5*x
^2+3))^(1/2)-243/8470*(-11*x^2+7)^(1/2)*((-2*x^2+1)/(5*x^2+3))^(1/2)*Ellip
ticPi(11^(1/2)*x/(5*x^2+3)^(1/2),5/11,2/77*1309^(1/2))*77^(1/2)/(-2*x^2+1)
^(1/2)/((-11*x^2+7)/(5*x^2+3))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \frac{\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\left(1089x\sqrt{\frac{1-2x^2}{7-11x^2}}+1815x^3\sqrt{\frac{1-2x^2}{7-11x^2}}+66i\sqrt{17}\sqrt{7-11x^2}\sqrt{\frac{3+5x^2}{7-11x^2}}E\left(i\operatorname{arcsinh}\left(\frac{2\sqrt{\frac{17}{3}}\cdots}{\sqrt{7-11\cdots}}\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*(1089*x*Sqrt[(1 - 2*x^2)/
(7 - 11*x^2)] + 1815*x^3*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)] + (66*I)*Sqrt[17]*
Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*EllipticE[I*ArcSinh[(2*Sqr
t[17/3]*x)/Sqrt[7 - 11*x^2]], -9/68] + 544*Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^
2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqrt[3]*x)/Sqrt[7 - 11*x^2]], -68/9] +
567*Sqrt[7 - 11*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*EllipticPi[-11/3, ArcS
in[(Sqrt[3]*x)/Sqrt[7 - 11*x^2]], -68/9]))/(726*Sqrt[1 - 2*x^2]*Sqrt[3 + 5
*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 321, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-2x^2}\sqrt{5x^2+3}}{\sqrt{7-11x^2}}\,dx$$

$$\Big\downarrow \ 430$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx+\frac{204}{55}\int \frac{1}{\sqrt{7-11x^2}\sqrt{1-2x^2}\sqrt{5x^2+3}}dx-\frac{81}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{1-2x^2}}dx-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 427}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx-\frac{81}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{1-2x^2}}dx+\frac{68\sqrt{3}\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx-\frac{81}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{1-2x^2}}dx+\frac{204\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\int \frac{1}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{\frac{68x^2}{7-11x^2}+3}}d\frac{x}{\sqrt{7-11x^2}}}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx-\frac{81}{110}\int \frac{\sqrt{5x^2+3}}{\sqrt{7-11x^2}\sqrt{1-2x^2}}dx+\frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 428}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx-\frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\int \frac{\sqrt{7}}{\sqrt{7-\frac{68x^2}{5x^2+3}}\sqrt{1-\frac{11x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{7}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}}+\frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx - \frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\int \frac{1}{\sqrt{7-\frac{68x^2}{5x^2+3}}\sqrt{1-\frac{11x^2}{5x^2+3}}\left(1-\frac{5x^2}{5x^2+3}\right)}d\frac{x}{\sqrt{5x^2+3}}}{110\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}} + \frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}} - \frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{412}$$

$$\frac{102}{11}\int \frac{\sqrt{1-2x^2}}{\sqrt{7-11x^2}\,(5x^2+3)^{3/2}}dx + \frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}} - \frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{68}{77}\right)}{110\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}} - \frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{429}$$

$$\frac{34\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}\int \frac{\sqrt{7}\sqrt{1-\frac{11x^2}{5x^2+3}}}{\sqrt{7-\frac{68x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7}\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}} + \frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}} - \frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{68}{77}\right)}{110\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}} - \frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{34\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}\int \frac{\sqrt{1-\frac{11x^2}{5x^2+3}}}{\sqrt{7-\frac{68x^2}{5x^2+3}}}d\frac{x}{\sqrt{5x^2+3}}}{11\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}} + \frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}} - \frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\,\text{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{68}{77}\right)}{110\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}} - \frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{68\sqrt{1-2x^2}\sqrt{\frac{5x^2+3}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),-\frac{68}{9}\right)}{55\sqrt{\frac{1-2x^2}{7-11x^2}}\sqrt{5x^2+3}}+\frac{\sqrt{17}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}E\left(\arcsin\left(\frac{2\sqrt{\frac{17}{7}}x}{\sqrt{5x^2+3}}\right)|\frac{77}{68}\right)}{11\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}}-\frac{243\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{5x^2+3}}\operatorname{EllipticPi}\left(\frac{5}{11},\arcsin\left(\frac{\sqrt{11}x}{\sqrt{5x^2+3}}\right),\frac{68}{77}\right)}{110\sqrt{77}\sqrt{1-2x^2}\sqrt{\frac{7-11x^2}{5x^2+3}}}-\frac{5\sqrt{7-11x^2}\sqrt{1-2x^2}x}{22\sqrt{5x^2+3}}$$

input
```
Int[(Sqrt[1 - 2*x^2]*Sqrt[3 + 5*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(-5*x*Sqrt[7 - 11*x^2]*Sqrt[1 - 2*x^2])/(22*Sqrt[3 + 5*x^2]) + (Sqrt[17]*S
qrt[1 - 2*x^2]*Sqrt[(7 - 11*x^2)/(3 + 5*x^2)]*EllipticE[ArcSin[(2*Sqrt[17/
7]*x)/Sqrt[3 + 5*x^2]], 77/68])/(11*Sqrt[7 - 11*x^2]*Sqrt[(1 - 2*x^2)/(3 +
 5*x^2)]) + (68*Sqrt[1 - 2*x^2]*Sqrt[(3 + 5*x^2)/(7 - 11*x^2)]*EllipticF[A
rcSin[(Sqrt[3]*x)/Sqrt[7 - 11*x^2]], -68/9])/(55*Sqrt[(1 - 2*x^2)/(7 - 11*
x^2)]*Sqrt[3 + 5*x^2]) - (243*Sqrt[7 - 11*x^2]*Sqrt[(1 - 2*x^2)/(3 + 5*x^2
)]*EllipticPi[5/11, ArcSin[(Sqrt[11]*x)/Sqrt[3 + 5*x^2]], 68/77])/(110*Sqr
t[77]*Sqrt[1 - 2*x^2]*Sqrt[(7 - 11*x^2)/(3 + 5*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{-2x^2+1}\,\sqrt{5x^2+3}}{\sqrt{-11x^2+7}}dx$$

input
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)*sqrt(-11*x^2 + 7)/(11*x^2 - 7),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}\sqrt{5x^2+3}}{\sqrt{7-11x^2}}\,dx$$

input
```
integrate((-2*x**2+1)**(1/2)*(5*x**2+3)**(1/2)/(-11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)*sqrt(5*x**2 + 3)/sqrt(7 - 11*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/sqrt(-11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3}\sqrt{-2\,x^2+1}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(5*x^2 + 3)*sqrt(-2*x^2 + 1)/sqrt(-11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}\,\sqrt{5\,x^2+3}}{\sqrt{7-11\,x^2}}\,dx$$

input
```
int(((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2))/(7 - 11*x^2)^(1/2),x)
```

output
```
int(((1 - 2*x^2)^(1/2)*(5*x^2 + 3)^(1/2))/(7 - 11*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-2x^2}\sqrt{3+5x^2}}{\sqrt{7-11x^2}}\,dx = -\left(\int \frac{\sqrt{5x^2+3}\,\sqrt{-2x^2+1}\,\sqrt{-11x^2+7}}{11x^2-7}dx\right)$$

input
```
int((-2*x^2+1)^(1/2)*(5*x^2+3)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt(5*x**2 + 3)*sqrt( - 2*x**2 + 1)*sqrt( - 11*x**2 + 7))/(11*x**
2 - 7),x)
```

3.507 $\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 340

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = -\frac{x\sqrt{7-11x^2}\sqrt{3-5x^2}}{11\sqrt{1+2x^2}}+\frac{\sqrt{\frac{7}{11}}\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1+2x^2}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right)|\frac{75}{77}\right)}{2\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1+2x^2}}}+\frac{\sqrt{\frac{11}{7}}\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1+2x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{75}{77}\right)}{4\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1+2x^2}}}-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{1+2x^2}}\,\text{EllipticPi}\left(\frac{6}{11},\arcsin\left(\frac{\sqrt{\frac{11}{3}}x}{\sqrt{1+2x^2}}\right),\frac{75}{77}\right)}{44\sqrt{77}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{1+2x^2}}}$$

output
```
-1/11*x*(-11*x^2+7)^(1/2)*(-5*x^2+3)^(1/2)/(2*x^2+1)^(1/2)+1/22*77^(1/2)*(
-11*x^2+7)^(1/2)*((-5*x^2+3)/(2*x^2+1))^(1/2)*EllipticE(1/3*33^(1/2)*x/(2*
x^2+1)^(1/2),5/77*231^(1/2))/(-5*x^2+3)^(1/2)/((-11*x^2+7)/(2*x^2+1))^(1/2
)+1/28*77^(1/2)*(-11*x^2+7)^(1/2)*((-5*x^2+3)/(2*x^2+1))^(1/2)*EllipticF(1
/3*33^(1/2)*x/(2*x^2+1)^(1/2),5/77*231^(1/2))/(-5*x^2+3)^(1/2)/((-11*x^2+7
)/(2*x^2+1))^(1/2)-59/3388*(-11*x^2+7)^(1/2)*((-5*x^2+3)/(2*x^2+1))^(1/2)*
EllipticPi(1/3*33^(1/2)*x/(2*x^2+1)^(1/2),6/11,5/77*231^(1/2))*77^(1/2)/(-
5*x^2+3)^(1/2)/((-11*x^2+7)/(2*x^2+1))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx$$
$$= \frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{7-11x^2}}\left(242x\sqrt{\frac{3-5x^2}{7-11x^2}}+484x^3\sqrt{\frac{3-5x^2}{7-11x^2}}+110i\sqrt{21-33x^2}\sqrt{\frac{1+2x^2}{7-11x^2}}E\left(i\operatorname{arcsinh}\left(\frac{5x}{\sqrt{7-11x^2}}\right)\middle|-\right.\right.}{}$$

input
```
Integrate[(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*(242*x*Sqrt[(3 - 5*x^2)/(
7 - 11*x^2)] + 484*x^3*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)] + (110*I)*Sqrt[21 -
33*x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*EllipticE[I*ArcSinh[(5*x)/Sqrt[7 -
11*x^2]], -2/75] + 775*Sqrt[14 - 22*x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*El
lipticF[ArcSin[(Sqrt[2/3]*x)/Sqrt[7 - 11*x^2]], -75/2] + 413*Sqrt[14 - 22*
x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*EllipticPi[-33/2, ArcSin[(Sqrt[2/3]*x)
/Sqrt[7 - 11*x^2]], -75/2]))/(484*Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {430, 427, 27, 321, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-5x^2}\sqrt{2x^2+1}}{\sqrt{7-11x^2}}dx$$

$$\Big\downarrow \text{430}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx+\frac{125}{44}\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{2x^2+1}}dx-\frac{59}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}dx-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{427}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx-\frac{59}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}dx+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{3-\frac{2x^2}{7-11x^2}}\sqrt{\frac{25x^2}{7-11x^2}+1}}d\frac{x}{\sqrt{7-11x^2}}}{44\sqrt{3}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx-\frac{59}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}dx+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\int \frac{1}{\sqrt{3-\frac{2x^2}{7-11x^2}}\sqrt{\frac{25x^2}{7-11x^2}+1}}d\frac{x}{\sqrt{7-11x^2}}}{44\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{321}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx-\frac{59}{44}\int \frac{\sqrt{2x^2+1}}{\sqrt{7-11x^2}\sqrt{3-5x^2}}dx+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{428}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\int \frac{\sqrt{21}}{\sqrt{7-\frac{25x^2}{2x^2+1}}\sqrt{3-\frac{11x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{44\sqrt{21}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\int \frac{1}{\sqrt{7-\frac{25x^2}{2x^2+1}}\sqrt{3-\frac{11x^2}{2x^2+1}}\left(1-\frac{2x^2}{2x^2+1}\right)}d\frac{x}{\sqrt{2x^2+1}}}{44\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{412}$$

$$\frac{25}{22}\int \frac{\sqrt{3-5x^2}}{\sqrt{7-11x^2}\,(2x^2+1)^{3/2}}dx+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\,\text{EllipticPi}\left(\frac{14}{25},\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right),\frac{77}{75}\right)}{220\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \text{429}$$

$$\frac{25\sqrt{\frac{3}{7}}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}\int \frac{\sqrt{\frac{7}{3}}\sqrt{3-\frac{11x^2}{2x^2+1}}}{\sqrt{7-\frac{25x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{22\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}}+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\,\text{EllipticPi}\left(\frac{14}{25},\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right),\frac{77}{75}\right)}{220\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{25\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}\int \frac{\sqrt{3-\frac{11x^2}{2x^2+1}}}{\sqrt{7-\frac{25x^2}{2x^2+1}}}d\frac{x}{\sqrt{2x^2+1}}}{22\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}}+\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\,\text{EllipticPi}\left(\frac{14}{25},\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right),\frac{77}{75}\right)}{220\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

$$\Big\downarrow \; 327$$

$$\frac{125\sqrt{3-5x^2}\sqrt{\frac{2x^2+1}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right),-\frac{75}{2}\right)}{44\sqrt{2}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{2x^2+1}}+\frac{5\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}E\left(\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right)|\frac{77}{75}\right)}{22\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}}-\frac{59\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{2x^2+1}}\,\text{EllipticPi}\left(\frac{14}{25},\arcsin\left(\frac{5x}{\sqrt{7}\sqrt{2x^2+1}}\right),\frac{77}{75}\right)}{220\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{7-11x^2}{2x^2+1}}}-\frac{\sqrt{7-11x^2}\sqrt{3-5x^2}x}{11\sqrt{2x^2+1}}$$

input
```
Int[(Sqrt[3 - 5*x^2]*Sqrt[1 + 2*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
-1/11*(x*Sqrt[7 - 11*x^2]*Sqrt[3 - 5*x^2])/Sqrt[1 + 2*x^2] + (5*Sqrt[3]*Sq
rt[3 - 5*x^2]*Sqrt[(7 - 11*x^2)/(1 + 2*x^2)]*EllipticE[ArcSin[(5*x)/(Sqrt[
7]*Sqrt[1 + 2*x^2])], 77/75])/(22*Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(1 + 2
*x^2)]) + (125*Sqrt[3 - 5*x^2]*Sqrt[(1 + 2*x^2)/(7 - 11*x^2)]*EllipticF[Ar
cSin[(Sqrt[2/3]*x)/Sqrt[7 - 11*x^2]], -75/2])/(44*Sqrt[2]*Sqrt[(3 - 5*x^2)
/(7 - 11*x^2)]*Sqrt[1 + 2*x^2]) - (59*Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(1
 + 2*x^2)]*EllipticPi[14/25, ArcSin[(5*x)/(Sqrt[7]*Sqrt[1 + 2*x^2])], 77/7
5])/(220*Sqrt[3]*Sqrt[3 - 5*x^2]*Sqrt[(7 - 11*x^2)/(1 + 2*x^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{-5x^2+3}\,\sqrt{2x^2+1}}{\sqrt{-11x^2+7}}dx$$

input
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(11*x^2 - 7),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{3-5x^2}\sqrt{2x^2+1}}{\sqrt{7-11x^2}}\,dx$$

input
```
integrate((-5*x**2+3)**(1/2)*(2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(3 - 5*x**2)*sqrt(2*x**2 + 1)/sqrt(7 - 11*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(-11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(-11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}\,\sqrt{3-5\,x^2}}{\sqrt{7-11\,x^2}}\,dx$$

input
```
int(((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2))/(7 - 11*x^2)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^(1/2)*(3 - 5*x^2)^(1/2))/(7 - 11*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1+2x^2}}{\sqrt{7-11x^2}}\,dx = -\left(\int \frac{\sqrt{2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{11x^2-7}dx\right)$$

input
```
int((-5*x^2+3)^(1/2)*(2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt(2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(11*x**
2 - 7),x)
```

3.508 $\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 313

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \frac{5x\sqrt{7-11x^2}\sqrt{1-2x^2}}{22\sqrt{3-5x^2}} + \frac{\sqrt{7}\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3-5x^2}}E\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right)|-\frac{2}{7}\right)}{22\sqrt{\frac{7-11x^2}{3-5x^2}}\sqrt{1-2x^2}} - \frac{3\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3-5x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{2}{7}\right)}{10\sqrt{7}\sqrt{\frac{7-11x^2}{3-5x^2}}\sqrt{1-2x^2}} + \frac{153\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{3-5x^2}}\,\text{EllipticPi}\left(-5,\arcsin\left(\frac{x}{\sqrt{3-5x^2}}\right),-\frac{2}{7}\right)}{110\sqrt{7}\sqrt{\frac{7-11x^2}{3-5x^2}}\sqrt{1-2x^2}}$$

output

```
5/22*x*(-11*x^2+7)^(1/2)*(-2*x^2+1)^(1/2)/(-5*x^2+3)^(1/2)+1/22*7^(1/2)*(-
11*x^2+7)^(1/2)*((-2*x^2+1)/(-5*x^2+3))^(1/2)*EllipticE(x/(-5*x^2+3)^(1/2)
,1/7*I*14^(1/2))/((-11*x^2+7)/(-5*x^2+3))^(1/2)/(-2*x^2+1)^(1/2)-3/70*(-11
*x^2+7)^(1/2)*((-2*x^2+1)/(-5*x^2+3))^(1/2)*EllipticF(x/(-5*x^2+3)^(1/2),1
/7*I*14^(1/2))*7^(1/2)/((-11*x^2+7)/(-5*x^2+3))^(1/2)/(-2*x^2+1)^(1/2)+153
/770*(-11*x^2+7)^(1/2)*((-2*x^2+1)/(-5*x^2+3))^(1/2)*EllipticPi(x/(-5*x^2+
3)^(1/2),-5,1/7*I*14^(1/2))*7^(1/2)/((-11*x^2+7)/(-5*x^2+3))^(1/2)/(-2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx$$
$$= \frac{\sqrt{7-11x^2}\sqrt{\frac{3-5x^2}{7-11x^2}}\left(121x\sqrt{\frac{3-5x^2}{7-11x^2}}-242x^3\sqrt{\frac{3-5x^2}{7-11x^2}}+33\sqrt{7-11x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}E\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right)|\frac{2}{9}\right)-}{242}$$

input
```
Integrate[(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(Sqrt[7 - 11*x^2]*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*(121*x*Sqrt[(3 - 5*x^2)/(
7 - 11*x^2)] - 242*x^3*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)] + 33*Sqrt[7 - 11*x^2
]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*EllipticE[ArcSin[(Sqrt[3]*x)/Sqrt[7 - 11*
x^2]], 2/9] - 31*Sqrt[7 - 11*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*EllipticF
[ArcSin[(Sqrt[3]*x)/Sqrt[7 - 11*x^2]], 2/9] + 119*Sqrt[7 - 11*x^2]*Sqrt[(1
 - 2*x^2)/(7 - 11*x^2)]*EllipticPi[-11/3, ArcSin[(Sqrt[3]*x)/Sqrt[7 - 11*x
^2]], 2/9]))/(242*Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {431, 427, 27, 321, 428, 27, 412, 429, 27, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx$$
$$\Big\downarrow \text{431}$$
$$-\frac{8}{121}\int \frac{1}{\sqrt{7-11x^2}\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{51}{242}\int \frac{\sqrt{7-11x^2}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+ \frac{7}{11}\int \frac{\sqrt{1-2x^2}}{(7-11x^2)^{3/2}\sqrt{3-5x^2}}dx+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \; 427$$

$$\frac{51}{242}\int \frac{\sqrt{7-11x^2}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{\left(7-11x^2\right)^{3/2}\sqrt{3-5x^2}}dx-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{121\sqrt{3}\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{51}{242}\int \frac{\sqrt{7-11x^2}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{\left(7-11x^2\right)^{3/2}\sqrt{3-5x^2}}dx-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{1}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{121\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \; 321$$

$$\frac{51}{242}\int \frac{\sqrt{7-11x^2}}{\sqrt{3-5x^2}\sqrt{1-2x^2}}dx+\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{\left(7-11x^2\right)^{3/2}\sqrt{3-5x^2}}dx-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \; 428$$

$$\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{\left(7-11x^2\right)^{3/2}\sqrt{3-5x^2}}dx+\frac{119\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{\sqrt{3}}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}\left(\frac{11x^2}{7-11x^2}+1\right)}d\frac{x}{\sqrt{7-11x^2}}}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{(7-11x^2)^{3/2}\sqrt{3-5x^2}}dx+\frac{357\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\int \frac{1}{\sqrt{1-\frac{3x^2}{7-11x^2}}\sqrt{3-\frac{2x^2}{7-11x^2}}\left(\frac{11x^2}{7-11x^2}+1\right)}d\frac{x}{\sqrt{7-11x^2}}}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \text{412}$$

$$\frac{7}{11}\int \frac{\sqrt{1-2x^2}}{(7-11x^2)^{3/2}\sqrt{3-5x^2}}dx-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{119\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticPi}\left(-\frac{11}{3},\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \text{429}$$

$$\frac{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}\int \frac{\sqrt{3}\sqrt{1-\frac{3x^2}{7-11x^2}}}{\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{11\sqrt{3}\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}}-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{119\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticPi}\left(-\frac{11}{3},\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}\int \frac{\sqrt{1-\frac{3x^2}{7-11x^2}}}{\sqrt{3-\frac{2x^2}{7-11x^2}}}d\frac{x}{\sqrt{7-11x^2}}}{11\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}}-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{119\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\operatorname{EllipticPi}\left(-\frac{11}{3},\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{8\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{363\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}E\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{7-11x^2}}\right)|\frac{9}{2}\right)}{11\sqrt{2}\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}}+\frac{119\sqrt{3-5x^2}\sqrt{\frac{1-2x^2}{7-11x^2}}\,\text{EllipticPi}\left(-\frac{11}{3},\arcsin\left(\frac{\sqrt{3}x}{\sqrt{7-11x^2}}\right),\frac{2}{9}\right)}{242\sqrt{\frac{3-5x^2}{7-11x^2}}\sqrt{1-2x^2}}+\frac{\sqrt{3-5x^2}\sqrt{1-2x^2}x}{2\sqrt{7-11x^2}}$$

input
```
Int[(Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/Sqrt[7 - 11*x^2],x]
```

output
```
(x*Sqrt[3 - 5*x^2]*Sqrt[1 - 2*x^2])/(2*Sqrt[7 - 11*x^2]) + (Sqrt[(3 - 5*x^
2)/(7 - 11*x^2)]*Sqrt[1 - 2*x^2]*EllipticE[ArcSin[(Sqrt[2/3]*x)/Sqrt[7 - 1
1*x^2]], 9/2])/(11*Sqrt[2]*Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)])
 - (8*Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*EllipticF[ArcSin[(Sqr
t[3]*x)/Sqrt[7 - 11*x^2]], 2/9])/(363*Sqrt[(3 - 5*x^2)/(7 - 11*x^2)]*Sqrt[
1 - 2*x^2]) + (119*Sqrt[3 - 5*x^2]*Sqrt[(1 - 2*x^2)/(7 - 11*x^2)]*Elliptic
Pi[-11/3, ArcSin[(Sqrt[3]*x)/Sqrt[7 - 11*x^2]], 2/9])/(242*Sqrt[(3 - 5*x^2
)/(7 - 11*x^2)]*Sqrt[1 - 2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 431
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Simp[e*((b*e - a*f)/(2*f))   Int[Sqrt[c + d*x^2]/(Sqrt[a +
b*x^2]*(e + f*x^2)^(3/2)), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*f^2)
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[(b*e -
 a*f)*((d*e - 2*c*f)/(2*f^2))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt
[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c
]
```

Maple [F]

$$\int \frac{\sqrt{-5x^2+3}\,\sqrt{-2x^2+1}}{\sqrt{-11x^2+7}}dx$$

input
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm
="fricas")
```

output
```
integral(-sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)*sqrt(-11*x^2 + 7)/(11*x^2 - 7)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{1-2x^2}\sqrt{3-5x^2}}{\sqrt{7-11x^2}}\,dx$$

input
```
integrate((-5*x**2+3)**(1/2)*(-2*x**2+1)**(1/2)/(-11*x**2+7)**(1/2),x)
```

output
```
Integral(sqrt(1 - 2*x**2)*sqrt(3 - 5*x**2)/sqrt(7 - 11*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(-11*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{-2\,x^2+1}\sqrt{-5\,x^2+3}}{\sqrt{-11\,x^2+7}}\,dx$$

input
```
integrate((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(-2*x^2 + 1)*sqrt(-5*x^2 + 3)/sqrt(-11*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = \int \frac{\sqrt{1-2\,x^2}\,\sqrt{3-5\,x^2}}{\sqrt{7-11\,x^2}}\,dx$$

input
```
int(((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2))/(7 - 11*x^2)^(1/2),x)
```

output
```
int(((1 - 2*x^2)^(1/2)*(3 - 5*x^2)^(1/2))/(7 - 11*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3-5x^2}\sqrt{1-2x^2}}{\sqrt{7-11x^2}}\,dx = -\left(\int \frac{\sqrt{-2x^2+1}\,\sqrt{-5x^2+3}\,\sqrt{-11x^2+7}}{11x^2-7}dx\right)$$

input
```
int((-5*x^2+3)^(1/2)*(-2*x^2+1)^(1/2)/(-11*x^2+7)^(1/2),x)
```

output
```
- int((sqrt( - 2*x**2 + 1)*sqrt( - 5*x**2 + 3)*sqrt( - 11*x**2 + 7))/(11*
x**2 - 7),x)
```

3.509 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 580

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx &= \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f\sqrt{e+fx^2}} \\
&- \frac{c\sqrt{-be+af}(3de-2cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}ef^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{(adf(de-2cf)-b(3d^2e^2-6cdef+2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{de(3bde-3bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}
$$

output
```
1/2*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)-1/2*c*(a*f-b*e)^
(1/2)*(-2*c*f+3*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellip
ticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))
^(1/2))/a^(1/2)/e/f^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*
(a*d*f*(-2*c*f+d*e)-b*(2*c^2*f^2-6*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*
(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)
^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x
^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*d*e*(-a*d*f-3*b*c*f+3*b*d*
e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^
(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(x^{2} d +c \right)^{\frac{3}{2}}}{\left(f\,x^{2}+e \right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*
f*x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.510 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 481

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{a}b(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{cf^2\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{a}bde\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{cf^2\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

output
```
a^(1/2)*(a*f-b*e)^(1/2)*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*El
lipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/e/f/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)-a^(1/2)*b*(
-c*f+d*e)*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticF((a*f-b
*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/c/f
^2/(a*f-b*e)^(1/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)+a^(1/2)
*b*d*e*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/c/f^2/(a*f-b*e)^(1/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {432, 428, 412, 429, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 432}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$$\Big\downarrow \text{ 428}$$

$$\frac{bc\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{af\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$\downarrow$ 429

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\int\frac{\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 324

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\int\frac{1}{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}+\frac{(de-cf)\int\frac{x^2}{(fx^2+e)\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{\ldots}}}{c}\ldots\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 320

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\int\frac{x^2}{(fx^2+e)\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{EllipticF}\left(\arctan\left(\ldots\right)\ldots\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}\sqrt{\ldots}}\ldots\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 388

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$

$$\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}}-\frac{a\int\frac{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}}{\left(\frac{(de-cf)x^2}{c(fx^2+e)}+1\right)^{3/2}}d\frac{x}{\sqrt{fx^2+e}}}{be-af}\right)}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{E}\cdots}{\sqrt{de-cf}\cdots}\cdots\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 313

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-$$

$$\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}}+\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-\cdots}{c(e+f\cdots}}}\cdots\right.}{\cdots}\cdots\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*(((d*
e - c*f)*((a*x*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))])/((b*e - a*f)*S
qrt[e + f*x^2]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]) - (a*Sqrt[c]*S
qrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*EllipticE[ArcTan[(Sqrt[d*e - c*
f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b
*e - a*f)*Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*
(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f
*x^2))])))/c + (Sqrt[c]*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*Ellipt
icF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e +
 f*x^2)))/(a*(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x
^2)/(c*(e + f*x^2))])))/(e*f*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))])) + (b*c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x
^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sq
rt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*f*Sqrt[d*e - c*f]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 432
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Simp[b/f   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sq
rt[e + f*x^2]), x], x] - Simp[(b*e - a*f)/f   Int[Sqrt[c + d*x^2]/(Sqrt[a +
 b*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*
x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.511 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{c(be-af)}{a(de-cf)}\right)}{e\sqrt{-de+cf}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
c^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((c*f-d*e
)^(1/2)*x/c^(1/2)/(f*x^2+e)^(1/2),(c*(-a*f+b*e)/a/(-c*f+d*e))^(1/2))/e/(c*
f-d*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cfx}}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{c(-be+af)}{a(-de+cf)}\right)}{e\sqrt{-de+cf}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[A
rcSin[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (c*(-(b*e) + a*f)
)/(a*(-(d*e) + c*f))]])/(e*Sqrt[-(d*e) + c*f]*Sqrt[c + d*x^2]*Sqrt[(e*(a +
b*x^2))/(a*(e + f*x^2))])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 568 vs. 2(148) = 296.

Time = 0.58 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {429, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 429}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\int \frac{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}}{\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\left(\int \frac{1}{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}+\frac{(be-af)\int \frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{a}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\left(\frac{(be-af)\int \frac{x^2}{\left(fx^2+e\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{a}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+\right.}{a\left(\frac{x^2(de-cf)}{e+fx^2}+\right.}}}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\left(\frac{(be-af)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}}-\frac{a\int \frac{\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}}{\left(\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1\right)^{3/2}}d\frac{x}{\sqrt{fx^2+e}}}{be-af}\right)}{a}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+}}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\left(\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}}+\frac{(be-af)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a\left(e+fx^2\right)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c\left(e+fx^2\right)}+1}}-\frac{a\sqrt{}}{}\right.}{}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*(((b*e - a*f)*((a*x
*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))])/((b*e - a*f)*Sqrt[e + f*x^2]
*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]) - (a*Sqrt[c]*Sqrt[1 + ((b*e
- a*f)*x^2)/(a*(e + f*x^2))]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]
*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b*e - a*f)*Sqrt
[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*(c + ((d*e - c
*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))])))/a +
 (Sqrt[c]*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*EllipticF[ArcTan[(Sq
rt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c
*f)))])/(Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*(
c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*
x^2))])))/(e*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f^2*x^6 + (2*d
*e*f + c*f^2)*x^4 + c*e^2 + (d*e^2 + 2*c*e*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,{(f\,x^2+e)}^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**2 + 2*c*e*f
*x**2 + c*f**2*x**4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)
```

3.512 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 354

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{dx\sqrt{a+bx^2}}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{a(de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{c(be-af)}{a(de-cf)}\right)}{c^{3/2}e(-de+cf)^{3/2}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{ad\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),\frac{c(be-af)}{a(de-cf)}\right)}{c^{3/2}(-de+cf)^{3/2}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

output
```
d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+a*(c*f+d*
e)*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticE((c*f-d*e)^(1/
2)*x/c^(1/2)/(f*x^2+e)^(1/2),(c*(-a*f+b*e)/a/(-c*f+d*e))^(1/2))/c^(3/2)/e/
(c*f-d*e)^(3/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)-a*d*(d*x^2
+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticF((c*f-d*e)^(1/2)*x/c^(1
/2)/(f*x^2+e)^(1/2),(c*(-a*f+b*e)/a/(-c*f+d*e))^(1/2))/c^(3/2)/(c*f-d*e)^(
3/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f^2*x^8 + 2*
(d^2*e*f + c*d*f^2)*x^6 + (d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 +
2*(c*d*e^2 + c^2*e*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.513 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 565

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{dx\sqrt{a+bx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}$$
$$+\frac{d(bc(de-5cf)-2ad(de-3cf))x\sqrt{a+bx^2}}{3c^2(bc-ad)(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{-be+af}\left(ad\left(2d^2e^2-7cdef-3c^2f^2\right)-bc\left(d^2e^2-6cdef-3c^2f^2\right)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-b}}{\sqrt{a}\sqrt{}}\right.\right.}{3\sqrt{a}c(bc-ad)e(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{\left(adf(de-9cf)-b\left(d^2e^2-6cdef-3c^2f^2\right)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}c\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/3*d
*(b*c*(-5*c*f+d*e)-2*a*d*(-3*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(-
c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*f-b*e)^(1/2)*(a*d*(-3*c^
2*f^2-7*c*d*e*f+2*d^2*e^2)-b*c*(-3*c^2*f^2-6*c*d*e*f+d^2*e^2))*(b*x^2+a)^(
1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(
f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(-a*d+b*c)/e/(
-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a*d*f*(-9
*c*f+d*e)-b*(-3*c^2*f^2-6*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(
-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(a*f-b*e)^(1/2)/(-c*f+d*e)^3/(d*x
^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{5}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f^2*x^10 + (
2*d^3*e*f + 3*c*d^2*f^2)*x^8 + (d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*x^6 +
 c^3*e^2 + (3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*x^4 + (3*c^2*d*e^2 + 2*c^
3*e*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{5/2}\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{\frac{5}{2}}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

3.514 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 691

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{5(bde-bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8f^2\sqrt{e+fx^2}}+\frac{bdx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f\sqrt{e+fx^2}}$$

$$+\frac{c\sqrt{-be+af}(be(15de-13cf)-af(13de-8cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{8\sqrt{a}ef^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{\sqrt{-be+af}(adf(3de-8cf)-b(15d^2e^2-33cdef+16c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-b}}{\sqrt{a}\sqrt{}}\right.\right.}{8\sqrt{a}f^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{3e(a^2d^2f^2-6abdf(de-cf)+b^2(5d^2e^2-6cdef+c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\right.\right.}{8\sqrt{a}f^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-5/8*(-a*d*f-b*c*f+b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^
(1/2)+1/4*b*d*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)+1/8*c*
(a*f-b*e)^(1/2)*(b*e*(-13*c*f+15*d*e)-a*f*(-8*c*f+13*d*e))*(b*x^2+a)^(1/2)
*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^
2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/f^3/(d*x^2+c)^(1/2
)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/8*(a*f-b*e)^(1/2)*(a*d*f*(-8*c*f+3*d*e
)-b*(16*c^2*f^2-33*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f
+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)+3/8*e*(a^2*d^2*f^2-6*a*b*d*f*(-c*f+d*e)+b^2*(c^2*f^2-6*c*d*e*
f+5*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((
a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(
-a*f+b*e))^(1/2))/a^(1/2)/f^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)
/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(x^2d+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.515 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 551

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx &= \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f\sqrt{e+fx^2}} \\
&- \frac{c(3be-2af)\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}ef^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{b\sqrt{-be+af}(3de-4cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{be(3bde-bcf-3adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}
$$

output
```
1/2*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)-1/2*c*(-2*a*f+3*
b*e)*(a*f-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellip
ticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))
^(1/2))/a^(1/2)/e/f^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*
b*(a*f-b*e)^(1/2)*(-4*c*f+3*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c
/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^
(1/2)-1/2*b*e*(-3*a*d*f-b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a
*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*
x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}\sqrt{x^{2}d+c}}{(f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*
f*x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.516 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 502

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = -\frac{c(-be+af)^{3/2}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}ef(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{\sqrt{-be+af}(bde-2bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{b^2e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-c*(a*f-b*e)^(3/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellipti
cE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(
1/2))/a^(1/2)/e/f/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/
2)+(a*f-b*e)^(1/2)*(a*d*f-2*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f
*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*
f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x
^2+a)/a/(f*x^2+e))^(1/2)+b^2*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),
(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(
1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}}{\sqrt{x^{2} d+c}\,(f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx\right)b + \left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx\right)a$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e**2 + 2*
c*e*f*x**2 + c*f**2*x**4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)*b
+ int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**2 + 2*c*e
*f*x**2 + c*f**2*x**4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)*a
```

3.517 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 397

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{-be+af}(2bce-a(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}e(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{2(bc-ad)\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
-(a*f-b*e)^(1/2)*(2*b*c*e-a*(c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+
d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2
+a)/a/(f*x^2+e))^(1/2)+2*(-a*d+b*c)*(a*f-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1
/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2
)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f^2*x^8 +
2*(d^2*e*f + c*d*f^2)*x^6 + (d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2
+ 2*(c*d*e^2 + c^2*e*f)*x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.518 $\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 521

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx=-\frac{(bc-ad)x\sqrt{a+bx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}$$
$$+\frac{2(ad(de-3cf)+bc(de+cf))x\sqrt{a+bx^2}}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{-be+af}\left(2bce(de+3cf)+a\left(2d^2e^2-7cdef-3c^2f^2\right)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-}{c(be-}\right.}{3\sqrt{a}ce(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{\sqrt{-be+af}(ad(de-9cf)+2bc(de+3cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}c(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(
1/2)+2/3*(a*d*(-3*c*f+d*e)+b*c*(c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-c*f+d*e)
^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/3*(a*f-b*e)^(1/2)*(2*b*c*e*(3*c*f+d*e
)+a*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^
2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d
*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/e/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^
2+a)/a/(f*x^2+e))^(1/2)-1/3*(a*f-b*e)^(1/2)*(a*d*(-9*c*f+d*e)+2*b*c*(3*c*f
+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)
^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/c/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}}{(x^{2}d+c)^{\frac{5}{2}}\,(f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f^2*x^10 +
 (2*d^3*e*f + 3*c*d^2*f^2)*x^8 + (d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*x^6
 + c^3*e^2 + (3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*x^4 + (3*c^2*d*e^2 + 2*
c^3*e*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

3.519 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 789

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{5c(de-cf)(c+dx^2)^{5/2}\sqrt{e+fx^2}}$$

$$+\frac{2(ad(2de-5cf)+bc(de+2cf))x\sqrt{a+bx^2}}{15c^2(de-cf)^2(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

$$+\frac{(2b^2c^2(d^2e^2-9cdef-4c^2f^2)-a^2d^2(8d^2e^2-29cdef+45c^2f^2)+abcd(3d^2e^2-5cdef+50c^2f^2))\,x\sqrt{a+}}{15c^3(bc-ad)(de-cf)^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{-be+af}(2b^2c^2e(d^2e^2-10cdef-15c^2f^2)-a^2d(8d^3e^3-33cd^2e^2f+58c^2def^2+15c^3f^3)+abc(3d^3e^3}{15\sqrt{a}c^2(bc-ad)e(de-cf)^4\sqrt{c+}}$$

$$-\frac{2\sqrt{-be+af}(bc(d^2e^2-10cdef-15c^2f^2)+2ad(d^2e^2-4cdef+15c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}(}{15\sqrt{a}c^2(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/5*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(5/2)/(f*x^2+e)^(
1/2)+2/15*(a*d*(-5*c*f+2*d*e)+b*c*(2*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-c*f
+d*e)^2/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/15*(2*b^2*c^2*(-4*c^2*f^2-9*c*d*
e*f+d^2*e^2)-a^2*d^2*(45*c^2*f^2-29*c*d*e*f+8*d^2*e^2)+a*b*c*d*(50*c^2*f^2
-5*c*d*e*f+3*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)/(-c*f+d*e)^3/(d*x^
2+c)^(1/2)/(f*x^2+e)^(1/2)+1/15*(a*f-b*e)^(1/2)*(2*b^2*c^2*e*(-15*c^2*f^2-
10*c*d*e*f+d^2*e^2)-a^2*d*(15*c^3*f^3+58*c^2*d*e*f^2-33*c*d^2*e^2*f+8*d^3*
e^3)+a*b*c*(15*c^3*f^3+85*c^2*d*e*f^2-7*c*d^2*e^2*f+3*d^3*e^3))*(b*x^2+a)^
(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/
(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c^2/(-a*d+b*c)/
e/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-2/15*(a*f-b
*e)^(1/2)*(b*c*(-15*c^2*f^2-10*c*d*e*f+d^2*e^2)+2*a*d*(15*c^2*f^2-4*c*d*e*
f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(1/2)/c^2/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{7}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*f^2*x^12 +
 2*(d^4*e*f + 2*c*d^3*f^2)*x^10 + (d^4*e^2 + 8*c*d^3*e*f + 6*c^2*d^2*f^2)*
x^8 + 4*(c*d^3*e^2 + 3*c^2*d^2*e*f + c^3*d*f^2)*x^6 + c^4*e^2 + (6*c^2*d^2
*e^2 + 8*c^3*d*e*f + c^4*f^2)*x^4 + 2*(2*c^3*d*e^2 + c^4*e*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{7/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{7}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

3.520 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 925

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \frac{(33a^2d^2f^2-68abdf(de-cf)+b^2(35d^2e^2-38cdef+3c^2f^2))\,x\sqrt{a+bx^2}}{48df^3\sqrt{e+fx^2}}$$
$$-\frac{b(7bde-7bcf-13adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{24f^2\sqrt{e+fx^2}}+\frac{b^2dx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{6f\sqrt{e+fx^2}}$$
$$+\frac{c\sqrt{-be+af}(2abdef(95de-82cf)-3a^2df^2(27de-16cf)-b^2e(105d^2e^2-100cdef+3c^2f^2))\sqrt{a+bx^2}}{48\sqrt{a}def^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{\sqrt{-be+af}(3a^2df^2(5de-16cf)-2abf(60d^2e^2-145cdef+72c^2f^2)+b^2e(105d^2e^2-240cdef+127c^2}{48\sqrt{a}f^5\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{e(5a^3d^3f^3-45a^2bd^2f^2(de-cf)+15ab^2df(5d^2e^2-6cdef+c^2f^2)-b^3(35d^3e^3-45cd^2e^2f+9c^2def^2+}{16\sqrt{a}df^5\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(}{a(}}}$$

output
```
1/48*(33*a^2*d^2*f^2-68*a*b*d*f*(-c*f+d*e)+b^2*(3*c^2*f^2-38*c*d*e*f+35*d^
2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f^3/(f*x^2+e)^(1/2)-1/24*b*(-1
3*a*d*f-7*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)
^(1/2)+1/6*b^2*d*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)+1/4
8*c*(a*f-b*e)^(1/2)*(2*a*b*d*e*f*(-82*c*f+95*d*e)-3*a^2*d*f^2*(-16*c*f+27*
d*e)-b^2*e*(3*c^2*f^2-100*c*d*e*f+105*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+
c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),
(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/e/f^4/(d*x^2+c)^(1/2)/(e*(b*x
^2+a)/a/(f*x^2+e))^(1/2)-1/48*(a*f-b*e)^(1/2)*(3*a^2*d*f^2*(-16*c*f+5*d*e)
-2*a*b*f*(72*c^2*f^2-145*c*d*e*f+60*d^2*e^2)+b^2*e*(127*c^2*f^2-240*c*d*e*
f+105*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF(
(a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2
))/a^(1/2)/f^5/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/16*e*(5*a
^3*d^3*f^3-45*a^2*b*d^2*f^2*(-c*f+d*e)+15*a*b^2*d*f*(c^2*f^2-6*c*d*e*f+5*d
^2*e^2)-b^3*(c^3*f^3+9*c^2*d*e*f^2-45*c*d^2*e^2*f+35*d^3*e^3))*(b*x^2+a)^(
1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/
(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)
/d/f^5/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}(x^2d+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*x^2 +
e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.521 $\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 700

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{b(5bde-bcf-9adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8df^2\sqrt{e+fx^2}}+\frac{b^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f\sqrt{e+fx^2}}$$

$$-\frac{c\sqrt{-be+af}\left(25abdef-8a^2df^2-b^2e(15de-cf)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{8\sqrt{a}def^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{3b\sqrt{-be+af}(af(5de-8cf)-be(5de-7cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-c}{c(be-a}\right.}{8\sqrt{a}f^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{be(15a^2d^2f^2-10abdf(3de-cf)+b^2(15d^2e^2-6cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\text{a}\right.}{8\sqrt{a}df^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/8*b*(-9*a*d*f-b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f^2/(f
*x^2+e)^(1/2)+1/4*b^2*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2
)-1/8*c*(a*f-b*e)^(1/2)*(25*a*b*d*e*f-8*a^2*d*f^2-b^2*e*(-c*f+15*d*e))*(b*
x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a
^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/e/f^3/
(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-3/8*b*(a*f-b*e)^(1/2)*(a*f
*(-8*c*f+5*d*e)-b*e*(-7*c*f+5*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+
e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e
)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)+1/8*b*e*(15*a^2*d^2*f^2-10*a*b*d*f*(-c*f+3*d*e)+b^2*(-c^2*f^2-6*c
*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellipt
icPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*
e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/f^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(
b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2d+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.522 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 634

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2df\sqrt{e+fx^2}}$$
$$+\frac{c\sqrt{-be+af}\,(4abdef-2a^2df^2-b^2e(3de-cf))\,\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}def^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{\sqrt{-be+af}\,(2a^2df^2-b^2e(3de-5cf)+2abf(de-3cf))\,\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right.\right.}{2\sqrt{a}f^3(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{b^2e(3bde+bcf-5adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}df^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/2*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f/(f*x^2+e)^(1/2)+1/2*c*(a*f-b
*e)^(1/2)*(4*a*b*d*e*f-2*a^2*d*f^2-b^2*e*(-c*f+3*d*e))*(b*x^2+a)^(1/2)*(e*
(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)
^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/e/f^2/(-c*f+d*e)/(d*x^
2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*(a*f-b*e)^(1/2)*(2*a^2*d*f^
2-b^2*e*(-5*c*f+3*d*e)+2*a*b*f*(-3*c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*
(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*
(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*b^2*e*(-5*a*d*f+b*c*f+3*b*d*e)*(b*x^2+a)^
(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)
/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/d/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{2}}}{\sqrt{x^{2} d+c}\,(f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
(3*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*x - 5*int((sq
rt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c*d*e**2*f +
2*a**2*c*d*e*f**2*x**2 + a**2*c*d*f**3*x**4 + a**2*d**2*e**2*f*x**2 + 2*a*
*2*d**2*e*f**2*x**4 + a**2*d**2*f**3*x**6 + a*b*c**2*e**2*f + 2*a*b*c**2*e
*f**2*x**2 + a*b*c**2*f**3*x**4 + 3*a*b*c*d*e**3 + 8*a*b*c*d*e**2*f*x**2 +
 7*a*b*c*d*e*f**2*x**4 + 2*a*b*c*d*f**3*x**6 + 3*a*b*d**2*e**3*x**2 + 7*a*
b*d**2*e**2*f*x**4 + 5*a*b*d**2*e*f**2*x**6 + a*b*d**2*f**3*x**8 + b**2*c*
*2*e**2*f*x**2 + 2*b**2*c**2*e*f**2*x**4 + b**2*c**2*f**3*x**6 + 3*b**2*c*
d*e**3*x**2 + 7*b**2*c*d*e**2*f*x**4 + 5*b**2*c*d*e*f**2*x**6 + b**2*c*d*f
**3*x**8 + 3*b**2*d**2*e**3*x**4 + 6*b**2*d**2*e**2*f*x**6 + 3*b**2*d**2*e
*f**2*x**8),x)*a**2*b**3*d**2*e*f**2 - 5*int((sqrt(e + f*x**2)*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c*d*e**2*f + 2*a**2*c*d*e*f**2*x**2 + a
**2*c*d*f**3*x**4 + a**2*d**2*e**2*f*x**2 + 2*a**2*d**2*e*f**2*x**4 + a**2
*d**2*f**3*x**6 + a*b*c**2*e**2*f + 2*a*b*c**2*e*f**2*x**2 + a*b*c**2*f**3
*x**4 + 3*a*b*c*d*e**3 + 8*a*b*c*d*e**2*f*x**2 + 7*a*b*c*d*e*f**2*x**4 + 2
*a*b*c*d*f**3*x**6 + 3*a*b*d**2*e**3*x**2 + 7*a*b*d**2*e**2*f*x**4 + 5*a*b
*d**2*e*f**2*x**6 + a*b*d**2*f**3*x**8 + b**2*c**2*e**2*f*x**2 + 2*b**2*c*
*2*e*f**2*x**4 + b**2*c**2*f**3*x**6 + 3*b**2*c*d*e**3*x**2 + 7*b**2*c*d*e
**2*f*x**4 + 5*b**2*c*d*e*f**2*x**6 + b**2*c*d*f**3*x**8 + 3*b**2*d**2*e**
3*x**4 + 6*b**2*d**2*e**2*f*x**6 + 3*b**2*d**2*e*f**2*x**8),x)*a**2*b**...
```

3.523 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 607

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{cd(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{-be+af}\,(4abcdef-b^2ce(de+cf)-a^2df(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(de-c}{c(be-a}\right.}{\sqrt{a}def(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{(-be+af)^{3/2}(bde-3bcf+2adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{b^3e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}df^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1
/2)-(a*f-b*e)^(1/2)*(4*a*b*c*d*e*f-b^2*c*e*(c*f+d*e)-a^2*d*f*(c*f+d*e))*(b
*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/
a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/e/f/(
-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-(a*f-b*e)^(3/2
)*(2*a*d*f-3*b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*
EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+
b*e))^(1/2))/a^(1/2)/f^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^
2+e))^(1/2)+b^3*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Elliptic
Pi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)
/c/(-a*f+b*e))^(1/2))/a^(1/2)/d/f^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*
x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{2}}}{(x^{2} d+c)^{\frac{3}{2}} (f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2} (e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.524 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 555

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{3cd(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

$$-\frac{(bc-ad)(2ad(de-3cf)+bc(5de-cf))x\sqrt{a+bx^2}}{3c^2d(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{-be+af}(8b^2c^2e^2-abce(3de+13cf)-a^2(2d^2e^2-7cdef-3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3\sqrt{a}ce(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{(bc-ad)\sqrt{-be+af}(8bce+ade-9acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}c(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e
)^(1/2)-1/3*(-a*d+b*c)*(2*a*d*(-3*c*f+d*e)+b*c*(-c*f+5*d*e))*x*(b*x^2+a)^(
1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*f-b*e)^(1/2
)*(8*b^2*c^2*e^2-a*b*c*e*(13*c*f+3*d*e)-a^2*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^
2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c
/e/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(-a*d+
b*c)*(a*f-b*e)^(1/2)*(-9*a*c*f+a*d*e+8*b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a
*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{2}}}{(x^{2}d+c)^{\frac{5}{2}}\,(f\,x^{2}+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^3*f^2*x^10 + (2*d^3*e*f + 3*c*d^2*f^2)*x^8 + (d^3*e^2 + 6*c*
d^2*e*f + 3*c^2*d*f^2)*x^6 + c^3*e^2 + (3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^
2)*x^4 + (3*c^2*d*e^2 + 2*c^3*e*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

3.525 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 784

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{5cd(de-cf)(c+dx^2)^{5/2}\sqrt{e+fx^2}}$$

$$-\frac{(bc-ad)(2ad(2de-5cf)+bc(7de-cf))x\sqrt{a+bx^2}}{15c^2d(de-cf)^2(c+dx^2)^{3/2}\sqrt{e+fx^2}}$$

$$+\frac{(abcd(7d^2e^2-35cdef-20c^2f^2)+2b^2c^2(4d^2e^2+9cdef-c^2f^2)+a^2d^2(8d^2e^2-29cdef+45c^2f^2))\,x\sqrt{a \cdots}}{15c^3d(de-cf)^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{-be+af}(8b^2c^2e^2(de+5cf)+abce(7d^2e^2-38cdef-65c^2f^2)+a^2(8d^3e^3-33cd^2e^2f+58c^2def^2+15 \cdots}{15\sqrt{a}c^2e(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{\sqrt{-be+af}(8b^2c^2e(de+5cf)+3abc(d^2e^2-18cdef-15c^2f^2)+4a^2d(d^2e^2-4cdef+15c^2f^2))\sqrt{a+b \cdots}}{15\sqrt{a}c^2(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/5*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(5/2)/(f*x^2+e
)^(1/2)-1/15*(-a*d+b*c)*(2*a*d*(-5*c*f+2*d*e)+b*c*(-c*f+7*d*e))*x*(b*x^2+a
)^(1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/15*(a*b*c*d*(
-20*c^2*f^2-35*c*d*e*f+7*d^2*e^2)+2*b^2*c^2*(-c^2*f^2+9*c*d*e*f+4*d^2*e^2)
+a^2*d^2*(45*c^2*f^2-29*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d/(-c*f+
d*e)^3/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e^2
*(5*c*f+d*e)+a*b*c*e*(-65*c^2*f^2-38*c*d*e*f+7*d^2*e^2)+a^2*(15*c^3*f^3+58
*c^2*d*e*f^2-33*c*d^2*e^2*f+8*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f
+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c^2/e/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(
b*x^2+a)/a/(f*x^2+e))^(1/2)-1/15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e*(5*c*f+d*e)+
3*a*b*c*(-15*c^2*f^2-18*c*d*e*f+d^2*e^2)+4*a^2*d*(15*c^2*f^2-4*c*d*e*f+d^2
*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)
^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/c^2/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{7}{2}}(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^4*f^2*x^12 + 2*(d^4*e*f + 2*c*d^3*f^2)*x^10 + (d^4*e^2 + 8*c
*d^3*e*f + 6*c^2*d^2*f^2)*x^8 + 4*(c*d^3*e^2 + 3*c^2*d^2*e*f + c^3*d*f^2)*
x^6 + c^4*e^2 + (6*c^2*d^2*e^2 + 8*c^3*d*e*f + c^4*f^2)*x^4 + 2*(2*c^3*d*e
^2 + c^4*e*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{7/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{7}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(3/2),x)
```

3.526 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1188

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx = \text{Too large to display}$$

output
```
1/7*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(7/2)/(f*x^2+e
)^(1/2)-1/35*(-a*d+b*c)*(2*a*d*(-7*c*f+3*d*e)+b*c*(-c*f+9*d*e))*x*(b*x^2+a
)^(1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2)+1/105*(a*b*c*d*
(-56*c^2*f^2-53*c*d*e*f+13*d^2*e^2)+4*b^2*c^2*(-c^2*f^2+11*c*d*e*f+2*d^2*e
^2)+3*a^2*d^2*(35*c^2*f^2-27*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d/(
-c*f+d*e)^3/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/105*(a^2*b*c*d^2*(-525*c^3*f
^3-14*c^2*d*e*f^2-53*c*d^2*e^2*f+16*d^3*e^3)-12*a^3*d^3*(-35*c^3*f^3+33*c^
2*d*e*f^2-18*c*d^2*e^2*f+4*d^3*e^3)+8*b^3*c^3*(c^3*f^3-13*c^2*d*e*f^2-13*c
*d^2*e^2*f+d^3*e^3)+a*b^2*c^2*d*(112*c^3*f^3+469*c^2*d*e*f^2-14*c*d^2*e^2*
f+9*d^3*e^3))*x*(b*x^2+a)^(1/2)/c^4/d/(-a*d+b*c)/(-c*f+d*e)^4/(d*x^2+c)^(1
/2)/(f*x^2+e)^(1/2)+1/105*(a*f-b*e)^(1/2)*(8*b^3*c^3*e^2*(-35*c^2*f^2-14*c
*d*e*f+d^2*e^2)+a*b^2*c^2*e*(455*c^3*f^3+707*c^2*d*e*f^2-19*c*d^2*e^2*f+9*
d^3*e^3)+a^2*b*c*(-105*c^4*f^4-1022*c^3*d*e*f^3+23*c^2*d^2*e^2*f^2-64*c*d^
3*e^3*f+16*d^4*e^4)-3*a^3*d*(-35*c^4*f^4-194*c^3*d*e*f^3+165*c^2*d^2*e^2*f
^2-80*c*d^3*e^3*f+16*d^4*e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(
-a*f+b*e))^(1/2))/a^(1/2)/c^3/(-a*d+b*c)/e/(-c*f+d*e)^5/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/105*(a*f-b*e)^(1/2)*(8*b^2*c^2*e*(-35*c^2*
f^2-14*c*d*e*f+d^2*e^2)+3*a^2*d*(-175*c^3*f^3+78*c^2*d*e*f^2-39*c*d^2*e^2*
f+8*d^3*e^3)+a*b*c*(315*c^3*f^3+511*c^2*d*e*f^2-71*c*d^2*e^2*f+13*d^3*e...
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(x^2d+c)^{\frac{9}{2}}(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^5*f^2*x^14 + (2*d^5*e*f + 5*c*d^4*f^2)*x^12 + (d^5*e^2 + 10*
c*d^4*e*f + 10*c^2*d^3*f^2)*x^10 + 5*(c*d^4*e^2 + 4*c^2*d^3*e*f + 2*c^3*d^
2*f^2)*x^8 + c^5*e^2 + 5*(2*c^2*d^3*e^2 + 4*c^3*d^2*e*f + c^4*d*f^2)*x^6 +
 (10*c^3*d^2*e^2 + 10*c^4*d*e*f + c^5*f^2)*x^4 + (5*c^4*d*e^2 + 2*c^5*e*f)
*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(9/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{9}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(9/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{9/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(9/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{9/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{9}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(9/2)/(f*x^2+e)^(3/2),x)
```

3.527 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 320

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a}f\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{e\sqrt{-be+af}(de-cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\sqrt{a}d\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{c\sqrt{-be+af}(de-cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

output
```
-a^(1/2)*f*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/e/
(a*f-b*e)^(1/2)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
+a^(1/2)*d*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticF((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/c/
(a*f-b*e)^(1/2)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^2*x^8 + (2
*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e*f)*
x^4 + a*c*e^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**2 + 2*a*c
*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*
e*f*x**6 + b*d*f**2*x**8),x)
```

3.528 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 420

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = -\frac{d^2x\sqrt{a+bx^2}}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{(adf(de+cf)-b(d^2e^2+c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}(bc-ad)e\sqrt{-be+af}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{2df\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-d^2*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(
1/2)-(a*d*f*(c*f+d*e)-b*(c^2*f^2+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-
c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(-a*d+b*c)/e/(a*f-b*e)^(1/2)/(-c*f+d
*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-2*d*f*(b*x^2+a)^(1/2
)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x
^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-b*e)^(1/2)/(-c
*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^2*f^2*x^10 +
 (2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^8 + (b*d^2*e^2 + 2*(2*b*c*d + a*d
^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^6 + a*c^2*e^2 + (a*c^2*f^2 + (2*b*c*d +
 a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*x^4 + (2*a*c^2*e*f + (b*c^2 + 2*a*c
*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.529 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 637

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = -\frac{d^2x\sqrt{a+bx^2}}{3c(bc-ad)(de-cf)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}} + \frac{2d^2(ad(de-3cf)-2bc(de-2cf))x\sqrt{a+bx^2}}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{(a^2d^2f(2d^2e^2-7cdef-3c^2f^2)-abd(2d^3e^3-3cd^2e^2f-9c^2def^2-6c^3f^3)+b^2c(4d^3e^3-9cd^2e^2f-3c^3}{3\sqrt{a}c(bc-ad)^2e\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a}{a(e}}} + \frac{d(adf(de-9cf)-b(d^2e^2-9c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}c(bc-ad)\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*d^2*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+
e)^(1/2)+2/3*d^2*(a*d*(-3*c*f+d*e)-2*b*c*(-2*c*f+d*e))*x*(b*x^2+a)^(1/2)/c
^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/3*(a^2*d^2*
f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-a*b*d*(-6*c^3*f^3-9*c^2*d*e*f^2-3*c*d^2
*e^2*f+2*d^3*e^3)+b^2*c*(-3*c^3*f^3-9*c*d^2*e^2*f+4*d^3*e^3))*(b*x^2+a)^(1
/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f
*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(-a*d+b*c)^2/e/
(a*f-b*e)^(1/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/
2)+1/3*d*(a*d*f*(-9*c*f+d*e)-b*(-9*c^2*f^2+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(
1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(-a*d+b*c)/(a*f-b*e)^(1/
2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{5}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^3*f^2*x^12 +
 (2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^10 + (b*d^3*e^2 + 2*(3*b*c*d^2
 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^8 + ((3*b*c*d^2 + a*d^3)*e^2 +
 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*a*c^2*d)*f^2)*x^6 + a*c^3*e^2 + (a
*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2 + 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^4 + (2
*a*c^3*e*f + (b*c^3 + 3*a*c^2*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{5}{2}}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{5/2}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{\frac{5}{2}}\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

3.530 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 434

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2x\sqrt{c+dx^2}}{a(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{e+fx^2}} - \frac{c(abcf^2-a^2df^2-b^2e(de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{a^{3/2}(bc-ad)e(-be+af)^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{a^{3/2}(-be+af)^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
b^2*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1
/2)-c*(a*b*c*f^2-a^2*d*f^2-b^2*e*(-c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*
(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(-a*d+b*c)/e/(a*f-b*e)^(3/2)/(-c*f
+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-(-a*d*f-b*c*f+b*d*e)
*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)
*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(a*f
-b*e)^(3/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f^2*x^10 +
 (2*b^2*d*e*f + (b^2*c + 2*a*b*d)*f^2)*x^8 + (b^2*d*e^2 + 2*(b^2*c + 2*a*b
*d)*e*f + (2*a*b*c + a^2*d)*f^2)*x^6 + a^2*c*e^2 + (a^2*c*f^2 + (b^2*c + 2
*a*b*d)*e^2 + 2*(2*a*b*c + a^2*d)*e*f)*x^4 + (2*a^2*c*e*f + (2*a*b*c + a^2
*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.531 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 614

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx=\frac{b^2x}{a(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$+\frac{d(abd^2e-a^2d^2f+b^2c(de-cf))\,x\sqrt{a+bx^2}}{ac(bc-ad)^2(be-af)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$+\frac{\left(b^3ce(de-cf)^2+a^3d^2f^2(de+cf)-2a^2bdf(d^2e^2+c^2f^2)+ab^2(d^3e^3+c^3f^3)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\Big(\text{arcs}}{a^{3/2}(bc-ad)^2e(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{\left(2a^2d^2f^2-b^2(de-cf)^2-abdf(de+cf)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{a^{3/2}(bc-ad)(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1
/2)+d*(a*b*d^2*e-a^2*d^2*f+b^2*c*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c/(-a*d+b
*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+(b^3*c*e*(-c*f
+d*e)^2+a^3*d^2*f^2*(c*f+d*e)-2*a^2*b*d*f*(c^2*f^2+d^2*e^2)+a*b^2*(c^3*f^3
+d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^
(3/2)/(-a*d+b*c)^2/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^
2+a)/a/(f*x^2+e))^(1/2)+(2*a^2*d^2*f^2-b^2*(-c*f+d*e)^2-a*b*d*f*(c*f+d*e))
*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)
*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(-a*
d+b*c)/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+
e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^2*f^2*x^12
 + 2*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^10 + (b^2*d^2*e^2 + 4*(b^2*
c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2)*x^8 + 2*((b^2*c*
d + a*b*d^2)*e^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e*f + (a*b*c^2 + a^2*c*
d)*f^2)*x^6 + a^2*c^2*e^2 + (a^2*c^2*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^4 + 2*(a^2*c^2*e*f + (a*b*c^2 + a^2*c*
d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)),
x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.532 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 967

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{b^2x}{a(bc-ad)(be-af)\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}$$
$$+\frac{d(abd^2e-a^2d^2f+3b^2c(de-cf))\,x\sqrt{a+bx^2}}{3ac(bc-ad)^2(be-af)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}$$
$$+\frac{d(ab^2cd^2e(7de-11cf)+2a^3d^3f(de-3cf)+3b^3c^2(de-cf)^2-a^2bd^2(2d^2e^2+cdef-11c^2f^2))\,x\sqrt{a+b}}{3ac^2(bc-ad)^3(be-af)(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$+\frac{(3b^4c^2e(de-cf)^3-a^4d^3f^2(2d^2e^2-7cdef-3c^2f^2)+a^3bd^2f(4d^3e^3-7cd^2e^2f-12c^2def^2-9c^3f^3)-a}{3a^{3/2}c(bc-ad)^3e}$$
$$-\frac{(a^3d^3f^2(de-9cf)+3b^3c(de-cf)^3-a^2bd^2f(2d^2e^2-3cdef-15c^2f^2)+ab^2d(d^3e^3+3cd^2e^2f-9c^2de}{3a^{3/2}c(bc-ad)^2(-be+af)^{3/2}(de-cf)^3\sqrt{c+}}$$

output
```
b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1
/2)+1/3*d*(a*b*d^2*e-a^2*d^2*f+3*b^2*c*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c/(
-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/3*d*(a
*b^2*c*d^2*e*(-11*c*f+7*d*e)+2*a^3*d^3*f*(-3*c*f+d*e)+3*b^3*c^2*(-c*f+d*e)
^2-a^2*b*d^2*(-11*c^2*f^2+c*d*e*f+2*d^2*e^2))*x*(b*x^2+a)^(1/2)/a/c^2/(-a*
d+b*c)^3/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/3*(3*b^
4*c^2*e*(-c*f+d*e)^3-a^4*d^3*f^2*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)+a^3*b*d^
2*f*(-9*c^3*f^3-12*c^2*d*e*f^2-7*c*d^2*e^2*f+4*d^3*e^3)-a^2*b^2*d*(-9*c^4*
f^4-24*c^2*d^2*e^2*f^2+7*c*d^3*e^3*f+2*d^4*e^4)+a*b^3*c*(-3*c^4*f^4-12*c*d
^3*e^3*f+7*d^4*e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellip
ticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))
^(1/2))/a^(3/2)/c/(-a*d+b*c)^3/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^3/(d*x^2+c)^(1
/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a^3*d^3*f^2*(-9*c*f+d*e)+3*b^3*c*
(-c*f+d*e)^3-a^2*b*d^2*f*(-15*c^2*f^2-3*c*d*e*f+2*d^2*e^2)+a*b^2*d*(-3*c^3
*f^3-9*c^2*d*e*f^2+3*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-
c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/c/(-a*d+b*c)^2/(a*f-b*e)^(3/2)/(-c*f
+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}(x^2d+c)^{\frac{5}{2}}(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^3*f^2*x^14
 + (2*b^2*d^3*e*f + (3*b^2*c*d^2 + 2*a*b*d^3)*f^2)*x^12 + (b^2*d^3*e^2 + 2
*(3*b^2*c*d^2 + 2*a*b*d^3)*e*f + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*f^2
)*x^10 + ((3*b^2*c*d^2 + 2*a*b*d^3)*e^2 + 2*(3*b^2*c^2*d + 6*a*b*c*d^2 + a
^2*d^3)*e*f + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*f^2)*x^8 + a^2*c^3*e^2
 + ((3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*e^2 + 2*(b^2*c^3 + 6*a*b*c^2*d +
 3*a^2*c*d^2)*e*f + (2*a*b*c^3 + 3*a^2*c^2*d)*f^2)*x^6 + (a^2*c^3*f^2 + (b
^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*e^2 + 2*(2*a*b*c^3 + 3*a^2*c^2*d)*e*f)
*x^4 + (2*a^2*c^3*e*f + (2*a*b*c^3 + 3*a^2*c^2*d)*e^2)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(d\,x^2+c\right)^{\frac{5}{2}}\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

3.533 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 666

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2x\sqrt{c+dx^2}}{3a(bc-ad)(be-af)\,(a+bx^2)^{3/2}\,\sqrt{e+fx^2}}$$
$$+\frac{2b^2(b^2ce-2abde-3abcf+4a^2df)\,x\sqrt{c+dx^2}}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\sqrt{e+fx^2}}$$
$$+\frac{c(6a^3bcdf^3-3a^4d^2f^3-2b^4ce^2(de-cf)+ab^3e(4d^2e^2+3cdef-7c^2f^2)-3a^2b^2f(3d^2e^2-3cdef+c^2f^2)}{3a^{5/2}(bc-ad)^2e(-be+af)^{5/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+}{a(e+}}}$$
$$-\frac{(3a^3d^2f^2-2b^3ce(de-cf)-3a^2bdf(2de-cf)+3ab^2(d^2e^2+cdef-2c^2f^2))\,\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{Ellipti}}{3a^{5/2}(bc-ad)(-be+af)^{5/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*b^2*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e
)^(1/2)+2/3*b^2*(4*a^2*d*f-3*a*b*c*f-2*a*b*d*e+b^2*c*e)*x*(d*x^2+c)^(1/2)/
a^2/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2)+1/3*c*(6*a^3
*b*c*d*f^3-3*a^4*d^2*f^3-2*b^4*c*e^2*(-c*f+d*e)+a*b^3*e*(-7*c^2*f^2+3*c*d*
e*f+4*d^2*e^2)-3*a^2*b^2*f*(c^2*f^2-3*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*
(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2
+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/(-a*d+b*c)^2/e/(a*f-b
*e)^(5/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(
3*a^3*d^2*f^2-2*b^3*c*e*(-c*f+d*e)-3*a^2*b*d*f*(-c*f+2*d*e)+3*a*b^2*(-2*c^
2*f^2+c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*El
lipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(5/2)/(-a*d+b*c)/(a*f-b*e)^(5/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(
e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2 d+c}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d*f^2*x^12 +
 (2*b^3*d*e*f + (b^3*c + 3*a*b^2*d)*f^2)*x^10 + (b^3*d*e^2 + 2*(b^3*c + 3*
a*b^2*d)*e*f + 3*(a*b^2*c + a^2*b*d)*f^2)*x^8 + ((b^3*c + 3*a*b^2*d)*e^2 +
 6*(a*b^2*c + a^2*b*d)*e*f + (3*a^2*b*c + a^3*d)*f^2)*x^6 + a^3*c*e^2 + (a
^3*c*f^2 + 3*(a*b^2*c + a^2*b*d)*e^2 + 2*(3*a^2*b*c + a^3*d)*e*f)*x^4 + (2
*a^3*c*e*f + (3*a^2*b*c + a^3*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.534 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 946

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{b^2x}{3a(bc-ad)(be-af)(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{2b^2(b^2ce+5a^2df-3ab(de+cf))x}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{d(6a^3bd^3ef-3a^4d^3f^2+2b^4c^2e(de-cf)-ab^3c(7d^2e^2-cdef-6c^2f^2)-a^2b^2d(3d^2e^2-11cdef+11c^2f}{3a^2c(bc-ad)^3(be-af)^2(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$-\frac{(2b^5c^2e^2(de-cf)^2+3a^5d^3f^3(de+cf)-7ab^4ce(de-cf)^2(de+cf)-9a^4bd^2f^2(d^2e^2+c^2f^2)+9a^3b^2df}{3a^{5/2}(bc-ad)^3e(-be}$$

$$-\frac{2(3a^4d^3f^3-b^4ce(de-cf)^2-3a^3bd^2f^2(de+cf)+3ab^3(de-cf)^2(de+cf)-3a^2b^2df(d^2e^2-3cdef+}{3a^{5/2}(bc-ad)^2(-be+af)^{5/2}(de-cf)^2\sqrt{c+dx}}$$

output
```
1/3*b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e
)^(1/2)+2/3*b^2*(b^2*c*e+5*a^2*d*f-3*a*b*(c*f+d*e))*x/a^2/(-a*d+b*c)^2/(-a
*f+b*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(6*a^3*b*d
^3*e*f-3*a^4*d^3*f^2+2*b^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-6*c^2*f^2-c*d*e*f+7*
d^2*e^2)-a^2*b^2*d*(11*c^2*f^2-11*c*d*e*f+3*d^2*e^2))*x*(b*x^2+a)^(1/2)/a^
2/c/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1
/3*(2*b^5*c^2*e^2*(-c*f+d*e)^2+3*a^5*d^3*f^3*(c*f+d*e)-7*a*b^4*c*e*(-c*f+d
*e)^2*(c*f+d*e)-9*a^4*b*d^2*f^2*(c^2*f^2+d^2*e^2)+9*a^3*b^2*d*f*(c^3*f^3+d
^3*e^3)-3*a^2*b^3*(c^4*f^4-4*c^3*d*e*f^3+8*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+d
^4*e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*
e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5
/2)/(-a*d+b*c)^3/e/(a*f-b*e)^(5/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+
a)/a/(f*x^2+e))^(1/2)-2/3*(3*a^4*d^3*f^3-b^4*c*e*(-c*f+d*e)^2-3*a^3*b*d^2*
f^2*(c*f+d*e)+3*a*b^3*(-c*f+d*e)^2*(c*f+d*e)-3*a^2*b^2*d*f*(c^2*f^2-3*c*d*
e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a
*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))
/a^(5/2)/(-a*d+b*c)^2/(a*f-b*e)^(5/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x
^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^2*f^2*x^14
 + (2*b^3*d^2*e*f + (2*b^3*c*d + 3*a*b^2*d^2)*f^2)*x^12 + (b^3*d^2*e^2 + 2
*(2*b^3*c*d + 3*a*b^2*d^2)*e*f + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^2
)*x^10 + ((2*b^3*c*d + 3*a*b^2*d^2)*e^2 + 2*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2
*b*d^2)*e*f + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^2)*x^8 + a^3*c^2*e^2
 + ((b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^2 + 2*(3*a*b^2*c^2 + 6*a^2*b*c
*d + a^3*d^2)*e*f + (3*a^2*b*c^2 + 2*a^3*c*d)*f^2)*x^6 + (a^3*c^2*f^2 + (3
*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2 + 2*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f)
*x^4 + (2*a^3*c^2*e*f + (3*a^2*b*c^2 + 2*a^3*c*d)*e^2)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx=\int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}(d\,x^2+c)^{\frac{3}{2}}(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

3.535 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 715

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \frac{(de-cf)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef^2\left(e+fx^2\right)^{3/2}} + \frac{d^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f^2\sqrt{e+fx^2}}$$

$$- \frac{c(af(13d^2e^2-6cdef-4c^2f^2)-be(15d^2e^2-10cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right.\right.}{6\sqrt{a}e^2f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$- \frac{(adf(3d^2e^2-8cdef+2c^2f^2)-b(15d^3e^3-30cd^2e^2f+10c^2def^2+2c^3f^3))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticF}}{6\sqrt{a}ef^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$- \frac{d^2e(5bde-5bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticPi}\left(-\frac{af}{be-af}, \arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right), \frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-c*f+d*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^(3/2)+1
/2*d^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)-1/6*c*(a*f*(-
4*c^2*f^2-6*c*d*e*f+13*d^2*e^2)-b*e*(-2*c^2*f^2-10*c*d*e*f+15*d^2*e^2))*(b
*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/
a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^3
/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/6*(a*d*
f*(2*c^2*f^2-8*c*d*e*f+3*d^2*e^2)-b*(2*c^3*f^3+10*c^2*d*e*f^2-30*c*d^2*e^2
*f+15*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF(
(a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2
))/a^(1/2)/e/f^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))
^(1/2)-1/2*d^2*e*(-a*d*f-5*b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(
f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/
(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^4/(a*f-b*e)^(1/2)/
(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2))/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,(x^2d+c)^{\frac{5}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(f^3*x^6 + 3*e*f^2*x^4 + 3*e^2*f*x^2 + e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(5/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{5/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{5}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

3.536 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 586

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = -\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef\,(e+fx^2)^{3/2}}$$
$$+\frac{c(2af(de+cf)-be(3de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2f^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{(acdf^2+b(3d^2e^2-3cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}ef^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{bd^2e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^(3/2)+1/3*
c*(2*a*f*(c*f+d*e)-b*e*(c*f+3*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+
e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e
)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*
(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a*c*d*f^2+b*(-c^2*f^2-3*c*d*e*f+3*d^2*e^
2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e
/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+b*d^2
*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^
(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}}}{\left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.537 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 388

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e(e+fx^2)^{3/2}} + \frac{c(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(f*x^2+e)^(3/2)+1/3*c*(-2*a*c*f+a*
d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^
(1/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2
+e))^(1/2)-1/3*c*(-a*d+b*c)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*
f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*
x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{\left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^3*x^6 + 3*e*f^
2*x^4 + 3*e^2*f*x^2 + e^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.538 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 420

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx = -\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e(de-cf)(e+fx^2)^{3/2}} - \frac{c(2af(2de-cf)-be(3de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2\sqrt{-be+af}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{(bc-ad)(3de-cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^(3/2)-1/3*
c*(2*a*f*(-c*f+2*d*e)-b*e*(-c*f+3*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f
+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)^2/(d*x^2
+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(-a*d+b*c)*(-c*f+3*d*e)*(b*x
^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^
(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*
e)^(1/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f^3*x^8 + (3*d
*e*f^2 + c*f^3)*x^6 + 3*(d*e^2*f + c*e*f^2)*x^4 + c*e^3 + (d*e^3 + 3*c*e^2
*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.539 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 532

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx =$$

$$-\frac{fx\sqrt{a+bx^2}}{3e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}+\frac{d(3de+cf)x\sqrt{a+bx^2}}{3ce(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{(af(3d^2e^2+7cdef-2c^2f^2)-be(3d^2e^2+6cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)\Big|\frac{a(de-}{c(be-}\right.}{3\sqrt{a}e^2\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{(adf(9de-cf)-b(3d^2e^2+6cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)+1/3*
d*(c*f+3*d*e)*x*(b*x^2+a)^(1/2)/c/e/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)
^(1/2)+1/3*(a*f*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)-b*e*(-c^2*f^2+6*c*d*e*f+3
*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^
(1/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)-1/3*(a*d*f*(-c*f+9*d*e)-b*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(b*
x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a
^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b
*e)^(1/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f^3*x^10 + (
3*d^2*e*f^2 + 2*c*d*f^3)*x^8 + (3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*x^6 +
 c^2*e^3 + (d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*x^4 + (2*c*d*e^3 + 3*c^2*
e^2*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{\frac{3}{2}}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.540 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 798

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \frac{dx\sqrt{a+bx^2}}{3c(de-cf)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}$$

$$+\frac{d(bc(de-7cf)-2ad(de-4cf))x\sqrt{a+bx^2}}{3c^2(bc-ad)(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$

$$-\frac{f(ad(2d^2e^2-9cdef-c^2f^2)-bc(d^2e^2-8cdef-c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2(bc-ad)e(de-cf)^3\,(e+fx^2)^{3/2}}$$

$$-\frac{(b^2ce(d^3e^3-9cd^2e^2f-9c^2def^2+c^3f^3)+2a^2df(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3)-ab(2d^4e^4-9cd^3e^3}{3\sqrt{a}c(bc-ad)e^2\sqrt{-be+af}(de-cf}$$

$$-\frac{(adf(d^2e^2-18cdef+c^2f^2)-b(d^3e^3-9cd^2e^2f-9c^2def^2+c^3f^3))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\right.}{3\sqrt{a}ce\sqrt{-be+af}(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*d*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2)+1/3*d
*(b*c*(-7*c*f+d*e)-2*a*d*(-4*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(-
c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)-1/3*f*(a*d*(-c^2*f^2-9*c*d*e*f+
2*d^2*e^2)-b*c*(-c^2*f^2-8*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)/c^2/(-a*d+b*c)/e/(-c*f+d*e)^3/(f*x^2+e)^(3/2)-1/3*(b^2*c*e*(c^3*f^3-9
*c^2*d*e*f^2-9*c*d^2*e^2*f+d^3*e^3)+2*a^2*d*f*(c^3*f^3-5*c^2*d*e*f^2-5*c*d
^2*e^2*f+d^3*e^3)-a*b*(2*c^4*f^4-9*c^3*d*e*f^3-18*c^2*d^2*e^2*f^2-9*c*d^3*
e^3*f+2*d^4*e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Elliptic
E((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1
/2))/a^(1/2)/c/(-a*d+b*c)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)^4/(d*x^2+c)^(1/2)
/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a*d*f*(c^2*f^2-18*c*d*e*f+d^2*e^2)-b
*(c^3*f^3-9*c^2*d*e*f^2-9*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+
c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),
(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/e/(a*f-b*e)^(1/2)/(-c*f+d*e)^
4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}}{\left(x^2 d+c\right)^{\frac{5}{2}}\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{5}{2}}\left(fx^2+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f^3*x^12 + 3
*(d^3*e*f^2 + c*d^2*f^3)*x^10 + 3*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3)*
x^8 + (d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*x^6 + c^3*e^3 +
3*(c*d^2*e^3 + 3*c^2*d*e^2*f + c^3*e*f^2)*x^4 + 3*(c^2*d*e^3 + c^3*e^2*f)*
x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{5}{2}}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(5/2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

3.541 $\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 688

$$
\begin{aligned}
&\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \frac{(be-af)(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef^2(e+fx^2)^{3/2}} \\
&+ \frac{bdx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f^2\sqrt{e+fx^2}} \\
&- \frac{c\sqrt{-be+af}(be(15de-4cf)-4af(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{6\sqrt{a}e^2f^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{(2a^2cdf^3+abf(9d^2e^2-14cdef-2c^2f^2)-b^2e(15d^2e^2-24cdef+4c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}}{6\sqrt{a}ef^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&- \frac{bde(5bde-3bcf-3adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}
$$

output
```
1/3*(-a*f+b*e)*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e
)^(3/2)+1/2*b*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)-1/6*
c*(a*f-b*e)^(1/2)*(b*e*(-4*c*f+15*d*e)-4*a*f*(c*f+d*e))*(b*x^2+a)^(1/2)*(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e
)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^3/(d*x^2+c)^(1/2)
/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/6*(2*a^2*c*d*f^3+a*b*f*(-2*c^2*f^2-14*c
*d*e*f+9*d^2*e^2)-b^2*e*(4*c^2*f^2-24*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)
*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^
2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/f^4/(a*f-b*e)^(1/2
)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*b*d*e*(-3*a*d*f-3*b*
c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a
*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-
a*f+b*e))^(1/2))/a^(1/2)/f^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/
a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}(x^2d+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)/(f^3*x^6 + 3*e*f^2*x^4 + 3*e^2*f*x^2 + e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^{3/2}}{\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(d\,x^2+c\right)^{\frac{3}{2}}}{\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.542 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 627

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = -\frac{(be-af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef(e+fx^2)^{3/2}}$$
$$+\frac{c\sqrt{-be+af}(af(de-2cf)+be(3de-2cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2f^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(a^2cdf^3-abcf^2(de+cf)-b^2e(3d^2e^2-6cdef+2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx}}\right.\right.}{3\sqrt{a}ef^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{b^2de\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-a*f+b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^(3/2)+1/3*
c*(a*f-b*e)^(1/2)*(a*f*(-2*c*f+d*e)+b*e*(-2*c*f+3*d*e))*(b*x^2+a)^(1/2)*(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e
)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^2/(-c*f+d*e)/(d*x
^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(a^2*c*d*f^3-a*b*c*f^2*(c*
f+d*e)-b^2*e*(2*c^2*f^2-6*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a
*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/f^3/(a*f-b*e)^(1/2)/(-c*f+d*e)/
(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+b^2*d*e*(b*x^2+a)^(1/2)*(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+
e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(a
*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{2}}\sqrt{x^{2}d+c}}{(f\,x^{2}+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.543 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 420

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \frac{(be-af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e(de-cf)\,(e+fx^2)^{3/2}}$$
$$+\frac{2c\sqrt{-be+af}(bce-2ade+acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)(2bce-3ade+acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-a*f+b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)^(3
/2)+2/3*c*(a*f-b*e)^(1/2)*(a*c*f-2*a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+
c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),
(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2
)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(-a*d+b*c)*(a*c*f-3*a*d*e+2*b*c*e)*(
b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x
/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f
-b*e)^(1/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{x^2d+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f^3*x^8 + (3
*d*e*f^2 + c*f^3)*x^6 + 3*(d*e^2*f + c*e*f^2)*x^4 + c*e^3 + (d*e^3 + 3*c*e
^2*f)*x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.544 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 526

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \frac{(be-af)x\sqrt{a+bx^2}}{3e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$
$$-\frac{d(4bce-3ade-acf)x\sqrt{a+bx^2}}{3ce(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{-be+af}(2bce(3de+cf)-a(3d^2e^2+7cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(de-}{c(be-}\right.}{3\sqrt{a}e^2(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)(af(9de-cf)-2be(3de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-a*f+b*e)*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3
/2)-1/3*d*(-a*c*f-3*a*d*e+4*b*c*e)*x*(b*x^2+a)^(1/2)/c/e/(-c*f+d*e)^2/(d*x
^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*f-b*e)^(1/2)*(2*b*c*e*(c*f+3*d*e)-a*(-2
*c^2*f^2+7*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(
-a*f+b*e))^(1/2))/a^(1/2)/e^2/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)+1/3*(-a*d+b*c)*(a*f*(-c*f+9*d*e)-2*b*e*(c*f+3*d*e))*(b*x^
2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(
1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e
)^(1/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f^3*x^10 +
 (3*d^2*e*f^2 + 2*c*d*f^3)*x^8 + (3*d^2*e^2*f + 6*c*d*e*f^2 + c^2*f^3)*x^6
 + c^2*e^3 + (d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^2)*x^4 + (2*c*d*e^3 + 3*c^
2*e^2*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.545 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 714

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{3c(de-cf)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}$$

$$+\frac{2(ad(de-4cf)+bc(de+2cf))x\sqrt{a+bx^2}}{3c^2(de-cf)^2\sqrt{c+dx^2}(e+fx^2)^{3/2}}$$

$$+\frac{f(2bce(de+3cf)+a(2d^2e^2-9cdef-c^2f^2))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2e(de-cf)^3(e+fx^2)^{3/2}}$$

$$+\frac{2\sqrt{-be+af}(bce(d^2e^2+6cdef+c^2f^2)+a(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\Big(\text{ar}}{3\sqrt{a}ce^2(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{(a^2df(d^2e^2-18cdef+c^2f^2)-2b^2ce(d^2e^2+6cdef+c^2f^2)-ab(d^3e^3-17cd^2e^2f-17c^2def^2+c^3f^3))\sqrt{}}{3\sqrt{a}ce\sqrt{-be+af}(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx}}}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(
3/2)+2/3*(a*d*(-4*c*f+d*e)+b*c*(2*c*f+d*e))*x*(b*x^2+a)^(1/2)/c^2/(-c*f+d*
e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)+1/3*f*(2*b*c*e*(3*c*f+d*e)+a*(-c^2*f^
2-9*c*d*e*f+2*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/e/(-c*f+d*e)
^3/(f*x^2+e)^(3/2)+2/3*(a*f-b*e)^(1/2)*(b*c*e*(c^2*f^2+6*c*d*e*f+d^2*e^2)+
a*(c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2)
,(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/e^2/(-c*f+d*e)^4/(d*x^2+c)^(
1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a^2*d*f*(c^2*f^2-18*c*d*e*f+d^2*
e^2)-2*b^2*c*e*(c^2*f^2+6*c*d*e*f+d^2*e^2)-a*b*(c^3*f^3-17*c^2*d*e*f^2-17*
c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Elli
pticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e)
)^(1/2))/a^(1/2)/c/e/(a*f-b*e)^(1/2)/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^
2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(x^2d+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f^3*x^12 +
 3*(d^3*e*f^2 + c*d^2*f^3)*x^10 + 3*(d^3*e^2*f + 3*c*d^2*e*f^2 + c^2*d*f^3
)*x^8 + (d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3)*x^6 + c^3*e^3
+ 3*(c*d^2*e^3 + 3*c^2*d*e^2*f + c^3*e*f^2)*x^4 + 3*(c^2*d*e^3 + c^3*e^2*f
)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(3/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(3/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

3.546 $\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 812

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = -\frac{(be-af)^2(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef^3(e+fx^2)^{3/2}} - \frac{b(9bde-5bcf-9adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8f^3\sqrt{e+fx^2}} + \frac{b^2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f^2\sqrt{e+fx^2}} - \frac{c\sqrt{-be+af}(abef(115de-24cf)-5b^2e^2(21de-11cf)-16a^2f^2(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\Big(\text{arcsi}}{24\sqrt{a}e^2f^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{\sqrt{-be+af}(8a^2cdf^3+abf(45d^2e^2-80cdef-8c^2f^2)-b^2e(105d^2e^2-195cdef+64c^2f^2))\sqrt{a+bx^2}\sqrt{}}{24\sqrt{a}ef^5\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{be(15a^2d^2f^2-10abdf(5de-3cf)+b^2(35d^2e^2-30cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\Big(-\frac{af}{be-a}}{8\sqrt{a}f^5\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-a*f+b*e)^2*(-c*f+d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^3/(f*x^
2+e)^(3/2)-1/8*b*(-9*a*d*f-5*b*c*f+9*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/f^3/(f*x^2+e)^(1/2)+1/4*b^2*d*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/
(f*x^2+e)^(1/2)-1/24*c*(a*f-b*e)^(1/2)*(a*b*e*f*(-24*c*f+115*d*e)-5*b^2*e^
2*(-11*c*f+21*d*e)-16*a^2*f^2*(c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f
*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*
f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a
/(f*x^2+e))^(1/2)-1/24*(a*f-b*e)^(1/2)*(8*a^2*c*d*f^3+a*b*f*(-8*c^2*f^2-80
*c*d*e*f+45*d^2*e^2)-b^2*e*(64*c^2*f^2-195*c*d*e*f+105*d^2*e^2))*(b*x^2+a)
^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)
/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/f^5/(d*x^2+c
)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/8*b*e*(15*a^2*d^2*f^2-10*a*b*d*f
*(-3*c*f+5*d*e)+b^2*(3*c^2*f^2-30*c*d*e*f+35*d^2*e^2))*(b*x^2+a)^(1/2)*(e*
(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e
)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^5/(a*
f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx$$

$$\downarrow 434$$

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}(x^2d+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.547 $\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 716

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \frac{(be-af)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef^2(e+fx^2)^{3/2}} + \frac{b^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f^2\sqrt{e+fx^2}}$$
$$-\frac{c\sqrt{-be+af}\,(b^2e^2(15de-13cf)-2abef(5de-3cf)-2a^2f^2(de-2cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\Big(\arcsin\Big(}{6\sqrt{a}e^2f^3(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{\sqrt{-be+af}\,(2a^2cdf^3-2abcf^2(de+cf)-b^2e(15d^2e^2-33cdef+16c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}}{6\sqrt{a}ef^4(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{b^2e(5bde-bcf-5adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-a*f+b*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^(3/2)+1
/2*b^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)-1/6*c*(a*f-b*
e)^(1/2)*(b^2*e^2*(-13*c*f+15*d*e)-2*a*b*e*f*(-3*c*f+5*d*e)-2*a^2*f^2*(-2*
c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b
*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(
1/2)/e^2/f^3/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/
6*(a*f-b*e)^(1/2)*(2*a^2*c*d*f^3-2*a*b*c*f^2*(c*f+d*e)-b^2*e*(16*c^2*f^2-3
3*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ell
ipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e
))^(1/2))/a^(1/2)/e/f^4/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)-1/2*b^2*e*(-5*a*d*f-b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*
f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^4/(a*f-b*e)^(1/2
)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2d+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.548 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 658

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = -\frac{(be-af)^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ef(de-cf)\,(e+fx^2)^{3/2}}$$

$$-\frac{c(-be+af)^{3/2}(be(3de-5cf)+2af(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2f^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{\sqrt{-be+af}(a^2df^2(3de-cf)+abf(3d^2e^2-8cdef+c^2f^2)+b^2e(3d^2e^2-9cdef+8c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c}{c(e}}}{3\sqrt{a}ef^3(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{b^3e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-a*f+b*e)^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(-c*f+d*e)/(f*x^2+
e)^(3/2)-1/3*c*(a*f-b*e)^(3/2)*(b*e*(-5*c*f+3*d*e)+2*a*f*(-c*f+2*d*e))*(b*
x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a
^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/f^2/
(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(a*f-b*e)
^(1/2)*(a^2*d*f^2*(-c*f+3*d*e)+a*b*f*(c^2*f^2-8*c*d*e*f+3*d^2*e^2)+b^2*e*(
8*c^2*f^2-9*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^
(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/
(-a*f+b*e))^(1/2))/a^(1/2)/e/f^3/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)
/a/(f*x^2+e))^(1/2)+b^3*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*
EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-
c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/
(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 434$$
$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{2}}}{\sqrt{x^{2} d+c}\,(f\,x^{2}+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.549 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 560

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = -\frac{(be-af)^2x\sqrt{a+bx^2}}{3ef(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$

$$-\frac{(8abcdef-a^2df(3de+cf)-b^2ce(de+3cf))\,x\sqrt{a+bx^2}}{3cef(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{-be+af}\,(8b^2c^2e^2-abce(13de+3cf)+a^2(3d^2e^2+7cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-}}{\sqrt{a}}\right.\right.}{3\sqrt{a}e^2(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{(bc-ad)\sqrt{-be+af}\,(8bce-9ade+acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(-a*f+b*e)^2*x*(b*x^2+a)^(1/2)/e/f/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+
e)^(3/2)-1/3*(8*a*b*c*d*e*f-a^2*d*f*(c*f+3*d*e)-b^2*c*e*(3*c*f+d*e))*x*(b*
x^2+a)^(1/2)/c/e/f/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+1/3*(a*f-b
*e)^(1/2)*(8*b^2*c^2*e^2-a*b*c*e*(3*c*f+13*d*e)+a^2*(-2*c^2*f^2+7*c*d*e*f+
3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(1/2)/e^2/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/
3*(-a*d+b*c)*(a*f-b*e)^(1/2)*(a*c*f-9*a*d*e+8*b*c*e)*(b*x^2+a)^(1/2)*(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(
1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(-c*f+d*e)^3/(d*x^2+c)^(
1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{2}}}{(x^{2}d+c)^{\frac{3}{2}}\,(f\,x^{2}+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^2*f^3*x^10 + (3*d^2*e*f^2 + 2*c*d*f^3)*x^8 + (3*d^2*e^2*f +
6*c*d*e*f^2 + c^2*f^3)*x^6 + c^2*e^3 + (d^2*e^3 + 6*c*d*e^2*f + 3*c^2*e*f^
2)*x^4 + (2*c*d*e^3 + 3*c^2*e^2*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.550 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 732

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{3cd(de-cf)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}$$
$$-\frac{(bc-ad)(2ad(de-4cf)+bc(5de+cf))x\sqrt{a+bx^2}}{3c^2d(de-cf)^2\sqrt{c+dx^2}(e+fx^2)^{3/2}}$$
$$-\frac{f(2b^2c^2e(3de+cf)-abcde(3de+13cf)-a^2d(2d^2e^2-9cdef-c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2de(de-cf)^3(e+fx^2)^{3/2}}$$
$$-\frac{\sqrt{-be+af}(8b^2c^2e^2(de+cf)-abce(3d^2e^2+26cdef+3c^2f^2)-2a^2(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3))}{3\sqrt{a}ce^2(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)\sqrt{-be+af}(8bce(de+cf)+a(d^2e^2-18cdef+c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3\sqrt{a}ce(de-cf)^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e
)^(3/2)-1/3*(-a*d+b*c)*(2*a*d*(-4*c*f+d*e)+b*c*(c*f+5*d*e))*x*(b*x^2+a)^(1
/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)-1/3*f*(2*b^2*c^2*e*
(c*f+3*d*e)-a*b*c*d*e*(13*c*f+3*d*e)-a^2*d*(-c^2*f^2-9*c*d*e*f+2*d^2*e^2))
*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/d/e/(-c*f+d*e)^3/(f*x^2+e)^(3/2)-1/
3*(a*f-b*e)^(1/2)*(8*b^2*c^2*e^2*(c*f+d*e)-a*b*c*e*(3*c^2*f^2+26*c*d*e*f+3
*d^2*e^2)-2*a^2*(c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3))*(b*x^2+a)^(
1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(
f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/e^2/(-c*f+d*e)
^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(-a*d+b*c)*(a*f-b*e
)^(1/2)*(8*b*c*e*(c*f+d*e)+a*(c^2*f^2-18*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)
*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^
2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/e/(-c*f+d*e)^4/(d*
x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{\left(x^2d+c\right)^{\frac{5}{2}}\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}\left(fx^2+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^3*f^3*x^12 + 3*(d^3*e*f^2 + c*d^2*f^3)*x^10 + 3*(d^3*e^2*f +
 3*c*d^2*e*f^2 + c^2*d*f^3)*x^8 + (d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
 + c^3*f^3)*x^6 + c^3*e^3 + 3*(c*d^2*e^3 + 3*c^2*d*e^2*f + c^3*e*f^2)*x^4
+ 3*(c^2*d*e^3 + c^3*e^2*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{5/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

3.551 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1028

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x\sqrt{a+bx^2}}{5cd(de-cf)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}$$

$$-\frac{(bc-ad)(4ad(de-3cf)+bc(7de+cf))x\sqrt{a+bx^2}}{15c^2d(de-cf)^2(c+dx^2)^{3/2}(e+fx^2)^{3/2}}$$

$$+\frac{(abcd(7d^2e^2-47cdef-56c^2f^2)+4b^2c^2(2d^2e^2+9cdef+c^2f^2)+a^2d^2(8d^2e^2-35cdef+75c^2f^2))\,x\sqrt{a}}{15c^3d(de-cf)^3\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$

$$+\frac{f(abcde(7d^2e^2-50cdef-85c^2f^2)+8b^2c^2e(d^2e^2+6cdef+c^2f^2)+a^2d(8d^3e^3-39cd^2e^2f+90c^2def^2+}{15c^3de(de-cf)^4(e+fx^2)^{3/2}}$$

$$+\frac{\sqrt{-be+af}(8b^2c^2e^2(d^2e^2+10cdef+5c^2f^2)+abce(7d^3e^3-53cd^2e^2f-195c^2def^2-15c^3f^3)+a^2(8d^4e^{}}{15\sqrt{a}c^2e^2(de-cf)^5}$$

$$-\frac{\sqrt{-be+af}(8b^2c^2e(d^2e^2+10cdef+5c^2f^2)+a^2d(4d^3e^3-21cd^2e^2f+150c^2def^2-5c^3f^3)+abc(3d^3e^3}{15\sqrt{a}c^2e(de-cf)^5\sqrt{c+}}$$

output
```
1/5*(-a*d+b*c)^2*x*(b*x^2+a)^(1/2)/c/d/(-c*f+d*e)/(d*x^2+c)^(5/2)/(f*x^2+e
)^(3/2)-1/15*(-a*d+b*c)*(4*a*d*(-3*c*f+d*e)+b*c*(c*f+7*d*e))*x*(b*x^2+a)^(
1/2)/c^2/d/(-c*f+d*e)^2/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2)+1/15*(a*b*c*d*(-56
*c^2*f^2-47*c*d*e*f+7*d^2*e^2)+4*b^2*c^2*(c^2*f^2+9*c*d*e*f+2*d^2*e^2)+a^2
*d^2*(75*c^2*f^2-35*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)/c^3/d/(-c*f+d*e)
^3/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)+1/15*f*(a*b*c*d*e*(-85*c^2*f^2-50*c*d*e
*f+7*d^2*e^2)+8*b^2*c^2*e*(c^2*f^2+6*c*d*e*f+d^2*e^2)+a^2*d*(5*c^3*f^3+90*
c^2*d*e*f^2-39*c*d^2*e^2*f+8*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c
^3/d/e/(-c*f+d*e)^4/(f*x^2+e)^(3/2)+1/15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e^2*(5
*c^2*f^2+10*c*d*e*f+d^2*e^2)+a*b*c*e*(-15*c^3*f^3-195*c^2*d*e*f^2-53*c*d^2
*e^2*f+7*d^3*e^3)+a^2*(-10*c^4*f^4+65*c^3*d*e*f^3+108*c^2*d^2*e^2*f^2-43*c
*d^3*e^3*f+8*d^4*e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ell
ipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e
))^(1/2))/a^(1/2)/c^2/e^2/(-c*f+d*e)^5/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)-1/15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e*(5*c^2*f^2+10*c*d*e*f+d^2*e
^2)+a^2*d*(-5*c^3*f^3+150*c^2*d*e*f^2-21*c*d^2*e^2*f+4*d^3*e^3)+a*b*c*(5*c
^3*f^3-175*c^2*d*e*f^2-89*c*d^2*e^2*f+3*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^
2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2
),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c^2/e/(-c*f+d*e)^5/(d*x^2+c)^
(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx=\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{7/2}\left(e+fx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{7/2}\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[(a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{\left(x^2d+c\right)^{\frac{7}{2}}\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{7/2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{7}{2}}\left(fx^2+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
f*x^2 + e)/(d^4*f^3*x^14 + (3*d^4*e*f^2 + 4*c*d^3*f^3)*x^12 + 3*(d^4*e^2*f
 + 4*c*d^3*e*f^2 + 2*c^2*d^2*f^3)*x^10 + (d^4*e^3 + 12*c*d^3*e^2*f + 18*c^
2*d^2*e*f^2 + 4*c^3*d*f^3)*x^8 + c^4*e^3 + (4*c*d^3*e^3 + 18*c^2*d^2*e^2*f
 + 12*c^3*d*e*f^2 + c^4*f^3)*x^6 + 3*(2*c^2*d^2*e^3 + 4*c^3*d*e^2*f + c^4*
e*f^2)*x^4 + (4*c^3*d*e^3 + 3*c^4*e^2*f)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{7/2}\left(e+fx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(7/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^{7/2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{7}{2}}\left(fx^2+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{7}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((b*x^2 + a)^(5/2)/((d*x^2 + c)^(7/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{7/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{7/2}(e+fx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}}{(d\,x^2+c)^{\frac{7}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x)
```

output

```
int((b*x^2+a)^(5/2)/(d*x^2+c)^(7/2)/(f*x^2+e)^(5/2),x)
```

3.552 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 449

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e(be-af)(de-cf)\,(e+fx^2)^{3/2}} + \frac{2cf(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(adf(3de-cf)-b(3d^2e^2-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e
)^(3/2)+2/3*c*f*(b*e*(-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*(e*(
d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^
(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(a*f-b*e)^(3/2)/(-c*f
+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(a*d*f*(-c*f+3
*d*e)-b*(-c^2*f^2+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1
/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-
a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 434}$$
$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^3*x^10 + (
3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c + a*d
)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^4
 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.553 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 643

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = -\frac{d^2x\sqrt{a+bx^2}}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$
$$+\frac{f(adf(3de+cf)-b(3d^2e^2+c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c(bc-ad)e(be-af)(de-cf)^2\,(e+fx^2)^{3/2}}$$
$$-\frac{(a^2df^2(3d^2e^2+7cdef-2c^2f^2)+b^2e(3d^3e^3+9c^2def^2-4c^3f^3)-abf(6d^3e^3+9cd^2e^2f+3c^2def^2-2c^3}{3\sqrt{a}(bc-ad)e^2(-be+af)^{3/2}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(}{a(}}}$$
$$-\frac{f(adf(9de-cf)-b(9d^2e^2-c^2f^2))\,\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e(-be+af)^{3/2}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-d^2*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(
3/2)+1/3*f*(a*d*f*(c*f+3*d*e)-b*(c^2*f^2+3*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/c/(-a*d+b*c)/e/(-a*f+b*e)/(-c*f+d*e)^2/(f*x^2+e)^(3/2)-1/3*(a
^2*d*f^2*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)+b^2*e*(-4*c^3*f^3+9*c^2*d*e*f^2+
3*d^3*e^3)-a*b*f*(-2*c^3*f^3+3*c^2*d*e*f^2+9*c*d^2*e^2*f+6*d^3*e^3))*(b*x^
2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(
1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(-a*d+b*c)
/e^2/(a*f-b*e)^(3/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e)
)^(1/2)-1/3*f*(a*d*f*(-c*f+9*d*e)-b*(-c^2*f^2+9*d^2*e^2))*(b*x^2+a)^(1/2)*
(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2
+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(3/2)/(-c
*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{3}{2}} \left(f\,x^{2}+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^2*f^3*x^12 +
 (3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^10 + (3*b*d^2*e^2*f + 3*(2*b*c*
d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^8 + (b*d^2*e^3 + a*c^2*f^3 + 3
*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*a*c*d)*e*f^2)*x^6 + a*c^2*e^3 + (3
*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e^3 + 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^4 + (3
*a*c^2*e^2*f + (b*c^2 + 2*a*c*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,{(d\,x^2+c)}^{3/2}\,{(f\,x^2+e)}^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.554 $\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1002

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx =$$

$$-\frac{d^2x\sqrt{a+bx^2}}{3c(bc-ad)(de-cf)\,(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}$$

$$-\frac{2d^2(bc(2de-5cf)-ad(de-4cf))x\sqrt{a+bx^2}}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$

$$-\frac{f(a^2d^2f(2d^2e^2-9cdef-c^2f^2)-abd(2d^3e^3-5cd^2e^2f-11c^2def^2-2c^3f^3)+b^2c(4d^3e^3-11cd^2e^2f-}{3c^2(bc-ad)^2e(be-af)(de-cf)^3\,(e+fx^2)^{3/2}}$$

$$+\frac{2(a^3d^2f^2(d^3e^3-5cd^2e^2f-5c^2def^2+c^3f^3)-2a^2bdf(d^4e^4-4cd^3e^3f-6c^2d^2e^2f^2-4c^3def^3+c^4f^4)-2}{3\sqrt{c}}$$

$$+\frac{(a^2d^2f^2(d^2e^2-18cdef+c^2f^2)-2abdf(d^3e^3-9cd^2e^2f-9c^2def^2+c^3f^3)+b^2(d^4e^4-18c^2d^2e^2f^2+c^4f}{3\sqrt{a}c(bc-ad)e(-be+af)^{3/2}(de-cf)^4\sqrt{c+dx^2}}$$

output
```
-1/3*d^2*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+
e)^(3/2)-2/3*d^2*(b*c*(-5*c*f+2*d*e)-a*d*(-4*c*f+d*e))*x*(b*x^2+a)^(1/2)/c
^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)-1/3*f*(a^2*d^
2*f*(-c^2*f^2-9*c*d*e*f+2*d^2*e^2)-a*b*d*(-2*c^3*f^3-11*c^2*d*e*f^2-5*c*d^
2*e^2*f+2*d^3*e^3)+b^2*c*(-c^3*f^3-11*c*d^2*e^2*f+4*d^3*e^3))*x*(b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/e/(-a*f+b*e)/(-c*f+d*e)^3/(f*x^2+e)
^(3/2)+2/3*(a^3*d^2*f^2*(c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+d^3*e^3)-2*a^
2*b*d*f*(c^4*f^4-4*c^3*d*e*f^3-6*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+d^4*e^4)-2*
b^3*c*e*(c^4*f^4-3*c^3*d*e*f^3-3*c*d^3*e^3*f+d^4*e^4)+a*b^2*(c^5*f^5-c^4*d
*e*f^4-12*c^3*d^2*e^2*f^3-12*c^2*d^3*e^3*f^2-c*d^4*e^4*f+d^5*e^5))*(b*x^2+
a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/
2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/c/(-a*d+b*c)
^2/e^2/(a*f-b*e)^(3/2)/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+
e))^(1/2)+1/3*(a^2*d^2*f^2*(c^2*f^2-18*c*d*e*f+d^2*e^2)-2*a*b*d*f*(c^3*f^3
-9*c^2*d*e*f^2-9*c*d^2*e^2*f+d^3*e^3)+b^2*(c^4*f^4-18*c^2*d^2*e^2*f^2+d^4*
e^4))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^
(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)
/c/(-a*d+b*c)/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^4/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/
a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\left(x^{2} d+c\right)^{\frac{5}{2}} \left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d^3*f^3*x^14 +
 (3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^12 + 3*(b*d^3*e^2*f + (3*b*c*
d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)*x^10 + (b*d^3*e^3 + 3*(3*b*c
*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^
3)*x^8 + a*c^3*e^3 + (a*c^3*f^3 + (3*b*c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a
*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*d)*e*f^2)*x^6 + 3*(a*c^3*e*f^2 + (b*c^2
*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*c^2*d)*e^2*f)*x^4 + (3*a*c^3*e^2*f + (b*c
^3 + 3*a*c^2*d)*e^3)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(5/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{5/2}\,{\left(f\,x^2+e\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{5/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{5}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(5/2)/(f*x^2+e)^(5/2),x)
```

3.555 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 651

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \frac{b^2x\sqrt{c+dx^2}}{a(bc-ad)(be-af)\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}$$
$$-\frac{f(abcf^2-a^2df^2-3b^2e(de-cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a(bc-ad)e(be-af)^2(de-cf)\,(e+fx^2)^{3/2}}$$
$$+\frac{c(ab^2cef^2(9de-7cf)-3b^3e^2(de-cf)^2+2a^3df^3(2de-cf)-a^2bf^2(9d^2e^2-3cdef-2c^2f^2))\,\sqrt{a+bx^2}}{3a^{3/2}(bc-ad)e^2(-be+af)^{5/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(3b^2e(de-cf)^2+a^2df^2(3de-cf)-abf(6d^2e^2-3cdef-c^2f^2))\,\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{3a^{3/2}e(-be+af)^{5/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
b^2*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3
/2)-1/3*f*(a*b*c*f^2-a^2*d*f^2-3*b^2*e*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/a/(-a*d+b*c)/e/(-a*f+b*e)^2/(-c*f+d*e)/(f*x^2+e)^(3/2)+1/3*c*(a
*b^2*c*e*f^2*(-7*c*f+9*d*e)-3*b^3*e^2*(-c*f+d*e)^2+2*a^3*d*f^3*(-c*f+2*d*e
)-a^2*b*f^2*(-2*c^2*f^2-3*c*d*e*f+9*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a
*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(-a*d+b*c)/e^2/(a*f-b*e)^(5/2)/(-
c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(3*b^2*e*(-
c*f+d*e)^2+a^2*d*f^2*(-c*f+3*d*e)-a*b*f*(-c^2*f^2-3*c*d*e*f+6*d^2*e^2))*(b
*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/
a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/e/(a*f-
b*e)^(5/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{x^2d+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f^3*x^12 +
 (3*b^2*d*e*f^2 + (b^2*c + 2*a*b*d)*f^3)*x^10 + (3*b^2*d*e^2*f + 3*(b^2*c
+ 2*a*b*d)*e*f^2 + (2*a*b*c + a^2*d)*f^3)*x^8 + (b^2*d*e^3 + a^2*c*f^3 + 3
*(b^2*c + 2*a*b*d)*e^2*f + 3*(2*a*b*c + a^2*d)*e*f^2)*x^6 + a^2*c*e^3 + (3
*a^2*c*e*f^2 + (b^2*c + 2*a*b*d)*e^3 + 3*(2*a*b*c + a^2*d)*e^2*f)*x^4 + (3
*a^2*c*e^2*f + (2*a*b*c + a^2*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.556 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 960

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \frac{b^2x}{a(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$
$$+\frac{d(abd^2e-a^2d^2f+b^2c(de-cf))\,x\sqrt{a+bx^2}}{ac(bc-ad)^2(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}$$
$$+\frac{f(3b^3ce(de-cf)^2+a^3d^2f^2(3de+cf)-2a^2bdf(3d^2e^2+c^2f^2)+ab^2(3d^3e^3+c^3f^3))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac(bc-ad)^2e(be-af)^2(de-cf)^2\,(e+fx^2)^{3/2}}$$
$$-\frac{(3b^4ce^2(de-cf)^3-a^4d^2f^3(3d^2e^2+7cdef-2c^2f^2)+a^3bdf^2(9d^3e^3+12cd^2e^2f+7c^2def^2-4c^3f^3)+a}{3a^{3/2}(bc-ad)^2e}$$
$$+\frac{(3b^3e(de-cf)^3+a^3d^2f^3(9de-cf)-a^2bdf^2(15d^2e^2+3cdef-2c^2f^2)+ab^2f(3d^3e^3+9cd^2e^2f-3c^2de}{3a^{3/2}(bc-ad)e(-be+af)^{5/2}(de-cf)^3\sqrt{c+}}$$

output
```
b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3
/2)+d*(a*b*d^2*e-a^2*d^2*f+b^2*c*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c/(-a*d+b
*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)+1/3*f*(3*b^3*c
*e*(-c*f+d*e)^2+a^3*d^2*f^2*(c*f+3*d*e)-2*a^2*b*d*f*(c^2*f^2+3*d^2*e^2)+a*
b^2*(c^3*f^3+3*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/(-a*d+b*c)^
2/e/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x^2+e)^(3/2)-1/3*(3*b^4*c*e^2*(-c*f+d*e)^
3-a^4*d^2*f^3*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)+a^3*b*d*f^2*(-4*c^3*f^3+7*c
^2*d*e*f^2+12*c*d^2*e^2*f+9*d^3*e^3)+a*b^3*e*(-7*c^4*f^4+12*c^3*d*e*f^3+3*
d^4*e^4)-a^2*b^2*f*(-2*c^4*f^4-7*c^3*d*e*f^3+24*c^2*d^2*e^2*f^2+9*d^4*e^4)
)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2
)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(-a
*d+b*c)^2/e^2/(a*f-b*e)^(5/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)+1/3*(3*b^3*e*(-c*f+d*e)^3+a^3*d^2*f^3*(-c*f+9*d*e)-a^2*b*
d*f^2*(-2*c^2*f^2+3*c*d*e*f+15*d^2*e^2)+a*b^2*f*(-c^3*f^3-3*c^2*d*e*f^2+9*
c*d^2*e^2*f+3*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*El
lipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(3/2)/(-a*d+b*c)/e/(a*f-b*e)^(5/2)/(-c*f+d*e)^3/(d*x^2+c)^(1/
2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 434}$$

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,(x^2d+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d^2*f^3*x^14
 + (3*b^2*d^2*e*f^2 + 2*(b^2*c*d + a*b*d^2)*f^3)*x^12 + (3*b^2*d^2*e^2*f +
 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3)*x^10 +
 (b^2*d^2*e^3 + 6*(b^2*c*d + a*b*d^2)*e^2*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*e*f^2 + 2*(a*b*c^2 + a^2*c*d)*f^3)*x^8 + a^2*c^2*e^3 + (a^2*c^2*f^3
+ 2*(b^2*c*d + a*b*d^2)*e^3 + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f + 6*
(a*b*c^2 + a^2*c*d)*e*f^2)*x^6 + (3*a^2*c^2*e*f^2 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*e^3 + 6*(a*b*c^2 + a^2*c*d)*e^2*f)*x^4 + (3*a^2*c^2*e^2*f + 2*(a
*b*c^2 + a^2*c*d)*e^3)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}(d\,x^2+c)^{\frac{3}{2}}(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.557 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 995

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = \frac{b^2x\sqrt{c+dx^2}}{3a(bc-ad)(be-af)\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}$$

$$+\frac{2b^2(b^2ce+5a^2df-2ab(de+2cf))\,x\sqrt{c+dx^2}}{3a^2(bc-ad)^2(be-af)^2\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}$$

$$-\frac{f(2a^3bcdf^3-a^4d^2f^3-2b^4ce^2(de-cf)+ab^3e(4d^2e^2+5cdef-9c^2f^2)-a^2b^2f(11d^2e^2-11cdef+c^2f^2 \ldots}{3a^2(bc-ad)^2e(be-af)^3(de-cf)\left(e+fx^2\right)^{3/2}}$$

$$+\frac{2c(b^5ce^3(de-cf)^2-a^5d^2f^4(2de-cf)-ab^4e^2(de-cf)^2(2de+5cf)+a^4bdf^3(6d^2e^2-cdef-2c^2f^2)- \ldots}{3a^{5/2}(bc-ad)^2 \ldots}$$

$$-\frac{(2b^4ce^2(de-cf)^2+a^4d^2f^3(3de-cf)-3ab^3e(de-cf)^2(de+3cf)-a^3bdf^2(9d^2e^2-3cdef-2c^2f^2)+ \ldots}{3a^{5/2}(bc-ad)e(-be+af)^{7/2} \ldots}$$

output
```
1/3*b^2*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(f*x^2+e
)^(3/2)+2/3*b^2*(b^2*c*e+5*a^2*d*f-2*a*b*(2*c*f+d*e))*x*(d*x^2+c)^(1/2)/a^
2/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2)-1/3*f*(2*a^3*b
*c*d*f^3-a^4*d^2*f^3-2*b^4*c*e^2*(-c*f+d*e)+a*b^3*e*(-9*c^2*f^2+5*c*d*e*f+
4*d^2*e^2)-a^2*b^2*f*(c^2*f^2-11*c*d*e*f+11*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a^2/(-a*d+b*c)^2/e/(-a*f+b*e)^3/(-c*f+d*e)/(f*x^2+e)^(3/2)+2
/3*c*(b^5*c*e^3*(-c*f+d*e)^2-a^5*d^2*f^4*(-c*f+2*d*e)-a*b^4*e^2*(-c*f+d*e)
^2*(5*c*f+2*d*e)+a^4*b*d*f^3*(-2*c^2*f^2-c*d*e*f+6*d^2*e^2)-a^3*b^2*c*f^3*
(-c^2*f^2-8*c*d*e*f+12*d^2*e^2)+a^2*b^3*e*f*(-5*c^3*f^3+12*c^2*d*e*f^2-12*
c*d^2*e^2*f+6*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*El
lipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(5/2)/(-a*d+b*c)^2/e^2/(a*f-b*e)^(7/2)/(-c*f+d*e)^2/(d*x^2+c)
^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(2*b^4*c*e^2*(-c*f+d*e)^2+a^4*d
^2*f^3*(-c*f+3*d*e)-3*a*b^3*e*(-c*f+d*e)^2*(3*c*f+d*e)-a^3*b*d*f^2*(-2*c^2
*f^2-3*c*d*e*f+9*d^2*e^2)+a^2*b^2*f*(-c^3*f^3+3*c^2*d*e*f^2-9*c*d^2*e^2*f+
9*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(5/2)/(-a*d+b*c)/e/(a*f-b*e)^(7/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2
+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 434$$
$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\sqrt{x^2d+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d*f^3*x^14 +
 (3*b^3*d*e*f^2 + (b^3*c + 3*a*b^2*d)*f^3)*x^12 + 3*(b^3*d*e^2*f + (b^3*c
+ 3*a*b^2*d)*e*f^2 + (a*b^2*c + a^2*b*d)*f^3)*x^10 + (b^3*d*e^3 + 3*(b^3*c
 + 3*a*b^2*d)*e^2*f + 9*(a*b^2*c + a^2*b*d)*e*f^2 + (3*a^2*b*c + a^3*d)*f^
3)*x^8 + a^3*c*e^3 + (a^3*c*f^3 + (b^3*c + 3*a*b^2*d)*e^3 + 9*(a*b^2*c + a
^2*b*d)*e^2*f + 3*(3*a^2*b*c + a^3*d)*e*f^2)*x^6 + 3*(a^3*c*e*f^2 + (a*b^2
*c + a^2*b*d)*e^3 + (3*a^2*b*c + a^3*d)*e^2*f)*x^4 + (3*a^3*c*e^2*f + (3*a
^2*b*c + a^3*d)*e^3)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.558 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1446

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3*b^2*x/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e
)^(3/2)+2/3*b^2*(6*a^2*d*f-4*a*b*c*f-3*a*b*d*e+b^2*c*e)*x/a^2/(-a*d+b*c)^2
/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2)+1/3*d*(6*a^3
*b*d^3*e*f-3*a^4*d^3*f^2+2*b^4*c^2*e*(-c*f+d*e)-a*b^3*c*(-8*c^2*f^2+c*d*e*
f+7*d^2*e^2)-a^2*b^2*d*(13*c^2*f^2-13*c*d*e*f+3*d^2*e^2))*x*(b*x^2+a)^(1/2
)/a^2/c/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/
2)+1/3*f*(2*b^5*c^2*e^2*(-c*f+d*e)^2+a^5*d^3*f^3*(c*f+3*d*e)-a*b^4*c*e*(-c
*f+d*e)^2*(9*c*f+7*d*e)-3*a^4*b*d^2*f^2*(c^2*f^2+3*d^2*e^2)+3*a^3*b^2*d*f*
(c^3*f^3+3*d^3*e^3)-a^2*b^3*(c^4*f^4-14*c^3*d*e*f^3+28*c^2*d^2*e^2*f^2-14*
c*d^3*e^3*f+3*d^4*e^4))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)
^3/e/(-a*f+b*e)^3/(-c*f+d*e)^2/(f*x^2+e)^(3/2)+1/3*(2*b^6*c^2*e^3*(-c*f+d*
e)^3-a*b^5*c*e^2*(-c*f+d*e)^3*(10*c*f+7*d*e)-a^6*d^3*f^4*(-2*c^2*f^2+7*c*d
*e*f+3*d^2*e^2)+a^5*b*d^2*f^3*(-6*c^3*f^3+11*c^2*d*e*f^2+15*c*d^2*e^2*f+12
*d^3*e^3)-3*a^4*b^2*d*f^2*(-2*c^4*f^4-3*c^3*d*e*f^3+15*c^2*d^2*e^2*f^2+6*d
^4*e^4)-a^2*b^4*e*(-10*c^5*f^5+30*c^4*d*e*f^4-45*c^3*d^2*e^2*f^3+45*c^2*d^
3*e^3*f^2-15*c*d^4*e^4*f+3*d^5*e^5)+a^3*b^3*f*(-2*c^5*f^5-23*c^4*d*e*f^4+4
5*c^3*d^2*e^2*f^3+12*d^5*e^5))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(
-a*f+b*e))^(1/2))/a^(5/2)/(-a*d+b*c)^3/e^2/(a*f-b*e)^(7/2)/(-c*f+d*e)^3/(d
*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(2*b^5*c*e^2*(-c*f+d*...
```

Mathematica [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx=\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$
$$\Big\downarrow \text{434}$$
$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}dx$$

input
```
Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(x^2d+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^3*d^2*f^3*x^16
 + (3*b^3*d^2*e*f^2 + (2*b^3*c*d + 3*a*b^2*d^2)*f^3)*x^14 + (3*b^3*d^2*e^2
*f + 3*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^2 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*
d^2)*f^3)*x^12 + (b^3*d^2*e^3 + 3*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f + 3*(b^3
*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 + (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*f^3)*x^10 + ((2*b^3*c*d + 3*a*b^2*d^2)*e^3 + 3*(b^3*c^2 + 6*a*b^2*c*
d + 3*a^2*b*d^2)*e^2*f + 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^2 + (
3*a^2*b*c^2 + 2*a^3*c*d)*f^3)*x^8 + a^3*c^2*e^3 + (a^3*c^2*f^3 + (b^3*c^2
+ 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3 + 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)
*e^2*f + 3*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f^2)*x^6 + (3*a^3*c^2*e*f^2 + (3*a*
b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^3 + 3*(3*a^2*b*c^2 + 2*a^3*c*d)*e^2*f)*
x^4 + (3*a^3*c^2*e^2*f + (3*a^2*b*c^2 + 2*a^3*c*d)*e^3)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^{3/2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{2}}\,(d\,x^2+c)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^(5/2),x)
```

3.559 $\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\begin{aligned}\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx &= \frac{2(bc-ad)^2(be-af)x}{ab^3\sqrt[4]{a+bx^2}} \\ &- \frac{2(15b^3c^2e-40a^3d^2f-30ab^2c(2de+cf)+36a^2bd(de+2cf))\,x}{15ab^3\sqrt[4]{a+bx^2}} \\ &+ \frac{2d(3bde+6bcf-5adf)x(a+bx^2)^{3/4}}{15b^3}+\frac{2d^2fx^3(a+bx^2)^{3/4}}{9b^2} \\ &+ \frac{2(15b^3c^2e-40a^3d^2f-30ab^2c(2de+cf)+36a^2bd(de+2cf))\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{a}b^{7/2}\sqrt[4]{a+bx^2}}\end{aligned}$$

output

```
2*(-a*d+b*c)^2*(-a*f+b*e)*x/a/b^3/(b*x^2+a)^(1/4)-2/15*(15*b^3*c^2*e-40*a^
3*d^2*f-30*a*b^2*c*(c*f+2*d*e)+36*a^2*b*d*(2*c*f+d*e))*x/a/b^3/(b*x^2+a)^(
1/4)+2/15*d*(-5*a*d*f+6*b*c*f+3*b*d*e)*x*(b*x^2+a)^(3/4)/b^3+2/9*d^2*f*x^3
*(b*x^2+a)^(3/4)/b^2+2/15*(15*b^3*c^2*e-40*a^3*d^2*f-30*a*b^2*c*(c*f+2*d*e
)+36*a^2*b*d*(2*c*f+d*e))*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/
2)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/4}} dx = \frac{x\left(2(45b^3c^2e-60a^3d^2f+2a^2bd(27de+54cf-5dfx^2)+ab^2(-45c^2f+18cd\right.}{}$$

input
```
Integrate[((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(x*(2*(45*b^3*c^2*e - 60*a^3*d^2*f + 2*a^2*b*d*(27*d*e + 54*c*f - 5*d*f*x^
2) + a*b^2*(-45*c^2*f + 18*c*d*(-5*e + f*x^2) + d^2*x^2*(9*e + 5*f*x^2)))
+ 3*(-15*b^3*c^2*e + 40*a^3*d^2*f + 30*a*b^2*c*(2*d*e + c*f) - 36*a^2*b*d*
(d*e + 2*c*f))*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b
*x^2)/a)]))/(45*a*b^3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {401, 27, 403, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2 (e+fx^2)}{(a+bx^2)^{5/4}} dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{2x(c+dx^2)^2 (be-af)}{ab\sqrt[4]{a+bx^2}} - \frac{2\int \frac{(dx^2+c)(d(9be-10af)x^2+c(be-2af))}{2\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x\left(c+dx^2\right)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\int \frac{\left(dx^2+c\right)\left(d(9be-10af)x^2+c(be-2af)\right)}{\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \ 403$$

$$\frac{2x\left(c+dx^2\right)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2\int \frac{d\left(60dfa^2-2b(27de+29cf)a+45b^2ce\right)x^2+c\left(20dfa^2-18b(de+cf)a+9b^2ce\right)}{2\sqrt[4]{bx^2+a}}dx}{9b}+\frac{2dx\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(9be-10af)}{9b}}{ab}$$

$$\Big\downarrow \ 27$$

$$\frac{2x\left(c+dx^2\right)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{\int \frac{d\left(60dfa^2-2b(27de+29cf)a+45b^2ce\right)x^2+c\left(20dfa^2-18b(de+cf)a+9b^2ce\right)}{\sqrt[4]{bx^2+a}}dx}{9b}+\frac{2dx\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)(9be-10af)}{9b}}{ab}$$

$$\Big\downarrow \ 299$$

$$\frac{2x\left(c+dx^2\right)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{3\left(-40a^3d^2f+36a^2bd(2cf+de)-30ab^2c(cf+2de)+15b^3c^2e\right)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}+\frac{2dx\left(a+bx^2\right)^{3/4}\left(60a^2df-58abcf-54abde+45b^2ce\right)}{5b}}{9b}+\frac{2dx\left(a+bx^2\right)^{3/4}(c\cdots}{9\cdots}}{ab}$$

$$\Big\downarrow \ 227$$

$$\frac{2x\left(c+dx^2\right)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{3\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3d^2f+36a^2bd(2cf+de)-30ab^2c(cf+2de)+15b^3c^2e\right)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}+\frac{2dx\left(a+bx^2\right)^{3/4}\left(60a^2df-58abcf-54abde+45b^2ce\right)}{5b}}{9b}+\frac{2dx\cdots}{\cdots}}{ab}$$

$$\Big\downarrow \ 225$$

$$\frac{\frac{2x(c+dx^2)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{3\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3d^2f+36a^2bd(2cf+de)-30ab^2c(cf+2de)+15b^3c^2e\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}+\frac{2dx\left(a+bx^2\right)^{3/4}\left(60a^2df-58abcf-54abde\right.}{5b}}{9b}}{ab}$$

$\downarrow$ 212

$$\frac{\frac{2x(c+dx^2)^2(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2dx\left(a+bx^2\right)^{3/4}\left(60a^2df-58abcf-54abde+45b^2ce\right)}{5b}+\frac{3\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)\left(-40a^3d^2f+36a^2bd(2cf+de)-30ab^2c(\right.}{5b\sqrt[4]{a+bx^2}}}{9b}}{ab}$$

input
```
Int[((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2)^2)/(a*b*(a + b*x^2)^(1/4)) - ((2*d*(9*b*e - 1
0*a*f)*x*(a + b*x^2)^(3/4)*(c + d*x^2))/(9*b) + ((2*d*(45*b^2*c*e - 54*a*b
*d*e - 58*a*b*c*f + 60*a^2*d*f)*x*(a + b*x^2)^(3/4))/(5*b) + (3*(15*b^3*c^
2*e - 40*a^3*d^2*f - 30*a*b^2*c*(2*d*e + c*f) + 36*a^2*b*d*(d*e + 2*c*f))*
(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4)))/(9*b
))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\left(x^{2} d+c\right)^{2} \left(f\,x^{2}+e\right)}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{\left(c+dx^2\right)^2\left(e+fx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{{\left(dx^{2}+c\right)}^{2}{\left(fx^{2}+e\right)}}{{\left(bx^{2}+a\right)}^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d^2*f*x^6 + (d^2*e + 2*c*d*f)*x^4 + c^2*e + (2*c*d*e + c^2*f)*x^
2)*(b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx=\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x**2+c)**2*(f*x**2+e)/(b*x**2+a)**(5/4),x)
```

output
```
Integral((c + d*x**2)**2*(e + f*x**2)/(a + b*x**2)**(5/4), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx=\int \frac{(dx^2+c)^2(fx^2+e)}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*(f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx=\int \frac{(dx^2+c)^2(fx^2+e)}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*(f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(d\,x^2+c)^2\,(f\,x^2+e)}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((c + d*x^2)^2*(e + f*x^2))/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^2(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \left(\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)d^2f + 2\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)cdf + \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)d^2e + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^2f + 2\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)cde + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^2e$$

input
```
int((d*x^2+c)^2*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int(x**6/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*d**2*f +
2*int(x**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c*d*f +
 int(x**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*d**2*e +
 int(x**2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c**2*f +
 2*int(x**2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c*d*e
+ int(1/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c**2*e
```

3.560 $\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{2(bc-ad)(be-af)x}{ab^2\sqrt[4]{a+bx^2}} - \frac{2(5b^2ce+12a^2df-10ab(de+cf))\,x}{5ab^2\sqrt[4]{a+bx^2}} + \frac{2dfx(a+bx^2)^{3/4}}{5b^2} + \frac{2(5b^2ce+12a^2df-10ab(de+cf))\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}$$

output

```
2*(-a*d+b*c)*(-a*f+b*e)*x/a/b^2/(b*x^2+a)^(1/4)-2/5*(5*b^2*c*e+12*a^2*d*f-
10*a*b*(c*f+d*e))*x/a/b^2/(b*x^2+a)^(1/4)+2/5*d*f*x*(b*x^2+a)^(3/4)/b^2+2/
5*(5*b^2*c*e+12*a^2*d*f-10*a*b*(c*f+d*e))*(1+b*x^2/a)^(1/4)*EllipticE(sin(
1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{x\left(2(5b^2ce+6a^2df+ab(-5de-5cf+dfx^2))+(-5b^2ce-12a^2df+10ab(de\right.}{5ab^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(x*(2*(5*b^2*c*e + 6*a^2*d*f + a*b*(-5*d*e - 5*c*f + d*f*x^2)) + (-5*b^2*c
*e - 12*a^2*d*f + 10*a*b*(d*e + c*f))*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {401, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{2\int \frac{d(5be-6af)x^2+c(be-2af)}{2\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\int \frac{d(5be-6af)x^2+c(be-2af)}{\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \ 299$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2dx(a+bx^2)^{3/4}(5be-6af)}{5b}-\frac{(2ad(5be-6af)-5bc(be-2af))\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}}{ab}$$

$$\Big\downarrow \ 227$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2dx(a+bx^2)^{3/4}(5be-6af)}{5b}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}(2ad(5be-6af)-5bc(be-2af))\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}}{ab}$$

$$\Big\downarrow \ 225$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2dx(a+bx^2)^{3/4}(5be-6af)}{5b}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}(2ad(5be-6af)-5bc(be-2af))\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}}{ab}$$

$$\Big\downarrow \ 212$$

$$\frac{2x(c+dx^2)(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\frac{2dx(a+bx^2)^{3/4}(5be-6af)}{5b}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)(2ad(5be-6af)-5bc(be-2af))}{5b\sqrt[4]{a+bx^2}}}{ab}$$

input
```
Int[((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2))/(a*b*(a + b*x^2)^(1/4)) - ((2*d*(5*b*e - 6*a
*f)*x*(a + b*x^2)^(3/4))/(5*b) - ((2*a*d*(5*b*e - 6*a*f) - 5*b*c*(b*e - 2*
a*f))*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*Elli
pticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4))
)/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

Maple [F]

$$\int \frac{(x^2d+c)\,(f\,x^2+e)}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^2)\,(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(fx^2+e)}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*(b*x^2 + a)^(3/4)/(b^2*x^4 + 2*
a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{cex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}} + \frac{cfx^3\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}} + \frac{dex^3\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}} + \frac{dfx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

input
```
integrate((d*x**2+c)*(f*x**2+e)/(b*x**2+a)**(5/4),x)
```

output
```
c*e*x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4) + c*f*x
**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)) + d*e
*x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)) + d
*f*x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))
```

Maxima [F]

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(fx^2+e)}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(fx^2+e)}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(d\,x^2+c)\,(f\,x^2+e)}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((c + d*x^2)*(e + f*x^2))/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)(e+fx^2)}{(a+bx^2)^{5/4}}\,dx = \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)df + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)cf + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)de + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)ce$$

input
```
int((d*x^2+c)*(f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int(x**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*d*f + int
(x**2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c*f + int(x*
*2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*d*e + int(1/((a
 + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c*e
```

3.561 $\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx=\frac{2fx}{b\sqrt[4]{a+bx^2}}+\frac{2(be-2af)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
2*f*x/b/(b*x^2+a)^(1/4)+2*(-2*a*f+b*e)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2
*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx=\frac{2(be-af)x+(-be+2af)x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{ab\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(e + f*x^2)/(a + b*x^2)^(5/4),x]
```

output
```
(2*(b*e - a*f)*x + (-(b*e) + 2*a*f)*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(a*b*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {298, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{2x(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{(be-2af)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-2af)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{ab\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{225}$$

$$\frac{2x(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-2af)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{ab\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2x(be-af)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-2af)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{ab\sqrt[4]{a+bx^2}}$$

input
```
Int[(e + f*x^2)/(a + b*x^2)^(5/4),x]
```

output
```
(2*(b*e - a*f)*x)/(a*b*(a + b*x^2)^(1/4)) - ((b*e - 2*a*f)*(1 + (b*x^2)/a)
^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a*b*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [F]

$$\int \frac{f\,x^{2}+e}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(f*x^2 + e)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx = \frac{ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}} + \frac{fx^3\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/4),x)
```

output
```
e*x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4) + f*x**3*
hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4))
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx=\int \frac{f\,x^2+e}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

input
```
int((e + f*x^2)/(a + b*x^2)^(5/4),x)
```

output
```
int((e + f*x^2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}}\,dx = \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)f + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4),x)
```

output
```
int(x**2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*f + int(1
/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*e
```

3.562 $\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 257

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{2(be-af)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}(de-cf)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}(-bc+ad)^{3/2}x} - \frac{\sqrt[4]{a}(de-cf)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}(-bc+ad)^{3/2}x}$$

output
```
2*(-a*f+b*e)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))
),2^(1/2))/a^(1/2)/b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/4)+a^(1/4)*(-c*f+d*e)*(
-b*x^2/a)^(1/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b
*c)^(1/2),I)/d^(1/2)/(a*d-b*c)^(3/2)/x-a^(1/4)*(-c*f+d*e)*(-b*x^2/a)^(1/2)
*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)/d^(
1/2)/(a*d-b*c)^(3/2)/x
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.42 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.37

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{x\left(-\frac{d(-be+af)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(3ac\left(-be(c+2dx^2)+ad(e+\cdots\right.\right.}{(c+dx^2)(\cdots}\right.}{\cdots}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

output
```
(x*(-((d*(-(b*e) + a*f)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])/c) + (6*(3*a*c*(-(b*e*(c + 2*d*x^2)) + a*d
*(e + 2*f*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (
b*e - a*f)*x^2*(c + d*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)
])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
+ b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(3*a*(-(
b*c) + a*d)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$$

$$\Big\downarrow \; 402$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{2\int \frac{d(be-af)x^2+bce+ade-2acf}{2\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\int \frac{d(be-af)x^2+bce+ade-2acf}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 405$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{a(de-cf)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+(be-af)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 227$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{a(de-cf)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-af)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}$$

$$\Big\downarrow \ 225$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{a(de-cf)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-af)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}$$

$$\Big\downarrow \ 212$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{a(de-cf)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)}dx+\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-af)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}$$

$$\Big\downarrow \ 310$$

$$\frac{\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(de-cf)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{\sqrt[4]{\frac{bx^2}{a}+1}(be-af)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{a(bc-ad)}}{}$$

$\Big\downarrow$ 993

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(de-cf)\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{x}+\frac{\sqrt[4]{\frac{bx^2}{a}+1}(\ldots}{\ldots}}{a(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(de-cf)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{x}+\frac{\sqrt[4]{\frac{bx^2}{a}\ldots}}{\ldots}}{a(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

output
```
(2*(b*e - a*f)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)) - (((b*e - a*f)*(1 + (
b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4) + (2*a*(d*e - c*f
)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c)
+ a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) +
a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(a*(b*
c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{f\,x^2+e}{\left(b\,x^2+a\right)^{\frac{5}{4}}\left(x^2 d+c\right)}dx$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{e+fx^2}{(a+bx^2)^{\frac{5}{4}}(c+dx^2)}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)}\,dx=\int \frac{f\,x^2+e}{{\left(b\,x^2+a\right)}^{5/4}\,\left(d\,x^2+c\right)} \,d x$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)}\,dx=\left(\int \frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} a c +\left(b\,x^{2}+a\right)^{\frac{1}{4}} a d\,x^{2}+\left(b\,x^{2}+a\right)^{\frac{1}{4}} b c\,x^{2}+\left(b\,x^{2}+a\right)^{\frac{1}{4}} b d\,x}\right.$$
$$+\left(\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} a c +\left(b\,x^{2}+a\right)^{\frac{1}{4}} a d\,x^{2}+\left(b\,x^{2}+a\right)^{\frac{1}{4}} b c\,x^{2}+\left(b\,x^{2}+a\right)^{\frac{1}{4}} b d\,x^{4}}d x \right) e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

output
```
int(x**2/((a + b*x**2)**(1/4)*a*c + (a + b*x**2)**(1/4)*a*d*x**2 + (a + b*
x**2)**(1/4)*b*c*x**2 + (a + b*x**2)**(1/4)*b*d*x**4),x)*f + int(1/((a + b
*x**2)**(1/4)*a*c + (a + b*x**2)**(1/4)*a*d*x**2 + (a + b*x**2)**(1/4)*b*c
*x**2 + (a + b*x**2)**(1/4)*b*d*x**4),x)*e
```

3.563 $\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 358

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = -\frac{(de-cf)x}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)}$$
$$+\frac{\sqrt{b}(4bce+ade-5acf)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c(bc-ad)^2\sqrt[4]{a+bx^2}}$$
$$-\frac{\sqrt[4]{a}(bc(7de-3cf)-2ad(de+cf))\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{5/2}x}$$
$$+\frac{\sqrt[4]{a}(bc(7de-3cf)-2ad(de+cf))\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{5/2}x}$$

output
```
-1/2*(-c*f+d*e)*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)+1/2*b^(1/2)*(-5*a
*c*f+a*d*e+4*b*c*e)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a
^(1/2))),2^(1/2))/a^(1/2)/c/(-a*d+b*c)^2/(b*x^2+a)^(1/4)-1/4*a^(1/4)*(b*c*
(-3*c*f+7*d*e)-2*a*d*(c*f+d*e))*(-b*x^2/a)^(1/2)*EllipticPi((b*x^2+a)^(1/4
)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)/c/d^(1/2)/(a*d-b*c)^(5/2)/x+
1/4*a^(1/4)*(b*c*(-3*c*f+7*d*e)-2*a*d*(c*f+d*e))*(-b*x^2/a)^(1/2)*Elliptic
Pi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)/c/d^(1/2)/(a
*d-b*c)^(5/2)/x
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.59 (sec) , antiderivative size = 987, normalized size of antiderivative = 2.76

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

output
```
(x*(-72*a*b^2*c^4*e*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]
 - 144*a^2*b*c^3*d*e*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)
] + 36*a^3*c^2*d^2*e*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)
] + 144*a^2*b*c^4*f*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]
 + 36*a^3*c^3*d*f*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 6*c*(a*d*(d*e - c*f)*(a + b*x^2) + 4*b*c*(b*e - a*f)*(c + d*x^2))*(6*a*c*
AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Appell
F1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4,
1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + 5*a*b*c*d*f*x^2*(1 + (b*x^2)/a)^(1
/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*
a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Ap
pellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5
/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + 4*b^2*c*d*e*x^2*(1 + (b*x^2)/a
)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a
*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3
/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + a*b*d^2*e*x^2*(1 + (b*x^2
)/a)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/
c)]*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(
4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*Appe...
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {402, 27, 402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(c+dx^2)(bc-ad)}-\frac{2\int \frac{-3d(be-af)x^2+bce+ade-2acf}{2\sqrt[4]{bx^2+a}(dx^2+c)^2}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(c+dx^2)(bc-ad)}-\frac{\int \frac{-3d(be-af)x^2+bce+ade-2acf}{\sqrt[4]{bx^2+a}(dx^2+c)^2}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(c+dx^2)(bc-ad)}-\frac{\frac{\int \frac{-2d(de+cf)a^2+8bc(de-cf)a+bd(4bce+ade-5acf)x^2+4b^2c^2e}{2\sqrt[4]{bx^2+a}(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx(a+bx^2)^{3/4}(-5acf+ade+4bce)}{2c(c+dx^2)(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(c+dx^2)(bc-ad)}-\frac{\frac{\int \frac{bd(4bce+ade-5acf)x^2+2\left(-d(de+cf)a^2+4bc(de-cf)a+2b^2c^2e\right)}{\sqrt[4]{bx^2+a}(dx^2+c)}dx}{4c(bc-ad)}-\frac{dx(a+bx^2)^{3/4}(-5acf+ade+4bce)}{2c(c+dx^2)(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{405}$$

$$\frac{\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b(-5acf+ade+4bce)\int\frac{1}{\sqrt[4]{bx^2+a}}dx+a(bc(7de-3cf)-2ad(cf+de))\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-5acf+ade+4bce)}{2c\left(c+dx^2\right)(bc-ad)}}{a(bc-ad)}}{}$$

$\Big\downarrow$ 227

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(-5acf+ade+4bce)\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}+a(bc(7de-3cf)-2ad(cf+de))\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-5acf+ade+4bce)}{2c\left(c+dx^2\right)(bc-ad)}}{a(bc-ad)}$$

$\Big\downarrow$ 225

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\frac{b\sqrt[4]{\frac{bx^2}{a}+1}(-5acf+ade+4bce)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}+a(bc(7de-3cf)-2ad(cf+de))\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-5acf+ade+4bce)}{2c\left(c+dx^2\right)(bc-ad)}}{a(bc-ad)}$$

$\Big\downarrow$ 212

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)(bc-ad)}-\frac{\frac{a(bc(7de-3cf)-2ad(cf+de))\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)(-5acf+ade+4bce)}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-5acf+ade+4bce)}{2c\left(c+dx^2\right)(bc-ad)}}{a(bc-ad)}$$

$\Big\downarrow$ 310

$$\frac{\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}-\frac{\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(bc(7de-3cf)-2ad(cf+de))\int\frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}}{a(bc-ad)}$$

$\Big\downarrow$ 993

$$\frac{\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}-\frac{\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(bc(7de-3cf)-2ad(cf+de))\left(\frac{\int\frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int\frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}\right)}{x}+}{4c(bc-ad)}}{a(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}-\frac{\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}(bc(7de-3cf)-2ad(cf+de))\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}\right)}{x}}{4c(bc-ad)}}{a(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

output
```
(2*(b*e - a*f)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) - (-1/2*(d
*(4*b*c*e + a*d*e - 5*a*c*f)*x*(a + b*x^2)^(3/4))/(c*(b*c - a*d)*(c + d*x^
2)) + ((b*(4*b*c*e + a*d*e - 5*a*c*f)*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b
*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/S
qrt[b]))/(a + b*x^2)^(1/4) + (2*a*(b*c*(7*d*e - 3*c*f) - 2*a*d*(d*e + c*f)
)*Sqrt[-((b*x^2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c)
+ a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) +
a*d]) - (a^(1/4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(4*c*(
b*c - a*d)))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{f\,x^{2}+e}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(x^{2} d+c\right)^{2}}d x$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\int \frac{f\,x^2+e}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)^2}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx=\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a\,c^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,acd\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,a\,d^2x^4+(b\,x^2+a\right.$$
$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a\,c^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,acd\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,a\,d^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,b\,c^2x^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,bcd\,x^4\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

output
```
int(x**2/((a + b*x**2)**(1/4)*a*c**2 + 2*(a + b*x**2)**(1/4)*a*c*d*x**2 +
(a + b*x**2)**(1/4)*a*d**2*x**4 + (a + b*x**2)**(1/4)*b*c**2*x**2 + 2*(a +
 b*x**2)**(1/4)*b*c*d*x**4 + (a + b*x**2)**(1/4)*b*d**2*x**6),x)*f + int(1
/((a + b*x**2)**(1/4)*a*c**2 + 2*(a + b*x**2)**(1/4)*a*c*d*x**2 + (a + b*x
**2)**(1/4)*a*d**2*x**4 + (a + b*x**2)**(1/4)*b*c**2*x**2 + 2*(a + b*x**2)
**(1/4)*b*c*d*x**4 + (a + b*x**2)**(1/4)*b*d**2*x**6),x)*e
```

3.564 $\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 493

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3} \, dx = -\frac{(de-cf)x}{4c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)^2}$$
$$-\frac{(bc(15de-7cf)-2ad(3de+cf))x}{16c^2(bc-ad)^2\sqrt[4]{a+bx^2}\,(c+dx^2)}$$
$$+\frac{\sqrt{b}(32b^2c^2e+abc(19de-43cf)-2a^2d(3de+cf))\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{16\sqrt{a}c^2(bc-ad)^3\sqrt[4]{a+bx^2}}$$
$$+\frac{\sqrt[4]{a}(7b^2c^2(11de-3cf)+4a^2d^2(3de+cf)-4abcd(11de+7cf))\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\cdots\right.\right.}{32c^2\sqrt{d}(-bc+ad)^{7/2}x}$$
$$-\frac{\sqrt[4]{a}(7b^2c^2(11de-3cf)+4a^2d^2(3de+cf)-4abcd(11de+7cf))\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\cdots\right.\right.}{32c^2\sqrt{d}(-bc+ad)^{7/2}x}$$

output
```
-1/4*(-c*f+d*e)*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)^2-1/16*(b*c*(-7*c
*f+15*d*e)-2*a*d*(c*f+3*d*e))*x/c^2/(-a*d+b*c)^2/(b*x^2+a)^(1/4)/(d*x^2+c)
+1/16*b^(1/2)*(32*b^2*c^2*e+a*b*c*(-43*c*f+19*d*e)-2*a^2*d*(c*f+3*d*e))*(1
+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1
/2)/c^2/(-a*d+b*c)^3/(b*x^2+a)^(1/4)+1/32*a^(1/4)*(7*b^2*c^2*(-3*c*f+11*d*
e)+4*a^2*d^2*(c*f+3*d*e)-4*a*b*c*d*(7*c*f+11*d*e))*(-b*x^2/a)^(1/2)*Ellipt
icPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)/c^2/d^(1/
2)/(a*d-b*c)^(7/2)/x-1/32*a^(1/4)*(7*b^2*c^2*(-3*c*f+11*d*e)+4*a^2*d^2*(c*
f+3*d*e)-4*a*b*c*d*(7*c*f+11*d*e))*(-b*x^2/a)^(1/2)*EllipticPi((b*x^2+a)^(
1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)/c^2/d^(1/2)/(a*d-b*c)^(7/2
)/x
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 1540, normalized size of antiderivative = 3.12

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^3),x]
```

output
```
(x*(-576*a*b^3*c^5*e*(c + d*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] - 1728*a^2*b^2*c^4*d*e*(c + d*x^2)*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] + 900*a^3*b*c^3*d^2*e*(c + d*x^2)*AppellF1[1
/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 216*a^4*c^2*d^3*e*(c + d*x^
2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 1152*a^2*b^2*c
^5*f*(c + d*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
540*a^3*b*c^4*d*f*(c + d*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] - 72*a^4*c^3*d^2*f*(c + d*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*
x^2)/a), -((d*x^2)/c)] + 6*c*(4*a*c*d*(-(b*c) + a*d)*(-(d*e) + c*f)*(a + b
*x^2) - a*d*(2*a*d*(3*d*e + c*f) + b*c*(-19*d*e + 11*c*f))*(a + b*x^2)*(c
+ d*x^2) + 32*b^2*c^2*(b*e - a*f)*(c + d*x^2)^2)*(6*a*c*AppellF1[1/2, 1/4,
 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/
a), -((d*x^2)/c)])) + 6*a^2*b*d^3*e*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)^
2*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*a*c*AppellF1[1
/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/
4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -(
(b*x^2)/a), -((d*x^2)/c)])) + 43*a*b^2*c^2*d*f*x^2*(1 + (b*x^2)/a)^(1/4)*(
c + d*x^2)^2*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*a*c
*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Ap...
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {402, 27, 402, 27, 402, 27, 405, 227, 225, 212, 310, 993, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}(c+dx^2)^2(bc-ad)}-\frac{2\int \frac{-7d(be-af)x^2+bce+ade-2acf}{2\sqrt[4]{bx^2+a}(dx^2+c)^3}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}-\frac{\int \frac{-7d(be-af)x^2+bce+ade-2acf}{\sqrt[4]{bx^2+a}(dx^2+c)^3}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\int \frac{-2d(3de+cf)a^2+16bc(de-cf)a-3bd(8bce+ade-9acf)x^2+8b^2c^2e}{2\sqrt[4]{bx^2+a}\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-9acf+ade+8bce)}{4c(c+dx^2)^2(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\int \frac{2\left(-d(3de+cf)a^2+8bc(de-cf)a+4b^2c^2e\right)-3bd(8bce+ade-9acf)x^2}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)^2}dx}{8c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}(-9acf+ade+8bce)}{4c(c+dx^2)^2(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\frac{\int \frac{4d^2(3de+cf)a^3-10bcd(5de+3cf)a^2+32b^2c^2(3de-2cf)a+bd\left(-2d(3de+cf)a^2+bc(19de-43cf)a+32b^2c^2e\right)x^2+32b^3c^3e}{2\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}\left(-2a^2d(cf+3de)+\right.}{2c\left(c+dx^2\right)(b}}{8c(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\frac{\int \frac{bd\left(-2d(3de+cf)a^2+bc(19de-43cf)a+32b^2c^2e\right)x^2+2\left(2d^2(3de+cf)a^3-5bcd(5de+3cf)a^2+16b^2c^2(3de-2cf)a+16b^3c^3e\right)}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}\left(-2a^2d(cf+3de\right.}{2c\left(c+dx^2\right.}}{8c(bc-ad)}}{a(bc-ad)}$$

$$\Big\downarrow \text{405}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{a\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx+b\left(-2a^2d(cf+3de)+abc(19de-43cf)+32b^2c^2e\right)\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{4c(bc-ad)}-\frac{dx}{\cdots}}{8c(bc-ad)}}{a(bc-ad)}$$

$\downarrow$ 227

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{a\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-2a^2d(cf+3de)+abc(19de-43cf)+32b^2c^2e\right)\int\frac{\cdots}{\sqrt[4]{\frac{bx^2}{a}\cdots}}}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}}{8c(bc-ad)}}{a(bc-ad)}$$

$\downarrow$ 225

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{a\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(-2a^2d(cf+3de)+abc(19de-43cf)+32b^2c^2e\right)\left(\frac{\cdots}{\sqrt[4]{\frac{bx^2}{a}\cdots}}\cdots\right.}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}}{8c(bc-ad)}}{a(bc-ad)}$$

$\downarrow$ 212

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{a\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\int\frac{1}{\sqrt[4]{bx^2+a}\left(dx^2+c\right)}dx+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)\left(-2\cdots\right.}{\sqrt[4]{a+bx^2}}}{4c(bc-ad)}}{8c(bc-ad)}}{a(bc-a\cdots}$$

$\downarrow$ 310

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-$$

$$\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{bx^2+a}{a}}\left(bc-ad+d\left(bx^2+a\right)\right)}d\sqrt[4]{bx^2+a}}{x}+\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{4c(bc-ad)}}{8c(bc-ad)}$$

$$\Big\downarrow \textbf{993}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-$$

$$\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\left(\frac{\int \frac{1}{\left(\sqrt{ad-bc}+\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2+a}{a}}}d\sqrt[4]{bx^2+a}}{2\sqrt{d}}-\frac{\int \frac{1}{\left(\sqrt{ad-bc}-\sqrt{d}\sqrt{bx^2+a}\right)\sqrt{1-\frac{bx^2}{\cdots}}}\cdots}{2\sqrt{d}}\right.}{x}\cdots}{4c(bc-ad)}$$

$$\Big\downarrow \textbf{1542}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}-$$

$$\frac{\frac{2a\sqrt{-\frac{bx^2}{a}}\left(4a^2d^2(cf+3de)-4abcd(7cf+11de)+7b^2c^2(11de-3cf)\right)\left(\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt{d}\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\text{arcs}\cdots\right.}{2\sqrt{d}\sqrt{a\cdots}}\right.}{x}\cdots}{4c(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^3),x]
```

output
```
(2*(b*e - a*f)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)^2) - (-1/4*
(d*(8*b*c*e + a*d*e - 9*a*c*f)*x*(a + b*x^2)^(3/4))/(c*(b*c - a*d)*(c + d*
x^2)^2) + (-1/2*(d*(32*b^2*c^2*e + a*b*c*(19*d*e - 43*c*f) - 2*a^2*d*(3*d*
e + c*f))*x*(a + b*x^2)^(3/4))/(c*(b*c - a*d)*(c + d*x^2)) + ((b*(32*b^2*c
^2*e + a*b*c*(19*d*e - 43*c*f) - 2*a^2*d*(3*d*e + c*f))*(1 + (b*x^2)/a)^(1
/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4) + (2*a*(7*b^2*c^2*(11*d*e - 3
*c*f) + 4*a^2*d^2*(3*d*e + c*f) - 4*a*b*c*d*(11*d*e + 7*c*f))*Sqrt[-((b*x^
2)/a)]*((a^(1/4)*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSi
n[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d]) - (a^(1/
4)*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/
4)/a^(1/4)], -1])/(2*Sqrt[d]*Sqrt[-(b*c) + a*d])))/x)/(4*c*(b*c - a*d)))/(
8*c*(b*c - a*d)))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{f\,x^{2}+e}{\left(b\,x^{2}+a\right)^{\frac{5}{4}} \left(x^{2} d+c\right)^{3}}d x$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/4)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}\,(c+dx^2)^3}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^3}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^3), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^3}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3}\,dx=\int \frac{f\,x^2+e}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)^3}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^3),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^3}\,dx=\left(\int \frac{}{(b\,x^2+a)^{\frac{1}{4}}\,a\,c^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,c^2d\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,ac\,d^2x^4+(b\,x^2\right.$$

$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a\,c^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,c^2d\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,ac\,d^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,a\,d^3x^6+(b\,x^2+a)^{\frac{1}{4}}\,b\,c^3\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^3,x)
```

output

```
int(x**2/((a + b*x**2)**(1/4)*a*c**3 + 3*(a + b*x**2)**(1/4)*a*c**2*d*x**2
 + 3*(a + b*x**2)**(1/4)*a*c*d**2*x**4 + (a + b*x**2)**(1/4)*a*d**3*x**6 +
 (a + b*x**2)**(1/4)*b*c**3*x**2 + 3*(a + b*x**2)**(1/4)*b*c**2*d*x**4 + 3
*(a + b*x**2)**(1/4)*b*c*d**2*x**6 + (a + b*x**2)**(1/4)*b*d**3*x**8),x)*f
 + int(1/((a + b*x**2)**(1/4)*a*c**3 + 3*(a + b*x**2)**(1/4)*a*c**2*d*x**2
 + 3*(a + b*x**2)**(1/4)*a*c*d**2*x**4 + (a + b*x**2)**(1/4)*a*d**3*x**6 +
 (a + b*x**2)**(1/4)*b*c**3*x**2 + 3*(a + b*x**2)**(1/4)*b*c**2*d*x**4 + 3
*(a + b*x**2)**(1/4)*b*c*d**2*x**6 + (a + b*x**2)**(1/4)*b*d**3*x**8),x)*e
```

3.565 $\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx = \frac{2(be-af)x}{a(bc-ad)\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}} - \frac{(bce+ade-2acf)x\sqrt[4]{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{(bc-ad)x^2}{a(c+dx^2)}\right)}{ac(bc-ad)\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}}$$

output
```
2*(-a*f+b*e)*x/a/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)^(1/4)-(-2*a*c*f+a*d*
e+b*c*e)*x*(c*(b*x^2+a)/a/(d*x^2+c))^(1/4)*hypergeom([1/4, 1/2],[3/2],-(-a
*d+b*c)*x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)^(1/4)
```

Mathematica [A] (warning: unable to verify)

Time = 10.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx = \frac{x\left(ce(3c+2dx^2)\,\text{Hypergeometric2F1}\left(1,\frac{5}{4},\frac{5}{2},\frac{(bc-ad)x^2}{c(a+bx^2)}\right)+\frac{c^2fx^2(a+bx^2)\sqrt[4]{\cdots}}{\cdots}\right.}{3c^3(\cdots}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^(5/4)),x]
```

output
```
(x*(c*e*(3*c + 2*d*x^2)*Hypergeometric2F1[1, 5/4, 5/2, ((b*c - a*d)*x^2)/(
c*(a + b*x^2))] + (c^2*f*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeomet
ric2F1[5/4, 3/2, 5/2, ((-(b*c) + a*d)*x^2)/(a*(c + d*x^2))])/(a*(1 + (d*x^
2)/c)^(5/4)) + ((b*c - a*d)*e*x^2*(c + d*x^2)*Hypergeometric2F1[2, 9/4, 7/
2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(3*c^3*(a + b*x^2)^(5
/4)*(c + d*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {402, 27, 294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}(bc-ad)}-\frac{2\int \frac{bce+ade-2acf}{2\sqrt[4]{bx^2+a}(dx^2+c)^{5/4}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}(bc-ad)}-\frac{(-2acf+ade+bce)\int \frac{1}{\sqrt[4]{bx^2+a}(dx^2+c)^{5/4}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{294}$$

$$\frac{2x(be-af)}{a\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}(bc-ad)}-\frac{x\sqrt[4]{\frac{c(a+bx^2)}{a(c+dx^2)}}(-2acf+ade+bce)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{(bc-ad)x^2}{a(dx^2+c)}\right)}{ac\sqrt[4]{a+bx^2}\sqrt[4]{c+dx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^(5/4)),x]
```

output
```
(2*(b*e - a*f)*x)/(a*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)^(1/4)) - ((
b*c*e + a*d*e - 2*a*c*f)*x*((c*(a + b*x^2))/(a*(c + d*x^2)))^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, -(((b*c - a*d)*x^2)/(a*(c + d*x^2)))])/(a*c*(b*c
 - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
+ p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q
+ 1) + 1, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{f\,x^{2}+e}{\left(b\,x^{2}+a\right)^{\frac{5}{4}} \left(x^{2} d+c\right)^{\frac{5}{4}}}dx$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x)
```

Fricas [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)^(3/4)*(f*x^2 + e)/(b^2*d^2*x^8 + 2*
(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 +
2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx = \int \frac{e+fx^2}{(a+bx^2)^{\frac{5}{4}}(c+dx^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(5/4)/(d*x**2+c)**(5/4),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(5/4)*(c + d*x**2)**(5/4)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^(5/4)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^{\frac{5}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(5/4)*(d*x^2 + c)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx=\int \frac{f\,x^2+e}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)^{5/4}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^(5/4)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(5/4)*(c + d*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{5/4}(c+dx^2)^{5/4}}\,dx=\left(\int \frac{x^2}{(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ac+(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ad\,x^2+(d\,x^2+c\right.$$

$$+\left(\int \frac{1}{(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ac+(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ad\,x^2+(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,bc\,x^2+(d\,x^2+c)^{\frac{1}{4}}\right.$$

input
```
int((f*x^2+e)/(b*x^2+a)^(5/4)/(d*x^2+c)^(5/4),x)
```

output
```
int(x**2/((c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*a*c + (c + d*x**2)**(1/4
)*(a + b*x**2)**(1/4)*a*d*x**2 + (c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*b
*c*x**2 + (c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*b*d*x**4),x)*f + int(1/(
(c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*a*c + (c + d*x**2)**(1/4)*(a + b*x
**2)**(1/4)*a*d*x**2 + (c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*b*c*x**2 +
(c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*b*d*x**4),x)*e
```

3.566 $\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx = \frac{2(be-af)x(c+dx^2)^{3/4}}{5a(bc-ad)\,(a+bx^2)^{5/4}} + \frac{(3bce-5ade+2acf)x\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{5/4}(c+dx^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(c+dx^2)}\right)}{5ac(bc-ad)\,(a+bx^2)^{5/4}}$$

output
```
2/5*(-a*f+b*e)*x*(d*x^2+c)^(3/4)/a/(-a*d+b*c)/(b*x^2+a)^(5/4)+1/5*(2*a*c*f
-5*a*d*e+3*b*c*e)*x*(c*(b*x^2+a)/a/(d*x^2+c))^(5/4)*(d*x^2+c)^(3/4)*hyperg
eom([1/2, 5/4],[3/2],-(-a*d+b*c)*x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/(b*x^2+a)
^(5/4)
```

Mathematica [A] (warning: unable to verify)

Time = 10.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.40

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx = \frac{x\left(\frac{16fx^2(a+bx^2)^3\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{4},\frac{5}{2},\frac{(-bc+ad)x^2}{a(c+dx^2)}\right)}{a^2\left(1+\frac{dx^2}{c}\right)^{5/4}} + \frac{e(c+dx^2)\,\text{Gamma}(\ldots}{\ldots}\right.}{4\ldots}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(9/4)*(c + d*x^2)^(1/4)),x]
```

output
```
(x*((16*f*x^2*(a + b*x^2)^3*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/2, 9
/4, 5/2, ((-(b*c) + a*d)*x^2)/(a*(c + d*x^2))])/(a^2*(1 + (d*x^2)/c)^(5/4)
) + (e*(c + d*x^2)*Gamma[1/4]*(5*c*(a + b*x^2)*(3*c + 2*d*x^2)*Hypergeomet
ric2F1[1, 9/4, 5/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 9*(b*c - a*d)*x^2
*(c + d*x^2)*Hypergeometric2F1[2, 13/4, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x
^2))]))/(c^3*Gamma[9/4])))/(48*(a + b*x^2)^(13/4)*(c + d*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {402, 27, 294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{2x\left(c+dx^2\right)^{3/4}(be-af)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}-\frac{2\int -\frac{3bce-5ade+2acf}{2(bx^2+a)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2acf-5ade+3bce)\int \frac{1}{(bx^2+a)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}+\frac{2x\left(c+dx^2\right)^{3/4}(be-af)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \text{294}$$

$$\frac{x\left(c+dx^2\right)^{3/4}\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{5/4}(2acf-5ade+3bce)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(dx^2+c)}\right)}{5ac\left(a+bx^2\right)^{5/4}(bc-ad)}+\frac{2x\left(c+dx^2\right)^{3/4}(be-af)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(9/4)*(c + d*x^2)^(1/4)),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2)^(3/4))/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) +
((3*b*c*e - 5*a*d*e + 2*a*c*f)*x*((c*(a + b*x^2))/(a*(c + d*x^2)))^(5/4)*(
c + d*x^2)^(3/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -(((b*c - a*d)*x^2)/(a*(
c + d*x^2)))])/(5*a*c*(b*c - a*d)*(a + b*x^2)^(5/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
+ p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q
+ 1) + 1, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{f\,x^2+e}{\left(b\,x^2+a\right)^{\frac{9}{4}}\left(x^2 d+c\right)^{\frac{1}{4}}}dx$$

input
```
int((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x)
```

output
```
int((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x)
```

Fricas [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)^(3/4)*(f*x^2 + e)/(b^3*d*x^8 + (b^3
*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a^3
*d)*x^2), x)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\int \frac{e+fx^2}{(a+bx^2)^{\frac{9}{4}}\sqrt[4]{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(9/4)/(d*x**2+c)**(1/4),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(9/4)*(c + d*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(9/4)*(d*x^2 + c)^(1/4)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(9/4)*(d*x^2 + c)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{{\left(b\,x^2+a\right)}^{9/4}\,{\left(d\,x^2+c\right)}^{1/4}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(9/4)*(c + d*x^2)^(1/4)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(9/4)*(c + d*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{9/4}\sqrt[4]{c+dx^2}}\,dx=\left(\int \frac{x^2}{(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(d\,x^2+c)^{\frac{1}{4}}}\right.$$
$$+\left(\int \frac{1}{(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(d\,x^2+c)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(9/4)/(d*x^2+c)^(1/4),x)
```

output

```
int(x**2/((c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*a**2 + 2*(c + d*x**2)**(
1/4)*(a + b*x**2)**(1/4)*a*b*x**2 + (c + d*x**2)**(1/4)*(a + b*x**2)**(1/4
)*b**2*x**4),x)*f + int(1/((c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*a**2 +
2*(c + d*x**2)**(1/4)*(a + b*x**2)**(1/4)*a*b*x**2 + (c + d*x**2)**(1/4)*(
a + b*x**2)**(1/4)*b**2*x**4),x)*e
```

3.567 $\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \frac{2(be-af)x(c+dx^2)^{7/4}}{9a(bc-ad)(a+bx^2)^{9/4}} + \frac{(7bce-9ade+2acf)x\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{9/4}(c+dx^2)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(c+dx^2)}\right)}{9ac(bc-ad)(a+bx^2)^{9/4}}$$

output
```
2/9*(-a*f+b*e)*x*(d*x^2+c)^(7/4)/a/(-a*d+b*c)/(b*x^2+a)^(9/4)+1/9*(2*a*c*f
-9*a*d*e+7*b*c*e)*x*(c*(b*x^2+a)/a/(d*x^2+c))^(9/4)*(d*x^2+c)^(7/4)*hyperg
eom([1/2, 9/4],[3/2],-(-a*d+b*c)*x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/(b*x^2+a)
^(9/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.91 (sec) , antiderivative size = 1207, normalized size of antiderivative = 7.18

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*x^2)^(3/4)*(e + f*x^2))/(a + b*x^2)^(13/4),x]
```

output
```
(-2*x*(63*a*b^3*c^4*e*(a + b*x^2)^2*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] - 18*a^2*b^2*c^3*d*e*(a + b*x^2)^2*AppellF1[1/2, 1/4, 1
/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 81*a^3*b*c^2*d^2*e*(a + b*x^2)^2*Ap
pellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 18*a^2*b^2*c^4*f*
(a + b*x^2)^2*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 1
8*a^3*b*c^3*d*f*(a + b*x^2)^2*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -
((d*x^2)/c)] - (c + d*x^2)*(5*a^2*(b*c - a*d)^2*(b*e - a*f) + a*(-(b*c) +
a*d)*(-7*b^2*c*e + 5*a^2*d*f + a*b*(4*d*e - 2*c*f))*(a + b*x^2) + 3*b^2*c*
(7*b*c*e - 9*a*d*e + 2*a*c*f)*(a + b*x^2)^2)*(6*a*c*AppellF1[1/2, 1/4, 1/4
, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1/4, 5/2, -((b*x^2)/
a), -((d*x^2)/c)])) + 14*b^3*c^2*d*e*x^2*(a + b*x^2)^2*(1 + (b*x^2)/a)^(1/
4)*(1 + (d*x^2)/c)^(1/4)*AppellF1[3/2, 1/4, 1/4, 5/2, -((b*x^2)/a), -((d*x
^2)/c)]*(6*a*c*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] -
x^2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*Ap
pellF1[3/2, 5/4, 1/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + 4*a*b^2*c^2*d*f
*x^2*(a + b*x^2)^2*(1 + (b*x^2)/a)^(1/4)*(1 + (d*x^2)/c)^(1/4)*AppellF1[3/
2, 1/4, 1/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*a*c*AppellF1[1/2, 1/4, 1/
4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2
, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1/4, 5/2, -((b*x...
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {401, 27, 402, 27, 294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{2x(c+dx^2)^{3/4}(be-af)}{9ab(a+bx^2)^{9/4}}-\frac{2\int -\frac{d(4be+5af)x^2+c(7be+2af)}{2(bx^2+a)^{9/4}\sqrt[4]{dx^2+c}}dx}{9ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{d(4be+5af)x^2+c(7be+2af)}{(bx^2+a)^{9/4}\sqrt[4]{dx^2+c}}dx}{9ab}+\frac{2x(c+dx^2)^{3/4}(be-af)}{9ab(a+bx^2)^{9/4}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{2x(c+dx^2)^{3/4}(bc(2af+7be)-ad(5af+4be))}{5a(a+bx^2)^{5/4}(bc-ad)}-\frac{2\int -\frac{3bc(7bce-9ade+2acf)}{2(bx^2+a)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}}{9ab}+\frac{2x(c+dx^2)^{3/4}(be-af)}{9ab(a+bx^2)^{9/4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3bc(2acf-9ade+7bce)\int \frac{1}{(bx^2+a)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}+\frac{2x(c+dx^2)^{3/4}(bc(2af+7be)-ad(5af+4be))}{5a(a+bx^2)^{5/4}(bc-ad)}}{9ab}+\frac{2x(c+dx^2)^{3/4}(be-af)}{9ab(a+bx^2)^{9/4}}$$

$$\Big\downarrow \text{ 294}$$

$$\frac{\frac{3bx(c+dx^2)^{3/4}\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{5/4}(2acf-9ade+7bce)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(dx^2+c)}\right)}{5a(a+bx^2)^{5/4}(bc-ad)}+\frac{2x(c+dx^2)^{3/4}(bc(2af+7be)-ad(5af+4be))}{5a(a+bx^2)^{5/4}(bc-ad)}}{9ab}$$

$$\frac{2x(c+dx^2)^{3/4}(be-af)}{9ab(a+bx^2)^{9/4}}$$

input
```
Int[((c + d*x^2)^(3/4)*(e + f*x^2))/(a + b*x^2)^(13/4),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2)^(3/4))/(9*a*b*(a + b*x^2)^(9/4)) + ((2*(b*c*(
7*b*e + 2*a*f) - a*d*(4*b*e + 5*a*f))*x*(c + d*x^2)^(3/4))/(5*a*(b*c - a*d
)*(a + b*x^2)^(5/4)) + (3*b*(7*b*c*e - 9*a*d*e + 2*a*c*f)*x*((c*(a + b*x^2
))/(a*(c + d*x^2)))^(5/4)*(c + d*x^2)^(3/4)*Hypergeometric2F1[1/2, 5/4, 3/
2, -(((b*c - a*d)*x^2)/(a*(c + d*x^2)))])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/
4)))/(9*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
 + p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q
 + 1) + 1, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{3}{4}}(f\,x^2+e)}{(b\,x^2+a)^{\frac{13}{4}}}dx$$

input
```
int((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x)
```

output
```
int((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{4}}(fx^2+e)}{(bx^2+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x, algorithm="fricas"
)
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)^(3/4)*(f*x^2 + e)/(b^4*x^8 + 4*a*b^
3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(3/4)*(f*x**2+e)/(b*x**2+a)**(13/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{4}}(fx^2+e)}{(bx^2+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x, algorithm="maxima"
)
```

output
```
integrate((d*x^2 + c)^(3/4)*(f*x^2 + e)/(b*x^2 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{4}}(fx^2+e)}{(bx^2+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(3/4)*(f*x^2 + e)/(b*x^2 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \int \frac{(d\,x^2+c)^{3/4}\,(f\,x^2+e)}{(b\,x^2+a)^{13/4}}\,dx$$

input
```
int(((c + d*x^2)^(3/4)*(e + f*x^2))/(a + b*x^2)^(13/4),x)
```

output
```
int(((c + d*x^2)^(3/4)*(e + f*x^2))/(a + b*x^2)^(13/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{3/4}(e+fx^2)}{(a+bx^2)^{13/4}}\,dx = \left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{4}}}\right.$$
$$+\left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}\,x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,b^3x^6}dx\right) f$$

input
```
int((d*x^2+c)^(3/4)*(f*x^2+e)/(b*x^2+a)^(13/4),x)
```

output
```
int((c + d*x**2)**(3/4)/((a + b*x**2)**(1/4)*a**3 + 3*(a + b*x**2)**(1/4)*
a**2*b*x**2 + 3*(a + b*x**2)**(1/4)*a*b**2*x**4 + (a + b*x**2)**(1/4)*b**3
*x**6),x)*e + int(((c + d*x**2)**(3/4)*x**2)/((a + b*x**2)**(1/4)*a**3 + 3
*(a + b*x**2)**(1/4)*a**2*b*x**2 + 3*(a + b*x**2)**(1/4)*a*b**2*x**4 + (a
+ b*x**2)**(1/4)*b**3*x**6),x)*f
```

3.568 $\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \frac{2(be-af)x(c+dx^2)^{11/4}}{13a(bc-ad)(a+bx^2)^{13/4}} + \frac{(11bce-13ade+2acf)x\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{13/4}(c+dx^2)^{11/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(c+dx^2)}\right)}{13ac(bc-ad)(a+bx^2)^{13/4}}$$

output
```
2/13*(-a*f+b*e)*x*(d*x^2+c)^(11/4)/a/(-a*d+b*c)/(b*x^2+a)^(13/4)+1/13*(2*a
*c*f-13*a*d*e+11*b*c*e)*x*(c*(b*x^2+a)/a/(d*x^2+c))^(13/4)*(d*x^2+c)^(11/4
)*hypergeom([1/2, 13/4],[3/2],-(-a*d+b*c)*x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/
(b*x^2+a)^(13/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 12.19 (sec) , antiderivative size = 1290, normalized size of antiderivative = 7.68

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*x^2)^(7/4)*(e + f*x^2))/(a + b*x^2)^(17/4),x]
```

output
```
(-2*x*(693*a*b^4*c^5*e*(a + b*x^2)^3*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2
)/a), -((d*x^2)/c)] - 126*a^2*b^3*c^4*d*e*(a + b*x^2)^3*AppellF1[1/2, 1/4,
 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 819*a^3*b^2*c^3*d^2*e*(a + b*x^2)
^3*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 126*a^2*b^3*
c^5*f*(a + b*x^2)^3*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c
)] + 126*a^3*b^2*c^4*d*f*(a + b*x^2)^3*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] - (c + d*x^2)*(45*a^3*(-(b*c) + a*d)^3*(-(b*e) + a*f
) + 5*a^2*(b*c - a*d)^2*(11*b^2*c*e - 18*a^2*d*f + a*b*(5*d*e + 2*c*f))*(a
 + b*x^2) + a*(-(b*c) + a*d)*(-77*b^3*c^2*e + 45*a^3*d^2*f + 2*a*b^2*c*(18
*d*e - 7*c*f) + 10*a^2*b*d*(2*d*e - c*f))*(a + b*x^2)^2 + 21*b^3*c^2*(11*b
*c*e - 13*a*d*e + 2*a*c*f)*(a + b*x^2)^3)*(6*a*c*AppellF1[1/2, 1/4, 1/4, 3
/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2, -(
(b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1/4, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])) + 154*b^4*c^3*d*e*x^2*(a + b*x^2)^3*(1 + (b*x^2)/a)^(1/4)
*(1 + (d*x^2)/c)^(1/4)*AppellF1[3/2, 1/4, 1/4, 5/2, -((b*x^2)/a), -((d*x^2
)/c)]*(6*a*c*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^
2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*Appe
llF1[3/2, 5/4, 1/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + 28*a*b^3*c^3*d*f*
x^2*(a + b*x^2)^3*(1 + (b*x^2)/a)^(1/4)*(1 + (d*x^2)/c)^(1/4)*AppellF1[3/2
, 1/4, 1/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*a*c*AppellF1[1/2, 1/4, ...
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {401, 27, 401, 27, 402, 27, 294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{2x(c+dx^2)^{7/4}(be-af)}{13ab(a+bx^2)^{13/4}}-\frac{2\int -\frac{(dx^2+c)^{3/4}(d(4be+9af)x^2+c(11be+2af))}{2(bx^2+a)^{13/4}}dx}{13ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(dx^2+c)^{3/4}(d(4be+9af)x^2+c(11be+2af))}{(bx^2+a)^{13/4}}dx}{13ab}+\frac{2x(c+dx^2)^{7/4}(be-af)}{13ab(a+bx^2)^{13/4}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{2x(c+dx^2)^{3/4}(bc(2af+11be)-ad(9af+4be))}{9ab(a+bx^2)^{9/4}}-\frac{2\int -\frac{d\left(45dfa^2+20bdea+8bcfa+44b^2ce\right)x^2+c(7bc(11be+2af)+2ad(4be+9af))}{2\left(bx^2+a\right)^{9/4}\sqrt[4]{dx^2+c}}dx}{9ab}}{13ab}+\frac{2x(c+dx^2)^{7/4}(be-af)}{13ab(a+bx^2)^{13/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{d\left(45dfa^2+20bdea+8bcfa+44b^2ce\right)x^2+c(7bc(11be+2af)+2ad(4be+9af))}{\left(bx^2+a\right)^{9/4}\sqrt[4]{dx^2+c}}dx}{9ab}+\frac{2x(c+dx^2)^{3/4}(bc(2af+11be)-ad(9af+4be))}{9ab(a+bx^2)^{9/4}}}{13ab}+\frac{2x(c+dx^2)^{7/4}(be-af)}{13ab(a+bx^2)^{13/4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-45a^3d^2f-10a^2bd(2de-cf)-2ab^2c(18de-7cf)+77b^3c^2e\right)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}-\frac{2\int -\frac{21b^2c^2(11bce-13ade+2acf)}{2\left(bx^2+a\right)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}}{9ab}+\frac{2x\left(c+dx^2\right)^{3/4}(bc(2af+11be)-ad(9a}{9ab(a+bx^2)^{9/4}}}{13ab}$$

$$\frac{2x\left(c+dx^2\right)^{7/4}(be-af)}{13ab\left(a+bx^2\right)^{13/4}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{21b^2c^2(2acf-13ade+11bce)\int \frac{1}{\left(bx^2+a\right)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}+\frac{2x\left(c+dx^2\right)^{3/4}\left(-45a^3d^2f-10a^2bd(2de-cf)-2ab^2c(18de-7cf)+77b^3c^2e\right)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}}{9ab}+\frac{2x\left(c+dx^2\right)^{3/4}}{}}{13ab}$$

$$\frac{2x\left(c+dx^2\right)^{7/4}(be-af)}{13ab\left(a+bx^2\right)^{13/4}}$$

$\downarrow$ 294

$$\frac{\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-45a^3d^2f-10a^2bd(2de-cf)-2ab^2c(18de-7cf)+77b^3c^2e\right)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}+\frac{21b^2cx\left(c+dx^2\right)^{3/4}\left(\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}\right)^{5/4}(2acf-13ade+11bce)\text{ Hypergeometric2F1}\left(\frac{1}{2}\right.}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}}{9ab}}{13ab}$$

$$\frac{2x\left(c+dx^2\right)^{7/4}(be-af)}{13ab\left(a+bx^2\right)^{13/4}}$$

input
```
Int[((c + d*x^2)^(7/4)*(e + f*x^2))/(a + b*x^2)^(17/4),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2)^(7/4))/(13*a*b*(a + b*x^2)^(13/4)) + ((2*(b*c
*(11*b*e + 2*a*f) - a*d*(4*b*e + 9*a*f))*x*(c + d*x^2)^(3/4))/(9*a*b*(a +
b*x^2)^(9/4)) + ((2*(77*b^3*c^2*e - 45*a^3*d^2*f - 2*a*b^2*c*(18*d*e - 7*c
*f) - 10*a^2*b*d*(2*d*e - c*f))*x*(c + d*x^2)^(3/4))/(5*a*(b*c - a*d)*(a +
 b*x^2)^(5/4)) + (21*b^2*c*(11*b*c*e - 13*a*d*e + 2*a*c*f)*x*((c*(a + b*x^
2))/(a*(c + d*x^2)))^(5/4)*(c + d*x^2)^(3/4)*Hypergeometric2F1[1/2, 5/4, 3
/2, -(((b*c - a*d)*x^2)/(a*(c + d*x^2)))])/(5*a*(b*c - a*d)*(a + b*x^2)^(5
/4)))/(9*a*b))/(13*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
+ p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q
+ 1) + 1, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{7}{4}}(f\,x^2+e)}{(b\,x^2+a)^{\frac{17}{4}}}dx$$

input
```
int((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x)
```

output
```
int((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \int \frac{(dx^2+c)^{\frac{7}{4}}(fx^2+e)}{(bx^2+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x, algorithm="fricas"
)
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*(b*x^2 + a)^(3/4)*(d*x^2 + c)^(
3/4)/(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(7/4)*(f*x**2+e)/(b*x**2+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \int \frac{(dx^2+c)^{\frac{7}{4}}(fx^2+e)}{(bx^2+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x, algorithm="maxima"
)
```

output
```
integrate((d*x^2 + c)^(7/4)*(f*x^2 + e)/(b*x^2 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \int \frac{(dx^2+c)^{\frac{7}{4}}(fx^2+e)}{(bx^2+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(7/4)*(f*x^2 + e)/(b*x^2 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \int \frac{(d\,x^2+c)^{7/4}\,(f\,x^2+e)}{(b\,x^2+a)^{17/4}}\,dx$$

input
```
int(((c + d*x^2)^(7/4)*(e + f*x^2))/(a + b*x^2)^(17/4),x)
```

output
```
int(((c + d*x^2)^(7/4)*(e + f*x^2))/(a + b*x^2)^(17/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{7/4}(e+fx^2)}{(a+bx^2)^{17/4}}\,dx = \left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a^3b\,x^2+6\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b^2x^4+4\,(b\,x^2+a\right.$$

$$+\left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}\,x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a^3b\,x^2+6\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b^2x^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^3x^6+(b\,x^2+a)^{\frac{1}{4}}\,b^4x^8}d\right.$$

$$+\left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}\,x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a^3b\,x^2+6\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b^2x^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^3x^6+(b\,x^2+a)^{\frac{1}{4}}\,b^4x^8}d\right.$$

$$+\left(\int \frac{(d\,x^2+c)^{\frac{3}{4}}\,x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a^3b\,x^2+6\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b^2x^4+4\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^3x^6+(b\,x^2+a)^{\frac{1}{4}}\,b^4x^8}d\right.$$

input
```
int((d*x^2+c)^(7/4)*(f*x^2+e)/(b*x^2+a)^(17/4),x)
```

output
```
int((c + d*x**2)**(3/4)/((a + b*x**2)**(1/4)*a**4 + 4*(a + b*x**2)**(1/4)*
a**3*b*x**2 + 6*(a + b*x**2)**(1/4)*a**2*b**2*x**4 + 4*(a + b*x**2)**(1/4)
*a*b**3*x**6 + (a + b*x**2)**(1/4)*b**4*x**8),x)*c*e + int(((c + d*x**2)**
(3/4)*x**4)/((a + b*x**2)**(1/4)*a**4 + 4*(a + b*x**2)**(1/4)*a**3*b*x**2
+ 6*(a + b*x**2)**(1/4)*a**2*b**2*x**4 + 4*(a + b*x**2)**(1/4)*a*b**3*x**6
 + (a + b*x**2)**(1/4)*b**4*x**8),x)*d*f + int(((c + d*x**2)**(3/4)*x**2)/
((a + b*x**2)**(1/4)*a**4 + 4*(a + b*x**2)**(1/4)*a**3*b*x**2 + 6*(a + b*x
**2)**(1/4)*a**2*b**2*x**4 + 4*(a + b*x**2)**(1/4)*a*b**3*x**6 + (a + b*x*
*2)**(1/4)*b**4*x**8),x)*c*f + int(((c + d*x**2)**(3/4)*x**2)/((a + b*x**2
)**(1/4)*a**4 + 4*(a + b*x**2)**(1/4)*a**3*b*x**2 + 6*(a + b*x**2)**(1/4)*
a**2*b**2*x**4 + 4*(a + b*x**2)**(1/4)*a*b**3*x**6 + (a + b*x**2)**(1/4)*b
**4*x**8),x)*d*e
```

3.569 $\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \frac{2(be-af)x(c+dx^2)^{15/4}}{17a(bc-ad)(a+bx^2)^{17/4}} + \frac{(15bce-17ade+2acf)x\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{17/4}(c+dx^2)^{15/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{17}{4},\frac{3}{2},-\frac{(bc-ad)x^2}{a(c+dx^2)}\right)}{17ac(bc-ad)(a+bx^2)^{17/4}}$$

output
```
2/17*(-a*f+b*e)*x*(d*x^2+c)^(15/4)/a/(-a*d+b*c)/(b*x^2+a)^(17/4)+1/17*(2*a
*c*f-17*a*d*e+15*b*c*e)*x*(c*(b*x^2+a)/a/(d*x^2+c))^(17/4)*(d*x^2+c)^(15/4
)*hypergeom([1/2, 17/4],[3/2],-(-a*d+b*c)*x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/
(b*x^2+a)^(17/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 12.58 (sec) , antiderivative size = 1387, normalized size of antiderivative = 8.26

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*x^2)^(11/4)*(e + f*x^2))/(a + b*x^2)^(21/4),x]
```

output
```
(-2*x*(10395*a*b^5*c^6*e*(a + b*x^2)^4*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] - 1386*a^2*b^4*c^5*d*e*(a + b*x^2)^4*AppellF1[1/2, 1
/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 11781*a^3*b^3*c^4*d^2*e*(a + b
*x^2)^4*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 1386*a^
2*b^4*c^6*f*(a + b*x^2)^4*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*
x^2)/c)] + 1386*a^3*b^3*c^5*d*f*(a + b*x^2)^4*AppellF1[1/2, 1/4, 1/4, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] - (c + d*x^2)*(585*a^4*(b*c - a*d)^4*(b*e - a
*f) + 45*a^3*(b*c - a*d)^3*(15*b^2*c*e - 39*a^2*d*f + 2*a*b*(11*d*e + c*f)
)*(a + b*x^2) + 5*a^2*(b*c - a*d)^2*(165*b^3*c^2*e - 351*a^3*d^2*f + 9*a^2
*b*d*(5*d*e + 4*c*f) + a*b^2*c*(83*d*e + 22*c*f))*(a + b*x^2)^2 + a*(-(b*c
) + a*d)*(-1155*b^4*c^3*e + 585*a^4*d^3*f + 10*a^2*b^2*c*d*(26*d*e - 11*c*
f) + 90*a^3*b*d^2*(2*d*e - c*f) - 22*a*b^3*c^2*(-22*d*e + 7*c*f))*(a + b*x
^2)^3 + 231*b^4*c^3*(15*b*c*e - 17*a*d*e + 2*a*c*f)*(a + b*x^2)^4)*(6*a*c*
AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(a*d*Appell
F1[3/2, 1/4, 5/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4
, 1/4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + 2310*b^5*c^4*d*e*x^2*(a + b*x^
2)^4*(1 + (b*x^2)/a)^(1/4)*(1 + (d*x^2)/c)^(1/4)*AppellF1[3/2, 1/4, 1/4, 5
/2, -((b*x^2)/a), -((d*x^2)/c)]*(6*a*c*AppellF1[1/2, 1/4, 1/4, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] - x^2*(a*d*AppellF1[3/2, 1/4, 5/4, 5/2, -((b*x^2)/a)
, -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1/4, 5/2, -((b*x^2)/a), -((d*x...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(168) = 336.

Time = 0.71 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.70, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {401, 27, 401, 27, 401, 27, 402, 27, 294}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{11/4}\left(e+fx^2\right)}{\left(a+bx^2\right)^{21/4}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}-\frac{2\int -\frac{\left(dx^2+c\right)^{7/4}\left(d(4be+13af)x^2+c(15be+2af)\right)}{2\left(bx^2+a\right)^{17/4}}dx}{17ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(dx^2+c\right)^{7/4}\left(d(4be+13af)x^2+c(15be+2af)\right)}{\left(bx^2+a\right)^{17/4}}dx}{17ab}+\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{2x\left(c+dx^2\right)^{7/4}(bc(2af+15be)-ad(13af+4be))}{13ab\left(a+bx^2\right)^{13/4}}-\frac{2\int -\frac{\left(dx^2+c\right)^{3/4}\left(d\left(117dfa^2+36bdea+8bcfa+60b^2ce\right)x^2+c(11bc(15be+2af)+2ad(4be+13af))\right)}{2\left(bx^2+a\right)^{13/4}}dx}{13ab}}{17ab}+\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\left(dx^2+c\right)^{3/4}\left(d\left(117dfa^2+36bdea+8bcfa+60b^2ce\right)x^2+c(11bc(15be+2af)+2ad(4be+13af))\right)}{\left(bx^2+a\right)^{13/4}}dx}{13ab}+\frac{2x\left(c+dx^2\right)^{7/4}(bc(2af+15be)-ad(13af+4be))}{13ab\left(a+bx^2\right)^{13/4}}}{17ab}+\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$$\Big\downarrow \text{ 401}$$

$$\frac{\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-117a^3d^2f-18a^2bd(2de-cf)-2ab^2c(26de-11cf)+165b^3c^2e\right)}{9ab\left(a+bx^2\right)^{9/4}}-\frac{2\int -\frac{d\left(585d^2fa^3+36bd(5de+4cf)a^2+4b^2c(83de+22cf)a+660b^3c^2e\right)x^2+c\left(234\right.}{2\left(bx^2+a\right)^{9/4}\sqrt[4]{dx}}}{9ab}}{13ab}}{17ab}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{d\left(585d^2fa^3+36bd(5de+4cf)a^2+4b^2c(83de+22cf)a+660b^3c^2e\right)x^2+c\left(234d^2fa^3+18bd(4de+11cf)a^2+22b^2c(8de+7cf)a+1155b^3c^2e\right)}{\left(bx^2+a\right)^{9/4}\sqrt[4]{dx^2+c}}dx}{9ab}+\frac{2x\left(c+dx^2\right)^{3/4}\left(-1\right.}{}}{13ab}}{17ab}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-585a^4d^3f-90a^3bd^2(2de-cf)-10a^2b^2cd(26de-11cf)-22ab^3c^2(22de-7cf)+1155b^4c^3e\right)}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}-\frac{2\int -\frac{231b^3c^3(15bce-17ade+2acf)}{2\left(bx^2+a\right)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}}{9ab}+\frac{2x\left(c+dx^2\right.}{}}{13ab}}{17ab}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{231b^3c^3(2acf-17ade+15bce)\int \frac{1}{\left(bx^2+a\right)^{5/4}\sqrt[4]{dx^2+c}}dx}{5a(bc-ad)}+\frac{2x\left(c+dx^2\right)^{3/4}\left(-585a^4d^3f-90a^3bd^2(2de-cf)-10a^2b^2cd(26de-11cf)-22ab^3c^2(22de-7cf)+1\right.}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}}{9ab}}{13ab}}{17ab}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

$\downarrow$ 294

$$\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-117a^3d^2f-18a^2bd(2de-cf)-2ab^2c(26de-11cf)+165b^3c^2e\right)}{9ab\left(a+bx^2\right)^{9/4}}+\frac{\frac{2x\left(c+dx^2\right)^{3/4}\left(-585a^4d^3f-90a^3bd^2(2de-cf)-10a^2b^2cd(26de-11cf)-22ab^3\cdots\right.}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}\cdots}{13ab}}{\cdots}$$

$$\frac{2x\left(c+dx^2\right)^{11/4}(be-af)}{17ab\left(a+bx^2\right)^{17/4}}$$

input
```
Int[((c + d*x^2)^(11/4)*(e + f*x^2))/(a + b*x^2)^(21/4),x]
```

output
```
(2*(b*e - a*f)*x*(c + d*x^2)^(11/4))/(17*a*b*(a + b*x^2)^(17/4)) + ((2*(b*
c*(15*b*e + 2*a*f) - a*d*(4*b*e + 13*a*f))*x*(c + d*x^2)^(7/4))/(13*a*b*(a
 + b*x^2)^(13/4)) + ((2*(165*b^3*c^2*e - 117*a^3*d^2*f - 2*a*b^2*c*(26*d*e
 - 11*c*f) - 18*a^2*b*d*(2*d*e - c*f))*x*(c + d*x^2)^(3/4))/(9*a*b*(a + b*
x^2)^(9/4)) + ((2*(1155*b^4*c^3*e - 585*a^4*d^3*f - 10*a^2*b^2*c*d*(26*d*e
 - 11*c*f) - 22*a*b^3*c^2*(22*d*e - 7*c*f) - 90*a^3*b*d^2*(2*d*e - c*f))*x
*(c + d*x^2)^(3/4))/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (231*b^3*c^2*(15
*b*c*e - 17*a*d*e + 2*a*c*f)*x*((c*(a + b*x^2))/(a*(c + d*x^2)))^(5/4)*(c
+ d*x^2)^(3/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -(((b*c - a*d)*x^2)/(a*(c
+ d*x^2)))])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)))/(9*a*b))/(13*a*b))/(17*a
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
 + p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeBb*c - a*d, 0] && EqQ[2*(p + q
 + 1) + 1, 0]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(x^2d+c)^{\frac{11}{4}} (f\,x^2+e)}{(b\,x^2+a)^{\frac{21}{4}}}dx$$

input
```
int((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x)
```

output
```
int((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \int \frac{(dx^2+c)^{\frac{11}{4}}(fx^2+e)}{(bx^2+a)^{\frac{21}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x, algorithm="fricas
")
```

output
```
integral((d^2*f*x^6 + (d^2*e + 2*c*d*f)*x^4 + c^2*e + (2*c*d*e + c^2*f)*x^
2)*(b*x^2 + a)^(3/4)*(d*x^2 + c)^(3/4)/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b
^4*x^8 + 20*a^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**(11/4)*(f*x**2+e)/(b*x**2+a)**(21/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \int \frac{(dx^2+c)^{\frac{11}{4}}(fx^2+e)}{(bx^2+a)^{\frac{21}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x, algorithm="maxima
")
```

output
```
integrate((d*x^2 + c)^(11/4)*(f*x^2 + e)/(b*x^2 + a)^(21/4), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \int \frac{(dx^2+c)^{\frac{11}{4}}(fx^2+e)}{(bx^2+a)^{\frac{21}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(11/4)*(f*x^2 + e)/(b*x^2 + a)^(21/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \int \frac{(d\,x^2+c)^{11/4}(f\,x^2+e)}{(b\,x^2+a)^{21/4}}\,dx$$

input
```
int(((c + d*x^2)^(11/4)*(e + f*x^2))/(a + b*x^2)^(21/4),x)
```

output
```
int(((c + d*x^2)^(11/4)*(e + f*x^2))/(a + b*x^2)^(21/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^2)^{11/4}(e+fx^2)}{(a+bx^2)^{21/4}}\,dx = \text{Too large to display}$$

input
```
int((d*x^2+c)^(11/4)*(f*x^2+e)/(b*x^2+a)^(21/4),x)
```

output

```
int((c + d*x**2)**(3/4)/((a + b*x**2)**(1/4)*a**5 + 5*(a + b*x**2)**(1/4)*
a**4*b*x**2 + 10*(a + b*x**2)**(1/4)*a**3*b**2*x**4 + 10*(a + b*x**2)**(1/
4)*a**2*b**3*x**6 + 5*(a + b*x**2)**(1/4)*a*b**4*x**8 + (a + b*x**2)**(1/4
)*b**5*x**10),x)*c**2*e + int(((c + d*x**2)**(3/4)*x**6)/((a + b*x**2)**(1
/4)*a**5 + 5*(a + b*x**2)**(1/4)*a**4*b*x**2 + 10*(a + b*x**2)**(1/4)*a**3
*b**2*x**4 + 10*(a + b*x**2)**(1/4)*a**2*b**3*x**6 + 5*(a + b*x**2)**(1/4)
*a*b**4*x**8 + (a + b*x**2)**(1/4)*b**5*x**10),x)*d**2*f + 2*int(((c + d*x
**2)**(3/4)*x**4)/((a + b*x**2)**(1/4)*a**5 + 5*(a + b*x**2)**(1/4)*a**4*b
*x**2 + 10*(a + b*x**2)**(1/4)*a**3*b**2*x**4 + 10*(a + b*x**2)**(1/4)*a**
2*b**3*x**6 + 5*(a + b*x**2)**(1/4)*a*b**4*x**8 + (a + b*x**2)**(1/4)*b**5
*x**10),x)*c*d*f + int(((c + d*x**2)**(3/4)*x**4)/((a + b*x**2)**(1/4)*a**
5 + 5*(a + b*x**2)**(1/4)*a**4*b*x**2 + 10*(a + b*x**2)**(1/4)*a**3*b**2*x
**4 + 10*(a + b*x**2)**(1/4)*a**2*b**3*x**6 + 5*(a + b*x**2)**(1/4)*a*b**4
*x**8 + (a + b*x**2)**(1/4)*b**5*x**10),x)*d**2*e + int(((c + d*x**2)**(3/
4)*x**2)/((a + b*x**2)**(1/4)*a**5 + 5*(a + b*x**2)**(1/4)*a**4*b*x**2 + 1
0*(a + b*x**2)**(1/4)*a**3*b**2*x**4 + 10*(a + b*x**2)**(1/4)*a**2*b**3*x*
*6 + 5*(a + b*x**2)**(1/4)*a*b**4*x**8 + (a + b*x**2)**(1/4)*b**5*x**10),x
)*c**2*f + 2*int(((c + d*x**2)**(3/4)*x**2)/((a + b*x**2)**(1/4)*a**5 + 5*
(a + b*x**2)**(1/4)*a**4*b*x**2 + 10*(a + b*x**2)**(1/4)*a**3*b**2*x**4 +
10*(a + b*x**2)**(1/4)*a**2*b**3*x**6 + 5*(a + b*x**2)**(1/4)*a*b**4*x*...
```

3.570 $\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx = ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
e*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+
b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*f*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2
,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx = \frac{1}{3}x(a+bx^2)^p (c +dx^2)^q \left(\frac{9ace \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcp \operatorname{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+ \right.} + fx^2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(x*(a + b*x^2)^p*(c + d*x^2)^q*((9*a*c*e*AppellF1[1/2, -p, -q, 3/2, -((b*x
^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d
*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)
])) + (f*x^2*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 +
 (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx^2)(a+bx^2)^p (c+dx^2)^q\, dx$$

$$\Big\downarrow \; 406$$

$$e\int (bx^2+a)^p (dx^2+c)^q dx + f\int x^2 (bx^2+a)^p (dx^2+c)^q dx$$

$$\Big\downarrow \textbf{334}$$

$$e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx+f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx$$

$$\Big\downarrow \textbf{334}$$

$$e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+ f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx$$

$$\Big\downarrow \textbf{333}$$

$$f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \textbf{395}$$

$$f(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \textbf{395}$$

$$f(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \textbf{394}$$

$$ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+ \frac{1}{3}fx^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(e*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a),
-((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (f*x^3*(a + b*x^2)^
p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3
*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(x^{2} d+c\right)^{q} \left(f\,x^{2}+e\right)dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)\,dx=\int \left(fx^{2}+e\right)\left(bx^{2}+a\right)^{p}\left(dx^{2}+c\right)^{q}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Giac [F]

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (b\,x^2+a)^p \, (d\,x^2+c)^q \, (f\,x^2+e) \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
(2*(c + d*x**2)**q*(a + b*x**2)**p*a*d*f*p*x + 2*(c + d*x**2)**q*(a + b*x*
*2)**p*b*c*f*q*x + 2*(c + d*x**2)**q*(a + b*x**2)**p*b*d*e*p*x + 2*(c + d*
x**2)**q*(a + b*x**2)**p*b*d*e*q*x + 3*(c + d*x**2)**q*(a + b*x**2)**p*b*d
*e*x + 2*(c + d*x**2)**q*(a + b*x**2)**p*b*d*f*p*x**3 + 2*(c + d*x**2)**q*
(a + b*x**2)**p*b*d*f*q*x**3 + (c + d*x**2)**q*(a + b*x**2)**p*b*d*f*x**3
- 16*int(((c + d*x**2)**q*(a + b*x**2)**p*x**2)/(4*a*c*p**2 + 8*a*c*p*q +
8*a*c*p + 4*a*c*q**2 + 8*a*c*q + 3*a*c + 4*a*d*p**2*x**2 + 8*a*d*p*q*x**2
+ 8*a*d*p*x**2 + 4*a*d*q**2*x**2 + 8*a*d*q*x**2 + 3*a*d*x**2 + 4*b*c*p**2*
x**2 + 8*b*c*p*q*x**2 + 8*b*c*p*x**2 + 4*b*c*q**2*x**2 + 8*b*c*q*x**2 + 3*
b*c*x**2 + 4*b*d*p**2*x**4 + 8*b*d*p*q*x**4 + 8*b*d*p*x**4 + 4*b*d*q**2*x*
*4 + 8*b*d*q*x**4 + 3*b*d*x**4),x)*a**2*d**2*f*p**3*q - 8*int(((c + d*x**2
)**q*(a + b*x**2)**p*x**2)/(4*a*c*p**2 + 8*a*c*p*q + 8*a*c*p + 4*a*c*q**2
+ 8*a*c*q + 3*a*c + 4*a*d*p**2*x**2 + 8*a*d*p*q*x**2 + 8*a*d*p*x**2 + 4*a*
d*q**2*x**2 + 8*a*d*q*x**2 + 3*a*d*x**2 + 4*b*c*p**2*x**2 + 8*b*c*p*q*x**2
 + 8*b*c*p*x**2 + 4*b*c*q**2*x**2 + 8*b*c*q*x**2 + 3*b*c*x**2 + 4*b*d*p**2
*x**4 + 8*b*d*p*q*x**4 + 8*b*d*p*x**4 + 4*b*d*q**2*x**4 + 8*b*d*q*x**4 + 3
*b*d*x**4),x)*a**2*d**2*f*p**3 - 32*int(((c + d*x**2)**q*(a + b*x**2)**p*x
**2)/(4*a*c*p**2 + 8*a*c*p*q + 8*a*c*p + 4*a*c*q**2 + 8*a*c*q + 3*a*c + 4*
a*d*p**2*x**2 + 8*a*d*p*q*x**2 + 8*a*d*p*x**2 + 4*a*d*q**2*x**2 + 8*a*d*q*
x**2 + 3*a*d*x**2 + 4*b*c*p**2*x**2 + 8*b*c*p*q*x**2 + 8*b*c*p*x**2 + 4...
```

3.571 $\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 197

$$\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx = -\frac{(be-af)x(a+bx^2)^{1+p} (c+dx^2)^{-\frac{3}{2}-p}}{2a(bc-ad)(1+p)}$$
$$-\frac{(acf+2ade(1+p)-bce(3+2p))x(a+bx^2)^{1+p} \left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{-1-p} (c+dx^2)^{-\frac{3}{2}-p} \text{Hypergeometric2F1}\left(\ldots\right.}{2ac(bc-ad)(1+p)}$$

output
```
-1/2*(-a*f+b*e)*x*(b*x^2+a)^(p+1)*(d*x^2+c)^(-3/2-p)/a/(-a*d+b*c)/(p+1)-1/
2*(a*c*f+2*a*d*e*(p+1)-b*c*e*(3+2*p))*x*(b*x^2+a)^(p+1)*(c*(b*x^2+a)/a/(d*
x^2+c))^(-1-p)*(d*x^2+c)^(-3/2-p)*hypergeom([1/2, -1-p],[3/2],-(-a*d+b*c)*
x^2/a/(d*x^2+c))/a/c/(-a*d+b*c)/(p+1)
```

Mathematica [F]

$$\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx = \int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^(-5/2 - p)*(e + f*x^2),x]
```

output
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^(-5/2 - p)*(e + f*x^2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {406, 296, 294, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^2\right)\left(a+bx^2\right)^p\left(c+dx^2\right)^{-p-\frac{5}{2}}dx$$

$$\Big\downarrow \text{406}$$

$$e\int \left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}dx+f\int x^2\left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}dx$$

$$\Big\downarrow \text{296}$$

$$e\left(\frac{\left(\frac{bc}{(p+1)(bc-ad)}+2\right)\int \left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{-p-\frac{5}{2}}dx}{2a}-\frac{bx\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{-p-\frac{3}{2}}}{2a(p+1)(bc-ad)}\right)+ f\int x^2\left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}dx$$

$$\Big\downarrow \text{294}$$

$$f\int x^2\left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}dx+ e\left(\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{-p-\frac{3}{2}}\left(\frac{bc}{(p+1)(bc-ad)}+2\right)\left(\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},-p-1,\frac{3}{2},-\frac{(bc-ad)x}{a(dx^2+c}\right.}{2ac}\right.$$

$$\Big\downarrow \text{395}$$

$$f\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}dx+ e\left(\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{-p-\frac{3}{2}}\left(\frac{bc}{(p+1)(bc-ad)}+2\right)\left(\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},-p-1,\frac{3}{2},-\frac{(bc-ad)x}{a(dx^2+c}\right.}{2ac}\right.$$

$$\Big\downarrow \text{395}$$

$$\frac{f\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(\frac{dx^2}{c}+1\right)^{p+\frac{1}{2}}\left(c+dx^2\right)^{-p-\frac{1}{2}}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^{-p-\frac{5}{2}}dx}{c^2}+$$
$$e\left(\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{-p-\frac{3}{2}}\left(\frac{bc}{(p+1)(bc-ad)}+2\right)\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},-p-1,\frac{3}{2},-\frac{(bc-ad)x}{a(dx^2+c}\right.}{2ac}\right.$$

$$\Big\downarrow \text{394}$$

$$\frac{fx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(\frac{dx^2}{c}+1\right)^{p-1}\left(c+dx^2\right)^{-p-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{c\left(\frac{bx^2}{a}-\frac{dx^2}{c}\right)}{dx^2+c}\right)}{3c^2}+$$
$$e\left(\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{-p-\frac{3}{2}}\left(\frac{bc}{(p+1)(bc-ad)}+2\right)\left(\frac{c(a+bx^2)}{a(c+dx^2)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},-p-1,\frac{3}{2},-\frac{(bc-ad)x}{a(dx^2+c}\right.}{2ac}\right.$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^(-5/2 - p)*(e + f*x^2),x]
```

output
```
e*(-1/2*(b*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^(-3/2 - p))/(a*(b*c - a*d)*(1
 + p)) + ((2 + (b*c)/((b*c - a*d)*(1 + p)))*x*(a + b*x^2)^(1 + p)*((c*(a +
 b*x^2))/(a*(c + d*x^2)))^(-1 - p)*(c + d*x^2)^(-3/2 - p)*Hypergeometric2F
1[1/2, -1 - p, 3/2, -(((b*c - a*d)*x^2)/(a*(c + d*x^2)))])/(2*a*c)) + (f*x
^3*(a + b*x^2)^p*(c + d*x^2)^(-1/2 - p)*(1 + (d*x^2)/c)^(-1 + p)*Hypergeom
etric2F1[3/2, -p, 5/2, -((c*((b*x^2)/a - (d*x^2)/c))/(c + d*x^2))])/(3*c^2
*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 294
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*((a + b*x^2)^p/(c*(c*((a + b*x^2)/(a*(c + d*x^2))))^p*(c + d*x^2)^(1/2
 + p)))*Hypergeometric2F1[1/2, -p, 3/2, (-(b*c - a*d))*(x^2/(a*(c + d*x^2)))
], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q
 + 1) + 1, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(x^{2} d +c\right)^{-\frac{5}{2}-p} \left(f\,x^{2}+e\right)d x$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^{-\frac{5}{2}-p} \left(e+fx^2\right)\,dx=\int \left(fx^2+e\right)\left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^(-p - 5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^{-\frac{5}{2}-p} \left(e+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**(-5/2-p)*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^{-\frac{5}{2}-p} \left(e+fx^2\right)\,dx=\int \left(fx^2+e\right)\left(bx^2+a\right)^p\left(dx^2+c\right)^{-p-\frac{5}{2}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^(-p - 5/2), x)
```

Giac [F]

$$\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^{-p-\frac{5}{2}} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^(-p - 5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx = \int \frac{(b\,x^2+a)^p\,(f\,x^2+e)}{(d\,x^2+c)^{p+\frac{5}{2}}} \, dx$$

input
```
int(((a + b*x^2)^p*(e + f*x^2))/(c + d*x^2)^(p + 5/2),x)
```

output
```
int(((a + b*x^2)^p*(e + f*x^2))/(c + d*x^2)^(p + 5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (a+bx^2)^p (c+dx^2)^{-\frac{5}{2}-p} (e+fx^2) \, dx \\ &= \left(\int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^{p+\frac{1}{2}}\,c^2+2\,(d\,x^2+c)^{p+\frac{1}{2}}\,cd\,x^2+(d\,x^2+c)^{p+\frac{1}{2}}\,d^2x^4}dx\right)e \\ &\quad+\left(\int \frac{(b\,x^2+a)^p\,x^2}{(d\,x^2+c)^{p+\frac{1}{2}}\,c^2+2\,(d\,x^2+c)^{p+\frac{1}{2}}\,cd\,x^2+(d\,x^2+c)^{p+\frac{1}{2}}\,d^2x^4}dx\right)f\end{aligned}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^(-5/2-p)*(f*x^2+e),x)
```

output
```
int((a + b*x**2)**p/((c + d*x**2)**((2*p + 1)/2)*c**2 + 2*(c + d*x**2)**((
2*p + 1)/2)*c*d*x**2 + (c + d*x**2)**((2*p + 1)/2)*d**2*x**4),x)*e + int((
(a + b*x**2)**p*x**2)/((c + d*x**2)**((2*p + 1)/2)*c**2 + 2*(c + d*x**2)**
((2*p + 1)/2)*c*d*x**2 + (c + d*x**2)**((2*p + 1)/2)*d**2*x**4),x)*f
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:  #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file